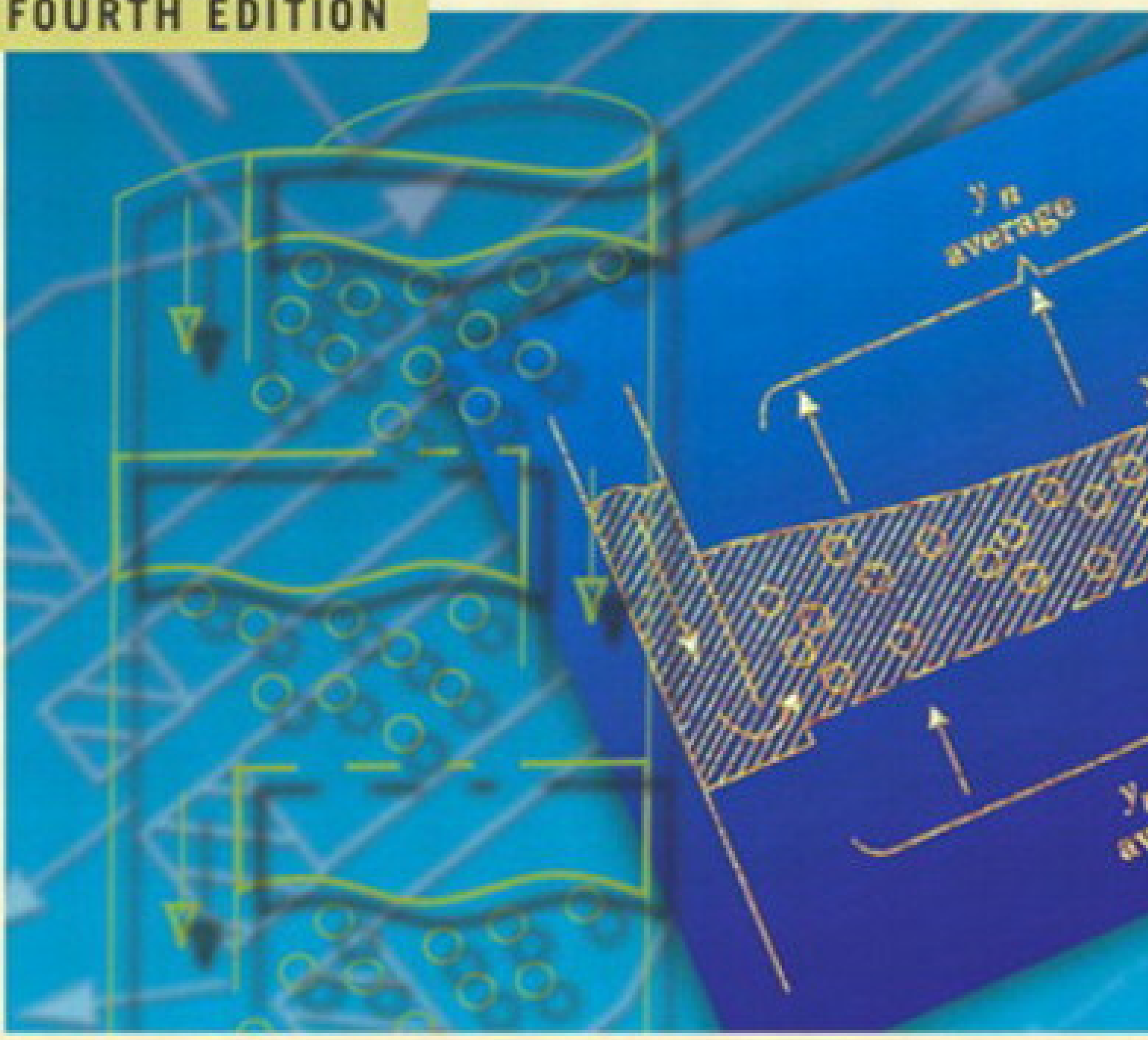
Transport Processes and Separation Process Principles

(INCLUDES UNIT OPERATIONS)

FOURTH EDITION



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Chapter 4. Principles of Steady-State Heat Transfer

INTRODUCTION AND MECHANISMS OF HEAT TRANSFER

Introduction to Steady-State Heat Transfer

The transfer of energy in the form of heat occurs in many chemical and other types of processes. Heat transfer often occurs in combination with other separation processes, such as drying of lumber or foods, alcohol distillation, burning of fuel, and evaporation. The heat transfer occurs because of a temperature-difference driving force and heat flows from the high to the low-temperature region.

In Section 2.3 we derived an equation for a general property balance of momentum, thermal energy, or mass at unsteady state by writing Eq. (2.3-7). Writing a similar equation but specifically for heat transfer,

Equation 4.1-1.

$$\begin{pmatrix}\text{rate of}\\ \text{heat in}\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{generation of heat}\end{pmatrix} = \begin{pmatrix}\text{rate of}\\ \text{heat out}\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{accumulation of heat}\end{pmatrix}$$

Assuming the rate of transfer of heat occurs only by conduction, we can rewrite Eq. (2.3-14), which is *Fourier's law,* as

Equation 4.1-2.

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

Making an unsteady-state heat balance for the x direction only on the element of volume or control volume in Fig. 4.1-1 by using Eqs. (4.1-1) and (4.1-2), with the cross-sectional area being A m^2,

Equation 4.1-3.

$$q_{x|x} + \dot{q}(\Delta x \cdot A) = q_{x|x+\Delta x} + \rho c_p \frac{\partial T}{\partial t}(\Delta x \cdot A)$$

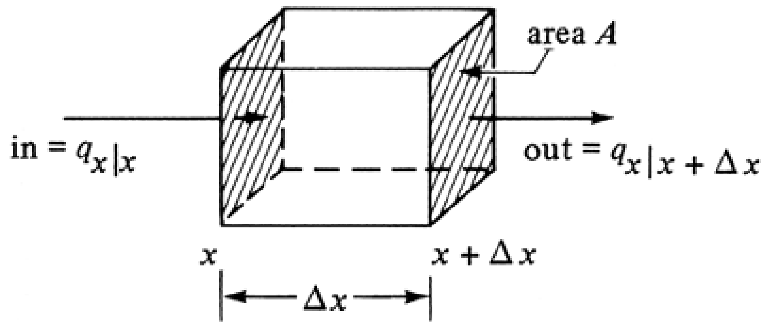


Figure 4.1-1. Unsteady-state balance for heat transfer in control volume.

where $\dot{q}$ is rate of heat generated per unit volume. Assuming no heat generation and also assuming steady-state heat transfer, where the rate of accumulation is zero, Eq. (4.1-3) becomes

Equation 4.1-4.

$$q_{x|x} = q_{x|x+\Delta x}$$

This means the rate of heat input by conduction = the rate of heat output by conduction; or q_x is a constant with time for steady-state heat transfer.

In this chapter we are concerned with a control volume where the rate of accumulation of heat is zero and we have steady-state heat transfer. The rate of heat transfer is then constant with time, and the temperatures at various points in the system do not change with time. To solve problems in steady-state heat transfer, various mechanistic expressions in the form of differential equations for the different modes of heat transfer such as Fourier's law are integrated. Expressions for the temperature profile and heat flux are then obtained in this chapter.

In Chapter 5 the conservation-of-energy equations (2.7-2) and (4.1-3) will be used again when the rate of accumulation is not zero and unsteady-state heat transfer occurs. The mechanistic expression for Fourier's law in the form of a partial differential equation will be used where the temperature at various points and the rate of heat transfer change with time. In Section 5.6 a general differential equation of energy change will be derived and integrated for various specific cases to determine the temperature profile and heat flux.

Basic Mechanisms of Heat Transfer

Heat transfer may occur by any one or more of the three basic mechanisms of heat transfer: conduction, convection, and radiation.

Conduction

In conduction, heat can be conducted through solids, liquids, and gases. The heat is conducted by the transfer of the energy of motion between adjacent molecules. In a gas the "hotter" molecules, which have greater energy and motions, impart energy to the adjacent molecules at lower energy levels. This type of transfer is present to some extent in all solids, gases, or liquids in which a temperature gradient exists. In conduction, energy can also be transferred by "free" electrons, which is quite important in metallic solids. Examples of heat transfer mainly by conduction are heat transfer through walls of exchangers or a refrigerator, heat treatment of steel forgings, freezing of the ground during the winter, and so on.

Convection

The transfer of heat by convection implies the transfer of heat by bulk transport and mixing of macroscopic elements of warmer portions with cooler portions of a gas or liquid. It also often refers to the energy exchange between a solid surface and a fluid. A distinction must be made between forced-convection heat transfer, where a fluid is forced to flow past a solid surface by a pump, fan, or other mechanical means, and natural or free convection, where warmer or cooler fluid next to the solid surface causes a circulation because of a density difference resulting from the temperature differences in the fluid. Examples of heat transfer by convection are loss of heat from a car radiator where the air is being circulated by a fan, cooking of foods in a vessel being stirred, cooling of a hot cup of coffee by blowing over the surface, and so on.

Radiation

Radiation differs from heat transfer by conduction and convection in that no physical medium is needed for its propagation. Radiation is the transfer of energy through space by means of electromagnetic waves in much the same way as electromagnetic light waves transfer light. The same laws that govern the transfer of light govern the radiant transfer of heat. Solids and liquids tend to absorb the radiation being transferred through them, so that radiation is important primarily in transfer through space or gases. The most important example of radiation is the transport of heat to the earth from the sun. Other examples are cooking of food when passed below red-hot electric heaters, heating of fluids in coils of tubing inside a combustion furnace, and so on.

Fourier's Law of Heat Conduction

As discussed in Section 2.3 for the general molecular transport equation, all three main types of rate-transfer processes—momentum transfer, heat transfer, and mass transfer—are characterized by the same general type of equation. The transfer of electric current can also be included in this category. This basic equation is as follows:

Equation 2.3-1.

$$\text{rate of a transfer process} = \frac{\text{driving force}}{\text{resistance}}$$

This equation states what we know intuitively: that in order to transfer a property such as heat or mass, we need a driving force to overcome a resistance.

The transfer of heat by conduction also follows this basic equation and is written as Fourier's law for heat conduction in fluids or solids:

Equation 4.1-2.

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

where q_x is the heat-transfer rate in the x direction in watts (W), A is the cross-sectional area normal to the direction of flow of heat in m^2, T is temperature in K, x is distance in m, and k is the thermal conductivity in W/m · K in the SI system. The quantity q_x/A is called the heat flux in W/m^2. The quantity dT/dx is the temperature gradient in the x direction. The minus sign in Eq. (4.1-2) is required because if the heat flow is positive in a given direction, the temperature decreases in this direction.

The units in Eq. (4.1-2) may also be expressed in the cgs system, with q_x in cal/s, A in cm^2, k in cal/s · °C · cm, T in °C, and x in cm. In the English system, q_x is in btu/h, A in ft^2, T in °F, x in ft, k in btu/h · °F · ft, and q_x/A in btu/h · ft^2. From Appendix A.1, the conversion factors are, for thermal conductivity,

Equation 4.1-5.

$$1\ \text{btu/h} \cdot \text{ft} \cdot {}^\circ\text{F} = 4.1365 \times 10^{-3}\ \text{cal/s} \cdot \text{cm} \cdot {}^\circ\text{C}$$

Equation 4.1-6.

$$1\ \text{btu/h} \cdot \text{ft} \cdot {}^\circ\text{F} = 1.73073\ \text{W/m} \cdot \text{K}$$

For heat flux and power,

Equation 4.1-7.

$$1\ \text{btu/h} \cdot \text{ft}^2 = 3.1546\ \text{W/m}^2$$

Equation 4.1-8.

$$1\ \text{btu/h} = 0.29307\ \text{W}$$

Fourier's law, Eq. (4.1-2), can be integrated for the case of steady-state heat transfer through a flat wall of constant cross-sectional area *A,* where the inside temperature is T_1 at point 1 and T_2 at point 2, a distance of $x_2 - x_1$ m away. Rearranging Eq. (4.1-2),

Equation 4.1-9.

$$\frac{q_x}{A} \int_{x_1}^{x_2} dx = -k \int_{T_1}^{T_2} dT$$

Integrating, assuming that k is constant and does not vary with temperature and dropping the subscript x on q_x for convenience,

Equation 4.1-10.

$$\frac{q}{A} = \frac{k}{x_2 - x_1}(T_1 - T_2)$$

EXAMPLE 4.1-1. Heat Loss Through an Insulating Wall

Calculate the heat loss per m^2 of surface area for an insulating wall composed of 25.4-mm-thick fiber insulating board, where the inside temperature is 352.7 K and the outside temperature is 297.1 K.

Solution: From Appendix A.3, the thermal conductivity of fiber insulating board is 0.048 W/m · K. The thickness $x_2 - x_1 = 0.0254$ m. Substituting into Eq. (4.1-10),

$$\frac{q}{A} = \frac{k}{x_2 - x_1}(T_1 - T_2) = \frac{0.048}{0.0254}(352.7 - 297.1)$$

$$= 105.1\ \text{W/m}^2$$

$$= (105.1\ \text{W/m}^2)\frac{1}{(3.1546\ \text{W/m}^2)/(\text{btu/h} \cdot \text{ft}^2)} = 33.30\ \text{btu/h} \cdot \text{ft}^2$$

Thermal Conductivity

The defining equation for thermal conductivity is given as Eq. (4.1-2), and with this definition, experimental measurements have been made to determine the thermal conductivity of different materials. In Table 4.1-1 thermal conductivities are given for a few materials for the purpose of comparison. More-detailed data are given in Appendix A.3 for inorganic and organic materials and Appendix A.4 for food and biological materials. As seen in Table 4.1-1, gases have quite low values of thermal conductivity, liquids intermediate values, and solid metals very high values.

Table 4.1-1. Thermal Conductivities of Some Materials at 101.325 kPa (1 Atm) Pressure (k in W/m · K)

Substance	*Temp. (K)*	*k*	*Ref.*	*Substance*	*Temp. (K)*	*k*	*Ref.*
Gases				Solids			
Air	273	0.0242	(K2)	Ice	273	2.25	(C1)
	373	0.0316		Fire claybrick	473	1.00	(P1)
H_2	273	0.167	(K2)	Paper	—	0.130	(M1)
n-Butane	273	0.0135	(P2)	Hard rubber	273	0.151	(M1)
Liquids				Cork board	303	0.043	(M1)
Water	273	0.569	(P1)	Asbestos	311	0.168	(M1)
	366	0.680		Rock wool	266	0.029	(K1)
Benzene	303	0.159	(P1)	Steel	291	45.3	(P1)
	333	0.151			373	45	
Biological materials and foods				Copper	273	388	(P1)
					373	377	
Olive oil	293	0.168	(P1)	Aluminum	273	202	(P1)
	373	0.164					
Lean beef	263	1.35	(C1)				
Skim milk	275	0.538	(C1)				
Applesauce	296	0.692	(C1)				
Salmon	277	0.502	(C1)				
	248	1.30					

Gases

In gases the mechanism of thermal conduction is relatively simple. The molecules are in continuous random motion, colliding with one another and exchanging energy and momentum. If a molecule moves from a high-temperature region to a region of lower temperature, it transports kinetic energy to this region and gives up this energy through collisions with lower-energy molecules. Since smaller molecules move faster, gases such as hydrogen should have higher thermal conductivities, as shown in Table 4.1-1.

Theories to predict thermal conductivities of gases are reasonably accurate and are given elsewhere (R1). The thermal conductivity increases approximately as the square root of the absolute temperature and is independent of pressure up to a few atmospheres. At very low pressures (vacuum), however, the thermal conductivity approaches zero.

Liquids

The physical mechanism of conduction of energy in liquids is somewhat similar to that of gases, where higher-energy molecules collide with lower-energy molecules. However, the molecules are packed so closely together that molecular force fields exert a strong effect on the energy exchange. Since an adequate molecular theory of liquids is not available, most correlations to predict the thermal conductivities are empirical. Reid et al. (R1) discuss these in detail. The thermal conductivity of liquids varies moderately with temperature and often can be expressed as a linear variation.

Equation 4.1-11.

$$k = a + bT$$

where a and b are empirical constants. Thermal conductivities of liquids are essentially independent of pressure.

Water has a high thermal conductivity compared to organic-type liquids such as benzene. As shown in Table 4.1-1, the thermal conductivities of most unfrozen foodstuffs, such as skim milk and applesauce, which contain large amounts of water have thermal conductivities near that of pure water.

Solids

The thermal conductivity of homogeneous solids varies quite widely, as may be seen for some typical values in Table 4.1-1. The metallic solids of copper and aluminum have very high thermal conductivities, while some insulating nonmetallic materials such as rock wool and corkboard have very low conductivities.

Heat or energy is conducted through solids by two mechanisms. In the first, which applies primarily to metallic solids, heat, like electricity, is conducted by free electrons which move through the metal lattice. In the second mechanism, present in all solids, heat is conducted by the transmission of energy of vibration between adjacent atoms.

Thermal conductivities of insulating materials such as rock wool approach that of air since the insulating materials contain large amounts of air trapped in void spaces. Superinsulations to insulate cryogenic materials such as liquid hydrogen are composed of multiple layers of highly reflective materials separated by evacuated insulating spacers. Values of thermal conductivity are considerably lower than for air alone.

Ice has a thermal conductivity much greater than water. Hence, the thermal conductivities of frozen foods such as lean beef and salmon given in Table 4.1-1 are much higher than for unfrozen foods.

Convective Heat-Transfer Coefficient

It is well known that a hot piece of material will cool faster when air is blown or forced past the object. When the fluid outside the solid surface is in forced or natural convective motion, we express the rate of heat transfer from the solid to the fluid, or vice versa, by the following equation:

Equation 4.1-12.

$$q = hA(T_w - T_f)$$

where q is the heat-transfer rate in W, A is the area in m^2, T_w is the temperature of the solid surface in K, T_f is the average or bulk temperature of the fluid flowing past in K, and h is the convective heat-transfer coefficient in $W/m^2 \cdot K$. In English units, h is in $btu/h \cdot ft^2 \cdot °F$.

The coefficient h is a function of the system geometry, fluid properties, flow velocity, and temperature difference. In many cases, empirical correlations are available to predict this coefficient, since it often cannot be predicted theoretically. Since we know that when a fluid flows past a surface there is a thin, almost stationary layer or film of fluid adjacent to the wall which presents most of the resistance to heat transfer, we often call the coefficient h a *film coefficient.*

In Table 4.1-2 some order-of-magnitude values of *h* for different convective mechanisms of free or natural convection, forced convection, boiling, and condensation are given. Water gives the highest values of the heat-transfer coefficients.

Table 4.1-2. Approximate Magnitude of Some Heat-Transfer Coefficients

	Range of Values of h	
Mechanism	**btu/h · ft² · °F**	**W/m² · K**
Condensing steam	1000–5000	5700–28000
Condensing organics	200–500	1100–2800
Boiling liquids	300–5000	1700–28000
Moving water	50–3000	280–17000
Moving hydrocarbons	10–300	55–1700
Still air	0.5–4	2.8–23
Moving air	2–10	11.3–55

To convert the heat-transfer coefficient *h* from English to SI units,

$$1 \text{ btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} = 5.6783 \text{ W/m}^2 \cdot \text{K}$$

CONDUCTION HEAT TRANSFER

Conduction Through a Flat Slab or Wall

In this section Fourier's equation (4.1-2) will be used to obtain equations for one-dimensional steady-state conduction of heat through some simple geometries. For a flat slab or wall where the cross-sectional area *A* and *k* in Eq. (4.1-2) are constant, we obtained Eq. (4.1-10), which we rewrite as

Equation 4.2-1.

$$\frac{q}{A} = \frac{k}{x_2 - x_1}(T_1 - T_2) = \frac{k}{\Delta x}(T_1 - T_2)$$

This is shown in Fig. 4.2-1, where $\Delta x = x_2 - x_1$. Equation (4.2-1) indicates that if *T* is substituted for T_2 and *x* for X_2, the temperature varies linearly with distance, as shown in Fig. 4.2-1b.

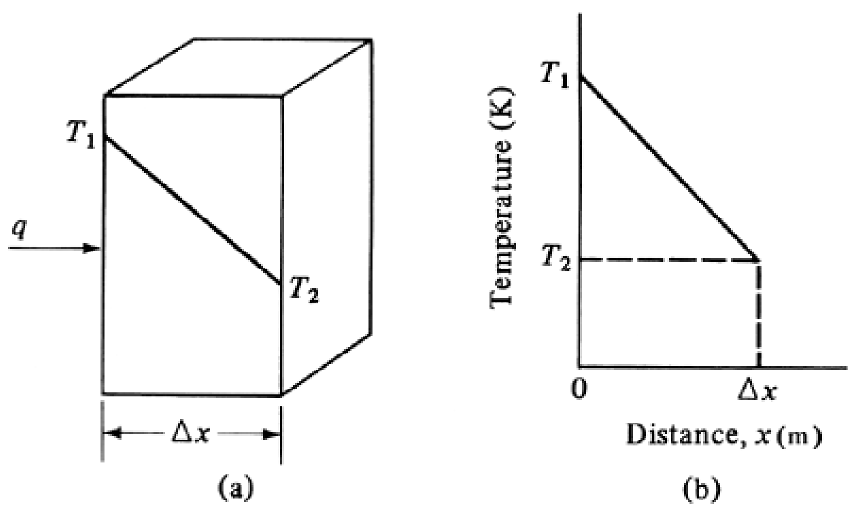


Figure 4.2-1. Heat conduction in a flat wall: (a) geometry of wall, (b) temperature plot.

If the thermal conductivity is not constant but varies linearly with temperature, then substituting Eq. (4.1-11) into Eq. (4.1-2) and integrating,

Equation 4.2-2.

$$\frac{q}{A} = \frac{a + b\dfrac{T_1 + T_2}{2}}{\Delta x}(T_1 - T_2) = \frac{k_m}{\Delta x}(T_1 - T_2)$$

where

Equation 4.2-3.

$$k_m = a + b\frac{T_1 + T_2}{2}$$

This means that the mean value of k (i.e., k_m) to use in Eq. (4.2-2) is the value of k evaluated at the linear average of T_1 and T_2.

As stated in the introduction to transport processes in Eq. (2.3-1), the rate of a transfer process equals the driving force over the resistance. Equation (4.2-1) can be rewritten in that form:

Equation 4.2-4.

$$q = \frac{T_1 - T_2}{\Delta x/kA} = \frac{T_1 - T_2}{R} = \frac{\text{driving force}}{\text{resistance}}$$

where $R = \Delta x/kA$ and is the resistance in K/W or h · °F/btu.

Conduction Through a Hollow Cylinder

In many instances in the process industries, heat is being transferred through the walls of a thick-walled cylinder, such as a pipe that may or may not be insulated. Consider the hollow cylinder in Fig. 4.2-2 with an inside radius of r_1, where the temperature is T_1, an outside radius of r_2 having a temperature of T_2, and a length of L m. Heat is flowing radially from the inside surface to the outside. Rewriting Fourier's law, Eq. (4.1-2), with distance dr instead of dx,

Equation 4.2-5.

$$\frac{q}{A} = -k\frac{dT}{dr}$$

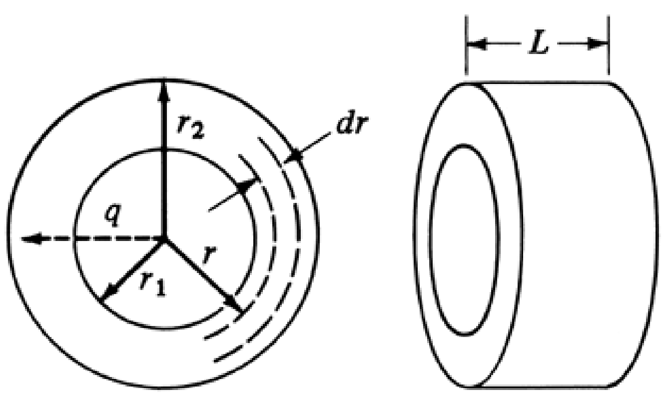


Figure 4.2-2. Heat conduction in a cylinder.

The cross-sectional area normal to the heat flow is

Equation 4.2-6.

$$A = 2\pi rL$$

Substituting Eq. (4.2-6) into (4.2-5), rearranging, and integrating,

Equation 4.2-7.

$$\frac{q}{2\pi L}\int_{r_1}^{r_2}\frac{dr}{r} = -k\int_{T_1}^{T_2} dT$$

Equation 4.2-8.

$$q = k\frac{2\pi L}{\ln(r_2/r_1)}(T_1 - T_2)$$

Multiplying numerator and denominator by (r_2 - r_1),

Equation 4.2-9.

$$q = kA_{\text{lm}}\frac{T_1 - T_2}{r_2 - r_1} = \frac{T_1 - T_2}{(r_2 - r_1)/(kA_{\text{lm}})} = \frac{T_1 - T_2}{R}$$

where

Equation 4.2-10.

$$A_{\text{lm}} = \frac{(2\pi Lr_2) - (2\pi Lr_1)}{\ln(2\pi Lr_2/2\pi Lr_1)} = \frac{A_2 - A_1}{\ln(A_2/A_1)}$$

Equation 4.2-11.

$$R = \frac{r_2 - r_1}{kA_{\text{lm}}} = \frac{\ln(r_2/r_1)}{2\pi kL}$$

The log mean area is A_{lm}. In engineering practice, if $A_2/A_1 < 1.5/1$, the linear mean area of $(A_1 + A_2)/2$ is within 1.5% of the log mean area. From Eq. (4.2-8), if *r* is substituted for r_2 and *T* for T_2, the temperature is seen to be a linear function of ln *r* instead of *r* as in the case of a flat wall. If the thermal conductivity varies with temperature as in Eq. (4.1-10), it can be shown that the mean value to use in a cylinder is still k_m of Eq. (4.2-3).

EXAMPLE 4.2-1. Length of Tubing for Cooling Coil

A thick-walled cylindrical tubing of hard rubber having an inside radius of 5 mm and an outside radius of 20 mm is being used as a temporary cooling coil in a bath. Ice water is flowing rapidly inside, and the inside wall temperature is 274.9 K. The outside surface temperature is 297.1 K. A total of 14.65 W must be removed from the bath by the cooling coil. How many m of tubing are needed?

Solution: From Appendix A.3, the thermal conductivity at 0°C (273 K) is $k = 0.151$ W/m · K. Since data at other temperatures are not available, this value will be used for the range of 274.9 to 297.1 K.

$$r_1 = \frac{5}{1000} = 0.005 \text{ m} \qquad r_2 = \frac{20}{1000} = 0.02 \text{ m}$$

The calculation will be done first for a length of 1.0 m of tubing. Solving for the areas A_1, A_2, and A_{lm} in Eq. (4.2-10),

$$A_1 = 2\pi L r_1 = 2\pi(1.0)(0.005) = 0.0314 \text{ m}^2 \qquad A_2 = 0.1257 \text{ m}^2$$

$$A_{\text{lm}} = \frac{A_2 - A_1}{\ln(A_2/A_1)} = \frac{0.1257 - 0.0314}{\ln(0.1257/0.0314)} = 0.0680 \text{ m}^2$$

Substituting into Eq. (4.2-9) and solving,

$$q = kA_{\text{lm}}\frac{T_1 - T_2}{r_2 - r_1} = 0.151(0.0682)\left(\frac{274.9 - 297.1}{0.02 - 0.005}\right)$$

$$= -15.2 \text{ W } (51.9 \text{ btu/h})$$

The negative sign indicates that the heat flow is from r_2 on the outside to r_1 on the inside. Since 15.2 W is removed for a 1-m length, the needed length is

$$\text{length} = \frac{14.65 \text{ W}}{15.2 \text{ W/m}} = 0.964 \text{ m}$$

Note that the thermal conductivity of rubber is quite small. Generally, metal cooling coils are used, since the thermal conductivity of metals is quite high. The liquid film resistances in this case are quite small and are neglected.

Conduction Through a Hollow Sphere

Heat conduction through a hollow sphere is another case of one-dimensional conduction. Using Fourier's law for constant thermal conductivity with distance *dr,* where *r* is the radius of the sphere,

Equation 4.2-5.

$$\frac{q}{A} = -k\frac{dT}{dr}$$

The cross-sectional area normal to the heat flow is

Equation 4.2-12.

$$A = 4\pi r^2$$

Substituting Eq. (4.2-12) into (4.2-5), rearranging, and integrating,

Equation 4.2-13.

$$\frac{q}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -k\int_{T_1}^{T_2} dt$$

Equation 4.2-14.

$$q = \frac{4\pi k(T_1 - T_2)}{1/r_1 - 1/r_2} = \frac{T_1 - T_2}{(1/r_1 - 1/r_2)/4\pi k}$$

It can easily be shown that the temperature varies hyperbolically with the radius. (See Problem 4.2-5.)

CONDUCTION THROUGH SOLIDS IN SERIES

Plane Walls in Series

In the case where there is a multilayer wall of more than one material present, as shown in Fig. 4.3-1, we proceed as follows. The temperature profiles in the three materials *A, B,* and *C are* shown. Since the heat flow *q* must be the same in each layer, we can write Fourier's equation for each layer as

Equation 4.3-1.

$$q = \frac{k_A A}{\Delta x_A}(T_1 - T_2) = \frac{k_B A}{\Delta x_B}(T_2 - T_3) = \frac{k_C A}{\Delta x_C}(T_3 - T_4)$$

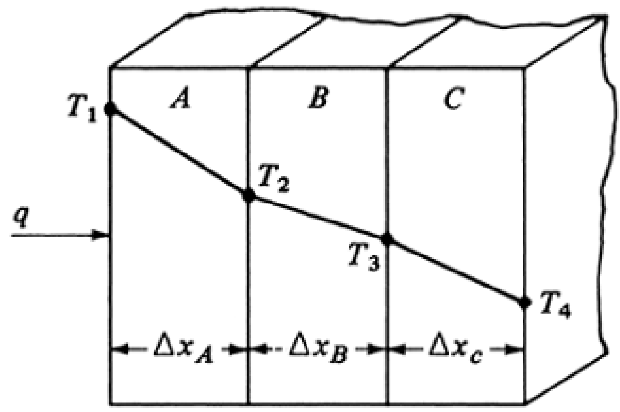


Figure 4.3-1. Heat flow through a multilayer wall.

Solving each equation for ΔT,

Equation 4.3-2.

$$T_1 - T_2 = q\frac{\Delta x_A}{k_A A} \qquad T_2 - T_3 = q\frac{\Delta x_B}{k_B A} \qquad T_3 - T_4 = q\frac{\Delta x_C}{k_C A}$$

Adding the equations for $T_1 - T_2$, $T_2 - T_3$, and $T_3 - T_4$, the internal temperatures T_2 and T_3 drop out and the final rearranged equation is

Equation 4.3-3.

$$q = \frac{T_1 - T_4}{\Delta x_A/(k_A A) + \Delta x_B/(k_B A) + \Delta x_C/(k_C A)} = \frac{T_1 - T_4}{R_A + R_B + R_C}$$

where the resistance $R_A = \Delta x_A/k_A A$, and so on.

Hence, the final equation is in terms of the overall temperature drop, $T_1 - T_4$, and the total resistance, $R_A + R_B + R_C$.

EXAMPLE 4.3-1. Heat Flow Through an Insulated Wall of a Cold Room

A cold-storage room is constructed of an inner layer of 12.7 mm of pine, a middle layer of 101.6 mm of cork board, and an outer layer of 76.2 mm of concrete. The wall surface temperature is 255.4 K inside the cold room and 297.1 K at the outside surface of the concrete. Use conductivities from Appendix A.3 for pine, 0.151; for cork board, 0.0433; and for concrete, 0.762 W/m · K. Calculate the heat loss in W for 1 m^2 and the temperature at the interface between the wood and cork board.

Solution. Calling T_1 = 255.4, T_4 = 297.1 K, pine as material *A,* cork as *B,* and concrete as *C,* a tabulation of the properties and dimensions is as follows:

$$k_A = 0.151 \qquad k_B = 0.0433 \qquad k_C = 0.762$$

$$\Delta x_A = 0.0127 \text{ m} \qquad \Delta x_B = 0.1016 \text{ m} \qquad \Delta x_C = 0.0762 \text{ m}$$

The resistances for each material are, from Eq. (4.3-3), for an area of 1 m^2,

$$R_A = \frac{\Delta x_A}{k_A A} = \frac{0.0127}{0.151(1)} = 0.0841 \text{ K/W}$$

$$R_B = \frac{\Delta x_B}{k_B A} = \frac{0.1016}{0.0433(1)} = 2.346 \text{ K/W}$$

$$R_C = \frac{\Delta x_C}{k_C A} = \frac{0.0762}{0.762(1)} = 0.100 \text{ K/W}$$

Substituting into Eq. (4.3-3),

$$q = \frac{T_1 - T_4}{R_A + R_B + R_C} = \frac{255.4 - 297.1}{0.0841 + 2.346 + 0.100}$$

$$= \frac{-41.7}{2.530} = -16.48 \text{ W } (-56.23 \text{ btu/h})$$

Since the answer is negative, heat flows in from the outside.

To calculate the temperature T_2 at the interface between the pine wood and cork,

$$q = \frac{T_1 - T_2}{R_A}$$

Substituting the known values and solving,

$$-16.48 = \frac{255.4 - T_2}{0.0841} \quad \text{and} \quad T_2 = 256.79 \text{ K at the interface}$$

An alternative procedure for calculating T_2 is to use the fact that the temperature drop is proportional to the resistance:

Equation 4.3-4.

$$T_1 - T_2 = \frac{R_A}{R_A + R_B + R_C}(T_1 - T_4)$$

Substituting,

$$255.4 - T_2 = \frac{0.0841(255.4 - 297.1)}{2.530} = -1.39 \text{ K}$$

Hence, T_2 = 256.79 K, as calculated before.

Multilayer Cylinders

In the process industries, heat transfer often occurs through multilayers of cylinders, as for example when heat is being transferred through the walls of an insulated pipe. Figure 4.3-2 shows a pipe with two layers of insulation around it, that is, a total of three concentric hollow cylinders. The temperature drop is $T_1 - T_2$ across material *A,* $T_2 - T_3$ across *B,* and $T_3 - T_4$ across *C.*

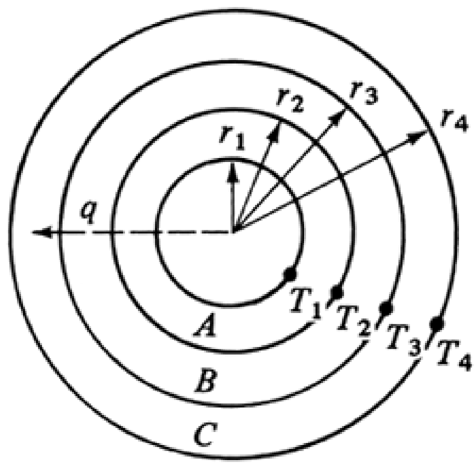


Figure 4.3-2. Radial heat flow through multiple cylinders in series.

The heat-transfer rate *q* will, of course, be the same for each layer, since we are at steady state. Writing an equation similar to Eq. (4.2-9) for each concentric cylinder,

Equation 4.3-5.

$$q = \frac{T_1 - T_2}{(r_2 - r_1)/(k_A A_{A\,\text{lm}})} = \frac{T_2 - T_3}{(r_3 - r_2)/(k_B A_{B\,\text{lm}})} = \frac{T_3 - T_4}{(r_4 - r_3)/(k_C A_{C\,\text{lm}})}$$

where

Equation 4.3-6.

$$A_{A\,\text{lm}} = \frac{A_2 - A_1}{\ln(A_2/A_1)} \qquad A_{B\,\text{lm}} = \frac{A_3 - A_2}{\ln(A_3/A_2)} \qquad A_{C\,\text{lm}} = \frac{A_4 - A_3}{\ln(A_4/A_3)}$$

Using the same method of combining the equations to eliminate T_2 and T_3 as was done for the flat walls in series, the final equations are

Equation 4.3-7.

$$q = \frac{T_1 - T_4}{(r_2 - r_1)/(k_A A_{A\,\text{lm}}) + (r_3 - r_2)/(k_B A_{B\,\text{lm}}) + (r_4 - r_3)/(k_C A_{C\,\text{lm}})}$$

Equation 4.3-8.

$$q = \frac{T_1 - T_4}{R_A + R_B + R_C} = \frac{T_1 - T_4}{\sum R}$$

Hence, the overall resistance is again the sum of the individual resistances in series.

EXAMPLE 4.3-2. Heat Loss from an Insulated Pipe

A thick-walled tube of stainless steel *(A)* having a $k = 21.63$ W/m · K with dimensions of 0.0254 m ID and 0.0508 m OD is covered with a 0.0254-m-thick layer of an insulation (*B*), $k = 0.2423$ W/m · K. The inside-wall temperature of the pipe is 811 K and the outside surface of the insulation is at 310.8 K. For a 0.305-m length of pipe, calculate the heat loss and also the temperature at the interface between the metal and the insulation.

Solution: Calling T_1 = 811 K, T_2 the interface, and T_3 = 310.8 K, the dimensions are

$$r_1 = \frac{0.0254}{2} = 0.0127 \text{ m} \qquad r_2 = \frac{0.0508}{2} = 0.0254 \text{ m} \qquad r_3 = 0.0508 \text{ m}$$

The areas are as follows for L = 0.305 m:

$$A_1 = 2\pi L r_1 = 2\pi(0.305)(0.0127) = 0.0243 \text{ m}^2$$

$$A_2 = 2\pi L r_2 = 2\pi(0.305)(0.0254) = 0.0487 \text{ m}^2$$

$$A_3 = 2\pi L r_3 = 2\pi(0.305)(0.0508) = 0.0974 \text{ m}^2$$

From Eq. (4.3-6), the log mean areas for the stainless steel *(A)* and insulation *(B)* are

$$A_{A\,\text{lm}} = \frac{A_2 - A_1}{\ln(A_2/A_1)} = \frac{0.0487 - 0.0243}{\ln(0.0487/0.0243)} = 0.0351 \text{ m}^2$$

$$A_{B\,\text{lm}} = \frac{A_3 - A_2}{\ln(A_3/A_2)} = \frac{0.0974 - 0.0487}{\ln(0.0974/0.0487)} = 0.0703 \text{ m}^2$$

From Eq. (4.3-7) the resistances are

$$R_A = \frac{r_2 - r_1}{k_A A_{A\,\text{lm}}} = \frac{0.0127}{21.63(0.0351)} = 0.01673 \text{ K/W}$$

$$R_B = \frac{r_3 - r_2}{k_B A_{B\,\text{lm}}} = \frac{0.0254}{0.2423(0.0703)} = 1.491 \text{ K/W}$$

Hence, the heat-transfer rate is

$$q = \frac{T_1 - T_3}{R_A + R_B} = \frac{811 - 310.8}{0.01673 + 1.491} = 331.7 \text{ W (1132 btu/h)}$$

To calculate the temperature T_2,

$$q = \frac{T_1 - T_2}{R_A} \quad \text{or} \quad 331.7 = \frac{811 - T_2}{0.01673}$$

Solving, $811 - T_2 = 5.5$ K and $T_2 = 805.5$ K. Only a small temperature drop occurs across the metal wall because of its high thermal conductivity.

Conduction Through Materials in Parallel

Suppose that two plane solids *A* and *B* are placed side by side in parallel, and the direction of heat flow is perpendicular to the plane of the exposed surface of each solid. Then the total heat flow is the sum of the heat flow through solid *A* plus that through *B.* Writing Fourier's equation for each solid and summing,

Equation 4.3-9.

$$q_T = q_A + q_B = \frac{k_A A_A}{\Delta x_A}(T_1 - T_2) + \frac{k_B A_B}{\Delta x_B}(T_3 - T_4)$$

where q_T is total heat flow, T_1 and T_2 are the front and rear surface temperatures for solid *A,* and T_3 and T_4 are those for solid *B.*

If we assume that $T_1 = T_3$ (front temperatures the same for *A* and *B*) and $T_2 = T_4$ (equal rear temperatures),

Equation 4.3-10.

$$q_T = \frac{T_1 - T_2}{\Delta x_A / k_A A_A} + \frac{T_1 - T_2}{\Delta x_B / k_B A_B} = \left(\frac{1}{R_A} + \frac{1}{R_B}\right)(T_1 - T_2)$$

An example would be an insulated wall *(A)* of a brick oven where steel reinforcing members (*B*) are in parallel and penetrate the wall. Even though the area A_B of the steel would be small compared to the insulated brick area A_A, the higher conductivity of the metal (which could be several hundred times larger than that of the brick) could allow a large portion of the heat lost to be conducted by the steel.

Another example is a method of increasing heat conduction to accelerate the freeze-drying of meat. Spikes of metal in the frozen meat conduct heat more rapidly into the insides of the meat.

It should be mentioned that in some cases some two-dimensional heat flow can occur if the thermal conductivities of the materials in parallel differ markedly. Then the results using Eq. (4.3-10) would be affected somewhat.

Combined Convection and Conduction and Overall Coefficients

In many practical situations the surface temperatures (or boundary conditions at the surface) are not known, but there is a fluid on both sides of the solid surfaces. Consider the plane wall in Fig. 4.3-3a, with a hot fluid at temperature T_1 on the inside surface and a cold fluid at T_4 on the outside surface. The convective coefficient on the outside is h_o W/m$^2 \cdot$ K and h_i on the inside. (Methods for predicting the convective *h* will be given later, in Section 4.4 of this chapter.)

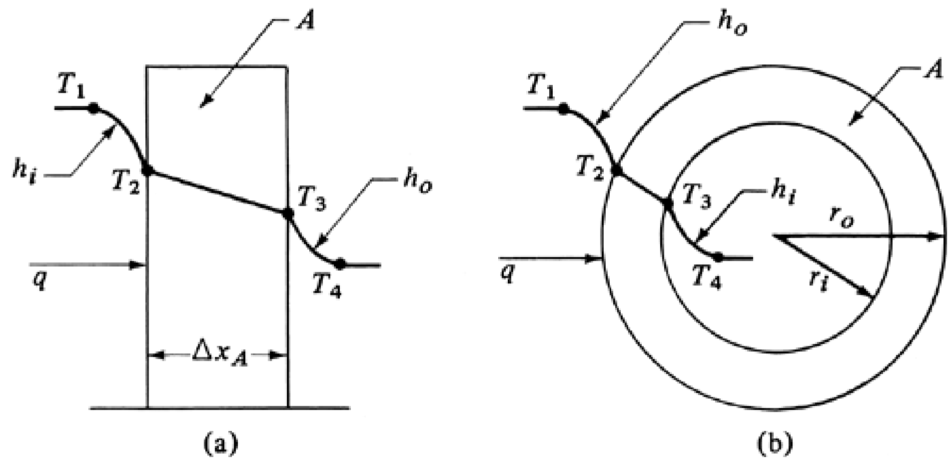


Figure 4.3-3. Heat flow with convective boundaries: (a) plane wall, (b) cylindrical wall.

The heat-transfer rate using Eqs. (4.1-12) and (4.3-1) is given as

Equation 4.3-11.

$$q = h_i A(T_1 - T_2) = \frac{k_A A}{\Delta x_A}(T_2 - T_3) = h_o A(T_3 - T_4)$$

Expressing $1/h_i A$, $\Delta x_A/k_A A$, and $1/h_o A$ as resistances and combining the equations as before,

Equation 4.3-12.

$$q = \frac{T_1 - T_4}{1/h_i A + \Delta x_A/k_A A + 1/h_o A} = \frac{T_1 - T_4}{\sum R}$$

The overall heat transfer by combined conduction and convection is often expressed in terms of an overall heat-transfer coefficient U defined by

Equation 4.3-13.

$$q = UA\, \Delta T_{\text{overall}}$$

where $\Delta T_{\text{overall}} = T_1 - T_4$ and U is

Equation 4.3-14.

$$U = \frac{1}{1/h_i + \Delta x_A/k_A + 1/h_o} \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \left(\frac{\text{btu}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}}\right)$$

A more important application is heat transfer from a fluid outside a cylinder, through a metal wall, to a fluid inside the tube, as often occurs in heat exchangers. In Fig. 4.3-3b, such a case is shown.

Using the same procedure as before, the overall heat-transfer rate through the cylinder is

Equation 4.3-15.

$$q = \frac{T_1 - T_4}{1/h_i A_i + (r_o - r_i)/k_A A_{A\,\text{lm}} + 1/h_o A_o} = \frac{T_1 - T_4}{\sum R}$$

where A_i represents $2\pi L r_i$, the inside area of the metal tube; $A_{A\,\text{lm}}$ the log mean area of the metal tube; and A_0 the outside area.

The overall heat-transfer coefficient U for the cylinder may be based on the inside area A_i or the outside area A_0 of the tube. Hence,

Equation 4.3-16.

$$q = U_i A_i (T_1 - T_4) = U_o A_o (T_1 - T_4) = \frac{T_1 - T_4}{\sum R}$$

Equation 4.3-17.

$$U_i = \frac{1}{1/h_i + (r_o - r_i) A_i / k_A A_{A\,\mathrm{lm}} + A_i / A_o h_o}$$

Equation 4.3-18.

$$U_o = \frac{1}{A_o / A_i h_i + (r_o - r_i) A_o / k_A A_{A\,\mathrm{lm}} + 1/h_o}$$

EXAMPLES 4.3-3. Heat Loss by Convection and Conduction and Overall U

Saturated steam at 267°F is flowing inside a $\frac{3}{4}$ -in. steel pipe having an ID of 0.824 in. and an OD of 1.050 in. The pipe is insulated with 1.5 in. of insulation on the outside. The convective coefficient for the inside steam surface of the pipe is estimated as $h_i = 1000$ btu/h · ft^2 · °F, and the convective coefficient on the outside of the lagging is estimated as $h_o = 2$ btu/h · ft^2 · °F. The mean thermal conductivity of the metal is 45 W/m · K or 26 btu/h · ft · °F and 0.064 W/m · K or 0.037 btu/h · ft · °F for the insulation.

a. Calculate the heat loss for 1 ft of pipe using resistances if the surrounding air is at 80°F

b. Repeat, using the overall U_i based on the inside area A_i.

