PART I- TRANSPORT PROCESSES: MOMENTUM, HEAT, AND MASS

(1.2-1)

Eq.(1.3-4) $K = 353.2 = °C + 273.15$ $°C = 80.05 = \boxed{80°C}$

Eq.(1.3-2) $°C = 80.0 = (1/1.8)(°F - 32)$ $°F = \boxed{176°F}$

Eq.(1.3-3) $°R = °F + 460 = 176 + 460$ $°R = \boxed{636°R}$

(1.2-2)

Eq.(1.3-2) $°C = (1/1.8)(°F - 32) = (1/1.8)(155 - 32) = \boxed{68.33°C}$

Eq.(1.3-4) $K = °C + 273.15 = 68.33 + 273.15 = \boxed{341.5\ K}$

Eq.(1.3-3) $°R = °F + 460 = 155 + 460 = \boxed{615\ °R}$

(1.3-1)

$MW(O_2) = 32.00$, $MW(N_2) = 28.02$

$MW(air) = 0.21(32.00) + 0.79(28.02) = \boxed{28.9\ kg/kg\ mol}$

(1.3-2)

$$\underset{MW\ 28.01}{2CO} + \underset{32.00}{O_2} \rightarrow \underset{44.01}{2CO_2}$$

$$\text{Moles } CO = \frac{56.0\ kg}{28.01\ kg/kg\ mol} = 2.00\ kg\ mol$$

Moles $O_2 = 1.00$, $1.00(32.00) = \boxed{32.0\ kg\ O_2}$ $2.00(44.0) = \boxed{88.0\ kg\ CO_2}$

(1.3-3)

Gas	g	MW	g mol	Mol frac
N_2	20	28.02	0.7138	0.1648 mol frac
O_2	83	32.00	2.5938	0.5990
CO_2	45	44.01	1.0225	0.2362
Total	148 g		4.3301 g mol	1.0000

$$\text{average mol wt} = \frac{148\ g}{4.3301\ g\ mol} = \boxed{34.2\ g/g\ mol \quad 34.2\ kg/kg\ mol}$$

(1.4-1)

$P = 2.4 \text{ mm Hg}$

From Appendix A.1

$1 \text{ atm} = 760 \text{ mm Hg}$

$$P = \frac{2.4 \text{ mm Hg}}{760 \text{ mm Hg/atm}} = \boxed{3.16 \times 10^{-3} \text{ atm}}$$

$1 \text{ atm} = 33.90 \times 12 \text{ in. } H_2O \text{ (at } 4°C)$

$$P = \frac{2.4}{760} \text{ atm} \left(33.90 \times 12 \frac{\text{in. } H_2O}{\text{atm}}\right) = \boxed{1.285 \text{ in. } H_2O}$$

$1 \mu m = 10^{-3} \text{ mm}$

$$P = \frac{2.4 \text{ mm}}{10^{-3} \text{ mm}/\mu m} = \boxed{2{,}400 \ \mu m \text{ Hg}}$$

$1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa}$

$$P = \frac{2.4}{760} \text{ atm} \left(1.01325 \times 10^5 \frac{\text{Pa}}{\text{atm}}\right) = \boxed{320 \text{ Pa}}$$

(1.4-2)

$V_1 = 65.0 \text{ ft}^3$, $T_1 = 460 + 90 = 550 °R$, $T_2 = 460 + 65 = 525 °R$

From Appendix A.1, $1 \text{ atm} = 14.696 \text{ psia}$

$p_1 = 29.0 + 14.696 = 43.696 \text{ psia}$, $p_2 = 75.0 + 14.696 = 89.696 \text{ psia}$

$p_1 = 43.696/14.696 = 2.975 \text{ atm}$, $p_2 = 89.696/14.696 = 6.104 \text{ atm}$

Eq.(1.4-2)

$$V_2 = V_1 \frac{p_1}{p_2} = 65.0 \frac{(2.975)}{(6.104)} = \boxed{30.24 \text{ ft}^3}$$

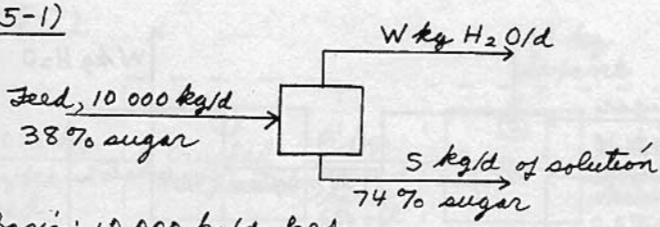
$R = 0.7302 \text{ ft}^3 \cdot \text{atm/lb mol} \cdot °R$, MW of $N_2 = 28.01$

Eq.(1.4-1)

$$n = pV/RT = (2.975)(65.0)/(0.7302)(550) = \boxed{0.4815 \text{ lb mol}}$$

$$\text{Density, } \rho = (0.4815)(28.01)/30.24 = \boxed{0.446 \text{ lb}_m/\text{ft}^3}$$

(1.5-1)



Basis: 10 000 kg/d feed

Total balance

$$10\,000 = S + W$$

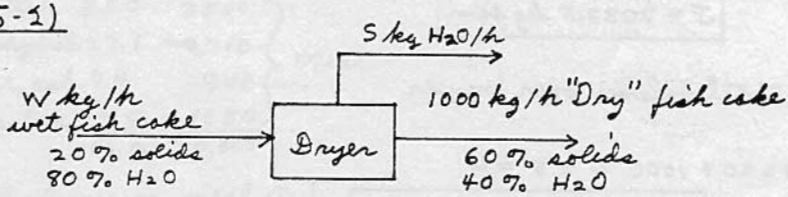
Solids balance

$$0.38(10\,000) = 0.74S + (0)W$$

$$W = 10\,000 - 5135 = \boxed{4865 \text{ kg } H_2O/d}$$

$$\boxed{S = 5135 \text{ kg/d of 74\% solution}}$$

(1.5-2)



Basis: 1000 kg/h "dry" fish cake

Solids balance

$$0.20W = S(0) + 0.60(1000)$$

$$\boxed{W = 3000 \text{ kg/h wet cake feed}}$$

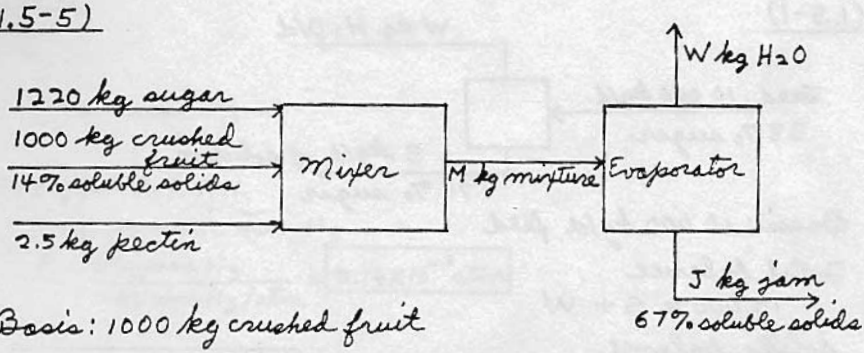
Total balance

$$W = S + 1000$$

$$3000 = S + 1000$$

$$\boxed{S = 2000 \text{ kg } H_2O/h}$$

(1.5-5)



Basis: 1000 kg crushed fruit

Soluble solids balance over whole process

$$1220 + 0.14(1000) + 2.5 = 0.67J + W(0)$$

$$\boxed{J = 2033.5 \text{ kg jam}}$$

Total balance over mixer

$$1220 + 1000 + 2.5 = M$$

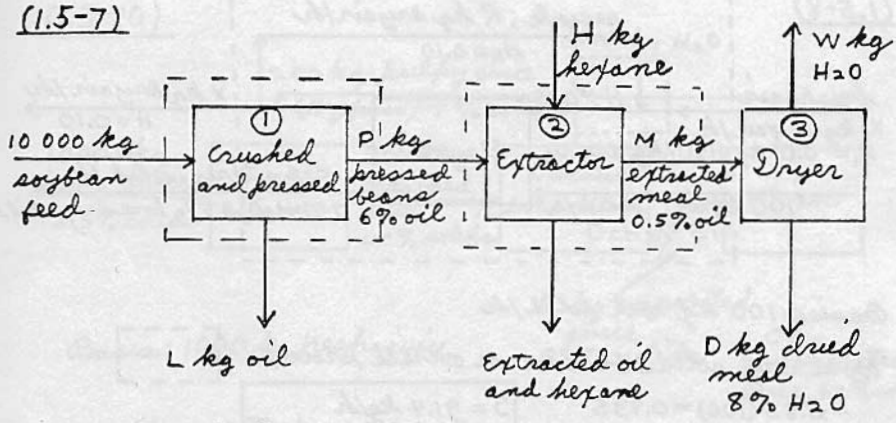
$$\boxed{M = 2222.5 \text{ kg mixture}}$$

Total balance over evaporator

$$2222.5 = W + 2033.5$$

$$\boxed{W = 189 \text{ kg } H_2O}$$

(1.5-7)



soybean feed

	%	kg
Oil	18.0	1800
Protein	35.0	3500
Carbohydrate	27.1	2710
Fiber, ash	9.4	940
Moisture	10.5	1050
	100.0	10,000

Meal

(a) Balance on meal ①

$0.82(10\,000) = 0.94P$

$\boxed{P = 8723 \text{ kg pressed beans}}$ $L = 1277$ kg oil

Total balance

$10,000 = L + P$

(b) Balance on meal ②

$0.94(8723) = 0.995M$

$\boxed{M = 8241 \text{ kg extracted meal}}$

kg oil $= 0.005(8241) = 41$ kg oil in meal

(c) Composition of M

Oil	41 kg
Protein	3500
Carbohydrate	2710
Fiber, ash	940
H_2O	1050
M =	8241 kg

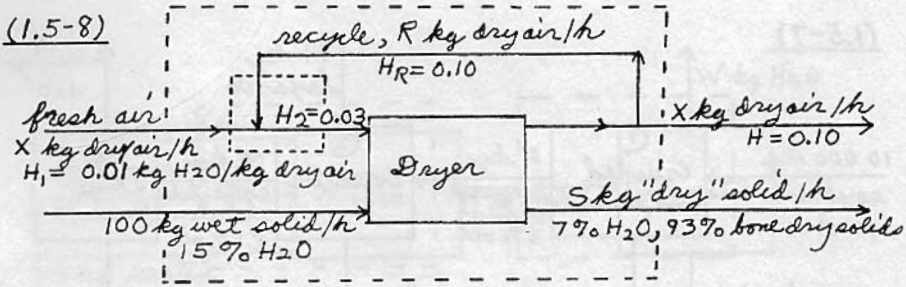
Balance on meal (less H_2O) ③

$8241 - 1050 = 0.92D$

$\boxed{D = 7816 \text{ kg dried meal}}$

$$\text{Wt\%} = \frac{3500}{7816} \times 100 = \boxed{44.8 \text{ wt\% protein}}$$

(1.5-8)



Basis: 100 kg wet solid/h

Bone dry solids balance on overall process

$0.85(100) = 0.93S$

$\boxed{S = 91.4 \text{ kg/h "dried" product}}$

Water balance on overall process

$0.01X + 0.15(100) = 0.10X + 0.07(91.4)$

$\boxed{X = 95.6 \text{ kg/h dry air in fresh air}}$

Water balance on inlet point

$0.01X + 0.10R = 0.03(R + X)$

$0.01(95.6) + 0.10R = 0.03(R + 95.6)$

$\boxed{R = 27.3 \text{ kg/h dry air in recycle air}}$

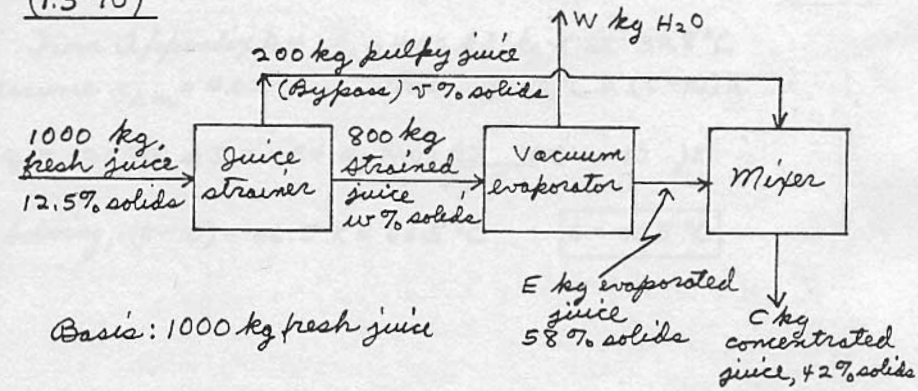
Overall check balance on H_2O

$0.01(95.6) + 0.15(100) = 0.07(91.4) + 0.10(95.6)$

$0.956 + 15.00 = 6.40 + 9.56$

$15.956 \cong 15.96$ Hence, a close check.

(1.5-10)



Basis: 1000 kg fresh juice

Total balance on overall process

$$1000 = W + C$$

Solids balance on overall process

$$0.125(1000) = W(0) + 0.42C$$

$$W = 1000 - 298$$

$$W = 702 \text{ kg } H_2O$$

$$\boxed{C = 298 \text{ kg concentrated juice}}$$

Balance on evaporator

Total

$$800 = 702 + E \qquad E = 98 \text{ kg evaporated juice}$$

Solids

$$\frac{w}{100}(800) = 702(0) + 0.58\,(98)$$

$$\boxed{w = 7.10\% \text{ solids in strained juice}}$$

Solids balance on mixer

$$\frac{v}{100}(200) + 0.58\overset{(E)}{(98)} = 0.42\,\overset{(C)}{(298)}$$

$$\boxed{v = 34.2\% \text{ solids in pulpy juice bypassed}}$$

(1.6-1)

$$\text{kg mol } CO_2 = \frac{250}{44.0} = 5.680$$

$$C_{pm}(298-373K) = 38.73 \text{ kJ/kg mol}\cdot K \qquad \text{Table 1.6-1}$$
$$(298-623K) = 43.06$$

$$q(373-623K) = q_1(298-623) - q_2(298-373)$$

$$q = \frac{5.680}{1000}\left[43.06(623-298) - 38.73(373-298)\right]$$

$$\boxed{q = 62.98 \text{ kJ}}$$

$$q = \frac{62.98 \text{ kJ} \times 1000}{4.184 \frac{J}{cal}} = \boxed{15\,050 \text{ cal}}$$

$$q = \frac{62.98 \text{ kJ}}{1.05506 \text{ kJ/btu}} = \boxed{59.7 \text{ btu}}$$

(1.6-3)

From Appendix A.4, $c_p = 4.02\ kJ/kg \cdot K$ at 32.8 °C

assume $c_{pm} = 4.02$ Note: $(t-10)°C = (t-10)K$

$$q = 121\,300\ kJ = 454\ kg\left(4.02\ \frac{kJ}{kg \cdot K}\right)(t - 10)K$$

Solving, $(t-10) = 66.5\ K = 66.5\ °C$ $\boxed{t = 76.5\ °C}$

(1.6-4)

(a) A.2-9 $P = 30$ psia Liquid H_2O

$$\Delta H = H_{240°F} - H_{40°F}$$
$$= 208.44 - 8.02 = \boxed{200.42\ btu/lb_m}$$

(b) $P = 24.97$ psia

$H_{240°F}$ (vapor) = 1160.7

$H_{40°F}$ (liquid) = 8.02

$$\boxed{\Delta H = 1152.7\ btu/lb_m}$$

(c) $P = 1.0$ atm abs

$H_{212°F}$ (vapor) = 1150.5

$H_{40°F}$ (liquid) = 28.1

$$\boxed{\Delta H = -1122.4\ btu/lb_m}$$

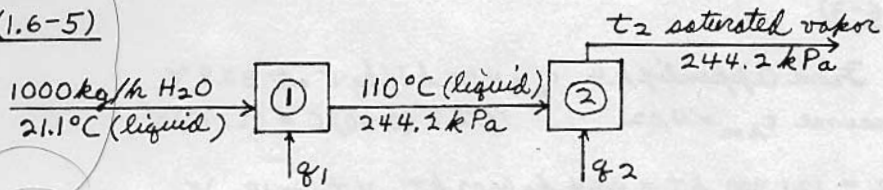
(d) $P = 1.0$ atm abs

$H_{212°F}$ (vapor) = 1150.5

$H_{212°F}$ (liquid) = 180.2

$$\boxed{\Delta H = -970.3\ btu/lb_m \quad (-2256.9\ kJ/kg)}$$

(1.6-5)



From Table A.2-9, $t_2 = 126.6\,°C$ (linear interpolation)

h (liq. at 21.1°C) = 88.56 kJ/kg, h (liq. at 110°C) = 461.30 kJ/kg

H (sat'd vapor at 126.6°C) = 2715.7 kJ/kg

First stage ①

$$q_1 = (461.30 - 88.56\ \text{kJ/kg})\left(1000\ \frac{\text{kg}}{\text{h}}\right) = \boxed{3.7274 \times 10^5\ \text{kJ/h}}$$

Both stages ① and ②

$$q_1 + q_2 = (2715.7 - 88.56)(1000) = \boxed{2.6271 \times 10^6\ \text{kJ/h}}$$

(1.6-7) Table A.3-1 (4.0 g mol NH_3)

$$4\,NH_3(g) + 5\,O_2(g) \longrightarrow 4\,NO(g) + 6\,H_2O(g)$$

$$4(-46.19\ \text{kJ/g mol}) \quad 5(0) \quad 4(+90.374) \quad 6(-241.826)$$

Eq (1.6-7)

$$\Delta H^\circ = \Sigma \Delta H^\circ_{f\,(\text{products})} - \Sigma \Delta H^\circ_{f\,(\text{reactants})}$$

$$= [4(+90.374) + 6(-241.826)] - [4(-46.19) + 5(0)]$$

$$\boxed{\Delta H^\circ = -904.7\ \text{kJ/g mol} \quad (-216.24\ \text{kcal/g mol})}$$

(1.7-1)

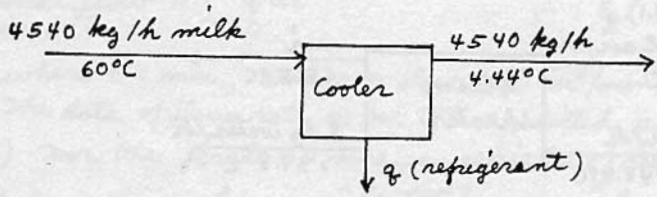


Table A.4-1

c_{pm} (milk) = 3.85 kJ/kg·K

Heat Balance: Datum = 25°C

Input

H(milk) = 4540 (3.85)(60 − 25) = 611 765 kJ/h

Output

H(milk) = 4540 (3.85)(4.44 − 25) = −359 368 kJ/h

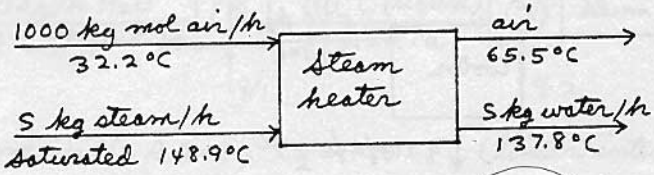
q

Input = Output

611 765 = −359 368 + q

$q = 971\,133 \text{ kJ/h} = \frac{971{,}133}{3600} = \boxed{269.6 \text{ kW } (920\,000 \text{ Btu/h})}$

(1.7-4)



Datum: 25°C

(air c_{pm} = 29.20 kJ/kg mol·K for 25–32.2°C Table 16-1)

Heat in

$H(\text{air}) = (1000)(29.20)(32.2-25.0) = 210\,240$ kJ/h

$H(\text{steam}) = S(2745.1) = 2745.1\,S$ kJ/h
(Table A.2-9) 148.9°C

Heat out

$H(\text{air}) = (1000)(29.25)(65.5-25.0) = 1\,184\,625$ kJ/h

$H(\text{water}) = S(579.70) = 579.70\,S$ kJ/h
137.8°C

In = Out

$210\,240 + 2745.1S = 1\,184\,625 + 579.70S$

$\boxed{S = 450 \text{ kg steam/h}}$

(1.8-2)

$$\text{Total flow in } m^3 = \int_{t=0}^{t=t_F} q\,dt \qquad \text{Eq.(1.8-1)}$$

where t = min., and q = flow rate m^3/min.

The data of flow rate q vs. t are plotted as shown.

(a) For the first 60 min., a series of rectangles are drawn as shown. The total area under the curve is the sum of the rectangles A through F.

$$\text{Total area} = 680(10) + 740(10) + 805(10) + 853(10) + 884(10) + 884(10)$$
$$= \boxed{48\,460\ m^3 \text{ for } 60 \text{ min.}}$$

For 120 min., use rectangles A through L

$$\text{Total area} = 48{,}460 + 840(10) + 763(10) + 700(10) + 650(10) + 620(10) + 620(10)$$
$$= \boxed{90\,390\ m^3 \text{ for } 120 \text{ min.}}$$

(b) Simpson's method

Eq.(1.8-3) $\quad h = 10$ min. $\qquad m = \dfrac{b-a}{h} = \dfrac{120-0}{10} = 12$

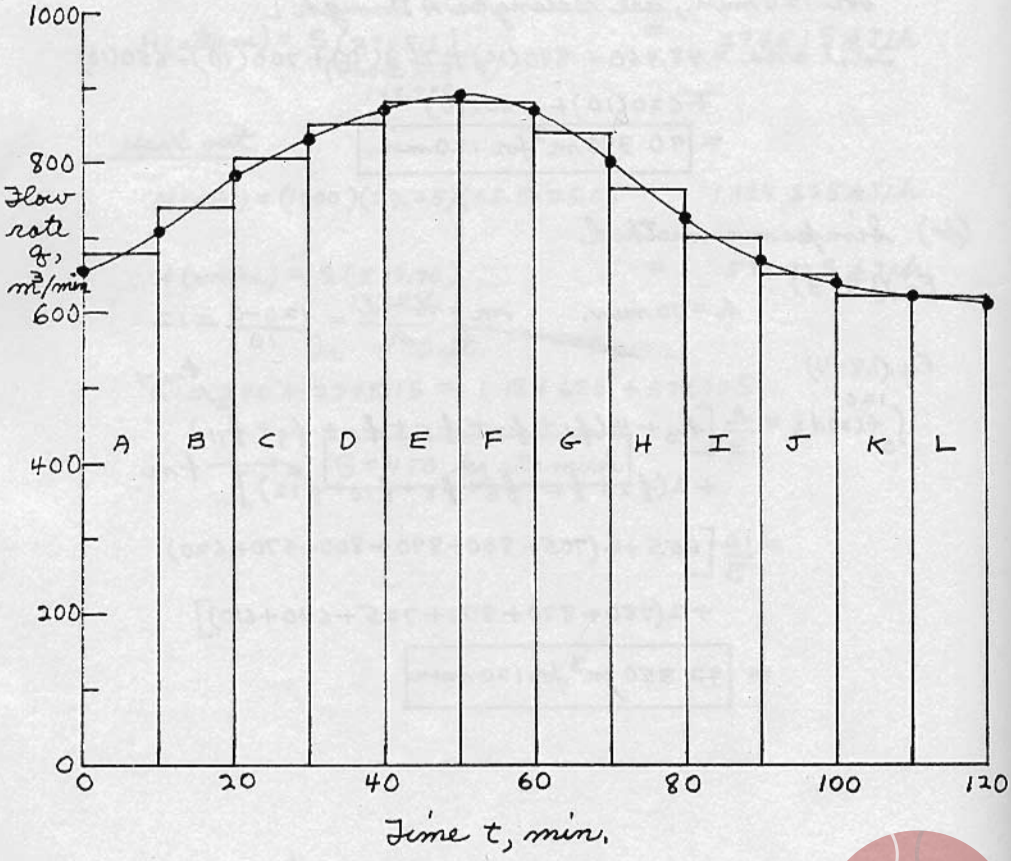
Eq.(1.8-4)

$$\int_0^{120} f(x)\,dx = \frac{h}{3}\Big[h_0 + 4(f_1 + f_3 + f_5 + f_7 + f_9 + \overset{f_{m-1}}{f_{11}}) + 2(f_2 + f_4 + f_6 + f_8 + f_{10} + \underset{f_m}{f_{12}})\Big]$$

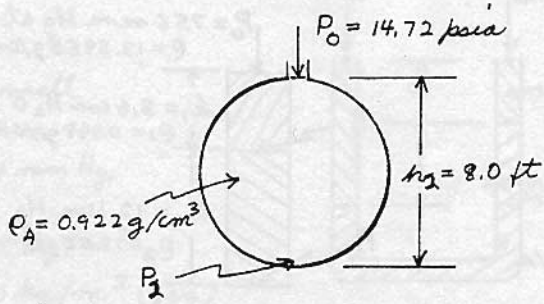
$$= \frac{10}{3}\Big[655 + 4(705 + 830 + 890 + 800 + 670 + 620) + 2(780 + 870 + 870 + 725 + 640 + 610)\Big]$$

$$= \boxed{92\,350\ m^3 \text{ for } 120 \text{ min}}$$

(1.8-2) - p. 2



(2.2-1)



English units

$\rho_A = 0.922(62.43) = 57.56$ lb$_m$/ft^3

Eq. (2.2-5)

$$P_2 = h_2 \rho_A \frac{g}{g_c} + P_0$$

$$= 8.0(57.56)\frac{(32.174)}{(32.174)}(1/144) + 14.72 = \boxed{17.92 \text{ psia}}$$

SI units $\quad P_2 = h_2 \rho_A g + P_0$

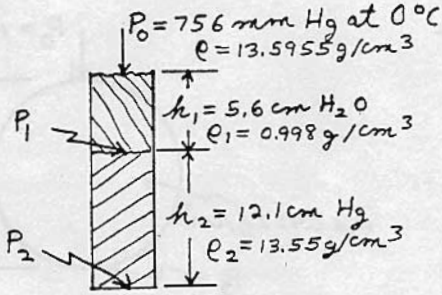
$P_0 = 14.72(6.89476) = 101.5\times10^3$ N/m^2

$h_2 = 8.0(1/3.2808) = 2.438$ m

$\rho_A = 0.922(1000) = 922$ kg/m^3

$$P_2 = 2.438(922)(9.80665) + 101.5\times10^3 = 123.5\times10^3 = \boxed{123.5 \text{ kN/m}^2}$$

(2.2-2)



$10\ dyn/cm^2 = 1\ N/m^2$

$1\ dyn = 1\ g \cdot cm/s^2$

$$P_2 = P_1 + P_0 = h_1\rho_1 + h_2\rho_2 + P_0$$

$$P_2 = \left[(5.6\,cm)\left(0.998\,\frac{g\ mass}{cm^3}\right) + (12.1\,cm)\left(13.55\,\frac{g\ mass}{cm^3}\right) + (75.6\,cm)\left(13.5955\,\frac{g\ mass}{cm^3}\right)\right]\left(980\,\frac{cm}{s^2}\right)$$

$$P_2 = 1.175 \times 10^6\,\frac{g \cdot cm}{cm^2 \cdot s^2} = \boxed{1.175 \times 10^6\ dyn/cm^2}$$

Table A.1-8

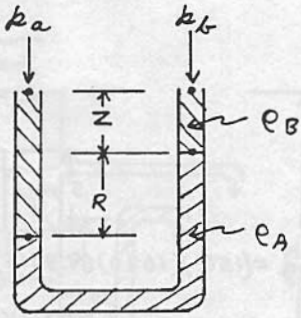
$1\ psia = 6.8947 \times 10^4\ dyn/cm^2$

$$P_2 = \frac{1.175 \times 10^6\ dyn/cm^2}{6.8947 \times 10^4\ \frac{dyn/cm^2}{psia}} = \boxed{17.0\ psia}$$

$$P_2 = 17.0\ psia - 14.7\ psia = \boxed{2.3\ psig}$$

$$P_2 = \frac{1.175 \times 10^6\ dyn/cm^2}{10\ \frac{dyn/cm^2}{N/m^2}} = 1.175 \times 10^5\ N/m^2 = \boxed{117.5\ kN/m^2}$$

(2.2-4)



Z very small

$R = 0.415\ m$

$p_b = 754\ mm\ Hg$

$p_a = p_a$

$\rho_A = 1000\ kg/m^3$

$\rho_B = 1.30\ kg/m^3$ (gas)

Eq. (2.2-14)

$$p_a - p_b = R(\rho_A - \rho_B)g$$

$$= (0.415)(1000 - 1.3)(9.80665)$$

$$p_a - p_b = 4064\ N/m^2 = 4.064\ kN/m^2$$

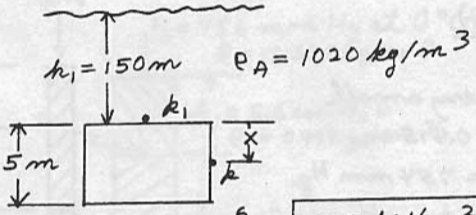
$$p_b = \frac{754}{760}(101.325) = 100.52\ kN/m^2$$

$$p_a = 100.52 + 4.064$$

$$\boxed{p_a = 104.6\ kN/m^2}$$

(15.17 psia)

(2.2-6)



$$k_1 = h_1 \rho_A g = (150)(1020)(9.8066) = 1.500 \times 10^6 = \boxed{1500\ kN/m^2}$$

$$k = h_1 \rho_A g + x \rho_A g = (150+x)(1020)(9.8066)$$

$$= (150+x)(10003) = \boxed{10.00\,(150+x)\ kN/m^2}$$

(2.2-7)

See Fig. 2.2-5b for diagram

$$k_1 = k_A + h_1 \rho_A g$$

$$k_2 = k_B + h_3 \rho_B g + h_2 \rho_C g$$

By principles of hydrostatics,

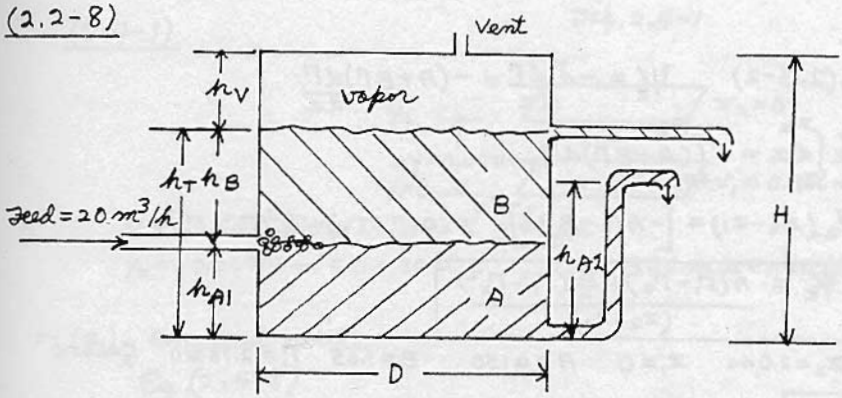
$$k_1 = k_2$$

$$k_A + h_1 \rho_A g = k_B + h_3 \rho_B g + h_2 \rho_C g$$

solving,

$$\boxed{k_A - k_B = -h_1 \rho_A g + h_3 \rho_B g + h_2 \rho_C g}$$

(2.2-8)



$\rho_A = 1050\ kg/m^3$, $\rho_B = 875\ kg/m^3$

$10\ m^3/h$ of A, $10\ m^3/h$ of B

vol. of liquid in tank for 25 min. $= \frac{20\ m^3}{60\ min.}(25\ min.) = 8.333\ m^3$

$8.333\ m^3 = \frac{2}{3} V_{tank}$ $\quad V_{tank} = 12.50\ m^3$ $\quad D = H$

$12.50\ m^3 = (\pi D^2/4)H = (\pi D^2/4)D$

Solving, $\boxed{D = H = 2.515\ m}$

$h_V = h_B = h_{A1} = 2.515/3 = 0.8383\ m$

$\boxed{h_{A1} = 0.838\ m \text{ level of A}}$ $\quad h_T = h_{A1} + h_B = \boxed{1.677\ m \text{ level of B}}$

Eq. (2.2-21)

$$h_{A1} = \frac{h_{A2} - h_T \rho_B/\rho_A}{1 - \rho_B/\rho_A}$$

$$0.838 = \frac{h_{A2} - 1.677(875/1050)}{1 - 875/1050}$$

Solving,

$$\boxed{h_{A2} = 1.537\ m}$$

(2.3-1)

Eq. (2.3-2) $\quad \Psi_z = -\delta \dfrac{d\Gamma}{dz} = -(A+B\Gamma)\dfrac{d\Gamma}{dz}$

$$\Psi_z \int_{z_1}^{z_2} dz = -\int_{\Gamma_1}^{\Gamma_2} (A+B\Gamma)\,d\Gamma$$

$$\Psi_z (z_2 - z_1) = \left[-A\Gamma - \frac{B\Gamma^2}{2}\right]_{\Gamma_1}^{\Gamma_2} = A(\Gamma_1 - \Gamma_2) + \frac{B}{2}(\Gamma_1^2 - \Gamma_2^2)$$

(a) $$\boxed{\Psi_z = \frac{A(\Gamma_1 - \Gamma_2) + \frac{B}{2}(\Gamma_1^2 - \Gamma_2^2)}{(z_2 - z_1)}}$$

$z_2 = 2.0\,m \quad z_1 = 0 \quad A = 0.150 \quad B = 1.65 \quad \Gamma_1 = 2.78\times10^{-2} \quad \Gamma_2 = 1.5\times10^{-2}$

$$\Psi_z = \frac{0.150(2.78\times10^{-2} - 1.50\times10^{-2}) + \frac{1.65}{2}\left[(2.78\times10^{-2})^2 - (1.50\times10^{-2})^2\right]}{(2.0 - 1.0)}$$

$$\boxed{\Psi_z = 1.186\times10^{-3}\ \text{amount property}/s\cdot m^2}$$

(b) $\Gamma = \Gamma_2 \quad z = z_2 = 1.0\,m$

$$\Psi_z = \frac{A(\Gamma_1 - \Gamma) + B/2(\Gamma_1^2 - \Gamma^2)}{z - z_1}$$

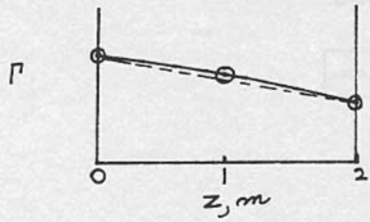
$$1.186\times10^{-3} = \frac{0.150(2.78\times10^{-2} - \Gamma) + \frac{1.65}{2}\left[(2.78\times10^{-2})^2 - \Gamma^2\right]}{(1.0 - 0)}$$

Rearranging,

$$0.825\Gamma^2 + 0.150\Gamma - 3.6216\times10^{-3} = 0$$

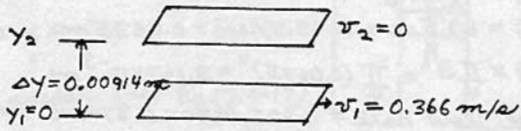
Solving, $\boxed{\Gamma = 2.158\times10^{-2}\ \text{amount property}/m^3}$

Plot below shows a slight curvature since δ varies.
(If the plot were a straight line, $\Gamma = \dfrac{(2.78+1.50)10^{-2}}{2} = 2.140\times10^{-2}$)



(2.4-1)

Fig. 2.4-1



$\mu = 4.0 \times 10^{-2}$ Pa·s at 303 K = 40 cp

$y_2 = 0.00914\,m = 0.030\,ft$ $\qquad v_2 = 0.366\,m/s = 1.20\,ft/s$

(a) English units.

Eq. (2.4-8)

$$\tau = \frac{\mu(v_1 - v_2)}{g_c(y_2 - y_1)} = \frac{40(6.7197\times10^{-4})(1.20-0)}{32.174\,(0.030-0)} = \boxed{3.34\times10^{-2}\,\frac{lb_f}{ft^2}}$$

$$= \frac{(cp)\left(\frac{lb_m/ft\cdot s}{cp}\right)(ft/s)}{\frac{lb_m\cdot ft}{lb_f\cdot s^2}\,(ft)}$$

Shear stress

$$\text{shear rate} = \frac{\Delta v}{\Delta y} = \frac{1.20\,ft/s}{0.03\,ft} = \boxed{40.0\,s^{-1}\text{ shear rate}}$$

(b) SI units

Eq. (1H-5)

$$\tau = \frac{\mu(v_1 - v_2)}{(y_2 - y_1)} = \frac{(4.0\times10^{-2}\,kg/m\cdot s)(0.366-0\,m/s)}{(0.00914-0\,m)}$$

$$= 1.602\,\frac{kg\cdot m/s^2}{m^2} = \boxed{1.602\,N/m^2}$$

(c) 1 = soybean $\qquad$ 2 = glycerol

$\mu_2 = 1.069\,kg/m\cdot s = 1069\,cp \qquad \mu_1 = 4.0\times10^{-2}\,kg/m\cdot s$

$$\tau_1 = \frac{\mu_1(\Delta v_1)}{\Delta y_1} = \frac{\mu_2(\Delta v_2)}{\Delta y_2} = \tau_2$$

$\Delta y_1 = \Delta y_2$

$$\frac{\mu_1\,\Delta v_1}{\cancel{\Delta y_1}} = \frac{\mu_2\,\Delta v_2}{\cancel{\Delta y_2}}$$

$4.0\times10^{-2}(0.366-0) = 1.069\,(\Delta v_2)$

$\Delta v_2 = \boxed{0.01369\,m/s \quad (0.0449\,ft/s)}$

$$\text{Shear rate} = \frac{\Delta v}{\Delta y} = \frac{0.01369}{0.00914} = \boxed{1.50\,s^{-1}\text{ shear rate}}$$

(2.5-1)

$\rho = 1030\ kg/m^3 \quad \mu = 2.12\ cp = 2.12\times10^{-3}\ Pa\cdot s \quad m = 0.605\ kg/s$

$D = 63.5\ mm = 63.5/1000 = 0.0635\ m$

$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.0635)^2 = 3.167\times10^{-3}\ m^2$

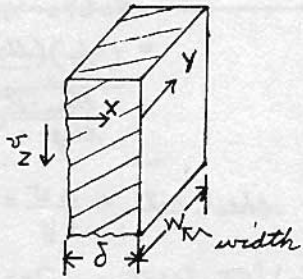
vol. flow rate $= 0.605/1030 = 5.874\times10^{-4}\ m^3/s \quad v = \frac{5.874\times10^{-4}}{3.167\times10^{-3}} = 0.1855\ m/s$

(a) $N_{Re} = \frac{Dv\rho}{\mu} = \frac{0.0635(0.1855)(1030)}{2.12\times10^{-3}} = \boxed{5723 \quad \text{Turbulent flow}}$

(b) $\frac{2100}{5723}(5.874\times10^{-4}) = \boxed{2.155\times10^{-4}\ m^3/s}$

$\frac{2100}{5723}(0.1855) = \boxed{v = 6.806\times10^{-2}\ m/s}$

(2.6-1)



(a) Max. vel. at $x=0$

$v_z = \frac{\rho g \delta^2}{2\mu}\left[1-(x/\delta)^2\right]$

$\boxed{v_{z\,max} = \rho g \delta^2/2\mu}$

(b) Eq. (2.6-17)

$v_{z\,av} = \frac{1}{A}\iint_A v_z\, dA \qquad A = W\delta \qquad dA = dx\,dy$

$v_{z\,av} = \frac{1}{W\delta}\int_{y=0}^{y=W}\int_{x=0}^{x=\delta} v_z\, dx\,dy$

$= \frac{1}{W\delta}\int_{y=0}^{y=W}\int_{x=0}^{x=\delta}\frac{\rho g \cancel{\delta^2}}{2\mu}\left(\frac{\delta^2-x^2}{\cancel{\delta^2}}\right)dx\,dy$

$= \frac{1}{W\delta}\frac{\rho g}{2\mu}\left[y\right]_{y=0}^{y=W}\left[\delta^2 x - \frac{x^3}{3}\right]_{x=0}^{x=\delta}$

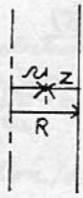
$= \frac{\rho g}{W\delta 2\mu}(W-0)(\delta^3-\delta^3/3)$

$\boxed{v_{z\,av} = \frac{\rho g \delta^2}{3\mu}} \qquad \frac{v_{z\,av}}{v_{z\,max}} = \frac{\frac{\rho g\delta^2}{3\mu}}{\frac{\rho g \delta^2}{2\mu}} = \frac{2}{3}$

$\boxed{v_{z\,av} = \frac{2}{3}v_{z\,max}}$

(2.6-3)

$$v = v_{max}\left[(R-r)/R\right]^{1/7}$$



Eq. (2.6-17) $\quad v_{av} = \frac{1}{A}\iint_A v\,dA$

$A = \pi R^2 \quad dA = r\,dr\,d\theta \quad$ subst. into Eq. (2.6-17)

$$v_{av} = \frac{v_{max}}{\pi R^2}\int_0^{2\pi}\int_{r=0}^{r=R}\left(\frac{R-r}{R}\right)^{1/7} r\,dr\,d\theta$$

Let $z = R - r$ = distance from wall

$dz = -dr \qquad r = R - z$

B.C.

at wall, $r = R$

$z = R - R = 0$

at center, $r = 0$

$z = R - 0 = R$

subst. into above eq. for v_{av}.

$$v_{av} = \frac{v_{max}}{\pi R^2}\int_0^{2\pi}\int_{z=R}^{z=0}\left(\frac{z}{R}\right)^{1/7}(R-z)(-dz)\,d\theta$$

$$= \frac{v_{max}}{\pi R^2}\int_0^{2\pi}\int_{z=R}^{z=0}\frac{(z^{1/7})(z-R)}{R^{1/7}}(dz)\,d\theta$$

$$= \frac{v_{max}}{\pi R^2}\int_0^{2\pi}\int_{z=R}^{z=0}\frac{(z^{8/7} - Rz^{1/7})}{R^{1/7}}(dz)\,d\theta$$

$$= \frac{v_{max}}{\pi R^2 R^{1/7}}\int_0^{2\pi}\left[\frac{7}{15}z^{15/7} - R\frac{7}{8}z^{8/7}\right]_{z=R}^{z=0} d\theta$$

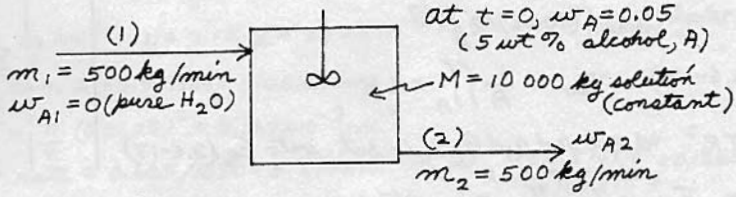
$$= \frac{v_{max}}{\pi R^2 R^{1/7}}\int_0^{2\pi}\left(0 - \frac{7}{15}R^{15/7} + \frac{7}{8}R^{15/7}\right) d\theta$$

$$= \frac{v_{max}}{\pi R^2}(2\pi)\left(-\frac{7}{15}R^2 + \frac{7}{8}R^2\right)$$

$$= v_{max}\frac{49}{120}(2)$$

$$\boxed{v_{av} = \frac{49}{60}v_{max} = 0.817\,v_{max}}$$

(2.6-5)



Total mass balance

M = mass in tank at time $t = 10\,000$ kg (constant)

Eq. (2.6-5)

$$\frac{dM}{dt} = 0$$

Eq. (2.6-2)

$$m_1 = m_2 = 500 \text{ kg/min}$$

alcohol (A) balance (Calculate time for $w_{A2} = 0.01$)

Eq. (2.6-8)

$$m_{i2} - m_{i1} + \frac{dM_i}{dt} = R_i$$

$$m_{A2} - m_{A1} + \frac{d(Mw_A)}{dt} = 0 \quad \text{no reaction}$$

$$m_{A1} = 500(w_{A1}) = 500(0) = 0$$

$$m_{A2} = 500(w_{A2})$$

$$M_i = Mw_{A2} = 10000\,w_{A2}$$

Substituting into above equation

$$500(w_{A2}) - 0 + \frac{d(10000\,w_{A2})}{dt} = 0$$

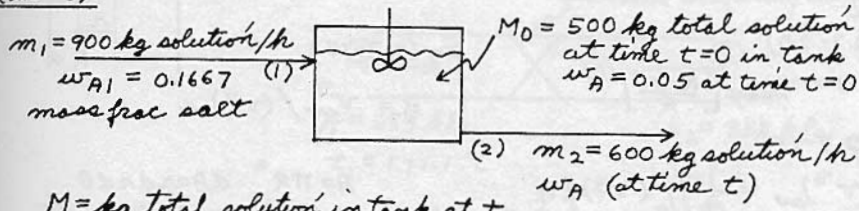
$$500\,w_{A2} + 10\,000\frac{dw_{A2}}{dt} = 0$$

$$\frac{-10\,000}{500}\int_{w_{A2}=0.05}^{w_{A2}=0.01} \frac{dw_{A2}}{w_{A2}} = \int_{t=0}^{t=t_F} dt$$

$$20 \ln \frac{0.05}{0.01} = t_F - 0$$

$$\boxed{t_F = 32.2 \text{ min}}$$

(2.6-6)



M = kg total solution in tank at t

Overall mass balance

Eq. (2.6-5) $$\frac{\partial}{\partial t}\iiint_V \rho\, dV = \frac{dM}{dt}$$

Eq. (2.6-7)
$$\iint_A v\rho\cos\alpha\, dA = m_2 - m_1 = 600-900 = -300 \text{ kg/h}$$

Substituting above 2 equations into Eq. (2.6-6)
$$-300 + \frac{dM}{dt} = 0$$
$$\int_{M_0=500}^{M} dM = 300\int_{t=0}^{t} dt$$
$$M = 300t + 500$$

Salt (A) balance $\qquad w_A$ = concentration in tank at time t

Eq. (2.6-5) $$\frac{\partial}{\partial t}\iiint_V \rho\, dV = \frac{dM_A}{dt} = \frac{d(Mw_A)}{dt} = w_A\frac{dM}{dt} + M\frac{dw_A}{dt}$$

Eq. (2.6-7)
$$\iint_A v\rho\cos\alpha\, dA = w_A(600) - \overset{0.1667}{w_{A1}}(900) = 600w_A - 150.0$$

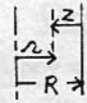
Substituting above 2 equations into Eq. (2.6-6)
$$600w_A - 150 + w_A\frac{dM}{dt} + M\frac{dw_A}{dt} = 0$$

Substitute $M = 300t + 500$ into above,
$$600w_A - 150 + w_A\frac{d(300t+500)}{dt} + (300t+500)\frac{dw_A}{dt} = 0$$
$$600w_A - 150 + w_A\left(\frac{300dt+0}{dt}\right) + (300t+500)\frac{dw_A}{dt} = 0$$
$$\int_{w_A=0.05}^{w_A}\frac{dw_A}{(30-180w_A)} = \int_{t=0}^{t=t}\frac{dt}{(60t+100)}\ ; \quad \boxed{w_A = \frac{1}{6} - \frac{7}{60}\left(\frac{10}{6t+10}\right)^3}$$

When $t = 2h$

$$w_A = \frac{1}{6} - \frac{7}{60}\left(\frac{10}{6(2)+10}\right)^3 \qquad \boxed{w_A = 0.1557} \quad t = 2.0h$$

(2.7-1)



Eq.(2.7-20)

$$v = v_{max}\left[\frac{R-r}{R}\right]^{1/7}$$

Eq.(2.7-14)

$$\alpha = v_{av}^3 / (v^3)_{av}$$

Eq.(2.7-15)

$$(v^3)_{av} = \frac{1}{A}\iint_A (v^3)\,dA \qquad A = \pi R^2 \quad dA = r\,dr\,d\theta$$

subst. values for A and dA into Eq.(2.7-15) and v from Eq.(2.7-20)

$$(v^3)_{av} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \left\{v_{max}\left[\frac{R-r}{R}\right]^{1/7}\right\}^3 r\,dr\,d\theta$$

$$= \frac{2\pi}{\pi R^2}\,\frac{v_{max}^3}{R^{3/7}}\int_0^R (R-r)^{3/7}\,r\,dr$$

Set $z = R - r$ = distance from wall

$dz = -dr \qquad r = R - z$

B.C.

at wall: $r = R$, $z = R - R = 0$

at center: $r = 0$, $z = R - 0 = R$

subst. into eq. for $(v^3)_{av}$

$$(v^3)_{av} = \left(\frac{2v_{max}^3}{R^{17/7}}\right)\int_R^0 z^{3/7}(R-z)(-dz) = \left(\frac{2v_{max}^3}{R^{17/7}}\right)\int_R^0 \left(z^{10/7} - Rz^{3/7}\right)dz$$

$$= \left(\frac{2v_{max}^3}{R^{17/7}}\right)\left[\frac{7}{17}z^{17/7} - \frac{7}{10}Rz^{10/7}\right]_R^0$$

$$= \left(\frac{2v_{max}^3}{R^{17/7}}\right)\left(0 - \left[\frac{7}{17}R^{17/7} - \frac{7}{10}R^{17/7}\right]\right)$$

$$\boxed{(v^3)_{av} = \frac{98}{170}v_{max}^3}$$

From Problem (2.6-3),

$$v_{av} = \frac{98}{120}v_{max}$$

(see Problem (2.6-3) for derivation)

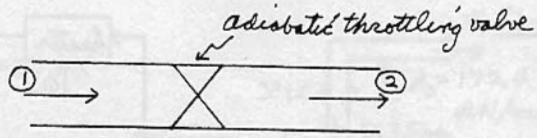
$$\boxed{v_{av}^3 = \frac{(98)^3}{(120)^3}v_{max}^3}$$

substituting $(v^3)_{av}$ and v_{av}^3 into Eq.(2.7-14),

$$\alpha = \frac{v_{av}^3}{(v^3)_{av}} = \frac{(98)^3 v_{max}^3/(120)^3}{98\,v_{max}^3/170} = \frac{(98)^2(170)}{(120)^3}$$

$$\boxed{\alpha = 0.9448}$$

(2.7-3)



$p_1 = 689\ kPa$ $\qquad$ $p_2 = 359\ kPa$

$t_1 = 171.1\ °C$ $\qquad$ $t_2 = ?$

Neglect kinetic energy and potential energy terms

Eq.(2.7-21)

$$\cancel{Q} = \cancel{(z_2 - z_1)g} + \cancel{\frac{(v_2^2 - v_1^2)}{2\alpha}} + (H_2 - H_1)$$

$Q = 0$, since adiabatic.

$H_2 = H_1$

For $t_1 = 171.1°C$ and $p_1 = 689\ kPa$, from Table A.2-9, steam is superheated (H_1).

Interpolation (linear) of Tables A.2-9 and A.2-10 is as follows;

		t	H	
Table A.2-10	400 kPa	150°C	2752.8 kJ/kg	
		171.1°C	2798.2	← Interpolated
		200°C	2860.5	
	700 kPa	165°C	2763.5	(Table A.2-9)
		171.1°C	2777.7	← Interpolated
		200°C	2844.8	(Table A.2-10)
	171.1°C	400 kPa	2798.2	
		689 kPa	2778.5	← Interpolated
		700 kPa	2777.7	

$H_1 = 2778.5\ kJ/kg = H_2$

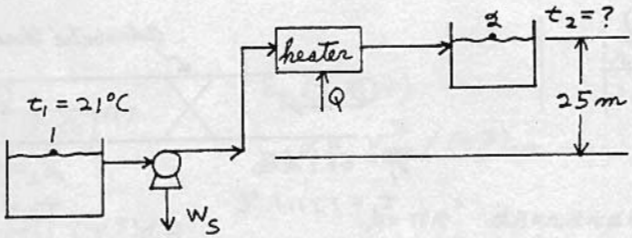
Interpolation for t_2.

		150°C		200°C
Table A.2-10	150 kPa	2772.6		2872.9
	359 kPa	2756.0 ←	Interpolated	→ 2862.5
	400 kPa	2752.8		2860.5

	150°C	160.6°C	200°C
359 kPa	2756.0	2778.5	2862.5
		Interpolated	

$$\boxed{t_2 = 160.6°C}$$

(2.7-6)



From A.2 steam tables,

$H_1 = 88.14\ kJ/kg = 88.14 \times 10^3\ J/kg$ (H_2O at 21°C)

$\rho_1 = 1/(0.0010020) = 998.0\ kg/m^3$

$m_1 = m_2 = (40\ m^3/h)(998\ kg/m^3)(1/3600\ s/h) = 11.089\ kg/s$

$z_1 = 0,\ z_2 = 25\ m$

$W_S = -(8.5 \times 10^3\ J/s)(1/11.089\ kg/s) = -0.7665 \times 10^3\ J/kg$

(negative since work is done on fluid)

$Q = +(225 \times 10^3\ J/s)(1/11.089\ kg/s) = +20.29 \times 10^3\ J/kg$

(Q is + since fluid takes on heat)

$(v_2^2/2\alpha - v_1^2/2\alpha) = 0$ (neglect kinetic energy changes)

Substituting into Eq. (2.7-10)

$$H_2 - H_1 + \frac{1}{2\alpha}(v_{2av}^2 - v_{1av}^2) + g(z_2 - z_1) = Q - W_S$$

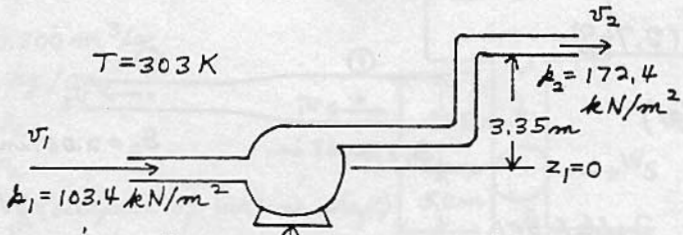
$$H_2 - 88.14 \times 10^3 + 0 + 9.80665(25-0) = 20.29 \times 10^3 - (-0.7665 \times 10^3)$$

$$H_2 = 108.95 \times 10^3\ J/kg = 108.95\ kJ/kg$$

From A.2-9,

$$\boxed{t_2 = 25.97°C}$$

(2.7-7)



$v_1 = v_2$ (same pipe diameter)

Setting datum $z_1 = 0$, $z_2 = 3.35\ m$

For turbulent flow $\alpha = 1.0$

$$\frac{1}{2\alpha}(v_2^2 - v_1^2) = \frac{1}{2(1.0)}(v_2^2 - v_1^2) = 0$$

$$z_2 g = (3.35\ m)(9.806\ m/s^2) = (3.35)(9.806)\ J/kg$$

The liquid is incompressible and Eq. (2.7-28) is used.

Table A.4-2

$\rho = 919\ kg/m^3$ (soybean oil)

$$\frac{p_1}{\rho} = \frac{103.4 \times 1000}{919} \qquad \frac{p_2}{\rho} = \frac{172.4 \times 1000}{919}$$

Eq. (2.7-28)

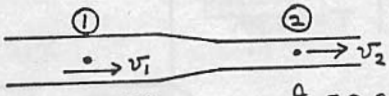
$$\Sigma F = -W_S + \frac{1}{2\alpha}(v_1^2 - v_2^2) + g(z_1 - z_2) + \frac{p_1 - p_2}{\rho}$$

$$= -(-209.2) + 0 + 9.806(0 - 3.35) + \frac{(103.4 - 172.4)1000}{919}$$

$$\boxed{\Sigma F = 101.3\ J/kg}$$

(2.7-9)

(a)



$D_1 = 3.068$ in. $\quad D_2 = 2.067$ in.

Table A.5-1

$A_1 = 4.769 \times 10^{-3}\ m^2$

$A_2 = 2.165 \times 10^{-3}\ m^2$

$\rho = 998\ kg\ H_2O/m^3 \quad v_1 = 1.676\ m/s \quad p_1 = 68.9\ kPa$

$p_2 = p_2$

assume no friction losses.

$$v_2 = v_1 \frac{A_1}{A_2} = (1.676)\frac{(4.769 \times 10^{-3})}{(2.165 \times 10^{-3})} = 3.692\ m/s$$

Eq. (2.7-32)

$$z_1 g + \frac{v_1^2}{2} + \frac{p_1}{\rho} = z_2 g + \frac{v_2^2}{2} + \frac{p_2}{\rho}$$

$z_1 = z_2$

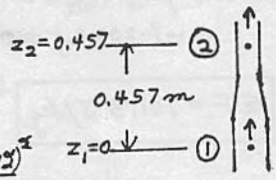
$$0 + \frac{(1.676)^2}{2} + \frac{(p_1 - p_2)}{998} = \frac{(3.692)^2}{2}$$

$(p_1 - p_2) = 5395\ Pa = 5.395\ kPa$

$(68.9 - p_2) = 5.4\ kPa$

$$\boxed{p_2 = 63.5\ kPa}$$

(b) vertical pipe

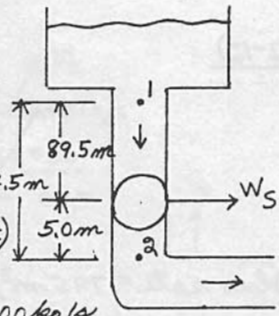


Eq. (2.7-32)

$$0 + \frac{(1.676)^2}{2} + \frac{(p_1 - p_2)}{998} = 0.457(9.8016) + \frac{(3.692)^2}{2}$$

$$\boxed{p_2 = 59.1\ kPa \quad (8.57\ psia)}$$

(2.7-11)



Flow rate = $0.800\ m^3/s$

$\rho_{H_2O} = 1000\ kg/m^3$

$p_1 = 172.4\ kPa$

$p_2 = 89.6\ kPa$

658 kW = (output of turbine shaft)

η_t = turbine efficiency = 0.89

$m_1 = m_2 = 0.800\ m^3/s\ (1000\ kg/m^3) = 800\ kg/s$

$$W = -\eta_t W_S$$

$$W = \frac{-658\times10^3\ J/s}{800\ kg/s} = -822.5\ J/kg$$

$$W_S = \frac{W}{-\eta_t} = \frac{-822.5}{-0.89} = 924.2\ J/kg$$

Eq. (2.7-28)

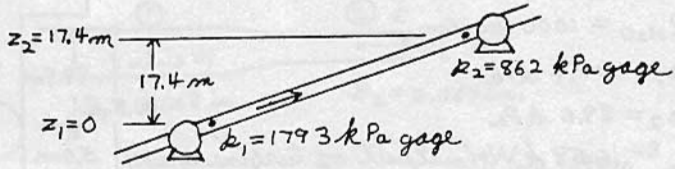
$$\frac{1}{2\alpha}(v_{2av}^2 - v_{1av}^2) + g(z_2 - z_1) + \frac{(p_2 - p_1)}{\rho} + \Sigma F + W_S = 0$$

$$0 + 9.806(0 - 94.5) + \frac{(89.6\times10^3 - 172.4\times10^3)}{1000} + \Sigma F + 924.2 = 0$$

Solving,

$$\boxed{\Sigma F = 85.3\ J/kg}$$

(2.7-12)



Flow rate $= 795\ m^3/d$ $\quad \rho_{oil} = 769\ kg/m^3$

Eq. (2.7-28)

$$z_1 g + \frac{v_1^2}{2\alpha} + \frac{k_1}{\rho_1} - W_s = z_2 g + \frac{v_2^2}{2\alpha} + \frac{k_2}{\rho_2} + \Sigma F$$

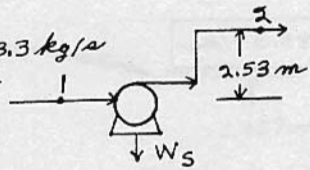
$$\frac{v_1^2}{2\alpha} = \frac{v_2^2}{2\alpha}$$

$$W_s = 0$$

$$0 + \cancel{\frac{v_1^2}{2\alpha}} + \frac{1793 \times 1000}{769} - 0 = 17.4 \times 9.806 + \cancel{\frac{v_2^2}{2\alpha}} + \frac{862 \times 1000}{769} + \Sigma F$$

$$\boxed{\Sigma F = 1040\ J/kg}$$

(2.7-13)



$m = (0.1133\ m^3/s)(1000\ kg/m^3) = 113.3\ kg/s$

$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(0.305)^2 = 0.07306\ m^2$

$v_1 = 0.1133/0.07306 = 1.550\ m/s$

$A_2 = \frac{\pi}{4}(0.254)^2 = 0.05067\ m^2$ $\quad v_2 = 0.1133/0.05067 = 2.236\ m/s$

$k_1 = -20.7$ kPa below atm $\quad k_2 = 289.6$ kPa gage

If $\mu \cong 1 \times 10^{-3}\ kg/m \cdot s$, $N_{Re} = \frac{D v \rho}{\mu} = \frac{(0.305)(1.550)(1000)}{1 \times 10^{-3}} = 4.73 \times 10^5$

Hence, the flow is turbulent and $\alpha = 1.0$.

Eq. (2.7-28)

$$\frac{1}{2\alpha}(v_{2av}^2 - v_{1av}^2) + g(z_2 - z_1) + \frac{k_2 - k_1}{\rho} + \Sigma F + W_s = 0$$

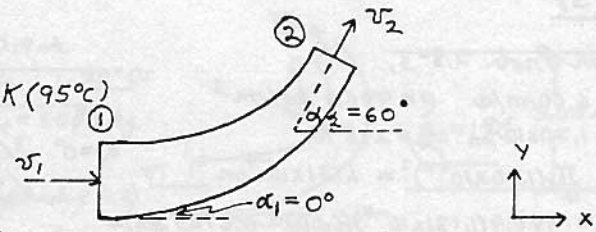
$$\frac{1}{2}(2.236^2 - 1.550^2) + 9.806(2.53 - 0) + \frac{(289.6 + 20.7)10^3}{1000} + 0 + W_s = 0$$

$W_s = -336.4\ J/kg$ $\quad$ Pump input $= 336.4 \times 113.3 \times \frac{1}{1000} = \boxed{38.11\ kW}$

(2.8-2)

Water at 363K (95°C)

$\Sigma F = v_2^2/5$



$\rho = 962\ kg/m^3$ (A.2-3)

$p_2 = 1.115 \times 10^5\ N/m^2$ gage

ΣF (fric loss) $= v_2^2/5$

Neglect gravity forces.

Flow rate $= 0.0566\ m^3/s$

$D_1 = 0.1016\ m$ $\qquad D_2 = 0.0762\ m$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.1016)^2 = 8.107 \times 10^{-3}\ m^2$, $A_2 = \frac{\pi}{4}(0.0762)^2 = 4.560 \times 10^{-3}\ m^2$

$v_1 = \frac{0.0566}{8.107 \times 10^{-3}} = 6.982\ m/s$, $\qquad v_2 = \frac{0.0566}{4.560 \times 10^{-3}} = 12.41\ m/s$

Mechanical energy balance

Eq. (2.7-28) $\qquad$ assume turbulent flow ($N_{Re} > 2100$)

$$\frac{1}{2\alpha}(v_2^2 - v_1^2) + \cancel{g(z_2 - z_1)} + \frac{(p_2 - p_1)}{\rho} + \Sigma F + \cancel{W_s} = 0$$

$$\frac{1}{2(1.0)}(12.41^2 - 6.982^2) + \frac{p_1 - p_2}{962} + \frac{(12.41)^2}{5} = 0$$

$p_2 = 1.115 \times 10^5$, $\qquad p_1 = 1.9176 \times 10^5\ N/m^2$

$m = \rho_1 v_1 A_1 = (962)(6.982)(8.107 \times 10^{-3}) = 54.45\ kg/s$

Momentum balance $\qquad \cos\alpha_2 = \cos 60° = 0.5$, $\cos\alpha_1 = \cos 0° = 1.0$

Eq. (2.8-28)

$$R_x = m v_2 \cos\alpha_2 - m v_1 \cos\alpha_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1 \cos\alpha_1$$

$$= (54.45)(12.41)(0.5) - (54.45)(6.982) + (1.115 \times 10^5)(4.560 \times 10^{-3})(0.5) - (1.9176 \times 10^5)(8.107 \times 10^{-3})$$

$\boxed{R_x = -1343.7\ N \ (-302.1\ lb_f)}$ Force on fluid

$\sin 60° = 0.866$, $\sin 0° = 0$

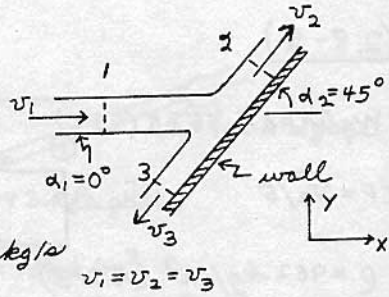
Eq. (2.8-29) $\quad R_y = m v_2 \sin\alpha_2 + p_2 A_2 \sin\alpha_2 + \cancel{m_t g} - m v_1 \sin\alpha_1 - p_1 A_1 \sin\alpha_1$ (neglect)

$$R_y = (54.45)(12.41)(0.866) + (1.115 \times 10^5)(4.560 \times 10^{-3})(0.866) + 0 - 0 - 0$$

$\boxed{R_y = +1026\ N \ (+230.6\ lb_f)}$ Force on fluid

$\boxed{-R_x = +1344\ N,\ -R_y = -1026\ N}$ Forces on bend

(2.8-5)



From Prob. 2.8-3,

$v_1 = 6.00\ m/s \quad \rho = 996.9\ kg/m^3$

$D_1 = 1.20\times10^{-2}\ m \quad t = 298\ K$

$A_1 = \frac{\pi}{4}(1.20\times10^{-2})^2 = 1.131\times10^{-4}\ m^2$

$m_1 = 996.9(1.131\times10^{-4})(6.00) = 0.6765\ kg/s$

Eq. (2.8-41)

$$m_2 = \frac{m_1}{2}(1+\cos\alpha_2) = \frac{0.6765}{2}(1+0.7071) = \boxed{0.5774\ kg/s = m_2}$$

$$m_3 = \frac{m_1}{2}(1-\cos\alpha_2) = \frac{0.6765}{2}(1-0.7071) = \boxed{0.09907\ kg/s = m_3}$$

Force on wall

Method I

Normal force on wall $= m_1 v_1 \sin\alpha_2 = 0.6765(6.00)(0.7071)$

$$\boxed{\text{Normal force on wall} = 2.870\ N}$$

Method II

Eq. (2.8-28)

pressure forces

$$R_x = m_2 v_2 \cos\alpha_2 - m_1 v_1 \cos\alpha_1 + m_3 v_3(-\cos\alpha_2) + 0$$

$$= (0.5774)(6.00)(0.7071) - (0.6765)(6.00)(1.0) + (0.09907)(6.00)(-0.7071)$$

$$= -2.030\ N$$

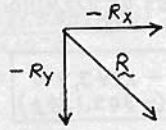
Eq. (2.8-29)

$$R_y = m_2 v_2 \sin\alpha_2 - m_1 v_1 \sin\alpha_1 + m_3 v_3(-\sin\alpha_2) + 0$$

$$= (0.5774)(6.00)(0.7071) - 0 + (0.09907)(6.00)(-0.7071)$$

$$= +2.030\ N$$

$$\boxed{\begin{array}{l} -R_x = +2.030\ N \\ -R_y = -2.030\ N \end{array} \text{ Force on Wall}}$$



Eq. (2.8-32)

$$|\underset{\sim}{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.030)^2 + (2.030)^2} = \boxed{2.870\ N = |\underset{\sim}{R}|}$$

(2.8-8)

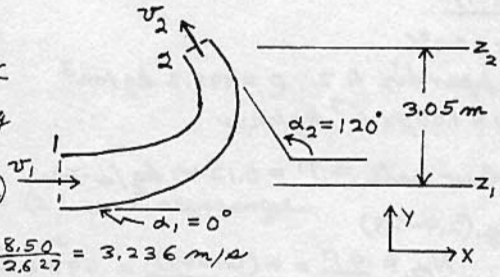
$D_1 = 1.829\,m \qquad D_2 = 1.219\,m$

Flow rate $= 8.50\,m^3/s \qquad 20°C$

$p_1 = 276\times10^3\,Pa \qquad m_t = 8500\,kg$

$\Sigma F = 0.5\,v_2^2/2$

$\rho = 998.2\,kg/m^3$ (Appendix A.2)



$A_1 = \frac{\pi}{4}(1.829)^2 = 2.627\,m^2 \qquad v_1 = \frac{8.50}{2.627} = 3.236\,m/s$

$A_2 = \frac{\pi}{4}(1.219)^2 = 1.167\,m^2 \qquad v_2 = \frac{8.50}{1.167} = 7.284\,m/s$

$m = v_1 A_1 \rho_1 = 3.236(2.627)(998.2) = 8486\,kg/s$

Flow is turbulent (N_{Re} calc. $>10^5$)

Eq.(2.7-28) Mech. energy balance

$$\frac{1}{2\alpha}(v_2^2 - v_1^2) + g(z_2 - z_1) + (p_2 - p_1)/\rho + \Sigma F + \cancel{W_s} = 0$$

$$\frac{1}{2(1.0)}(7.284^2 - 3.236^2) + 9.806(3.05 - 0) + (p_2 - 276\times10^3)/9982 + \frac{0.5}{2}(7.284)^2 + 0 = 0$$

solving, $p_2 = 211.7\times10^3\,Pa = 211.7\,kPa$

$\alpha_1 = 0° \quad \sin 0° = 0 \quad \cos 0° = 1.0$

$\alpha_2 = 120° \quad \sin 120° = 0.8660 \quad \cos 120° = -0.50$

Momentum balance

Eq.(2.8-28)

$$R_x = m v_2 \cos\alpha_2 - m v_1 \cos\alpha_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1 \cos\alpha_1$$

$$= 8486(7.284)(-0.5) - 8486(3.236)(1.0) + 211.7\times10^3(1.167)(-0.50) - 276\times10^3(2.627)(1.0)$$

$$\boxed{R_x = -906.95\times10^3\,N}$$

Eq.(2.8-29)

$$R_y = m v_2 \sin\alpha_2 - m v_1 \sin\alpha_1 + p_2 A_2 \sin\alpha_2 - p_1 A_1 \sin\alpha_1 + m_t g$$

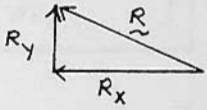
$$= 8486(7.284)(0.8660) - 0 + 211.7\times10^3(1.167)(0.8660) - 0 + 8500(9.806)$$

$$\boxed{R_y = +350.83\times10^3\,N}$$

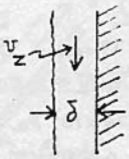
Eq.(2.8-32)

$$|\underset{\sim}{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-906.95\times10^3)^2 + (+350.83\times10^3)^2}$$

$$\boxed{\underset{\sim}{R} = 972.4\times10^3\,N}$$ Resultant force on control volume fluid



(2.9-1)



$t = 20°C$

appendix A.2 $\rho = 998.2\ kg/m^3$

$\mu = 1.005 \times 10^{-3}\ kg/m \cdot s$

Flow rate $= \Gamma = 0.1240\ kg/s \cdot m$

Eq. (2.9-29)

$$N_{Re} = \frac{4\Gamma}{\mu} = \frac{4(0.1240)}{1.005 \times 10^{-3}} = 493.6$$ Hence, laminar flow

Eq. (2.9-30)

$$\Gamma = \frac{\rho^2 \delta^3 g}{3\mu}$$

Substituting, $$0.1240 = \frac{(998.2)^2 \delta^3 (9.8066)}{3(1.005 \times 10^{-3})}$$

$$\boxed{\delta = 3.370 \times 10^{-4}\ m}$$ Film thickness

Eq. (2.9-27)

$$v_{z\,av} = \frac{\rho g \delta^2}{3\mu} = \frac{998.2(9.8066)(3.370 \times 10^{-4})^2}{3(1.005 \times 10^{-3})}$$

$$\boxed{v_{z\,av} = 0.3687\ m/s}$$

alternate: $$\Gamma\ kg/m \cdot s = \left(\rho \frac{kg}{m^3}\right)(\delta\ m)\left(v_{z\,av} \frac{cm}{s}\right)$$

$$0.1240 = 998.2(3.370 \times 10^{-4})(v_{z\,av})$$

$$v_{z\,av} = 0.3687\ m/s$$

(2.9-3)

(1) $\tau_{rx} = K\left(\frac{-dv_x}{dr}\right)^n$

Eq.(2.9-6)

(2) $\tau_{rx} = \frac{(k_0-k_L)}{2L} r$

(3) Substitute (1) into (2) and rearrange.

$$\frac{(k_0-k_L)}{2KL} r = \left(\frac{-dv_x}{dr}\right)^n$$

Raise both sides to 1/n power.

(4) $$\left(\frac{k_0-k_L}{2KL}\right)^{1/n} r^{1/n} = -\frac{dv_x}{dr}$$

Rearrange Eq.(4)

(5) $$\left(\frac{k_0-k_L}{2KL}\right)^{1/n} \int_0^r r^{1/n}\, dr = -\int_{v_{max}}^{v}$$

Integrate

(6) $$\left(\frac{k_0-k_L}{2KL}\right)^{1/n} \frac{r^{(n+1)/n}}{(n+1)/n} = v_{max} - v$$

But $v=0$ at $r=R_0$ (no slip at wall).

(7) $$\left(\frac{k_0-k_L}{2KL}\right)^{1/n} \frac{R_0^{(n+1)/n}}{(n+1)/n} = v_{max} - 0$$

Substitute Eq.(7) into (6).

(8) $$\left(\frac{n}{n+1}\right)\left(\frac{k_0-k_L}{2KL}\right)^{1/n} r^{(n+1)/n} = \left(\frac{n}{n+1}\right)\left(\frac{k_0-k_L}{2KL}\right)^{1/n} R_0^{(n+1)/n} - v$$

Solve for v

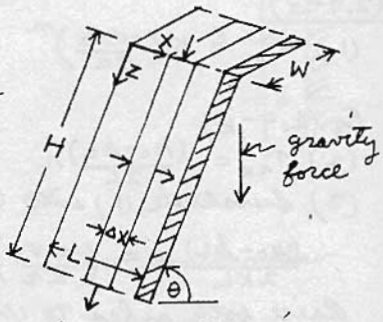
(9) $$v = \left(\frac{n}{n+1}\right)\left(\frac{k_0-k_L}{2KL}\right)^{1/n} \left[R_0^{(n+1)/n} - r^{(n+1)/n}\right]$$

Multiply and divide Eq.(9) by $R_0^{(n+1)/n}$

$$\boxed{v = \left(\frac{n}{n+1}\right)\left(\frac{k_0-k_L}{2KL}\right)^{1/n} (R_0)^{(n+1)/n} \left[1 - \left(\frac{r}{R_0}\right)^{(n+1)/n}\right]}$$

(2.9-4)

Momentum balance in z direction over system Δx thick, $z=0$ to $z=H$, and W m thick.



