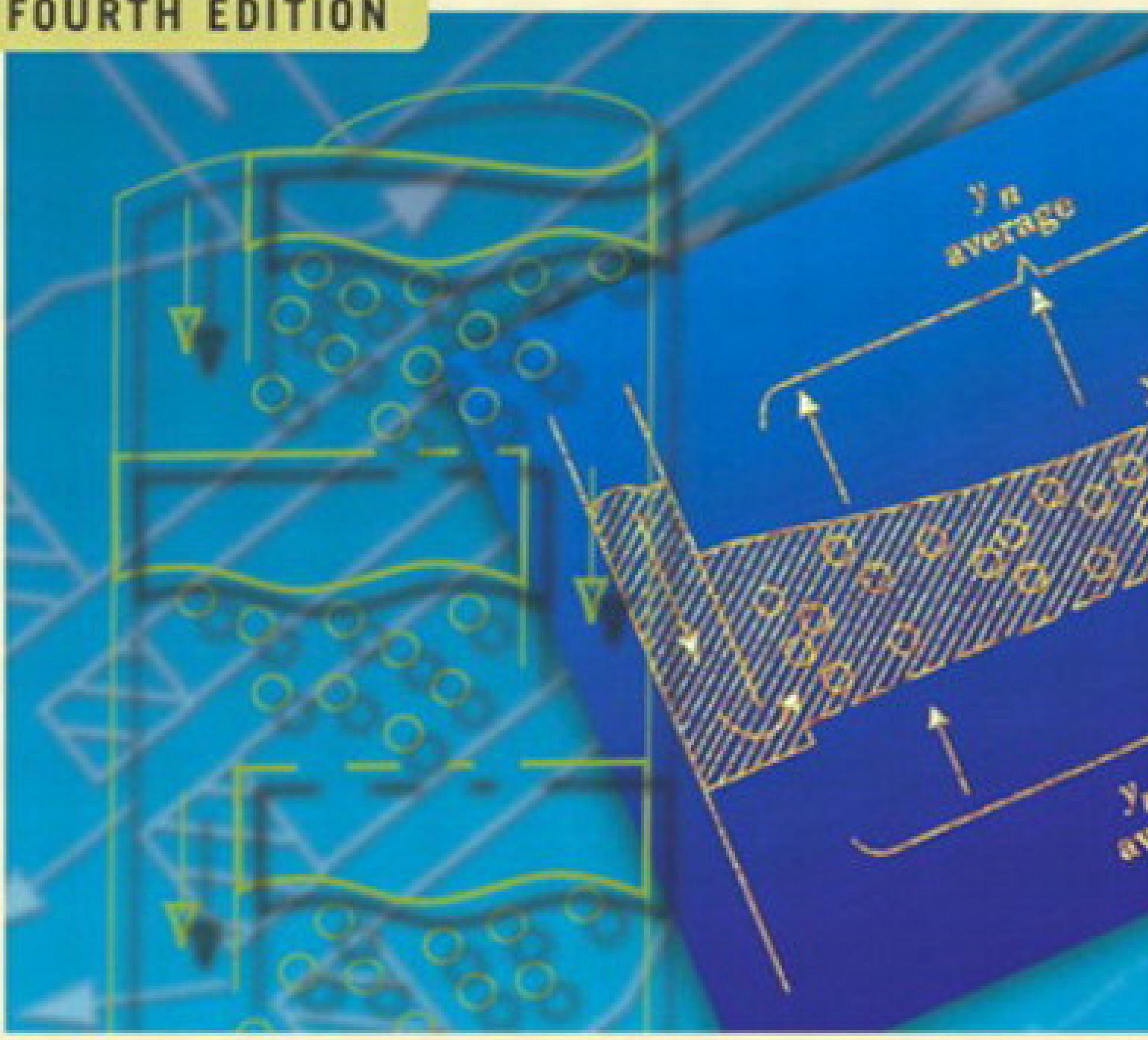
Transport Processes and Separation Process Principles

(INCLUDES UNIT OPERATIONS)

FOURTH EDITION



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Chapter 2. Principles of Momentum Transfer and Overall Balances

INTRODUCTION

The flow and behavior of fluids is important in many of the separation processes in process engineering. A fluid may be defined as a substance that does not permanently resist distortion and, hence, will change its shape. In this text gases, liquids, and vapors are considered to have the characteristics of fluids and to obey many of the same laws.

In the process industries, many of the materials are in fluid form and must be stored, handled, pumped, and processed, so it is necessary that we become familiar with the principles that govern the flow of fluids as well as with the equipment used. Typical fluids encountered include water, air, CO_2, oil, slurries, and thick syrups.

If a fluid is inappreciably affected by changes in pressure, it is said to be *incompressible.* Most liquids are incompressible. Gases are considered to be *compressible* fluids. However, if gases are subjected to small percentage changes in pressure and temperature, their density changes will be small and they can be considered to be incompressible.

Like all physical matter, a fluid is composed of an extremely large number of molecules per unit volume. A theory such as the kinetic theory of gases or statistical mechanics treats the motions of molecules in terms of statistical groups and not in terms of individual molecules. In engineering we are mainly concerned with the bulk or macroscopic behavior of a fluid rather than the individual molecular or microscopic behavior.

In momentum transfer we treat the fluid as a continuous distribution of matter, or a "continuum." This treatment as a continuum is valid when the smallest volume of fluid contains a number of molecules large enough that a statistical average is meaningful and the macroscopic properties of the fluid, such as density, pressure, and so on, vary smoothly or continuously from point to point.

The study of *momentum transfer,* or *fluid mechanics* as it is often called, can be divided into two branches: *fluid statics,* or fluids at rest, and *fluid dynamics,* or fluids in motion. In Section 2.2 we treat fluid statics; in the remaining sections of Chapter 2 and in Chapter 3, fluid dynamics. Since in fluid dynamics momentum is being transferred, the term "momentum transfer" or "transport" is usually used. In Section 2.3 momentum transfer is related to heat and mass transfer.

FLUID STATICS

Force, Units, and Dimensions

In a static fluid an important property is the pressure in the fluid. Pressure is familiar as a surface force exerted by a fluid against the walls of its container. Also, pressure exists at any point in a volume of a fluid.

In order to understand *pressure,* which is defined as force exerted per unit area, we must first discuss a basic law of Newton's. This equation for calculation of the force exerted by a mass under the influence of gravity is

Equation 2.2-1.

$$F = mg \qquad \text{(SI units)}$$

$$F = \frac{mg}{g_c} \qquad \text{(English units)}$$

where in SI units F is the force exerted in newtons N (kg · m/s^2), m the mass in kg, and g the standard acceleration of gravity, 9.80665 m/s^2.

In English units, F is in lb_f, m in lb_m, g is 32.1740 ft/s^2, and g_c (a gravitational conversion factor) is 32.174 lb_m · ft/lb_f · s^2. The use of the conversion factor g_c means that g/g_c has a value of 1.0 lb_f/lb_m and that 1 lb_m conveniently gives a force equal to 1 lb_f. Often when units of pressure are given, the word "force" is omitted, as in lb/in.^{2} (psi) instead of lb_f/in.^{2}. When the mass m is given in g mass, F is g force, g = 980.665 cm/s^2, and g_c = 980.665 g mass · cm/g force · s^2. However, the units g force are seldom used.

Another system of units sometimes used in Eq. (2.2-1) is that where the g_c is omitted and the force ($F = mg$) is given as lb_m · ft/s^2, called *poundals*. Then 1 lb_m acted on by gravity will give a force of 32.174 poundals (lb_m · ft/s^2). Or if 1 g mass is used, the force ($F = mg$) is expressed in terms of dynes (g · cm/s^2). This is the centimeter–gram–second (cgs) systems of units.

Conversion factors for different units of force and of force per unit area (pressure) are given in Appendix A.1. Note that always in the SI system, and usually in the cgs system, the term g_c is not used.

EXAMPLE 2.2-1. Units and Dimensions of Force

Calculate the force exerted by 3 lb mass in terms of the following:

a. Lb force (English units).
b. Dynes (cgs units).
c. Newtons (SI units).

Solution: For part (a), using Eq. (2.2-1),

$$F\ \text{(force)} = m\frac{g}{g_c} = (3\ \text{lb}_m)\left(32.174\ \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{1}{32.174\ \frac{\text{lb}_m \cdot \text{ft}}{\text{lb}_f \cdot \text{s}^2}}\right) = 3\ \text{lb force (lb}_f)$$

For part (b),

$$F = mg = (3\ \text{lb}_m)\left(453.59\ \frac{\text{g}}{\text{lb}_m}\right)\left(980.665\ \frac{\text{cm}}{\text{s}^2}\right)$$

$$= 1.332 \times 10^6\ \frac{\text{g} \cdot \text{cm}}{\text{s}^2} = 1.332 \times 10^6\ \text{dyn}$$

As an alternative method for part (b), from Appendix A.1,

$$1 \text{ dyn} = 2.2481 \times 10^{-6} \text{ lb}_f$$

$$F = (3 \text{ lb}_f)\left(\frac{1}{2.2481 \times 10^{-6} \text{ lb}_f/\text{dyn}}\right) = 1.332 \times 10^6 \text{ dyn}$$

To calculate newtons in part (c),

$$F = mg = \left(3 \text{ lb}_m \times \frac{1 \text{ kg}}{2.2046 \text{ lb}_m}\right)\left(9.80665 \frac{\text{m}}{\text{s}^2}\right)$$

$$= 13.32 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 13.32 \text{ N}$$

As an alternative method, using values from Appendix A.1,

$$1 \frac{\text{g} \cdot \text{cm}}{\text{s}^2} (\text{dyn}) = 10^{-5} \frac{\text{kg} \cdot \text{m}}{\text{s}^2} (\text{newton})$$

$$F = (1.332 \times 10^6 \text{ dyn})\left(10^{-5} \frac{\text{newton}}{\text{dyn}}\right) = 13.32 \text{ N}$$

Pressure in a Fluid

Since Eq. (2.2-1) gives the force exerted by a mass under the influence of gravity, the force exerted by a mass of fluid on a supporting area, or force/unit area (pressure), also follows from this equation. In Fig. 2.2-1 a stationary column of fluid of height h_2 m and constant cross-sectional area A m^2, where $A = A_0 = A_1 = A_2$, is shown. The pressure above the fluid is P_0 N/m^2; that is, this could be the pressure of the atmosphere above the fluid. The fluid at any point, say h_1, must support all the fluid above it. It can be shown that the forces at any given point in a nonmoving or static fluid must be the same in all directions. Also, for a fluid at rest, the force/unit area, or pressure, is the same at all points with the same elevation. For example, at h_1 m from the top, the pressure is the same at all points shown on the cross-sectional area A_1.

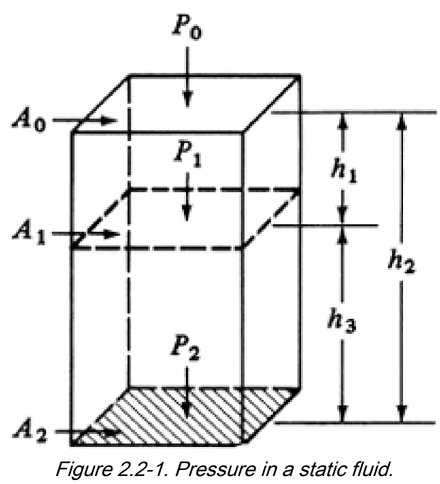

Figure 2.2-1. Pressure in a static fluid.

The use of Eq. (2.2-1) will be shown in calculating the pressure at different vertical points in Fig. 2.2-1. The total mass of fluid for h_2 m height and density ρ kg/m^3 is

Equation 2.2-2.

$$\text{total kg fluid} = (h_2 \text{ m})(A \text{ m}^2)\left(\rho \frac{\text{kg}}{\text{m}^3}\right) = h_2 A \rho \text{ kg}$$

Substituting into Eq. (2.2-2), the total force F of the fluid on area A_1 due to the fluid only is

Equation 2.2-3.

$$F = (h_2 A \rho \text{ kg})(g \text{ m/s}^2) = h_2 A \rho g \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \text{ (N)}$$

The pressure P is defined as force/unit area:

Equation 2.2-4.

$$P = \frac{F}{A} = (h_2 A \rho g)\frac{1}{A} = h_2 \rho g \text{ N/m}^2 \quad \text{or} \quad \text{Pa}$$

This is the pressure on A_2 due to the mass of the fluid above it. However, to get the total pressure P_2 on A_2, the pressure P_0 on the top of the fluid must be added:

Equation 2.2-5.

$$P_2 = h_2 \rho g + P_0 \text{ N/m}^2 \quad \text{or} \quad \text{Pa}$$

Equation (2.2-5) is the fundamental equation for calculating the pressure in a fluid at any depth. To calculate P_1,

Equation 2.2-6.

$$P_1 = h_1 \rho g + P_0$$

The pressure difference between points 2 and 1 is

Equation 2.2-7.

$$P_2 - P_1 = (h_2 \rho g + P_0) - (h_1 \rho g + P_0) = (h_2 - h_1)\rho g \qquad \text{(SI units)}$$

$$P_2 - P_1 = (h_2 - h_1)\rho \frac{g}{g_c} \qquad \text{(English units)}$$

Since it is the vertical height of a fluid that determines the pressure in a fluid, the shape of the vessel does not affect the pressure. For example, in Fig. 2.2-2, the pressure P_1 at the bottom of all three vessels is the same and is equal to $h_1 \rho g + P_0$.

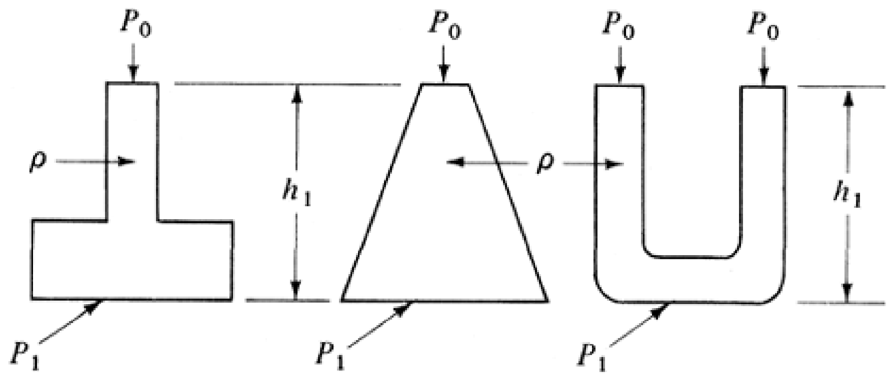

Figure 2.2-2. Pressure in vessels of various shapes.

EXAMPLE 2.2-2. Pressure in Storage Tank

A large storage tank contains oil having a density of 917 kg/m^3 (0.917 g/cm^3). The tank is 3.66 m (12.0 ft) tall and is vented (open) to the atmosphere of 1 atm abs at the top. The tank is filled with oil to a depth of 3.05 m (10 ft) and also contains 0.61 m (2.0 ft) of water in the bottom of the tank. Calculate the pressure in Pa and psia 3.05 m from the top of the tank and at the bottom. Also calculate the gage pressure at the tank bottom.

Solution: First a sketch is made of the tank, as shown in Fig. 2.2-3. The pressure P_0 = 1 atm abs = 14.696 psia (from Appendix A.1). Also,

$$P_0 = 1.01325 \times 10^5 \text{ Pa}$$

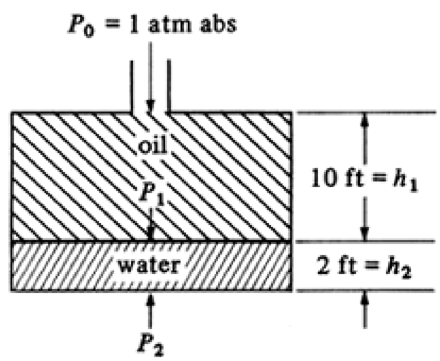

Figure 2.2-3. Storage tank in Example 2.2-2.

From Eq. (2.2-6), using English and then SI units,

$$P_1 = h_1 \rho_{\text{oil}} \frac{g}{g_c} + P_0 = (10 \text{ ft})\left(0.917 \times 62.43 \frac{\text{lb}_\text{m}}{\text{ft}^3}\right)\left(1.0 \frac{\text{lb}_\text{f}}{\text{lb}_\text{m}}\right)\left(\frac{1}{144 \text{ in.}^2/\text{ft}^2}\right)$$
$$+ 14.696 \text{ lb}_\text{f}/\text{in.}^2 = 18.68 \text{ psia}$$

$$P_1 = h_1 \rho_{\text{oil}} g + P_0 = (3.05 \text{ m})\left(917 \frac{\text{kg}}{\text{m}^3}\right)\left(9.8066 \frac{\text{m}}{\text{s}^2}\right) + 1.0132 \times 10^5$$
$$= 1.287 \times 10^5 \text{ Pa}$$

To calculate P_2 at the bottom of the tank, ρ_{water} = 1.00 g/cm^3 and

$$P_2 = h_2 \rho_{\text{water}} \frac{g}{g_c} + P_1 = (2.0)(1.00 \times 62.43)(1.0)(\tfrac{1}{144}) + 18.68$$

$$= 19.55 \text{ psia}$$

$$= h_2 \rho_{\text{water}} g + P_1 = (0.61)(1000)(9.8066) + 1.287 \times 10^5$$

$$= 1.347 \times 10^5 \text{ Pa}$$

The gage pressure at the bottom is equal to the absolute pressure P_2 minus 1 atm pressure:

$$P_{\text{gage}} = 19.55 \text{ psia} - 14.696 \text{ psia} = 4.85 \text{ psig}$$

Head of a Fluid

Pressures are given in many different sets of units, such as psia, dyn/cm^2, and newtons/m^2, as given in Appendix A.1. However, a common method of expressing pressures is in terms of head in m or feet of a particular fluid. This height or head in m or feet of the given fluid will exert the same pressure as the pressures it represents. Using Eq. (2.2-4), which relates pressure P and height h of a fluid, and solving for h, which is the head in m,

Equation 2.2-8.

$$h(\text{head}) = \frac{P}{\rho g} \text{ m} \qquad (\text{SI})$$

$$h = \frac{P g_c}{\rho g} \text{ ft} \qquad (\text{English})$$

EXAMPLE 2.2-3. Conversion of Pressure to Head of a Fluid

Given the pressure of 1 standard atm as 101.325 kN/m^2 (Appendix A.1), do as follows:

a. Convert this pressure to head in m water at 4°C.
b. Convert this pressure to head in m Hg at 0°C.

Solution: For part (a), the density of water at 4°C in Appendix A.2 is 1.000 g/cm^3. From A.1, a density of 1.000 g/cm^3 equals 1000 kg/m^3. Substituting these values into Eq. (2.2-8),

$$h(\text{head}) = \frac{P}{\rho g} = \frac{101.325 \times 10^3}{(1000)(9.80665)}$$

$$= 10.33 \text{ m of water at } 4°\text{C}$$

For part (b), the density of Hg in Appendix A.1 is 13.5955 g/cm^3. For equal pressures P from different fluids, Eq. (2.2-8) can be rewritten as

Equation 2.2-9.

$$P = \rho_{\text{Hg}} h_{\text{Hg}} g = \rho_{\text{H}_2\text{O}} h_{\text{H}_2\text{O}} g$$

Solving for h_{Hg} in Eq. (2.2-9) and substituting known values,

$$h_{Hg}(\text{head}) = \frac{\rho_{H_2O}}{\rho_{Hg}} h_{H_2O} = \left(\frac{1.000}{13.5955}\right)(10.33) = 0.760 \text{ m Hg}$$

Devices to Measure Pressure and Pressure Differences

In chemical and other industrial processing plants, it is often important to measure and control the pressure in a vessel or process and/or the liquid level in a vessel. Also, since many fluids are flowing in a pipe or conduit, it is necessary to measure the rate at which the fluid is flowing. Many of these flow meters depend upon devices for measuring a pressure or pressure difference. Some common devices are considered in the following paragraphs.

Simple U-tube manometer

The U-tube manometer is shown in Fig. 2.2-4a. The pressure p_a N/m^2 is exerted on one arm of the U tube and p_b on the other arm. Both pressures p_a and p_b could be pressure taps from a fluid meter, or p_a could be a pressure tap and p_b the atmospheric pressure. The top of the manometer is filled with liquid *B*, having a density of ρ_B kg/m^3, and the bottom with a more dense fluid *A*, having a density of ρ_A kg/m^3. Liquid *A* is immiscible with *B*. To derive the relationship between p_a and p_b, p_a is the pressure at point 1 and p_b at point 5. The pressure at point 2 is

Equation 2.2-10.

$$p_2 = p_a + (Z + R)\rho_B g \text{ N/m}^2$$

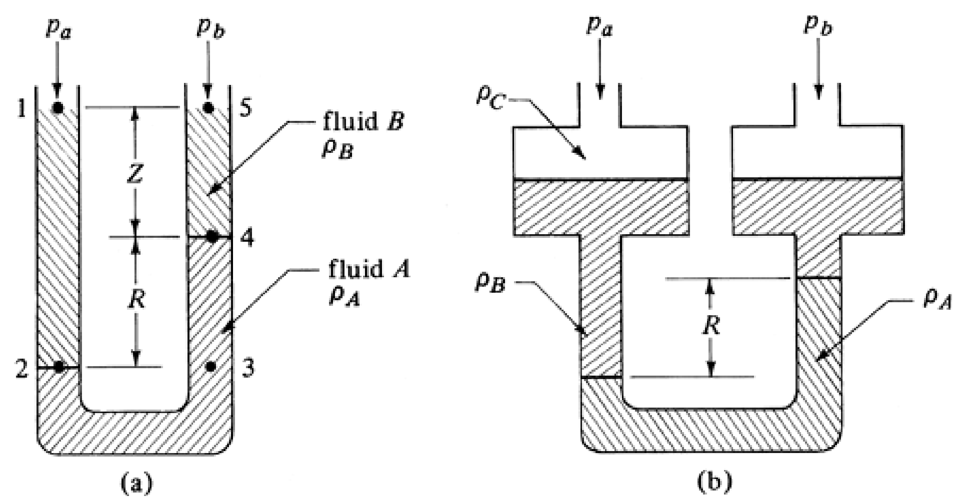

Figure 2.2-4. Manometers to measure pressure differences: (a) U tube; (b) two-fluid U tube.

where *R* is the reading of the manometer in m. The pressure at point 3 must be equal to that at 2 by the principles of hydrostatics:

Equation 2.2-11.

$$p_3 = p_2$$

The pressure at point 3 also equals the following:

Equation 2.2-12.

$$p_3 = p_b + Z\rho_B g + R\rho_A g$$

Equating Eq. (2.2-10) to (2.2-12) and solving,

Equation 2.2-13.

$$p_a + (Z + R)\rho_B g = p_b + Z\rho_B g + R\rho_A g$$

Equation 2.2-14.

$$p_a - p_b = R(\rho_A - \rho_B)g \qquad \text{(SI)}$$

$$p_a - p_b = R(\rho_A - \rho_B)\frac{g}{g_c} \qquad \text{(English)}$$

The reader should note that the distance Z does not enter into the final result nor do the tube dimensions, provided that p_a and p_b are measured in the same horizontal plane.

EXAMPLE 2.2-4. Pressure Difference in a Manometer

A manometer, as shown in Fig. 2.2-4a, is being used to measure the head or pressure drop across a flow meter. The heavier fluid is mercury, with a density of 13.6 g/cm^3, and the top fluid is water, with a density of 1.00 g/cm^3. The reading on the manometer is R = 32.7 cm. Calculate the pressure difference in N/m^2 using SI units.

Solution: Converting R to m,

$$R = \frac{32.7}{100} = 0.327 \text{ m}$$

Also converting ρ_A and ρ_B to kg/m^3 and substituting into Eq. (2.2-14),

$$\begin{aligned} p_a - p_b = R(\rho_A - \rho_B)g &= (0.327 \text{ m})[(13.6 - 1.0)(1000 \text{ kg/m}^3)](9.8066 \text{ m/s}^2) \\ &= 4.040 \times 10^4 \text{ N/m}^2 \text{ (5.85 psia)} \end{aligned}$$

Two-fluid U tube

In Fig. 2.2-4b a two-fluid U tube is shown, which is a sensitive device for measuring small heads or pressure differences. Let A m^2 be the cross-sectional area of each of the large reservoirs and a m^2 be the cross-sectional area of each of the tubes forming the U. Proceeding and making a pressure balance as for the U tube,

Equation 2.2-15.

$$p_a - p_b = (R - R_0)\left(\rho_A - \rho_B + \frac{a}{A}\rho_B - \frac{a}{A}\rho_C\right)g$$

where R_0 is the reading when $p_a = p_b$, R is the actual reading, ρ_A is the density of the heavier fluid, and ρ_B is the density of the lighter fluid. Usually, a/A is made sufficiently small as to be negligible, and also R_0 is often adjusted to zero; then

Equation 2.2-16.

$$p_a - p_b = R(\rho_A - \rho_B)g \qquad \text{(SI)}$$

$$p_a - p_b = R(\rho_A - \rho_B)\frac{g}{g_c} \qquad \text{(English)}$$

If ρ_A and ρ_B are close to each other, the reading R is magnified.

EXAMPLE 2.2-5. Pressure Measurement in a Vessel

The U-tube manometer in Fig. 2.2-5a is used to measure the pressure p_A in a vessel containing a liquid with a density ρ_A. Derive the equation relating the pressure p_A and the reading on the manometer as shown.

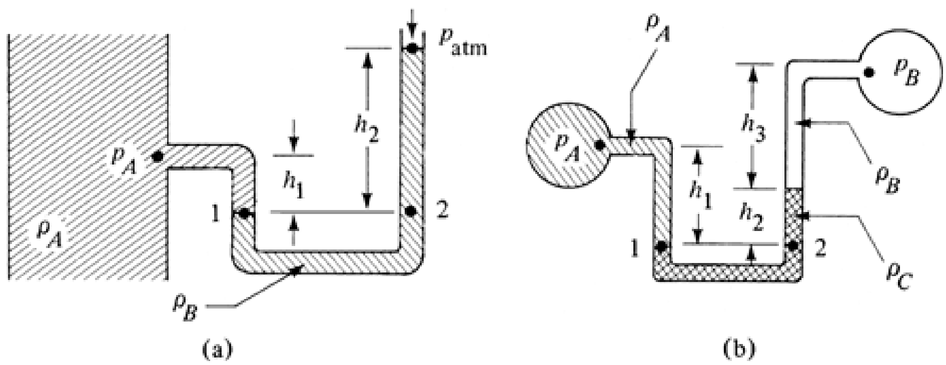

Figure 2.2-5. Measurements of pressure in vessels: (a) measurement of pressure in a vessel, (b) measurement of differential pressure.

Solution:

At point 2 the pressure is

Equation 2.2-17.

$$p_2 = p_{\text{atm}} + h_2 \rho_B g \text{ N/m}^2$$

At point 1 the pressure is

Equation 2.2-18.

$$p_1 = p_A + h_1 \rho_A g$$

Equating $p_1 = p_2$ by the principles of hydrostatics and rearranging,

Equation 2.2-19.

$$p_A = p_{\text{atm}} + h_2 \rho_B g - h_1 \rho_A g$$

Another example of a U-tube manometer is shown in Fig. 2.2-5b. In this case the device is used to measure the pressure difference between two vessels.

Bourdon pressure gage

Although manometers are used to measure pressures, the most common pressure-measuring device is the mechanical Bourdon-tube pressure gage. A coiled hollow tube in the gage tends to straighten out when subjected to internal pressure, and the degree of straightening depends on the pressure difference between the inside and outside pressures. The tube is connected to a pointer on a calibrated dial.

Gravity separator for two immiscible liquids

In Fig. 2.2-6 a continuous gravity separator (decanter) is shown for the separation of two immiscible liquids *A* (heavy liquid) and *B* (light liquid). The feed mixture of the two liquids enters at one end of the separator vessel, and the liquids flow slowly to the other end and separate into two distinct layers. Each liquid flows through a separate overflow line as shown. Assuming the frictional resistance to the flow of the liquids is essentially negligible, the principles of fluid statics can be used to analyze the performance.

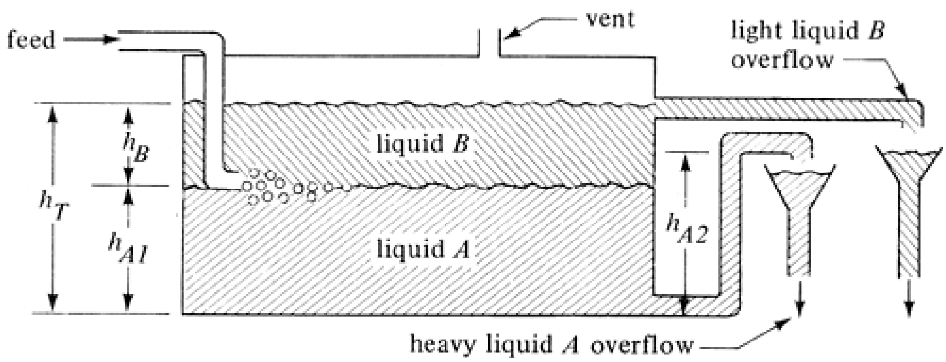

Figure 2.2-6. Continuous atmospheric gravity separator for immiscible liquids.

In Fig. 2.2-6, the depth of the layer of heavy liquid *A* is h_{A1} m and that of *B* is h_B. The total depth $h_T = h_{A1} + h_B$ and is fixed by position of the overflow line for *B*. The heavy liquid *A* discharges through an overflow leg h_{A2} m above the vessel bottom. The vessel and the overflow lines are vented to the atmosphere. A hydrostatic balance gives

Equation 2.2-20.

$$h_B \rho_B g + h_{A1} \rho_A g = h_{A2} \rho_A g$$

Substituting $h_B = h_T - h_{A1}$ into Eq. (2.2-20) and solving for h_{A1},

Equation 2.2-21.

$$h_{A1} = \frac{h_{A2} - h_T \rho_B / \rho_A}{1 - \rho_B / \rho_A}$$

This shows that the position of the interface or height h_{A1} depends on the ratio of the densities of the two liquids and on the elevations h_{A2} and h_T of the two overflow lines. Usually, the height h_{A2} is movable so that the interface level can be adjusted.

GENERAL MOLECULAR TRANSPORT EQUATION FOR MOMENTUM, HEAT, AND MASS TRANSFER

General Molecular Transport Equation and General Property Balance

Introduction to transport processes

In molecular transport processes in general we are concerned with the transfer or movement of a given property or entity by molecular movement through a system or medium which can be a fluid (gas or liquid) or a solid. This property that is being transferred can be mass, thermal energy (heat), or momentum. Each molecule of a system has a given quantity of the property mass, thermal energy, or momentum associated with it. When a difference in concentration of the property exists for any of these properties from one region to an adjacent region, a net transport of this property occurs. In dilute fluids such as gases, where the molecules are relatively far apart, the rate of transport of the property should be relatively fast, since few molecules are present to block the transport or interact. In dense fluids such as liquids, the molecules are close together, and transport or diffusion proceeds more slowly. The molecules in solids are even more closely packed than in liquids and molecular migration is even more restricted.

General molecular transport equation

All three of the molecular transport processes of momentum, heat or thermal energy, and mass are characterized in the elementary sense by the same general type of transport equation. First we start by noting the following:

Equation 2.3-1.

$$\text{rate of transfer process} = \frac{\text{driving force}}{\text{resistance}}$$

This states what is quite obvious—that we need a driving force to overcome a resistance in order to transport a property. This is similar to Ohm's law in electricity, where the rate of flow of electricity is proportional to the voltage drop (driving force) and inversely proportional to the resistance.

We can formalize Eq. (2.3-1) by writing an equation as follows for molecular transport or diffusion of a property:

Equation 2.3-2.

$$\psi_z = -\delta \frac{d\Gamma}{dz}$$

where ψ_z is defined as the flux of the property as amount of property being transferred per unit time per unit cross-sectional area perpendicular to the z direction of flow in amount of property/s · m^2, δ is a proportionality constant called diffusivity in m^2/s, Γ is concentration of the property in amount of property/m^3, and z is the distance in the direction of flow in m.

If the process is at steady state, then the flux ψ_z is constant. Rearranging Eq. (2.3-2) and integrating,

Equation 2.3-3.

$$\psi_z \int_{z_1}^{z_2} dz = -\delta \int_{\Gamma_1}^{\Gamma_2} d\Gamma$$

Equation 2.3-4.

$$\psi_z = \frac{\delta(\Gamma_1 - \Gamma_2)}{z_2 - z_1}$$

A plot of the concentration Γ versus z is shown in Fig. 2.3-1a and is a straight line. Since the flux is in the direction 1 to 2 of decreasing concentration, the slope $d\Gamma/dz$ is negative, and the negative sign in Eq. (2.3-2) gives a positive flux in the direction 1 to 2. In Section 2.3B the specialized equations for momentum, heat, and mass transfer will be shown to be the same as Eq. (2.3-4) for the general property transfer.

