

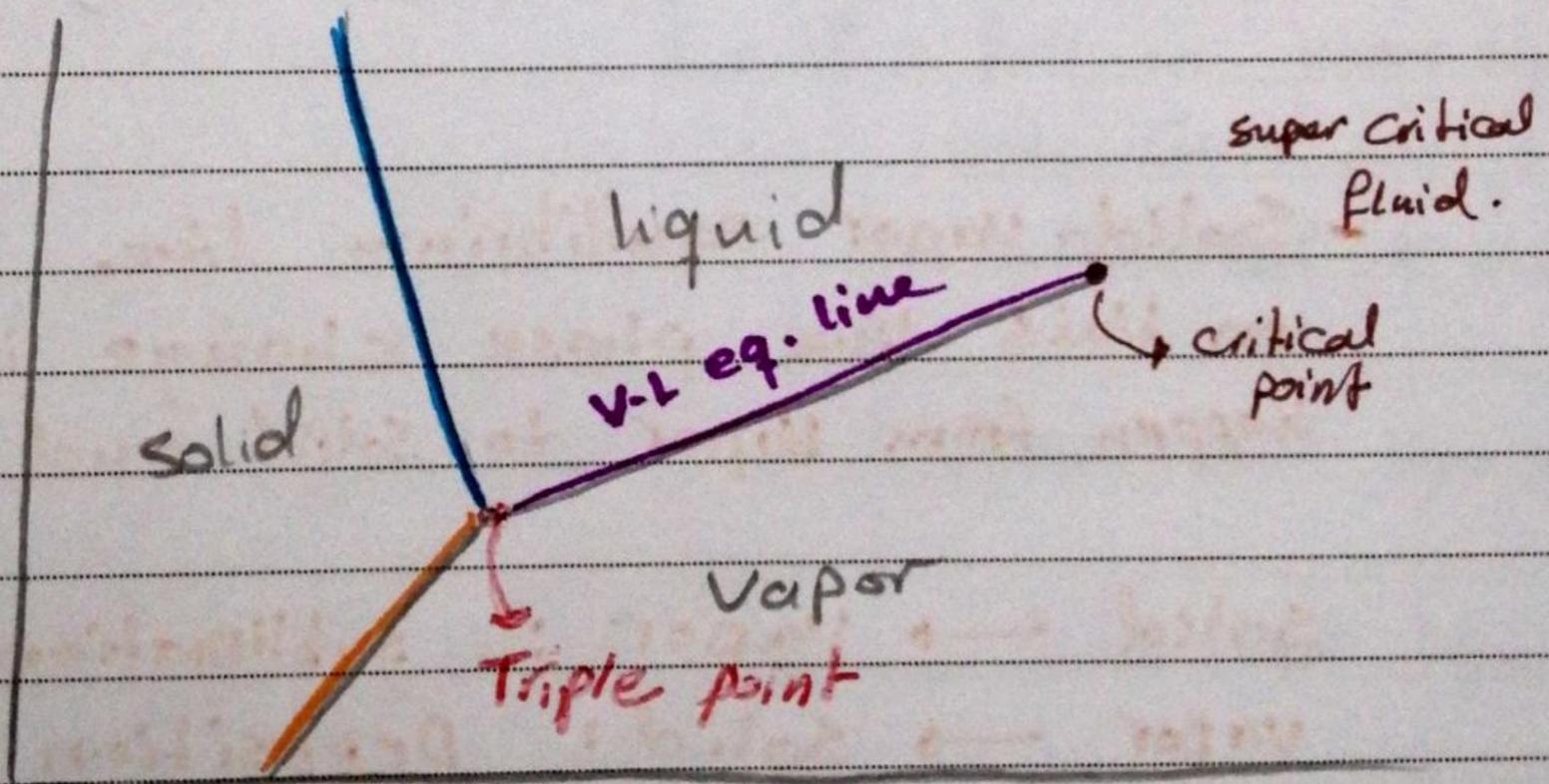
## CH6: Multiphase systems:

### Phase equilibrium:

The concentrations of all species in each phase no longer change with time.

### 6.1: Single Component phase diagram.

- Phase diagram of  $H_2O$ .