Momentum flux by molec. transport (out − in) times area HW

(1) net efflux $= HW(\tau_{xz})|_{x+\Delta x} - HW(\tau_{xz})|_x$

Momentum flux by convection (out − in) times area $\Delta x W$

(2) net efflux $= \Delta x W(\rho v_z)|_{z=L} - \Delta x W(\rho v_z)|_{z=0} = 0$

Gravitational force on fluid

(3) grav. force $= \Delta x W H(\rho g)\sin\theta$

substituting into conservation Eq. (2.8-3) at steady state,

(4) $\Delta x W H(\rho g)\sin\theta = HW(\tau_{xz})|_{x+\Delta x} - HW(\tau_{xz})|_x + 0$

Rearranging and letting $\Delta x \to 0$,

(5) $$\frac{d\tau_{xz}}{dx} = \rho g \sin\theta$$

Integrating

at free liquid surface, $x=0$, and $\tau_{xz}=0$

(6) $$\int_{\tau_{xz}=0}^{\tau_{xz}=\tau_{xz}} d\tau_{xz} = \rho g \sin\theta \int_{x=0}^{x=x} dx$$

(7) $$\tau_{xz} = \rho g x \sin\theta$$

For Newton's law,

(8) $$\tau_{xz} = -\mu\, dv_z/dx$$

Combining (7) and (8) and integrating,

(9) $$v_z = -\left(\frac{\rho g \sin\theta}{2\mu}\right)x^2 + C_1$$

at $x=L$, $v_z=0$ Hence, $C_1 = \left(\frac{\rho g \sin\theta}{2\mu}\right)L^2$

(10) $$\boxed{v_z = \frac{\rho g \sin\theta\, L^2}{2\mu}\left[1-\left(\frac{x}{L}\right)^2\right]}$$

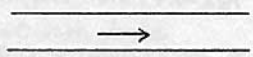
at $x=0$, $v_z = v_{z\,max}$

$$\boxed{v_{z\,max} = \frac{\rho g L^2 \sin\theta}{2\mu}} \qquad v_z = v_{z\,max}\left[1-\left(\frac{x}{L}\right)^2\right]$$

(2.10-2)

$t = 293\,K$ $\quad D = 0.0525\,m$ $\quad \Delta L = 76.2\,m$

$v = 1.22\,m/s$



From Appendix A.4

For olive oil,

$\rho = 919\,kg/m^3$ $\quad \mu = 84\times10^{-3}\,kg/m\cdot s = 84\times10^{-3}\,Pa\cdot s$

$$N_{Re} = \frac{D v \rho}{\mu}$$

$$= \frac{0.0525(1.22)(919)}{84\times10^{-3}}$$

$= 700.7$ Hence, the flow is laminar

Eq.(2.10-7)

$$f = \frac{16}{N_{Re}}$$

$$= \frac{16}{700.7}$$

$$= 0.02283$$

Eq.(2.10-5)

$$\Delta p_f = 4 f \rho \frac{\Delta L}{D} \frac{v^2}{2}$$

$$= 4(0.02283)(919)\frac{(76.2)}{(0.0525)}\frac{(1.22)^2}{2}$$

$$\boxed{\Delta p_f = 90\,650\ N/m^2 = 90.65\ kN/m^2}$$

(2.10-3)

$\Delta L = 61\ m$

$\rho = 801\ kg/m^3$

$\mu = 1.49 \times 10^{-3}\ Pa \cdot s = 1.49 \times 10^{-3}\ kg/m \cdot s$ $\qquad v = 4.57\ m/s$

$D = 40.89/1000 = 0.04089\ m$ (A.5-1: 1.5 in. nominal size)

(a) Steel pipe

$$N_{Re} = \frac{D v \rho}{\mu} = \frac{(0.04089)(4.57)(801)}{1.49 \times 10^{-3}} = 100\,460$$

Hence, flow is turbulent.

Fig. 2.10-3

$\varepsilon = 4.6 \times 10^{-5}\ m$ $\qquad \frac{\varepsilon}{D} = \frac{4.6 \times 10^{-5}}{0.04089} = 0.001125$

From Fig. 2.10-3 for $N_{Re} = 100\,460$ and $\varepsilon/D = 0.001125$

$f = 0.0056$ (Steel pipe)

$f = 0.0044$ (Smooth tube)

Eq. (2.10-6)

$$F_f = 4f\frac{\Delta L v^2}{D\ 2} = \frac{4(0.0056)(61.0)(4.57)^2}{(0.04089)(2)}$$

$$\boxed{F_f = 348.9\ J/kg \quad (116.7\ ft\ lb_f/lb_m)}$$

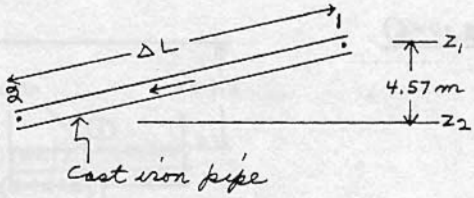
(b) Smooth tube

$$F_f = \frac{4(0.0044)(61.0)(4.57)^2}{(0.04089)(2)}$$

$$\boxed{F_f = 274.2\ J/kg \quad (91.7\ ft\ lb_f/lb_m)}$$

$\boxed{21.4\%\ \text{reduction}}$

(2.10-4)



$D = 0.156\ m$

$\Delta L = 305\ m$

Water at 293 K (20°C)

Appendix A.2

$\rho = 998.2\ kg/m^3$ $\qquad \mu = 1.005 \times 10^{-3}\ kg/m \cdot s$

$F_f = (z_1 - z_2)g = 4.57(9.8066) = 44.81\ J/kg$

For cast iron pipe, Fig. 2.10-3, $\varepsilon = 2.6 \times 10^{-4}\ m$

$N_{Re} = D v \rho / \mu$

Trial 1. assume $v = 1.7\ m/s$

$$N_{Re} = \frac{0.156(1.7)(998.2)}{1.005 \times 10^{-3}}$$

$= 2.634 \times 10^5$ Hence, turbulent flow

$$\frac{\varepsilon}{D} = \frac{2.6 \times 10^{-4}}{0.156} = 0.001667$$

From Fig. 2.10-3, $f = 0.0056$

Eq. (2.10-6)

$$F_f = 4f \frac{\Delta L}{D} \frac{v^2}{2}$$

$$44.81 = 4(0.0056)\frac{(305)}{(0.156)}\frac{v^2}{(2)}$$

Solving, $v = 1.431\ m/s$

Trial 2. assume $v = 1.40\ m/s$

$$N_{Re} = \frac{0.156(1.40)(998.2)}{1.005 \times 10^{-3}}$$

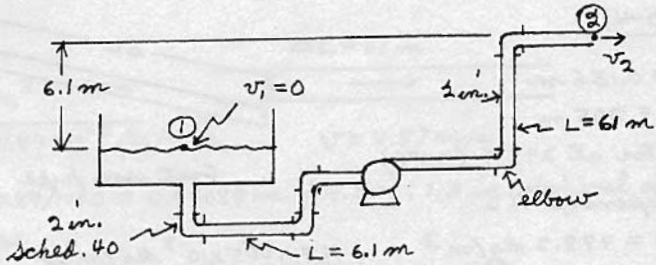
$= 2.169 \times 10^5$

Fig. 2.10-3, $f = 0.0058$

$$44.81 = 4(0.0058)\frac{(305)}{(0.156)}\frac{v^2}{(2)}$$

$\boxed{v = 1.406\ m/s}$ Hence, Trial 2 is correct and the v assumed is correct.

(2.10-6)



$t = 82.2°C$ (water)

$q = 0.379\ m^3/min = 6.317 \times 10^{-3}\ m^3/s$

appendix A.2

$\rho = 970.4\ kg/m^3 \quad \mu = 0.3474 \times 10^{-3}\ Pa \cdot s$

appendix A.5

$D(2\ in) = 0.0525\ m \qquad A = 21.65 \times 10^{-4}\ m^2$

$v_2 = \dfrac{6.317 \times 10^{-3}}{21.65 \times 10^{-4}} = 2.918\ m/s \qquad p_1 = 101.325\ kPa = p_2$

$N_{Re} = \dfrac{D v_2 \rho}{\mu} = \dfrac{0.0525(2.918)(970.4)}{0.3474 \times 10^{-3}} = 4.279 \times 10^5$ Hence, turbulent flow.

Eq. (2.7-28) Mechanical energy balance

$$\frac{1}{2\underset{1.0}{\alpha}}(v_2^2 - \underset{0}{\cancel{v_1^2}}) + g(z_2 - \underset{0}{\cancel{z_1}}) + \underset{0}{\cancel{\frac{p_2 - p_1}{\rho}}} + \Sigma F + W_s = 0$$

$$\frac{v_2^2}{2} + g z_2 + \Sigma F + W_s = 0$$

Calculation of ΣF

1. Contraction loss - tank to pipe Eq. (2.10-16)

$K_c = 0.55(1 - \underset{0}{\cancel{A_2/A_1}}) = 0.55$

$h_c = K_c v_2^2/2 = 0.55(2.918)^2/2 = 2.333\ J/kg$

2. Friction loss in 5 elbows

Table 2.10-1, $K_f = 0.75$

Eq. (2.10-17)

$h_f = K_f \dfrac{v_2^2}{2}(\overset{5\ elbows}{5}) = 0.75(2.918)^2(5)/2 = 15.965\ J/kg$

3. Friction loss in straight pipe ($\Delta L = 6.1 + 61 = 67.1\ m$)

$N_{Re} = \dfrac{D v_2 \rho}{\mu} = 4.279 \times 10^5$

Fig. 2.10-3 $\quad \varepsilon/D = 4.6 \times 10^{-5}/0.0525 = 0.000876$

(2.10-6)-p.2

Fig. 2.10-3 $\quad f = 0.0048$

Eq.(2.10-6)

$$F_f = 4f\frac{\Delta L}{D}\frac{v_2^2}{2} = 4(0.0048)\frac{(67.1)}{(0.0525)}\frac{(2.918)^2}{2}$$

$$= 104.47 \text{ J/kg}$$

Total loss, $\Sigma F = 2.333 + 15.965 + 104.47 = 122.77$

(a) $\boxed{\Sigma F = 122.8 \text{ J/kg}}$

Eq.(2.7-28)

$$\frac{v_2^2}{2} + gz_2 + \Sigma F + W_s = 0$$

$$\frac{(2.918)^2}{2} + 9.806(6.1) + 122.77 + W_s = 0, \quad W_s = -186.85$$

(b) $\boxed{W_s = -186.9 \text{ J/kg}}$

$$m = (6.317\times10^{-3} \text{ m}^3/\text{s})(970.4 \text{ kg/m}^3) = 6.130 \text{ kg/s}$$

Eq.(2.7-30)

$$W_s = -\eta W_p$$

$$-186.85 = -0.75 W_p$$

$$W_p = 249.13 \text{ J/kg}$$

$$kW = 249.13\frac{(6.130)}{1000} = \boxed{1.527 \text{ kW}}$$

(2.10-7)

appendix A.5

$D = 102.3/1000 = 0.1023\ m$ $\quad A = 82.19\times10^{-4}\ m^2$

$M = MW\ N_2 = 28.02$

$\Delta L = 3000\ m$

appendix A.3

$\mu = 1.77\times10^{-5}\ Pa\cdot s$ $\quad k_1 = 200\ kPa = 2.00\times10^5\ Pa$

Total mass flow rate $= 7.40\times10^{-2}\ kg/s$

$$G = \frac{7.40\times10^{-2}}{82.19\times10^{-4}} = 9.004\ kg/s\cdot m^2$$

Fig. 2.10-3

$\varepsilon = 4.6\times10^{-5}\ m$

$$\frac{\varepsilon}{D} = \frac{4.6\times10^{-5}}{0.1023} = 4.5\times10^{-4} = 0.00045$$

$$N_{Re} = \frac{DG}{\mu} = \frac{0.1023(9.004)}{1.77\times10^{-5}} = 5.204\times10^4 \quad \text{(Turbulent flow)}$$

$f = 0.0053$

Eq.(2.10-10)

assume pressure drop is less than 10% of k_1.

$$k_1^2 - k_2^2 = \frac{4f\Delta L G^2 RT}{DM}$$

$$(2.00\times10^5)^2 - k_2^2 = \frac{4(0.0053)(3000)(9.004)^2(8314.3)(298)}{0.1023(28.02)}$$

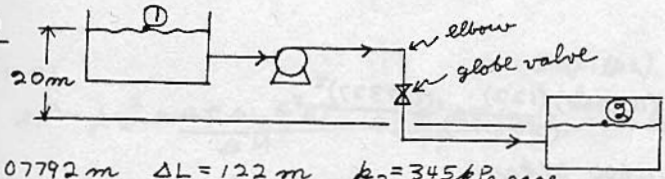
$$4.000\times10^{10} - k_2^2 = 0.4457\times10^{10}$$

$k_2 = 1.885\times10^5\ Pa$ $\quad \boxed{k_2 = 188.5\ kPa}$

$$\%\ \text{press. drop} = \frac{(2.000\times10^5 - 1.885\times10^5)}{2.000\times10^5}\times100 = 5.75\ \%$$

Hence, Eq.(2.10-10) can be used.

(2.10-9)



$D = 0.07792\ m \quad \Delta L = 122\ m \quad p_2 = 345\ kPa\ gage$

$\rho = 833\ kg/m^3 \quad \mu = 3.3\times10^{-3}\ Pa\cdot s \quad q = 3.494\times10^{-3}\ m^3/s$

$A = \frac{\pi}{4}(0.07792)^2 = 47.69\times10^{-4}\ m^2 \quad m = 3.494\times10^{-3}(833) = 2.911\ kg/s$

$v = 3.494\times10^{-3}/47.69\times10^{-4} = 0.7327\ m/s$

$$N_{Re} = \frac{D v \rho}{\mu} = \frac{0.07792(0.7327)(833)}{3.3\times10^{-3}} = 1.441\times10^4 \text{ (Hence, turbulent)}$$

<u>Calculation of ΣF frictional losses</u>

1. Globe valve, half open

$K_f = 9.5$ (Table 2.10-1)

Eq.(2.10-17) $\quad h_f = K_f \frac{v_1^2}{2} = 9.5\frac{(0.7327)^2}{2} = 2.550\ J/kg$

2. Two elbows (90°)

$K_f = 0.75$ (Table 2.10-1)

$h_f = 0.75\frac{(0.7327)^2}{2}(2) = 0.403\ J/kg$ ← 2 elbows

3. Contraction loss from open tank

Eq.(2.10-16) $\quad h_c = 0.55\left(1-\frac{A_2}{A_1}\right)\frac{v_2^2}{2\alpha} = 0.55(1-0)\frac{(0.7327)^2}{2(1.0)}$

$= 0.148\ J/kg$

4. Expansion loss to pressurized tank

Eq.(2.10-15) $\quad h_{ex} = \left(1-\frac{A_1}{A_2}\right)^2\frac{v_1^2}{2\alpha} = (1-0)^2\frac{(0.7327)^2}{2(1.0)} = 0.268\ J/kg$

5. Frictional loss in straight pipe

For commercial steel pipe, Fig. 2.10-3,

$\varepsilon = 4.6\times10^{-5}\ m \quad \frac{\varepsilon}{D} = \frac{4.6\times10^{-5}}{0.07792} = 0.000595$

$N_{Re} = 1.441\times10^4 \quad f = 0.0069$

Eq.(2.10-6)

$F_f = 4f\frac{\Delta L}{D}\frac{v^2}{2}$

(2.10-9) - p.2

$$F_f = 4(0.0069)\frac{(122)}{(0.07792)}\frac{(0.7327)^2}{2}$$

$$= 11.600 \text{ J/kg}$$

$$\Sigma F = 2.550 + 0.403 + 0.148 + 0.268 + 11.600$$

$$= 14.969 \text{ J/kg}$$

Eq. (2.7-28) Mechanical energy balance

$$\frac{1}{2\alpha}(v_2^2 - v_1^2) + g(z_2 - z_1) + \frac{(p_1 - p_2)}{\rho} + \Sigma F + W_s = 0$$

$$v_1 = 0,\ v_2 = 0,\ p_1 = 0 \text{ (gage)}$$

$$0 + (9.806)(-20-0) + \frac{(345\times10^3 - 0)}{833} + 14.969 + W_s = 0$$

$$0 - 196.12 + 414.17 + 14.969 + W_s = 0$$

$$W_s = -202.98 \text{ J/kg}$$

Eq. (2.7-30)

$$W_s = -\eta W_p$$

$$-202.98 = -(0.65)W_p$$

$$W_p = 312.3 \text{ J/kg}$$

$$\text{Pump kW} = m W_p = 2.911\frac{(312.3)}{1000} = \boxed{0.909 \text{ kW}}$$

(2.11-1)

Eq. (2.11-9)

$$(1) \quad k_1^2 - k_2^2 = \frac{4f\Delta L G^2 RT}{\theta M} + \frac{2G^2RT}{M}\overbrace{(\ln k_1 - \ln k_2)}^{\ln(k_1/k_2)}$$

For maximum flow, $\frac{\partial G}{\partial k_2} = 0$ for constant k_1 and f.

Taking the partial of (1) with respect to k_2,

$$(2) \quad 0 - 2k_2\frac{\partial k_2}{\partial k_2} = \left(\frac{4f\Delta LRT}{\theta M}\right)\overset{0}{\cancel{2G}}\frac{\partial G}{\partial k_2} + \left(\frac{2RT}{M}\right)G^2\left(0 - \frac{1}{k_2}\frac{\partial k_2}{\partial k_2}\right)$$

$$+ (\ln k_1 - \ln k_2)\overset{0}{\left(\frac{2RT}{M}\right)}2G\frac{\partial G}{\partial k_2}$$

Set $\partial G/\partial k_2 = 0$ for $G = G_{max}$.

$$(3) \quad -2k_2(1) = -\frac{2RTG^2}{M}\left(\frac{1}{k_2}\right)(1)$$

Solve for G and call G_{max}.

$$(4) \quad G_{max}^2 = k_2^2\frac{M}{RT}$$

Now, $\frac{1}{\rho_2} = V_2 = \frac{RT}{k_2 M}$ or $\frac{M}{RT} = \frac{1}{k_2 V_2}$

Solve (4) for G_{max}

$$(5) \quad \boxed{G_{max} = \sqrt{\frac{M k_2^2}{RT}}} \quad \text{Eq. (2.11-11)}$$

Also,

$$(6) \quad G_{max} = \sqrt{\frac{1}{k_2 V_2}k_2^2} = \sqrt{\frac{k_2}{V_2}}$$

Eq. (2.11-3)

$$(7) \quad G = v\rho \quad \text{or} \quad G_{max} = v_{max}\rho_2 = v_{max}/V_2$$

Eq. (2.11-6)

$$(8) \quad kV = \frac{1}{M}RT \quad \text{or} \quad k = \frac{RT}{MV}, \quad \frac{k_2}{V_2} = \frac{RT}{MV_2^2}$$

Substitute (7) and (8) to (5) and (6).

$$(9) \quad \frac{v_{max}}{V_2} = \sqrt{\frac{k_2}{V_2}} = \sqrt{\frac{RT}{MV_2^2}}$$

$$(10) \quad v_{max} = V_2\sqrt{\frac{RT}{MV_2^2}} = \sqrt{\frac{\cancel{V_2^2}RT}{M\cancel{V_2^2}}}$$

$$(11) \quad \boxed{v_{max} = \sqrt{\frac{RT}{M}} = \sqrt{k_2 V_2}} \quad \text{Eq. (2.11-12)}$$

(2.11-2)

$\Delta L = 305\,m \quad G = 41.0\,kg/m^2 \cdot s \quad p_1 = 345 \times 10^3\,Pa \quad T = 288.8\,K$

From appendix A.5, $D = 0.05250\,m \quad A = 21.65 \times 10^{-4}\,m^2$

$\mu = 1.04 \times 10^{-5}\,kg/m \cdot s$

$$N_{Re} = \frac{DG}{\mu} = \frac{0.05250(41.0)}{1.04 \times 10^{-5}} = 2.0697 \times 10^5$$

Fig. 2.10-3 $\quad \varepsilon = 4.6 \times 10^{-5}\,m \quad \frac{\varepsilon}{D} = \frac{4.6 \times 10^{-5}}{0.05250} = 0.000876 \quad f = 0.0051$

Eq. (2.11-9)

$$p_1^2 - p_2^2 = \frac{4 f \Delta L G^2 RT}{DM} + \frac{2G^2RT}{M} \ln \frac{p_1}{p_2}$$

Trial-and-error. For first trial, neglect last term in Eq. (2.11-9).

$$(345 \times 10^3)^2 - p_2^2 = \frac{4(0.0051)(305)(41.0)^2(8314.34)(288.8)}{0.05250(16.0)} + \sim 0$$

$$(345 \times 10^3)^2 - p_2^2 = 2.9898 \times 10^{10}$$

Solving, $p_2 = 298.5 \times 10^3\,Pa$

Recalculating Eq. (2.11-9) using $p_2 = 298.5 \times 10^3\,Pa$ for the last term,

$$(345 \times 10^3)^2 - p_2^2 = 2.9898 \times 10^{10} + \frac{2(41.0)^2(8314.34)(288.8)}{(16.0)} \ln \frac{345 \times 10^3}{298.5 \times 10^3}$$

$$= 2.9898 \times 10^{10} + 0.00731 \times 10^{10}$$

Solving, $p_2 = 298.4 \times 10^3\,Pa$ Hence, trial is correct.

(a) $\boxed{p_2 = 298.4\,kPa}$

Eq. (2.11-12)

(b) $$v_{mp} = \sqrt{\frac{RT}{M}} = \sqrt{\frac{8314.34(288.8)}{(16.0)}} = \boxed{387.4\,m/s = v_{mp}}$$

actual velocity

Eq. (2.11-13)

$$v_2 = \frac{RTG}{p_2 M} = \frac{8314.34(288.8)(41.0)}{298.4 \times 10^3 (16.0)}$$

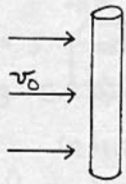
$$\boxed{v_2 = 20.62\,m/s}$$

(3.1-1)

$t = 25\,°C$ air at 101.3 kPa

$v_0 = 10\ m/s$ $\quad D_p = \frac{90}{1000} = 9.0\times10^{-2}\ m$

$L = 1.0\ m$



From appendix A.3

$\rho = 1.187\ kg/m^3$ $\quad \mu = 1.845\times10^{-5}\ kg/m\cdot s$

Eq. (3.1-3)

$$N_{Re} = \frac{D_p v_0 \rho}{\mu} = \frac{(9.0\times10^{-2})(10)(1.187)}{1.845\times10^{-5}} = 5.790\times10^4$$

Fig. 3.1-2

$\boxed{C_D = 1.3}$

Eq. (3.1-2)

$$A_p = L D_p = (1.0)(9.0\times10^{-2}) = 0.090\ m^2$$

$$F_D = C_D \frac{v_0^2}{2} \rho A_p = 1.3\frac{(10)^2}{2}(1.187)(0.090) = 6.944$$

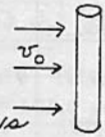
$\boxed{F_D = 6.94\ N}$

(3.1-2)

$t = 25°C$ air at 101.3 kPa $\quad D_p = 1.0\ m$ $\quad L = 30\ m$

From A.3 $\quad \rho = 1.187\ kg/m^3$ $\quad \mu = 1.845\times10^{-5}\ kg/m\cdot s$

$$v_0 = \left(50\frac{miles}{h}\right)\left(5280\frac{ft}{mile}\right)\left(\frac{1\ m}{3.2808\ ft}\right)\left(\frac{1}{3600\ s/h}\right) = 22.35\ m/s$$



Eq. (3.1-3)

$$N_{Re} = \frac{D_p v_0 \rho}{\mu} = \frac{1.0(22.35)(1.187)}{1.845\times10^{-5}} = 1.438\times10^6$$

Fig. 3.1-2

$\boxed{C_D = 0.33}$

Eq. (3.1-2)

$$A_p = L D_p = 30(1.0) = 30.0\ m^2$$

$$F_D = C_D \frac{v_0^2}{2} \rho A_p = 0.33\frac{(22.35)^2}{2}(1.187)(30.0)$$

$\boxed{F_D = 2935\ N}$

(3.1-3)

$p = 1\,atm \quad T = 37.8\,°C \quad D_p = 0.042\,m \quad v_0 = 2.30\times10^{-4}\,m/s$

Appendix A.3 $\rho = 1.137\,kg/m^3 \quad \mu = 1.90\times10^{-5}\,kg/m\cdot s$

Eq. (3.1-3)

$$N_{Re} = \frac{D_p v_0 \rho}{\mu} = \frac{(0.042)(2.30\times10^{-4})(1.137)}{1.90\times10^{-5}} = 0.5781$$

It is in Stokes' law region ($N_{Re} < 1.0$)

Eq. (3.1-5)

$$C_D = \frac{24}{N_{Re}} = \frac{24}{0.5781} = 41.52$$

$$A_p = \frac{\pi D_p^2}{4} = \frac{\pi}{4}(0.042)^2 = 1.385\times10^{-3}\,m^2$$

Eq. (3.1-2)

$$F_D = C_D \frac{v_0^2}{2}\rho A_p = 41.52\frac{(2.30\times10^{-4})^2}{2}(1.137)(1.385\times10^{-3})$$

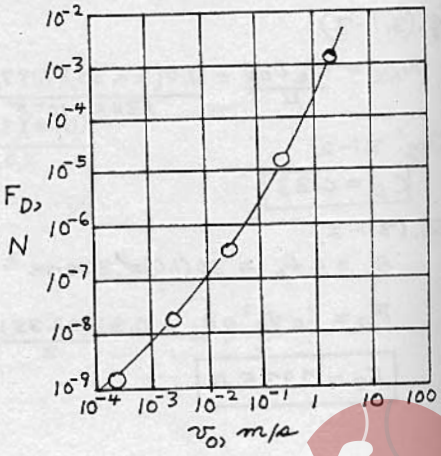
(a) $\boxed{F_D = 1.729\times10^{-9}\,N}$

(b) $v_0 = 2.30\times10^{-3} \qquad N_{Re} = \frac{(0.042)(2.30\times10^{-3})(1.137)}{1.90\times10^{-5}} = 5.781$

Fig. 3.1-2 $\quad C_D = 6.2$

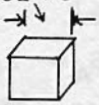
$$F_D = 6.2\frac{(2.30\times10^{-3})^2}{2}(1.137)(1.385\times10^{-3}) = 2.582\times10^{-8}\,N$$

v_0	N_{Re}	C_D	F_D, N
2.3×10^{-4}	0.5781	41.52	1.729×10^{-9}
2.3×10^{-3}	5.781	6.2	2.582×10^{-8}
2.3×10^{-2}	57.81	1.5	6.248×10^{-7}
2.3×10^{-1}	578.1	0.53	2.208×10^{-5}
2.3	5781	0.41	1.708×10^{-3}



(3.1-5)

$D = 0.02\ m$



(a) Take $1.0\ m^3$ of packed bed as a basis.

Bulk density of packed bed $= 980\ kg/m^3$

Density of solid cubes $= 1500\ kg/m^3$

Mass of bed $= (980\ kg/m^3)(1.0\ m^3) = 980\ kg$ of solid cubes

Volume of solid cubes $= (980\ kg)/(1500\ kg/m^3) = 0.6533\ m^3$

Eq. (3.1-6)

$$\varepsilon = \frac{\text{volume voids}}{\text{total volume bed}} = \frac{\text{total vol} - \text{solid vol}}{\text{total vol}} = \frac{1.0 - 0.6533}{1.0}$$

$$\boxed{\varepsilon = 0.3467}$$

Surface area of particle

$$S_p = 6(D^2)\ m^2$$

Volume of particle

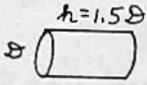
$$v_p = D^3\ m^3$$

Eq. (3.1-7) $$a_v = \frac{S_p}{v_p} = \frac{6D^2}{D^3} = \frac{6}{D}$$

Eq. (3.1-9) $$D_p = \frac{6}{a_v} = \frac{6}{6/D} = D \qquad \boxed{D_p = 0.020\ m}$$

Eq. (3.10-10) $$a = \frac{6(1-\varepsilon)}{D_p} = \frac{6(1-0.3467)}{0.020} \qquad \boxed{a = 196.0\ m^{-1}}$$

(b) $\boxed{\varepsilon = 0.3467}$



$$S_p = 2\left(\frac{\pi D^2}{4}\right) + \pi D(\overset{h}{1.5D}) = 2.0\pi D^2$$

$$v_p = \left(\frac{\pi D^2}{4}\right)(\overset{h}{1.5D}) = \frac{1.5}{4}\pi D^3$$

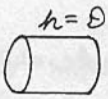
Eq. (3.1-7) $$a_v = \frac{S_p}{v_p} = \frac{2.0\pi D^2}{\frac{1.5}{4}\pi D^3} = \frac{8}{1.5D}$$

Eq. (3.1-9) $$D_p = \frac{6}{a_v} = \frac{6}{\frac{8}{1.5D}} = \frac{9}{8}D = \frac{9}{8}(0.02) \qquad \boxed{D_p = 0.0225\ m}$$

Eq. (3.1-10) $$a = \frac{6(1-\varepsilon)}{D_p} = \frac{6(1-0.3467)}{0.0225} \qquad \boxed{a = 174.2\ m^{-1}}$$

(3.1-6)

(a) $S_p = 2\left(\pi \frac{D^2}{4}\right) + \pi D(\overset{h}{D}) = 1.5\pi D^2$



$v_p = \frac{\pi D^2}{4}(\overset{h}{D}) = \frac{\pi D^3}{4}$

Eq. (3.1-7) $\quad a_v = \frac{S_p}{v_p} = \frac{1.5\pi D^2}{\frac{\pi D^3}{4}} = \frac{6}{D}$

Eq. (3.1-9) $\quad D_p = \frac{6}{a_v} = \frac{6}{6/D} = D \qquad \boxed{D_p = D}$

(b)

assume 1.0 m^3 volume of bed

let n = number of particles in bed

volume of solids in bed $= (1-\varepsilon)(1.0\,m^3) = n\,\frac{\pi D^3}{4}$

$$\boxed{n = \frac{4(1-\varepsilon)}{\pi D^3}}$$

(3.1-7)

Eq. (3.1-20)

$$\Delta p = \frac{150\mu v' \Delta L}{D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho (v')^2 \Delta L}{D_p}\frac{(1-\varepsilon)}{\varepsilon^3}$$

Divide above by ΔL and multiply by $\frac{\varepsilon^3 D_p}{\rho (v')^2 (1-\varepsilon)}$

$$\Delta p \frac{1}{\Delta L}\frac{\varepsilon^3 D_p}{\rho (v')^2(1-\varepsilon)} = \frac{150\mu v' \Delta L}{D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\varepsilon^3 D_p}{\Delta L \rho (v')^2 (1-\varepsilon)}$$

$$+ \frac{1.75\rho (v')^2 \Delta L}{D_p}\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\varepsilon^3 D_p}{\Delta L \rho (v')^2 (1-\varepsilon)}$$

Cancel like terms and multiply 1st term by ρ/ρ

$$\underbrace{\frac{\Delta p\, \rho}{\rho \rho (v')^2}}_{(G')^2}\frac{D_p}{\Delta L}\frac{\varepsilon^3}{1-\varepsilon} = 150\frac{\mu(1-\varepsilon)}{D_p \underbrace{v'\rho}_{G'}} + 1.75$$

Eq. (3.1-15)

$$N_{Re,p} = \frac{D_p G'}{(1-\varepsilon)\mu}$$

substitute into above.

Eq. (3.1-21)

$$\boxed{\frac{\Delta p\, \rho}{(G')^2}\frac{D_p}{\Delta L}\frac{\varepsilon^3}{(1-\varepsilon)} = \frac{150}{N_{Re,p}} + 1.75}$$

(3.1-8)

$G' = 2.45\ kg/m^2 \cdot s \quad \Delta L = 3.66\ m \quad \varepsilon = 0.40$

$D_p = D_{cylinder} = 0.0127\ m$ (see Problem 3.1-6 for proof)

Appendix A.3 air at 394.3 K $\quad \mu = 2.27 \times 10^{-5}\ Pa \cdot s$

Inlet $p_1 = 2.20(1.01325 \times 10^5) = 2.229 \times 10^5\ Pa \quad M(air) = 28.97$

Eq. (3.1-15)

$$N_{Re,p} = \frac{D_p G'}{(1-\varepsilon)\mu} = \frac{0.0127(2.45)}{(1-0.4)(2.27 \times 10^{-5})} = 2285$$

Eq. (3.1-21)

$$\frac{\Delta p\, \rho}{(G')^2} \frac{D_p}{\Delta L} \frac{\varepsilon^3}{(1-\varepsilon)} = \frac{150}{N_{Re,p}} + 1.75 \qquad \Delta p = p_1 - p_2$$

To use for gases, the ρ at $(p_1 + p_2)/2$ is used.
This is trial-and-error since p_2 is unknown.

Trial 1

assume $\Delta p = 0.15 \times 10^5\ Pa$

$0.15 \times 10^5 = p_1 - p_2 = 2.229 \times 10^5 - p_2$

$p_2 = 2.079 \times 10^5\ Pa$

$$p_{av} = \frac{p_1 + p_2}{2} = \frac{2.229 \times 10^5 + 2.079 \times 10^5}{2} = 2.154 \times 10^5$$

Eq. (3.1-22)

$$\rho_{av} = \frac{M}{RT} p_{av} = \frac{28.97(2.154 \times 10^5)}{8314.34(394.3)} = 1.903\ kg/m^3$$

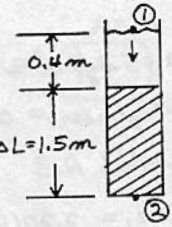
substituting into Eq. (3.1-21)

$$\frac{\Delta p\,(1.903)}{(2.45)^2} \frac{0.0127}{3.66} \frac{(0.40)^3}{(1-0.40)} = \frac{150}{2285} + 1.75$$

$$\boxed{\Delta p = 0.1547 \times 10^5\ Pa}$$

This is close to assumed 0.15×10^5 Pa so a second trial is not needed.

(3.1-9)



Assume $v_1 = v_2$ (after leaves bed)

Assume contraction and expansion losses entering and leaving bed are negligible. This is because v_1 and v_2 will be small and $v^2/2$ losses will be small.

From A.2, $\rho = 997\ kg/m^3$ $\mu = 0.9142\times10^{-3}\ Pa\cdot s$ 24°C

$\varepsilon = 0.42$ $D_p = 0.0060\ m$ $p_1 = p_2 = 1\ atm$ pressure

Eq. (2.7-28) Mechanical energy balance

$$\frac{1}{2\alpha}(\overset{0}{v_2^2} - \overset{0}{v_1^2}) + g(z_2 - z_1) + \overset{0}{\frac{p_1 - p_2}{\rho}} + \Sigma F + \overset{0}{W_s} = 0$$

$$\Sigma F = -g(z_2 - z_1)$$

$$\Delta p = -g(z_2 - z_1)\rho = -9.8066(-1.9 - 0)997 = 1.858\times10^4\ Pa$$ Pressure drop in bed

Eq. (3.1-15)

$$N_{Re,p} = \frac{D_p G'}{\mu(1-\varepsilon)} = \frac{0.0060\,G'}{0.9142\times10^{-3}(1-0.42)} = 11.316\,G'$$

Eq. (3.1-21)

$$\frac{\Delta p\,\rho}{(G')^2}\,\frac{D_p}{\Delta L}\,\frac{\varepsilon^3}{1-\varepsilon} = \frac{150}{N_{Re,p}} + 1.75$$

$$\frac{1.858\times10^4(997)}{(G')^2}\,\frac{0.0060}{1.50}\,\frac{(0.42)^3}{1-0.42} = \frac{150}{11.316\,G'} + 1.75$$

Multiply eq. by $(G')^2$.

$$9.465\times10^3 = 13.255G' + 1.75(G')^2$$

$$1.75(G')^2 + 13.255G' - 9.465\times10^3 = 0$$

$$G' = \frac{-13.255 + \sqrt{(13.255)^2 - 4(9465\times1.75)}}{2(1.75)}$$

$$G' = 69.85\ kg/m^2\cdot s$$

$$G' = 69.85 = v'\rho = v'(997)$$

$$\boxed{v' = 7.006\times10^{-2}\ m/s}$$

Note: $\frac{v^2}{2} = \frac{(7.0\times10^{-2})^2}{2} = 0.0025$

This is small compared to ΣF.

(3.1-12)

$D_p = \frac{0.10}{1000} = 0.00010\ m$ $\quad \phi_s = 0.86$ $\quad \rho_p = 1200\ kg/m^3$

$p = 2.0265 \times 10^5\ Pa$ $\quad$ air at 25°C $\quad \varepsilon_{mf} = 0.43$

A.3 $\quad \rho = 2.374\ kg/m^3$ $\quad \mu = 1.845 \times 10^{-5}\ Pa \cdot s$ $\quad$ Diam bed = 0.6 m

(a) $A_{bed} = \frac{\pi}{4}(0.6)^2 = 0.2827\ m^2$ $\quad$ Bed has 350 kg solids

Volume solids $= 350\ kg / 1200\ kg/m^3 = 0.2917\ m^3$

For $\varepsilon_1 = 0$, $\quad L_1 = \frac{0.2917\ m^3}{0.2827\ m^2} = 1.032\ m$

Eq. (3.1-30) $\quad L_2 = L_{mf}$ $\quad \varepsilon_2 = \varepsilon_{mf} = 0.43$ $\quad \varepsilon_1 = 0$

$$\frac{L_1}{L_2} = \frac{1-\varepsilon_2}{1-\varepsilon_1} \qquad \frac{1.032}{L_{mf}} = \frac{1-0.43}{1-0}$$

$$\boxed{L_{mf} = 1.810\ m}$$

(b) Eq. (3.1-32)

$$\Delta p = L_{mf}(1-\varepsilon_{mf})(\rho_p - \rho)g$$

$$= 1.810(1-0.43)(1200-2.374)(9.80665)$$

$$\boxed{\Delta p = 0.1212 \times 10^5\ Pa}$$

(c) Eq. (3.1-36)

$$\frac{1.75(N_{Re,mf})^2}{\phi_s\ \varepsilon_{mf}^3} + \frac{150(1-\varepsilon_{mf})}{\phi_s^2\ \varepsilon_{mf}^3}(N_{Re,mf}) - \frac{D_p^3\ \rho\ (\rho_p - \rho)g}{\mu^2} = 0$$

$$\frac{1.75(N_{Re,mf})^2}{0.86(0.43)^3} + \frac{150(1-0.43)}{(0.86)^2(0.43)^3}(N_{Re,mf}) - \frac{(0.0001)^3(2.374)(1200-2.3)(9.8066)}{(1.845\times10^{-5})^2} = 0$$

$$25.59(N_{Re,mf})^2 + 1454(N_{Re,mf}) - 0.8191 \times 10^2 = 0$$

$$N_{Re,mf} = 0.05628$$

Eq. (3.1-35)

$$N_{Re,mf} = 0.05628 = \frac{D_p\ v'_{mf}\ \rho}{\mu} = \frac{(0.0001)(v'_{mf})(2.374)}{(1.845\times10^{-5})}$$

(3.1-12) - p.2

$$\boxed{v'_{mf} = 0.004374 \text{ m/s}}$$

(d)

Use $4 \times v'_{mf}$

$$v' = 4(0.004374) = 0.01750 \text{ m/s}$$

Eq. (3.1-39)

$$v' = K_1 \frac{\varepsilon^3}{(1-\varepsilon)}$$

Use mf conditions

$$v'_{mf} = 0.004374 = K_1 \frac{(0.43)^3}{(1-0.43)}$$

$$K_1 = 0.03136 \text{ m/s}$$

For $v' = 0.01750$

$$0.01750 = K_1 \frac{\varepsilon^3}{(1-\varepsilon)} = 0.03136 \frac{\varepsilon^3}{(1-\varepsilon)}$$

Trial- and -error

assume $\varepsilon = 0.57$

$$0.01750 = 0.03136 \frac{(0.57)^3}{(1-0.57)} = 0.01351$$

assume $\varepsilon = 0.60$

$$0.01750 = 0.03136 \frac{(0.60)^3}{(1-0.60)} = 0.01693$$

assume $\varepsilon = 0.604$

$$0.01750 = 0.03136 \frac{(0.604)^3}{(1-0.604)} = 0.01745$$

$$\boxed{\varepsilon = 0.604}$$

(3.1-13)

$D_p = \frac{4.42}{1000} = 0.00442\ m \qquad \phi_s = 1.00$ (spheres)

$\rho_p = 1603\ kg/m^3$ water at 20.2°C

From A.2,

$\rho = 997.5\ kg/m^3 \quad \mu = 1.00 \times 10^{-3}\ Pa \cdot s$

Data for ε_{mf} are not available. Hence, use the approximate equation.

Eq. (3.1-38)

$$N_{Re,mf} = \left[(33.7)^2 + \frac{0.0408 D_p^3 \rho (\rho_p - \rho) g}{\mu^2}\right]^{1/2} - 33.7$$

$$= \left[(33.7)^2 + \frac{0.0408(0.00442)^3(997.5)(1603-997.5)(9.8066)}{(1.00 \times 10^{-3})^2}\right]^{1/2} - 33.7$$

$$= \left[1135.7 + 20870\right]^{1/2} - 33.7$$

$$N_{Re,mf} = 114.6$$

Eq. (3.1-35)

$$N_{Re,mf} = \frac{D_p v'_{mf} \rho}{\mu}$$

$$114.6 = \frac{(0.00442)(v'_{mf})(997.5)}{(1.00 \times 10^{-3})}$$

$$\boxed{v'_{mf} = 0.02599\ m/s \text{ predicted}}$$

$v'_{mf} = 0.02307\ m/s$ experimental

(3.2-1)

$D = \frac{102.3}{1000} = 0.1023\ m$ $\quad C_p = 0.98$ $\quad \Delta h = \frac{78}{1000} = 0.078\ m\ CCl_4$

$t = 20°C$

appendix A.2 for water

$\rho = 998.23\ kg/m^3$ $\quad \mu = 1.005 \times 10^{-3}\ Pa \cdot s$

$A = \frac{\pi}{4}(0.1023)^2 = 8.219 \times 10^{-3}\ m^2$

From ref. (P1), $\rho_A(CCl_4) = 1595\ kg/m^3$ at 20°C

(a)

Eq. (3.2-3)

$$\Delta p = \Delta h(\rho_A - \rho)g$$
$$= 0.078(1595 - 998.23)(9.80665)$$
$$= 456.5\ Pa$$

Eq. (3.2-2)

$$v = C_p\sqrt{\frac{2(p_2 - p_1)}{\rho}} = 0.98\sqrt{\frac{2(456.5)}{998.23}} = 0.9372\ m/s$$

$$\boxed{v_{max} = 0.9372\ m/s}$$

$$N_{Re} = \frac{D v_{max} \rho}{\mu} = \frac{0.1023(0.9372)(998.23)}{1.005 \times 10^{-3}} = 95\,230$$

Fig. 2.10-2

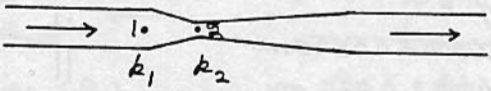
$$v_{av}/v_{max} = 0.825$$
$$v_{av} = 0.825(0.9372)$$
$$\boxed{v_{av} = 0.773\ m/s}$$

(b) $A = 8.219 \times 10^{-3}\ m^2$

$$q = v_{av} A = 0.773(8.219 \times 10^{-3})$$

$$\boxed{q = 6.35 \times 10^{-3}\ m^3/s}$$

(3.2-4)



$D_1 = 102.3\ mm = 0.1023\ m$

$D_2 = 38.9\ mm = 0.0389\ m$

$\rho = 999\ kg/m^3$

$C_v = 0.98$

$p_1 - p_2 = 156.9\ kPa$

$\left(\frac{D_2}{D_1}\right)^4 = \left(\frac{0.0389}{0.1023}\right)^4 = 0.0209$

Eq. (3.2-7)

$$v_2 = \frac{C_v}{\left[1-(D_2/D_1)^4\right]^{\frac{1}{2}}}\sqrt{\frac{2(p_1-p_2)}{\rho}}$$

$$= \frac{0.98}{\left[1-0.0209\right]^{\frac{1}{2}}}\sqrt{\frac{2(156.9\times1000)}{999}}$$

$v_2 = 17.54\ m/s$

Eq (3.2-8)

$$\text{Flow rate} = v_2\frac{\pi D_2^2}{4} = (17.54)\frac{(\pi)}{4}(0.0389)^2$$

$$\boxed{\text{Flow rate} = 0.0208\ m^3/s}$$

$$= (0.0208)\left(264.17\ \frac{gal}{m^3}\right)(60\ s/m)$$

$$\boxed{\text{Flow rate} = 330\ GPM}$$

(Check of N_{Re} at point 1)

$$vol = 0.0208\ m^3/s = v_1\frac{\pi D_1^2}{4} = v_1\frac{\pi}{4}(0.1023)^2$$

$v_1 = 2.531\ m/s$

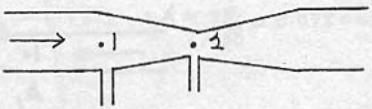
assume $\mu = 1.0\times10^{-3}\ Pa\cdot s$ ($t\ °C$ not given)

$$N_{Re} = \frac{D_1 v_1 \rho}{\mu} = \frac{0.1023(2.531)(999)}{1.0\times10^{-3}} = 2.586\times10^5$$

This is $> 10^4$

Hence, C_v is constant.

(3.2-5)



appendix A.5

$D_1 = 52.5/1000 = 0.0525\ m$

$D_2 = 20/1000 = 0.020\ m$

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.020)^2 = 3.142 \times 10^{-4}\ m^2$ $\quad C_v = 0.98$

Water at 20°C, A.2

$\mu = 1.0050 \times 10^{-3}\ Pa \cdot s$ $\quad \rho = 998.2\ kg/m^3$

$\rho_A (Hg) = 13600\ kg/m^3$ appendix A.1

$\Delta h = 214/1000\ m$ of Hg

Eq. (3.2-3)

$$\Delta p = \Delta h (\rho_A - \rho) g = \frac{214}{1000}(13600 - 998.2)(9.80665)$$

$$\Delta p = p_1 - p_2 = 26470\ Pa$$

$$D_2/D_1 = \frac{0.020}{0.0525} = 0.3810$$

Eq. (3.2-7)

$$v_2 = \frac{C_v}{\sqrt{1-(D_2/D_1)^4}} \sqrt{\frac{2(p_1 - p_2)}{\rho}}$$

$$= \frac{0.98}{\sqrt{1-(0.381)^4}} \sqrt{\frac{2(26470)}{998.2}}$$

$$v_2 = 7.21\ m/s$$

Flow rate $= v_2 A_2 = (7.21)(3.142 \times 10^{-4})$

$$\boxed{\text{Flow rate} = 2.265 \times 10^{-3}\ m^3/s}$$

(Check of N_{Re})

$$v_1 = v_2 \left(\frac{D_2}{D_1}\right)^2 = 7.21(0.3810)^2 = 1.047\ m/s$$

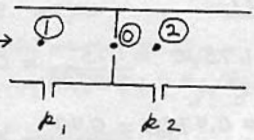
$$N_{Re1} = \frac{D_1 v_1 \rho}{\mu} = \frac{(0.0525)(1.047)(998.2)}{1.005 \times 10^{-3}} = 5.46 \times 10^4$$

This is $> 10^4$. Hence, C_v is constant.

(3.2-6)

$\rho = 900\ kg/m^3$ $\quad t = 20°C$

$\mu = 6\ cp = 6.0\times10^{-3}\ Pa\cdot s$



4 in. sched 40 pipe (D_1) $\quad C_0 = 0.61$

Appendix A.5

$D_1 = 102.3/1000 = 0.1023\ m$ $\quad D_0 = ?$

$\Delta p = p_1 - p_2 = 0.93\times10^5\ Pa$

Trial-and-error.

assume $D_0 = 50\ mm = 50/1000 = 0.050\ m$

$$\frac{D_0}{D_1} = \frac{0.050}{0.1023} = 0.489$$

Eq. (3.2-10)

$$v_0 = \frac{C_0}{\sqrt{1-(D_0/D_1)^4}}\sqrt{\frac{2(p_1-p_2)}{\rho}}$$

$$= \frac{0.61}{\sqrt{1-(0.489)^4}}\sqrt{\frac{2(0.93\times10^5)}{900}}$$

$v_0 = 9.03\ m/s$

$q = 0.0174\ m^3/s = v_0 A_0 = (9.03)\frac{\pi}{4}(D_0^2)$

$D_0 = 0.04953\ m$ $\quad \boxed{D_0 = 49.5\ mm}$

Hence, a 2nd trial is not needed.

Check on N_{Re0}

$$N_{Re0} = \frac{D_0 v_0 \rho}{\mu} = \frac{(0.0495)(9.03)(900)}{(6.0\times10^{-3})} = 6.705\times10^4$$

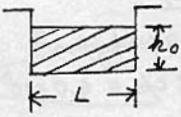
This is above N_{Re0} of 1×10^4 and C_0 is constant.

Permanent pressure loss. $\quad D_0/D_1 = 49.5/0.1023 = 0.484$

For D_0/D_1 of 0.5, permanent loss is 73% of $p_1 - p_2$ across orifice (see text)

$0.73(0.93\times10^5) = \boxed{0.679\times10^5\ Pa \text{ permanent pressure loss}}$

(3.2-7)



$L = 1.75\,ft = \frac{1.75}{3.2808} = 0.5334\ m$

$h_0 = 0.47\,ft = \frac{0.47}{3.2808} = 0.1433\ m$

Eq. (3.2-12)

$$q = 0.415\,(L - 0.2h_0)\,h_0^{1.5}\sqrt{2g}$$

English units

$$q = 0.415(1.75 - 0.2\times0.47)(0.47)^{1.5}\sqrt{2\times32.174}$$

$$\boxed{q = 1.776\ ft^3/s}$$

SI units

$$q = 0.415(0.5334 - 0.2\times0.1433)(0.1433)^{1.5}\sqrt{2\times9.80665}$$

$$\boxed{q = 0.0503\ m^3/s}$$

(3.3-1)

$q = 60\ gal/min \quad \rho = 62.4\ lb_m/ft^3$

(a) Fig. 3.3-2 For 60 gal/min, brake hp = 0.80 hp

$\eta = 0.58$, H (head) = 31 ft

$$m = (60\ gal/min)\left(\frac{1}{60\ s/min}\right)\left(\frac{1}{7.481\ gal/ft^3}\right)(62.4\ lb_m/ft^3)$$
$$= 8.34\ lb_m/s$$

Eq. (3.3-4) $\quad W_s = -Hg/g_c = -31\ ft\cdot lb_f/lb_m$

Eq. (3.3-2) $\quad$ brake hp $= -\frac{W_s m}{\eta\cdot550} = \frac{(31)(8.34)}{0.58(550)} = 0.810$ hp

$0.810\times0.7457 = 0.604$ kW $\quad$ $\boxed{\text{brake hp} = 0.81\ hp \quad 0.60\ kW}$

(b) $\rho = 0.85(62.43) = 53.1\ lb_m/ft^3$

brake hp $\propto \rho \qquad$ brake hp $= 0.810\,(53.1)/62.4 = 0.689$ hp

$0.689\times0.7457 = 0.513$ kW $\quad$ $\boxed{\text{brake hp} = 0.69\ hp \quad 0.51\ kW}$

(3.3-2)

At point 1 (suction)

$v_1 = 0 \quad t = 352.6\ K$

$p_1 = 749.3$ mm Hg

$v_2 = 38.1$ m/s

$p_2 = 800.1$ mm Hg

$\eta = 0.65 \quad M(MW) = 30.7$ kg/kg mol

vol gas = 56.6 m^3/min (294.3 K and 760 mm Hg)

Assume incompressible flow since $\Delta p = 800.1 - 749.3 = 50.8$ mm

$\frac{50.8}{749.3} \times 100 < 10\%$

Use average ρ in mechanical energy balance Eq. (2.7-28).

$$\rho_1 = \left(30.1 \frac{kg}{kg\,mol}\right)\left(\frac{1\ kg\,mol}{22.414\ m^3}\right)\left(\frac{273.2}{352.6}\right)\left(\frac{749.3}{760}\right) = 1.046\ kg/m^3$$

$$\rho_2 = 1.046\left(\frac{800.1}{749.3}\right) = 1.117\ kg/m^3$$

$$\rho_{av} = \frac{\rho_1 + \rho_2}{2} = \frac{1.046 + 1.117}{2} = 1.082\ kg/m^3$$

$$m = \left(56.6 \frac{m^3}{min}\right)\left(\frac{1\ min}{60\ s}\right)\left(\frac{1\ kg\,mol}{22.414\ m^3}\right)\left(\frac{273.2}{294.3}\right)\left(30.7 \frac{kg}{kg\,mol}\right) = 1.199\ kg/s$$

Pressure head developed

$$= \frac{p_2 - p_1}{\rho_{av}} = \frac{(800.1 - 749.3)}{760}(1.01325 \times 10^5)\frac{1}{(1.082)} = 6259\ J/kg$$

Velocity head developed

$$= \frac{(v_2^2 - v_1^2)}{2} = \frac{(38.1)^2 - 0}{2} = 725.8\ J/kg$$

Eq. (2.7-28)

$$\frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1) + \frac{(p_2 - p_1)}{\rho_{av}} + \Sigma F + W_s = 0$$

$$725.8 + 0 + 6259 + 0 + W_s = 0$$

$$W_s = -6985\ J/kg$$

Eq. (3.3-2)

$$\text{brake kW} = \frac{-W_s m}{\eta(1000)} = \frac{6985(1.199)}{0.65(1000)}$$

$$\text{brake kW} = \boxed{12.88\ kW}$$

(3.4-1)

$\mu = 1.5 \times 10^{-3}$ Pa·s
$\rho = 969$ kg/m^3

$D_t = 0.91$ m
$D_a = 0.305$ m 180 rpm $N = \frac{180}{60} = 3/s$
4 vertical baffles $J = 0.076$ m

$$\frac{D_t}{J} = \frac{0.91}{0.076} = 12.0$$

Use curve 2, Fig. 3.4-4

Eq. (3.4-1)

$$N'_{Re} = \frac{D_a^2 N \rho}{\mu} = \frac{(0.305)^2(3.0)(969)}{1.5 \times 10^{-3}}$$

$$= 180{,}300$$

curve 2, $\boxed{N_p = 2.5}$

Eq. (3.4-2)

$$N_p = \frac{P}{\rho N^3 D_a^5}$$

$$2.5 = \frac{P}{(969)(3.0)^3(0.305)^5}$$

$P = 172.0$ W

$$\boxed{P = 0.172 \text{ kW} \quad (0.231 \text{ hp})}$$

(3.4-2)

$D_a = 0.203\ m \quad D_t = 0.61\ m \quad H = 0.61\ m \quad W = 0.0405\ m$

$J = 0.051\ m \quad N = 275/60 = 4.583\ rev/s$ 6 flat blades + disk

$\frac{D_t}{J} = \frac{0.61}{0.051} = \frac{11.96}{1} \left(\frac{12}{1}\right) \qquad \frac{D_a}{W} = \frac{0.203}{0.0405} = \frac{5.01}{1} \left(\frac{5}{1}\right)$

(a)

Use curve 1, Fig. 3.4-4 $\qquad \rho = 909\ kg/m^3 \quad \mu = 0.020\ Pa \cdot s$

$$N'_{Re} = \frac{D_a^2 N \rho}{\mu} = \frac{(0.203)^2(4.583)(909)}{0.020} = 8584$$

$N_p = 5.0$

Eq. (3.4-2)

$$N_p = \frac{P}{\rho N^3 D_a^5} \qquad 5.0 = \frac{P}{909(4.583)^3(0.203)^5}$$

$P = 150.8\ W \qquad \boxed{P = 0.1508\ kW}$

$$V_1 = (\pi D_t^2/4)(H) = \frac{\pi(0.61)^3}{4} = 0.1783\ m^3$$

$$\frac{P_1}{V_1} = \frac{0.1508}{0.1783} = \boxed{0.845\ kW/m^3}$$

(b)

$V_2 = 100(0.1783) = 17.83\ m^3$

Eq. (3.4-8)

$$R = \left(\frac{V_2}{V_1}\right)^{1/3} = \left(\frac{17.83}{0.1783}\right)^{1/3} = 4.642$$

$D_{T2} = 4.642\ D_{T1} = 4.642(0.61) = \boxed{2.832\ m = D_{T2}}$

$D_{a2} = 4.642(0.203) = \boxed{0.942\ m = D_{a2}}$

Equal mass transfer, $n = 2/3$

Eq. (3.4-10)

$$N_2 = N_1\left(\frac{1}{R}\right)^{2/3} = 4.583\left(\frac{1}{4.642}\right)^{2/3} = 1.647\ rev/s$$

Eq. (3.4-1)

$$N'_{Re} = \frac{D_a^2 N \rho}{\mu} = \frac{(0.942)^2(1.647)(909)}{0.020} = 6.642 \times 10^4$$

$$N_p = 5.0 = \frac{P_2}{909(1.647)^3(0.942)^5}$$

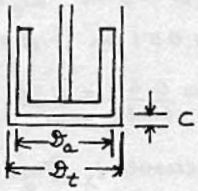
$P_2 = 15062 \qquad \boxed{P_2 = 15.06\ kW}$

$$\frac{P_2}{V_2} = \frac{15.062}{17.83} = \boxed{0.845\ kW/m^3}$$

(3.4-4)

$D_t = 0.90\ m \qquad \rho = 980\ kg/m^3$

$\frac{D_a}{D_t} = 0.90 \qquad \mu = 100\ Pa \cdot s$



$D_a = 0.90\,(0.90) = 0.81\ m$

$\frac{C}{D_t} = 0.05 \qquad C = 0.05\,(0.9) = 0.045\ m$

$N = 50\ rpm / 60 \frac{s}{min} = 0.8333\ rev/s$

Eq.(3.4-1)

$$N'_{Re} = \frac{D_a^2 N \rho}{\mu} = \frac{(0.81)^2(0.8333)(980)}{100}$$

$N'_{Re} = 5.358$

This is < 100, so Eq. (3.4-3) is valid.

Eq.(3.4-3)

$$N_p = 215\,(N'_{Re})^{-0.955} = \frac{215}{(5.358)^{0.955}}$$

$N_p = 43.28$

Eq.(3.4-2)

$$N_p = \frac{P}{\rho N^3 D_a^5}$$

$$43.28 = \frac{P}{980(0.8333)^3(0.81)^5}$$

$P = 8557\ W \qquad \boxed{P = 8.56\ kW}$

(3.4-7)

(a) $D_a = 0.61\,m$, $N = 1.50\,rev/s$, $D_t = 1.83\,m$, $\rho = 929\,kg/m^3$, $\mu = 0.01\,Pa\cdot s$, $H_t = 1.83\,m$

$$N'_{Re} = \frac{D_a^2 N \rho}{\mu} = 5.185\times10^4 \quad , N_p = 5$$

$$P = N_p \rho N^3 D_a^5 = 1324\,J/s = 1.324\,KW\ (1.77\,hp)$$

$$V_1 = \frac{\pi D_T^2}{4}(H_t) = \frac{\pi}{4}(1.83)^2(1.83) = 4.813\,m^3$$

Eq. (3.4-16) $$f_T = \frac{t_T (N D_a^2)^{2/3} g^{1/6} D_a^{1/2}}{H_t^{1/2} D_t^{3/2}}$$

From Fig. 3.4-5 for $N'_{Re} = 5.185\times10^4$, $f_t = 4.1$

$$f_T = 4.1 = \frac{t_T (1.50\times0.61^2)^{2/3}(9.806)^{1/6}(0.61)^{1/2}}{(1.83)^{1/2}(1.83)^{3/2}}$$

$$\boxed{t_T = 17.7\,s}$$

(b) For geometric scaleup,

$$V_2 = 10.0\,m^3 = \frac{\pi}{4}(D_T^2)(H) = \frac{\pi}{4}(D_T^2)(H) = \frac{\pi}{4}D_T^3 \qquad (H_T = D_T)$$

$$D_T = 2.335\,m$$

Eq. (3.4-17) for same power/volume,

$$t_{T2} = t_{T1}(D_{a2}/D_{a1})^{11/18} \qquad \text{Note: } D_{a2}/D_{a1} = D_{T2}/D_{T1} = R$$

$$t_{T2} = 17.7(2.335/1.83)^{11/18}$$

$$\boxed{t_{T2} = 20.5\,s}$$

(3.5-1)

23.9 °C $V = 1.018\ m/s$ $L = 6.10\ m$ (smooth tube)

$D = 0.01267\ m$ $\rho = 976\ kg/m^3$

Flow properties of fluid. $K = 6.00\ N \cdot s^{0.454}$ $n = 0.454$

(a)

Eq. (3.5-6) $n = n' = 0.454$

Eq. (3.5-7)

$$K' = K\left(\frac{3n'+1}{4n'}\right)^{n'}$$

$$= 6.00\left(\frac{3\times 0.454+1}{4\times 0.454}\right)^{0.454} = 6.7606$$

Eq. (3.5-9)

$$\Delta p = \frac{K' 4L}{D}\left(\frac{8V}{D}\right)^{n'}$$

$$= \frac{6.7606(4)(6.10)}{(0.01267)}\left(\frac{8\times 1.018}{0.01267}\right)^{0.454} = 245\,160\ N/m^2$$

$$\boxed{\Delta p = 245.2\ kN/m^2 \quad 5120\ lbf/ft^2}$$

(b)

Eq. (3.5-11)

$$N_{Re,gen} = \frac{D^{n'} V^{2-n'} \rho}{K' 8^{n'-1}} = \frac{(0.01267)^{0.454}(1.018)^{2-0.454}(976)}{(6.7606)(8)^{0.454-1}}$$

$$\boxed{N_{Re,gen} = 63.56}$$

Hence, flow is laminar.

Eq. (3.5-12)

$$f = \frac{16}{N_{Re,gen}} = \frac{16}{63.56} = 0.25173$$

Eq. (3.5-13)

$$\Delta p = 4 f \rho \frac{L}{D}\frac{V^2}{2}$$

$$= 4(0.25173)(976)\frac{6.10}{0.01267}\frac{(1.018)^2}{2}$$

$$= 245\,170\ N/m^2$$

$$\boxed{\Delta p = 245.2\ kN/m^2}$$

(3.5-3)

Apple sauce Table 3.5-1

$\rho = 1100\ kg/m^3$ $n' = 0.645 = n$ $K = 0.500\ N \cdot s^{n'}/m^2$

$\mathcal{D} = 50.8\ mm = 0.0508\ m$ $L = 3.05\ m$ $v = V = 4.57\ m/s$

(a) Smooth tube

Eq. (3.5-7) $$K' = K\left(\frac{3n'+1}{4n'}\right)^{n'} = 0.500\left(\frac{3\times 0.645+1}{4\times 0.645}\right)^{0.645} = 0.5435$$

Eq. (3.5-11)

$$N_{Re,gen} = \frac{\mathcal{D}^{n'} V^{2-n'} \rho}{K' 8^{n'-1}}$$

$$= \frac{(0.0508)^{0.645}(4.57)^{2-0.645}(1100)}{(0.5435)(8)^{0.645-1}} = 4855$$

Hence, the flow is turbulent. Using Fig. 3.5-3,

$$\boxed{f = 0.0073}$$

Eq. (3.5-13)

$$\Delta p = 4 f \rho \frac{L}{\mathcal{D}} \frac{V^2}{2} = 4(0.0073)(1100)\frac{(3.05)}{(0.0508)}\frac{(4.57)^2}{2}$$

$$\Delta p = 2.014\times 10^4\ N/m^2 = \boxed{20.14\ kN/m^2}$$

(b) Commercial pipe

$\varepsilon = 0.000046\ m$

$$\frac{\varepsilon}{\mathcal{D}} = \frac{0.000046}{0.0508} = 0.00090$$

Fig. 2.10-3

$$\boxed{f = 0.0100}$$

Eq. (3.5-13)

$$\Delta p = 4(0.0100)(1100)\frac{(3.05)}{(0.0508)}\frac{(4.57)^2}{2}$$

$$\Delta p = 2.759\times 10^4\ N/m^2 = \boxed{27.59\ kN/m^2}$$

(3.5-4)

$D_t = 0.304\ m \qquad D_a = 0.151\ m \qquad N = 5\ rev/s$

Flow properties of liquid

$n = 0.53 \qquad K = 26.49\ N \cdot s^{n'}/m^2 \qquad \rho = 975\ kg/m^3$

Eq. (3.5-6)

$$n' = n = 0.53$$

Eq. (3.5-34)

$$\mu_a = (11N)^{n-1} K = (11 \times 5)^{0.53-1.0} (26.49)$$

$$\boxed{\mu_a = 4.028\ Pa \cdot s}$$

Eq. (3.5-20)

$$N'_{Re,n} = \frac{D_a^2 N \rho}{\mu_a} = \frac{(0.151)^2 (5)(975)}{4.028}$$

$$\boxed{N'_{Re,n} = 27.60}$$

Fig. 3.5-4

$$\boxed{N_p = 3.1}$$

$$N_p = 3.1 = \frac{P}{\rho N^3 D_a^5} = \frac{P}{975(5)^3(0.151)^5}$$

$$P = 29.66W \qquad \boxed{P = 0.02966\ kW}$$

$$Vol = \frac{\pi}{4}(0.304)^2 = 2.207 \times 10^{-2}\ m^3$$

$$\frac{kW}{vol} = \frac{0.02966}{2.207 \times 10^{-2}} = \boxed{1.344\ kW/m^3}$$

(3.5-5)

Appendix A.1-7

$1\,dyne = 1\,g\cdot\frac{cm}{s^2} = 10^{-5}\,kg\cdot\frac{m}{s^2} = 10^{-5}\,N$

T torque, $1\,dyne\text{-}cm = 1\,g\cdot\frac{cm}{s^2}\cdot cm = 10^{-5}\,kg\cdot\frac{m}{s^2}\cdot cm \times \frac{1}{100\,cm/m}$

$1\,dyne\text{-}cm = 10^{-7}\,kg\cdot m^2/s^2$, $\omega = 2\pi\,(RPM)/60$ rad/s

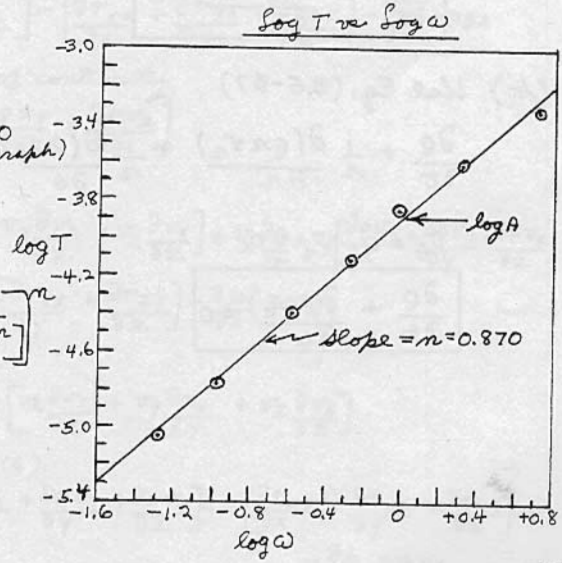
RPM	ω	$\log\omega$	Torque, dyne-cm	Torque, T, $kg\cdot m^2/s^2$	$\log T$
0.5	0.0524	-1.281	86.2	8.62×10^{-6}	-5.065
1.0	0.1047	-0.980	168.9	1.689×10^{-5}	-4.772
2.5	0.2618	-0.582	402.5	4.025×10^{-5}	-4.395
5.0	0.5236	-0.2810	754	7.54×10^{-5}	-4.123
10	1.047	0.0199	1365	1.366×10^{-4}	-3.865
20	2.094	0.3210	2379	2.379×10^{-4}	-3.624
50	5.236	0.7189	4636	4.636×10^{-4}	-3.334

$\omega = 2\pi\,(0.5)/60 = 0.0524$ rad/s

Eq.(3.5-23) $T = A\omega^n$ $\log T = \log A + n\log\omega$

Plot $\log T$ vs $\log\omega$

slope = $\boxed{n = 0.870}$



$\log T = \log A + n\log\omega$

When $\log\omega = 0$, $\log T = -3.90$ (from Graph)

$\log A + 0 = \log T = -3.90$

$A = 1.259\times10^{-4}$

Eq.(3.5-24)

$$A = 2\pi R_b^2 L K\left[\frac{2}{n\left[1-(R_b/R_c)^{2/n}\right]}\right]^n$$

$R_b = \frac{25.15}{100(2)} = 0.02515/2\ m$

$R_c = \frac{27.62}{100(2)} = 0.02762/2\ m$

$L = 92.39/100 = 0.09239\ m$

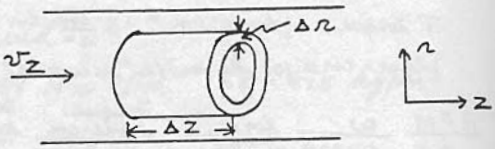
substituting,

$$1.259\times10^{-4} = 2\pi\left(\frac{0.02515}{2}\right)^2(0.09239)K\left[\frac{2}{0.870\left[1-\left(\frac{0.02515/2}{0.02762/2}\right)^{2/0.870}\right]}\right]^{0.870}$$

$$\boxed{K = 0.1594\,\frac{N\cdot s^n}{m^2} = 0.1594\,\frac{kg\cdot s^{n-2}}{m^2}}$$

(3.6-1)

(a) Shell balance



Eq. (3.6-17)

Rate mass in - rate mass out = rate of accumulation of mass

$$\left[(\rho v_z)_z - (\rho v_z)_{z+\Delta z}\right] 2\pi r \Delta r = \frac{\partial \rho}{\partial t} 2\pi r \Delta r \Delta z$$

Divide by $r \Delta r \Delta z$ and let $\Delta z \to 0$.

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_z)}{\partial z} = 0}$$

(b) Use Eq. (3.6-27)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

$v_\theta = 0,\ v_r = 0$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_z)}{\partial z} = 0}$$

(3.7-1)

Derive Eq.(3.7-13).

$$(1)\quad \rho\frac{Dv_x}{Dt} = -\left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] + \rho g_x - \frac{\partial p}{\partial x}$$

Use definition of substantial time derivative Eq.(3.6-2) but for v_x.

$$(2)\quad \frac{Dv_x}{Dt} = \frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}$$

Eq.(3.7-7)

$$(3)\quad \frac{\partial\rho v_x}{\partial t} = \left[\frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_y v_x)}{\partial y} + \frac{\partial(\rho v_z v_x)}{\partial z}\right] - \left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

Perform the differentiation on (3).

$$(4)\quad \rho\frac{\partial v_x}{\partial t} + v_x\frac{\partial\rho}{\partial t} = -\left[\rho v_x\frac{\partial v_x}{\partial x} + v_x\frac{\partial\rho v_x}{\partial x} + \rho v_y\frac{\partial v_x}{\partial y} + v_x\frac{\partial\rho v_y}{\partial y} + \rho v_z\frac{\partial v_x}{\partial z} + v_x\frac{\partial\rho v_z}{\partial z}\right] - \left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

Eq.(3.6-20) Equation of continuity

$$(5)\quad \frac{\partial\rho}{\partial t} = -\left[\frac{\partial\rho v_x}{\partial x} + \frac{\partial\rho v_y}{\partial y} + \frac{\partial\rho v_z}{\partial z}\right]$$

Rearrange Eq.(4).

$$(6)\quad \rho\frac{\partial v_x}{\partial t} + \rho\left[v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right] + v_x\frac{\partial\rho}{\partial t} + v_x\left[\frac{\partial\rho v_x}{\partial x} + \frac{\partial\rho v_y}{\partial y} + \frac{\partial\rho v_z}{\partial z}\right] = -\left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

Multiply Eq.(2) by ρ.

$$(7)\quad \rho\frac{Dv_x}{Dt} = \rho\frac{\partial v_x}{\partial t} + \rho\left[v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right]$$

Substitute Eq.(7) into (6).

$$(8)\quad \rho\frac{Dv_x}{Dt} + v_x\frac{\partial\rho}{\partial t} + v_x\left[\frac{\partial\rho v_x}{\partial x} + \frac{\partial\rho v_y}{\partial y} + \frac{\partial\rho v_z}{\partial z}\right] = -\left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] - \frac{\partial p}{\partial x} + \rho g_x$$

Multiply Eq.(5) by v_x

$$(9)\quad v_x\frac{\partial\rho}{\partial t} = -v_x\left[\frac{\partial\rho v_x}{\partial x} + \frac{\partial\rho v_x}{\partial y} + \frac{\partial\rho v_x}{\partial z}\right]$$

Substitute Eq.(9) into (8) and cancel like terms.

$$\boxed{\rho\frac{Dv_x}{Dt} + \cancel{v_x\frac{\partial\rho}{\partial t}} - \cancel{v_x\frac{\partial\rho}{\partial t}} = -\left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] + \rho g_x - \frac{\partial p}{\partial x}}$$

(3.8-1)

Eq.(3.8-17)

$$v_z = \frac{1}{4\mu}\frac{dp}{dz}(r^2 - r_0^2)$$

Eq.(2.6-17)

$$v_{z\,av} = \frac{1}{A}\iint_A v_z \, dA$$

$A = \pi r_0^2$ $\quad dA = r\,dr\,d\theta$ (polar coordinates)

substitute to Eq.(2.6-17).

$$v_{z\,av} = \frac{1}{\pi r_0^2}\int_0^{2\pi}\int_0^{r_0}\left[\frac{1}{4\mu}\frac{dp}{dz}(r^2 - r_0^2)\right] r\,dr\,d\theta$$

$$= \frac{1}{4\mu\pi r_0^2}\frac{dp}{dz}\int_0^{2\pi} d\theta \int_0^{r_0}(r^2 r\,dr - r_0^2 r\,dr)$$

$$= \frac{1}{4\mu\pi r_0^2}\frac{dp}{dz}(2\pi)\left[\frac{r^4}{4} - \frac{r_0^2 r^2}{2}\right]_0^{r_0}$$

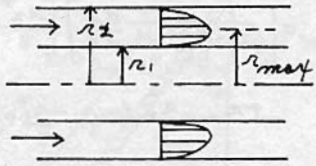
$$= \frac{1}{4\mu\pi r_0^2}\frac{dp}{dz}(2\pi)\left[-\frac{r_0^4}{4}\right]$$

Eq.(3.8-19)

$$\boxed{v_{z\,av} = -\frac{r_0^2}{8\mu}\frac{dp}{dz}}$$

(3.8-2)

Eq (3.8-16)



(1) $\frac{1}{\mu}\frac{dp}{dz} = \text{const.} = \frac{d^2 v_z}{dr^2} + \frac{1}{r}\frac{dv_z}{dr} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$

B.C. $dv_z/dr = 0$ at $r = r_{max}$.

Rearrange (1) and integrate.

(2) $\int d\left(r\frac{dv_z}{dr}\right) = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\int r\,dr$

(3) $r\frac{dv_z}{dr} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{2} + K_1$

Substitute B.C. into (3) and solve for K.

(4) $r(0) = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r_{max}^2}{2} + K_1$

(5) $r\frac{dv_z}{dr} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{2} - \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r_{max}^2}{2} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\left[\frac{r^2}{2} - \frac{r_{max}^2}{2}\right]$ (3.6-62)

B.C. $r = r_1$, $v_z = 0$ at inner wall. Integrating (5),

(6) $\int_{v_z=0}^{v_z} dv_z = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\int_{r_1}^{r}\left[\frac{r}{2}dr - \frac{r_{max}^2}{2}\frac{dr}{r}\right]$

(7) $v_z = \left(\frac{1}{2\mu}\frac{dp}{dz}\right)\left[\frac{r^2}{2} - \frac{r_1^2}{2} - r_{max}^2 \ln(r/r_1)\right]$ (3.6-63)

B.C. $r = r_2$, $v_z = 0$ at outer wall. Integrating (5) again,

(8) $v_z = \left(\frac{1}{2\mu}\frac{dp}{dz}\right)\left[\frac{r^2}{2} - \frac{r_2^2}{2} - r_{max}^2 \ln(r/r_2)\right]$ (3.6-64)

Equate (7) to (8) and solve for r_{max}.

(9) $\frac{r^2}{2} - \frac{r_1^2}{2} - r_{max}^2 \ln(r/r_1) = \frac{r^2}{2} - \frac{r_2^2}{2} - r_{max}^2 \ln(r/r_2)$

(10) $\boxed{r_{max} = \sqrt{\frac{1}{\ln(r_2/r_1)}(r_2^2 - r_1^2)/2}}$ (3.6-65)

Using Eq. (2.9-10) where $dA = r\,dr\,d\theta$ and $A = \pi(r_2^2 - r_1^2)$

(11) $v_{z\,av} = \frac{1}{A}\iint_A v_z\,dA = \frac{1}{\pi(r_2^2 - r_1^2)}\iint_A v_z\,r\,dr\,d\theta$

Substitute (7) to (11).