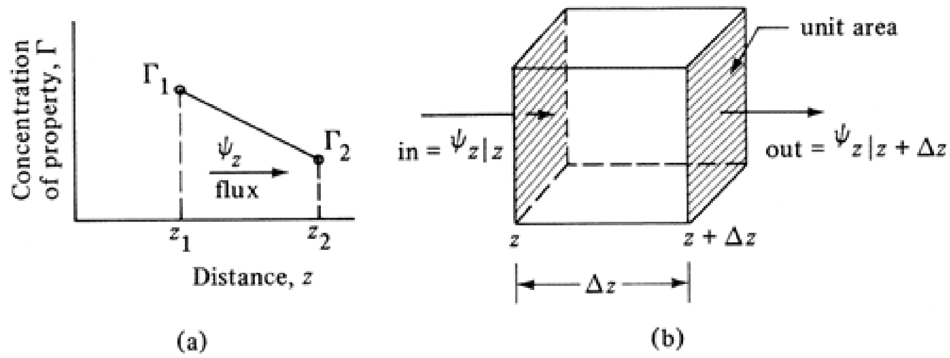

Figure 2.3-1. Molecular transport of a property: (a) plot of concentration versus distance for steady state, (b) unsteady-state general property balance.

EXAMPLE 2.3-1. Molecular Transport of a Property at Steady State

A property is being transported by diffusion through a fluid at steady state. At a given point 1 the concentration is 1.37×10^{-2} amount of property/m^3 and 0.72×10^{-2} at point 2 at a distance $z_2 = 0.40$ m. The diffusivity $\delta = 0.013$ m^2/s and the cross-sectional area is constant.

a. Calculate the flux.
b. Derive the equation for Γ as a function of distance.
c. Calculate Γ at the midpoint of the path.

Solution: For part (a), substituting into Eq. (2.3-4),

$$\psi_z = \frac{\delta(\Gamma_1 - \Gamma_2)}{z_2 - z_1} = \frac{(0.013)(1.37 \times 10^{-2} - 0.72 \times 10^{-2})}{0.40 - 0}$$

$$= 2.113 \times 10^{-4} \text{ amount of property/s} \cdot \text{m}^2$$

For part (b), integrating Eq. (2.3-2) between Γ_1 and Γ and z_1 and z and rearranging,

Equation 2.3-5.

$$\psi_z \int_{z_1}^{z_2} dz = -\delta \int_{\Gamma_1}^{\Gamma_2} d\Gamma$$

Equation 2.3-6.

$$\Gamma = \Gamma_1 + \frac{\psi_z}{\delta}(z_1 - z)$$

For part (c), using the midpoint $z = 0.20$ m and substituting into Eq. (2.3-6),

$$\Gamma = 1.37 \times 10^{-2} + \frac{2.113 \times 10^{-4}}{0.013}(0 - 0.2)$$

$$= 1.045 \times 10^{-2} \text{ amount of property/m}^3$$

General property balance for unsteady state

In calculating the rates of transport in a system using the molecular transport equation (2.3-2), it is necessary to account for the amount of this property being transported in the entire system. This is done by writing a general property balance or conservation equation for the property (momentum, thermal energy, or mass) at unsteady state. We start by writing an equation for the *z* direction only, which accounts for all the property entering by molecular transport, leaving, being generated, and accumulating in a system shown in Fig. 2.3-1b, which is an element of volume $\Delta z(1)$ m^3 fixed in space.

Equation 2.3-7.

$$\begin{pmatrix}\text{rate of}\\ \text{property in}\end{pmatrix} + \begin{pmatrix}\text{rate of generation}\\ \text{of property}\end{pmatrix} = \begin{pmatrix}\text{rate of}\\ \text{property out}\end{pmatrix} + \begin{pmatrix}\text{rate of accumulation}\\ \text{of property}\end{pmatrix}$$

The rate of input is $(\psi_{z|z}) \cdot 1$ amount of property/s and the rate of output is $(\psi_{z|z+\Delta z}) \cdot 1$, where the cross-sectional area is 1.0 m^2. The rate of generation of the property is $R(\Delta z \cdot 1)$, where R is rate of generation of property/s · m^3. The accumulation term is

Equation 2.3-8.

$$\text{rate of accumulation of property} = \frac{\partial \Gamma}{\partial t}(\Delta z \cdot 1)$$

Substituting the various terms into Eq. (2.3-7),

Equation 2.3-9.

$$(\psi_{z|z}) \cdot 1 + R(\Delta z \cdot 1) = (\psi_{z|z+\Delta z}) \cdot 1 + \frac{\partial \Gamma}{\partial t}(\Delta z \cdot 1)$$

Dividing by Δz and letting Δz go to zero,

Equation 2.3-10.

$$\frac{\partial \Gamma}{\partial t} + \frac{\partial \psi_z}{\partial z} = R$$

Substituting Eq. (2.3-2) for ψ_z into (2.3-10) and assuming that δ is constant,

Equation 2.3-11.

$$\frac{\partial \Gamma}{\partial t} - \delta \frac{\partial^2 \Gamma}{\partial z^2} = R$$

For the case where no generation is present,

Equation 2.3-12.

$$\frac{\partial \Gamma}{\partial t} = \delta \frac{\partial^2 \Gamma}{\partial z^2}$$

This final equation relates the concentration of the property Γ to position *z* and time *t*.

Equations (2.3-11) and (2.3-12) are general equations for the conservation of momentum, thermal energy, or mass and will be used in many sections of this text. The equations consider here only molecular transport occurring and not other transport mechanisms such as convection and so on, which will be considered when the specific conservation equations are derived in later sections of this text for momentum, energy, and mass.

Introduction to Molecular Transport

The kinetic theory of gases gives us a good physical interpretation of the motion of individual molecules in fluids. Because of their kinetic energy the molecules are in rapid random movement, often colliding with each other. Molecular transport or molecular diffusion of a property such as momentum, heat, or mass occurs in a fluid because of these random movements of individual molecules. Each individual molecule containing the property being transferred moves randomly in all directions, and there are fluxes in all directions. Hence, if there is a concentration gradient of the property, there will be a net flux of the property from high to low concentration. This occurs because equal numbers of molecules diffuse in each direction between the high-concentration and low-concentration regions.

Momentum transport and Newton's law

When a fluid is flowing in the x direction parallel to a solid surface, a velocity gradient exists where the velocity v_x in the x direction decreases as we approach the surface in the z direction. The fluid has x-directed momentum and its concentration is $v_x\rho$ momentum/m^3, where the momentum has units of kg · m/s. Hence, the units of $v_x\rho$ are (kg · m/s)/m^3. By random diffusion of molecules there is an exchange of molecules in the z direction, an equal number moving in each direction ($+z$ and $-z$ directions) between the faster-moving layer of molecules and the slower adjacent layer. Hence, the x-directed momentum has been transferred in the z direction from the faster- to the slower-moving layer. The equation for this transport of momentum is similar to Eq. (2.3-2) and is Newton's law of viscosity written as follows for constant density ρ:

Equation 2.3-13.

$$\tau_{zx} = -\nu \frac{d(v_x \rho)}{dz}$$

where τ_{zx} is flux of x-directed momentum in the z direction, (kg · m/s)/s · m^2; ν is μ/ρ, the momentum diffusivity in m^2/s; z is the distance of transport or diffusion in m; ρ is the density in kg/m^3; and μ is the viscosity in kg/m · s.

Heat transport and Fourier's law

Fourier's law for molecular transport of heat or heat conduction in a fluid or solid can be written as follows for constant density ρ and heat capacity c_p:

Equation 2.3-14.

$$\frac{q_z}{A} = -\alpha \frac{d(\rho c_p T)}{dz}$$

where q_z/A is the heat flux in J/s · m^2, α is the thermal diffusivity in m^2/s, and $\rho c_p T$ is the concentration of heat or thermal energy in J/m^3. When there is a temperature gradient in a fluid, equal numbers of molecules diffuse in each direction between the hot and the colder region. In this way energy is transferred in the z direction.

Mass transport and Fick's law

Fick's law for molecular transport of mass in a fluid or solid for constant total concentration in the fluid is

Equation 2.3-15.

$$J^*_{Az} = -D_{AB}\frac{dc_A}{dz}$$

where J^*_{AZ} is the flux of *A* in kg mol $A/s \cdot m^2$, D_{AB} is the molecular diffusivity of the molecule *A* in *B* in m^2/s, and c_A is the concentration of *A* in kg mol A/m^3. In a manner similar to momentum and heat transport, when there is a concentration gradient in a fluid, equal numbers of molecules diffuse in each direction between the high- and low-concentration regions and a net flux of mass occurs.

Hence, Eqs. (2.3-13), (2.3-14), and (2.3-15) for momentum, heat, and mass transfer are all similar to each other and to the general molecular transport equation (2.3-2). All equations have a flux on the left-hand side of each equation, a diffusivity in m^2/s, and the derivative of the concentration with respect to distance. All three of the molecular transport equations are mathematically identical. Thus, we state that we have an analogy or similarity among them. It should be emphasized, however, that even though there is a mathematical analogy, the actual physical mechanisms occurring may be totally different. For example, in mass transfer two components are often being transported by relative motion through one another. In heat transport in a solid, the molecules are relatively stationary and the transport is done mainly by the electrons. Transport of momentum can occur by several types of mechanisms. More-detailed considerations of each of the transport processes of momentum, energy, and mass are presented in this and succeeding chapters.

VISCOSITY OF FLUIDS

Newton's Law and Viscosity

When a fluid is flowing through a closed channel such as a pipe or between two flat plates, either of two types of flow may occur, depending on the velocity of this fluid. At low velocities the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. There are no cross currents perpendicular to the direction of flow, nor eddies or swirls of fluid. This regime or type of flow is called *laminar flow.* At higher velocities eddies form, which leads to lateral mixing. This is called *turbulent flow.* The discussion in this section is limited to laminar flow.

A fluid can be distinguished from a solid in this discussion of viscosity by its behavior when subjected to a stress (force per unit area) or applied force. An elastic solid deforms by an amount proportional to the applied stress. However, a fluid, when subjected to a similar applied stress, will continue to deform, that is, to flow at a velocity that increases with increasing stress. A fluid exhibits resistance to this stress. Viscosity is that property of a fluid which gives rise to forces that resist the relative movement of adjacent layers in the fluid. These *viscous forces* arise from forces existing between the molecules in the fluid and are similar in character to the *shear forces* in solids.

The ideas above can be clarified by a more quantitative discussion of viscosity. In Fig. 2.4-1 a fluid is contained between two infinite (very long and very wide) parallel plates. Suppose that the bottom plate is moving parallel to the top plate and at a constant velocity Δv_z m/s faster relative to the top plate because of a steady force *F* newtons being applied. This force is called the *viscous drag,* and it arises from the viscous forces in the fluid. The plates are Δy m apart. Each layer of liquid moves in the *z* direction. The layer immediately adjacent to the bottom plate is carried along at the velocity of this plate. The layer just above is at a slightly slower velocity, each layer moving at a slower velocity as we go up in the *y* direction. This velocity profile is linear, with *y* direction as shown in Fig. 2.4-1. An analogy to a fluid is a deck of playing cards, where, if the bottom card is moved, all the other cards above will slide to some extent.

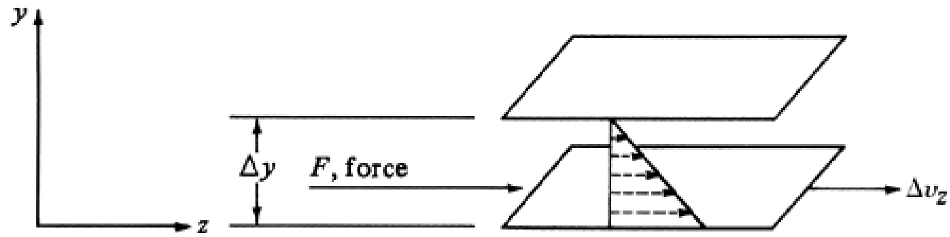

Figure 2.4-1. Fluid shear between two parallel plates.

It has been found experimentally for many fluids that the force F in newtons is directly proportional to the velocity Δv_z in m/s and to the area A in m^2 of the plate used, and inversely proportional to the distance Δy in m. Or, as given by Newton's law of viscosity when the flow is laminar,

Equation 2.4-1.

$$\frac{F}{A} = -\mu \frac{\Delta v_z}{\Delta y}$$

where μ is a proportionality constant called the *viscosity* of the fluid, in Pa · s or kg/m · s. If we let Δy approach zero, then, using the definition of the derivative,

Equation 2.4-2.

$$\tau_{yz} = -\mu \frac{dv_z}{dy} \qquad \text{(SI units)}$$

where $\tau_{yz} = F/A$ and is the shear stress or force per unit area in newtons/m^2 (N/m^2). In the cgs system, F is in dynes, μ in g/cm · s, v_z in cm/s, and y in cm. We can also write Eq. (2.2-2) as

Equation 2.4-3.

$$\tau_{yz} g_c = -\mu \frac{dv_z}{dy} \qquad \text{(English units)}$$

where τ_{yz} is in units of lb$_f$/ft^2.

The units of viscosity in the cgs system are g/cm · s, called *poise* or centipoise (cp). In the SI system, viscosity is given in Pa · s (N · s/m^2 or kg/m · s):

$$1 \text{ cp} = 1 \times 10^{-3} \text{ kg/m} \cdot \text{s} = 1 \times 10^{-3} \text{ Pa} \cdot \text{s} = 1 \times 10^{-3} \text{ N} \cdot \text{s/m}^2 \qquad \text{(SI)}$$

$$1 \text{ cp} = 0.01 \text{ poise} = 0.01 \text{ g/cm} \cdot \text{s}$$

$$1 \text{ cp} = 6.7197 \times 10^{-4} \text{ lb}_m/\text{ft} \cdot \text{s}$$

Other conversion factors for viscosity are given in Appendix A.1. Sometimes the viscosity is given as μ/ρ, kinematic viscosity, in m^2/s or cm^2/s, where ρ is the density of the fluid.

EXAMPLE 2.4-1. Calculation of Shear Stress in a Liquid

Referring to Fig. 2.4-1, the distance between plates is Δy = 0.5 cm, Δv_z = 10 cm/s, and the fluid is ethyl alcohol at 273 K having a viscosity of 1.77 cp (0.0177 g/cm · s).

a. Calculate the shear stress τ_{yz} and the velocity gradient or shear rate dv_z/dy using cgs units.
b. Repeat, using lb force, s, and ft units (English units).
c. Repeat, using SI units.

Solution: We can substitute directly into Eq. (2.4-1) or we can integrate Eq. (2.4-2). Using the latter method, rearranging Eq. (2.4-2), calling the bottom plate point 1, and integrating,

Equation 2.4-4.

$$\tau_{yz} \int_{y_1=0}^{y_2=0.5} dy = -\mu \int_{v_1=10}^{v_2=0} dv_z$$

Equation 2.4-5.

$$\tau_{yz} = \mu \frac{v_1 - v_2}{y_2 - y_1}$$

Substituting the known values,

Equation 2.4-6.

$$\tau_{yz} = \mu \frac{v_1 - v_2}{y_2 - y_1} = \left(0.0177 \frac{\text{g}}{\text{cm} \cdot \text{s}}\right) \frac{(10 - 0)\ \text{cm/s}}{(0.5 - 0)\ \text{cm}}$$
$$= 0.354 \frac{\text{g} \cdot \text{cm/s}^2}{\text{cm}^2} = 0.354 \frac{\text{dyn}}{\text{cm}^2}$$

To calculate the shear rate dv_z/dy, since the velocity change is linear with y,

Equation 2.4-7.

$$\text{shear rate} = \frac{dv_z}{dy} = \frac{\Delta v_z}{\Delta y} = \frac{(10 - 0)\ \text{cm/s}}{(0.5 - 0)\ \text{cm}} = 20.0\ \text{s}^{-1}$$

For part (b), using lb force units and the viscosity conversion factor from Appendix A.1,

$$\mu = 1.77\ \text{cp}(6.7197 \times 10^{-4}\ \text{lb}_m/\text{ft} \cdot \text{s})/\text{cp}$$
$$= 1.77(6.7197 \times 10^{-4})\ \text{lb}_m/\text{ft} \cdot \text{s}$$

Integrating Eq. (2.4-3),

Equation 2.4-8.

$$\tau_{yz} = \frac{\mu\ \text{lb}_m/\text{ft} \cdot \text{s}}{g_c \dfrac{\text{lb}_m \cdot \text{ft}}{\text{lb}_f \cdot \text{s}^2}} \frac{(v_1 - v_2)\text{ft/s}}{(y_2 - y_1)\ \text{ft}}$$

Substituting known values into Eq. (2.4-8) and converting Δv_z to ft/s and Δy to ft, $\tau_{yz} = 7.39 \times 10^{-4}\ \text{lb}_f/\text{ft}^2$. Also, $dv_z/dy = 20\ \text{s}^{-1}$.

For part (c), $\Delta y = 0.5/100 = 0.005$ m, $\Delta v_z = 10/100 = 0.1$ m/s, and $\mu = 1.77 \times 10^{-3}$ kg/m · s = 1.77×10^{-3} Pa · s. Substituting into Eq. (2.4-5),

$$\tau_{yz} = (1.77 \times 10^{-3})(0.10)/0.005 = 0.0354\ \text{N/m}^2$$

The shear rate will be the same at $20.0\ \text{s}^{-1}$.

Momentum Transfer in a Fluid

The shear stress τ_{yz} in Eqs. (2.4-1)–(2.4-3) can also be interpreted as a *flux of z-directed momentum in the y direction,* which is the rate of flow of momentum per unit area. The units of momentum are mass times velocity in kg · m/s. The shear stress can be written

Equation 2.4-9.

$$\tau_{yz} = \frac{\text{kg} \cdot \text{m/s}}{\text{m}^2 \cdot \text{s}} = \frac{\text{momentum}}{\text{m}^2 \cdot \text{s}}$$

This gives an amount of momentum transferred per second per unit area.

This can be shown by considering the interaction between two adjacent layers of a fluid in Fig. 2.4-1 which have different velocities, and hence different momentum, in the *z* direction. The random motions of the molecules in the faster-moving layer send some of the molecules into the slower-moving layer, where they collide with the slower-moving molecules and tend to speed them up or increase their momentum in the *z* direction. Also, in the same fashion, molecules in the slower layer tend to retard those in the faster layer. This exchange of molecules between layers produces a transfer or flux of *z*-directed momentum from high-velocity to low-velocity layers. The negative sign in Eq. (2.4-2) indicates that momentum is transferred down the gradient from high- to low-velocity regions. This is similar to the transfer of heat from high- to low-temperature regions.

Viscosities of Newtonian Fluids

Fluids that follow Newton's law of viscosity, Eqs. (2.4-1)–(2.4-3), are called *Newtonian fluids.* For a Newtonian fluid, there is a linear relation between the shear stress τ_{yz} and the velocity gradient dv_z/dy (rate of shear). This means that the viscosity μ is a constant and is independent of the rate of shear. For non-Newtonian fluids, the relation between τ_{yz} and dv_z/dy is not linear; that is, the viscosity μ does not remain constant but is a function of shear rate. Certain liquids do not obey this simple law of Newton's. These are primarily pastes, slurries, high polymers, and emulsions. The science of the flow and deformation of fluids is often called *rheology.* A discussion of non-Newtonian fluids will not be given here but will be included in Section 3.5.

The viscosity of gases, which are Newtonian fluids, increases with temperature and is approximately independent of pressure up to a pressure of about 1000 kPa. At higher pressures, the viscosity of gases increases with increase in pressure. For example, the viscosity of N_2 gas at 298 K approximately doubles in going from 100 kPa to about 5×10^4 kPa (R1). In liquids, the viscosity decreases with increasing temperature. Since liquids are essentially incompressible, the viscosity is not affected by pressure.

In Table 2.4-1 some experimental viscosity data are given for some typical pure fluids at 101.32 kPa. The viscosities for gases are the lowest and do not differ markedly from gas to gas, being about 5×10^{-6} to 3×10^{-5} Pa · s. The viscosities for liquids are much greater. The value for water at 293 K is about 1×10^{-3} and for glycerol 1.069 Pa · s. Hence, there are great differences between viscosities of liquids. More complete tables of viscosities are given for water in Appendix A.2, for inorganic and organic liquids and gases in Appendix A.3, and for biological and food liquids in Appendix A.4. Extensive data are available in other references (P1, R1, W1, L1). Methods of estimating viscosities of gases and liquids when experimental data are not available are summarized elsewhere (R1). These estimation methods for gases at pressures below 100 kPa are reasonably accurate, with an error within about ±5%, but the methods for liquids are often quite inaccurate.

Table 2.4-1. Viscosities of Some Gases and Liquids at 101.32 kPa Pressure

Gases				Liquids			
Substance	*Temp., K*	*Viscosity (Pa · s) 10^3 or (kg/m · s) 10^3*	*Ref.*	*Substance*	*Temp., K*	*Viscosity (Pa · s) 10^3 or (kg/m · s) 10^3*	*Ref.*
Air	293	0.01813	N1	Water	293	1.0019	S1
CO_2	273	0.01370	R1		373	0.2821	S1
	373	0.01828	R1	Benzene	278	0.826	R1
CH_4	293	0.01089	R1				
				Glycerol	293	1069	L1
SO_2	373	0.01630	R1	Hg	293	1.55	R2
				Olive oil	303	84	E1

TYPES OF FLUID FLOW AND REYNOLDS NUMBER

Introduction and Types of Fluid Flow

The principles of the statics of fluids, treated in Section 2.2, are almost an exact science. On the other hand, the principles of the motions of fluids are quite complex. The basic relations describing the motions of a fluid are the equations for the overall balances of mass, energy, and momentum, which will be covered in the following sections.

These overall or macroscopic balances will be applied to a finite enclosure or control volume fixed in space. We use the term “overall” because we wish to describe these balances from outside the enclosure. The changes inside the enclosure are determined in terms of the properties of the streams entering and leaving and the exchanges of energy between the enclosure and its surroundings.

When making overall balances on mass, energy, and momentum we are not interested in the details of what occurs inside the enclosure. For example, in an overall balance, average inlet and outlet velocities are considered. However, in a differential balance, the velocity distribution inside an enclosure can be obtained by the use of Newton's law of viscosity.

In this section we first discuss the two types of fluid flow that can occur: laminar flow and turbulent flow. Also, the Reynolds number used to characterize the regimes of flow is considered. Then in Sections 2.6, 2.7, and 2.8, the overall mass balance, energy balance, and momentum balance are covered together with a number of applications. Finally, a discussion is given in Section 2.9 on the methods of making a shell balance on an element to obtain the velocity distribution in the element and the pressure drop.

Laminar and Turbulent Flow

The type of flow occurring in a fluid in a channel is important in fluid dynamics problems. When fluids move through a closed channel of any cross section, either of two distinct types of flow can be observed, according to the conditions present. These two types of flow can commonly be seen in a flowing open stream or river. When the velocity of flow is slow, the flow patterns are smooth. However, when the velocity is quite high, an unstable pattern is observed, in which eddies or small packets of fluid particles are present, moving in all directions and at all angles to the normal line of flow.

The first type of flow, at low velocities, where the layers of fluid seem to slide by one another without eddies or swirls being present, is called *laminar flow,* and Newton's law of viscosity holds, as discussed in Section 2.4A. The second type of flow, at higher velocities, where eddies are present giving the fluid a fluctuating nature, is called *turbulent flow.*

The existence of laminar and turbulent flow is most easily visualized by the experiments of Reynolds. His experiments are shown in Fig. 2.5-1. Water was allowed to flow at steady state through a transparent pipe with the flow rate controlled by a valve at the end of the pipe. A fine, steady stream of dyed water was introduced from a fine jet as shown and its flow pattern observed. At low rates of water flow, the dye pattern was regular and formed a single line or stream similar to a thread, as shown in Fig. 2.5-1a. There was no lateral mixing of the fluid, and it flowed in streamlines down the tube. By putting in additional jets at other points in the pipe cross section, it was shown that there was no mixing in any parts of the tube and the fluid flowed in straight parallel lines. This type of flow is called laminar or *viscous flow*.

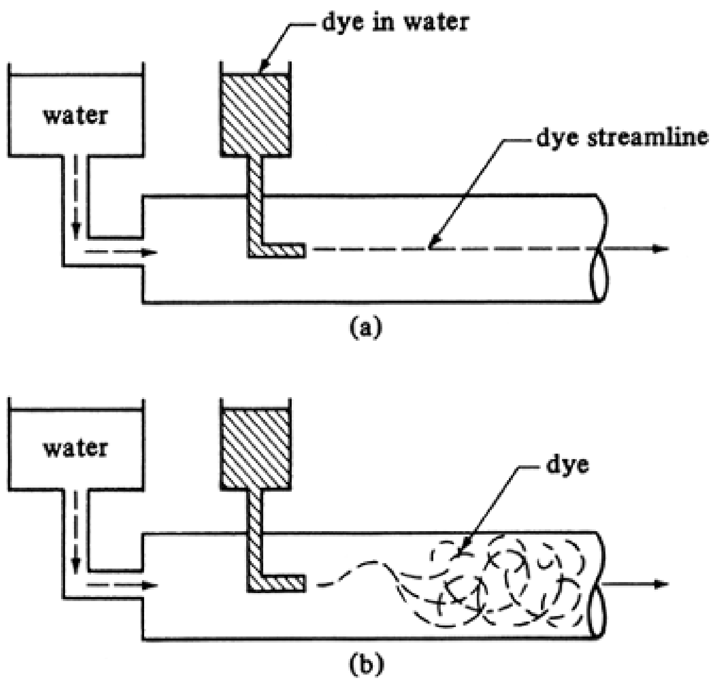

Figure 2.5-1. Reynolds' experiment for different types of flow: (a) laminar flow; (b) turbulent flow.

As the velocity was increased, it was found that at a definite velocity the thread of dye became dispersed and the pattern was very erratic, as shown in Fig. 2.5-1b. This type of flow is known as turbulent flow. The velocity at which the flow changes is known as the *critical velocity.*

Reynolds Number

Studies have shown that the transition from laminar to turbulent flow in tubes is not only a function of velocity but also of density and viscosity of the fluid and the tube diameter. These variables are combined into the Reynolds number, which is dimensionless:

Equation 2.5-1.

$$N_{Re} = \frac{Dv\rho}{\mu}$$

where N_{Re} is the Reynolds number, D the diameter in m, ρ the fluid density in kg/m^3, μ the fluid viscosity in Pa · s, and μ the average velocity of the fluid in m/s (where average velocity is defined as the volumetric rate of flow divided by the cross-sectional area of the pipe). Units in the cgs system are D in cm, ρ in g/cm^3, μ in g/cm · s, and v in cm/s. In the English system D is in ft, ρ in lb$_m$/ft^3, μ in lb$_m$/ft · s, and v in ft/s.

The instability of the flow that leads to disturbed or turbulent flow is determined by the ratio of the kinetic or inertial forces to the viscous forces in the fluid stream. The inertial forces are proportional to ρv^2 and the viscous forces to $\mu v/D$, and the ratio $\rho v^2/(\mu v/D)$ is the Reynolds number $Dv\rho/\mu$. Further explanation and derivation of dimensionless numbers are given in Section 3.11.

For a straight circular pipe, when the value of the Reynolds number is less than 2100, the flow is always laminar. When the value is over 4000, the flow will be turbulent, except in very special cases. In between—called the *transition region*—the flow can be viscous or turbulent, depending upon the apparatus details, which cannot be predicted.

EXAMPLE 2.5-1. Reynolds Number in a Pipe

Water at 303 K is flowing at the rate of 10 gal/min in a pipe having an inside diameter (ID) of 2.067 in. Calculate the Reynolds number using both English units and SI units.

Solution: From Appendix A.1, 7.481 gal = 1 ft^3. The flow rate is calculated as

$$\text{flow rate} = \left(10.0\,\frac{\text{gal}}{\text{min}}\right)\left(\frac{1\ \text{ft}^3}{7.481\ \text{gal}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= 0.0223\ \text{ft}^3/\text{s}$$

$$\text{pipe diameter, } D = \frac{2.067}{12} = 0.172\ \text{ft}$$

$$\text{cross-sectional area of pipe} = \frac{\pi D^2}{4} = \frac{\pi(0.172)^2}{4} = 0.0233\ \text{ft}^2$$

$$\text{velocity in pipe, } v = \left(0.0223\,\frac{\text{ft}^3}{s}\right)\left(\frac{1}{0.0233\ \text{ft}^2}\right) = 0.957\ \text{ft/s}$$

From Appendix A.2, for water at 303 K (30°C),

$$\text{density, } \rho = 0.996(62.43)\ \text{lb}_m/\text{ft}^3$$

$$\text{viscosity, } \mu = (0.8007\ \text{cp})\left(6.7197 \times 10^{-4}\,\frac{\text{lb}_m/\text{ft}\cdot\text{s}}{\text{cp}}\right)$$

$$= 5.38 \times 10^{-4}\ \text{lb}_m/\text{ft}\cdot\text{s}$$

Substituting into Eq. (2.5-1).

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{(0.172\ \text{ft})(0.957\ \text{ft/s})(0.996 \times 62.43\ \text{lb}_m/\text{ft}^3)}{5.38 \times 10^{-4}\ \text{lb}_m/\text{ft}\cdot\text{s}}$$

$$= 1.905 \times 10^4$$

Hence, the flow is turbulent. Using SI units,

$$\rho = (0.996)(1000\ \text{kg/m}^3) = 996\ \text{kg/m}^3$$

$$D = (2.067\ \text{in.})(1\ \text{ft}/12\ \text{in.})(1\ \text{m}/3.2808\ \text{ft}) = 0.0525\ \text{m}$$

$$v = \left(0.957\ \frac{\text{ft}}{\text{s}}\right)(1\ \text{m}/3.2808\ \text{ft}) = 0.2917\ \text{m/s}$$

$$\mu = (0.8007\ \text{cp})\left(1 \times 10^{-3}\frac{\text{kg/m}\cdot\text{s}}{\text{cp}}\right) = 8.007 \times 10^{-4}\frac{\text{kg}}{\text{m}\cdot\text{s}}$$

$$= 8.007 \times 10^{-4}\ \text{Pa}\cdot\text{s}$$

$$N_{\text{Re}} = \frac{Dv\rho}{\mu} = \frac{(0.0525\ \text{m})(0.2917\ \text{m/s})(996\ \text{kg/m}^3)}{8.007 \times 10^{-4}\ \text{kg/m}\cdot\text{s}} = 1.905 \times 10^4$$

OVERALL MASS BALANCE AND CONTINUITY EQUATION

Introduction and Simple Mass Balances

In fluid dynamics fluids are in motion. Generally, they are moved from place to place by means of mechanical devices such as pumps or blowers, by gravity head, or by pressure, and flow through systems of piping and/or process equipment. The first step in the solution of flow problems is generally to apply the principles of the conservation of mass to the whole system or to any part of the system. First, we will consider an elementary balance on a simple geometry, and later we shall derive the general mass-balance equation.

Simple mass or material balances were introduced in Section 1.5, where

Equation 1.5-1.

$$\text{input} = \text{output} + \text{accumulation}$$

Since, in fluid flow, we are usually working with rates of flow and usually at steady state, the rate of accumulation is zero and we obtain

Equation 2.6-1.

$$\text{rate of input} = \text{rate of output (steady state)}$$

In Fig. 2.6-1 a simple flow system is shown, where fluid enters section 1 with an average velocity v_1 m/s and density ρ_1 kg/m^3. The cross-sectional area is A_1 m^2. The fluid leaves section 2 with average velocity v_2. The mass balance, Eq. (2.6-1), becomes

Equation 2.6-2.

$$m = \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

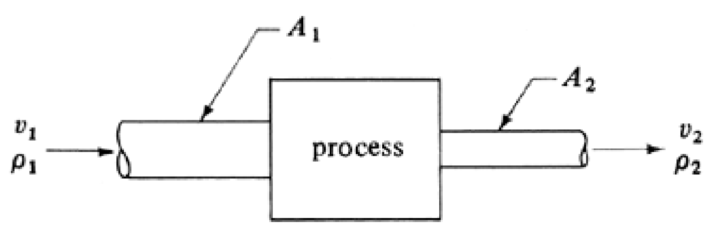

Figure 2.6-1. Mass balance on flow system.

where m = kg/s. Often, $v\rho$ is expressed as $G = v\rho$, where G is mass velocity or mass flux in kg/s · m^2. In English units, v is in ft/s, ρ in lb$_m$/ft^3, A in ft^2, m in lb$_m$/s, and G in lb$_m$/s · ft^2.

EXAMPLE 2.6-1. Flow of Crude Oil and Mass Balance

A petroleum crude oil having a density of 892 kg/m^3 is flowing through the piping arrangement shown in Fig. 2.6-2 at a total rate of 1.388 × 10^{-3} m^3/s entering pipe 1.

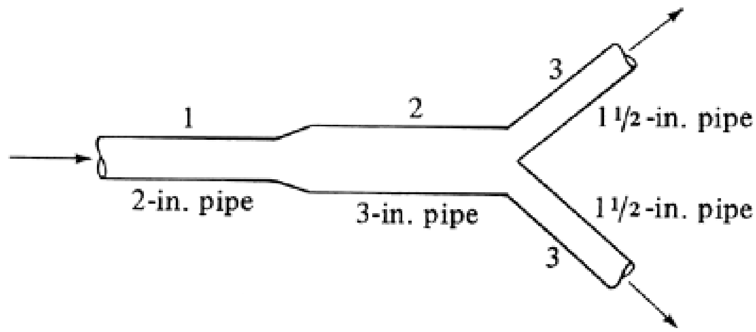

Figure 2.6-2. Piping arrangement for Example 2.6-1.