- Triple point: point at which solid, liquid and vapor can all coexist.

- critical point ( $T_c, P_c$ ): Above and to right of it only one phase is exist. (no phase separation).

- Vapor-liquid equilibrium line:  
on this line phase change is  
happen from Liquid to vapor or opposite.

Liquid  $\rightarrow$  vapor : evaporation

Vapor  $\rightarrow$  liquid : Condensation

- For given p on this line  $\rightarrow$  T (boiling point)  
Can obtain

**Remark** boiling is happen when Vapor pressure  
equal total pressure (pressure on system).

- T at  $p = 1 \text{ atm} \Rightarrow$  Normal boiling point

- Solid-vapor equilibrium line

on this line phase change is  
happen from Vapor to solid and opposite.

Solid  $\rightarrow$  vapor : sublimation.

vapor  $\rightarrow$  solid : Deposition

- For given point on the line

p: vapor pressure

T: sublimation point.

- Solid-liquid equilibrium line:

phase change from:

Solid  $\rightarrow$  liquid : melting

liquid  $\rightarrow$  solid : freezing

Note: phase change can only happen on the equilibrium lines.

Example: At given  $(T, P) \rightarrow$  if it fall on liquid region  $\rightarrow$  subcooled liquid

$\rightarrow$  if it fall on vapor-liquid equilibrium line, there are 3 probabilities

$\rightarrow$  Saturated vapor

$\rightarrow$  Vapor and liquid mixture

$\rightarrow$  Saturated liquid.

phase  
\* هذا يمكن حدوث تغير في  
equilibrium line  
على خط التوازن

\* عند وجود المادة في احدى المناطق الثلاث  
تكون one phase  
equilibrium line  
\* عند وجود المادة على  
line of equilibrium  
لا يمكن الجزم بالاحتمال  
هل هي سائلة او بخارية  
Sat. -  
Sat. -  
mix of two phases

Saturated

$\rightarrow$  All the matter on one phase but on the equilibrium line.

Note: Read p. 242. last paragraph.

- Estimation of vapor pressure:

↳ vapor pressure of a species is a measure of its volatility

vapor pressure ( $P^*(T)$ ) → it is function in T.

to estimate it:

↳ Capeyron equation

$$\frac{dP^*}{dT} = \frac{\Delta \hat{H}_v}{T(\hat{V}_g - \hat{V}_l)}$$

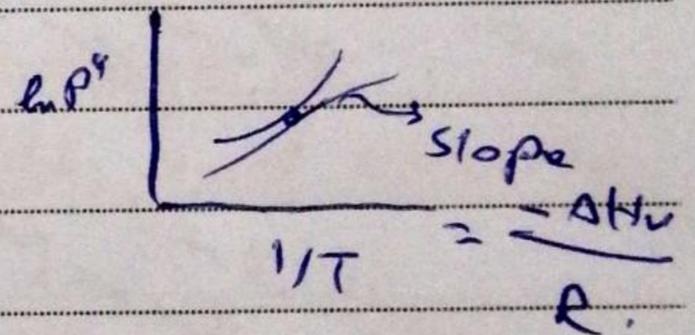
$\Delta \hat{H}_v$ : latent heat of vaporization

$\hat{V}$ : molar volume of (g) gas

(l) liquid

replace  $\hat{V}_g$  with  $\frac{RT}{P^*}$

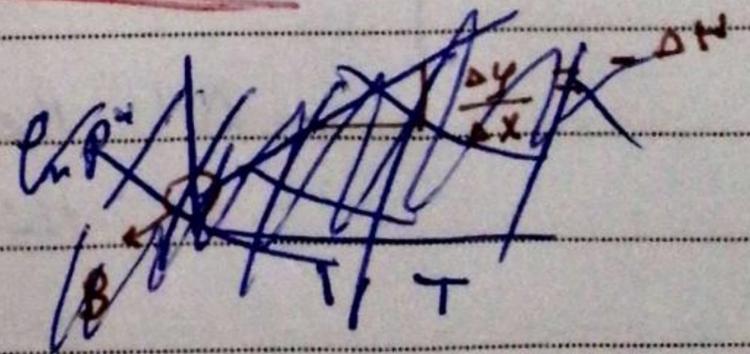
$$\Rightarrow \frac{d(\ln P^*)}{d(1/T)} = \frac{-\Delta \hat{H}_v}{R}$$