Solution: Calling r_i the inside radius of the steel pipe, r_1 the outside radius of the pipe, and r_0 the outside radius of the lagging, then

$$r_i = \frac{0.412}{12}\ \mathrm{ft} \qquad r_1 = \frac{0.525}{12}\ \mathrm{ft} \qquad r_o = \frac{2.025}{12}\ \mathrm{ft}$$

For 1 ft of pipe, the areas are as follows:

$$A_i = 2\pi L r_i = 2\pi(1)\left(\frac{0.412}{12}\right) = 0.2157\ \mathrm{ft}^2$$

$$A_1 = 2\pi L r_1 = 2\pi(1)\left(\frac{0.525}{12}\right) = 0.2750\ \mathrm{ft}^2$$

$$A_o = 2\pi L r_o = 2\pi(1)\left(\frac{2.025}{12}\right) = 1.060\ \mathrm{ft}^2$$

From Eq. (4.3-6), the log mean areas for the steel *(A)* pipe and lagging (*B*) are

$$A_{A\,\mathrm{lm}} = \frac{A_1 - A_i}{\ln(A_1/A_i)} = \frac{0.2750 - 0.2157}{\ln(0.2750/0.2157)} = 0.245$$

$$A_{B\,\mathrm{lm}} = \frac{A_o - A_1}{\ln(A_o/A_1)} = \frac{1.060 - 0.2750}{\ln(1.060/0.2750)} = 0.583$$

From Eq. (4.3-15) the various resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{1000(0.2157)} = 0.00464$$

$$R_A = \frac{r_1 - r_i}{k_A A_{A\,\text{lm}}} = \frac{(0.525 - 0.412)/12}{26(0.245)} = 0.00148$$

$$R_B = \frac{r_o - r_1}{k_B A_{B\,\text{lm}}} = \frac{(2.025 - 0.525)/12}{0.037(0.583)} = 5.80$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{2(1.060)} = 0.472$$

Using an equation similar to Eq. (4.3-15),

Equation 4.3-19.

$$q = \frac{T_i - T_o}{R_i + R_A + R_B + R_o} = \frac{267 - 80}{0.00464 + 0.00148 + 5.80 + 0.472}$$

$$= \frac{267 - 80}{6.278} = 29.8 \text{ btu/h}$$

For part (b), the equation relating U_i to q is Eq. (4.3-16), which can be equated to Eq. (4.3-19):

Equation 4.3-20.

$$q = U_i A_i (T_i - T_o) = \frac{T_i - T_o}{\sum R}$$

Solving for U_i,

Equation 4.3-21.

$$U_i = \frac{1}{A_i \sum R}$$

Substituting known values,

$$U_i = \frac{1}{0.2157(6.278)} = 0.738 \frac{\text{btu}}{\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}}$$

Then to calculate q,

$$q = U_i A_i (T_i - T_o) = 0.738(0.2157)(267 - 80) = 29.8 \text{ btu/h } (8.73 \text{ W})$$

Conduction with Internal Heat Generation

In certain systems heat is generated inside the conducting medium; that is, a uniformly distributed heat source is present. Examples of this are electric-resistance heaters and nuclear fuel rods. Also, if a chemical reaction is occurring uniformly in a medium, a heat of reaction is given off. In the agricultural and sanitation fields, compost heaps and trash heaps in which biological activity is occurring will have heat given off.

Other important examples are in food processing, where the heat of respiration of fresh fruits and vegetables is present. These heats of generation can be as high as 0.3 to 0.6 W/kg or 0.5 to 1 btu/ h · lb_m.

Heat generation in plane wall

In Fig. 4.3-4 a plane wall is shown with internal heat generation. Heat is conducted only in the x direction. The other walls are assumed to be insulated. The temperature T_w in K at $x = L$ and $x = -L$ is held constant. The volumetric rate of heat generation is $\dot{q}$ W/m^3 and the thermal conductivity of the medium is k W/m · K.

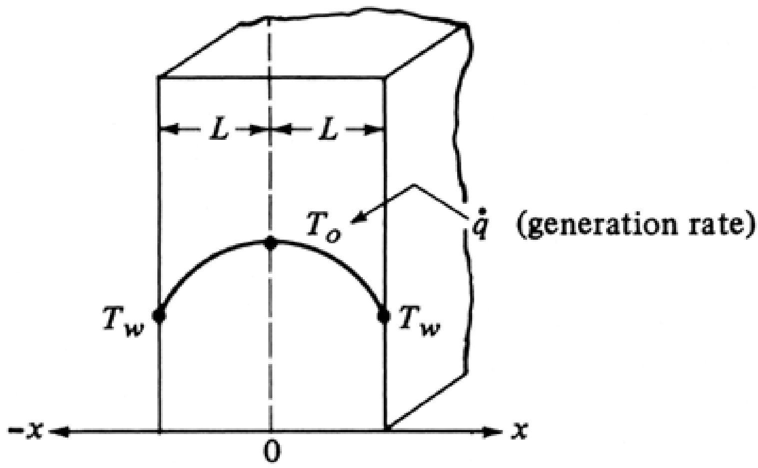


Figure 4.3-4. Plane wall with internal heat generation at steady state.

To derive the equation for this case of heat generation at steady state, we start with Eq. (4.1-3) but drop the accumulation term:

Equation 4.3-22.

$$q_{x|x} + \dot{q}(\Delta x \cdot A) = q_{x|x+\Delta x} + 0$$

where A is the cross-sectional area of the plate. Rearranging, dividing by Δx, and letting Δx approach zero,

Equation 4.3-23.

$$\frac{-dq_x}{dx} + \dot{q} \cdot A = 0$$

Substituting Eq. (4.1-2) for q_x,

Equation 4.3-24.

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

Integration gives the following for $\dot{q}$ constant:

Equation 4.3-25.

$$T = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2$$

where C_1 and C_2 are integration constants. The boundary conditions are at $x = L$ or $-L$, $T = T_w$, and at $x = 0$, $T = T_0$ (center temperature). Then, the temperature profile is

Equation 4.3-26.

$$T = -\frac{\dot{q}}{2k}x^2 + T_0$$

The center temperature is

Equation 4.3-27.

$$T_0 = \frac{\dot{q}L^2}{2k} + T_w$$

The total heat lost from the two faces at steady state is equal to the total heat generated, $\dot{q}_T$, in W:

Equation 4.3-28.

$$\dot{q}_T = \dot{q}(2LA)$$

where A is the cross-sectional area (surface area at T_w) of the plate.

Heat generation in cylinder

In a similar manner an equation can be derived for a cylinder of radius R with uniformly distributed heat sources and constant thermal conductivity. The heat is assumed to flow only radially, that is, the ends are neglected or insulated. The final equation for the temperature profile is

Equation 4.3-29.

$$T = \frac{\dot{q}}{4k}(R^2 - r^2) + T_w$$

where r is distance from the center. The center temperature T_0 is

Equation 4.3-30.

$$T_0 = \frac{\dot{q}R^2}{4k} + T_w$$

EXAMPLE 4.3-4. Heat Generation in a Cylinder

An electric current of 200 A is passed through a stainless-steel wire having a radius R of 0.001268 m. The wire is $L = 0.91$ m long and has a resistance R of 0.126 ohms. The outer surface temperature T_w is held at 422.1 K. The average thermal conductivity is $k = 22.5$ W/m · K. Calculate the center temperature.

***Solution*:** First the value of $\dot{q}$ must be calculated. Since power = I^2R, where I is current in amps and R is resistance in ohms,

Equation 4.3-31.

$$I^2R = \text{watts} = \dot{q}\pi R^2 L$$

Substituting known values and solving,

$$(200)^2(0.126) = \dot{q}\pi(0.001268)^2(0.91)$$

$$\dot{q} = 1.096 \times 10^9 \text{ W/m}^3$$

Substituting into Eq. (4.3-30) and solving, $T_0 = 441.7$ K.

Critical Thickness of Insulation for a Cylinder

In Fig. 4.3-5 a layer of insulation is installed around the outside of a cylinder whose radius r_1 is fixed and with a length L. The cylinder has a high thermal conductivity and the inner temperature T_1 at point r_1 outside the cylinder is fixed. An example is the case where the cylinder is a metal pipe with saturated steam inside. The outer surface of the insulation at T_2 is exposed to an environment at T_0 where convective heat transfer occurs. It is not obvious if adding more insulation with a thermal conductivity of k **will** decrease the heat-transfer rate.

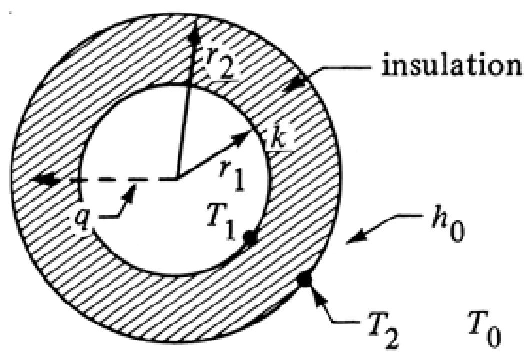


Figure 4.3-5. Critical radius for insulation of cylinder or pipe.

At steady state the heat-transfer rate q through the cylinder and the insulation equals the rate of convection from the surface:

Equation 4.3-32.

$$q = h_o A(T_2 - T_0)$$

As insulation is added, the outside area, which is $A = 2\pi r_2 L$, increases, but T_2 decreases. However, it is not apparent whether q increases or decreases. To determine this, an equation similar to Eq. (4.3-15) with the resistance of the insulation represented by Eq. (4.2-11) is written using the two resistances:

Equation 4.3-33.

$$q = \frac{2\pi L(T_1 - T_0)}{\dfrac{\ln(r_2/r_1)}{k} + \dfrac{1}{r_2 h_o}}$$

To determine the effect of the thickness of insulation on q, we take the derivative of q with respect to r_2, equate this result to zero, and obtain the following for maximum heat flow:

Equation 4.3-34.

$$\frac{dq}{dr_2} = \frac{-2\pi L(T_1 - T_0)(1/r_2 k - 1/r_2^2 h_o)}{\left[\dfrac{\ln(r_2/r_1)}{k} + \dfrac{1}{r_2 h_o}\right]^2} = 0$$

Solving,

Equation 4.3-35.

$$(r_2)_{cr} = \frac{k}{h_o}$$

where $(r_2)_{cr}$ is the value of the critical radius when the heat-transfer rate is a maximum. Hence, if the outer radius r_2 is less than the critical value, adding more insulation will actually increase the heat-transfer rate *q.* Also, if the outer radius is greater than the critical, adding more insulation will decrease the heat-transfer rate. Using values of *k* and h_o typically encountered, the critical radius is only a few mm. As a result, adding insulation on small electrical wires could increase the heat loss. Adding insulation to large pipes decreases the heat-transfer rate.

EXAMPLE 4.3-5. Insulating an Electrical Wire and Critical Radius

An electric wire having a diameter of 1.5 mm and covered with a plastic insulation (thickness = 2.5 mm) is exposed to air at 300 K and $h_o = 20$ W/m^2 · K. The insulation has a *k* of 0.4 W/m · K. It is assumed that the wire surface temperature is constant at 400 K and is not affected by the covering.

a. Calculate the value of the critical radius.
b. Calculate the heat loss per m of wire length with no insulation.
c. Repeat (b) for insulation being present.

Solution:
For part (a), using Eq. (4.3-35),

$$(r_2)_{cr} = \frac{k}{h_o} = \frac{0.4}{20} = 0.020 \text{ m} = 20 \text{ mm}$$

For part (b), $L = 1.0$ m, $r_2 = 1.5/(2 \times 1000) = 0.75 \times 10^{-3}$ m, $A = 2\pi r_2 L$. Substituting into Eq. (4.3-32),

$$q = h_o A(t_2 - T_0) = (20)(2\pi \times 0.75 \times 10^{-3} \times 1)(400 - 300) = 9.42 \text{ W}$$

For part (c) with insulation, $r_1 = 1.5/(2 \times 1000) = 0.75 \times 10^{-3}$ m, $r_2 = (2.5 + 1.5/2)/1000 = 3.25 \times 10^{-3}$ m. Substituting into Eq. (4.3-33),

$$q = \frac{2\pi(1.0)(400 - 300)}{\dfrac{\ln(3.25 \times 10^{-3}/0.75 \times 10^{-3})}{0.4} + \dfrac{1}{(3.25 \times 10^{-3})(20)}} = 32.98 \text{ W}$$

Hence, adding insulation greatly increases the heat loss.

Contact Resistance at an Interface

In the equations derived in this section for conduction through solids in series (see Fig. 4.3-1), it has been assumed that the adjacent touching surfaces are at the same temperature, that is, that completely perfect contact is made between the surfaces. For many engineering designs in industry, this assumption is reasonably accurate. However, in cases such as in nuclear power plants, where very high heat fluxes are present, a significant drop in temperature may be present at the interface. This interface resistance, called *contact resistance,* occurs when the two solids do not fit tightly together and a thin layer of stagnant fluid is trapped between the two surfaces. At some points the solids touch at peaks in the surfaces and at other points the fluid occupies the open space.

This interface resistance is a complex function of the roughness of the two surfaces, the pressure applied to hold the surfaces in contact, the interface temperature, and the interface fluid. Heat transfer takes place by conduction, radiation, and convection across the trapped fluid and also by conduction through the points of contact of the solids. No completely reliable empirical correlations or theories are available to predict contact resistances for all types of materials. See references (C7, R2) for detailed discussions.

The equation for the contact resistance is often given as follows:

Equation 4.3-36.

$$q = h_c A \, \Delta T = \frac{\Delta T}{1/h_c A} = \frac{\Delta T}{R_c}$$

where h_c is the contact-resistance coefficient in $W/m^2 \cdot K$, ΔT the temperature drop across the contact resistance in K, and R_c the contact resistance. The contact resistance R_c can be added to the other resistances in Eq. (4.3-3) to include this effect for solids in series. For contact between two ground-metal surfaces, h_c values on the order of magnitude of about 0.2×10^4 to 1×10^4 $W/m^2 \cdot K$ have been obtained.

An approximation of the maximum contact resistance can be obtained if the maximum gap Δx between the surfaces can be estimated. Then, assuming that the heat transfer across the gap is by conduction only through the stagnant fluid, h_c is estimated as

Equation 4.3-37.

$$h_c = \frac{k}{\Delta x}$$

If any actual convection, radiation, or point-to-point contact is present, it will reduce this assumed resistance.

STEADY-STATE CONDUCTION AND SHAPE FACTORS

Introduction and Graphical Method for Two-Dimensional Conduction

In previous sections of this chapter we discussed steady-state heat conduction in one direction. In many cases, however, steady-state heat conduction is occurring in two directions, that is, two-dimensional conduction is occurring. The two-dimensional solutions are more involved and in most cases analytical solutions are not available. One important approximate method for solving such problems is to use a numerical method discussed in detail in Section 4.15. Another important approximate method is the graphical method, which is a simple method that can provide reasonably accurate answers for the heat-transfer rate. This method is particularly applicable to systems having isothermal boundaries.

In the graphical method we first note that for one-dimensional heat conduction through a flat slab (see Fig. 4.2-1) the direction of the heat flux or flux lines is always perpendicular to the isotherms. The graphical method for two-dimensional conduction is also based on the requirement that the heat-flux lines and isotherm lines intersect each other at right angles while forming a network of curvilinear squares. This means, as shown in Fig. 4.4-1, that we can sketch the isotherms and also the flux lines until they intersect at right angles (are perpendicular to each other). With care and experience we can obtain reasonably accurate results. General steps to use in this graphical method are as follows:

1. Draw a model to scale of the two-dimensional solid. Label the isothermal boundaries. In Fig. 4.4-1, T_1 and T_2 are isothermal boundaries.
2. Select a number N that is the number of equal temperature subdivisions between the isothermal boundaries. In Fig. 4.4-1, $N = 4$ subdivisions between T_1 and T_2. Sketch in the isotherm lines and the heat-flow or -flux lines so that they are perpendicular to each other at the intersections. Note that isotherms are perpendicular to adiabatic (insulated) boundaries and also lines of symmetry.
3. Keep adjusting the isotherm and flux lines until for each curvilinear square the condition $\Delta x = \Delta y$ is satisfied.

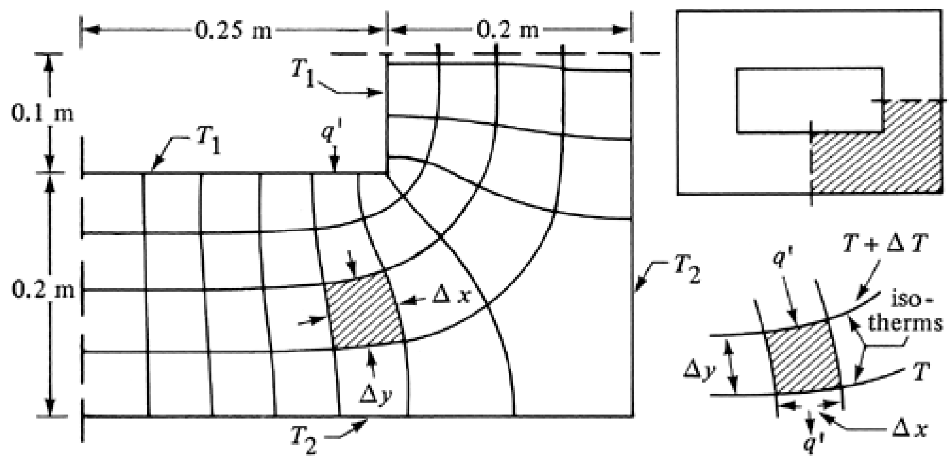


Figure 4.4-1. Graphical curvilinear-square method for two-dimensional heat conduction in a rectangular flue.

In order to calculate the heat flux using the results of the graphical plot, we first assume unit depth of the material. The heat flow q' through the curvilinear section shown in Fig. 4.4-1 is given by Fourier's law:

Equation 4.4-1.

$$q' = -kA\frac{dT}{dy} = k(\Delta x \cdot 1)\frac{\Delta T}{\Delta y}$$

This heat flow q' *will* be the same through each curvilinear square within this heat-flow lane. Since $\Delta x = \Delta y$, each temperature subdivision ΔT is equal. This temperature subdivision can be expressed in terms of the overall temperature difference $T_1 - T_2$ and N, the number of equal subdivisions:

Equation 4.4-2.

$$\Delta T = \frac{T_1 - T_2}{N}$$

Also, the heat flow q' through each lane is the same, since $\Delta x = \Delta y$ in the construction and in Eq. (4.4-1). Hence, the total heat transfer q through all of the lanes is

Equation 4.4-3.

$$q = Mq' = Mk\,\Delta T$$

where M is the total number of heat-flow lanes as determined by the graphical procedure. Substituting Eq. (4.4-2) into (4.4-3),

Equation 4.4-4.

$$q = \frac{M}{N} k(T_1 - T_2)$$

EXAMPLE 4.4-1. Two-Dimensional Conduction by Graphical Procedure

Determine the total heat transfer through the walls of the flue shown in Fig. 4.4-1 if T_1 = 600 K, T_2 = 400 K, k = 0.90 W/m · K, and L (length of flue) = 5 m.

Solution: In Fig. 4.4-1, N = 4 temperature subdivisions and M = 9.25. The total heat-transfer rate through the four identical sections with a depth or length L of 5 m is obtained by using Eq. (4.4-4):

$$q = 4\left[\frac{M}{N} kL(T_1 - T_2)\right] = 4\left[\frac{9.25}{4}(0.9)(5.0)(600 - 400)\right]$$

$$= 8325 \text{ W}$$

Shape Factors in Conduction

In Eq. (4.4-4) the factor *M/N is* called the conduction shape factor *S,* where

Equation 4.4-5.

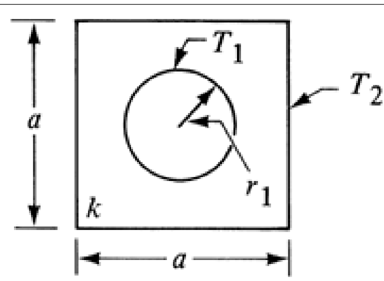
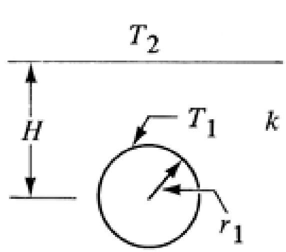
$$S = \frac{M}{N}$$

Equation 4.4-6.

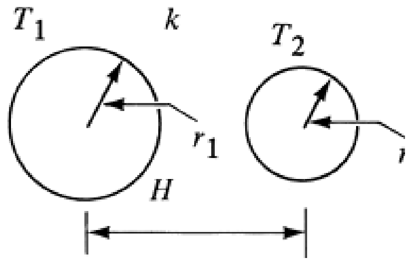
$$q = kS(T_1 - T_2)$$

This shape factor *S* has units of m and is used in two-dimensional heat conduction where only two temperatures are involved. The shape factors for a number of geometries have been obtained and some are given in Table 4.4-1.

*Table 4.4-1. Conduction Shape Factors for q = kS(T_1 − T_2)**

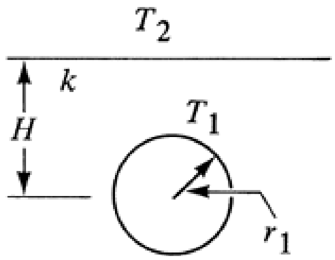
Cylinder of length *L* in a square		$S = \dfrac{2\pi L}{\ln(0.54\, a/r_1)}$
Horizontal buried cylinder of length *L*		$S = \dfrac{2\pi L}{\ln(2H/r_1)}$ $(H > 3r_1)$

Two parallel cylinders of length ***L***



$$S = \frac{2\pi L}{\cosh^{-1}\left[\dfrac{H^2 - r_1^2 - r_2^2}{2r_1 r_2}\right]}$$

Sphere buried



$$S = \frac{4\pi r_1}{1 - r_1/2H}$$

*The thermal conductivity of the medium is k.

For a three-dimensional geometry such as a furnace, separate shape factors are used to obtain the heat flow through the edge and corner sections. When each of the interior dimensions is greater than one-fifth of the wall thickness, the shape factors are as follows for a uniform wall thickness T_w:

Equation 4.4-7.

$$S_{\text{wall}} = \frac{A}{T_w} \qquad S_{\text{edge}} = 0.54L \qquad S_{\text{corner}} = 0.15T_w$$

where A is the inside area of wall and L the length of inside edge. For a completely enclosed geometry, there are six wall sections, 12 edges, and eight corners. Note that for a single flat wall, $q = kS_{wall}(T_1 - T_2) = k(A/T_w)(T_1 - T_2)$, which is the same as Eq. (4.2-1) for conduction through a single flat slab.

For a long, hollow cylinder of length L such as that in Fig. 4.2-2,

Equation 4.4-8.

$$S = \frac{2\pi L}{\ln(r_2/r_1)}$$

For a hollow sphere, from Eq. (4.2-14),

Equation 4.4-9.

$$S = \frac{4\pi r_2 r_1}{r_2 - r_1}$$

FORCED CONVECTION HEAT TRANSFER INSIDE PIPES

Introduction and Dimensionless Numbers

In most situations involving a liquid or a gas in heat transfer, convective heat transfer usually occurs as well as conduction. In most industrial processes where heat transfer is occurring, heat is being transferred from one fluid through a solid wall to a second fluid. In Fig. 4.5-1 heat is being transferred from the hot flowing fluid to the cold flowing fluid. The temperature profile is shown.

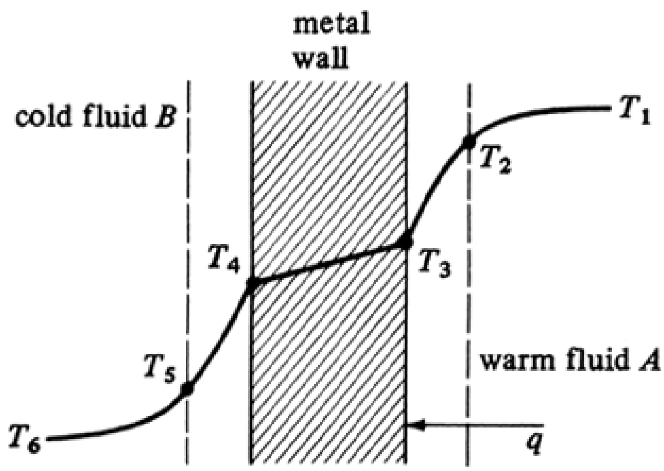


Figure 4.5-1. Temperature profile for heat transfer by convection from one fluid to another

The velocity gradient, when the fluid is in turbulent flow, is very steep next to the wall in the thin viscous sublayer where turbulence is absent. Here the heat transfer is mainly by conduction, with a large temperature difference of $T_2 - T_3$ in the warm fluid. As we move farther away from the wall, we approach the turbulent region, where rapidly moving eddies tend to equalize the temperature. Hence, the temperature gradient is less and the difference $T_1 - T_2$ is small. The average temperature of fluid A is slightly less than the peak value T_1. A similar explanation can be given for the temperature profile in the cold fluid.

The convective coefficient for heat transfer through a fluid is given by

Equation 4.5-1.

$$q = hA(T - T_w)$$

where h is the convective coefficient in W/m^2 · K, A is the area in m^2, T is the bulk or average temperature of the fluid in K, T_w is the temperature of the wall in contact with the fluid in K, and q is the heat-transfer rate in W. In English units, q is in btu/h, h in btu/h · ft^2 · °F, A in ft^2, and T and T_w in °F.

The type of fluid flow, whether laminar or turbulent, of the individual fluid has a great effect on the heat-transfer coefficient *h,* which is often called a *film coefficient,* since most of the resistance to heat transfer is in a thin film close to the wall. The more turbulent the flow, the greater the heat-transfer coefficient.

There are two main classifications of convective heat transfer. The first is free or *natural convection,* where the motion of the fluid results from the density changes in heat transfer. The buoyant effect produces a natural circulation of the fluid, so it moves past the solid surface. In the second type, *forced convection,* the fluid is forced to flow by pressure differences, a pump, a fan, and so on.

Most of the correlations for predicting film coefficients h are semiempirical in nature and are affected by the physical properties of the fluid, the type and velocity of flow, the temperature difference, and the geometry of the specific physical system. Some approximate values for convective coefficients were presented in Table 4.1-2. In the following correlations, either SI or English units can be used, since the equations are dimensionless.

To correlate these data for heat-transfer coefficients, dimensionless numbers such as the Reynolds and Prandtl numbers are used. The Prandtl number is the ratio of the shear component of diffusivity for momentum μ/ρ to the diffusivity for heat $k/\rho c_p$ and physically relates the relative thicknesses of the hydrodynamic layer and thermal boundary layer:

Equation 4.5-2.

$$N_{\mathrm{Pr}} = \frac{\mu/\rho}{k/\rho c_p} = \frac{c_p \mu}{k}$$

Values of the N_{Pr} for gases are given in Appendix A.3 and range from about 0.5 to 1.0. Values for liquids range from about 2 to well over 10^4. The dimensionless Nusselt number, N_{Nu}, is used to relate data for the heat-transfer coefficient h to the thermal conductivity k of the fluid and a characteristic dimension D:

Equation 4.5-3.

$$N_{\mathrm{Nu}} = \frac{hD}{k}$$

For example, for flow inside a pipe, D is the diameter.

Heat-Transfer Coefficient for Laminar Flow Inside a Pipe

Certainly, the most important convective heat-transfer process industrially is that of cooling or heating a fluid flowing inside a closed circular conduit or pipe. Different types of correlations for the convective coefficient are needed for laminar flow (N_{Re} below 2100), for fully turbulent flow (N_{Re} above 6000), and for the transition region (N_{Re} between 2100 and 6000).

For laminar flow of fluids inside horizontal tubes or pipes, the following equation of Sieder and Tate (S1) can be used for $N_{\mathrm{Re}} < 2100$:

Equation 4.5-4.

$$(N_{\mathrm{Nu}})_a = \frac{h_a D}{k} = 1.86\left(N_{\mathrm{Re}}\, N_{\mathrm{Pr}}\, \frac{D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

where D = pipe diameter in m, L = pipe length before mixing occurs in the pipe in m, μ_b = fluid viscosity at bulk average temperature in Pa · s, μ_w = viscosity at the wall temperature, c_p = heat capacity in J/kg · K, k = thermal conductivity in W/m · K, h_a = average heat-transfer coefficient in W/m^2 · K, and N_{Nu} = dimensionless Nusselt number. All the physical properties are evaluated at the bulk fluid temperature except μ_w. The Reynolds number is

Equation 4.5-5.

$$N_{\mathrm{Re}} = \frac{Dv\rho}{\mu}$$

and the Prandtl number,

Equation 4.5-6.

$$N_{\text{Pr}} = \frac{c_p \mu}{k}$$

This equation holds for ($N_{\text{Re}} N_{\text{Pr}} D/L$) > 100. If used down to ($N_{\text{Re}} N_{Pr} D/L$) > 10, it still holds to ±20% (B1). For ($N_{\text{Re}} N_{Pr} D/L$) < 100, another expression is available (P1).

In laminar flow the average coefficient h_a depends strongly on heated length. The average (arithmetic mean) temperature drop ΔT_a is used in the equation to calculate the heat-transfer rate q:

Equation 4.5-7.

$$q = h_a A \, \Delta T_a = h_a A \, \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

where T_w is the wall temperature in K, T_{bi} the inlet bulk fluid temperature, and T_{bo} the outlet bulk fluid temperature.

For large pipe diameters and large temperature differences ΔT between pipe wall and bulk fluid, natural convection effects can increase h (P1). Equations are also available for laminar flow in vertical tubes.

Heat-Transfer Coefficient for Turbulent Flow Inside a Pipe

When the Reynolds number is above 6000, the flow is fully turbulent. Since the rate of heat transfer is greater in the turbulent region, many industrial heat-transfer processes are in the turbulent region.

The following equation has been found to hold for tubes but is also used for pipes. It holds for a N_{Re} > 6000, a N_{Pr} between 0.7 and 16 000, and L/D > 60.

Equation 4.5-8.

$$N_{\text{Nu}} = \frac{h_L D}{k} = 0.027 N_{\text{Re}}^{0.8} \, N_{\text{Pr}}^{1/3} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

where h_L is the heat-transfer coefficient based on the log mean driving force ΔT_{lm} (see Section 4.5H). The fluid properties except for μ_w are evaluated at the mean bulk temperature. If the bulk fluid temperature varies from the inlet to the outlet of the pipe, the mean of the inlet and outlet temperatures is used. For an L/D < 60, where the entry is an abrupt contraction, an approximate correction is provided by multiplying the right-hand side of Eq. (4.5-8) by a correction factor given in Section 4.5F.

The use of Eq. (4.5-8) may be trial and error, since the value of h_L must be known in order to evaluate T_w, and hence μ_w, at the wall temperature. Also, if the mean bulk temperature increases or decreases in the tube length L because of heat transfer, the bulk temperature at length L must be estimated in order to have a mean bulk temperature of the entrance and exit to use.

The heat-transfer coefficient for turbulent flow is somewhat greater for a pipe than for a smooth tube. This effect is much less than in fluid friction, and it is usually neglected in calculations. Also, for liquid metals that have Prandtl numbers << 1, other correlations must be used to predict the heat-transfer coefficient. (See Section 4.5G.) For shapes of tubes other than circular, the equivalent diameter can be used, as discussed in Section 4.5E.

For air at 1 atm total pressure, the following simplified equation holds for turbulent flow in pipes:

Equation 4.5-9.

$$h_L = \frac{3.52v^{0.8}}{D^{0.2}} \qquad \text{(SI)}$$

$$h_L = \frac{0.5v_s^{0.8}}{(D')^{0.2}} \qquad \text{(English)}$$

where D is in m, v in m/s, and h_L in W/m^2 · K for SI units; and D' is in in., v_s in ft/s, and h_L in btu/h · ft^2 · °F for English units.

Water is often used in heat-transfer equipment. A simplified equation to use for a temperature range of T = 4−105°C (40−220°F) is

Equation 4.5-10.

$$h_L = 1429(1 + 0.0146T\text{°C})\frac{v^{0.8}}{D^{0.2}} \qquad \text{(SI)}$$

$$h_L = 150(1 + 0.011T\text{°F})\frac{v_s^{0.8}}{(D')^{0.2}} \qquad \text{(English)}$$

For organic liquids, a very simplified equation to use for approximations is as follows (P3):

Equation 4.5-11.

$$h_L = 423\frac{v^{0.8}}{D^{0.2}} \qquad \text{(SI)}$$

$$h_L = 60\frac{v_s^{0.8}}{(D')^{0.2}} \qquad \text{(English)}$$

For flow inside helical coils and N_{Re} above 10^4, the predicted film coefficient for straight pipes should be increased by the factor $(1 + 3.5D/D_{coil})$.

EXAMPLE 4.5-1. Heating of Air in Turbulent Flow

Air at 206.8 kPa and an average of 477.6 K is being heated as it flows through a tube of 25.4 mm inside diameter at a velocity of 7.62 m/s. The heating medium is 488.7 K steam condensing on the outside of the tube. Since the heat-transfer coefficient of condensing steam is several thousand W/m^2 · K and the resistance of the metal wall is very small, it will be assumed that the surface wall temperature of the metal in contact with the air is 488.7 K. Calculate the heat-transfer coefficient for an $L/D > 60$ and also the heat-transfer flux q/A.

Solution: From Appendix A.3, for physical properties of air at 477.6 K (204.4°C), $\mu_b = 2.60 \times 10^{-5}$ Pa · s, k = 0.03894 W/m, N_{Pr} = 0.686. At 488.7 K (215.5°C), μ_w = 2.64 × KT^5 Pa · s.

$$\mu_b = 2.60 \times 10^{-5}\ \text{Pa}\cdot\text{s} = 2.60 \times 10^{-5}\ \text{kg/m}\cdot\text{s}$$

$$\rho = (28.97)\left(\frac{1}{22.414}\right)\left(\frac{206.8}{101.33}\right)\left(\frac{273.2}{477.6}\right) = 1.509\ \text{kg/m}^3$$

The Reynolds number calculated at the bulk fluid temperature of 477.6 K is

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{0.0254(7.62)(1.509)}{2.6 \times 10^{-5}} = 1.122 \times 10^4$$

Hence, the flow is turbulent and Eq. (4.5-8) will be used. Substituting into Eq. (4.5-8),

$$N_{Nu} = \frac{h_L D}{k} = 0.027 N_{Re}^{0.8} N_{Pr}^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$\frac{h_L(0.0254)}{0.03894} = 0.027(1.122 \times 10^4)^{0.8}(0.686)^{1/3}\left(\frac{0.0260}{0.0264}\right)^{0.14}$$

Solving, h_L = 63.2 W/m^2 · K (11.13 btu/h · ft^2 · °F). To solve for the flux q/A,

$$\frac{q}{A} = h_L(T_w - T) = 63.2(488.7 - 477.6)$$

$$= 701.1 \text{ W/m}^2 \text{ (222.2 btu/h} \cdot \text{ft}^2\text{)}$$

Heat-Transfer Coefficient for Transition Flow Inside a Pipe

In the transition region for a N_{Re} between 2100 and 6000, the empirical equations are not well defined, just as in the case of fluid friction factors. No simple equation exists for accomplishing a smooth transition from heat transfer in laminar flow to that in turbulent flow, that is, a transition from Eq. (4.5-4) at a N_{Re} = 2100 to Eq. (4.5-8) at a N_{Re} = 6000.

The plot in Fig. 4.5-2 represents an approximate relationship to use between the various heat-transfer parameters and the Reynolds number between 2100 and 6000. For below a N_{Re}, of 2100, the curves represent Eq. (4.5-4), and above 10^4, Eq. (4.5-8). The mean ΔT_a of Eq. (4.5-7) should be used with the h_a in Fig. 4.5-2.

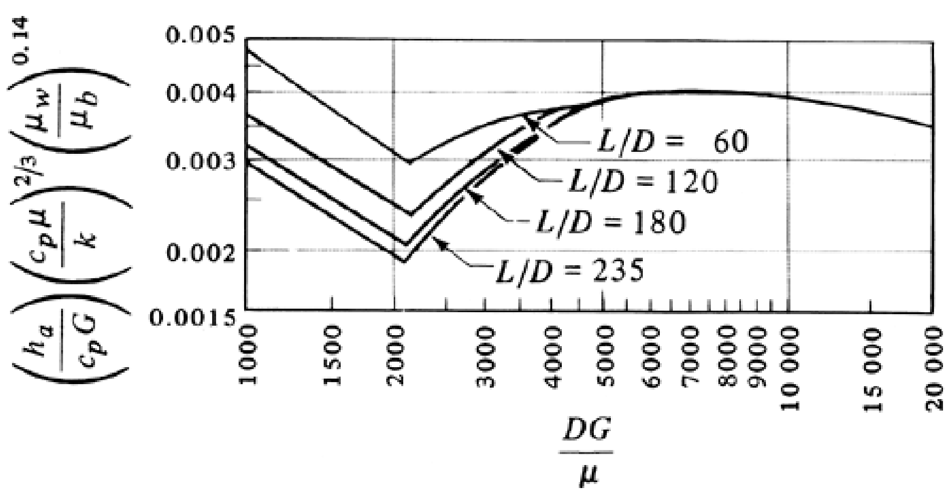


Figure 4.5-2. Correlation of heat-transfer parameters for transition region for Reynolds numbers between 2100 and 6000. (From R. H. Perry and C. H. Chilton, Chemical Engineers' Handbook. 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.)

Heat-Transfer Coefficient for Noncircular Conduits

A heat-transfer system often used is one in which fluids flow at different temperatures in concentric pipes. The heat-transfer coefficient of the fluid in the annular space can be predicted by using the same equations as for circular pipes. However, the equivalent diameter defined in Section 2.10G must be used. For an annular space, D_{eq} is the ID of the outer pipe D_1 minus the OD of the inner pipe D_2. For other geometries, an equivalent diameter can also be used.

EXAMPLE 4.5-2. Water Heated by Steam and Trial-and-Error Solution

Water is flowing in a horizontal 1-in. schedule 40 steel pipe at an average temperature of 65.6°C and a velocity of 2.44 m/s. It is being heated by condensing steam at 107.8°C on the outside of the pipe wall. The steam-side coefficient has been estimated as $h_0 = 10\,500$ W/m^2 · K.

a. Calculate the convective coefficient h_i for water inside the pipe.

b. Calculate the overall coefficient U_i based on the inside surface area.

c. Calculate the heat-transfer rate q for 0.305 m of pipe with the water at an average temperature of 65.6°C.

Solution: From Appendix A.5, the various dimensions are $D_i = 0.0266$ m and $D_0 = 0.0334$ m. For water at a bulk average temperature of 65.6°C, from Appendix A.2, $N_{Pr} = 2.72$, $\rho = 0.980(1000) = 980$ kg/m^3, $k = 0.633$ W/m · K, and $\mu = 4.32 \times 10^{-4}$ Pa · S = 4.32×10^{-4} kg/m · s.

The temperature of the inside metal wall is needed and will be assumed as about one-third the difference between 65.6 and 107.8, or 80°C = T_w, for the first trial. Hence, μ_w at 80°C = 3.56×10^{-4} Pa · s.

First, the Reynolds number of the water is calculated at the bulk average temperature:

$$N_{Re} = \frac{D_i v \rho}{\mu} = \frac{0.0266(2.44)(980)}{4.32 \times 10^{-4}} = 1.473 \times 10^5$$

Hence, the flow is turbulent. Using Eq. (4.5-8) and substituting known values,

$$\frac{h_L D}{k} = 0.027 N_{Re}^{0.8} N_{Pr}^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$\frac{h_L(0.0266)}{0.663} = 0.027(1.473 \times 10^5)^{0.8}(2.72)^{1/3}\left(\frac{4.32 \times 10^{-4}}{3.56 \times 10^{-4}}\right)^{0.14}$$

Solving, $h_L = h_i = 13\,324$ W/m^2 · K.

For part (b), the various areas are as follows for 0.305-m pipe:

$$A_i = \pi D_i L = \pi(0.0266)(0.305) = 0.0255 \text{ m}^2$$

$$A_{lm} = \pi \frac{(0.0266 + 0.0334)0.305}{2} = 0.0287 \text{ m}^2$$

$$A_o = \pi(0.0334)(0.305) = 0.0320 \text{ m}^2$$

The k for steel is 45.0 W/m · K. The resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(13\,324)0.0255} = 0.002943$$

$$R_m = \frac{r_o - r_i}{k A_{\text{lm}}} = \frac{0.0334 - 0.0266}{2} \frac{1}{45.0(0.0287)} = 0.002633$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(10\,500)(0.0320)} = 0.002976$$

$$\sum R = 0.002943 + 0.002633 + 0.002976 = 0.008552$$

The overall temperature difference is (107.8 - 65.6)°C = 42.2°C = 42.2 K. The temperature drop across the water film is

$$\text{temperature drop} = \frac{R_i}{\sum R}(42.2) = \left(\frac{0.002943}{0.008852}\right)(42.2) = 14.5\text{ K} = 14.5°\text{C}$$

Hence, T_w = 65.6 + 14.5 = 80.1°C. This is quite close to the original estimate of 80°C. The only physical property changing in the second estimate would be μ_w. This would have a negligible effect on h_i, and a second trial is not necessary.

For part (b), the overall coefficient is, by Eq. (4.3-16),

$$q = U_i A_i (T_o - T_i) = \frac{T_o - T_i}{\sum R}$$

$$U_i = \frac{1}{A_i \sum R} = \frac{1}{0.0255\,(0.008552)} = 4586\text{ W/m}^2 \cdot \text{K}.$$

For part (c), with the water at an average temperature of 65.6°C,

$$T_o - T_i = 107.8 - 65.6 = 42.2°\text{C} = 42.2\text{ K}$$

$$q = U_i A_i (T_o - T_i) = 4586(0.0255)(42.2) = 4935\text{ W}$$

Entrance-Region Effect on Heat-Transfer Coefficient

Near the entrance of a pipe where the fluid is being heated, the temperature profile is not fully developed and the local coefficient h is greater than the fully developed heat-transfer coefficient h_L for turbulent flow. At the entrance itself, where no temperature gradient has been established, the value of h is infinite. The value of h drops rapidly and is approximately the same as h_L at $L/D \cong 60$, where L is the entrance length. These relations for turbulent flow inside a pipe are as follows where the entrance is an abrupt contraction:

Equation 4.5-12.

$$\frac{h}{h_L} = 1 + \left(\frac{D}{L}\right)^{0.7} \qquad 2 < \frac{L}{D} < 20$$

Equation 4.5-13.

$$\frac{h}{h_L} = 1 + 6\left(\frac{D}{L}\right) \qquad 20 < \frac{L}{D} < 60$$

where h is the average value for a tube of finite length L and h_L is the value for a very long tube.

Liquid-Metals Heat-Transfer Coefficient

Liquid metals are sometimes used as heat-transfer fluids in cases where a fluid is needed over a wide temperature range at relatively low pressures. Liquid metals are often used in nuclear reactors and have high heat-transfer coefficients as well as a high heat capacity per unit volume. The high heat-transfer coefficients are due to the very high thermal conductivities and, hence, low Prandtl numbers. In liquid metals in pipes, heat transfer by conduction is very important in the entire turbulent core because of the high thermal conductivity and is often more important than the convection effects.

For fully developed turbulent flow in tubes with uniform heat flux, the following equation can be used (L1):

Equation 4.5-14.

$$N_{\mathrm{Nu}} = \frac{h_L D}{k} = 0.625 N_{\mathrm{Pe}}^{0.4}$$

where the Peclet number $N_{\mathrm{Pe}} = N_{\mathrm{Re}} N_{\mathrm{Pr}}$. This holds for $L/D > 60$ and N_{Pe} between 100 and 10^4. For constant wall temperatures,

Equation 4.5-15.

$$N_{\mathrm{Nu}} = \frac{h_L D}{k} = 5.0 + 0.025 N_{\mathrm{Pe}}^{0.8}$$

for $L/D > 60$ and $N_{Pe} > 100$. All physical properties are evaluated at the average bulk temperature.

EXAMPLE 4.5-3. Liquid-Metal Heat Transfer Inside a Tube

A liquid metal flows at a rate of 4.00 kg/s through a tube having an inside diameter of 0.05 m. The liquid enters at 500 K and is heated to 505 K in the tube. The tube wall is maintained at a temperature of 30 K above the fluid bulk temperature and constant heat flux is also maintained. Calculate the required tube length. The average physical properties are as follows: $\mu = 7.1 \times 10^{-4}$ Pa · s, $\rho = 7400$ kg/m^3, $c_p = 120$ J/kg · K, $k = 13$ W/m · K.

Solution. The area is $A = \pi D^2/4 = \pi(0.05)^2/4 = 1.963 \times 10^{-3}$ m^2. Then $G = 4.0/1.963 \times 10^{-3} = 2.038 \times 10^3$ kg/m^2 · s. The Reynolds number is

$$N_{\mathrm{Re}} = \frac{DG}{\mu} = \frac{0.05(2.038 \times 10^3)}{7.1 \times 10^{-4}} = 1.435 \times 10^5$$

$$N_{\mathrm{Pr}} = \frac{c_p \mu}{k} = \frac{120(7.1 \times 10^{-4})}{13} = 0.00655$$

Using Eq. (4.5-14),

$$h_L = \frac{k}{D}(0.625)N_{\mathrm{Pe}}^{0.4} = \frac{13}{0.05}(0.625)(1.435 \times 10^5 \times 0.00655)^{0.4}$$

$$= 2512 \text{ W/m}^2 \cdot \text{K}$$

Using a heat balance,

Equation 4.5-16.

$$q = mc_p \, \Delta T$$

$$= 4.00(120)(505 - 500) = 2400 \text{ W}$$

Substituting into Eq. (4.5-1),

$$\frac{q}{A} = \frac{2400}{A} = h_L(T_w - T) = 2512(30) = 75\,360 \text{ W/m}^2$$

Hence, A = 2400/75 360 = 3.185 × 10^{-2} m^2. Then,

$$A = 3.185 \times 10^{-2} = \pi DL = \pi(0.05)(L)$$

Solving, L = 0.203 m.