The flow divides equally in each of pipes 3. The steel pipes are schedule 40 pipe (see Appendix A.5 for actual dimensions). Calculate the following, using SI units:

a. The total mass flow rate *m* in pipe 1 and pipes 3.

b. The average velocity *v* in 1 and 3.

c. The mass velocity *G* in 1.

Solution: From Appendix A.5, the dimensions of the pipes are as follows: 2-in. pipe: D_1 (ID) = 2.067 in.; cross-sectional area

$$A_1 = 0.02330 \text{ ft}^2 = 0.02330(0.0929) = 2.165 \times 10^{-3} \text{ m}^2$$

$1\frac{1}{2}$-in. pipe: D_3 (ID) = 1.610 in.; cross-sectional area

$$A_3 = 0.01414 \text{ ft}^2 = 0.01414(0.0929) = 1.313 \times 10^{-3} \text{ m}^2$$

The total mass flow rate is the same through pipes 1 and 2 and is

$$m_1 = (1.388 \times 10^{-3} \text{ m}^3\text{/s})(892 \text{ kg/m}^3) = 1.238 \text{ kg/s}$$

Since the flow divides equally in each of pipes 3,

$$m_3 = \frac{m_1}{2} = \frac{1.238}{2} = 0.619 \text{ kg/s}$$

For part (b), using Eq. (2.6-2) and solving for *v*,

$$v_1 = \frac{m_1}{\rho_1 A_1} = \frac{1.238 \text{ kg/s}}{(892 \text{ kg/m}^3)(2.165 \times 10^{-3} \text{ m}^2)} = 0.641 \text{ m/s}$$

$$v_3 = \frac{m_3}{\rho_3 A_3} = \frac{0.619}{(892)(1.313 \times 10^{-3})} = 0.528 \text{ m/s}$$

For part (c),

$$G_1 = v_1\rho_1 = \frac{m_1}{A_1} = \frac{1.238}{2.165 \times 10^{-3}} = 572 \frac{\text{kg}}{\text{s} \cdot \text{m}^2}$$

Control Volume for Balances

The laws for the conservation of mass, energy, and momentum are all stated in terms of a system; these laws give the interaction of a system with its surroundings. A *system* is defined as a collection of fluid of fixed identity. However, in flow of fluids, individual particles are not easily identifiable. As a result, attention is focused on a given space through which the fluid flows rather than on a given mass of fluid. The method used, which is more convenient, is to select a control volume, which is a region fixed in space through which the fluid flows.

In Fig. 2.6-3 the case of a fluid flowing through a conduit is shown. The control surface shown as a dashed line is the surface surrounding the control volume. In most problems part of the control surface will coincide with some boundary, such as the wall of the conduit. The remaining part of the control surface is a hypothetical surface through which the fluid can flow, shown as point 1 and point 2 in Fig. 2.6-3. The control-volume representation is analogous to the open system of thermodynamics.

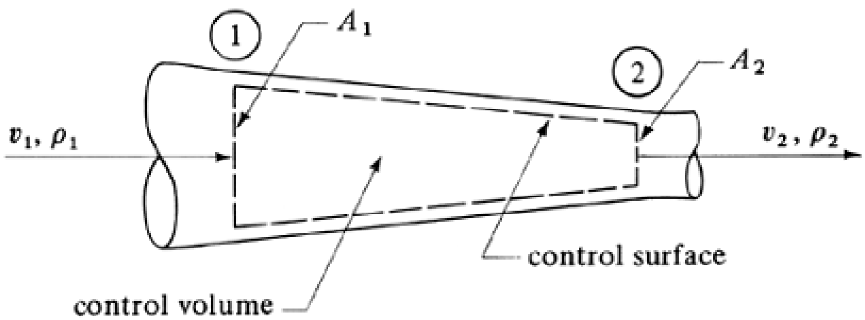

Figure 2.6-3. Control volume for flow through a conduit.

Overall Mass-Balance Equation

In deriving the general equation for the overall balance of the property mass, the law of conservation of mass may be stated as follows for a control volume where no mass is being generated.

Equation 2.6-3.

$$\begin{pmatrix}\text{rate of mass output}\\ \text{from control volume}\end{pmatrix} - \begin{pmatrix}\text{rate of mass input}\\ \text{from control volume}\end{pmatrix} + \begin{pmatrix}\text{rate of mass accumulation}\\ \text{in control volume}\end{pmatrix} = 0 \quad (\text{rate of mass generation})$$

We now consider the general control volume fixed in space and located in a fluid flow field, as shown in Fig. 2.6-4. For a small element of area dA m^2 on the control surface, the rate of mass efflux from this element = $(\rho v)(dA \cos \alpha)$, where $(dA \cos \alpha)$ is the area dA projected in a direction normal to the velocity vector v, α is the angle between the velocity vector v and the outward-directed unit normal vector n to dA, and ρ is the density in kg/m^3. The quantity ρv has units of kg/s · m^2 and is called *a flux* or *mass velocity G.*

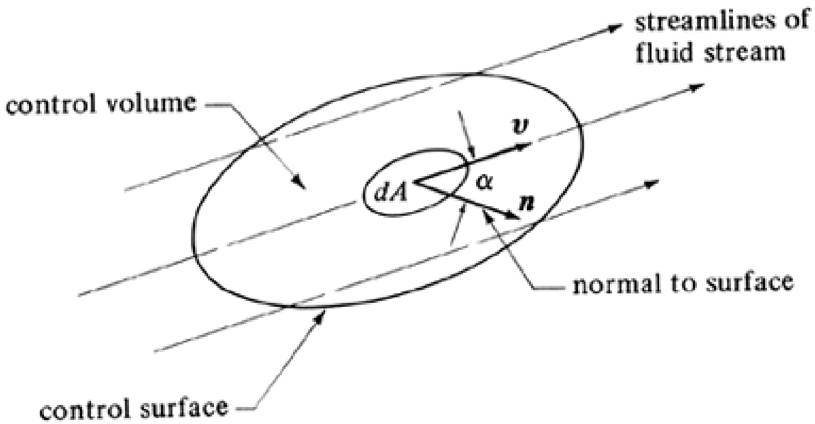

Figure 2.6-4. Flow through a differential area dA on a control surface.

From vector algebra we recognize that $(\rho v)(dA \cos \alpha)$ is the scalar or dot product $\rho(v \cdot n)\, dA$. If we now integrate this quantity over the entire control surface A, we have the net outflow of mass across the control surface, or the net mass efflux in kg/s from the entire control volume V:

Equation 2.6-4.

$$\begin{pmatrix}\text{net mass efflux}\\ \text{from control volume}\end{pmatrix} = \iint_A v\rho \cos\alpha \, dA = \iint_A \rho(\boldsymbol{v}\cdot\boldsymbol{n})\, dA$$

We should note that if mass is entering the control volume, that is, flowing inward across the control surface, the next efflux of mass in Eq. (2.6-4) is negative, since $\alpha > 90°$ and $\cos \alpha$ is negative. Hence, there is a net influx of mass. If $\alpha < 90°$, there is a net efflux of mass.

The rate of accumulation of mass within the control volume V can be expressed as follows:

Equation 2.6-5.

$$\begin{pmatrix}\text{rate of mass accumulation}\\ \text{in control volume}\end{pmatrix} = \frac{\partial}{\partial t}\iiint_V \rho\, dV = \frac{dM}{dt}$$

where M is the mass of fluid in the volume in kg. Substituting Eqs. (2.6-4) and (2.6-5) into (2.6-3), we obtain the general form of the overall mass balance:

Equation 2.6-6.

$$\iint_A \rho(\boldsymbol{v}\cdot\boldsymbol{n})\, dA + \frac{\partial}{\partial t}\iiint_V \rho\, dV = 0$$

The use of Eq. (2.6-6) can be shown for a common situation of steady-state one-dimensional flow, where all the flow inward is normal to A_1 and outward normal to A_2, as shown in Fig. 2.6-3. When the velocity v_2 leaving (Fig. 2.6-3) is normal to A_2, the angle α_2 between the normal to the control surface and the direction of the velocity is 0°, and $\cos \alpha_2 = 1.0$. Where v_1 is directed inward, $\alpha_1 > \pi/2$, and for the case in Fig. 2.6-3, α_1 is 180° ($\cos \alpha_1 = -1.0$). Since α_2 is 0° and α_1 is 180°, using Eq. (2.6-4),

Equation 2.6-7.

$$\iint_A v\rho \cos\alpha \, dA = \iint_{A_2} v\rho \cos\alpha_2 \, dA + \iint_{A_1} v\rho \cos\alpha_1 \, dA$$

$$= v_2\rho_2 A_2 - v_1\rho_1 A_1$$

For steady state, $dM/dt = 0$ in Eq. (2.6-5), and Eq. (2.6-6) becomes

Equation 2.6-2.

$$m = \rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

which is Eq. (2.6-2), derived earlier.

In Fig. 2.6-3 and Eqs. (2.6-3)–(2.6-7) we were not concerned with the composition of any of the streams. These equations can easily be extended to represent an overall mass balance for component i in a multicomponent system. For the case shown in Fig. 2.6-3, we combine Eqs. (2.6-5), (2.6-6), and (2.6-7), add a generation term, and obtain

Equation 2.6-8.

$$m_{i2} - m_{i1} + \frac{dM_i}{dt} = R_i$$

where m_{i2} is the mass flow rate of component i leaving the control volume and R_i is the rate of generation of component i in the control volume in kg per unit time. (Diffusion fluxes are neglected here or are assumed negligible.) In some cases, of course, $R_i = 0$ for no generation. Often it is more convenient to use Eq. (2.6-8) written in molar units.

EXAMPLE 2.6-2. Overall Mass Balance in Stirred Tank

Initially, a tank holds 500 kg of salt solution containing 10% salt. At point (1) in the control volume in Fig. 2.6-5, a stream enters at a constant flow rate of 10 kg/h containing 20% salt. A stream leaves at point (2) at a constant rate of 5 kg/h. The tank is well stirred. Derive an equation relating the weight fraction w_A of the salt in the tank at any time t in hours.

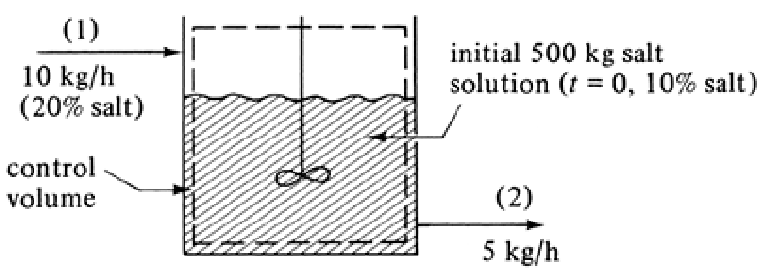

Figure 2.6-5. Control volume for flow in a stirred tank for Example 2.6-2.

Solution: First we make a total mass balance using Eq. (2.6-7) for the net total mass efflux from the control volume:

Equation 2.6-9.

$$\iint_A v\rho \cos\alpha \, dA = m_2 - m_1 = 5 - 10 = -5 \text{ kg solution/h}$$

From Eq. (2.6-5), where M is total kg of solution in control volume at time t,

Equation 2.6-5.

$$\frac{\partial}{\partial t}\iiint_V \rho\, dV = \frac{dM}{dt}$$

Substituting Eqs. (2.6-5) and (2.6-9) into (2.6-6), and then integrating,

Equation 2.6-10.

$$-5 + \frac{dM}{dt} = 0$$

Equation 2.6-11.

$$\int_{M=500}^{M} dM = 5\int_{t=0}^{t} dt$$

$$M = 5t + 500$$

Equation (2.6-11) relates the total mass *M* in the tank at any time to *t*.

Next, making a component *A* salt balance, let w_A = weight fraction of salt in tank at time *t* and also the concentration in the stream m_2 leaving at time *t*. Again using Eq. (2.6-7) but for a salt balance,

Equation 2.6-12.

$$\iint_A v\rho \cos\alpha\, dA = (5)w_A - 10(0.20) = 5w_A - 2 \text{ kg salt/h}$$

Using Eq. (2.6-5) for a salt balance,

Equation 2.6-13.

$$\frac{\partial}{\partial t}\iiint_V \rho\, dV = \frac{d}{dt}(Mw_A) = \frac{M\,dw_A}{dt} + w_A\frac{dM}{dt} \text{ kg salt/h}$$

Substituting Eqs. (2.6-12) and (2.6-13) into (2.6-6),

Equation 2.6-14.

$$5w_A - 2 + M\frac{dw_A}{dt} + w_A\frac{dM}{dt} = 0$$

Substituting the value for *M* from Eq. (2.6-11) into (2.6-14), separating variables, integrating, and solving for w_A,

Equation 2.6-15.

$$5w_A - 2 + (500 + 5t)\frac{dw_A}{dt} + w_A\frac{d(500 + 5t)}{dt} = 0$$

$$5w_A - 2 + (500 + 5t)\frac{dw_A}{dt} + 5w_A = 0$$

$$\int_{w_A=0.10}^{w_A}\frac{dw_A}{2 - 10w_A} = \int_{t=0}^{t}\frac{dt}{500 + 5t}$$

$$-\frac{1}{10}\ln\left(\frac{2 - 10w_A}{1}\right) = \frac{1}{5}\ln\left(\frac{500 + 5t}{500}\right)$$

Equation 2.6-16.

$$w_A = -0.1\left(\frac{100}{100 + t}\right)^2 + 0.20$$

Note that Eq. (2.6-8) for component *i* could have been used for the salt balance with $R_i = 0$ (no generation).

Average Velocity to Use in Overall Mass Balance

In solving the case in Eq. (2.6-7), we assumed a constant velocity v_1 at section 1 and constant v_2 at section 2. If the velocity is not constant but varies across the surface area, an average or bulk velocity is defined by

Equation 2.6-17.

$$v_{av} = \frac{1}{A}\iint_A v\,dA$$

for a surface over which *v* is normal to *A* and the density *ρ* is assumed constant.

EXAMPLE 2.6-3. Variation of Velocity Across Control Surface and Average Velocity

For the case of incompressible flow (*ρ* is constant) through a circular pipe of radius *R*, the velocity profile is parabolic for laminar flow as follows:

Equation 2.6-18.

$$v = v_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where v_{max} is the maximum velocity at the center where $r = 0$ and *v* is the velocity at a radial distance *r* from the center. Derive an expression for the average or bulk velocity v_{av} to use in the overall mass-balance equation.

Solution: The average velocity is represented by Eq. (2.6-17). In Cartesian coordinates *dA* is *dx dy*. However, using polar coordinates, which are more appropriate for a pipe, $dA = r dr d\theta$, where *θ* is the angle in polar coordinates. Substituting Eq. (2.6-18), $dA = r dr d\theta$, and $A = \pi R^2$ into Eq. (2.6-17) and integrating,

Equation 2.6-19.

$$\begin{aligned} v_{av} &= \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R v_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right] r\,dr\,d\theta \\ &= \frac{v_{max}}{\pi R^4}\int_0^{2\pi}\int_0^R (R^2 - r^2) r\,dr\,d\theta \\ &= \frac{v_{\max}}{\pi R^4}(2\pi - 0)\left(\frac{R^4}{2} - \frac{R^4}{4}\right) \end{aligned}$$

Equation 2.6-20.

$$v_{av} = \frac{v_{\max}}{2}$$

In this discussion overall or macroscopic mass balances were made because we wish to describe these balances from outside the enclosure. In this section on overall mass balances, some of the equations presented may have seemed quite obvious. However, the purpose was to develop the methods which should be helpful in the next sections. Overall balances will also be made on energy and momentum in the next sections. These overall balances do not tell us the details of what happens inside. However, in Section 2.9 a shell momentum balance will be made in order to obtain these details, which will give us the velocity distribution and pressure drop. To further study these details of the processes occurring inside the enclosure, differential balances rather than shell balances can be written; these are discussed later in Sections 3.6–3.9 on differential equations of continuity and momentum transfer, Sections 5.6 and 5.7 on differential equations of energy change and boundary-layer flow, and Section 7.5B on differential equations of continuity for a binary mixture.

OVERALL ENERGY BALANCE

Introduction

The second property to be considered in the overall balances on a control volume is energy. We shall apply the principle of the conservation of energy to a control volume fixed in space in much the same manner as the principle of conservation of mass was used to obtain the overall mass balance. The energy-conservation equation will then be combined with the first law of thermodynamics to obtain the final overall energy-balance equation.

We can write the first law of thermodynamics as

Equation 2.7-1.

$$\Delta E = Q - W$$

where E is the total energy per unit mass of fluid, Q is the heat *absorbed* per unit mass of fluid, and W is the work of all kinds done per unit mass of fluid *upon* the surroundings. In the calculations, each term in the equation must be expressed in the same type of units, such as J/kg (SI), btu/lb_m, or ft · lb_f/lb_m (English).

Since mass carries with it associated energy due to its position, motion, or physical state, we will find that each of these types of energy will appear in the energy balance. In addition, we can also transport energy across the boundary of the system without transferring mass.

Derivation of Overall Energy-Balance Equation

The entity balance for a conserved quantity such as energy is similar to Eq. (2.6-3) and is as follows for a control volume:

Equation 2.7-2.

$$\text{rate of entity output} - \text{rate of entity input} + \text{rate of entity accumulation} = 0$$

The energy E present within a system can be classified in three ways:

1. *Potential energy* zg of a unit mass of fluid is the energy present because of the position of the mass in a gravitational field g, where z is the relative height in meters from a reference plane. The units for zg in the SI system are m · m/s^2. Multiplying and dividing by kg mass, the units can be expressed as (kg · m/s^2) · (m/kg), or J/kg. In English units the potential energy is zg/g_c in ft · lb_f/lb_m.

2. *Kinetic energy* $v^2/2$ of a unit mass of fluid is the energy present because of translational or rotational motion of the mass, where v is the velocity in m/s relative to the boundary of the system at a given point. Again, in the SI system the units of $v^2/2$ are J/kg. In the English system the kinetic energy is $v^2/2g_c$ in ft · lb_f/lb_m.
3. *Internal energy* U of a unit mass of a fluid is all of the other energy present, such as rotational and vibrational energy in chemical bonds. Again the units are in J/kg or ft · lb_f/lb_m.

The total energy of the fluid per unit mass is then

Equation 2.7-3.

$$E = U + \frac{v^2}{2} + zg \qquad \text{(SI)}$$

$$E = U + \frac{v^2}{2g_c} + \frac{zg}{g_c} \qquad \text{(English)}$$

The rate of accumulation of energy within the control volume V in Fig. 2.6-4 is

Equation 2.7-4.

$$\begin{pmatrix}\text{rate of energy accumulation}\\ \text{in control volume}\end{pmatrix} = \frac{\partial}{\partial t}\iiint_V \left(U + \frac{v^2}{2} + zg\right)\rho\, dV$$

Next we consider the rate of energy input and output associated with mass in the control volume. The mass added or removed from the system carries internal, kinetic, and potential energy. In addition, energy is transferred when mass flows into and out of the control volume. Net work is done by the fluid as it flows into and out of the control volume. This pressure–volume work per unit mass fluid is pV. The contribution of shear work is usually neglected. The pV term and U term are combined using the definition of enthalpy, H:

Equation 2.7-5.

$$H = U + pV$$

Hence, the total energy carried with a unit mass is $(H + v^2/2 + zg)$.

For a small area dA on the control surface in Fig. 2.6-4, the rate of energy efflux is $(H + v^2/2 + zg)(\rho v)(dA \cos\alpha)$, where $(dA \cos\alpha)$ is the area dA projected in a direction normal to the velocity vector v and α is the angle between the velocity vector v and the outward-directed unit normal vector n. We now integrate this quantity over the entire control surface to obtain

Equation 2.7-6.

$$\begin{pmatrix}\text{net energy efflux}\\ \text{from control volume}\end{pmatrix} = \iint_A \left(H + \frac{v^2}{2} + zg\right)(\rho v)\cos\alpha\, dA$$

Now we have accounted for all energy associated with mass in the system and moving across the boundary in the entity balance, Eq. (2.7-2). Next we take into account heat and work energy which transfers across the boundary and is not associated with mass. The term q is the heat transferred per unit time across the boundary to the fluid because of a temperature gradient. Heat absorbed by the system is positive by convention.

The work $\dot{W}$, which is energy per unit time, can be divided into $\dot{W}_s$, purely mechanical shaft work identified with a rotating shaft crossing the control surface, and the pressure–volume work, which has been included in the enthalpy term *H* in Eq. (2.7-6). By convention, work done by the fluid upon the surroundings, that is, work out of the system, is positive.

To obtain the overall energy balance, we substitute Eqs. (2.7-4) and (2.7-6) into the entity balance Eq. (2.7-2) and equate the resulting equation to $q - \dot{W}_s$:

Equation 2.7-7.

$$\iint_A \left(H + \frac{v^2}{2} + zg\right)(\rho v)\cos\alpha\, dA + \frac{\partial}{\partial t}\iiint_V \left(U + \frac{v^2}{2} + zg\right)\rho\, dV = q - \dot{W}_S$$

Overall Energy Balance for Steady-State Flow System

A common special case of the overall or macroscopic energy balance is that of a steady-state system with one-dimensional flow across the boundaries, a single inlet, a single outlet, and negligible variation of height *z*, density *ρ*, and enthalpy *H* across either inlet or outlet area. This is shown in Fig. 2.7-1. Setting the accumulation term in Eq. (2.7-7) equal to zero and integrating,

Equation 2.7-8.

$$H_2 m_2 - H_1 m_1 + \frac{m_2 (v_2^3)_{av}}{2 v_{2\,av}} - \frac{m_1 (v_1^3)_{av}}{2 v_{1\,av}} + g m_2 z_2 - g m_1 z_1 = q - \dot{W}_S$$

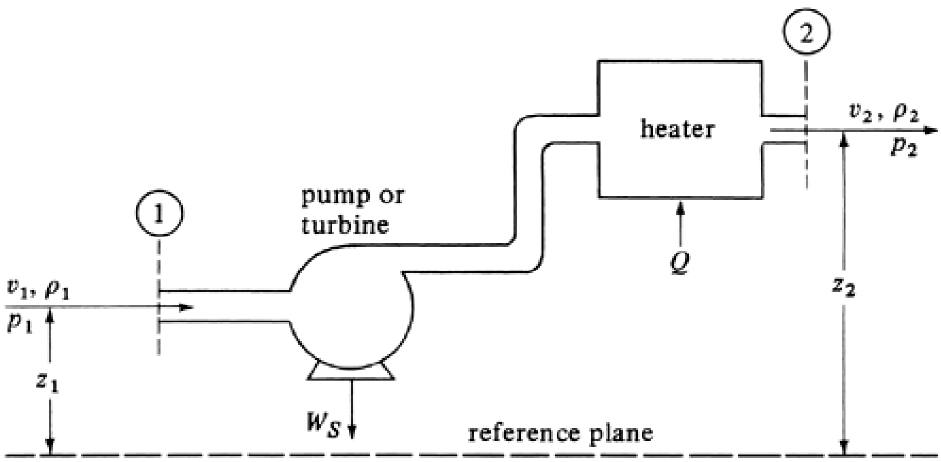

Figure 2.7-1. Steady-state flow system for a fluid.

For steady state, $m_1 = \rho_1 v_{1\,av} A_1 = m_2 = m$. Dividing through by *m* so that the equation is on a unit mass basis,

Equation 2.7-9.

$$H_2 - H_1 + \frac{1}{2}\left[\frac{(v_2^3)_{av}}{v_{2\,av}} - \frac{(v_1^3)_{av}}{v_{1\,av}}\right] + g(z_2 - z_1) = Q - W_S \qquad \text{(SI)}$$

The term $(v^3)_{av}/(2v_{av})$ can be replaced by $v_{av}^2/2\alpha$, where α is the kinetic-energy velocity correction factor and is equal to $v_{av}^3/(v^3)_{av}$. The term α has been evaluated for various flows in pipes and is $\frac{1}{2}$ for laminar flow and close to 1.0 for turbulent flow. (See Section 2.7D.) Hence, Eq. (2.7-9) becomes

Equation 2.7-10.

$$H_2 - H_1 + \frac{1}{2\alpha}(v_{2av}^2 - v_{1av}^2) + g(z_2 - z_1) = Q - W_S \qquad \text{(SI)}$$

$$H_2 - H_1 + \frac{1}{2\alpha g_c}(v_{2av}^2 - v_{1av}^2) + \frac{g}{g_c}(z_2 - z_1) = Q - W_S \qquad \text{(English)}$$

Some useful conversion factors from Appendix A.1 are as follows:

$$1\ \text{btu} = 778.17\ \text{ft}\cdot\text{lb}_f = 1055.06\ \text{J} = 1.05506\ \text{kJ}$$

$$1\ \text{hp} = 550\ \text{ft}\cdot\text{lb}_f/\text{s} = 0.7457\ \text{kW}$$

$$1\ \text{ft}\cdot\text{lb}_f/\text{lb}_m = 2.9890\ \text{J/kg}$$

$$1\ \text{J} = 1\ \text{N}\cdot\text{m} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$$

Kinetic-Energy Velocity Correction Factor α

Introduction

In obtaining Eq (2.7-8) it was necessary to integrate the kinetic-energy term,

Equation 2.7-11.

$$\text{kinetic energy} = \iint_A \left(\frac{v^2}{2}\right)(\rho v)\cos\alpha\, dA$$

which appeared in Eq. (2.7-7). To do this we first take ρ as a constant and cos α = 1.0. Then multiplying the numerator and denominator by $v_{av}A$, where v_{av} is the bulk or average velocity, and noting that $m = \rho v_{av}A$, Eq. (2.7-11) becomes

Equation 2.7-12.

$$\frac{\rho}{2}\iint_A (v^3)dA = \frac{\rho v_{av}A}{2v_{av}A}\iint_A (v^3)\,dA = \frac{m}{2v_{av}}\,\frac{1}{A}\iint_A (v^3)\,dA$$

Dividing through by m so that Eq. (2.7-12) is on a unit mass basis,

Equation 2.7-13.

$$\left(\frac{1}{2v_{av}}\right)\frac{1}{A}\iint_A (v^3)\,dA = \frac{(v^3)_{av}}{2v_{av}} = \frac{v_{av}^2}{2\alpha}$$

where α is defined as

Equation 2.7-14.

$$\alpha = \frac{v_{av}^3}{(v^3)_{av}}$$

and $(v^3)_{av}$ is defined as follows:

Equation 2.7-15.

$$(v^3)_{av} = \frac{1}{A} \iint_A (v^3)\, dA$$

The local velocity v varies across the cross-sectional area of a pipe. To evaluate $(v^3)_{av}$ and, hence, the value of α, we must have an equation relating v as a function of position in the cross-sectional area.

Laminar flow

In order to determine the value of α for laminar flow, we first combine Eqs. (2.6-18) and (2.6-20) for laminar flow to obtain v as a function of position r:

Equation 2.7-16.

$$v = 2v_{av}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Substituting Eq. (2.7-16) into (2.7-15) and noting that $A = \pi R^2$ and $dA = r\, dr\, d\theta$ (see Example 2.6-3), Eq. (2.7-15) becomes

Equation 2.7-17.

$$(v^3)_{av} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \left[2v_{av}\left(1 - \frac{r^2}{R^2}\right)\right]^3 r\, dr\, d\theta$$

$$= \frac{(2\pi)2^3 v_{av}^3}{\pi R^2}\int_0^R \frac{(R^2 - r^2)^3}{R^6} r\, dr = \frac{16 v_{av}^3}{R^8}\int_0^R (R^2 - r^2)^3 r\, dr$$

Integrating Eq. (2.7-17) and rearranging,

Equation 2.7-18.

$$(v^3)_{av} = \frac{16 v_{av}^3}{R^8}\int_0^R (R^6 - 3r^2R^4 + 3r^4R^2 - r^6) r\, dr$$

$$= \frac{16 v_{av}^3}{R^8}\left(\frac{R^8}{2} - \frac{3}{4}R^8 + \frac{1}{2}R^8 - \frac{1}{8}R^8\right)$$

$$= 2v_{av}^3$$

Substituting Eq. (2.7-18) into (2.7-14),

Equation 2.7-19.

$$\alpha = \frac{v_{av}^3}{(v^3)_{av}} = \frac{v_{av}^3}{2v_{av}^3} = 0.50$$

Hence, for laminar flow the value of α to use in the kinetic-energy term of Eq. (2.7-10) is 0.50.

Turbulent flow

For turbulent flow a relationship is needed between v and position. This can be approximated by the following expression:

Equation 2.7-20.

$$v = v_{max}\left(\frac{R - r}{R}\right)^{1/7}$$

where r is the radial distance from the center. Eq. (2.7-20) is substituted into Eq. (2.7-15) and the resultant integrated to obtain the value of $(v^3)_{av}$. Next, Eq. (2.7-20) is substituted into Eq. (2.6-17) and this equation integrated to obtain v_{av} and $(v_{av})^3$. Combining the results for $(v^3)_{av}$ and $(v_{av})^3$ into Eq. (2.7-14), the value of a α is 0.945. (See Problem 2.7-1 for solution.) The value of α for turbulent flow varies from about 0.90 to 0.99. In most cases (except for precise work) the value of α is taken to be 1.0.

Applications of Overall Energy-Balance Equation

The total energy balance, Eq. (2.7-10), in the form given is not often used when appreciable enthalpy changes occur or appreciable heat is added (or subtracted), since the kinetic- and potential-energy terms are usually small and can be neglected. As a result, when appreciable heat is added or subtracted or large enthalpy changes occur, the methods for doing heat balances described in Section 1.7 are generally used. Examples will be given to illustrate this and other cases.

EXAMPLE 2.7-1. Energy Balance on Steam Boiler

Water enters a boiler at 18.33°C and 137.9 kPa through a pipe at an average velocity of 1.52 m/s. Exit steam at a height of 15.2 m above the liquid inlet leaves at 137.9 kPa, 148.9°C, and 9.14 m/s in the outlet line. At steady state, how much heat must be added per kg mass of steam? The flow in the two pipes is turbulent.

Solution: The process flow diagram is shown in Fig. 2.7-2. Rearranging Eq. (2.7-10) and setting α = 1 for turbulent flow and $W_S = 0$ (no external work),

Equation 2.7-21.

$$Q = (z_2 - z_1)g + \frac{v_2^2 - v_1^2}{2} + (H_2 - H_1)$$

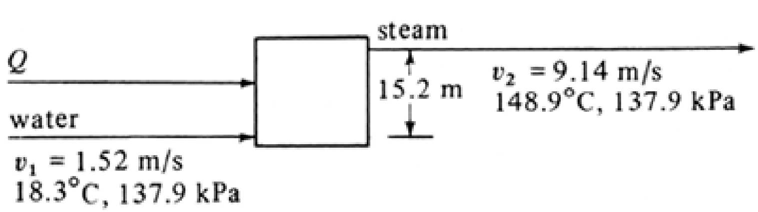

Figure 2.7-2. Process flow diagram for Example 2.7-1.

To solve for the kinetic-energy terms,

$$\frac{v_1^2}{2} = \frac{(1.52)^2}{2} = 1.115 \text{ J/kg}$$

$$\frac{v_2^2}{2} = \frac{(9.14)^2}{2} = 41.77 \text{ J/kg}$$

Taking the datum height z_1 at point 1, z_2 = 15.2 m. Then,

$$z_2 g = (15.2)(9.80665) = 149.1 \text{ J/kg}$$

From Appendix A.2, steam tables in SI units, H_1 at 18.33°C = 76.97 kJ/kg, H_2 of superheated steam at 148.9°C = 2771.4 kJ/kg, and

$$H_2 - H_1 = 2771.4 - 76.97 = 2694.4 \text{ kJ/kg} = 2.694 \times 10^6 \text{ J/kg}$$

Substituting these values into Eq. (2.7-21),

$$Q = (149.1 - 0) + (41.77 - 1.115) + 2.694 \times 10^6$$

$$Q = 189.75 + 2.694 \times 10^6 = 2.6942 \times 10^6 \text{ J/kg}$$

Hence, the kinetic-energy and potential-energy terms totaling 189.75 J/kg are negligible compared to the enthalpy change of 2.694×10^6 J/kg. This 189.75 J/kg would raise the temperature of liquid water about 0.0453°C, a negligible amount.

EXAMPLE 2.7-2. Energy Balance on a Flow System with a Pump

Water at 85.0°C is being stored in a large, insulated tank at atmospheric pressure, as shown in Fig. 2.7-3. It is being pumped at steady state from this tank at point 1 by a pump at the rate of 0.567 m^3/min. The motor driving the pump supplies energy at the rate of 7.45 kW. The water passes through a heat exchanger, where it gives up 1408 kW of heat. The cooled water is then delivered to a second large open tank at point 2, which is 20 m above the first tank. Calculate the final temperature of the water delivered to the second tank. Neglect any kinetic-energy changes, since the initial and final velocities in the tanks are essentially zero.

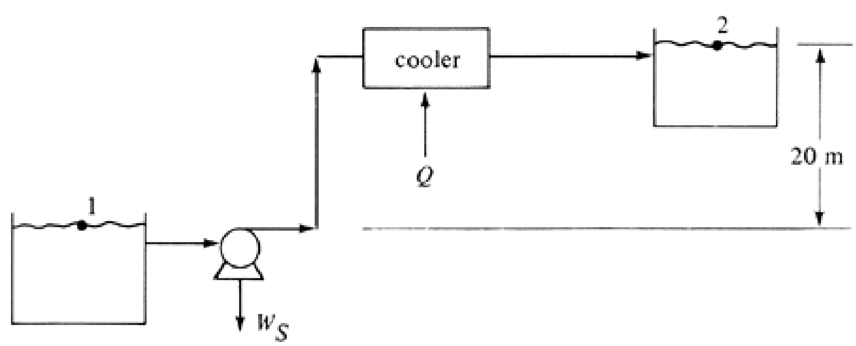

Figure 2.7-3. Process flow diagram for energy balance for Example 2.7-2.