if  $\Delta \hat{H}_v$  constant

↳ Clausius - Capeyron equation

$$\ln P^* = -\frac{\Delta \hat{H}_v}{RT} + B$$



\* if only two point is available

divide two equations or (-)

$$\frac{\ln P_2^*}{\ln P_1^*} = \dots$$

## ↳ Antoine equation

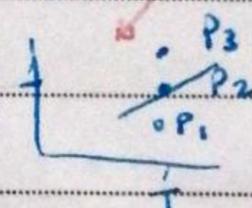
$$\log_{10} P^{\text{sat}} = A - \frac{B}{C + T}$$

$T$  is in absolute  
T

A, B, C: Constant  
for each component.

Also charts may be used.

For Liquid:  $P_{(T)}^{\text{sat}} = P$  → boiling material  $P_2$  where  $T$  is absolute  
 Vapor System  $P_{(T)} < P$  → Liquid  $P_1$   
 $P_{(T)} > P$  → vapor  $P_3$



## 6.2: The Gibbs phase rule:

↳ The degrees of freedom for multiphase system

$$DF = 2 + C - \Pi$$

where

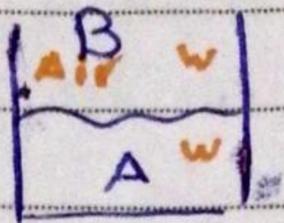
C: number of chemical species

$\Pi$ : number of phase in a system at equilibrium

if  $DF = 0$  All intensive property of the system can be obtained.

intensive property: properties that does not depend on the size of the system.

### 6.3: Gas-liquid systems: one condensable component?



B: gas phase (vapor phase)  
A: liquid phase

if only  $w$  can transfer between two phases then:  $P_w = y_w P = P_w^*(T)$

Partial pressure of  $w$       vapor fraction of  $w$       total pressure  
 $P = P_w + P_{air}$

#### Notes:

- Saturation: the gas phase contain the maximum amount of condensable material ( $w$ )

- At equilibrium:  $P_i = P_i^* = y_i P \rightarrow$  Saturation

$P_i^*(T) = y_i P_{tot}$   
 ↳ constant for given  $(T)$       ↑ or ↓      must      why

if we try to increase  $P_i$  by  $y_i \uparrow$  or  $P \uparrow$  } both lead to Condensation

- Quantity of  $w$  in gas phase may be below saturation  
 $P_i = y_i P < P_i^*(T) \rightarrow$  under Saturation  
 ↳ This case: Superheated vapor

No. ....

- Dew point: The temperature that saturated vapor start to condensate -

P. 251, P. 252, P. 253.

A stream of air at 100°C and 5260 mm Hg contains 10.0% water by volume.

1. Calculate the dew point and degrees of superheat of the air.
2. Calculate the percentage of the vapor that condenses and the final composition of the gas phase if the air is cooled to 80°C at constant pressure.
3. Calculate the percentage condensation and the final gas-phase composition if, instead of being cooled, the air is compressed isothermally to 8500 mm Hg.
4. Suppose the process of part 2 is run, the product gas is analyzed, and the mole fraction of water differs considerably from the calculated value. What could be responsible for the disparity between calculated and measured values? (List several possibilities.)

*even that.*  
 $P_{H_2O} = P_i \times (100\%)$   
 $P_{H_2O} = 760$   
 $P_{H_2O} = y_{H_2O} P_{tot} = P_i \times (100\%)$   
**always**

**SOLUTION**

$$p_{H_2O} = y_{H_2O} P = (0.100)(5260 \text{ mm Hg}) = 526 \text{ mm Hg}$$

$$p_{H_2O}^*(100^\circ\text{C}) = 760 \text{ mm Hg} > p_{H_2O} \implies \text{the vapor is superheated (see Inequality 6.3-2)}$$