(12) $v_{z\,av} = \frac{1}{\pi(r_2^2 - r_1^2)}\int_0^{2\pi} d\theta \int_{r=r_1}^{r=r_2}\left(\frac{1}{2\mu}\frac{dp}{dz}\right)\left[\frac{r^2}{2} - \frac{r_1^2}{2} - r_{max}^2(\ln r/r_1)\right]r\,dr$

(13) $v_{z\,av} = \frac{1}{\pi(r_2^2 - r_1^2)}\frac{(2\pi)}{1}\left(\frac{1}{2\mu}\frac{dp}{dz}\right)\int_{r=r_1}^{r=r_2}\left[\frac{r^3}{2} - r\frac{r_1^2}{2} - r_{max}^2\,r\ln r + r_{max}^2(\ln r_1)r\right]dr$

(14) $v_{z\,av} = \left[\frac{1}{(r_2^2 - r_1^2)}\frac{1}{\mu}\frac{dp}{dz}\right]\left[\frac{1}{4}\frac{r^4}{2} - \frac{r^2}{2}\frac{r_1^2}{2} - r_{max}^2\left[\frac{r^2}{2}\ln r - \frac{r^2}{4}\right] + r_{max}^2(\ln r_1)\frac{r^2}{2}\right]_{r_1}^{r_2}$

(3.8-2) - p.2

$$(15)\quad v_{zav} = \left[\frac{1}{(r_2^2-r_1^2)}\frac{1}{\mu}\frac{dp}{dz}\right]\left[\frac{r_2^4}{8} - \frac{r_2^2 r_1^2}{4} - r_{max}^2\left(\frac{r_2^2}{2}\ln r_2 - \frac{r_2^2}{4}\right)\right.$$

$$\left. + r_{max}^2\frac{r_2^2}{2}\ln r_1 - \frac{r_1^4}{8} + \frac{r_1^4}{4} + r_{max}^2\left(\frac{r_1^2}{2}\ln r_1 - \frac{r_1^2}{4}\right) - r_{max}^2\frac{r_1^2}{2}\ln r_1\right]$$

$$(16)\quad v_{zav} = \left[\frac{1}{(r_2^2-r_1^2)}\frac{1}{\mu}\frac{dp}{dz}\right]\left[\frac{r_2^4}{8} - \frac{r_2^2 r_1^2}{4} + r_{max}^2\frac{r_2^2}{2}\ln(r_1/r_2)\right.$$

$$\left. + r_{max}^2\frac{r_2^2}{4} + \frac{r_1^4}{8} + r_{max}^2\frac{r_1^2}{4}\right]$$

Substitute (10) into (16).

$$(17)\quad v_{zav} = \left[\frac{1}{(r_2^2-r_1^2)}\frac{1}{\mu}\frac{dp}{dz}\right]\left[\frac{r_2^4}{8} - \frac{r_2^2 r_1^2}{4} + \frac{(r_2^2-r_1^2)}{2\ln(r_2/r_1)}\frac{r_2^2}{2}\ln(r_1/r_2)\right.$$

$$\left. + \frac{(r_2^2-r_1^2)}{2\ln(r_2/r_1)}\frac{r_2^2}{4} + \frac{r_1^4}{8} + \frac{(r_2^2-r_1^2)}{2\ln(r_2/r_1)}\frac{r_1^2}{4}\right]$$

$$(18)\quad v_{zav} = \left[\frac{1}{(r_2^2-r_1^2)}\frac{1}{\mu}\frac{dp}{dz}\right]\left[\left(\frac{r_2^4}{8} - \frac{r_2^2 r_1^2}{4} + \frac{r_1^4}{8}\right) - (r_2^2-r_1^2)\frac{r_2^2}{4}\right.$$

$$\left. + \frac{(r_2^2-r_1^2)}{\ln(r_2/r_1)}\frac{r_2^2}{8} + \frac{(r_2^2-r_1^2)}{\ln(r_2/r_1)}\frac{r_1^2}{8}\right]$$

Factor and cancel $(r_2^2-r_1^2)$.

$$(19)\quad v_{zav} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\left[\frac{1}{8}(r_2^2-r_1^2) - \frac{r_2^2}{4} + \left(\frac{r_2^2}{8} - \frac{r_1^2}{8}\right)\frac{1}{\ln(r_2/r_1)}\right]$$

$$(20)\quad \boxed{v_{zav} = -\left(\frac{1}{8\mu}\frac{dp}{dz}\right)\left[r_2^2 + r_1^2 - \frac{(r_2^2-r_1^2)}{\ln(r_2/r_1)}\right]}$$

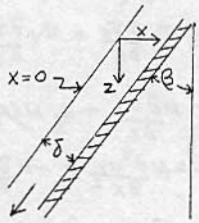
Integrate (20),

$$(21)\quad v_{zav}\int_{z=0}^{z=L} dz = -\left(\frac{1}{8\mu}\right)\left[r_2^2 + r_1^2 - \frac{(r_2^2-r_1^2)}{\ln(r_2/r_1)}\right]\int_{p=p_0}^{p=p_L} dp$$

$$(22)\quad \boxed{v_{zav} = \left(\frac{p_0-p_L}{8\mu L}\right)\left[r_2^2 + r_1^2 - \frac{(r_2^2-r_1^2)}{\ln(r_2/r_1)}\right]}$$

(3.8-4)

(a) Eq. (3.7-38) for z component



$$\rho\left(\cancelto{0}{\frac{\partial v_z}{\partial t}} + v_x\cancelto{0}{\frac{\partial v_z}{\partial x}} + v_y\cancelto{0}{\frac{\partial v_z}{\partial y}} + v_z\cancelto{0}{\frac{\partial v_z}{\partial z}}\right)$$
$$= \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \cancelto{0}{\frac{\partial^2 v_z}{\partial y^2}} + \cancelto{0}{\frac{\partial^2 v_z}{\partial z^2}}\right) - \cancelto{0}{\frac{\partial p}{\partial z}} + \rho g_z$$

(1) $\mu \frac{\partial^2 v_z}{\partial x^2} = -\rho g_z$

But $g_z = g\cos\beta$

(2) $\mu \frac{\partial^2 v_z}{\partial x^2} = -\rho g \cos\beta$

Integrating (2),

(3) $\mu \frac{\partial v_z}{\partial x} = -\rho g \cos\beta\, x + C_1$ ← constant

B.C. $\frac{\partial v_z}{\partial x} = 0$, at $x = 0$

Hence, $C_1 = 0$

(4) $\mu \frac{\partial v_z}{\partial x} = -\rho g \cos\beta\, x + 0$

B.C. $v_z = 0$, at $x = \delta$

(5) $\mu v_z = -\rho g \cos\beta \frac{x^2}{2} + C_2 = 0$

Hence, $C_2 = \rho g \cos\beta \frac{\delta^2}{2}$

(6) $\mu v_z = -\rho g \cos\beta \frac{x^2}{2} + \rho g \cos\beta \frac{\delta^2}{2}$

Solving for v_z, $\boxed{v_z = \frac{\rho g \delta^2}{2\mu}\cos\beta\left(1 - \frac{x^2}{\delta^2}\right)}$

(b) Maximum velocity at $x = 0$

(7) $\boxed{v_{z_{max}} = \frac{\rho g \delta^2}{2\mu}\cos\beta}$

Using Eq. (2.9-26) where W is distance in y direction,

(8) $v_{z_{av}} = \frac{1}{A}\iint_{A'} v_z\, dA = \frac{1}{W\delta}\int_0^W\int_0^\delta v_z\, dx\, dy = \frac{W}{W\delta}\int_0^\delta v_z\, dx$

Substituting (6) into (8) and integrating, $\boxed{v_{z_{av}} = \frac{\rho g \delta^2}{3\mu}\cos\beta}$

(c) Eq. (3.7-19) $\tau_{xz} = -\mu\left(\frac{\partial v_z}{\partial x} + \cancelto{0}{\frac{\partial v_x}{\partial z}}\right)$

(9) $\tau_{xz} = -\mu \frac{\partial v_z}{\partial x}$

(10) Subst. (6) into (9)

$\tau_{xz} = -\mu\left(-\frac{\rho g x}{\mu}\cos\beta\right)$

$\boxed{\tau_{xz} = \rho g x \cos\beta}$

(3.8-6) Eq. (3.7-10)

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right] + \rho g_x - \frac{\partial p}{\partial x}$$

(1) $\tau_{xx} = -2\mu\frac{\partial v_x}{\partial x} + \frac{2}{3}\mu\cancel{(\nabla\cdot v)}$ Eq. (3.7-14) $(\nabla\cdot v)=0$ ρ, μ are const.

(2) $\frac{\partial \tau_{xx}}{\partial x} = -2\mu\frac{\partial^2 v_x}{\partial x^2} = -\mu\frac{\partial^2 v_x}{\partial x^2} - \mu\frac{\partial\left(\frac{\partial v_x}{\partial x}\right)}{\partial x}$

(3) $\tau_{yx} = -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$ Eq. (3.7-17)

(4) $\frac{\partial \tau_{yx}}{\partial y} = -\mu\frac{\partial^2 v_x}{\partial y^2} - \mu\frac{\partial\left(\frac{\partial v_y}{\partial x}\right)}{\partial y} = -\mu\frac{\partial^2 v_x}{\partial y^2} - \mu\frac{\partial\left(\frac{\partial v_y}{\partial y}\right)}{\partial x}$ (can interchange order of differentiation)

(5) $\tau_{zx} = -\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)$ Eq. (3.7-19)

(6) $\frac{\partial \tau_{zx}}{\partial z} = -\mu\frac{\partial\left(\frac{\partial v_z}{\partial x}\right)}{\partial z} - \mu\frac{\partial^2 v_x}{\partial z^2} = -\mu\frac{\partial\left(\frac{\partial v_z}{\partial z}\right)}{\partial x} - \mu\frac{\partial^2 v_x}{\partial z^2}$

Hence, substitute (2), (4) and (6) into (7).

(7) $\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right] = -\mu\frac{\partial^2 v_x}{\partial x^2} - \mu\frac{\partial\left(\frac{\partial v_x}{\partial x}\right)}{\partial x} - \mu\frac{\partial^2 v_x}{\partial y^2} - \mu\frac{\partial\left(\frac{\partial v_y}{\partial y}\right)}{\partial x} - \mu\frac{\partial\left(\frac{\partial v_z}{\partial z}\right)}{\partial x} - \mu\frac{\partial^2 v_x}{\partial z^2}$

Rearranging (7)

(8) $-\mu\frac{\partial^2 v_x}{\partial x^2} - \mu\frac{\partial^2 v_x}{\partial y^2} - \mu\frac{\partial^2 v_x}{\partial z^2} - \mu\frac{\partial\left(\frac{\partial v_x}{\partial x}\right)}{\partial x} - \mu\frac{\partial\left(\frac{\partial v_y}{\partial y}\right)}{\partial x} - \mu\frac{\partial\left(\frac{\partial v_z}{\partial z}\right)}{\partial z} =$

$$-\mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) - \mu\frac{\partial}{\partial x}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]$$

(9) But $\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right] = (\nabla\cdot v) = 0$ Eq. (3.6-24)

Combining (7), (8), and (9)

(10) $\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right] = -\mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right)$

Substituting (10) into Eq. (3.7-10), gives Eq. (3.7-36)

$$\boxed{\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x - \frac{\partial p}{\partial x}}$$

(3.9-1)

$$\phi = C(x^2 - y^2)$$

Eq. (3.9-5) for potential function is

(1) $v_x = \frac{\partial \phi}{\partial x} = C\,2x\frac{\partial x}{\partial x} + 0 = 2Cx$

(2) $v_y = \frac{\partial \phi}{\partial y} = 0 - C\,2y\frac{\partial y}{\partial y} = -2Cy$

$$\boxed{\begin{aligned} v_x &= 2Cx \\ v_y &= -2Cy \end{aligned}}$$

Laplace's Eq. (3.9-9)

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 = 2C\frac{\partial x}{\partial x} - 2C\frac{\partial y}{\partial y} = 0$$ Hence, satisfies this Eq.

(3.9-3)

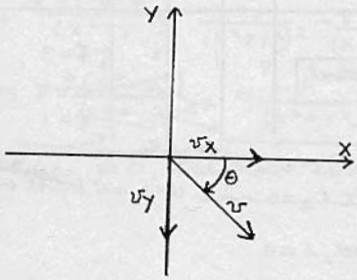
$$\psi = Bxy$$

$v_x = \frac{\partial \psi}{\partial y} = Bx\frac{\partial y}{\partial y}$ Hence, $\boxed{v_x = Bx}$ Eq. (3.9-1)

$v_y = -\frac{\partial \psi}{\partial x} = -By\frac{\partial x}{\partial x}$ $\boxed{v_y = -By}$

$v_x = 50(x) = 50(1) = 50$ cm/s

$v_y = -50(y) = -50(1) = -50$ cm/s



$\tan\theta = 1/1 = 1.0$ Hence, $\boxed{\theta = 45°}$

$v^2 = v_x^2 + v_y^2 = (50)^2 + (-50)^2$

$$\boxed{v = 70.7 \text{ cm/s}}$$

(3.9-4)

$v_x = U, \; v_y = 0$

(a) Stream function ψ Eq. (3.9-1)

$$v_x = \frac{\partial \psi}{\partial y}, \quad v_y = -\frac{\partial \psi}{\partial x}$$

$$v_y = -\frac{\partial \psi}{\partial x} = 0. \quad \text{Hence, } \psi = f(y)$$

$$v_x = \frac{\partial \psi}{\partial y} = U. \quad \text{Hence, } \boxed{\psi = Uy + C} \text{ where } \boxed{C = \text{constant}}$$

To plot the streamlines, C can be ignored since integration constants have no effect on velocities in the flow.

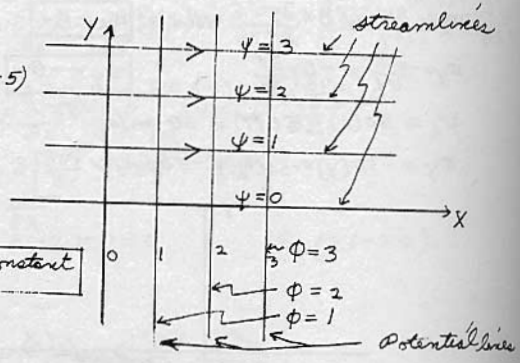
Hence, use $\psi = Uy + 0$

set $\psi = 0$, hence, $y = 0$

set $\psi = 1$. also set $U = 1$. Hence, $y = 1$

set $\psi = 2$. Hence, $y = 2$

set $\psi = 3$. Hence, $y = 3$.



(b) Potential function ϕ Eq. (3.9-5)

$$v_y = \frac{\partial \phi}{\partial y}, \quad v_x = \frac{\partial \phi}{\partial x}$$

$$v_x = \frac{\partial \phi}{\partial x} = U$$

Hence, integrating,

$$\boxed{\phi = Ux + C_2} \text{ where } \boxed{C_2 = \text{constant}}$$

$$v_y = \frac{\partial \phi}{\partial y} = 0, \text{ Hence, } \phi = f(x)$$

To plot the potential lines, set $C_2 = 0$ and $U = 1$ as before in (a)

set $\phi = 0$, $\phi = 0 = 1x + 0$; Hence, $x = 0$

set $\phi = 1$, $\phi = 1 = 1x + 0$; Hence, $x = 1$

set $\phi = 2$, $\phi = 2 = 1x + 0$; Hence, $x = 2$

(3.9-6)

$$\psi = \frac{1}{(x^2+y^2)} \quad \text{stream function}$$

By Eq. (3.9-1)

$$(1)\quad v_x = \frac{\partial \psi}{\partial y} = \boxed{\frac{-2y}{(x^2+y^2)^2} = v_x}$$

$$(2)\quad v_y = -\frac{\partial \psi}{\partial x} = \frac{-(-2x)}{(x^2+y^2)^2} = \boxed{\frac{2x}{(x^2+y^2)^2} = v_y}$$

Eq. (3.9-6) defines vorticity of a fluid

$$\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = 2\omega_z$$

If $2\omega_z = 0$, the flow is irrotational and a potential function exists.

Hence, using (1),

$$(3)\quad \frac{\partial v_x}{\partial y} = \frac{(x^2+y^2)^2(-2) + (2y)(2)(x^2+y^2)(2y)}{(x^2+y^2)^4} = \frac{-2}{(x^2+y^2)^2} + \frac{8y^2}{(x^2+y^2)^3}$$

also, using (2),

$$(4)\quad \frac{\partial v_y}{\partial x} = \frac{(x^2+y^2)^2(2) - (2x)(2)(x^2+y^2)(2x)}{(x^2+y^2)^4} = \frac{2}{(x^2+y^2)^2} - \frac{8x^2}{(x^2+y^2)^3}$$

Substituting (3) and (4) into (3.9-6),

$$\frac{2}{(x^2+y^2)^2} - \frac{8x^2}{(x^2+y^2)^3} - \frac{(-2)}{(x^2+y^2)^2} - \frac{8y^2}{(x^2+y^2)^3} =$$

$$\frac{4}{(x^2+y^2)^2} - \frac{8\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{3}\,2}} = \frac{-4}{(x^2+y^2)^2} = 2\omega_z$$

Since, $2\omega_z \neq 0$, the $\boxed{\text{flow is rotational}}$

(3.9-7)

Given: $v_x = ax^2 - ay^2 \qquad v_y = -2axy$

For potential function ϕ to exist, Eq (3.9-6),

(1) $$\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = 2\omega_z = 0 \text{ for potential function to exist.}$$

(2) $$\frac{\partial v_x}{\partial y} = 0 - 2ay \qquad \frac{\partial v_y}{\partial x} = -2ay$$

Substituting (2) into (1),

(3) $$\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = -2ay - (-2ay) = 0$$

$\boxed{\text{Hence, } \phi \text{ exists.}}$

From Eq. (3.9-5),

(4) $$v_x = \frac{\partial \phi}{\partial x}, \quad v_y = \frac{\partial \phi}{\partial y}$$

(5) $$v_x = \frac{\partial \phi}{\partial x} = ax^2 - ay^2$$

Integrating (5),

(6) $$\phi = \frac{ax^3}{3} - ay^2x + f(y)$$

Differentiating (6),

(7) $$\frac{\partial \phi}{\partial y} = 0 - ax2y + f'(y)$$

Equate (7) to (4) and to v_y,

(8) $$\frac{\partial \phi}{\partial y} = -ax2y + f'(y) = v_y = -2axy$$

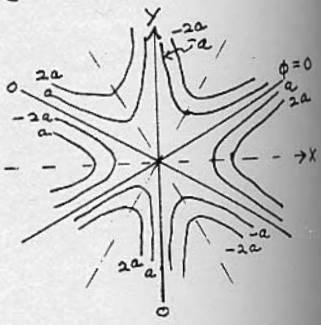
(9) $$-\cancel{ax2y} + f'(y) = -\cancel{2a}xy$$

(10) Hence, $f'(y) = 0$ and $f(y) = \text{constant} = C$

Hence,

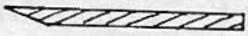
$$\boxed{\phi = \frac{ax^3}{3} - axy^2 + C}$$

To plot this eq, let $C = 0$



(3.10-1) $v_\infty = 0.914\ m/s$

Water at 20°C



$L = 0.305\ m$

(a) $\mu = 1.005 \times 10^{-3}\ Pa \cdot s$ $\rho = 998.2\ kg/m^3$

$$N_{Re,L} = \frac{L v_\infty \rho}{\mu} = \frac{(0.305)(0.914)(998.2)}{1.005 \times 10^{-3}}$$

$\boxed{N_{Re,L} = 2.77 \times 10^5}$ Laminar since $N_{Re,L} < 5 \times 10^5$

(b) Boundary layer thickness.

Eq (3.10-6)

$x = 0.152\ m$ $\delta = \dfrac{5.0x}{\sqrt{N_{Re,x}}} = \dfrac{5.0(0.152)}{\sqrt{\frac{2.77 \times 10^5}{2}}} = 0.00204\ m$

$N_{Re,x} = \dfrac{2.77 \times 10^5}{2}$

$\boxed{\delta = 0.00204\ m \quad (x = 0.152\ m)}$

$x = 0.305$ $\delta = \dfrac{5.0(0.305)}{\sqrt{2.77 \times 10^5}} = 0.0029\ m$

$\boxed{\delta = 0.0029\ m \quad (x = 0.305\ m)}$

(c) Total drag on plate $0.305\ m \times 0.305\ m$.

$b = 0.305\ m,\ L = 0.305\ m$

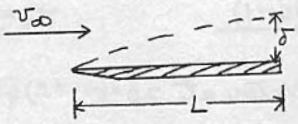
Eq. (3.10-10)

$$F_D = 0.664\, b \sqrt{\mu \rho v_\infty^3 L}$$

$$= (0.664)(0.305)\sqrt{(1.005 \times 10^{-3})(998.2)(0.914)^3(0.305)}$$

$\boxed{F_D = 0.0979\ N}$

(3.10-2)



air at 294.3 K $\quad p = 101.3$ kPa

A.3 $\quad \mu = 1.820 \times 10^{-5}$ kg/m·s $\quad b = 0.30$ m

$\rho = 1.207$ kg/m^3 $\quad L = 0.30$ m $\quad v_\infty = 6.1$ m/s

$$N_{Re,L} = \frac{L v_\infty \rho}{\mu} = \frac{0.3(6.1)(1.207)}{1.820 \times 10^{-5}}$$

$$N_{Re,L} = 1.214 \times 10^5$$

The flow is laminar since $N_{Re,L} < 5 \times 10^5$

Boundary layer thickness

Eq.(3.10-6) $\quad x = L = 0.30$ m $\quad N_{Re,x} = N_{Re,L}$

$$\delta = \frac{5.0x}{\sqrt{N_{Re,x}}} = \frac{5.0(0.30)}{\sqrt{1.214 \times 10^5}}$$

$$\boxed{\delta = 0.00431 \text{ m}}$$

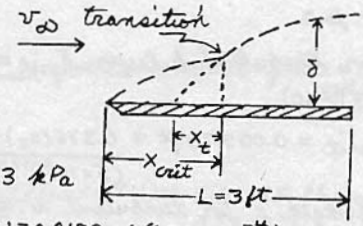
Total drag Eq.(3.10-10)

$$F_D = 0.664 b \sqrt{\mu \rho v_\infty^3 L}$$

$$= 0.664(0.30)\sqrt{(1.820 \times 10^{-5})(1.207)(6.1)^3(0.30)}$$

$$\boxed{F_D = 7.70 \times 10^{-3} \text{ N}}$$

(3.10-4)



$v_\infty = 100\ ft/s$

$N_{Re,x} = 5\times10^5$ (critical)

air at 293 K (20°C) $p = 101.3\ kPa$

A.3 $\rho = 0.0754\ lb_m/ft^3$ $\quad \mu = 0.0182\ cp\,(6.7197\times10^{-4}) = 1.223\times10^{-5}\ \frac{lb_m}{ft\cdot s}$

$$N_{Re,x} = \frac{x\,v_\infty\,\rho}{\mu}$$

(a) $$N_{Re,x} = \frac{x(100)(0.0754)}{1.223\times10^{-5}} = 5\times10^5$$

$$\boxed{x_{crit} = \begin{matrix}0.811\ ft\\ 0.2742\ m\end{matrix}}$$

(b) $L = 0.5\ ft = x$ $\quad N_{Re,x} = \frac{0.5(100)(0.0754)}{1.223\times10^{-5}} = 3.083\times10^5$

This is in laminar region.

Eq.(3.10-6)

$$\delta = \frac{5.0\,x}{\sqrt{N_{Re,x}}} = \frac{5.0(0.5)}{\sqrt{3.083\times10^5}} = \boxed{\begin{matrix}0.00450\ ft\\ 0.001372\ m\end{matrix}}$$

$L = 3.0\ ft = x$ $\quad N_{Re,x} = \frac{3.0(100)(0.0754)}{1.223\times10^{-5}} = 1.850\times10^6 = N_{Re,L}$

This is in turbulent region.

Eq.(3.10-60) Method I to calculate δ at L=3ft

$$\delta = \frac{0.376\,L}{N_{Re,L}^{1/5}} = \frac{0.376\,(3.0)}{(1.85\times10^6)^{1/5}} = \boxed{\begin{matrix}0.0629\ ft\\ 0.01917\ m\end{matrix}}$$ approximate

This value of δ is approximate since the turbulent b. layer is assumed to extend to x=0. Actually x_{crit} = 0.811 ft of 3.0 ft. Hence, a large fraction is in laminar region.

Method II to calcute δ at L = 3 ft

This method is still approximate but it is more accurate.

1. Calculate δ_{crit} for laminar b. layer for $N_{Re,x} = 5\times10^5$ at $x_{crit} = 0.811$ ft. Eq.(3.10-6)

$$\delta_{crit} = \frac{5(0.811)}{\sqrt{5\times10^5}} = 0.00573\ ft$$

(3.10-4) - p.2

2. For turbulent b. layer, $\delta_{crit} = 0.00573\ ft$, $N_{Re,x} = 5\times10^5$

Eq.(3.10-60)

$$\delta_{crit} = 0.00573\ ft = \frac{0.376(x_t)}{(5\times10^5)^{1/5}} \qquad x_t = 0.2103\ ft$$

3. Length x for turbulent b. layer equation

$$x = 3.0 - x_{crit} + x_t = 3.0 - 0.811 + 0.210 = 2.389\ ft$$

$$\delta_{3.0ft} = \frac{0.376(2.389)}{(1.850\times10^6)^{1/5}} = \boxed{\begin{matrix}0.0501\ ft\\ 0.01527\ m\end{matrix}}$$

Drag coefficient C_D for $L = 0.5\ ft$

Eq.(3.10-12) Laminar flow, $N_{Re,L} = 3.083\times10^5$

$$C_D = \frac{1.328}{N_{Re,L}^{1/2}} = \frac{1.328}{(3.083\times10^5)^{1/2}} = \boxed{0.00239}$$

Drag coefficient C_D and total drag F_D for $L = 3.0\ ft$

Eq.(3.10-61) $C_D = \dfrac{0.072}{N_{Re,L}^{1/5}}$ This eq. cannot be used since it assumes that the turbulent b. layer extends to $x=0$. Since a substantial fraction of the total length of $L = 3.0\ ft$ is laminar ($x_{crit} = 0.811\ ft$), the following approximate method is used (P1).

1. Calculate C_D and F_D for $L = 3.0\ ft$, using turbulent Eq.(3.10-61).

$$C_{D turb} = \frac{0.072}{N_{Re,L}^{1/5}} = \frac{0.072}{(1.85\times10^6)^{1/5}} = 0.004013$$

(This would be accurate if laminar length of plate is small)

Eq.(3.10-11) $F_{D turb} = C_D \dfrac{v_\infty^2}{2}\dfrac{\rho}{g_c}A \qquad A = bL = (1.0\times3.0)$

$$F_{D turb} = 0.004013\frac{(100)^2}{2}\frac{(0.0754)(1.0\times3.0)}{32.174} = \begin{matrix}0.1411\ lb_f\\ 0.6276\ N\end{matrix}$$

2. Calculate $C_{D crit}$ and $F_{D crit}$ for turbulent length $x_{crit} = 0.811\ ft$,

$$C_{D crit} = \frac{0.072}{(5\times10^5)^{1/5}} = 0.005219$$

$$F_{D crit} = 0.005219\frac{(100)^2}{2}\frac{(0.0754)(1.0\times0.811)}{32.174} = \begin{matrix}0.0496\ lb_f\\ 0.2206\ N\end{matrix}$$

(3.10-4) - p.3

3. Calculate $C_{D\,lamin}$ and $F_{D\,lamin}$ for laminar length $x_{crit} = 0.811$ ft.

$$C_{D\,lamin} = \frac{1.328}{(5\times10^5)^{1/5}} = 0.001878$$

$$F_{D\,lamin} = 0.001878\frac{(100)^2}{2}\frac{(0.0754)}{32.174}(1.0\times0.811) = \begin{matrix}0.01785\ lb_f \\ 0.0794\ N\end{matrix}$$

4. Resultant C_D and F_D for $L = 3.0$ ft

$$F_D = F_{D\,turb} - F_{D\,crit} + F_{D\,lamin}$$
$$= 0.1411 - 0.0496 + 0.01785$$
$$F_D = \boxed{\begin{matrix}0.1094\ lb_f \\ 0.4864\ N\end{matrix}}$$

$$F_D = 0.1094 = C_D\frac{(100)^2}{2}\frac{(0.0754)}{32.174}(1.0\times3.0)$$

$$\boxed{C_D = 0.00311}$$

(3.11-1)

F (force) is a function of v, ρ, μ, L

Item	Units
F	$\frac{ML}{t^2}$
v	L/t
ρ	M/L^3
μ	M/Lt
L	L

$q = 5$, total number of variables

$u = 3$, number of fundamental units

$q - u = 5 - 3 = 2$, number of π's

Hence, $\pi_1 = f(\pi_2)$

Select a core group of $u(3)$ variables v, ρ, and L common to both groups.

$$\pi_1 = L^a v^b \rho^c F \qquad \pi_2 = L^d v^e \rho^f \mu$$

For π_1

$$1 = L^a \left(\frac{L}{t}\right)^b \left(\frac{M}{L^3}\right)^c \frac{ML}{t^2}$$

Equate exponents on both sides of equation.

(L) $0 = a + b - 3c + 1$

(M) $0 = c + 1$

(t) $0 = -b - 2$

Solving, $a = -2, b = -2, c = -1$

$$\pi_1 = L^{-2} v^{-2} \rho^{-1} F$$

$$\boxed{\pi_1 = \frac{F/L^2}{\rho v^2} = \frac{p}{\rho v^2} = N_{Eu}}$$

For π_2

$$1 = L^d \left(\frac{L}{t}\right)^e \left(\frac{M}{L^3}\right)^f \frac{M}{Lt}$$

(L) $0 = d + e - 3f - 1$

(M) $0 = f + 1$

(t) $0 = -e - 1$

Solving, $d = -1, e = -1, f = -1$

$$\pi_2 = L^{-1} v^{-1} \rho^{-1} \mu$$

$$\boxed{\pi_2 = \frac{\mu}{L v \rho} = \frac{1}{N_{Re}}}$$

$$\boxed{N_{Eu} = f(N_{Re})}$$

(3.11-2)

$\mathcal{D}$ (bubble diameter) a function of d, ρ, σ, μ, g

Item	Units
$\mathcal{D}$	L
d	L
ρ	M/L^3
σ	M/t^2
μ	M/Lt
g	L/t^2

$q = 6$, total number of variables

$u = 3$, number of fundamental units

$q - u = 6 - 3 = 3$, number of π's

$\pi_1 = f(\pi_2, \pi_3)$

select core variables d, ρ, and g ($u = 3$) common to all 3 groups.

$$\pi_1 = d^a \rho^b g^c \mathcal{D}^1$$

$$1 = L^a \left(\frac{M}{L^3}\right)^b \left(\frac{L}{t^2}\right)^c L$$

(L) $0 = a - 3b + c + 1$

(M) $0 = b$

(t) $0 = -2c$

solving, $a = -1, b = 0, c = 0$

$$\boxed{\pi_1 = \frac{\mathcal{D}}{d}}$$

$$\pi_2 = d^d \rho^e g^f \sigma$$

$$1 = L^d \left(\frac{M}{L^3}\right)^e \left(\frac{L}{t^2}\right)^f \frac{M}{t^2}$$

(L) $0 = d - 3e + f$

(M) $0 = e + 1$

(t) $0 = -2f - 2$

solving, $d = -2, e = -1, f = -1$

$$\boxed{\pi_2 = \frac{\sigma}{\rho d^2 g}}$$

$$\pi_3 = d^g \rho^h g^i \mu$$

$$1 = L^g \left(\frac{M}{L^3}\right)^h \left(\frac{L}{t^2}\right)^i \frac{M}{Lt}$$

(L) $0 = g - 3h + i - 1$

(M) $0 = h + 1$

(t) $0 = -2i - 1$

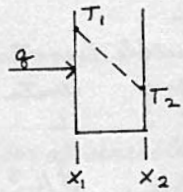
solving, $g = -3/2, h = -1, i = -\frac{1}{2}$

$$\pi_3 = \frac{\mu}{\rho d^{3/2} g^{1/2}}$$

Square the right hand side to remove fractional exponents.

$$\boxed{\pi_3 = \frac{\mu^2}{\rho^2 d^3 g}}$$

(4.1-1)



$T_1 = 299.9\,K \qquad T_2 = 276.5\,K$

$x_2 - x_1 = \frac{25.4}{1000} = 0.0254\,m$

$k = 0.0433\,W/m\cdot K$

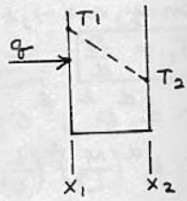
Eq.(4.1-10)

$$\frac{q}{A} = \frac{k}{x_2 - x_1}(T_1 - T_2)$$

$$\frac{q}{A} = \frac{0.0433}{0.0254}(299.9 - 276.5)$$

$$\boxed{q/A = 39.9\,W/m^2}$$

(4.1-2)



$T_1 = 318.4\,K \qquad T_2 = 303.2\,K$

$x_2 - x_1 = \frac{25}{1000} = 0.0250\,m$

$q/A = 35.1\,W/m^2$

Eq.(4.1-10)

$$\frac{q}{A} = \frac{k}{x_2 - x_1}(T_1 - T_2)$$

$$35.1 = \frac{k}{0.0250}(318.4 - 303.2)$$

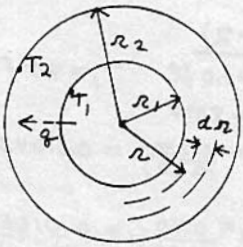
$$\boxed{k = 0.05773\,W/m\cdot K}$$

$$k = 0.0577\,\frac{W}{m\cdot K} \times \frac{1}{1.73075\,(W/m\cdot K)/(btu/h\cdot ft\cdot {}^\circ F)}$$

appendix A.1

$$\boxed{k = 0.03336\,btu/h\cdot ft\cdot {}^\circ F}$$

(4.2-1)



Eq.(4.1-11)

$k = a + bT$

Conduction through a hollow cylinder. Eq.(4.2-7)

$$\frac{q}{2\pi L}\int_{r_1}^{r_2}\frac{dr}{r} = -k\int_{T_1}^{T_2} dT$$

Substitute Eq.(4.1-11) into (4.2-7) and put k under $\int$ sign.

$$\frac{q}{2\pi L}\int_{r_1}^{r_2}\frac{dr}{r} = -\int_{T_1}^{T_2}(a+bT)\,dT$$

$$\frac{q}{2\pi L}\ln(r_2/r_1) = a(T_1-T_2) + b(T_1-T_2)\frac{(T_1+T_2)}{2}$$

$$= \left[a + b\frac{(T_1+T_2)}{2}\right](T_1-T_2)$$

$$\frac{q}{2\pi L}\ln(r_2/r_1) = k_m(T_1-T_2)$$

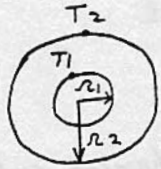
$$\boxed{k_m = a + b\frac{(T_1+T_2)}{2}} \qquad \text{Eq.(4.2-3)}$$

(4.2-2)

$L = 1.0\ ft \qquad T_1 = 40°F$

$T_2 = 80°F$

$r_1 = \frac{0.25}{12(2)} = 0.0104\ ft$



$r_2 = \frac{0.40}{12(2)} = 0.01665\ ft$

Mean thermal conductivity, k_m

$k = a + bT$

$k = 7.75 + 7.78 \times 10^{-3}\ T\ °F$

Eq. (4.2-3) holds for cylinders also (See Problem 4.2-1)

$k_m = a + b\frac{(T_1+T_2)}{2} = 7.75 + 7.78 \times 10^{-3}\left(\frac{40+80}{2}\right)$

$k_m = 8.22\ btu/h \cdot ft \cdot °F$

$A_1 = 2\pi r_1 L = 2\pi(0.0104) = 0.0653\ ft^2$

$A_2 = 2\pi r_2 L = 2\pi(0.01665) = 0.1046\ ft^2$

Eq. (4.2-10)

$A_{lm} = \frac{A_2 - A_1}{\ln(A_2/A_1)} = \frac{0.1046 - 0.0653}{\ln(0.1046/0.0653)} = 0.0838\ ft^2$

Eq. (4.2-8)

$q = k_m A_{lm} \frac{(T_1 - T_2)}{(r_2 - r_1)}$

$= (8.22)(0.0838)\frac{(80-40)}{0.01665 - 0.0104} = 4410\ btu/h$

$q = \frac{4410}{3600} = \boxed{1.225\ btu/s \quad 1292\ W}$

(4.2-3)

$k = (15.23\ W/m \cdot K)\frac{1}{(1.73073\ W/m \cdot K)/(btu/h \cdot ft \cdot °F)}$ ← appendix A.1

$k = 8.80\ btu/h \cdot ft \cdot °F$

$q = k A_{lm} \frac{(T_1 - T_2)}{(r_2 - r_1)} = (8.80)(0.0838)\frac{(80-40)}{0.01665 - 0.0104}$

$\boxed{q = 1.311\ btu/s \quad 1383\ W}$

(4.2-4)

$$k = (a + bT + cT^3)$$

$$\frac{q}{A} = -k\frac{dT}{dx} = -(a+bT+cT^3)\frac{dT}{dx}$$

$$\frac{q}{A}\int_{x_1}^{x_2} dx = -\int_{T_1}^{T_2}(a+bT+cT^3)\,dT$$

$$\frac{q}{A}(x_2-x_1) = \left[a(T_1-T_2)+\frac{b}{2}(T_1^2-T_2^2)+\frac{c}{4}(T_1^4-T_2^4)\right]$$

$$\boxed{\frac{q}{A} = \frac{\left[a(T_1-T_2)+\frac{b}{2}(T_1^2-T_2^2)+\frac{c}{4}(T_1^4-T_2^4)\right]}{(x_2-x_1)}}$$

(4.2-5)

(1) $\frac{q}{A} = -k\frac{dT}{dr}$

(2) $A = 4\pi r^2$ Substitute (2) to (1) and integrate.

(3) $\frac{q}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -k\int_{T_1}^{T_2}dT$

(4) $q = \frac{4\pi k(T_1-T_2)}{(1/r_1-1/r_2)}$ Eq. (4.2-14)

Integrate (3) between r_1 and r and T_1 and T.

(5) $\frac{q}{4\pi}\int_{r_1}^{r}\frac{dr}{r^2} = -k\int_{T_1}^{T}dT$

(6) $q = \frac{4\pi k(T_1-T)}{(1/r_1-1/r)}$

Equate (4) to (6).

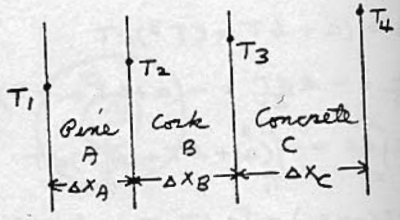
(7) $\frac{4\pi k(T_1-T_2)}{(1/r_1-1/r_2)} = \frac{4\pi k(T_1-T)}{(1/r_1-1/r)}$

Cancel like terms and rearrange.

(8) $\frac{T-T_1}{T_2-T_1} = \frac{1/r_1-1/r}{1/r_1-1/r_2} = \frac{\frac{r-r_1}{r_1 r}}{\frac{r_2-r_1}{r_1 r_2}} = r_2\frac{\left(1-\frac{r_1}{r}\right)}{r_2-r_1}$

$$\boxed{\frac{T-T_1}{T_2-T_1} = \frac{r_2}{(r_2-r_1)}\left(1-\frac{r_1}{r}\right)}$$

(4.3-1)



$A = 39.0\ m^2$

$T_1 = -17.8°C \quad T_4 = 29.4°C$

$\Delta x_A = 0.0191\ m \quad \Delta x_B = ?$

$\Delta x_C = 0.0508\ m$

$k_A = 0.151\ W/m\cdot K \quad k_B = 0.0433 \quad k_C = 0.762$

Eq.(4.3-3)

$$R_A = \frac{\Delta x_A}{k_A A} = \frac{0.0191}{(0.151)(39.0)} = 0.003243\ K/W$$

$$R_B = \frac{\Delta x_B}{k_B A} = \frac{\Delta x_B}{(0.0433)(39.0)}\ K/W$$

$$R_C = \frac{\Delta x_C}{k_C A} = \frac{0.0508}{(0.762)(39.0)} = 0.001709\ K/W$$

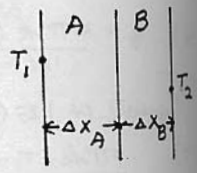
$$q = \frac{T_1 - T_4}{R_A + R_B + R_C} = \frac{-17.8 - 29.4}{0.003243 + R_B + 0.001709} = 586\ W$$

$$R_B = 0.075596 = \frac{\Delta x_B}{(0.0433)(39.0)}\ K/W$$

$\Delta x_B = 0.1277\ m$

$$\boxed{\Delta x_B = 0.128\ m \quad (0.419\ ft)}$$

(4.3-2)



$\Delta x_A = 0.244\ m \quad k_A = 1.30\ W/m\cdot K$

$k_B = 0.346\ W/m\cdot K \quad T_1 = 1588\ K \quad T_2 = 299\ K$

$q/A = 1830\ W/m^2 \quad A = 1.0\ m^2$

Eq.(4.3-3)

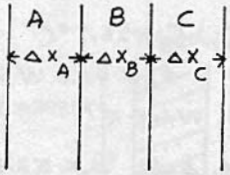
$$R_A = \frac{\Delta x_A}{k_A A} = \frac{0.244}{1.30\,(1.0)} = 0.1877\ K/W$$

$$R_B = \frac{\Delta x_B}{k_B A} = \frac{\Delta x_B}{0.346\,(1.0)} = 2.890\,\Delta x_B\ K/W$$

$$q = 1830 = \frac{T_1 - T_2}{R_A + R_B} = \frac{1588 - 299}{0.1877 + 2.890\,\Delta x_B}$$

$$\boxed{\Delta x_B = 0.179\ m}$$

(4.3-3)



$\Delta x_A = 6.35/1000 = 0.00635\ m$

$\Delta x_B = 0.00635\ m$

$\Delta x_C = 0.00635\ m$

$A = 0.914 \times 1.83 = 1.673\ m^2$

$k_A = k_C = 0.869\ W/m \cdot K$

$k_B = 0.026\ W/m \cdot K$

$\Delta T = 27.8\ K = 27.8\ °C$

Eq.(4.3-3)

$$R_A = \frac{\Delta x_A}{k_A A} = \frac{0.00635}{0.869(1.673)} = 0.004368\ K/W$$

$$R_B = \frac{\Delta x_B}{k_B A} = \frac{0.00635}{0.026(1.673)} = 0.145984$$

$$R_C = \frac{\Delta x_C}{k_C A} = \frac{0.00635}{0.869(1.673)} = 0.004368$$

$$q = \frac{\Delta T}{R_A + R_B + R_C}$$

$$= \frac{27.8}{0.004368 + 0.145984 + 0.004368}$$

$$\boxed{q = 179.7\ W}$$

(4.3-4)

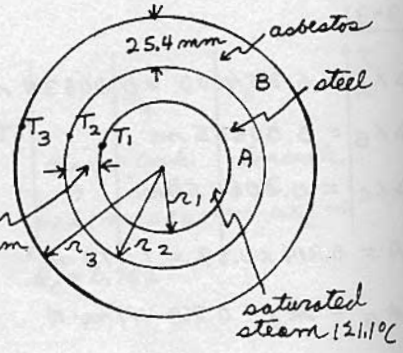
$T_1 = 121.1°C \quad T_3 = 26.7°C$

$L = 30.5 m \quad k_A = 45.0 \ W/m \cdot K$

$k_B = 0.182 \ W/m \cdot K$

A.5 2 in. pipe $D_1 = 52.50$ mm

wall thickness $= 3.91$ mm



$r_2 - r_1 = 3.91/1000 = 0.00391 \ m$

$r_3 - r_2 = 25.4/1000 = 0.0254 \ m$

$D_1 = 52.50/1000 = 0.0525 \ m$

$D_2 = 52.50 + 2(3.91) = 60.32 \ mm = 0.06032 \ m$

$D_3 = 60.32 + 2(25.4) = 111.12 \ mm = 0.11112 \ m$

$A_1 = \pi D_1 L = \pi(0.0525)(30.5) = 5.030 \ m^2$

$A_2 = \pi D_2 L = \pi(0.06032)(30.5) = 5.780$

$A_3 = \pi D_3 L = \pi(0.11112)(30.5) = 10.647$

Eq.(4.3-6)

$$A_{Alm} = \frac{A_2 - A_1}{\ln A_2/A_1} = \frac{5.780 - 5.030}{\ln 5.780/5.030} = 5.396 \ m^2$$

$$A_{Blm} = \frac{A_3 - A_2}{\ln A_3/A_2} = \frac{10.647 - 5.780}{\ln 10.647/5.780} = 7.967 \ m^2$$

Eq.(4.3-7)

$$R_A = \frac{r_2 - r_1}{k_A A_{Alm}} = \frac{0.00391}{45.0(5.396)} = 0.00001610 \ K/W$$

$$R_B = \frac{r_3 - r_2}{k_B A_{Blm}} = \frac{0.0254}{0.182(7.967)} = 0.017517 \ K/W$$

$$q = \frac{T_1 - T_3}{R_A + R_B} = \frac{121.1 - 26.7}{0.00001610 + 0.017517} = \boxed{5384 \ W}$$

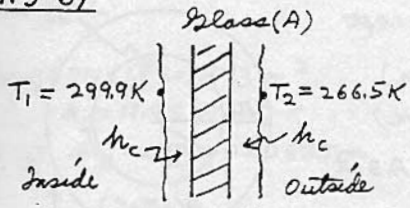
Steam tables A.2

By interpolation, $h = 2199.5 \ kJ/kg$ steam

$$5384 \ \frac{J}{s} = \left(2199.5 \times 10^3 \ \frac{J}{kg}\right)\left(m \ \frac{kg}{h}\right)\left(\frac{1}{3600 \ s/h}\right)$$

$$m = \boxed{8.81 \ kg/h}$$

(4.3-6)



$h_c = 8.52\ W/m^2 \cdot K$

$A_A = 0.557\ m^2$

$\Delta x_A = 0.00318\ m$

$k_A = 0.692\ W/m \cdot K$

$$R_A = \frac{\Delta x_A}{k_A A_A} = \frac{0.00318}{(0.692)(0.557)}$$

$$= 0.00825\ K/W$$

$$R_{A\,conv} = 2\frac{1}{h_c A_A} = \frac{2}{(8.5)(0.557)}$$

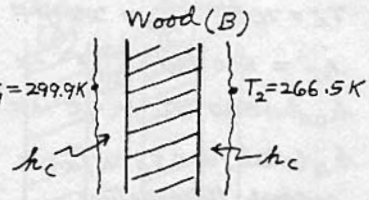
$$= 0.4224\ K/W$$

Eq (4.3-12)

$$q_{glass} = \frac{T_1 - T_2}{R_A + R_{A\,conv}}$$

$$= \frac{299.9 - 266.5}{0.00825 + 0.4224}$$

$$\boxed{q_{glass} = 77.6\,W \quad (265\ btu/h)}$$



$h_c = 8.52\ W/m^2 \cdot K$

$A_B = 2.44 \times 3.05 - 0.557 = 6.885\ m^2$

$\Delta x_B = 0.0254\ m$

$k_B = 0.1505\ W/m \cdot K$

$$R_B = \frac{\Delta x_B}{k_B A_B} = \frac{0.0254}{(0.1505)(6.885)}$$

$$= 0.02451\ K/W$$

$$R_{B\,conv} = 2\frac{1}{h_c A_B} = \frac{2}{(8.5)(6.885)}$$

$$= 0.03417\ K/W$$

Eq (4.3-12)

$$q_{wood} = \frac{T_1 - T_2}{R_B + R_{B\,conv}}$$

$$= \frac{299.9 - 266.5}{0.02451 + 0.03417}$$

$$\boxed{q_{wood} = 569.2\,W \quad (1942\ btu)}$$

$$\boxed{Total = 646.8\,W \quad (2207\ btu/h)}$$

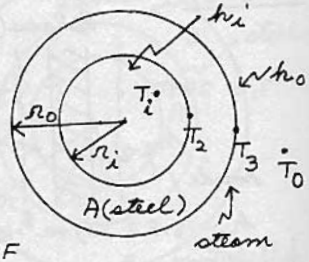
(4.3-8)

$T_i = 70°F \quad T_o = 220°F \quad L = 1.0\ ft$

$h_i = 500\ btu/h \cdot ft^2 \cdot °F$ (water)

$h_o = 1500\ btu/h \cdot ft^2 \cdot °F$ (steam)

k_A (steel) $= 45.1\ W/m \cdot K$ from A.3 at average temperature.



$$k_A = 45.1 \times \frac{1}{1.73073} = 26.06\ btu/h \cdot ft \cdot °F$$

A.1

A.5 For 2 in. pipe, $D_i = 2.067$ in. $\quad D_o = 2.375$ in.

$r_i = 2.067/2 = 1.0335$ in. $\quad r_o = 2.375/2 = 1.1875$ in.

$$A_i = 2\pi(L) r_i = 2\pi(1.0)\left(\frac{1.0355}{12}\right) = 0.5411\ ft^2$$

$$A_o = 2\pi(1.0)\left(\frac{1.1875}{12}\right) = 0.6218\ ft^2$$

Eq.(4.3-6) $\quad A_{A\,lm} = \dfrac{A_o - A_i}{\ln(A_o/A_i)} = \dfrac{0.6218 - 0.5411}{\ln(0.6218/0.5411)} = 0.5805\ ft^2$

Eq.(4.3-12)

$$R_i = \frac{1}{h_i A_i} = \frac{1}{500(0.5411)} = 0.003696$$

$$R_A = \frac{r_o - r_i}{k_A A_{A\,lm}} = \frac{1.1875 - 1.0335}{26.06(0.5805)12} = 0.0008483$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{1500(0.6218)} = 0.0010722$$

(a) $$q = \frac{T_i - T_o}{R_i + R_A + R_o} = \frac{220 - 270}{0.003696 + 0.0008483 + 0.0010722} = \frac{220 - 270}{0.005616}$$

$$\boxed{q = 26\,710\ btu/h \quad (7.828\ kW)}$$

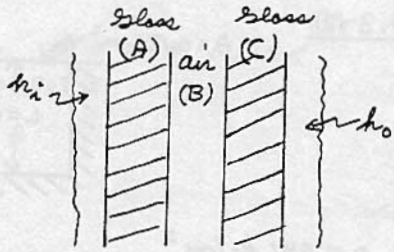
(b) Eq.(4.3-16)

$$q = U_o A_o (T_i - T_o) = U_i A_i (T_i - T_o) = \frac{(T_i - T_o)}{\Sigma R}$$

$$U_i = \frac{1}{A_i \Sigma R} = \frac{1}{0.5411(0.005616)} = \boxed{329.1\ btu/h \cdot ft^2 \cdot °F \quad (1869\ W/m^2 \cdot K)}$$

(c) $$U_o = \frac{1}{A_o \Sigma R} = \frac{1}{(0.6218)(0.005616)} = \boxed{286.4\ btu/h \cdot ft^2 \cdot °F \quad (1626\ W/m^2 \cdot K)}$$

(4.3-10)



$A = 0.914 \times 1.83 = 1.673\ m^2$

$h_i = h_o = 11.35\ W/m^2 \cdot K$

$\Delta x_A = \Delta x_B = \Delta x_C = 0.00635\ m$

$k_A = k_C = 0.869\ W/m \cdot K$

$k_B = 0.026\ W/m \cdot K$

$\Delta T_{overall} = 27.8\ K$

$$R_{fi} = \frac{1}{h_i A_i} = \frac{1}{(11.35)(1.673)} = 0.052663\ K/W$$

$$R_A = \frac{\Delta x_A}{k_A A} = \frac{0.00635}{(0.869)(1.673)} = 0.004368$$

$$R_B = \frac{\Delta x_B}{k_B A} = \frac{0.00635}{(0.026)(1.673)} = 0.145984$$

$$R_C = \frac{\Delta x_C}{k_C A} = \frac{0.00635}{(0.869)(1.673)} = 0.004368$$

$$R_{fo} = \frac{1}{h_o A_o} = \frac{1}{(11.35)(1.673)} = 0.052663$$

$$\Sigma R = 0.260046\ K/W$$

Eq.(4.3-12)

$$q = \frac{\Delta T}{\Sigma R} = \frac{27.8}{0.260046} = \boxed{106.7\ W}$$

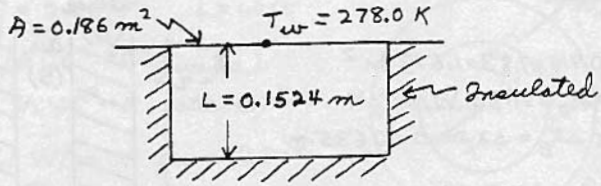
Eq.(4.3-13)

$$q = UA\Delta T = \frac{\Delta T}{\Sigma R}$$

$$U = \frac{1}{A \Sigma R} = \frac{1}{(1.673)(0.260046)}$$

$$\boxed{U = 2.29\ W/m^2 \cdot K \quad (0.403\ Btu/h \cdot ft^2 \cdot {}^\circ F)}$$

(4.3-12)



$\rho = 641\ kg/m^3$

$k = 0.346\ W/m \cdot K = 0.346\ J/s \cdot m \cdot K$

$$\dot{q} = 0.070 \frac{kJ}{kg \cdot h} \left(641 \frac{kg}{m^3}\right) = 44.87 \frac{kJ}{h \cdot m^3}$$

Eq. (4.3-27)

$$T_0 = \frac{\dot{q} L^2}{2k} + T_w = \frac{(44.87 \times 1000)(0.1524)^2}{(2)(0.346)(3600)} + 278.0$$

$$T_0 = 0.418 + 278.0$$

$$\boxed{T_0 = 278.42\ K}$$

Eq. (4.3-28)

$$\dot{q}_T = \dot{q}\ (volume)$$

$$\dot{q}_T = \frac{(44.87 \times 1000)}{3600}(0.1524 \times 0.186)$$

$$\boxed{\dot{q}_T = 0.353\ W}$$

(1.22 btu/h)

(4.3-13)

$D = 0.00508\,m \qquad L = 2.44\,m \qquad k = 22.5\ W/m\cdot K$

$T_W = 427.6\ K \qquad R = \frac{D}{2} = 0.00508/2$

$R = 0.0843$ ohms $\qquad I = 200\ A$

Eq.(4.3-31)

(1) Watts $= I^2R = (250)^2(0.0843)\ W$

(2) Watts $= \dot{q}\pi\frac{D^2}{4}L = \dot{q}\pi\frac{(0.00508)^2}{4}(2.44)$

Equate (1) to (2) and solve for $\dot{q}$.

$(250)^2(0.0843) = \dot{q}\pi\frac{(0.00508)^2}{4}(2.44)$

$\dot{q} = 10.65\times10^7\ W/m^3$

Eq.(4.3-30)

$T_0 = \frac{\dot{q}R^2}{4k} + T_W = \frac{10.65\times10^7}{4(22.5)}\frac{(0.00508)^2}{4} + 427.6$

$\boxed{T_0 = 435.23\ K}$

(4.3-14)

(a) $k = 0.08\ W/m\cdot K \qquad h_0 = 30\ W/m^2\cdot K$

Eq.(4.3-35)

$(r_2)_{cr} = \frac{k}{h_0} = \frac{0.08}{30} = \boxed{2.667\times10^{-3}\,m = 2.667\,mm}$

$r_2(pipe) = \frac{30\ mm}{2\times1000\ mm/m} = 1.50\times10^{-2}\,m$

$A = 2\pi r_2 L = 2(\pi)(1.50\times10^{-2})(1.0) \qquad T_2 = 400\ K \quad T_0 = 300\ K$

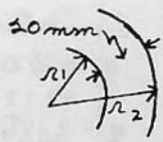
Eq.(4.3-32)

$q = h_0 A(T_2 - T_0) = 30(2)(\pi)(1.50\times10^{-2})(1.0)(400-300) = \boxed{282.7\ W}$

(b)

$r_1 = \frac{30}{2}\ mm = 1.50\times10^{-2}\,m$

$r_2 = \frac{30}{2} + 20 = 35\ mm = 3.5\times10^{-2}\,m$



Eq.4.3-33

$q = \frac{2\pi L(T_1 - T_0)}{\frac{\ln(r_2/r_1)}{k} + \frac{1}{r_2 h_0}} = \frac{2\pi(1.0)(400-300)}{\ln(3.5\times10^{-2}/1.5\times10^{-2}) + 1/(3.5\times10^{-2}\times30)}$

$\boxed{q = 54.4\ W}$

(4.4-1)

$N = 5$ $\quad k = 0.9\ W/m \cdot K$ $\quad L = 5.0\,m$

(a) From plot below, $M = 11.5$ $\quad T_1 = 600\,K$ $\quad T_2 = 400\,K$

Eq (4.4-4)

$$q = \frac{M}{N} k (T_1 - T_2)$$

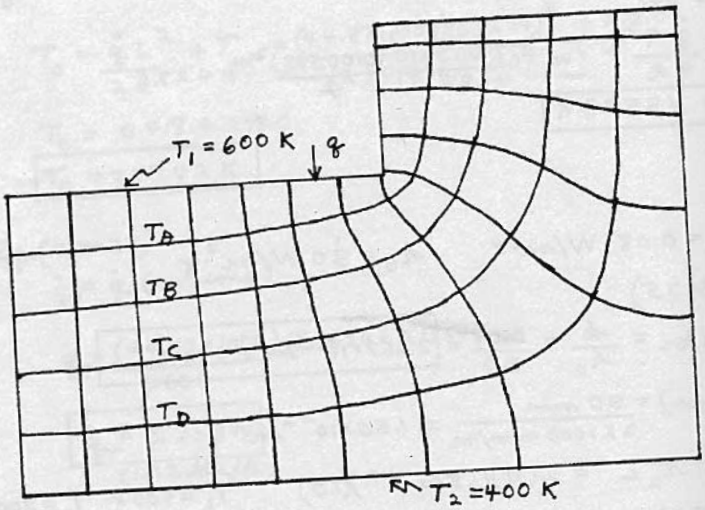
For 4 identical sections and $L = 5.0\,m$

$$q = (\overset{\text{sections}}{4})(\underset{L}{5.0})\left(\frac{11.5}{5}\right)(0.9)(600-400)$$

Eq (4.4-5)

$$S = \frac{M}{N} = \frac{11.5}{5} = \boxed{2.30}$$

$$\boxed{q = 8280\ W}$$



(b) Derivation of equations.

$q' = k\,\Delta T$ $\quad q = \frac{M}{N} k (T_1 - T_2)$ $\quad \frac{q}{M} = q'$ $\quad k$ constant

$$q' = k_m \frac{(T_1 - T_2)}{N} = \frac{q}{M} = \frac{1}{M} \cdot \frac{M}{N} k (T_1 - T_2) = k \frac{(T_1 - T_2)}{N}$$

$$k_m = \text{average } k = k_0 \left[1 + a\left(\frac{T_1 + T_2}{2}\right)\right]$$

(4.4-1) - p.2

$$k = 0.85[1 + 0.0004T]$$

$$k_m = 0.85\left[1 + 0.0004\left(\frac{600+400}{2}\right)\right] = 1.020$$

For 1 lane, 1 section, and $L = 1.0\ m$,

$$q = 11.5\left(\frac{1.020}{5}\right)(600-400) = 469.20\ W$$

$$q' = \frac{q}{M} = \frac{469.20}{11.5} = 40.80\ W$$

Solve for $T_1 (600) \rightarrow T_A$

$$q' = k_m \Delta T = 40.80 = 0.85\left[1 + 0.0004\left(\frac{600+T_A}{2}\right)\right](600 - T_A)$$

Rearranging and solving for T_A,

$$-1.70\times10^{-4} T_A^2 - 0.850 T_A + 530.40 = 0$$

$$T_A = \frac{+0.850 - \sqrt{(0.850)^2 - (4)(-1.70\times10^{-4})(530.40)}}{2(-1.70\times10^{-4})} = \boxed{561.03\ K}$$

Solve for T_B

$$40.80 = 0.85\left[1 + 0.0004\left(\frac{561.03+T_B}{2}\right)\right](561.03 - T_B)$$

Solving, $\boxed{T_B = 521.56\ K}$

Solve for T_C

$$40.80 = 0.85\left[1 + 0.0004\left(\frac{521.56+T_C}{2}\right)\right](521.56 - T_C)$$

$$\boxed{T_C = 481.59\ K}$$

Solve for T_D

$$40.80 = 0.85\left[1 + 0.0004\left(\frac{481.59+T_D}{2}\right)\right](481.59 - T_D)$$

$$\boxed{T_D = 441.06\ K}$$

Overall q

$$q = 4L\frac{M}{N}k_m(T_1 - T_2)$$

$$= 4(5)\left(\frac{11.5}{5}\right)(1.020)(600-400)$$

$$\boxed{q = 9384\ W}$$

Isotherm	Part (a) Const. k	Part (b) Variable k
T_1	600 K	600 K
T_A	560	561.03
T_B	520	521.56
T_C	480	481.59
T_D	440	441.06
T_2	400	400

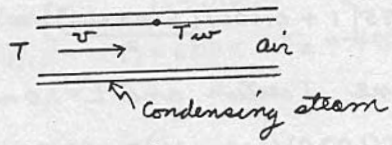
(4.5-1)

$D_i = 0.0381\ m$

$v = 6.71\ m/s$

$P = 138\ kPa\ (20\ psia)$

$T_w = 204.4°C = 477.6\ K$



$T = 449.9\ K$

A.3

Properties of air at mean bulk $T = 449.9\ K$

$\mu_b = 2.50 \times 10^{-5}\ Pa \cdot s \qquad k = 0.03721\ W/m \cdot K$

$N_{Pr} = 0.687 \qquad \rho = 0.785\left(\frac{138}{101.32}\right) = 1.069\ kg/m^3$

$\mu_w\ (\text{at } T_w = 477.6\ K) = 2.60 \times 10^{-5}\ Pa \cdot s$

$$N_{Re} = \frac{D v \rho}{\mu_b} = \frac{(0.0381)(6.71)(1.069)}{(2.50 \times 10^{-5})} = 10{,}903$$

Eq. (4.5-8)

$$\frac{h D}{k} = 0.023\, N_{Re}^{0.8}\, N_{Pr}^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$\frac{h(0.0381)}{(0.03721)} = 0.023(10{,}903)^{0.8}(0.687)^{1/3}\left(\frac{2.50 \times 10^{-5}}{2.60 \times 10^{-5}}\right)^{0.14}$$

$$\boxed{h = 33.45\ W/m^2 \cdot K \quad (5.89\ Btu/h \cdot ft^2 \cdot °F)}$$

(Zept has error of 33.78)

Eq. (4.5-1)

$$\frac{q}{A} = h(T_w - T) = 33.45(477.6 - 449.9)$$

$$\boxed{q/A = 927\ W/m^2}$$

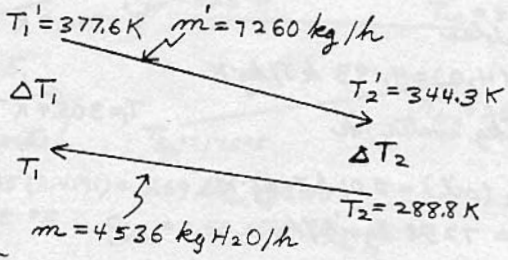
(4.5-3)

$c'_{pm} = 2.85\ kJ/kg\cdot K$

$c_{pm}(H_2O) = 4.181$

(A.2-5)

$U_0 = 653\ W/m^2\cdot K$



(a) Countercurrent flow

$$q' = m' c'_{pm}(T_1' - T_2') = 7260(2.85)(377.6 - 344.3)$$
$$= 689\ 010\ kJ/h$$

$$q' = q = m c_{pm}(T_1 - T_2) = 4536(4.181)(T_1 - 288.8) = 689{,}010$$

$$\boxed{T_1 = 325.2\ K}$$

Eq. (4.5-27)

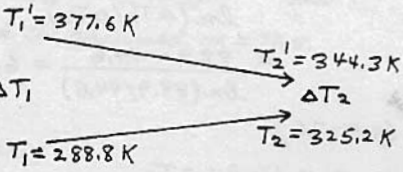
$$\Delta T_2 = 344.3 - 288.8 = 55.5\ K$$
$$\Delta T_1 = 377.6 - 325.2 = 52.4\ K$$
$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = \frac{55.5 - 52.4}{\ln(55.5/52.4)} = 53.9\ K$$

Eq. (4.5-26)

$$q = U_0 A_0 \Delta T_{lm} = (653) A_0 (53.9) = \frac{689\ 010 \times 10^3}{(3600)}$$

$$\boxed{A_0 = 5.43\ m^2}$$

(b) Cocurrent flow



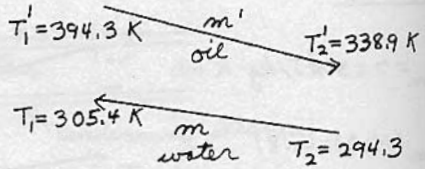
$$\Delta T_1 = 377.6 - 288.8 = 88.8\ K$$
$$\Delta T_2 = 344.3 - 325.2 = 19.1\ K$$

$$\Delta T_{lm} = \frac{88.8 - 19.1}{\ln(88.8/19.1)} = 45.35\ K$$

$$q = 653(A_0)(45.35) = \frac{689\ 010 \times 10^3}{(3600)}$$

$$\boxed{A_0 = 6.46\ m^2}$$

(4.5-5)



A.3 Tables

$c_{pm}(H_2O) = 4.183$ kJ/kg·K

m = kg water/h

c'_{pm} (oil) = 2.01 kJ/kg·K

m' = 7258 kg oil/h

$A_i = 5.11\ m^2$

Eq. (4.5-2)

$$q = m'\, c'_{pm}(T_1' - T_2')$$

$$= 7258(2.01)(394.3 - 338.9)$$

$$= 8.082 \times 10^5 \text{ kJ/h}$$

$$q = m\, c_{pm}(T_1 - T_2)$$

$$8.082 \times 10^5 = m(4.183)(305.4 - 294.3)$$

$$\boxed{m = 17\,420 \text{ kg } H_2O/h}$$

Eq. (4.5-27)

$$\Delta T_1 = 394.3 - 305.4 = 88.9 \text{ K}$$

$$\Delta T_2 = 338.9 - 294.3 = 44.6 \text{ K}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$= \frac{88.9 - 44.6}{\ln(88.9/44.6)} = 64.22 \text{ K}$$

Eq (4.5-26)

$$q = U_i A_i' \Delta T_{lm}$$

$$\frac{8.082 \times 10^5 (10^3)}{3600} \frac{J}{s} = U_i (5.11)(64.22)$$

$$\boxed{U_i = 686 \text{ W/m}^2\cdot\text{K}}$$

(4.5-6)

Similar to Ex. 4.5-5.

Oil physical properties

$T_w = 325°F$, $\mu_w = 2.35\,(2.4191) = 5.69\ lb_m/ft\cdot h$

$k_m = 0.083\ btu/h\cdot ft\cdot °F$ $\quad c_{pm} = 0.50\ btu/lb_m\cdot ft\cdot °F$

$T_b = \frac{250+175}{2} = 212.5°F$ $\quad \mu_b = 4.74\,(2.4191) = 11.50\ lb_m/ft\cdot h$

(From plot of μ vs. T°F)

$D_i = 0.0303\ ft$ $\quad L = 15\ ft$

$A = \frac{\pi D_i^2}{4} = \frac{\pi}{4}(0.0303)^2 = 0.000722\ ft^2$ $\quad N_{Pr} = \frac{c_p \mu}{k} = \frac{(0.50)(11.5)}{0.083} = 69.3$

$A_{surf} = \pi D_i L = \pi(0.0303)(15) = 1.4279\ ft^2$

Trial and error solution

Trial 1 $\quad$ assume $m = 77.5\ lb_m/h$

$G = \frac{77.5}{0.000722} = 107{,}208\ \frac{lb_m}{ft^2\cdot h}$ $\quad N_{Re} = \frac{D_i G}{\mu_b} = \frac{(0.0303)(107{,}208)}{11.50} = 282.9$

Eq. (4.5-4) $\quad h_a = \frac{k}{D}(1.86)\left(N_{Re} N_{Pr} \frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$

$h_a = \frac{0.083}{0.0303}(1.86)\left(282.9 \times 69.3 \times \frac{0.0303}{15}\right)^{1/3}\left(\frac{11.50}{5.69}\right)^{0.14} = 19.173\ btu/ft^2\cdot h\cdot °F$

Eq. (4.5-28) $\quad q = m\, c_{pm}(T_{bo} - T_{bi}) = m(0.50)(250-175) = 37.50\, m$

Eq. (4.5-7) $\quad \Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2} = \frac{(325-175)+(325-250)}{2} = 112.5°F$

$q = 37.50(m) = h_a A \Delta T_a = (19.173)(1.4279)(112.5)$

$m = 82.13$ This does not check the assumed $m = 77.5$.

Trial 2 $\quad$ assume $m = 83.6$

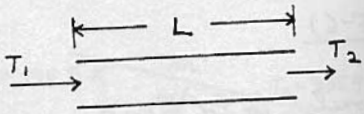
$h_a = 19.65$

$q = 37.5(m) = (19.65)(1.4279)(112.5)$

$m = 84.2\ lb_m/h$ (close enough)

$$\boxed{m = 84.2\ lb_m/h \quad (38.2\ kg/h)}$$

(4.5-8)



$D = \frac{35}{1000} = 0.035\ m$

$m = 2.0\ kg/s \qquad T_1 = 425\ °C$

$T_2 = 430°C \qquad k = 15.6\ W/m\cdot K \qquad C_p = 149\ J/kg\cdot K$

$\mu = 1.34 \times 10^{-3}\ Pa\cdot s$

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.035)^2}{4} = 9.621 \times 10^{-4}\ m^2$$

$$G = \frac{m}{A} = \frac{2.0}{9.621 \times 10^{-4}} = 2.079 \times 10^3\ kg/m^2\cdot s$$

$$N_{Re} = \frac{DG}{\mu} = \frac{0.035(2.079 \times 10^3)}{1.34 \times 10^{-3}} = 5.430 \times 10^4$$

$$N_{Pr} = \frac{C_p \mu}{k} = \frac{149(1.34 \times 10^{-3})}{15.6} = 0.01280$$

Eq.(4.5-14)

$$h_L = \frac{k}{D}(0.625) N_{Pe}^{0.4}$$

$$N_{Pe} = N_{Re} N_{Pr} = (5.430 \times 10^4)(0.01280)$$

$$h_L = \frac{15.6}{0.035}(0.625)[(5.430 \times 10^4)(0.0128)]^{0.4}$$

$$h_L = 3817\ W/m^2\cdot K$$

Eq.(4.5-16)

$$q = m C_p \Delta T = 2.0(149)(430-425) = 1490\ W$$

Eq.(4.5-1)

$$\frac{q}{A} = \frac{1490}{A} = h_L(T_w - T) = (3817)(25)$$

$$A = 0.01561\ m^2 = \pi D L = \pi(0.035)(L)$$

$$\boxed{L = 0.1420\ m}$$

(4.6-1)

air

$P = 101.3\ kPa$

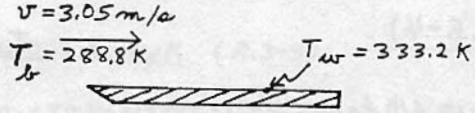
A.3-3

$\rho = 1.137\ kg/m^3\ (311.0K)$

$k = 0.02700\ W/m \cdot K$

$N_{Pr} = 0.705$

$\mu = 1.90 \times 10^{-5}\ Pa \cdot s$



$$T_f = \frac{T_b + T_w}{2} = \frac{288.8 + 333.2}{2} = 311.0\ K$$

$$N_{Re,L} = \frac{L v \rho}{\mu} = \frac{(0.305)(3.05)(1.137)}{1.90 \times 10^{-5}} = 5.567 \times 10^4$$

Eq.(4.6-2)

$$\frac{hL}{k} = 0.664\, N_{Re,L}^{0.5}\, N_{Pr}^{1/3}$$

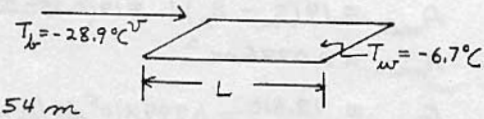
$$h = \frac{(0.02700)}{(0.305)}(0.664)(5.567 \times 10^4)^{0.5}\,(0.705)^{1/3}$$

$$\boxed{h = 12.35\ W/m^2 \cdot K}$$

(4.6-2)

$v = 0.61\ m/s \quad P = 1\ atm$

A.3 (air)



$L = 0.254\ m$

$$t_f = \frac{T_b + T_w}{2} = \frac{-28.9 + (-6.7)}{2} = -17.8°C \qquad \rho = 1.379\ kg/m^3$$

$k = 0.0225\ W/m \cdot K \qquad N_{Pr} = 0.720 \qquad \mu = 1.62 \times 10^{-5}\ kg/m \cdot s$

$$N_{Re,L} = \frac{L v \rho}{\mu} = \frac{(0.254)(0.61)(1.379)}{1.62 \times 10^{-5}} = 1.319 \times 10^4$$

Eq.(4.6-2)

$$h = \frac{k}{L}(0.664)\, N_{Re,L}^{0.5}\, N_{Pr}^{1/3}$$

$$= \frac{0.0225}{0.254}(0.664)(1.319 \times 10^4)^{0.5}(0.720)^{1/3}$$

$$\boxed{h = 6.05\ W/m^2 \cdot K}$$

(4.6-4)

$T_w = 93.33°C$

$m = 13\,610$ kg air/h

$P = 1.0$ atm

$D_t = 0.0127\,m = 0.50$ in.

$L = 0.61\,m$ long tubes

$S_p = S_n = 0.75$ in

$T_{b\,out}$

$T_{b\,in} = 15.6°C$

Tubes in-line, square pattern.

6 transverse rows in direction of flow

Number of tubes = 19×6

Trial and error solution

Trial 1 Assume $T_{b\,out} = 31.1°C$

$$T_{b\,av} = \frac{15.6 + 31.1}{2} = 23.35°C \qquad T_f = \frac{T_w + T_b}{2} = \frac{93.33 + 23.35}{2}$$

$$T_f = 58.34°C$$

Table A.3-3 (58.34°C)

$\mu_f = 2.00 \times 10^{-5}$ Pa·s $\quad k = 0.02866$ W/m·K

$N_{Pr} = 0.703$

$$A_{min} = 19(S_n - D_t)L = 19(0.75 - 0.50)\text{in.}\left(\frac{1}{12 \times 3.2808}\frac{m}{in}\right)(0.61\,m)$$

$A_{min} = 0.0736\,m^2$

$$G_{max} = \frac{13\,610}{0.0736} = 1.849 \times 10^5\ kg/h \cdot m^2 = \frac{1.849 \times 10^5}{3600} = 51.36\ kg/s \cdot m^2$$

$$N_{Re} = \frac{D_t G_{max}}{\mu} = \frac{(0.0127)(51.36)}{(2.00 \times 10^{-5})} = 32\,610$$

$S_p/D_t = 0.75/0.5 = 1.5 = S_n/D_t$

Table 4.6-2 $\quad C = 0.278,\ m = 0.620$

Eq. (4.6-1)

$$h = \frac{k}{D} C N_{Re}^{m} N_{Pr}^{1/3} = \frac{0.02866}{0.0127}(0.278)(32\,610)^{0.62}(0.703)^{1/3}$$

$h = 350.5\ W/m^2 \cdot K$

From Table 4.6-3, for 6 transverse tubes in line, use $0.94h$.

(4.6-4) - p.2

At $T_{bav} = 23.35\,°C$, $c_p = 1.0048\ kJ/kg\cdot K$ (A.3-3)

$A_{tubes} = (19\times 6)(\pi)(D)(L) = (19\times 6)(\pi)(0.0127)(0.61) = 2.774\ m^2$

$q = (0.94h)\,A\,(T_w - T_b) = (0.94)(350.5)(2.774)(93.3-23.35)$

$q = 63\,931\ W$

$q = 63\,931\ \frac{J}{s} = m\,c_p\,\Delta T = \frac{13\,610}{3600}(1.0048)(\Delta T)10^3$

$\Delta T = 16.91\ K$

$T_{bout} = 15.6 + 16.91 = 32.51\,°C$

Trial 2 assume $T_{bout} = 32.5\,°C$

Physical properties will not change significantly from 31.1 to 32.5 °C.