Solution: From Appendix A.2, steam tables, H_1 (85°C) = 355.90×10^3 J/kg and ρ_1 = 1/0.0010325 = 968.5 kg/m^3. Then, for steady state,

$$m_1 = m_2 = (0.567)(968.5)(\tfrac{1}{60}) = 9.152 \text{ kg/s}$$

Also, z_1 = 0 and z_2 = 20 m. The work done by the fluid is W_S, but in this case work is done on the fluid and W_S is negative:

$$W_S = -(7.45 \times 10^3 \text{ J/s})(1/9.152 \text{ kg/s}) = -0.8140 \times 10^3 \text{ J/kg}$$

The heat added to the fluid is also negative since it gives up heat and is

$$Q = -(1408 \times 10^3 \text{ J/s})(1/9.152 \text{ kg/s}) = -153.8 \times 10^3 \text{ J/kg}$$

Setting $(v_1^2 - v_2^2)/2 = 0$ and substituting into Eq. (2.7-10),

$$H_2 - 355.90 \times 10^3 + 0 + 9.80665(20 - 0) = (-153.8 \times 10^3) - (-0.814 \times 10^3)$$

Solving, H_2 = 202.71 × 10^3 J/kg. From the steam tables this corresponds to t_2 = 48.41°C. Note that in this example, W_S and $g(z_2 - z_1)$ are very small compared to Q.

EXAMPLE 2.7-3. Energy Balance in Flow Calorimeter

A flow calorimeter is being used to measure the enthalpy of steam. The calorimeter, which is a horizontal insulated pipe, consists of an electric heater immersed in a fluid flowing at steady state. Liquid water at 0°C at a rate of 0.3964 kg/min enters the calorimeter at point 1. The liquid is vaporized completely by the heater, where 19.63 kW is added, and steam leaves point 2 at 250°C and 150 kPa absolute. Calculate the exit enthalpy H_2 of the steam if the liquid enthalpy at 0°C is set arbitrarily as 0. The kinetic-energy changes are small and can be neglected. (It will be assumed that pressure has a negligible effect on the enthalpy of the liquid.)

***Solution*:** For this case, W_S = 0 since there is no shaft work between points 1 and 2. Also, $(v_2^2/2\alpha - v_1^2/2\alpha) = 0$ and $g(z_2 - z_1) = 0$. For steady state, $m_1 = m_2 = 0.3964/60 = 6.607 \times 10^{-3}$ kg/s. Since heat is added to the system,

$$Q = +\frac{19.63 \text{ kJ/s}}{6.607 \times 10^{-3} \text{ kg/s}} = 2971 \text{ kJ/kg}$$

The value of H_1 = 0. Equation (2.7-10) becomes

$$H_2 - H_1 + 0 + 0 = Q - 0$$

The final equation for the calorimeter is

Equation 2.7-22.

$$H_2 = Q + H_1$$

Substituting Q = 2971 kJ/kg and H_1 = 0 into Eq. (2.7-22), H_2 = 2971 kJ/kg at 250°C and 150 kPa, which is close to the value from the steam table of 2972.7 kJ/kg.

Overall Mechanical-Energy Balance

A more useful type of energy balance for flowing fluids, especially liquids, is a modification of the total energy balance to deal with mechanical energy. Engineers are often concerned with this special type of energy, called *mechanical energy,* which includes the work term, kinetic energy, potential energy, and the flow work part of the enthalpy term. Mechanical energy is a form of energy that is either work or a form that can be directly converted into work. The other terms in the energy-balance equation (2.7-10), heat terms and internal energy, do not permit simple conversion into work because of the second law of thermodynamics and the efficiency of conversion, which depends on the temperatures. Mechanical-energy terms have no such limitation and can be converted almost completely into work. Energy converted to heat or internal energy is lost work or a loss in mechanical energy caused by frictional resistance to flow.

It is convenient to write an energy balance in terms of this loss, ΣF, which is the sum of all frictional losses per unit mass. For the case of steady-state flow, when a unit mass of fluid passes from inlet to outlet, the batch work done by the fluid, W', is expressed as

Equation 2.7-23.

$$W' = \int_{V_1}^{V_2} p\,dV - \sum F \qquad \left(\sum F > 0\right)$$

This work W' differs from the W of Eq. (2.7-1), which also includes kinetic- and potential-energy effects. Writing the first law of thermodynamics for this case, where ΔE becomes ΔU,

Equation 2.7-24.

$$\Delta U = Q - W'$$

The equation defining enthalpy, Eq. (2.7-5), can be written as

Equation 2.7-25.

$$\Delta H = \Delta U + \Delta pV = \Delta U + \int_{V_1}^{V_2} p\,dV + \int_{p_1}^{p_2} V\,dp$$

Substituting Eq. (2.7-23) into (2.7-24) and then combining the resultant with Eq. (2.7-25), we obtain

Equation 2.7-26.

$$\Delta H = Q + \sum F + \int_{p_1}^{p_2} V\,dp$$

Finally, we substitute Eq. (2.7-26) into (2.7-10) and $1/\rho$ for V, to obtain the overall mechanical-energy-balance equation:

Equation 2.7-27.

$$\frac{1}{2\alpha}\left[v_{2\text{av}}^2 - v_{1\text{av}}^2\right] + g(z_2 - z_1) + \int_{p_1}^{p_2} \frac{dp}{\rho} + \sum F + W_S = 0$$

For English units the kinetic- and potential-energy terms of Eq. (2.7-27) are divided by g_c.

The value of the integral in Eq. (2.7-27) depends on the equation of state of the fluid and the path of the process. If the fluid is an incompressible liquid, the integral becomes $(p_2 - p_1)/\rho$ and Eq. (2.7-27) becomes

Equation 2.7-28.

$$\frac{1}{2\alpha}\left(v_{2\text{av}}^2 - v_{1\text{av}}^2\right) + g(z_2 - z_1) + \frac{p_2 - p_1}{\rho} + \sum F + W_S = 0$$

EXAMPLE 2.7-4. Mechanical-Energy Balance on Pumping System

Water with a density of 998 kg/m^3 is flowing at a steady mass flow rate through a uniform-diameter pipe. The entrance pressure of the fluid is 68.9 kN/m^2 abs in the pipe, which connects to a pump that actually supplies 155.4 J/kg of fluid flowing in the pipe. The exit pipe from the pump is the same diameter as the inlet pipe. The exit section of the pipe is 3.05 m higher than the entrance, and the exit pressure is 137.8 kN/m^2 abs. The Reynolds number in the pipe is above 4000 in the system. Calculate the frictional loss ΣF in the pipe system.

Solution: First a flow diagram of the system is drawn (Fig. 2.7-4), with 155.4 J/kg mechanical energy added to the fluid. Hence, $W_S = -155.4$, since the work done by the fluid is positive.

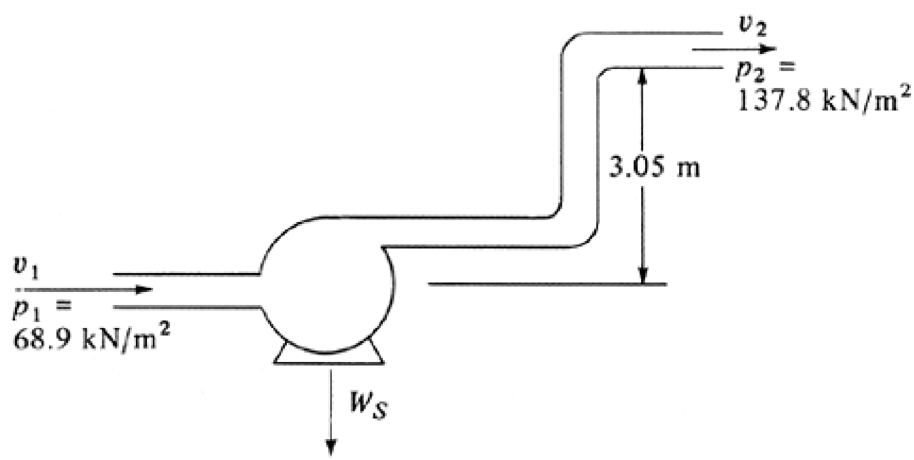

Figure 2.7-4. Process flow diagram for Example 2.7-4.

Setting the datum height $z_1 = 0$, $z_2 = 3.05$ m. Since the pipe is of constant diameter, $v_1 = v_2$. Also, for turbulent flow α = 1.0 and

$$\frac{1}{2(1)}(v_2^2 - v_1^2) = 0$$

$$z_2 g = (3.05 \text{ m})(9.806 \text{ m/s}^2) = 29.9 \text{ J/kg}$$

Since the liquid can be considered incompressible, Eq. (2.7-28) is used:

$$\frac{p_1}{\rho} = \frac{68.9 \times 1000}{998} = 69.0 \text{ J/kg}$$

$$\frac{p_2}{\rho} = \frac{137.8 \times 1000}{998} = 138.0 \text{ J/kg}$$

Using Eq. (2.7-28) and solving for ΣF, the frictional losses,

Equation 2.7-29.

$$\sum F = -W_S + \frac{1}{2\alpha}(v_1^2 - v_2^2) + g(z_1 - z_2) + \frac{p_1 - p_2}{\rho}$$

Substituting the known values, and solving for the frictional losses,

$$\sum F = -(-155.4) + 0 - 29.9 + 69.0 - 138.0$$

$$= 56.5 \text{ J/kg} \left(18.9 \frac{\text{ft}\cdot\text{lb}_f}{\text{lb}_m}\right)$$

EXAMPLE 2.7-5. Pump Horsepower in Flow System

A pump draws 69.1 gal/min of a liquid solution having a density of 114.8 lb_m/ft^3 from an open storage feed tank of large cross-sectional area through a 3.068-in.-ID suction line. The pump discharges its flow through a 2.067-in.-ID line to an open overhead tank. The end of the discharge line is 50 ft above the level of the liquid in the feed tank. The friction losses in the piping system are Σ *F* = 10.0 ft-lb force/lb mass. What pressure must the pump develop and what is the horsepower of the pump if its efficiency is 65% (η = 0.65)? The flow is turbulent.

***Solution*:** First, a flow diagram of the system is drawn (Fig. 2.7-5). Equation (2.7-28) will be used. The term W_S in Eq. (2.7-28) becomes

Equation 2.7-30.

$$W_S = -\eta W_p$$

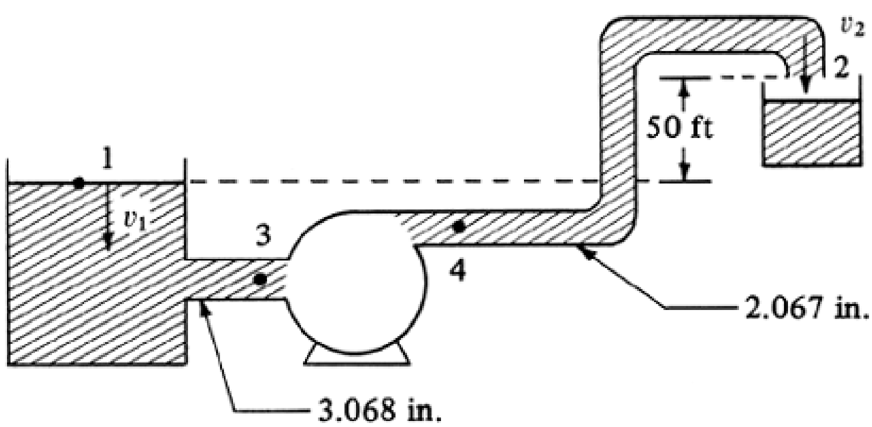

Figure 2.7-5. Process flow diagram for Example 2.7-5.

where $-W_S$ = mechanical energy actually delivered to the fluid by the pump or net mechanical work, η = fractional efficiency, and W_p is the energy or shaft work delivered to the pump.

From Appendix A.5, the cross-sectional area of the 3.068-in. pipe is 0.05134 ft^2 and of the 2.067-in. pipe, 0.0233 ft^2. The flow rate is

$$\text{flow rate} = \left(69.1 \frac{\text{gal}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{1 \text{ ft}^3}{7.481 \text{ gal}}\right) = 0.1539 \text{ ft}^3/\text{s}$$

$$v_2 = \left(0.1539 \frac{\text{ft}^3}{\text{s}}\right)\left(\frac{1}{0.0233 \text{ ft}^2}\right) = 6.61 \text{ ft/s}$$

v_1 = 0, since the tank is very large. Then $v_1^2/2g_c = 0$ The pressure p_1 = 1 atm and p_2 = 1 atm. Also, α = 1.0 since the flow is turbulent. Hence,

$$\frac{p_1}{\rho} - \frac{p_2}{\rho} = 0$$

$$\frac{v_2^2}{2g_c} = \frac{(6.61)^2}{2(32.174)} = 0.678 \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

Using the datum of $z_1 = 0$, we have

$$z_2 \frac{g}{g_c} = (50.0) \frac{32.174}{32.174} = 50.0 \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

Using Eq. (2.4-28), solving for W_S, and substituting the known values,

$$W_S = z_1 \frac{g}{g_c} - z_2 \frac{g}{g_c} + \frac{v_1^2}{2g_c} - \frac{v_2^2}{2g_c} + \frac{p_1 - p_2}{\rho} - \sum F$$

$$= 0 - 50.0 + 0 - 0.678 + 0 - 10 = -60.678 \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

Using Eq. (2.7-30) and solving for W_p,

$$W_p = -\frac{W_S}{\eta} = \frac{60.678}{0.65} \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}} = 93.3 \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

$$\text{mass flow rate} = \left(0.1539 \frac{\text{ft}^3}{\text{s}}\right)\left(114.8 \frac{\text{lb}_\text{m}}{\text{ft}^3}\right) = 17.65 \frac{\text{lb}_\text{m}}{\text{s}}$$

$$\text{pump horsepower} = \left(17.65 \frac{\text{lb}_\text{m}}{\text{s}}\right)\left(93.3 \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}\right)\left(\frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb}_\text{f}/\text{s}}\right)$$

$$= 3.00 \text{ hp}$$

To calculate the pressure the pump must develop, Eq. (2.7-28) must be written over the pump itself between points 3 and 4 as shown on the diagram:

$$v_3 = \left(0.1539 \frac{\text{ft}^3}{\text{s}}\right)\left(\frac{1}{0.05134 \text{ ft}^2}\right) = 3.00 \text{ ft/s}$$

$$v_4 = v_2 = 6.61 \text{ ft/s}$$

Since the difference in level between z_3 and z_4 of the pump itself is negligible, it will be neglected. Rewriting Eq. (2.7-28) between points 3 and 4 and substituting known values ($\Sigma F = 0$, since this is for the piping system),

Equation 2.7-31.

$$\frac{p_4 - p_3}{\rho} = z_3\frac{g}{g_c} - z_4\frac{g}{g_c} + \frac{v_3^2}{2g_c} - \frac{v_4^2}{2g_c} - W_S - \sum F$$

$$= 0 - 0 + \frac{(3.00)^2}{2(32.174)} - \frac{(6.61)^2}{2(32.174)} + 60.678 - 0$$

$$= 0 - 0 + 0.140 - 0.678 + 60.678 = 60.14\,\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}_m}$$

$$p_4 - p_3 = \left(60.14\,\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}_m}\right)\left(114.8\,\frac{\text{lb}_m}{\text{ft}^3}\right)\left(\frac{1}{144\ \text{in.}^2/\text{ft}^2}\right)$$

$$= 48.0\ \text{lb force/in.}^2\ \text{(psia pressure developed by pump) (331 kPa)}$$

Bernoulli Equation for Mechanical-Energy Balance

In the special case where no mechanical energy is added ($W_S = 0$) and for no friction ($\Sigma F = 0$), then Eq. (2.7-28) becomes the Bernoulli equation, Eq. (2.7-32), for turbulent flow, which is of sufficient importance to deserve further discussion:

Equation 2.7-32.

$$z_1 g + \frac{v_1^2}{2} + \frac{p_1}{\rho} = z_2 g + \frac{v_2^2}{2} + \frac{p_2}{\rho}$$

This equation covers many situations of practical importance and is often used in conjunction with the mass-balance equation (2.6-2) for steady state:

Equation 2.6-2.

$$m = \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Several examples of its use will be given.

EXAMPLE 2.7-6. Rate of Flow from Pressure Measurements

A liquid with a constant density ρ kg/m^3 is flowing at an unknown velocity v_1 m/s through a horizontal pipe of cross-sectional area A_1 m^2 at a pressure p_1 N/m^2, and then it passes to a section of the pipe in which the area is reduced gradually to A_2 m^2 and the pressure is p_2. Assuming no friction losses, calculate the velocities v_1 and v_2 if the pressure difference ($p_1 - p_2$) is measured.

***Solution*:** In Fig. 2.7-6, the flow diagram is shown with pressure taps to measure p_1 and p_2. From the mass-balance continuity equation (2.6-2), for constant ρ where $\rho_1 = \rho_2 = \rho$,

Equation 2.7-33.

$$v_2 = \frac{v_1 A_1}{A_2}$$

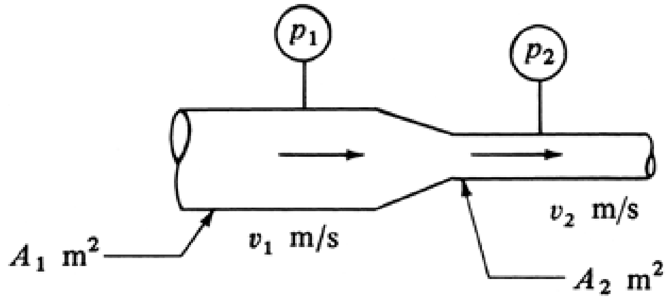

Figure 2.7-6. Process flow diagram for Example 2.7-6.

For the items in the Bernoulli equation (2.7-32), for a horizontal pipe,

$$z_1 = z_2 = 0$$

Then Eq. (2.7-32) becomes, after substituting Eq. (2.7-33) for v_2,

Equation 2.7-34.

$$0 + \frac{v_1^2}{2} + \frac{p_1}{\rho} = 0 + \frac{v_1^2 A_1^2/A_2^2}{2} + \frac{p_2}{\rho}$$

Rearranging,

Equation 2.7-35.

$$p_1 - p_2 = \frac{\rho v_1^2[(A_1/A_2)^2 - 1]}{2}$$

Equation 2.7-36.

$$v_1 = \sqrt{\frac{p_1 - p_2}{\rho}\frac{2}{[(A_1/A_2)^2 - 1]}} \qquad \text{(SI)}$$

$$v_1 = \sqrt{\frac{p_1 - p_2}{\rho}\frac{2g_c}{[(A_1/A_2)^2 - 1]}} \qquad \text{(English)}$$

Performing the same derivation but in terms of v_2,

Equation 2.7-37.

$$v_2 = \sqrt{\frac{p_1 - p_2}{\rho}\frac{2}{[1 - (A_2/A_1)^2]}}$$

EXAMPLE 2.7-7. Rate of Flow from a Nozzle in a Tank

A nozzle of cross-sectional area A_2 is discharging to the atmosphere and is located in the side of a large tank, in which the open surface of the liquid in the tank is H m above the center line of the nozzle. Calculate the velocity v_2 in the nozzle and the volumetric rate of discharge if no friction losses are assumed.

Solution: The process flow is shown in Fig. 2.7-7, with point 1 taken in the liquid at the entrance to the nozzle and point 2 at the exit of the nozzle.

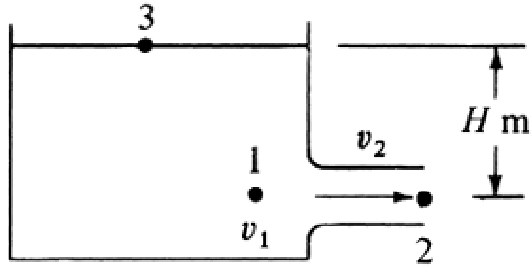

Figure 2.7-7. Nozzle flow diagram for Example 2.7-7.

Since A_1 is very large compared to A_2, $v_1 \cong 0$. The pressure p_1 is greater than 1 atm (101.3 kN/m^2) by the head of fluid of H m. The pressure p_2, which is at the nozzle exit, is at 1 atm. Using point 2 as a datum, $z_2 = 0$ and $z_1 = 0$ m. Rearranging Eq. (2.7-32),

Equation 2.7-38.

$$z_1 g + \frac{v_1^2}{2} + \frac{p_1 - p_2}{\rho} = z_2 g + \frac{v_2^2}{2}$$

Substituting the known values,

Equation 2.7-39.

$$0 + 0 + \frac{p_1 - p_2}{\rho} = 0 + \frac{v_2^2}{2}$$

Solving for v_2,

Equation 2.7-40.

$$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho}} \quad \text{m/s}$$

Since $p_1 - p_3 = H\rho g$ and $p_3 = p_2$ (both at 1 atm),

Equation 2.7-41.

$$H = \frac{p_1 - p_2}{\rho g} \quad \text{m}$$

where H is the head of liquid with density ρ. Then Eq. (2.4-40) becomes

Equation 2.7-42.

$$v_2 = \sqrt{2gH}$$

The volumetric flow rate is

Equation 2.7-43.

$$\text{flow rate} = v_2 A_2 \quad \text{m}^3/\text{s}$$

To illustrate the fact that different points can be used in the balance, points 3 and 2 will be used. Writing Eq. (2.7-32),

Equation 2.7-44.

$$z_2 g + \frac{v_2^2}{2} + \frac{p_2 - p_3}{\rho} = z_3 g + \frac{v_3^2}{2}$$

Equation 2.7-45.

Since $p_2 = p_3 = 1$ atm, $v_3 = 0$, and $z_2 = 0$,

$$v_2 = \sqrt{2gz_3} = \sqrt{2gH}$$

OVERALL MOMENTUM BALANCE

Derivation of General Equation

A momentum balance can be written for the control volume shown in Fig. 2.6-3 which is somewhat similar to the overall mass-balance equation. Momentum, in contrast to mass and energy, is a vector quantity. The total linear momentum vector *P* of the total mass *M* of a moving fluid having a velocity of *v* is

Equation 2.8-1.

$$\boldsymbol{P} = M\boldsymbol{v}$$

The term *Mv* is the momentum of this moving mass *M* enclosed at a particular instant in the control volume shown in Fig. 2.6-4. The units of *Mv* are kg · m/s in the SI system.

Starting with Newton's second law, we will develop the integral momentum-balance equation for linear momentum. Angular momentum will not be considered here. *Newton's law* may be stated: The time rate of change of momentum of a system is equal to the summation of all forces acting on the system and takes place in the direction of the net force:

Equation 2.8-2.

$$\sum \boldsymbol{F} = \frac{d\boldsymbol{P}}{dt}$$

where *F* is force. In the SI system *F* is in newtons (N) and 1 N = 1 kg · m/s^2. Note that in the SI system g_c is not needed, but it is needed in the English system.

The equation for the conservation of momentum with respect to a control volume can be written as follows:

Equation 2.8-3.

$$\begin{pmatrix}\text{sum of forces acting}\\ \text{on control volume}\end{pmatrix} = \begin{pmatrix}\text{rate of momentum}\\ \text{out of control volume}\end{pmatrix} - \begin{pmatrix}\text{rate of momentum}\\ \text{into control volume}\end{pmatrix} + \begin{pmatrix}\text{rate of accumulation of momentum}\\ \text{in control volume}\end{pmatrix}$$

This is in the same form as the general mass-balance equation (2.6-3), with the sum of the forces as the generation rate term. Hence, momentum is not conserved, since it is generated by external forces on the system. If external forces are absent, momentum is conserved.

Using the general control volume shown in Fig. 2.6-4, we shall evaluate the various terms in Eq. (2.8-3), using methods very similar to the development of the general mass balance. For a small element of area *dA* on the control surface, we write

Equation 2.8-4.

$$\text{rate of momentum efflux} = \boldsymbol{v}(\rho v)(dA \cos\alpha)$$

Note that the rate of mass efflux is (ρv)(dA cos α). Also, note that (dA cos α) is the area dA projected in a direction normal to the velocity vector v, and α is the angle between the velocity vector v and the outward-directed-normal vector n. From vector algebra the product in Eq. (2.8-4) becomes

Equation 2.8-5.

$$\boldsymbol{v}(\rho v)(dA \cos \alpha) = \rho \boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{n})\, dA$$

Integrating over the entire control surface A,

Equation 2.8-6.

$$\begin{pmatrix}\text{net momentum efflux}\\ \text{from control volume}\end{pmatrix} = \iint_A \boldsymbol{v}(\rho v)\cos \alpha\, dA = \iint_A \rho \boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{n})\, dA$$

The net efflux represents the first two terms on the right-hand side of Eq. (2.8-3).

Similarly to Eq. (2.6-5), the rate of accumulation of linear momentum within the control volume V is

Equation 2.8-7.

$$\begin{pmatrix}\text{rate of accumulation of momentum}\\ \text{in control volume}\end{pmatrix} = \frac{\partial}{\partial t}\iiint_V \rho \boldsymbol{v}\, dV$$

Substituting Equations (2.8-2), (2.8-6), and (2.8-7) into (2.8-3), the overall linear momentum balance for a control volume becomes

Equation 2.8-8.

$$\sum \boldsymbol{F} = \iint_A \rho \boldsymbol{v}(\boldsymbol{v} \cdot \boldsymbol{n})\, dA + \frac{\partial}{\partial t}\iiint_V \rho \boldsymbol{v}\, dV$$

We should note that Σ **F** in general may have a component in any direction, and that F is the force the surroundings exert on the control-volume fluid. Since Eq. (2.8-8) is a vector equation, we may write the component scalar equations for the x, y, and z directions:

Equation 2.8-9.

$$\sum F_x = \iint_A v_x \rho v \cos \alpha\, dA + \frac{\partial}{\partial t}\iiint_V \rho v_x\, dV \qquad \text{(SI)}$$

$$\sum F_x = \iint_A v_x \frac{\rho}{g_c} v \cos \alpha\, dA + \frac{\partial}{\partial t}\iiint_V \frac{\rho}{g_c} v_x\, dV \qquad \text{(English)}$$

Equation 2.8-10.

$$\sum F_y = \iint_A v_y \rho v \cos \alpha\, dA + \frac{\partial}{\partial t}\iiint_V \rho v_y\, dV$$

Equation 2.8-11.

$$\sum F_z = \iint_A v_z \rho v \cos \alpha\, dA + \frac{\partial}{\partial t}\iiint_V \rho v_z\, dV$$

The force term ΣF_x in Eq. (2.8-9) is composed of the sum of several forces. These are given as follows:

1. ***Body force.*** The body force F_{xg} is the x-directed force caused by gravity acting on the total mass M in the control volume. This force, F_{xg}, is Mg_x. It is zero if the x direction is horizontal.
2. ***Pressure force.*** The force F_{xp} is the x-directed force caused by the pressure forces acting on the surface of the fluid system. When the control surface cuts through the fluid, the pressure is taken to be directed inward and perpendicular to the surface. In some cases part of the control surface may be a solid, and this wall is included inside the control surface. Then there is a contribution to F_{xp} from the pressure on the outside of this wall, which typically is atmospheric pressure. If gage pressure is used, the integral of the constant external pressure over the entire outer surface can be automatically ignored.
3. ***Friction force.*** When the fluid is flowing, an x-directed shear or friction force F_{xs} is present, which is exerted on the fluid by a solid wall when the control surface cuts between the fluid and the solid wall. In some or many cases, this frictional force may be negligible compared to the other forces and is neglected.
4. ***Solid surface force.*** In cases where the control surface cuts through a solid, there is present force R_x, which is the x component of the resultant of the forces acting on the control volume at these points. This occurs typically when the control volume includes a section of pipe and the fluid it contains. This is the force exerted by the solid surface on the fluid.

The force terms of Eq. (2.8-9) can then be represented as

Equation 2.8-12.

$$\sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x$$

Similar equations can be written for the y and z directions. Then Eq. (2.8-9) becomes, for the x direction,

Equation 2.8-13.

$$\sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x$$

$$= \iint_A v_x \rho v \cos\alpha \, dA + \frac{\partial}{\partial t} \iiint_V \rho v_x \, dV$$

Overall Momentum Balance in Flow System in One Direction

A quite common application of the overall momentum-balance equation is the case of a section of a conduit with its axis in the x direction. The fluid will be assumed to be flowing at steady state in the control volume shown in Fig. 2.6-3 and in Fig. 2.8-1. Since $v = v_x$, Eq. (2.8-13) for the x direction becomes as follows:

Equation 2.8-14.

$$\sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x = \iint_A v_x \rho v_x \cos\alpha \, dA$$

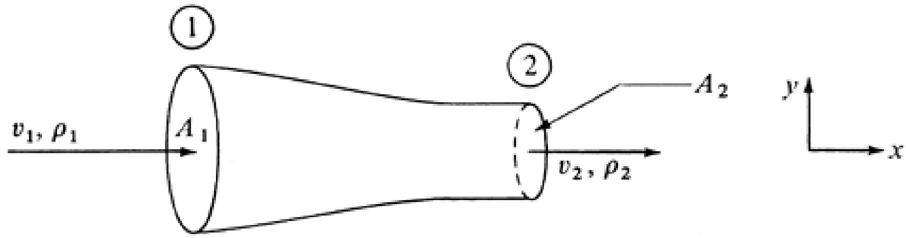

Figure 2.8-1. Flow through a horizontal nozzle in the x direction only.

Integrating, with cos α = ±1.0 and $\rho A = m/v_{av}$,

Equation 2.8-15.

$$F_{xg} + F_{xp} + F_{xs} + R_x = m\frac{(v_{x2}^2)_{av}}{v_{x\,2av}} - m\frac{(v_{x1}^2)_{av}}{v_{x1\,av}}$$

where, if the velocity is not constant and varies across the surface area,

Equation 2.8-16.

$$(v_x^2)_{av} = \frac{1}{A}\iint_A v_x^2\,dA$$

The ratio $(v_x^2)_{av}/v_{x\,av}$ is replaced by v_{xav}/β where β, which is the momentum velocity correction factor, has a value of 0.95 to 0.99 for turbulent flow and $\frac{3}{4}$ for laminar flow. For most applications in turbulent flow, $(v_x)_{av}^2/v_{x\,av}$ is replaced by $v_{x\,av}$, the average bulk velocity. Note that the subscript x on v_x and F_x can be dropped, since $v_x = v$ and $F_x = F$ for one-directional flow.

The term F_{xp}, which is the force caused by the pressures acting on the surface of the control volume, is

Equation 2.8-17.

$$F_{xp} = p_1A_1 - p_2A_2$$

The friction force will be neglected in Eq. (2.8-15), so $F_{xs} = 0$. The body force $F_{xg} = 0$ since gravity is acting only in the y direction. Substituting F_{xp} from Eq. (2.8-17) into (2.8-15), replacing $(v_x^2)_{av}/v_{x\,av}$ by v/β (where $v_{x\,av} = v$), setting $\beta = 1.0$, and solving for R_x in Eq. (2.8-15),

Equation 2.8-18.

$$R_x = mv_2 - mv_1 + p_2A_2 - p_1A_1$$

where R_x is the force exerted by the solid on the fluid. The force of the fluid on the solid (reaction force) is the negative of this, or $-R_x$.

EXAMPLE 2.8-1. Momentum Velocity Correction Factor β for Laminar Flow

The momentum velocity correction factor β is defined as follows for flow in one direction, where the subscript x is dropped:

Equation 2.8-19.

$$\frac{(v^2)_{av}}{v_{av}} = \frac{v_{av}}{\beta}$$

Equation 2.8-20.

$$\beta = \frac{(v_{av})^2}{(v^2)_{av}}$$

Determine β for laminar flow in a tube.

Solution: Using Eq. (2.8-16),

Equation 2.8-21.

$$(v^2)_{av} = \frac{1}{A}\iint_A v^2\, dA$$

Substituting Eq. (2.7-16) for laminar flow into Eq. (2.8-21) and noting that $A = \pi R^2$ and $dA = r\, dr\, d\theta$, we obtain (see Example 2.6-3)

Equation 2.8-22.

$$(v^2)_{av} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \left[2v_{av}\left(1 - \frac{r^2}{R^2}\right)\right]^2 r\, dr\, d\theta$$
$$= \frac{(2\pi)2^2 v_{av}^2}{\pi R^2}\int_0^R \frac{(R^2 - r^2)^2}{R^4} r\, dr$$

Integrating Eq. (2.8-22) and rearranging,

Equation 2.8-23.

$$(v^2)_{av} = \frac{8v_{av}^2}{R^6}\left(\frac{R^6}{2} - \frac{R^6}{2} + \frac{R^6}{6}\right) = \tfrac{4}{3}v_{av}^2$$

Substituting Eq. (2.8-23) into (2.8-20), $\beta = \frac{3}{4}$.

EXAMPLE 2.8-2. Momentum Balance for Horizontal Nozzle

Water is flowing at a rate of 0.03154 m^3/s through a horizontal nozzle shown in Fig. 2.8-1 and discharges to the atmosphere at point 2. The nozzle is attached at the upstream end at point 1 and frictional forces are considered negligible. The upstream ID is 0.0635 m and the downstream 0.0286 m. Calculate the resultant force on the nozzle. The density of the water is 1000 kg/m^3.