From Equation 6.3-3

$$p_{H_2O} = p_{H_2O}^*(T_{dp}) = 526 \text{ mm Hg}$$

Table B.3

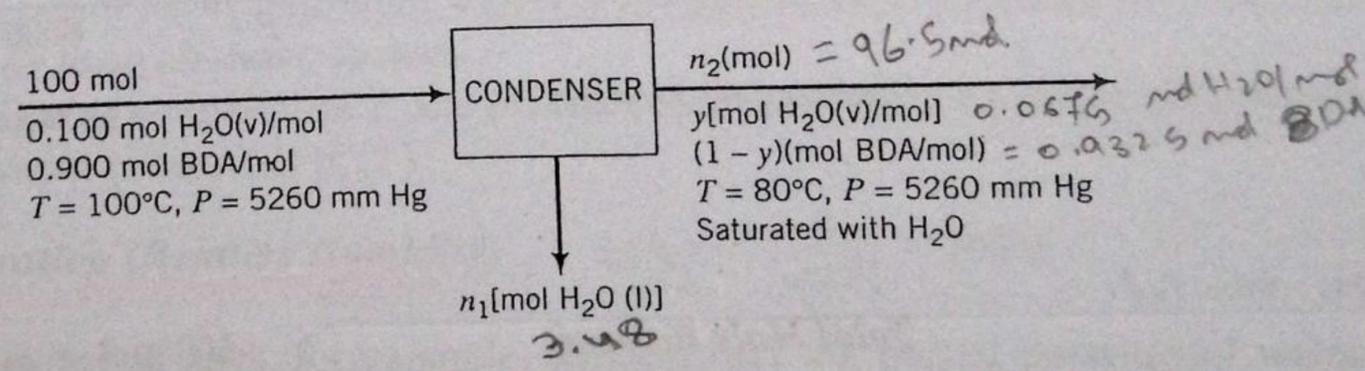
$$T_{dp} = 90^\circ\text{C}$$

and the air has  $100^\circ\text{C} - 90^\circ\text{C} = 10^\circ\text{C}$  of superheat

2. Since the air becomes saturated at 90°C, further cooling must lead to condensation. Since the products are liquid water in equilibrium with a gas phase, the water vapor in the gas must remain saturated.

On the following flowchart, the symbol BDA stands for **bone-dry air**, a term used to signify the water-free component of an air-water vapor mixture.

**Basis: 100 mol Feed Gas**



Let us first do the degree-of-freedom analysis. Three unknown variables appear on the chart:  $n_1$ ,  $n_2$ , and  $y$ . Since only two species are involved in the process, we can only write two independent material balances, leaving us one equation short. If we fail to observe that the outlet at the condenser outlet is saturated with water, solution of the problem would be impossible; however, the saturation condition supplies the needed third equation, Raoult's law.

The solution outline is as follows: apply Raoult's law at the outlet to determine  $y$ , the fraction of water in the outlet gas; then use a dry air balance to determine  $n_2$  and a total balance or a water balance to determine the final unknown,  $n_1$ .

**Raoult's Law at Outlet**  $yP = p_{H_2O}^*(T)$

↓

$$y = \frac{p_{H_2O}^*(80^\circ\text{C})}{P} = \frac{355 \text{ mm Hg}}{5260 \text{ mm Hg}} = 0.0675 \frac{\text{mol H}_2\text{O}}{\text{mol}}$$

Multiphase Systems

Balance on Dry Air

$$\frac{100 \text{ mol}}{\text{mol}} \left| \frac{0.900 \text{ mol BDA}}{\text{mol}} \right. = n_2(1 - y)$$

$$\Downarrow y = 0.0675$$

$$n_2 = 96.5 \text{ mol}$$

Total Mole Balance

$$100 \text{ mol} = n_1 + n_2$$

$$\Downarrow n_2 = 96.5 \text{ mol}$$

$$n_1 = 3.5 \text{ mol H}_2\text{O condensed}$$

Percentage Condensation

$$\frac{3.5 \text{ mol H}_2\text{O condensed}}{(0.100 \times 100) \text{ mol H}_2\text{O fed}} \times 100\% = \boxed{35\%}$$

... when  $P$  is high enough for the in

The mechanism of evaporation of a liquid depends on the relative values of the vapor pressure of the liquid and the total system pressure. If evaporation takes place at a temperature such that  $p^* < P$ , the process involves transfer of molecules from the surface of the liquid to the gas above the surface, while if  $p^* = P$ , vapor bubbles form throughout the entire liquid, but predominantly at the heated container walls: that is, the liquid boils. The temperature at which  $p^* = P$  is the **boiling point** of the liquid at the given pressure.