$T_{bav} = \frac{15.6+32.5}{2} = 24.05\,°C$ $\quad T_f = \frac{93.33+24.05}{2} = 58.69\,°C$

h will remain approximately constant

$q = (0.94)(350.5)(2.774)(93.3-24.05) = 63\,291\ W$

$q = 63\,291 = \frac{13\,610}{3600}(1.0048)(\Delta T)10^3$

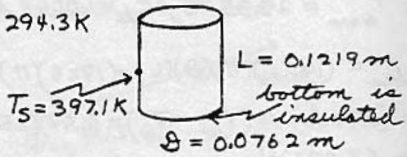
$\Delta T = 16.66\ K$

$T_{bout} = 15.6 + 16.66 = \boxed{32.26\,°C}$

(4.7-2)

$T = 294.3K$

$\Delta T = (397.1 - 294.3) = 102.8K$



$A_{side} = \pi D L = \pi (0.0762) = 0.02918 m^2$

$A_{top} = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.0762) = 0.004560 m^2$

$L^3 \Delta T = (0.1219)^3 (102.8) = 0.186$. This is < 4.7. Hence, can use for $N_{Gr} N_{Pr}$ of 10^4 to 109 (see text)

Table 4.7-2

For the top, the equivalent L to use is $0.9 \times D_{cyl}$

$L = 0.9(0.0762) = 0.06858 m$

$$h_{top} = 1.32\left(\frac{\Delta T}{L}\right)^{1/4} = 1.32\left(\frac{102.8}{0.06858}\right)^{1/4} = 8.213 W/m^2 \cdot K$$

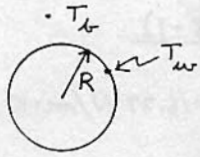
$$h_{side} = 1.37\left(\frac{\Delta T}{L}\right)^{1/4} = 1.37\left(\frac{102.8}{0.1219}\right)^{1/4} = 7.383 W/m^2 \cdot K$$

$q_{top} = hA\Delta T = (8.213)(0.004560)(102.8) = 3.850 W$

$q_{side} = hA\Delta T = (7.383)(0.02918)(102.8) = 22.147 W$

$$\boxed{q_{total} = 26.0 W (88.6 btu/h)}$$

(4.7-4)



$T_w = 21.4\,°C \qquad T_b = 4.4\,°C$

$D = 0.102\ m \qquad R = 0.051\ m$

Can use (M1) $L = R = 0.051\ m$ and flat plate correlation (see text).

$\Delta T = T_w - T_b = 21.1 - 4.4 = 16.7\,°C = 16.7\ K$

$L^3 \Delta T = (0.051)^3 (16.7) = 2.22 \times 10^{-3}$

Hence, can use Table 4.7-2.

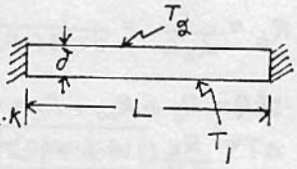
$h = 1.37(\Delta T / L)$

$= 1.37(16.7/0.051)^{1/4} = 5.828\ W/m^2 \cdot K$

$A = 4\pi R^2 = 4\pi(0.05)^2 = 0.03268\ m^2$

$q = hA\Delta T = 5.828(0.03268)(16.7) = \boxed{3.181\ W}$

(4.7-5)



$T_1 = 394.3\ K \qquad T_2 = 366.5\ K$

$\delta = 0.030\ m \quad L = 0.4\ m \quad k = 0.03219\ W/m \cdot K$

From Example 4.7-3,

$$N_{Gr,\delta} N_{Pr} = 2.372 \times 10^4$$

Eq. (4.7-13)

$$h = \frac{k}{\delta}(0.061)(N_{Gr,\delta} N_{Pr})^{1/3}$$

$$h = \frac{(0.03219)}{(0.030)}(0.061)\left[2.372 \times 10^4\right]^{1/3}$$

$$h = 1.881\ W/m^2 \cdot K$$

$$A = (0.6 \times 0.4) = 0.24\ m^2$$

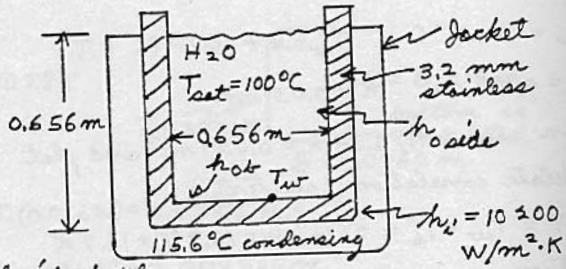
Eq (4.7-7)

$$q = hA\Delta T = 1.881(0.24)(394.3 - 366.5)$$

$$\boxed{q = 12.54\ W}$$

(4.8-1)

$k = 16.27\ W/m\cdot K$



Predict h for vertical side of kettle

Trial 1

Assume $T_w = 107.8°C$ $\quad \Delta T = T_w - T_{sat} = 107.8 - 100 = 7.8°C = 7.8\ K$

Eq.(4.8-4)

$$h_o = 7.95(\Delta T)^3 = 7.95(7.8)^3 = 3773\ W/m^2\cdot K$$

$$R_i = \frac{1}{h_i A} = \frac{1}{10{,}200(1.0)} = 9.80\times10^{-5}$$

$$R_w = \frac{\Delta x}{kA} = \frac{3.2/1000}{16.27(1.0)} = 19.66\times10^{-5}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{3773(1.0)} = 26.50\times10^{-5}$$

$$\Sigma R = R_i + R_w + R_o = 9.80\times10^{-5} + 19.66\times10^{-5} + 26.50\times10^{-5} = 55.96\times10^{-5}$$

$$\Delta T = \frac{R_o}{\Sigma R}(115.6-100) = \frac{26.50\times10^{-5}}{55.96\times10^{-5}}(15.6) = 7.39°C = 7.39\ K$$

Trial 2

Assume $T_w = 107.65$ $\quad \Delta T = 107.65 - 100 = 7.65\ K$

$$h_o = 7.95(7.65)^3 = \boxed{3560\ W/m^2\cdot K}$$

$$R_o = \frac{1}{3560(1.0)} = 28.09\times10^{-5} \qquad \Sigma R = 57.55\times10^{-5}$$

$$q/A = h_o\Delta T = 3560(7.65)/1000 = 27.23\ kW/m^2$$

Hence, Eq.(4.8-4) is valid to use.

$$\Delta T = \frac{28.09\times10^{-5}}{57.55\times10^{-5}}(15.6) = 7.62°K$$ Hence, 7.65 K is close enough!

$$A_{sides} = \pi DL = \pi(0.656)(0.656) = 1.3519\ m^2$$

$$q = \frac{\Sigma\Delta T}{\Sigma R}(A) = \frac{(115.6-100)(1.3519)}{(57.55\times10^{-5})1000} = \boxed{36.64\ kW}$$

(4.8-1) - p.1

Bottom of kettle h

From Eq. 4.8-1 (see Left), assume $T_w = 108.2°C$

$\Delta T = T_w - T_{sat} = 108.2 - 100 = 8.2°C = 8.2\,K$

Eq. (4.8-2)

$h_o = 5.56(\Delta T)^3 = 5.56(8.2)^3 = 3066\ W/m^2 \cdot K$

$R_{o\,btm} = \frac{1}{h_o A} = \frac{1}{(3066)(1.0)} = 32.62 \times 10^{-5}$

$R_i = 9.80 \times 10^{-5}$

$R_w = 19.66 \times 10^{-5}$

$\Sigma R = 32.62 \times 10^{-5} + 9.80 \times 10^{-5} + 19.66 \times 10^{-5} = 62.08 \times 10^{-5}$

$\Delta T = \frac{R_o}{\Sigma R}(115.6 - 100) = \frac{32.62 \times 10^{-5}}{62.08 \times 10^{-5}}(15.6) = 8.2°C$, Hence, checks.

$q/A = h_o \Delta T = \frac{3066}{1000}(8.2) = 25.14\ kW/m^2$

Hence, Eq. (4.8-2) can be used.

$A_{btm} = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.656)^2 = 0.3380\ m^2$

$q = \frac{\Sigma \Delta T}{\Sigma R}(A) = \frac{(115.6-100)(0.3380)}{(62.08 \times 10^{-5})(1000)} = \boxed{8.494\ kW}$

Total heat flow $= 36.64 + 8.49 = \boxed{45.13\ kW\ total}$

(4.8-3)

SI Units

$\Delta T = 3.33\ K \quad h_{fg} = 2.283\times10^6\ J/kg \quad \mu_\ell = 3.24\times10^{-4}\ Pa\cdot s$

$\rho_\ell = 966.7\ kg/m^3 \quad D = 0.0254\ m \quad L = 1.22\ m \quad k_\ell = 0.675\ W/m\cdot K$

Eq.(4.8-20)

$$N_{Nu} = 1.13\left(\frac{\rho_\ell^2 g h_{fg} L^3}{\mu_\ell k_\ell \Delta T}\right)^{1/4}$$

$$= 1.13\left[\frac{(966.7)^2(9.8066)(2.283\times10^6)(1.22)^3}{(3.24\times10^{-4})(0.675)(3.33)}\right]^{1/4} = 17\,060$$

$$N_{Nu} = 17\,060 = \frac{hL}{k_\ell} = \frac{h(1.22)}{0.675} \qquad \boxed{h = 9438\ W/m^2\cdot K}$$

Eq.(4.8-25)

$q = hA\Delta T = h_{fg}\dot{m} \qquad A = \pi DL = \pi(0.0254)(1.22) = 0.09735\ m^2$

$9438(0.09735)(3.33) = 2.284\times10^6(\dot{m}) \qquad \dot{m} = 1.340\times10^{-3}\ kg/s$

Eq.(4.8-21)

$$N_{Re} = \frac{4\dot{m}}{\pi D \mu_\ell} = \frac{4(1.340\times10^{-3})}{\pi(0.0254)(3.24\times10^{-4})} = \boxed{207.2}$$

English Units

$\Delta T = 6°F \quad h_{fg} = 982.3\ btu/lb_m \quad \mu_\ell = 0.784\ lb_m/ft\cdot h$

$\rho_\ell = 60.3\ lb_m/ft^3 \quad D = 1.0/12\ ft \quad L = 4.0\ ft \quad k_\ell = 0.390\ btu/ft\cdot h\cdot °F$

$$N_{Nu} = 1.13\left[\frac{(60.3)^2(32.174)(3600)^2(982.3)(4.0)^3}{(0.784)(0.390)(6)}\right] = 17060$$

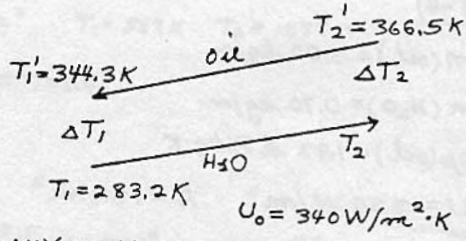
$$N_{Nu} = 17060 = \frac{hL}{k_\ell} = \frac{h(4.0)}{0.390} \qquad \boxed{h = 1663\ btu/h\cdot ft^2\cdot °F}$$

$q = hA\Delta T = h_{fg}\dot{m} \qquad A = \pi DL = \pi(1.0/12)(4.0) = 1.047\ ft^2$

$1663(1.047)(6.0) = 982.3(\dot{m}) \qquad \dot{m} = 10.64\ lb_m/h$

$$N_{Re} = \frac{4\dot{m}}{\pi D \mu_\ell} = \frac{4(10.64)}{\pi(1/12)(0.785)} = \boxed{207.2}$$ Laminar flow.

(4.9-2)



Oil heat balance

$m_{oil} = 5.04\ kg/s$

$c_{pm} = 2.09\ kJ/kg \cdot K$

$$q = m\, c_{pm}\,(T_2' - T_1') = (5.04)(2.09)(366.5 - 344.3)$$
$$= 233.85\ kJ/s$$

Water heat balance

$m_{H_2O} = 2.02\ kg/s$

$c_{pm} = 4.185\ kJ/kg \cdot K$

$$q = 233.85 \times 10^3 = (2.02)(4.185 \times 10^3)(T_2 - 283.2)$$
$$T_2 = 310.9\,K$$
$$\Delta T_2 = 366.5 - 310.9 = 55.6\,°K \qquad \Delta T_1 = 344.3 - 283.2 = 61.4\,K$$

Eq. (4.9-1)

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{61.4 - 55.6}{\ln(61.4/55.6)} = 58.31\,K$$

$T_{hi} = T_2' = 366.5\,K \qquad T_{ho} = T_1' = 344.3\,K$

$T_{ci} = T_1 = 283.2\,K \qquad T_{co} = T_2 = 310.9\,K$

Fig. 4.9-4a

$$Z = \frac{T_{hi} - T_{ho}}{T_{co} - T_{ci}} = \frac{366.5 - 344.3}{310.9 - 283.2} = 0.801$$

$$Y = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} = \frac{310.9 - 283.2}{366.5 - 283.2} = 0.333$$

$F_T = 0.97$

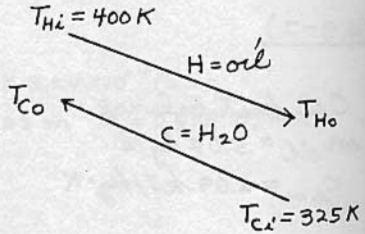
Eq. (4.9-6) $\quad \Delta T_m = F_T\, \Delta T_{lm} = (0.97 \times 58.31)\,K$

Eq. (4.9-5) $\quad q = U_o A_o \Delta T_m$

$$233.85 \times 10^3 = (340)(A_o)(0.97 \times 58.31)$$

$$\boxed{A_o = 12.16\ m^2}$$

(4.9-4)



$m(\text{oil}) = 3.00\ kg/s$

$m(H_2O) = 0.70\ kg/s$

$c_p(\text{oil}) = 1.92\ kJ/kg\cdot K$

$U = 350\ W/m^2 \qquad A = 12.9\ m^2$

assume water outlet $T_{Co} \cong 374\,K$

$T_{av} = \dfrac{374 + 325}{2} = 349\ K$ From A.2 for H_2O at 349 K,

$c_p = 4.196\ kJ/kg\cdot K$

<u>Oil</u> $(mc_p)_H = C_H = (3.00)(1.92\times10^3) = 5760\ W/K$

<u>H_2O</u> $(mc_p)_C = C_C = (0.70)(4.196\times10^3) = 2937\ W/K = C_{min}$

Eq. (4.9-19)

$NTU = UA/C_{min} = 350(12.9)/2937 = 1.537$

Fig. 4.10-7a, $\varepsilon = 0.70$

Eq. (4.9-10)

$q = \varepsilon C_{min}(T_{Hi} - T_{Ci}) = 0.70(2937)(400-325) = \boxed{154190\ W}$

Eq. (4.9-7)

$q = 154190 = (mc_p)_C(T_{Co} - T_{Ci}) = 2936(T_{Co} - 325)$

$T_{Co} = 377.5\ K$ (close enough to assumed 374 K)

$q = 154190 = (mc_p)_H(T_{Hi} - T_{Ho}) = 5760(400 - T_{Ho})$

$\boxed{T_{Ho} = 373.2\ K}$

as a check on the above.

$$\Delta T_{lm} = \frac{(373.2-325)-(400-377.5)}{\ln[(373.2-325)/(400-377.5)]} = 33.74\ K$$

$q = UA\Delta T_{lm} = 350(12.9)(33.74)$

$= 152320\ W$ This is a close check.

(4.10-1)

$A_1 = \pi D L = \pi(0.0254)(0.61)\ m^2$ $\quad T_1 = 588\,K \quad T_2 = 1088\,K$

ε_2 at $T_2 \cong \alpha_{12} = 0.60$ $\quad \varepsilon_1$ at $T_1 = 0.46$

Eq. (4.9-5)

$$q = A_1 \sigma \left[\varepsilon_1 T_1^4 - \alpha_{12} T_2^4\right]$$

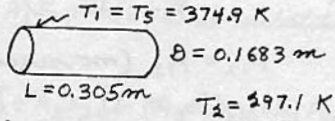
$$= \left[\pi(0.0254)(0.61)\right](5.676\times10^{-8})\left[0.46(588)^4 - 0.60(1088)^4\right]$$

$$\boxed{q = -2171\ W\ (-2410\ btu/h)}$$

(4.10-3)

$\varepsilon = 0.79$

$\Delta T = 374.9 - 297.1 = 77.8\ K$



Use simplified Eq. in Table 4.7-2 for natural convection.

$$h_c = 1.32(\Delta T/D)^{1/4} = 1.32(77.8/0.1683)^{1/4} = 6.121\ W/m^2\cdot K$$

For radiation, use Eq. (4.10-10).

$$h_r = \varepsilon\,(5.676)\left[\frac{(T_1/100)^4 - (T_2/100)^4}{T_1 - T_2}\right]$$

$$= 0.79(5.676)\left[\frac{(374.9/100)^4 - (297.1/100)^4}{374.9 - 297.1}\right]$$

$$h_r = 6.895\ W/m^2\cdot K$$

$$h_c + h_r = 6.121 + 6.895 = 13.016\ W/m^2\cdot K$$

$$A = \pi D L = \pi(0.1683)(0.305) = 0.1613\ m^2$$

Eq. (4.10-9)

$$q = (h_c + h_r) A_1 \Delta T$$

$$= (13.016)(0.1613)(77.8)$$

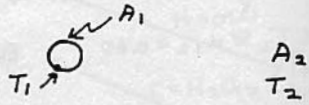
$$\boxed{q = 163.3\ W \quad 557\ btu/h}$$

(4.11-2)

$T_1 = T_s = 283.2\ K$ (surface)

$T_2 = 0\ K$

$\varepsilon_1 = 0.10$



Eq. (4.11-40)

$$q_{12} = \mathcal{F}_{12} A_1 \sigma (T_1^4 - T_2^4)$$

Eq. (4.11-41)

$$\mathcal{F}_{12} = \frac{1}{\frac{1}{\bar{F}_{12}} + \left(\frac{1}{\varepsilon_1} - 1\right) + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$\bar{F}_{12} = F_{12}$ (no reradiating walls) (See text)

$A_1/A_2 = 0$ $\quad$ $\bar{F}_{12} = F_{12} = 1.0$ (all radiation leaving A_1 is intercepted by A_2)

$$\mathcal{F}_{12} = \frac{1}{\frac{1}{1} + \left(\frac{1}{\varepsilon_1} - 1\right) + 0\left(\frac{1}{\varepsilon_2} - 1\right)} = \varepsilon_1$$

$$\frac{q_{12}}{A} = \varepsilon_1 \sigma (T_1^4 - T_2^4)$$

$$= (0.10)(5.676 \times 10^{-8})\left[(283.2)^4 - (0)^4\right]$$

$$\boxed{q_{12}/A = 36.5\ W/m^2}$$

(4.11-3)

Find view factor F_{12} between A_1 and A_2.

Conservation of energy requires:

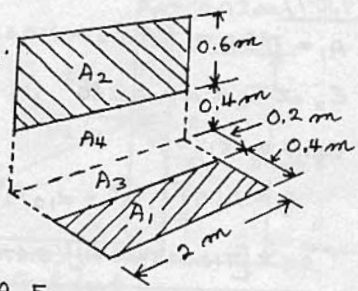


Fig. P4.11-3

(1) $A_3 F_{3(24)} = A_3 F_{34} + A_3 F_{32}$

(2) $A_{(13)} F_{(13)(24)} = A_1 F_{14} + A_1 F_{12} + A_3 F_{34} + A_3 F_{32}$

(3) $A_{(13)} F_{(13)4} = A_1 F_{14} + A_3 F_{34}$

Solve for $A_3 F_{32}$ in (1) and solve for $A_1 F_{14}$ in (3) and substitute into (2) and cancel terms.

(4) $A_{(13)} F_{(13)(24)} = (A_{(13)} F_{(13)4} - A_3 F_{34}) + A_1 F_{12} + A_3 F_{34} + (A_3 F_{3(24)} - A_3 F_{34})$

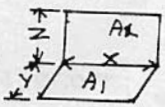
Solve for $A_1 F_{12}$.

$$\boxed{A_1 F_{12} = A_{(13)} F_{(13)(24)} + A_3 F_{34} - A_3 F_{3(24)} - A_{(13)} F_{(13)4}}$$

(4.11-6)

Dimensions Fig. P4.11-3

$Y = y/x \qquad Z = z/x$ Use Fig. 4.11-8



$F_{(13)(24)}$ $\quad Y = \frac{0.6}{2} = 0.3 \quad Z = \frac{1.0}{2} = 0.5$ From Fig. 4.11-8

$F_{(13)4}$ $\quad Y = \frac{0.6}{2} = 0.3 \quad Z = \frac{0.4}{2} = 0.2$

$F_{3(24)}$ $\quad Y = \frac{0.2}{2} = 0.1 \quad Z = \frac{1.0}{2} = 0.5$

F_{34} $\quad Y = \frac{0.2}{2} = 0.1 \quad Z = \frac{0.4}{2} = 0.2$

$$\boxed{\begin{aligned} F_{(13)(24)} &= 0.30 \\ F_{(13)4} &= 0.19 \\ F_{3(24)} &= 0.41 \\ F_{34} &= 0.36 \end{aligned}}$$

From Prob. (4.11-3),

$$F_{12} = \frac{1}{A_1}\left(A_{(13)} F_{(13)(24)} + A_3 F_{34} - A_3 F_{3(24)} - A_{(13)} F_{(13)4}\right)$$

$A_1 = 2(0.4) = 0.8\ m^2 \quad A_2 = 2(0.6) = 1.2 \quad A_3 = 2(0.2) = 0.4 \quad A_4 = 2(0.4) = 0.8$

$A_{(13)} = 2(0.6) = 1.2 \quad A_{(24)} = 2(1.0) = 2.0$

$$F_{12} = \frac{1}{0.8}\left(1.2(0.30) + 0.4(0.36) - 0.4(0.41) - 1.2(0.19)\right) \qquad \boxed{F_{12} = 0.140}$$

(4.10-1)

$A_1 = \pi D L = \pi(0.0254)(0.61)\ m^2 \qquad T_1 = 588\,K \quad T_2 = 1088\,K$

$\varepsilon_2 \text{ at } T_2 \cong \alpha_{12} = 0.60 \qquad \varepsilon_1 \text{ at } T_1 = 0.46$

Eq. (4.9-5)

$$q = A_1 \sigma \left[\varepsilon_1 T_1^4 - \alpha_{12} T_2^4\right]$$

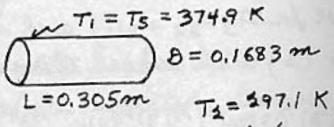
$$= \left[\pi(0.0254)(0.61)\right](5.676\times10^{-8})\left[0.46(588)^4 - 0.60(1088)^4\right]$$

$$\boxed{q = -2171\ W\ (-2410\ btu/h)}$$

(4.10-3)

$\varepsilon = 0.79$

$\Delta T = 374.9 - 297.1 = 77.8\ K$



Use simplified Eq. in Table 4.7-2 for natural convection.

$$h_c = 1.32(\Delta T/D)^{1/4} = 1.32(77.8/0.1683)^{1/4} = 6.121\ W/m^2\cdot K$$

For radiation, use Eq. (4.10-10).

$$h_r = \varepsilon\,(5.676)\left[\frac{(T_1/100)^4 - (T_2/100)^4}{T_1 - T_2}\right]$$

$$= 0.79(5.676)\left[\frac{(374.9/100)^4 - (297.1/100)^4}{374.9 - 297.1}\right]$$

$$h_r = 6.895\ W/m^2\cdot K$$

$$h_c + h_r = 6.121 + 6.895 = 13.016\ W/m^2\cdot K$$

$$A = \pi D L = \pi(0.1683)(0.305) = 0.1613\ m^2$$

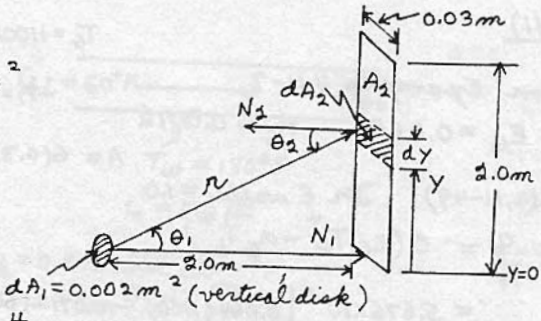
Eq. (4.10-9)

$$q = (h_c + h_r) A_1 \Delta T$$

$$= (13.016)(0.1613)(77.8)$$

$$\boxed{q = 163.3\ W \quad 557\ btu/h}$$

(4.11-10)



$$dA_2 = (0.03)(dy)\ m^2$$

$$r^2 = y^2 + (2.0)^2$$

$$\theta_2 = \theta_1$$

$$\cos\theta = \frac{2}{r} = \frac{2}{\sqrt{y^2+2^2}}$$

$$(\cos\theta)^2 = \frac{2^2}{y^2+2^2} = \frac{4}{y^2+4} = \cos\theta_1 \cos\theta_2$$

Eq.(4.11-36)

$$F_{12} = \frac{1}{A_1}\int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

Integrate dA_1 to get A_1 since A_1 is very small compared to A_2 (other terms inside the integral can be assumed constant).

$$F_{12} = \frac{A_1}{A_1}\int_{A_2} \frac{(\cos\theta)^2 \overbrace{(0.03)\,dy}^{dA_2}}{\pi r^2}$$

$$= \int_{A_2} \frac{(\cos\theta)^2 (0.03)\,dy}{\pi (y^2+2^2)}$$

$$= \frac{0.03}{\pi}\int_{y=0}^{y=2.0} \frac{4}{(y^2+4)(y^2+4)}\,dy$$

$$= \frac{0.12}{\pi}\int_{y=0}^{y=2.0} \frac{dy}{(y^2+4)^2}$$

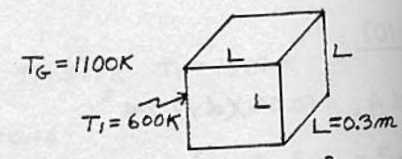
$$= \frac{0.12}{\pi}\left[\frac{y}{2(4)(4+y^2)} + \frac{1}{2(4)(2)}\tan^{-1}\frac{y}{2}\right]_0^2$$

$$= \frac{0.12}{\pi}\left[\frac{2}{2(4)(4+4)} + \frac{1}{2(4)(2)}\tan^{-1}\frac{2}{2} - 0 - \frac{1}{2(4)(2)}\tan^{-1}0\right]$$

$$= \frac{0.12}{\pi}\left[\frac{2}{64} + \frac{1}{16}\left(\frac{\pi}{4}\right) - 0 - 0\right]$$

$$\boxed{F_{12} = 0.00307}$$

(4.11-11)



From Example 4.11-7,

$\varepsilon_G = 0.064 \qquad \alpha_G = 0.0712$

$A = 6(0.3 \times 0.3) = 0.540\ m^2$

Eq (4.11-49) For ε walls $= 1.0$

$$\frac{q}{A} = \sigma(\varepsilon_G T_G^4 - \alpha_G T_1^4)$$

$$= 5.676 \times 10^{-8}\left[0.064(1100)^4 - 0.0712(600)^4\right]$$

$$= 4.795 \times 10^3\ W/m^2 = 4.795\ kW/m^2$$

For ε walls $= 0.75$

Eq. (4.11-50)

$$\varepsilon' = (\varepsilon + 1.0)/2 = (0.75 + 1.0)/2 = 0.875$$

Eq. (4.11-51)

$$\frac{q}{A} = \sigma\varepsilon'(\varepsilon_G T_G^4 - \alpha_G T_1^4) = \varepsilon'\left[\sigma(\varepsilon T_G^4 - \alpha_G T_1^4)\right]$$

$$= 0.875[4.795]$$

(a) $\frac{q}{A} = 4.196\ kW/m^2$

$$q_{rad} = 4.196(0.540) = \boxed{2.266\ kW}$$

(b) $\frac{q_{convec}}{A} = h_c(T_G - T_1)$

$$= 8.0(1100 - 600) = 4000\ W/m^2 = 4.00\ kW/m^2$$

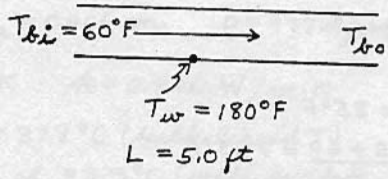
$$q_{convec} = 4.00(0.540) = 2.160\ kW$$

$$q_{total} = q_{rad} + q_{convec} =$$

$$= 2.266 + 2.160$$

$$q_{total} = \boxed{4.426\ kW}$$

(4.12-1)



$m = 300\ lb_m/h$

$D_i = 1.0/12\ ft$

$\rho = 69.9\ lb_m/ft^3$ $\quad c_p = 0.875\ btu/lb_m \cdot °F$

$k = 0.320\ btu/h \cdot ft \cdot °F$ $\quad n = n' = 0.458$

$K = 0.146\ lb_f \cdot s^n \cdot ft^{-2}$ at 70°F and 0.0417 at 190°F.

Eq (4.12-3)

$$N_{Gz} = \frac{m c_p}{kL} = \frac{(300)(0.875)}{(0.320)(5.0)} = 164.1$$

Hence, can use Eq.(4.12-1). Trial and error solution.

Trial 1

Assume $T_{bo} = 100°F$

$$T_{bmean} = \frac{T_{bo} + T_{bi}}{2} = \frac{100+60}{2} = 80°F$$

Plot data for K of K on log scale vs T°F on linear scale.

$K_{80°F} = 0.130\ lb_f \cdot s^n \cdot ft^{-2} = K_b \qquad K_w = K_{180°F} = 0.0464$

Eq.(4.12-2) $\quad \delta = \dfrac{3n'+1}{4n'} = \dfrac{3(0.458)+1}{4(0.458)} = 1.296$

Eq.(4.12-1)

$$\frac{h_a D}{k} = 1.75\,\delta^{1/3}\, N_{Gz}^{1/3} \left(\frac{\gamma_b}{\gamma_w}\right)^{0.14}$$

Eq.(4.12-4) $\quad \dfrac{\gamma_b}{\gamma_w} = \dfrac{K_b}{K_w}$

$$\frac{h_a(1.0/12)}{0.320} = 1.75(1.296)^{1/3}(164.1)^{1/3}\left(\frac{0.130}{0.0464}\right)^{0.14}$$

$$h_a = 46.33\ btu/h \cdot ft^2 \cdot °F$$

Eq.(4.12-5) $\quad \Delta T_a = [(T_w - T_{bi}) + (T_w - T_{bo})]/2$

$$= [(180-60) + (180 - T_{bo})]/2 = 150 - 0.500 T_{bo}$$

$$q = m c_p (T_{bo} - T_{bi}) = h_a (\pi D L)(\Delta T_a)$$

$$(300)(0.875)(T_{bo} - 60) = 46.33(\pi)(1/12)(5.0)(150 - 0.500 T_{bo})$$

(4.12-1) - p.2

$T_{bo} = 84.85°F$

Trial 2

assume $T_{bo} = 85°F$

$T_{b\,mean} = \frac{85+60}{2} = 72.5°F$

$K_b = K_{72.5°F} = 0.141$ (From plot of K vs T°F)

$$\frac{h_a(1.0/12)}{0.320} = 1.75(1.296)^{1/3}(164.1)\left(\frac{0.141}{0.0464}\right)^{0.14}$$

$h_a = 46.86$ btu/h·ft²·°F
(266.1 W/m²·K)

$$(300)(0.875)(T_{bo}-60) = 46.86(\pi)(1/12)(5.0)(150-0.500T_{bo})$$

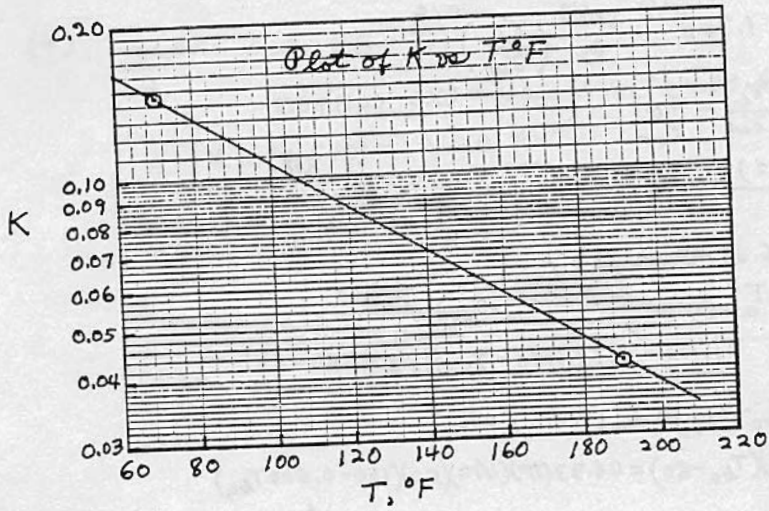
$T_{bo} = 85.09°F$

Hence, trial 2 is fine and $\boxed{T_{bo} = 85.1°F\ (302.7K)}$

$$q = m\,c_p\,(T_{bo}-T_{bi})$$

$$= (300)(0.875)(85.1-60)$$

$$\boxed{q = 6589 \text{ btu/h } (1931 W)}$$



(4.13-1)

$D_t = 1.22\ m \qquad D_a = 0.406\ m \qquad \rho = 977\ kg/m^3$

$C_p = 2.72\ kJ/kg\cdot K \qquad k = 0.346\ W/m\cdot K$

$\mu = 0.100\ Pa\cdot s$ at 37.8 °C (bulk liquid T)

$\mu_w = 7.5\times10^{-3}\ Pa\cdot s$ at 93.3°C (wall surface T)

$N = 150/60\ rev/s$

$$N'_{Re} = \frac{D_a^2 N \rho}{\mu} = \frac{(0.406)^2(150/60)(977)}{0.100} = 4026$$

$$N_{Pr} = \frac{C_p \mu}{k} = \frac{2.72\times10^3(0.100)}{0.346} = 786.1$$

Eq.(4.13-1)

$$\frac{h D_t}{k} = a(N'_{Re})^b (N_{Pr})^{1/3}\left(\frac{\mu}{\mu_w}\right)^m$$

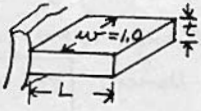
$a = 0.36,\ b = 2/3,\ m = 0.21$

$$\frac{h(1.22)}{0.346} = 0.36(4026)^{2/3}(786.1)^{1/3}\left(\frac{0.100}{7.5\times10^{-3}}\right)^{0.21}$$

$$\boxed{h = 410.9\ W/m^2\cdot K}$$

(4.13-3)

$L = 0.080\ m \qquad t = 0.0030\ m \qquad T_0 = 450\ K$ (tube base)



$T_\infty = 300\ K \qquad h = 25\ W/m^2\cdot K \qquad k = 230\ W/m\cdot K$

Eq.(4.13-16) $L_c = L + t/2 = 0.080 + 0.0030/2 = 0.0815\ m$

$$L_c\left(\frac{h}{kt}\right)^{1/2} = 0.0815\left(\frac{25}{230(0.0030)}\right)^{1/2} = 0.491$$

Fig. 4.13-3a $\boxed{\eta_f = 0.86}$

Eq.(4.13-18) $A_f = 2(L_c \times w) = 2(0.0815)(1.0) = 0.163\ m^2$

Eq.(4.13-17)

$$q = \eta_f h A_f (T_0 - T_\infty) = 0.86(25)(0.163)(450-300)$$

$$\boxed{q = 525.7\ W}$$

(4.14-2)

Variables ρ, c_p, L, k, t, z

Dimensions of the variables (units of variables)

$\rho = \frac{M}{L^3}$ $\quad c_p = \frac{L^2}{t^2 T}$ $\quad L = L$ $\quad k = \frac{ML}{t^3 T}$ $\quad t = t$ $\quad z = L$

Total number of variables $q = 6$

Fundamental units or dimensions $u = 4$ (M, L, t, T)

The number of π's $= q - u = 6 - 4 = 2$ dimensionless groups.

$$\pi_1 = f(\pi_2)$$

Select the 4 variables ρ, c_p, L, and k to be common to π_1 and π_2.

$$\pi_1 = \rho^a c_p^b L^c k^d t$$

$$1 = \left(\frac{M}{L^3}\right)^a \left(\frac{L^2}{t^2 T}\right)^b (L)^c \left(\frac{ML}{t^3 T}\right)^d t$$

(L) $0 = -3a + 2b + c + d$

(M) $0 = a + d$

(t) $0 = -2b - 3d + 1$

(T) $0 = -b - d$

Solving, $a = -1$, $b = -1$, $c = -2$, $d = 1$.

Hence, $\boxed{\pi_1 = \frac{kt}{\rho c_p L^2}}$

$$\pi_2 = \rho^a c_p^b L^c k^d z$$

$$1 = \left(\frac{M}{L^3}\right)^a \left(\frac{L^2}{t^2 T}\right)^b (L)^c \left(\frac{ML}{t^3 T}\right)^d L$$

(L) $0 = -3a \quad 2b + c + 1$

(M) $0 = a + d$

(t) $0 = -2b - 3d$

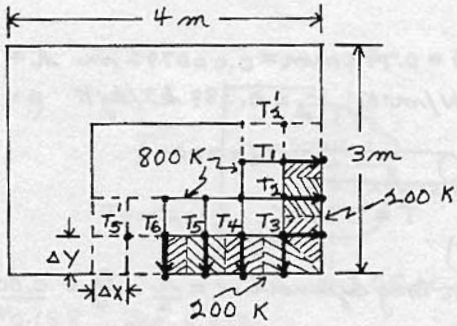
(T) $0 = -b - d$

Solving, $a = 0$, $b = 0$, $c = -1$, $d = 0$

$\boxed{\pi_2 = \frac{z}{L}}$ $\qquad \boxed{\frac{z}{L} = f\left(\frac{kT}{\rho c_p L^2}\right)}$

(4.15-3)

$\Delta x = \Delta y = 0.5\,m$
$k = 1.4\ W/m\cdot K$



Trial 1

Set

$T_6 = 500 \quad T_2 = T_2' = 460$
$T_5 = T_5' = 490 \quad T_1 = 490$
$T_4 = 480$
$T_3 = 350$

Eq. (4.5-12) $\quad T_{n,m} = \dfrac{T_{n-1,m} + T_{n+1,m} + T_{n,m+1} + T_{n,m-1}}{4}$

$T_6 = (T_5' + T_5 + 800 + 200)/4 = (490 + 490 + 800 + 200)/4 = 495.0$

$T_5 = (T_6 + T_4 + 800 + 200)/4 = (495.0 + 480 + 800 + 200)/4 = 493.8$

$T_4 = (T_5 + T_3 + 800 + 200)/4 = (493.8 + 350 + 800 + 200)/4 = 461.0$

$T_3 = (T_4 + T_2 + 200 + 200)/4 = (461.0 + 460 + 200 + 200)/4 = 330.3$

$T_2 = (T_3 + T_1 + 800 + 200)/4 = (330.3 + 490 + 800 + 200)/4 = 455.1$

$T_1 = (T_2 + T_2' + 800 + 200)/4 = (455.1 + 455.1 + 800 + 200)/4 = 477.6$

Trial 2 $\quad T_6 = 496.9,\ T_5 = 489.5,\ T_4 = 455.0,\ T_3 = 327.5,\ T_2 = 451.3,\ T_1 = 475.7$

Trial 3 $\quad T_6 = 494.7,\ T_5 = 487.4,\ T_4 = 453.7,\ T_3 = 326.2,\ T_2 = 450.4,\ T_1 = 475.2$

Trial 4

Eq. (4.5-11) $\quad \bar{q}_{n,m} = T_{n-1,m} + T_{n+1,m} + T_{n,m+1} + T_{n,m-1} - 4T_{n,m}$

$T_6 = 493.7,\ T_5 = 486.8,\ T_4 = 453.2,\ T_3 = 325.9,\ T_2 = 450.3,\ T_1 = 475.1$

Check of residuals.

$\bar{q}_6 = 486.8 + 486.8 + 800 + 200 - 4(493.7) = -1.2$

$\bar{q}_5 = 493.7 + 453.2 + 800 + 200 - 4(486.8) = -0.3$

$\bar{q}_4 = 486.8 + 325.9 + 800 + 200 - 4(453.2) = -0.1$

$\bar{q}_3 = 453.2 + 450.3 + 200 + 200 - 4(325.9) = -0.1$

$\bar{q}_2 = 325.9 + 475.1 + 800 + 200 - 4(450.3) = -0.2$

$\bar{q}_1 = 450.3 + 450.3 + 800 + 200 - 4(475.1) = +0.2$

The residuals are small, so no further trials made.

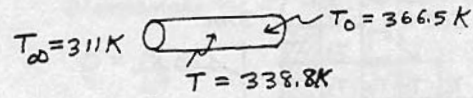
$$q(\text{outside}) = \overset{\text{4 sections}}{4}(k)\Big[\tfrac{1}{2}\tfrac{\Delta y}{\Delta x}(T_1 - 200) + \tfrac{\Delta y}{\Delta x}(T_2 - 200) + \tfrac{\Delta y}{\Delta x}(T_3 - 200)$$
$$+ \tfrac{\Delta x}{\Delta y}(T_3 - 200) + \tfrac{\Delta x}{\Delta y}(T_4 - 200) + \tfrac{\Delta x}{\Delta y}(T_5 - 200) + \tfrac{1}{2}\tfrac{\Delta x}{\Delta y}(T_6 - 200)\Big]$$
$$= 4(1.4)\Big[\tfrac{1}{2}(475.1 - 200) + (450.3 - 200) + (325.9 - 200)$$
$$+ (325 - 200) + (453.2 - 200) + (486.8 - 200) + \tfrac{1}{2}(493.7 - 200)\Big]$$

$$\boxed{q = 7428\ W}$$

(5.2-1)

$D = 0.792\,mm = 0.000792\,m \quad h = 85.2\,W/m^2\cdot K$

$k = 374\,W/m\cdot K \quad c_p = 0.389\,kJ/kg\cdot K \quad \rho = 8890\,kg/m^3$



Eq (5.2-5)

For long cylinder $x_1 = \frac{r}{2} = \frac{D}{4} = \frac{0.000792}{4}$

Eq.(5.2-4)

$$N_{Bi} = \frac{hx_1}{k} = \frac{(85.2)(0.000792/4)}{374} = 4.53\times10^{-5}$$

$4.53\times10^{-5} = N_{Bi} << 0.1$ Hence, can use Eq.(5.2-3)

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-\left(\frac{hA}{c_p\rho V}\right)t}$$

$$\frac{V}{A} = x_1 = 0.000792/4$$

(a) $$\frac{hA}{c_p\rho V} = \frac{(85.2)(4/0.000792)}{(0.389\times10^3)(8890)} = 0.1224\,s^{-1}$$

$$\frac{338.8-311}{366.5-311} = 0.50 = e^{-0.1224t}$$

$$\boxed{t = 5.66\,s}$$

(b) $$\frac{hA}{c_p\rho V} = \frac{(11.36)(4/0.000792)}{(0.389\times10^3)(8890)} = 0.01659\,s^{-1}$$

$$0.50 = e^{-0.01659t}$$

$$\boxed{t = 41.8\,s}$$

(c) $A = \pi DL = \pi(0.000792(1.0) = 2.488\times10^{-3}\,m^2$

$V = \frac{\pi D^2}{4}L = \pi(0.000792)^2(1.0)/4 = 4.927\times10^{-7}\,m^3$

Eq.(5.2-9)

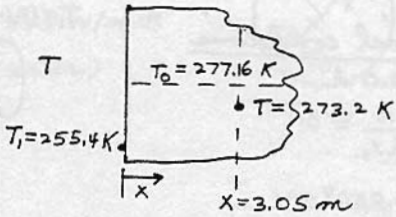
$$Q = c_p\rho V(T_0-T_\infty)\left[1-e^{-\left(\frac{hA}{c_p\rho V}\right)t}\right]$$

$$=(0.389\times1000)(8890)(4.927\times10^{-7})(366.5-311.0)\left[1-e^{-0.1224(5.66)}\right]$$

$$\boxed{Q = 47.28\,J}$$

(5.3-2)

$T_0 = 277.6\,K$
$\alpha = 5.16\times10^{-7}\,m^2/s$
$k = 1.384\ W/m\cdot K$
$h = 8.52\ W/m^2\cdot K$



$$\frac{T-T_0}{T_1-T_0} = \frac{273.2-277.6}{255.4-277.6} = 0.198 \quad \text{Fig. 5.3-3}$$

$$\frac{x}{2\sqrt{\alpha t}} = \frac{3.05}{2\sqrt{5.16\times10^{-7}\,t}} = \frac{2123}{\sqrt{t}} \qquad \frac{h\sqrt{\alpha t}}{k} = \frac{8.52\sqrt{5.16\times10^{-7}t}}{1.384} = 4.422\times10^{-3}\sqrt{t}$$

Trial and error solution

Trial 1 Assume $t = 1000\,h \times 3600\,\frac{s}{h} = 3.6\times10^6\,s$

$$\frac{h\sqrt{\alpha t}}{k} = 4.422\times10^{-3}\sqrt{3.6\times10^6} = 8.39$$

From Fig. 5.3-3 for $\frac{T-T_0}{T_1-T_0} = 0.198$, $\frac{x}{2\sqrt{\alpha t}} = 0.88$

$$\frac{x}{2\sqrt{\alpha t}} = \frac{2123}{\sqrt{t}} = 0.88 \qquad \text{Hence, } t = 5.825\times10^6\,s = 1620\,h$$

This assumed t does not check calculated t.

Trial 2 Assume $t = 1545\,h \times 3600\,\frac{s}{h} = 5.562\times10^6\,s$

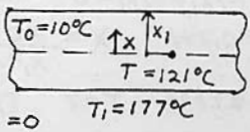
$$\frac{h\sqrt{\alpha t}}{k} = 4.422\times10^{-3}\sqrt{5.562\times10^6} = 10.43$$

From Fig. 5.3-3 ($\frac{h\sqrt{\alpha t}}{k}$ line of close to ∞), $\frac{x}{2\sqrt{\alpha t}} = 0.90$

$$\frac{2123}{\sqrt{t}} = 0.90 \qquad \boxed{t = 5.562\times10^6\,s = 1545\,h} \quad \text{(Trial 2 checks)}$$

(5.3-5)

$x_1 = 0.0254/2 = 0.0127\ m \quad h = 25.6\ W/m^2\cdot K$
$k = 0.69\ W/m\cdot K \quad \alpha = 5.85\times10^{-4}\,m^2/h$
$m = k/hx_1 = 0.69/(25.6\times0.0127) = 2.12 \quad n = x/x_1 = 0$

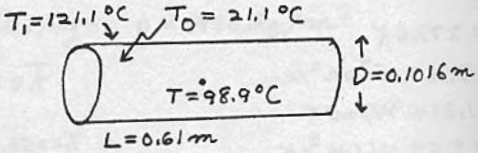


Use Fig 5.3-6 for T at center.

$$\frac{T_1-T}{T_1-T_0} = \frac{177-121}{177-10} = 0.335 \qquad \text{From chart, } X = 2.90 = \frac{\alpha t}{x_1^2}$$

$$2.90 = \frac{(5.85\times10^{-4})\,t}{(0.0127)^2} \qquad \boxed{t = 0.80\,h}$$

(5.3-8)



Original conditions

$t = 2.0\,h$

$m = \dfrac{k}{hx_1} \cong 0$

$x_1 = 0.0508\,m$

$$Y = \frac{T_1 - T}{T_1 - T_0} = \frac{121.1 - 98.9}{121.1 - 21.1} = 0.222$$

$n = x/x_1 = 0$ (center)

Fig. 5.3-8 (also Fig. 5.3-7)

$$X = 0.37 = \frac{\alpha t}{x_1^2} = \frac{\alpha(2.0)}{(0.0508)^2}$$

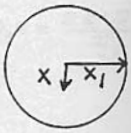
New conditions

$$x_1 = \frac{0.1397}{2} = 0.06985\,m$$

$$X = 0.37 = \frac{\alpha t}{(0.06985)^2}$$

Equate the X values $\dfrac{\alpha(2.0)}{(0.0508)^2} = \dfrac{\alpha t}{(0.06985)^2}$ $\boxed{t = 3.78\,h}$

(5.3-9)



$x_1 = 0.102/2 = 0.051\,m$ $\quad x = 0 \quad h = 11.4\,W/m^2 \cdot K$

$t = 6.0\,h \quad \alpha = 4.65 \times 10^{-4}\,m^2/h \quad k = 0.431\,W/m \cdot K$

$n = x/x_1 = 0/x_1 = 0 \quad m = k/(hx_1) = 0.431/(11.4)(0.051)$

$m = 0.741 \quad X = \dfrac{\alpha t}{x_1^2} = \dfrac{4.65 \times 10^{-4}(6.0)}{(0.051)^2} = 1.073$

$T_0 = 21.1\,°C \quad T = T \quad T_1 = -3.9\,°C$

From Fig. 5.3-10, $Y = 0.05 = \dfrac{T_1 - T}{T_1 - T_0} = \dfrac{-3.9 - T}{-3.9 - 21.1}$

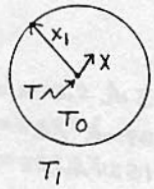
$$\boxed{T = -2.65\,°C}$$

(5.3-10)

$x_1 = 0.0508/2 = 0.0254\ m \quad T_i = 300\ K \quad T = 366.5\ K$

$T_0 = 1033\ K \quad h = 710\ W/m^2 \cdot K \quad k = 45\ W/m \cdot K$

$\alpha = 0.0325\ m^2/h \quad n = x/x_1 = 0\ (\text{center})$



$$m = \frac{k}{h x_1} = \frac{45}{710(0.0254)} = 2.495$$

$$\frac{T_i - T}{T_i - T_0} = \frac{300 - 366.5}{300 - 1033} = 0.0907$$

Fig. 5.3-10 $\quad X = 2.25 = \frac{\alpha t}{x_1^2} = \frac{0.0325(t)}{(0.0254)^2}$ $\quad \boxed{t = 0.045\ h}$

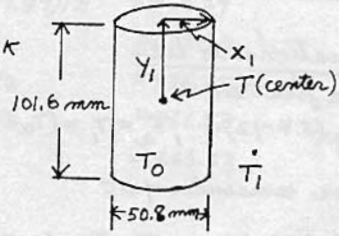
(5.3-11)

$T_0 = 204.4\ °C \quad T_i = 93.3\ °C \quad h = 568\ W/m^2 \cdot K$

$t = 60\ s. \quad x_1 = 0.0508/2 = 0.0254\ m$

$\alpha = 9.44 \times 10^{-5}\ m^2/s \quad k = 207.7\ W/m \cdot K$

$y_1 = 0.1016/2 = 0.0508\ m$



x-direction of cylinder

$n = x/x_1 = 0/x_1 = 0 \qquad m = k/h x_1 = 207.7/(568)(0.0254) = 14.40$

$$X = \frac{\alpha t}{x_1^2} = \frac{9.44 \times 10^{-5}(60)}{(0.0254)^2} = 8.78$$

Fig. 5.3-8 $\qquad Y_x = 0.3$

Y-direction of flat plate

$n = y/y_1 = 0 \qquad m = k/h y_1 = 207.7/(568)(0.0508) = 7.20$

$$X = \frac{\alpha t}{y_1^2} = \frac{9.44 \times 10^{-5}(60)}{(0.0508)^2} = 2.195$$

Fig. 5.3-6 $\qquad Y_y = 0.77$

$$\frac{T_i - T_{x,y}}{T_i - T_0} = \frac{93.3 - T_{x,y}}{93.3 - 204.4} = Y_x Y_y = 0.3(0.77)$$

$$\boxed{T_{xy} = 119.0\ °C}$$

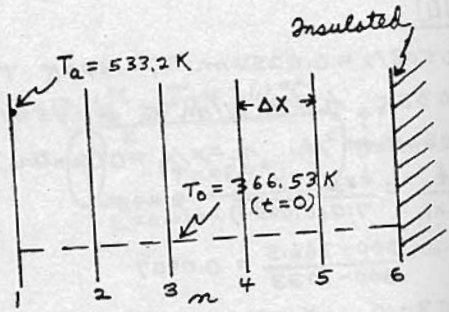
(5.4-1)

$t = 0.875\ h$

$\alpha = 0.0929\ m^2/h$

$\Delta x = 0.1524\ m$

Total thickness $= 0.762\ m$



Eq.(5.4-3) $\quad M = 2 = \dfrac{(\Delta x)^2}{\alpha\,\Delta t} = \dfrac{(0.1524)^2}{(0.0929)(\Delta t)}$

$\boxed{\Delta t = 0.1250\ h}$ $\quad \dfrac{0.875}{0.125} = \boxed{7.0 \text{ time increments}}$ $\quad \dfrac{0.762\ m}{0.1524} = \boxed{5 \text{ slices}}$

Equations to use

$T_0 = {}_0T_1$ (initial T at $n=1$)

For first Δt

Eq.(5.4-12) $\quad {}_1T_a = {}_1T_1 = (T_a + {}_0T_1)/2 \quad (n=1)$

For remainder of Δt

Eq.(5.4-13) $\quad T_1 = T_a \quad (n=1)$

For all Δt

Eq.(5.4-14) $\quad {}_{t+\Delta t}T_n = ({}_tT_{n-1} + {}_tT_{n+1})/2 \quad (n = 2 \text{ to } 5)$

Eq.(5.4-10), for $t=6$ and $M=2$ $\quad (n=6)$

$${}_{t+\Delta t}T_n = {}_tT_{n-1} \quad \text{or} \quad {}_{t+\Delta t}T_6 = {}_tT_5$$

Calculations

$1\Delta t$

$T_1 = (T_a + {}_0T_1)/2 = (533.2 + 366.53)/2 = 449.87$ K

$t+\Delta t$

$T_2 = ({}_tT_1 + {}_tT_3)/2 = (449.87 + 366.53)/2 = 408.20$

$T_3 = ({}_tT_2 + {}_tT_4)/2 = (366.53 + 366.53)/2 = 366.53$

$T_4 = ({}_tT_3 + {}_tT_5)/2 = (366.53 + 366.53)/2 = 366.53$

$T_5 = ({}_tT_4 + {}_tT_6)/2 = (366.53 + 366.53)/2 = 366.53$

$T_6 = ({}_tT_5) = 366.53$

(5.4-1)-p.2

2Δt

$_{t+2\Delta t}T_1 = T_a$ $= 533.2$

$T_2 = (_{t+\Delta t}T_1 + _{t+\Delta t}T_3)/2 = (533.2+366.53)/2 = 449.86$

$T_3 = (T_2 + T_4)/2 = (408.2+366.53)/2 = 387.37$

$T_4 = (T_3 + T_5)/2 = (366.53+366.53)/2 = 366.53$

$T_5 = (T_4 + T_6)/2 = (366.53+366.53)/2 = 366.53$

$T_6 = (_{t+\Delta t}T_5) = 366.53$

3Δt

$_{t+3\Delta t}T_1 = T_a$ $= 533.2$

$T_2 = (_{t+2\Delta t}T_1 + _{t+2\Delta t}T_3)/2 = 533.2+387.37)/2 = 460.29$

$T_3 = (T_2 + T_4)/2 = (449.86+366.53)/2 = 408.20$

$T_4 = (T_3 + T_5)/2 = (387.37+366.53)/2 = 376.95$

$T_5 = (T_4 + T_6)/2 = (366.53+366.53)/2 = 366.53$

$T_6 = (_{t+2\Delta t}T_5) = 366.53$

4Δt

$_{t+4\Delta t}T_1 = T_a$ $= 533.2$

$T_2 = (_{t+3\Delta t}T_1 + _{t+3\Delta t}T_3)/2 = (533.20+408.20)/2 = 470.70$

$T_3 = (T_2 + T_4)/2 = (460.29+376.95)/2 = 418.62$

$T_4 = (T_3 + T_5)/2 = (408.20+366.50)/2 = 387.37$

$T_5 = (T_4 + T_6)/2 = (376.95+366.53)/2 = 371.74$

$T_6 = (_{t+3\Delta t}T_5) = 366.53$

5Δt

$_{t+5\Delta t}T_1 = T_a$ $= 533.2$

$T_2 = (_{t+4\Delta t}T_1 + _{t+4\Delta t}T_3)/2 = (533.20+418.62)/2 = 475.91$

$T_3 = (T_2 + T_4)/2 = (470.70+387.37)/2 = 429.04$

$T_4 = (T_3 + T_5)/2 = (418.62+371.74)/2 = 395.18$

$T_5 = (T_4 + T_6)/2 = (387.37+366.53)/2 = 376.95$

$T_6 = (_{t+4\Delta t}T_5) = 371.74$

(5.4-1) - p.3

$6\Delta t$

$$_{t+6\Delta t}T_1 = T_a \qquad = 533.20$$

$$T_2 = (_{t+5\Delta t}T_1 + _{t+5\Delta t}T_3)/2 = (533.20 + 429.04)/2 \qquad = 481.12$$

$$T_3 = (T_2 + T_4)/2 = (475.91 + 395.18)/2 \qquad = 435.55$$

$$T_4 = (T_3 + T_5)/2 = (429.04 + 376.95)/2 \qquad = 402.99$$

$$T_5 = (T_4 + T_6)/2 = (395.18 + 371.74)/2 \qquad = 383.46$$

$$T_6 = (_{t+5\Delta t}T_5) \qquad = 376.95$$

$7\Delta t$ (Final)

$$_{t+7\Delta t}T_1 = T_a \qquad = 533.20\ K$$

$$T_2 = (_{t+6\Delta t}T_1 + _{t+6\Delta t}T_3)/2 = (533.20 + 435.55)/2 \qquad = 484.38\ K$$

$$T_3 = (T_2 + T_4)/2 = (481.12 + 402.99)/2 \qquad = 442.06\ K$$

$$T_4 = (T_3 + T_5)/2 = (435.55 + 383.46)/2 \qquad = 409.51\ K$$

$$T_5 = (T_4 + T_6)/2 = (402.99 + 376.95)/2 \qquad = 389.97\ K$$

$$T_6 = (_{t+6\Delta t}T_5) \qquad = 383.46\ K$$

(5.4-9)

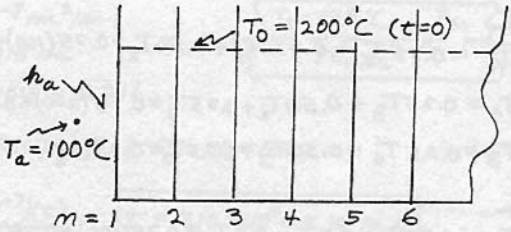
$h = 250\ W/m^2 \cdot K$

$\alpha = 4 \times 10^{-5}\ m^2/s$

$k = 20\ W/m \cdot K$

$t = 50\ s$ total time

$\Delta x = 0.040\ m$



Eq. (5.4-8)

$N = h\Delta x/k = 250(0.040)/20 = 0.50$

Eq. (5.4-9)

$M \geq 2N + 2 = 2(0.5) + 2 = 3.0$ Use $M = 4.0$

Eq. (5.4-3)

$M = 4.0 = (\Delta x)^2/\alpha \Delta t = (0.040)^2/(4 \times 10^{-5})\Delta t$

$\boxed{\Delta t = 10.0\ s}$ $50s/10.0s/\text{increment} = \boxed{5.0\ \Delta t \text{ time increments}}$

Since $M = 4.0$, use $T_a = 100°C$ for all time increments.

Equations to use.

Eq. (5.4-7) for $n = 1$

$$_{t+\Delta t}T_1 = \frac{1}{M}\left[2N\,_tT_a + [M-(2N+2)]\,_tT_1 + 2\,_tT_2\right]$$

$$= \frac{1}{4}\left[2(0.5)\,_tT_a + [4-(2\times 0.5+2)\,_tT_1 + 2\,_tT_2\right]$$

$$= 0.25\,T_a + 0.25\,_tT_1 + 0.50\,_tT_2$$

Eq. (5.4-2) for $n = 2, 3, \ldots$

$$_{t+\Delta t}T_n = \frac{1}{M}\left[_tT_{n+1} + (M-2)\,_tT_n + \,_tT_{n-1}\right]$$

$$= \frac{1}{4}\left[_tT_{n+1} + (4-2)\,_tT_n + \,_tT_{n-1}\right]$$

$$= 0.25\,_tT_{n+1} + 0.50\,_tT_n + 0.25\,_tT_{n-1}$$

Calculations

$1\Delta t$

$_{t+\Delta t}T_1 = 0.25(100) + 0.25(200) + 0.50(200) = 175.0\ °C$

$T_2 = 0.25\,_tT_3 + 0.50\,_tT_2 + 0.25\,_tT_1$

$= 0.25(200) + 0.50(200) + 0.25(200) = 200.0$

$T_3 = \qquad = 200.0$

(5.4-9) - p.2

$2\Delta t$

$_{t+2\Delta t}T_1 = 0.25T_a + 0.25T_1 + 0.50T_2 = 0.25(100) + 0.25(175) + 0.50(200) = 168.75$

$T_2 = 0.25T_3 + 0.50T_2 + 0.25T_1 = 0.25(200) + 0.5(200) + 0.25(175) = 193.75$

$T_3 = 0.25T_4 + 0.50T_3 + 0.25T_2 = 0.25(200) + 0.5(200) + 0.25(200) = 200.00$

$3\Delta t$

$_{t+3\Delta t}T_1 = 0.25T_a + 0.25T_1 + 0.5T_2 = 0.25(100) + 0.25(168.75) + 0.5(193.75) = 164.07$

$T_2 = 0.25T_3 + 0.50T_2 + 0.25T_1 = 0.25(200) + 0.5(193.75) + 0.25(168.75) = 189.06$

$T_3 = 0.25T_4 + 0.50T_3 + 0.25T_2 = 0.25(200) + 0.50(200) + 0.25(193.75) = 198.44$

$T_4 = 200.00$

$4\Delta t$

$_{t+4\Delta t}T_1 = 0.25T_a + 0.25T_1 + 0.50T_2 = 0.25(100) + 0.25(164.07) + 0.5(189.06) = 160.55$

$T_2 = 0.25T_3 + 0.5T_2 + 0.25T_1 = 0.25(198.44) + 0.5(189.06) + 0.25(164.07) = 185.16$

$T_3 = 0.25T_4 + 0.5T_3 + 0.25T_2 = 0.25(200) + 0.5(198.44) + 0.25(189.06) = 196.49$

$T_4 = 0.25T_5 + 0.5T_4 + 0.25T_3 = 0.25(200) + 0.5(200) + 0.25(198.44) = 199.61$

$T_5 = 200.00$

$5\Delta t$ (Final)

$_{t+5\Delta t}T_1 = 0.25T_a + 0.25T_1 + 0.50T_2 = 0.25(100) + 0.25(160.55) + 0.5(185.16) = 157.72^\circ C$

$T_2 = 0.25T_3 + 0.5T_2 + 0.25T_1 = 0.25(196.49) + 0.5(185.16) + 0.25(160.55) = 181.84$

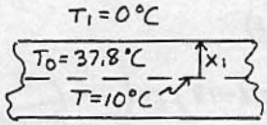
$T_3 = 0.25T_4 + 0.5T_3 + 0.25T_2 = 0.25(199.61) + 0.5(196.49) + 0.25(185.16) = 194.44$

$T_4 = 0.25T_5 + 0.5T_4 + 0.25T_3 = 0.25(200) + 0.5(199.61) + 0.25(196.49) = 198.93$

$T_5 = 0.25T_6 + 0.5T_5 + 0.25T_4 = 0.25(200) + 0.5(200) + 0.25(199.61) = 199.90$

$T_6 = 200.00$

(5.5-1)



$h = 22.7\ W/m^2 \cdot K \quad \alpha = 1.334 \times 10^{-7}\ m^2/s$

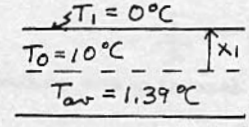
$k = 0.498\ W/m \cdot K \quad x_1 = 0.1015\ m$

$m = k/h x_1 = 0.498/(22.7 \times 0.1015) = 0.216$

$Y = \dfrac{T_1 - T}{T_1 - T_0} = \dfrac{0-10}{0-37.8} = 0.265$ Fig. 5.3-6 $\quad X = 0.92$

$X = 0.92 = \dfrac{\alpha t}{x_1^2} = \dfrac{(1.334 \times 10^{-7})(t)}{(0.1015)^2}$ $\quad \boxed{t = 7.105 \times 10^4\ s = 19.74\ h}$

(5.5-3)



$x_1 = 0.102/2 = 0.0510\ m; \quad k = 0.571\ W/m \cdot K$

$\rho = 1052\ kg/m^3 \quad c_p = 4.02 \times 10^3\ J/kg \cdot K$

$\alpha = \dfrac{k}{\rho c_p} = \dfrac{0.571(3600)}{1052(4.02 \times 10^3)} = 4.86 \times 10^{-4}\ m^2/h$

$E_a = \dfrac{T_1 - T_{av}}{T_1 - T_0} = \dfrac{0-1.39}{0-10} = 0.139$ Fig. 5.3-13 $\quad X = 0.70$ (slab)

$X = 0.70 = \dfrac{\alpha t}{x_1^2} = \dfrac{4.86 \times 10^{-4} t}{(0.0510)^2}$ $\quad \boxed{t = 3.75\ h}$

(5.5-4)

Same as Example 5.5-2 but $h = 142\ W/m^2 \cdot K$

$\lambda = 251.2 \times 10^3\ J/kg \quad a = 0.0635\ m \quad T_f = 270.4\ K$

$T_1 = 244.3\ K \quad \rho = 1057\ kg/m^3 \quad k = 1.038\ W/m \cdot K$

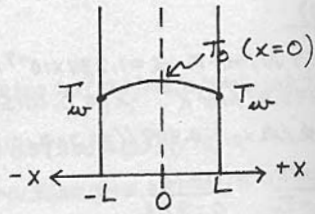
Eq. (5.5-11)

$$t = \frac{\lambda \rho}{T_f - T_1}\left(\frac{a}{2h} + \frac{a^2}{8k}\right)$$

$$= \frac{251.2 \times 10^3 (1057)}{270.4 - 244.3}\left(\frac{0.0635}{2(142)} + \frac{(0.0635)^2}{8(1.038)}\right)$$

$$\boxed{t = 7213\ s = 2.00\ h}$$

(5.6-1)



Eq.(5.6-18)

(1) $\rho c_p \frac{\partial T}{\partial t} = k\nabla^2 T + \dot{q}$

(2) $\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}}{\rho c_p}$

at steady state $\partial T/\partial t = 0$. also, no conduction in y and z directions, so $\partial^2 T/\partial y^2 = 0$ and $\partial^2 T/\partial z^2 = 0$

(3) $\frac{\partial^2 T}{\partial x^2} = -\frac{\dot{q}}{k}$

This becomes an ordinary differential equation.

(4) $\frac{d^2 T}{dx^2} = -\frac{\dot{q}}{k}$ Eq (4.3-24)

Integration of (4) gives

(5) $T = \frac{-\dot{q}x^2}{2k} + C_1 x + C_2$

at $x = 0$, $dT/dx = 0$.

(6) $\frac{dT}{dx} = 0 = \frac{-\dot{q}2x}{2k} + C_1 + 0$ Hence, $C_1 = 0$

(7) $T = \frac{-\dot{q}x^2}{2k} + C_2$

at $x = L$ or $-L$, $T = T_w$.

(8) $T_w = \frac{-\dot{q}L^2}{2k} + C_2$

(9) $C_2 = T_w + \frac{\dot{q}L^2}{2k}$

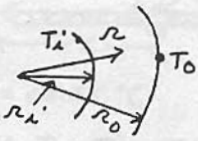
(10) subst (9) to (7).

$$\boxed{T = \frac{\dot{q}(L^2 - x^2)}{2k} + T_w}$$

at $x = 0$, $T = T_0$

(11) $$\boxed{T_0 = \frac{\dot{q}L^2}{2k} + T_w}$$

(5.6-4)



Start with Eq (5.6-21) but with heat flow only in r direction.

(1) $$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + 0 + 0\right]$$

For steady state $\partial T/\partial t = 0$ and the equation becomes an ordinary differential equation.

(2) $$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

Integrating once,

(3) $$r^2\frac{dT}{dr} = K_1 \qquad (K_1 \text{ a constant})$$

Integrating again

(4) $$T = -K_1\frac{1}{r} + K_2 \qquad (K_2 \text{ a constant})$$

B.C. 1: $r = r_o$, $T = T_o$

B.C. 2: $r = r_i$, $T = T_i$

Substituting B.C. 1 and 2 into (4) and solving for K_1 and K_2,

(5) $$T = \left(\frac{T_i - T_o}{1/r_i - 1/r_o}\right)\frac{1}{r} + T_i + \left(\frac{T_i - T_o}{1/r_o - 1/r_i}\right)\frac{1}{r_i}$$

Rearranging,

(a) $$\boxed{T = T_i - \left(\frac{T_i - T_o}{1/r_i - 1/r_o}\right)\left(\frac{1}{r_i} - \frac{1}{r}\right)}$$

For the heat flow q in W,

(6) $$q = -k\frac{dT}{dr}(A) = -k\frac{dT}{dr}(4\pi r^2)$$

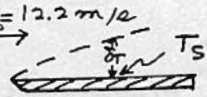
Rearranging and integrating,

$$q\int_{r_i}^{r_o}\frac{dr}{r^2} = -4\pi k\int_{T_i}^{T_o} dT$$

$$\boxed{q = \frac{4\pi k(T_i - T_o)}{1/r_i - 1/r_o}}$$

(5.7-1)

$T_\infty = 294.3\,K$ $\quad v_\infty = 12.2\,m/s$ $\quad T_s = 383\,K$



(a)

air at $T_f = (T_s + T_\infty)/2 = (383 + 294.3)/2 = 338.7\,K$

From A.3-3 for air,

$N_{Pr} = 0.702 \qquad k = 0.02925\,W/m\cdot K \qquad \mu = 2.03\times10^{-5}\,Pa\cdot s$

$\rho = 1.043\,kg/m^3$

$$N_{Re,L} = 5\times10^5 = \frac{x\,v_\infty\,\rho}{\mu} = \frac{x(12.2)(1.043)}{2.03\times10^{-5}}$$

$$\boxed{x = 0.798\,m}$$

Hydrodynamic boundary layer

Eq.(3.10-6)

$$\delta = 5.0\sqrt{\frac{\nu x}{\rho v_\infty}} = 5.0\sqrt{\frac{2.03\times10^{-5}(0.798)}{1.043(12.2)}}$$

$$\boxed{\delta = 5.642\times10^{-3}\,m}$$

Eq.(5.7-10)

$$\frac{\delta}{\delta_T} = N_{Pr}^{1/3} \qquad \frac{5.642\times10^{-3}}{\delta_T} = (0.702)^{1/3}$$

$$\boxed{\delta_T = 6.145\times10^{-3}\,m}$$

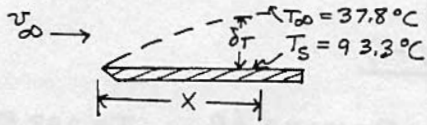
(b) Eq.(5.7-15)

$$h = \frac{k}{L}(0.664)N_{Re,L}^{1/2}\,N_{Pr}^{1/3}$$

$$= \frac{0.02925}{0.798}(0.664)(5\times10^5)^{1/2}(0.702)^{1/3}$$

$$\boxed{h = 15.296\,W/m^2\cdot K}$$

(5.7-1)



$x = 0.61\ m \quad v_\infty = 3.05\ m/s$

$b = 1\ m$ (width)

air at $T_f = (T_s + T_\infty)/2 = (93.3 + 37.8)/2 = 65.55\ °C$

A.3-3

$N_{Pr} = 0.702 \quad k = 0.02925\ W/m \cdot K \quad \rho = 1.043\ kg/m^3$

$\mu = 2.03 \times 10^{-5}\ kg/m \cdot s$

$$N_{Re,L} = \frac{x v_\infty \rho}{\mu} = \frac{0.61(3.05)(1.043)}{2.03 \times 10^{-5}} = 9.559 \times 10^4$$

Hence, since $N_{Re,L} < 5 \times 10^5$, have laminar flow.