Solution: First, the mass flow and average or bulk velocities at points 1 and 2 are calculated. The area at point 1 is $A_1 = (\pi/4)(0.0635)^2 = 3.167 \times 10^{-3}$ m^2 and $A_2 = (\pi/4)(0.0286)^2 = 6.424 \times 10^{-4}$ m^2. Then,

$$m_1 = m_2 = m = (0.03154)(1000) = 31.54 \text{ kg/s}$$

The velocity at point 1 is $v_1 = 0.03154/(3.167 \times 10^{-3}) = 9.96$ m/s, and $v_2 = 0.03154/(6.424 \times 10^{-4}) = 49.1$ m/s.

To evaluate the upstream pressure p_1 we use the mechanical-energy-balance equation (2.7-28) assuming no frictional losses and turbulent flow. (This can be checked by calculating the Reynolds number.) This equation then becomes, for $\alpha = 1.0$,

Equation 2.8-24.

$$\frac{v_1^2}{2} + \frac{p_1}{\rho} = \frac{v_2^2}{2} + \frac{p_2}{\rho}$$

Setting p_2 = 0 gage pressure, ρ = 1000 kg/m^3, v_1 = 9.96 m/s, v_2 = 49.1 m/s, and solving for p_1,

$$p_1 = \frac{(1000)(49.1^2 - 9.96^2)}{2} = 1.156 \times 10^6 \text{ N/m}^2 \qquad \text{(gage pressure)}$$

For the x direction, the momentum-balance equation (2.8-18) is used. Substituting the known values and solving for R_x,

$$R_x = 31.54(49.10 - 9.96) + 0 - (1.156 \times 10^6)(3.167 \times 10^{-3})$$
$$= -2427 \text{ N } (-546 \text{ lb}_f)$$

Since the force is negative, it is acting in the negative x direction, or to the left. This is the force of the nozzle on the fluid. The force of the fluid on the solid is $-R_x$, or +2427 N.

Overall Momentum Balance in Two Directions

Another application of the overall momentum balance is shown in Fig. 2.8-2 for a flow system with fluid entering a conduit at point 1 inclined at an angle of α_1 relative to the horizontal x direction and leaving a conduit at point 2 at an angle α_2. The fluid will be assumed to be flowing at steady state and the frictional force F_{xs} will be neglected. Then Eq. (2.8-13) for the x direction becomes as follows for no accumulation:

Equation 2.8-25.

$$F_{xg} + F_{xp} + R_x = \iint_A v_x \rho v \cos \alpha \, dA$$

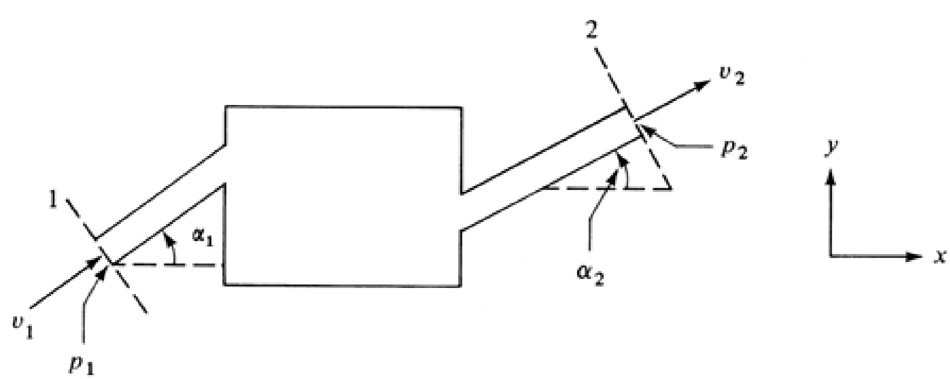

Figure 2.8-2. Overall momentum balance for flow system with fluid entering at point 1 and leaving at 2.

Integrating the surface (area) integral,

Equation 2.8-26.

$$F_{xg} + F_{xp} + R_x = m\frac{(v_2^2)_{av}}{v_{2av}} \cos \alpha_2 - m\frac{(v_1^2)_{av}}{v_{1av}} \cos \alpha_1$$

The term $(v^2)_{av}/v_{av}$ can again be replaced by v_{av}/β, with β being set at 1.0. From Fig. 2.8-2, the term F_{xp} is

Equation 2.8-27.

$$F_{xp} = p_1 A_1 \cos\alpha_1 - p_2 A_2 \cos\alpha_2$$

Then Eq. (2.8-26) becomes as follows after solving for R_x:

Equation 2.8-28.

$$R_x = mv_2 \cos\alpha_2 - mv_1 \cos\alpha_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1 \cos\alpha_1$$

The term F_{xg} = 0 in this case.

For R_y the body force F_{yg} is in the negative y direction and $F_{yg} = -m_t g$, where m_t is the total mass fluid in the control volume. Replacing cos α by sin α, the equation for the y direction becomes

Equation 2.8-29.

$$R_y = mv_2 \sin\alpha_2 - mv_1 \sin\alpha_1 + p_2 A_2 \sin\alpha_2 - p_1 A_1 \sin\alpha_1 + m_t g$$

EXAMPLE 2.8-3. Momentum Balance in a Pipe Bend

Fluid is flowing at steady state through a reducing pipe bend, as shown in Fig. 2.8-3. Turbulent flow will be assumed with frictional forces negligible. The volumetric flow rate of the liquid and the pressure p_2 at point 2 are known, as are the pipe diameters at both ends. Derive the equations to calculate the forces on the bend. Assume that the density ρ is constant.

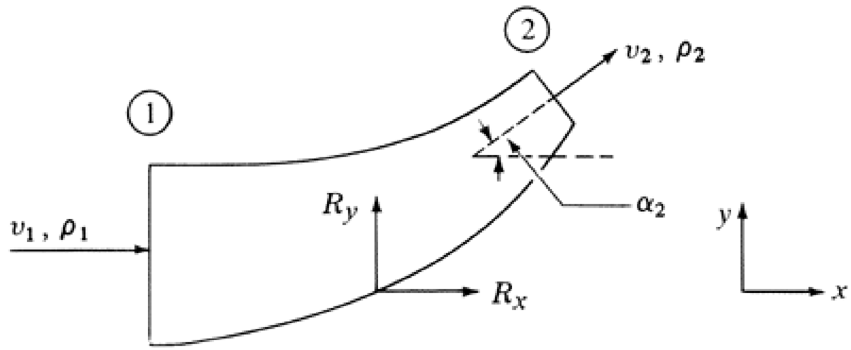

Figure 2.8-3. Flow through a reducing bend in Example 2.8-3.

***Solution*:** The velocities v_1 and v_2 can be obtained from the volumetric flow rate and the areas. Also, $m = \rho_1 v_1 A_1 = \rho_2 v_2 A_2$. As in Example 2.8-2, the mechanical-energy-balance equation (2.8-24) is used to obtain the upstream pressure, p_1. For the x direction, Eq. (2.8-28) is used for the momentum balance. Since $\alpha_1 = 0°$, cos α_1 = 1.0. Equation (2.8-28) becomes

Equation 2.8-30.

$$R_x = mv_2 \cos\alpha_2 - mv_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1 \qquad \text{(SI)}$$

$$R_x = \frac{m}{g_c} v_2 \cos\alpha_2 - \frac{m}{g_c} v_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1 \qquad \text{(English)}$$

For the y direction, the momentum-balance equation (2.8-29) is used, where sin α_1 = 0:

Equation 2.8-31.

$$R_y = mv_2 \sin\alpha_2 + p_2 A_2 \sin\alpha_2 + m_t g \qquad \text{(SI)}$$

where m_t is total mass fluid in the pipe bend. The pressures at points 1 and 2 are gage pressures since the atmospheric pressures acting on all surfaces cancel. The magnitude of the resultant force of the bend acting on the control volume fluid is

Equation 2.8-32.

$$|\boldsymbol{R}| = \sqrt{R_x^2 + R_y^2}$$

The angle this makes with the vertical is $\theta = \arctan(R_x/R_y)$. Often the gravity force F_{yg} is small compared to the other terms in Eq. (2.8-31) and is neglected.

EXAMPLE 2.8-4. Friction Loss in a Sudden Enlargement

A mechanical-energy loss occurs when a fluid flows from a small pipe to a large pipe through an abrupt expansion, as shown in Fig. 2.8-4. Use the momentum balance and mechanical-energy balance to obtain an expression for the loss for a liquid. (*Hint:* Assume that $p_0 = p_1$ and $v_0 = v_1$. Make a mechanical-energy balance between points 0 and 2 and a momentum balance between points 1 and 2. It will be assumed that p_1 and p_2 are uniform over the cross-sectional area.)

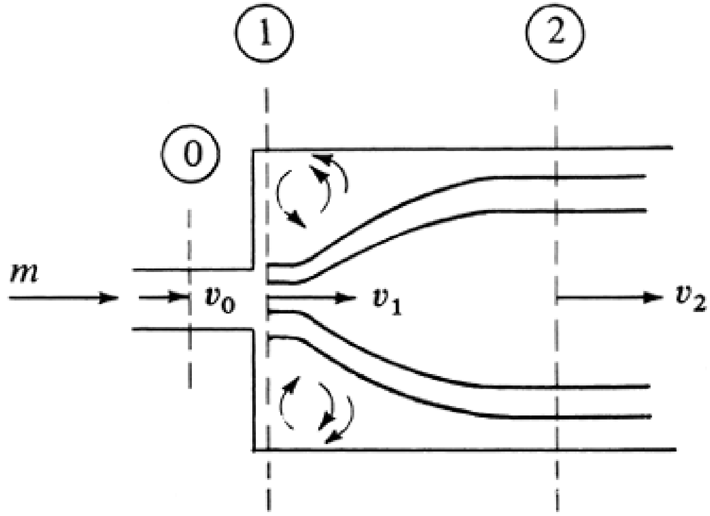

Figure 2.8-4. Losses in expansion flow.

Solution: The control volume is selected so that it does not include the pipe wall and R_x drops out. The boundaries selected are points 1 and 2. The flow through plane 1 occurs only through an area A_0. The frictional drag force will be neglected, and all the loss is assumed to be from eddies in this volume. Making a momentum balance between points 1 and 2 using Eq. (2.8-18) and noting that $p_0 = p_1$, $v_1 = v_0$, and $A_1 = A_2$,

Equation 2.8-33.

$$p_1 A_2 - p_2 A_2 = m v_2 - m v_1$$

The mass flow rate is $m = v_0 \rho A_0$ and $v_2 = (A_0/A_2) v_0$. Substituting these terms into Eq. (2.8-33) and rearranging gives us

Equation 2.8-34.

$$v_0^2 \frac{A_0}{A_2}\left(1 - \frac{A_0}{A_2}\right) = \frac{p_2 - p_1}{\rho}$$

Applying the mechanical-energy-balance equation (2.7-28) to points 1 and 2,

Equation 2.8-35.

$$\frac{v_0^2 - v_2^2}{2} - \sum F = \frac{p_2 - p_1}{\rho}$$

Finally, combining Eqs. (2.8-34) and (2.8-35),

Equation 2.8-36.

$$\sum F = \frac{v_0^2}{2}\left(1 - \frac{A_0}{A_2}\right)^2$$

Overall Momentum Balance for Free Jet Striking a Fixed Vane

When a free jet impinges on a fixed vane as in Fig. 2.8-5, the overall momentum balance can be applied to determine the force on the smooth vane. Since there are no changes in elevation or pressure before and after impact, there is no loss in energy, and application of the Bernoulli equation shows that the magnitude of the velocity is unchanged. Losses due to impact are neglected. The frictional resistance between the jet and the smooth vane is also neglected. The velocity is assumed to be uniform throughout the jet upstream and downstream. Since the jet is open to the atmosphere, the pressure is the same at all ends of the vane.

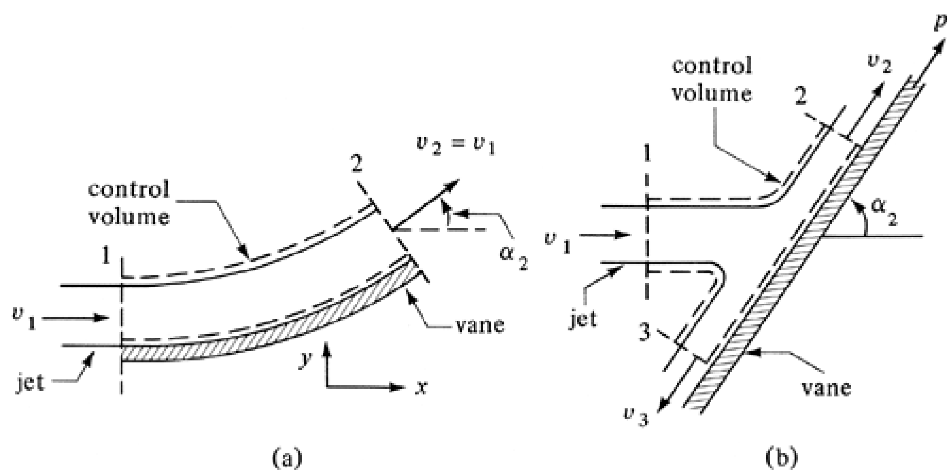

Figure 2.8-5. Free jet impinging on a fixed vane: (a) smooth, curved vane, (b) smooth, flat vane.

In making a momentum balance for the control volume shown for the curved vane in Fig. 2.8-5a, Eq. (2.8-28) is written as follows for steady state, where the pressure terms are zero, $v_1 = v_2$, $A_1 = A_2$, and $m = v_1 A_1 \rho_1 = v_2 A_2 v_2$:

Equation 2.8-37.

$$R_x = mv_2 \cos \alpha_2 - mv_1 + 0 = mv_1(\cos \alpha_2 - 1)$$

Using Eq. (2.8-29) for the *y* direction and neglecting the body force,

Equation 2.8-38.

$$R_y = mv_2 \sin \alpha_2 - 0 = mv_1 \sin \alpha_2$$

Hence, R_x and R_y are the force components of the vane on the control volume fluid. The force components on the vane are $-R_x$ and $-R_y$.

EXAMPLE 2.8-5. Force of Free Jet on a Curved, Fixed Vane

A jet of water having a velocity of 30.5 m/s and a diameter of 2.54×10^{-2} m is deflected by a smooth, curved vane as shown in Fig. 2.8-5a, where $\alpha_2 = 60°$. What is the force of the jet on the vane? Assume that $\rho = 1000$ kg/m^3.

Solution. The cross-sectional area of the jet is $A_1 = \pi(2.54 \times 10^{-2})^2/4 = 5.067 \times 10^{-4}$ m^2. Then, $m = v_1 A_1 \rho_1 = 30.5 \times 5.067 \times 10^{-4} \times 1000 = 15.45$ kg/s. Substituting into Eqs. (2.8-37) and (2.8-38),

$$R_x = 15.45 \times 30.5(\cos 60° - 1) = -235.6 \text{ N } (-52.97 \text{ lb}_f)$$

$$R_y = 15.45 \times 30.5 \sin 60° = 408.1 \text{ N } (91.74 \text{ lb}_f)$$

The force on the vane is $-R_x = +235.6$ N and $-R_y = -408.1$ N. The resultant force is calculated using Eq. (2.8-32).

In Fig. 2.8-5b a free jet at velocity v_1 strikes a smooth, inclined flat plate and the flow divides into two separate streams whose velocities are all equal ($v_1 = v_2 = v_3$) since there is no loss in energy. It is convenient to make a momentum balance in the p direction parallel to the plate. No force is exerted on the fluid by the flat plate in this direction; that is, there is no tangential force. Then, the initial momentum component in the p direction must equal the final momentum component in this direction. This means $\Sigma F_p = 0$. Writing an equation similar to Eq. (2.8-26), where m_1 is kg/s entering at 1 and m_2 leaves at 2 and m_3 at 3,

Equation 2.8-39.

$$\sum F_p = 0 = m_2 v_2 - m_1 v_1 \cos \alpha_2 - m_3 v_3$$

$$0 = m_2 v_1 - m_1 v_1 \cos \alpha_2 - m_3 v_1$$

By the continuity equation,

Equation 2.8-40.

$$m_1 = m_2 + m_3$$

Combining and solving,

Equation 2.8-41.

$$m_2 = \frac{m_1}{2}(1 + \cos \alpha_2), \qquad m_3 = \frac{m_1}{2}(1 - \cos \alpha_2)$$

The resultant force exerted by the plate on the fluid must be normal to it. This means the resultant force is simply $m_1 v_1 \sin \alpha_2$. Alternatively, the resultant force on the fluid can be calculated by determining R_x and R_y from Eqs. (2.8-28) and (2.8-29) and then using Eq. (2.8-32). The force on the bend is the opposite of this.

SHELL MOMENTUM BALANCE AND VELOCITY PROFILE IN LAMINAR FLOW

Introduction

In Section 2.8 we analyzed momentum balances using an overall, macroscopic control volume. From this we obtained the total or overall changes in momentum crossing the control surface. This overall momentum balance did not tell us the details of what happens inside the control volume. In the present section we analyze a small control volume and then shrink this control volume to differential size. In doing this we make a shell momentum balance using the momentum-balance concepts of the preceding section, and then, using the equation for the definition of viscosity, we obtain an expression for the velocity profile inside the enclosure and the pressure drop. The equations are derived for flow systems of simple geometry in laminar flow at steady state.

In many engineering problems a knowledge of the complete velocity profile is not needed, but a knowledge of the maximum velocity, the average velocity, or the shear stress on a surface is needed. In this section we show how to obtain these quantities from the velocity profiles.

Shell Momentum Balance Inside a Pipe

Engineers often deal with the flow of fluids inside a circular conduit or pipe. In Fig. 2.9-1 we have a horizontal section of pipe in which an incompressible Newtonian fluid is flowing in one-dimensional, steady-state, laminar flow. The flow is fully developed; that is, it is not influenced by entrance effects and the velocity profile does not vary along the axis of flow in the x direction.

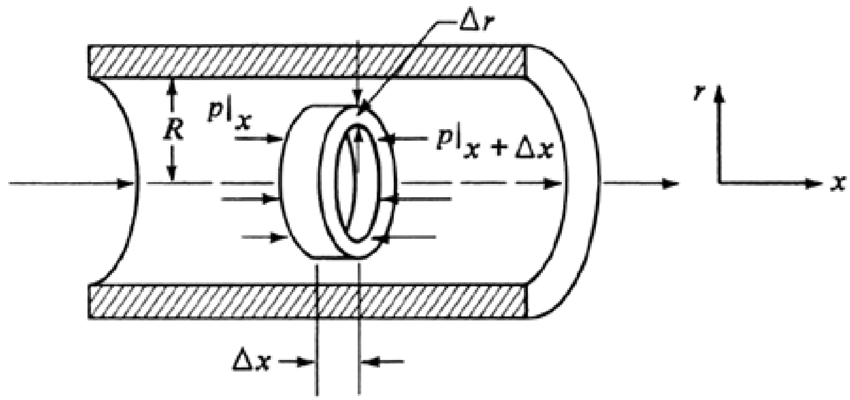

Figure 2.9-1. Control volume for shell momentum balance on a fluid flowing in a circular tube.

The cylindrical control volume is a shell with an inside radius r, thickness Δr, and length Δx. At steady state the conservation of momentum, Eq. (2.8-3), becomes as follows: sum of forces acting on control volume = rate of momentum out − rate of momentum into volume. The pressure forces become, from Eq. (2.8-17),

Equation 2.9-1.

$$\text{pressure forces} = pA|_x - pA|_{x+\Delta x} = p(2\pi r\ \Delta r)|_x - p(2\pi r\ \Delta r)|_{x+\Delta x}$$

The shear force or drag force acting on the cylindrical surface at the radius r is the shear stress τ_{rx} times the area $2\pi r\,\Delta x$. However, this can also be considered as the rate of momentum flow into the cylindrical surface of the shell as described by Eq. (2.4-9). Hence, the net rate of momentum efflux is the rate of momentum out − rate of momentum in and is

Equation 2.9-2.

$$\text{net efflux} = (\tau_{rx}\, 2\pi r\, \Delta x)|_{r+\Delta r} - (\tau_{rx}\, 2\pi r\, \Delta x)|_r$$

The net convective momentum flux across the annular surface at x and $x + \Delta x$ is zero, since the flow is fully developed and the terms are independent of x. This is true since v_x at x is equal to v_x at $x + \Delta x$.

Equating Eq. (2.9-1) to (2.9-2) and rearranging,

Equation 2.9-3.

$$\frac{(r\tau_{rx})|_{r+\Delta r} - (r\tau_{rx})|_r}{\Delta r} = \frac{r(p|_x - p|_{x+\Delta r})}{\Delta x}$$

In fully developed flow, the pressure gradient ($\Delta p/\Delta x$) is constant and becomes ($\Delta p/L$), where Δp is the pressure drop for a pipe of length L. Letting Δr approach zero, we obtain

Equation 2.9-4.

$$\frac{d(r\tau_{rx})}{dr} = \left(\frac{\Delta p}{L}\right) r$$

Separating variables and integrating,

Equation 2.9-5.

$$\tau_{rx} = \left(\frac{\Delta p}{L}\right)\frac{r}{2} + \frac{C_1}{r}$$

The constant of integration C_1 must be zero if the momentum flux is not infinite at $r = 0$. Hence,

Equation 2.9-6.

$$\tau_{rx} = \left(\frac{\Delta p}{2L}\right) r = \frac{p_0 - p_L}{2L}\, r$$

This means that the momentum flux varies linearly with the radius, as shown in Fig. 2.9-2, and the maximum value occurs at $r = R$ at the wall.

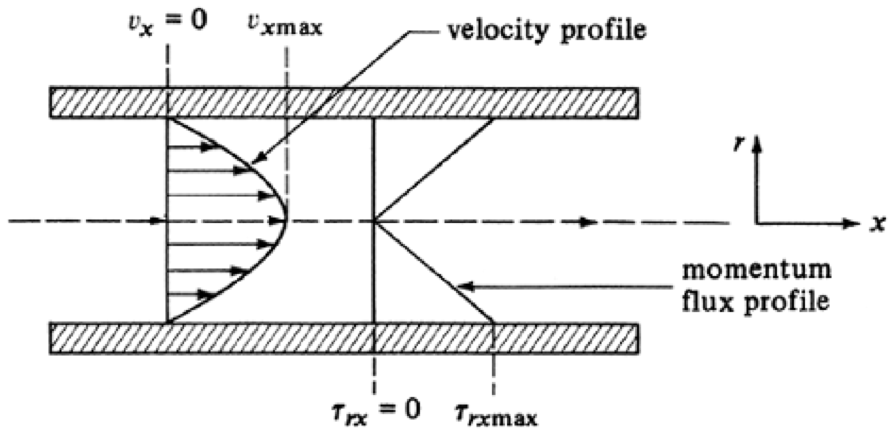

Figure 2.9-2. Velocity and momentum flux profiles for laminar flow in a pipe.

Substituting Newton's law of viscosity,

Equation 2.9-7.

$$\tau_{rx} = -\mu \frac{dv_x}{dr}$$

into Eq. (2.9-6), we obtain the following differential equation for the velocity:

Equation 2.9-8.

$$\frac{dv_x}{dr} = -\frac{p_0 - p_L}{2\mu L} r$$

Integrating using the boundary condition that at the wall, $v_x = 0$ at $r = R$, we obtain the equation for the velocity distribution:

Equation 2.9-9.

$$v_x = \frac{p_0 - p_L}{4\mu L} R^2 \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

This result shows us that the velocity distribution is parabolic, as shown in Fig. 2.9-2.

The average velocity v_{xav} for a cross section is found by summing up all the velocities over the cross section and dividing by the cross-sectional area, as in Eq. (2.6-17). Following the procedure given in Example 2.6-3, where $dA = r dr d\theta$ and $A = \pi R^2$,

Equation 2.9-10.

$$v_{x\,\mathrm{av}} = \frac{1}{A} \iint_A v_x \, dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R v_x \, r \, dr \, d\theta = \frac{1}{\pi R^2} \int_0^R v_x \, 2\pi r \, dr$$

Combining Eqs. (2.9-9) and (2.9-10) and integrating,

Equation 2.9-11.

$$v_{x\,\mathrm{av}} = \frac{(p_0 - p_L) R^2}{8\mu L} = \frac{(p_0 - p_L) D^2}{32\mu L}$$

where diameter $D = 2R$. Hence, Eq. (2.9-11), which is the *Hagen–Poiseuille equation,* relates the pressure drop and average velocity for laminar flow in a horizontal pipe.

The maximum velocity for a pipe is found from Eq. (2.9-9) and occurs at $r = 0$:

Equation 2.9-12.

$$v_{x\,\max} = \frac{p_0 - p_L}{4\mu L} R^2$$

Combining Eqs. (2.9-11) and (2.9-12), we find that

Equation 2.9-13.

$$v_{x\,\text{av}} = \frac{v_{x\,\max}}{2}$$

Also, dividing Eq. (2.9-9) by (2.9-11),

Equation 2.9-14.

$$\frac{v_x}{v_{x\,\text{av}}} = 2\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Shell Momentum Balance for Falling Film

We now use an approach similar to that used for laminar flow inside a pipe for the case of flow of a fluid as a film in laminar flow down a vertical surface. Falling films have been used to study various phenomena in mass transfer, coatings on surfaces, and so on. The control volume for the falling film is shown in Fig. 2.9-3a, where the shell of fluid considered is Δx thick and has a length of L in the vertical z direction. This region is sufficiently far from the entrance and exit regions so that the flow is not affected by these regions. This means the velocity $v_z(x)$ does not depend on position z.

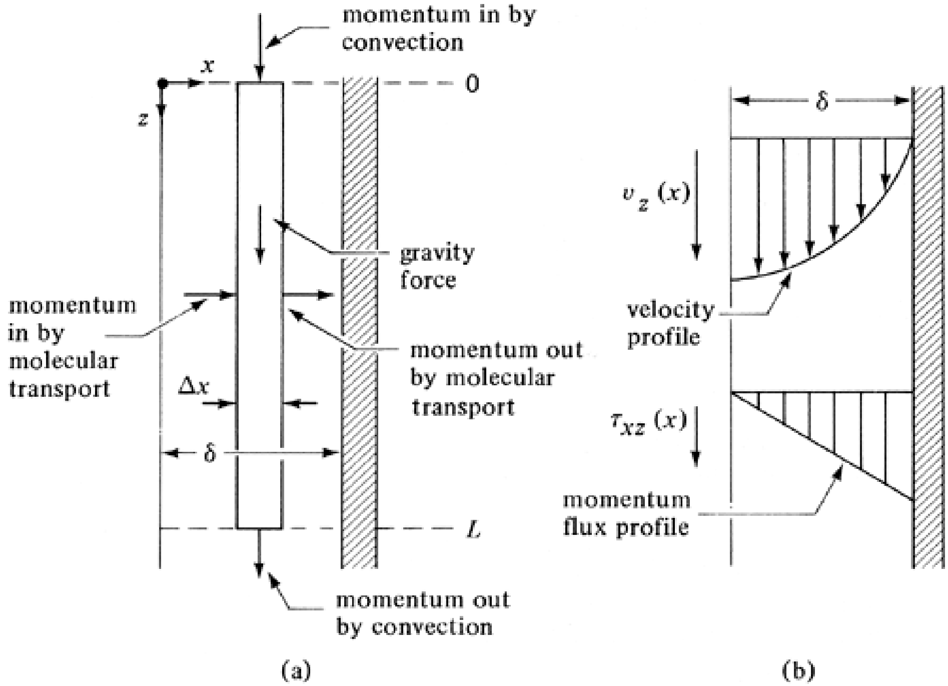

Figure 2.9-3. Vertical laminar flow of a liquid film: (a) shell momentum balance for a control volume Δx thick; (b) velocity and momentum flux profiles.

To start we set up a momentum balance in the z direction over a system Δx thick, bounded in the z direction by the planes $z = 0$ and $z = L$, and extending a distance W in the y direction. First, we consider the momentum flux due to molecular transport. The rate of momentum out − rate of momentum in is the momentum flux at point $x + \Delta x$ minus that at x times the area LW:

Equation 2.9-15.

$$\text{net efflux} = LW(\tau_{xz})|_{x+\Delta x} - LW(\tau_{xz})|_x$$

The net convective momentum flux is the rate of momentum entering the area ΔxW at $z = L$ minus that leaving at $z = 0$. This net efflux is equal to 0, since v_z at $z = 0$ is equal to v_z at $z = L$ for each value of x:

Equation 2.9-16.

$$\text{net efflux} = \Delta xWv_z(\rho v_z)|_{z=L} - \Delta xWv_z(\rho v_z)|_{z=0} = 0$$

The gravity force acting on the fluid is

Equation 2.9-17.

$$\text{gravity force} = \Delta xWL(\rho g)$$

Then, using Eq. (2.8-3) for the conservation of momentum at steady state,

Equation 2.9-18.

$$\Delta xWL(\rho g) = LW(\tau_{xz})|_{x+\Delta x} - LW(\tau_{xz})|_x + 0$$

Rearranging Eq. (2.9-18) and letting $\Delta x \to 0$,

Equation 2.9-19.

$$\frac{\tau_{xz}|_{x+\Delta x} - \tau_{xz}|_x}{\Delta x} = \rho g$$

Equation 2.9-20.

$$\frac{d}{dx}\tau_{xz} = \rho g$$

Integrating using the boundary conditions at $x = 0$, $\tau_{xz} = 0$ at the free liquid surface and at $x = x$, $\tau_{xz} = \tau_{xz}$,

Equation 2.9-21.

$$\tau_{xz} = \rho g x$$

This means the momentum-flux profile is linear, as shown in Fig. 2.9-3b, and the maximum value is at the wall. For a Newtonian fluid using Newton's law of viscosity,

Equation 2.9-22.

$$\tau_{xz} = -\mu \frac{dv_z}{dx}$$

Combining Eqs. (2.9-21) and (2.9-22) we obtain the following differential equation for the velocity:

Equation 2.9-23.

$$\frac{dv_z}{dx} = -\left(\frac{\rho g}{\mu}\right)x$$

Separating variables and integrating gives

Equation 2.9-24.

$$v_z = -\left(\frac{\rho g}{2\mu}\right)x^2 + C_1$$

Using the boundary condition that $v_z = 0$ at $x = \delta$, $C_1 = (\rho g/2\mu)\delta^2$. Hence, the velocity-distribution equation becomes

Equation 2.9-25.

$$v_z = \frac{\rho g \delta^2}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

This means the velocity profile is parabolic, as shown in Fig. 2.9-3b. The maximum velocity occurs at $x = 0$ in Eq. (2.9-25) and is

Equation 2.9-26.

$$v_{z\,\max} = \frac{\rho g \delta^2}{2\mu}$$

The average velocity can be found by using Eq. (2.6-17):

Equation 2.9-27.

$$v_{z\,\text{av}} = \frac{1}{A}\iint_A v_z\,dA = \frac{1}{W\delta}\int_0^W\int_0^\delta v_z\,dx\,dy = \frac{W}{W\delta}\int_0^\delta v_z\,dx$$

Substituting Eq. (2.9-25) into (2.9-27) and integrating,

Equation 2.9-28.

$$v_{z\,\text{av}} = \frac{\rho g \delta^2}{3\mu}$$

Combining Eqs. (2.9-26) and (2.9-28), we obtain $v_{z\,\text{av}} = \frac{2}{3}v_{z\,\text{max}}$. The volumetric flow rate q is obtained by multiplying the average velocity $v_{z\text{av}}$ times the cross-sectional area δW:

Equation 2.9-29.

$$q = \frac{\rho g \delta^3 W}{3\mu}\ \text{m}^3/\text{s}$$

Often in falling films, the mass rate of flow per unit width of wall Γ in kg/s · m is defined as $\Gamma = \rho\delta v_{z\text{av}}$ and a Reynolds number is defined as

Equation 2.9-30.

$$N_{\text{Re}} = \frac{4\Gamma}{\mu} = \frac{4\rho\delta v_{z\,\text{av}}}{\mu}$$

Laminar flow occurs for $N_{\text{Re}} < 1200$. Laminar flow with rippling present occurs above a N_{Re} of 25.

EXAMPLE 2.9-1. Falling Film Velocity and Thickness

An oil is flowing down a vertical wall as a film 1.7 mm thick. The oil density is 820 kg/m^3 and the viscosity is 0.20 Pa · s. Calculate the mass flow rate per unit width of wall, Γ, needed and the Reynolds number. Also calculate the average velocity.