Log Mean Temperature Difference and Varying Temperature Drop

Equations (4.5-1) and (4.3-12) as written apply only when the temperature drop $(T_i - T_0)$ is constant for all parts of the heating surface. Hence, the equation

Equation 4.5-17.

$$q = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o) = UA(\Delta T)$$

only holds at one point in the apparatus when the fluids are being heated or cooled. However, as the fluids travel through the heat exchanger, they become heated or cooled and either T_i or T_0 or both vary. Then ($T_i - T_0$) or ΔT varies with position, and some mean ΔT_m must be used over the whole apparatus.

In a typical heat exchanger, a hot fluid inside a pipe is cooled from T'_1 to T'_2 by a cold fluid which is flowing on the outside in a double pipe countercurrently (in the reverse direction) and is heated from T_2 to T_1, as shown in Fig. 4.5-3a. The ΔT shown is varying with distance. Hence, ΔT in Eq. (4.5-17) varies as the area A goes from 0 at the inlet to A at the outlet of the exchanger.

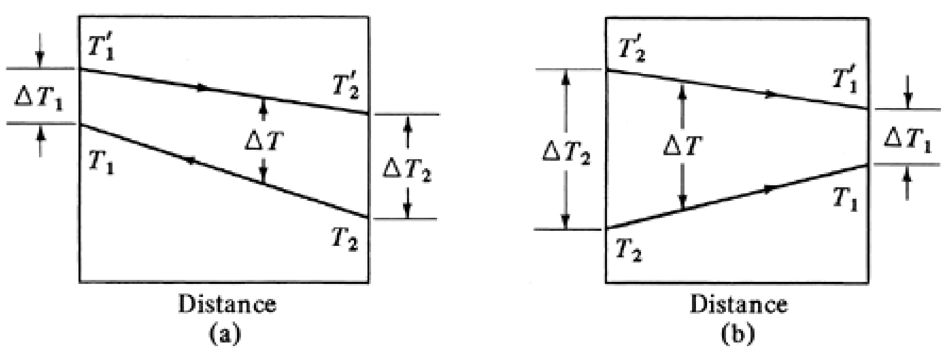


Figure 4.5-3. Temperature profiles for one-pass double-pipe heat exchangers: (a) counter-current flow; (b) cocurrent or parallel flow.

For countercurent flow of the two fluids as in Fig. 4.5-3a, the heat-transfer rate is

Equation 4.5-18.

$$q = UA \, \Delta T_m$$

where ΔT_m is a suitable mean temperature difference to be determined. For a *dA* area, a heat balance on the hot and the cold fluids gives

Equation 4.5-19.

$$dq = -m'c'_p\, dT' = mc_p\, dT$$

where *m* is flow rate in kg/s. The values of *m, m',* c_p, c'_p, and *U* are assumed constant. Also,

Equation 4.5-20.

$$dq = U(T' - T)\, dA$$

From Eq. (4.5-19), $dT' = -dq/m'c'_p$ and $dT = dq/mc_p$. Then,

Equation 4.5-21.

$$dT' - dT = d(T' - T) = -dq\left(\frac{1}{m'c'_p} + \frac{1}{mc_p}\right)$$

Substituting Eq. (4.5-20) into (4.5-21),

Equation 4.5-22.

$$\frac{d(T' - T)}{T' - T} = -U\left(\frac{1}{m'c'_p} + \frac{1}{mc_p}\right) dA$$

Integrating between points 1 and 2,

Equation 4.5-23.

$$\ln\left(\frac{T'_2 - T_2}{T'_1 - T_1}\right) = -UA\left(\frac{1}{m'c'_p} + \frac{1}{mc_p}\right)$$

Making a heat balance between the inlet and outlet,

Equation 4.5-24.

$$q = m'c'_p(T'_1 - T'_2) = mc_p(T_2 - T_1)$$

Solving for $m'c'_p$ and mc_p in Eq. (4.5-24) and substituting into Eq. (4.5-23),

Equation 4.5-25.

$$q = \frac{UA[(T'_2 - T_2) - (T'_1 - T_1)]}{\ln[(T'_2 - T_2)/(T'_1 - T_1)]}$$

Comparing Eqs. (4.5-18) and (4.5-25), we see that ΔT_m is the log mean temperature difference ΔT_{lm}. Hence, in the case where the overall heat-transfer coefficient *U* is constant throughout the equipment and the heat capacity of each fluid is constant, the proper temperature driving force to use over the entire apparatus is the log mean driving force,

Equation 4.5-26.

$$q = UA\,\Delta T_{lm}$$

where

Equation 4.5-27.

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

It can be also shown that for parallel flow, as pictured in Fig. 4.5-3b, the log mean temperature difference should be used. In some cases, where steam is condensing, T'_1 and T'_2 may be the same. The equations still hold for this case. When U varies with distance or other complicating factors occur, other references should be consulted (B2, P3, W1).

EXAMPLE 4.5-4. Heat-Transfer Area and Log Mean Temperature Difference

A heavy hydrocarbon oil which has a c_{pm} = 2.30 kJ/kg · K is being cooled in a heat exchanger from 371.9 K to 349.7 K and flows inside the tube at a rate of 3630 kg/h. A flow of 1450 kg water/h enters at 288.6 K for cooling and flows outside the tube.

a. Calculate the water outlet temperature and heat-transfer area if the overall U_i = 340 W/m^2 · K and the streams are countercurrent.

b. Repeat for parallel flow.

Solution:

Assume a c_{pm} = 4.187 kJ/kg · K for water. The water inlet T_2 = 288.6 K, outlet = T_1; oil inlet T'_1 = 371.9, outlet T'_2 = 349.7 K. Calculating the heat lost by the oil,

$$q = \left(3630\frac{\text{kg}}{\text{h}}\right)\left(2.30\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(371.9 - 349.7)\ \text{K}$$

$$= 185\ 400\ \text{kJ/h} \quad \text{or} \quad 51\ 490\ \text{W}\ (175\ 700\ \text{btu/h})$$

By a heat balance, the q must also equal the heat gained by the water:

$$q = 185\ 400\ \text{kJ/h} = \left(1450\frac{\text{kg}}{\text{h}}\right)\left(4.187\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(T_1 - 288.6)\text{K}$$

Solving, T_1 = 319.1 K.

To solve for the log mean temperature difference, $\Delta T_2 = T'_2 - T_2$ = 349.7 − 288.6 = 61.1 K, $\Delta T_1 = T'_1 - T_1$ = 371.9 - 319.1 = 52.8 K. Substituting into Eq. (4.5-27),

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = \frac{61.1 - 52.8}{\ln(61.1/52.8)} = 56.9\ \text{K}$$

Using Eq. (4.5-26),

$$q = U_i A_i\, \Delta T_{\text{lm}}$$

$$51\ 490 = 340(A_i)(56.9)$$

Solving, A_i = 2.66 m^2.

For part (b), the water outlet is still T_1 = 319.1 K. Referring to Fig. 4.5-3b, ΔT_2 = 371.9 − 288.6 = 83.3K and ΔT_1 = 349.7 - 319.1 = 30.6 K. Again, using Eq. (4.5-27) and solving, ΔT_{lm} = 52.7 K. Substituting into Eq. (4.5-26), A_i = 2.87 m^2. This is a larger area than for counterflow. This occurs because counterflow gives larger temperature driving forces, and it is usually preferred over parallel flow for this reason.

EXAMPLE 4.5-5. Laminar Heat Transfer and Trial and Error

A hydrocarbon oil at 150°F enters inside a pipe with an inside diameter of 0.0303 ft and a length of 15 ft with a flow rate of 80 lb_m/h. The inside pipe surface is assumed constant at 350°F, since steam is condensing outside the pipe wall, and has a very large heat-transfer coefficient. The properties of the oil are c_{pm} = 0.50 btu/lb_m · °F and k_m = 0.083 btu/h · ft · °F. The viscosity of the oil varies with temperature as follows: 150°F, 6.50 cp; 200°F, 5.05 cp; 250°F, 3.80 cp; 300°F, 2.82 cp; 350°F, 1.95 cp. Predict the heat-transfer coefficient and the oil outlet temperature, T_{bo}.

Solution. This is a trial-and-error solution since the outlet temperature of the oil T_{bo} is unknown. The value of T_{bo} = 250°F will be assumed and checked later. The bulk mean temperature of the oil to use for the physical properties is (150 + 250)/2 or 200°F. The viscosity at 200°F is

$$\mu_b = 5.05(2.4191) = 12.23 \frac{\text{lb}_\text{m}}{\text{ft} \cdot \text{h}}$$

At the wall temperature of 350°F,

$$\mu_w = 1.95(2.4191) = 4.72 \frac{\text{lb}_\text{m}}{\text{ft} \cdot \text{h}}$$

The cross-section area of the pipe A is

$$A = \frac{\pi D_i^2}{4} = \frac{\pi(0.0303)^2}{4} = 0.000722 \text{ ft}^2$$

$$G = \frac{m}{A} = \frac{80 \text{ lb}_\text{m}/\text{h}}{0.000722 \text{ ft}^2} = 111\,000 \frac{\text{lb}_\text{m}}{\text{ft}^2 \cdot \text{h}}$$

The Reynolds number at the bulk mean temperature is

$$N_{\text{Re}} = \frac{D_i v \rho}{\mu} = \frac{D_i G}{\mu} = \frac{0.0303(111\,000)}{12.23} = 275.5$$

The Prandtl number is

$$N_{\text{Pr}} = \frac{c_p \mu}{k} = \frac{0.50(12.23)}{0.083} = 73.7$$

Since the N_{Re} is below 2100, the flow is in the laminar region and Eq. (4.5-4) will be used. Even at the outlet temperature of 250°F, the flow is still laminar. Substituting,

$$(N_{\text{Nu}})_a = \frac{h_a D}{k} = 1.86\left(N_{\text{Re}} N_{\text{Pr}} \frac{D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$\frac{h_a(0.0303)}{0.083} = 1.86\left[275.5(73.7)\left(\frac{0.0303}{15.0}\right)\right]^{1/3} \left(\frac{12.23}{4.72}\right)^{0.14}$$

Solving, h_a = 20.1 btu/h · ft^2 · °F (114 W/m^2 · K). Next, making a heat balance on the oil,

Equation 4.5-28.

$$q = mc_{pm}(T_{bo} - T_{bi}) = 80.0(0.50)(T_{bo} - 150)$$

Using Eq. (4.5-7)

Equation 4.5-7.

$$q = h_a A \, \Delta T_a$$

For ΔT_a,

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

$$= \frac{(350 - 150) + (350 - T_{bo})}{2}$$

$$= 275 - 0.5T_{bo}$$

Equating Eq. (4.5-28) to (4.5-7) and substituting,

$$80.0(0.50)(T_{bo} - 150) = hA \, \Delta T_a$$

$$= 20.1[\pi(0.303)(15)](275 - 0.5T_{bo})$$

Solving, T_{bo} = 255°F.

This is higher than the assumed value of 250°F. For the second trial, the mean bulk temperature of the oil would be (150 + 255)/2 or 202.5°F. The new viscosity is 5.0 cp compared with 5.05 for the first estimate. This only affects the $(\mu_b/\mu_w)^{0.14}$ factor in Eq. (4.5-4), since the viscosity effect in the $(N_{\text{Re}})(N_{\text{Pr}})$ factor cancels out. The heat-transfer coefficient will change by less than 0.2%, which is negligible. Hence, the outlet temperature of T_1 = 255°F (123.9°C) is correct.

HEAT TRANSFER OUTSIDE VARIOUS GEOMETRIES IN FORCED CONVECTION

Introduction

In many cases a fluid is flowing over completely immersed bodies such as spheres, tubes, plates, and so on, and heat transfer is occurring between the fluid and the solid only. Many of these shapes are of practical interest in process engineering. The sphere, cylinder, and flat plate are perhaps of greatest importance, with heat transfer between these surfaces and a moving fluid frequently encountered.

When heat transfer occurs during immersed flow, the flux is dependent on the geometry of the body, the position on the body (front, side, back, etc.), the proximity of other bodies, the flow rate, and the fluid properties. The heat-transfer coefficient varies over the body. The average heat-transfer coefficient is given in the empirical relationships to be discussed in the following sections.

In general, the average heat-transfer coefficient on immersed bodies is given by

Equation 4.6-1.

$$N_{\text{Nu}} = C N_{\text{Re}}^{m} N_{\text{Pr}}^{1/3}$$

where *C* and *m* are constants that depend on the various configurations. The fluid properties are evaluated at the film temperature $T_f = (T_w + T_b)/2$, where T_w is the surface or wall temperature and T_b the average bulk fluid temperature. The velocity in the N_{Re} is the undisturbed free stream velocity *v* of the fluid approaching the object.

Flow Parallel to Flat Plate

When the fluid is flowing parallel to a flat plate and heat transfer is occurring between the whole plate of length *L* m and the fluid, the N_{Nu} is as follows for a $N_{Re,L}$ below 3×10^5 in the laminar region and a $N_{Pr} > 0.7$:

Equation 4.6-2.

$$N_{Nu} = 0.664 N_{Re,L}^{0.5} N_{Pr}^{1/3}$$

where $N_{Re,L} = Lv\rho/\mu$ and $N_{Nu} = hL/k$.

For the completely turbulent region at a $N_{Re,L}$ above 3×10^5 (K1, K3) and $N_{Pr} > 0.7$,

Equation 4.6-3.

$$N_{Nu} = 0.0366 N_{Re,L}^{0.8} N_{Pr}^{1/3}$$

However, turbulence can start at a $N_{Re,L}$ below 3×10^5 if the plate is rough (K3), and then Eq. (4.6-3) will hold and give a N_{Nu} greater than that given by Eq. (4.6-2). For a $N_{Re,L}$ below about 2×10^4, Eq. (4.6-2) gives the larger value of N_{Nu}.

EXAMPLE 4.6-1. Cooling a Copper Fin

A smooth, flat, thin fin of copper extending out from a tube is 51 mm by 51 mm square. Its temperature is approximately uniform at 82.2°C. Cooling air at 15.6°C and 1 atm abs flows parallel to the fin at a velocity of 12.2 m/s.

a. For laminar flow, calculate the heat-transfer coefficient, *h.*
b. If the leading edge of the fin is rough so that all of the boundary layer or film next to the fin is completely turbulent, calculate *h.*

***Solution*:** The fluid properties will be evaluated at the film temperature $T_f = (T_w + T_b)/2$:

$$T_f = \frac{T_w + T_b}{2} = \frac{82.2 + 15.6}{2} = 48.9°\text{C (322.1 K)}$$

The physical properties of air at 48.9°C from Appendix A.3 are $k = 0.0280$ W/m · K, $\rho = 1.097$ kg/m^3, $\mu = 1.95 \times 10^{-5}$ Pa · s, $N_{Pr} = 0.704$. The Reynolds number is, for $L = 0.051$ m,

$$N_{Re,L} = \frac{Lv\rho}{\mu} = \frac{(0.051)(12.2)(1.097)}{1.95 \times 10^{-5}} = 3.49 \times 10^4$$

Substituting into Eq. (4.6-2),

$$N_{Nu} = \frac{hL}{k} = 0.664 N_{Re,L}^{0.5} N_{Pr}^{1/3}$$

$$\frac{h(0.051)}{(0.0280)} = 0.664(3.49 \times 10^4)^{0.5}(0.704)^{1/3}$$

Solving, $h = 60.7$ W/m^2 · K (10.7 btu/h · ft^2 · °F).

For part (b), substituting into Eq. (4.6-3) and solving, $h = 77.2$ W/m^2 · K (13.6 btu/h · ft^2 · °F).

Cylinder with Axis Perpendicular to Flow

Often a cylinder containing a fluid inside is being heated or cooled by a fluid flowing perpendicular to its axis. The equation for predicting the average heat-transfer coefficient of the outside of the cylinder for gases and liquids is (K3, P3) Eq. (4.6-1), with C and m as given in Table 4.6-1. The $N_{Re} = Dv\rho/\mu$, where D is the outside tube diameter and all physical properties are evaluated at the film temperature T_f. The velocity is the undisturbed free stream velocity approaching the cylinder.

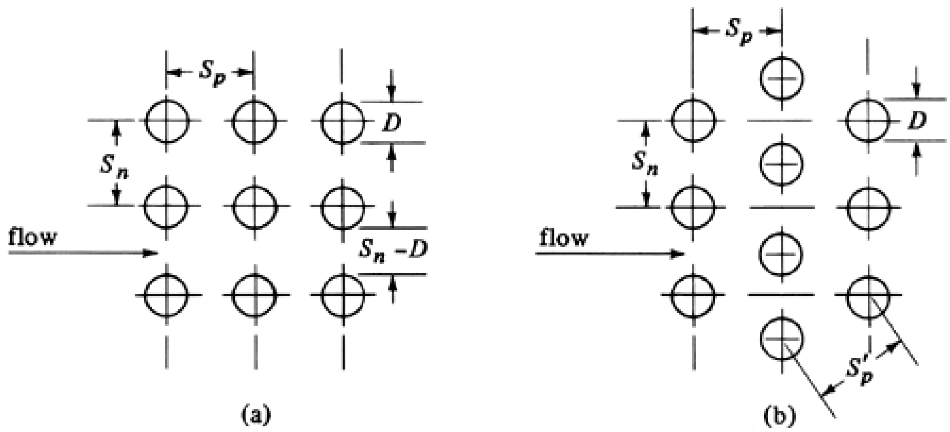


Figure 4.6-1. Nomenclature for banks of tubes in Table 4.6-2: (a) in-line tube rows, (b) staggered tube rows.

Table 4.6-1. Constants for Use in Eq. (4.6-1) for Heat Transfer to Cylinders with Axis Perpendicular to Flow (N_{Pr} > 0.6)

N_{Re}	m	C
1–4	0.330	0.989
4–40	0.385	0.911
40–4 × 10^3	0.466	0.683
4 × 10^3–4 × 10^4	0.618	0.193
4 × 10^4–2.5 × 10^5	0.805	0.0266

Flow Past Single Sphere

When a single sphere is being heated or cooled by a fluid flowing past it, the following equation can be used to predict the average heat-transfer coefficient for a $N_{Re} = Dv\rho/\mu$ of 1 to 70 000 and a N_{Pr} of 0.6 to 400:

Equation 4.6-4.

$$N_{Nu} = 2.0 + 0.60 N_{Re}^{0.5} N_{Pr}^{1/3}$$

The fluid properties are evaluated at the film temperature T_f. A somewhat more accurate correlation, which takes into account the effects of natural convection at these lower Reynolds numbers, is available for a N_{Re} range 1–17 000 from other sources (S2).

EXAMPLE 4.6-2. Cooling of a Sphere

Using the same conditions as Example 4.6-1, where air at 1 atm abs pressure and 15.6°C is flowing at a velocity of 12.2 m/s, predict the average heat-transfer coefficient for air flowing past a sphere having a diameter of 51 mm and an average surface temperature of 82.2°C. Compare this with the value of h = 77.2 W/m^2 · K for the flat plate in turbulent flow.

Solution: The physical properties at the average film temperature of 48.9°C are the same as for Example 4.6-1. The N_{Re} is

$$N_{\text{Re}} = \frac{Dv\rho}{\mu} = \frac{(0.051)(12.2)(1.097)}{1.95 \times 10^{-5}} = 3.49 \times 10^4$$

Substituting into Eq. (4.6-4) for a sphere,

$$N_{\text{Nu}} = \frac{hD}{k} = \frac{h(0.051)}{0.0280} = 2.0 + 0.60 N_{\text{Re}}^{0.5} N_{\text{Pr}}^{1/3}$$

$$= 2.0 + (0.60)(3.49 \times 10^4)^{0.5}(0.704)^{1/3}$$

Solving, h = 56.1 W/m^2 · K (9.88 btu/h · ft^2 · °F). This value is somewhat smaller than the value of h = 77.2 W/m^2 · K (13.6 btu/h · ft^2 · °F) for a flat plate.

Flow Past Banks of Tubes or Cylinders

Many types of commercial heat exchangers are constructed with multiple rows of tubes, where the fluid flows at right angles to the bank of tubes. An example is a gas heater in which a hot fluid inside the tubes heats a gas passing over the outside of the tubes. Another example is a cold liquid stream inside the tubes being heated by a hot fluid on the outside.

Figure 4.6-1 shows the arrangement for banks of tubes in-line and banks of tubes staggered, where D is tube OD in m (ft), S_n is distance m (ft) between the centers of the tubes normal to the flow, and S_p that parallel to the flow. The open area to flow for in-line tubes is ($S_n - D$) and ($S_p - D$); for staggered tubes it is ($S_n - D$) and ($S'_p - D$). Values of C and m to be used in Eq. (4.6-1) for a Reynolds-number range of 2000 to 40 000 for heat transfer to banks of tubes containing more than 10 transverse rows in the direction of flow are given in Table 4.6-2. For less than 10 rows, Table 4.6-3 gives correction factors.

Table 4.6-2. Values of C and m To Be Used in Eq. (4.6-1) for Heat Transfer to Banks of Tubes Containing More Than 10 Transverse Rows

	$\frac{S_n}{D} = \frac{S_p}{D} = 1.25$		$\frac{S_n}{D} = \frac{S_p}{D} = 1.50$		$\frac{S_n}{D} = \frac{S_p}{D} = 2.0$	
Arrangement	**C**	**m**	**C**	**m**	**C**	**m**
In-line	**0.386**	**0.592**	**0.278**	**0.620**	**0.254**	**0.632**
Staggered	**0.575**	**0.556**	**0.511**	**0.562**	**0.535**	**0.556**

***Source:* E. D. Grimison, *Trans. ASME*, 59, 583 (1937).**

Table 4.6-3. Ratio of h for N Transverse Rows Deep to h for 10 Transverse Rows Deep (for Use with Table 4.6-2)

N	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Ratio for staggered tubes	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99	1.00
Ratio for in-line tubes	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99	1.00

Source: **W. M. Kays and R. K. Lo, *Stanford Univ. Tech. Rept. 15,* Navy Contract N6-ONR-251 T.O.6, 1952.**

For cases where S_n/D and S_p/D are not equal to each other, the reader should consult Grimison (G1) for more data. In baffled exchangers where there is normal leakage where all the fluid does not flow normal to the tubes, the average values of h obtained should be multiplied by about 0.6 (P3). The Reynolds number is calculated using the minimum area open to flow for the velocity. All physical properties are evaluated at T_f.

EXAMPLE 4.6-3. Heating Air by a Bank of Tubes

Air at 15.6°C and 1 atm abs flows across a bank of tubes containing four transverse rows in the direction of flow and 10 rows normal to the flow at a velocity of 7.62 m/s as the air approaches the bank of tubes. The tube surfaces are maintained at 57.2°C. The outside diameter of the tubes is 25.4 mm and the tubes are in-line to the flow. The spacing S_n of the tubes normal to the flow is 38.1 mm and S_p is also 38.1 mm parallel to the flow. For a 0.305-m length of the tube bank, calculate the heat-transfer rate.

Solution: Referring to Fig. 4.6-1a,

$$\frac{S_n}{D} = \frac{38.1}{25.4} = \frac{1.5}{1} \qquad \frac{S_p}{D} = \frac{38.1}{25.4} = \frac{1.5}{1}$$

Since the air is heated in passing through the four transverse rows, an outlet bulk temperature of 21.1°C will be assumed. The average bulk temperature is then

$$T_b = \frac{15.6 + 21.1}{2} = 18.3°\text{C}$$

The average film temperature is

$$T_f = \frac{T_w + T_b}{2} = \frac{57.2 + 18.3}{2} = 37.7°\text{C}$$

From Appendix A.3, for air at 37.7°C,

$$k = 0.02700 \text{ W/m}\cdot\text{K} \qquad N_{\text{Pr}} = 0.705$$

$$c_p = 1.0048 \text{ kJ/kg}\cdot\text{K} \qquad \rho = 1.137 \text{ kg/m}^3$$

$$\mu = 1.90 \times 10^{-5} \text{ Pa}\cdot\text{s}$$

The ratio of the minimum-flow area to the total frontal area is $(S_n - D)/S_n$. The maximum velocity in the tube banks is then

$$v_{\max} = \frac{vS_n}{S_n - D} = \frac{7.62(0.0381)}{(0.0381 - 0.0254)} = 22.86 \text{ m/s}$$

$$N_{\text{Re}} = \frac{Dv_{\max}\rho}{\mu} = \frac{0.0254(22.86)(1.137)}{1.90 \times 10^{-5}} = 3.47 \times 10^4$$

For $S_n/D = S_p/D = 1.5/1$, the values of C and m from Table 4.6-2 are 0.278 and 0.620, respectively. Substituting into Eq. (4.6-1) and solving for h,

$$h = \frac{k}{D} C N_{\mathrm{Re}}^{m} N_{\mathrm{Pr}}^{1/3} = \left(\frac{0.02700}{0.0254}\right)(0.278)(3.47 \times 10^4)^{0.62}(0.705)^{1/3}$$

$$= 171.8\ \mathrm{W/m^2 \cdot K}$$

This h is for 10 rows. For only four rows in the transverse direction, the h must be multiplied by 0.90, as given in Table 4.6-3.

Since there are 10 × 4 or 40 tubes, the total heat-transfer area per 0.305 m length is

$$A = 40\pi DL = 40\pi(0.0254)(0.305) = 0.973\ \mathrm{m^2}$$

The total heat-transfer rate q using an arithmetic average temperature difference between the wall and the bulk fluid is

$$q = hA(T_w - T_b) = (0.90 \times 171.8)(0.973)(57.2 - 18.3) = 5852\ \mathrm{W}$$

Next, a heat balance on the air is made to calculate its temperature rise ΔT using the calculated q. First, the mass flow rate of air m must be calculated. The total frontal area of the tube-bank assembly of 10 rows of tubes each 0.305 m long is

$$A_t = 10S_n(1.0) = 10(0.0381)(0.305) = 0.1162\ \mathrm{m^2}$$

The density of the entering air at 15.6°C is $\rho = 1.224\ \mathrm{kg/m^3}$. The mass flow rate m is

$$m = \upsilon\rho A_t(3600) = 7.62(1.224)(0.1162) = 1.084\ \mathrm{kg/s}$$

For the heat balance, the mean c_p of air at 18.3°C is 1.0048 kJ/kg · K, and then

$$q = 5852 = mc_p\,\Delta T = 1.084(1.0048 \times 10^3)\,\Delta T$$

Solving, $\Delta T = 5.37$°C.

Hence, the calculated outlet bulk gas temperature is 15.6 + 5.37 = 20.97°C, which is close to the assumed value of 21.1°C. If a second trial were to be made, the new average T_b to use would be (15.6 + 20.97)/2 or 18.28°C.

Heat Transfer for Flow in Packed Beds

Correlations for heat-transfer coefficients for packed beds are useful in designing fixed-bed systems such as catalytic reactors, dryers for solids, and pebble-bed heat exchangers. In Section 3.1C the pressure drop in packed beds was considered and discussions of the geometry factors in these beds were given. For determining the rate of heat transfer in packed beds for a differential length dz in m,

Equation 4.6-5.

$$dq = h(aS\,dz)(T_1 - T_2)$$

where a is the solid-particle surface area per unit volume of bed in $\mathrm{m^{-1}}$, S the empty cross-sectional area of bed in $\mathrm{m^2}$, T_1 the bulk gas temperature in K. and T_2 the solid surface temperature.

For the heat transfer of gases in beds of spheres (G2, G3) and a Reynolds number range of 10–10 000,

Equation 4.6-6.

$$\varepsilon J_H = \varepsilon \frac{h}{c_p v' \rho}\left(\frac{c_p \mu}{k}\right)_f^{2/3} = \frac{2.876}{N_{\mathrm{Re}}} + \frac{0.3023}{N_{\mathrm{Re}}^{0.35}}$$

where v' is the superficial velocity based on the cross section of the empty container in m/s [see Eq. (3.1-11)], ε is the void fraction, $N_{\mathrm{Re}} = D_p G'/\mu_f$, and $G' = v'\rho$ is the superficial mass velocity in kg/m$^2 \cdot$ s. The subscript f indicates properties evaluated at the film temperature, with others at the bulk temperature. This correlation can also be used for a fluidized bed. An alternate equation to use in place of Eq. (4.6-6) for fixed and fluidized beds is Eq. (7.3-36) for a Reynolds-number range of 10–4000. The term J_H is called the Colburn J factor and is defined as in Eq. (4.6-6) in terms of h.

Equations for heat transfer to noncircular cylinders such as hexagons and so forth are given elsewhere (H1, J1, P3).

NATURAL CONVECTION HEAT TRANSFER

Introduction

Natural convection heat transfer occurs when a solid surface is in contact with a gas or liquid which is at a different temperature from the surface. Density differences in the fluid arising from the heating process provide the buoyancy force required to move the fluid. Free or natural convection is observed as a result of the motion of the fluid. An example of heat transfer by natural convection is a hot radiator used for heating a room. Cold air encountering the radiator is heated and rises in natural convection because of buoyancy forces. The theoretical derivation of equations for natural convection heat-transfer coefficients requires the solution of motion and energy equations.

An important heat-transfer system occurring in process engineering is that in which heat is being transferred from a hot vertical plate to a gas or liquid adjacent to it by natural convection. The fluid is not moving by forced convection but only by natural or free convection. In Fig. 4.7-1 the vertical flat plate is heated and the free convection boundary layer is formed. The velocity profile differs from that in a forced convection system in that the velocity at the wall is zero and also is zero at the other edge of the boundary layer, since the free stream velocity is zero for natural convection. The boundary layer initially is laminar as shown, but at some distance from the leading edge it starts to become turbulent. The wall temperature is T_w K and the bulk temperature T_b.

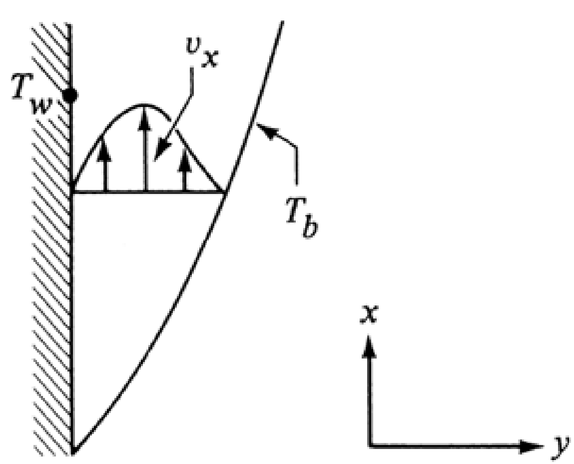


Figure 4.7-1. Boundary-layer velocity profile for natural convection heat transfer from a heated, vertical plate.

The differential momentum-balance equation is written for the x and y directions for the control volume ($dx\,dy \cdot 1$). The driving force is the buoyancy force in the gravitational field and is due to the density difference of the fluid. The momentum balance becomes

Equation 4.7-1.

$$\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) = g(\rho_b - \rho) + \mu \frac{\partial^2 v_x}{\partial y^2}$$

where ρ_b is the density at the bulk temperature T_b and ρ the density at T. The density difference can be expressed in terms of the volumetric coefficient of expansion β and substituted back into Eq. (4.7-1):

Equation 4.7-2.

$$\beta = \frac{\rho_b - \rho}{\rho(T - T_b)}$$

For gases, $\beta = 1/T$. The energy-balance equation can be expressed as follows:

Equation 4.7-3.

$$\rho c_p\left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y}\right) = k \frac{\partial^2 T}{\partial y^2}$$

The solutions of these equations have been obtained by using integral methods of analysis discussed in Section 3.10. Results have been obtained for a vertical plate, which is the simplest case and serves to introduce the dimensionless Grashof number discussed below. However, in other physical geometries the relations are too complex and empirical correlations have been obtained. These are discussed in the following sections.

Natural Convection from Various Geometries

Natural convection from vertical planes and cylinders

For an isothermal vertical surface or plate with height L less than 1 m (P3), the average natural convection heat-transfer coefficient can be expressed by the following general equation:

Equation 4.7-4.

$$N_{\mathrm{Nu}} = \frac{hL}{k} = a\left(\frac{L^3\rho^2 g\beta\,\Delta T}{\mu^2}\,\frac{c_p\mu}{k}\right)^m = a(N_{\mathrm{Gr}}\,N_{\mathrm{Pr}})^m$$

where a and m are constants from Table 4.7-1, N_{Gr} the Grashof number, ρ density in kg/m^3, μ viscosity in kg/m · s, ΔT the positive temperature difference between the wall and bulk fluid or vice versa in K, k the thermal conductivity in W/m · K, c_p the heat capacity in J/kg · K, β the volumetric coefficient of expansion of the fluid in 1/K [for gases β is $1/(T_f$ K)], and g is 9.80665 m/s^2. All the physical properties are evaluated at the film temperature $T_f = (T_w + T_b)/2$. In general, for a vertical cylinder with length L m, the same equations can be used as for a vertical plate. In English units β is $1/(T_f\,°\mathrm{F} + 460)$ in 1/°R and g is $32.174 \times (3600)^2$ ft/h^2.

Table 4.7-1. Constants for Use with Eq. (4.7-4) for Natural Convection

Physical Geometry	$N_{Gr}N_{Pr}$	*a*	*m*	*Ref.*
Vertical planes and cylinders				
[vertical height L < 1 m (3 ft)]				
	$<10^4$	1.36	$\frac{1}{5}$	(P3)
	10^4-10^9	0.59	$\frac{1}{4}$	(M1)
	$>10^9$	0.13	$\frac{1}{3}$	(M1)
Horizontal cylinders				
[diameter D used for L and D < 0.20 m (0.66 ft)]				
	$<10^{-5}$	0.49	0	(P3)
	10^{-5}-10^{-3}	0.71	$\frac{1}{25}$	(P3)
	10^{-3}-1	1.09	$\frac{1}{10}$	(P3)
	1-10^4	1.09	$\frac{1}{5}$	(P3)
	10^4-10^9	0.53	$\frac{1}{4}$	(M1)
	$>10^9$	0.13	$\frac{1}{3}$	(P3)
Horizontal plates				
Upper surface of heated plates or ower surface of cooled plates	10^5-2×10^7	0.54	$\frac{1}{4}$	(M1)
	2×10^7-3×10^{10}	0.14	$\frac{1}{3}$	(M1)
Lower surface of heated plates or upper surface of cooled plates	10^5-10^{11}	0.58	$\frac{1}{5}$	(F1)

The Grashof number can be interpreted physically as a dimensionless number that represents the ratio of the buoyancy forces to the viscous forces in free convection and plays a role similar to that of the Reynolds number in forced convection.

EXAMPLE 4.7-1. Natural Convection from Vertical Wall of an Oven

A heated vertical wall 1.0 ft (0.305 m) high of an oven for baking food with the surface at 450°F (505.4 K) is in contact with air at 100°F (311 K). Calculate the heat-transfer coefficient and the heat transfer/ft (0.305 m) width of wall. Note that heat transfer for radiation will not be considered. Use English and SI units.

Solution. The film temperature is

$$T_f = \frac{T_w + T_b}{2} = \frac{450 + 100}{2} = 275°\text{F} = \frac{505.4 + 311}{2} = 408.2 \text{ K}$$

The physical properties of air at 275°F are k = 0.0198 btu/h · ft · °F, 0.0343 W/m · K; ρ = 0.0541 lb_m/ft^3, 0.867 kg/m^3; N_{Pr} = 0.690; μ = (0.0232 cp) × (2.4191) = 0.0562 lb_m/ft · h = 2.32 × 10^{-5} Pa · s; β = 1/408.2 = 2.45 × 10–3 K^{-1}, β = 1/(460 + 275) = 1.36 × 10^{-3} °R^{-1}; $\Delta T = T_w - T_b$ = 450 - 100 = 350°F (194.4 K). The Grashof number is, in English units,

$$N_{Gr} = \frac{L^3\rho^2 g\beta\,\Delta T}{\mu^2} = \frac{(1.0)^3(0.0541)^2(32.174)(3600)^2(1.36\times10^{-3})(350)}{(0.0562)^2}$$

$$= 1.84\times10^8$$

In SI units,

$$N_{Gr} = \frac{(0.305)^3(0.867)^2(9.806)(2.45\times10^{-3})(194.4)}{(2.32\times10^{-5})^2} = 1.84\times10^8$$

The Grashof numbers calculated using English and SI units must, of course, be the same as shown:

$$N_{Gr}N_{Pr} = (1.84\times10^8)(0.690) = 1.270\times10^8$$

Hence, from Table 4.7-1, $a = 0.59$ and $m = \frac{1}{4}$ for use in Eq. (4.7-4). Solving for h in Eq. (4.7-4) and substituting known values,

$$h = \frac{k}{L}a(N_{Gr}\,N_{Pr})^m = \left(\frac{0.0198}{1.0}\right)(0.59)(1.270\times10^8)^{1/4} = 1.24 \text{ btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$h = \left(\frac{0.0343}{0.305}\right)(0.59)(1.27\times10^8)^{1/4} = 7.03 \text{ W/m}^2\cdot\text{K}$$

For a 1-ft width of wall, $A = 1\times1 = 1.0$ ft^2 (0.305 × 0.305 m^2). Then

$$q = hA(T_w - T_b) = (1.24)(1.0)(450-100) = 433 \text{ btu/h}$$

$$q = 7.03(0.305\times0.305)(194.4) = 127.1 \text{ W}$$

A considerable amount of heat will also be lost by radiation. This will be considered in Section 4.10.

Simplified equations for the natural convection heat transfer from air to vertical planes and cylinders at 1 atm abs pressure are given in Table 4.7-2. In SI units the equation for the range of $N_{Gr}N_{Pr}$ of 10^4 to 10^9 is the one usually encountered, and this holds for ($L^3\,\Delta T$) values below about 4.7 m^3 · K and film temperatures between 255 and 533 K. To correct the value of h to pressures other than 1 atm, the values of h in Table 4.7-2 can be multiplied by $(p/101.32)^{1/2}$ for $N_{Gr}N_{Pr}$ 10^4 to 10^9 and by $(p/101.32)^{2/3}$ for $N_{Gr}N_{Pr} > 10^9$, where p = pressure in kN/m^2. In English units the range of $N_{Gr}N_{Pr}$ of 10^4 to 10^9 is encountered when ($L^3\,\Delta T$) is less than about 300 ft^3 · °F. The value of h can be corrected to pressures other than 1.0 atm abs by multiplying the h at 1 atm by $p^{1/2}$ for $N_{Gr}N_{Pr}$ of 10^4 to 10^9 and by $p^{2/3}$ for $N_{Gr}N_{Pr}$ above 10^9, where p = atm abs pressure. Simplified equations are also given for water and organic liquids.

Table 4.7-2. Simplified Equations for Natural Convection from Various Surfaces

Physical Geometry	$N_{Gr}N_{Pr}$	Equation: h = btu/h · ft² · °F, L = ft, ΔT = °F, D = ft	Equation: h = W/m² · K, L = m, ΔT = K, D = m	Ref.
		Air at 101.32 pa (1 atm) abs pressure		
Vertical planes and cylinders	10^4-10^9	$h = 0.28(\Delta T/L)^{1/4}$	$h = 1.37(\Delta T/L)^{1/4}$	(P1)
	$>10^9$	$h = 0.18(\Delta T)^{1/3}$	$h = 1.24\,\Delta T^{1/3}$	(P1)

		Equation		
		h = btu/h · ft2 · °F	$h = W/m^2 \cdot K$	
		L = ft, ΔT = °F	L = m, ΔT = K	
Physical Geometry	$N_{Gr}N_{Pr}$	D = ft	D = m	**Ref.**
Horizontal cylinders	10^3-10^9	$h = 0.27(\Delta T/D)^{1/4}$	$h = 1.32(\Delta T/D)^{1/4}$	(M1)
	$>10^9$	$h = 0.18(\Delta T)^{1/3}$	$h = 1.24\ \Delta T^{1/3}$	(M1)
Horizontal plates				
Heated plate facing upward or cooled plate facing downward	10^5-2×10^7	h = 0.2T$(\Delta T/L)^{1/4}$	$h = 1.32(\Delta T/L)^{1/4}$	(M1)
	2×10^7-3×10^{10}	$h = 0.22(\Delta T)^{1/3}$	$h = 1.52\ \Delta T^{1/3}$	(M1)
Heated plate facing downward or cooled plate facing upward	3×10^5-3×10^{10}	$h = 0.12(\Delta T/L)^{1/4}$	$h = 0.59(\Delta T/L)^{1/4}$	(M1)
		Water at 70°F (294 K)		
Vertical planes and cylinders	10^4-10^9	$h = 26(\Delta T/L)^{1/4}$	h = 12T$(\Delta T/L)^{1/4}$	(P1)
		Organic liquids at 70°F (294 K)		
Vertical planes and cylinders	10^4-10^9	$h = 12(\Delta T/L)^{1/4}$	$h = 59(\Delta T/L)^{1/4}$	(P1)

EXAMPLE 4.7-2. Natural Convection and Simplified Equation

Repeat Example 4.7-1 but use the simplified equation.

Solution: The film temperature of 408.2 K is in the range 255–533 K. Also,

$$L^3\ \Delta T = (0.305)^3(194.4) = 5.5$$

This is slightly greater than the value of 4.7 given as the approximate maximum for use of the simplified equation. However, in Example 4.7-1 the value of $N_{Gr}N_{Pr}$ is below 10^9, so the simplified equation from Table 4.7-2 will be used:

$$h = 1.37\left(\frac{\Delta T}{L}\right)^{1/4} = 1.37\left(\frac{194.4}{0.305}\right)^{1/4}$$

$$= 6.88\ \text{W/m}^2 \cdot \text{K}\ (1.21\ \text{btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})$$

The heat-transfer rate q is

$$q = hA(T_w - T_b) = 6.88(0.305 \times 0.305)(194.4) = 124.4\ \text{W}\ (424\ \text{btu/h})$$

This value is reasonably close to the value of 127.1 W for Example 4.7-1.

Natural convection from horizontal cylinders

For a horizontal cylinder with an outside diameter of D m, Eq. (4.7-4) is used with the constants given in Table 4.7-1. The diameter D is used for L in the equation. Simplified equations are given in Table 4.7-2. The usual case for pipes is for the $N_{Gr}N_{Pr}$ range 10^4 to 10^9 (M1).

Natural convection from horizontal plates

For horizontal flat plates Eq. (4.7-4) is also used with the constants given in Table 4.7-1 and simplified equations in Table 4.7-2. The dimension L to be used is the length of a side of a square plate, the linear mean of the two dimensions for a rectangle, and 0.9 times the diameter of a circular disk.

Natural convection in enclosed spaces

Free convection in enclosed spaces occurs in a number of processing applications. One example is in an enclosed double window in which two layers of glass are separated by a layer of air for energy conservation. The flow phenomena inside these enclosed spaces are complex, since a number of different types of flow patterns can occur. At low Grashof numbers the heat transfer is mainly by conduction across the fluid layer. As the Grashof number is increased, different flow regimes are encountered.

The system for two vertical plates of height L m containing the fluid with a gap of δ m is shown in Fig. 4.7-2, where the plate surfaces are at temperatures T_1 and T_2. The Grashof number is defined as

Equation 4.7-5.

$$N_{\mathrm{Gr},\delta} = \frac{\delta^3 \rho^2 g \beta (T_1 - T_2)}{\mu^2}$$

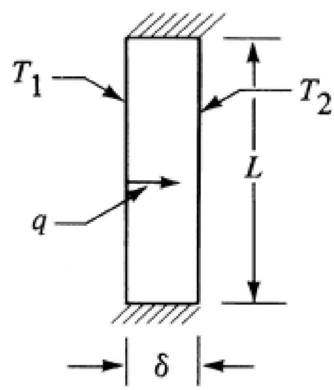


Figure 4.7-2. Natural convection in enclosed vertical space.

The Nusselt number is defined as

Equation 4.7-6.

$$N_{\mathrm{Nu},\delta} = \frac{h\delta}{k}$$

The heat flux is calculated from

Equation 4.7-7.

$$\frac{q}{A} = h(T_1 - T_2)$$

The physical properties are all evaluated at the mean temperature between the two plates. For gases enclosed between vertical plates and $L/\delta > 3$ (H1, J1, K1, P1),

Equation 4.7-8.

$$N_{Nu,\delta} = \frac{h\delta}{k} = 1.0 \qquad (N_{Gr,\delta} N_{Pr} < 2 \times 10^3)$$

Equation 4.7-9.

$$N_{Nu,\delta} = 0.20 \frac{(N_{Gr,\delta} N_{Pr})^{1/4}}{(L/\delta)^{1/9}} \qquad (6 \times 10^3 < N_{Gr,\delta} N_{Pr} < 2 \times 10^5)$$

Equation 4.7-10.

$$N_{Nu,\delta} = 0.073 \frac{(N_{Gr,\delta} N_{Pr})^{1/3}}{(L/\delta)^{1/9}} \qquad (2 \times 10^5 < N_{Gr,\delta} N_{Pr} < 2 \times 10^7)$$

For liquids in vertical plates,

Equation 4.7-11.

$$N_{Nu,\delta} = \frac{h\delta}{k} = 1.0 \qquad (N_{Gr,\delta} N_{Pr} < 1 \times 10^3)$$

Equation 4.7-12.

$$N_{Nu,\delta} = 0.28 \frac{(N_{Gr,\delta} N_{Pr})^{1/4}}{(L/\delta)^{1/4}} \qquad (1 \times 10^3 < N_{Gr,\delta} N_{Pr} < 1 \times 10^7)$$

For gases or liquids in a vertical annulus, the same equations hold as for vertical plates.

For gases in horizontal plates with the lower plate hotter than the upper,

Equation 4.7-13.

$$N_{Nu,\delta} = 0.21(N_{Gr,\delta} N_{Pr})^{1/4} \qquad (7 \times 10^3 < N_{Gr,\delta} N_{Pr} < 3 \times 10^5)$$

Equation 4.7-14.

$$N_{Nu,\delta} = 0.061(N_{Gr,\delta} N_{Pr})^{1/3} \qquad (N_{Gr,\delta} N_{Pr} > 3 \times 10^5)$$

For liquids in horizontal plates with the lower plate hotter than the upper (G5),

Equation 4.7-15.

$$N_{Nu,\delta} = 0.069(N_{Gr,\delta} N_{Pr})^{1/3} N_{Pr}^{0.074} \qquad (1.5 \times 10^5 < N_{Gr,\delta} N_{Pr} < 1 \times 10^9)$$

EXAMPLE 4.7-3. Natural Convection in Enclosed Vertical Space

Air at 1 atm abs pressure is enclosed between two vertical plates where L = 0.6 m and δ = 30 mm. The plates are 0.4 m wide. The plate temperatures are T_1 = 394.3 K and T_2 = 366.5 K. Calculate the heat-transfer rate across the air gap.