Eq. (3.10-6)

$$\delta = 5.0\sqrt{\frac{\mu x}{\rho v_\infty}} = 5.0\sqrt{\frac{2.03 \times 10^{-5}(0.61)}{1.043(3.05)}}$$

(a) $\boxed{\delta = 9.865 \times 10^{-3}\ m}$ hydrodynamic b. layer

Eq. (5.7-10)

$$\frac{\delta}{\delta_T} = N_{Pr}^{1/3} \qquad \frac{9.865 \times 10^{-3}}{\delta_T} = (0.702)^{1/3}$$

$\boxed{\delta_T = 1.110 \times 10^{-2}\ m}$ thermal b. layer

(b)

Eq. (5.7-15)

$$h = \frac{k}{L}(0.664) N_{Re,L}^{1/2} N_{Pr}^{1/3}$$

$$= \frac{(0.02925)}{(0.61)}(0.664)(9.559 \times 10^4)^{1/2}(0.702)^{1/3}$$

$h = 8.749\ W/m^2 \cdot K$

$A = bL = 1.0(0.61)\ m^2$

$q = hA(T_s - T_\infty)$

$= 8.749(1.0 \times 0.61)(93.3 - 37.8)$

$\boxed{q = 296.2\ W}$

(6.1-1)

$P = 101.32\ kPa \qquad T = 298\ K$

$p_{A1} = 60.79\ kPa \qquad p_{A2} = 20.26\ kPa = 20.26 \times 10^3\ Pa$

$z_2 - z_1 = 0.020\ m$

$\mathcal{D}_{AB}\ (CH_4 \text{ and } He) = 0.675 \times 10^{-4}\ m^2/s$ (Table 6.2-1)

$R = 8314\ m^3 \cdot Pa/kg\ mol \cdot K$ (Table A.1-1)

Eq. (6.1-13)

$$J_{Az}^* = \frac{\mathcal{D}_{AB}(p_{A1} - p_{A2})}{RT(z_2 - z_1)}$$

$$J_{Az}^* = \frac{0.675 \times 10^{-4}(60.79 - 20.26) \times 10^3}{8314(298)(0.020 - 0)}$$

$$\boxed{J_{Az}^* = 5.52 \times 10^{-5}\ \frac{kg\ mol\ A}{s \cdot m^2}}$$

(6.2-1)

$\mathcal{D}_{AB} = 0.687 \times 10^{-4}\ m^2/s$ (Table 6.2-1) $\quad z_2 - z_1 = 0.1\ m$

(a) Eq. (6.1-13) $\qquad p_{A1} = 0.060\ atm \quad p_{A2} = 0.020\ atm$

$$J_{Az}^* = \frac{\mathcal{D}_{AB}(p_{A1} - p_{A2})}{RT(z_2 - z_1)} = \frac{0.687 \times 10^{-4}(0.060 - 0.020)}{82.06 \times 10^{-3}(298)(0.10 - 0)}$$

$$= \boxed{1.124 \times 10^{-6}\ kg\ mol/s \cdot m^2}$$

(b) $J_{Bz}^* = -J_{Az}^* = \boxed{-1.124 \times 10^{-6}\ kg\ mol/s \cdot m^2}$

(c) Solve for p_A at $z = 0.05\ m$

$$J_{Az}^* = 1.124 \times 10^{-6} = \frac{0.687 \times 10^{-4}(0.060 - p_A)}{82.06 \times 10^{-3}(298)(0.05 - 0)}$$

$$\boxed{p_A = 0.040\ atm}$$

Alternate method. p_A a linear function of z.

$$p_A = \frac{0.06 + 0.02}{2} = 0.040\ atm$$

(6.2-3)

$P = 1.013 \times 10^5$ Pa $T = 298$ K

$p_{A1} = 1.333 \times 10^4$ Pa $p_{A2} = 6.666 \times 10^3$ Pa

$z_2 - z_1 = 20$ mm $= 0.020$ m $\mathcal{D}_{AB} = 2.30 \times 10^{-5}$ m²/s

(a) Diffusion of NH_3(A) through non-diffusing N_2(B) gas

Eq. (6.2-21)

$$p_{BM} = \frac{(p_{B2} - p_{B1})}{\ln \frac{p_{B2}}{p_{B1}}}$$

$$p_{B1} = P - p_{A1} = 10.132 \times 10^4 - 1.333 \times 10^4 = 8.799 \times 10^4 \text{ Pa}$$

$$p_{B2} = P - p_{A2} = 10.132 \times 10^4 - 0.666 \times 10^4 = 9.466 \times 10^4 \text{ Pa}$$

$$p_{BM} = \frac{9.466 \times 10^4 - 8.799 \times 10^4}{\ln \frac{9.466 \times 10^4}{8.799 \times 10^4}} = 9.128 \times 10^4 \text{ Pa}$$

Eq. (6.2-22)

$$N_A = \frac{\mathcal{D}_{AB} P}{RT(z_2 - z_1) p_{BM}} (p_{A1} - p_{A2})$$

$$= \frac{(2.30 \times 10^{-5})(10.13 \times 10^4)(1.333 - 0.666) 10^4}{(8314)(298)(0.020)(9.128 \times 10^4)}$$

$$\boxed{N_A = 3.44 \times 10^{-6} \text{ kg mol/s} \cdot \text{m}^2}$$

(b) Equimolar counterdiffusion

Eq. (6.1-13)

$$N_A = J_A^* = \frac{\mathcal{D}_{AB}(p_{A1} - p_{A2})}{RT(z_2 - z_1)}$$

$$= \frac{(2.30 \times 10^{-5})(1.333 - 0.666) 10^4}{(8314)(298)(0.020)}$$

$$N_A = 3.10 \times 10^{-6} \text{ kg mol/s} \cdot \text{m}^2$$

The flux is greater in case (a)

(6.2-5)

$p_{A1} = \frac{1}{760} = 1.316\times10^{-3}$ atm $\quad p_{A2} = 0$

$T = 52.6 + 273 = 325.6$ K

$P = 1.00$ atm abs $= 1.0132\times10^{5}$ Pa

$\mathcal{D}_{AB} = 5.16\times10^{-6}$ m²/s at 52.6 °C and 1.0 atm

$r_1 = 10$ mm $= 0.010$ m

Eq. (6.2-21)

$$p_{BM} = \frac{p_{B2} - p_{B1}}{\ln(p_{B2}/p_{B1})}$$

$p_{B1} = 1.00 - 1.316\times10^{-3} = 0.9987$ atm

$p_{B2} = 1.00 - 0 = 1.00$ atm

$$p_{BM} = \frac{1.00 - 0.9987}{\ln 1.00/0.9987} = 0.9993 \text{ atm}$$

Eq. (6.2-45)

$\mathcal{D}_{AB} \propto T^{1.75}$

$$\mathcal{D}_{AB} = 5.16\times10^{-6}\left(\frac{325.6}{273}\right)^{1.75}$$

$\mathcal{D}_{AB} = 7.024\times10^{-6}$ m²/s

Eq. (6.2-28)

$$N_{A1} = \frac{\mathcal{D}_{AB} P (p_{A1} - p_{A2})}{RTr_1 p_{BM}}$$

$$= \frac{(7.024\times10^{-6})(1.0132\times10^{5})(1.316\times10^{-3} - 0)(1.0132\times10^{5})}{(8314)(325.6)(0.010)(0.9993\times1.0132\times10^{5})}$$

$$\boxed{N_{A1} = 3.46\times10^{-8} \text{ kgmol/s}\cdot\text{m}^2}$$

(6.2-6)

$A = CH_4 \quad M_A = 16.04 \qquad B = CH_3CH_2OH \quad M_B = 46.1 \quad T = 298$ K $\quad P = 1.0$ atm

Table 6.2-2

$$\Sigma v_A = \overset{1C}{1(16.5)} + \overset{4H}{4(1.98)} = 24.42$$

$$\Sigma v_B = \overset{2C}{2(16.5)} + \overset{6H}{6(1.98)} + \overset{1O}{5.48} = 50.36$$

Eq. (6.2-45)

$$\mathcal{D}_{AB} = \frac{1.00\times10^{-7} T^{1.75}\left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{1/2}}{P(\Sigma v_A^{1/3} + \Sigma v_B^{1/3})^2} = \frac{1.00\times10^{-7}(298)^{1.75}\left(\frac{1}{16.04} + \frac{1}{46.1}\right)^{1/2}}{1.0(24.42^{1/3} + 50.36^{1/3})^2}$$

(a) $\mathcal{D}_{AB} = \boxed{1.43\times10^{-5} \text{ m}^2/\text{s} (298\text{K})}$

$$\mathcal{D}_{AB} = 1.43\times10^{-5}\left(\frac{373}{298}\right)^{1.75} = \boxed{2.12\times10^{-5} \text{ m}^2/\text{s} (373\text{ K})}$$

(b) $\mathcal{D}_{AB} \propto \frac{1}{P}$ $\quad \mathcal{D}_{AB} = 1.43\times10^{-5}(1/2) = \boxed{0.715\times10^{-5} \text{ m}^2/\text{s} (298\text{K}, 2\text{ atm})}$

(6.2-7)

$z_2 - z_1 = 0.11\,m \qquad P = 1.00\,atm \qquad T = 298\,K$

$p_{A1}(N_2) = \frac{80}{760} = 0.1052\,atm \qquad p_{A2} = \frac{10}{760} = 0.0132\,atm$

Eq (6.2-45)

$M_A = 28.0 \qquad M_B(CO) = 28.0$

Table 6.2-2

$v_A = 17.9 \qquad v_A^{1/3} = 17.9^{1/3} = 2.61$

$v_B = 18.9 \qquad v_B^{1/3} = 18.9^{1/3} = 2.66$

$$\mathcal{D}_{AB} = \frac{1.00\times10^{-7}\,T^{1.75}\,(1/M_A + 1/M_B)^{1/2}}{P\left[(\Sigma v_A)^{1/3} + (\Sigma v_B)^{1/3}\right]^2}$$

$$= \frac{1.00\times10^{-7}\,(298)^{1.75}\,(1/28 + 1/28)^{0.5}}{1.00\left[2.61 + 2.66\right]^2} = \boxed{2.05\times10^{-5}\,m^2/s}$$

(a) Equimolar counterdiffusion

Eq. (6.1-13)

$$N_A = J_A^* = \frac{\mathcal{D}_{AB}(p_{A1} - p_{A2})}{RT\,(z_2 - z_1)} = \frac{2.05\times10^{-5}\,(0.1052 - 0.0132)}{82.06\times10^{-3}\,(298)(0.11 - 0)}$$

$$\boxed{N_A = 7.02\times10^{-7}\,kg\,mol/s\cdot m^2}$$

(b) at 200°C (473 K)

$$\mathcal{D}_{AB} = 2.05\times10^{-5}\left(\frac{473}{298}\right)^{1.75} = 4.60\times10^{-5}\,m^2/s$$

$$N_A = \frac{4.60\times10^{-5}\,(0.1052 - 0.0132)}{82.06\times10^{-3}\,(473)(0.11 - 0)}$$

$$\boxed{N_A = 9.92\times10^{-7}\,kg\,mol/s\cdot m^2 \text{ (flux increases)}}$$

(c) at P = 3.0 atm and 298 K

$$\mathcal{D}_{AB} \propto \frac{1}{P} \qquad \mathcal{D}_{AB} = 2.05\times10^{-5}\left(\frac{1.0}{3.0}\right) = 0.683\times10^{-5}\,m^2/s$$

$$N_A = \frac{0.683\times10^{-5}\,(0.1052 - 0.0132)}{82.06\times10^{-3}\,(298)(0.11 - 0)}$$

$$\boxed{N_A = 2.34\times10^{-7}\,kg\,mol/s\cdot m^2}$$

(6.2-9)

(a) For pseudo steady-state,

Eq.(6.2-32)

(1) $\frac{\bar{N}_A}{4\pi r_1^2} = N_{A1} = \frac{\mathcal{D}_{AB}P}{RTr_1}\frac{(p_{A1}-p_{A2})}{p_{BM}}$

at time t, the drop starts to evaporate and $r_1 = r$ (variable)

(2) Mass at time $t = \frac{4}{3}\pi r^3 \rho_A$

(3) kg mol at time $t = m = \frac{4}{3}\pi r^3 \frac{\rho_A}{M_A}$

In dt lose dm kg mol by diffusion.

(4) $\frac{dm}{dt} = -\frac{4}{3}\pi 3r^2 \frac{\rho_A}{M_A}\frac{dr}{dt}$

Divide both sides by $4\pi r^2$.

(5) $\frac{1}{4\pi r^2}\frac{dm}{d} = \frac{\bar{N}_A}{4\pi r^2} = -\frac{\rho_A}{M_A}\frac{dr}{dt}$

Equate (1) to (5)

(6) $\frac{\bar{N}_A}{4\pi r^2} = \frac{\mathcal{D}_{AB}P}{RTr}\frac{(p_{A1}-p_{A2})}{p_{BM}} = -\frac{\rho_A}{M_A}\frac{dr}{dt}$

Rearrange and integrate.

(7) $\frac{\mathcal{D}_{AB}P}{RT}\frac{(p_{A1}-p_{A2})}{p_{BM}}\int_0^{t_F} dt = -\frac{\rho_A}{M_A}\int_{r_1}^{0} r\,dr = \frac{\rho_A}{M_A}\frac{r_1^2}{2}$

Eq.(6.2-34)

$$\boxed{t_F = \frac{\rho_A r_1^2 RT p_{BM}}{2M_A \mathcal{D}_{AB}P(p_{A1}-p_{A2})}}$$

(b) $\mathcal{D}_{AB} = 0.086\times10^{-4}\ m^2/s$ Table 6.2-1 $\rho_A = 866\ kg/m^3$

$r_1 = 2/1000 = 0.002\ m$ $T = 273+25.9 = 298.9\ K$ $M_A = 92.13$

$p_{A1} = 3.84\times10^3\ Pa$ $p_{A2} = 0$ $p_{B1} = 1.01325\times10^5 - 0.0384\times10^5 = 0.9748\times10^5\ Pa$

$p_{B2} = 1.01325\times10^5$ $p_{BM} \cong \frac{p_{B1}+p_{B2}}{2} = \frac{(0.9748+1.01325)10^5}{2} = 0.994\times10^5\ Pa$

$$t_F = \frac{866(0.002)^2(8314)(298.9)(0.994\times10^5)}{2(92.13)(0.086\times10^{-4})(1.01325\times10^5)(3.84\times10^3-0)}$$

$$\boxed{t_F = 1388\ s}$$

(6.3-1)

$\mathcal{D}_{AB} = 2.50 \times 10^{-9}\ m^2/s \qquad z_2 - z_1 = 0.002\ m$

12.0% HCl (A) $\qquad$ 6.0% HCl

$\rho_1 = 1060.7\ kg/m^3 \qquad \rho_2 = 1030.3\ kg/m^3$

$M_A = 36.47 \qquad M_B(H_2O) = 18.02$

assume 100 kg solution.

$$x_{A1} = \frac{12.0/36.47}{12.0/36.47 + 88.0/18.02} = \frac{0.329}{5.21} = 0.0632$$

$$x_{B1} = 1 - 0.0632 = 0.9368$$

$$M_1 = \frac{100}{5.21} = 19.18\ kg/kg\ mol\ solution$$

$$x_{A2} = \frac{6.0/36.47}{6.0/36.47 + 94.0/18.02} = \frac{0.1645}{5.385} = 0.03055$$

$$x_{B2} = 1.0 - 0.03055 = 0.9694$$

$$M_2 = \frac{100}{5.385} = 18.60$$

Eq.(6.3-4) $$x_{BM} = \frac{x_{B2} - x_{B1}}{\ln(x_{B2}/x_{B1})} = \frac{0.9694 - 0.9368}{\ln(0.9694/0.9368)} = 0.953$$

Eq.(6.3-2)

$$c_{av} = \frac{\rho_1/M_1 + \rho_2/M_2}{2} = \frac{1060.7/19.18 + 1030.3/18.60}{2}$$

$$= 55.4\ kg\ mol/m^3$$

Eq.(6.3-3)

$$N_A = \frac{\mathcal{D}_{AB}\, c_{av}\,(x_{A1} - x_{A2})}{(z_2 - z_1)\, x_{BM}} = \frac{(2.50 \times 10^{-9})(55.4)(0.0632 - 0.03055)}{(0.002)(0.953)}$$

$$\boxed{N_A = 2.372 \times 10^{-6}\ kg\ mol\ A/s \cdot m^2}$$

(6.3-4)

$T = 293\,K$ $\quad CH_3OH(A)$ $\quad H_2O(B)$ $\quad M_B = 18.02$

Appendix A.2 $\quad \mu_B = 1.005 \times 10^{-3}\ kg/m \cdot s$

Table 6.3-2

$$V_A = \overset{1C}{0.0148} + \overset{4H}{4(0.0037)} + \overset{1O}{1(0.0074)} = 0.0370\ m^3/kg\ mol$$

Eq. (6.3-9)

$$\mathcal{D}_{AB} = \frac{1.173 \times 10^{-16} (\phi_B M_B)^{1/2} T}{\mu_B V_A^{0.6}} \qquad \phi_B = 2.6$$

(a)

$$= \frac{1.173 \times 10^{-6} (2.6 \times 18.02)^{1/2} (293)}{1.005 \times 10^{-3} (0.0370)^{0.6}}$$

$$\boxed{\mathcal{D}_{AB} = 1.692 \times 10^{-9}\ m^2/s}$$

(b) $T = 288\,K$ $\quad$ A.2 $\quad \mu_B = 1.1404 \times 10^{-3}\ kg/m \cdot s$

$$\mathcal{D}_{AB} \propto \frac{T}{\mu_B} \qquad \mathcal{D}_{AB\,exp} = 1.26 \times 10^{-9}\ m^2/s\ (288\ K)$$

$$\mathcal{D}_{AB} = 1.26 \times 10^{-9} \left(\frac{293}{288}\right)\left(\frac{1.1404}{1.005}\right)$$

$$\boxed{\mathcal{D}_{AB} = 1.455 \times 10^{-9}\ m^2/s}$$

(6.4-1)

Table 6.4-1 $\quad M_A = 482{,}700 \qquad T = 298\,K$

$$\boxed{\mathcal{D}_{AB}(exp.) = 4.01 \times 10^{-11}\ m^2/s}$$

A.2 $\quad \mu = 0.8937 \times 10^{-3}\ kg/m \cdot s \quad (298\ K)$

Eq. (6.4-1)

$$\mathcal{D}_{AB} = \frac{9.40 \times 10^{-15}\, T}{\mu\, (M_A)^{1/3}}$$

$$= \frac{9.40 \times 10^{-15} (298)}{0.8937 \times 10^{-3} (482{,}700)^{1/3}}$$

$$\boxed{\mathcal{D}_{AB} = 3.995 \times 10^{-11}\ m^2/s}$$

(6.4-4)

$\mathcal{D}_{AB} = 1.21\times10^{-5}\ cm^2/s$, $C_p = 8.2\ g/100mL$, $\mathcal{D}_p = 0.091\times10^{-5}\ cm^2/s$

(a)

$$C_p = \frac{8.2\ g}{100\ cm^3}\times\frac{1}{1000\ g/kg}\times\left(\frac{100\ cm}{m}\right)^3 = 82.0\ kg/m^3$$

Eq.(6.4-2) for blockage but no binding

$$\mathcal{D}_{AP} = \mathcal{D}_{AB}(1-1.81\times10^{-3}C_p) = \mathcal{D}_{AB}(1-1.81\times10^{-3}\times82.0) = \mathcal{D}_{AB}(0.8516)$$

$$\boxed{\mathcal{D}_{AP}/\mathcal{D}_{AB} = 0.8516}$$

(b) Eq.(6.4-4) for blockage and binding

$$\mathcal{D}_{AP} = \left[\mathcal{D}_{AB}(1-1.81\times10^{-3}C_p)\left(\frac{\%\ free\ A}{100}\right) + \mathcal{D}_p\left(\frac{\%\ bound\ A}{100}\right)\right]$$

$$= \left[1.21\times10^{-5}(0.8516)\left(\frac{66.66}{100}\right) + 0.091\times10^{-5}\left(\frac{33.33}{100}\right)\right]$$

$$= \left[0.6869\times10^{-5} + 0.0303\times10^{-5}\right].$$

$$\boxed{\mathcal{D}_{AP} = 0.7172\times10^{-5}\ cm^2/s}$$

$$\mathcal{D}_{AP}/\mathcal{D}_{AB} = \frac{0.7172\times10^{-5}}{1.21\times10^{-5}} = \boxed{0.5927\ \text{Predicted}}\ \boxed{0.616\ \text{Exp.}}$$

(c) Eq.(6.4-3)

$$N_A = \frac{\mathcal{D}_{AP}(C_{A1}-C_{A2})}{(z_2-z_1)}$$

$$= \frac{\left(0.7172\times10^{-5}\ \frac{cm^2}{s}\right)\left(\frac{0.05\ g}{1000\ cm^3}-0\right)}{(1.5\times10^{-4}\ cm)}$$

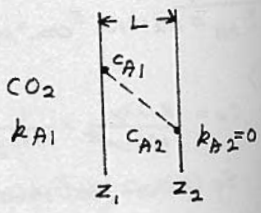
$$\boxed{N_A = 2.392\times10^{-6}\ g/s\cdot cm^2}$$

(6.5-1)

$L = 30/1000 = 0.030\ m$ $\quad A = 4.0 \times 10^{-4}\ m^2$

$T = 25\,°C$ $\quad p_{A1} = 2.0$ atm abs

$S = \dfrac{0.90\ m^3\ (STP)}{m^3\ rubber \cdot atm}$ $\quad \mathcal{D}_{AB} = 0.11 \times 10^{-9}\ m^2/s$



Eq.(6.5-7)

$$c_{A1} = \frac{S p_{A1}}{22.414} = \frac{0.90(2.0)}{22.414} = 8.031 \times 10^{-2}\ kg\,mol/m^3\ rubber$$

Eq.(6.5-2)

$$N_A = \frac{\mathcal{D}_{AB}(c_{A1} - c_{A2})}{z_2 - z_1}$$

$$N_A A = \frac{A \mathcal{D}_{AB}(c_{A1} - c_{A2})}{z_2 - z_1}$$

$$= \frac{4 \times 10^{-4}(0.11 \times 10^{-9})(8.031 \times 10^{-2} - 0)}{(0.030 - 0)}$$

$$\boxed{N_A A = 1.178 \times 10^{-13}\ kg\,mol/s}$$

alternate Eq.(6.5-8)

$$N_A = \frac{\mathcal{D}_{AB} S (p_{A1} - p_{A2})}{22.414(z_2 - z_1)}$$

(6.5-3)

(a) Table 6.5-1 $\quad P_M = 0.342(10^{-10})\ \dfrac{m^3\ solute\ (STP)}{s \cdot m^2 \cdot atm/m}$ $\quad L = 0.020\ m$

$\mathcal{D}_{AB} = 0.85 \times 10^{-9}\ m^2/s$ $\quad S = 0.040\ \dfrac{m^3\ solute\ (STP)}{m^3\ solid \cdot atm}$

Eq.(6.5-9)

$$P_M = \mathcal{D}_{AB} S$$

$$\mathcal{D}_{AB} = \frac{P_M}{S} = \frac{0.342 \times 10^{-10}}{0.040} = \boxed{8.55 \times 10^{-10}\ m^2/s}$$

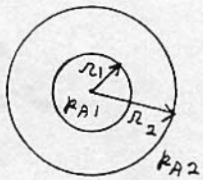
(b) Eq.(6.5-8)

$$N_A = \frac{P_M(p_{A1} - p_{A2})}{22.414(z_2 - z_1)}$$ $\quad p_{A1} = 1.5$ atm $\quad p_{A2} = 0$

$$= \frac{0.342 \times 10^{-10}(1.50)}{22.414(0.020 - 0)}$$

$$N_A = \boxed{1.144 \times 10^{-10}\ kg\,mol/s \cdot m^2}$$

(6.5-4)



$r_2 = \frac{11.0}{1000 \times 2} = 0.0055\ m$, $r_1 = \frac{3.0}{1000 \times 2} = 0.0015\ m$

$L = 1.00\ m$ $T = 27 + 273 = 300K$ H_2 gas

$p_{A1} = 2.0\ atm$ $p_{A2} = 0$

Table 6.5-1 H_2 in neoprene

$\mathcal{D}_{AB} = 0.180 \times 10^{-9}\ m^2/s$ $S = 0.053 \frac{m^3(STP)}{m^3\ solid \cdot atm}$

Eq.(6.5-5)

$$c_{A1} = \frac{S}{22.414} p_{A1}$$

Eq.(6.5-4)

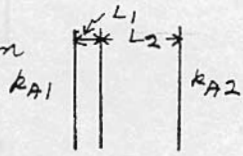
$$\bar{N}_A = \mathcal{D}_{AB}(c_{A1} - c_{A2}) \frac{2\pi L}{\ln(r_2/r_1)}$$

substitute (6.5-5) to (6.5-4).

$$\bar{N}_A = \frac{\mathcal{D}_{AB} S (p_{A1} - p_{A2}) 2\pi L}{22.414 \ln(r_2/r_1)}$$

$$= \frac{0.18 \times 10^{-9}(0.053)(2.0-0)(2\pi)(1.00)}{22.414 \ln(0.0055/0.0015)} = \boxed{4.117 \times 10^{-12}\ kg\ mol\ H_2/s}$$

(6.5-5)



N_2 gas $T = 30 + 273 = 303\ K$ $p_{A1} = 2.0\ atm$

$p_{A2} = 0$ $L_1 = 0.001\ m$ $L_2 = 0.008\ m$

Table 6.5-1

P_{M1} nylon $= \frac{0.0152(10^{-12})\ m^3(STP)}{s \cdot m^2 \cdot atm/m}$ P_{M2}(polyeth) $= 1.52(10^{-12})$

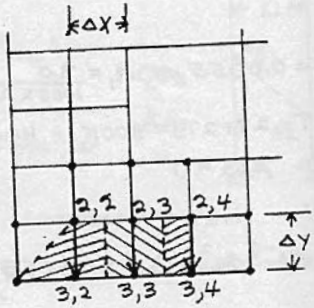
Eq.(6.5-12)

$$N_A = \frac{(p_{A1} - p_{A2})}{22.414(L_1/P_{M1} + L_2/P_{M2})}$$

$$= \frac{(2.0-0)}{22.414(0.001m/(0.0152 \times 10^{-12}) + 0.008/(1.52 \times 10^{-12})}$$

$$\boxed{N_A = 1.256 \times 10^{-12}\ kg\ mol/m^2 \cdot s}$$

(6.6-1)



$\Delta x = \Delta y$

Writing Eq.(6.6-5)

$$N = \mathcal{D}_{AB}\frac{(\Delta x \cdot 1)}{\Delta y}\left[(c_{2,2}-c_{3,2})+(c_{2,3}-c_{3,3})+\frac{1}{2}(c_{2,4}-c_{3,4})\right]$$

$$= 1.00\times10^{-9}\left[(3.47-2.72)+(4.24-3.06)+\frac{1}{2}(4.41-3.16)\right](10^{-3})$$

$$\boxed{N = 2.555\times10^{-12}\ \text{kg mol/s}}$$

Close check with Example 6.6-1.

(7.1-1)

$K = 0.50 \quad c_0 = 1.0 \times 10^{-2}$ kg mol A/m^3

$c_{Li} = 0.10$ kg mol A/m^3 $\quad k_c = 2.0 \times 10^{-7}$ m/s

$K = c_{Li}/c_i$ $\quad$ semi infinite solid

$t = 3 \times 10^4$ s $\quad \mathcal{D}_{AB} = 4.0 \times 10^{-9}$ m^2/s

Use Eq. 5.3-3.

$$\frac{K k_c}{\mathcal{D}_{AB}} \sqrt{\mathcal{D}_{AB} t} = \frac{(0.5)(2.0 \times 10^{-7})}{(4.0 \times 10^{-9})} \sqrt{(4.0 \times 10^{-9})(3 \times 10^4)} = 0.274$$

For $x = 0.01$ m from surface,

$$\frac{x}{2\sqrt{\mathcal{D}_{AB} t}} = \frac{0.01}{2\sqrt{(4.0 \times 10^{-9})(3 \times 10^4)}} = 0.457$$

From Eq. 5.3-3, $1 - Y = 0.093$

$$1 - Y = 0.093 = \frac{c - c_0}{c_i/K - c_0} = \frac{c - 1.0 \times 10^{-2}}{0.1/0.5 - 1.0 \times 10^{-2}}$$

$$\boxed{c = 2.78 \times 10^{-2} \text{ kg mol/m}^3 (x = 0.01)}$$

For $x = 0$ m (in solid)

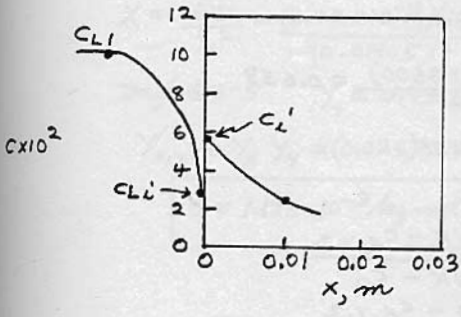
$$1 - Y = 0.25 = \frac{c - 1.0 \times 10^{-2}}{0.1/0.5 - 1.0 \times 10^{-2}}$$

$$c = \boxed{c_i = 5.75 \times 10^{-2} \text{ kg mol/m}^3 (x = 0)}$$

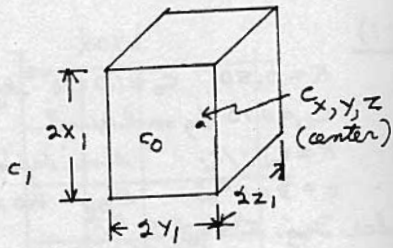
In liquid at $x = 0$

$$K = 0.5 = \frac{c_{Li}' (\text{liquid})}{c_i (\text{solid})} = \frac{c_{Li}'}{5.75 \times 10^{-2}}$$

$$\boxed{c_{Li}' = 2.87 \times 10^{-2} \text{ kg mol/m}^3}$$



(7.1-3)



$$x_1 = \frac{10.16}{1000 \times 2} = 5.08 \times 10^{-3}\ m$$

$$y_1 = \frac{7.62}{1000 \times 2} = 3.81 \times 10^{-3}\ m$$

$$z_1 = \frac{10.16}{1000 \times 2} = 5.08 \times 10^{-3}\ m$$

$c_1 = 0$ $\quad c_0 = 0.10\ kg\ mol/m^3$ $\quad K = 1.0$

Table 6.4-2

$\mathcal{D}_{AB} = 4.72 \times 10^{-10}\ m^2/s$

x-direction

$x = 0 \qquad n = x/x_1 = 0$

$$m = \frac{\mathcal{D}_{AB}}{K k_c x_1} \cong 0 \ (k_c \text{ very large})$$

$$X = \frac{\mathcal{D}_{AB} t}{x_1^2} = \frac{4.72 \times 10^{-10}(10 \times 3600)}{(5.08 \times 10^{-3})^2} = 0.658$$

From Fig. 5.3-5, $Y_x = 0.275$

y-direction

$y = 0 \qquad n = y/y_1 = 0 \qquad m = 0$

$$X = \frac{\mathcal{D}_{AB} t}{y_1^2} = \frac{4.72 \times 10^{-10}(10 \times 3600)}{(3.81 \times 10^{-3})^2} = 1.171$$

From Fig. 5.3-5, $Y_y = 0.082$

z-direction

$z = 0 \qquad n = z/z_1 = 0 \qquad m = 0$

$$X = \frac{\mathcal{D}_{AB} t}{z_1^2} = \frac{4.72 \times 10^{-10}(10 \times 3600)}{(5.08 \times 10^{-3})^2} = 0.658$$

From Fig. 5.3-5, $Y_z = 0.275$

Eq. (7.1-17)

$$Y_{x,y,z} = Y_x Y_y Y_z = \frac{c_1/K - c_{x,y,z}}{c_1/K - c_0}$$

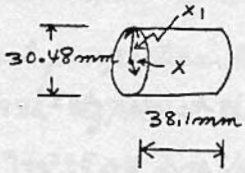
$$0.275(0.082)(0.275) = \frac{0 - c_{x,y,z}}{0 - 0.10}$$

$$\boxed{c_{x,y,z} = 6.20 \times 10^{-4}\ \frac{kg\ mol}{m^3}}$$

(7.1-5)

$T = 278\,K \qquad c_0 = 0.10$ kg mol urea/m^3

$\mathcal{D} = 4.72\times10^{-10}\,m^2/s \qquad t = 100\,h \qquad c_1 = 0$



(a) Radial diffusion only

$x = 0$ (center)

$x_1 = \frac{0.03048}{2} = 0.01542\,m$

$$X = \frac{\mathcal{D}t}{x_1^2} = \frac{(4.72\times10^{-10})(100\times3600)}{(0.01542)^2} = 0.715$$

$n = \frac{x}{x_1} = \frac{0}{x_1} = 0 \qquad m = 0$

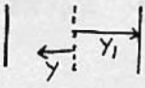
Fig. 5.3-7

$$Y_x = 0.025\,(\text{off graph}) = \frac{c_1 - c}{c_1 - c_0} = \frac{0 - c}{0 - 0.10}$$

$$\boxed{c = 2.50\times10^{-3}\,\text{kg mol}/m^3}$$

(b) axial and radial diffusion

Two flat plates (axial)



$Y_1 = \frac{0.0381}{2} = 0.01905\,m$

$y = 0$ (center) $\quad n = \frac{y}{Y_1} = \frac{0}{Y_1} = 0 \qquad m = 0$

$$X = \frac{\mathcal{D}t}{Y_1^2} = \frac{(4.72\times10^{-10})(100\times3600)}{(0.01905)^2} = 0.466$$

Fig. 5.3-5 $\qquad Y_y = 0.45$

$$Y_{x,y} = Y_x\,Y_y = (0.025)(0.45) = \frac{0 - c}{0 - 0.1}$$

$$\boxed{c = 1.125\times10^{-3}\,\text{kg mol}/m^3}$$

(7.2-2)

(a) Convert k_c' to k_y and k_G

Eq. (7.2-10) and Table 7.2-1

$$N_A = k_y(y_{A1} - y_{A2}) = \frac{k_c'}{x_{BM}}(c_{A1} - c_{A2}) = k_G(p_{A1} - p_{A2})$$

$$N_A = k_y\left(\frac{c_{A1}}{c} - \frac{c_{A2}}{c}\right) = \frac{k_c'}{x_{BM}}(c_{A1} - c_{A2}) = k_G(c_{A1}RT - c_{A2}RT)$$

$$\boxed{\frac{k_y}{c} = \frac{k_c'}{x_{BM}} = k_G RT}$$

(b) Convert k_L to k_x and k_x'

$$N_A = k_L(c_{A1} - c_{A2}) = k_x(x_{A1} - x_{A2}) = \frac{k_x'}{x_{BM}}(x_{A1} - x_{A2})$$

$$N_A = k_L(c_{A1} - c_{A2}) = k_x\left(\frac{c_{A1}}{c} - \frac{c_{A2}}{c}\right) = \frac{k_x'}{x_{BM}}\left(\frac{c_{A1}}{c} - \frac{c_{A2}}{c}\right)$$

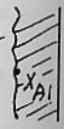
$$\boxed{k_L = \frac{k_x}{c} = \frac{k_x'}{x_{BM}c}}$$

(c) Convert k_G to k_y and k_c

$$N_A = k_G(p_{A1} - p_{A2}) = k_y(y_{A1} - y_{A2}) = k_c(c_{A1} - c_{A2})$$

$$N_A = k_G(p_{A1} - p_{A2}) = k_y\left(\frac{p_{A1}}{P} - \frac{p_{A2}}{P}\right) = k_c\left(\frac{p_{A1}}{RT} - \frac{p_{A2}}{RT}\right) \qquad \boxed{k_G = \frac{k_y}{P} = \frac{k_c}{RT}}$$

(7.2-3)



$p_{A2} = 0.05$ atm

$P = 1.5$ atm

$p_{A1} = 609(x_{A1}) = 609(2\times10^{-5}) = 0.01218$ atm

$y_{A1} = 0.01218/1.5 = 0.00812$ $\quad y_{A2} = 0.05/1.5 = 0.03333$

$y_{B1} = 1 - 0.00812 = 0.99188$ $\quad y_{B2} = 1 - 0.03333 = 0.9667$

$y_{BM} \cong (y_{B1} + y_{B2})/2 = 0.99188 + 0.9667 = 0.979$ $\quad T = 303$ K

Table 7.2-1 $\quad k_c' = 9.567\times10^{-4}$ m/s

$$N_A = k_y(y_{A1} - y_{A2}) = \left(\frac{k_c'}{y_{BM}}\frac{P}{RT}\right)(y_{A1} - y_{A2})$$

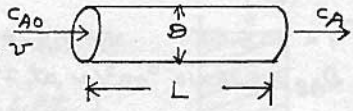
$$= \frac{9.567\times10^{-4}(1.5)}{0.979(82.06\times10^{-3})(303)}(0.00812 - 0.03333)$$

$$\boxed{N_A = -1.485\times10^{-6}\ \text{kg mol/s}\cdot\text{m}^2}$$

(7.3-2)

$T = 26.1°C \quad D = 6.35/1000 = 0.00635\ m$

$v = 0.0305\ m/s \quad L = 1.22\ m$



Physical properties from Example 7.3-2.

$\rho = 996\ kg/m^3 \quad \mu = 8.71 \times 10^{-4}\ Pa \cdot s$

$\mathcal{D}_{AB} = 1.245 \times 10^{-9}\ m^2/s \quad N_{Sc} = 702 \quad C_{Ai} = 0.02948\ kg\ mol/m^3$ (Solubility)

$$N_{Re} = \frac{D v \rho}{\mu} = \frac{0.00635(0.0305)(996)}{8.71 \times 10^{-4}}$$

$N_{Re} = 221.4$

Hence, flow is laminar.

$$\left(\frac{W}{\mathcal{D}_{AB} \rho L}\right) = N_{Re} N_{Sc} \frac{D}{L} \frac{\pi}{4}$$

$$= (221.4)(702)\left(\frac{0.00635}{1.22}\right)\left(\frac{\pi}{4}\right)$$

$$= 635.3$$

Eq. (7.3-24) for liquids

$$\frac{C_A - C_{A0}}{C_{Ai} - C_{A0}} = 5.5\left(\frac{W}{\mathcal{D}_{AB} \rho L}\right)^{-2/3}$$

$$\frac{C_A - 0}{0.02948 - 0} = 5.5(635.3)^{-2/3} = 0.0744$$

$$\boxed{C_A = 2.193 \times 10^{-3}\ kg\ mol/m^3}$$

(7.3-4)

$T = 325.6\ K \qquad v = 1.524\ m/s \qquad P = 202.6\ kPa\ (2\ atm)$

$\mathcal{D}_{AB} = 5.16\times10^{-6}\ m^2/s$ at 273 K and 101.3 kPa

(a) air parallel to a plate 0.152 m long (L)

Correct $\mathcal{D}_{AB}$ to 202.6 kPa and 325.6 K by Eq. (6.2-45)

$$\mathcal{D}_{AB} = \left(\frac{325.6}{273}\right)^{1.75}\left(\frac{101.3}{202.6}\right)(5.16\times10^{-6}) = 3.512\times10^{-6}\ m^2/s$$

For air, A.3,

$$\rho = 2(1.086)\ kg/m^3 \qquad \mu = 1.97\times10^{-5}\ Pa\cdot s \qquad p_{BM} = 202.6\times10^3\ Pa$$

$$N_{Sc} = \frac{\mu}{\rho\mathcal{D}_{AB}} = \frac{1.97\times10^{-5}}{2(1.086)(3.512\times10^{-6})} = 2.583 \qquad (2.583)^{2/3} = 1.882$$

$$N_{Re,L} = \frac{Lv\rho}{\mu} = \frac{(0.152)(1.524)(2\times1.086)}{1.97\times10^{-5}} = 25\,550$$

$$G_M = \frac{v\rho}{M_{av}} = \frac{1.524(2\times1.086)}{29(air)} = 0.1142\ kg\ mol/s\cdot m^2$$

Eq. (7.3-28) $\quad J_D = 0.036\,(N_{Re,L})^{-0.2} = 0.036(25\,550)^{-0.2} = 0.00473$

Eq. (7.3-5) $\quad J_D = \dfrac{k_G p_{BM}}{G_M}(N_{Sc})^{2/3} = 0.00473 = \dfrac{k_G(202.6\times10^3)(1.882)}{0.1142}$

$$\boxed{k_G = 1.416\times10^{-9}\ kg\ mol/s\cdot m^2\cdot Pa \quad (0.1058\ lb\ mol/h\cdot ft^2\cdot atm)}$$

(b) single sphere $\quad D_p = 0.0127\ m$

$$N_{Re} = \frac{D_p v\rho}{\mu} = \frac{(0.0127)(1.524)(2\times1.086)}{1.97\times10^{-5}} = 2132$$

Eq. (7.3-33)

$$N_{Sh} = 2.0 + 0.552 N_{Re}^{0.53} N_{Sc}^{1/3}$$

$$= 2.0 + 0.552(2132)^{0.53}(2.583)^{1/3} = 46.0$$

Eq. (7.3-3)

$$N_{Sh} = 46.0 = k_c'\frac{D_p}{\mathcal{D}_{AB}} = \frac{k_G \cancel{p_{BM}} RT D_p}{\cancel{P}\,\mathcal{D}_{AB}}$$

Table 7.2-1

$k_c' c = k_G p_{BM}$, $c = P/RT$

$P \cong p_{BM}$ (dilute)

$$46.0 = k_G\frac{(8314.3)(325.6)(0.0127)}{(3.512\times10^{-6})}$$

$$\boxed{k_G = 4.70\times10^{-9}\ kg\ mol/s\cdot m^2\cdot Pa \quad (0.3507\ lb\ mol/h\cdot ft^2\cdot atm)}$$

(7.3-5)

$c_{A1} = 0$ (pure water) $\quad T = 26.1\ °C$

$c_{Ai} = 0.00184$ lb mol/ft^3 (solubility)

$c_{A2} = 1.80 \times 10^{-4}$ lb mol/ft^3

$A = 0.129$ ft^2 surface area of bed of spheres

$V = 0.0701$ ft^3/h water flow

Mass transfer rate in packed bed.

Eq. (7.3-42)

$$N_A A = A k_c \frac{(c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})}{\ln\left(\frac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}\right)}$$

$$= 0.129 (k_c) \frac{[(0.00184 - 0) - (0.00184 - 0.000180)]}{\ln\left(\frac{0.00184 - 0}{0.00184 - 0.000180}\right)}$$

Material balance.

Eq. (7.3-43)

$$N_A A = V(c_{A2} - c_{A1})$$

$$= 0.0701(0.000180 - 0) = 1.262 \times 10^{-5} \text{ lb mol/h}$$

Equate (7.3-42) to (7.3-43).

$$1.262 \times 10^{-5} = 0.129 (k_c) \frac{[(0.00184 - 0) - (0.00184 - 0.000180)]}{\ln\left(\frac{0.00184 - 0}{0.00184 - 0.000180}\right)}$$

$$\boxed{k_c = 0.0559 \text{ ft/h}}$$

(7.3-6)

$\varepsilon = 0.499$ $\quad v' = 0.02198$ m/s (superficial vel.)

$N_{Sc} = 124.1$ $\quad T = 26.5\,°C$ $\quad \rho = 13\,530$ kg/m^3

$\mu = 1.577 \times 10^{-3}$ Pa·s

$D_p = 2.096$ mm $= 2.096/1000 = 0.002096$ m

$$N_{Re} = \frac{D_p v' \rho}{\mu} = \frac{(0.002096)(0.02198)(13\,530)}{1.577 \times 10^{-3}} = 395.3$$

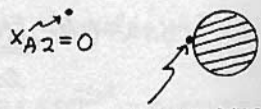
Eq. (7.3-38)

$$J_D = \frac{0.250}{\varepsilon} N_{Re}^{-0.31} = \frac{0.250}{0.499}(395.3)^{-0.31}$$

(a) $\boxed{J_D = 0.07843}$

M_A (Pb) = 207.19

M_B (Hg) = 200.59



$x_{A1} = 0.0172$ wt frac (solubility of Pb(A) in Hg (B))

Assume 100 kg saturated solution:
1.721 kg A and 98.279 kg B

$$x_{A1} \text{ (mol frac)} = \frac{\frac{1.721}{207.19}}{\frac{1.721}{207.19} + \frac{98.279}{200.59}} = 0.01667$$

$x_{B1} = 1.000 - 0.01667 = 0.98333$

$x_{B2} = 1.000 - 0 = 1.000$

$$x_{BM} \cong \frac{x_{B1} + x_{B2}}{2} = \frac{0.9833 + 1.000}{2} = 0.9916$$

Table 7.2-1

$k_c' C = k_c x_{BM} C$ or $k_c = \dfrac{k_c'}{x_{BM}}$

Eq. (7.3-5)

$$J_D = \frac{k_c'}{v'} N_{Sc}^{2/3}$$

$$0.07843 = \frac{k_c'}{0.02198}(124.1)^{2/3}$$

$k_c' = 6.929 \times 10^{-5}$ m/s

(b) $k_c = \dfrac{k_c'}{x_{BM}} = \dfrac{6.929 \times 10^{-5}}{0.9916}$ $\quad \boxed{k_c = 6.986 \times 10^{-5} \text{ m/s}}$

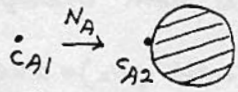
(7.4-1)

$t = 37°C \qquad D_k = \frac{2}{3}\,\mu m = 6.67 \times 10^{-7}\,m$

$D_{AB} = 3.25 \times 10^{-9}\,m^2/s \qquad P = 1.0$ atm air

From Eq. 7.4-1,

solubility of O_2 from air in water $= 2.26 \times 10^{-4}$ kg mol O_2/m^3



$c_{A1} = 2.26 \times 10^{-4}$ kg mol O_2/m^3

$c_{A2} = 0$ (fast reaction at surface)

$\Delta\rho = \rho_c - \rho_k = 1.0 - 1.0 = 0$

Eq. (7.4-1)

$$k_L' = \frac{2 D_{AB}}{D_k} + 0.31 N_{Sc}^{-2/3} \left(\frac{\Delta\rho \mu_c g}{\rho_c^2} \right)^{1/3}$$

$$k_L = \frac{2(3.25 \times 10^{-9})}{6.67 \times 10^{-5}} + 0$$

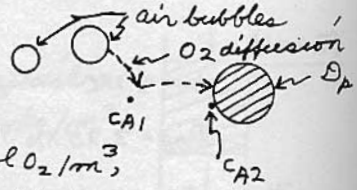
$$\boxed{k_L = 9.75 \times 10^{-3}\,m/s}$$

$$N_A = k_L (c_{A1} - c_{A2}) = (9.75 \times 10^{-3})(2.26 \times 10^{-4} - 0)$$

$$\boxed{N_A = 2.20 \times 10^{-6}\,kg\ mol\ O_2/s \cdot m^2}$$

(7.4-2)

(a)



For maximum rate of mass transfer to the microorganisms, c_{A1} = saturation = 2.26×10^{-4} kg mol O_2/m^3, $c_{A2} = 0$ (microorganism utilizes the O_2 faster than it is supplied to its surface.)

Physical property data from Example 7.4-1.

$D_p = 6.67\times10^{-7}$ m $\quad c_{A1}$(solub. of O_2) $= 2.26\times10^{-4}$ kg mol O_2/m^3

$\rho_c = 994$ kg/m^3 $\quad \rho_p = 1100$ kg/m^3 $\quad \Delta\rho = 1100-994 = 106$ kg/m^3

$\mathcal{D}_{AB} = 3.25\times10^{-9}$ m^2/s $\quad N_{Sc} = 215 \quad \mu_c = 6.947\times10^{-4}$ Pa·s

Eq. (7.4-1)

$$k_L' = \frac{2\mathcal{D}_{AB}}{D_p} + \frac{0.31}{N_{Sc}^{2/3}}\left[\frac{\Delta\rho\,\mu_c\,g}{\rho_c^2}\right]$$

$$= \frac{2(3.25\times10^{-9})}{6.67\times10^{-7}} + \frac{0.31}{215^{2/3}}\left[\frac{106(6.947\times10^{-4})(9.8066)}{(994)^2}\right]^{1/3}$$

$$\boxed{k_L' = 9.82\times10^{-3}\ \text{m/s}}$$

(1) Vol. solids $= \left(\frac{5.0}{1000}\text{ kg}\right)\left(\frac{1}{1100\text{ kg/m}^3}\right) = 4.545\times10^{-6}\ \text{m}^3 = V$

Let n = number of spheres $\qquad$ area sphere $= \pi D_p^2$

(2) $V = n\pi D_p^3/6 \qquad$ vol. sphere $= \frac{\pi D_p^3}{6}$

(3) area $= a\ \text{m}^2 = n\pi D_p^2$ or $n = a/\pi D_p^2$

Combine (2) and (3)

(4) $V = n\pi D_p^3/6 = (a/\pi D_p^2)\pi D_p^3/6 = a D_p/6$

(5) $A = a = 6V/D_p = 6(4.545\times10^{-6})/(6.67\times10^{-7}) = 40.88$ m^2 surface area (A)

Assume $k_L = k_L'$ for dilute solutions.

$N_A = k_L(c_{A1} - c_{A2})$

$N_A a = N_A A = a k_L(c_{A1} - c_{A2}) = 40.88(9.82\times10^{-3})(2.26\times10^{-4} - 0)$

$$\boxed{N_A A = 9.07\times10^{-5}\ \text{kg mol } O_2/\text{s}}$$

(b) Experimental $N_A A = 6.30\times10^{-6}$ kg mol/s $\quad$ assume as zero.

$6.30\times10^{-6} = a k_L(c_{A1} - c_{A2}) = 40.88(9.82\times10^{-3})(c_{A1} - 0)$

$c_{A1} = 1.57\times10^{-5}$ kg mol O_2/m^3

$\frac{1.57\times10^{-5}}{2.26\times10^{-4}}(100) = \boxed{6.95\ \%\ \text{of saturation}}$

(7.5-1)

Prove

(1) $N_A + N_B = c v_M$

From Table 7.5-1, $N_A = c_A v_A$, $N_B = c_B v_B$. Also, using Eq. (7.5-2) for $v_M = \frac{c_A}{c} v_A + \frac{c_B}{c} v_B$,

(2) $c_A v_A + c_B v_B = c\left[\frac{c_A}{c} v_A + \frac{c_B}{c} v_B\right]$

Cancelling c,

(3) $c_A v_A + c_B v_B = c_A v_A + c_B v_B$

(7.5-2)

Prove

(1) $j_A = n_A - w_A(n_A + n_B)$

From Table 7.5-1, $j_A = \rho_A(v_A - v)$, $n_A = \rho_A v_A$, and $n_B = \rho_B v_A$.

Also, from Eq. (7.5-1) for $v = \frac{\rho_A}{\rho} v_A + \frac{\rho_B}{\rho} v_B$. Also, $w_A = \rho_A/\rho$.

(2) $\rho_A(v_A - v) = \rho_A v_A - \frac{\rho_A}{\rho}(\rho_A v_A + \rho_B v_B)$

(3) $\rho_A v_A - \rho_A\left[\frac{\rho_A}{\rho} v_A + \frac{\rho_B}{\rho} v_B\right] = \rho_A v_A - \frac{\rho_A}{\rho}(\rho_A v_A + \rho_B v_B)$

(7.5-3)

(1) $j_A = -\rho D_{AB}\, dw_A/dz$

(2) $w_A = \dfrac{x_A M_A}{x_A M_A + (1-x_A) M_B}$

Differentiate both sides.

(3) $dw_A = \dfrac{[x_A M_A + (1-x_A) M_B] M_A dx_A - x_A M_A (M_A dx_A - M_B dx_A)}{[x_A M_A + (1-x_A) M_B]^2}$

Cancel out like terms in numerator. Substitute $M = [x_A M_A + (1-x_A) M_B]$ where M = average molecular weight.

(4) $dw_A = \dfrac{M_A M_B\, dx_A}{M^2}$

Substituting (4) to (1),

(5) $j_A = -\rho D_{AB} \dfrac{M_A M_B}{M^2} dx_A/dz$

Substitute $M = \rho/c$ to (5)

(6) $j_A = -\dfrac{\rho D_{AB} M_A M_B\, dx_A/dz}{\rho^2/c^2}$

(7) $j_A = -\dfrac{c^2}{\rho} M_A M_B D_{AB} \dfrac{dx_A}{dz}$

(7.5-4)

Show

$$c(v_A - v_B) = \frac{-c\,\mathcal{D}_{AB}\,dx_A}{x_A x_B\,dz}$$

(1) $N_A = J_A^* + c_A v_M$

Table 7.5-1

$N_A = c_A v_A \qquad J_A^* = -c\,\mathcal{D}_{AB}\dfrac{dx_A}{dz}$

Eq. (7.5-2)

$$v_M = \frac{c_A}{c} v_A + \frac{c_B}{c} v_B$$

Substitute into original equation (1) for N_A

(2) $$c_A v_A = -c\,\mathcal{D}_{AB}\frac{dx_A}{dz} + c_A\left[\frac{c_A}{c} v_A + \frac{c_B}{c} v_B\right]$$

Rearrange as follows.

$$c_A v_A = \frac{c}{c} c_A v_A = \frac{(c_A + c_B)}{c} c_A v_A = \frac{c_A c_A}{c} v_A + \frac{c_A c_B}{c} v_A$$

Substitute into Eq. (2).

$$\frac{c_A c_A}{\cancel{c}} v_A + \frac{c_A c_B}{c} v_A = -c\,\mathcal{D}_{AB}\frac{dx_A}{dz} + \frac{c_A c_A}{\cancel{c}} v_A + \frac{c_A c_B}{c} v_B$$

Rearrange.

$$\frac{c_A c_B}{c}(v_A - v_B) = -c\,\mathcal{D}_{AB}\frac{dx_A}{dz}$$

Multiply left side by c/c.

$$c\,\frac{c_A}{c}\,\frac{c_B}{c}(v_A - v_B) = c\,x_A x_B\,(v_A - v_B) = -c\,\mathcal{D}_{AB}\frac{dx_A}{dz}$$

$$\boxed{c(v_A - v_B) = \frac{-c\,\mathcal{D}_{AB}}{x_A x_B}\frac{dx_A}{dz}}$$

(7.5-7)

$t = 1\times10^5\ s \qquad k' = 1\times10^{-4}\ s^{-1} \qquad \mathcal{D}_{AB} = 1\times10^{-9}\ m^2/s$

$c_{A0} = 1.0\ kg\ mol/m^3$

Eq. (7.5-39)

$$\frac{c_A}{c_{A0}} = \frac{1}{2}\left(e^{-z\sqrt{k'/\mathcal{D}_{AB}}}\right)\operatorname{erfc}\left(\frac{z}{2\sqrt{t\mathcal{D}_{AB}}} - \sqrt{k't}\right)$$

$$+ \frac{1}{2}\left(e^{z\sqrt{k'/\mathcal{D}_{AB}}}\right)\operatorname{erfc}\left(\frac{z}{2\sqrt{t\mathcal{D}_{AB}}} + \sqrt{k't}\right)$$

$\sqrt{k't} = \sqrt{1\times10^{-4}\times10^5} = \sqrt{10} \qquad \sqrt{t\mathcal{D}_{AB}} = \sqrt{1\times10^5\times1\times10^{-9}} = 10^{-2}$

$\sqrt{k'/\mathcal{D}_{AB}} = \sqrt{\frac{1\times10^{-4}}{1\times10^{-9}}} = \sqrt{10^5}$

$\operatorname{erfc}(-x) = 1 + \operatorname{erf} x$

$\operatorname{erfc}(x) = 1 - \operatorname{erf} x$

$z = 0\ m$

$$\frac{c_A}{c_{A0}} = \frac{1}{2}\left(e^{-0}\right)\operatorname{erfc}\left(0 - \sqrt{10}\right)^{(1+0)} + \frac{1}{2}\left(e^{-0}\right)\operatorname{erfc}\left(0 + \sqrt{10}\right)^{(1-0)} = 1.000$$

$$\boxed{c_A = c_{A0} = 1.00\ kg\ mol/m^3}$$

$z = 0.004\ m$ (4 mm)

$$\frac{c_A}{c_{A0}} = \frac{1}{2}\left(e^{-0.004\sqrt{10^5}}\right)\operatorname{erfc}\left(\frac{0.004}{2\times10^{-2}} - \sqrt{10}\right)$$

$$+ \frac{1}{2}\left(e^{+0.004\sqrt{10^5}}\right)\operatorname{erfc}\left(\frac{0.004}{2\times10^{-2}} + \sqrt{10}\right) = 0.2782$$

$$\boxed{c_A = 0.2782\ kg\ mol/m^3}$$

$z = 0.0100\ m$ (10 mm)

$$\frac{c_A}{c_{A0}} = \frac{1}{2}\left(e^{-0.010\sqrt{10^5}}\right)\operatorname{erfc}\left(\frac{0.010}{2\times10^{-2}} - \sqrt{10}\right)$$

$$+ \frac{1}{2}\left(e^{+0.010\sqrt{10^5}}\right)\operatorname{erfc}\left(\frac{0.010}{2\times10^{-2}} + \sqrt{10}\right)$$

$$\boxed{c_A = 0.0423\ kg\ mol/m^3}$$

Eq. (7.5-40)

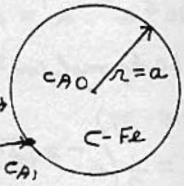
$$Q = c_{A0}\sqrt{\mathcal{D}_{AB}/k'}\left[\left(k't + \tfrac{1}{2}\right)\operatorname{erf}\sqrt{k't} + \sqrt{\frac{k't}{\pi}}\ e^{-k't}\right]$$

$$= 1.00\sqrt{\frac{1\times10^{-9}}{1\times10^{-4}}}\left[\left(10 + \tfrac{1}{2}\right)\operatorname{erf}\sqrt{10} + \sqrt{\frac{10}{\pi}}\ e^{-10}\right]$$

$$\boxed{Q = 0.03320\ kg\ mol\ absorbed/m^2}$$

(7.5-10)

$t = 2.0\ s$



$C + \frac{1}{2} O_2(g) \rightarrow CO(g)$ Instant. reac. at surface

$c_{A1} = 0$ (concentration of C (carbon) at surface)

$D_{AB} = 7.5 \times 10^{-9}\ m^2/s$ $\quad c_{A0} = 4.0$ wt %. $\quad c_{Aav} = 0.1$ wt %.

Eq. 5.3-13

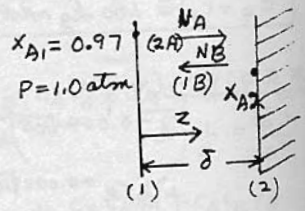
$$E_S = \text{fraction unremoved} = \frac{c_{A1} - c_{Aav}}{c_{A1} - c_{A0}} = \frac{0 - 0.1}{0 - 4.0} = 0.025$$

From Eq. for sphere, $\frac{Dt}{a^2} = 0.318$

$$\frac{Dt}{a^2} = \frac{7.5 \times 10^{-9}(2.0)}{a^2} = 0.318 \qquad a = 2.17 \times 10^{-4}\ m = \boxed{0.217\ mm}$$

(7.5-11)

$2A \rightarrow B \qquad N_A = -2N_B$



Eq. (6.2-4)

(1) $N_A = -D_{AB}c\frac{dx_A}{dz} + x_A(N_A + N_B)$ $\quad (1.0N_A - \frac{1}{2}N_A)$

(2) $N_A = -cD_{AB}c\frac{dx_A}{dz} + x_A(\frac{1}{2}N_A)$

(3) $N_A(1 - \frac{1}{2}x_A) = -D_{AB}c\frac{dx_A}{dz}$

(4) $N_A\int_{z=0}^{z=\delta} dz = -D_{AB}c\int_{x_{A1}}^{x_{A2}} \frac{dx_A}{(1 - \frac{1}{2}x_A)}$

(5)

(a) $$\boxed{N_A = \frac{2D_{AB}c}{\delta}\ln\left[\frac{1 - \frac{1}{2}x_{A2}}{1 - \frac{1}{2}x_{A1}}\right] \quad (x_{A2} = 0)}$$

$\delta = 0.0013\ m \qquad D_{AB} = 0.2 \times 10^{-4}\ m^2/s \qquad T = 298\ K \qquad c = P/RT$

(b) $$N_A = \frac{2(0.2 \times 10^{-4})}{0.0013}\frac{(1.0)}{82.06 \times 10^{-3}(298)}\ln\left[\frac{1 - 0}{1 - \frac{1}{2}(0.97)}\right] = \boxed{8.35 \times 10^{-4}\ kg\ mol/s \cdot m^2}$$

(c) Slow reaction

$N_{Az} = \delta = k_1' c_{A2} = k_1' c\, x_{A2} \qquad x_{A2} = \frac{N_A}{k_1' c} = \frac{N_A RT}{k_1' P}$

$k_1' = 0.53 \times 10^{-2}\ m/s$

$$\boxed{N_A = \frac{2D_{AB}c}{\delta}\ln\left[\frac{1 - \frac{1}{2}\frac{N_A RT}{k_1' P}}{1 - \frac{1}{2}x_{A1}}\right]}$$

Trial and error to solve for N_A.

$\boxed{N_A = 1.766 \times 10^{-4}\ kg\ mol/s \cdot m^2}$ $\qquad \boxed{x_{A2} = 0.8148}$

(7.6-2)

He(A) Ar(B) $T = 298\ K$ $x_{A1} = 0.8$ $x_{A2} = 0.2$

$L = 15\ mm = 0.015\ m$ $\mathcal{D}_{AB} = 7.29 \times 10^{-5}\ m^2/s$

$\bar{r} = 1000\ \mathring{A} = 1.0 \times 10^{-7}\ m$ $P = 1.013 \times 10^5\ Pa$

Eq. (7.6-3)

$$\mathcal{D}_{KA} = 97.0\,\bar{r}\sqrt{\frac{T}{M_A}} = 97.0\,(1.0 \times 10^{-7})\sqrt{\frac{298}{4.002}}$$

(a) $$\boxed{\mathcal{D}_{KA} = 8.37 \times 10^{-5}\ m^2/s}$$

(b) $$\alpha = 1.0 - \sqrt{\frac{M_A}{M_B}} = 1.0 - \sqrt{\frac{4.002}{39.95}} = 0.6835$$

Eq. (7.6-12)

$$N_A = \frac{\mathcal{D}_{AB} P}{\alpha RTL} \ln\left[\frac{1 - \alpha x_{A2} + \mathcal{D}_{AB}/\mathcal{D}_{KA}}{1 - \alpha x_{A1} + \mathcal{D}_{AB}/\mathcal{D}_{KA}}\right]$$

$$= \frac{(7.29 \times 10^{-5})(1.013 \times 10^5)}{(0.6835)(8314.3)(298)(0.015)} \ln\left[\frac{1 - 0.6835(0.2) + \frac{7.29 \times 10^{-5}}{8.37 \times 10^{-5}}}{1 - 0.6835(0.8) + \frac{7.29 \times 10^{-5}}{8.37 \times 10^{-5}}}\right]$$

$$\boxed{N_A = 7.843 \times 10^{-5}\ kgmol/s \cdot m^2}$$

(c) Use Eq. (7.6-15)

$$x_{Aav} = \frac{0.2 + 0.8}{2} = 0.5$$

$$\mathcal{D}''_{NA} = \frac{1}{(1 - \alpha x_{Aav})/\mathcal{D}_{AB} + 1/\mathcal{D}_{KA}}$$

$$= \frac{}{(1 - 0.6835 \times 0.5)/7.29 \times 10^{-5} + 1/8.37 \times 10^{-5}} = 4.766 \times 10^{-5}\ m^2/s$$

Eq. (7.6-14)

$$N_A = \frac{\mathcal{D}''_{NA} P}{RTL}(x_{A1} - x_{A2})$$

$$= \frac{(4.766 \times 10^{-5})(1.013 \times 10^5)(0.8 - 0.2)}{(8314.3)(298)(0.015)}$$

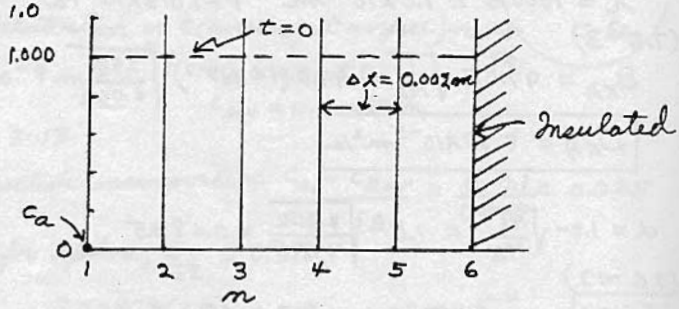
$$\boxed{N_A = 7.794 \times 10^{-5}\ kgmol/s \cdot m^2}$$

(7.7-1)

$\mathcal{D}_{AB} = 1.0 \times 10^{-10}\ m^2/s$ $\quad c_{original} = 1.00$

$c_a = 0$ $\quad t = 12 \times 10^4\ s$ $\quad \Delta x = 0.002\ m$ $\quad L = 0.01\ m$

$K = 1.0$



$$M = 2.0 = \frac{(\Delta x)^2}{\mathcal{D}_{AB}\,\Delta t} = \frac{(0.002)^2}{(1.0 \times 10^{-10})(\Delta t)}$$

$$\Delta t = 2.0 \times 10^4\ s$$

$$\text{Number of } \Delta t \text{ steps} = \frac{12 \times 10^4\ s}{2.0 \times 10^4\ s} = 6.0$$

Equations to use

Eq (7.7-12) $\qquad {}_0c_1 = 1.00 \times 10^{-3}$

$(n=1)$ $\quad {}_1c_a = \dfrac{c_a/K + {}_0c_1}{2} = \dfrac{0/1 + 1.00}{2} = 0.500$ (First Δt only)

Eq. (7.7-13) Next Δt, etc. $\quad c_1 = \dfrac{c_a}{K} = \dfrac{0}{K} = 0$

Eq. (7.7-5)

$(n = 2,3,4,5)$ $\quad {}_{t+\Delta t}c_n = \dfrac{{}_tc_{n-1} + {}_tc_{n+1}}{2}$

Eq. (7.7-9) Insulated end $M = 2$, $f = n = 6$

$(n=6)$ $\quad {}_{t+\Delta t}c_6 = \dfrac{{}_tc_{6-1} + {}_tc_{6+1}}{2} = {}_tc_5$

Calculations

For $1\Delta t$

$(n=1)$ $\quad {}_{t+1\Delta t}c_1 = \dfrac{c_a + 1.00}{2} = \dfrac{0 + 1.00}{2} = 0.500$

$(n = 2,3,4,5)$

$${}_{t+\Delta t}c_2 = ({}_0c_1 + {}_0c_3)/2 = (0.500 + 1.00)/2 = 0.75$$

$$c_3 = ({}_0c_2 + {}_0c_4)/2 = (1.00 + 1.00)/2 = 1.00$$

$$c_4 = ({}_0c_3 + {}_0c_5)/2 = (1.00 + 1.00)/2 = 1.00$$

(7.7-1) - p.2

$$c_5 = ({}_0c_4 + {}_0c_6)/2 = (1.00 + 1.00)/2 = 1.00$$

$(n=6)$

$$_{t+\Delta t}c_6 = {}_0c_5 = 1.00$$

For $2\Delta t$

$(n=1)$ $_{t+2\Delta t}c_1 = c_a/K = 0$

$(n=2,3,4,5)$

$$_{t+2\Delta t}c_2 = ({}_1c_1 + {}_1c_3)/2 = (0.0 + 1.00)/2 = 0.500$$

$$c_3 = ({}_1c_2 + {}_1c_4)/2 = (0.750 + 1.000)/2 = 0.875$$

$$c_4 = ({}_1c_3 + {}_1c_5)/2 = (1.00 + 1.00) = 1.00$$

$$c_5 = ({}_1c_4 + {}_1c_6)/2 = (1.00 + 1.00) = 1.00$$

$(n=6)$ $_{t+2\Delta t}c_6 = {}_1c_5 = 1.00$

For $3\Delta t$

$(n=1)$ $_{t+3\Delta t}c_1 = 0$

$(n=2,3,4,5)$

$$c_2 = ({}_2c_1 + {}_2c_3)/2 = (0 + 0.875)/2 = 0.4375$$

$$c_3 = ({}_2c_2 + {}_2c_4)/2 = (0.500 + 1.00)/2 = 0.750$$

$$c_4 = ({}_2c_3 + {}_2c_5)/2 = (0.875 + 1.00)/2 = 0.9375$$

$$c_5 = ({}_2c_4 + {}_2c_6)/2 = (1.00 + 1.00)/2 = 1.00$$

$(n=6)$ $_{t+3\Delta t}c_6 = {}_2c_5 = 1.00$

For $4\Delta t$

$(n=1)$ $_{t+4\Delta t}c_1 = 0$

$(n=2,3,4,5)$

$$c_2 = ({}_3c_1 + {}_3c_3)/2 = (0 + 0.750)/2 = 0.3750$$

$$c_3 = ({}_3c_2 + {}_3c_4)/2 = (0.4375 + 0.9375)/2 = 0.6875$$

$$c_4 = ({}_3c_3 + {}_3c_5)/2 = (0.750 + 1.000)/2 = 0.8750$$

$$c_5 = ({}_3c_4 + {}_3c_6)/2 = (0.9375 + 1.000)/2 = 0.9688$$

$(n=6)$ $_{t+4\Delta t}c_6 = {}_3c_5 = 1.00$

For $5\Delta t$

$(n=1)$ $_{t+5\Delta t}c_1 = 0$

$(n=2,3,4,5)$

$$c_2 = ({}_4c_1 + {}_4c_3)/2 = (0 + 0.6875)/2 = 0.3438$$

$$c_3 = ({}_4c_2 + {}_4c_4)/2 = (0.375 + 0.875)/2 = 0.625$$

$$c_4 = ({}_4c_3 + {}_4c_5)/2 = (0.6875 + 0.9688)/2 = 0.8281$$

$$c_5 = ({}_4c_4 + {}_4c_6)/2 = (0.875 + 1.000)/2 = 0.9375$$

(7.7-1) - p.3

$(n=6)$ $\quad {}_{t+5\Delta t}C_6 = {}_4C_5 = 0.9688$

For $6\Delta t$ (Final)

$(n=1)$ $\quad {}_{t+6\Delta t}C_1 = 0$

$C_2 = ({}_5C_1 + {}_5C_3)/2 = (0 + 0.625)/2 = 0.3125$

$C_3 = ({}_5C_2 + {}_5C_4)/2 = (0.3438 + 0.8281)/2 = 0.5859$

$C_4 = ({}_5C_3 + {}_5C_5)/2 = (0.625 + 0.9375)/2 = 0.7813$

$C_5 = ({}_5C_4 + {}_5C_6)/2 = (0.8281 + 0.9688)/2 = 0.8984$

$(n=6)$ $\quad {}_{t+6\Delta t}C_6 = {}_5C_5 = 0.9375$

n	C, kg-mol/m^3	x, m
1	0	0
2	0.3125	0.002
3	0.5859	0.004
4	0.7813	0.006
5	0.8984	0.008
6	0.9375	0.010

(7.7-2)

$\Delta x = 0.0005$ m $\quad L = 0.01$ m $\quad \frac{0.01}{0.0005} = 20$ Δx slices $\quad n = 1$ to 21

$M = 2.0 = \frac{(\Delta x)^2}{\mathcal{D}_{AB}\Delta t} = \frac{(0.0005)^2}{(1.0\times10^{-10})\Delta t}$ $\quad \Delta t = 1250$ s $\quad$ No. of steps $= \frac{12\times10^4}{1250} = 96.0$

Equations to use

$(n=1)$ $\quad {}_1C_a = (C_a/K + {}_0C_1)/2 = (0/1 + 1.00)/2 = 0.500$ (1st Δt only)

$C_1 = C_a/K = 0/1 = 0$ (rest Δt's)

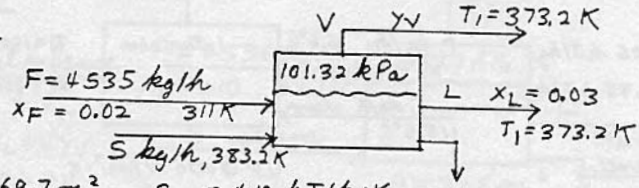
$(n = 2, 3, 4, \ldots)$ $\quad {}_{t+\Delta t}C_n = ({}_tC_{n-1} + {}_tC_{n+1})/2$

$(n=21)$ $\quad {}_{t+\Delta t}C_{21} = ({}_tC_{21-1} + {}_tC_{21+1})/2 = {}_tC_{20}$

n	C	n	C	n	C	n	C	n	C
1	0	6	0.3896	11	0.6905	16	0.8643	21	0.9187
2	0.0812	7	0.4590	12	0.7356	17	0.8841		
3	0.1615	8	0.5242	13	0.7755	18	0.8994		
4	0.2402	9	0.5847	14	0.8101	19	0.9101		
5	0.3165	10	0.6402	15	0.8397	20	0.9165		

PART II - UNIT OPERATIONS

(8.4-1)



$A = 69.7\ m^2$ $\quad c_{pF} = 4.10$ kJ/kg·K

$F = L + V$

$Fx_F = Lx_L + Vy_V$

$4535(0.02) = L(0.03) + V(0)$

$$\boxed{\begin{array}{l} L = 3023 \text{ kg/h} \\ V = 1512 \text{ kg/h} \end{array}}$$

Heat balance. Datum $= T_1 = 373.2$ K

Table A.2 $\quad \lambda_{383.2K} = 2230.2$ kJ/kg $\quad \lambda_{373.2} = 2257.06$ kJ/kg

$Fc_{pF}(311 - T_1) + S\lambda_S = L(373.2 - T_1)c_{pL} + V\lambda_V$

$(4535)(4.10)(311.0 - 373.2) + S(2230.2) = 0 + 1512(2257.06)$

$$\boxed{S = 2049 \text{ kg steam/h}}$$

$q = S\lambda_S = UA\Delta T$

$q = 2049(2230.2) \times 10^3 \times \frac{1}{3600} = U(69.7)(383.2 - 373.2)$

$$\boxed{U = 1823\ W/m^2 \cdot K}$$

(8.4-2)

$F = 6804$ kg/h $\quad S\lambda = 2049(2230.2)$

Heat balance. Datum $= T_1 = 373.2$ K

$6804(4.10)(311.0 - 373.2) + 2049(2230.2) = 0 + V(2257.06)$

$V = 1256$ kg/h $\quad L = 6804 - 1256 = 5548$ kg/h

$6804(0.02) = 5548(x_L) + V(0)$

$x_L = 0.0245$

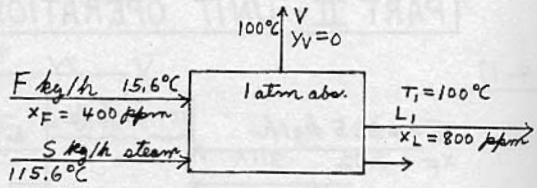
$$\boxed{\begin{array}{l} V = 1256 \text{ kg/h} \\ L = 5548 \text{ kg/h} \\ x_L = 2.45\ \% \end{array}}$$

(8.4-4)

A.2-9

$h_{100°C} = 2257.06$ kJ/kg

$h_{115.6°C} = 2214.85$ kJ/kg



$U = 2270\ W/m^2 \cdot K$

$A = 83.6\ m^2$

Material balance

$F(400) = L_1(800) + V(0)$ (x_F)

$L_1 = 0.5F$

$F = L_1 + V = 0.5F + V$

$V = 0.5F$

Heat balance: datum 100°C

$F(4.188)(15.6-100.0) + S(2214.85) = L_1(100-100) + V(2257.06)$ (C_{pL})

$-353.47F + S(2214.85) = 2257.06V$ kJ/h

Rate equation

$q = UA\Delta T = 2270(83.6)(115.6-100.0)(3600/1000)$ kJ/h

$q = 10\,657\,595$ kJ/h

Substitute q for S(2214.85) in heat balance.

$-353.47F + 10\,657\,595 = 2257.06(0.5F)$ (V)

$$\boxed{\begin{array}{l} F = 7191 \text{ kg/h} \\ V = 3595 \text{ kg/h distilled } H_2O \end{array}}$$

(8.4-5)

(a) For 172.4 kPa (Table A.2-9), b. pt. $H_2O = 115.6°C$

Fig. 8.4-2

For 30% NaOH, b. pt. = 130.6°C

$\boxed{\text{b. pt.} = 130.6°C}$

B. pt. rise = 130.6 − 115.6 = $\boxed{15°C \text{ b. pt. rise}}$

(b) For 3.45 kPa (Table A.2-9), b. pt. H_2O = 26.7°C

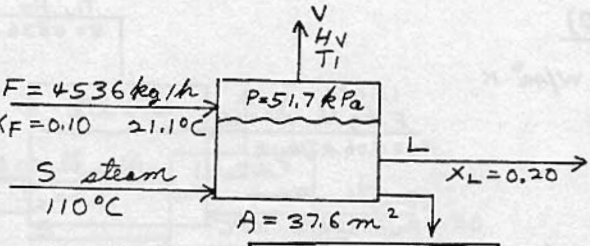
Fig. 8.4-2

For 60% NaOH, b. pt. = 81.7°C

$\boxed{\text{b. pt.} = 81.7°C}$

B. pt. rise = 81.7 − 26.7 = $\boxed{55°C \text{ b. pt. rise}}$

(8.4-8)



Material balance

$F = L + V$

$Fx_F = Lx_L + Vy_V$

$4536 = L + V$

$4536(0.10) = L(0.20) + V(0)$

$\boxed{L = 2268 \text{ kg/h} \quad V = 2268 \text{ kg/h}}$

at 51.7 kPa, b.pt. H_2O = 82.2°C (Table A.2-9)

Fig. 8.4-2

B.pt. 20% NaOH = 88.9°C

B.pt. rise = 88.9 − 82.2 = 6.7°C

Enthalpies (Fig. 8.4-3)

10% NaOH at 21.1°C h_F = 74.4 kJ/kg

20% NaOH at 88.9°C h_L = 321.0 kJ/kg

Steam λ (saturated at 110°C) = 2230.2 kJ/kg (A.2-9)

H_V (superheated at 88.9°C and 51.7 kPa) = 2660.6 (A.2-10 interpolated)

alternate calculation of H_V (see Eq. 8.4-3).

$H_V = H_V$ (satd at 82.2°C, 51.7 kPa) + 1.884 (ΔT superheat)

$H_V = 2647.3 + 1.884(88.9 - 82.2) = 2660$ kJ/kg (close check!)