Solution: The film thickness is $\delta = 0.0017$ m. Substituting Eq. (2.9-28) into the definition of Γ,

Equation 2.9-31.

$$\Gamma = \rho\delta v_{z\,\text{av}} = \frac{(\rho\delta)\rho g\delta^2}{3\mu} = \frac{\rho^2\delta^3 g}{3\mu}$$

$$= \frac{(820)^2(1.7\times10^{-3})^3(9.806)}{3\times0.20} = 0.05399\ \text{kg/s}\cdot\text{m}$$

Using Eq. (2.9-30),

$$N_{\text{Re}} = \frac{4\Gamma}{\mu} = \frac{4(0.05399)}{0.20} = 1.080$$

Hence, the film is in laminar flow. Using Eq. (2.9-28),

$$v_{z\,av} = \frac{\rho g \delta^2}{3\mu} = \frac{820(9.806)(1.7 \times 10^{-3})^2}{3(0.20)} = 0.03873 \text{ m/s}$$

DESIGN EQUATIONS FOR LAMINAR AND TURBULENT FLOW IN PIPES

Velocity Profiles in Pipes

One of the most important applications of fluid flow is flow inside circular conduits, pipes, and tubes. Appendix A.5 gives sizes of commercial standard steel pipe. Schedule 40 pipe in the different sizes is the standard usually used. Schedule 80 has a thicker wall and will withstand about twice the pressure of schedule 40 pipe. Both have the same outside diameter so that they will fit the same fittings. Pipes of other metals have the same outside diameters as steel pipe to permit interchanging parts of a piping system. Sizes of tubing are generally given by the outside diameter and wall thickness. Perry and Green (P1) give detailed tables of various types of tubing and pipes.

When fluid is flowing in a circular pipe and the velocities are measured at different distances from the pipe wall to the center of the pipe, it has been shown that in both laminar and turbulent flow, the fluid in the center of the pipe is moving faster than the fluid near the walls. These measurements are made at a reasonable distance from the entrance to the pipe. Figure 2.10-1 is a plot of the relative distance from the center of the pipe versus the fraction of maximum velocity v'/v_{max}, where v' is local velocity at the given position and v_{max} the maximum velocity at the center of the pipe. For viscous or laminar flow, the velocity profile is a true parabola, as derived in Eq. (2.9-9). The velocity at the wall is zero.

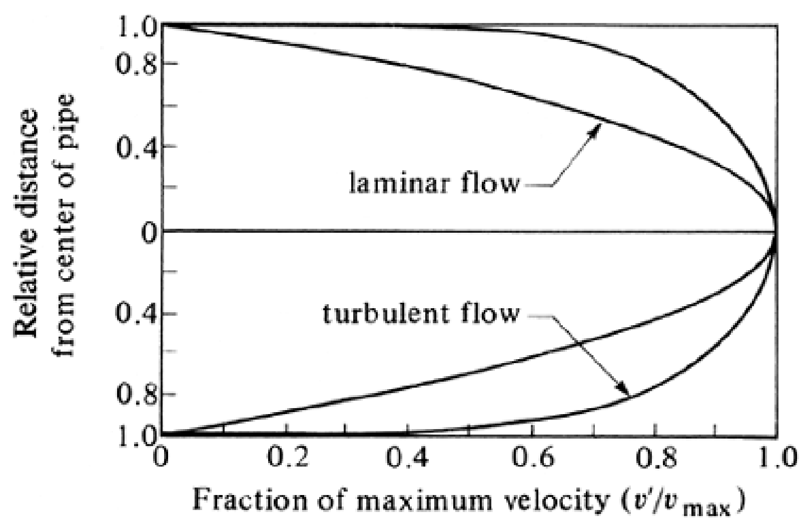

Figure 2.10-1. Velocity distribution of a fluid across a pipe.

In many engineering applications the relation between the average velocity v_{av} in a pipe and the maximum velocity v_{max} is useful, since in some cases only the v_{max} at the center point of the tube is measured. Hence, from only one point measurement this relationship between v_{max} and v_{av} can be used to determine v_{av}. In Fig. 2.10-2 experimentally measured values of v_{av}/v_{max} are plotted as a function of the Reynolds numbers $Dv_{av}\rho/\mu$ and $Dv_{max}\rho/\mu$.

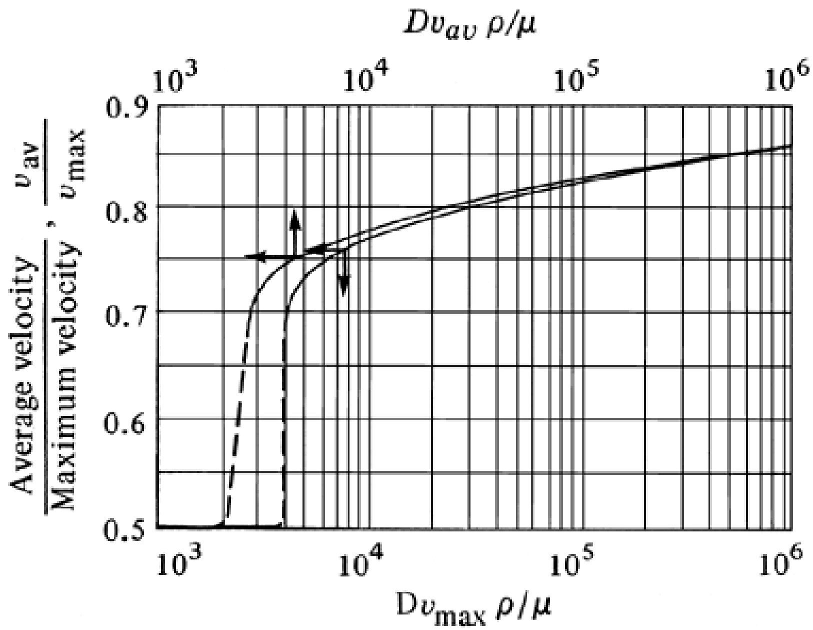

Figure 2.10-2. Ratio v_{av}/v_{max} as a function of Reynolds number for pipes.

The average velocity over the whole cross section of the pipe is precisely 0.5 times the maximum velocity at the center as given by the shell momentum balance in Eq. (2.9-13) for laminar flow. On the other hand, for turbulent flow, the curve is somewhat flattened in the center (see Fig. 2.10-1) and the average velocity is about 0.8 times the maximum. This value of 0.8 varies slightly, depending upon the Reynolds number, as shown in the correlation in Fig. 2.10-2. (*Note:* See Problem 2.6-3, where a value of 0.817 is derived using the $\frac{1}{7}$-power law.)

Pressure Drop and Friction Loss in Laminar Flow

Pressure drop and loss due to friction

When the fluid is in steady-state laminar flow in a pipe, then for a Newtonian fluid the shear stress is given by Eq. (2.4-2), which is rewritten for change in radius dr rather than distance dy, as follows:

Equation 2.10-1.

$$\tau_{rz} = -\mu \frac{dv_z}{dr}$$

Using this relationship and making a shell momentum balance on the fluid over a cylindrical shell, the Hagen–Poiseuille equation (2.9-11) for laminar flow of a liquid in circular tubes is obtained. This can be written as

Equation 2.10-2.

$$\Delta p_f = (p_1 - p_2)_f = \frac{32\mu v(L_2 - L_1)}{D^2}$$

where p_1 is upstream pressure at point 1, N/m^2; p_2 is pressure at point 2; v is average velocity in tube, m/s; D is inside diameter, m; and $(L_2 - L_1)$ or ΔL is length of straight tube, m. For English units, the right-hand side of Eq. (2.10-2) is divided by g_c.

The quantity $(p_1 - p_2)_f$ or Δp_f is the pressure loss due to skin friction. Then, for constant ρ, the friction loss F_f is

Equation 2.10-3.

$$F_f = \frac{(p_1 - p_2)_f}{\rho} = \frac{\text{N} \cdot \text{m}}{\text{kg}} \quad \text{or} \quad \frac{\text{J}}{\text{kg}} \qquad \text{(SI)}$$

$$F_f = \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m} \qquad \text{(English)}$$

This is the mechanical-energy loss due to skin friction for the pipe in N · m/kg of fluid and is part of the ΣF term for frictional losses in the mechanical-energy-balance equation (2.7-28). This term $(p_1 - p_2)_f$ for skin-friction loss is different from the $(p_1 - p_2)$ term, owing to velocity head or potential head changes in Eq. (2.7-28). That part of ΣF which arises from friction within the channel itself by laminar or turbulent flow is discussed in Sections 2.10B and 2.10C. The part of friction loss due to fittings (valves, elbows, etc.), bends, and the like, which sometimes constitute a large part of the friction, is discussed in Section 2.10F. Note that if Eq. (2.7-28) is applied to steady flow in a straight, horizontal tube, we obtain $(p_1 - p_2)/\rho = \Sigma F$.

One of the uses of Eq. (2.10-2) is in the experimental measurement of the viscosity of a fluid by measuring the pressure drop and volumetric flow rate through a tube of known length and diameter. Slight corrections for kinetic energy and entrance effects are usually necessary in practice. Also, Eq. (2.10-2) is often used in the metering of small liquid flows.

EXAMPLE 2.10-1. Metering of Small Liquid Flows

A small capillary with an inside diameter of 2.22×10^{-3} m and a length 0.317 m is being used to continuously measure the flow rate of a liquid having a density of 875 kg/m^3 and $\mu = 1.13 \times 10^{-3}$ Pa · s. The pressure-drop reading across the capillary during flow is 0.0655 m water (density 996 kg/m^3). What is the flow rate in m^3/s if end-effect corrections are neglected?

Solution: Assuming that the flow is laminar, Eq. (2.10-2) will be used. First, to convert the height h of 0.0655 m water to a pressure drop using Eq. (2.2-4),

$$\Delta p_f = h\rho g = (0.0655 \text{ m})\left(996 \frac{\text{kg}}{\text{m}^3}\right)\left(9.80665 \frac{\text{m}}{\text{s}^2}\right)$$

$$= 640 \text{ kg} \cdot \text{m/s}^2 \cdot \text{m}^2 = 640 \text{ N/m}^2$$

Substituting into Eq. (2.10-2) the values $\mu = 1.13 \times 10^{-3}$ Pa · s, $L_2 - L_1 = 0.317$ m, $D = 2.22 \times 10^{-3}$ m, and $\Delta p_f = 640$ N/m^2, and solving for v,

Equation 2.10-2.

$$\Delta p_f = \frac{32\mu v(L_2 - L_1)}{D^2}$$

$$640 = \frac{32(1.13 \times 10^{-3})(v)(0.317)}{(2.22 \times 10^{-3})^2}$$

$$v = 0.275 \text{ m/s}$$

The volumetric rate is then

$$\text{volumetric flow rate} = v\pi \frac{D^2}{4} = \frac{0.275(\pi)(2.22 \times 10^{-3})^2}{4}$$

$$= 1.066 \times 10^{-6} \text{ m}^3/\text{s}$$

Since it was assumed that laminar flow is occurring, the Reynolds number will be calculated to check this:

$$N_{\text{Re}} = \frac{Dv\rho}{\mu} = \frac{(2.22 \times 10^{-3})(0.275)(875)}{1.13 \times 10^{-3}} = 473$$

Hence, the flow is laminar as assumed.

Use of friction factor for friction loss in laminar flow

A common parameter used in laminar and especially in turbulent flow is the *Fanning friction factor, f,* which is defined as the drag force per wetted surface unit area (shear stress τ_s at the surface) divided by the product of density times velocity head, or $\frac{1}{2}\rho v^2$. The force is Δp_f times the cross-sectional area πR^2 and the wetted surface area is $2\pi R \Delta L$. Hence, the relation between the pressure drop due to friction and f is as follows for laminar and turbulent flow:

Equation 2.10-4.

$$f = \frac{\tau_s}{\rho v^2/2} = \frac{\Delta p_f \pi R^2}{2\pi R \, \Delta L} \bigg/ \frac{\rho v^2}{2}$$

Rearranging, this becomes

Equation 2.10-5.

$$\Delta p_f = 4f\rho \frac{\Delta L}{D} \frac{v^2}{2} \qquad \text{(SI)}$$

$$\Delta p_f = 4f\rho \frac{\Delta L}{D} \frac{v^2}{2g_c} \qquad \text{(English)}$$

Equation 2.10-6.

$$F_f = \frac{\Delta p_f}{\rho} = 4f \frac{\Delta L}{D} \frac{v^2}{2} \qquad \text{(SI)}$$

$$F_f = 4f \frac{\Delta L}{D} \frac{v^2}{2g_c} \qquad \text{(English)}$$

For laminar flow only, combining Eqs. (2.10-2) and (2.10-5),

Equation 2.10-7.

$$f = \frac{16}{N_{\text{Re}}} = \frac{16}{Dv\rho/\mu}$$

Equations (2.10-2), (2.10-5), (2.10-6), and (2.10-7) for laminar flow hold up to a Reynolds number of 2100. Beyond that, at a N_{Re} value above 2100, Eqs. (2.10-2) and (2.10-7) do not hold for turbulent flow. For turbulent flow, Eqs. (2.10-5) and (2.10-6), however, are used extensively along with empirical methods for predicting the friction factor *f,* as discussed in the next section.

EXAMPLE 2.10-2. Use of Friction Factor in Laminar Flow

Assume the same known conditions as in Example 2.10-1 except that the velocity of 0.275 m/s is known and the pressure drop Δp_f is to be predicted. Use the Fanning friction factor method.

***Solution*:** The Reynolds number is, as before,

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{(22.2 \times 10^{-3}\ \text{m})(0.275\ \text{m/s})(875\ \text{kg/m}^3)}{1.13 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} = 473$$

From Eq. (2.10-7) the friction factor *f* is

$$f = \frac{16}{N_{Re}} = \frac{16}{473} = 0.0338 \qquad \text{(dimensionless)}$$

Using Eq. (2.10-5) with ΔL = 0.317 m, v = 0.275 m/s, D = 2.22 × 10^{-3} m, and ρ = 875 kg/m^3,

$$\Delta p_f = 4f\rho \frac{\Delta L}{D}\frac{v^2}{2} = \frac{4(0.0338)(875)(0.317)(0.275)^2}{(22.2 \times 10^{-3})(2)} = 640\ \text{N/m}^2$$

This, of course, agrees with the value in Example 2.10-1.

When the fluid is a gas and not a liquid, the Hagen–Poiseuille equation (2.10-2) can be written as follows for laminar flow:

Equation 2.10-8.

$$m = \frac{\pi D^4 M(p_1^2 - p_2^2)}{128(2RT)\mu(L_2 - L_1)} \qquad \text{(SI)}$$

$$m = \frac{\pi D^4 g_c M(p_1^2 - p_2^2)}{128(2RT)\mu(L_2 - L_1)} \qquad \text{(English)}$$

where m = kg/s, M = molecular weight in kg/kg mol, T = absolute temperature in K, and R = 8314.3 N · m/kg mol · K. In English units, R = 1545.3 ft · lb_f/lb mol · °R.

Pressure Drop and Friction Factor in Turbulent Flow

In turbulent flow, as in laminar flow, the friction factor also depends on the Reynolds number. However, it is not possible to predict theoretically the Fanning friction factor *f* for turbulent flow as was done for laminar flow. The friction factor must be determined empirically (experimentally), and it not only depends upon the Reynolds number but also on surface roughness of the pipe. In laminar flow the roughness has essentially no effect.

Dimensional analysis also shows the dependence of the friction factor on these factors. In Sections 3.11 and 4.14, methods of obtaining the dimensionless numbers and their importance are discussed.

A large number of experimental data on friction factors for smooth pipe and pipes of varying degrees of equivalent roughness have been obtained and the data correlated. For design purposes, to predict the friction factor f and, hence, the frictional pressure drop for round pipe, the friction-factor chart in Fig. 2.10-3 can be used. It is a log–log plot of f versus N_{Re}. This friction factor f is then used in Eqs. (2.10-5) and (2.10-6) to predict the friction loss Δp_f or F_f.

Equation 2.10-5.

$$\Delta p_f = 4f\rho \frac{\Delta L}{D}\frac{v^2}{2} \qquad \text{(SI)}$$

$$\Delta p_f = 4f\rho \frac{\Delta L}{D}\frac{v^2}{2g_c} \qquad \text{(English)}$$

Equation 2.10-6.

$$F_f = \frac{\Delta p_f}{\rho} = 4f\frac{\Delta L}{D}\frac{v^2}{2} \qquad \text{(SI)}$$

$$F_f = 4f\frac{\Delta L}{D}\frac{v^2}{2g_c} \qquad \text{(English)}$$

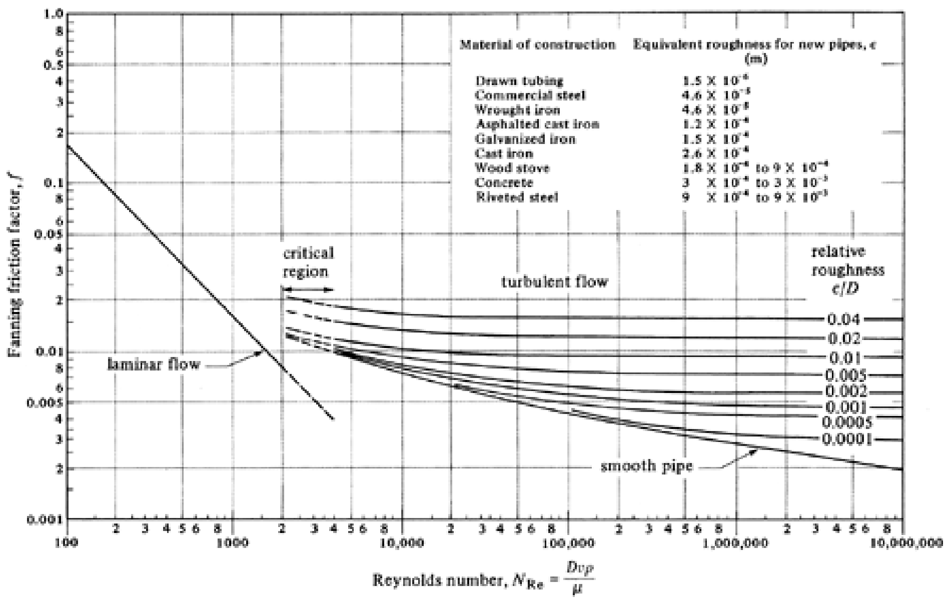

Figure 2.10-3. Friction factors for fluids inside pipes. [Based on L. F. Moody, Trans. A.S.M.E., ***66,*** *671 (1944): Mech. Eng.* ***69,*** *1005 (1947). With permission.]*

For the region with a Reynolds number below 2100, the line is the same as Eq. (2.10-7). For a Reynolds number above 4000 for turbulent flow, the lowest line in Fig. 2.10-3 represents the friction-factor line for smooth pipes and tubes, such as glass tubes and drawn copper and brass tubes. The other lines, for higher friction factors, represent lines for different relative roughness factors, ε/D, where D is the inside pipe diameter in m and ε is a roughness parameter, which represents the average height in m of roughness projections from the wall (M1). In Fig. 2.10-3, values for the equivalent roughness of new pipes are given (M1). The most common pipe, commercial steel, has a roughness of $\varepsilon = 4.6 \times 10^{-5}$ m (1.5×10^{-4} ft).

The reader should be cautioned about using friction factors f from other sources. The Fanning friction factor f in Eq. (2.10-6) is the one used here. Others use a friction factor that may be four times larger.

EXAMPLE 2.10-3. Use of Friction Factor in Turbulent Flow

A liquid is flowing through a horizontal straight pipe at 4.57 m/s. The pipe used is commercial steel, schedule 40, 2-in. nominal diameter. The viscosity of the liquid is 4.46 cp and the density 801 kg/m^3. Calculate the mechanical-energy friction loss F_f in J/kg for a 36.6-m section of pipe.

Solution: The following data are given: From Appendix A.5, $D = 0.0525$ m, $v = 4.57$ m/s, $\rho = 801$ kg/m^3, $\Delta L = 36.6$ m, and

$$\mu = (4.46 \text{ cp})(1 \times 10^{-3}) = 4.46 \times 10^{-3} \text{ kg/m} \cdot \text{s}$$

The Reynolds number is calculated as

$$N_{\text{Re}} = \frac{Dv\rho}{\mu} = \frac{0.0525(4.57)(801)}{4.46 \times 10^{-3}} = 4.310 \times 10^4$$

Hence, the flow is turbulent. For commercial steel pipe from the table in Fig. 2.10-3, the equivalent roughness is 4.6×10^{-5} m:

$$\frac{\varepsilon}{D} = \frac{4.6 \times 10^{-5} \text{ m}}{0.0525 \text{ m}} = 0.00088$$

For a N_{Re} of 4.310×10^4, the friction factor from Fig. 2.10-3 is $f = 0.0060$. Substituting into Eq. (2.10-6), the friction loss is

$$F_f = 4f \frac{\Delta L}{D} \frac{v^2}{2} = \frac{4(0.0060)(36.6)(4.57)^2}{(0.0525)(2)} = 174.8 \frac{\text{J}}{\text{kg}} \left(58.5 \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}} \right)$$

In problems involving the friction loss F_f in pipes, F_f is usually the unknown, with the diameter D, velocity v, and pipe length ΔL known. Then a direct solution is possible, as in Example 2.10-3. However, in some cases, the friction loss F_f is already set by the available head of liquid. Then if the volumetric flow rate and pipe length are set, the unknown to be calculated is the diameter. This solution must be by trial and error, since the velocity y appears in both N_{Re} and f, which are unknown. In another case, with F_f again being already set, the diameter and pipe length are specified. This solution is also by trial and error, to calculate the velocity. Example 2.10-4 indicates the method to be used to calculate the pipe diameter with F_f set. Others (M2) give a convenient chart to aid in these types of calculations.

EXAMPLE 2.10-4. Trial-and-Error Solution to Calculate Pipe Diameter

Water at 4.4°C is to flow through a horizontal commercial steel pipe having a length of 305 m at the rate of 150 gal/min. A head of water of 6.1 m is available to overcome the friction loss F_f. Calculate the pipe diameter.

Solution: From Appendix A.2, the density $\rho = 1000$ kg/m^3 and the viscosity μ is

$$\mu = (1.55 \text{ cp})(1 \times 10^{-3}) = 1.55 \times 10^{-3} \text{ kg/m} \cdot \text{s}$$

$$\text{friction loss } F_f = (6.1 \text{ m})g = (6.1)(9.80665) = 59.82 \text{ J/kg}$$

$$\text{flow rate} = \left(150 \frac{\text{gal}}{\text{min}}\right)\left(\frac{1 \text{ ft}^3}{7.481 \text{ gal}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)(0.028317 \text{ m}^3/\text{ft}^3)$$

$$= 9.46 \times 10^{-3} \text{ m}^3/\text{s}$$

$$\text{area of pipe} = \frac{\pi D^2}{4} \text{ m}^2 \quad (D \text{ is unknown})$$

$$\text{velocity } v = (9.46 \times 10^{-3} \text{ m}^3/\text{s})\left(\frac{1}{\pi D^2/4 \text{ m}^2}\right) = \frac{0.01204}{D^2} \text{ m/s}$$

The solution is by trial and error since v appears in N_{Re} and f. Assume that D = 0.089 m for the first trial.

$$N_{Re} = \frac{Dv\rho}{\mu} = (0.089)\frac{0.01204(1000)}{(0.089)^2(1.55 \times 10^{-3})} = 8.730 \times 10^4$$

For commercial steel pipe and using Fig. 2.10-3, $\varepsilon = 4.6 \times 10^{-5}$ m. Then,

$$\frac{\varepsilon}{D} = \frac{4.6 \times 10^{-5} \text{ m}}{0.089 \text{ m}} = 0.00052$$

From Fig. 2.10-3 for $N_{Re} = 8.730 \times 10^4$ and $\varepsilon/D = 0.00052$, $f = 0.0051$. Substituting into Eq. (2.10-6),

$$F_f = 59.82 = 4f \frac{\Delta L}{D} \frac{v^2}{2} = \frac{4(0.0051)(305)}{D(2)} \frac{(0.01204)^2}{D^4}$$

Solving for D, D = 0.0945 m. This does not agree with the assumed value of 0.089 m.

For the second trial, D will be assumed to be 0.0945 m.

$$N_{Re} = (0.0945)\frac{0.01204}{(0.0945)^2}\frac{1000}{1.55 \times 10^{-3}} = 8.220 \times 10^4$$

$$\frac{\varepsilon}{D} = \frac{4.6 \times 10^{-5}}{0.0945} = 0.00049$$

From Fig. 2.10-3, $f = 0.0052$. It can be seen that f does not change much with N_{Re} in the turbulent region:

$$F_f = 59.82 = \frac{4(0.0052)(305)}{D(2)} \frac{(0.01204)^2}{D^4}$$

Solving, D = 0.0954 m or 3.75 in. Hence, the solution agrees closely with the assumed value of D.

Pressure Drop and Friction Factor in Flow of Gases

The equations and methods discussed in this section for turbulent flow in pipes hold for incompressible liquids. They also hold for a gas if the density (or the pressure) changes by less than 10%. Then an average density, ρ_{av} in kg/m^3, should be used and the errors involved will be less than the uncertainty limits in the friction factor *f*. For gases, Eq. (2.10-5) can be rewritten as follows for laminar and turbulent flow:

Equation 2.10-9.

$$(p_1 - p_2)_f = \frac{4f\ \Delta L G^2}{D 2\rho_{av}}$$

where ρ_{av} is the density at $p_{av} = (p_1 + p_2)/2$. Also, the N_{Re} used is DG/μ, where G is kg/m$^2 \cdot$ s and is a constant independent of the density and velocity variations for the gas. Equation (2.10-5) can also be written for gases as

$$p_1^2 - p_2^2 = \frac{4f\ \Delta L G^2 RT}{DM} \qquad \text{(SI)}$$

Equation 2.10-10.

$$p_1^2 - p_2^2 = \frac{4f\ \Delta L G^2 RT}{g_c DM} \qquad \text{(English)}$$

where R is 8314.3 J/kg mol · K or 1545.3 ft · lb$_f$/lb mol · °R and M is molecular weight.

The derivation of Eqs. (2.10-9) and (2.10-10) applies only to cases with gases where the relative pressure change is small enough that large changes in velocity do not occur. If the exit velocity becomes large, the kinetic-energy term, which has been omitted, becomes important. For pressure changes above about 10%, compressible flow is occurring, and the reader should refer to Section 2.11. In adiabatic flow in a uniform pipe, the velocity in the pipe cannot exceed the velocity of sound.

EXAMPLE 2.10-5. Flow of Gas in Line and Pressure Drop

Nitrogen gas at 25°C is flowing in a smooth tube having an inside diameter of 0.010 m at the rate of 9.0 kg/s · m^2. The tube is 200 m long and the flow can be assumed to be isothermal. The pressure at the entrance to the tube is 2.0265×10^5 Pa. Calculate the outlet pressure.

Solution: The viscosity of the gas from Appendix A.3 is $\mu = 1.77 \times 10^{-5}$ Pa · s at T = 298.15 K. Inlet gas pressure $p_1 = 2.0265 \times 10^5$ Pa, G = 9.0 kg/s · m^2, D = 0.010 m, M = 28.02 kg/kg mol, ΔL = 200 m, and R = 8314.3 J/kg mol · K. Assuming that Eq. (2.10-10) holds for this case and that the pressure drop is less than 10%, the Reynolds number is

$$N_{Re} = \frac{DG}{\mu} = \frac{0.010(9.0)}{1.77 \times 10^{-5}} = 5085$$

Hence, the flow is turbulent. Using Fig. 2.10-3, f = 0.0090 for a smooth tube.

Substituting into Eq. (2.10-10),

$$p_1^2 - p_2^2 = \frac{4f\,\Delta L G^2 RT}{DM}$$

$$(2.0265 \times 10^5)^2 - p_2^2 = \frac{4(0.0090)(200)(9.0)^2(8314.3)(298.15)}{0.010(28.02)}$$

$$4.1067 \times 10^{10} - p_2^2 = 0.5160 \times 10^{10}$$

Solving, $p_2 = 1.895 \times 10^5$ Pa. Hence, Eq. (2.10-10) can be used, since the pressure drop is less than 10%.

Effect of Heat Transfer on Friction Factor

The friction factor f in Fig. 2.10-3 is given for isothermal flow, that is, no heat transfer. When a fluid is being heated or cooled, the temperature gradient will cause a change in physical properties of the fluid, especially the viscosity. For engineering practice the following method of Sieder and Tate (P1, S3) can be used to predict the friction factor for nonisothermal flow for liquids and gases:

1. Calculate the mean bulk temperature t_a as the average of the inlet and outlet bulk fluid temperatures.
2. Calculate the N_{Re} using the viscosity μ_a at t_a and use Fig. 2.10-3 to obtain f.
3. Using the tube wall temperature t_w, determine μ_w at t_w.
4. Calculate ψ for the appropriate case:

Equation 2.10-11.

$$\psi = \left(\frac{\mu_a}{\mu_w}\right)^{0.17} \qquad \text{(heating)} N_{Re} > 2100$$

Equation 2.10-12.

$$\psi = \left(\frac{\mu_a}{\mu_w}\right)^{0.11} \qquad \text{(cooling)} N_{Re} > 2100$$

Equation 2.10-13.

$$\psi = \left(\frac{\mu_a}{\mu_w}\right)^{0.38} \qquad \text{(heating)} N_{Re} < 2100$$

Equation 2.10-14.

$$\psi = \left(\frac{\mu_a}{\mu_w}\right)^{0.23} \qquad \text{(cooling)} N_{Re} < 2100$$

5. The final friction factor is obtained by dividing f from step 2 by ψ from step 4.

Hence, when the liquid is being heated, ψ is greater than 1.0 and the final f decreases. The reverse occurs on cooling the liquid.

Friction Losses in Expansion, Contraction, and Pipe Fittings

Skin-friction losses in flow through straight pipe are calculated by using the Fanning friction factor. However, if the velocity of the fluid is changed in direction or magnitude, additional friction losses occur. This results from additional turbulence which develops because of vortices and other factors. Methods for estimating these losses are discussed below.

Sudden enlargement losses

If the cross section of a pipe enlarges very gradually, very little or no extra losses are incurred. If the change is sudden, it results in additional losses due to eddies formed by the jet expanding in the enlarged section. This friction loss can be calculated by the following for turbulent flow in both sections. The following equation was derived in Example 2.8-4 as Eq. (2.8-3.6):

Equation 2.10-15.

$$h_{ex} = \frac{(v_1 - v_2)^2}{2\alpha} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{v_1^2}{2\alpha} = K_{ex}\frac{v_1^2}{2\alpha}\ \frac{\text{J}}{\text{kg}}$$

where h_{ex} is the friction loss in J/kg, K_{ex} is the expansion-loss coefficent and equals $(1 - A_1/A_2)^2$, v_1 is the upstream velocity in the smaller area in m/s, v_2 is the downstream velocity, and α = 1.0. If the flow is laminar in both sections, the factor α in the equation becomes $\frac{1}{2}$. For English units the right-hand side of Eq. (2.10-15) is divided by g_c. Also, h = ft · lb_f/lb_m.

Sudden contraction losses

When the cross section of the pipe is suddenly reduced, the stream cannot follow around the sharp corner, and additional frictional losses due to eddies occur. For turbulent flow, this is given by

Equation 2.10-16.

$$h_c = 0.55\left(1 - \frac{A_2}{A_1}\right)\frac{v_2^2}{2\alpha} = K_c\frac{v_2^2}{2\alpha}\ \frac{\text{J}}{\text{kg}}$$

where h_c is the friction loss, α = 1.0 for turbulent flow, v_2 is the average velocity in the smaller or downstream section, and K_c is the contraction-loss coefficient (P1) and approximately equals 0.55 $(1 - A_2/A_1)$. For laminar flow, the same equation can be used with α = $\frac{1}{2}$ (S2). For English units the right side is divided by g_c.

Losses in fittings and valves

Pipe fittings and valves also disturb the normal flow lines in a pipe and cause additional friction losses. In a short pipe with many fittings, the friction loss from these fittings could be greater than in the straight pipe. The friction loss for fittings and valves is given by the following equation:

Equation 2.10-17.

$$h_f = K_f\frac{v_1^2}{2}$$

where K_f is the loss factor for the fitting or valve and v_1 is the average velocity in the pipe leading to the fitting. Experimental values for K_f are given in Table 2.10-1 for turbulent flow (P1) and in Table 2.10-2 for laminar flow.

Table 2.10-1. Friction Loss for Turbulent Flow Through Valves and Fittings

Type of Fitting or Valve		*Frictional Loss, Numberof Velocity Heads, K_f*	*Frictional Loss, Equivalent Length of Straight Pipe in Pipe Diameters, L_e/D*
Elbow, 45°		0.35	17
Elbow, 90°		0.75	35
Tee		1	50
Return bend		1.5	75
Coupling		0.04	2
Union		0.04	2
Gate valve			
	Wide open	0.17	9
	Half open	4.5	225
Globe valve			
	Wide open	6.0	300
	Half open	9.5	475
Angle valve, wide open		2.0	100
Check valve			
	Ball	70.0	3500
	Swing	2.0	100
Water meter, disk		7.0	350

Source: **R. H. Perry and C. H. Chilton, *Chemical Engineers' Handbook*, 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.**

Table 2.10-2. Friction Loss for Laminar Flow Through Valves and Fittings (K1)

	Frictional Loss, Number of Velocity Heads, K_f Reynolds Number					
Type of Fitting or Valve	***50***	***100***	***200***	***400***	***1000***	***Turbulent***
Elbow, 90°	**17**	**7**	**2.5**	**1.2**	**0.85**	**0.75**
Tee	**9**	**4.8**	**3.0**	**2.0**	**1.4**	**1.0**
Globe valve	**28**	**22**	**17**	**14**	**10**	**6.0**
Check valve, swing	**55**	**17**	**9**	**5.8**	**3.2**	**2.0**

As an alternative method, some texts and references (B1) give data for losses in fittings as an equivalent pipe length in pipe diameters. These data, also given in Table 2.10-1, are presented as L_e/D, where L_e is the equivalent length of straight pipe in m having the same frictional loss as the fitting, and D is the inside pipe diameter in m. The K values in Eqs. (2.10-15) and (2.10-16) can be converted to L_e/D values by multiplying the K by 50 (P1). The L_e values for the fittings are simply added to the length of the straight pipe to get the total length of equivalent straight pipe to use in Eq. (2.10-6).