***Solution*:** The mean temperature between the plates is used to evaluate the physical properties: $T_f = (T_1 + T_2)/2$ = (394.3 + 366.5)/2 = 380.4 K. Also, δ = 30/1000 = 0.030 m. From Appendix A.3, ρ = 0.9295 kg/m^3, μ = 2.21 × 10^{-5} Pa · s, k = 0.03219 W/m · K, N_{Pr} = 0.693, $\beta = 1/T_f$ = 1/380.4 = 2.629 × 10–3 K^{-1}.

$$N_{\mathrm{Gr},\delta} = \frac{(0.030)^3(0.9295)^2(9.806)(2.629 \times 10^{-3})(394.3 - 366.5)}{(2.21 \times 10^{-5})^2}$$

$$= 3.423 \times 10^4$$

Also, $N_{\mathrm{Gr},\delta}N_{\mathrm{Pr}} = (3.423 \times 10^4)0.693 = 2.372 \times 10^4$. Using Eq. (4.7-9),

$$h = \frac{k}{\delta}\frac{(0.20)(N_{\mathrm{Gr},\delta}N_{\mathrm{Pr}})^{1/4}}{(L/\delta)^{1/9}} = \frac{0.03219(0.20)(2.352 \times 10^4)^{1/4}}{0.030(0.6/0.030)^{1/9}}$$

$$= 1.909\ \mathrm{W/m^2 \cdot K}$$

The area $A = (0.6 \times 0.4) = 0.24\ \mathrm{m}^2$. Substituting into Eq. (4.7-7),

$$q = hA(T_1 - T_2) = 1.909(0.24)(394.3 - 366.5) = 12.74\ \mathrm{W}$$

Natural convection from other shapes

For spheres, blocks, and other types of enclosed air spaces, references elsewhere (H1, K1, M1, P1, P3) should be consulted. In some cases when a fluid is forced over a heated surface at low velocity in the laminar region, combined forced convection plus natural convection heat transfer occurs. For further discussion of this, see (H1, K1, M1).

BOILING AND CONDENSATION

Boiling

Mechanisms of boiling

Heat transfer to a boiling liquid is very important in evaporation and distillation as well as in other kinds of chemical and biological processing, such as petroleum processing, control of the temperature of chemical reactions, evaporation of liquid foods, and so on. The boiling liquid is usually contained in a vessel with a heating surface of tubes or vertical or horizontal plates which supply the heat for boiling. The heating surfaces can be heated electrically or by a hot or condensing fluid on the other side of the heated surface.

In boiling, the temperature of the liquid is the boiling point of this liquid at the pressure in the equipment. The heated surface is, of course, at a temperature above the boiling point. Bubbles of vapor are generated at the heated surface and rise through the mass of liquid. The vapor accumulates in a vapor space above the liquid level and is withdrawn.

Boiling is a complex phenomenon. Suppose we consider a small heated horizontal tube or wire immersed in a vessel containing water boiling at 373.2 K (100°C). The heat flux is q/A W/m^2; $\Delta T = T_w - 373.2$ K, where T_w is the tube or wire wall temperature; and h is the heat-transfer coefficient in W/m$^2 \cdot$ K. Starting with a low ΔT, the q/A and h values are measured. This is repeated at higher values of ΔT and the data obtained are plotted as q/A versus ΔT, as shown in Fig. 4.8-1.

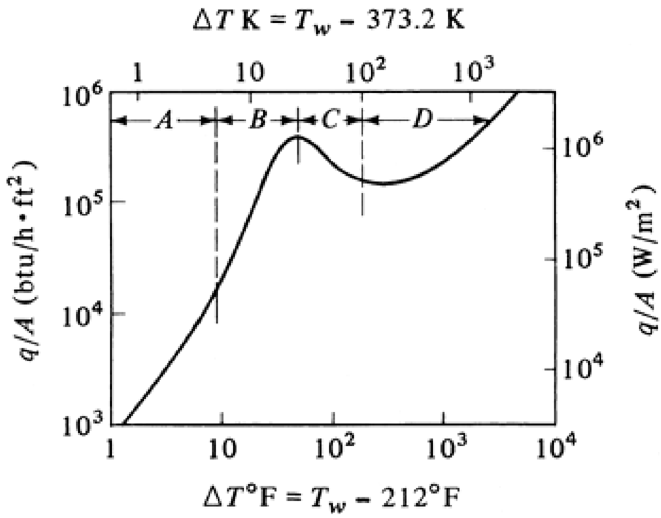


Figure 4.8-1. Boiling mechanisms for water at atmospheric pressure, heat flux vs. temperature drop: (A) natural convection, (B) nucleate boiling, (C) transition boiling, (D) film boiling.

In the first region *A* of the plot in Fig. 4.8-1, at low temperature drops, the mechanism of boiling is essentially that of heat transfer to a liquid in natural convection. The variation of *h* with $\Delta T^{0.25}$ is approximately the same as that for natural convection to horizontal plates or cylinders. The very few bubbles formed are released from the surface of the metal and rise without appreciably disturbing the normal natural convection.

In the region *B* of nucleate boiling for a ΔT of about 5–25 K (9–45°F), the rate of bubble production increases so that the velocity of circulation of the liquid increases. The heat-transfer coefficient *h* increases rapidly and is proportional to ΔT^2 to ΔT^3 in this region.

In the region *C* of transition boiling, many bubbles are formed so quickly that they tend to coalesce and form a layer of insulating vapor. Increasing the ΔT increases the thickness of this layer and the heat flux and *h* drop as ΔT is increased. In the region *D* of film boiling, bubbles detach themselves regularly and rise upward. At higher ΔT values radiation through thevapor layer next to the surface helps increase the *q/A* and *h.* Similar-shaped curves are obtained for other shapes of surfaces (M1).

The curve of *h* versus ΔT has approximately the same shape as in Fig. 4.8-1. The values of *h* are quite large. At the beginning of region *B* in Fig. 4.8-1 for nucleate boiling, *h* has a value of about 5700–11 400 W/m^2 · K, or 1000–2000 btu/h · ft^2 · °F, and at the end of this region *h* has a peak value of almost 57 000 W/m^2 · K, or 10 000 btu/hr · ft^2 · °F. These values are quite high, and in most cases the percent resistance of the boiling film is only a few percent of the overall resistance to heat transfer.

The regions of commercial interest are the nucleate and film-boiling regions (P3). Nucleate boiling occurs in kettle-type and natural-circulation reboilers.

Nucleate boiling

In the nucleate-boiling region, the heat flux is affected by ΔT, pressure, nature and geometry of the surface and system, and physical properties of the vapor and liquid. Equations have been derived by Rohesenow et al. (**P1**). They apply to single tubes or flat surfaces and are quite complex.

Simplified empirical equations for estimating the boiling heat-transfer coefficients for water boiling on the outside of submerged surfaces at 1.0 atm abs pressure have been developed (J2).

For a horizontal surface (SI and English units),

Equation 4.8-1.

$$h, \text{btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} = 151(\Delta T\ {}^\circ\text{F})^{1/3} \qquad q/A, \text{btu/h} \cdot \text{ft}^2, <5000$$

$$h, \text{W/m}^2 \cdot \text{K} = 1043(\Delta T\ \text{K})^{1/3} \qquad q/A, \text{kW/m}^2, <16$$

Equation 4.8-2.

$$h, \text{btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} = 0.168(\Delta T\ {}^\circ\text{F})^3 \qquad 5000 < q/A, \text{btu/h} \cdot \text{ft}^2, <75\,000$$

$$h, \text{W/m}^2 \cdot \text{K} = 5.56(\Delta T\ \text{K})^3 \qquad 16 < q/A, \text{kW/m}^2, <240$$

For a vertical surface,

Equation 4.8-3.

$$h, \text{btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} = 87(\Delta T\ {}^\circ\text{F})^{1/7} \qquad q/A, \text{btu/h} \cdot \text{ft}^2, <1000$$

$$h, \text{W/m}^2 \cdot \text{K} = 537(\Delta T\ \text{K})^{1/7} \qquad q/A, \text{kW/m}^2, <3$$

Equation 4.8-4.

$$h, \text{btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} = 0.240(\Delta T\ {}^\circ\text{F})^3 \qquad 1000 < q/A, \text{btu/h} \cdot \text{ft}^2, <20\,000$$

$$h, \text{W/m}^2 \cdot \text{K} = 7.95(\Delta T\ \text{K})^3 \qquad 3 < q/A, \text{kW/m}^2, <63$$

where $\Delta T = T_w - T_{sat}$ K or °F.

If the pressure is p atm abs, the values of h at 1 atm given above are multiplied by $(p/1)^{0.4}$. Equations (4.8-1) and (4.8-3) are in the natural convection region.

For forced convection boiling inside tubes, the following simplified relation can be used (J3):

Equation 4.8-5.

$$h = 2.55(\Delta T\ \text{K})^3 e^{p/1551}\ \text{W/m}^2 \cdot \text{K} \qquad \text{(SI)}$$

$$h = 0.077(\Delta T\ {}^\circ\text{F})^3 e^{p/225}\ \text{btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F} \qquad \text{(English)}$$

where p in this case is in kPa (SI units) and psia (English units).

Film boiling

In the film-boiling region, the heat-transfer rate is low in view of the large temperature drop used, which is not utilized effectively. Film boiling has been subjected to considerable theoretical analysis. Bromley (B3) gives the following equation to predict the heat-transfer coefficient in the film-boiling region on a horizontal tube:

Equation 4.8-6.

$$h = 0.62\left[\frac{k_v^3 \rho_v(\rho_l - \rho_v) g (h_{fg} + 0.4 c_{pv}\,\Delta T)}{D\mu_v\,\Delta T}\right]^{1/4}$$

where k_v is the thermal conductivity of the vapor in W/m · K, ρ_v the density of the vapor in kg/m^3, ρ_l the density of the liquid in kg/m^3, h_{fg} the latent heat of vaporization in J/kg, $\Delta T = T_w - T_{sat}$, T_{sat} the temperature of saturated vapor in K, D the outside tube diameter in m, μ_v the viscosity of the vapor in Pa · s, and g the acceleration of gravity in m/s^2. The physical properties of the vapor are evaluated at the film temperature of $T_f = (T_w + T_{sat})/2$, and h_{fg} at the saturation temperature. If the temperature difference is quite high, some additional heat transfer occurs by radiation (H1).

EXAMPLE 4.8-1. Rate of Heat Transfer in a Jacketed Kettle

Water is being boiled at 1 atm abs pressure in a jacketed kettle with steam condensing in the jacket at 115.6°C. The inside diameter of the kettle is 0.656 m and the height is 0.984 m. The bottom is slightly curved but it will be assumed to be flat. Both the bottom and the sides up to a height of 0.656 m are jacketed. The kettle surface for heat transfer is 3.2-mm stainless steel with a k of 16.27 W/m · K. The condensing-steam coefficient h_i inside the jacket has been estimated as 10 200 W/m^2 · K. Predict the boiling heat-transfer coefficient h_o for the bottom surface of the kettle.

Solution: A diagram of the kettle is shown in Fig. 4.8-2. The simplified equations will be used for the boiling coefficient h_o. The solution is trial and error, since the inside metal surface temperature T_w is unknown. Assuming that T_w = 110°C,

$$\Delta T = T_w - T_{sat} = 110 - 100 = 10°\text{C} = 10\text{ K}$$

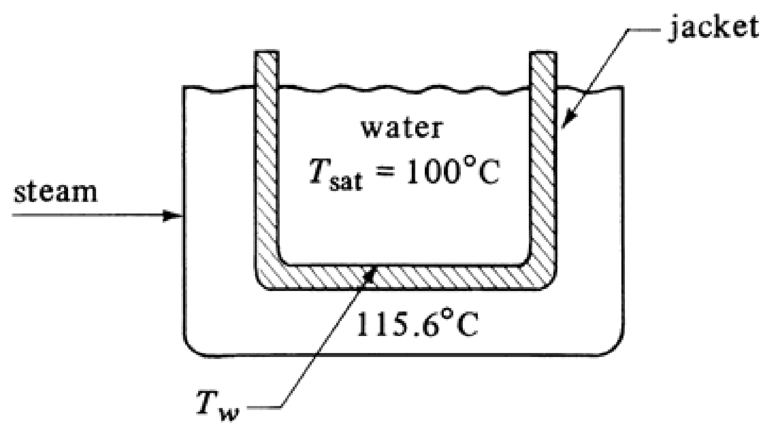


Figure 4.8-2. Steam-jacketed kettle and boiling water for Example 4.8-1.

Substituting into Eq. (4.8-2),

$$h_o = 5.56(\Delta T)^3 = 5.56(10)^3 = 5560\text{ W/m}^2\cdot\text{K}$$

$$\frac{q}{A} = h\,\Delta T = 5560(10) = 55\,600\text{ W/m}^2$$

To check the assumed T_w, the resistance R_i of the condensing steam, R_w of the metal wall, and R_o of the boiling liquid must be calculated. Assuming equal areas of the resistances for A = 1 m^2, then by Eq. (4.3-12),

$$R_i = \frac{1}{h_i A} = \frac{1}{10\,200(1)} = 9.80 \times 10^{-5}$$

$$R_w = \frac{\Delta x}{kA} = \frac{3.2/1000}{16.27(1.0)} = 19.66 \times 10^{-5}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{5560(1)} = 17.98 \times 10^{-5}$$

$$\sum R = 9.80 \times 10^{-5} + 19.66 \times 10^{-5} + 17.98 \times 10^{-5}$$

$$= 47.44 \times 10^{-5}$$

The temperature drop across the boiling film is then

$$\Delta T = \frac{R_o}{\sum R}(115.6 - 100) = \frac{17.98 \times 10^{-5}}{47.44 \times 10^{-5}}(15.6) = 5.9°\text{C}$$

Hence, $T_w = 100 + 5.9 = 105.9$°C. This is lower than the assumed value of 110°C.

For the second trial, $T_w = 108.3$°C will be used. Then, $\Delta T = 108.3 - 100 = 8.3$°C and, from Eq. (4.8-2), the new $h_o = 3180$. Calculating the new $R_o = 31.44 \times 10^{-5}$, and

$$\Delta T = \left(\frac{31.44 \times 10^{-5}}{60.90 \times 10^{-5}}\right)(115.6 - 100) = 8.1°\text{C}$$

and

$$T_w = 100 + 8.1 = 108.1°\text{C}$$

This value is reasonably close to the assumed value of 108.3°C, so no further trials will be made.

Condensation

Mechanisms of condensation

Condensation of a vapor to a liquid and vaporization of a liquid to a vapor both involve a change of phase of a fluid with large heat-transfer coefficients. Condensation occurs when a saturated vapor such as steam comes in contact with a solid whose surface temperature is below the saturation temperature, to form a liquid such as water.

Normally, when a vapor condenses on a surface such as a vertical or horizontal tube or other surface, a film of condensate is formed on the surface and flows over the surface by the action of gravity. It is this film of liquid between the surface and the vapor that forms the main resistance to heat transfer. This is called *film-type condensation.*

Another type of condensation, *dropwise condensation,* can occur, where small drops are formed on the surface. These drops grow and coalesce, and the liquid flows from the surface. During this condensation, large areas of tube are devoid of any liquid and are exposed directly to the vapor. Very high rates of heat transfer occur on these bare areas. The average coefficient can be as high as 110 000 $W/m^2 \cdot K$ (20 000 $btu/h \cdot ft^2 \cdot °F$), which is five to 10 times larger than film-type coefficients. Film-condensation coefficients are normally much greater than those in forced convection and are on the order of magnitude of several thousand $W/m^2 \cdot K$ or more.

Dropwise condensation occurs on contaminated surfaces and when impurities are present. Film-type condensation is more dependable and more common. Hence, for normal design purposes, film-type condensation is assumed.

Film-condensation coefficients for vertical surfaces

Film-type condensation on a vertical wall or tube can be analyzed analytically by assuming laminar flow of the condensate film down the wall. The film thickness is zero at the top of the wall or tube and increases in thickness as it flows downward because of condensation. Nusselt (H1, W1) assumed that the heat transfer from the condensing vapor at T_{sat} K, through this liquid film, and to the wall at T_w K was by conduction. Equating this heat transfer by conduction to that from condensation of the vapor, a final expression can be obtained for the average heat-transfer coefficient over the whole surface.

In Fig. 4.8-3a, vapor at T_{sat} is condensing on a wall whose temperature is T_w K. The condensate is flowing downward in laminar flow. Assuming unit thickness, the mass of the element with liquid density ρ_l in Fig. 4.8-3b is $(\delta - y)(dx \cdot 1)\rho_l$. The downward force on this element is the gravitational force minus the buoyancy force, or $(\delta - y)(dx) \times (\rho_l - \rho_v)g$, where ρ_v is the density of the saturated vapor. This force is balanced by the viscous-shear force at the plane y of $\mu_l(dv/dy)(dx \cdot 1)$. Equating these forces,

Equation 4.8-7.

$$(\delta - y)(dx)(\rho_l - \rho_v)g = \mu_l\left(\frac{dv}{dy}\right)(dx)$$

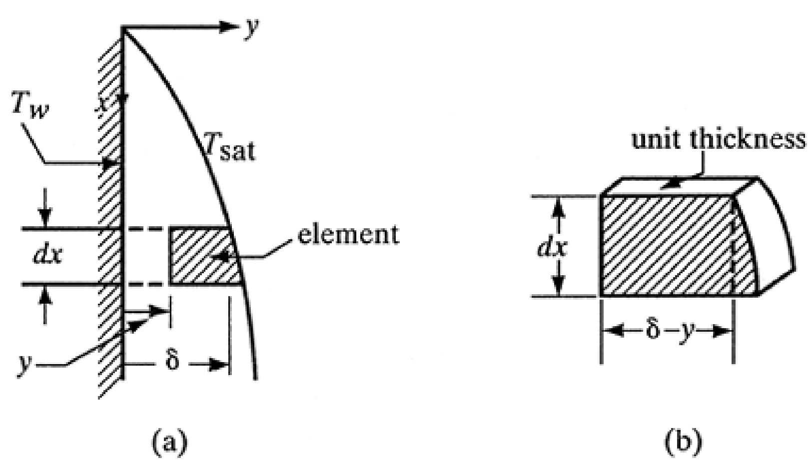


Figure 4.8-3. Film condensation on a vertical plate: (a) increase in film thickness with position, (b) balance on element of condensate.

Integrating and using the boundary condition that $v = 0$ at $y = 0$,

Equation 4.8-8.

$$v = \frac{g(\rho_l - \rho_v)}{\mu_l}(\delta y - y^2/2)$$

The mass flow rate of film condensate at any point x for unit depth is

Equation 4.8-9.

$$m = \int_0^\delta \rho_l v \, dy = \int_0^\delta \rho_l \frac{g(\rho_l - \rho_v)}{\mu_l}(\delta y - y^2/2)\, dy$$

Integrating,

Equation 4.8-10.

$$m = \frac{\rho_l g(\rho_l - \rho_v)\delta^3}{3\mu_l}$$

At the wall, for area ($dx \cdot 1$) m^2, the rate of heat transfer is as follows if a linear temperature distribution is assumed in the liquid between the wall and the vapor:

Equation 4.8-11.

$$q_x = -k_l(dx \cdot 1)\left.\frac{dT}{dy}\right|_{y=0} = k_l\, dx\, \frac{T_{\text{sat}} - T_w}{\delta}$$

In a dx distance, the rate of heat transfer is q_x. Also, in this dx distance, the increase in mass from condensation is dm. Using Eq. (4.8-10),

Equation 4.8-12.

$$dm = d\left[\frac{\rho_l g(\rho_l - \rho_v)\delta^3}{3\mu_l}\right] = \frac{\rho_l g(\rho_l - \rho_v)\delta^2\, d\delta}{\mu_l}$$

Making a heat balance for dx distance, the mass flow rate dm times the latent heat h_{fg} must equal the q_x from Eq. (4.8-11):

Equation 4.8-13.

$$h_{fg}\frac{\rho_l g(\rho_l - \rho_v)\delta^2\, d\delta}{\mu_l} = k_l\, dx\, \frac{T_{\text{sat}} - T_w}{\delta}$$

Integrating, with $\delta = 0$ at $x = 0$ and $\delta = \delta$ at $x = x$,

Equation 4.8-14.

$$\delta = \left[\frac{4\mu_l k_l x(T_{\text{sat}} - T_w)}{g h_{fg}\rho_l(\rho_l - \rho_v)}\right]^{1/4}$$

Using the local heat-transfer coefficient h_x at x, a heat balance gives

Equation 4.8-15.

$$h_x(dx \cdot 1)(T_{\text{sat}} - T_w) = k_l(dx \cdot 1)\frac{T_{\text{sat}} - T_w}{\delta}$$

This gives

Equation 4.8-16.

$$h_x = \frac{k_l}{\delta}$$

Combining Eqs. (4.8-14) and (4.8-16),

Equation 4.8-17.

$$h_x = \left[\frac{\rho_l(\rho_l - \rho_v) g h_{fg} k_l^3}{4\mu_l x (T_{\text{sat}} - T_w)} \right]^{1/4}$$

By integrating over the total length *L,* the average value of *h* is obtained as follows:

Equation 4.8-18.

$$h = \frac{1}{L} \int_0^L h_x \, dx = \tfrac{4}{3} h_{x=L}$$

Equation 4.8-19.

$$h = 0.943 \left[\frac{\rho_l(\rho_l - \rho_v) g h_{fg} k_l^3}{\mu_l L (T_{\text{sat}} - T_w)} \right]^{1/4}$$

However, for laminar flow, experimental data are about 20% above Eq. (4.8-19).

Hence, the final recommended expression for vertical surfaces in laminar flow is (M1)

Equation 4.8-20.

$$N_{\text{Nu}} = \frac{hL}{k_l} = 1.13 \left(\frac{\rho_l(\rho_l - \rho_v) g h_{fg} L^3}{\mu_l k_l \Delta T} \right)^{1/4}$$

where ρ_l is the density of liquid in kg/m^3 and ρ_v that of the vapor, g is 9.8066 m/s^2, L is the vertical height of the surface or tube in m, μ_l is the viscosity of liquid in Pa · s, k_l is the liquid thermal conductivity in W/m · K, $\Delta T = T_{\text{sat}} - T_w$ in K, and h_{fg} is the latent heat of condensation in J/kg at T_{sat}. All physical properties of the liquid except h_{fg} are evaluated at the film temperature $T_f = (T_{sat} + T_W)/2$. For long vertical surfaces the flow at the bottom can be turbulent. The Reynolds number is defined as

Equation 4.8-21.

$$N_{\text{Re}} = \frac{4m}{\pi D \mu_l} = \frac{4\Gamma}{\mu_l} \qquad \text{(vertical tube, diameter } D\text{)}$$

Equation 4.8-22.

$$N_{\text{Re}} = \frac{4m}{W \mu_l} = \frac{4\Gamma}{\mu_l} \qquad \text{(vertical plate, width } W\text{)}$$

where m is total kg mass/s of condensate at tube or plate bottom and $\Gamma = m/\pi D$ or m/W. The N_{Re} should be below about 1800 for Eq. (4.8-20) to hold. The reader should note that some references define N_{Re} as Γ/μ. Then this N_{Re} should be below 450.

For turbulent flow for $N_{\text{Re}} > 1800$ (M1),

Equation 4.8-23.

$$N_{\mathrm{Nu}} = \frac{hL}{k_l} = 0.0077\left(\frac{g\rho_l^2 L^3}{\mu_l^2}\right)^{1/3}(N_{\mathrm{Re}})^{0.4}$$

Solution of this equation is by trial and error, since a value of N_{Re} must first be assumed in order to calculate *h*.

EXAMPLE 4.8-2. Condensation on a Vertical Tube

Steam saturated at 68.9 kPa (10 psia) is condensing on a vertical tube 0.305 m (1.0 ft) long having an OD of 0.0254 m (1.0 in.) and a surface temperature of 86.11°C (187°F). Calculate the average heat-transfer coefficient using English and SI units.

Solution: From Appendix A.2,

$$T_{\mathrm{sat}} = 193\text{°F } (89.44\text{°C}) \qquad T_w = 187\text{°F } (86.11\text{°C})$$

$$T_f = \frac{T_w + T_{\mathrm{sat}}}{2} = \frac{187 + 193}{2} = 190\text{°F } (87.8\text{°C})$$

$$\text{latent heat } h_{fg} = 1143.3 - 161.0 = 982.3 \text{ btu/lb}_\mathrm{m}$$

$$= 2657.8 - 374.6 = 2283.2 \text{ kJ/kg} = 2.283 \times 10^6 \text{ J/kg}$$

$$\rho_l = \frac{1}{0.01657} = 60.3 \text{ lb}_\mathrm{m}/\text{ft}^3 = 60.3(16.018) = 966.7 \text{ kg/m}^3$$

$$\rho_v = \frac{1}{40.95} = 0.0244 \text{ lb}_\mathrm{m}/\text{ft}^3 = 0.391 \text{ kg/m}^3$$

$$\mu_l = (0.324 \text{ cp})(2.4191) = 0.784 \text{ lb}_\mathrm{m}/\text{ft}\cdot\text{h} = 3.24 \times 10^{-4} \text{ Pa}\cdot\text{s}$$

$$k_l = 0.390 \text{ btu/ft}\cdot\text{h}\cdot\text{°F} = (0.390)(1.7307) = 0.675 \text{ W/m}\cdot\text{K}$$

$$L = 1 \text{ ft} = 0.305 \text{ m} \qquad \Delta T = T_{\mathrm{sat}} - T_w = 193 - 187 = 6\text{°F } (3.33 \text{ K})$$

Assuming a laminar film, using Eq. (4.8-20) in English as well as SI units, and neglecting ρ_v as compared to ρ_l,

$$N_{\mathrm{Nu}} = 1.13\left(\frac{\rho_l^2 g h_{fg} L^3}{\mu_l k_l \Delta T}\right)^{1/4}$$

$$= 1.13\left[\frac{(60.3)^2(32.174)(3600)^2(982.3)(1.0)^3}{(0.784)(0.390)(6)}\right]^{1/4} = 6040$$

$$N_{\mathrm{Nu}} = 1.13\left[\frac{(966.7)^2(9.806)(2.283 \times 10^6)(0.305)^3}{(3.24 \times 10^{-4})(0.675)(3.33)}\right]^{1/4} = 6040$$

$$N_{\mathrm{Nu}} = \frac{hL}{k_l} = \frac{h(1.0)}{0.390} = 6040 \qquad \text{SI units: } \frac{h(0.305)}{0.675} = 6040$$

Solving, $h = 2350$ btu/h · ft^2 · °F = 13 350 W/m^2 · K.

Next, the N_{Re} will be calculated to see if laminar flow occurs as assumed. To calculate the total heat transferred for a tube of area

Equation 4.8-24.

$$A = \pi D L = \pi(1/12)(1.0) = \pi/12 \text{ ft}^2, \qquad A = \pi(0.0254)(0.305) \text{ m}^2$$

$$q = hA\,\Delta T$$

However, this q must also equal that obtained by condensation of m lb_m/h or kg/s. Hence,

Equation 4.8-25.

$$q = hA\,\Delta T = h_{fg} m$$

Substituting the values given and solving for m,

$$2350(\pi/12)(193 - 187) = 982.3(m) \qquad m = 3.77 \text{ lb}_m/\text{h}$$

$$13.350(\pi)(0.0254)(0.305)(3.33) = 2.284 \times 10^6(m) \qquad m = 4.74 \times 10^{-4} \text{ kg/s}$$

Substituting into Eq. (4.8-21),

$$N_{Re} = \frac{4m}{\pi D \mu_l} = \frac{4(3.77)}{\pi(1/12)(0.784)} = 73.5 \qquad N_{Re} = \frac{4(4.74 \times 10^{-4})}{\pi(0.0254)(3.24 \times 10^{-4})} = 73.5$$

Hence, the flow is laminar as assumed.

Film-condensation coefficients outside horizontal cylinders

The analysis of Nusselt can also be extended to the practical case of condensation outside a horizontal tube. For a single tube the film starts out with zero thickness at the top of the tube and increases in thickness as it flows around to the bottom and then drips off. If there is a bank of horizontal tubes, the condensate from the top tube drips onto the one below; and so on.

For a vertical tier of N horizontal tubes placed one below the other with outside tube diameter D (M1),

Equation 4.8-26.

$$N_{Nu} = \frac{hD}{k_l} = 0.725\left(\frac{\rho_l(\rho_l - \rho_v) g h_{fg} D^3}{N \mu_l k_l\,\Delta T}\right)^{1/4}$$

In most practical applications, the flow is in the laminar region and Eq. (4.8-26) holds (C3, M1).

HEAT EXCHANGERS

Types of Exchangers

Introduction

In the process industries the transfer of heat between two fluids is generally done in heat exchangers. The most common type is one in which the hot and cold fluids do not come into direct contact with each other but are separated by a tube wall or a flat or curved surface. The transfer of heat from the hot fluid to the wall or tube surface is accomplished by convection, through the tube wall or plate by conduction, and then by convection to the cold fluid. In the preceding sections of this chapter we have discussed the calculation procedures for these various steps. Now we will discuss some of the types of equipment used and overall thermal analyses of exchangers. Complete, detailed design methods have been highly developed and will not be considered here.

Double-pipe heat exchanger

The simplest exchanger is the double-pipe or concentric-pipe exchanger. This is shown in Fig. 4.9-1, where one fluid flows inside one pipe and the other fluid flows in the annular space between the two pipes. The fluids can be in cocurrent or countercurrent flow. The exchanger can be made from a pair of single lengths of pipe with fittings at the ends or from a number of pairs interconnected in series. This type of exchanger is useful mainly for small flow rates.

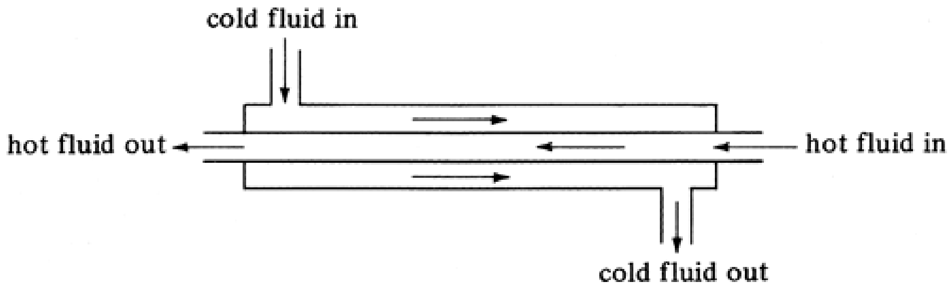


Figure 4.9-1. Flow in a double-pipe heat exchanger.

Shell-and-tube exchanger

If larger flows are involved, a shell-and-tube exchanger is used, which is the most important type of exchanger in use in the process industries. In these exchangers the flows are continuous. Many tubes in parallel are used, where one fluid flows inside these tubes. The tubes, arranged in a bundle, are enclosed in a single shell and the other fluid flows outside the tubes in the shell side. The simplest shell-and-tube exchanger is shown in Fig. 4.9-2a for one shell pass and one tube pass, or a 1-1 counterflow exchanger. The cold fluid enters and flows inside through all the tubes in parallel in one pass. The hot fluid enters at the other end and flows counterflow across the outside of the tubes. Cross-baffles are used so that the fluid is forced to flow perpendicular across the tube bank rather than parallel with it. The added turbulence generated by this cross-flow increases the shell-side heat-transfer coefficient.

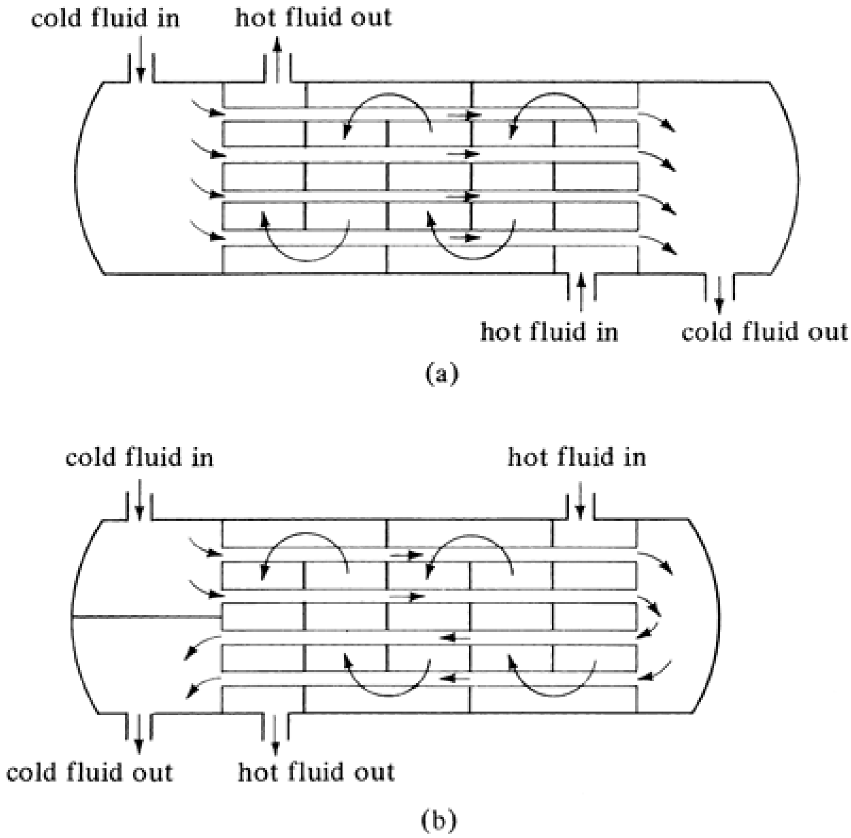


Figure 4.9-2. Shell-and-tube heat exchangers: (a) 1 shell pass and 1 tube pass (1-1 exchanger); (b) 1 shell pass and 2 tube passes (1–2 exchanger).

In Fig. 4.9-2b a 1-2 parallel-counterflow exchanger is shown. The liquid on the tube side flows in two passes as shown and the shell-side liquid flows in one pass. In the first pass of the tube side, the cold fluid is flowing counterflow to the hot shell-side fluid; in the second pass of the tube side, the cold fluid flows in parallel (cocurrent) with the hot fluid. Another type of exchanger has two shell-side passes and four tube passes. Other combinations of number of passes are also used sometimes, with the 1-2 and 2-4 types being the most common.

Cross-flow exchanger

When a gas such as air is being heated or cooled, a common device used is the cross-flow heat exchanger shown in Fig. 4.9-3a. One of the fluids, which is a liquid, flows inside through the tubes, and the exterior gas flows across the tube bundle by forced or sometimes natural convection. The fluid inside the tubes is considered to be unmixed, since it is confined and cannot mix with any other stream. The gas flow outside the tubes is mixed, since it can move about freely between the tubes, and there will be a tendency for the gas temperature to equalize in the direction normal to the flow. For the unmixed fluid inside the tubes, there will be a temperature gradient both parallel and normal to the direction of flow.

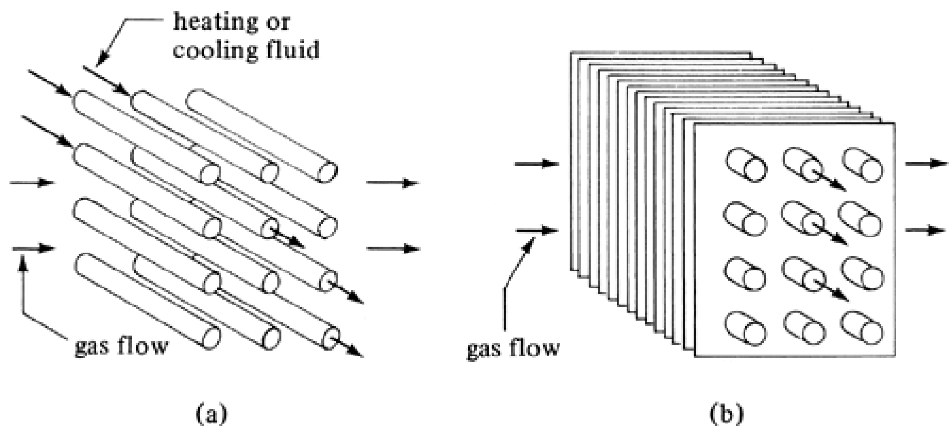


Figure 4.9-3. Flow patterns of cross-flow heat exchangers: (a) one fluid mixed (gas) and one fluid unmixed; (b) both fluids unmixed.

A second type of cross-flow heat exchanger shown in Fig. 4.9-3b is typically used in air-conditioning and space-heating applications. In this type the gas flows across a finned-tube bundle and is unmixed, since it is confined in separate flow channels between the fins as it passes over the tubes. The fluid in the tubes is unmixed.

Discussions of other types of specialized heat-transfer equipment will be deferred to Section 4.13. The remainder of this section deals primarily with shell-and-tube and cross-flow heat exchangers.

Log-Mean-Temperature-Difference Correction Factors

In Section 4.5H it was shown that when the hot and cold fluids in a heat exchanger are in true countercurrent flow or in cocurrent (parallel) flow, the log mean temperature difference should be used:

Equation 4.9-1.

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

where ΔT_2 is the temperature difference at one end of the exchanger and ΔT_1 at the other end. This ΔT_{1m} holds for a double-pipe heat exchanger and a 1-1 exchanger with one shell pass and one tube pass in parallel or counterflow.

In cases where a multiple-pass heat exchanger is involved, it is necessary to obtain a different expression for the mean temperature difference, depending on the arrangement of the shell and tube passes. Considering first the one-shell-pass, two-tube-pass exchanger in Fig. 4.9-2b, the cold fluid in the first tube pass is in counterflow with the hot fluid. In the second tube pass, the cold fluid is in parallel flow with the hot fluid. Hence, the log mean temperature difference, which applies either to parallel or to counterflow but not to a mixture of both types, as in a 1-2 exchanger, cannot be used to calculate the true mean temperature drop without a correction.

The mathematical derivation of the equation for the proper mean temperature to use is quite complex. The usual procedure is to use a correction factor F_T which is so defined that when it is multiplied by the ΔT_{1m}, the product is the correct mean temperature drop ΔT_m to use. In using the correction factors F_T, it is immaterial whether the warmer fluid flows through the tubes or the shell (K1). The factor F_T has been calculated (B4) for a 1-2 exchanger and is shown in Fig. 4.9-4a. Two dimensionless ratios are used as follows:

Equation 4.9-2.

$$Z = \frac{T_{hi} - T_{ho}}{T_{co} - T_{ci}}$$

Equation 4.9-3.

$$Y = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}}$$

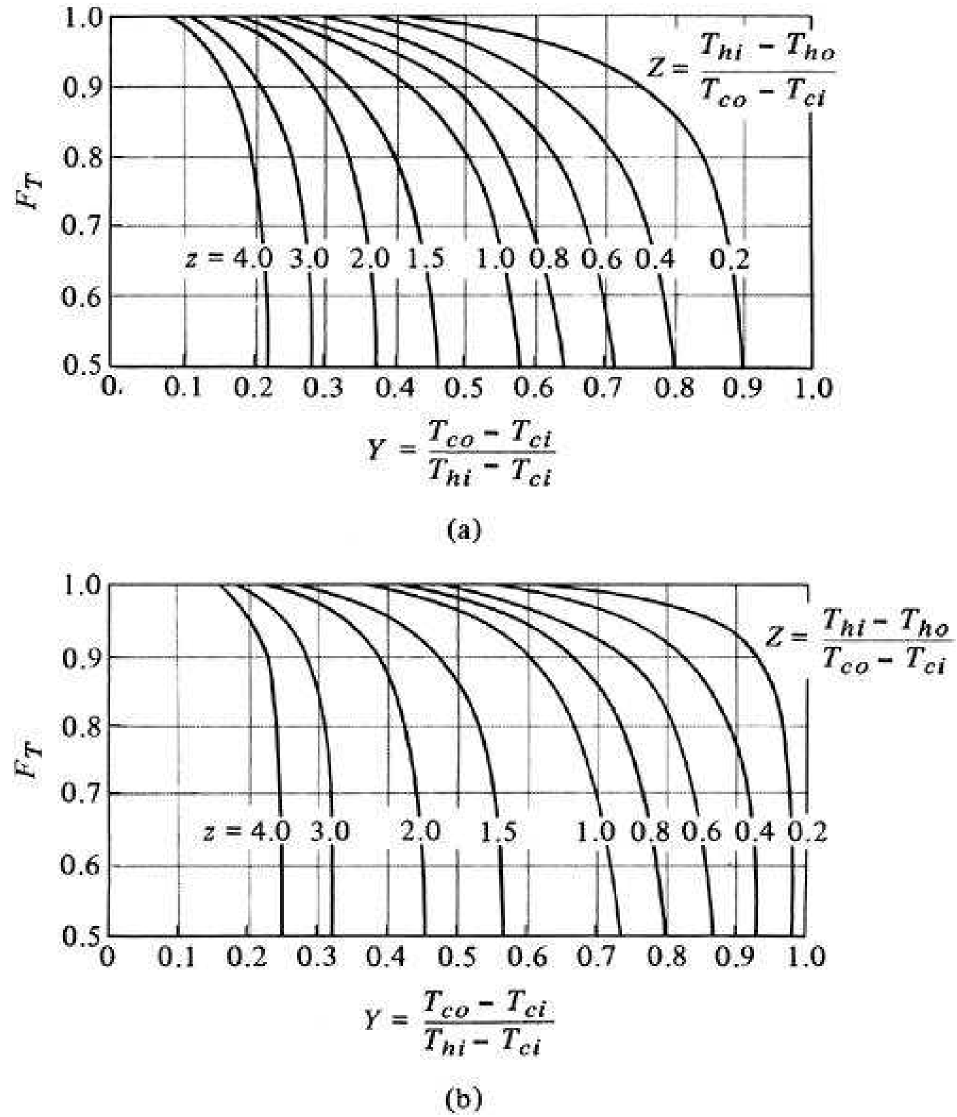


*Figure 4.9-4. Correction factor F_T to log mean temperature difference: (a) 1-2 and 1-4 exchangers, (b) 2-4 exchangers. [From R. A. Bowman, A. C. Mueller, and W. M. Nagle, Trans. A.S.M.E., **62**, 284, 285 (1940). With permission.]*

where T_{hi} = inlet temperature of hot fluid in K (°F), T_{ho} = outlet of hot fluid, T_{ci} inlet of cold fluid, and T_{co} = outlet of cold fluid.

In Fig. 4.9-4b, the factor F_T (B4) for a 2-4 exchanger is shown. In general, it is not recommended to use a heat exchanger for conditions under which $F_T < 0.75$. Another shell-and-tube arrangement should be used. Correction factors for two types of cross-flow exchanger are given in Fig. 4.9-5. Other types are available elsewhere (B4, P1).

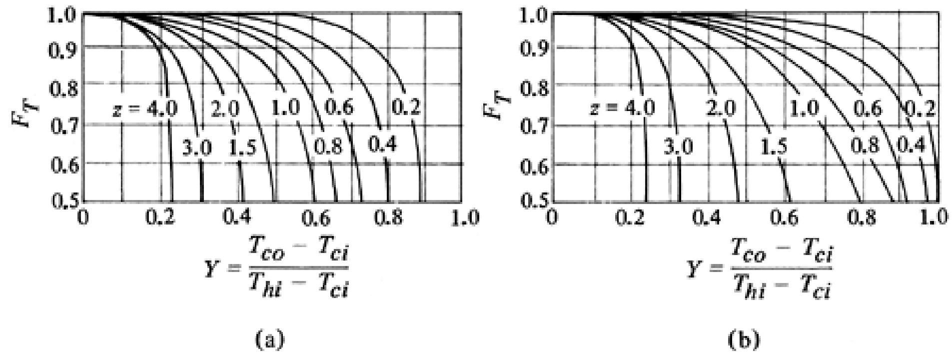


*Figure 4.9-5. Correction factor F_T to log mean temperature difference for cross-flow exchangers [$Z = (T_{hi} - T_{ho})/(T_{co} - T_{ci})$]: (a) single pass, shell fluid mixed, other fluid unmixed, (b) single pass, both fluids unmixed. [From R. A. Bowman, A. C. Mueller, and W. M. Nagle, Trans. A.S.M.E., **62**, 288, 289 (1940). With permission.]*

Using the nomenclature of Eqs. (4.9-2) and (4.9-3), the ΔT_{1m} of Eq. (4.9-1) can be written as

Equation 4.9-4.

$$\Delta T_{\text{lm}} = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln[(T_{hi} - T_{co})/(T_{ho} - T_{ci})]}$$

Then the equation for an exchanger is

Equation 4.9-5.

$$q = U_i A_i \, \Delta T_m = U_o A_o \, \Delta T_m$$

where

Equation 4.9-6.

$$\Delta T_m = F_T \, \Delta T_{\text{lm}}$$

EXAMPLE 4.9-1. Temperature Correction Factor for a Heat Exchanger

A 1-2 heat exchanger containing one shell pass and two tube passes heats 2.52 kg/s of water from 21.1 to 54.4°C by using hot water under pressure entering at 115.6 and leaving at 48.9°C. The outside surface area of the tubes in the exchanger is $A_o = 9.30 \text{ m}^2$.

a. Calculate the mean temperature difference ΔT_m in the exchanger and the overall heat-transfer coefficient U_o.
b. For the same temperatures but using a 2-4 exchanger, what would be the ΔT_m?