Heat balance: datum 0°C

$Fh_F + S\lambda = Lh_L + VH_V$

$4536(74.4) + S(2230.2) = 2268(321.0) + 2268(2660)$

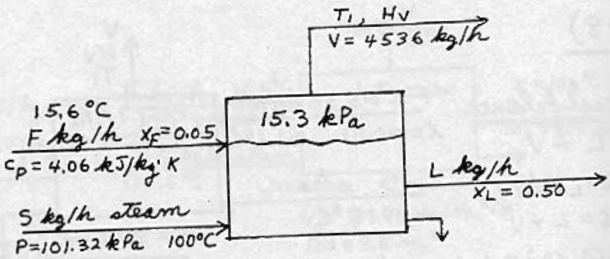
$\boxed{S = 2880 \text{ kg steam/h}}$

$q = S\lambda = 2880(2230.2) \times 10^3 \times \frac{1}{3600} = UA\Delta T = U(37.6)(110 - 88.9)$

$\boxed{U = 2249 \text{ W/m}^2\cdot\text{K}}$

(8.4-10)

$U = 1988\ W/m^2 \cdot K$



Material balance

(1) $F = L + 4536$

(2) $F(0.05) = L(0.50) + 4536(0)$

Solving (2), $L = 0.10F$

Substituting $L = 0.10F$ into (1),

(3) $F = 0.10F + 4536$

$F = 5040$ kg/h $\qquad L = 504$ kg/h

From Table A.2-9,

$T_1 = 54.3$ °C (saturated, no b.pt. rise, at 15.3 kPa)

$H_V(54.3°C) = 2372.4$ kJ/kg

Heat balance: datum = 54.3 °C

(4) $F h_F + S h_S = L h_L + V H_V$

$5040(\overset{c_p}{4.06})(15.6 - 54.3) + S(\overset{h_S\ (100°C)}{2257.1}) = 0 + 4536(2372.4)$

$$\boxed{S = 5119 \text{ kg steam/h}}$$

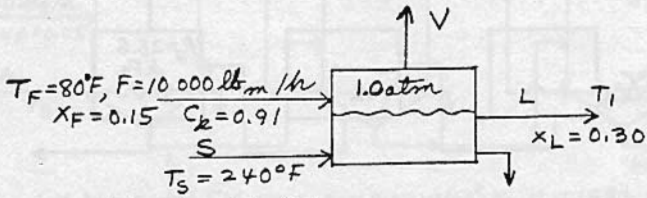
Heat transfer rate

$$q = S h_S = 5119(2257.1)\frac{kJ}{h}\left(10^3 \frac{J}{kJ}\right)\left(\frac{1}{3600\ s/h}\right) = 5119(2257.1)\frac{(10^3)}{3600}\ W$$

$$q = 5119(2257.1)\frac{(10^3)}{3600} = UA\Delta T = 1988(A)(100.0 - 54.3)$$

$$\boxed{A = 35.35\ m^2}$$

(8.4-12)



$U = 350\ Btu/h \cdot ft^2 \cdot °F$

Material balance

$F = L + V$

$10\,000 = L + V$

$Fx_F = Lx_L + Vy_V$

$10\,000(0.15) = L(0.30) + V(0)$

$$\boxed{\begin{aligned} L &= 5000\ lb_m/h \\ V &= 5000\ lb_m/h \end{aligned}}$$

From Eq. 8.5-1,

$BPR = 3.2x + 11.2x^2 = 3.2(0.30) + 11.2(0.30)^2 = 1.96°F$

$\boxed{BPR = 2.0°F}$ (1.1°C)

$T_1 =$ T boiling point at 1 atm + BPR

$T_1 = 212°F + 2°F = 214°F$

Heat balance: datum = $T_1 = 214°F$

$h_F = C_{pF}(T_F - T_1) = 0.91(80-214)$

λ_S (latent heat at $T_S = 240°F$) = 952.3 Btu/lb_m

H_V (superheated vapor at 214°F, 1 atm abs). See Eq. 8.4-3 in text.

$H_V = 2257.1$ (saturated λ at 100°C) + 1.884 (ΔT K superheat) SI units

$H_V = 970.3$ (saturated λ at 212°F) + 0.45 (ΔT superheat) English.

$= 970.3 + 0.45(2) = 971.2$ Btu/lb_m

$Fh_F + S\lambda_S = Lh_L + VH_V$

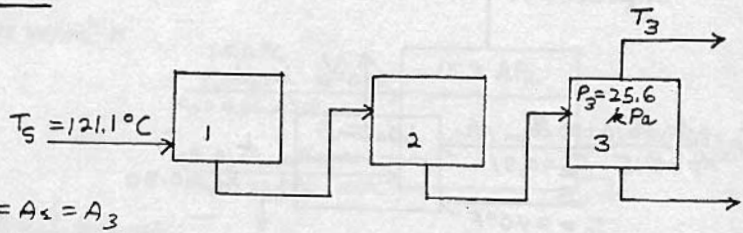
$10\,000(0.91)(80-214) + S(952.3) = 5000(0) + 5000(971.2)$

$\boxed{S = 6\,380\ lb_m \text{ steam}/h\ (2894\ kg/h)}$

$q = 6{,}380(952.3) = UA\Delta T = 350(A)(240-214)$

$\boxed{A = 667\ ft^2\ (62.0\ m^2)}$

(8.5-1)



$A_1 = A_2 = A_3$

at $P_3 = 25.6$ kPa, $T_3 = 65.6°C$ (Table A.2-9, saturated)

Eq. (8.5-5)

$$\Sigma \Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3 = T_S - T_3 = 121.1 - 65.6 = 55.5°C$$

$$U_1 = 2840 \, W/m^2 \cdot K, \; U_2 = 1988, \; U_3 = 1420$$

Eq. (8.5-6)

$$\Delta T_1 = \Sigma \Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3} = \frac{55.5(1/2840)}{1/2840 + 1/1988 + 1/1420}$$

$$\Delta T_1 = 12.53°C$$

$$\Delta T_2 = \frac{55.5(1/1988)}{1/2840 + 1/1988 + 1/1420} = 17.90°C$$

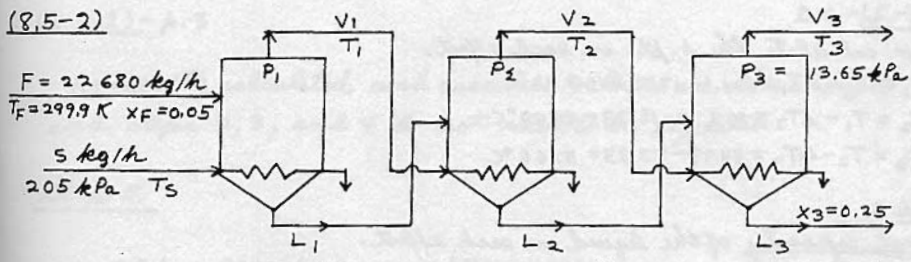
$$\Delta T_3 = \frac{55.5(1/1420)}{1/2840 + 1/1988 + 1/1420} = 25.07°C$$

$$T_1 = T_S - \Delta T_1 = 121.1 - 12.5 = \boxed{108.6°C = T_1}$$

$$T_2 = T_1 - \Delta T_2 = 108.6 - 17.9 = \boxed{90.7°C = T_2}$$

$$T_3 = T_2 - \Delta T_3 = 90.7 - 25.1 = \boxed{65.6°C = T_3}$$

(8.5-2)



$c_p(\text{liq.}) = 4.19 - 2.35x$ kJ/kg·K $\quad U_1 = 3123$ W/m²·K, $U_2 = 1987$, $U_3 = 1136$

Step 1.

For 13.65 kPa, sat. temp. $T_3 = 51.7°C$ (A.2)

For $P_S = 205$ kPa, $T_S = 121.1°C$ $\qquad$ Neglect BPR.

Step 2.

Overall and solids balances

$F = 22\,680 = L_3 + (V_1 + V_2 + V_3)$

$Fx_F = L_3x_3 + 0$

$22\,680(0.05) = L_3(0.25)$

$L_3 = 4536$ kg/h $\qquad (V_1+V_2+V_3) = 22\,680 - 4536 = 18\,144$ kg/h

Assume equal vaporization in each effect. $V_1 = V_2 = V_3 = 6048$ kg/h

Total material balances on (1), (2), and (3) evaporators.

(1) $F = 22\,680 = V_1 + L_1 = 6048 + L_1$ $\qquad L_1 = 16\,632$

(2) $L_1 = 16\,632 = V_2 + L_2 = 6048 + L_2$ $\qquad L_2 = 10\,584$

(3) $L_2 = 10\,584 = V_3 + L_3 = 6048 + L_3$ $\qquad L_3 = 4536$

Solids balance on (1), (2), and (3)

(1) $22\,680(0.05) = 0 + 16\,632(x_1)$ $\qquad x_1 = 0.068$

(2) $16\,632(0.068) = 0 + 10\,584(x_2)$ $\qquad x_2 = 0.107$

(3) $10\,584(0.107) = 0 + 4536(x_3)$ $\qquad x_3 = 0.250$ (check balance)

Step 3.

Eq. (8.5-5) (no BPR) $\qquad \Sigma\Delta T = T_S - T_3 = 121.1 - 51.7 = 69.4°C$

Eq. (8.5-6)

$$\Delta T_1 = \Sigma\Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3} = \frac{(69.4)(1/3123)}{1/3123 + 1/1987 + 1/1136} = 13.05°C$$

$$\Delta T_2 = \frac{(69.4)(1/1987)}{1/3123 + 1/1987 + 1/1136} = \qquad 20.50$$

$$\Delta T_3 = \frac{(69.4)(1/1136)}{1/3123 + 1/1987 + 1/1136} = \qquad 35.85$$

69.40

since have cold feed, increase ΔT_1 and lower ΔT_2 and ΔT_3.

$\Delta T_1 = 17.78°C \qquad \Delta T_2 = 18.33 \qquad \Delta T_3 = 33.33$

(8.5-2) - k.1

To calculate the b.pt. in each effect,

$T_1 = T_S - \Delta T_1 = 121.1 - 17.78 = 103.32\,°C$

$T_2 = T_1 - \Delta T_2 = 103.32 - 18.33 = 84.99\,°C$

$T_3 = T_2 - \Delta T_3 = 84.99 - 33.33 = 51.66\,°C$

Step 4.

Heat capacity of the liquid in each effect.

F: $c_p = 4.19 - 2.35(0.05) = 4.072$ kJ/kg·K

L_1: $c_p = 4.19 - 2.35(0.068) = 4.030$

L_2: $c_p = 4.19 - 2.35(0.107) = 3.939$

L_3: $c_p = 4.19 - 2.35(0.250) = 3.602$

Temperatures and latent heats by interpolation from A.2-9.

$T_F = 299.9 - 273.2 = 26.7\,°C$

$T_S = 121.1\,°C$ $\quad \lambda_S = 2199.5$ kJ/kg, $\quad T_1 = 103.32\,°C$ $\quad \lambda_1 = 2248.1$

$T_2 = 84.99\,°C$ $\quad \lambda_2 = 2296.0$ $\quad T_3 = 51.66\,°C$ $\quad \lambda_3 = 2378.8$

Material balance for V_1, V_2, V_3.

$V_1 = F - L_1 = 22\,680 - L_1$, $\quad V_2 = L_1 - L_2$, $\quad V_3 = L_2 - L_3 = L_2 - 4536$

Heat balance on each effect.

Use as datum the temperature of liquid leaving the effect since no BPR.

On (1), datum $= T_1 = 103.32\,°C$

(1) $F c_{pF}(T_F - T_1) + S\lambda_S = L_1 c_{p1}(T_1 - T_1) + V_1\lambda_1$

$22\,680(4.072)(26.7 - 103.32) + S(2199.5) = 0 + (22\,680 - L_1)(2248.1)$

On (2), datum $= T_2 = 84.99\,°C$

(2) $L_1 c_{p1}(T_1 - T_2) + V_1\lambda_1 = L_2 c_{p2}(T_2 - T_2) + V_2\lambda_2$

$L_1(4.030)(103.32 - 84.99) + (22\,680 - L_1)(2248.1) = 0 + (L_1 - L_2)(2296.0)$

On (3), datum $= T_3 = 51.66\,°C$

(3) $L_2 c_{p2}(T_2 - T_3) + V_2\lambda_2 = L_3 c_{p3}(T_3 - T_3) + V_3\lambda_3$

$L_2(3.939)(84.99 - 51.66) + (L_1 - L_2)(2296.0) = 0 + (L_2 - 4536)(2378.8)$

Solving (2) and (3) simultaneously,

$L_1 = 17\,050$ kg/h $\quad L_2 = 10\,991$ kg/h

Substituting into (1) to calculate S, $\boxed{S = 8972 \text{ kg/h steam}}$

	Calculated	Assumed
S	8972	
L_1	17 050	16 632
L_2	10 991	10 584
L_3	4536	4536

Hence, the calculated and assumed values are close enough and steps 2, 3, and 4 do not need to be repeated.

Step 5.

$$q_1 = S h_S = 8972 \frac{kg}{h}(2199.5 \frac{kJ}{kg})(1000 \frac{J}{kJ})(1/3600 \frac{s}{h}) = 5.482 \times 10^6 \text{ W}$$

$$q_2 = V_1 h_1 = (\overset{F}{22\,680} - \overset{L_1}{17\,050})(2248.1)(1000/3600) = 3.516 \times 10^6 \text{ W}$$

$$q_3 = V_2 h_2 = (\overset{L_1}{17\,050} - \overset{L_2}{10\,991})(2296.0)(1000/3600) = 3.864 \times 10^6 \text{ W}$$

$$A_1 = \frac{q_1}{U_1 \Delta T_1} = \frac{5.482 \times 10^6}{3123\,(17.78)} = 98.7 \text{ m}^2$$

$$A_2 = \frac{q_2}{U_2 \Delta T_2} = \frac{3.516 \times 10^6}{1987\,(18.33)} = 96.5 \text{ m}^2$$

$$A_3 = \frac{q_3}{U_3 \Delta T_3} = \frac{3.864 \times 10^6}{1136\,(33.33)} = 102.1 \text{ m}^2$$

$$\boxed{\text{average } A = 99.1 \text{ m}^2}$$

The areas differ from the average by less than 3% and a second trial is not necessary.

(9.3-2)

$H = 0.021$ kg H_2O/kg dry air at 32.2 °C

$P = 101.325$ kPa

(a) Table A.2-9 $\quad p_{AS} = 4.824$ kPa

Eq. (9.3-2)

$$H_S = \frac{18.02}{28.97}\frac{p_{AS}}{P-p_{AS}} = \frac{18.02}{28.97}\frac{4.824}{101.325-4.824}$$

$$H_S = 0.0311$$

Eq. (9.3-3)

$$H_p = 100\frac{H}{H_S} = 100\frac{(0.021)}{(0.0311)} = \boxed{67.5\%}$$

(b) $$H = 0.021 = \frac{18.02}{28.97}\frac{p_A}{P-p_A} = \frac{18.02}{28.97}\frac{p_A}{101.325-p_A}$$

$$p_A = 3.309 \text{ kPa}$$

Eq. (9.3-4)

$$H_R = 100\frac{p_A}{p_{AS}} = 100\frac{(3.309)}{(4.824)} = \boxed{68.6\%}$$

(9.3-3)

T dry bulb = 65.6 °C = 65.6 + 273.2 = 338.8 K = 150 °F = 610 °R

Dew point of 15.6 °C is temp. at sat. This intersects vertically the 100% line at $\boxed{H = 0.0113 \text{ kg } H_2O/\text{kg air} = 0.0113 \text{ lb}_m H_2O/\text{lb}_m \text{ air}}$

at $H = 0.0113$ and $T = 65.6$, $\boxed{H_p = 5.3\%}$ By linear interpolation between 0 and 10% line.

Humid volume Eq. (9.3-7)

SI $\quad v_H = (2.83\times10^{-3} + 4.56\times10^{-3}\times \overset{H}{0.0113})(\overset{T\,K}{338.8}) = \boxed{0.976 \text{ m}^3/\text{kg dry air}}$

English $\quad v_H = (0.0252 + 0.0405\times \overset{H}{0.0113})(\overset{T\,°R}{610}) = \boxed{15.65 \text{ ft}^3/\text{lb}_m \text{ dry air}}$

Humid heat Eq. (9.3-6)

SI $\quad c_S = 1.005 + 1.88(\overset{H}{0.0113}) = \boxed{1.026 \text{ kJ/kg dry air}}$

English $\quad c_S = 0.24 + 0.45(\overset{H}{0.0113}) = \boxed{0.245 \text{ Btu/lb}_m \text{ dry air}}$

(9.3-4)

$t_{dry\ bulb} = 57.2°C$
$H = 0.030\ kg\ H_2O/kg\ dry\ air$

Plot point H on chart of 0.030 and $t = 57.2°C$.
Read off chart $\boxed{H_p = 23\%}$

For H_s, saturation humidity, go vertically and at 100%, $\boxed{H_s = 0.130\ kg\ H_2O/kg\ dry\ air}$

For dew point, go horizontally to left to 100% line, $\boxed{t = 31.7°C \text{ dew point}}$

Humid heat Eq.(9.3-6)

$$c_s = 1.005 + 1.88\,H = 1.005 + 1.88\,(0.030)$$

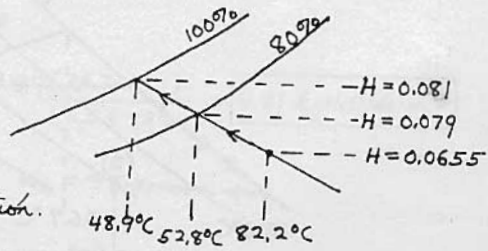
$$\boxed{c_s = 1.0614\ kJ/kg\ dry\ air \cdot K \quad (0.254\ btu/lb_m\ dry\ air\ °F)}$$

Humid volume Eq.(9.3-7) $T = 330.4\ K$

$$v_H = (2.83\times10^{-3} + 4.56\times10^{-3}H)\,T$$
$$= (2.83\times10^{-3} + 4.56\times10^{-3}\times0.030)(330.4)$$
$$\boxed{v_H = 0.980\ m^3/kg\ dry\ air}$$

(9.3-5)

$H = 0.0655$
$T = 82.2°C$



(a) Leaves at 80% saturation.

$$\boxed{\begin{array}{l} T = 52.8°C \\ H = 0.079\ kg\ H_2O/kg\ dry\ air \end{array}}$$

(b) Leaves at 100% saturation.

$$\boxed{\begin{array}{l} T = 48.9°C \\ H = 0.081\ kg\ H_2O/kg\ dry\ air \end{array}}$$

(9.3-7)

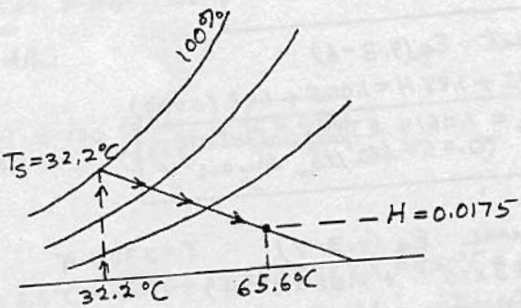
$T_{dry\,bulb} = 65.6\,°C$

$T_{wet\,bulb} = 32.2\,°C$

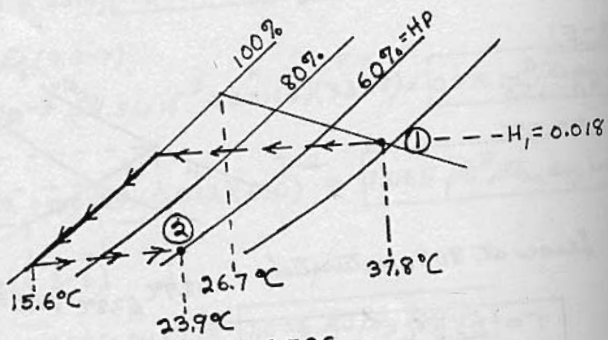
Go vertically at $T_s = 32.2\,°C$. and then follow downward and to right along adiabatic humidification line and to $T_{dry\,bulb} = 65.6\,°C$.

Read off value of

$$\boxed{H = 0.0175\ kg\,H_2O/kg\,dry\,air}$$



(9.3-9)

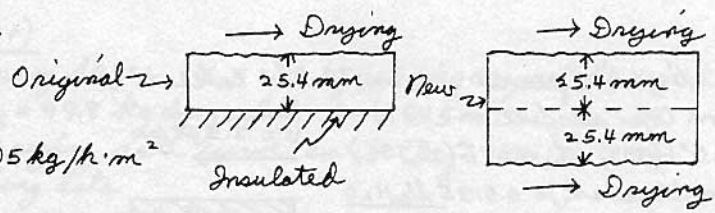


(a) $T_1 = 37.8\,°C$ T_1 wet bulb $= 26.7\,°C$

$$\boxed{H_1 = 0.018\ kg\,H_2O/kg\,dry\,air,\ H_{P1} = 41\%}$$

(b) $$\boxed{H_2 = 0.0115\ kg\,H_2O/kg\,dry\,air,\ H_{P2} = 60\%}$$

(9.6-1)



$R_c = 2.05\ kg/h \cdot m^2$

The value of L_s/A is the same for both original and new cases. Also, thickness for drying is the same.

Eq.(9.6-2)

$$\tau = \frac{L_s}{A R_c}(X_1 - X_2) = \frac{24.4}{2.05}(0.45 - 0.30) = \boxed{1.785\ h}$$

(9.6-2)

(a) From Example 9.6-3, $v_1 = 6.10\ m/s$

$R_{c1} = 3.39\ kg/h \cdot m^2$ $\quad v_2 = 3.05\ m/s$ (new velocity)

Eq.(9.6-9) $\quad h \propto G^{0.8} \propto v^{0.8}$

Eq.(9.6-8) $\quad R_c \propto h \qquad$ Hence, $R_c \propto v^{0.8}$

$$R_{c2} = R_{c1}\left(\frac{v_2}{v_1}\right)^{0.8} = 3.39\left(\frac{3.05}{6.10}\right)^{0.8} = \boxed{1.947\ kg\ H_2O/h \cdot m^2}$$

(b) $T_1 = 65.6°C \qquad H_1 = 0.010 \qquad T_{W1} = 28.9°C$

$T_2 = 76.7°C \qquad H_2 = H_1 = 0.010$

From humidity chart, T_{W2} is found as 31.1 °C.

Eq.(9.6-8)

$$R_c \propto (T - T_W)$$

$$R_{c2} = R_{c1}\frac{(T_2 - T_{W2})}{(T_1 - T_{W1})} = 3.39\frac{(76.7 - 31.1)}{(65.6 - 28.9)} = \boxed{4.21\ kg H_2O/h \cdot m^2}$$

(c) $x_1 = 25.4$ mm thick $\qquad x_2 = 38.1$ mm

Eq(9.6-2)

$$t = \frac{L_s}{A R_c}(X_1 - X_2)$$

A is kept constant. Increasing thickness x increases L_s kg dry solid. R_c constant since in constant-rate period.

$t \propto \frac{x_2}{x_1} \propto \frac{38.1}{25.4} \propto 1.5 \qquad \boxed{t_2 \text{ is } 1.50 \text{ times longer}}$

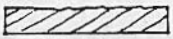
(9.6-4)

Pan 1.0'×1.0' square, 1.0 in. thick cake

Dry on top surface

$T = 120°F (48.9°C)$ $T_w = 80°F (26.7°C)$ $v = 2.5\ ft/s$

H (humidity chart) $= 0.0135 \dfrac{lb_m H_2O}{lb\ dry\ air}$



Eq. (9.3-7)

$$v_H = (0.0252 + 0.0405 \times 0.0135)(460 + 120°F) = 14.90 \frac{ft^3}{lb_m\ dry\ air}$$

$$\rho = \frac{1.00\ lb_m\ dry\ air + 0.0135\ lb_m\ H_2O}{14.90} = 0.0680\ lb_m/ft^3$$

$$G = v\rho = 2.5(3600)(0.0680) = 612\ lb_m/h \cdot ft^2$$

Eq. (9.6-9)

$$h = 0.0128 G^{0.8} = 0.0128(612)^{0.8} = 2.171\ btu/h \cdot ft^2 \cdot °F$$

$\lambda_w (80°F) = 1048.3\ btu/lb_m$, Table A.2-9

Eq. (9.6-8)

$$R_c = \frac{h}{\lambda_w}(T - T_w) = \frac{2.171}{1048.3}(120 - 80°F) = 0.08284\ lb_m H_2O/h \cdot ft^2$$

ρ_s (dry solid) $= 120\ lb_m/ft^3$

volume of solid $= 1.0 \times 1.0 \times \frac{1}{12} = \frac{1}{12}\ ft^3$

$L_s = 120 \times \frac{1}{12} = 10.00\ lb_m$ solid dry

$A = 1.0 \times 1.0 = 1.00\ ft^2$

$X_1 = 0.20\ lb_m H_2O / lb_m$ dry solid

$X_2 = X_c = 0.09$

Eq. (9.6-2)

$$\tau = \frac{L_s}{A R_c}(X_1 - X_2) = \frac{10.00}{1.0(0.08284)}(0.20 - 0.09) = 13.27\ h$$

$$\boxed{\tau = 13.3\ h}$$

(9.7-1)

Tray dryer $X_1 = 0.40$ kg H_2O/kg dry solid, $X_2 = 0.02$
$L_S = 99.8$ kg dry solid $A = 4.645\ m^2$
Drying rate curve as in Fig. 9.5-1b.
Drying data

X kg/kg	R $\frac{kg\ H_2O}{h \cdot m^2}$	1/R	X	R	1/R
0.40	1.51	0.663	0.050	0.37	2.703
0.195	1.51	0.663	0.040	0.27	3.703
0.15	1.21	0.826	0.030	0.181	5.525
0.10	0.90	1.111	0.020	0.107	9.346
0.065	0.71	1.408			

(a) Similar to Ex. 9.7-1

Constant Rate Period $X_C = X_2 = 0.195$, $R_C = 1.51$

Eq. (9.6-2) $t = \frac{L_S}{A R_C}(X_1 - X_C) = \frac{99.8}{4.645(1.51)}(0.40 - 0.195)$ $\boxed{t = 2.91\ h}$

Falling Rate Period

Plot 1/R from $X_C = 0.195$ to $X_2 = 0.02$

area $= A_0 + A_1 + A_2 + A_3$

$= 5.35(0.02) + 1.5(0.024) + 1.18(0.056) + 0.84(0.075) = 0.296$

Eq. (9.6-1)

$$t = \frac{L_S}{A}\int_{0.195}^{0.02}\frac{dX}{R} = \frac{99.8}{4.645}(0.296) \qquad \boxed{t = 6.36\ h}$$

Total time $= 2.91 + 6.36 = \boxed{9.27\ h \text{ total}}$

(b) For falling rate period use straight line through origin.

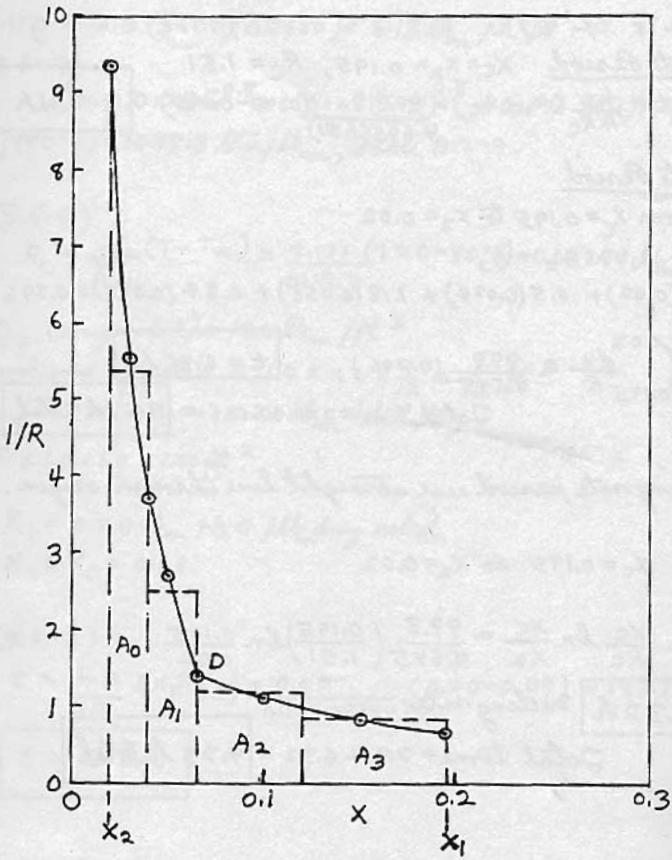
Eq. (9.7-8) $X_C = 0.195$ to $X_2 = 0.02$

$$t = \frac{L_S}{A}\frac{X_C}{R_C}\ln\frac{X_C}{X_2} = \frac{99.8}{4.645}\left(\frac{0.195}{1.51}\right)\ln\frac{0.195}{0.02}$$

$\boxed{t = 6.32\ h}$ Falling rate

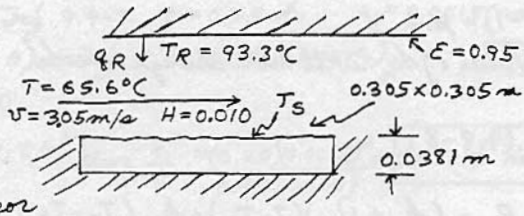
Total time $= 2.91 + 6.32 = \boxed{9.23\ h \text{ total}}$

(9.7-1) - k.1



(9.8-1)

Keep T_S below 32.2°C



Solution is trial and error

$T_R = 93.3°C.$

Eq. (9.3-7)

$$v_H = (2.83\times10^{-3} + 4.56\times10^{-3}H)T$$
$$= (2.83\times10^{-3} + 4.56\times10^{-3}\times 0.010)(273+65.6)$$
$$= 0.974\ m^3/kg\ dry\ air$$
$$\rho = \frac{1.00+0.010}{0.974} = 1.037\ kg/m^3$$

$$G = v\rho = 3.05(1.037)(3600) = 11{,}386\ kg/h\cdot m^2$$

Eq. (9.6-9)

$$h_c = 0.0204G^{0.8} = 0.0204(11{,}386)^{0.8} = 35.87\ W/m^2\cdot K$$

Trial 1 assume $T_S = 32.2°C + 273.2 = 305.4\ K$

$T_R = 93.3 + 273.2 = 366.5\ K$ $\varepsilon = 0.95$

Eq. (4.9-10)

$$h_r = \varepsilon(5.676)\frac{\left[\left(\frac{366.5}{100}\right)^4 - \left(\frac{305.4}{100}\right)^4\right]}{366.5-305.4} = 8.24\ W/m^2\cdot K$$

Eq (9.8-5) $U_K = 0$ (no conduction, bottom insulated)

Eq. (9.3-6)

$$c_S = (1.005 + 1.88\times0.01)10^3 = 1.024\times10^3\ J/kg\cdot K$$

Eq (9.8-9)

$$\frac{(H_S - H)}{c_S}\lambda_S = (1 + U_K/h_c)(T-T_S) + h_R/h_c(T_R - T_S)$$

For $T_S = 32.2°C$ (305.4 K), H_S (Eq. 9.3-1) = 0.031, $\lambda_S = 2425.3\times10^3$ J/kg (Table A.2-9)

$$\frac{(0.031-0.01)}{1.024\times10^3}2425.3\times10^3 = (1+0)(65.6-T_S) + \frac{8.24}{35.87}(93.3-T_S)$$

$T_S = 30.3°C$

Trial 2 assume $T_S = 31.7°C$, $H_S = 0.0305$, $\lambda_S = 2426.5\times10^3$

$$\frac{(0.0305-0.01)}{1.024\times10^3}2426.5\times10^3 = (1+0)(65.6-T_S) + \frac{8.24}{35.87}(93.3-T_S)$$

$\boxed{T_S = 31.3°C}$ close enough to assumed T_S

(9.8-1) - p.2

Value of h_r will not change much.

Eq.(9.8-8)

$$R_C = \frac{(h_c + U_K)(T - T_S) + h_r(T_R - T_S)}{\lambda_S}$$

$$= \frac{(35.87 + 0)(65.6 - 31.3) + 8.24(93.3 - 31.7)}{2426.5 \times 10^3}$$

$$\boxed{R_C = 2.583\ kg H_2O/h \cdot m^2}$$

(9.9-1)

(a) $\mathcal{D}_L = 2.97 \times 10^{-6}\ m^2/h$ $\quad X_1 = X_{t1} - X^* = 0.22 - 0.04 = 0.18\ kg H_2O/kg$ dry wood

$X(\text{final}) = 0.13 - 0.04 = 0.09$ $\quad x_1 = 25.4/(2 \times 1000) = 0.0127\ m$

$X/X_1 = E_a = 0.09/0.18 = 0.50$

Fig. 5.3-13 $\quad \frac{\mathcal{D}_L t}{x_1^2} = 0.20$

$$t = \frac{x_1^2(0.20)}{\mathcal{D}_L} = \frac{(0.0127)^2(0.20)}{2.97 \times 10^{-6}}$$

$$\boxed{t = 10.9\ h}$$

(b) $X_1 = 0.29 - 0.04 = 0.25$

$X = 0.09 - 0.04 = 0.05$

$x_1 = 12.7/(2 \times 1000) = 0.00635\ m$

$X/X_1 = E_a = 0.05/0.25 = 0.20$

Fig. 5.3-13 $\quad \frac{\mathcal{D}_L t}{x_1^2} = 0.56$

$$t = \frac{x_1^2(0.56)}{\mathcal{D}_L} = \frac{(0.00635)^2(0.56)}{2.97 \times 10^{-6}}$$

$$\boxed{t = 7.60\ h}$$

(9.9-2)

$X/X_c = 0.5$ From Fig. 9.9-2, $t = 0.45\ h$ $x_1 = 0.0015\ m$

From Fig. 9.9-1 (Curve A) or Fig. 5.3-13 (slab curve),

$(D_L t/x_1^2)_{theor} = 0.20$

Eq. (9.9-12) $D_L = (D_L t/x_1^2)_{theor} \dfrac{x_1^2}{t} = \dfrac{(0.20)(0.0015)^2}{0.45(3600)} = \boxed{2.78\times10^{-10}\ m^2/s}$

(9.10-2)

(a) Constant-rate period

First by writing a mass balance similar to the heat balance of Eq. (9.10-2),

(1) $A\,dN = G\,A\,dH$

Writing a mass transfer equation similar to Eq. (9.10-3)

(2) $A\,dN = G\,A\,dH = k_y M_B a\,A\,dz\,(H_w - H)$

Rearranging and integrating,

(3) $k_y M_B a z = \ln\left(\dfrac{H_w - H_1}{H_w - H_2}\right)$ $z = x_1$ = bed thickness

Eq. (9.6-11) (constant rate period) with $\rho_s/a = L_s/A$ from Eq. (9.10-6)

(4) $t = \dfrac{\rho_s \lambda_w (X_1 - X_c)}{a\,h\,(T - T_w)} = \dfrac{\rho_s (X_1 - X_c)}{a\,k_y M_B (H_w - H)}$ $(X_2 = X_c)$

Writing an equation for $(H_w - H)_{LM}$ similar to Eq. (9.10-10)

(5) $(H_w - H)_{LM} = \dfrac{(H_w - H_1)(1 - e^{-k_y M_B a z/G})}{k_y M_B a z/G}$ (since H of gas varies in bed)

Substitute $(H_w - H_1)$ from Eq. (5) into (4) and set $x_1 = z$.

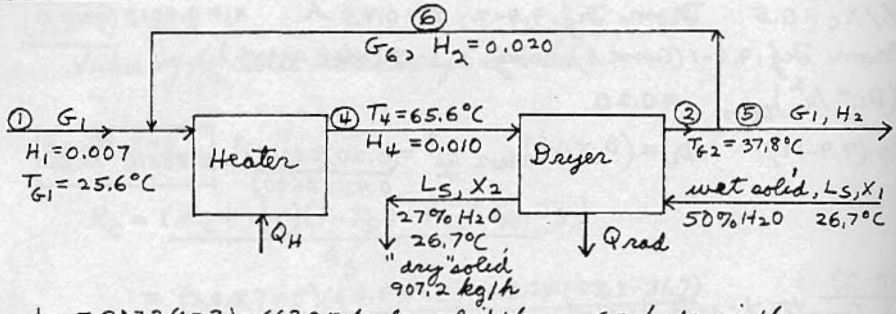
$$\boxed{t = \frac{\rho_s x_1 (X_1 - X_c)}{G(H_w - H_1)(1 - e^{-k_y M_B a x_1/G})}}$$

(b) Falling-rate period

In a similar manner writing an equation for the falling-rate period like Eq. (9.7-8) and (9.10-8)

$$\boxed{t = \frac{\rho_s x_1 X_c \ln(X_c/X)}{G(H_w - H_1)(1 - e^{-k_y M_B a x_1/G})}}$$

(9.10-6)



$L_S = 907.2(0.73) = 662.25$ kg dry solid/h G = kg dry air/h

$X_1 = \frac{0.5}{1.0-0.5} = 1.00$ kg H_2O/kg dry solid $X_2 = \frac{0.27}{1.0-0.27} = 0.370$

Material balance for H_2O on heater Eq. (9.10-29)

$$G_1 H_1 + G_6 H_2 = (G_1 + G_6) H_4$$

(1) $G_1(0.007) + G_6(0.020) = (G_1 + G_6)(0.010)$

Material balance for H_2O on dryer Eq. (9.10-30)

$$(G_1 + G_6)H_4 + L_S X_1 = (G_1 + G_6)H_2 + L_S X_2$$

(2) $(G_1 + G_6)(0.010) + 662.25(1.00) = (G_1 + G_6)(0.020) + 662.25(0.370)$

Solving (1) and (2), $\boxed{G_1 = 32\,094 \text{ kg fresh dry air/h}}$

$G_6 = 9628$ kg/h $\frac{9628}{32\,094 + 9628} \times 100 = \boxed{23.08\% \text{ recycled}}$

Heat balance on heater, datum = 0°C = T_0

Eq. (9.3-6) $c_S = (1.005 + 1.88H)$ kJ/kg dry air·K

Eq. (9.10-24) $H'_G = c_S(T_G - T_0) + H\lambda_0 = \left[(1.005 + 1.88H)(T_G - T_0) + H(2501)\right] \frac{\text{kJ}}{\text{kg dry air}}$

(3) $G_1 H'_{G1} + G_6 H'_{G6} + Q_H = (G_1 + G_6) H'_{G4}$

$32\,094\left[(1.005 + 1.88 \times 0.007)(25.6 - 0) + 0.007(2501)\right] +$

$9628\left[(1.005 + 1.88 \times 0.020)(37.8 - 0) + 0.020(2501)\right] + Q_H$

$= (32\,094 + 9628)\left[(1.005 + 1.88 \times 0.010)(65.6 - 0) + 0.010(2501)\right]$

$Q_H = 1\,586\,068$ kJ/h $= 1\,586\,068/3600 = \boxed{Q_H = 440.6 \text{ kW}}$

Heat balance on dryer, datum = 0°C = T_0

Eq. (9.10-25) $H'_{S1} = c_{pS}(T_S - T_0) + X c_{pA}(T_{S1} - T_0)$

(Since dry solid in at 26.7°C and out at 26.7°C, heat content $c_{pS}(T_S - T_0)$ drops out.)

(9.10-6) - p.2

Eq.(9.10-26)

$$(G_1+G_6)H'_{G4} + L_S H'_{S1} = (G_1+G_6)H'_{G2} + L_S H'_{S2} + Q_{rad}$$

$$(32\,094+9628)\left[(1.005+1.88\times0.010)(65.6-0)+0.010(2501)\right]$$

$$+662.25\left[0+1.00(4.187)(26.7-0)\right] =$$

$$(32\,094+9628)\left[(1.005+1.88\times0.020)(37.8-0)+0.020(2501)\right]$$

$$+662.25\left[0+0.37(4.187)(26.7-0)\right] + Q_{rad}$$

$$Q_{rad} = 160\,944\ kJ/h = 160\,944/3600 = \boxed{Q_{rad} = 44.7\ kW}$$

(9.12-1)

$F_0 = 2.50$ min $\quad z = 10°C \quad T_1 = 98.9°C \quad T_2 = 110.0°C \quad t_1 = 30$ min

Eq.(9.12-14) and (9.12-15)

$$F_0 = 2.50 = t_1 \cdot 10^{(T_1-121.1)/z} + t_2 \cdot 10^{(T_2-121.1)/z}$$

$$2.50 = (30)\cdot 10^{(98.9-121.1)/10} + t_2 \cdot 10^{(110-121.1)/10} \qquad \boxed{t_2 = 29.9\ \text{min}}$$

(9.12-3)

$F_0 = 2.45$ min $\quad z = 9.94°C \quad T = 112.8°C$

Eq.(9.12-14)

$$F_0 = 2.45 = t\cdot 10^{(T-121.1)/z} = t\cdot 10^{(112.8-121.1)/9.94}$$

$$\boxed{t = 16.76\ \text{min}}$$

(9.12-7)

$T_1 = 65.6°C \quad T = 62.8°C \quad z = 5°C \quad F_0 = 9.00$ min

Eq.(9.12-19)

$$F^{5°C}_{65.6°C} = 9.00 = t\cdot 10^{(T-T_1)/z} = t\cdot 10^{(62.8-65.6)/5} \qquad \boxed{t = 32.7\ \text{min}}$$

(9.12-8)

$F_0 = 9.0$ min $\quad N_0/N = 10^{15}$

Eq.(9.12-18) $\quad F_0 = 9.0 = D_{T1}\log_{10}(N_0/N) = D_{T1}\log_{10}(10^{15})$

$D_{T1} = 9/15$

For $\frac{N_0}{N} = 10^{16}$, $\quad F_0 = (9/15)\log(10^{16}) \qquad \boxed{F_0 = 9.60\ \text{min}}$

(10.2-2)

Conc. CO_2 in $H_2O = 0.90\times10^{-4}$ kg CO_2/kg H_2O

$$x_A \text{ (mol frac } CO_2) = \frac{(0.90\times10^{-4})/44.0}{(0.90\times10^{-4})/44.0 + 1.0/18.02} = 3.68\times10^{-5}$$

Appendix A.3, 303 K $\quad H = 0.186\times10^4$ atm/mol frac

$$p_A = Hx_A = 0.186\times10^4(3.68\times10^{-5}) = \boxed{0.0684 \text{ atm}}$$
$$= 0.0684(1.01325\times10^5) = \boxed{6.93\times10^3 \text{ Pa}}$$

(10.3-1)

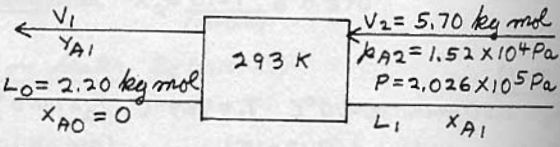


Fig. 10.2-1 $\quad p_A(\text{atm}) = Hx_A = 29.6\,x_A$ (Up to $x_A \cong 0.005$)

$$P = \frac{2.026\times10^5}{1.0132\times10^5} = 2.00 \text{ atm} \qquad y_{A2} = \frac{1.52\times10^4}{2.026\times10^5} = 0.0750$$

$$y_{A1} = \frac{p_{A1}}{P} = \frac{H}{P}x_{A1} = \frac{29.6}{2.0}x_{A1} = 14.8\,x_{A1}$$

Eq. (10.3-4) $\quad L' = L_0 = 2.20 \qquad V' = V_2(1-y_{A2}) = 5.70(1-0.075) = 5.272$

$$L'\frac{x_{A0}}{1-x_{A0}} + V'\frac{y_{A2}}{1-y_{A2}} = L'\frac{x_{A1}}{1-x_{A1}} + V'\frac{y_{A1}}{1-y_{A1}}$$

$$\frac{2.20(0)}{1-0} + \frac{5.272(0.075)}{1-0.075} = \frac{2.20\,x_{A1}}{1-x_{A1}} + \frac{5.272\,y_{A1}}{1-y_{A1}}$$

$$0 + \frac{5.272(0.075)}{1-0.075} = \frac{2.20\,x_{A1}}{1-x_{A1}} + \frac{5.272(14.8\,x_{A1})}{1-14.8\,x_{A1}}$$

Solving, $\boxed{x_{A1} = 0.00495}$

$$y_{A1} = 14.8(0.00495) = \boxed{0.0733 = y_{A1}}$$

$$L_1 = \frac{L'}{1-x_{A1}} = \frac{2.20}{1-0.00495} = \boxed{2.211 \text{ kg mol}}$$

$$V_1 = \frac{V'}{1-y_{A1}} = \frac{5.272}{1-0.0733} = \boxed{5.69 \text{ kg mol}}$$

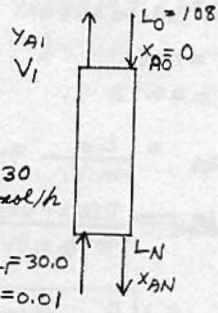
(10.3-1)

$L_0 = 108$ kg mol H_2O/h Other conditions as in Ex. 10.3-2.

$V_{N+1} = 30.0$ kg mol gas/h 300 K

$P = 101.3$ kPa

$y_A = 2.53 x_A$ (equilibrium relation)



amount entering acetone $= y_{AN+1} V_{N+1} = 0.01(30) = 0.30$ kg mol/h

amount leaving in $V_1 = 0.10(0.3) = 0.03$ kg mol/h

amount leaving in $L_N = 0.90(0.30) = 0.27$ kg mol/h

$V' = 0.99(30) = 29.70$ kg mol/h

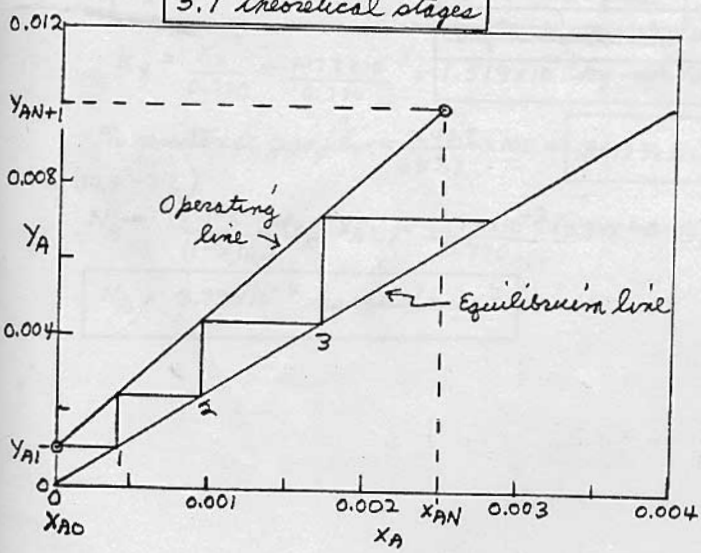
$V_1 = 29.70 + 0.030 = 29.73$ kg mol/h

$y_{A1} = \dfrac{0.030}{29.73} = 0.00101$

$L_N = 108 + 0.27 = 108.27$ $x_{AN} = \dfrac{0.27}{108.27} = 0.002494$

Since L varies from 108.0 to 108.27 and V from 30.00 to 29.73, the operating line is plotted as a straight line. The equilibrium line $y_A = 2.53 x_A$ is plotted. Step off stages.

3.7 theoretical stages



(10.3-2) - p. 2

Analytical Kremser equation.

$Y_A = m x_A \qquad m = 2.53 \qquad x_{A0} = 0 \qquad Y_{A1} = 0.00101$

$x_{AN} = 0.002494 \qquad Y_{AN+1} = 0.010$

$L_0 = 108 \qquad L_N = 108.27$

$V_1 = 29.73 \qquad V_{N+1} = 30.0$

$$A_{top} = \frac{L_0}{m V_1} = \frac{108}{2.53(29.73)} = 1.436$$

$$A_{btm} = \frac{L_N}{m V_{N+1}} = \frac{108.27}{2.53(30.0)} = 1.426$$

$$A_{av} = \sqrt{A_{top} A_{btm}} = \sqrt{(1.436)(1.426)} = 1.431$$

Eq. (10.3-25)

$$N = \frac{\log\left[\frac{Y_{N+1} - m x_0}{Y_1 - m x_0}\left(1 - \frac{1}{A}\right) + \frac{1}{A}\right]}{\log A}$$

$$= \frac{\log\left[\frac{0.010 - 0}{0.00101 - 0}\left(1 - \frac{1}{1.431}\right) + \frac{1}{1.431}\right]}{\log 1.431}$$

$$\boxed{N = 3.64 \text{ theoretical stages}}$$

(10.4-1)

$k_y' = 1.465 \times 10^{-3}$ kg mol/s·m²·mol frac $\quad y_{AG} = 0.380 \quad x_{AL} = 0.100$

$k_x' = 1.967 \times 10^{-3}$ kg mol/s·m²·mol frac $\quad y_{Ai} = 0.197 \quad x_{Ai} = 0.257$

Data from Ex. 10.4-1. $\quad x_A^* = 0.348$ (Eq. 10.4-4)

Eq. (10.4-26)

$$\frac{1}{K_x'/(1-x_A)_{*M}} = \frac{1}{m'' k_y'/(1-y_A)_{iM}} + \frac{1}{k_x'/(1-x_A)_{iM}}$$

$$m'' \text{ (slope between O and M)} = \frac{y_{AG} - y_{Ai}}{x_A^* - x_{Ai}} = \frac{0.380 - 0.197}{0.348 - 0.257} = 2.01$$

Eq (10.4-27)

$$(1-x_A)_{*M} = \frac{(1-x_{AL}) - (1-x_A^*)}{\ln (1-x_{AL})/(1-x_A^*)} = \frac{(1-0.100)-(1-0.348)}{\ln (1-0.100)/(1-0.348)} = 0.770$$

$(1-y_A)_{iM} = 0.709 \qquad (1-x_A)_{iM} = 0.820$ (Ex. 10.4-1)

Eq. (10.4-26)

$$\frac{1}{K_x'/(0.770)} = \frac{1}{(2.01)(1.465\times10^{-3})/(0.709)} + \frac{1}{(1.967\times10^{-3})/(0.820)}$$

$$\frac{1}{K_x'/0.770} = 240.8 + 416.9 = 657.7$$

$$\boxed{K_x' = 1.173 \times 10^{-3} \text{ kg mol/s·m²·mol frac}}$$

$$K_x = \frac{K_x'}{0.770} = \frac{1.173\times10^{-3}}{0.770} = \boxed{1.519\times10^{-3} \text{ kg mol/s·m²·mol frac}}$$

$$\% \text{ resistance gas film} = \frac{240.8}{657.7} \times 100 = \boxed{36.7\% \text{ resistance}}$$

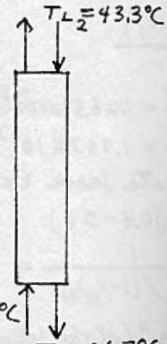
Eq (10.4-22)

$$N_A = \frac{K_x'}{(1-x)_{*M}}(x_A^* - x_{AL}) = \frac{1.173\times10^{-3}}{0.770}(0.348 - 0.100)$$

$$\boxed{N_A = 3.78\times10^{-4} \text{ kg mol/s·m²}}$$

(10.5-1)

$H_G = 0.533$ m Neglect liquid phase resistance. Use H_y^* values.



$T_{G1} = 23.9°C$ (wet bulb = 21.1°C). From H chart,

$H_1 = 0.0145$ kg H_2O/kg dry air

Eq.(9.3-9)

$$H_{y1} = (1.005 + 1.88 \times 0.0145)10^3(23.9-0) + 2.5014\times10^6(0.0145)$$
$$= 60.94\times10^3 \text{ J/kg dry air}$$

Plot $H_{y1} = 60.94\times10^3$ at $T_{L1} = 26.7°C$. Equil. data from Table 10.5-1.

Plot minimum line and read off $H_{y2} = 191\times10^3$ at $T_{L2} = 43.3°C$

Eq.(10.5-2)

$$G(H_{y2} - H_{y1}) = L c_L (T_{L2} - T_{L1})$$

$$G_{min}(191 - 60.94)10^3 = L(4.188\times10^3)(43.3 - 26.7)$$

$$L/G_{min} = 1.871$$

Use $1.50\,G_{min}$

$$\frac{L}{G} = \frac{1.871}{1.50} = 1.248$$

Eq.(10.5-2)

$$(H_{y2} - 60.94\times10^3) = \overset{L/G}{(1.248)}(4.188\times10^3)(43.3 - 26.7)$$

$$H_{y2} = 147.68\times10^3$$

Plot new point $H_{y2} = 147.68\times10^3$ and $T_{L2} = 43.3$ and draw operating line. Read off H_y^* and H_y values at a given T_L.

T_L	H_y^*	H_y	$H_y^* - H_y$	$1/(H_y^* - H_y)$
26.7°C	84.0×10^3	60.94×10^3	23.06×10^3	4.337×10^{-5}
30	100.0	78.0	22.0	4.545
34	122.8	99.0	23.8	4.200
36	136.0	109.3	26.7	3.745
38	150.0	119.7	30.3	3.300
40	167.0	130.0	37.0	2.703
42	185.0	140.3	44.7	2.237
43.3	197.2	147.68	49.52	2.019

(10.5-1) - p.2

Plot $1/(H_Y^* - H_Y)$ vs. H_Y. Graphically integrate.

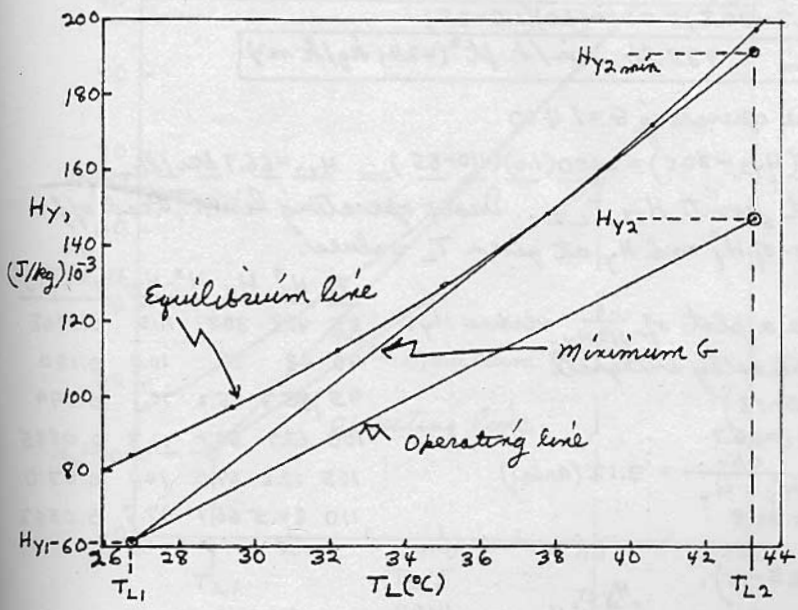
Eq. (10.5-16)

$$z = H_G \int_{H_{Y1} = 60.94 \times 10^3}^{H_{Y2} = 147.68 \times 10^3} \frac{dH_Y}{(H_Y^* - H_Y)}$$

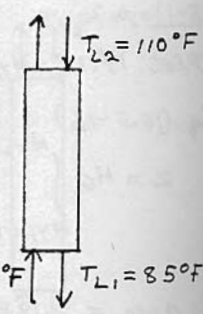
use H_Y^* for H_{Yi}

area = 3.188

$$z = H_G\,(\text{area}) = 0.533(3.188) = \boxed{1.70\ m}$$



(10.5-2)



$L = 2000\ lb_m/h \cdot ft^2 \quad G = 1400\ lb_m\ air/h \cdot ft^2$

$K_G a = 6.90\ lb\ mol/h \cdot ft^3 \cdot atm$

$T_{wet\ bulb\ inlet\ air} = 75°F$

$H_1 = 0.0165\ lb\ water/lb\ dry\ air$ (Eq. 9.3-2)

Eq. (9.3-8).

$H_y = (0.24 + 0.45H)(T - T_0\,°F) + H\lambda_0$

$H_{y1} = (0.24 + 0.45 \times 0.0165)(85 - 32) + 1075.4(0.0165) = 30.8\ \frac{btu}{lb_m\ dry\ air}$

(a) Minimum air rate

Saturated enthalpies from Table 10.5-1 are plotted as H_y vs T_L.

The point $H_{y1} = 30.8$ and $T_{L1} = 85°F$ is plotted.

The line for minimum G is plotted where the operating line becomes tangent as shown. At $T_{L2} = 110°F$, a value of $H_{y2} = 84.3$ is read off mins operating line.

Eq. (10.5-2)

$G(H_{y2} - H_{y1}) = L c_L (T_{L2} - T_{L1}) \qquad c_L = 1.00\ btu/lb_m \cdot °F$

$G(84.3 - 30.8) = 2000(1.0)(110 - 85)$

$$\boxed{G_{min} = 935\ lb_m\ air/h \cdot ft^2\ (4241\ kg/h \cdot m^2)}$$

(b) Use operating $G = 1400$

$1400(H_{y2} - 30.8) = 2000(1.0)(110 - 85) \qquad H_{y2} = 66.7\ btu/lb_m$

Plot point H_{y2}, T_{L2}. Draw operating line. Read off values of H_y^* and H_y at given T_L values.

T_L	H_y^*	H_y	$H^* - H_y$	$\frac{1}{H^* - H_y}$
85	41.8	30.8	11.0	0.0908
90	48	38	10.0	0.100
95	55.3	45.2	10.1	0.099
100	63.7	52.4	11.3	0.0885
105	73.6	59.5	14.1	0.0710
110	84.5	66.7	17.8	0.0562

Make a plot of $\frac{1}{H^* - H_y}$ versus H_y.

Graphically integrate

Eq. (10.5-18)

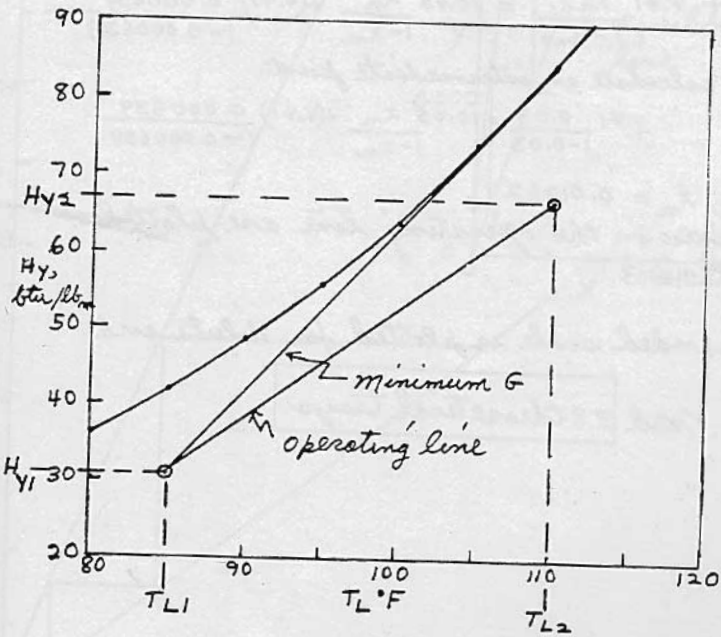
$$\int_{H_{y1} = 30.8}^{H_{y2} = 66.7} \frac{dH_y}{H_y^* - H_y} = 3.12\ (area)$$

Eq. (10.5-18)

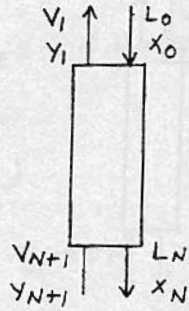
$$z = \frac{G}{M_B K_G a P} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_y^* - H_y} = \frac{1400}{29(6.90)(1.00)}(3.12)$$

$$\boxed{z = 21.8\ ft\ (6.64\ m)}$$

(10.5-2)-pg. 2


90
80
70
Hy2
60
Hy, btu/lbm
50
40
Hy1 30
20
80
90
100
110
120
TL1
TL °F
TL2
minimum G
operating line

(10.6-3)



$V' = \frac{128}{29} = 4.41$ kg mol inert air/h·m²

$L' = \frac{188}{18} = 10.45$ kg mol inert water/h·m²

$X_0 = 0 \qquad Y_{N+1} = 0.06 \qquad (20°C)\ 293\ K$

Moles NH_3 inlet $= \frac{0.06}{0.94}(4.41) = 0.282$

outlet $= 0.01(0.282) = 0.00282$

$$Y_1 = \frac{0.00282}{4.41 + 0.00282} = \boxed{0.000639 = Y_1}$$

Moles NH_3 in outlet water $= 0.282 - 0.00282 = 0.2792$

$$X_N = \frac{0.2792}{10.45 + 0.2792} = \boxed{0.0260 = X_N}$$

Eq. (10.6-2) Operating line

$$L' \frac{X_0}{1-X_0} + V' \frac{Y_{n+1}}{1-Y_{n+1}} = L' \frac{X_n}{1-X_n} + V' \frac{Y_1}{1-Y_1}$$

$$0 + 4.41 \frac{Y_{n+1}}{1-Y_{n+1}} = 10.45 \frac{X_n}{1-X_n} + (4.41) \frac{0.000639}{1-0.000639}$$

To plot, calculate an intermediate point.

$Y_{n+1} = 0.03 \qquad 4.41 \frac{0.03}{1-0.03} = 10.45 \frac{X_n}{1-X_n} + (4.41) \frac{0.000639}{1-0.000639}$

Solving, $X_n = 0.01262$

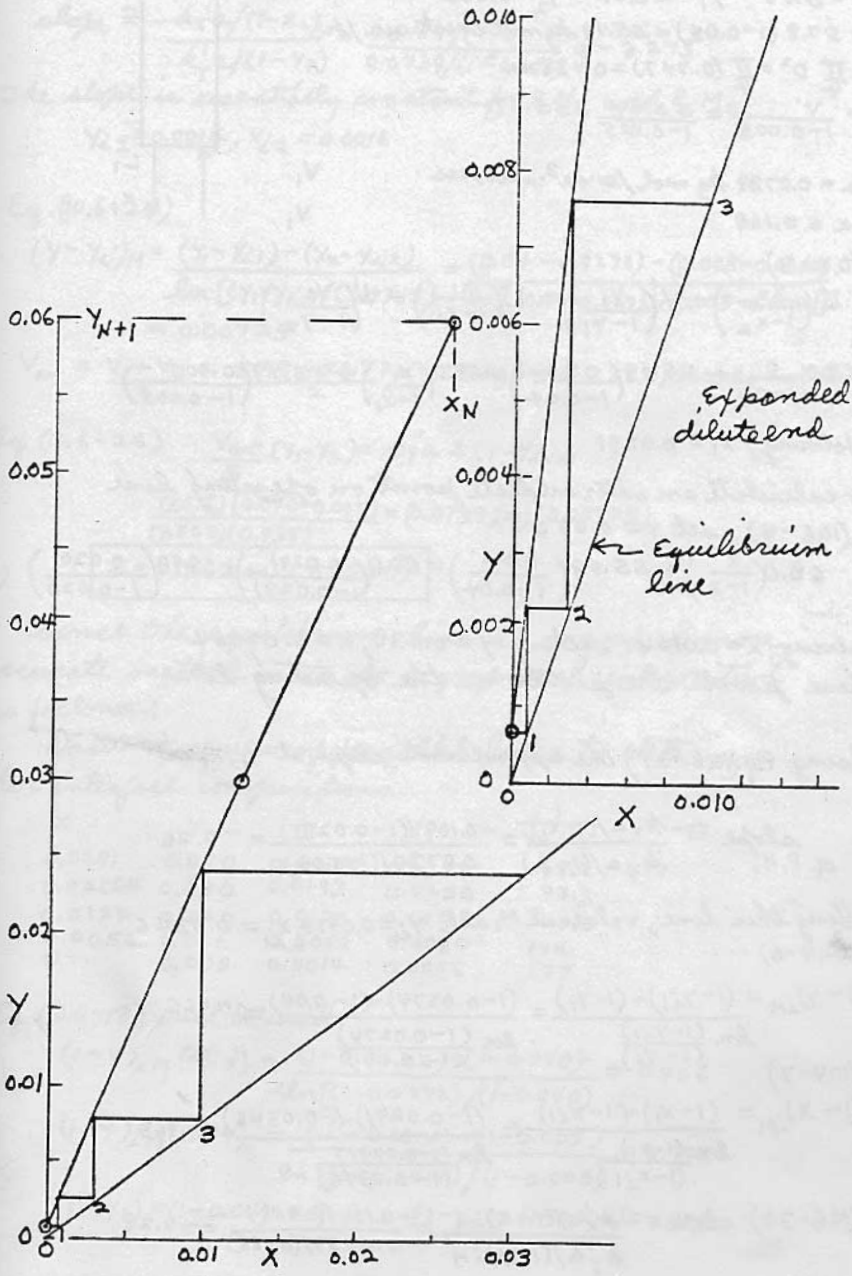
The 3 points on the operating line are plotted.

Equil. data A.3

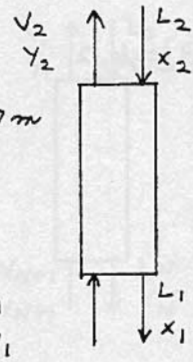
An expanded scale is plotted for dilute end.

Need $\boxed{3.8 \text{ theoretical trays}}$

(10.6-3) - p. 2


0.06
Y_{N+1}
X_N
0.05
0.04
0.03
0.02
Y
0.01
0
0
0.01
X
0.02
0.03
1
2
3
0.010
0.008
0.006
0.004
Y
0.002
0
0
0.010
X
1
2
3
Expanded dilute end
Equilibrium line

(10.6-6)



$L_2 = L' = 68.0$ kg mol/h $\quad x_2 = 0 \quad 20°C$

$V_1 = 57.8 \quad Y_1 = 0.04 \quad Y_2 = 0.005 \quad D = 0.747\ m$

$V' = 57.8(1-0.04) = 55.49$ kg mol inert air/h

$S = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.747)^2 = 0.438\ m^2$

$V_2 = \frac{V'}{1-0.005} = \frac{55.49}{1-0.005} = 55.77$

$k'_y a = 0.0739$ kg mol/s·m³·mol frac

$k'_x a = 0.169$

Eq.(10.6-3)

$$L'\left(\frac{X_2}{1-X_2}\right) + V'\left(\frac{Y_1}{1-Y_1}\right) = L'\left(\frac{X_1}{1-X_1}\right) + V'\left(\frac{Y_2}{1-Y_2}\right)$$

$$68.0\,\frac{0}{1-0} + 55.49\left(\frac{0.04}{1-0.04}\right) = 68.0\left(\frac{X_1}{1-X_1}\right) + 55.49\left(\frac{0.005}{1-0.005}\right)$$

Solving, $X_1 = 0.0291$

To calculate an intermediate point on operating line Eq.(10.6-4), set $Y = 0.020$.

$$68.0\left(\frac{X}{1-X}\right) + 55.49\left(\frac{0.04}{1-0.04}\right) = 68.0\left(\frac{0.0291}{1-0.0291}\right) + 55.49\left(\frac{0.020}{1-0.020}\right)$$

Solving, $X = 0.0124$. Setting $Y = 0.030$, $X = 0.02064$

These points are plotted as the operating line.

Using Eq.(10.6-31) the approximate slope at Y_1, X_1 is

$$\text{slope of } P_1M_1 \cong -\frac{k'_x a/(1-X_1)}{k'_y a/(1-Y_1)} = -\frac{0.169/(1-0.0291)}{0.0739/(1-0.04)} = -2.26$$

Plotting this line, values at M_1 are $Y_{i1} = 0.0274$, $X_{i1} = 0.0346$

Eq.(10.4-6)

$$(1-Y)_{iM} = \frac{(1-Y_{i1})-(1-Y_1)}{\ln\frac{(1-Y_{i1})}{(1-Y_1)}} = \frac{(1-0.0274)-(1-0.04)}{\ln\frac{(1-0.0274)}{(1-0.04)}} = 0.966$$

Eq.(10.4-7)

$$(1-X)_{iM} = \frac{(1-X_1)-(1-X_{i1})}{\ln\frac{(1-X_1)}{(1-X_{i1})}} = \frac{(1-0.0291)-(1-0.0346)}{\ln\frac{(1-0.0291)}{(1-0.0346)}} = 0.968$$

Eq.(10.6-30) $\text{slope} = -\frac{k'_x a/(1-X)_{iM}}{k'_y a/(1-Y)_{iM}} = \frac{-0.169/(0.968)}{0.0739/(0.966)} = -2.28$

(10.6-6) - p.2

Replotting P_1M_1, $Y_{i1} = 0.0272$, $X_{i1} = 0.0346$

For the slope at point 2

$$\text{slope} \cong -\frac{k_x' a/(1-X_2)}{k_y' a/(1-Y_2)} = \frac{-0.169/(1-0)}{0.0739/(1-0.005)} = -2.275$$

The slope is essentially constant for P_1M_1 and P_2M_2

$Y_{i2} = 0.0014$, $X_{i2} = 0.0016$

Eq. (10.6-24)

$$(Y-Y_i')_M = \frac{(Y_1-Y_{i1})-(Y_2-Y_{i2})}{\ln[(Y_1-Y_{i1})/(Y_2-Y_{i2})]} = \frac{(0.04-0.0272)-(0.005-0.0014)}{\ln[(0.04-0.0272)/(0.005-0.0014)]}$$

$$= 0.00725$$

$$V_{av} = \frac{V_1+V_2}{2} = \frac{57.8+55.77}{2} = 56.8 \text{ kg mol/h} = 56.8/3600 \text{ kg mol/s}$$

Eq. (10.6-26) $\quad \frac{V_{av}}{S}(Y_1-Y_2) = k_y' a\, z\, (Y-Y_i)_M$

$$\frac{(56.8)(0.040-0.005)}{(3600)(0.438)} = (0.0739)(z)(0.00725)$$

(a) $\boxed{z = 2.362 \text{ m } (7.75 \text{ ft})}$

Since the equilibrium line is curved a more accurate method is to do a graphical integration as follows:

Draw a series of parallel lines to determine other interface compositions.

X	Y	Y_i'	X_i'	$\frac{1}{Y-Y_i'}$
0.0291	0.040	0.0272	0.0346	78.2
0.02064	0.030	0.0193	0.0250	93.3
0.0124	0.020	0.0120	0.0158	125.0
0.0056	0.012	0.0062	0.0080	172.
0	0.005	0.0014	0.0016	177

Eq. (10.6-18) will be used.

$$(1-Y)_{iM}(\text{at } 1) = \frac{(1-0.0272)-(1-0.040)}{\ln[(1-0.0272)/(1-0.040)]} = 0.966$$

$$(1-Y)_{iM}(\text{at } 2) = \frac{(1-0.0014)-(1-0.005)}{\ln[(1-0.0014)/(1-0.005)]} = 0.996$$

$(1-Y_1) = (1-0.04) = 0.960 \qquad (1-Y_2) = (1-0.005) = 0.995$

(10.6-6) - p. 3

at point 1, $\frac{(1-Y)_{iM}}{(1-Y_1)} = \frac{0.966}{0.960} = 1.006$ at 2, $\frac{(1-Y)_{iM}}{(1-Y_2)} = \frac{0.996}{0.995} = 1.001$

Hence, an average of $(1.006+1.001)/2 = 1.003$ will be used

For the value of $\int_{Y_2}^{Y_1} \frac{dY}{Y-Y_i}$, a plot of $1/(Y-Y_i)$ is made and the area $= 4.595$.

Eq(10.6-18)

$$z = \left[\frac{V}{k'_y a S}\frac{(1-Y)_{iM}}{(1-Y)}\right]_{av} \int_{Y_2}^{Y_1} \frac{dY}{Y-Y_i}$$

$$z = \frac{(56.8)}{(3600)(0.0739)(0.438)}(1.003)(4.595) = \boxed{2.25\,m}$$

This is a more accurate method but the dilute method still gives a close answer.

(b) Use $K'_y a$ for design

Eq.(10.4-24)

$$\frac{1}{K'_y a/(1-Y)_{*M}} = \frac{1}{k'_y a/(1-Y)_{iM}} + \frac{m'}{k'_x a/(1-X)_{iM}}$$

To calculate slope m'_1 for line $E_1 M_1$ (See Fig. 10.4-2)

Eq.(10.4-13)

$$m'_1 = \frac{Y_i - Y^*}{X_i - X} = \frac{0.0272 - 0.0227}{0.0346 - 0.0291} = 0.8182$$

$(1-Y)_{iM} = 0.966$ $\quad (1-X)_{iM} = \frac{(1-X)-(1-X_i)}{\ln[(1-X)/(1-X_i)]} = \frac{(1-0.0291)-(1-0.0346)}{\ln[(1-0.0291)/(1-0.0346)]}$

$(1-X)_{iM} = 0.968$

$$(1-Y)_{*M} = \frac{(1-Y^*)-(1-Y)}{\ln[(1-Y^*)/(1-Y)]} = \frac{(1-0.0227)-(1-0.040)}{\ln[(1-0.0227)/(1-0.040)]} = 0.968$$

Eq(10.4-24)

$$\frac{1}{K'_y a/0.968} = \frac{1}{0.0739/0.966} + \frac{0.8182}{0.169/0.968} \qquad K'_y a = 5.451$$

Repeating at point 2 $\quad m'_2 = \frac{0.0014-0}{0.0016-0} = 0.875$

$(1-Y)_{iM} = 0.997 \quad (1-X)_{iM} = 0.999 \quad (1-Y)_{*M} = 0.998$

$$\frac{(10.6-6)-0.4}{\frac{1}{K'_y a/0.998}} = \frac{1}{0.0739/0.997} + \frac{0.875}{0.169/0.999} \qquad K'_y a = 5.347$$

Using an average, $K'_y a = (5.451 + 5.347)/2 = 5.40$

Eq. (10.6-25)

$$(Y-Y^*)_M = \frac{(Y_1-Y_1^*)-(Y_2-Y_2^*)}{\ln[(Y_1-Y_1^*)/(Y_2-Y_2^*)]} = \frac{(0.040-0.0227)-(0.005-0)}{\ln[(0.040-0.0227)/(0.005-0)]}$$

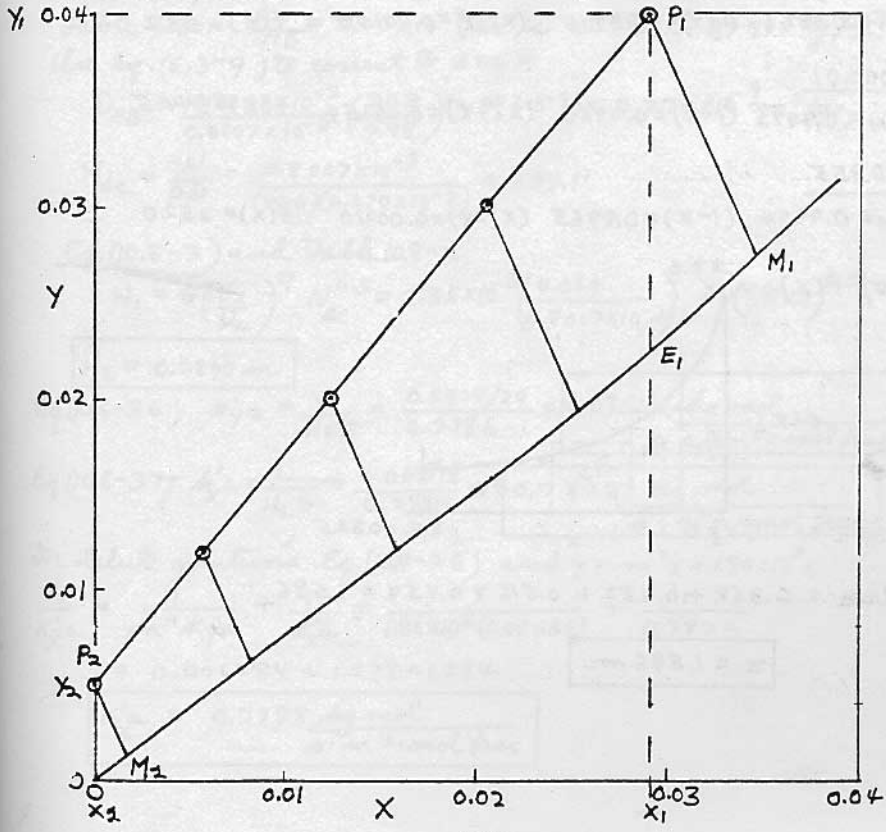
$$= 0.00991$$

Eq. (10.6-28)

$$\frac{V}{S}(Y_1-Y_2) = K'_y a\, z\, (Y-Y^*)_M$$

$$\frac{56.8}{3600(0.438)}(0.040-0.005) = 5.40\, z\, (0.00991)$$

$\boxed{z = 2.356\ m}$ This is close to part (a).



(10.7-1)

Use data of Example 10.6-15.