↳ only for one condensable pt

1. If water vapor is in equilibrium with liquid water, must the vapor be saturated? Can a vapor be saturated if there is no liquid present in the system? <sup>yes</sup> yes
2. The vapor pressure of acetone is 200 mm Hg at 22.7°C. Liquid acetone is kept in a sealed flask at 22.7°C, and the gas above the liquid contains air and acetone vapor at a pressure of 960 mm Hg. What is (a) the partial pressure of acetone in the gas, (b) the partial pressure of N<sub>2</sub>, and (c) the mole fraction of acetone in the gas? What assumption have you made in answering the preceding question? How would you determine the boiling point of the acetone, assuming a constant total pressure of 960 mm Hg?
3. Suppose you have a curve of  $p_{\text{H}_2\text{O}}^*$  versus  $T$ , and you are given the temperature and pressure ( $T_0$  and  $P_0$ ) of a mixture of water and noncondensable gases.
  - a. Define the dew point of the gas. Would the vapor be saturated or superheated if  $T_0 > T_{\text{dp}}$ ? If  $T_0 = T_{\text{dp}}$ ?
  - b. If you are told that the gas is saturated, how would you calculate the mole fraction of water in it? What would happen to the vapor if you (i) heated the gas isobarically (at constant pressure), (ii) cooled the gas isobarically, (iii) compressed the gas isothermally (at constant temperature), and (iv) expanded the gas isothermally?  $\uparrow$   $p_{\text{H}_2\text{O}} = p_{\text{H}_2\text{O}}^*(T_0)$
  - c. If you are given the mole fraction of water in the gas, how would you calculate the dew point of the gas?
  - d. If you are given the degrees of superheat of the gas, how would you calculate the mole fraction of water in the gas?

Several quantities besides those introduced in the previous section are commonly used to describe the state and composition of a gas containing a single condensable vapor. In the definitions to be given, the term **"saturation"** refers to any gas-vapor combination, while **"humidity"** refers specifically to an air-water system.

Suppose a gas at temperature  $T$  and pressure  $P$  contains a vapor whose partial pressure is  $p_i$  and whose vapor pressure is  $p_i^*(T)$ .

**Relative Saturation (Relative Humidity)**  $s_r(h_r) = \frac{p_i}{p_i^*(T)} \times 100\%$  (6.3-4)  
 if it was 100% → sat

A relative humidity of 40%, for example, signifies that the partial pressure of water vapor equals  $\frac{4}{10}$  of the vapor pressure of water at the system temperature.

**Molal Saturation (Molal Humidity)**

$$s_m(h_m) = \frac{p_i}{P - p_i} = \frac{\text{moles of vapor}}{\text{moles of vapor-free (dry) gas}} \quad (6.3-5)$$

(Can you prove the second equality?)

**Absolute Saturation (Absolute Humidity)**

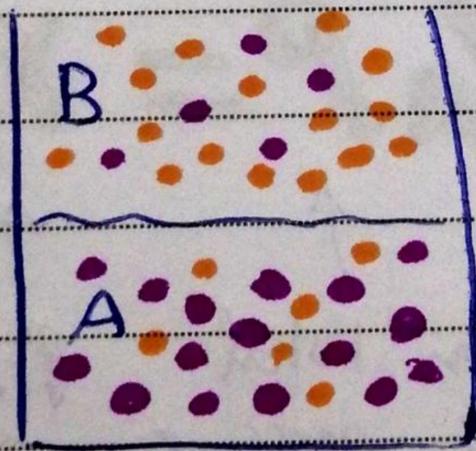
$$s_a(h_a) = \frac{p_i M_i}{(P - p_i) M_{\text{dry}}} = \frac{\text{mass of vapor}}{\text{mass of dry gas}} \quad (6.3-6)$$

where  $M_i$  is the molecular weight of the vapor and  $M_{\text{dry}}$  is the average molecular weight of the dry (vapor-free) gas.