Solution: The temperatures are as follows:

$$T_{hi} = 115.6°\text{C} \qquad T_{ho} = 48.9°\text{C} \qquad T_{ci} = 21.1°\text{C} \qquad T_{co} = 54.4°\text{C}$$

First making a heat balance on the cold water, assuming a c_{pm} of water of 4187 J/kg · K and $T_{co} - T_{ci}$ = (54.4 − 21.1)°C = 33.3°C = 33.3 K,

$$q = mc_{pm}(T_{co} - T_{ci}) = (2.52)(4187)(54.4 - 21.1) = 348\,200 \text{ W}$$

The log mean temperature difference using Eq. (4.9-4) is

$$\Delta T_{\text{lm}} = \frac{(115.6 - 54.4) - (48.9 - 21.1)}{\ln[(115.6 - 54.4)/(48.9 - 21.1)]} = 42.3°\text{C} = 42.3\ \text{K}$$

Next, substituting into Eqs. (4.9-2) and (4.9-3),

Equation 4.9-2.

$$Z = \frac{T_{hi} - T_{ho}}{T_{co} - T_{ci}} = \frac{115.6 - 48.9}{54.4 - 21.1} = 2.00$$

Equation 4.9-3.

$$Y = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} = \frac{54.4 - 21.1}{115.6 - 21.1} = 0.352$$

From Fig. 4.9-4a, F_T = 0.74. Then, by Eq. (4.9-6),

Equation 4.9-6.

$$\Delta T_m = F_T\, \Delta T_{\text{lm}} = 0.74(42.3) = 31.3°\text{C} = 31.3\ \text{K}$$

Rearranging Eq. (4.9-5) to solve for U_o and substituting the known values, we have

$$U_o = \frac{q}{A_o\, \Delta T_m} = \frac{348\ 200}{(9.30)(31.3)} = 1196\ \text{W/m}^2 \cdot \text{K}\ (211\ \text{bu/h} \cdot \text{ft}^2 \cdot °\text{F})$$

For part (b), using a 2-4 exchanger and Fig. 4.9-4b, F_T = 0.94. Then,

$$\Delta T_m = F_T\, \Delta T_{\text{lm}} = 0.94(42.3) = 39.8°\text{C} = 39.8\ \text{K}$$

Hence, in this case the 2-4 exchanger utilizes more of the available temperature driving force.

Heat-Exchanger Effectiveness

Introduction

In the preceding section the log mean temperature difference was used in the equation $q = UA\, \Delta T_{1m}$ in the design of heat exchangers. This form is convenient when the inlet and outlet temperatures of the two fluids are known or can be determined by a heat balance. Then the surface area can be determined if U is known. However, when the temperatures of the fluids leaving the exchanger are not known and a given exchanger is to be used, a tedious trial-and-error procedure is necessary. To solve these cases, a method called the heat-exchanger effectiveness ε is used which does not involve any of the outlet temperatures.

The heat-exchanger effectiveness is defined as the ratio of the actual rate of heat transfer in a given exchanger to the maximum possible amount of heat transfer if an infinite heat-transfer area were available. The temperature profile for a counterflow heat exchanger is shown in Fig. 4.9-6.

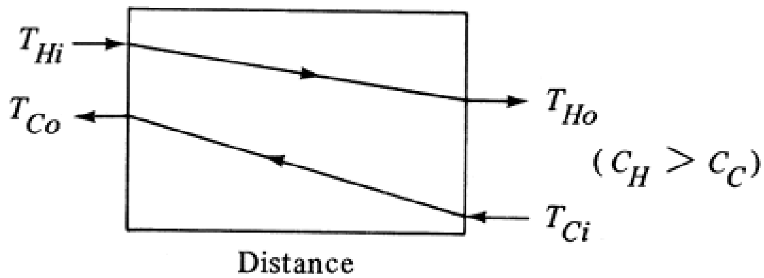


Figure 4.9-6. Temperature profile for countercurrent heat exchanger.

Derivation of effectiveness equation

The heat balance for the cold (*C*) and hot (*H*) fluids is

Equation 4.9-7.

$$q = (mc_p)_H(T_{Hi} - T_{Ho}) = (mc_p)_C(T_{Co} - T_{Ci})$$

Calling $(mc_p)_H = C_H$ and $(mc_p)_c = C_C$, then in Fig. 4.9-6, $C_H > C_C$, and the cold fluid undergoes a greater temperature change than the hot fluid. Hence, we designate C_C as C_{min} or minimum heat capacity. Then, if there is an infinite area available for heat transfer, $T_{Co} = T_{Hi}$. Then the effectiveness ε is

Equation 4.9-8.

$$\varepsilon = \frac{C_H(T_{Hi} - T_{Ho})}{C_C(T_{Hi} - T_{Ci})} = \frac{C_{max}(T_{Hi} - T_{Ho})}{C_{min}(T_{Hi} - T_{Ci})}$$

If the hot fluid is the minimum fluid, $T_{Ho} = T_{Ci}$, and

Equation 4.9-9.

$$\varepsilon = \frac{C_C(T_{Co} - T_{Ci})}{C_H(T_{Hi} - T_{Ci})} = \frac{C_{max}(T_{Co} - T_{Ci})}{C_{min}(T_{Hi} - T_{Ci})}$$

In both equations the denominators are the same and the numerator gives the actual heat transfer:

Equation 4.9-10.

$$q = \varepsilon C_{min}(T_{Hi} - T_{Ci})$$

Note that Eq. (4.9-10) uses only inlet temperatures, which is an advantage when inlet temperatures are known and it is desired to predict the outlet temperatures for a given existing exchanger.

For the case of a single-pass, counterflow exchanger, combining Eqs. (4.9-8) and (4.9-9),

Equation 4.9-11.

$$\varepsilon = \frac{C_H(T_{Hi} - T_{Ho})}{C_{min}(T_{Hi} - T_{Ci})} = \frac{C_C(T_{Co} - T_{Ci})}{C_{min}(T_{Hi} - T_{Ci})}$$

We consider first the case when the cold fluid is the minimum fluid. Rewriting Eq. (4.5-25) using the present nomenclature,

Equation 4.9-12.

$$q = C_C(T_{Co} - T_{Ci}) = UA\frac{(T_{Ho} - T_{Ci}) - (T_{Hi} - T_{Co})}{\ln[(T_{Ho} - T_{Ci})/(T_{Hi} - T_{Co})]}$$

Combining Eq. (4.9-7) with the left side of Eq. (4.9-11) and solving for T_{Hi},

Equation 4.9-13.

$$T_{Hi} = T_{Ci} + \frac{1}{\varepsilon}(T_{Co} - T_{Ci})$$

Subtracting T_{Co} from both sides,

Equation 4.9-14.

$$T_{Hi} - T_{Co} = T_{Ci} - T_{Co} + \frac{1}{\varepsilon}(T_{Co} - T_{Ci}) = \left(\frac{1}{\varepsilon} - 1\right)(T_{Co} - T_{Ci})$$

From Eq. (4.9-7) for $C_{\min} = C_C$ and $C_{\max} = C_H$,

Equation 4.9-15.

$$T_{Ho} = T_{Hi} - \frac{C_{\min}}{C_{\max}}(T_{Co} - T_{Ci})$$

This can be rearranged to give the following:

Equation 4.9-16.

$$T_{Ho} - T_{Ci} = T_{Hi} - T_{Ci} - \frac{C_{\min}}{C_{\max}}(T_{Co} - T_{Ci})$$

Substituting Eq. (4.9-13) into (4.9-16),

Equation 4.9-17.

$$T_{Ho} - T_{Ci} = \frac{1}{\varepsilon}(T_{Co} - T_{Ci}) - \frac{C_{\min}}{C_{\max}}(T_{Co} - T_{Ci})$$

Finally, substituting Eqs. (4.9-14) and (4.9-17) into (4.9-12), rearranging, taking the antilog of both sides, and solving for ε,

Equation 4.9-18.

$$\varepsilon = \frac{1 - \exp\left[-\frac{UA}{C_{\min}}\left(1 - \frac{C_{\min}}{C_{\max}}\right)\right]}{1 - \frac{C_{\min}}{C_{\max}}\exp\left[-\frac{UA}{C_{\min}}\left(1 - \frac{C_{\min}}{C_{\max}}\right)\right]}$$

We define NTU as the number of transfer units as follows:

Equation 4.9-19.

$$\text{NTU} = \frac{UA}{C_{\min}}$$

The same result would have been obtained if $C_H = C_{\min}$. For parallel flow we obtain

Equation 4.9-20.

$$\varepsilon = \frac{1 - \exp\left[\frac{-UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]}{1 + \frac{C_{\min}}{C_{\max}}}$$

In Fig. 4.9-7, Eqs. (4.9-18) and (4.9-20) have been plotted in convenient graphical form. Additional charts are available for different shell-and-tube and cross-flow arrangements (K1).

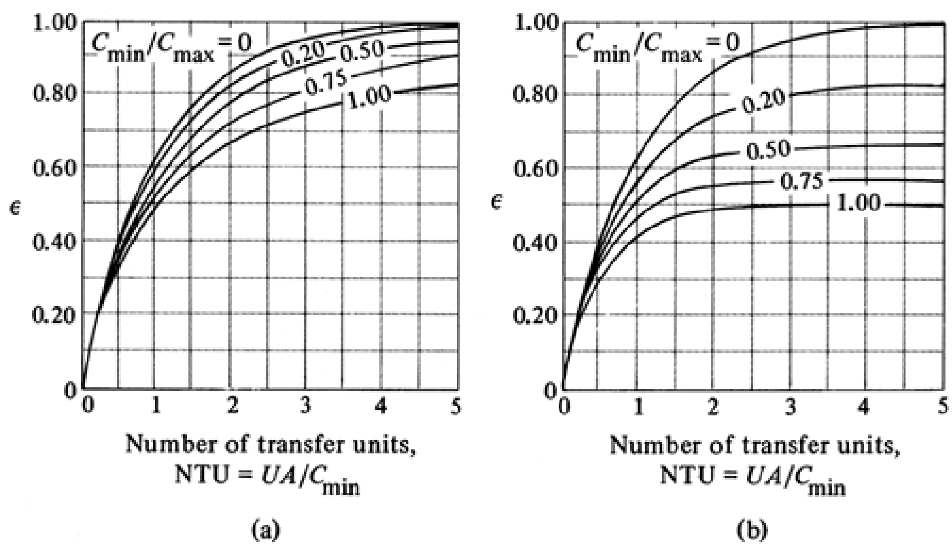


Figure 4.9-7. Heat-exchanger effectiveness ε: (a) counterflow exchanger, (b) parallel flow exchanger.

EXAMPLE 4.9-2. Effectiveness of Heat Exchanger

Water flowing at a rate of 0.667 kg/s enters a countercurrent heat exchanger at 308 K and is heated by an oil stream entering at 383 K at a rate of 2.85 kg/s (c_p = 1.89 kJ/kg · K). The overall U = 300 W/m^2 · K and the area A = 15.0 m^2. Calculate the heat-transfer rate and the exit water temperature.

Solution: Assuming that the exit water temperature is about 370 K, the c_p for water at an average temperature of (308 + 370)/2 = 339 K is 4.192 kJ/kg · K (Appendix A.2). Then, $(mc_p)_H = C_H$ = 2.85(1.89 × 10^3) = 5387 W/K and $(mc_p)_C = C_C$ = 0.667(4.192 × 10^3) = 2796 W/K = $C_{\min}$. Since C_C is the minimum, $C_{\min}/C_{\max}$ = 2796/5387 = 0.519.

Using Eq. (4.9-19), NTU = $UA/C_{\min}$ = 300(15.0)/2796 = 1.607. Using Fig. (4.9-7a) for a counterflow exchanger, ε = 0.71. Substituting into Eq. (4.9-10),

$$q = \varepsilon C_{\min}(T_{Hi} - T_{Ci}) = 0.71(2796)(383 - 308) = 148\,900 \text{ W}$$

Using Eq. (4.9-7),

$$q = 148\,900 = 2796(T_{Co} - 308)$$

Solving, $T_{Co} = 361.3$ K.

Fouling Factors and Typical Overall *U* Values

In actual practice, heat-transfer surfaces do not remain clean. Dirt, soot, scale, and other deposits form on one or both sides of the tubes of an exchanger and on other heat-transfer surfaces. These deposits offer additional resistance to the flow of heat and reduce the overall heat-transfer coefficient *U*. In petroleum processes coke and other substances can deposit. Silting and deposits of mud and other materials can occur. Corrosion products which could constitute a serious resistance to heat transfer may form on the surfaces. Biological growth such as algae can occur with cooling water and in the biological industries.

To avoid or lessen these fouling problems, chemical inhibitors are often added to minimize corrosion, salt deposition, and algae growth. Water velocities above 1 m/s are generally used to help reduce fouling. Large temperature differences may cause excessive deposition of solids on surfaces and should be avoided if possible.

The effect of such deposits and fouling is usually taken care of in design by adding a term for the resistance of the fouling on the inside and outside of the tube in Eq. (4.3-17) as follows:

Equation 4.9-21.

$$U_i = \frac{1}{1/h_i + 1/h_{di} + (r_o - r_i)A_i/k_A A_{A\,\text{lm}} + A_i/A_o h_o + A_i/A_o h_{do}}$$

where h_{di} is the fouling coefficient for the inside and h_{do} the fouling coefficient for the outside of the tube in W/m^2 · K. A similar expression can be written for U_0 using Eq. (4.3-18).

Fouling coefficients recommended for use in designing heat-transfer equipment are available in many references (P3, N1). A short tabulation of some typical fouling coefficients is given in Table 4.9-1.

Table 4.9-1. Typical Fouling Coefficients (P3, N1)

	h_d (W/m^2 · K)	h_d (btu/h · ft^2 · °F)
Distilled and seawater	**11 350**	**2000**
City water	**5680**	**1000**
Muddy water	**1990–2840**	**350–500**
Gases	**2840**	**500**
Vaporizing liquids	**2840**	**500**
Vegetable and gas oils	**1990**	**350**

In order to perform preliminary estimates of sizes of shell-and-tube heat exchangers, typical values of overall heat-transfer coefficients are given in Table 4.9-2. These values should be useful as a check on the results of the design methods described in this chapter.

Table 4.9-2. Typical Values of Overall Heat-Transfer Coefficients in Shell-and-Tube Exchangers (H1, P3, W1)

	U (W/m^2· K)	*U* (btu/h · ft^2 · °F)
Water to water	1140–1700	200–300
Water to brine	570–1140	100–200
Water to organic liquids	570–1140	100–200
Water to condensing steam	1420–2270	250–400
Water to gasoline	340–570	60–100

	U (W/m²· K)	*U* (btu/h · ft² · °F)
Water to gas oil	140–340	25–60
Water to vegetable oil	110–285	20–50
Gas oil to gas oil	110–285	20–50
Steam to boiling water	1420–2270	250–400
Water to air (finned tube)	110–230	20–40
Light organics to light organics	230–425	40–75
Heavy organics to heavy organics	55–230	10–40

INTRODUCTION TO RADIATION HEAT TRANSFER

Introduction and Basic Equation for Radiation

Nature of radiant heat transfer

In the preceding sections of this chapter we have studied conduction and convection heat transfer. In conduction, heat is transferred from one part of a body to another, and the intervening material is heated. In convection, heat is transferred by the actual mixing of materials and by conduction. In radiant heat transfer, the medium through which the heat is transferred usually is not heated. Radiation heat transfer is the transfer of heat by electromagnetic radiation.

Thermal radiation is a form of electromagnetic radiation similar to X rays, light waves, gamma rays, and so on, differing only in wavelength. It obeys the same laws as light: It travels in straight lines, can be transmitted through space and vacuum, and so on. It is an important mode of heat transfer and is especially important where large temperature differences occur, as, for example, in a furnace with boiler tubes, in radiant dryers, or in an oven baking food. Radiation often occurs in combination with conduction and convection. An elementary discussion of radiant heat transfer will be given here, with a more advanced and comprehensive discussion being given in Section 4.11.

In an elementary sense the mechanism of radiant heat transfer is composed of three distinct steps or phases:

1. The thermal energy of a hot source, such as the wall of a furnace at T_1, is converted into energy in the form of electromagnetic-radiation waves.
2. These waves travel through the intervening space in straight lines and strike a cold object at T_2, such as a furnace tube containing water to be heated.
3. The electromagnetic waves that strike the body are absorbed by the body and converted back to thermal energy or heat.

Absorptivity and black bodies

When thermal radiation (such as light waves) falls upon a body, part is absorbed by the body in the form of heat, part is reflected back into space, and part may actually be transmitted through the body. For most cases in process engineering, bodies are opaque to transmission, so this will be neglected. Hence, for opaque bodies,

Equation 4.10-1.

$$\alpha + \rho = 1.0$$

where α is absorptivity or fraction absorbed and ρ is reflectivity or fraction reflected.

A *black body* is defined as one that absorbs all radiant energy and reflects none. Hence, $\rho = 0$ and $\alpha = 1.0$ for a black body. Actually, in practice there are no perfect black bodies, but a close approximation is a small hole in a hollow body, as shown in Fig. 4.10-1. The inside surface of the hollow body is blackened by charcoal. The radiation enters the hole and impinges on the rear wall; part is absorbed there and part is reflected in all directions. The reflected rays impinge again, part is absorbed, and the process continues. Hence, essentially all of the energy entering is absorbed and the area of the hole acts as a perfect black body. The surface of the inside walls is "rough" and rays are scattered in all directions, unlike a mirror, where they are reflected at a definite angle.

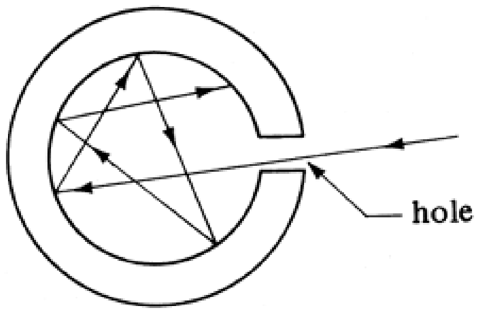


Figure 4.10-1. Concept of a perfect black body.

As stated previously, a black body absorbs all radiant energy falling on it and reflects none. Such a black body also emits radiation, depending on its temperature, and does not reflect any. The ratio of the emissive power of a surface to that of a black body is called *emissivity* ε and is 1.0 for a black body. Kirchhoff's law states that at the same temperature T_1, α_1 and ε_1 of a given surface are the same, or

Equation 4.10-2.

$$\alpha_1 = \varepsilon_1$$

Equation (4.10-2) holds for any black or nonblack solid surface.

Radiation from a body and emissivity

The basic equation for heat transfer by radiation from a perfect black body with an emissivity $\varepsilon = 1.0$ is

Equation 4.10-3.

$$q = A\sigma T^4$$

where q is heat flow in W, A is m^2 surface area of body, σ is a constant 5.676×10^{-8} W/m$^2 \cdot$ K^4 (0.1714×10^{-8} btu/h $\cdot$ ft$^2 \cdot$ °R^4), and T is temperature of the black body in K (°R).

For a body that is not a black body and has an emissivity $\varepsilon < 1.0$, the emissive power is reduced by ε, or

Equation 4.10-4.

$$q = A\varepsilon\sigma T^4$$

Substances that have emissivities of less than 1.0 are called *gray bodies* when the emissivity is independent of the wavelength. All real materials have an emissivity $\varepsilon < 1$.

Since the emissivity ε and absorptivity α of a body are equal at the same temperature, the emissivity, like absorptivity, is low for polished metal surfaces and high for oxidized metal surfaces. Typical values are given in Table 4.10-1 but do vary some with temperature. Most non-metallic substances have high values. Additional data are tabulated in Appendix A.3.

Table 4.10-1. Total Emissivity, ε, of Various Surfaces

Surface	T(K)	T(°F)	Emissivity, ε
Polished aluminum	500	440	0.039
	850	1070	0.057
Polished iron	450	350	0.052
Oxidized iron	373	212	0.74
Polished copper	353	176	0.018
Asbestos board	296	74	0.96
Oil paints, all colors	373	212	0.92–0.96
Water	273	32	0.95

Radiation to a Small Object from Surroundings

In the case of a small gray object of area A_1 m^2 at temperature T_1 in a large enclosure at a higher temperature T_2, there is a net radiation to the small object. The small body emits an amount of radiation to the enclosure given by Eq. (4.10-4) as $A_1\varepsilon_1\sigma T_1^4$. The emissivity ε_1 of this body is taken at T_1. The small body also absorbs an amount of energy from the surroundings at T_2 given by $A_1\alpha_{12}\sigma T_2^4$. The α_{12} is the absorptivity of body 1 for radiation from the enclosure at T_2. The value of α_{12} is approximately the same as the emissivity of this body at T_2. The net heat of absorption is then, by the Stefan-Boltzmann equation,

Equation 4.10-5.

$$q = A_1\varepsilon_1\sigma T_1^4 - A_1\alpha_{12}\sigma T_2^4 = A_1\sigma(\varepsilon_1 T_1^4 - \alpha_{12}T_2^4)$$

A further simplification of Eq. (4.10-5) is usually made for engineering purposes by using only one emissivity for the small body, at temperature T_2. Thus,

Equation 4.10-6.

$$q = A_1\varepsilon\sigma(T_1^4 - T_2^4)$$

EXAMPLE 4.10-1. Radiation to a Metal Tube

A small oxidized horizontal metal tube with an OD of 0.0254 m (1 in.), 0.61 m (2 ft) long, and with a surface temperature at 588 K (600°F) is in a very large furnace enclosure with fire-brick walls and the surrounding air at 1088 K (1500°F). The emissivity of the metal tube is 0.60 at 1088 K and 0.46 at 588 K. Calculate the heat transfer to the tube by radiation using SI and English units.

Solution: Since the large-furnace surroundings are very large compared to the small enclosed tube, the surroundings, even if gray, when viewed from the position of the small body appear black, and Eq. (4.10-6) is applicable. Substituting given values into Eq. (4.10-6) with an ε of 0.6 at 1088 K,

$$A_1 = \pi DL = \pi(0.0254)(0.61)\ \text{m}^2 = \pi(1/12)(2.0)\ \text{ft}^2$$

$$q = A_1\varepsilon\sigma(T_1^4 - T_2^4) = [\pi(0.0254)(0.61)](0.6)(5.676\times10^{-8})[(588)^4 - (1088)^4]$$

$$= -2130\ \text{W}$$

$$= [\pi(1/12)(2)](0.6)(0.1714\times10^{-8})[(1060)^4 - (1960)^4] = -7270\ \text{btu/h}$$

Other examples of small objects in large enclosures occurring in the process industries are a loaf of bread in an oven receiving radiation from the walls around it, a package of meat or food radiating heat to the walls of a freezing enclosure, a hot ingot of solid iron cooling and radiating heat in a large room, and a thermometer measuring the temperature in a large duct.

Combined Radiation and Convection Heat Transfer

When radiation heat transfer occurs from a surface, it is usually accompanied by convective heat transfer, unless the surface is in a vacuum. When the radiating surface is at a uniform temperature, we can calculate the heat transfer for natural or forced convection using the methods described in the previous sections of this chapter. The radiation heat transfer is calculated by the Stefan-Boltzmann equation (4.10-6). Then the total rate of heat transfer is the sum of convection plus radiation.

As discussed before, the heat-transfer rate by convection and the convective coefficient are given by

Equation 4.10-7.

$$q_{\text{conv}} = h_c A_1 (T_1 - T_2)$$

where q_{conv} is the heat-transfer rate by convection in W, h_c the natural or forced convection coefficient in W/m^2 · K, T_1 the temperature of the surface, and T_2 the temperature of the air and the enclosure. A radiation heat-transfer coefficient h_r in W/m^2 · K can be defined as

Equation 4.10-8.

$$q_{\text{rad}} = h_r A_1 (T_1 - T_2)$$

where q_{rad} is the heat-transfer rate by radiation in W. The total heat transfer is the sum of Eqs. (4.10-7) and (4.10-8),

Equation 4.10-9.

$$q = q_{\text{conv}} + q_{\text{rad}} = (h_c + h_r) A_1 (T_1 - T_2)$$

To obtain an expression for h_r, we equate Eq. (4.10-6) to (4.10-8) and solve for h_r:

Equation 4.10-10.

$$h_r = \frac{\varepsilon\sigma(T_1^4 - T_2^4)}{T_1 - T_2} = \varepsilon(5.676)\frac{(T_1/100)^4 - (T_2/100)^4}{T_1 - T_2} \qquad \text{(SI)}$$

$$h_r = \varepsilon(0.1714)\frac{(T_1/100)^4 - (T_2/100)^4}{T_1 - T_2} \qquad \text{(English)}$$

A convenient chart giving values of h_r in English units calculated from Eq. (4.10-10) with ε = 1.0 is given in Fig. 4.10-2. To use values from this figure, the value obtained from the figure should be multiplied by ε to give the value of h_r to use in Eq. (4.10-9). If the air temperature is not the same as T_2 of the enclosure, Eqs. (4.10-7) and (4.10-8) must be used separately and not combined together as in (4.10-9).

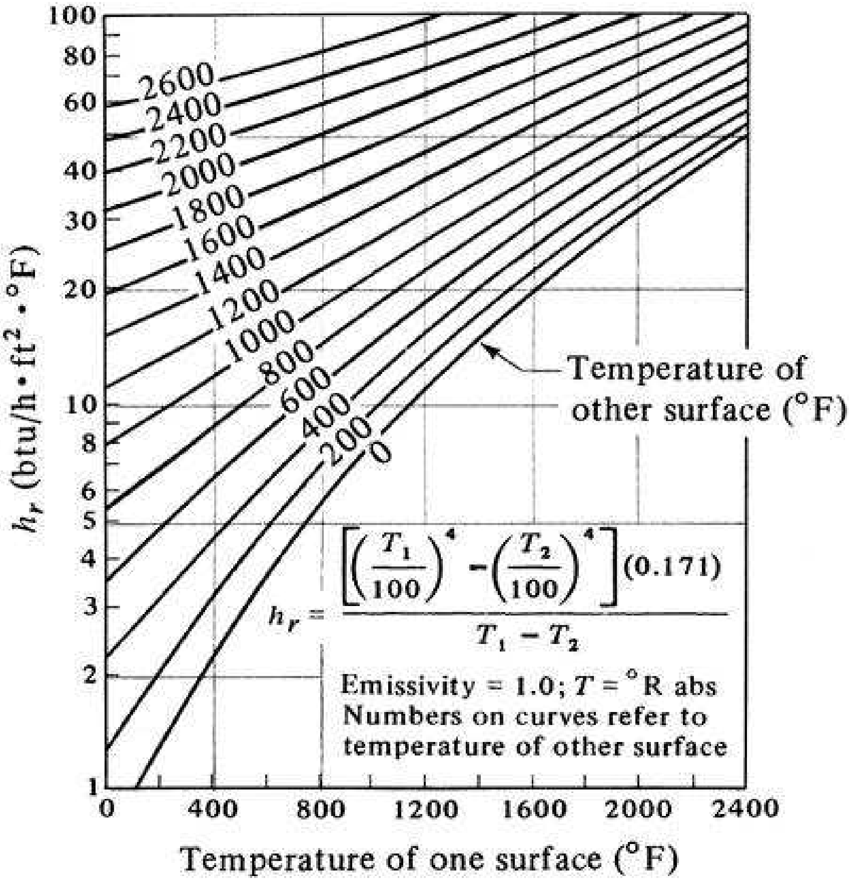


Figure 4.10-2. Radiation heat-transfer coefficient as a function of temperature. (From R. H. Perry and C. H. Chilton, Chemical Engineers' Handbook, 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.)

EXAMPLE 4.10-2. Combined Convection Plus Radiation from a Tube

Recalculate Example 4.10-1 for combined radiation plus natural convection to the horizontal 0.0254-m tube.

***Solution*:** The area A of the tube = $\pi(0.0254)(0.61) = 0.0487\ m^2$. For the natural convection coefficient to the 0.0254-m horizontal tube, the simplified equation from Table 4.7-2 will be used as an approximation even though the film temperature is quite high:

$$h_c = 1.32\left(\frac{\Delta T}{D}\right)^{1/4}$$

Substituting the known values,

$$h_c = 1.32\left(\frac{1088 - 588}{0.0254}\right)^{1/4} = 15.64\ \text{W/m}^2\cdot\text{K}$$

Using Eq. (4.10-10) and $\varepsilon = 0.6$,

$$h_r = (0.60)(5.676)\frac{(1088/100)^4 - (588/100)^4}{1088 - 588} = 87.3\ \text{W/m}^2\cdot\text{K}$$

Substituting into Eq. (4.10-9),

$$q = (h_c + h_r)A_1(T_1 - T_2) = (15.64 + 87.3)(0.0487)(588 - 1088)$$

$$= -2507 \text{ W}$$

Hence, the heat loss of -2130 W for radiation is increased to only -2507 W when natural convection is also considered. In this case, because of the large temperature difference, radiation is the most important factor.

Perry and Green (P3, pp. 10-14) give a convenient table of natural convection plus radiation coefficients ($h_c + h_r$) from single horizontal oxidized steel pipes as a function of the outside diameter and temperature difference. The coefficients for insulated pipes are about the same as those for a bare pipe (except that lower surface temperatures are involved for the insulated pipes), since the emissivity of cloth insulation wrapping is about that of oxidized steel, approximately 0.8. A more detailed discussion of radiation will be given in Section 4.11.

Effect of Radiation on Temperature Measurement of a Gas

When a temperature sensor or probe (thermometer, thermocouple, etc.) is used to measure the temperature of a gas flowing in an enclosure, significant errors can occur. Radiation heat exchange will take place between the sensor and the wall and convection heat transfer between the sensor and the gas. The sensor will indicate a temperature between the true gas and wall surface temperatures. This is shown in Fig. 4.10-3, where the wall temperature T_w is less than the true gas temperature T_g.

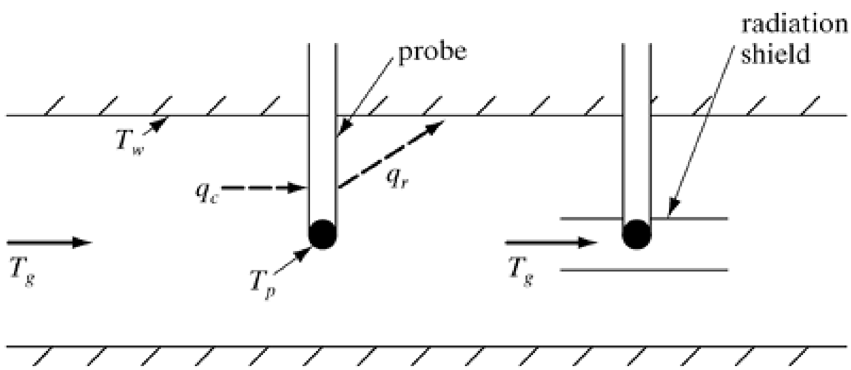


Figure 4.10-3. Temperature measurement of a gas showing radiation and convection heat transfer for a bare probe and a shielded probe.

The equations for the heat transfer q_c by convection to the probe and radiation q_r from the probe to the wall are as follows for $T_w < T_g$:

Equation 4.10-11.

$$q_c = h_c A_p(T_g - T_p) = q_r = \varepsilon A_p(5.676)\left(\frac{T_p}{100}\right)^4 - \left(\frac{T_w}{100}\right)^4$$

where A_p is the area of the tube in m^2 and ε is the emissivity of the probe.

EXAMPLE 4.10-3. Effect of Radiation on Temperature Measurement in a Gas

A thermocouple is measuring the temperature of hot air flowing in a pipe whose walls are at T_w = 400 K (260°F). The true gas temperature T_g = 465 K (377°F). Calculate the temperature T_p indicated by the thermocouple. The emissivity of the probe is assumed as ε = 0.6 and the convection heat-transfer coefficient h_c = 40 W/m^2 · K.

Solution: Substituting into Eq. (4.10-11) for convection, q_c, and for radiation, q_r,

$$q_c = h_c A_p (T_g - T_p) = 40A_p(465 - T_p)$$

$$q_r = 0.6A_p(5.676)[(T_p/100)^4 - (400/100)^4]$$

Equating $q_c = q_r$, canceling out the term A_p, and solving by trial and error, T_p = 451.4 K. Hence, the thermocouple reading of T_p = 451.4 K (352.5°F) is 13.6 K (24.5°F) lower than the true gas temperature of 465 K (377°F).

Probes with a radiation shield as shown in Fig. 4.10-3 are often used to reduce radiation errors. The shield will have a temperature which is closer to the gas temperature than is the wall. Since the probe now radiates heat to a surface which is closer to the gas temperature, the radiation loss is less. It can be shown that with one shield, the radiation heat loss will be halved. Multiple shields can be used to further reduce the error. Using a polished surface on the probe to reduce the emissivity lowers the radiation heat loss. This also reduces the measurement error.

ADVANCED RADIATION HEAT-TRANSFER PRINCIPLES

Introduction and Radiation Spectrum

Introduction

This section will cover some basic principles together with some advanced topics on radiation that were not covered in Section 4.10. The exchange of radiation between two surfaces depends upon the size, shape, and relative orientation of these two surfaces and also upon their emissivities and absorptivities. In the cases to be considered the surfaces are separated by nonabsorbing media such as air. When gases such as CO_2 and H_2O vapor are present, some absorption by the gases occurs, which is not considered until later in this section.

Radiation spectrum and thermal radiation

Energy can be transported in the form of electromagnetic waves, which travel at the speed of light. Bodies may emit many forms of radiant energy, such as gamma rays, thermal energy, radio waves, and so on. In fact, there is a continuous spectrum of electromagnetic radiation. This electromagnetic spectrum is divided into a number of wavelength ranges, such as cosmic rays ($\lambda < 10^{-13}$ m), gamma rays ($\lambda = 10^{-13}$ to 10–10 m), thermal radiation ($\lambda = 10^{-7}$ to 10^{-4} m), and so on. The electromagnetic radiation produced solely because of the temperature of the emitter is called thermal radiation and exists between the wavelengths of 10^{-7} and 10^{-4} m. This portion of the electromagnetic spectrum is of importance in radiant thermal heat transfer. Electromagnetic waves having wavelengths between 3.8×10^{-7} and 7.6×10^{-7} m, called *visible radiation,* can be detected by the human eye. This visible radiation lies within the thermal radiation range.

When different surfaces are heated to the same temperature, they do not all emit or absorb the same amount of thermal radiant energy. A body that absorbs and emits the maximum amount of energy at a given temperature is called a *black body.* A black body is a standard to which other bodies can be compared.

Planck's law and emissive power

When a black body is heated to a temperature T, photons are emitted from the surface which have a definite distribution of energy. Planck's equation relates the monochromatic emissive power $E_{B\lambda}$ in W/m^3 at a temperature T in K and a wavelength λ in m:

Equation 4.11-1.

$$E_{B\lambda} = \frac{3.7418 \times 10^{-16}}{\lambda^5[e^{1.4388\times10^{-2}/\lambda T} - 1]}$$

A plot of Eq. (4.11-1) is given in Fig. 4.11-1 and shows that the energy given off increases with *T*. Also, for a given *T*, the emissive power reaches a maximum value at a wavelength that decreases as the temperature *T* increases. At a given temperature the radiation emitted extends over a spectrum of wavelengths. The visible-light spectrum occurs in the low λ region. The sun has a temperature of about 5800 K and the solar spectrum straddles this visible range.

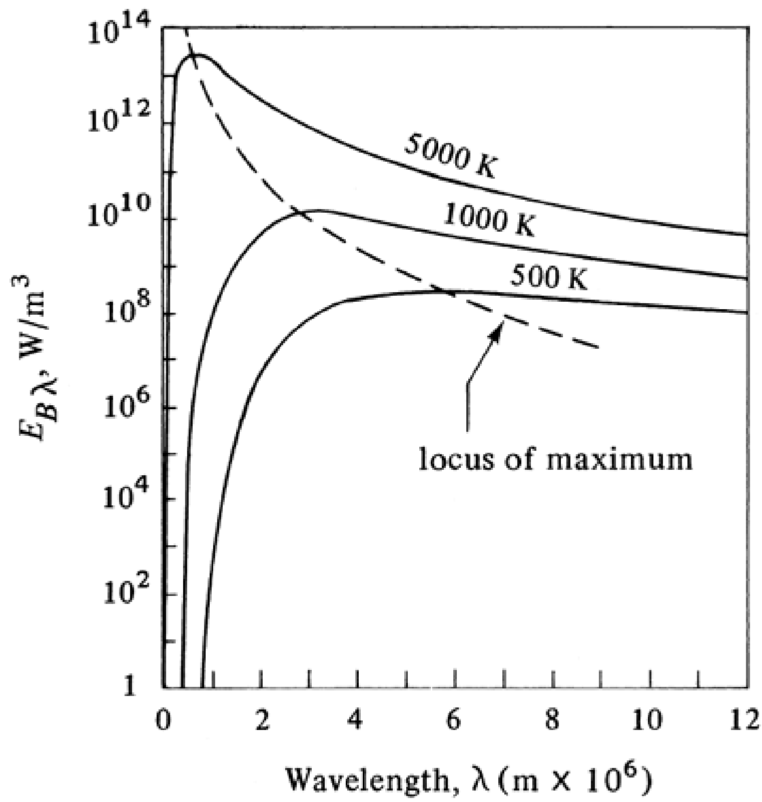


Figure 4.11-1. Spectral distribution of total energy emitted by a black body at various temperatures of the black body.

For a given temperature, the wavelength at which the black-body emissive power is a maximum can be determined by differentiating Eq. (4.11-1) with respect to λ at constant *T* and setting the result equal to zero. The result is as follows and is known as *Wien's displacement law:*

Equation 4.11-2.

$$\lambda_{max} T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$$

The locus of the maximum values is shown in Fig. 4.11-1.

Stefan-Boltzmann law

The total emissive power is the total amount of radiation energy per unit area leaving a surface with temperature *T* over all wavelengths. For a black body, the total emissive power is given by the integral of Eq. (4.11-1) at a given *T* over all wavelengths, or the area under the curve in Fig. 4.11-1:

Equation 4.11-3.

$$E_B = \int_0^\infty E_{B\lambda}\, d\lambda$$

This gives

Equation 4.11-4.

$$E_B = \sigma T^4$$

The result is the Stefan-Boltzmann law with $\sigma = 5.676 \times 10^{-8}$ W/m$^2 \cdot$ K^4. The units of E_B are W/m^2.

Emissivity and Kirchhoff's law

An important property in radiation is the emissivity of a surface. The *emissivity* ε of a surface is defined as the total emitted energy of the surface divided by the total emitted energy of a black body at the same temperature:

Equation 4.11-5.

$$\varepsilon = \frac{E}{E_B} = \frac{E}{\sigma T^4}$$

Since a black body emits the maximum amount of radiation, ε is always <1.0.

We can derive a relationship between the absorptivity α_1 and emissitivy ε_1 of a material by placing this material in an isothermal enclosure and allowing the body and enclosure to reach the same temperature at thermal equilibrium. If G is the irradiation on the body, the energy absorbed must equal the energy emitted:

Equation 4.11-6.

$$\alpha_1 G = E_1$$

If this body is removed and replaced by a black body of equal size, then at equilibrium,

Equation 4.11-7.

$$\alpha_2 G = E_B$$

Dividing Eq. (4.11-6) by (4.11-7),

Equation 4.11-8.

$$\frac{\alpha_1}{\alpha_2} = \frac{E_1}{E_B}$$

But α_2 = 1.0 for a black body. Hence, since $E_1/E_B = \varepsilon_1$,

Equation 4.11-9.

$$\alpha_1 = \frac{E_1}{E_B} = \varepsilon_1$$

This is Kirchhoff's law, which states that at thermal equilibrium $\alpha = \varepsilon$ of a body. When a body is not at equilibrium with its surroundings, the result is not valid.

Concept of gray body

A gray body is defined as a surface for which the monochromatic properties are constant over all wavelengths. For a gray surface,

Equation 4.11-10.

$$\varepsilon_\lambda = \text{const.}, \qquad \alpha_\lambda = \text{const.}$$

Hence, the total absorptivity *a* and the monochromatic absorptivity α_λ of a gray surface are equal, as are ε and ε_λ:

Equation 4.11-11.

$$\alpha = \alpha_\lambda, \qquad \varepsilon = \varepsilon_\lambda$$

Applying Kirchhoff's law to a gray body, $\alpha_\lambda = \varepsilon_\lambda$ and

Equation 4.11-12.

$$\alpha = \varepsilon$$

As a result, the total absorptivity and emissivity are equal for a gray body even if the body is not in thermal equilibrium with its surroundings.

Gray bodies do not exist in practice; the concept of a gray body is an idealized one. The absorptivity of a surface actually varies with the wavelength of the incident radiation. Engineering calculations can often be based on the assumption of a gray body with reasonable accuracy. The a is assumed constant even with a variation in λ of the incident radiation. Also, in actual systems, various surfaces may be at different temperatures. In these cases, α for a surface is evaluated by determining the emissivity not at the actual surface temperature but at the temperature of the source of the other radiating surface or emitter, since this is the temperature the absorbing surface would reach if the absorber and emitter were at thermal equilibrium. The temperature of the absorber has only a slight effect on the absorptivity.

Derivation of View Factors in Radiation for Various Geometries

Introduction

The concepts and definitions presented in Section 4.11A form a sufficient foundation so that the net radiant exchange between surfaces can be determined. If two surfaces are arranged so that radiant energy can be exchanged, a net flow of energy will occur from the hotter surface to the colder surface. The size, shape, and orientation of two radiating surfaces or a system of surfaces are factors in determining the net heat-flow rate between them. To simplify the discussion we assume that the surfaces are separated by a nonabsorbing medium such as air. This assumption is adequate for many engineering applications. However, in cases such as a furnace, the presence of CO_2 and H_2O vapor make such a simplification impossible because of their high absorptivities.

The simplest geometrical configuration will be considered first, that of radiation exchange between parallel, infinite planes. This assumption implies that there are no edge effects in the case of finite surfaces. First, the simplest case will be treated, in which the surfaces are black bodies, and then more complicated geometries and gray bodies will be treated.

View factor for infinite parallel black planes

If two parallel and infinite black planes at T_1 and T_2 are radiating toward each other, plane 1 emits σT_1^4 radiation to plane 2, which is all absorbed. Also, plane 2 emits σT_2^4 radiation to plane 1, which is all absorbed. Then for plane 1, the net radiation is from plane 1 to 2,

Equation 4.11-13.

$$q_{12} = A_1\sigma(T_1^4 - T_2^4)$$

In this case all the radiation from 1 to 2 is intercepted by 2; that is, the fraction of radiation leaving 1 that is intercepted by 2 is F_{12}, which is 1.0. The factor F_{12} is called the geometric view factor or simply view factor. Hence,

Equation 4.11-14.

$$q_{12} = F_{12}A_1\sigma(T_1^4 - T_2^4)$$

where F_{12} is fraction of radiation leaving surface 1 in all directions which is intercepted by surface 2. Also,

Equation 4.11-15.

$$q_{21} = F_{21}A_2\sigma(T_1^4 - T_2^4)$$

In the case of parallel plates, $F_{12} = F_{21} = 1.0$ and the geometric factor is simply omitted.

View factor for infinite parallel gray planes

If both of the parallel plates A_1 and A_2 are gray, with emissivities and absorptivities of $\varepsilon_1 = \alpha_1$ and $\varepsilon_2 = \alpha_2$, respectively, we can proceed as follows. Since each surface has an unobstructed view of the other, the view factor is 1.0. In unit time, surface A_1 emits $\varepsilon_1 A_1\sigma T_1^4$ radiation to A_2. Of this, the fraction ε_2 (where $\alpha_2 = \varepsilon_2$) is absorbed:

Equation 4.11-16.

$$\text{absorbed by} A_2 = \varepsilon_2(\varepsilon_1 A_1\sigma T_1^4)$$

Also, the fraction $(1 - \varepsilon_2)$ or the amount $(1 - \varepsilon_2)(\varepsilon_1 A_1\sigma T_1^4)$ is reflected back to A_1. Of this amount A_1 reflects back to A_2 a fraction $(1 - \varepsilon_1)$ or an amount $(1 - \varepsilon_1)(1 - \varepsilon_2)(\varepsilon_1 A_1\sigma T_1^4)$. The surface A_2 absorbs the fraction ε_2, or

Equation 4.11-17.

$$\text{absorbed by} A_2 = \varepsilon_2(1 - \varepsilon_1)(1 - \varepsilon_2)(\varepsilon_1 A_1\sigma T_1^4)$$

The amount reflected back to A_1 from A_2 is $(1 - \varepsilon_2)(1 - \varepsilon_1)(1 - \varepsilon_2)(\varepsilon_1 A_1\sigma T_1^4)$. Then A_1 absorbs ε_1 of this and reflects back to A_2 an amount $(1 - \varepsilon_1)(1 - \varepsilon_2)(1 - \varepsilon_1)(1 - \varepsilon_2) \times (\varepsilon_1 A_1\sigma T_1^4)$. The surface A_2 then absorbs

Equation 4.11-18.

$$\text{absorbed by} A_2 = \varepsilon_2(1 - \varepsilon_1)(1 - \varepsilon_2)(1 - \varepsilon_1)(1 - \varepsilon_2)(\varepsilon_1 A_1\sigma T_1^4)$$

This continues, and the total amount absorbed at A_2 is the sum of Eqs. (4.11-16), (4.11-17), (4.11-18), and so on:

Equation 4.11-19.

$$q_{1\to 2} = A_1\sigma T_1^4[\varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_2(1 - \varepsilon_1)(1 - \varepsilon_2) + \varepsilon_1\varepsilon_2(1 - \varepsilon_1)^2(1 - \varepsilon_2)^2 + \cdots]$$

The result is a geometric series (M1):

Equation 4.11-20.

$$q_{1\to 2} = A_1 \sigma T_1^4 \frac{\varepsilon_1 \varepsilon_2}{1-(1-\varepsilon_1)(1-\varepsilon_2)} = A_1 \sigma T_1^4 \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

Repeating the above for the amount absorbed at A_1 which comes from A_2,

Equation 4.11-21.

$$q_{2\to 1} = A_1 \sigma T_2^4 \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

The net radiation is the difference of Eqs. (4.11-20) and (4.11-21):

Equation 4.11-22.

$$q_{12} = A_1 \sigma (T_1^4 - T_2^4) \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

If $\varepsilon_1 = \varepsilon_2 = 1.0$ for black bodies, Eq. (4.11-22) becomes Eq. (4.11-13).