Eq. (10.6-15)

$$z = \int_{x_2}^{x_1} \frac{L\,dx}{\frac{k_x' a S}{(1-x)_{iM}}(1-x)(x_i'-x)} = \int f(x)\,dx$$

$x_2 = 0$

Table 10.7-1 $\quad x_i = 0.00046 \quad L = 0.04200 \quad k_x' a = 0.848 \quad S = 0.0929$

$x = 0$

$$(1-x)_{iM} = \frac{(1-0)-(1-0.00046)}{\ln(1-0)/(1-0.00046)} = 0.9998$$

$(1-x) = (1-0) = 1.00 \quad (x_i'-x) = 0.00046 - 0 = 0.00046$

$$f(x) = \frac{0.04200}{\frac{0.848(0.0929)(1.00)(0.00046)}{(0.9998)}} = 1159$$

$x = 0.000332$

$(1-x)_{iM} = 0.9993 \quad (1-x) = 0.99967 \quad (x_i'-x) = 0.000698 \quad f(x) = 763$

$x = 0.000855$

$(1-x)_{iM} = 0.9987 \quad (1-x) = 0.99915 \quad (x_i'-x) = 0.00100 \quad f(x) = 532$

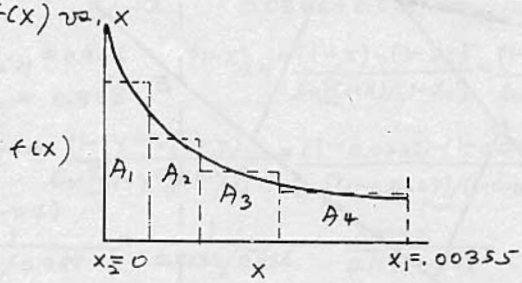
$x = 0.00201$

$(1-x)_{iM} = 0.9972 \quad (1-x) = 0.9980 \quad (x_i'-x) = 0.00154 \quad f(x) = 344.5$

$x_1 = 0.00355$

$(1-x)_{iM} = 0.9954 \quad (1-x) = 0.9965 \quad (x_i'-x) = 0.00210 \quad f(x) = 252.0$

Plot of $f(x)$ vs. x



Area $= 0.364 + 0.252 + 0.516 + 0.454 = 1.586$

$$\boxed{z = 1.586\ m}$$

(10.8-1)

CO_2 at 30°C in air-water A.3-17 at 30°C for CO_2

$G_y = (1.6)(0.339) = 0.5424\ kg/s\cdot m^2$ $y_A = 1.86\times10^3 x_A$

$G_x = (1.6)(2.543) = 4.069$

Table 6.2-1 at 3°C, $\mathcal{D}_{AB} = 0.142\times10^{-4}\ m^2/s$. Correct to 303 K by Eq.(6.2-45) $\mathcal{D}_{AB} = 0.142\times10^{-4}\left(\frac{303}{276.2}\right)^{1.75} = 0.167\times10^{-4}$

For air, $\mu = 1.86\times10^{-5}\ kg/m\cdot s$ $\rho = 1.168\ kg/m^3$

$$N_{Sc} = \frac{\mu}{\rho\mathcal{D}} = \frac{1.86\times10^{-5}}{(1.168)(0.167\times10^{-4})} = 0.954 \text{ (gas phase)}$$

Eq. (10.8-1) and Table 10.8-1

$$H_G = \alpha G_y^{\beta} G_x^{\gamma} N_{Sc}^{0.5} = 0.557(0.5424)^{0.32}(4.069)^{-0.51}(0.954)^{0.5}$$

$$\boxed{H_G = 0.2186\ m}$$

For water, $\mu = 0.8007\times10^{-3}$ at 30°C $= 0.8937\times10^{-3}$ at 25°C (298 K)

Table 6.3-1 $\mathcal{D}_{AB} = 2.00\times10^{-9}\ m^2/s$ at 25°C $\rho = 996\ kg/m^3$

Use Eq. (6.3-9) to correct to 303 K

$$\mathcal{D}_{AB} = \frac{0.8937\times10^{-3}}{0.8007\times10^{-3}}\left(\frac{303}{298}\right)(2.00\times10^{-9}) = 2.270\times10^{-9}\ m^2/s$$

$$N_{Sc} = \frac{\mu}{\rho\mathcal{D}} = \frac{0.8007\times10^{-3}}{(996)(2.270\times10^{-9})} = 354.1$$

Eq. (10.8-3) and Table 10.8-2

$$H_L = \theta\left(\frac{G_x}{\mu_L}\right)^{\eta} N_{Sc}^{0.5} = 2.35\times10^{-3}\left(\frac{4.069}{0.8007\times10^{-3}}\right)^{0.22}(354.1)^{0.5}$$

$$\boxed{H_L = 0.2890\ m}$$

Eq (10.6-36) $k'_y a = \frac{V}{H_G S} = \frac{0.5424/29}{0.2186} = \boxed{0.08556\ \frac{kg\ mol}{s\cdot m^3\cdot mol\ frac}}$

Eq (10.6-37) $k'_x a = \frac{L}{H_L S} = \frac{4.069/18}{0.2890} = \boxed{0.7822\ \frac{kg\ mol}{s\cdot m^3\cdot mol\ frac}}$

For dilute solutions Eq. (10.4-26) and $y = m''x = 1.86\times10^3 x$

$$\frac{1}{K'_x a} = \frac{1}{m'' k'_y a} + \frac{1}{k'_x a} = \frac{1}{1.86\times10^3(0.08556)} + \frac{1}{0.7822}$$

$$= 0.006284 + 1.278 = 1.284$$

$$\boxed{K'_x a = 0.7788\ \frac{kg\ mol}{s\cdot m^3\cdot mol\ frac}}$$

(11.1-3)

Eq.(11.1-3) $P_A x_A + P_B(1-x_A) = P$ Eq.(11.1-4) $y_A = \frac{p_A}{P} = \frac{P_A x_A}{P}$

Solving, $x_A = \frac{P - P_B}{P_A - P_B}$

68.7°C

$x_A = \frac{101.32-16.1}{101.32-16.1} = 1.000$ $y_A = \frac{101.32(1.0)}{101.32} = 1.000$

79.4°C

$x_A = \frac{101.32-23.1}{136.7-23.1} = 0.689$ $y_A = \frac{136.7(0.689)}{101.32} = 0.930$

93.3°C $x_A = 0.401$ $y_A = 0.781$

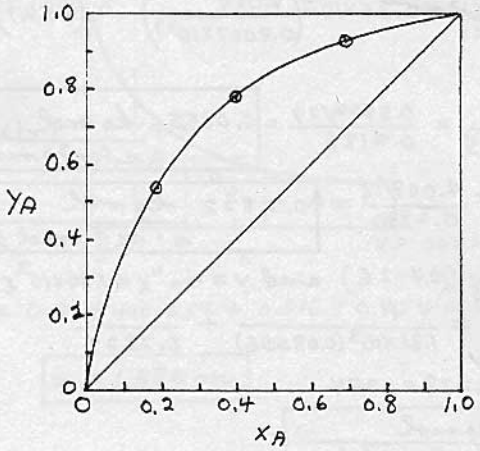
107.2°C $x_A = 0.192$ $y_A = 0.538$

125.7°C $x_A = 0.00$ $y_A = 0.000$

(a)

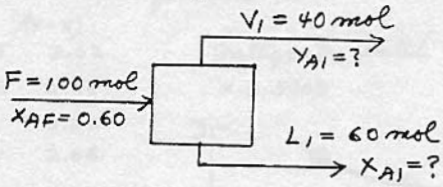
T °C	x_A	y_A
68.7	1.000	1.000
79.4	0.689	0.930
93.3	0.401	0.781
107.2	0.192	0.538
125.7	0.000	0.000

(b) Boiling point diagram.



(11.2-1)

Equilibrium data in Ex. (11.3-2)
A = n-pentane, B = n-heptane



Balance on A

$$F x_{AF} = V_1 y_{A1} + L_1 x_{A1}$$

$$100(0.60) = 40 y_{A1} + 60 x_{A1} \quad (x_{A1} \text{ and } y_{A1} \text{ in equilibrium})$$

Trial and error solution

Assume $x_{A1} = 0.40$

$$100(0.60) = 40 y_{A1} + 60(0.40)$$

$y_{A1} = 0.900$ Plot point $x_{A1} = 0.40$ and $y_{A1} = 0.90$ on equilibrium diagram.

Assume $x_{A1} = 0.30$

$$60 = 40 y_{A1} + 60(0.30)$$

$y_{A1} = 1.05$ Plot this point.

Assume $x_{A1} = 0.50$

$$60 = 40 y_{A1} + 60(0.50)$$

$y_{A1} = 0.75$ Plot this point.

Draw a straight line through the 3 points.

At the intersection with the equilibrium curve

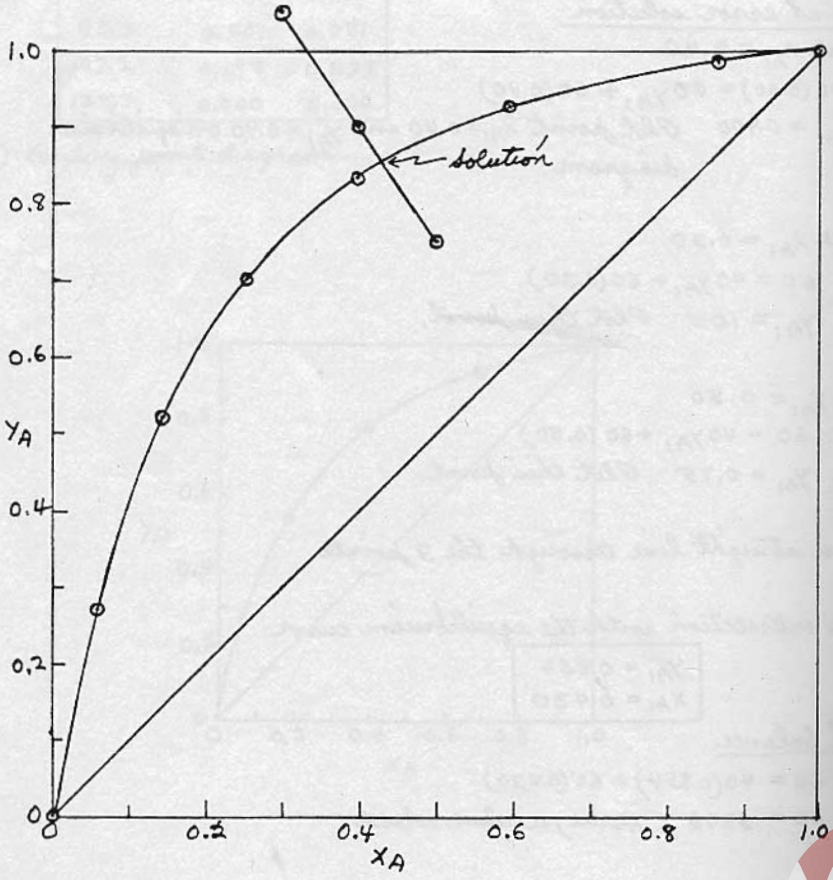
$$\boxed{\begin{aligned} y_{A1} &= 0.854 \\ x_{A1} &= 0.430 \end{aligned}}$$

Check balance

$$60 = 40(0.854) + 60(0.430)$$

$60 = 59.96$ Hence, a close check.

(11.2-1) - p.2



(11.3-2)

(a) $L_1 = 100$ kg mol $L_2 = 60$ $x_1 = 0.62$ $x_2 = ?$ $V = L_1 - L_2 = 40$

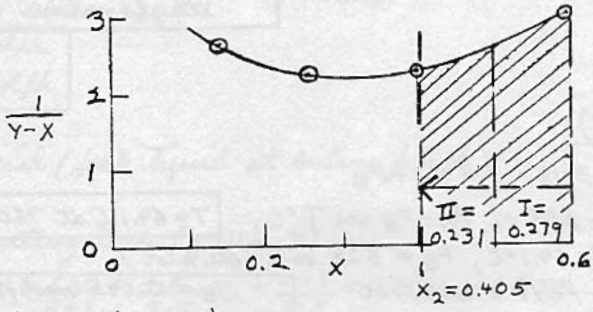
Eq (11.3-10) $\ln \frac{L_1}{L_2} = \ln \frac{100}{60} = 0.5108 = \int_{x_2}^{x_1=0.60} \frac{dx}{y-x}$

Equilibrium data Example 11.3-2.

x	y	1/(y-x)
0.594	0.925	3.02
0.398	0.836	2.28
0.254	0.701	2.24
0.145	0.521	2.66

Integrate until area under curve = 0.5108

$\boxed{x_2 = 0.405}$



Eq.(11.3-11) $L_1 x_1 = L_2 x_2 + (L_1 - L_2) y_{av}$

$100(0.6) = 60(0.405) + 40 y_{av}$

$\boxed{y_{av} = 0.892}$

(b) Equilibrium or flash distillation

$F = 100$ $V = 40$ $F - V = 60$

Eq.(11.3-6) $F x_F = V y + (F - V) x$

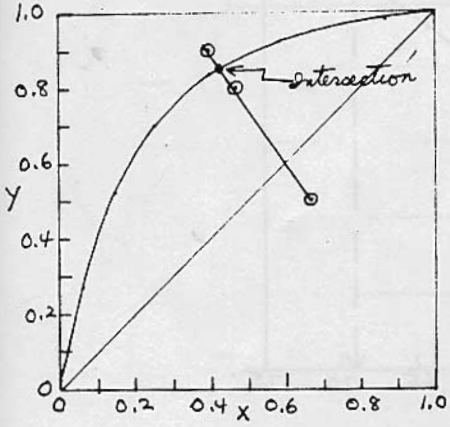
$100(0.6) = 40y + 60x$

Trial-and-error solution

Assume $y = 0.5$ $60 = 40(0.5) + 60x$ $x = 0.667$

$y = 0.8$ $60 = 40(0.8) + 60x$ $x = 0.467$

$y = 0.9$ $60 = 40(0.9) + 60x$ $x = 0.400$



Straight line intersects equilibrium curve at

$\boxed{\begin{array}{l} y = 0.854 \\ x = 0.430 \end{array}}$

Differential distillation gives higher y.

(11.3-4)

Eq. (11.3-13)

$P = P_A + P_B$

T°C	P_A	P_B	$P = P_A + P_B$
80.6	48.5	1.33	49.83
96.0	87.7	2.67	90.37
99.15	98.3	3.04	101.34
113.2	163.3	5.33	168.63

Hence, $P = 101.34$ (1 atm) at $\boxed{T = 99.15°C}$

$$y_B = \frac{P_B}{P} = \frac{3.04}{101.34} = \boxed{0.030 \text{ mole fraction ethylaniline}}$$

(11.3-5)

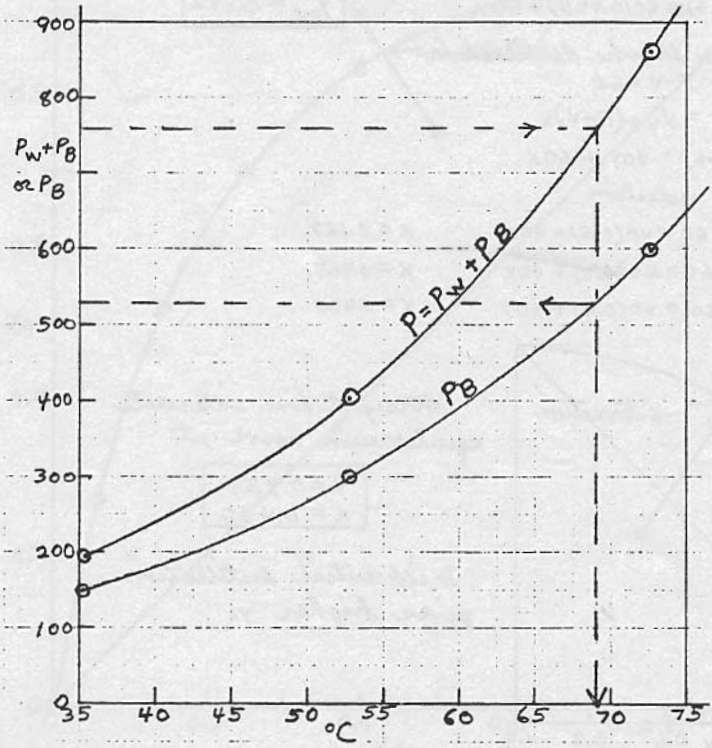
Eq. (11.3-13) $P = P_W + P_B$

From plot of $P_W + P_B$ vs. T°C, $\boxed{T = 69.1°C \text{ at } 760 \text{ mm}}$

at $T = 69.1°C$, $P_B = 528$ mm (see plot)

$y_B = P_B/P = 528/760$ $\boxed{y_B = 0.695 \text{ mol frac benzene}}$

Since $x_B = 0.50$ at start, $\boxed{\text{Benzene is first completely removed}}$



(11.4-2)

$F = 200$ mol/h $\quad x_F = 0.42 \quad x_D = 0.97 \quad x_W = 0.011 \quad R = 2.5$

$F = D + W$

$200 = D + W$

$Fx_F = Dx_D + Wx_W$

$200(0.42) = 0.97D + (200 - D)0.011$

$$\boxed{\begin{array}{l} D = 85.3 \text{ mol/h} \\ W = 114.7 \text{ mol/h} \end{array}}$$

q line is vertical (feed liquid at boiling point)

Enriching operating line, Eq. (11.4-8)

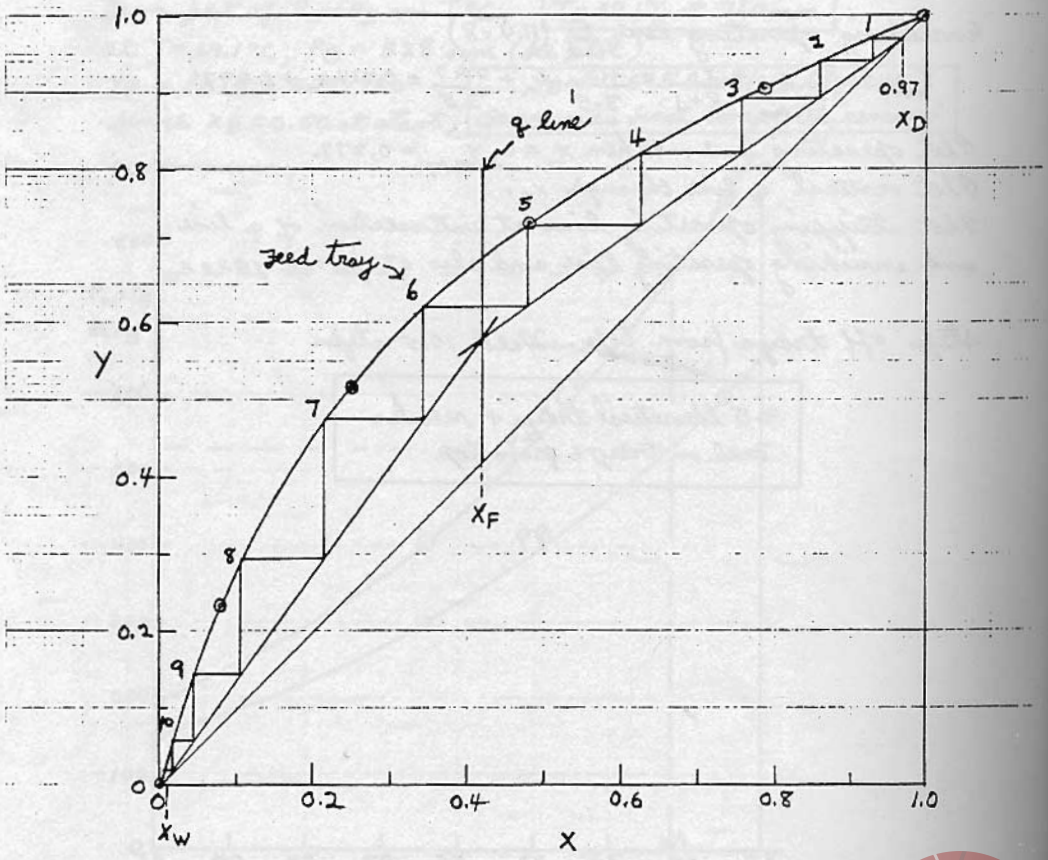
$$y_{n+1} = \frac{R}{R+1}x_n + \frac{x_D}{R+1} = \frac{2.5}{3.5}x_n + \frac{0.97}{3.5} = 0.714x_n + 0.277$$

Plot operating line. When $x_n = 0$, $y_{n+1} = 0.277$.
Plot vertical q line through x_F.
Plot stripping operating line at intersection of q line and enriching operating line and also at point $y = x = x_W$.

Step off trays from top. Need 10.5 steps

$$\boxed{\begin{array}{l} \text{9.5 theoretical trays + reboiler} \\ \text{Feed on tray 6 from top} \end{array}}$$

(11.4-2) - p.2



(11.4-4)

Determine R_m and minimum steps for Prob. (11.4-2)

$x_F = 0.42 \quad x_D = 0.97 \quad x_W = 0.011$

q line is vertical

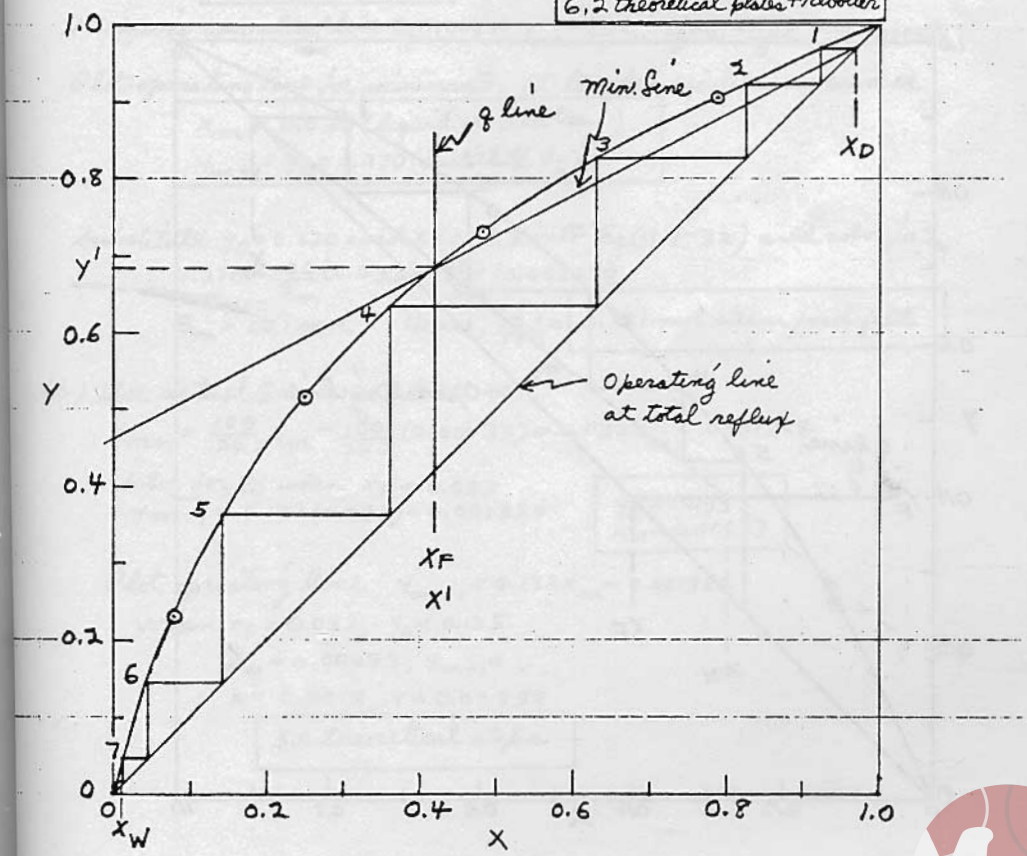
Plot enriching operating line for R_m as shown from point $x_D = y$ to point where line intersects the equilibrium line where q line crosses at $x' = 0.42$ and $y' = 0.682$

Eq. (11.4-24) $\dfrac{R_m}{R_m+1} = \dfrac{x_D - y'}{x_D - x'} = \dfrac{0.97-0.682}{0.97-0.42} = 0.524$

$\boxed{R_m = 1.10}$

at total reflux, stepping from top = 7.2 steps

6.2 theoretical plates + reboiler



(11.4-7)

$F = 100 \quad x_F = 0.40 \quad x_D = 0.90 \quad L/D = 4.0 = R$

Saturated vapor feed, $q = 0$; slope q line $= 0$, Eq. (11.4-19).

$L/D = 4.0, \quad L = 4.0D \quad V = F =$ all vapor

$V = L + D = 4.0D + D = 5.0D$

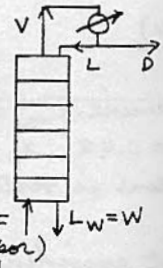
$F = L_W + D \qquad L_W = L$

$100 = 4.0D + D \qquad \boxed{D = 20.0 \text{ kg mol/h}}$

$F x_F = L_W x_W + D x_D \qquad \boxed{L_W = W = 4.0(20) = 80 \text{ kg mol/h}}$

$100(0.40) = 4.0(20)(x_W) + (20.0)(0.90)$

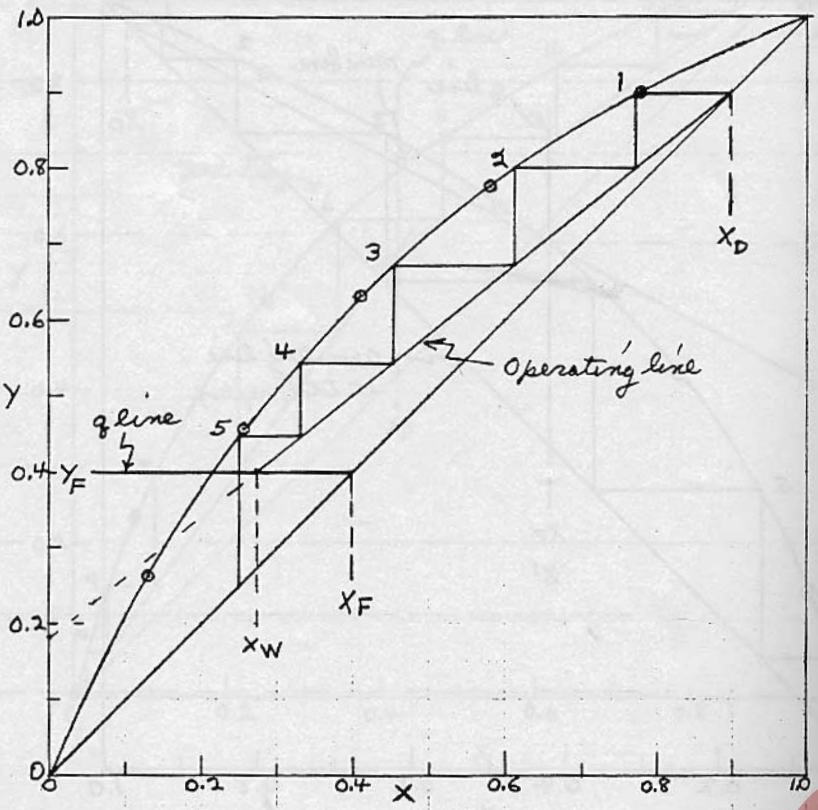
$\boxed{x_W = 0.275}$



Eq. (11.4-8)

$$y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1} = \frac{4.0}{4.0+1} x_n + \frac{0.90}{4.0+1} = 0.80 x_n + 0.180$$

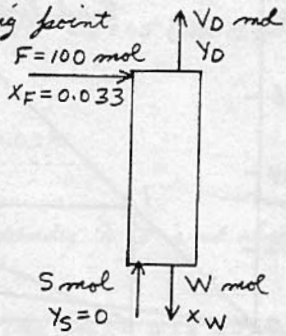
Plot operating line. Step off trays. $\boxed{\text{4.7 theoretical plates}}$



(11.4-9)

Assume $F = 100$ mol/h Feed liquid at boiling point ($q = 1.0$) $x_F = 0.033$

Overhead vapor contains 99% of alcohol in feed. $S = V_D$ (equimolar overflow)



(a) For minimum steam (infinite steps)

Eq. (11.4-29) $F + S = V_D + W$

But $S = V_D$, hence $F = 100 = W$ (equimolar)

$$F x_F + S y_S = V_D y_D + W x_W$$

$$100(0.033) + 0 = V_D y_D + 100 x_W$$

Now $V_D y_D$ = 99% of feed = $0.99(100)(0.033) = 3.267$ mol alcohol

Hence, $W x_W = 100 x_W = 0.01(3.30) = 0.033$

$$\boxed{x_W = 0.000330}$$

Stripping operating line Eq (11.4-32) $y_{m+1} = \frac{W}{S} x_m - \frac{W}{S} x_W = \frac{100}{S} x_m - \frac{100}{S}(0.00033)$

Plot operating line for minimum S. It touches equilibrium curve at

$$\boxed{\begin{array}{l} x_m = 0.033 \text{ (liquid on feed tray)} \\ y_{m+1} = y_D = 0.270 \text{ (distillate } V_D) \end{array}}$$

Substitute $y_D = 0.270$ and $x = 0.033$ into Eq (11.4-32) and solve for S_m

$$0.270 = \frac{100}{S_m}(0.033) - \frac{100}{S_m}(0.00033)$$

$S_m = 12.1$ mol Hence, $\frac{12.1}{100} = \boxed{0.121 \text{ mol steam/mol feed}}$

(b) Use actual $S = 2 \times 12.1 = 24.2$ mol

$$y_{m+1} = \frac{100}{24.2} x_m - \frac{100}{24.2}(0.00033) = 4.132 x_m - 0.001364$$

Solve for y_D when $x_F = 0.033$

$y_{m+1} = 4.132(0.033) - 0.001364$ $\boxed{\begin{array}{l} y_D = 0.135 \\ x_W = 0.00033 \end{array}}$

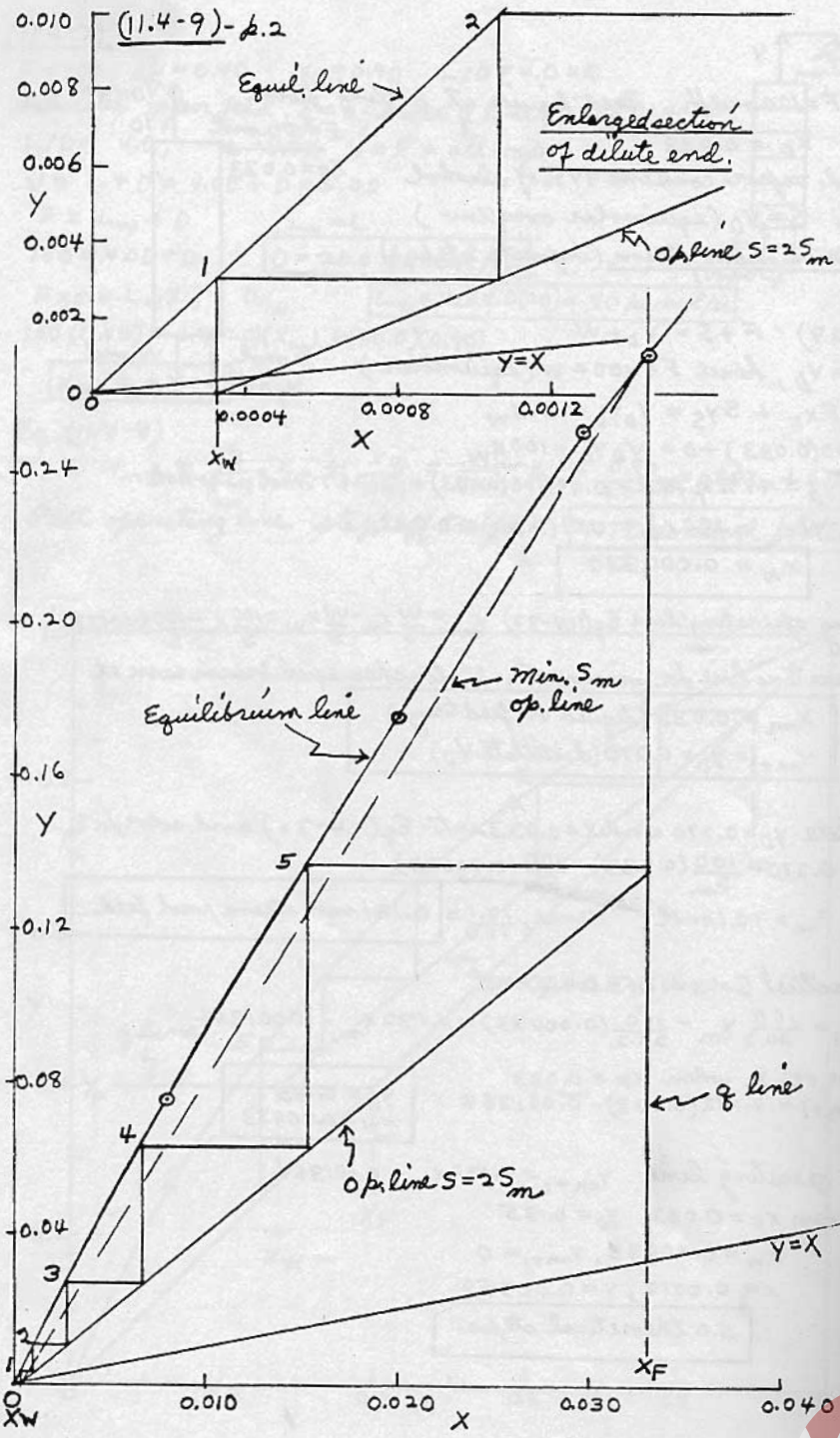
Plot operating line $y_{m+1} = 4.132 x_m - 0.001364$

When $x_F = 0.033$, $y_D = 0.135$

$x_W = 0.00033$, $y_{m+1} = 0$

$x = 0.0012$, $y = 0.00359$

$$\boxed{\text{5.0 theoretical steps}}$$


(11.4-9)-p.2
Equil. line
Enlarged section of dilute end.
Op. line S=2S_m
Y=X
Y
X
X_W
Equilibrium line
min. S_m op. line
q line
Op. line S=2S_m
Y=X
X_F
X_W
X

(11.6-1)

$x_F = 0.50$ saturated liq. at b. pt., hence $q = 1$ and vertical q line.

$x_D = 0.85$ wt. frac. $x_W = 0.03$ $F = 453.6$ kg/h $L/D = 1.5$ (top of tower)

(a) Overall material balance

$$F = 453.6 = D + W$$

Ethanol balance $0.50(453.6) = 0.85D + 0.03W$

$$\boxed{D = 260.0 \text{ kg/h}, \; W = 193.6 \text{ kg/h}}$$

(b) Plotting the enthalpy data from Appendix A-3 and also an x-y plot on next page,

at $x_D = 0.85$, H_1 (sat. vapor plot) $= 1303$ kJ/kg; $h_D = 228$ kJ/kg

Flows at top of tower.

$L/D = 1.50$, $L = 1.50(260.0) = 390.0$ kg/h

V_1 (top of tower) $= L + D = 390.0 + 260.0 = 650.0$ kg/h; $H_1 = 1303$ kJ/kg

To calculate points on operating line for tower it is trial-and-error.

Enriching section (Follow steps in section 11.6 B)

Trial 1

Step 1 assume $x_n = 0.50$. Assume $V_{n+1} = V_1$, $L_n = L$

Eq (11.6-8) to obtain an approx. value of y_{n+1}.

$$y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{D x_D}{V_{n+1}} = \frac{390.0}{650.0} x_n + \frac{260.0}{650.0}(0.85) = 0.640$$

Step 2

For $y_{n+1} = 0.640$, $H_{n+1} = 1640$; for $x_n = 0.50$, $h_n = 286$

Eq. (11.6-12) $V_{n+1} H_{n+1} = (V_{n+1} - D) h_n + V_1 H_1 - L h_D$

$$V_{n+1}(1640) = (V_{n+1} - 260.0)286 + 650.0(130.3) - 390.0(228)$$

$$V_{n+1} = 504.9 \text{ kg/h}$$

Eq (11.6-6) $V_{n+1} = L_n + D$, $504.9 = L_n + 260.0$, $L_n = 244.9$

Step 3

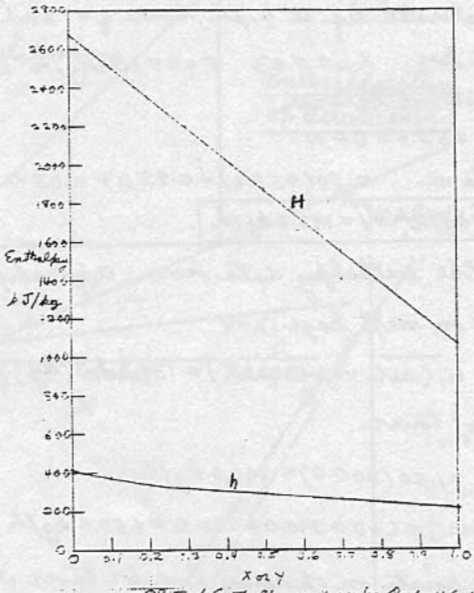
Eq (11.6-8) $y_{n+1} = \frac{244.9}{504.9}(0.50) + \frac{260.0}{504.9}(0.85) = 0.680$

Trial 2

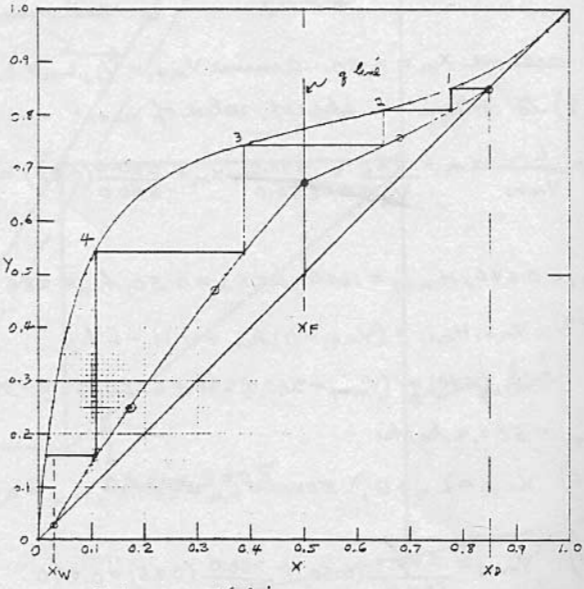
Step 2

For $y_{n+1} = 0.680$, $H_{n+1} = 1575$; for $x_n = 0.50$, $h_n = 286$

(11.6-1)-p.2



Plot of Enthalpy vs X or Y for Prob. 11.6-1



Plot of Operating Lines for Prob. 11.6-1

(11.6-1)-k.3

Eq.(11.6-12) $V_{n+1}(1575) = (V_{n+1} - 260.0)286 + 650.0(1303) - 390.0(228)$

$V_{n+1} = 530.4$

Eq.(11.6-6) $V_{n+1} = 530.4 = L_n + 260.0$, $L_n = 270.4$

Step 3

Eq.(11.6-8) $y_{n+1} = \frac{270.4}{530.4}(0.50) + \frac{260.0}{530.4}(0.85) = 0.672$

(a third trial gives $y_{n+1} = 0.673$). Plot pt. $x_n = 0.50$ and $y_{n+1} = 0.672$ on x-y diagram as one point on the operating line.

Assume $x_n = 0.68$

Trial 1

Step 1

$$y_{n+1} = \frac{L_n}{V_{n+1}} x_n + \frac{D x_D}{V_{n+1}} = \frac{390.0}{650.0}(0.68) + \frac{260.0}{650.0}(0.85) = 0.748$$

Step 2

$y_{n+1} = 0.748$, $H_{n+1} = 1468$ $x_n = 0.68$, $h_n = 260$

Eq.(11.6-12) $V_{n+1}(1468) = (V_{n+1} - 260.0)260 + 650(1303) - 390.0(228)$

$V_{n+1} = 571.5$

Eq.(11.6-6) $V_{n+1} = 571.5 = L_n + 260$, $L_n = 311.5$

Step 3

Eq.(11.6-8) $y_{n+1} = \frac{311.5}{571.5}(0.68) + \frac{260.0}{571.5}(0.85) = 0.757$

a second trial gives $y_{n+1} = 0.756$

Plot pt. $x_n = 0.68$ and $y_{n+1} = 0.756$ on x-y diagram. Then plot the enriching operating line which is curved slightly.

Stripping Section (Follow steps in Section 11.6C)

Calculate q_C from Eq.(11.6-10)

(c)

$q_C = V_1 H_1 - L h_D - D h_D = 650.0(1303) - 390.0(228) - 260(228)$

$x_W = 0.03$, $h_W = 395$; $x_F = 0.50$, $h_F = 286$ $\boxed{q_C = 698{,}750 \text{ kJ/h}}$

Calc. q_R from Eq.(11.6-17)

$q_R = D h_D + W h_W + q_C - F h_F$

(b) $= 260.0(228) + (193.6)(395) + 698{,}750 - 453.6(286) = \boxed{704{,}770 \text{ kJ/h} = q_R}$

Trial 1

assume $y_{m+1} = 0.47$ Assume $L_m = L = 390$ ($q = 1.0$), $V_m = V_1 = 650$

Step 1

Eq.(11.4-14) $L_m = L_n + qF = 390 + 1.0(453.6) = 843.6$

Eq.(11.4-15) $V_n = V_m - (1-q)F$, $650 = V_m - 0$, also assume $V_m = V_{m+1} = 650$

(11.6-1)-p.4

Eq. (11.6-15)

$$0.47 = Y_{m+1} = \frac{L_m}{V_{m+1}} x_m - \frac{W x_W}{V_{m+1}} = \frac{843.6}{650} x_m - \frac{193.6(0.03)}{650}$$

Solving, $x_m = 0.369$

Step 2

$x_m = 0.369$, $h_m = 305$; $Y_{m+1} = 0.470$, $H_{m+1} = 1{,}910$

Eq. (11.6-16)

$$V_{m+1} H_{m+1} = (V_{m+1} + W) h_m + q_R - W h_W$$

$$V_{m+1}(1{,}910) = (V_{m+1} + 193.6)(305) + 704{,}770 - 193.6(395)$$

$V_{m+1} = 428.3$ Eq. (11.6-13) $L_m = W + V_{m+1} = 193.6 + 428.3 = 621.9$

Step 3

Eq. (11.6-15) $0.470 = \frac{621.9}{428.3}(x_m) - \frac{193.6(0.03)}{428.3}$

Solving, $x_m = 0.333$

Trial 2

$x_m = 0.333$, $h_m = 310$; $Y_{m+1} = 0.470$, $H_{m+1} = 1910$

Eq. (11.6-16)

$$V_{m+1}(1910) = (V_{m+1} + 193.6)(310) + 704{,}770 - 193.6(395)$$

Solving, $V_{m+1} = 430.2$

Eq. (11.6-13)

$$L_m = W + V_{m+1} = 193.6 + 430.2 = 623.8$$

Eq. (11.6-15) $0.470 = \frac{623.8}{430.2} x_m - \frac{193.6}{430.2}(0.03)$

$x_m = 0.333$ Plot the point $x_m = 0.333$, $Y_{m+1} = 0.470$ on X-Y diagram.

Trial 1

Assume $Y_{m+1} = 0.25$

Eq. (11.6-15) $Y_{m+1} = 0.25 = \frac{843.6}{650} x_m - \frac{193.6}{650}(0.03)$; $x_m = 0.1995$

$x_m = 0.1995$, $h_m = 335$; $Y_{m+1} = 0.25$, $H_{m+1} = 2270$

Eq. (11.6-16) $V_{m+1}(2270) = (V_{m+1} + 193.6)(335) + 704{,}770 - 193.6(395)$

$V_{m+1} = 358.7$. Eq. (11.6-13) $L_m = 193.6 + 358.7 = 552.3$

Eq. (11.6-15) $0.25 = \frac{552.3}{358.7} x_m - \frac{193.6}{358.7}(0.03)$; $x_m = 0.1729$. Plot on X-Y plot. Plotting the operating line and stepping off the steps, we get 4.8+ steps. Hence, need $\boxed{\text{4.9 theoretical steps}}$

(11.7-1)

(a) Calculate dew point at 405.3 kPa. Use Eq. 11.7-2 for K values.

Trial 1 - assume 106°C

Comp	y_{iD}	K_i'	$\frac{K_i'}{K_c}=\alpha_i'$	y_i'/α_i'
A	0.40	3.50	5.036	0.0794
B	0.25	1.54	2.216	0.1128
C	0.20	0.695	1.000	0.2000
D	0.15	0.330	0.4748	0.3159
	1.00		$\Sigma y_i/\alpha_i=0.7081$	

$K_{c_{calc}}=\Sigma y_i/\alpha_i=0.7081$

This value is > 0.695 at 106°C.

Trial 2 - assume 107°C

Eq (11.7-8)

Comp	y_{iD}	K_i'	$\frac{K_i'}{K_c}=\alpha_i'$	y_i'/α_i'	$\frac{y_i'/\alpha_i'}{\Sigma y_i/\alpha_i'}=$ x_i'
A	0.40	3.55	4.931	0.0811	0.114
B	0.25	1.60	2.222	0.1125	0.158
C	0.20	0.720	1.000	0.2000	0.281
D	0.15	0.340	0.472	0.3178	0.447
	1.00		$\Sigma y_i/\alpha_i=0.7114$		1.000

Liquid composition

$K_{c_{calc}}=\Sigma y_i/\alpha_i'=0.7114$ Close to $K_{c\,assumed}=0.720$. $T_{dew\ point}=107°C$

(b) Partial vaporization, $f=0.40$ Trial 1 - Assume 85°C. Eq (11.7-11)

Comp.	x_{iF}	K_i'	α_i'	$K_c\alpha_i'-1$	$f(K_c\alpha_i'-1)+1$	$\frac{x_{iF}}{f(K_c\alpha_i'-1)+1}$
A	0.40	2.45	5.698	1.450	1.580	0.2532
B	0.25	1.01	2.349	0.010	1.004	0.2490
C	0.20	0.43	1.000	-0.570	0.772	0.2591
D	0.15	0.185	0.430	-0.815	0.674	0.2256
	1.00				$\Sigma x_i'=$	0.9869

Trial 2 - assume 82°

Comp	x_{iF}	K_i'	α_i'	$K_c\alpha_i'-1$	$f(K_c\alpha_i'-1)+1$	$\frac{x_{iF}}{f(K_c\alpha_i'-1)+1}$
A	0.40	2.34	5.850	1.340	1.536	0.2604
B	0.25	0.95	2.375	-0.050	0.980	0.2550
C	0.20	0.40	1.000	-0.600	0.760	0.2632
D	0.15	0.170	0.425	-0.830	0.668	0.2246
	1.00				$\Sigma x_i=$	1.0032

T = 82°C
x_i (liquid)
0.260
0.254
0.262
0.224
1.000

Vapor composition by material balance. F (feed) = 100 mol

	x_{iF}	$x_{iF}F$	$x_iL = L_i'$	$(x_iF-L_i')=V_i'$	$V_i'/40=$ y_i'
A	0.40	40.0	0.260(60) = 15.60	24.40	0.610
B	0.25	25.0	0.254(60) = 15.24	9.76	0.244 (Vapor)
C	0.20	20.0	0.262(60) = 15.72	4.28	0.107
D	0.15	15.0	0.224(60) = 13.44	1.56	0.039
		100.0	60.00	40.00	1.000

(11.7-4)

Recover 95% of nC_5 (B) in D and 95% of nC_6 (C) in W.
(Feed of Example 11.7-3 at boiling point.)

$F = 100$ mol/h $\quad x_{AF}(nC_4) = 0.40,\ x_{BF}(nC_5) = 0.25,\ x_{CF}(nC_6) = 0.20,$
$x_{DF}(nC_7) = 0.15.$

(a) Material balances.

Compon. B $\quad$ Eq. (11.7-16)

$$x_{BF}F = 0.25(100) = y_{BD}D + x_{BW}W$$
$$y_{BD}D = 0.95(25) = 23.75 \quad (95\%\ B \text{ in distillate})$$
$$x_{BW}W = 0.05(25) = 1.25$$

Compon. C

$$x_{CF}F = 0.20(100) = 20.0 = y_{CD}D + y_{CW}W$$
$$x_{CW}W = 0.95(20) = 19.0$$
$$y_{CD}D = 0.05(20) = 1.0$$

Assume no compon. D in distillate and no A in W for first trial.

Hence, $y_{AD}D = 0.40(100) = 40.0 \qquad x_{DW}W = 0.15(100) = 15.0$

Comp.	Feed F		Distillate, D		Bottoms, W	
	x_F	$x_F F$	$y_D D$	$y_D = x_D$	$x_W W$	x_W
A	0.40	40.0	40.00	0.6178	0	0
B (L)	0.25	25.0	23.75	0.3668	1.25	0.0355
C (H)	0.20	20.0	1.00	0.0154	19.00	0.5390
D	0.15	15.0	0.00	0	15.00	0.4255
	1.00	F = 100.0	D = 64.75	1.0000	W = 35.25	1.0000

(b) Top temp. of distillate (dew point)

Comp.	$y_i D$	Trial 1, assume 67°C K_i'	α_i'	y_i/α_i'	Trial 2, assume 66°C K_i'	α_i'	y_i/α_i'	$\frac{y_i/\alpha_i'}{\Sigma y_i/\alpha_i'} = x_i'$
A	0.6178	1.75	6.73	0.0918	1.72	6.745	0.0916	0.3635
B (L)	0.3668	0.65	2.50	0.1467	0.645	2.529	0.1450	0.5754
C (H)	0.0154	0.26	1.00	0.0154	0.255	1.00	0.0154	0.0611
D	0	0.10	0.385	0	0.095	0.373	0	0
		$K_{c_{calc}} = \Sigma y_i/\alpha_i' = 0.2539$			$K_{c_{calc}} = \Sigma y_i/\alpha_i' = 0.2520$			1.0000
								T = 66°C

(11.7-4) – p. 2

Bottom temp. of tower (boiling point)

	Trial 1, assume 133°C				Trial 2, 134°C			$\frac{\alpha_i x_i'}{\Sigma \alpha_i x_i'} =$
Comp	x_{iW}	K_i	α_i	$\alpha_i x_i'$	K_i	α_i	$\alpha_i x_i'$	y_i'
A	0	5.10	4.322	0	5.20	4.370	0	0
B(L)	0.0355	2.40	2.034	0.0722	2.45	2.059	0.0731	0.0869
C(H)	0.5390	1.18	1.000	0.5390	1.19	1.000	0.5390	0.6410
D	0.4255	0.63	0.534	0.2272	0.64	0.538	0.2288	0.2721
	1.0000		$\Sigma \alpha_i x_i' = 0.8384$			$\Sigma \alpha_i x_i' = 0.8409$		1.000
								T = 134°C

$$K_{c_{calc}} = \frac{1}{\Sigma \alpha_i x_i'} = 1.193 (134°C) \quad K_{c\,calc} = 1.189$$

(c) Minimum stages

$$\alpha_{LD} = 2.529\ (66°C) \quad \alpha_{LW} = 2.059\ (134°C)$$

$$\alpha_{L,av} = \sqrt{2.529 (2.059} = 2.282$$

Eq. (11.7-12)

$$N_m = \frac{\log[(x_{LD}D / x_{HD}D)(x_{HW}W / x_{LW}W)]}{\log(\alpha_{L,av})}$$

$$N_m = \frac{\log[(0.3668 \times 64.75 / 0.0154 \times 64.75)(0.5390 \times 35.25 / 0.0355 \times 35.25)]}{\log(2.282)}$$

$$N_m = 7.139 = \boxed{7.14 \text{ theoretical stages}}$$

Calculation of traces using N_m Fenské

	α_D	α_W	$\sqrt{\alpha_D \alpha_W}$
A	6.745	4.370	5.429
B (L)	2.529	2.059	2.282
C (H)	1.000	1.000	1.000
D	0.373	0.538	0.448

Component A Eq (11.7-14)

$$\frac{x_{AD}D}{x_{AW}W} = (\alpha_{A,av})^{N_m} \frac{x_{HD}D}{x_{HW}W} = (5.429)^{7.139} \left(\frac{0.0154 \times 64.75}{0.539 \times 35.25}\right) = 9229$$

Overall balance on A

$$x_{AF}F = 40 = x_{AD}D + x_{AW}W = 9229\, x_{AW}W + x_{AW}W$$

$$\boxed{\begin{aligned} x_{AW}W &= 0.0043 \text{ mol} \\ x_{AD}D &= 39.996 \text{ mol} \end{aligned}}$$

(11.7-4) - p.3

Component D

$$\frac{x_{DD}D}{x_{DW}W} = (0.448)^{7.139}\left(\frac{0.0154\times 64.75}{0.539\times 35.25}\right) = 1.710\times 10^{-4}$$

Overall balance on D

$$x_{DF}F = 15.00 = x_{DD}D + x_{DW}W = 1.710\times 10^{-4}x_{DW}W + x_{DW}W$$

$$\boxed{\begin{array}{l} x_{DW}W = 14.9974 \\ x_{DD}D = 0.0026 \end{array}}$$

New compositions

	Distillate D		Bottoms W		
comp.	$y_D = x_D$	$x_D D$ (moles)	x_W	$x_W W$ (moles)	Traces
A	0.61776	39.996	0.00012	0.0043	$x_{AW} = 1.2\times 10^{-4}$
B(L)	0.3668	23.750	0.0355	1.250	$y_{DD} = 4.0\times 10^{-5}$
C(H)	0.0154	1.000	0.5390	19.000	
D	0.00004	0.0026	0.4254	14.997	
	1.0000	D=64.749	1.0000	W=35.251	

No change in top and bottom temp. (traces too small)

(d) Underwood min. L/D

Eq. (11.7-19) $\quad 1-q = \dfrac{\sum \alpha_i x_{iF}}{\alpha_i - \theta}$ $\qquad q = 1.0$ (feed at b. pt.)

$$T_{av} = (T_{top} + T_{bottom})/2 = (66+134)/2 = 100°C$$

	x_{iF}	$K_i(100°C)$	α_i'	x_{iD}
A	0.40	3.15	5.25	0.6177
B	0.25	1.38	2.30	0.3668
C	0.20	0.600	1.00	0.0154
D	0.15	0.285	0.475	0.0004
	1.00			1.000

Trial-and-error solution

$$1-q = 1-1 = 0 = \frac{5.25(0.40)}{5.25-\theta} + \frac{2.30(0.25)}{2.30-\theta} + \frac{1.00(0.20)}{1.00-\theta} + \frac{0.475(0.15)}{0.475-\theta}$$

θ	$\frac{2.10}{5.25-\theta}$	$+\frac{0.575}{2.30-\theta}$	$+\frac{0.20}{1.00-\theta}$	$+\frac{0.0713}{0.475-\theta}$	$= \Sigma$(sum)
1.210	0.5198	0.5275	−0.9524	−0.0970	−0.0021
1.2096	0.5198	0.5273	−0.9542	−0.0971	−0.0042
1.2104	0.5199	0.5277	−0.9506	−0.0970	−0.0

Hence, $\theta = 1.2104$

(11.7-4)-p.4

Eq. (11.7-20)

$$R_m + 1 = \frac{\Sigma \alpha_i x_{iD}}{\alpha_i - \theta} = \frac{5.25(0.6177)}{5.25-1.2104} + \frac{2.30(0.3668)}{2.30-1.2104} + \frac{1.00(0.0154)}{1.00-1.2104} + \frac{0.475(0.00004)}{0.475-1.2104}$$

$R_m + 1 = 1.5039$ $\boxed{R_m = 0.504}$

2) Use $1.30 R_m$

$R = 1.30(0.504) = 0.6552$

Erbar-Maddox method

$$\frac{R}{R+1} = \frac{0.6552}{1+0.6552} = 0.3958$$

$$\frac{R_m}{R_m+1} = \frac{0.504}{1+0.504} = 0.3351$$

Fig. 11.7-3 $\frac{N_m}{N} = 0.425$

$N = \frac{N_m}{0.425} = \frac{7.14}{0.425} = 16.8$ $\boxed{N = 16.8 \text{ theoretical stages}}$

4) Feed tray location

Eq. (11.7-21)

$$\log \frac{N_e}{N_s} = 0.206 \log \left[\frac{x_{HF}}{x_{LF}} \frac{W}{D} \left(\frac{x_{LW}}{x_{HD}} \right)^2 \right]$$

$$= 0.206 \log \left[\frac{0.20}{0.25} \frac{35.251}{64.749} \left(\frac{0.0355}{0.0154} \right)^2 \right]$$

$$= 0.075075$$

$\frac{N_e}{N_s} = 1.188$ $N_e = 1.188 N_s$

$N = N_e + N_s = 16.8$

$1.188 N_s + N_s = 16.8$

$N_s = 7.68 = 7.7$

$N_e = 9.1$

$N_s = 7.7$ stages (strip)

$N_e = 9.1$ stages (enrich)

Feed on 9.1 stages from top

(12.1-1)

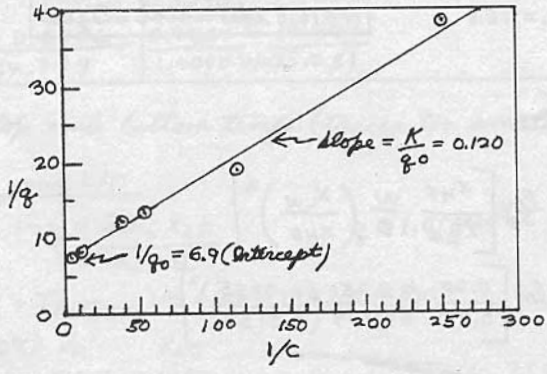
Data are as follows

q	c	$1/q$	$1/c$
0.026	0.0040	38.5	250
0.053	0.0087	18.9	114.9
0.075	0.019	13.33	52.6
0.082	0.027	12.19	37.0
0.123	0.094	8.13	10.63
0.129	0.195	7.75	5.13

First try Langmuir equation, Eq. (12.1-3) $q = \frac{q_o c}{K + c}$

Plot $1/q$ as follows.

(Note: a plot of $\log q$ vs $\log c$ for the Freundlich Eq. (12.1-2) where $q = Kc^n$ is not a straight line. Hence, data do not follow this Eq.)



The data give a straight line and follow the Langmuir equation.

$\frac{K}{q_o}$ = slope of line = 0.120

The intercept is $1/q_o = 6.9$. Hence, $q_o = \frac{1}{6.9} = 0.145$

$K = 0.120 q_o = 0.120(0.145) = 0.0174$

$$\boxed{q = \frac{0.145}{0.0174 + c}}$$

(12.3-1)

From Eq. 12.3-1, τ_t (Total capac. of bed) = 5.16 h, t_b = 6.95 h,

τ_u (Usable capac. to break pt) = 3.65 h, t_b (Break pt) = 3.65 h

H_{UNB} (Unused bed) = 4.1 cm, H_B (Length used bed) = 9.9 cm, A_1 = 3.65 h, A_2 = 1.51 h

(a)

For the new tower, t_b = 8.5 h

New H_B is proportional to t_b.

$H_B(\text{new}) = \frac{8.5\,h}{3.65\,h}(9.9\,cm) = 23.1\,cm$

Eq. (12.3-5)

$H_{T(new)} = H_{UNB} + H_{B(new)} = 4.1 + 23.1 = \boxed{27.2\,cm = H_T}$

New $t_t = A_1 + A_2 = 8.50 + 1.51 = 10.01\,h$

t_u (usable to break pt) = 8.5 h

$t_u/t_t = \frac{8.5}{10.01} = \boxed{0.849 \text{ frac. usable capac.}}$ $\boxed{\text{same diameter}}$

(b) New C.S. area $= \frac{2000}{754}$ (old C.S. area)

$\frac{\pi}{4} D^2 = \frac{2000}{754}\left(\frac{\pi}{4} 4^2\right)$ $\boxed{D = 6.52\,cm}$ $\boxed{\text{all other values as in (a)}}$

(12.4-2)

Existing: H_T = 0.40 m, t_b = 8.0 min, t_u/t_t = 0.65 frac. usable capac.

For a new tower, t_b = 13.0 min for the same flow rate.

Eq. (12.3-4)

H_{UNB} (length of unused bed) = (Unused frac.) H_T

$H_{UNB} = (1-0.65)0.40 = 0.14\,m$ (Mass transf. section constant)

Eq. (12.3-5)

H_T (total bed) = $H_{UNB} + H_B$ (used bed)

$0.40 = 0.14 + H_B$ $\quad H_B = 0.26\,m$

New H_B is prop. to t_b

$H_B = \frac{13.0}{8.0}(0.26) = 0.423\,m$

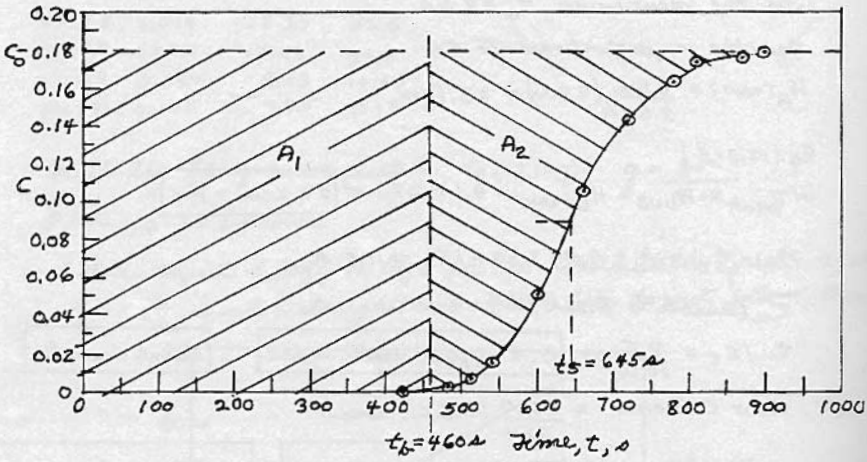
H_T (total new) =

$H_T = H_{UNB} + H_B = 0.14 + 0.423 = \boxed{0.563\,m = H_T}$

(12.4-3)

Given: $H_T = 30.5\ cm$, $D = 2.59\ cm$, $c_0 = 0.18\ M\ CuSO_4$, flow rate $= 1.37\ \frac{cm^3}{s}$, 99.3 g resin, $MW\ Cu^{2+} = 63.54$

Breakthrough data are plotted below.



From graph, break point time $\boxed{t_b = 460\ s}$

area above

$$\text{area } A_1 = \frac{82.8}{0.18 \leftarrow c_0} = 460\ s = t_u = \int_0^{t_b} (1 - c/c_0)\,dt \qquad \text{Eq (12.3-2)}$$

note that t_u is essentially the same as t_b.

$$\text{area } A_2 = \frac{34.34}{0.18 \leftarrow c_0} = 190.8\ s \qquad t_t = 460 + 190.8 = 650.8\ s$$

Using the approximate method at $c/c_0 = 0.5$, gives $t_t = 645\ s$ which is close to $t_t = 650.8\ s$.

Eq. (12.3-4)

$$H_{UNB}\ (\text{Unused bed}) = (1 - t_u/t_t)\,H_T$$

$1 - t_u/t_t$ = frac. of unused bed. Hence, frac bed used $= \frac{t_u}{t_t} = \frac{460}{650.8}$

$$460/650.8 = \boxed{0.707 \text{ frac bed used}}$$

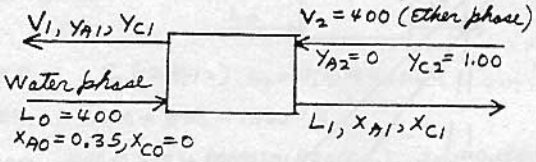
$$H_{UNB} = (1 - 0.707)(30.5\ cm) = \boxed{8.9\ cm = H_{UNB}}$$

Eq. (12.3-5) $\quad H_B = H_T - H_{UNB} = 30.5 - 8.9 = \boxed{21.6\ cm = H_B}$

$$\text{gm Cu ads. to } t_t = \frac{1.37\ cm^3/s}{1000\ cm^3/L} \times 0.18\ \frac{g\ mol}{L} \times 63.54\ \frac{g\ Cu}{g\ mol} \times 650.8\ s = 10.197\ g\ Cu$$

$$\frac{g\ Cu}{g\ solid} = \frac{10.197}{99.3} = \boxed{0.103\ g\ Cu/g\ solid \text{ sat. loading capac.}}$$

(12.5-2)



Eq. (12.5-12)

$$L_0 + V_2 = M = L_1 + V_1$$
$$400 + 400 = 800 = M$$

Eq. (12.5-13)

$$L_0 x_{A0} + V_2 y_{A2} = M x_{AM}$$
$$400(0.35) + 400(0) = 800 x_{AM}$$
$$x_{AM} = 0.175$$

Eq. (12.5-14)

$$L_0 x_{C0} + V_2 y_{C2} = M x_{CM}$$
$$400(0) + 400(1.00) = 800 x_{CM}$$
$$x_{CM} = 0.50$$

Plot point M on graph. Tie line through M gives L_1 and V_1.

L_1 Raffinate layer

$x_{A1} = 0.255$
$x_{C1} = 0.03$
$x_{B1} = 0.715$

V_1 Extract layer

$y_{A1} = 0.11$
$y_{C1} = 0.86$
$y_{B1} = 0.03$

Calculate L_1 and V_1 amounts.

Eq. (12.5-13)

$$L_1 x_{A1} + V_1 y_{A1} = M x_{AM}$$
$$L_1(0.255) + V_1(0.11) = 800(0.175)$$

Eq. (12.5-12)

$$L_1 + V_1 = M = 800$$

Solving,

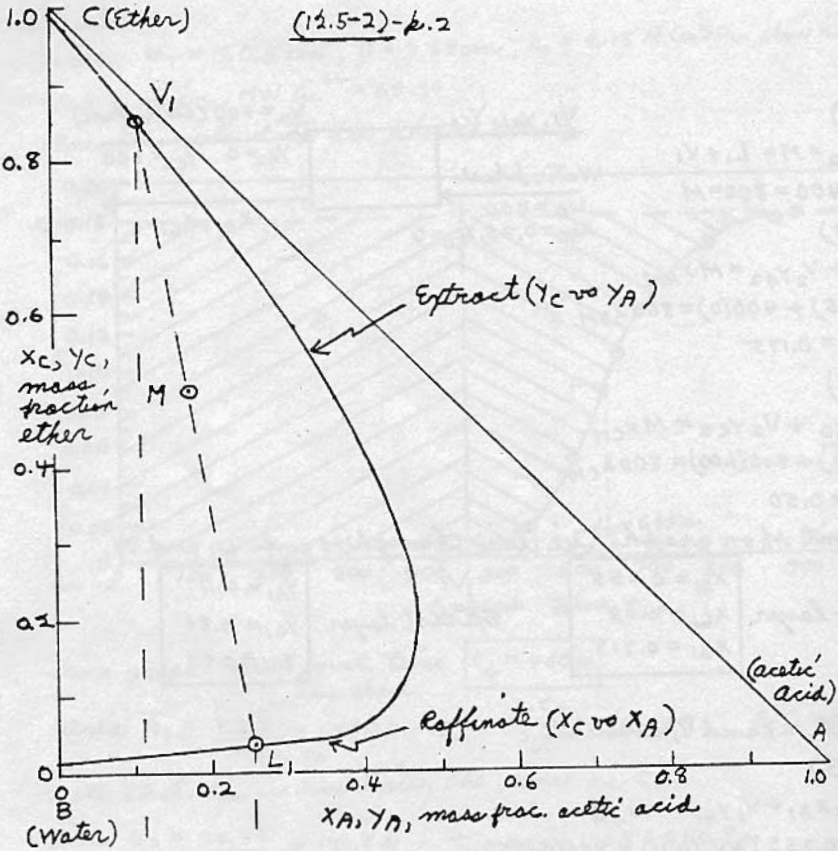
$$\boxed{\begin{aligned} L_1 &= 358 \text{ kg} \\ V_1 &= 442 \text{ kg} \end{aligned}}$$

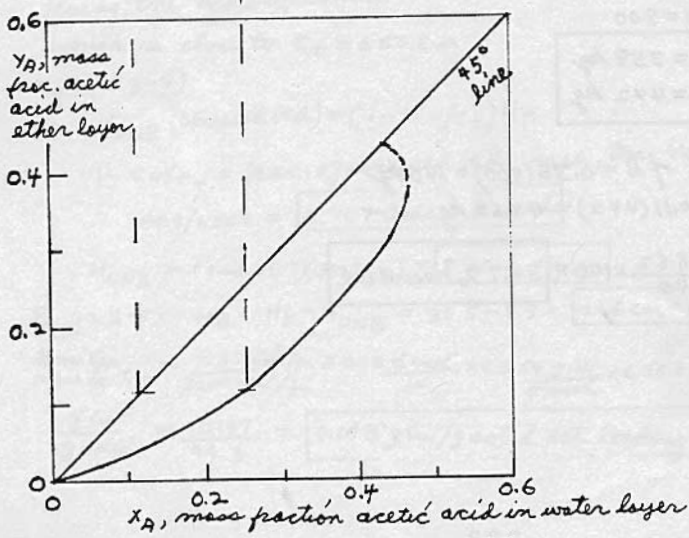
Original amount of A = 0.35(400) = 140 kg

Recovered in V_1 = 0.11(442) = 48.62 kg

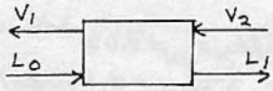
$$\% \text{ recovered} = \frac{48.62}{140} \times 100 = \boxed{34.7\% \text{ recovered}}$$

(12.5-2)-p.2





(12.5-4)



$L_0 = 1000$ kg $\quad x_{A0} = 0.235$ (acetone)

$x_{B0} = 0.765$ (H_2O), $x_{C0} = 0$ (MIK)

$V_2 = 500$ kg $\quad y_{C2} = 1.00 \quad y_{A2} = 0$ $\qquad$ Eq. (12.5-12) $\quad L_1 + V_1 = M = 1500$

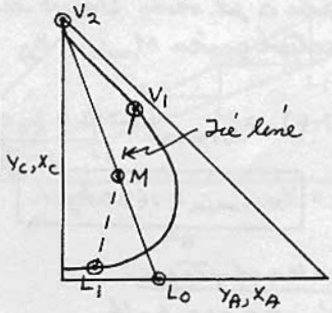
Eq. (12.5-12) $\quad L_0 + V_2 = M = L_1 + V_1 \qquad 1000 + 500 = 1500 = M$

Eq. (12.5-13) $\quad L_0 x_{A0} + V_2 y_{A2} = M x_{AM} \qquad 1000(0.235) + 500(0) = 1500(x_{AM}) \qquad x_{AM} = 0.1567$

Eq. (12.5-14) $\quad L_0 x_{C0} + V_2 y_{C2} = M x_{CM} \qquad 1000(0) + 500(1.0) = 1500(x_{CM}) \qquad x_{CM} = 0.333$

Plot points M, V_2, L_0 on graph.
Tie line through M gives L_1 and V_1 compositions.

Raffinate L_1 $\boxed{\begin{matrix} x_{A1} = 0.120 \\ x_{C1} = 0.012 \end{matrix}}$ Extract V_1 $\boxed{\begin{matrix} y_{A1} = 0.205 \\ y_{C1} = 0.748 \end{matrix}}$

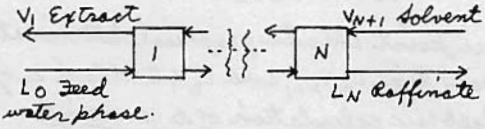


Eq. (12.5-13) $\quad L_1 x_{A1} + V_1 y_{A1} = M x_{AM}$

$L_1(0.120) + (1500 - L_1)(0.205) = 1500(0.1567)$

$\boxed{L_1 = 852 \text{ kg}, \; V_1 = 648 \text{ kg}}$

(12.7-2)



$L_0 = 200$ kg/h (feed)

$x_{A0} = 0.25$ acetic acid $\quad x_{C0} = 0$

$x_{B0} = 0.75$ H_2O $\qquad x_{AN} = 0.03$ in exit water phase.

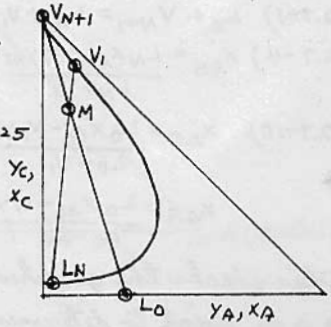
$V_{N+1} = 600$ kg/h (pure i pr. ether) $\quad y_{AN+1} = 0 \quad y_{CN+1} = 1.00$

Eq. (12.7-3)

$$x_{CM} = \frac{L_0 x_{C0} + V_{N+1} y_{CN+1}}{L_0 + V_{N+1}} = \frac{200(0) + 600(1.0)}{200 + 600} = 0.75$$

Eq. (12.7-4)

$$x_{AM} = \frac{L_0 x_{A0} + V_{N+1} y_{AN+1}}{L_0 + V_{N+1}} = \frac{200(0.25) + 600(0)}{200 + 600} = 0.0625$$



Plot points V_{N+1}, L_0, and M. L_N on phase b.
Locate V_1 by drawing line $L_N M$ to intersect extract phase boundary at V_1.

$\boxed{\begin{matrix} V_1 & y_{A1} = 0.070 \\ & y_{C1} = 0.905 \end{matrix}}$ $\qquad$ $\boxed{\begin{matrix} L_N & x_{AN} = 0.030 \\ & x_{CN} = 0.017 \end{matrix}}$

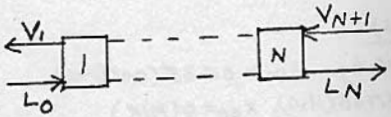
Eq. (12.7-1) $\quad L_0 + V_{N+1} = 200 + 600 = 800 = L_N + V_1 = M$

Eq. (12.7-2) $\quad L_0 x_{C0} + V_{N+1} y_{CN+1} = M x_{CM} = L_N x_{CN} + V_1 y_{C1}$

$200(0) + 600(1.00) = 800(0.75) = L_N(0.017) + (800 - L_N)(0.905)$

$\boxed{L_N = 139.6 \text{ kg/h} \quad V_1 = 660.4 \text{ kg/h}}$

(12.7-4)



$L_0 = 1000\ kg/h$, $x_{AN} = 0.02$

$x_{A0} = 0.30$ $\quad V_{N+1} = 2{,}500\ kg/h$

$x_{C0} = 0$ $\quad y_{AN+1} = 0$, $y_{CN+1} = 1.00$

(a) Minimum solvent

The line through L_0 gives $V_{1\,min}$. Line $L_N V_{N+1}$ and $L_0 V_{1\,min}$ (tie line) gives Δ pt. min. Draw line $L_N V_{1\,min}$ and $L_0 V_{N+1}$ and intersection gives M_{min}. $x_A = 0.114$

$$\text{Eq (12.7-4)} \quad x_{AM\,min} = 0.114 = \frac{L_0 x_{A0} + V_{N+1} y_{AN+1}}{L_0 + V_{N+1}} = \frac{1000(0.30) + V_{N+1}(0)}{1000 + V_{N+1}}$$

$$V_{N+1} = \boxed{V_{N+1\,min} = 1630\ kg/h}$$

(b) Number of stages

Use $V_{N+1} = 2500\ kg/h$

$$x_{AM} = \frac{1000(0.30) + 2500(0)}{1000 + 2500} = 0.0857 \qquad x_{CM} = \frac{1000(0) + 2500(1.0)}{1000 + 2500} = 0.714$$

This point checks graphical determination of M on lines $L_N V_1$ and $L_0 V_{N+1}$. Draw line $L_N V_{N+1}$ and $L_0 V_1$ to obtain Δ op. point graphically.