## 6.4: Multicomponent Gas-liquid systems

- Raoult's Law and Henry's law:

Note: The composition of the two phases at given temperature and pressure are dependent.



B: gas phase

A: Liquid phase

●: Molecules of Tol component

●: Molecules of Ben component

- Notice that there is differences in distribution of molecules of Tol and Ben. between the two phases A and B.

↳ This is natural.

The more volatility will be in gas phase in larger quantity than in liquid phase while the less volatility would be in liquid phase more.

The more volatility • Ben.  
The less volatility • Tol.

↳ The compositions of Ben. and Tol. in both phases are controlled by equilibrium relations.

- one is Raoult's law

$$P_A = y_A P = X_A P_A^*(T)$$

↑ partial pressure of A component  
↑ fraction of (A) in gas phase  
↑ total pressure  
↑ fraction of A in liquid phase  
↑ vapor pressure of component A at given T.

used for similar substances  $X_A \in [0, 1]$

- ملاحظة: عندما كان هناك مادة واحدة بالهالة السائلة (w)

كان القانون  $P_A = y_A P = P_A^*(T)$  حيث أن

$$X_A = 1$$

- Another one is Henry's law:

$$P_A = Y_A P = X_A H_A(T)$$

↑            ↑    ↑            ↑            ↑

at specified.

Henry's law constant for A component in specific solvent.

This used when

$x_A \rightarrow 0$  (diluted)  
only

- Concept of bubble and dew point

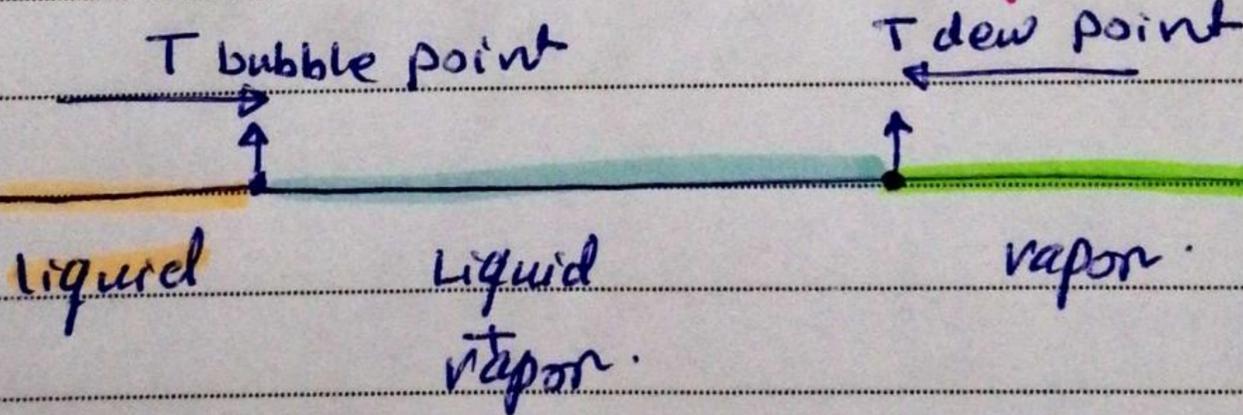
- For one component At given P.