EXAMPLE 4.11-1. Radiation Between Parallel Planes

Two parallel gray planes which are very large have emissivities of $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.7$; surface 1 is at 1100°F (866.5 K) and surface 2 at 600°F (588.8 K). Use English and SI units for the following:

a. What is the net radiation from 1 to 2?

b. If the surfaces are both black, what is the net radiation?

Solution: For part (a), using Eq. (4.11-22) and substituting the known values,

$$\frac{q_{12}}{A_1} = \frac{\sigma(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = (0.1714 \times 10^{-8}) \frac{(1100+460)^4 - (600+460)^4}{1/0.8 + 1/0.7 - 1}$$
$$= 4750 \text{ btu/h} \cdot \text{ft}^2$$

$$\frac{q_{12}}{A_1} = (5.676 \times 10^{-8}) \frac{(866.5)^4 - (588.8)^4}{1/0.8 + 1/0.7 - 1} = 15\,010 \text{ W/m}^2$$

For black surfaces in part (b), using Eq. (4.11-13),

$$q = 7960 \text{ btu/h} \cdot \text{ft}^2 \quad \text{or} \quad 25\,110 \text{ W/m}^2$$

Note the large reduction in radiation when surfaces with emissivities less than 1.0 are used.

Example 4.11-1 shows the large influence that emissivities less than 1.0 have on radiation. This fact is used to reduce radiation loss or gain from a surface by using planes as a radiation shield. For example, for two parallel surfaces of emissivity ε at T_1 and T_2, the interchange is, by Eq. (4.11-22),

Equation 4.11-23.

$$\frac{(q_{12})_0}{A} = \frac{\sigma(T_1^4 - T_2^4)}{2/\varepsilon - 1}$$

The subscript 0 indicates that there are no planes in between the two surfaces. Suppose that we now insert one or more radiation planes between the original surfaces. Then it can be shown that

Equation 4.11-24.

$$\frac{(q_{12})_N}{A} = \frac{1}{N+1}\frac{\sigma(T_1^4 - T_2^4)}{2/\varepsilon - 1}$$

where N is the number of radiation planes or shields between the original surfaces. Hence, a great reduction in radiation heat loss is obtained by using these shields.

Derivation of general equation for view factor between black bodies

Suppose that we consider radiation between two parallel black planes of finite size as shown in Fig. 4.11-2a. Since the planes are not infinite in size, some of the radiation from surface 1 does not strike surface 2, and vice versa. Hence, the net radiation interchange is less, since some is lost to the surroundings. The fraction of radiation leaving surface 1 in all directions which is intercepted by surface 2 is called F_{12} and must be determined for each geometry by taking differential surface elements and integrating over the entire surface.

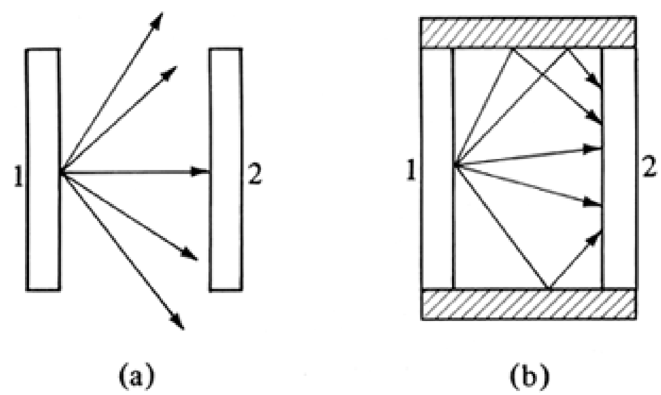


Figure 4.11-2. Radiation between two black surfaces: (a) two planes alone, (b) two planes connected by refractory reradiating walls.

Before we can derive a general relationship for the view factor between two finite bodies, we must consider and discuss two quantities, a solid angle and the intensity of radiation. A solid angle ω is a dimensionless quantity which is a measure of an angle in solid geometry. In Fig. 4.11-3a the differential solid angle $d\omega_1$ is equal to the normal projection of dA_2 divided by the square of the distance between the point P and area dA_2:

Equation 4.11-25.

$$d\omega_1 = \frac{dA_2 \cos\theta_2}{r^2}$$

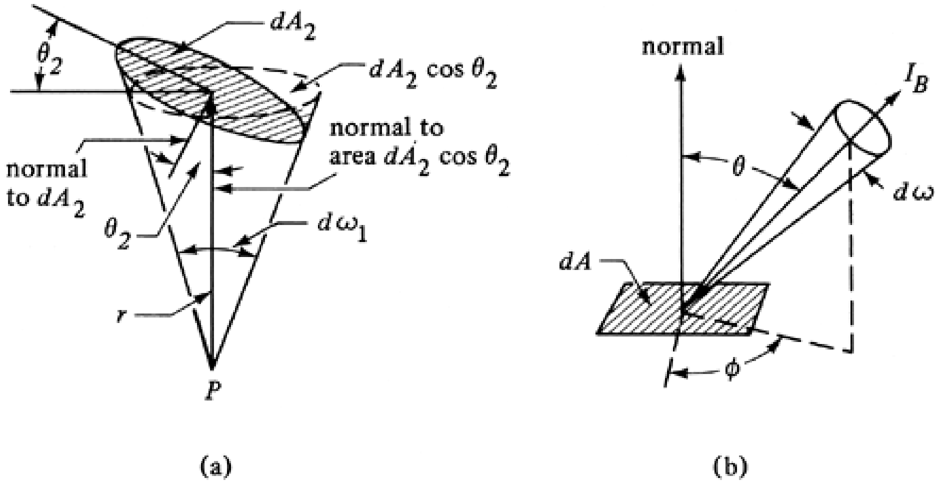


Figure 4.11-3. Geometry for a solid angle and intensity of radiation: (a) solid-angle geometry, (b) intensity of radiation from emitting area dA.

The units of a solid angle are steradian or sr. For a hemisphere the number of sr subtended by this surface is 2π.

The intensity of radiation for a black body, I_B, is the rate of radiation emitted per unit area projected in a direction normal to the surface and per unit solid angle in a specified direction as shown in Fig. 4.11-3b. The projection of *dA* on the line between centers is $dA \cos \theta$.

Equation 4.11-26.

$$I_B = \frac{dq}{dA \cos d\omega}$$

where q is in W and I_B is in $W/m^2 \cdot sr$. We assume that the black body is a diffuse surface which emits with equal intensity in all directions, that is, I = constant. The emissive power E_B which leaves a black-body plane surface is determined by integrating Eq. (4.11-26) over all solid angles subtended by a hemisphere covering the surface. The final result is as follows [see references (C3, H1, K1) for details]:

Equation 4.11-27.

$$E_B = \pi I_B$$

where E_B is in W/m^2.

In order to determine the radiation heat-transfer rates between two black surfaces, we must determine the general case for the fraction of the total radiant heat that leaves a surface and arrives on a second surface. Using only black surfaces, we consider the case shown in Fig. 4.11-4, in which radiant energy is exchanged between area elements dA_1 and dA_2. The line r is the distance between the areas, and the angles between this line and the normals to the two surfaces are θ_1 and θ_2. The rate of radiant energy that leaves dA_1 in the direction given by the angle θ_1 is $I_{B1}\, dA \cos \theta_1$. The rate that leaves dA_1 and arrives on dA_2 is given by Eq. (4.11-28):

Equation 4.11-28.

$$dq_{1\rightarrow 2} = I_{B1}\, dA \cos \theta_1\, d\omega_1$$

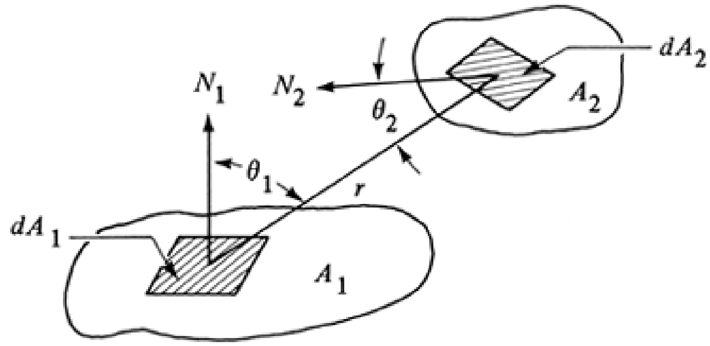


Figure 4.11-4. Area elements for radiation shape factor.

where $d\omega_1$ is the solid angle subtended by the area dA_2 as seen from dA_1. Combining Eqs. (4.11-25) and (4.11-28),

Equation 4.11-29.

$$dq_{1\to2} = \frac{I_{B1}\, dA \cos\theta_1 \cos\theta_2\, dA_2}{r^2}$$

From Eq. (4.11-27), $I_{B1} = E_{B1}/\pi$. Substituting E_{B1}/π for I_{B1} into Eq. (4.11-29),

Equation 4.11-30.

$$dq_{1\to2} = \frac{E_{B1} \cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

The energy leaving dA_2 and arriving at dA_1 is

Equation 4.11-31.

$$dq_{2\to1} = \frac{E_{B2} \cos\theta_2 \cos\theta_1\, dA_2\, dA_1}{\pi r^2}$$

Substituting σT_1^4 for E_{B1} and σT_2^4 for E_{B2} from Eq. (4.11-4) and taking the difference of Eqs. (4.11-30) and (4.11-31) for the net heat flow,

Equation 4.11-32.

$$dq_{12} = \sigma(T_1^4 - T_2^4)\frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

Performing the double integrations over surfaces A_1 and A_2 will yield the total net heat flow between the finite areas:

Equation 4.11-33.

$$q_{12} = \sigma(T_1^4 - T_2^4)\int_{A_2}\int_{A_1}\frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

Equation (4.11-33) can also be written as

Equation 4.11-34.

$$q_{12} = A_1F_{12}\sigma(T_1^4 - T_2^4) = A_2F_{21}\sigma(T_1^4 - T_2^4)$$

where F_{12} is a geometric shape factor or view factor and designates the fraction of the total radiation leaving A_1 which strikes A_2, and F_{21} represents the fraction leaving A_2 which strikes A_1. Also, the following relation exists:

Equation 4.11-35.

$$A_1F_{12} = A_2F_{21}$$

which is valid for black surfaces and nonblack surfaces. The view factor F_{12} is then

Equation 4.11-36.

$$F_{12} = \frac{1}{A_1}\int_{A_2}\int_{A_1}\frac{\cos\theta_1\cos\theta_2\,dA_1\,dA_2}{\pi r^2}$$

Values of the view factor can be calculated for a number of geometrical arrangements.

View factors between black bodies for various geometries

A number of basic relationships between view factors are given below.

The reciprocity relationship given by Eq. (4.11-35) is

Equation 4.11-35.

$$A_1F_{12} = A_2F_{21}$$

This relationship can be applied to any two surfaces *i* and *j*:

Equation 4.11-37.

$$A_iF_{ij} = A_jF_{ji}$$

If surface A_1 can only see surface A_2, then $F_{12} = 1.0$.

If surface A_1 sees a number of surfaces $A_2, A_3, \ldots$, and all the surfaces form an enclosure, then the enclosure relationship is

Equation 4.11-38.

$$F_{11} + F_{12} + F_{13} + \cdots = 1.0$$

If the surface A_1 cannot see itself (surface is flat or convex), $F_{11} = 0$.

EXAMPLE 4.11-2. View Factor from a Plane to a Hemisphere

Determine the view factors between a plane A_1 covered by a hemisphere A_2 as shown in Fig. 4.11-5.

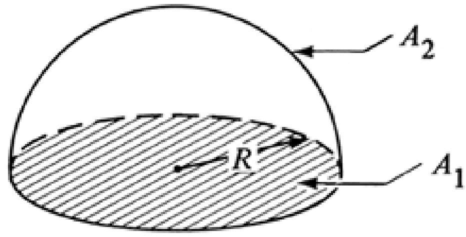


Figure 4.11-5. Radiant exchange between a flat surface and a hemisphere for Example 4.11-2.

Solution: Since surface A_1 sees only A_2, the view factor F_{12} = 1.0. Using Eq. (4.11-35),

Equation 4.11-35.

$$A_1 F_{12} = A_2 F_{21}$$

The area $A_1 = \pi R^2; A_2 = 2\pi R^2$. Substituting into Eq. (4.11-35) and solving for F_{21},

$$F_{21} = F_{12} = \frac{A_1}{A_2} = (1.0)\frac{\pi R^2}{2\pi R^2} = \frac{1}{2}$$

Using Eq. (4.11-38) for surface A_1, F_{11} = 1.0 - F_{12} = 1.0 - 1.0 = 0. Also, writing Eq. (4.11-38) for surface A_2,

Equation 4.11-39.

$$F_{22} + F_{21} = 1.0$$

Solving for F_{22}, $F_{22} = 1.0 - F_{21} = 1.0 - \frac{1}{2} = \frac{1}{2}$.

EXAMPLE 4.11-3. Radiation Between Parallel Disks

In Fig. 4.11-6 a small disk of area A_1 is parallel to a large disk of area A_2, with A_1 centered directly below A_2. The distance between the centers of the disks is R and the radius of A_2 is *a.* Determine the view factor for radiant heat transfer from A_1 to A_2.

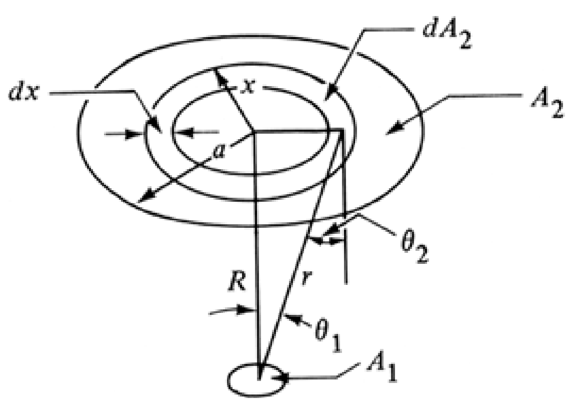


Figure 4.11-6. View factor for radiation from a small element to a parallel disk for Example 4.11-3.

Solution: The differential area for A_2 is taken as the circular ring of radius x so that $dA_2 = 2\pi x\, dx$. The angle $\theta_1 = \theta_2$. Using Eq. (4.11-36),

$$F_{12} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos \theta_1 \cos \theta_1 \, dA_1 (2\pi x \, dx)}{\pi r^2}$$

In this case the area A_1 is very small compared to A_2, so dA_1 can be integrated to A_1 and the other terms inside the integral can be assumed constant. From the geometry shown, $r = (R^2 + x^2)^{1/2}$, $\cos \theta_1 = R/(R^2 + x^2)^{1/2}$. Making these substitutions into the equation for F_{12},

$$F_{12} = \int_0^a \frac{2R^2 x \, dx}{(R^2 + x^2)^2}$$

Integrating,

$$F_{12} = \frac{a^2}{R^2 + a^2}$$

The integration of Eq. (4.11-36) has been performed for numerous geometrical configurations and values of F_{12} tabulated. Then,

Equation 4.11-34.

$$q_{12} = F_{12} A_1 \sigma (T_1^4 - T_2^4) = F_{21} A_2 \sigma (T_1^4 - T_2^4)$$

where F_{12} is the fraction of the radiation leaving A_1 which is intercepted by A_2 and F_{21} the fraction reaching A_1 from A_2. Since the flux from 1 to 2 must equal that from 2 to 1, Eq. (4.11-34) becomes Eq. (4.11-35) as given previously:

Equation 4.11-35.

$$A_1 F_{12} = A_2 F_{21}$$

Hence, one selects the surface whose view factor can be determined most easily. For example, the view factor F_{12} for a small surface A_1 completely enclosed by a large surface A_2 is 1.0, since all the radiation leaving A_1 is intercepted by A_2. In Fig. 4.11-7 the view factors F_{12} between parallel planes are given, and in Fig. 4.11-8 the view factors for adjacent perpendicular rectangles. View factors for other geometries are given elsewhere (H1, K1, P3, W1).

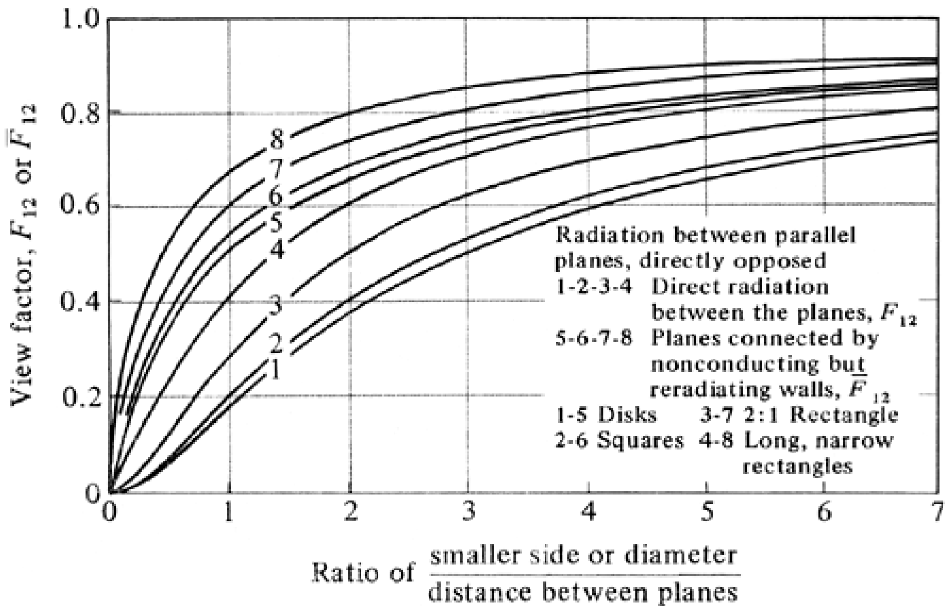


Figure 4.11-7. View factor between parallel planes directly opposed. (From W. H. McAdams, Heat Transmission, 3rd ed. New York McGraw-Hill Book Company, 1954. With permission.)

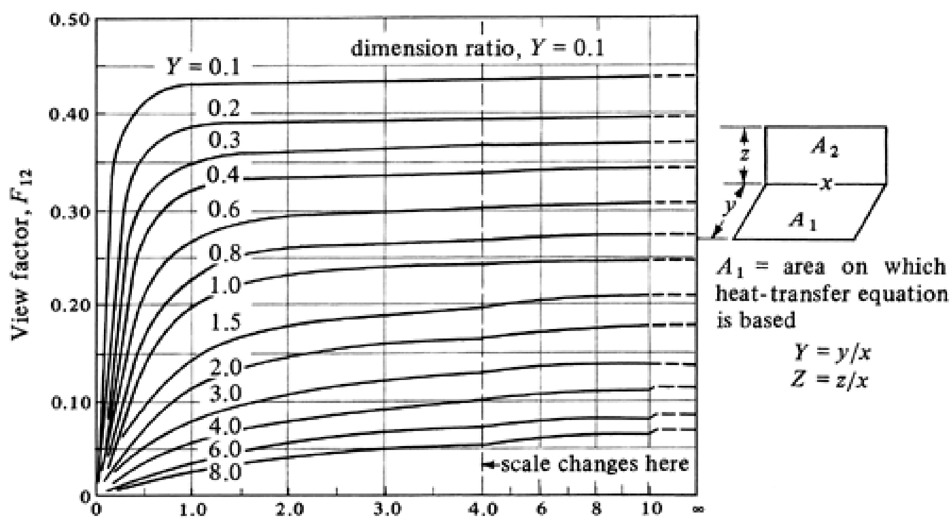


*Figure 4.11-8. View factor for adjacent perpendicular rectangles. [From H. C. Hottel, Mech. Eng., **52,** 699 (1930). With permission.]*

View Factors When Surfaces Are Connected by Reradiating Walls

If the two black-body surfaces A_1 and A_2 are connected by nonconducting (refractory) but reradiating walls as in Fig. 4.11-2b, a larger fraction of the radiation from surface 1 is intercepted by 2. This view factor is called $\bar{F}_{12}$. The case of two surfaces connected by the walls of an enclosure such as a furnace is a common example of this. The general equation for this case assuming a uniform refractory temperature has been derived (M1, C3) for two radiant sources A_1 and A_2 which are not concave, so they do not see themselves:

Equation 4.11-40.

$$\overline{F}_{12} = \frac{A_2 - A_1 F_{12}^2}{A_1 + A_2 - 2A_1 F_{12}} = \frac{1 - (A_1/A_2)F_{12}^2}{A_1/A_2 + 1 - 2(A_1/A_2)F_{12}}$$

Also, as before,

Equation 4.11-41.

$$A_1 \overline{F}_{12} = A_1 \overline{F}_{21}$$

Equation 4.11-42.

$$q_{12} = \overline{F}_{12} A_1 \sigma (T_1^4 - T_2^4)$$

The factor $\overline{F}_{12}$ for parallel planes is given in Fig. 4.11-7 and for other geometries can be calculated from Eq. (4.11-36). For view factors F_{12} and $\overline{F}_{12}$ for parallel tubes adjacent to a wall as in a furnace as well as for variation in refractory wall temperature, see elsewhere (M1, P3). If there are no reradiating walls,

Equation 4.11-43.

$$F_{12} = \overline{F}_{12}$$

View Factors and Gray Bodies

A general and more practical case, which is the same as for Eq. (4.11-40) but with the surfaces A_1 and A_2 being gray with emissivities ε_1 and ε_2, will be considered. Nonconducting reradiating walls are present as before. Since the two surfaces are now gray, there will be some reflection of radiation, which will decrease the net radiant exchange between the surfaces below that for black surfaces. The final equations for this case are

Equation 4.11-44.

$$q_{12} = \mathfrak{F}_{12} A_1 \sigma (T_1^4 - T_2^4)$$

Equation 4.11-45.

$$\mathfrak{F}_{12} = \frac{1}{\dfrac{1}{\overline{F}_{12}} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_1} - 1\right)}$$

where $\mathfrak{F}_{12}$ is the new view factor for two gray surfaces A_1 and A_2 which cannot see themselves and are connected by reradiating walls. If no refractory walls are present, F_{12} is used in place of $\overline{F}_{12}$ in Eq. (4.11-41). Again,

Equation 4.11-46.

$$A_1 \mathfrak{F}_{12} = A_2 \mathfrak{F}_{21}$$

EXAMPLE 4.11-4. Radiation Between Infinite Parallel Gray Planes

Derive Eq. (4.11-22) by starting with the general equation for radiation between two gray bodies A_1 and A_2 which are infinite parallel planes having emissivities ε_1 and ε_2, respectively.

***Solution*:** Since there are no reradiating walls, by Eq. (4.11-43) $\bar{F}_{12}$ becomes F_{12}. Also, since all the radiation from surface 1 is intercepted by surface 2, $F_{12} = 1.0$. Substituting into Eq. (4.11-45), noting that $A_1/A_2 = 1.0$,

$$\mathfrak{F}_{12} = \frac{1}{\dfrac{1}{\bar{F}_{12}} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_1} - 1\right)} = \frac{1}{\dfrac{1}{1} + 1\left(\dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_1} - 1\right)}$$

$$= \frac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

Then using Eq. (4.11-44),

$$q_{12} = \mathfrak{F}_{12} A_1 \sigma (T_1^4 - T_2^4) = A_1 \sigma (T_1^4 - T_2^4) \frac{1}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

This is identical to Eq. (4.11-22).

EXAMPLE 4.11-5. Complex View Factor for Perpendicular Rectangles

Find the view factor F_{12} for the configuration shown in Fig. 4.11-9 of the rectangle with area A_2 displaced from the common edge of rectangle A_1 and perpendicular to A_1. The temperature of A_1 is T_1 and that of A_2 and A_3 is T_2.

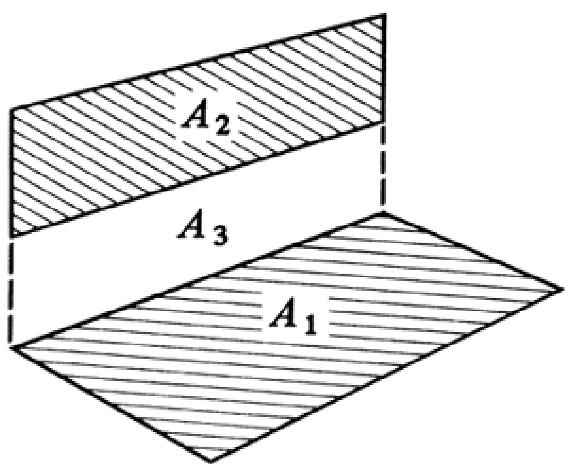


Figure 4.11-9. Configuration for Example 4.11-5.

***Solution*:** The area A_3 is a fictitious area between areas A_2 and A_1. Designate the area A_2 plus A_3 as $A_{(23)}$. The view factor $F_{1(23)}$ for areas A_1 and $A_{(23)}$ can be obtained from Fig. 4.11-8 for adjacent perpendicular rectangles. Also, F_{13} can be obtained from Fig. 4.11-8. The radiation interchange between A_1 and $A_{(23)}$ is equal to that intercepted by A_2 and by A_3:

Equation 4.11-47.

$$A_1 F_{1(23)} \sigma (T_1^4 - T_2^4) = A_1 F_{12} \sigma (T_1^4 - T_2^4) + A_1 F_{13} (T_1^4 - T_2^4)$$

Hence,

Equation 4.11-48.

$$A_1 F_{1(23)} = A_1 F_{12} + A_1 F_{13}$$

Solving for F_{12},

Equation 4.11-49.

$$F_{12} = F_{1(23)} - F_{13}$$

Methods similar to those used in this example can be employed to find the shape factors for a general orientation of two rectangles in perpendicular planes or parallel rectangles (C3, H1, K1).

EXAMPLE 4.11-6. Radiation to a Small Package

A small, cold package having an area A_1 and emissivity ε_1 is at temperature T_1. It is placed in a warm room with the walls at T_2 and an emissivity ε_2. Using Eq. (4.11-45), derive the view factor for this and the equation for the radiation heat transfer.

Solution: For the small surface A_1 completely enclosed by the enclosure A_2, $\bar{F}_{12} = F_{12}$ by Eq. (4.11-43), since there are no reradiating (refractory) walls. Also, $F_{12} = 1.0$, since all the radiation from A_1 is intercepted by the enclosure A_2 because A_1 does not have any concave surfaces and cannot "see" itself. Since A_2 is very large compared to A_1, $A_1/A_2 = 0$. Substituting into Eq. (4.11-45),

$$\mathfrak{F}_{12} = \frac{1}{\dfrac{1}{\bar{F}_{12}} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_1} - 1\right)} = \frac{1}{\dfrac{1}{1} + 0\left(\dfrac{1}{\varepsilon_2} - 1\right) + \dfrac{1}{\varepsilon_1} - 1} = \varepsilon_1$$

Substituting into Eq. (4.11-44),

$$q_{12} = \mathfrak{F}_{12} A_1 \sigma (T_1^4 - T_2^4) = \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4)$$

This is the same as Eq. (4.10-6) derived previously.

For solving complicated radiation problems involving more than four or five heat-transfer surfaces, matrix methods have been developed and are discussed in detail elsewhere (H1, K1).

Radiation in Absorbing Gases

Introduction to absorbing gases in radiation

As discussed in this section, solids and liquids emit radiation over a continuous spectrum. However, most gases that are monatomic or diatomic, such as He, Ar, H_2, O_2, and N_2, are virtually transparent to thermal radiation; that is, they emit practically no radiation and do not absorb radiation. Gases with a dipole moment and higher polyatomic gases emit significant amounts of radiation and also absorb radiant energy within the same bands in which they emit radiation. These gases include CO_2, H_2O, CO, SO_2, NH_3, and organic vapors.

For a particular gas, the width of the absorption or emission bands depends on the pressure and also the temperature. If an absorbing gas is heated, it radiates energy to the cooler surroundings. The net radiation heat-transfer rate between surfaces is decreased in these cases because the gas absorbs some of the radiant energy being transported between the surfaces.

Absorption of radiation by a gas

The absorption of radiation in a gas layer can be described analytically, since the absorption by a given gas depends on the number of molecules in the path of radiation. Increasing the partial pressure of the absorbing gas or the path length increases the amount of absorption. We define $I_{\lambda O}$ as the intensity of radiation at a particular wavelength before it enters the gas and $I_{\lambda L}$ as the intensity at the same wavelength after having traveled a distance of L in the gas. If the beam impinges on a gas layer of thickness dL, the decrease in intensity, dI_λ, is proportional to I_λ and dL:

Equation 4.11-50.

$$dI_\lambda = -\alpha_\lambda I_\lambda \, dL$$

where I_λ is in W/m^2. Integrating,

Equation 4.11-51.

$$I_{\lambda L} = I_{\lambda 0} e^{-\alpha_\lambda L}$$

The constant α_λ depends on the particular gas, its partial pressure, and the wavelength of radiation. This equation is called Beer's law. Gases frequently absorb only in narrow-wavelength bands.

Characteristic mean beam length of absorbing gas

The calculation methods for gas radiation are quite complicated. For the purpose of engineering calculations, Hottel (M1) has presented approximate methods for calculating radiation and absorption when gases such as CO_2 and water vapor are present. Thick layers of a gas absorb more energy than do thin layers. Hence, in addition to specifying the pressure and temperature of a gas, we must specify a characteristic length (mean beam length) of a gas mass to determine the emissivity and absorptivity of a gas. The mean beam length L depends on the specific geometry.

For a black differential receiving surface area dA located in the center of the base of a hemisphere of radius L containing a radiating gas, the mean beam length is L. The mean beam length has been evaluated for various geometries, as presented in Table 4.11-1. For other shapes, L can be approximated by

Equation 4.11-52.

$$L = 3.6\frac{V}{A}$$

Table 4.11-1. Mean Beam Length for Gas Radiation to Entire Enclosure Surface (M1, R2, P3)

Geometry of Enclosure	*Mean Beam Length, L*
Sphere, diameter D	$0.65D$
Infinite cylinder, diameter D	$0.95D$
Cylinder, length = diameter D	$0.60D$
Infinite parallel plates, separation distance D	$1.8D$
Hemisphere, radiation to element in base, radius R	R
Cube, radiation to any face, side D	$0.60D$
Volume surrounding bank of long tubes with centers on equilateral triangle, clearance = tube diameter D	$2.8D$

where V is volume of the gas in m^3, A is surface area of the enclosure in m^2, and L is in m.

Emissivity, absorptivity, and radiation of a gas

Gas emissivities have been correlated and Fig. 4.11-10 gives the gas emissivity ε_G of CO_2 at a total pressure of the system of 1.0 atm abs. The p_G is the partial pressure of CO_2 in atm and L is the mean beam length in m. The emissivity ε_G is defined as the ratio of the rate of energy transfer from the hemispherical body of gas to a surface element at the midpoint divided by the rate of energy transfer from a black hemisphere surface of radius L and temperature T_G to the same element.

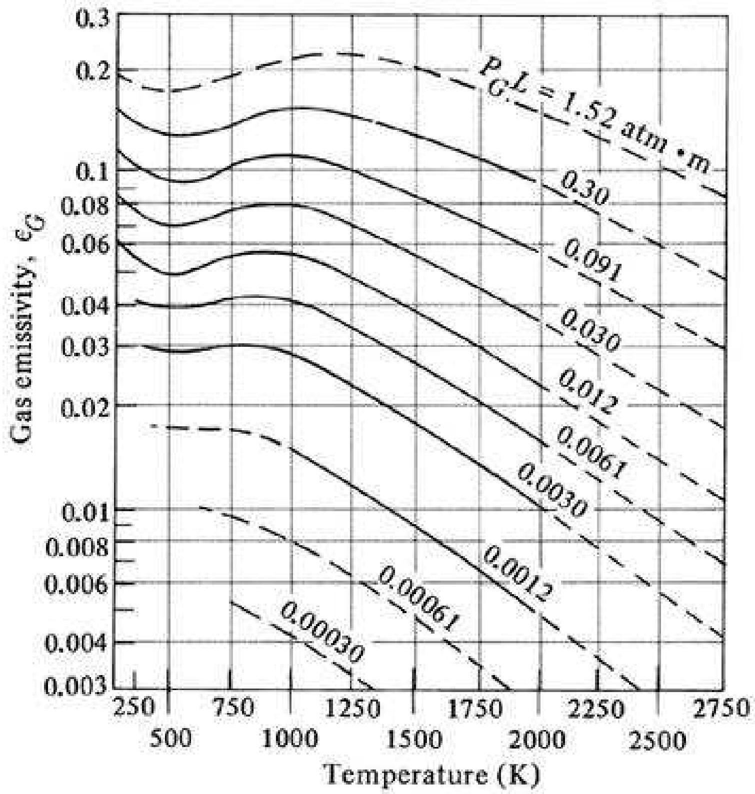


Figure 4.11-10. Total emissivity of the gas carbon dioxide at a total pressure of 1.0 atm. (From W. H. McAdams, Heat Transmission, 3rd ed. New York: McGraw-Hill Book Company, 1954. With permission.)

The rate of radiation emitted from the gas is $\sigma \varepsilon_G T_G^4$ in W/m^2 of receiving surface element, where ε_G is evaluated at T_G. If the surface element at the midpoint at T_1 is radiating heat back to the gas, the absorption rate of the gas will be $\sigma \alpha_G T_1^4$, where α_G is the absorptivity of the gas for black-body radiation from the surface at T_1. The α_G of CO_2 is determined from Fig. 4.11-10 at T_1, but instead of the parameter $p_G L$, the parameter $p_G L(T_1/T_G)$ is used. The resulting value from the chart is then multiplied by $(T_G/T_1)^{0.65}$ to give α_G. The net rate of radiant transfer between a gas at T_G and a black surface of finite area A_1 at T_1 is then

Equation 4.11-53.

$$q = \sigma A(\varepsilon_G T_G^4 - \alpha_G T_1^4)$$

When the total pressure is not 1.0 atm, a correction chart is available to correct the emissivity of CO_2. Charts are also available for water vapor (H1, K1, M1, P3). When both CO_2 and H_2O are present the total radiation is reduced somewhat, since each gas is somewhat opaque to radiation from the other gas. Charts for these interactions are also available (H1, K1, M1, P3).

EXAMPLE 4.11-7. Gas Radiation to a Furnace Enclosure

A furnace is in the form of a cube 0.3 m on a side inside, and these interior walls can be approximated as black surfaces. The gas inside at 1.0 atm total pressure and 1100 K contains 10 mol % CO_2 and the rest is O_2 and N_2. The small amount of water vapor present will be neglected. The walls of the furnace are maintained at 600 K by external cooling. Calculate the total heat transfer to the walls neglecting heat transfer by convection.

Solution: From Table 4.11-1, the mean beam length for radiation to a cube face is $L = 0.60D = 0.60(0.30) = 0.180$ m. The partial pressure of CO_2 is $p_G = 0.10(100) = 0.10$ atm. Then $p_G L = 0.10(0.180) = 0.0180$ atm m. From Fig. 4.11-10, $\varepsilon_G = 0.064$ at $T_G = 1100$ K.

To obtain α_G, we evaluate α_G at $T_1 = 600$ K and $p_G L(T_1/T_G) = (0.0180)(600/1100) = 0.00982$ atm · m. From Fig. 4.11-10, the uncorrected value of $\alpha_G = 0.048$. Multiplying this by the correction factor $(T_G/T_1)^{0.65}$, the final correction value is

$$\alpha_G = 0.048(1100/600)^{0.65} = 0.0712$$

Substituting into Eq. (4.11-53),

$$\begin{aligned}\frac{q}{A} &= \sigma(\varepsilon_G T_G^4 - \alpha_G T_1^4)\\ &= (5.676 \times 10^{-8})[0.064(1100)^4 - 0.0712(600)^4]\\ &= 4.795 \times 10^3 \text{ W/m}^2 = 4.795 \text{ kW/m}^2\end{aligned}$$

For six sides, $A = 6(0.3 \times 0.3) = 0.540$ m^2. Then,

$$q = 4.795(0.540) = 2.589 \text{ kW}$$

For the case where the walls of the enclosure are not black, some of the radiation striking the walls is reflected back to the other walls and into the gas. As an approximation when the emissivity of the walls is greater than 0.7, an effective emissivity ε' can be used:

Equation 4.11-54.

$$\varepsilon' = \frac{\varepsilon + 1.0}{2}$$

where ε is the actual emissivity of the enclosure walls. Then Eq. (4.11-53) is modified to give the following (M1):

Equation 4.11-55.

$$q = \sigma A \varepsilon'(\varepsilon_G T_G^4 - \alpha_G T_1^4)$$

Other approximate methods are available for gases containing suspended luminous flames, clouds of nonblack particles, refractory walls and absorbing gases present, and so on (M1, P3).

HEAT TRANSFER OF NON-NEWTONIAN FLUIDS

Introduction

Most of the studies on heat transfer with fluids have been done with Newtonian fluids. However, a wide variety of non-Newtonian fluids are encountered in the industrial chemical, biological, and food processing industries. To design equipment to handle these fluids, the flow-property constants (rheological constants) must be available or must be measured experimentally. Section 3.5 gave a detailed discussion of rheological constants for non-Newtonian fluids. Since many non-Newtonian fluids have high effective viscosities, they are often in laminar flow. Since the majority of non-Newtonian fluids are pseudoplastic fluids, which can usually be represented by the power law, Eq. (3.5-2), the discussion will be concerned with such fluids. For other fluids, the reader is referred to Skelland (S3).

Heat Transfer Inside Tubes

Laminar flow in tubes

A large portion of the experimental investigations have been concerned with heat transfer of non-Newtonian fluids in laminar flow through cylindrical tubes. The physical properties that are needed for heat-transfer coefficients are density, heat capacity, thermal conductivity, and the rheological constants K' and n' or K and n.

In heat transfer in a fluid in laminar flow, the mechanism is primarily one of conduction. However, for low flow rates and low viscosities, natural convection effects can be present. Since many non-Newtonian fluids are quite "viscous," natural convection effects are reduced substantially. For laminar flow inside circular tubes of power-law fluids, the equation of Metzner and Gluck (M2) can be used with highly "viscous" non-Newtonian fluids with negligible natural convection for horizontal or vertical tubes for the Graetz number $N_{Gz} > 20$ and $n' > 0.10$:

Equation 4.12-1.

$$(N_{\mathrm{Nu}})_a = \frac{h_a D}{k} = 1.75\delta^{1/3}(N_{\mathrm{Gz}})^{1/3}\left(\frac{\gamma_b}{\gamma_w}\right)^{0.14}$$

where

Equation 4.12-2.

$$\delta = \frac{3n' + 1}{4n'}$$

Equation 4.12-3.

$$N_{\mathrm{Gz}} = \frac{mc_p}{kL} = \frac{\pi}{4}\frac{\mathrm{D}v\rho}{\mu}\frac{\mathrm{c_p}\mu}{\mathrm{k}}\frac{\mathrm{D}}{\mathrm{L}} = \frac{\pi}{4}\mathrm{N_{Re}}N_{\mathrm{Pr}}\frac{D}{L}$$

The viscosity coefficients γ_b at temperature T_b and γ_w at T_w are defined as

Equation 4.12-4.

$$\frac{\gamma_b}{\gamma_w} = \frac{K'_b 8^{n'-1}}{K'_w 8^{n'-1}} = \frac{K'_b}{K'_w} = \frac{K_b}{K_w}$$

The nomenclature is as follows: k in W/m · K, c_p in J/kg · K, ρ in kg/m^3, flow rate m in kg/s, length of heated section of tube L in m, inside diameter D in m, the mean coefficient h_a in W/m^2 · K, and K and n' rheological constants (see Section 3.5). The physical properties and K_b are all evaluated at the mean bulk temperature T_b and K_w at the average wall temperature T_w.

The value of the rheological constant n' or n has been found not to vary appreciably over wide temperature ranges (S3). However, the rheological constant K' or K has been found to vary appreciably. A plot of log K' versus $1/T_{abs}$ (C1) or versus T°C (S3) can often be approximated by a straight line. Often data for the temperature effect on K are not available. Since the ratio K_b/K_w is taken to the 0.14 power, this factor can sometimes be neglected without causing large errors. For a value of the ratio of 2:1, the error is only about 10%. A plot of log viscosity versus $1/T$ for Newtonian fluids is also often a straight line. The value of h_a obtained from Eq. (4.12-1) is the mean value to use over the tube length L with the arithmetic temperature difference ΔT_a:

Equation 4.12-5.

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

when T_w is the average wall temperature for the whole tube and T_{bi} is the inlet bulk temperature and T_{bo} the outlet bulk temperature. The heat flux q is

Equation 4.12-6.

$$q = h_a A\,\Delta T_a = h_a(\pi DL)\,\Delta T_a$$

EXAMPLE 4.12-1. Heating a Non-Newtonian Fluid in Laminar Flow

A non-Newtonian fluid flowing at a rate of 7.56×10^{-2} kg/s inside a 25.4-mm-ID tube is being heated by steam condensing outside the tube. The fluid enters the heating section of the tube, which is 1.524 m long, at a temperature of 37.8°C. The inside wall temperature T_w is constant at 93.3°C. The mean physical properties of the fluid are $\rho = 1041$ kg/m^3, $c_{pm} = 2.093$ kJ/kg · K, and $k = 1.212$ W/m · K. The fluid is a power-law fluid having the following flow-property (rheological) constants: $n = n' = 0.40$, which is approximately constant over the temperature range encountered, and $K = 139.9$ N · s$^{n'}$/m^2 at 37.8°C and 62.5 at 93.3°C. For this fluid a plot of log K versus T°C is approximately a straight line. Calculate the outlet bulk temperature of the fluid if it is in laminar flow.

Solution: The solution is trial and error, since the outlet bulk temperature T_{bo} of the fluid must be known in order to calculate h_a from Eq. (4.12-2). Assuming $T_{bo} = 54.4$°C for the first trial, the mean bulk temperature T_b is (54.4 + 37.8)/2, or 46.1°C.

Plotting the two values of K given at 37.8 and 93.3°C as log K versus T°C and drawing a straight line through these two points, a value for K_b of 123.5 at $T_b = 46.1$°C is read from the plot. At $T_w = 93.3$°C, $K_w = 62.5$.

Next, δ is calculated using Eq. (4.12-2):

$$\delta = \frac{3n'+1}{4n'} = \frac{3(0.40)+1}{4(0.40)} = 1.375$$

Substituting into Eq. (4.12-3),

$$N_{Gz} = \frac{mc_p}{kL} = \frac{(7.56 \times 10^{-2})(2.093 \times 10^3)}{1.212(1.524)} = 85.7$$

From Eq. (4.12-4),

$$\frac{\gamma_b}{\gamma_w} = \frac{K_b}{K_w} = \frac{123.5}{62.5}$$

Then substituting into Eq. (4.12-1),

Equation 4.12-1.

$$\frac{h_a D}{k} = \frac{h_a(0.0254)}{1.212} = 1.75\delta^{1/3}(N_{Gz})^{1/3}\left(\frac{\gamma_b}{\gamma_w}\right)^{0.14}$$

$$= 1.75(1.375)^{1/3}(85.7)^{1/3}\left(\frac{123.5}{62.5}\right)^{0.14}$$

Solving, $h_a = 448.3\ \text{W/m}^2 \cdot \text{K}$.

By a heat balance, the value of q in W is as follows:

Equation 4.12-7.

$$q = mc_{pm}(T_{bo} - T_{bi})$$

This is equated to Eq. (4.12-6) to obtain

Equation 4.12-8.

$$q = mc_{pm}(T_{bo} - T_{bi}) = h_a(\pi DL)\Delta T_a$$

The arithmetic mean temperature difference ΔT_a by Eq. (4.12-5) is

$$\Delta T_a = \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

$$= \frac{(93.3 - 37.8) + (93.3 - T_{bo})}{2} = 74.4 - 0.5T_{bo}$$

Substituting the known values in Eq. (4.12-8) and solving for T_{bo},

$$(7.56 \times 10^{-2})(2.093 \times 10^3)(T_{bo} - 37.8)$$
$$= 448.3(\pi \times 0.0254 \times 1.524)(74.4 - 0.5T_{bo})$$
$$T_{bo} = 54.1°\text{C}$$

This value of 54.1°C is close enough to the assumed value of 54.5°C that a second trial is not needed. Only the value of K_b would be affected. Known values can be substituted into Eq. (3.5-11) for the Reynolds number to show that it is less than 2100 and that the flow is laminar.

For less "viscous" non-Newtonian power-law fluids in laminar flow, natural convection may affect the heat-transfer rates. Metzner and Gluck (M2) recommend use of an empirical correction to Eq. (4.12-1) for horizontal tubes.

Turbulent flow in tubes

For turbulent flow of power-law fluids through tubes. Clapp (C4) presents the following empirical equation for heat transfer:

Equation 4.12-9.

$$N_{Nu} = \frac{h_L D}{k} = 0.0041(N_{Re,gen})^{0.99}\left[\frac{K' c_p}{k}\left(\frac{8V}{D}\right)^{n-1}\right]^{0.4}$$

where $N_{Re,gen}$ is defined by Eq. (3.5-11) and h_L is the heat-transfer coefficient based on the log mean temperature driving force. The fluid properties are evaluated at the bulk mean temperature. Metzner and Friend (M3) also give equations for turbulent heat transfer.

Natural Convection

Acrivos (A1, S3) gives relationships for natural convection heat transfer to power-law fluids from various geometries of surfaces such as spheres, cylinders, and plates.

SPECIAL HEAT-TRANSFER COEFFICIENTS

Heat Transfer in Agitated Vessels

Introduction

Many chemical and biological processes are often carried out in agitated vessels. As discussed in Section 3.4, the liquids are generally agitated in cylindrical vessels with an impeller mounted on a shaft and driven by an electric motor. Typical agitators and vessel assemblies have been shown in Figs. 3.4-1 and 3.4-3. Often it is necessary to cool or heat the contents of the vessel during agitation. This is usually done by heat-transfer surfaces, which may be in the form of cooling or heating jackets in the wall of the vessel or coils of pipe immersed in the liquid.