Algebraic calculation of Δ op. point

$x_{AN} = 0.02$, $x_{CN} = 0.015$, $y_{A1} = 0.10$ (off graph), $x_{C1} = 0.863$

$$\text{Eq. (12.7-1)} \quad L_0 + V_{N+1} = L_N + V_1 = M \qquad 1000 + 2500 = 3500 = L_N + V_1$$

$$\text{Eq. (12.7-4)} \quad x_{AM} = \frac{L_N x_{AN} + V_1 y_{A1}}{L_N + V_1} \qquad \text{Solving, } L_N = 626 \quad V_1 = 2874$$

$$\text{Eq. (12.7-10)} \quad x_{\Delta C} = \frac{L_0 x_{C0} - V_1 y_{C1}}{L_0 - V_1} = \frac{1000(0) - 2874(0.863)}{1000 - 2874} = +1.32$$

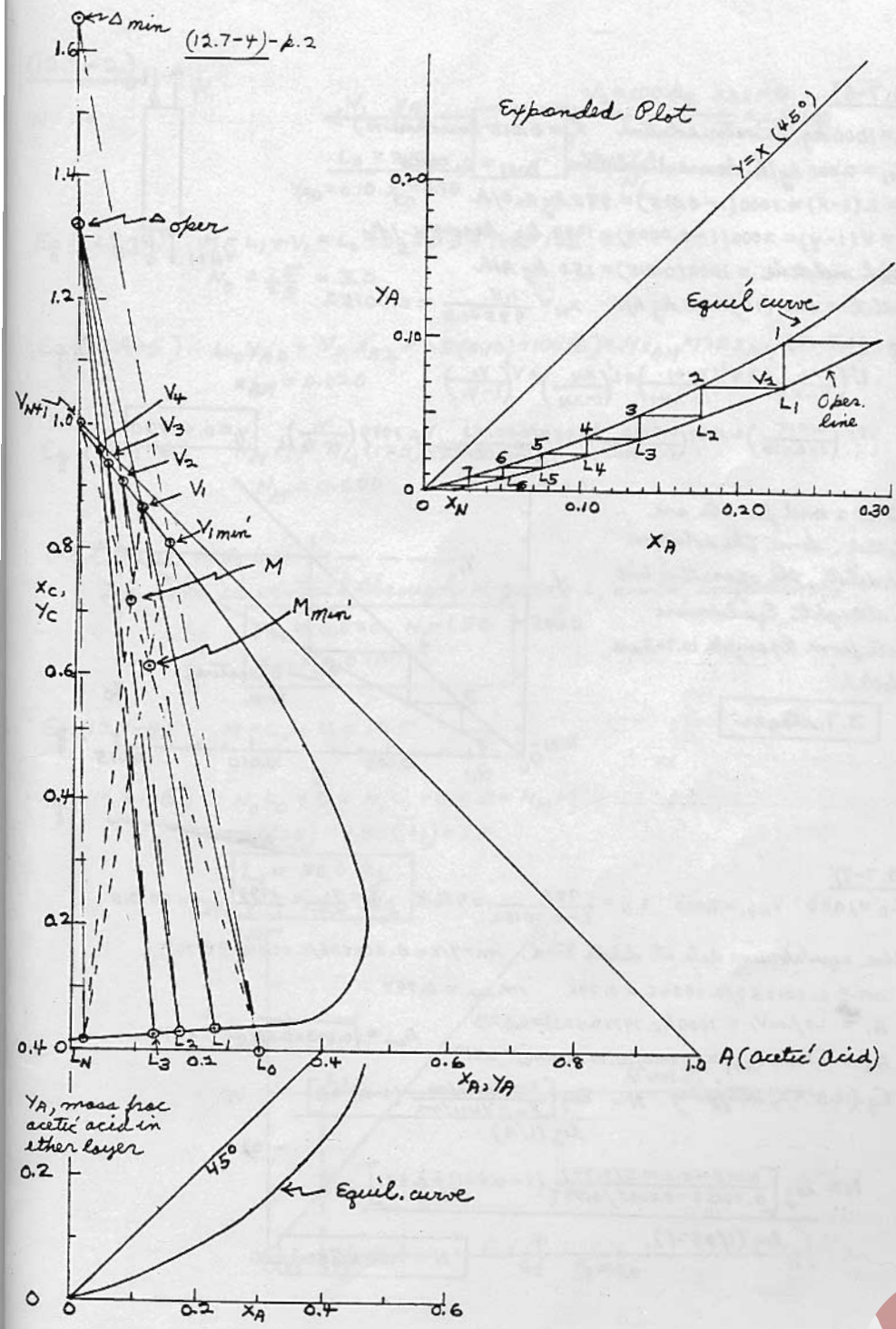
$$x_{\Delta A} = \frac{L_0 x_{A0} - V_1 y_{A1}}{L_0 - V_1} = \frac{1000(0.30) - 2874(0.10)}{1000 - 2874} = -0.0067$$

This checks the graphical Δ op. point.

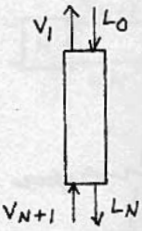
Use Δ op. point to determine number of stages. Plot stages up to V_4. Then, plot expanded scale as shown. The operating points on expanded scale are $L_1 V_2$, $L_2 V_3$, etc., determined by lines from L to Δ op. point.

$$\boxed{\text{Need } 7.5 \text{ stages}}$$

(12.7-4)-p.2


Δ min
Δ oper
V_{N+1}
V_4
V_3
V_2
V_1
V_{1min}
M
M_{min}
x_c, y_c
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.4 0
L_N
L_3
0.2
L_2
L_1
L_0
0.4
0.6
0.8
1.0
A (acetic acid)
x_A, y_A
Expanded Plot
y=x (45°)
Equil curve
Oper. line
y_A
0.20
0.10
0
x_N
0.10
0.20
0.30
x_A
y_A, mass frac acetic acid in ether layer
0.2
45°
Equil. curve
0
0.2
x_A
0.4
0.6

(12.7-6)



$L_0 = 1000$ kg/h water solution $\quad x_0 = 0.015$ (nicotine A)

$V_{N+1} = 2000$ kg/h kerosene solution $\quad y_{N+1} = 0.0005$

$L' = L(1-x) = 1000(1-0.015) = 985$ kg H_2O/h

$V' = V(1-y) = 2000(1-0.0005) = 1999$ kg kerosene/h

Inlet nicotine $= 1000(0.015) = 15.0$ kg A/h

Outlet $= 0.10\,(15) = 1.5$ kg A/h $\quad x_N = \dfrac{1.5}{985+1.5} = 0.00152$

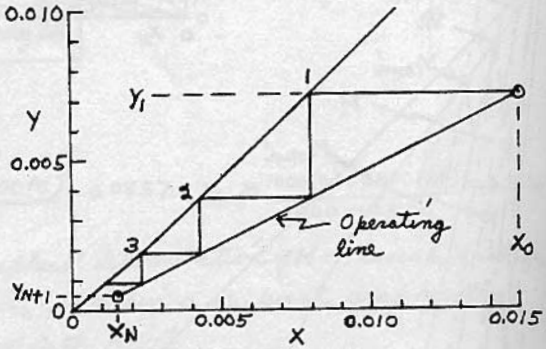
Eq. (12.7-12)

$$L'\left(\frac{x_0}{1-x_0}\right) + V'\left(\frac{y_{N+1}}{1-y_{N+1}}\right) = L'\left(\frac{x_N}{1-x_N}\right) + V'\left(\frac{y_1}{1-y_1}\right)$$

$$985\left(\frac{0.015}{1-0.015}\right) + 1999\left(\frac{0.0005}{1-0.0005}\right) = 985\left(\frac{0.00152}{1-0.00152}\right) + 1999\left(\frac{y_1}{1-y_1}\right), \quad \boxed{y_1 = 0.00720}$$

The 2 end points are plotted. Since the solutions are dilute, the operating line is straight. Equilibrium data from Example 12.7-3 are plotted.

$\boxed{3.7 \text{ stages}}$



(12.7-7)

$L_0 = 1000 \quad V_{N+1} = 2000 \quad L_N = \dfrac{985}{1-0.00152} = 986.5 \quad V_1 = \dfrac{y_1}{1-y_1} = \dfrac{1999}{1-0.00720} = 2013.5$

Use equilibrium data at dilute end. $m = y/x = 0.000806/0.001010 = 0.798$

$m = 0.001959/0.00246 = 0.796 \qquad m_{av} = 0.797$

$A_1 = L_0/mV_1 = 1000/(0.797 \times 2013.5) = 0.623$

$A_2 = L_N/mV_{N+1} = 986.5/(0.797 \times 2000) = 0.619 \qquad A_{av} = \sqrt{0.623 \times 0.619} = 0.621$

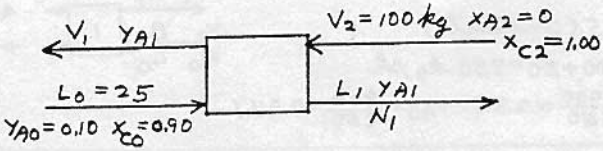
Eq. (10.3-22) stripping $\quad N = \dfrac{\log\left[\dfrac{x_0 - y_{N+1}/m}{x_N - y_{N+1}/m}(1-A) + A\right]}{\log(1/A)}$

$$N = \frac{\log\left[\dfrac{0.015 - 0.0005/0.797}{0.00152 - 0.0005/0.797}(1-0.621) + 0.621\right]}{\log(1/0.621)}$$

$\boxed{N = 4.00 \text{ stages}}$

(12.9-2)

$N = 1.5$



Eq. (12.9-4) $M = L_1 + V_1 = L_0 + V_2 = 25 + 100 = 125$

$N_0 = \frac{75}{25} = 3.0$

Eq. (12.9-5) $L_0 Y_{A0} + V_2 x_{A2} = 25(0.10) + 100(0) = M x_{AM} = 125 x_{AM}$

$x_{AM} = 0.020$

Eq. (12.9-6) $N_M M = N_M (125) = B = 75$

$N_M = 0.600$

Plot line MV_2L_0

The line is vertical through M giving L_1 and V_1 compositions

L_1	$Y_{A1} = 0.020$ $N_1 = 1.50$
V_1	$x_{A1} = 0.020$

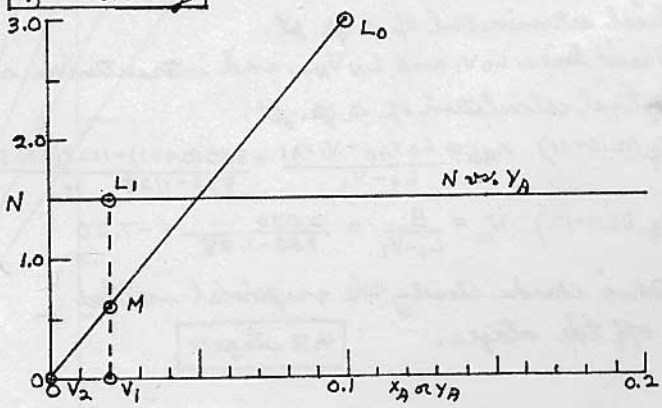
Eq. (12.9-4) $M = L_1 + V_1 = 125$

Eq. (12.9-6) $N_0 L_0 + 0 = N_1 L_1 + 0 = B = N_M M$

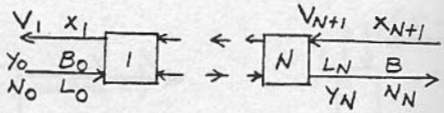
$3.0(25) = 1.50(L_1) = 75$

$L_1 = 50.0$ kg

$V_1 = 75.0$ kg



(12.10-1)



$N = 1.85$ (constant)

$L_0 = 800 + 50 = 850$ kg/h

$N_0 = \frac{2000}{850} = 2.36 \quad Y_{A0} = \frac{800}{850} = 0.941$

$B = N_0 L_0 = 2.36 \times 850 = 2000$ kg/h $= B_0$ (inert solids) $\quad V_{N+1} = 1310 + 20 = 1330$

$N_{N+1} = 0 \quad x_{AN+1} = \frac{20}{1330} = 0.015$

Plot point V_{N+1} at $N_{N+1} = 0$ and $x_{AN+1} = 0.015$. Plot L_0 at $N_0 = 2.36$, $Y_{A0} = 0.941$

Slope of line through $N = 0$ and $x_A = 0$ is $\frac{N_N}{Y_{AN}} \frac{\text{kg solid}}{\text{kg oil}} = \frac{2000}{120} = 16.67$

Line intersec. at L_N where $N_N = 1.85$ and $\boxed{Y_{AN} = 0.111}$

Calculate point M

Eq. (12.10-4) $L_0 + V_{N+1} = 850 + 1330 = 2180 = M$

Eq. (12.10-5) $L_0 Y_{A0} + V_{N+1} x_{AN+1} = 850(0.941) + 1330(0.015) = 2180 x_{AM}$

$\boxed{x_{AM} = 0.376}$

Eq. (12.10-6) $B = 2000 = N_M(2180)$ $\boxed{N_M = 0.918}$

Plot point M

Plot line $V_{N+1} M L_0$. Also plot line $L_M M$ intersecting at V_1 where

$\boxed{x_{A1} = 0.623}$

Eq. (12.10-4) $L_N + V_1 = M = 2180$

Eq. (12.10-5) $L_N Y_{AN} + V_1 x_{A1} = L_N(0.111) + V_1(0.623) = M x_{AM} = 2180(0.376)$

Solving, $\boxed{L_N = 1052 \text{ kg}, V_1 = 1128 \text{ kg}}$

Graphical determination of Δ op. pt.

Draw lines $L_0 V_1$ and $L_N V_{N+1}$ and intersection is Δ op. pt.

Analytical calculation of Δ op. pt.

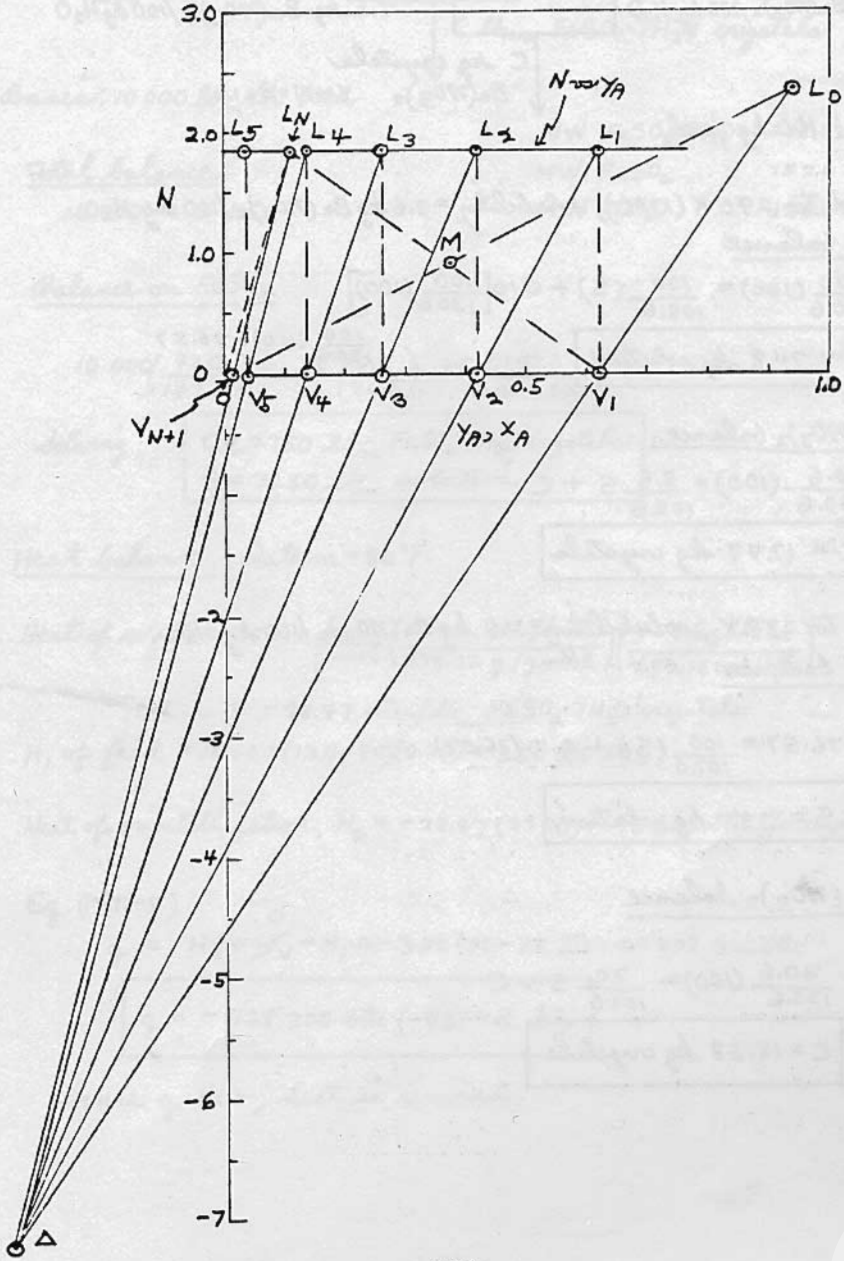
Eq. (12.10-11) $x_{A\Delta} = \frac{L_0 Y_{A0} - V_1 x_{A1}}{L_0 - V_1} = \frac{850(0.941) - 1128(0.623)}{850 - 1128} = -0.349$

Eq. (12.10-12) $N_\Delta = \frac{B}{L_0 - V_1} = \frac{2000}{850 - 1128} = -7.20$

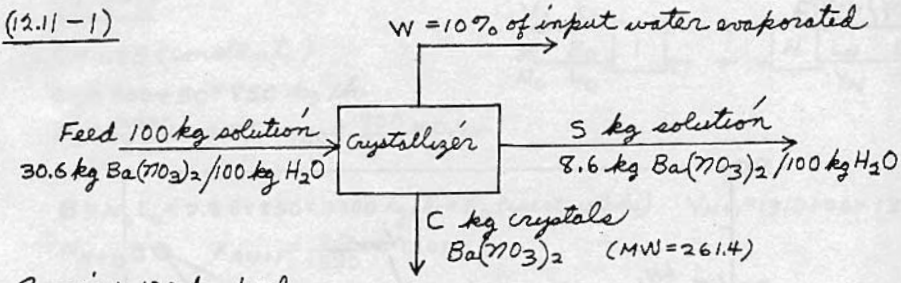
This checks closely the graphical method.

Step off the stages. $\boxed{4.3 \text{ stages}}$

(12.10-1) - p.2



(12.11 – 1)



Basis: 100 kg feed

(a) Cool to 290 K (17°C), solubility = 8.6 kg $Ba(NO_3)_2$/100 kg H_2O

H_2O balance

$$\frac{100}{130.6}(100) = \frac{100}{108.6}(S) + 0.10\left[\frac{100}{130.6}(100)\right]$$

$$\frac{100}{130.6}(100) = 76.57$$

$$\boxed{S = 74.8 \text{ kg solution}}$$

$Ba(NO_3)_2$ balance

$$\frac{30.6}{130.6}(100) = \frac{8.6}{108.6}S + C$$

$$\boxed{C = 17.47 \text{ kg crystals}}$$

(b) Cool to 283 K, solubility = 7.0 kg $Ba(NO_3)_2$/100 kg H_2O

H_2O balance

$$76.57 = \frac{100}{107.0}(S) + 0.10(76.57)$$

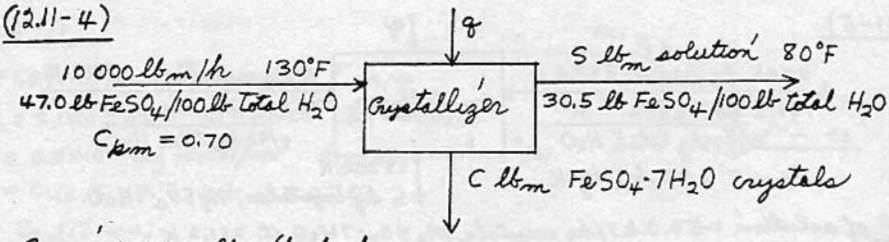
$$\boxed{S = 73.7 \text{ kg solution}}$$

$Ba(NO_3)_2$ balance

$$\frac{30.6}{130.6}(100) = \frac{7.0}{107.0}S + C$$

$$\boxed{C = 18.58 \text{ kg crystals}}$$

(12.11-4)



Basis: 10 000 lb_m/h feed

MW $FeSO_4 \cdot 7H_2O$	278.02
MW $FeSO_4$	152.0
MW $7H_2O$	126.0

Total balance

$$10\,000 = S + C$$

Balance on $FeSO_4$

$$10\,000\left(\frac{47.0}{147.0}\right) = S\left(\frac{30.5}{130.5}\right) + C\left(\frac{152.0}{278.02}\right)$$

Solving,

$$\boxed{\begin{array}{l} C = 2750\ lb_m\ FeSO_4 \cdot 7H_2O \text{ crystals} \\ S = 7250\ lb_m \text{ solution} \end{array}}$$

Heat balance. Datum = 80°F

$$\text{Heat of solution} = -4.4\ \frac{kcal}{g\ mol}\left(\frac{1000\ cal/kcal}{278.02\ g/g\ mol}\right)\left(\frac{453.6\ g/lb_m}{252.16\ cal/btu}\right)$$

$$= -28.47\ btu/lb_m\ FeSO_4 \cdot 7H_2O \text{ crystals}$$

H_1 of feed $= 10\,000(130-80)(0.70) = 350\,000$ btu

Heat of crystallization, $H_2 = -28.47(2750) = -78\,300$ btu (for crystals)

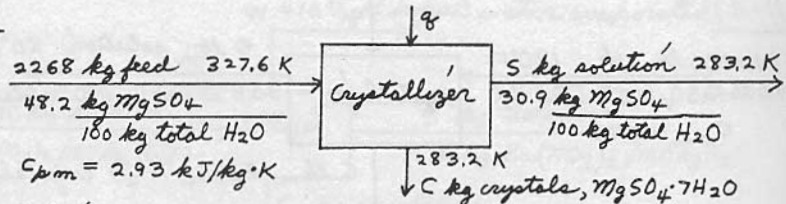
Eq. (12.11-4)

$$q = H_2 + \cancel{H_V}^{\,0} - H_1 = -350\,000 - 78\,300 = -428\,300 \text{ btu}$$

$$\boxed{q = -428\,300 \text{ btu } (-451\,900 \text{ kJ})}$$

Since q is −, heat is removed.

(12.11-5)



Heat of solution = -54.0 kJ/kg crystals $MgSO_4 \cdot 7H_2O$ at 291.2 K
(assume same value at 283.2 K.)
(From Example 12.11-2.)

MW $7H_2O$ = 126.11
MW $MgSO_4$ = 120.38
MW $MgSO_4 \cdot 7H_2O$ = 246.49

Material balance: basis 2268 kg feed solution

Water balance.

$$(1) \quad 2268\left(\frac{100}{148.2}\right) = S\left(\frac{100}{130.9}\right) + C\left(\frac{126.11}{246.49}\right)$$

Solids $MgSO_4$ balance.

$$(2) \quad 2268\left(\frac{48.2}{148.2}\right) = S\left(\frac{30.9}{130.9}\right) + C\left(\frac{120.38}{246.49}\right)$$

Solving (1) and (2) simultaneously,

$$\boxed{C = 801 \text{ kg} \quad S = 1467 \text{ kg}}$$

Heat balance: datum = 283.2 K

$$H_1(\text{feed}) = 2268(327.6 - 283.2)(2.93) = 295\,050 \text{ kJ}$$

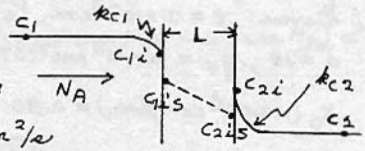
Heat of crystallization = −(−54.0 kJ/kg crystals)(801 kg crystals)
= + 43 254 kJ

$$H_2 = -43\,254$$

Eq. (12.11-4)

$$q = -H_1 + H_2$$
$$= -295\,050 - 43\,254$$
$$\boxed{q = -338\,304 \text{ kJ}}$$ (Heat is removed)

(13.2-1)



$L = 1.59\times10^{-5}\,m$ $\quad k_{c1} = 3.5\times10^{-5}\,m/s$

$k_{c2} = 2.1\times10^{-5}\,m/s$ $\quad c_1 = 2.0\times10^{-2}\,kg\,mol\,A/m^3$

$c_2 = 0.3\times10^{-2}\,kg\,mol\,A/m^3$ $\quad \mathcal{D}_{AB} = 3.5\times10^{-11}\,m^2/s$

$K' = c_{1is}/c_{1i} = c_{2is}/c_{2i} = 0.75$

(a) Eq. (13.2-4)

$$k_M = \mathcal{D}_{AB}K'/L = 3.5\times10^{-11}(0.75)/(1.59\times10^{-5}) = 1.651\times10^{-6}\,m/s$$

Eq (13.2-6)

$$N_A = \frac{c_1 - c_2}{1/k_{c1} + 1/k_M + 1/k_{c2}} = \frac{(2.0\times10^{-2} - 0.3\times10^{-2})}{\frac{1}{3.5\times10^{-5}} + \frac{1}{1.651\times10^{-6}} + \frac{1}{2.1\times10^{-5}}} = \frac{c_1 - c_2}{\text{Total resis.}}$$

$$= \frac{1.7\times10^{-2}}{0.2857\times10^{-5} + 6.061\times10^{-5} + 0.4762\times10^{-5}}$$ $\boxed{\text{Total resis.} = 6.823\times10^5\,s/m}$

$$\%\ \text{resis film} = \frac{0.2857 + 0.4762}{6.823}\times100 = \boxed{11.17\ \%\ \text{resis film}}$$

(b) $N_A = 1.7\times10^{-2}/6.823\times10^5 = \boxed{2.492\times10^{-8}\,kg\,mol\,A/s\cdot m^2}$

$A = 0.0100/(2.492\times10^{-8}\times3600) = \boxed{111.5\,m^2}$

(c) $k_c \propto v^{0.6} \propto 2.0^{0.6} \propto 1.516$ $\quad k_{c1} = 3.5\times10^{-5}(1.516) = 5.306\times10^{-5}\,m/s$

$k_{c2} = 2.1\times10^{-5}(1.516) = 3.184\times10^{-5}\,m/s$

Total resis $= 1/5.306\times10^{-5} + 6.061\times10^{5} + 1/3.184\times10^{-5}$

$= 0.1885\times10^5 + 6.061\times10^5 + 0.3141\times10^5 = 6.564\times10^5\,s/m$

$$\%\ \text{resis film} = \frac{0.1885 + 0.3141}{6.564}\times100 = \boxed{7.66\ \%\ \text{resis film}}$$

$N_A = 1.7\times10^{-2}/6.564\times10^5 = 2.590\times10^{-8}$

$$\%\ \text{increase} = (2.590 - 2.492)(100)/2.492 = \boxed{3.93\ \%\ \text{increase flux}}$$

(13.3-1)

$P_M'' = 6.50\times10^{-7}\,cm^3\,O_2(STP)/(s\cdot cm^2\cdot cmHg/mm)$ $\quad p_{A1} = 700\,mm$

$p_{A2} = 50\,mm$ $\quad L = 0.165\,mm = 0.165/1000 = 1.65\times10^{-4}\,m$

Eq. (6.5-11) $\quad P_M = 7.60\times10^{-4}P_M'' = 7.60\times10^{-4}(6.50\times10^{-7})$

$= 4.94\times10^{-10}\,m^3\,O_2(STP)/(s\cdot m^2\cdot atm/m)$

Eq. (6.5-9) $\quad P_M = \mathcal{D}_{AB}S$

Eq. (13.3-3)

$P_m = \mathcal{D}_{AB}S/22.414 = 4.94\times10^{-10}/22.414$

(a) $= 2.204\times10^{-11}\,kg\,mol/s\cdot m^2\cdot atm$

Eq (13.3-4)

$$N_A = \frac{p_{A1} - p_{A2}}{0 + 1/(P_m/L) + 0} = \frac{2.204\times10^{-11}\left(\frac{700}{760} - \frac{50}{760}\right)}{(1.65\times10^{-4})} = \boxed{1.142\times10^{-7}\,\frac{kg\,mol\,O_2}{s\cdot m^2}}$$

(b) $A = \frac{300}{22.414}\times\frac{1}{1000}\times\frac{1}{60}\times\frac{1}{1.142\times10^{-7}} = \boxed{1.953\,m^2}$

(13.4-2)

Given: $t = 2\times10^{-3}$ cm, $P_A' = 400\times10^{-10}$ cm^3(STP)·cm/(s·cm^2·cmHg),

$\alpha^* = 10$, $q_f = 2\times10^3$ cm^3(STP)/s, $x_f = 0.413$, $p_h = 80$ cmHg, $p_\ell = 20$ cmHg

x_0 (reject compos.) = 0.30

Eq. (13.4-10) $\quad y_p = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad a = 1 - \alpha^* = 1 - 10 = -9$

$$b = \frac{p_h}{p_\ell}(1 - x_0) - 1 + \alpha^* \frac{p_h}{p_\ell} x_0 + \alpha^* = \frac{80}{20}(1 - 0.30) - 1 + 10\frac{(80)}{20}(0.30) + 10 = 23.8$$

$$c = -\alpha^* \frac{p_h}{p_\ell} x_0 = -10\frac{(80)}{20}(0.30) = -12.0$$

$$y_p = \frac{-23.8 + \sqrt{(23.8)^2 - 4(-9)(-12)}}{2(-9)} = \boxed{0.678 = y_p \text{ permeate composition}}$$

Eq. (13.4-8) $\quad x_0 = \dfrac{x_f - \theta y_p}{(1-\theta)}$

$$0.30 = \frac{0.413 - \theta(0.678)}{(1-\theta)}$$

solving, $\boxed{\theta = 0.299 \text{ frac. permeated}}$

Eq. (13.4-9)

$$A_m = \frac{\theta q_f y_p}{(P_A'/t)(p_h x_0 - p_\ell y_p)}$$

$$= \frac{0.299(2\times10^3)(0.678)}{[(400\times10^{-10})/(2\times10^{-3})][80(0.30) - 20(0.678)]}$$

$$\boxed{A_m = 1.942\times10^6 \text{ cm}^2 \text{ area}}$$

(13.4-3)

Given: $x_f = 0.50$, $q_f = 2\times10^3\ cm^3\ (STP)/s$, $p_h = 40\ cm\ Hg$, $p_l = 10\ cm\ Hg$, $t = 1.5\times10^{-3}\ cm$, $P_A' = 40\times10^{-10}\ cm^3\ (STP)\cdot cm/(s\cdot cm^2\cdot cm\ Hg)$, $\alpha^* = 10$, $\theta = 0.529$

(a)

Eq (13.4-11)

$$y_p = \frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1}$$

$$a_1 = \theta + \frac{p_l}{p_h} - \frac{p_l}{p_h}\theta - \alpha^*\theta - \alpha^*\frac{p_l}{p_h} + \alpha^*\frac{p_l}{p_h}\theta$$

$$= 0.529 + \frac{10}{40} - \frac{10}{40}(0.529) - 10(0.529) - 10\frac{(10)}{40} + 10\frac{(10)}{40}(0.529) = -5.8205$$

$$b_1 = 1 - \theta - x_f - \frac{p_l}{p_h} + \frac{p_l}{p_h}\theta + \alpha^*\theta + \alpha^*\frac{p_l}{p_h} - \alpha^*\frac{p_l}{p_h}\theta + \alpha^* x_f$$

$$= 1 - 0.529 - 0.50 - \frac{10}{40} + \frac{10}{40}(0.529) + 10(0.529) + 10\frac{(10)}{40} - 10\frac{(10)}{40}(0.529) + 10(0.50)$$

$$= +11.3207$$

$$c_1 = -\alpha^* x_f = -10(0.50) = -5.00$$

$$y_p = \frac{-11.3207 + \sqrt{(11.3207)^2 - 4(-5.8205)(-5.00)}}{2(-5.8205)} = \boxed{0.678 = y_p \text{ permeate comp.}}$$

Eq. (13.4-8) $$x_0 = \frac{x_f - \theta y_p}{(1-\theta)} = \frac{0.50 - 0.529(0.678)}{(1-0.529)} = \boxed{0.300 = x_0 \text{ reject compos.}}$$

Eq (13.4-9)

$$A_m = \frac{\theta q_f y_p}{(P_A'/t)(p_h x_0 - p_l y_p)} = \frac{0.529(2\times10^3)(0.678}{(40\times10^{-10}/1.5\times10^{-3})[(40)(0.30) - (10)(0.678)]}$$

$$\boxed{A_m = 5.153\times10^7\ cm^2}$$

(b)

Eq. (13.4-12) $$x_{0M} = \frac{x_f\left[1 + (\alpha^*-1)\frac{p_l}{p_h}(1-x_f)\right]}{\alpha^*(1-x_f) + x_f}$$

$$= \frac{0.50\left[1 + (10-1)\frac{10}{40}(1-0.50)\right]}{10(1-0.50) + 0.50} = \boxed{0.1932 = x_{0M} \text{ min. reject conc.}}$$

(c) For $x_f = 0.60$ using Eq (13.4-12),

$$x_{0M} = \frac{0.60\left[1 + (10-1)\frac{10}{40}(1-0.60)\right]}{10(1-0.60) + 0.60}$$

$$\boxed{x_{0M} = 0.2478}$$

(13.4-4)

Given: $x_f = 0.413$, $\alpha^* = 10$, $p_\ell = 20$ cmHg, $p_h = 80$ cmHg, $x_0 = 0.30$

(a)

Eq. (13.4-12)

$$x_{OM} = \frac{x_f\left[1 + (\alpha^* - 1)\frac{p_\ell}{p_h}(1 - x_f)\right]}{\alpha^*(1 - x_f) + x_f}$$

$$= \frac{0.413\left[1 + (10-1)\frac{(20)}{80}(1 - 0.413)\right]}{10(1 - 0.413) + 0.413}$$

$$\boxed{x_{OM} = 0.1526}$$

(b) $\alpha^* = 5$

$$x_{OM} = \frac{0.413\left[1 + (5-1)\frac{20}{40}(1 - 0.413)\right]}{5(1 - 0.413) + 0.413}$$

$$\boxed{x_{OM} = 0.1958}$$

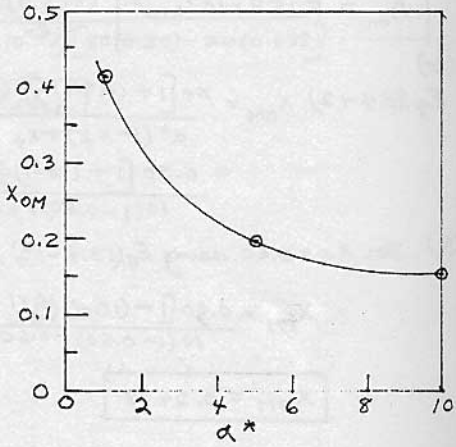
(c) $\alpha^* = 1.0$

This means no separation

$$x_{OM} = \frac{0.413\left[1 + (1-1)\frac{20}{40}(1 - 0.413)\right]}{1(1 - 0.413) + 0.413}$$

$$\boxed{x_{OM} = 0.413}$$

The values are plotted. Note that at high values of α^*, the value of x_{OM} levels off.



(13.7-1)

(a)

Eq (13.7-4)

$$qX = (q - dq)(X - dX) + Y\,dq$$

Multiplying out,

$$\cancel{qX} = \cancel{qX} - q\,dX - X\,dq + \cancel{dq\,dX} + Y\,dq$$

Cancelling and neglecting second order differentials $dq\,dX$,

$$Y\,dq = q\,dX + X\,dq$$

But $q\,dX + X\,dq = d(qX)$

Hence,

$$Y\,dq = d(qX) \qquad \text{Eq. (3.7-5)}$$

(b)

Eq. (13.7-10)

$$-q\,dX = -(1-X)\,d(qX) + X\,d\left[q(1-X)\right]$$

Performing the differentiations,

$$-q\,dX = -dqX + X\,\cancel{d(qX)} + X\,dq - X\,\cancel{d(qX)}$$

$$-\cancel{q}\,dX = -\cancel{q}\,dX - \cancel{X\,dq} + \cancel{X\,dq}$$

Hence, it is proved.

(c) Starting with Eq. (13.7-1),

$$q = q_0 + q'$$

$$qX = q_0 X_0 + q'Y \qquad (13.7\text{-}2)$$

Substitute q from Eq. (13.7-1) into (13.7-2)

$$X(q_0 + q') = q_0 X_0 + q'Y$$

Rearranging,

$$q_0(X - X_0) = q'(Y - X)$$

Multiply above by dY and rearrange.

$$q_0\,dY = \left(\frac{X - Y}{X - X_0}\right)(-q'\,dY) \qquad \text{Eq. (13.7-12)}$$

(13.9-1)

(a) Solution of 0.50 g mol NaCl / kg H_2O

Table A.2-3, ρ = 997.0 kg H_2O/m^3

$n = (2)(0.50 \times 10^{-3})$ kg mol (2 ions)

$$V_m = \frac{1.00 \text{ kg } H_2O}{997.0 \text{ kg } H_2O/m^3}$$

Eq. (13.9-1)

$$\pi = \frac{nRT}{V_m} = \frac{(2 \times 0.5 \times 10^{-3})(82.057 \times 10^{-3})(298.15)}{(1.0/997.0)}$$

$\boxed{\pi \text{ (Predicted)} = 24.39 \text{ atm}}$ π (Exp.) = 22.55 atm

(b) Solution of 1.0 g sucrose / kg H_2O

$$n = \frac{1.0 \text{ g} \times 10^{-3} \text{ kg}}{342.3 \text{ kg/kg mol}} = 2.921 \times 10^{-6} \text{ kg mol sucrose}$$

Eq. (13.9-1)

$$\pi = \frac{(2.921 \times 10^{-6})(82.057 \times 10^{-3})(298.15)}{(1.0/997.0)} = \boxed{0.0713 \text{ atm} = \pi \text{(Pred)}}$$

π (Exp) = 0.0714 atm

(c) Solution of 1.0 g $MgCl_2$ / kg H_2O

$$n = \frac{1.0 \times 10^{-3} \text{ kg}}{95.23 \text{ kg/kg mol}} (3) = 3.150 \times 10^{-5} \text{ kg mol} \quad (3 \text{ ions})$$

Eq. (13.9-1)

$$\pi = \frac{(3.150 \times 10^{-5})(82.05 \times 10^{-3})(298.15)}{(1/997.0)}$$

$\boxed{\pi \text{(Predicted)} = 0.768 \text{ atm}}$ π(Exp) = 0.660 atm

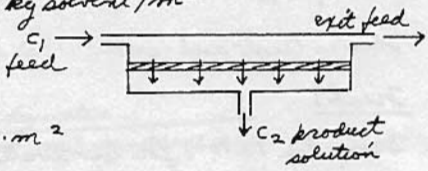
(13.9-2)

$A = 4.00 \times 10^{-3}\ m^2$ $T = 25°C$ $c_1 = 12\ kg\ NaCl/m^3$ ($\rho_1 = 1005.5\ kg/m^3$)

$c_2 = 0.468\ kg\ NaCl/m^3$ ($\rho_2 = 997.3\ kg/m^3$) $\Delta P = 56.0\ atm$

Flow rate $= 3.84 \times 10^{-8}\ m^3/s$

Since c_2 is very low (dilute), assume c_{w2} as the density of water (Table 12.4-1), $c_{w2} = 997\ kg\ solvent/m^3$



$$N_w = \frac{(3.84 \times 10^{-8})(997)}{4.00 \times 10^{-3}}$$

$$= 9.57 \times 10^{-3}\ kg\ solvent\ (water)/s \cdot m^2$$

Eq. (13.9-7)

$$N_s = N_w c_2 / c_{w2} = (9.57 \times 10^{-3})(0.468)/997 = 4.492 \times 10^{-6}\ \frac{kg\ solute\ NaCl}{s \cdot m^2}$$

Convert the concentrations so Table 12.4-1 can be used.

For c_1,

$1005.5\ kg\ solution/m^3 - 12\ kg\ NaCl/m^3 = 993.5\ kg\ H_2O$

$$\frac{12.0 \times 1000}{993.5 \times 58.45} = 0.2066\ g\ mol\ NaCl/kg\ H_2O \qquad \pi = 9.35\ atm$$

Table 12.4-1

For c_2,

$997.3 - 0.468 = 997\ kg\ H_2O$

$$\frac{0.468 \times 1000}{997 \times 58.45} = 0.00803\ g\ mole\ NaCl/kg\ H_2O \qquad \pi_2 = 0.38\ atm$$

$\Delta\pi = 9.35 - 0.38 = 8.97\ atm$

Eq. (13.9-2)

$$N_w = A_w(\Delta P - \Delta\pi) \qquad 9.57 \times 10^{-3} = A_w(56.0 - 8.97)$$

$$\boxed{A_w = 2.013 \times 10^{-4}\ kg\ solvent/s \cdot m^2 \cdot atm}$$

Eq. (13.9-5)

$$N_s = A_s(c_1 - c_2) \qquad 4.492 \times 10^{-6} = A_s(12.00 - 0.468)$$

$$\boxed{A_s = 3.895 \times 10^{-7}\ m/s}$$

Eq. (13.9-8)

$$R = (c_1 - c_2)/c_1 = (12.00 - 0.468)/12.0 \qquad \boxed{R = 0.961}$$

Eq. (13.9-10)

$$B = A_w/(A_s c_{w2}) = 2.013 \times 10^{-4}/(3.895 \times 10^{-7} \times 997) = \boxed{0.5184 = B}$$

Eq. (13.9-9)

$$R = \frac{B(\Delta P - \Delta\pi)}{1 + B(\Delta P - \Delta\pi)} = \frac{0.5184(56.0 - 8.97)}{1 + 0.5184(56.0 - 8.97)} = 0.961$$

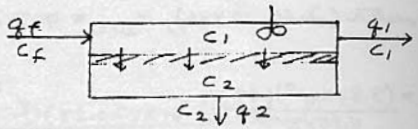
(13.10-3)

From Eq. 13.10-1, $c_f = 2.5$ kg NaCl/m^3, $\pi_1 = 1.97$ atm,

$A_w = 4.81 \times 10^{-4}$ kg/s·m^2·atm, $A_s = 4.42 \times 10^{-7}$ m/s, $\Delta P = 27.20$ atm

$q_2 = 100$ gal/h, $\theta = 0.10$ Process Flow Diag, Complete Mixing

$\theta = q_2/q_f$



This is trial-and-error.

Trial 1

From Eq. 13.10-1 for low water recovery or $\theta \cong 0$, $c_2 = 0.0875$.

Hence, assume $c_2 = 0.10$. Then $\pi_2 = 0.08$ atm (From Eq. 13.10-1)

Substituting into Eq. (13.10-5),

$$c_f = (1-\theta)c_1 + \theta c_2$$

$$2.50 = (1-0.10)c_1 + 0.10(0.10)$$

Solving, $c_1 = 2.767$ kg NaCl/m^3

Using the data from Eq. 13.10-3,

$$(2.767 \times 1000)/[(999 - 2.767) \times (58.45^{-1})] = 0.0475 \text{ g mol NaCl/kg } H_2O$$

From Table 13.9-1, $\pi_1 = 2.17$ atm $\Delta\pi = \pi_1 - \pi_2 = 2.17 - 0.08 = 2.09$ atm

Eq. (13.9-2), $N_w = A_w(\Delta P - \Delta\pi) = 4.81 \times 10^{-4}(27.20 - 2.09) = 1.208 \times 10^{-2}$ kg/s·m^2

From Eq. 13.10-1, $B = 1.092$ atm^{-1}

Eq. (13.9-9) $R = \dfrac{B(\Delta P - \Delta\pi)}{1 + B(\Delta P - \Delta\pi)} = \dfrac{1.092(27.2 - 2.09)}{1 + 1.092(27.2 - 2.09)} = 0.9648$

Eq. (13.9-8) $R = \dfrac{c_1 - c_2}{c_1}$ $0.9648 = \dfrac{(2.767 - c_2)}{2.767}$ $\boxed{c_2 = 0.0973 \text{ kg/m}^3}$

This is close enough to c_2 assumed of 0.10 and no further trials are needed.

Eq. (13.10-5)

$$2.50 = (1-0.10)c_1 + 0.10(0.0973)$$

(13.10-3) — p. 2

Solving, $\boxed{C_1 = 2.767\ kg/m^3}$

From Table A.1-6, $1\ m^3 = 264.17\ gal$

$$q_2 = 100\ gal/h = 100\ \frac{gal}{h} \times \frac{1}{3600\ s/h} \times \frac{1}{264.17\ gal/m^3} \times 997\ \frac{kg}{m^3}$$

$$q_2 = 0.1048\ kg\ water/s$$

$$N_w = \text{kg water flux in } \frac{kg\ water}{s \cdot m^2\ \text{membrane area}}$$

$$1.208 \times 10^{-4}\ \frac{kg\ water}{s \cdot m^2\ area} = \frac{0.1048\ kg\ water}{s \cdot A\ m^2}$$

Solving:

Membrane area = $\boxed{A = 8.68\ m^2}$

Eq. (13.10-3) for dilute solutions,

$$q_f = q_1 + q_2$$

$$q_f = q_1 + 100$$

Eq. (13.10-4)

$$C_f q_f = C_1 q_1 + C_2 q_2$$

$$2.50(q_f) = 2.767(q_1) + 0.0973(100)$$

Substituting $q_f = q_1 + 100$ into the above, and solving,

$$2.50(q_1 + 100) = 2.767(q_1) + 9.73$$

$\boxed{q_1 = 899.9\ gal/h}$ $\boxed{q_f = 999.9\ gal/h}$

(13.11-1)

Given: $A_w = 1.37\times10^{-2}$ kg/s·m²·atm, $\Delta P = 5$ psi

Since the solution is a small (0.9 wt % protein) concentration and the protein molecule will have a large MW, the osmotic pressure is low and will be neglected.

Use Eq (13.11-1)

$$N_w = A_w(\Delta P) = \left(1.37\times10^{-2}\frac{kg}{s\cdot m^2\cdot atm}\right)\left(\frac{5.0\ psi}{14.696\ psi/atm}\right)$$

$$\boxed{N_w = 4.66\times10^{-3}\ kg/s\cdot m^2}$$

Table A.1-6

$$N_w = 4.66\times10^{-3}\frac{kg\ water}{s\cdot m^2}\times\frac{1}{1000\frac{kg\ water}{m^3}}\times 264.17\frac{gal}{m^3}\times\frac{1}{\left(3.2808\frac{ft}{m}\right)^2}$$

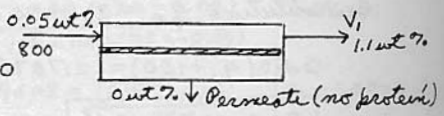
$$\times\ 3600\frac{s}{hr}\times 24\frac{hr}{day}$$

$$\boxed{N_w = 9.88\ gal/ft^2\cdot day}$$

(13.11-2)

Given: $A_w = 2.50\times10^{-2}$ kg/s·m²·atm, $\Delta P = 0.50$ atm

Feed of 800 kg, 0.05 wt % protein. To obtain 1.1 wt %. Area membrane = 9.90 m² = A_m. Assume constant rate of solvent flux.



Material Bal. $\frac{0.05}{100}(800) = \frac{1.1}{100}(V_1) + 0$

$$\boxed{V_1 = 36.4\ kg\ \text{final solution}}$$

Water removed = 800 − 36.4 = 763.6 kg

$$\frac{763.6\ kg}{(\theta\ sec)(9.90\ m^2)} = N_w = A_w\Delta P = 2.50\times10^{-2}\frac{kg}{s\cdot m^2\cdot atm}(0.50\ atm)\quad \text{Eq. (13.11-1)}$$

Solving, $\boxed{\theta = 6.171\times10^3\ s = 1.714\ h}$

(14.2-1)

t, s	$V \times 10^3$, m^3	$(t/V) \times 10^{-3}$
0	0	
17.3	0.5	34.6
41.3	1.0	41.3
72.0	1.5	48.0
108.3	2.0	54.15
152.1	2.5	60.84
201.1	3.0	67.03

Eq. (14.2-18)

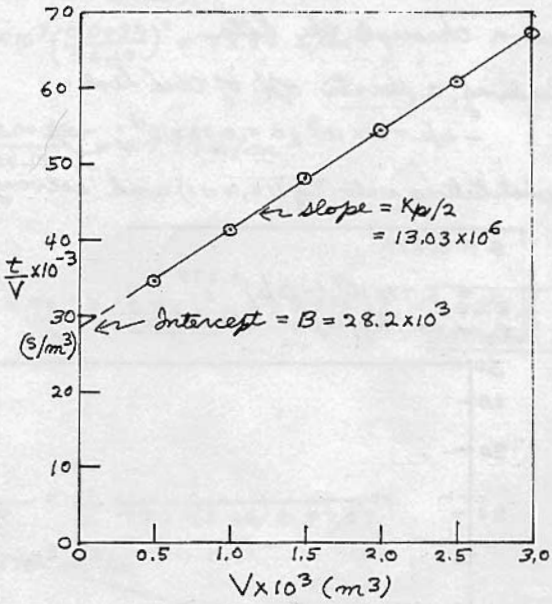
$$\frac{t}{V} = \frac{K_p}{2} V + B$$

Plot t/V vs. V. Draw a straight line through the data.

$$\boxed{B = 28.2 \times 10^3 \ s/m^3}$$

$K_p = 2 \times 13.03 \times 10^6 \ s/m^6$

$$\boxed{K_p = 26.06 \times 10^6 \ s/m^6}$$



$\mu_{H_2O} = 8.937 \times 10^{-4}$ Pa·s (298.2 K), $A = 0.0439 \ m^2$

$-\Delta p = 46.20 \ kN/m^2$ (6.70 psia), $c_s = 23.47$ kg solid/m^3 filtrate

Eq. (14.2-14)

$$K_p = 26.06 \times 10^6 = \frac{\mu \alpha c_s}{A^2(-\Delta p)} = \frac{8.937 \times 10^{-4} (\alpha)(23.47)}{(0.0439)^2 (46.20 \times 10^3)}$$

$$\boxed{\alpha = 1.106 \times 10^{11} \ m/kg \ (1.65 \times 10^{11} \ ft/lb_m)}$$

Eq. (14.2-15) $B = 28.2 \times 10^3 = \dfrac{\mu R_m}{A(-\Delta P)} = \dfrac{8.937 \times 10^{-4} R_m}{(0.0439)(46.20 \times 10^3)}$

$$\boxed{R_m = 6.40 \times 10^{10} \ m^{-1} \ (1.95 \times 10^{10} \ ft^{-1})}$$

(14.2-3)	$(-\Delta p)$, N/m^2	α, m/kg
Prob. (14.2-1)	46.2×10^3	1.106×10^{11}
Prob. (14.2-2)	194.4×10^3	1.61×10^{11}
Example (14.2-1)	338.0×10^3	1.863×10^{11}

Eq (14.2-11)

$$\alpha = \alpha_0 (-\Delta p)^s$$

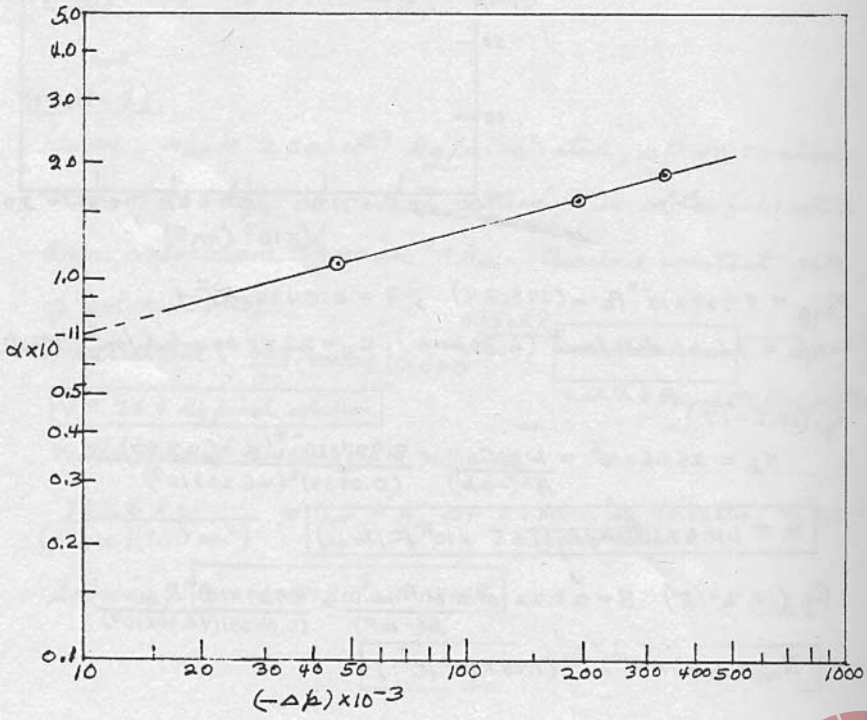
above data are plotted as below and a straight line drawn through the data.

Reading 2 points off of this line.

$$-\Delta p = 10 \times 10^3,\ \alpha = 0.72 \times 10^{11};\quad -\Delta p = 500 \times 10^3,\ \alpha = 2.10 \times 10^{11}$$

Substituting into Eq. (14.2-11) and solving,

$$\boxed{\begin{array}{l} s = 0.274 \\ \alpha = 5.77 \times 10^9 (-\Delta p)^{0.274} \\ \alpha \text{ in } m/kg,\ -\Delta p \text{ in } N/m^2 \text{ or } Pa \end{array}}$$



(14.2-4)

Use slurry of Problem (14.2-1) $\quad -\Delta P = 46.20 \times 10^3\ N/m^2$

$A = 30$ frames $(0.873\ m^2/\text{frame}) = 26.19\ m^2 \quad V_f = 2.26\ m^3$ filtrate

$V_{wash} = 0.283\ m^3$ clean press $= 30$ min

$K_p = 2.606 \times 10^7\ s/m^6$

$B = 28200\ s/m^3 \quad A = 0.0439\ m^2$

By Eq. (14.2-14) for new area,

$$K_p \propto \frac{1}{A^2} = 2.606 \times 10^7 \left(\frac{0.0439}{26.19}\right)^2 = 73.22\ s/m^6$$

By Eq. (14.2-15)

$$B \propto \frac{1}{A} = 28200\left(\frac{0.0439}{26.19}\right) = 47.27\ s/m^3$$

Filtration time

Eq. (14.2-17)

$$t = \frac{K_p}{2} V^2 + BV = \frac{73.22}{2}(2.26)^2 + 47.27(2.26) = \boxed{293.8\ s \text{ filtration time}}$$

Washing time

Eq. (14.2-20)

$$\left(\frac{dV}{dt}\right)_f = \frac{1}{4}\frac{1}{K_p V_f + B} = \frac{1}{4}\frac{1}{73.22(2.26) + 47.27}$$

Rate of washing $= 1.175 \times 10^{-3}\ m^3/s$

$V_{wash} = 0.283\ m^3$

$$t = \frac{0.283\ m^3}{1.175 \times 10^{-3}\ m^3/s} = \boxed{240.9\ s \text{ wash time}}$$

Total filter cycle time

$$t_{total} = t_{filter} + t_{wash} + t_{clean}$$

$$= \frac{293.8}{60} + \frac{240.9}{60} + 30.0$$

$$= 4.897 + 4.015 + 30.0$$

$$\boxed{t_{total} = 38.91 \text{ min } (2335\ s)}$$

(14.2-6)

$(-\Delta p) = 267\ kPa$ $\qquad V = 1.00\ m^3$

$A = 0.0414\ m^2$ $\qquad A_1$ (new area) $= 6.97\ m^2$

Filtration equation.

$$t = 10.25 \times 10^6 V^2 + 3.4 \times 10^3 V$$

Eq. (14.2-17)

$$t = \frac{K_p}{2} V^2 + BV$$

$$\frac{K_p}{2} = 10.25 \times 10^6\ s/m^6$$

$$B = 3.4 \times 10^3\ s/m^3$$

Eq. (14.2-14)

$$K_p \propto \frac{1}{A^2} \qquad K_p = (2)(10.25 \times 10^6)\left(\frac{0.0414}{6.97}\right)^2 = 7.233 \times 10^2\ s/m^6$$

Eq. (13.2-15)

$$B \propto \frac{1}{A} \qquad B = 3.4 \times 10^3 \left(\frac{0.0414}{6.97}\right)^2 = 20.20\ s/m^3$$

Time for filtration

(a) Eq. (14.2-17)

$$t = \frac{7.233 \times 10^2}{2}(1.00)^2 + 20.20(1.0)$$

$$\boxed{t = 381.8\ s \text{ filtration time}}$$

Time for washing.

Eq (14.2-19) $\qquad V_f = 1.00\ m^3 \qquad V_{wash} = 0.100\ m^3$

$$\left(\frac{dV}{dt}\right)_f = \frac{1}{K_p V_f + B}$$

$$= \frac{1}{7.233 \times 10^2 (1.00) + 20.20} = 1.345 \times 10^{-3}\ m^3/s$$

$$t = \frac{0.100\ m^3}{1.345 \times 10^{-3}\ m^3/s}$$

(b) $\boxed{t = 74.3\ s \text{ wash time}}$

(14.2-8)

From Example (14.2-4)

$c_x = 0.191$ kg solid/kg slurry, $m = 2$ kg wet cake/kg dry cake

$\rho = 996.9\ kg/m^3$, $(-\Delta p) = 67.0 \times 10^3\ N/m^2$, $t_c = 250\ s$

$\mu_c = 0.8937 \times 10^{-3}$ Pa·s, $c_s = 308.1$ kg solids/m^3 filtrate

$\alpha = 1.225 \times 10^{11}\ m/kg$, $\frac{V}{t_c} = 4.823 \times 10^{-4}\ m^3$ filtrate/s, $f = 0.33$

From Example (14.2-1),

$B = 6400\ s/m^3$, $R_m = 10.63 \times 10^{10}\ m^{-1}$

Eq. (14.2-26)

$$\frac{V}{A t_c} = \frac{-R_m/t_c + \left[R_m^2/t_c^2 + 2c_s\alpha(-\Delta p)f/(\mu_c t_c)\right]^{1/2}}{\alpha c_s}$$

$$\frac{4.823 \times 10^{-4}}{A} = \frac{-10.63 \times 10^{10}/250 + \left[\frac{(10.63 \times 10^{10})^2}{(250)^2} + \frac{2(308.1)(1.225 \times 10^{11})(67.0 \times 10^3)(0.33)}{(0.8937 \times 10^{-3})(250)}\right]^{1/2}}{(1.225 \times 10^{11})(308.1)}$$

$$\frac{4.823 \times 10^{-4}}{A} = \frac{2.3408 \times 10^9}{(1.225 \times 10^{11})(308.1)}$$

$$\boxed{A = 7.78\ m^2}$$

$A = 6.60\ m^2$ in Example 14.2-4

(14.3-1)

$D_p = 400\ \mu m = 400 \times 10^{-6}\ m$ 422K $\rho_s = 1030\ kg/m^3 = \rho_p$

$\mu = 0.0237 \times 10^{-3}\ Pa \cdot s$ $P = 101.32\ kPa$ $\rho_{air} = 0.8378\ kg/m^3$

Solution is trial-and-error since v_t is unknown. Hence, C_D cannot be directly evaluated.

The Reynolds no. is $N_{Re} = \dfrac{D_p v_t \rho}{\mu} = \dfrac{(400 \times 10^{-6}) v_t (0.8378)}{(0.0237 \times 10^{-3})}$

$N_{Re} = 14.14\, v_t$

For trial no. 1, assume $v_t = 1.097\ m/s$. $N_{Re} = 14.14(1.097) = 15.51$

Eq. (14.3-7) $v_t = \sqrt{\dfrac{4(\rho_p - \rho) g D_p}{3 C_D \rho}} = \sqrt{\dfrac{4(1030 - 0.84)(9.806)(4.0 \times 10^{-4})}{3 C_D (0.8378)}}$

Solving for C_D, $C_D = \dfrac{6.424}{v_t^2}$

Using $v_t = 1.097$, $C_D = \dfrac{6.424}{(1.097)^2} = 5.33$

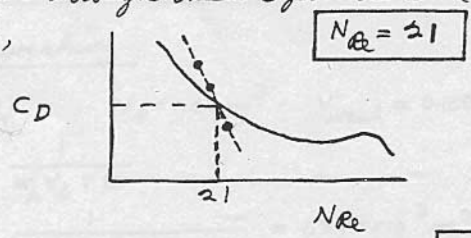
Trial no. 2, assume $v_t = 1.219\ m/s$.

$N_{Re} = 14.14(1.219) = 17.23$, $C_D = \dfrac{6.424}{(1.219)^2} = 4.32$

Trial no. 3, assume $v_t = 1.829\ m/s$

$N_{Re} = 14.14(1.829) = 25.86$, $C_D = \dfrac{6.424}{(1.829)^2} = 1.92$

Making a plot as in Fig. 14.3-2 in Ex. 14.3-1, a straight line is drawn through these 3 points and at the intersection,



$N_{Re} = 21 = 14.14\, v_t$ $v_t = 1.485$ or $\boxed{v_t = 1.49\ m/s}$

Distance of fall $= (1.49\ \frac{m}{s})(5.0\ s) = \boxed{7.45\ m \text{ fall}}$

(14.3-2)

$D_p = 60\ \mu m = 60\times10^{-6}\ m \quad T = 294.3\ K \quad \rho_p = 1280\ kg/m^3$

A.3 (air)

$\mu = 0.01828\times10^{-3}\ Pa\cdot s \quad \rho = 1.202\ kg/m^3 \quad P = 101.32\ kPa$

Eq. (14.3-9) Assume laminar flow.

$$v_t = \frac{g D_p^2(\rho_p-\rho)}{18\mu} = \frac{9.806(60\times10^{-6})^2(1280-1.202)}{18(0.01828\times10^{-3})}$$

$$\boxed{v_t = 0.1372\ m/s}$$

$$N_{Re} = \frac{D_p v_t \rho}{\mu} = \frac{(60\times10^{-6})(0.1372)(1.202)}{0.01828\times10^{-3}} = 0.541$$ Hence, laminar.

(14.3-5)

$D_p = 0.090\ mm = 9.0\times10^{-5}\ m \quad \rho_p = 2002\ kg/m^3 \quad 26.7°C$

Volume fraction solids in solution = 0.45

A.2 ρ (water) $= 994.7\ kg/m^3 \quad \mu = 0.861\times10^{-3}\ Pa\cdot s$

$\varepsilon_{liquid} = 1 - 0.45 = 0.55$

Eq. (14.3-14) $\rho_m = \varepsilon\rho + (1-\varepsilon)\rho_p$

$= 0.55(994.7) + (1-0.55)(2002) = 1448\ kg/m^3$

Eq. (14.3-13)

$$\psi_p = \frac{1}{10^{1.82(1-\varepsilon)}} = \frac{1}{10^{1.82(1-0.55)}} = 0.1517$$

Eq. (14.3-16)

$$v_t = \frac{g D_p^2(\rho_p-\rho)}{18\mu}(\varepsilon^2\psi_p)$$

$$= \frac{(9.807)(9.0\times10^{-5})^2(2002-994.7)}{18(0.861\times10^{-3})}(0.55^2\times0.1517)$$

$$\boxed{v_t = 2.369\times10^{-4}\ m/s}$$

Eq. (14.3-12) $\mu_m = \dfrac{\mu}{\psi_p} = \dfrac{0.861\times10^{-3}}{0.1517} = 5.676\times10^{-3}$

Eq. (14.3-17)

$$N_{Re} = \frac{D_p v_t \rho_m}{\mu_m \varepsilon} = \frac{(9.0\times10^{-5})(2.369\times10^{-4})(1448)}{(5.676\times10^{-3})(0.55)}$$

$$\boxed{N_{Re} = 9.89\times10^{-3}}$$ Hence, in laminar region.

(14.3-8)

Same as Ex. 13.3-3 but size range is 1.27×10^{-2} mm or 1.27×10^{-5} m to 5.08×10^{-2} mm or 5.08×10^{-5} m Silica (B)

Largest size $D_{p4} = 5.08\times10^{-5}$ m Galena (A) 20°C (293.2 K)

Smallest size $D_{p1} = 1.27\times10^{-5}$ m

$\rho_{pA} = 7500$ kg/m^3 $\rho_{pB} = 2650$ kg/m^3 $\rho = 998$ kg/m^3

$\mu = 1.005\times10^{-3}$ Pa·s $= 1.005\times10^{-3}$ kg/m·s

Assume Stokes' law settling

Eq. (14.3-29) $$v_{tA} = \frac{g D_{pA}^2(\rho_{pA}-\rho)}{18\mu}$$

The largest Reynolds number occurs for the largest diameter and density, $D_{pA} = 5.08\times10^{-5}$ m, $\rho_{pA} = 7500$ kg/m^3

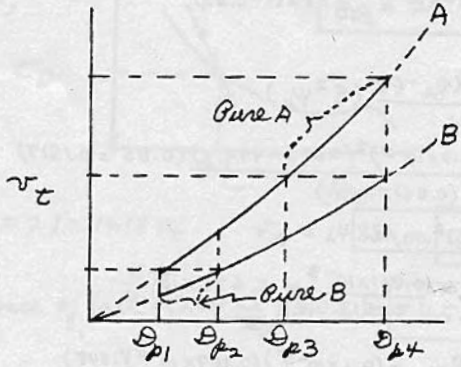
$$v_{tA} = \frac{9.807(5.08\times10^{-5})^2(7500-998)}{(18)(1.005\times10^{-3})} = 9.096\times10^{-3}\ \text{m/s}$$

Eq. (14.3-30)

$$N_{Re} = \frac{D_{pA} v_{tA}\rho}{\mu} = \frac{(5.08\times10^{-5})(9.096\times10^{-3})(998)}{1.005\times10^{-3}} = \boxed{0.4589 = N_{Re}}$$

Hence, since $N_{Re} < 1$ (see Text), in Stokes' law region.

Plot similar to Fig. 14.3-3 but for Stoke's (laminar) region and not Newton's turbulent region. By Eq. (14.3-9), $v_t \propto D_p^2$.



(14.3-8) - p. 2

Pure fraction of A is $D_{pA4} = 5.08 \times 10^{-5}$ m to D_{pA3}.

D_{pA3} and D_{pB4} have equal settling velocities by Eq (14.3-26)

$$\frac{D_{pA3}}{D_{pB4}} = \left(\frac{\rho_{pB} - \rho}{\rho_{pA} - \rho}\right)^{0.5}$$

$$\frac{D_{pA3}}{5.08 \times 10^{-5}} = \left(\frac{2650 - 998}{7500 - 998}\right)^{0.5}$$

$$D_{pA3} = 2.5606 \times 10^{-5} \text{ m}$$

Pure fraction of B is D_{pB2} to $D_{pB1} = 1.27 \times 10^{-5}$ m.
D_{pB2} and D_{pA1} have equal settling velocities by Eq (14.3-26)

$$\frac{D_{pA1}}{D_{pB2}} = \frac{1.27 \times 10^{-5}}{D_{pB2}} = \left(\frac{2650 - 998}{7500 - 998}\right)^{0.5}$$

$$D_{pB2} = 2.520 \times 10^{-5} \text{ m}$$

The three fractions recovered are as follows.

1. Pure A (galena)	$D_{pA3} = 2.561 \times 10^{-5}$ m to $D_{pA4} = 5.08 \times 10^{-5}$ m
2. Mixed A and B	$D_{pB2} = 2.520 \times 10^{-5}$ m to $D_{pB4} = 5.08 \times 10^{-5}$ m
	$D_{pA1} = 1.27 \times 10^{-5}$ m to $D_{pA3} = 2.561 \times 10^{-5}$ m
3. Pure B (silica)	$D_{pB1} = 1.27 \times 10^{-5}$ m to $D_{pB2} = 2.520 \times 10^{-5}$ m

(14.4-1)

$r_1 = 76.2\ mm = 76.2\times10^{-3}\ m \quad r_2 = 305\ mm = 305\times10^{-3}\ m$

$v = 53.34\ m/s$ for both centrifuges

Eq.(14.4-5)

$$N_1 = \frac{60v}{2\pi r_1} = \frac{60(53.34)}{2\pi(76.2\times10^{-3})} = \boxed{6684\ RPM = N_1}$$

$$N_2 = \frac{60v}{2\pi r_2} = \frac{60(53.34)}{2\pi(305\times10^{-3})} = \boxed{1670\ RPM = N_2}$$

Eq.(14.4-7)

$$\frac{F_c}{F_g} = 0.001118\, r_1 N_1^2 = 0.001118(76.2\times10^{-3})(6684)^2 = \boxed{3806\ g's\ bowl\ 1}$$

$$\frac{F_c}{F_g} = 0.001118(305\times10^{-3})(1670)^2 = \boxed{951\ g's\ in\ bowl\ 2}$$

(14.4-2)

(a) $N = 2000\ RPM \qquad g's = 455$

Eq.(14.4-7) $\frac{F_c}{F_g} = 455 = 0.001118\, r N^2 = 0.001118(r)(2000)^2 \quad \boxed{r = 0.1017\ m}$

(b) $r = 4(0.1017) = \boxed{0.4068\ m = r}$

(14.4-4)

$D_p = 5\times10^{-2}\ mm = 5\times10^{-5}\ m,\ \rho_p = 1050\ kg/m^3,\quad \rho = 1000\ kg/m^3$

$\mu = 1.20\times10^{-3}\ Pa\cdot s \quad N = 3000\ RPM \quad b = 100.1\ mm = 100.1\times10^{-3}\ m$

$r_1 = 5.00\ mm = 5.00\times10^{-3}\ m \quad r_2 = 30.0\ mm = 30.0\times10^{-3}\ m$

Eq.(14.4-4)

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(3000)}{60} = 314.2\ rad/s$$

Eq.(14.4-11)

$$q = \frac{\omega^2(\rho_p-\rho)D_p^2\left[\pi b(r_2^2-r_1^2)\right]}{18\mu \ln(r_2/r_1)}$$

$$= \frac{(314.2)^2(1050-1000)(5\times10^{-5})^2\left[\pi(100.1\times10^{-3})\left((30\times10^{-3})^2-(5.0\times10^{-3})^2\right)\right]}{18(1.20\times10^{-3})\ln\left[30\times10^{-3}/5\times10^{-3}\right]}$$

$$\boxed{q = 8.76\times10^{-5}\ m^3/s}$$

(14.4-6)

$\rho_H = 1032\ kg/m^3 \quad \rho_L = 865\ kg/m^3 \quad r_1 = 50.8\ mm \quad r_4 = 76.2\ mm$

Eq.(13.4-27)

$$r_2^2 = \frac{(\rho_H r_4^2 - \rho_L r_1^2)}{\rho_H - \rho_L} = \frac{[1032 \times (76.2)^2 - 865 \times (50.8)^2]}{1032 - 865}$$

$$\boxed{r_2 = 150.0\ mm}$$

(14.4-7)

(a) From Example 14.4-2: $\omega = 2410\ rad/s \quad r_2 = 0.02225\ m$

$r_1 = 0.00716\ m \quad b = 0.1970\ m \quad q_1 = q_c = 7.87 \times 10^{-7}\ m^3/s$

Eq.(14.4-19)

$$\Sigma_1 = \frac{\omega^2 [\pi b (r_2^2 - r_1^2)]}{2g \ln[2r_2/(r_1 + r_2)]}$$

$$= \frac{(2410)^2 [\pi (0.1970)((0.02225)^2 - (0.00716)^2)]}{2(9.8066) \ln[2(0.02225)/(0.00716 + 0.02225)]} = \boxed{\Sigma_1 = 196.3\ m^2}$$

(b) Eq.(14.4-4) $\quad \omega = 2\pi(26000)/60 = 2723\ rad/s \quad r_2 = 0.0445\ m$

$r_1 = 0.01432\ m \quad b = 0.394\ m \quad N = 26000\ RPM$

Eq.(14.4-19)

$$\Sigma_2 = \frac{(2723)^2 [\pi (0.394)((0.0445)^2 - (0.01432)^2)]}{2(9.8066) \ln[2(0.0445)/(0.01432 + 0.0445)]} = \boxed{\Sigma_2 = 2006\ m^2}$$

Eq.(14.4-21)

$$\frac{q_1}{\Sigma_1} = \frac{q_2}{\Sigma_2}$$

$$\frac{7.87 \times 10^{-7}}{196.3} = \frac{q_2}{2006}$$

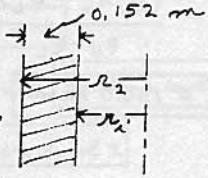
$$\boxed{q_2 = 80.42 \times 10^{-7}\ m^3/s}$$

(14.4-8)

$b = 0.457\ m$ $\quad r_2 = 0.381\ m$ $\quad N = 33.33$ rev/s (at 25°C) $\quad r_i = 0.2032\ m$

$c_s = 60.0$ kg solids/m^3 filtrate, $\varepsilon = 0.82$ $\quad \rho_s = 2002$ kg solids/m^3

cake thickness $= 0.152\ m$ $\quad \alpha = 6.38 \times 10^{10}\ m/kg$ $\quad R_m = 8.53 \times 10^{10}\ m^{-1}$



Volume of cake $= \pi b (r_2^2 - r_i^2)$

$r_i = 0.381 - 0.152 = 0.229\ m$

Volume of cake $= \pi(0.457)((0.381)^2 - (0.229)^2) = 0.1332\ m^3$

A.2 Water at 25°C $\quad \rho = 997\ kg/m^3$ $\quad \mu = 0.894 \times 10^{-3}$ Pa·s

$$\omega = \frac{2\pi N}{60} = 2\pi(33.33) = 209.4\ rad/s$$

m_c = vol. of cake solids $\times\ \rho_s = 0.1332\,(1.00 - 0.82)(2002) = 48.0\ kg$

$$A_2 = 2\pi r_2 b = 2\pi(0.381)(0.457) = 1.094\ m^2$$

$$\bar{A}_L = \frac{2\pi b(r_2 - r_i)}{\ln(r_2/r_i)} = \frac{2\pi(0.457)(0.381 - 0.229)}{\ln(0.381/0.229)} = 0.8573\ m^2$$

$$\bar{A}_a = \pi(r_i + r_2)b = \pi(0.229 + 0.381)(0.457) = 0.8758\ m^2$$

Eq. (14.4-34)

$$q = \frac{\rho\omega^2(r_2^2 - r_i^2)}{2\mu\left(\dfrac{m_c\alpha}{\bar{A}_L\bar{A}_a} + \dfrac{R_m}{A_2}\right)}$$

$$= \frac{997(209.4)^2\left((0.381)^2 - (0.2032)^2\right)}{2(0.894\times10^{-3})\left[\dfrac{48.0\,(6.38\times10^{10})}{0.8573\,(0.8758)} + \dfrac{8.53\times10^{10}}{1.094}\right]}$$

$$\boxed{q = 6.11 \times 10^{-4}\ m^3/s}$$

(14.5-1)

$D_F = 50.8$ mm $\quad D_P = 6.35$ mm $\quad P = 89.5$ kW

New $D_P = 3.18$ mm

Eq. (14.5-6) (English units).

$$\frac{P}{T} = 1.46\, E_i' \left(\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

Original case

(1) $\frac{89.5\,kW}{T} = 1.46 E_i' \left(\frac{1}{\sqrt{6.35}} - \frac{1}{\sqrt{50.8}} \right) = 1.46 E_i' (0.2565)$

New case

(2) $\frac{P}{T} = 1.46\, E_i' \left(\frac{1}{\sqrt{3.18}} - \frac{1}{\sqrt{50.8}} \right) = 1.46 E_i' (0.4205)$

Divide (2) by (1)

$$\frac{P/T}{89.5/T} = \frac{1.46 E_i' (0.2565)}{1.46 E_i' (0.4205)} \qquad \boxed{P = 146.7\ kW}$$

(Note that conversion factors from English to SI units would cancel out)

(14.5-2)

$T = 100/60 = 1.667$ T/min $\quad E_i' = 10.13 \quad D_F = 4/12 = 0.3333$ ft

$D_P = 1/(8 \times 12) = 0.01042$ ft

Eq. (14.5-6) English units

$$\frac{P}{T} = 1.46\, E_i' \left(\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}} \right)$$

$$\frac{P}{1.667} = 1.46 (10.13) \left(\frac{1}{\sqrt{0.01042}} - \frac{1}{\sqrt{0.3333}} \right)$$

(a) $\boxed{P = 198.7\ hp\ (148.2\ kW)}$

$D_P = 1000\ \mu m = 1000 \times 10^{-6} \times 3.2808 = 3.2808 \times 10^{-3}$ ft

$$\frac{P}{1.667} = 1.46\ (10.13) \left(\frac{1}{\sqrt{3.2808 \times 10^{-3}}} - \frac{1}{\sqrt{0.3333}} \right)$$

$\boxed{P = 387.6\ hp\ (289.1\ kW)}$