Frictional losses in mechanical-energy-balance equation

The frictional losses from the friction in the straight pipe (Fanning friction), enlargement losses, contraction losses, and losses in fittings and valves are all incorporated in the ΣF term of Eq. (2.7-28) for the mechanical-energy balance, so that

Equation 2.10-18.

$$\sum F = 4f\frac{\Delta L}{D}\frac{v^2}{2} + K_{ex}\frac{v_1^2}{2} + K_c\frac{v_2^2}{2} + K_f\frac{v_1^2}{2}$$

If all the velocities, v, v_1, and v_2, are the same, then by factoring, Eq. (2.10-18) becomes, for this special case,

Equation 2.10-19.

$$\sum F = \left(4f\frac{\Delta L}{D} + K_{ex} + K_c + K_f\right)\frac{v^2}{2}$$

The use of the mechanical-energy-balance equation (2.7-28) along with Eq. (2.10-18) will be shown in the following examples.

EXAMPLE 2.10-6. Friction Losses and Mechanical-Energy Balance

An elevated storage tank contains water at 82.2°C, as shown in Fig. 2.10-4. It is desired to have a discharge rate at point 2 of 0.223 ft^3/s. What must be the height *H* in ft of the surface of the water in the tank relative to the discharge point? The pipe used is commercial steel pipe, schedule 40, and the lengths of the straight portions of pipe are shown.

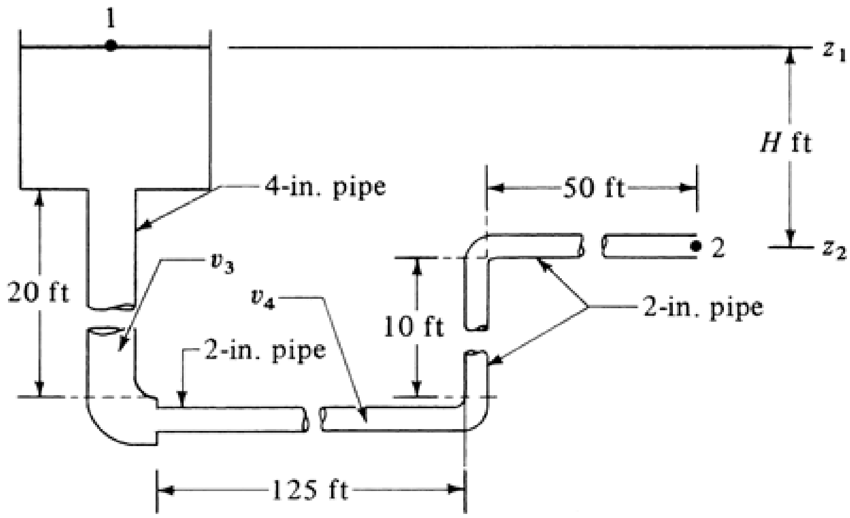

Figure 2.10-4. Process flow diagram for Example 2.10-6.

Solution: The mechanical-energy-balance equation (2.7-28) is written between points 1 and 2.

Equation 2.10-20.

$$z_1\frac{g}{g_c} + \frac{v_1^2}{2\alpha g_c} + \left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right) - W_S = z_2\frac{g}{g_c} + \frac{v_2^2}{2\alpha g_c} + \sum F$$

From Appendix A.2, for water, ρ = 0.970(62.43) = 60.52 lb_m/ft^3 and μ = 0.347 cp = 0.347(6.7197 × 10^{-4}) = 2.33 × 10^{-4} lb_m/ft · s. The diameters of the pipes are

$$\text{For 4-in. pipe:}\quad D_3 = \frac{4.026}{12} = 0.3353 \text{ ft};\qquad A_3 = 0.0884 \text{ ft}^2$$

$$\text{For 2-in. pipe:}\quad D_4 = \frac{2.067}{12} = 0.1722 \text{ ft};\qquad A_4 = 0.02330 \text{ ft}^2$$

The velocities in the 4-in. and 2-in. pipe are

$$v_3 = \frac{0.223 \text{ ft}^3/\text{s}}{0.0884 \text{ ft}^2} = 2.523 \text{ ft/s}\qquad \text{(4-in. pipe)}$$

$$v_4 = \frac{0.223}{0.02330} = 9.57 \text{ ft/s}\qquad \text{(2-in. pipe)}$$

The Σ F term for frictional losses in the system includes the following: (1) contraction loss at tank exit, (2) friction in the 4-in. straight pipe, (3) friction in 4-in. elbow, (4) contraction loss from 4-in. to 2-in. pipe, (5) friction in the 2-in. straight pipe, and (6) friction in the two 2-in. elbows. Calculations for the six items are as follows:

1. ***Contraction loss at tank exit.*** From Eq. (2.10-16), for contraction from A_1 to A_3 cross-sectional area, since A_1 of the tank is very large compared to A_3,

$$K_c = 0.55\left(1 - \frac{A_3}{A_1}\right) = 0.55(1 - 0) = 0.55$$

$$h_c = K_c \frac{v_3^2}{2g_c} = 0.55 \frac{(2.523)^2}{2(32.174)} = 0.054 \text{ ft} \cdot \text{lb}_f/\text{lb}_m$$

2. ***Friction in the 4-in. pipe.*** The Reynolds number is

$$N_{Re} = \frac{D_3 v_3 \rho}{\mu} = \frac{0.3353(2.523)(60.52)}{2.33 \times 10^{-4}} = 2.193 \times 10^5$$

Hence, the flow is turbulent. From Fig. 2.10-3, $\varepsilon = 4.6 \times 10^{-5}$ m (1.5×10^{-4} ft).

$$\frac{\varepsilon}{D_3} = \frac{0.00015}{0.3353} = 0.000448$$

Then, for N_{Re} = 219 300, the Fanning friction factor f = 0.0047. Substituting into Eq. (2.10-6) for ΔL = 20.0 ft of 4-in. pipe,

$$F_f = 4f \frac{\Delta L}{D} \frac{v^2}{2g_c} = 4(0.0047) \frac{20.0}{0.3353} \frac{(2.523)^2}{2(32.174)} = 0.111 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m}$$

3. ***Friction in 4-in. elbow.*** From Table 2.10-1, K_f = 0.75. Then, substituting into Eq. (2.10-17),

$$h_f = K_f \frac{v^2}{2g_c} = 0.75 \frac{(2.523)^2}{2(32.174)} = 0.074 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m}$$

4. ***Contraction loss from 4- to 2-in. pipe.*** Using Eq. (2.10-16) again for contraction from A_3 to A_4 cross-sectional area,

$$K_c = 0.55\left(1 - \frac{A_4}{A_3}\right) = 0.55\left(1 - \frac{0.02330}{0.0884}\right) = 0.405$$

$$h_c = K_c \frac{v_4^2}{2g_c} = 0.405 \frac{(9.57)^2}{2(32.174)} = 0.575 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m}$$

5. ***Friction in the 2-in pipe.*** The Reynolds number is

$$N_{Re} = \frac{D_4 v_4 \rho}{\mu} = \frac{0.1722(9.57)(60.52)}{2.33 \times 10^{-4}} = 4.280 \times 10^5$$

$$\frac{\varepsilon}{D} = \frac{0.00015}{0.1722} = 0.00087$$

The Fanning friction factor from Fig. 2.10-3 is f = 0.0048. The total length ΔL = 125 + 10 + 50 = 185 ft. Substituting into Eq. (2.10-6),

$$F_f = 4f\frac{\Delta L}{D}\frac{v^2}{2g_c} = 4(0.0048)\frac{185(9.57)^2}{(0.1722)(2)(32.174)} = 29.4\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}_m}$$

6. ***Friction in the two 2-in. elbows.*** For a K_f = 0.75 and two elbows,

$$h_f = 2K_f\frac{v^2}{2g_c} = \frac{2(0.75)(9.57)^2}{2(32.174)} = 2.136\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}_m}$$

The total frictional loss Σ F is the sum of items (1) through (6):

$$\sum F = 0.054 + 0.111 + 0.074 + 0.575 + 29.4 + 2.136$$

$$= 32.35 \text{ ft}\cdot\text{lb}_f/\text{lb}_m$$

Using as a datum level z_2, $z_1 = H$ ft, $z_2 = 0$. Since turbulent flow exists, α = 1.0. Also, $v_1 = 0$ and $v_2 = v_4 = 9.57$ ft/s. Since p_1 and p_2 are both at 1 atm abs pressure and $\rho_1 = \rho_2$,

$$\frac{p_1}{\rho} - \frac{p_2}{\rho} = 0$$

Also, since no pump is used, $W_S = 0$. Substituting these values into Eq. (2.10-20),

$$H\frac{g}{g_c} + 0 + 0 - 0 = 0 + \frac{1(9.57)^2}{2(32.174)} + 32.35$$

Solving, $H(g/g_c)$ = 33.77 ft · lb_f/lb_m (100.9 J/kg) and H is 33.77 ft (10.3 m) height of water level above the discharge outlet.

EXAMPLE 2.10-7. Friction Losses with Pump in Mechanical-Energy Balance

Water at 20°C is being pumped from a tank to an elevated tank at the rate of 5.0 × 10^{-3} m^3/s. All of the piping in Fig. 2.10-5 is 4-in. schedule 40 pipe. The pump has an efficiency of 65%. Calculate the kW power needed for the pump.

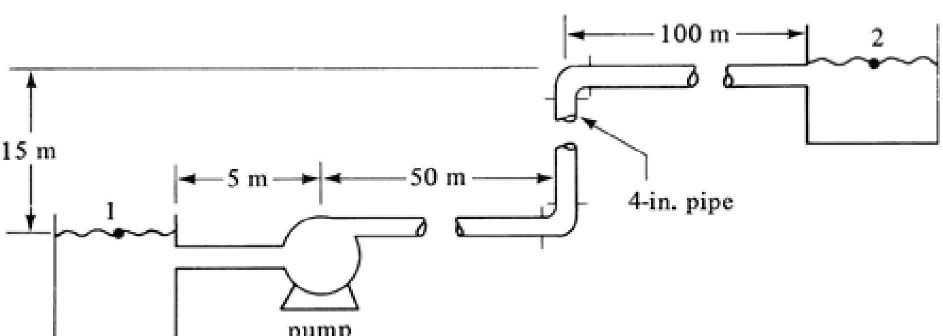

Figure 2.10-5. Process flow diagram for Example 2.10-7.

Solution: The mechanical-energy-balance equation (2.7-28) is written between points 1 and 2, with point 1 being the reference plane:

Equation 2.7-28.

$$\frac{1}{2\alpha}(v_{2\text{av}}^2 - v_{1\text{av}}^2) + g(z_2 - z_1) + \frac{p_2 - p_1}{\rho} + \sum F + W_S = 0$$

From Appendix A.2, for water, $\rho = 998.2\ kg/m^3$ and $\mu = 1.005 \times 10^{-3}$ Pa · s. For 4-in. pipe, from Appendix A. 5, $D = 0.1023$ m and $A = 8.219 \times 10^{-3}\ m^2$. The velocity in the pipe is $v = 5.0 \times 10^{-3}/(8.219 \times 10^{-3}) = 0.6083$ m/s. The Reynolds number is

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{0.1023(0.6083)(998.2)}{1.005 \times 10^{-3}} = 6.181 \times 10^4$$

Hence, the flow is turbulent.

The Σ F term for frictional losses includes the following: (1) contraction loss at tank exit, (2) friction in the straight pipe, (3) friction in the two elbows, and (4) expansion loss at the tank entrance.

1. ***Contraction loss at tank exit.*** From Eq. (2.10-16), for contraction from a large A_1 to a small A_2,

$$k_c = 0.55\left(1 - \frac{A_2}{A_1}\right) = 0.55(1 - 0) = 0.55$$

$$h_c = K_c \frac{v^2}{2\alpha} = (0.55)\frac{(0.6083)^2}{2(1.0)} = 0.102 \text{ J/kg}$$

2. ***Friction in the straight pipe.*** From Fig. 2.10-3, $\varepsilon = 4.6 \times 10^{-5}$ m and $\varepsilon/D = 4.6 \times 10^{-5}/0.1023 = 0.00045$. Then for $N_{Re} = 6.181 \times 10^4$, $f = 0.0051$. Substituting into Eq. (2.10-6) for $\Delta L = 5 + 50 + 15 + 100 = 170$ m,

$$F_f = 4f\frac{\Delta L}{D}\frac{v^2}{2} = 4(0.0051)\frac{170}{0.1023}\frac{(0.6083)^2}{2} = 6.272 \text{ J/kg}$$

3. ***Friction in the two elbows.*** From Table 2.10-1, $K_f = 0.75$. Then, substituting into Eq. (2.10-7) for two elbows,

$$h_f = 2K_f\frac{v^2}{2} = 2(0.75)\frac{(0.6083)^2}{2} = 0.278 \text{ J/kg}$$

4. ***Expansion loss at the tank entrance.*** Using Eq. (2.10-15),

$$K_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 = (1 - 0)^2 = 1.0$$

$$h_{ex} = K_{ex}\frac{v^2}{2} = 1.0\frac{(0.6083)^2}{2} = 0.185 \text{ J/kg}$$

The total frictional loss is Σ F:

$$\sum F = 0.102 + 6.272 + 0.278 + 0.185 = 6.837 \text{ J/kg}$$

Substituting into Eq. (2.7-28), where $(v_1^2 - v_2^2) = 0$ and $(p_2 - p_1) = 0$,

$$0 + 9.806(15.0 - 0) + 0 + 6.837 + W_S = 0$$

Solving, $W_S = -153.93$ J/kg. The mass flow rate is $m = 5.0 \times 10^{-3}(998.2) = 4.991$ kg/s. Using Eq. (2.7-30),

$$W_S = -\eta W_p$$

$$-153.93 = 0.65 W_p$$

Solving, $W_p = 236.8$ J/kg. The pump kW power is

$$\text{pump kW} = mW_p = \frac{4.991(236.8)}{1000} = 1.182 \text{ kW}$$

Friction Loss in Noncircular Conduits

The friction loss in long, straight channels or conduits of noncircular cross section can be estimated by using the same equations employed for circular pipes if the diameter in the Reynolds number and in the friction-factor equation (2.10-6) is taken as the equivalent diameter. The equivalent diameter D is defined as four times the hydraulic radius r_H. The hydraulic radius is defined as the ratio of the cross-sectional area of the channel to the wetted perimeter of the channel for turbulent flow only. Hence,

Equation 2.10-21.

$$D = 4r_H = 4\frac{\text{cross-sectional area of channel}}{\text{wetted perimeter of channel}}$$

For example, for a circular tube,

$$D = \frac{4(\pi D^2/4)}{\pi D} = D$$

For an annular space with outside diameter D_1 and inside D_2,

Equation 2.10-22.

$$D = \frac{4(\pi D_1^2/4 - \pi D_2^2/4)}{\pi D_1 + \pi D_2} = D_1 - D_2$$

For a rectangular duct of sides a and b ft,

Equation 2.10-23.

$$D = \frac{4(ab)}{2a + 2b} = \frac{2ab}{a + b}$$

For open channels and partly filled ducts in turbulent flow, the equivalent diameter and Eq. (2.10-6) are also used (P1). For a rectangle with depth of liquid y and width b,

Equation 2.10-24.

$$D = \frac{4(by)}{b + 2y}$$

For a wide, shallow stream of depth y,

Equation 2.10-25.

$$D = 4y$$

For laminar flow in ducts running full and in open channels with various cross-sectional shapes other than circular, equations are given elsewhere (P1).

Entrance Section of a Pipe

If the velocity profile at the entrance region of a tube is flat, a certain length of tube is necessary for the velocity profile to be fully established. This length for the establishment of fully developed flow is called the transition length or entry length. This is shown in Fig. 2.10-6 for laminar flow. At the entrance the velocity profile is flat; that is, the velocity is the same at all positions. As the fluid progresses down the tube, the thickness of the boundary layers increases until finally they meet at the center of the pipe and the parabolic velocity profile is fully established.

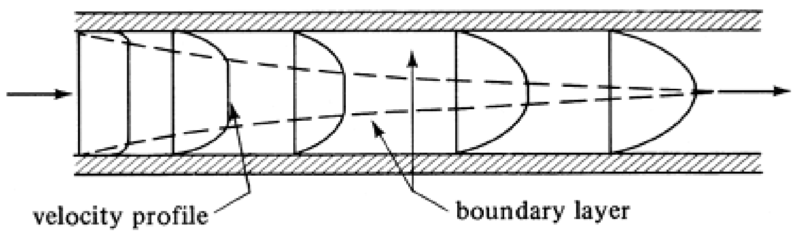

Figure 2.10-6. Velocity profiles near a pipe entrance for laminar flow.

The approximate entry length L_e of a pipe of diameter D for a fully developed velocity profile to be formed in laminar flow is (*L2*)

Equation 2.10-26.

$$\frac{L_e}{D} = 0.0575\ N_{Re}$$

For turbulent flow, no relation is available to predict the entry length for a fully developed turbulent velocity profile to form. As an approximation, the entry length is nearly independent of the Reynolds number and is fully developed after 50 diameters downstream.

EXAMPLE 2.10-8. Entry Length for a Fluid in a Pipe

Water at 20°C is flowing through a tube of diameter 0.010 m at a velocity of 0.10 m/s.

a. Calculate the entry length.
b. Calculate the entry length for turbulent flow.

***Solution*:** For part (a), from Appendix A.2, $\rho = 998.2\ kg/m^3$, $\mu = 1.005 \times 10^{-3}$ Pa · s. The Reynolds number is

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{0.010(0.10)(998.2)}{1.005 \times 10^{-3}} = 993.2$$

Using Eq. (2.10-26) for laminar flow,

$$\frac{L_e}{D} = \frac{L_e}{0.01} = 0.0575(993.2) = 57.1$$

Hence, $L_e = 0.571$ m.

For turbulent flow in part (b), $L_e = 50(0.01) = 0.50$ m.

The pressure drop or friction factor in the entry length is greater than in fully developed flow. For laminar flow the friction factor is highest at the entrance ($L2$) and then decreases smoothly to the fully developed flow value. For turbulent flow there will be some portion of the entrance over which the boundary layer is laminar and the friction-factor profile is difficult to express. As an approximation, the friction factor for the entry length can be taken as two to three times the value of the friction factor in fully developed flow.

Selection of Pipe Sizes

In large or complex process piping systems, the optimum size of pipe to use for a specific situation depends upon the relative costs of capital investment, power, maintenance, and so on. Charts are available for determining these optimum sizes (P1). However, for small installations, approximations are usually sufficiently accurate. Representative values for ranges of velocity in pipes are shown in Table 2.10-3. For stainless-steel pipes, recent data (D1) show that the velocities in Table 2.10-3 for process lines or pump discharge should be increased by 70%.

Table 2.10-3. Representative Ranges of Velocities in Steel Pipes

		Velocity	
Type of Fluid	**Type of Flow**	**ft/s**	**m/s**
Nonviscous liquid	Inlet to pump	2–3	0.6–0.9
	Process line or pump discharge	5.7	1.7
Viscous liquid	Inlet to pump	0.2–0.8	0.06–0.25
	Process line or pump discharge	3	0.9
Gas air	Process line	53	16
Steam 100 psig	Process line	38	11.6

COMPRESSIBLE FLOW OF GASES

Introduction and Basic Equation for Flow in Pipes

When pressure changes in gases occur which are greater than about 10%, the friction-loss equations (2.10-9) and (2.10-10) may be in error since compressible flow is occurring. Then the solution of the energy balance is more complicated because of the variation of the density or specific volume with changes in pressure. The field of compressible flow is very large and covers a very wide range of variation in geometry, pressure, velocity, and temperature. In this section we restrict our discussion to isothermal and adiabatic flow in uniform, straight pipes and do not cover flow in nozzles, which is discussed in some detail in other references (M2, P1).

The general mechanical-energy-balance equation (2.7-27) can be used as a starting point. Assuming turbulent flow, so that $\alpha = 1.0$, and no shaft work, so that $W_S = 0$, and writing the equation for a differential length dL, Eq. (2.7-27) becomes

Equation 2.11-1.

$$v\,dv + g\,dz + \frac{dp}{\rho} + dF = 0$$

For a horizontal duct, $dz = 0$. Using only the wall shear frictional term for dF and writing Eq. (2.10-6) in differential form,

Equation 2.11-2.

$$v\,dv + V\,dp + \frac{4fv^2\,dL}{2D} = 0$$

where $V = 1/\rho$. Assuming steady-state flow and a uniform pipe diameter, G is constant and

Equation 2.11-3.

$$G = v\rho = \frac{v}{V}$$

Equation 2.11-4.

$$dv = G\,dV$$

Substituting Eqs. (2.11-3) and (2.11-4) into (2.11-2) and rearranging,

Equation 2.11-5.

$$G^2\frac{dV}{V} + \frac{dp}{V} + \frac{2fG^2}{D}\,dL = 0$$

This is the basic differential equation that is to be integrated. To do this the relation between V and p must be known so that the integral of dp/V can be evaluated. This integral depends upon the nature of the flow, and two important conditions used are isothermal and adiabatic flow in pipes.

Isothermal Compressible Flow

To integrate Eq. (2.11-5) for isothermal flow, an ideal gas will be assumed, where

Equation 2.11-6.

$$pV = \frac{1}{M}RT$$

Solving for V in Eq. (2.11-6), and substituting it into Eq. (2.11-5) and integrating, assuming f is constant,

Equation 2.11-7.

$$G^2\int_1^2 \frac{dV}{V} + \frac{M}{RT}\int_1^2 p\,dp + 2f\frac{G^2}{D}\int_1^2 dL = 0$$

Equation 2.11-8.

$$G^2 \ln\frac{V_2}{V_1} + \frac{M}{2RT}(p_2^2 - p_1^2) + 2f\frac{G^2}{D}\Delta L = 0$$

Substituting p_1/p_2 for V_2/V_1 and rearranging,

Equation 2.11-9.

$$p_1^2 - p_2^2 = \frac{4f\,\Delta L G^2 RT}{DM} + \frac{2G^2RT}{M}\ln\frac{p_1}{p_2}$$

where M = molecular weight in kg mass/kg mol, R = 8314.34 N · m/kg mol · K, and T = temperature K. The quantity $RT/M = p_{av}/\rho_{av}$, where $p_{av} = (p_1 + p_2)/2$ and ρ_{av} is the average density at T and p_{av}. In English units, R = 1545.3 ft · lb$_f$/lb · mol · °R and the right-hand terms are divided by g_c. Equation (2.11-9) then becomes

Equation 2.11-10.

$$(p_1 - p_2)_f = \frac{4f\,\Delta L G^2}{2D\rho_{av}} + \frac{G^2}{\rho_{av}} \ln \frac{p_1}{p_2}$$

The first term on the right in Eqs. (2.11-9) and (2.11-10) represents the frictional loss as given by Eqs. (2.10-9) and (2.10-10). The last term in both equations is generally negligible in ducts of appreciable length unless the pressure drop is very large.

EXAMPLE 2.11-1. Compressible Flow of a Gas in a Pipe Line

Natural gas, which is essentially methane, is being pumped through a 1.016-m-ID pipeline for a distance of 1.609×10^5 m (D1) at a rate of 2.077 kg mol/s. It can be assumed that the line is isothermal at 288.8 K. The pressure p_2 at the discharge end of the line is 170.3×10^3 Pa absolute. Calculate the pressure p_1 at the inlet of the line. The viscosity of methane at 288.8 K is 1.04×10^{-5} Pa · s.

Solution: D = 1.016 m, $A = \pi D^2/4 = \pi(1.016)^2/4 = 0.8107\ \text{m}^2$. Then,

$$G = \left(2.077 \frac{\text{kg mol}}{\text{s}}\right)\left(16.0 \frac{\text{kg}}{\text{kg mol}}\right)\left(\frac{1}{0.8107\ \text{m}^2}\right) = 41.00 \frac{\text{kg}}{\text{s} \cdot \text{m}^2}$$

$$N_{\text{Re}} = \frac{DG}{\mu} = \frac{1.016(41.00)}{1.04 \times 10^{-5}} = 4.005 \times 10^6$$

From Fig. 2.10-3, $\varepsilon = 4.6 \times 10^5$ m.

$$\frac{\varepsilon}{D} = \frac{4.6 \times 10^{-5}}{1.016} = 0.0000453$$

The friction factor f = 0.0027.

In order to solve for p_1 in Eq. (2.11-9), trial and error must be used. Estimating p_1 at 620.5×10^3 Pa, R = 8314.34 N · m/kg mol · K, and $\Delta L = 1.609 \times 10^5$ m. Substituting into Eq. (2.11-9),

$$p_1^2 - p_2^2 = \frac{4(0.0027)(1.609 \times 10^5)(41.00)^2(8314.34)(288.8)}{1.016(16.0)} + \frac{2(41.00)^2(8314.34)(288.8)}{(16.0)} \ln \frac{620.5 \times 10^3}{170.3 \times 10^3}$$

$$= 4.375 \times 10^{11} + 0.00652 \times 10^{11} = 4.382 \times 10^{11} (\text{Pa})^2$$

Now, $P_2 = 170.3 \times 10^3$ Pa. Substituting this into the above and solving for p_1, $p_1 = 683.5 \times 10^3$ Pa. Substituting this new value of p_1 into Eq. (2.11-9) again and solving for p_1, the final result is $p_1 = 683.5 \times 10^3$ Pa. Note that the last term in Eq. (2.11-9) in this case is almost negligible.

When the upstream pressure p_1 remains constant, the mass flow rate G changes as the downstream pressure p_2 is varied. From Eq. (2.11-9), when $p_1 = p_2$, G = 0, and when $p_2 = 0$, G = 0. This indicates that at some intermediate value of p_2, the flow G must be a maximum. This means that the flow is a maximum when $dG/dp_2 = 0$. Performing this differentiation on Eq. (2.11-9) for constant p_1 and f and solving for G,

Equation 2.11-11.

$$G_{max} = \sqrt{\frac{M p_2^2}{RT}}$$

Using Eqs. (2.11-3) and (2.11-6),

Equation 2.11-12.

$$v_{max} = \sqrt{\frac{RT}{M}} = \sqrt{p_2 V_2}$$

This is the equation for the velocity of sound in the fluid at the conditions for isothermal flow. Thus, for isothermal compressible flow there is a maximum flow for a given upstream p_1, and further reduction of p_2 will not give any further increase in flow. Further details as to the length of pipe and the pressure at maximum flow conditions are discussed elsewhere (D1, M2, P1).

EXAMPLE 2.11-2. Maximum Flow for Compressible Flow of a Gas

For the conditions of Example 2.11-1, calculate the maximum velocity that can be obtained and the velocity of sound at these conditions. Compare with Example 2.11-1.

Solution: Using Eq. (2.11-12) and the conditions in Example 2.11-1,

$$v_{max} = \sqrt{\frac{RT}{M}} = \sqrt{\frac{8314(288.8)}{16.0}} = 387.4 \text{ m/s}$$

This is the maximum velocity obtainable if p_2 is decreased. This is also the velocity of sound in the fluid at the conditions for isothermal flow. To compare with Example 2.11-1, the actual velocity at the exit pressure p_2 is obtained by combining Eqs. (2.11-3) and (2.11-6) to give

Equation 2.11-13.

$$v_2 = \frac{RTG}{p_2 M}$$

$$= \frac{8314.34(288.8)(41.00)}{(170.3 \times 10^3)16.0} = 36.13 \text{ m/s}$$

Adiabatic Compressible Flow

When heat transfer through the wall of the pipe is negligible, the flow of gas in compressible flow in a straight pipe of constant cross section is adiabatic. Equation (2.11-5) has been integrated for adiabatic flow and details are given elsewhere (D1, M1, P1). Convenient charts for solving this case are also available (P1). The results for adiabatic flow often deviate very little from isothermal flow, especially in long lines. For very short pipes and relatively large pressure drops, the adiabatic flow rate is greater than the isothermal, but the maximum possible difference is about 20% (D1). For pipes of length about 1000 diameters or longer, the difference is generally less than 5%. Equation (2.11-8) can also be used when the temperature change over the conduit is small by using an arithmetic-average temperature.

Using the same procedures for finding maximum flow that were used in the isothermal case, maximum flow occurs when the velocity at the downstream end of the pipe is the sonic velocity for adiabatic flow. This is

Equation 2.11-14.

$$v_{max} = \sqrt{\gamma p_2 V_2} = \sqrt{\frac{\gamma RT}{M}}$$

where, $\gamma = c_p/c_v$, the ratio of heat capacities. For air, $\gamma = 1.4$. Hence, the maximum velocity for adiabatic flow is about 20% greater than for isothermal flow. The rate of flow may not be limited by the flow conditions in the pipe, in practice, but by the development of sonic velocity in a fitting or valve in the pipe. Hence, care should be used in the selection of fittings in such pipes for compressible flow. Further details as to the length of pipe and pressure at maximum flow conditions are given elsewhere (D1, M2, P1).

A convenient parameter often used in compressible-flow equations is the Mach number, N_{Ma}, which is defined as the ratio of v, the speed of the fluid in the conduit, to v_{max}, the speed of sound in the fluid at the actual flow conditions:

Equation 2.11-15.

$$N_{Ma} = \frac{v}{v_{max}}$$

At a Mach number of 1.0, the flow is sonic. At a value less than 1.0, the flow is subsonic, and supersonic at a number above 1.0.

PROBLEMS

2.2-1. ***Pressure in a Spherical Tank.*** Calculate the pressure in psia and kN/m^2 at the bottom of a spherical tank filled with oil having a diameter of 8.0 ft. The top of the tank is vented to the atmosphere having a pressure of 14.72 psia. The density of the oil is 0.922 g/cm^3.

A1: **Ans.** 17.92 $lb_f/in.^2$ (psia), 123.5 kN/m^2

2.2-2. ***Pressure with Two Liquids, Hg and Water.*** An open test tube at 293 K is filled at the bottom with 12.1 cm of Hg, and 5.6 cm of water is placed above the Hg. Calculate the pressure at the bottom of the test tube if the atmospheric pressure is 756 mm Hg. Use a density of 13.55 g/cm^3 for Hg and 0.998 g/cm^3 for water. Give the answer in terms of dyn/cm^2, psia, and kN/m^2. See Appendix A.1 for conversion factors.

A2: **Ans.** 1.175×10^6 dyn/cm^2, 17.0 psia, 2.3 psig, 117.5 kN/m^2

2.2-3. ***Head of a Fluid of Jet Fuel and Pressure.*** The pressure at the top of a tank of jet fuel is 180.6 kN/m^2. The depth of liquid in the tank is 6.4 m. The density of the fuel is 825 kg/m^3. Calculate the head of the liquid in m that corresponds to the absolute pressure at the bottom of the tank.

2.2-4. ***Measurement of Pressure.*** An open U-tube manometer similar to Fig. 2.2-4a is being used to measure the absolute pressure p_a in a vessel containing air. The pressure p_b is atmospheric pressure, which is 754 mm Hg. The liquid in the manometer is water having a density of 1000 kg/m^3. Assume that the density ρ_B is 1.30 kg/m^3 and that the distance Z is very small. The reading R is 0.415 m. Calculate p_a in psia and kPa.

A4: **Ans.** p_a = 15.17 psia, 104.6 kPa

2.2-5. ***Measurement of Small Pressure Differences.*** The two-fluid U-tube manometer is being used to measure the difference in pressure at two points in a line containing air at 1 atm abs pressure. The value of $R_0 = 0$ for equal pressures. The lighter fluid is a hydrocarbon with a density of 812 kg/m^3 and the heavier water has a density of 998 kg/m^3. The inside diameters of the U tube and reservoir are 3.2 mm and 54.2 mm, respectively. The reading R of the manometer is 117.2 mm. Calculate the pressure difference in mm Hg and pascal.

2.2-6. ***Pressure in a Sea Lab.*** A sea lab 5.0 m high is to be designed to withstand submersion to 150 m, measured from sea level to the top of the sea lab. Calculate the pressure on top of the sea lab and also the pressure variation on the side of the container measured as the distance x in m from the top of the sea lab downward. The density of seawater is 1020 kg/m^3.

A6: **Ans.** $p = 10.00(150 + x)$ kN/m^2

2.2-7. ***Measurement of Pressure Difference in Vessels.*** In Fig. 2.2-5b the differential manometer is used to measure the pressure difference between two vessels. Derive the equation for the pressure difference $p_A - p_B$ in terms of the liquid heights and densities.

2.2-8. ***Design of Settler and Separator for Immiscible Liquids.*** A vertical cylindrical settler–separator is to be designed for separating a mixture flowing at 20.0 m^3/h and containing equal volumes of a light petroleum liquid (ρ_B = 875 kg/m^3) and a dilute solution of wash water (ρ_A = 1050 kg/m^3). Laboratory experiments indicate a settling time of 15 min is needed to adequately separate the two phases. For design purposes use a 25-min settling time and calculate the size of the vessel needed, the liquid levels of the light and heavy liquids in the vessel, and the height h_{A2} of the heavy-liquid overflow. Assume that the ends of the vessel are approximately flat, that the vessel diameter equals its height, and that one-third of the volume is vapor space vented to the atmosphere. Use the nomenclature given in Fig. 2.2-6.