Vessel with heating jacke

In Fig. 4.13-1a, a vessel with a cooling or heating jacket is shown. When heating, the fluid entering is often steam, which condenses inside the jacket and leaves at the bottom. The vessel is equipped with an agitator and in most cases also with baffles (not shown).

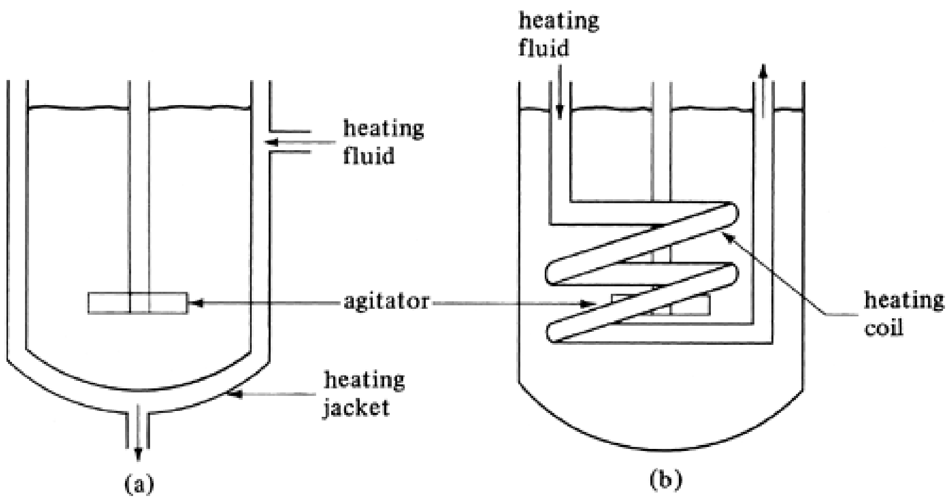


Figure 4.13-1. Heat transfer in agitated vessels: (a) vessel with heating jacket, (b) vessel with heating coils.

Correlations for the heat-transfer coefficient from the agitated Newtonian liquid inside the vessel to the jacket walls of the vessel have the following form:

Equation 4.13-1.

$$\frac{hD_t}{k} = a\left(\frac{D_a^2 N\rho}{\mu}\right)^b \left(\frac{c_p\mu}{k}\right)^{1/3} \left(\frac{\mu}{\mu_w}\right)^m$$

where h is the heat-transfer coefficient for the agitated liquid to the inner wall in W/m^2 · K, D_t is the inside diameter of the tank in m, k is thermal conductivity in W/m · K, D_a is diameter of agitator in m, N is rotational speed in revolutions per sec, ρ is fluid density in kg/m^3, and μ, is liquid viscosity in Pa · s. All the liquid physical properties are evaluated at the bulk liquid temperature except μ_w, which is evaluated at the wall temperature T_w. Below are listed some available correlations and the Reynolds-number range ($N'_{Re} = D_a^2 N\rho/\mu$).

1. Paddle agitator with no baffles (C5, U1)

$$a = 0.36, \quad b = \tfrac{2}{3}, \quad m = 0.21, \quad N'_{Re} = 300 \text{ to } 3 \times 10^5$$

2. Flat-blade turbine agitator with no baffles (B4)

$$a = 0.54, \quad b = \tfrac{2}{3}, \quad m = 0.14, \quad N'_{Re} = 30 \text{ to } 3 \times 10^5$$

3. Flat-blade turbine agitator with baffles (B4, B5)

$$a = 0.74, \quad b = \tfrac{2}{3}, \quad m = 0.14, \quad N'_{Re} = 500 \text{ to } 3 \times 10^5$$

4. Anchor agitator with no baffles (U1)

$$a = 1.0, \quad b = \tfrac{1}{2}, \quad m = 0.18, \quad N'_{Re} = 10 \text{ to } 300$$

$$a = 0.36, \quad b = \tfrac{2}{3}, \quad m = 0.18, \quad N'_{Re} = 300 \text{ to } 4 \times 10^4$$

5. Helical-ribbon agitator with no baffles (G4)

$$a = 0.633, \quad b = \tfrac{1}{2}, \quad m = 0.18, \quad N'_{Re} = 8 \text{ to } 10^5$$

Some typical overall U values for jacketed vessels for various process applications are tabulated in Table 4.13-1.

Table 4.13-1. Typical Overall Heat-Transfer Coefficients in Jacketed Vessels

Fluid in Jacket	Fluid in Vessel	Wall Material	Agitation	U: btu/h · ft^2 · °F	U: W/m^2 · K	Ref.
Steam	Water	Copper	None	150	852	(P1)
			Simple stirring	250	1420	
Steam	Paste	Cast iron	Double scrapers	125	710	(P1)
Steam	Boiling water	Copper	None	250	1420	(P1)
Steam	Milk	Enameled cast iron	None	200	1135	(P1)
			Stirring	300	1700	
Hot water	Cold water	Enameled cast iron	None	70	398	(P1)
Steam	Tomato purée	Metal	Agitation	30	170	(C1)

EXAMPLE 4.13-1. Heat-Transfer Coefficient in Agitated Vessel with Jacket

A jacketed 1.83-m-diameter agitated vessel with baffles is being used to heat a liquid which is at 300 K. The agitator is 0.61 m in diameter and is a flat-blade turbine rotating at 100 rpm. Hot water is in the heating jacket. The wall surface temperature is constant at 355.4 K. The liquid has the following bulk physical properties: ρ = 961 kg/m^3, c_p = 2500 J/kg · K, k = 0.173 W/m · K, and μ = 1.00 Pa · s at 300 K and 0.084 Pa · s at 355.4 K. Calculate the heat-transfer coefficient to the wall of the jacket.

Solution: The following are given:

$$D_t = 1.83 \text{ m} \qquad D_a = 0.61 \text{ m} \qquad N = 100/60 \text{ rev/s}$$

$$\mu(300 \text{ K}) = 1.00 \text{ Pa}\cdot\text{s} = 1.00 \text{ kg/m}\cdot\text{s}$$

$$\mu_w(355.4 \text{ K}) = 0.084 \text{ Pa}\cdot\text{s} = 0.084 \text{ kg/m}\cdot\text{s}$$

First, calculating the Reynolds number at 300 K,

$$N'_{\text{Re}} = \frac{D_a^2 N \rho}{\mu} = \frac{(0.61)^2(100/60)(961)}{1.00} = 596$$

The Prandtl number is

$$N_{\text{Pr}} = \frac{C_p \mu}{k} = \frac{2500(1.00)}{0.173} = 14\,450$$

Using Eq. (4.13-1) with $a = 0.74$, $b = \frac{2}{3}$, and $m = 0.14$,

Equation 4.13-1.

$$\frac{hD_t}{k} = 0.74(N'_{\text{Re}})^{2/3}(N_{\text{Pr}})^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Substituting and solving for h,

$$\frac{h(1.83)}{0.173} = 0.74(596)^{2/3}(14\,450)^{1/3}\left(\frac{1000}{84}\right)^{0.14}$$

$$h = 170.6 \text{ W/m}^2\cdot\text{K } (30.0 \text{ btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})$$

A correlation to predict the heat-transfer coefficient of a power-law non-Newtonian fluid in a jacketed vessel with a turbine agitator is also available elsewhere (C6).

Vessel with heating coils

In Fig. 4.13-1b, an agitated vessel with a helical heating or cooling coil is shown. Correlations for the heat-transfer coefficient to the outside surface of the coils in agitated vessels are listed below for various types of agitators.

For a paddle agitator with no baffles (C5),

Equation 4.13-2.

$$\frac{hD_t}{k} = 0.87\left(\frac{D_a^2 N \rho}{\mu}\right)^{0.62}\left(\frac{c_p \mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

This holds for a Reynolds-number range of 300 to 4×10^5.

For a flat-blade turbine agitator with baffles, see (O1).

When the heating or cooling coil is in the form of vertical tube baffles with a flat-blade turbine, the following correlation can be used (D1):

Equation 4.13-3.

$$\frac{hD_o}{k} = 0.09\left(\frac{D_a^2 N\rho}{\mu}\right)^{0.65}\left(\frac{c_p\mu}{k}\right)^{1/3}\left(\frac{D_a}{D_t}\right)^{1/3}\left(\frac{2}{n_b}\right)^{0.2}\left(\frac{\mu}{\mu_f}\right)^{0.4}$$

where D_0 is the outside diameter of the coil tube in *m*, n_b is the number of vertical baffle tubes, and μ_f is the viscosity at the mean film temperature.

Perry and Green (P3) give typical values of overall heat-transfer coefficients *U* for coils immersed in various liquids in agitated and nonagitated vessels.

Scraped-Surface Heat Exchangers

Liquid-solid suspensions, viscous aqueous and organic solutions, and numerous food products, such as margarine and orange juice concentrate, are often cooled or heated in a scraped-surface exchanger. This consists of a double-pipe heat exchanger with a jacketed cylinder containing steam or cooling liquid and an internal shaft rotating and fitted with wiper blades, as shown in Fig. 4.13-2.

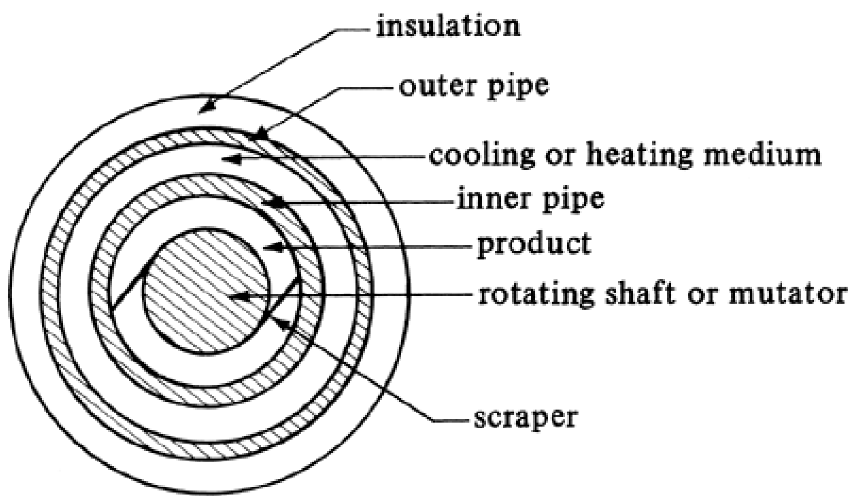


Figure 4.13-2. Scraped-surface heat exchanger.

The viscous liquid product flows at low velocity through the central tube between the rotating shaft and the inner pipe. The rotating scrapers or wiper blades continually scrape the surface of liquid, preventing localized overheating and giving rapid heat transfer. In some cases this device is also called a *votator heat exchanger*.

Skelland et al. (S4) give the following equation to predict the inside heat-transfer coefficient for the votator:

Equation 4.13-4.

$$\frac{hD}{k} = \alpha\left(\frac{c_p\mu}{k}\right)^{\beta}\left(\frac{(D-D_S)v\rho}{\mu}\right)^{1.0}\left(\frac{DN}{v}\right)^{0.62}\left(\frac{D_S}{D}\right)^{0.55}(n_B)^{0.53}$$

$$\alpha = 0.014 \quad \beta = 0.96 \qquad \text{for viscous liquids}$$

$$\alpha = 0.039 \quad \beta = 0.70 \qquad \text{for nonviscous liquids}$$

where D = diameter of vessel in m, D_S = diameter of rotating shaft in m, v = axial flow velocity of liquid in m/s, N = agitator speed in rev/s, and n_B = number of blades on agitator. Data cover a region of axial flow velocities of 0.076 to 0.38 m/min and rotational speeds of 100 to 750 rpm.

Typical overall heat-transfer coefficients in food applications are U = 1700 W/m^2 · K (300 btu/h · ft^2 · F) for cooling margarine with NH_3, 2270 (400) for heating applesauce with steam, 1420 (250) for chilling shortening with NH_3, and 2270 (400) for cooling cream with water (B6).

Extended Surface or Finned Exchangers

Introduction

The use of fins or extended surfaces on the outside of a heat-exchanger pipe wall to give relatively high heat-transfer coefficients in the exchanger is quite common. An automobile radiator is such a device, where hot water passes inside through a bank of tubes and loses heat to the air. On the outside of the tubes, extended surfaces receive heat from the tube walls and transmit it to the air by forced convection.

Two common types of fins attached to the outside of a tube wall are shown in Fig. 4.13-3. In Fig. 4.13-3a there are a number of longitudinal fins spaced around the tube wall and the direction of gas flow is parallel to the axis of the tube. In Fig. 4.13-3b the gas flows normal to the tubes containing many circular or transverse fins.

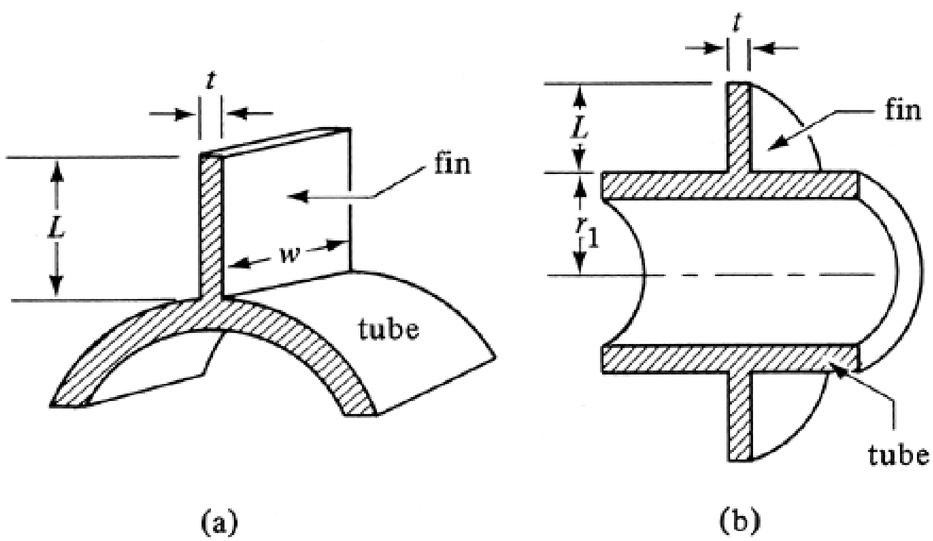


Figure 4.13-3. Two common types of fins on a section of circular tube: (a) longitudinal fin, (b) circular or transverse fin.

The qualitative effect of using extended surfaces can be shown approximately in Eq. (4.13-5) for a fluid inside a tube having a heat-transfer coefficient of h_i and an outside coefficient of h_0:

Equation 4.13-5.

$$\frac{1}{U_i A_i} = \sum R \simeq \frac{1}{h_i A_i} + R_{\text{metal}} + \frac{1}{h_o A_o}$$

The resistance R_{metal} of the wall can often be neglected. The presence of the fins on the outside increases A_0 and hence reduces the resistance $1/h_0A_0$ of the fluid on the outside of the tube. For example, if we have h_i for condensing steam, which is very large, and h_0 for air outside the tube, which is quite small, increasing A_0 greatly reduces $1/h_0A_0$. This in turn greatly reduces the total resistance, which increases the heat-transfer rate. If the positions of the two fluids are reversed, with air inside and steam outside, little increase in heat transfer could be obtained by using fins.

Equation (4.13-5) is only an approximation, since the temperature on the outside surface of the bare tube is not the same as that at the end of the fin because of the added resistance to heat flow by conduction from the fin tip to the base of the fin. Hence, a unit area of fin surface is not as efficient as a unit area of bare tube surface at the base of the fin. A fin efficiency η_f has been mathematically derived for various geometries of fins.

Derivation of equation for fin efficiency

We will consider a one-dimensional fin exposed to a surrounding fluid at temperature T_∞ as shown in Fig. 4.13-4. At the base of the fin the temperature is T_0 and at point x it is T. At steady state, the rate of heat conducted into the element at x is $q_{x|x}$ and is equal to the rate of heat conducted out plus the rate of heat lost by convection:

Equation 4.13-6.

$$q_{x|x} = q_{x|x+\Delta x} + q_c$$

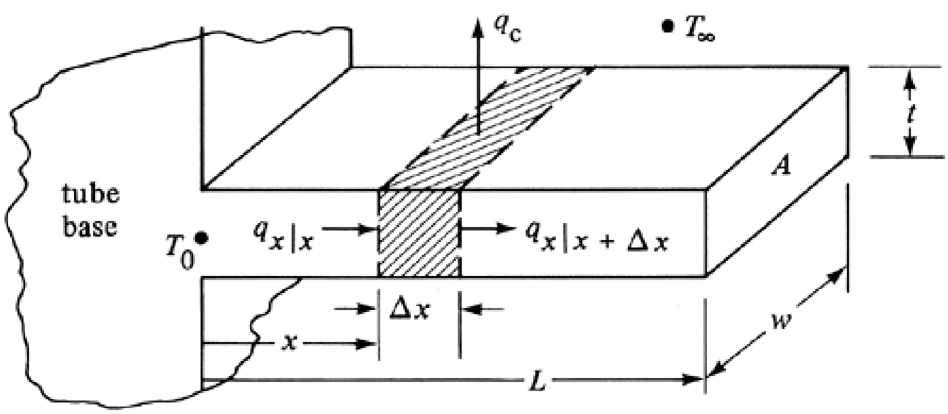


Figure 4.13-4. Heat balance for one-dimensional conduction and convection in a rectangular fin with constant cross-sectional area.

Substituting Fourier's equation for conduction and the convection equation,

Equation 4.13-7.

$$-kA \left.\frac{dT}{dx}\right|_x = -kA \left.\frac{dT}{dx}\right|_{x+\Delta x} + h(P\,\Delta x)(T - T_\infty)$$

where A is the cross-sectional area of the fin in m^2, P the perimeter of the fin in m, and ($P\Delta x$) the area for convection. Rearranging Eq. (4.13-7), dividing by Δx, and letting Δx approach zero,

Equation 4.13-8.

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0$$

Letting $\theta = T - T_\infty$, Eq. (4.13-8) becomes

Equation 4.13-9.

$$\frac{d^2\theta}{dx^2} = \frac{hP}{kA}\theta = 0$$

The first boundary condition is that $\theta = \theta_0 = T_0 - T_\infty$ at $x = 0$. For the second boundary condition needed to integrate Eq. (4.13-9), several cases can be considered, depending upon the physical conditions at $x = L$. In the first case, the end of the fin is insulated and $d\theta/dx = 0$ at $x = L$. In the second case, the fin loses heat by convection from the tip surface, so that $-k(dT/dx)_L = h(T_L - T_\infty)$. The solution using the second case is quite involved and will not be considered here. Using the first case, where the tip is insulated, integration of Eq. (4.13-9) gives

Equation 4.13-10.

$$\frac{\theta}{\theta_0} = \frac{\cosh[m(L - x)]}{\cosh mL}$$

where $m = (hP/kA)^{1/2}$.

The heat lost by the fin is expressed as

Equation 4.13-11.

$$q = -kA\left.\frac{dT}{dx}\right|_{x=0}$$

Differentiating Eq. (4.13-10) with respect to x and combining it with Eq. (4.13-11),

Equation 4.13-12.

$$q = (hPkA)^{1/2}(T_0 - T_\infty)\tanh mL$$

In the actual fin the temperature T in the fin decreases as the tip of the fin is approached. Hence, the rate of heat transfer per unit area decreases as the distance from the tube base is increased. To indicate this effectiveness of the fin in transferring heat, the fin efficiency η_f is defined as the ratio of the actual heat transferred from the fin to the heat transferred if the entire fin were at the base temperature T_0:

Equation 4.13-13.

$$\eta_f = \frac{(hPkA)^{1/2}(T_0 - T_\infty)\tanh mL}{h(PL)(T_0 - T_\infty)} = \frac{\tanh mL}{mL}$$

where PL is the entire surface area of fin. The expression for mL is

Equation 4.13-14.

$$mL = \left(\frac{hP}{kA}\right)^{1/2} L = \left[\frac{h(2w + 2t)}{k(wt)}\right]^{1/2} L$$

For fins which are thin, $2t$ is small compared to $2w$, and

Equation 4.13-15.

$$mL = \left(\frac{2h}{kt}\right)^{1/2} L$$

Equation (4.13-15) holds for a fin with an insulated tip. This equation can be modified to hold for the case where the fin loses heat from its tip. This can be done by extending the length of the fin by *t*/2, where the corrected length L_c to use in Eqs. (4.13-13)–(4.13-15) is

Equation 4.13-16.

$$L_c = L + \frac{t}{2}$$

The fin efficiency calculated from Eq. (4.13-13) for a longitudinal fin is shown in Fig. 4.13-5a. In Fig. 4.13-5b, the fin efficiency for a circular fin is presented. Note that the abscissa on the curves is $L_c(h/kt)^{1/2}$ and not $L_c(2h/kt)^{1/2}$ as in Eq. (4.13-15).

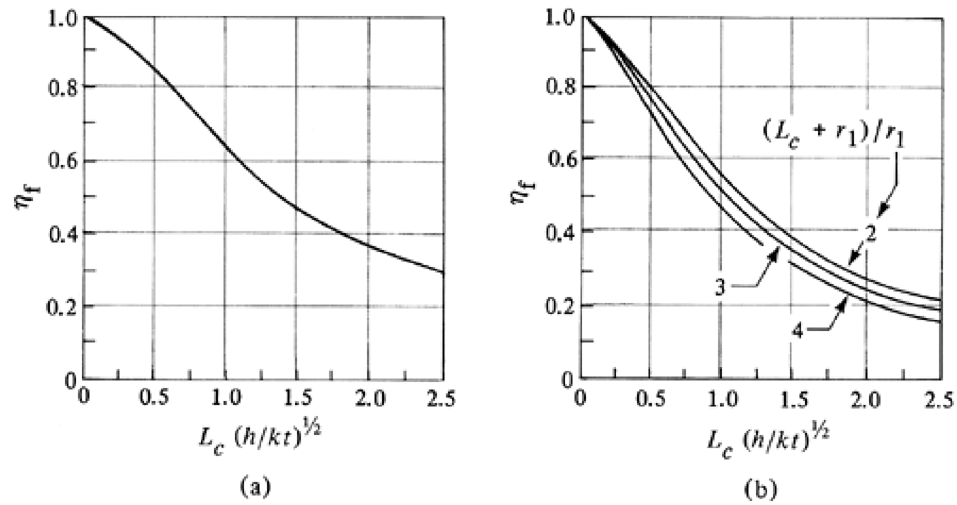


Figure 4.13-5. Fin efficiency η_f for various fins: (a) longitudinal or straight fins, (b) circular or transverse fins. (See Fig. 4.13-3 for the dimensions of the fins.)

EXAMPLE 4.13.-2. Fin Efficiency and Heat Loss from Fin

A circular aluminum fin as shown in Fig. 4.13-3b (k = 222 W/m · K) is attached to a copper tube having an outside radius of 0.04 m. The length of the fin is 0.04 m and the thickness is 2 mm. The outside wall or tube base is at 523.2 K and the external surrounding air at 343.2 K has a convective coefficient of 30 W/m^2 · K. Calculate the fin efficiency and rate of heat loss from the fin.

***Solution*:** The given data are T_0 = 523.2 K, T_∞ = 343.2 K, L = 0.04 m, r_1 = 0.04 m, t = 0.002 m, k = 222 W/m · K, h = 30 W/m^2 · K. By Eq. (4.13-16), $L_c = L + t/2$ = 0.040 + 0.002/2 = 0.041 m. Then,

$$L_c\left(\frac{h}{kt}\right)^{1/2} = (0.041)\left[\frac{30}{222(0.002)}\right]^{1/2} = 0.337$$

Also, $(L_c + r_1)/r_1$ = (0.041 + 0.040)/0.040 = 2.025. Using Fig. 4.13-5b, η_f = 0.89. The heat transfer from the fin itself is

Equation 4.13-17.

$$q_f = \eta_f h A_f (T_0 - T_\infty)$$

where A_f is the outside surface area (annulus) of the fin and is given by the following for both sides of the fin:

Equation 4.13-18.

$$A_f = 2\pi[(L_c + r_1)^2 - (r_1)^2] \qquad \text{(circular fin)}$$

$$A_f = 2\pi(L_c x w) \qquad \text{(longitudinal fin)}$$

Hence,

$$A_f = 2\pi[(0.041 + 0.040)^2 - (0.040)^2] = 3.118 \times 10^{-2}\,\text{m}^2$$

Substituting into Eq. (4.13-17),

$$q_f = 0.89(30)(3.118 \times 10^{-2})(523.2 - 343.2) = 149.9\,\text{W}$$

Overall heat-transfer coefficient for finned tubes

We consider here the general case similar to Fig. 4.3-3b, where heat transfer occurs from a fluid inside a cylinder or tube, through the cylinder metal wall A of thickness Δx_A, and then to the fluid outside the tube, where the tube has fins on the outside. The heat is transferred through a series of resistances. The total heat q leaving the outside of the tube is the sum of heat loss by convection from the base of the bare tube q_t and the loss by convection from the fins, q_f:

Equation 4.13-19.

$$q = q_t + q_f = h_o A_t (T_0 - T_\infty) + h_o A_f \eta_f (T_0 - T_\infty)$$

This can be written as a resistance since the paths are in parallel:

Equation 4.13-20.

$$q = (h_o A_t + h_o A_f \eta_f)(T_1 - T_\infty) = \frac{T_0 - T_\infty}{\dfrac{1}{h_o(A_t + A_f\eta_f)}} = \frac{T_0 - T_\infty}{R}$$

where A_t is the area of the bare tube between the fins, A_f the area of the fins, and h_0 the outside convective coefficient. The resistance in Eq. (4.3-20) can be substituted for the resistance ($1/h_0A_0$) in Eq. (4.3-15) for a bare tube to give the overall equation for a finned tube exchanger:

Equation 4.13-21.

$$q = \frac{T_4 - T_1}{1/h_i A_i + \Delta x_A / k_A A_{A\,\text{lm}} + 1/h_o(A_t + A_f\eta_f)} = \frac{T_4 - T_1}{\sum R}$$

where T_4 is the temperature of the fluid inside the tube and T_1 the outside fluid temperature. Writing Eq. (4.13-21) in the form of an overall heat-transfer coefficient U_i based on the inside area A_i, $q = U_iA_i(T_4 - T_1)$ and

Equation 4.13-22.

$$U_i = \frac{1}{1/h_i + \Delta x_A A_i / k_A A_{A\,\text{lm}} + A_i/h_o(A_t + A_f\eta_f)}$$

The presence of fins on the outside of the tube changes the characteristics of the fluid flowing past the tube (either flowing parallel to the longitudinal finned tube or transverse to the circular finned tube). Hence, the correlations for fluid flow parallel to or transverse to bare tubes cannot be used to predict the outside convective coefficient h_0. Correlations are available in the literature (K4, M1, P1, P3) for heat transfer to various types of fins.

DIMENSIONAL ANALYSIS IN HEAT TRANSFER

Introduction

As seen in many of the correlations for fluid flow and heat transfer, many dimensionless groups, such as the Reynolds number and Prandtl number, occur in these correlations. Dimensional analysis is often used to group the variables in a given physical situation into dimensionless parameters or numbers which can be useful in experimentation and correlating data.

An important way of obtaining these dimensionless groups is to use dimensional analysis of differential equations as described in Section 3.11. Another useful method is the Buckingham method, in which the listing of the significant variables in the particular physical problem is done first. Then we determine the number of dimensionless parameters into which the variables may be combined.

Buckingham Method

Heat transfer inside a pipe

The Buckingham theorem, given in Section 3.11, states that the function relationship among q quantities or variables whose units may be given in terms of u fundamental units or dimensions may be written as ($q-u$) dimensionless groups.

As an additional example to illustrate the use of this method, let us consider a fluid flowing in turbulent flow at velocity v inside a pipe of diameter D and undergoing heat transfer to the wall. We wish to predict the dimensionless groups relating the heat-transfer coefficient h to the variables D, ρ, μ, c_p, k, and v. The total number of variables is $q = 7$.

The fundamental units or dimensions are $u = 4$ and are mass M, length L, time t, and temperature T. The units of the variables in terms of these fundamental units are as follows:

$$h = \frac{M}{t^3 T} \qquad D = L \qquad \rho = \frac{M}{L^3} \qquad \mu = \frac{M}{Lt} \qquad c_p = \frac{L^2}{t^2 T} \qquad k = \frac{ML}{t^3 T} \qquad v = \frac{T}{t}$$

Hence, the number of dimensionless groups or π's can be assumed to be 7 – 4, or 3. Then

Equation 4.14-1.

$$\pi_1 = f(\pi_2, \pi_3)$$

We will choose the four variables D, k, μ, and v to be common to all the dimensionless groups. Then the three dimensionless groups are

Equation 4.14-2.

$$\pi_1 = D^a k^b \mu^c v^d \rho$$

Equation 4.14-3.

$$\pi_2 = D^e k^f \mu^g v^h c_p$$

Equation 4.14-4.

$$\pi_3 = D^i k^j \mu^k v^l h$$

For π_1, substituting the actual dimensions,

Equation 4.14-5.

$$M^0L^0t^0T^0 = 1 = L^a\left(\frac{ML}{t^3T}\right)^b\left(\frac{M}{LT}\right)^c\left(\frac{L}{T}\right)^d\left(\frac{M}{L^3}\right)$$

Summing for each exponent,

Equation 4.14-6.

$$\begin{aligned}
(L) \quad & 0 = a + b - c + d - 3 \\
(M) \quad & 0 = b + c + 1 \\
(t) \quad & 0 = -3b - c - d \\
(T) \quad & 0 = -b
\end{aligned}$$

Solving these equations simultaneously, $a = 1$, $b = 0$, $c = -1$, and $d = 1$.

Substituting these values into Eq. (4.14-2),

Equation 4.14-7.

$$\pi_1 = \frac{Dv\rho}{\mu} = N_{\mathrm{Re}}$$

Repeating for π_2 and π_3 and substituting the actual dimensions,

Equation 4.14-8.

$$\pi_2 = \frac{c_p\mu}{k} = N_{\mathrm{Pr}}$$

Equation 4.14-9.

$$\pi_3 = \frac{hD}{k} = N_{\mathrm{Nu}}$$

Substituting π_1, π_2, and π_3 into Eq. (4.14-1) and rearranging,

Equation 4.14-10.

$$\frac{hD}{k} = f\left(\frac{Dv\rho}{\mu}, \frac{c_p\mu}{k}\right)$$

This is in the form of the familiar equation for heat transfer inside pipes, Eq. (4.5-8).

This type of analysis is useful in empirical correlations of heat-transfer data. The importance of each dimensionless group, however, must be determined by experimentation (B1, M1).

Natural convection heat transfer outside a vertical plane

In the case of natural convection heat transfer from a vertical plane wall of length L to an adjacent fluid, different dimension-less groups should be expected as compared to forced convection inside a pipe, since velocity is not a variable. The buoyant force due to the difference in density between the cold and the heated fluid should be a factor. As seen in Eqs. (4.7-1) and (4.7-2), the buoyant force depends upon the variables β, g, ρ, and ΔT. Hence, the list of variables to be considered and their fundamental units are as follows:

$$L = L \qquad \rho = \frac{M}{L^3} \qquad \mu = \frac{M}{Lt} \qquad c_p = \frac{L^2}{t^2T} \qquad \beta = \frac{1}{T}$$

$$g = \frac{L}{t^2} \qquad \Delta T = T \qquad h = \frac{M}{t^3T} \qquad k = \frac{ML}{t^3T}$$

The number of variables is $q = 9$. Since $u = 4$, the number of dimensionless groups or π's is 9 - 4, or 5. Then $\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5)$.

We will choose the four variables $L, \mu, k,$ and g to be common to all the dimensionless groups:

$$\pi_1 = L^a\mu^b k^c g^d \rho \qquad \pi_2 = L^e\mu^f k^g g^h c_p \qquad \pi_3 = L^i\mu^j k^k g^l \beta$$

$$\pi_4 = L^m\mu^n k^o g^p\,\Delta T \qquad \pi_5 = L^q\mu^r k^s g^t h$$

For π_1, substituting the dimensions,

Equation 4.14-11.

$$1 = L^a\left(\frac{M}{Lt}\right)^b\left(\frac{ML}{t^3T}\right)^c\left(\frac{L}{t^2}\right)^d\left(\frac{M}{L^3}\right)$$

Solving for the exponents as before, $a = \frac{3}{2}$, $b = -1$, $c = 0$, and $d = \frac{1}{2}$. Then π_1 becomes

Equation 4.14-12.

$$\pi_1 = \frac{L^{3/2}\rho g^{1/2}}{\mu}$$

Taking the square of both sides to eliminate fractional exponents,

Equation 4.14-13.

$$\pi_1 = \frac{L^3\rho^2 g}{\mu^2}$$

Repeating for the other π equations,

$$\pi_1 = \frac{L^3\rho^2 g}{\mu^2} \qquad \pi_2 = \frac{c_p\mu}{k} = N_{\text{Pr}} \qquad \pi_3 = \frac{L\mu g\beta}{k}$$

$$\pi_4 = \frac{k\,\Delta T}{L\mu g} \qquad \pi_5 = \frac{hL}{k} = N_{\text{Nu}}$$

Combining the dimensionless groups π_1, π_3, and π_4 as follows,

Equation 4.14-14.

$$\pi_1\pi_3\pi_4 = \frac{L^3\rho^2 g}{\mu^2}\,\frac{L\mu g\beta}{k}\,\frac{k\,\Delta T}{L\mu g} = \frac{L^3\rho^2 g\beta\,\Delta T}{\mu^2} = N_{\text{Gr}}$$

Equation (4.14-14) is the Grashof group given in Eq. (4.7-4). Hence,

Equation 4.14-15.

$$N_{Nu} = f(N_{Gr}, N_{Pr})$$

NUMERICAL METHODS FOR STEADY-STATE CONDUCTION IN TWO DIMENSIONS

Analytical Equation for Conduction

In Section 4.4 we discussed methods for solving two-dimensional heat-conduction problems using graphical procedures and shape factors. In this section we consider analytical and numerical methods.

The equation for conduction in the *x* direction is as follows:

Equation 4.15-1.

$$q_x = -kA\frac{\partial T}{\partial x}$$

Now we shall derive an equation for steady-state conduction in two directions *x* and *y*. Referring to Fig. 4.15-1, a rectangular block Δx by Δy by *L* is shown. The total heat input to the block is equal to the output:

Equation 4.15-2.

$$q_{x|x} + q_{y|y} = q_{x|x+\Delta x} + q_{y|y+\Delta y}$$

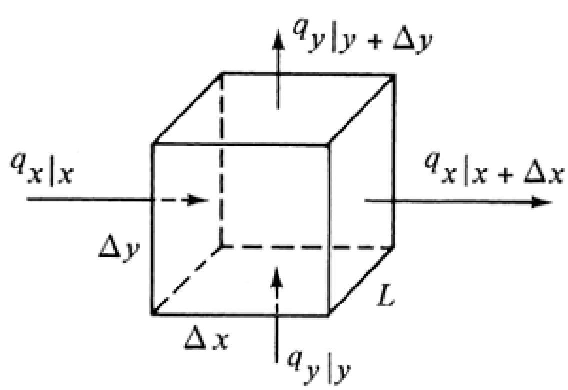


Figure 4.15-1. Steady-state conduction in two directions.

Now, from Eq. (4.15-1),

Equation 4.15-3.

$$q_{x|x} = -k(\Delta y L)\left.\frac{\partial T}{\partial x}\right|_x$$

Writing similar equations for the other three terms and substituting into Eq. (4.15-2),

Equation 4.15-4.

$$-k(\Delta yL)\left.\frac{\partial T}{\partial x}\right|_{x} - k(\Delta xL)\left.\frac{\partial T}{\partial y}\right|_{y} = -k(\Delta yL)\left.\frac{\partial T}{\partial x}\right|_{x+\Delta x} - k(\Delta xL)\left.\frac{\partial T}{\partial y}\right|_{y+\Delta y}$$

Dividing through by $\Delta x\,\Delta yL$ and letting Δx and Δy approach zero, we obtain the final equation for steady-state conduction in two directions:

Equation 4.15-5.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

This is called the *Laplace equation..* There are a number of analytical methods for solving this equation. In the method of separation of variables, the final solution is expressed as an infinite Fourier series (H1, G2, K1). We consider the case shown in Fig. 4.15-2. The solid is called a semi-infinite solid since one of its dimensions is ∞. The two edges or boundaries at $x = 0$ and $x = L$ are held constant at T_1 K. The edge at $y = 0$ is held at T_2. And at $y = \infty$, $T = T_1$. The solution relating T to position y and x is

Equation 4.15-6.

$$\frac{T - T_1}{T_2 - T_1} = \frac{4}{\pi}\left[\frac{1}{1}e^{-(\pi/L)y}\sin\frac{1\pi x}{L} + \frac{1}{3}e^{-(3\pi/L)y}\sin\frac{3\pi x}{L} + \cdots\right]$$

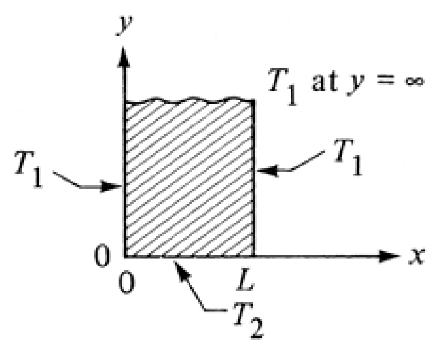


Figure 4.15-2. Steady-state heat conduction in two directions in a semi-infinite plate.

Other analytical methods are available and are discussed in many texts (C2, H1, G2, K1). A large number of such analytical solutions have been given in the literature. However, there are many practical situations where the geometry or boundary conditions are too complex for analytical solutions, so that finite-difference numerical methods are used. These are discussed in the next section.

Finite-Difference Numerical Methods

Derivation of the method

Since the advent of fast digital computers, solutions to many complex two-dimensional heat-conduction problems by numerical methods are readily possible. In deriving the equations we can start with the partial differential equation (4.15-5). Setting up the finite difference of $\partial^2 T/\partial x^2$,

Equation 4.15-7.

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial(\partial T/\partial x)}{\partial x} = \frac{\dfrac{T_{n+1,m} - T_{n,m}}{\Delta x} - \dfrac{T_{n,m} - T_{n-1,m}}{\Delta x}}{\Delta x}$$

$$= \frac{T_{n+1,m} - 2T_{n,m} + T_{n-1,m}}{(\Delta x)^2}$$

where the index *m* stands for a given value of *y*, *m* + 1 stands for *y* + 1 Δ*y*, and *n* is the index indicating the position of *T* on the *x* scale. This is shown in Fig. 4.15-3. The two-dimensional solid is divided into squares. The solid inside a square is imagined to be concentrated at the center of the square, and this concentrated mass is a "node." Each node is imagined to be connected to the adjacent nodes by a small conducting rod as shown.

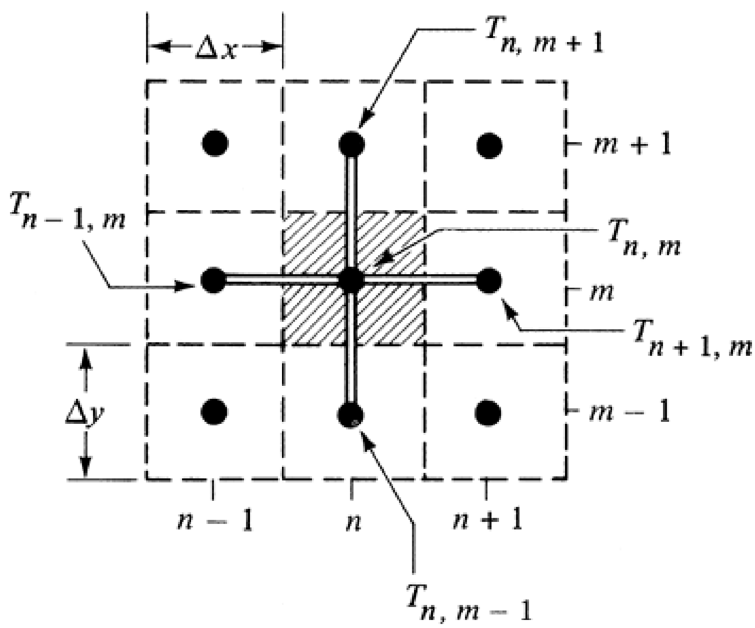


Figure 4.15-3. Temperatures and arrangement of nodes for two-dimensional steady-state heat conduction.

The finite difference of $\partial^2 T/\partial y^2$ is written in a similar manner:

Equation 4.15-8.

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{n,m+1} - 2T_{n,m} + T_{n,m-1}}{(\Delta y^2)}$$

Substituting Eqs. (4.15-7) and (4.15-8) into Eq. (4.15-5) and setting *Δx = Δy,*

Equation 4.15-9.

$$T_{n,m+1} + T_{n,m-1} + T_{n+1,m} + T_{n-1,m} - 4T_{n,m} = 0$$

This equation states that the net heat flow into any point or node is zero at steady state. The shaded area in Fig. 4.15-3 represents the area on which the heat balance was made. Alternatively, Eq. (4.15-9) can be derived by making a heat balance on this shaded area. The total heat in for unit thickness is

Equation 4.15-10.

$$\frac{k\,\Delta y}{\Delta x}(T_{n-1,m} - T_{n,m}) + \frac{k\,\Delta y}{\Delta x}(T_{n+1,m} - T_{n,m}) + \frac{k\,\Delta x}{\Delta y}(T_{n,m+1} - T_{n,m}) + \frac{k\,\Delta x}{\Delta y}(T_{n,m-1} - T_{n,m}) = 0$$

Rearranging, this becomes Eq. (4.15-9). In Fig. 4.15-3 the rods connecting the nodes act as fictitious heat-conducting rods.

To use the numerical method, Eq. (4.15-9) is written for each node or point. Hence, for *N* unknown nodes, *N* linear algebraic equations must be written and the system of equations solved for the various node temperatures. For a hand calculation using a modest number of nodes, the iteration method can be used to solve the system of equations.

Iteration method of solution

In using the iteration method, the right-hand side of Eq. (4.15-9) is set equal to a residual $\bar{q}_{n,m}$:

Equation 4.15-11.

$$\bar{q}_{n,m} = T_{n-1,m} + T_{n+1,m} + T_{n,m+1} + T_{n,m-1} - 4T_{n,m}$$

Since $\bar{q}_{n,m}$ = 0 at steady state, solving for $T_{n,m}$ in Eq. (4.15-11) or (4.15-9),

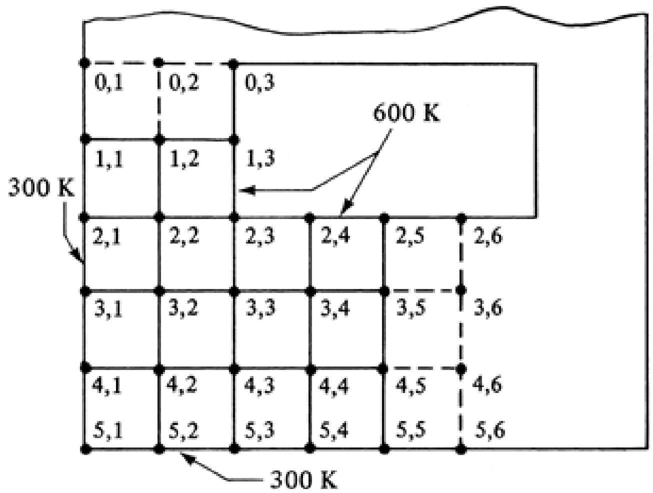
Equation 4.15-12.

$$T_{n,m} = \frac{T_{n-1,m} + T_{n+1,m} + T_{n,m+1} + T_{n,m-1}}{4}$$

Equations (4.15-11) and (4.15-12) are the final equations to be used. Their use is illustrated in the following example.

EXAMPLE 4.15-1. Steady-State Heat Conduction in Two Directions

Figure 4.15-4 shows a cross section of a hollow rectangular chamber with inside dimensions 4 × 2 m and outside dimensions 8 × 8 m. The chamber is 20 m long. The inside walls are held at 600 K and the outside at 300 K. The *k* is 1.5 W/m · K. For steady-state conditions find the heat loss per unit chamber length. Use grids 1 ×- 1 m.



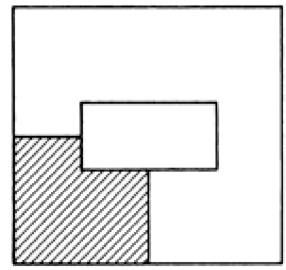


Figure 4.15-4. Square grid pattern for Example 4.15-1.

Solution: Since the chamber is symmetrical, one-fourth of the chamber (shaded part) will be used. Preliminary estimates will be made for the first approximation: $T_{1,2}$ = 450 K, $T_{2,2}$ = 400, $T_{3,2}$ = 400, $T_{3,3}$ = 400, $T_{3,4}$ = 450, $T_{3,5}$ = 500, $T_{4,2}$ = 325, $T_{4,3}$ = 350, $T_{4,4}$ = 375, and $T_{4,5}$ = 400. Note that $T_{0,2} = T_{2,2}$, $T_{3,6} = T_{3,4}$, and $T_{4,6} = T_{4,4}$ by symmetry.