$$T_{\text{boiling}} = T_{\text{Condensation}}$$

liquid      vapor

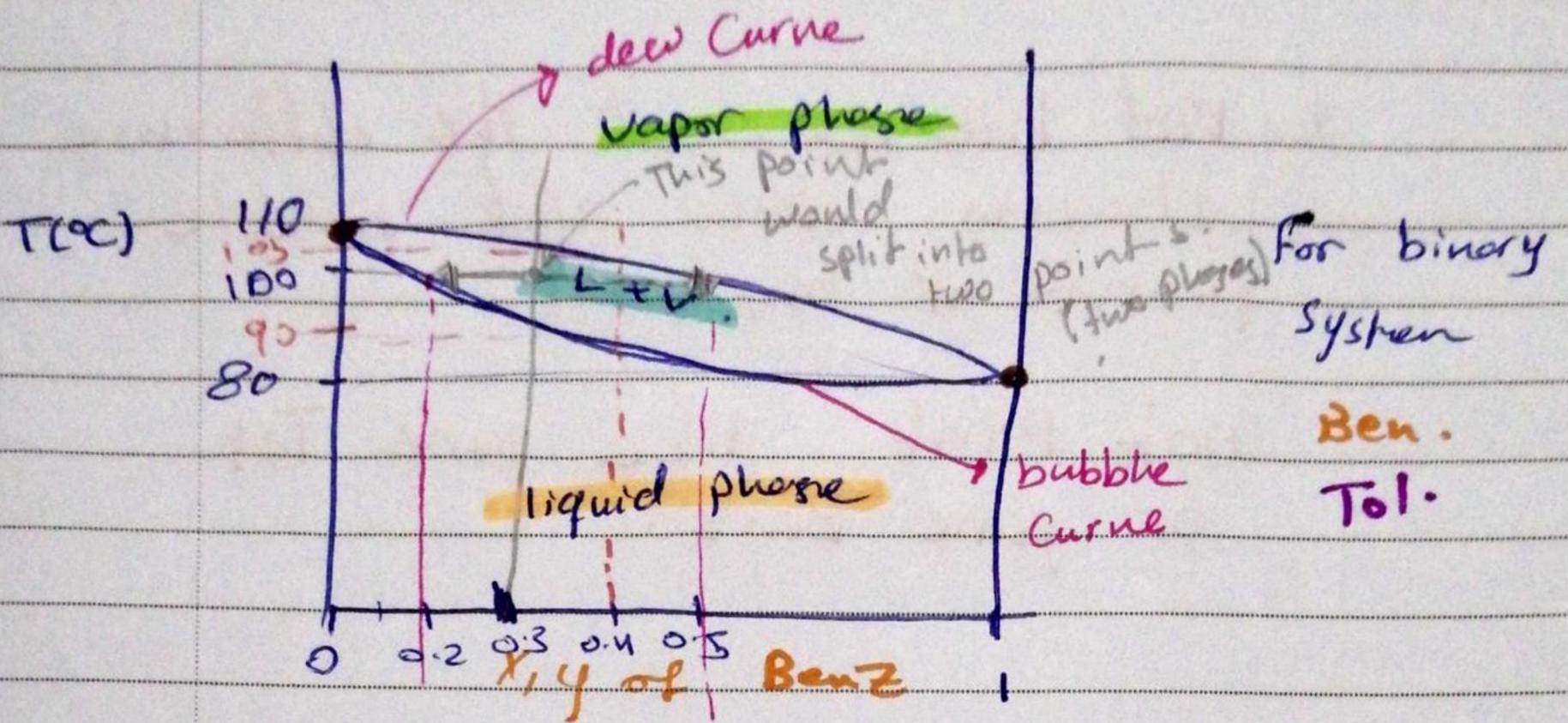
Such like water At  $P = 1 \text{ atm}$   
water transfer from liquid  $\rightarrow$  vapor  
and from vapor  $\rightarrow$  liquid at  
same temperature ( $100^\circ\text{C}$ )

- For more than one component:



-  $T_{\text{bubble point}}$ : The temperature at which  
first vapor bubble forms.

-  $T_{\text{dew point}}$ : The temperature at which  
the first liquid droplet forms.



At  $x_{Benz} = 0$   $\rightarrow$   $T = T_{boiling\ of\ Tol} = 110^\circ C$   
(pure Tol)

At  $x_{Benz} = 1$   $\rightarrow$   $T = T_{boiling\ of\ Benz} = 80^\circ C$   
(pure Benz)

(both are points only)

but Between them.

EX:  $x_{Benz} = 0.6$   $T = 70$

$\Rightarrow$  mixture in liquid phase

$x_{Tol} = 1 - 0.6 = 0.4$

$\rightarrow$   $z_{Benz} = 0.3$   $T = 100$  ( $z$  both liquid and vapor composition)  
From bubble curve  $\rightarrow$   $x_{Benz} = 0.2$   $x_{Tol} = 1 - 0.2 = 0.8$

From dew curve  $\rightarrow$   $y_{Benz} = 0.5$   $y_{Tol} = 1 - 0.5 = 0.5$

The red line represent the difference between dew and bubble point of mixture.

not point like pure Tol  
or pure Benz.