A8: **Ans.** h_{A2} = 1.537 m

2.3-1. ***Molecular Transport of a Property with a Variable Diffusivity.*** A property is being transported through a fluid at steady state through a constant cross-sectional area. At point 1 the concentration Γ_1 is 2.78×10^{-2} amount of property/m^3 and 1.50×10^{-2} at point 2 at a distance of 2.0 m away. The diffusivity depends on concentration Γ as follows:

$$\delta = A + B\Gamma = 0.150 + 1.65\Gamma$$

a. Derive the integrated equation for the flux in terms of Γ_1 and Γ_2. Then calculate the flux.

b. Calculate Γ at z = 1.0 m and plot Γ versus z for the three points.

A9: **Ans.** (a) $\psi_z = [A(\Gamma_1 - \Gamma_2) + (B/2)(\Gamma_1^2 - \Gamma_2^2)]/(z_2 - z_1)$

2.3-2. ***Integration of General-Property Equation for Steady State.*** Integrate the general-property equation (2.3-11) for steady state and no generation between the points Γ_1 at z_1 and Γ_2 at z_2. The final equation should relate Γ to *z*.

A10: **Ans.** $\Gamma = (\Gamma_2 - \Gamma_1)(z - z_1)/(z_2 - z_1) + \Gamma_1$

2.4-1. ***Shear Stress in Soybean Oil.*** Using Fig. 2.4-1, the distance between the two parallel plates is 0.00914 m, and the lower plate is being pulled at a relative velocity of 0.366 m/s greater than the top plate. The fluid used is soybean oil with viscosity 4×10^{-2} Pa · s at 303 K (Appendix A.4).

a. Calculate the shear stress τ and the shear rate using lb force, ft, and s units.
b. Repeat, using SI untis.
c. If glycerol at 293 K having a viscosity of 1.069 kg/m ·s is used instead of soybean oil, what relative velocity in m/s is needed using the same distance between plates so that the same shear stress is obtained as in part (a)? Also, what is the new shear rate?

A11: **Ans.** (a) Shear stress = 3.34×10^{-2} lb_f/ft^2, shear rate = 40.0 s^{-1}; (b) 1.60 N/m^2; (c) relative velocity = 0.01369 m/s, shear rate = 1.50 s^{-1}

2.4.-2 ***Shear Stress and Shear Rate in Fluids.*** Using Fig. 2.4-1, the lower plate is being pulled at a relative velocity of 0.40 m/s greater than the top plate. The fluid used is water at 24°C.

a. How far apart should the two plates be placed so that the shear stress τ is 0.30 N/m^2? Also, calculate the shear rate.
b. If oil with a viscosity of 2.0×10^{-2} Pa · s is used instead at the same plate spacing and velocity as in part (a), what are the shear stress and the shear rate?

2.5-1. ***Reynolds Number for Milk Flow.*** Whole milk at 293 K having a density of 1030 kg/m^3 and viscosity of 2.12 cp is flowing at the rate of 0.605 kg/s in a glass pipe having a diameter of 63.5 mm.

a. Calculate the Reynolds number. Is this turbulent flow?
b. Calculate the flow rate needed in m^3/s for a Reynolds number of 2100 and velocity in m/s.

A13: **Ans.** (a) N_{Re} = 5723, turbulent flow

2.5-2. ***Pipe Diameter and Reynolds Number.*** An oil is being pumped inside a 10.0-mm-diameter pipe at a Reynolds number of 2100. The oil density is 855 kg/m^3 and the viscosity is 2.1×10^{-2} Pa · s.

a. What is the velocity in the pipe?
b. It is desired to maintain the same Reynolds number of 2100 and the same velocity as in part (a) using a second fluid with a density of 925 kg/m^3 and a viscosity of 1.5×10^{-2} Pa · s. What pipe diameter should be used?

2.6-1. ***Average Velocity for Mass Balance in Flow Down Vertical Plate.*** For a layer of liquid flowing in laminar flow in the *z* direction down a vertical plate or surface, the velocity profile is

$$v_z = \frac{\rho g \delta^2}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

where δ is the thickness of the layer, x is the distance from the free surface of the liquid toward the plate, and v_z is the velocity at a distance x from the free surface.

a. What is the maximum velocity $v_{z\,max}$?
b. Derive the expression for the average velocity $v_{z\,av}$ and also relate it to $v_{z\,max}$.

A15: **Ans.** (a) $v_{z\,max} = \rho g \delta^2/2\mu$, (b) $v_{z\,av} = \frac{2}{3} v_{z\,max}$

2.6-2. ***Flow of Liquid in a Pipe and Mass Balance.*** A hydrocarbon liquid enters a simple flow system shown in Fig. 2.6-1 at an average velocity of 1.282 m/s, where $A_1 = 4.33 \times 10^{-3}$ m^2 and $\rho_1 = 902$ kg/m^3. The liquid is heated in the process and the exit density is 875 kg/m^3. The cross-sectional area at point 2 is 5.26×10^{-3} m^2. The process is steady state.

a. Calculate the mass flow rate m at the entrance and exit.
b. Calculate the average velocity v in 2 and the mass velocity G in 1.

A16: **Ans.** (a) $m_1 = m_2 = 5.007$ kg/s, (b) $G_1 = 1156$ kg/s · m^2

2.6-3. ***Average Velocity for Mass Balance in Turbulent Flow.*** For turbulent flow in a smooth, circular tube with a radius R, the velocity profile varies according to the following expression at a Reynolds number of about 10^5:

$$v = v_{max}\left(\frac{R - r}{R}\right)^{1/7}$$

where r is the radial distance from the center and v_{max} the maximum velocity at the center. Derive the equation relating the average velocity (bulk velocity) v_{av} to v_{max} for an incompressible fluid. (*Hint:* The integration can be simplified by substituting z for $R - r$.)

A17: **Ans.**

$$v_{av} = \left(\frac{49}{60}\right) v_{max} = 0.817 v_{max}$$

2.6-4. ***Bulk Velocity for Flow Between Parallel Plates.*** A fluid flowing in laminar flow in the x direction between two parallel plates has a velocity profile given by the following:

$$v_x = v_{x\,max}\left[1 - \left(\frac{y}{y_0}\right)^2\right]$$

where $2y_0$ is the distance between the plates, y is the distance from the center line, and v_x is the velocity in the x direction at position y. Derive an equation relating $v_{x\,av}$ (bulk or average velocity) to $v_{x\,max}$.

A18: **Ans.** $v_{x\,av} = \frac{2}{3} v_{x\,max}$

2.6-5. ***Overall Mass Balance for Dilution Process.*** A well-stirred storage vessel contains 10 000 kg of solution of a dilute methanol solution (w_A = 0.05 mass fraction alcohol). A constant flow of 500 kg/min of pure water is suddenly introduced into the tank and a constant rate of withdrawal of 500 kg/min of solution is started. These two flows are continued and remain constant. Assuming that the densities of the solutions are the same and that the total contents of the tank remain constant at 10000 kg of solution, calculate the time for the alcohol content to drop to 1.0 wt%.

A19: **Ans.** 32.2 min

2.6-6. ***Overall Mass Balance for Unsteady-State Process.*** A storage vessel is well stirred and contains 500 kg of total solution with a concentration of 5.0% salt. A constant flow rate of 900 kg/h of salt solution containing 16.67% salt is suddenly introduced into the tank and a constant withdrawal rate of 600 kg/h is also started. These two flows remain constant thereafter. Derive an equation relating the outlet withdrawal concentration as a function of time. Also, calculate the concentration after 2.0 h.

2.6-7 ***Mass Balance for Flow of Sucrose Solution.*** A 20 wt % sucrose (sugar) solution having a density of 1074 kg/m^3 is flowing through the same piping system as in Example 2.6-1 (Fig. 2.6-2). The flow rate entering pipe 1 is 1.892 m^3/h. The flow divides equally in each of pipes 3. Calculate the following:

a. The velocity in m/s in pipes 2 and 3.

b. The mass velocity G kg/m$^2 \cdot$ s in pipes 2 and 3.

2.7-1. ***Kinetic-Energy Velocity Correction Factor for Turbulent Flow.*** Derive the equation to determine the value of α, the kinetic-energy velocity correction factor, for turbulent flow. Use Eq. (2.7-20) to approximate the velocity profile and substitute this into Eq. (2.7-15) to obtain $(v^3)_{av}$. Then use Eqs. (2.7-20), (2.6-17), and (2.7-14) to obtain α.

A22: **Ans.** α = 0.9448

2.7-2. ***Flow Between Parallel Plates and Kinetic-Energy Correction Factor.*** The equation for the velocity profile of a fluid flowing in laminar flow between two parallel plates is given in Problem 2.6-4. Derive the equation to determine the value of the kinetic-energy velocity correction factor α. [*Hint:* First derive an equation relating v to v_{av}. Then derive the equation for $(v^3)_{av}$ and, finally, relate these results to α.]

2.7-3. ***Temperature Drop in Throttling Valve and Energy Balance.*** Steam is flowing through an adiabatic throttling valve (no heat loss or external work). Steam enters point 1 upstream of the valve at 689 kPa abs and 171.1°C and leaves the valve (point 2) at 359 kPa. Calculate the temperature t_2 at the outlet. [*Hint:* Use Eq. (2.7-21) for the energy balance and neglect the kinetic-energy and potential-energy terms as shown in Example 2.7-1. Obtain the enthalpy H_1 from Appendix A.2, steam tables. For H_2, linear interpolation of the values in the table will have to be done to obtain t_2.] Use SI units.

A24: **Ans.** t_2 = 160.6°C

2.7-4. ***Energy Balance on a Heat Exchanger and a Pump.*** Water at 93.3°C is being pumped from a large storage tank at 1 atm abs at a rate of 0.189 m^3/min by a pump. The motor that drives the pump supplies energy to the pump at the rate of 1.49 kW. The water is pumped through a heat exchanger, where it gives up 704 kW of heat and is then delivered to a large open storage tank at an elevation of 15.24 m above the first tank. What is the final temperature of the water to the second tank? Also, what is the gain in enthalpy of the water due to the work input? (*Hint:* Be sure to use the steam tables for the enthalpy of the water. Neglect any kinetic-energy changes but not potential-energy changes.)

A25: **Ans.** t_2 = 38.2°C, work input gain = 0.491 kJ/kg

2.7-5. ***Steam Boiler and Overall Energy Balance.*** Liquid water under pressure at 150 kPa enters a boiler at 24°C through a pipe at an average velocity of 3.5 m/s in turbulent flow. The exit steam leaves at a height of 25 m above the liquid inlet at 150°C and 150 kPa absolute, and the velocity in the outlet line is 12.5 m/s in turbulent flow. The process is steady state. How much heat must be added per kg of steam?

2.7-6. ***Energy Balance on a Flow System with a Pump and Heat Exchanger.*** Water stored in a large, well-insulated storage tank at 21.0°C and atmospheric pressure is being pumped at steady state from this tank by a pump at the rate of 40 m^3/h. The motor driving the pump supplies energy at the rate of 8.5 kW. The water is used as a cooling medium and passes through a heat exchanger, where 255 kW of heat is added to the water. The heated water then flows to a second large, vented tank, which is 25 m above the first tank. Determine the final temperature of the water delivered to the second tank.

2.7-7. ***Mechanical-Energy Balance in Pumping Soybean Oil.*** Soybean oil is being pumped through a uniform-diameter pipe at a steady mass-flow rate. A pump supplies 209.2 J/kg mass of fluid flowing. The entrance abs pressure in the inlet pipe to the pump is 103.4 kN/m^2. The exit section of the pipe downstream from the pump is 3.35 m above the entrance and the exit pressure is 172.4 kN/m^2. Exit and entrance pipes are the same diameter. The fluid is in turbulent flow. Calculate the friction loss in the system. See Appendix A.4 for the physical properties of soybean oil. The temperature is 303 K.

A28: **Ans.** $\sum F$ = 101.3 J/kg

2.7-8. ***Pump Horsepower in Brine System.*** A pump pumps 0.200 ft^3/s of brine solution having a density of 1.15 g/cm^3 from an open feed tank having a large cross-sectional area. The suction line has an inside diameter of 3.548 in. and the discharge line from the pump a diameter of 2.067 in. The discharge flow goes to an open overhead tank, and the open end of this line is 75 ft above the liquid level in the feed tank. If the friction losses in the piping system are 18.0 ft lb_f/lb_m, what pressure must the pump develop and what is the horsepower of the pump if the efficiency is 70%? The flow is turbulent.

2.7-9. ***Pressure Measurements from Flows.*** Water having a density of 998 kg/m^3 is flowing at the rate of 1.676 m/s in a 3.068-in.-diameter horizontal pipe at a pressure p_1 of 68.9 kPa abs. It then passes to a pipe having an inside diameter of 2.067 in.

a. Calculate the new pressure p_2 in the 2.067-in. pipe. Assume no friction losses.

b. If the piping is vertical and the flow is upward, calculate the new pressure p_2. The pressure tap for p_2 is 0.457 m above the tap for p_1.

A30: **Ans.** (a) p_2 = 63.5 kPa; (b) p_2 = 59.1 kPa

2.7-10. ***Draining Cottonseed Oil from a Tank.*** A cylindrical tank 1.52 m in diameter and 7.62 m high contains cottonseed oil having a density of 917 kg/m^3. The tank is open to the atmosphere. A discharge nozzle of inside diameter 15.8 mm and cross-sectional area A_2 is located near the bottom of the tank. The surface of the liquid is located at H = 6.1 m above the center line of the nozzle. The discharge nozzle is opened, draining the liquid level from H = 6.1 m to H = 4.57 m. Calculate the time in seconds to do this. [*Hint:* The velocity on the surface of the reservoir is small and can be neglected. The velocity v_2 m/s in the nozzle can be calculated for a given H by Eq. (2.7-36). However, H, and hence v_2, are varying. Set up an unsteady-state mass balance as follows: The volumetric flow rate in the tank is $(A_t\, dH)/dt$, where A_t is the tank cross section in m^2 and $A_t\, dH$ is the m^3 liquid flowing in dt s. This rate must equal the negative of the volumetric rate in the nozzle, or $-A_2 v_2$ m^3/s. The negative sign is present since dH is the negative of v_2. Rearrange this equation and integrate between H = 6.1 m at t = 0 and H = 4.57 m at $t = t_F$.]

A31: **Ans.** t_F = 1388 s

2.7-11. ***Friction Loss in Turbine Water Power System.*** Water is stored in an elevated reservoir. To generate power, water flows from this reservoir down through a large conduit to a turbine and then through a similar-sized conduit. At a point in the conduit 89.5 m above the turbine, the pressure is 172.4 kPa, and at a level 5 m below the turbine, the pressure is 89.6 kPa. The water flow rate is 0.800 m^3/s. The output of the shaft of the turbine is 658 kW. The water density is 1000 kg/m^3. If the efficiency of the turbine in converting the mechanical energy given up by the fluid to the turbine shaft is 89% (η_t = 0.89), calculate the friction loss in the turbine in J/kg. Note that in the mechanical-energy-balance equation, the W_s is equal to the output of the shaft of the turbine over η_t.

A32: **Ans.** $\sum F$ = 85.3 J/kg

2.7-12. ***Pipeline Pumping of Oil.*** A pipeline laid cross-country carries oil at the rate of 795 m^3/d. The pressure of the oil is 1793 kPa gage leaving pumping station 1. The pressure is 862 kPa gage at the inlet to the next pumping station, 2. The second station is 17.4 m higher than the first station. Calculate the lost work ($\sum F$ friction loss) in J/kg mass oil. The oil density is 769 kg/m^3.

2.7-13. ***Test of Centrifugal Pump and Mechanical-Energy Balance.*** A centrifugal pump is being tested for performance, and during the test the pressure reading in the 0.305-m-diameter suction line just adjacent to the pump casing is −20.7 kPa (vacuum below atmospheric pressure). In the discharge line with a diameter of 0.254 m at a point 2.53 m above the suction line, the pressure is 289.6 kPa gage. The flow of water from the pump is measured as 0.1133 m^3/s. (The density can be assumed as 1000 kg/m^3.) Calculate the kW input of the pump.

A34: **Ans.** 38.11 kW

2.7-14. ***Friction Loss in Pump and Flow System.*** Water at 20°C is pumped from the bottom of a large storage tank where the pressure is 310.3 kPa gage to a nozzle which is 15.25 m above the tank bottom and discharges to the atmosphere with a velocity in the nozzle of 19.81 m/s. The water flow rate is 45.4 kg/s. The efficiency of the pump is 80% and 7.5 kW are furnished to the pump shaft. Calculate the following:

a. The friction loss in the pump.

b. The friction loss in the rest of the process.

2.7-15. ***Power for Pumping in Flow System.*** Water is being pumped from an open water reservoir at the rate of 2.0 kg/s at 10°C to an open storage tank 1500 m away. The pipe used is schedule 40 3 $\frac{1}{2}$ in. pipe and the frictional losses in the system are 625 J/kg. The surface of the water reservoir is 20 m above the level of the storage tank. The pump has an efficiency of 75%.

a. What is the kW power required for the pump?

b. If the pump is not present in the system, will there be a flow?

A36: **Ans.** (a) 1.143 kW

2.8-1. ***Momentum Balance in a Reducing Bend.*** Water is flowing at steady state through the reducing bend in Fig. 2.8-3. The angle α_2 = 90° (a right-angle bend). The pressure at point 2 is 1.0 atm abs. The flow rate is 0.020 m^3/s and the diameters at points 1 and 2 are 0.050 m and 0.030 m, respectively. Neglect frictional and gravitational forces. Calculate the resultant forces on the bend in newtons and lb force. Use ρ = 1000 kg/m^3.

A37: **Ans.** $-R_x$ = +450.0 N, $-R_y$ = −565.8 N.

2.8-2. ***Forces on Reducing Bend.*** Water is flowing at steady state and 363 K at a rate of 0.0566 m^3/s through a 60° reducing bend (α_2 = 60°) in Fig. 2.8-3. The inlet pipe diameter is 0.1016 m and the outlet 0.0762 m. The friction loss in the pipe bend can be estimated as $v_2^2/5$ Neglect gravity forces. The exit pressure p_2 = 111.5 kN/m^2 gage. Calculate the forces on the bend in newtons.

A38: **Ans.** $-R_x = +1344$ N, $-R_y = -1026$ N

2.8-3. ***Force of Water Stream on a Wall.*** Water at 298 K discharges from a nozzle and travels horizontally, hitting a flat, vertical wall. The nozzle has a diameter of 12 mm and the water leaves the nozzle with a flat velocity profile at a velocity of 6.0 m/s. Neglecting frictional resistance of the air on the jet, calculate the force in newtons on the wall.

A39: **Ans.** $-R_x = 4.059$ N

2.8-4. ***Flow Through an Expanding Bend.*** Water at a steady-state rate of 0.050 m^3/s is flowing through an expanding bend that changes direction by 120°. The upstream diameter is 0.0762 m and the downstream is 0.2112 m. The upstream pressure is 68.94 kPa gage. Neglect energy losses within the elbow and calculate the downstream pressure at 298 K. Also calculate R_x and R_y.

2.8-5. ***Force of Stream on a Wall.*** Repeat Problem 2.8-3 for the same conditions except that the wall is inclined 45° to the vertical. The flow is frictionless. Assume no loss in energy. The amount of fluid splitting in each direction along the plate can be determined by using the continuity equation and a momentum balance. Calculate this flow division and the force on the wall.

A41: **Ans.** $m_2 = 0.5774$ kg/s, $m_3 = 0.09907$ kg/s, $-R_x = 2.030$ N, $-R_y = -2.030$ N (force on wall).

2.8-6. ***Momentum Balance for Free Jet on a Curved, Fixed Vane.*** A free jet having a velocity of 30.5 m/s and a diameter of 5.08×10^{-2} m is deflected by a curved, fixed vane as in Fig. 2.8-5a. However, the vane is curved downward at an angle of 60° instead of upward. Calculate the force of the jet on the vane. The density is 1000 kg/m^3.

A42: **Ans.** $-R_x = 942.8$ N, $-R_y = 1633$ N

2.8-7. ***Momentum Balance for Free Jet on a U-Type, Fixed Vane.*** A free jet having a velocity of 30.5 m/s and a diameter of 1.0×10^{-2} m is deflected by a smooth, fixed vane as in Fig. 2.8-5a. However, the vane is in the form of a U so that the exit jet travels in a direction exactly opposite to the entering jet. Calculate the force of the jet on the vane. Use $\rho = 1000$ kg/m^3.

A43: **Ans.** $-R_x = 146.1$ N, $-R_y = 0$

2.8-8. ***Momentum Balance on Reducing Elbow and Friction Losses.*** Water at 20°C is flowing through a reducing bend, where α_2 (see Fig. 2.8-3) is 120°. The inlet pipe diameter is 1.829 m, the outlet is 1.219 m, and the flow rate is 8.50 m^3/s. The exit point z_2 is 3.05 m above the inlet and the inlet pressure is 276 kPa gage. Friction losses are estimated as 0.5 $v_2^2/2$ and the mass of water in the elbow is 8500 kg. Calculate the forces R_x and R_y and the resultant force on the control-volume fluid.

2.8-9. ***Momentum Velocity Correction Factor β for Turbulent Flow.*** Determine the momentum velocity correction factor β for turbulent flow in a tube. Use Eq. (2.7-20) for the relationship between v and position.

2.9-1. ***Film of Water on Wetted-Wall Tower.*** Pure water at 20°C is flowing down a vertical wetted-wall column at a rate of 0.124 kg/s · m. Calculate the film thickness and the average velocity.

A46: **Ans.** $\delta = 3.370 \times 10^{-4}$ m, $v_{z\,av} = 0.3687$ m/s

2.9-2. ***Shell Momentum Balance for Flow Between Parallel Plates.*** A fluid of constant density is flowing in laminar flow at steady state in the horizontal x direction between two flat and parallel plates. The distance between the two plates in the vertical y direction is $2y_0$. Using a shell momentum balance, derive the equation for the velocity profile within this fluid and the maximum velocity for a distance L m in the x direction. [*Hint:* See the method used in Section 2.9B to derive Eq. (2.9-9). One boundary condition used is $dv_x/dy = 0$ at $y = 0$.]

A47:

$$v_x = \frac{p_0 - p_L}{2\mu L} y_0^2 \left[1 - \left(\frac{y}{y_0}\right)^2\right]$$

Ans.

2.9-3. ***Velocity Profile for Non-Newtonian Fluid.*** The stress rate of shear for a non-Newtonian fluid is given by

$$\tau_{rx} = K\left(-\frac{dv_x}{dr}\right)^n$$

where K and n are constants. Find the relation between velocity and radial position r for this incompressible fluid at steady state. [*Hint:* Combine the equation given here with Eq. (2.9-6). Then raise both sides of the resulting equation to the $1/n$ power and integrate.]

A48: **Ans.**

$$v_x = \frac{n}{n+1}\left(\frac{p_0 - p_L}{2KL}\right)^{1/n} (R_0)^{(n+1)/n}\left[1 - \left(\frac{r}{R_0}\right)^{(n+1)/n}\right]$$

2.9-4. ***Shell Momentum Balance for Flow Down an Inclined Plane.*** Consider the case of a Newtonian fluid in steady-state laminar flow down an inclined plane surface that makes an angle θ with the horizontal. Using a shell momentum balance, find the equation for the velocity profile within the liquid layer having a thickness L and the maximum velocity of the free surface. (*Hint:* The convective-momentum terms cancel for fully developed flow and the pressure-force terms also cancel, because of the presence of a free surface. Note that there is a gravity force on the fluid.)

A49: **Ans.** $v_{x\,\max} = \rho g L^2 \sin\theta / 2\mu$

2.10-1. ***Viscosity Measurement of a Liquid.*** One use of the Hagen–Poiseuille equation (2.10-2) is in determining the viscosity of a liquid by measuring the pressure drop and velocity of the liquid in a capillary of known dimensions. The liquid used has a density of 912 kg/m^3, and the capillary has a diameter of 2.222 mm and a length of 0.1585 m. The measured flow rate is 5.33×10^{-7} m^3/s of liquid and the pressure drop 131 mm of water (density 996 kg/m^3). Neglecting end effects, calculate the viscosity of the liquid in Pa · s.

A50: **Ans.** $\mu = 9.06 \times 10^{-3}$ Pa · s

2.10-2. ***Frictional Pressure Drop in Flow of Olive Oil.*** Calculate the frictional pressure drop in pascal for olive oil at 293 K flowing through a commercial pipe having an inside diameter of 0.0525 m and a length of 76.2 m. The velocity of the fluid is 1.22 m/s. Use the friction factor method. Is the flow laminar or turbulent? Use physical data from Appendix A.4.

2.10-3. ***Frictional Loss in Straight Pipe and Effect of Type of Pipe.*** A liquid having a density of 801 kg/m^3 and a viscosity of 1.49 × 10^{-3} Pa · s is flowing through a horizontal straight pipe at a velocity of 4.57 m/s. The commercial steel pipe is $1\frac{1}{2}$ in. nominal pipe size, schedule 40. For a length of pipe of 61 m, do as follows:

a. Calculate the friction loss F_f.

b. For a smooth tube of the same inside diameter, calculate the friction loss. What is the percent reduction of F_f for the smooth tube?

A52: **Ans.** (a) 348.9 J/kg; (b) 274.2 J/kg (91.7 ft · lb_f/lb_m), 21.4% reduction

2.10-4. ***Trial-and-Error Solution for Hydraulic Drainage.*** In a hydraulic project a cast-iron pipe having an inside diameter of 0.156 m and a 305-m length is used to drain wastewater at 293 K. The available head is 4.57 m of water. Neglecting any losses in fittings and joints in the pipe, calculate the flow rate in m^3/s. (*Hint:* Assume the physical properties of pure water. The solution is trial and error, since the velocity appears in N_{Re}, which is needed to determine the friction factor. As a first trial, assume that v = 1.7 m/s.)

2.10-5. ***Mechanical-Energy Balance and Friction Losses.*** Hot water is being discharged from a storage tank at the rate of 0.223 ft^3/s. The process flow diagram and conditions are the same as given in Example 2.10-6, except for different nominal pipe sizes of schedule 40 steel pipe as follows. The 20-ft-long outlet pipe from the storage tank is $1\frac{1}{2}$-in. pipe instead of 4-in. pipe. The other piping, which was 2-in. pipe, is now 2.5-in. pipe. Note that now a sudden expansion occurs after the elbow in the $1\frac{1}{2}$-in. pipe to a $2\frac{1}{2}$-in pipe.

2.10-6. ***Friction Losses and Pump Horsepower.*** Hot water in an open storage tank at 82.2°C is being pumped at the rate of 0.379 m^3/min from the tank. The line from the storage tank to the pump suction is 6.1 m of 2-in. schedule 40 steel pipe and it contains three elbows. The discharge line after the pump is 61 m of 2-in. pipe and contains two elbows. The water discharges to the atmosphere at a height of 6.1 m above the water level in the storage tank.

a. Calculate all frictional losses ΣF.

b. Make a mechanical-energy balance and calculate W_S of the pump in J/kg.

c. What is the kW power of the pump if its efficiency is 75%?

A55: **Ans.** (a) ΣF = 122.8 J/kg; (b) W_S = −186.9 J/kg; (c) 1.527 kW

2.10-7. ***Pressure Drop of a Flowing Gas.*** Nitrogen gas is flowing through a 4-in. schedule 40 commercial steel pipe at 298 K. The total flow rate is 7.40×10^{-2} kg/s and the flow can be assumed as isothermal. The pipe is 3000 m long and the inlet pressure is 200 kPa. Calculate the outlet pressure.

A56: **Ans.** $p_2 = 188.5$ kPa

2.10-8. ***Entry Length for Flow in a Pipe.*** Air at 10°C and 1.0 atm abs pressure is flowing at a velocity of 2.0 m/s inside a tube having a diameter of 0.012 m.

a. Calculate the entry length.
b. Calculate the entry length for water at 10°C and the same velocity.

2.10-9. ***Friction Loss in Pumping Oil to Pressurized Tank.*** An oil having a density of 833 kg/m^3 and a viscosity of 3.3×10^{-3} Pa · s is pumped from an open tank to a pressurized tank held at 345 kPa gage. The oil is pumped from an inlet at the side of the open tank through a line of commercial steel pipe having an inside diameter of 0.07792 m at the rate of 3.494×10^{-3} m^3/s. The length of straight pipe is 122 m, and the pipe contains two elbows (90°) and a globe valve half open. The level of the liquid in the open tank is 20 m above the liquid level in the pressurized tank. The pump efficiency is 65%. Calculate the kW power of the pump.

2.10-10. ***Flow in an Annulus and Pressure Drop.*** Water flows in the annulus of a horizontal, concentric-pipe heat exchanger and is being heated from 40°C to 50°C in the exchanger, which has a length of 30 m of equivalent straight pipe. The flow rate of the water is 2.90×10^{-3} m^3/s. The inner pipe is 1-in. schedule 40 and the outer is 2-in. schedule 40. What is the pressure drop? Use an average temperature of 45°C for bulk physical properties. Assume that the wall temperature is an average of 4°C higher than the average bulk temperature so that a correction can be made for the effect of heat transfer on the friction factor.

2.11-1. ***Derivation of Maximum Velocity for Isothermal Compressible Flow.*** Starting with Eq. (2.11-9), derive Eqs. (2.11-11) and (2.11-12) for the maximum velocity in isothermal compressible flow.

2.11-2. ***Pressure Drop in Compressible Flow.*** Methane gas is being pumped through a 305-m length of 52.5-mm-ID steel pipe at the rate of 41.0 kg/m^2 · s. The inlet pressure is $p_1 = 345$ kPa abs. Assume isothermal flow at 288.8 K.

a. Calculate the pressure p_2 at the end of the pipe. The viscosity is 1.04×10^{-5} Pa · s.
b. Calculate the maximum velocity that can be attained at these conditions and compare with the velocity in part (a).

A61: **Ans.** (a) $p_2 = 298.4$ kPa; (b) $v_{max} = 387.4$ m/s, $v_2 = 20.62$ m/s

2.11-3. ***Pressure Drop in Isothermal Compressible Flow.*** Air at 288 K and 275 kPa abs enters a pipe and is flowing in isothermal compressible flow in a commercial pipe having an ID of 0.080 m. The length of the pipe is 60 m. The mass velocity at the entrance to the pipe is 165.5 kg/m^2 · s. Assume 29 for the molecular weight of air. Calculate the pressure at the exit. Also, calculate the maximum allowable velocity that can be attained and compare with the actual.

REFERENCES

Bibliography

[ch02biblio01entry01] (B1) C. O., Bennett, and J. E. Meyers, *Momentum, Heat and Mass Transfer*, 3rd ed. New York: McGraw-Hill Book Company, 1982.

[ch02biblio01entry02] (D1) B. F. Dodge, *Chemical Engineering Thermodynamics*. New York: McGraw-Hill Book Company, 1944.

[ch02biblio01entry03] (D2) A. A. Durand,, et al. *Chem. Eng.*, **106** (May), 153 (1999).

[ch02biblio01entry04] (E1) R. L. Earle, *Unit Operations in Food Processing*. Oxford: Pergamon Press, Inc., 1966.

[ch02biblio01entry05] (K1) C. P., Kittridge, and D. S. Rowley, *Trans. A.S.M.E.*, **79**, 1759 (1957).

[ch02biblio01entry06] (L1) N. A. Lange, *Handbook of Chemistry*, 10th ed. New York: McGraw-Hill Book Company, 1967.

[ch02biblio01entry07] (L2) H. L. Langhaar, *Trans. A.S.M.E*, **64**, A-55 (1942).

[ch02biblio01entry08] (M1) L. F. Moody, *Trans. A.S.M.E.*, **66**, 671 (1944); *Mech. Eng.*, **69**, 1005 (1947).

[ch02biblio01entry09] (M2) W. L., McCabe, J. C., Smith, and P. Harriott, *Unit Operations of Chemical Engineering*, 4th ed. New York: McGraw-Hill Book Company, 1985.

[ch02biblio01entry10] (N1) National Bureau of Standards. *Tables of Thermal Properties of Gases*, Circular 464 (1955).

[ch02biblio01entry11] (P1) R. H., Perry, and D. Green, *Perry's Chemical Engineers' Handbook*, 6th ed. New York: McGraw-Hill Book Company, 1984.

[ch02biblio01entry12] (R1) R. C., Reid, J. M., Prausnitz, and T. K. Sherwood, *The Properties of Gases and Liquids*, 3rd ed. New York: McGraw-Hill Book Company, 1977.

[ch02biblio01entry13] (R2) *Reactor Handbook*, vol. 2, AECD-3646. Washington D.C.: Atomic Energy Commission, May 1955.

[ch02biblio01entry14] (S1) J. F., Swindells, J. R., Coe,, Jr., and T. B. Godfrey, J. *Res. Nat. Bur. Standards*, **48**, 1 (1952).

[ch02biblio01entry15] (S2) A.H. P. Skelland, *Non-Newtonian Flow and Heat Transfer*. New York: John Wiley & Sons, Inc., 1967.

[ch02biblio01entry16] (S3) E. N., Sieder, and G. E. Tate, *Ind. Eng. Chem.*, **28**, 1429 (1936).

[ch02biblio01entry17] (W1) R. C. Weast, *Handbook of Chemistry and Physics*, 48th ed. Boca Raton, Fla.: Chemical Rubber Co., Inc., 1967–1968.