To start the calculation, one can select any interior point, but it is usually better to start near a boundary. Using $T_{1,2}$, we calculate the residual $\bar{q}_{1,2}$ by Eq. (4.15-11):

$$\bar{q}_{1,2} = T_{1,1} + T_{1,3} + T_{0,2} + T_{2,2} - 4T_{1,2}$$

$$= 300 + 600 + 400 + 400 - 4(450) = -100$$

Hence, $T_{1,2}$ is not at steady state. Next, we set $\bar{q}_{1,2}$ to 0 and calculate a new value of $T_{1,2}$ by Eq. (4.15-12):

$$T_{1,2} = \frac{T_{1,1} + T_{1,3} + T_{0,2} + T_{2,2}}{4} = \frac{300 + 600 + 400 + 400}{4} = 425$$

This new value of $T_{1,2}$ of 425 K will replace the old one of 450 and be used to calculate the other nodes. Next,

$$\bar{q}_{2,2} = T_{2,1} + T_{2,3} + T_{1,2} + T_{3,2} - 4T_{2,2}$$

$$= 300 + 600 + 425 + 400 - 4(400) = 125$$

Setting $\bar{q}_{2,2}$ to zero and using Eq. (4.15-12),

$$T_{2,2} = \frac{T_{2,1} + T_{2,3} + T_{1,2} + T_{3,2}}{4} = \frac{300 + 600 + 425 + 400}{4} = 431$$

Continuing for all the rest of the interior nodes,

$$\bar{q}_{3,2} = 300 + 400 + 431 + 325 - 4(400) = -144$$

Using Eq. (4.15-12), $T_{3,2}$ = 364,

$$\bar{q}_{3,3} = 364 + 450 + 600 + 350 - 4(400) = 164$$

$$T_{3,3} = 441$$

$$\bar{q}_{3,4} = 441 + 500 + 600 + 375 - 4(450) = 116$$

$$T_{3,4} = 479$$

$$\bar{q}_{3,5} = 479 + 479 + 600 + 400 - 4(500) = -42$$

$$T_{3,5} = 489$$

$$\bar{q}_{4,2} = 300 + 350 + 364 + 300 - 4(325) = 14$$

$$T_{4,2} = 329$$

$$\bar{q}_{4,3} = 329 + 375 + 441 + 300 - 4(350) = 45$$

$$T_{4,3} = 361$$

$$\bar{q}_{4,4} = 361 + 400 + 479 + 300 - 4(375) = 40$$

$$T_{4,4} = 385$$

$$\bar{q}_{4,5} = 385 + 385 + 489 + 399 - 4(400) = -41$$

$$T_{4,5} = 390$$

Having completed one sweep across the grid map, we can start a second approximation, using, of course, the new values calculated. We can start again with $T_{1,2}$ or we can select the node with the largest residual. Starting with $T_{1,2}$ again,

$$\bar{q}_{1,2} = 300 + 600 + 431 + 431 - 4(425) = 62$$

$$T_{1,2} = 440$$

$$\bar{q}_{2,2} = 300 + 600 + 440 + 364 - 4(431) = -20$$

$$T_{2,2} = 426$$

This is continued until the residuals are as small as desired. The final values are as follows:

$$T_{1,2} = 441, \quad T_{2,2} = 432, \quad T_{3,2} = 384, \quad T_{3,3} = 461, \quad T_{3,4} = 485,$$

$$T_{3,5} = 490, \quad T_{4,2} = 340, \quad T_{4,3} = 372, \quad T_{4,4} = 387, \quad T_{4,5} = 391$$

To calculate the total heat loss from the chamber per unit chamber length, we use Fig. 4.15-5. For nodes $T_{2,4}$ to $T_{3,4}$ with $\Delta x = \Delta y$ and 1 m deep,

Equation 4.15-13.

$$q = \frac{kA\ \Delta T}{\Delta x} = \frac{k[\Delta x(1)]}{\Delta x}(T_{2,4} - T_{3,4}) = k(T_{2,4} - T_{3,4})$$

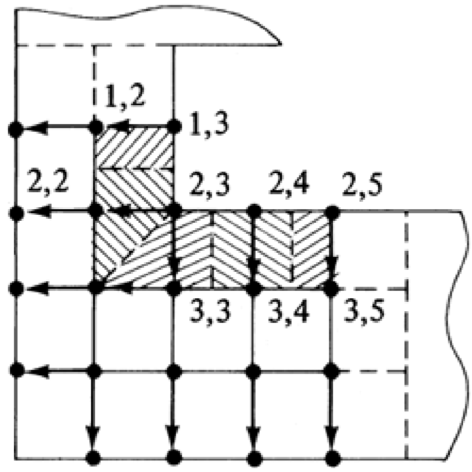


Figure 4.15-5. Drawing for calculation of total heat conduction.

The heat flux for nodes $T_{2,5}$ to $T_{3,5}$ and for $T_{1,3}$ to $T_{1,2}$ should be multiplied by $\frac{1}{2}$ because of symmetry. The total heat conducted is the sum of the five paths for one-fourth of the solid. For four duplicate parts,

Equation 4.15-14.

$$q_I = 4k[\tfrac{1}{2}(T_{1,3} - T_{1,2}) + (T_{2,3} - T_{2,2}) + (T_{2,3} - T_{3,3}) + (T_{2,4} - T_{3,4}) + \tfrac{1}{2}(T_{2,5} - T_{3,5})]$$
$$= 4(1.5)[\tfrac{1}{2}(600 - 441) + (600 - 432) + (600 - 461) + (600 - 485) + \tfrac{1}{2}(600 - 490)]$$
$$= 3340 \text{ W per 1.0 m deep}$$

Also, the total heat conducted can be calculated using the nodes at the outside, as shown in Fig. 4.15-5. This gives q_{II} = 3430 W. The average value is

$$q_{av} = \frac{3340 + 3430}{2} = 3385 \text{ W per 1.0 m deep}$$

If a larger number of nodes, that is, a smaller grid size, is used, a more accurate solution can be obtained. Using a grid size of 0.5 m instead of 1.0 m for Example 4.15-1, a q_{av} of 3250 W is obtained. If a very fine grid is used, more accuracy can be obtained, but a digital computer would be needed for the large number of calculations. Matrix methods are also available for solving a set of simultaneous equations on a computer. The iteration method used here is often called the Gauss-Seidel method. The simplest and most convenient method is to use a spreadsheet calculation with a computer. This avoids the complication of using complex matrix methods and so forth.

Equations for other boundary conditions

In Example 4.15-1 the conditions at the boundaries were such that the node points were known and constant. For the case where there is convection at the boundary to a constant temperature T_∞, a heat balance on the node *n, m* in Fig. 4.15-6a is as follows, where heat in = heat out (K1):

Equation 4.15-15.

$$\frac{k\,\Delta y}{\Delta x}(T_{n-1,m} - T_{n,m}) + \frac{k\,\Delta x}{2\,\Delta y}(T_{n,m+1} - T_{n,m}) + \frac{k\,\Delta x}{2\,\Delta y}(T_{n,m-1} - T_{n,m}) = h\,\Delta y(T_{n,m} - T_\infty)$$

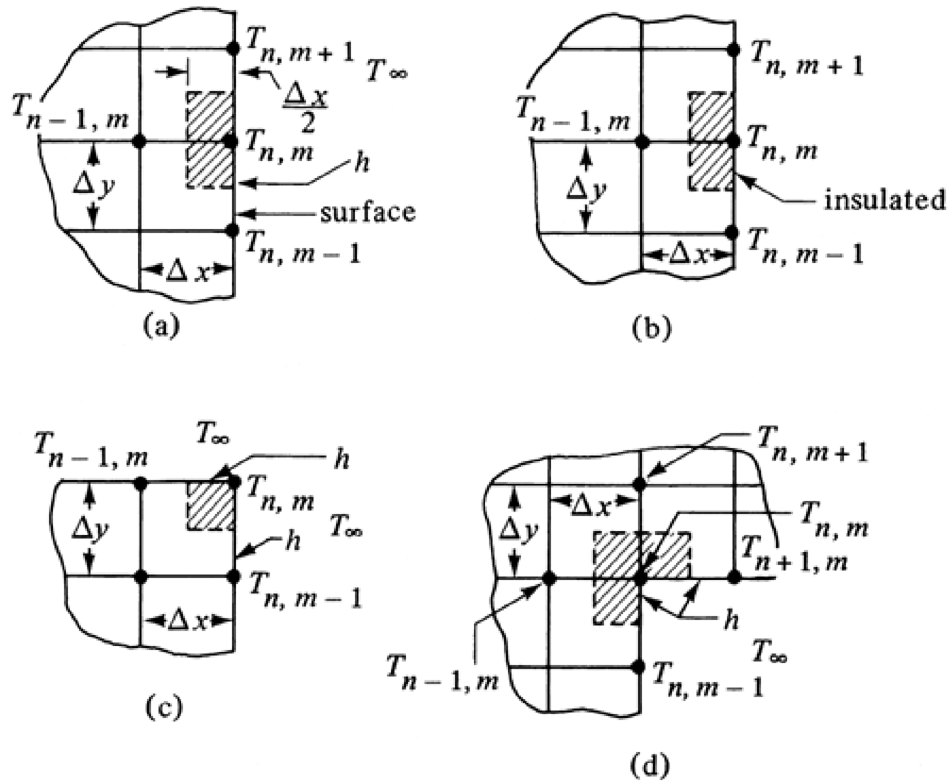


Figure 4.15-6. Other types of boundary conditions: (a) convection at a boundary, (b) insulated boundary, (c) exterior corner with convective boundary, (d) interior corner with convective boundary.

Setting $\Delta x = \Delta y$, rearranging, and setting the resultant equation = $\bar{q}_{n,m}$ residual, the following results:

a. For convection at a boundary,

Equation 4.15-16.

$$\frac{h\,\Delta x}{k}T_\infty + \tfrac{1}{2}(2T_{n-1,m} + T_{n,m+1} + T_{n,m-1}) - T_{n,m}\left(\frac{h\,\Delta x}{k} + 2\right) = \bar{q}_{n,m}$$

In a similar manner, for the cases in Fig. 4.15-6:

b. For an insulated boundary,

Equation 4.15-17.

$$\tfrac{1}{2}(T_{n,m+1} + T_{n,m-1}) + T_{n-1,m} - 2T_{n,m} = \bar{q}_{n,m}$$

c. For an exterior corner with convection at the boundary,

Equation 4.15-18.

$$\frac{h\,\Delta x}{k}T_\infty + \tfrac{1}{2}(T_{n-1,m} + T_{n,m-1}) - \left(\frac{h\,\Delta x}{k} + 1\right)T_{n,m} = \bar{q}_{n,m}$$

d. For an interior corner with convection at the boundary,

Equation 4.15-19.

$$\frac{h\,\Delta x}{k}T_\infty + T_{n-1,m} + T_{n,m+1} + \tfrac{1}{2}(T_{n+1,m} + T_{n,m-1}) - \left(3 + \frac{h\,\Delta x}{k}\right)T_{n,m} = \bar{q}_{n,m}$$

For curved boundaries and other types of boundaries, see (C3, K1). To use Eqs. (4.15-16)–(4.15-19), the residual $\bar{q}_{n,m}$ is first obtained using the proper equation. Then $\bar{q}_{n,m}$ is set equal to zero and $T_{n,m}$ solved for in the resultant equation.

PROBLEMS

4.1-1. ***Insulation in a Cold Room.*** Calculate the heat loss per m^2 of surface area for a temporary insulating wall of a food cold storage room where the outside temperature is 299.9 K and the inside temperature 276.5 K. The wall is composed of 25.4 mm of corkboard having a k of 0.0433 W/m · K.

A1: **Ans.** 39.9 W/m^2

4.1-2. ***Determination of Thermal Conductivity.*** In determining the thermal conductivity of an insulating material, the temperatures were measured on both sides of a flat slab of 25 mm of the material and were 318.4 and 303.2 K. The heat flux was measured as 35.1 W/m^2. Calculate the thermal conductivity in btu/h · ft · °F and in W/m · K.

4.2-1. ***Mean Thermal Conductivity in a Cylinder.*** Prove that if the thermal conductivity varies linearly with temperature as in Eq. (4.1-11), the proper mean value k_m to use in the cylindrical equation is given by Eq. (4.2-3) as in a slab.

4.2-2. ***Heat Removal of a Cooling Coil.*** A cooling coil of 1.0 ft of 304 stainless-steel tubing having an inside diameter of 0.25 in. and an outside diameter of 0.40 in. is being used to remove heat from a bath. The temperature at the inside surface of the tube is 40°F and is 80°F on the outside. The thermal conductivity of 304 stainless steel is a function of temperature:

$$k = 7.75 + 7.78 \times 10^{-3}T$$

where k is in btu/h · ft · °F and T is in °F. Calculate the heat removal in btu/s and watts.

A4: **Ans.** 1.225 btu/s, 1292 W

4.2-3. ***Removal of Heat from a Bath.*** Repeat Problem 4.2-2 but for a cooling coil made of 308 stainless steel having an average thermal conductivity of 15.23 W/m · K.

4.2-4. ***Variation of Thermal Conductivity.*** A flat plane of thickness Δx has one surface maintained at T_1 and the other at T_2. If the thermal conductivity varies according to temperature as

$$k = A + bT + cT^3$$

where *a, b,* and *c* are constants, derive an expression for the one-dimensional heat flux *q/A.*

4.2-5. ***Temperature Distribution in a Hollow Sphere.*** Derive Eq. (4.2-14) for the steady-state conduction of heat in a hollow sphere. Also, derive an equation which shows that the temperature varies hyperbolically with the radius *r.*

A7: Ans. $$\frac{T - T_1}{T_2 - T_1} = \frac{r_2}{r_2 - r_1}\left(1 - \frac{r_1}{r}\right)$$

4.3-1. ***Insulation Needed for Food Cold Storage Room.*** A food cold storage room is to be constructed of an inner layer of 19.1 mm of pine wood, a middle layer of cork board, and an outer layer of 50.8 mm of concrete. The inside wall surface temperature is -17.8°C and the outside surface temperature is 29.4°C at the outer concrete surface. The mean conductivities are for pine, 0.151; cork, 0.0433; and concrete, 0.762 W/m · K. The total inside surface area of the room to use in the calculation is approximately 39 m^2 (neglecting corner and end effects). What thickness of cork board is needed to keep the heat loss to 586 W?

A8: **Ans.** 0.128 m thickness

4.3-2. ***Insulation of a Furnace.*** A wall of a furnace 0.244 m thick is constructed of material having a thermal conductivity of 1.30 W/m · K. The wall will be insulated on the outside with material having an average *k* of 0.346 W/m · K, so the heat loss from the furnace will be equal to or less than 1830 W/m^2. The inner surface temperature is 1588 K and the outer 299 K. Calculate the thickness of insulation required.

A9: **Ans.** 0.179 m

4.3-3. ***Heat Loss Through Thermopane Double Window.*** A double window called ther-mopane is one in which two layers of glass are separated by a layer of dry, stagnant air. In a given window, each of the glass layers is 6.35 mm thick separated by a 6.35-mm space of stagnant air. The thermal conductivity of the glass is 0.869 W/m · K and that of air is 0.026 over the temperature range used. For a temperature drop of 27.8 K over the system, calculate the heat loss for a window 0.914 m X 1.83 m. *(Note:* This calculation neglects the effect of the convective coefficient on one outside surface of one side of the window, the convective coefficient on the other outside surface, and convection inside the window.)

4.3-4. ***Heat Loss from Steam Pipeline.*** A steel pipeline, 2-in. schedule 40 pipe, contains saturated steam at 121.1°C. The line is covered with 25.4 mm of insulation. Assuming that the inside surface temperature of the metal wall is at 121.1°C and the outer surface of the insulation is at 26.7°C, calculate the heat loss for 30.5 m of pipe. Also, calculate the kg of steam condensed per hour in the pipe due to the heat loss. The average k for steel from Appendix A.3 is 45 W/m · K and the k for the insulation is 0.182.

A11: **Ans.** 5384 W, 8.81 kg steam/h

4.3-5. ***Heat Loss with Trial-and-Error Solution.*** The exhaust duct from a heater has an inside diameter of 114.3 mm with ceramic walls 6.4 mm thick. The average k = 1.52 W/m · K. Outside this wall, an insulation of rock wool 102 mm thick is installed. The thermal conductivity of the rock wool is $k = 0.046 + 1.56 \times 10^{-4}\ T$°C (W/m · K). The inside surface temperature of the ceramic is T_1 = 588.7 K, and the outside surface temperature of the insulation is T_3 = 311 K. Calculate the heat loss for 1.5 m of duct and the interface temperature T_2 between the ceramic and the insulation. *[Hint:* The correct value of k_m for the insulation is that evaluated at the mean temperature of $(T_2 + T_3)/2$. Hence, for the first trial assume a mean temperature of, say, 448 K. Then calculate the heat loss and T_2. Using this new T_2, calculate a new mean temperature and proceed as before.]

4.3-6. ***Heat Loss by Convection and Conduction.*** A glass window with an area of 0.557 m^2 is installed in the wooden outside wall of a room. The wall dimensions are 2.44 × 3.05 m. The wood has a k of 0.1505 W/m · K and is 25.4 mm thick. The glass is 3.18 mm thick and has a k of 0.692. The inside room temperature is 299.9 K (26.7°C) and the outside air temperature is 266.5 K. The convection coefficient h_i on the inside wall of the glass and the wood is estimated as 8.5 W/m^2 · K; the outside h_0 is also estimated as 8.5 for both surfaces. Calculate the heat loss through the wooden wall, through the glass, and the total.

A13: **Ans.** 569.2 W (wood) (1942 btu/h), 77.6 W (glass) (265 btu/h), 646.8 W (total) (2207 btu/h)

4.3-7. ***Convection, Conduction, and Overall U.*** A gas at 450 K is flowing inside a 2-in. steel pipe, schedule 40. The pipe is insulated with 51 mm of lagging having a mean k = 0.0623 W/m · K. The convective heat-transfer coefficient of the gas inside the pipe is 30.7 W/m^2 · K and the convective coefficient on the outside of the lagging is 10.8. The air is at a temperature of 300 K.

a. Calculate the heat loss per unit length of 1 m of pipe using resistances.
b. Repeat, using the overall U_0 based on the outside area A_0.

4.3-8. ***Heat Transfer in Steam Heater.*** Water at an average of 70°F is flowing in a 2-in. steel pipe, schedule 40. Steam at 220°F is condensing on the outside of the pipe. The convective coefficient for the water inside the pipe is h = 500 btu/h · ft^2 · °F and the condensing steam coefficient on the outside is h = 1500.

a. Calculate the heat loss per unit length of 1 ft of pipe using resistances.
b. Repeat, using the overall U_i based on the inside area A_i.
c. Repeat, using U_o.

A15: **Ans.** (a) q = 26 710 btu/h (7.828 kW), (b) U_i = 329.1 btu/h · ft^2 · °F (1869 W/m^2 · K), (c) U_o = 286.4 btu/h · ft^2 · °F (1626 W/m^2 · K)

4.3-9. ***Heat Loss from Temperature Measurements.*** A steel pipe carrying steam has an outside diameter of 89 mm. It is lagged with 76 mm of insulation having an average k = 0.043 W/m · K. Two thermocouples, one located at the interface between the pipe wall and the insulation and the other at the outer surface of the insulation, give temperatures of 115°C and 32°C, respectively. Calculate the heat loss in W per m of pipe.

4.3-10. ***Effect of Convective Coefficients on Heat Loss in Double Window.*** Repeat Problem 4.3-3 for heat loss in the double window. However, include a convective coefficient of h = 11.35 W/m^2 · K on one outside surface of one side of the window and an h of 11.35 on the other outside surface. Also calculate the overall U.

A17: **Ans.** q = 106.7 W, U = 2.29 W/m^2 · K

4.3-11. ***Uniform Chemical Heat Generation.*** Heat is being generated uniformly by a chemical reaction in a long cylinder of radius 91.4 mm. The generation rate is constant at 46.6 W/m^3. The walls of the cylinder are cooled so that the wall temperature is held at 311.0 K. The thermal conductivity is 0.865 W/m · K. Calculate the center-line temperature at steady state.

A18: **Ans.** T_o = 311.112 K

4.3-12. ***Heat of Respiration of a Food Product.*** A fresh food product is held in cold storage at 278.0 K. It is packed in a container in the shape of a flat slab with all faces insulated except for the top flat surface, which is exposed to the air at 278.0 K. For estimation purposes the surface temperature will be assumed to be 278 K. The slab is 152.4 mm thick and the exposed surface area is 0.186 m^2. The density of the foodstuff is 641 kg/m^3. The heat of respiration is 0.070 kJ/kg · h and the thermal conductivity is 0.346 W/m · K. Calculate the maximum temperature in the food product at steady state and the total heat given off in W. *(Note:* It is assumed in this problem that there is no air circulation inside the foodstuff. Hence, the results will be conservative, since circulation during respiration will reduce the temperature.)

A19: **Ans.** 278.42 K, 0.353 W (1.22 btu/h)

4.3-13. ***Temperature Rise in Heating Wire.*** A current of 250 A is passing through a stainless-steel wire having a diameter of 5.08 mm. The wire is 2.44 m long and has a resistance of 0.0843 Ω. The outer surface is held constant at 427.6 K. The thermal conductivity is k = 22.5 W/m · K. Calculate the center-line temperature at steady state.

4.3-14. ***Critical Radius for Insulation.*** A metal steam pipe having an outside diameter of 30 mm has a surface temperature of 400 K and is to be insulated with an insulation having a thickness of 20 mm and a k of 0.08 W/m · K. The pipe is exposed to air at 300 K and a convection coefficient of 30 W/m^2 · K.

a. Calculate the critical radius and the heat loss per m of length for the bare pipe.

b. Calculate the heat loss for the insulated pipe assuming that the surface temperature of the pipe remains constant.

A21: **Ans.** (b) $q = 54.4$ W

4.4-1. ***Curvilinear-Squares Graphical Method.*** Repeat Example 4.4-1 but with the following changes:

a. Select the number of equal temperature subdivisions between the isothermal boundaries to be five instead of four. Draw in the curvilinear squares and determine the total heat flux. Also calculate the shape factor *S*. Label each isotherm with the actual temperature.

b. Repeat part (a), but in this case the thermal conductivity is not constant but $k = 0.85\ (1 + 0.00040\ T)$, where T is temperature in K. [*Note:* To calculate the overall q, the mean value of k at the mean temperature is used. The spacing of the isotherms is independent of how k varies with T (M1). However, the temperatures corresponding to the individual isotherms are a function of how the value of k depends upon T. Write the equation for q' for a given curvilinear section using the mean value of k over the temperature interval. Equate this to the overall value of q divided by M or q/M. Then solve for the isotherm temperature.]

4.4-2. ***Heat Loss from a Furnace.*** A rectangular furnace with inside dimensions of 1.0 × 1.0 × 2.0 m has a wall thickness of 0.20 m. The k of the walls is 0.95 W/m · K. The inside of the furnace is held at 800 K and the outside at 350 K. Calculate the total heat loss from the furnace.

A23: **Ans.** $q = 25\ 171$ W

4.4-3. ***Heat Loss from a Buried Pipe.*** A water pipe whose wall temperature is 300 K has a diameter of 150 mm and a length of 10 m. It is buried horizontally in the ground at a depth of 0.40 m measured to the center line of the pipe. The ground surface temperature is 280 K and $k = 0.85$ W/m · K. Calculate the loss of heat from the pipe.

A24: **Ans.** $q = 451.2$ W

4.5-1. ***Heating Air by Condensing Steam.*** Air is flowing through a tube having an inside diameter of 38.1 mm at a velocity of 6.71 m/s, average temperature of 449.9 K, and pressure of 138 kPa. The inside wall temperature is held constant at 204.4°C (477.6 K) by steam condensing outside the tube wall. Calculate the heat-transfer coefficient for a long tube and the heat-transfer flux.

A25: **Ans.** $h = 39.38$ W/m^2 · K (6.94 btu/h · ft^2 · °F)

4.5-2. ***Trial-and-Error Solution for Heating Water.*** Water is flowing inside a horizontal $1\frac{1}{4}$-in. schedule 40 steel pipe at 37.8°C and a velocity of 1.52 m/s. Steam at 108.3°C is condensing on the outside of the pipe wall and the steam coefficient is assumed constant at 9100 W/m^2 · K.

a. Calculate the convective coefficient h_i for the water. (Note that this is trial and error. A wall temperature on the inside must be assumed first.)

b. Calculate the overall coefficient U_i based on the inside area and the heat-transfer flux q/A_i in W/m^2.

4.5-3. ***Heat-Transfer Area and Use of Log Mean Temperature Difference.*** A reaction mixture having a c_{pm} = 2.85 kJ/kg · K is flowing at a rate of 7260 kg/h and is to be cooled from 377.6 K to 344.3 K. Cooling water at 288.8 K is available and the flow rate is 4536 kg/h. The overall U_o is 653 W/m^2 · K.

a. For counterflow, calculate the outlet water temperature and the area A_o of the exchanger.

b. Repeat for cocurrent flow.

A27: **Ans.** (a) T_1 = 325.2 K, A_o = 5.43 m^2; (b) A_o = 6.46 m^2

4.5-4. ***Heating Water with Hot Gases and Heat-Transfer Area.*** Water flowing at the rate of 13.85 kg/s is to be heated from 54.5 to 87.8°C in a heat exchanger by 54 430 kg/h of hot gas flowing counterflow and entering at 427°C (c_{pm} = 1.005 kJ/kg · K). The overall U_o = 69.1 W/m^2 · K. Calculate the exit-gas temperature and the heat-transfer area.

A28: **Ans.** T = 299.5°C

4.5-5. ***Cooling Oil and Overall U.*** Oil flowing at the rate of 7258 kg/h with a c_{pm} = 2.01 kJ/kg · K is cooled from 394.3 K to 338.9 K in a counterflow heat exchanger by water entering at 294.3 K and leaving at 305.4 K. Calculate the flow rate of the water and the overall U_i if the A_i is 5.11 m^2.

A29: **Ans.** 17 420 kg/h, U_i = 686 W/m^2 · K

4.5-6. ***Laminar Flow and Heating of Oil.*** A hydrocarbon oil having the same physical properties as the oil in Example 4.5-5 enters at 175°F inside a pipe having an inside diameter of 0.0303 ft and a length of 15 ft. The inside pipe surface temperature is constant at 325°F. The oil is to be heated to 250°F in the pipe. How many lb$_m$/h oil can be heated? *(Hint:* This solution is trial and error. One method is to assume a flow rate of, say, m = 75 lb mass/h. Calculate the N_{Re} and the value of h_a. Then make a heat balance to solve for q in terms of m. Equate this q to the q from the equation $q = h_a A \Delta T_a$. Solve for m. This is the new m to use for the second trial.)

A30: **Ans.** m = 84.2 lb$_m$/h (38.2 kg/h)

4.5-7. ***Heating Air by Condensing Steam.*** Air at a pressure of 101.3 kPa and 288.8 K enters inside a tube having an inside diameter of 12.7 mm and a length of 1.52 m with a velocity of 24.4 m/s. Condensing steam on the outside of the tube maintains the inside wall temperature at 372.1 K. Calculate the convection coefficient of the air. *(Note:* This solution is trial and error. First assume an outlet temperature of the air.)

4.5-8. ***Heat Transfer with a Liquid Metal.*** The liquid metal bismuth at a flow rate of 2.00 kg/s enters a tube having an inside diameter of 35 mm at 425°C and is heated to 430°C in the tube. The tube wall is maintained at a temperature of 25°C above the liquid bulk temperature. Calculate the tube length required. The physical properties are as follows (H1): k = 15.6 W/m · K, c_p = 149 J/kg · K, μ = 1.34 × 10^{-3} Pa · s.

4.6-1. ***Heat Transfer from a Flat Plate.*** Air at a pressure of 101.3 kPa and a temperature of 288.8 K is flowing over a thin, smooth, flat plate at 3.05 m/s. The plate length in the direction of flow is 0.305 m and its temperature is 333.2 K. Calculate the heat-transfer coefficient assuming laminar flow.

A33: **Ans.** h = 12.35 W/m^2 · K (2.18 btu/h · ft^2 · °F)

4.6-2. ***Chilling Frozen Meat.*** Cold air at -28.9°C and 1 atm is recirculated at a velocity of 0.61 m/s over the exposed top flat surface of a piece of frozen meat. The sides and bottom of this rectangular slab of meat are insulated and the top surface is 254 mm by 254 mm square. If the surface of the meat is at -6.7°C, predict the average heat-transfer coefficient to the surface. As an approximation, assume that either Eq. (4.6-2) or (4.6-3) can be used, depending on the $N_{\mathrm{Re},L}$.

A34: **Ans.** h = 6.05 W/m^2 · K

4.6-3. ***Heat Transfer to an Apple.*** It is desired to predict the heat-transfer coefficient for air being blown past an apple lying on a screen with large openings. The air velocity is 0.61 m/s at 101.32 kPa pressure and 316.5 K. The surface of the apple is at 277.6 K and its average diameter is 114 mm. Assume that it is a sphere.

4.6-4. ***Heating Air by a Steam Heater.*** A total of 13 610 kg/h of air at 1 atm abs pressure and 15.6°C is to be heated by passing over a bank of tubes in which steam at 100°C is condensing. The tubes are 12.7 mm OD, 0.61 m long, and arranged in-line in a square pattern with $S_p = S_n$ = 19.05 mm. The bank of tubes contains six transverse rows in the direction of flow and 19 rows normal to the flow. Assume that the tube surface temperature is constant at 93.33°C. Calculate the outlet air temperature.

4.7-1. ***Natural Convection from an Oven Wall.*** The oven wall in Example 4.7-1 is insulated so that the surface temperature is 366.5 K instead of 505.4 K. Calculate the natural convection heat-transfer coefficient and the heat-transfer rate per m of width. Use both Eq. (4.7-4) and the simplified equation. *(Note:* Radiation is being neglected in this calculation.) Use both SI and English units.

4.7-2. ***Losses by Natural Convection from a Cylinder.*** A vertical cylinder 76.2 mm in diameter and 121.9 mm high is maintained at 397.1 K at its surface. It loses heat by natural convection to air at 294.3 K. Heat is lost from the cylindrical side and the flat circular end at the top. Calculate the heat loss neglecting radiation losses. Use the simplified equations of Table 4.7-2 and those equations for the lowest range of $N_{\mathrm{Gr}} N_{\mathrm{Pr}}$. The equivalent L to use for the top flat surface is 0.9 times the diameter.

A38: **Ans.** q = 26.0 W

4.7-3. ***Heat Loss from a Horizontal Tube.*** A horizontal tube carrying hot water has a surface temperature of 355.4 K and an outside diameter of 25.4 mm. The tube is exposed to room air at 294.3 K. What is the natural convection heat loss for a 1-m length of pipe?

4.7-4. ***Natural Convection Cooling of an Orange.*** An orange 102 mm in diameter having a surface temperature of 21.1°C is placed on an open shelf in a refrigerator held at 4.4°C. Calculate the heat loss by natural convection, neglecting radiation. As an approximation, the simplified equation for vertical planes can be used with L replaced by the radius of the sphere (M1). For a more accurate correlation, see (S2).

4.7-5. ***Natural Convection in Enclosed Horizontal Space.*** Repeat Example 4.7-3 but for the case where the two plates are horizontal and the bottom plate is hotter than the upper plate. Compare the results.

A41: **Ans.** q = 18.64 W

4.7-6. ***Natural Convection Heat Loss in Double Window.*** A vertical double plate-glass window has an enclosed air-gap space of 10 mm. The window is 2.0 m high by 1.2 m wide. One window surface is at 25°C and the other at 10°C. Calculate the free convection heat-transfer rate through the air gap.

4.7-7. ***Natural Convection Heat Loss for Water in Vertical Plates.*** Two vertical square metal plates having dimensions of 0.40 X 0.40 m are separated by a gap of 12 mm and this enclosed space is filled with water. The average surface temperature of one plate is 65.6°C and the other plate is at 37.8°C. Calculate the heat-transfer rate through this gap.

4.7-8. ***Heat Loss from a Furnace.*** Two horizontal metal plates having dimensions of 0.8 X 1.0 m comprise the top of a furnace and are separated by a distance of 15 mm. The lower plate is at 400°C and the upper at 100°C, and air at 1 atm abs is enclosed in the gap. Calculate the heat-transfer rate between the plates.

4.8-1. ***Boiling Coefficient in a Jacketed Kettle.*** Predict the boiling heat-transfer coefficient for the vertical jacketed sides of the kettle given in Example 4.8-1. Then, using this coefficient for the sides and the coefficient from Example 4.8-1 for the bottom, predict the total heat transfer.

A45: **Ans.** T_w = 107.65°C, ΔT = 7.65 K, and h(vertical) = 3560 $W/m^2 \cdot K$

4.8-2. ***Boiling Coefficient on a Horizontal Tube.*** Predict the boiling heat-transfer coefficient for water under pressure boiling at 250°F for a horizontal surface of $\frac{1}{16}$-in.-thick stainless steel having a k of 9.4 btu/h · ft · °F. The heating medium on the other side of this surface is a hot fluid at 290°F having an h of 275 btu/h · ft^2 · °F. Use the simplified equations. Be sure to correct this h value for the effect of pressure.

4.8-3. ***Condensation on a Vertical Tube.*** Repeat Example 4.8-2 but for a vertical tube 1.22 m (4.0 ft) high instead of 0.305 m (1.0 ft) high. Use SI and English units.

A47: **Ans.** h = 9438 $W/m^2 \cdot K$, 1663 btu/h · ft^2 · °F; N_{Re} = 207.2 (laminar flow)

4.8-4. ***Condensation of Steam on Vertical Tubes.*** Steam at 1 atm abs pressure and 100°C is condensing on a bank of five vertical tubes each 0.305 m high and having an OD of 25.4 mm. The tubes are arranged in a bundle spaced far enough apart that they do not interfere with each other. The surface temperature of the tubes is 97.78°C. Calculate the average heat-transfer coefficient and the total kg condensate per hour.

A48: **Ans.** h = 15 240 W/m^2 · K

4.8-5. ***Condensation on a Bank of Horizontal Tubes.*** Steam at 1 atm abs pressure and 100°C is condensing on a horizontal tube bank with five layers of tubes (N = 5) placed one below the other. Each layer has four tubes (total tubes = 4 × 5 = 20) and the OD of each tube is 19.1 mm. The tubes are each 0.61 m long and the tube surface temperature is 97.78°C. Calculate the average heat-transfer coefficient and the kg condensate per second for the whole condenser. Make a sketch of the tube bank.

4.9-1. ***Mean Temperature Difference in an Exchanger.*** A 1-2 exchanger with one shell pass and two tube passes is used to heat a cold fluid from 37.8°C to 121.1°C by using a hot fluid entering at 315.6°C and leaving at 148.9°C. Calculate the ΔT_{lm} and the mean temperature difference ΔT_m in K.

A50: **Ans.** ΔT_{lm} = 148.9 K, ΔT_m = 131.8 K

4.9-2. ***Cooling Oil by Water in an Exchanger.*** Oil flowing at the rate of 5.04 kg/s (c_{pm} = 2.09 kJ/kg · K) is cooled in a 1-2 heat exchanger from 366.5 K to 344.3 K by 2.02 kg/s of water entering at 283.2 K. The overall heat-transfer coefficient U_o is 340 W/m^2 · K. Calculate the area required. *(Hint: A* heat balance must first be made to determine the outlet water temperature.)

4.9-3. ***Heat Exchange Between Oil and Water.*** Water is flowing at the rate of 1.13 kg/s in a 1-2 shell-and-tube heat exchanger and is heated from 45°C to 85°C by an oil having a heat capacity of 1.95 kJ/kg · K. The oil enters at 120°C and leaves at 85°C. Calculate the area of the exchanger if the overall heat-transfer coefficient is 300 W/m^2 · K.

4.9-4. ***Outlet Temperature and Effectiveness of an Exchanger.*** Hot oil at a flow rate of 3.00 kg/s (c_p = 1.92 kJ/kg · K) enters an existing counterflow exchanger at 400 K and is cooled by water entering at 325 K (under pressure) and flowing at a rate of 0.70 kg/s. The overall U = 350 W/m^2 · K and A = 12.9 m^2. Calculate the heat-transfer rate and the exit oil temperature.

4.10-1. ***Radiation to a Tube from a Large Enclosure.*** Repeat Example 4.10-1 but use the slightly more accurate Eq. (4.10-5) with two different emissivities.

A54: **Ans.** q = -2171 W (-7410 btu/h)

4.10-2. ***Baking a Loaf of Bread in an Oven.*** A loaf of bread having a surface temperature of 373 K is being baked in an oven whose walls and the air are at 477.4 K. The bread moves continuously through the large oven on an open chain belt conveyor. The emissivity of the bread is estimated as 0.85, and the loaf can be assumed to be a rectangular solid 114.3 mm high × 114.3 mm wide × 330 mm long. Calculate the radiation heat-transfer rate to the bread, assuming that it is small compared to the oven and neglecting natural convection heat transfer.

A55: **Ans.** q = 278.4 W (950 btu/h)

4.10-3. ***Radiation and Convection from a Steam Pipe.*** A horizontal oxidized steel pipe carrying steam and having an OD of 0.1683 m has a surface temperature of 374.9 K and is exposed to air at 297.1 K in a large enclosure. Calculate the heat loss for 0.305 m of pipe from natural convection plus radiation. For the steel pipe, use an ε of 0.79.

A56: **Ans.** q = 163.3 W (557 btu/h)

4.10-4. ***Radiation and Convection to a Loaf of Bread.*** Calculate the total heat-transfer rate to the loaf of bread in Problem 4.10-2, including the radiation plus natural convection heat transfer. For radiation first calculate a value of h_r. For natural convection, use the simplified equations for the lower $N_{Gr}N_{Pr}$ range. For the four vertical sides, the equation for vertical planes can be used with an L of 114.3 mm. For the top surface, use the equation for a cooled plate facing upward, and for the bottom, a cooled plate facing downward. The characteristic L for a horizontal rectangular plate is the linear mean of the two dimensions.

4.10-5. ***Heat Loss from a Pipe.*** A bare stainless-steel tube having an outside diameter of 76.2 mm and an ε of 0.55 is placed horizontally in air at 294.2 K. The pipe surface temperature is 366.4 K. Calculate the value of $h_c + h_r$ for convection plus radiation and the heat loss for 3 m of pipe.

4.11-1. ***Radiation Shielding.*** Two very large and parallel planes each have an emissivity of 0.7. Surface 1 is at 866.5 K and surface 2 is at 588.8 K. Use SI and English units.

a. What is the net radiation loss of surface 1?

b. To reduce this loss, two additional radiation shields also having an emissivity of 0.7 are placed between the original surfaces. What is the new radiation loss?

A59: **Ans.** (a) 13 565 W/m^2, 4304 btu/h · ft^2; (b) 4521 W/m^2, 1435 btu/h · ft^2

4.11-2. ***Radiation from a Craft in Space.*** A space satellite in the shape of a sphere is traveling in outer space, where its surface temperature is held at 283.2 K. The sphere "sees" only outer space, which can be considered as a black body with a temperature of 0 K. The polished surface of the sphere has an emissivity of 0.1. Calculate the heat loss per m^2 by radiation.

A60: **Ans.** $q_{12}/A_1 = 36.5\ W/m^2$

4.11-3. ***Radiation and Complex View Factor.*** Find the view factor F_{12} for the configuration shown in Fig. P4.11-3. The area A_4 and A_3 are fictitious areas (C3). The area $A_2 + A_4$ is called $A_{(24)}$ and $A_1 + A_3$ is called $A_{(13)}$. Areas $A_{(24)}$ and $A_{(13)}$ are perpendicular to each other.

[*Hint:* Follow the methods in Example 4.11-5. First, write an equation similar to Eq. (4.11-48) which relates the interchange between A_3 and $A_{(24)}$. Then relate the interchange between $A_{(13)}$ and $A_{(24)}$. Finally, relate $A_{(13)}$ and A_4.]

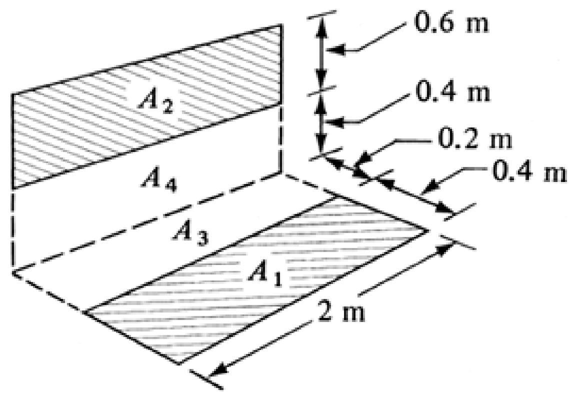


Figure P4.11-3. Geometric configuration for Problem 4.11-3.

A61: **Ans.** $A_1F_{12} = A_{(13)}F_{(13)(24)} + A_3F_{34} - A_3F_{3(24)} - A_{(13)}F_{(13)4}$

4.11-4. **Radiation Between Parallel Surfaces.** Two parallel surfaces each 1.83 × 1.83 m square are spaced 0.91 m apart. The surface temperature of A_1 is 811 K and that of A_2 is 533 K. Both are black surfaces.

a. Calculate the radiant heat transfer between the two surfaces.
b. Do the same as for part (a), but for the case where the two surfaces are connected by nonconducting reradiating walls.
c. Repeat part (b), but A_1 has an emissivity of 0.8 and A_2 an emissivity of 0.7.

4.11-5. ***Radiation Between Adjacent Perpendicular Plates.*** Two adjacent rectangles are perpendicular to each other. The first rectangle is 1.52 × 2.44 m and the second 1.83 × 2.44 m, with the 2.44-m side common to both. The temperature of the first surface is 699 K and that of the second is 478 K. Both surfaces are black. Calculate the radiant heat transfer between the two surfaces.

4.11-6. *View Factor for Complex Geometry.* Using the dimensions given in Fig. P4.11-3, calculate the individual view factors and also F_{12}.

4.11-7. *Radiation from a Surface to the Sky.* A plane surface having an area of 1.0 m^2 is insulated on the bottom side and is placed on the ground exposed to the atmosphere at night. The upper surface is exposed to air at 290 K, and the convective heat-transfer coefficient from the air to the plane is 12 W/m^2 · K. The plane radiates to the clear sky. The effective radiation temperature of the sky can be assumed as 80 K. If the plane is a black body, calculate the temperature of the plane at equilibrium.

Ans. T = 266.5 K = -6.7°C

4.11-8. ***Radiation and Heating of Planes.*** Two plane disks each 1.25 m in diameter are parallel and directly opposite each other. They are separated by a distance of 0.5 m. Disk 1 is heated by electrical resistance to 833.3 K. Both disks are insulated on all faces except the two faces directly opposite each other. Assume that the surroundings emit no radiation and that the disks are in space. Calculate the temperature of disk 2 at steady state and also the electrical energy input to disk 1. *(Hint:* The fraction of heat lost from area number 1 to space is $1 - F_{12}$.)

A66: **Ans.** $F_{12} = 0.45$, $T_2 = 682.5$ K

4.11-9. ***Radiation by Disks to Each Other and to Surroundings.*** Two disks each 2.0 m in diameter are parallel and directly opposite each other and are separated by a distance of 2.0 m. Disk 1 is held at 1000 K by electric heating and disk 2 at 400 K by cooling water in a jacket at the rear of the disk. The disks radiate only to each other and to the surrounding space at 300 K. Calculate the electric heat input and also the heat removed by the cooling water.

4.11-10. ***View Factor by Integration.*** A small black disk is vertical, with an area of 0.002 m^2, and radiates to a vertical black plane surface that is 0.03 m wide and 2.0 m high and is opposite and parallel to the small disk. The disk source is 2.0 m away from the vertical plane and placed opposite the bottom of the plane. Determine F_{12} by integration of the view-factor equation.

A68: **Ans.** $F_{12} = 0.00307$

4.11-11. ***Gas Radiation to Gray Enclosure.*** Repeat Example 4.11-7 but with the following changes:

a. The interior walls are not black surfaces but gray surfaces with an emissivity of 0.75.

b. The same conditions as part (a) with gray walls, but in addition heat is transferred by natural convection to the interior walls. Assume an average convective coefficient of 8.0 $W/m^2 \cdot K$.

Ans. (b) q(convection + radiation) = 4.426 W

4.11-12. ***Gas Radiation and Convection to a Stack.*** A furnace discharges hot flue gas at 1000 K and 1 atm abs pressure containing 5% CO_2 into a stack having an inside diameter of 0.50 m. The inside walls of the refractory lining are at 900 K and the emissivity of the lining is 0.75. The convective heat-transfer coefficient of the gas has been estimated as 10 $W/m^2 \cdot K$. Calculate the rate of heat transfer q/A from the gas by radiation plus convection.

4.12-1. ***Laminar Heat Transfer of a Power-Law Fluid.*** A non-Newtonian power-law fluid banana purée flowing at a rate of 300 lb_m/h inside a 1.0-in.-ID tube is being heated by a hot fluid flowing outside the tube. The banana purée enters the heating section of the tube, which is 5 ft long, at a temperature of 60°F. The inside wall temperature is constant at 180°F. The fluid properties as given by Charm (C1) are $\rho = 69.9$ lb_m/ft^3, $c_p = 0.875$ $btu/lb_m \cdot °F$, and $k = 0.320$ $btu/h \cdot ft \cdot °F$. The fluid has the following rheological constants: $n = n' = 0.458$, which can be assumed constant, and $K = 0.146$ $lb_f \cdot s^n \cdot ft^{-2}$ at 70°F and 0.0417 at 190°F A plot of log K versus T°F can be assumed to be a straight line. Calculate the outlet bulk temperature of the fluid in laminar flow.

4.12-2. ***Heating a Power-Law Fluid in Laminar Flow.*** A non-Newtonian power-law fluid having the same physical properties and rheological constants as the fluid in Example 4.12-1 is flowing in laminar flow at a rate of 6.30×10^{-2} kg/s inside a 25.4-mm-ID tube. It is being heated by a hot fluid outside the tube. The fluid enters the heating section of the tube at 26.7°C and leaves the heating section at an outlet bulk temperature of 46.1°C. The inside wall temperature is constant at 82.2°C. Calculate the length of tube needed in m. *(Note:* In this case the unknown tube length L appears in the equation for h_a and in the heat-balance equation.)

Ans. L = 1.722 m

4.13-1. ***Heat Transfer in a Jacketed Vessel with a Paddle Agitator.*** A vessel with a paddle agitator and no baffles is used to heat a liquid at 37.8°C. A steam-heated jacket furnishes the heat. The vessel's inside diameter is 1.22 m; the agitator diameter is 0.406 m and it is rotating at 150 rpm. The wall surface temperature is 93.3°C. The physical properties of the liquid are ρ = 977 kg/m^3, c_p = 2.72 kJ/kg · K, k = 0.346 W/m · K, and μ = 0.100 kg/m · s at 37.8°C and 7.5 × 10^{-3} at 93.3°C. Calculate the heat-transfer coefficient to the wall of the jacket.

4.13-2. ***Heat Loss from Circular Fins.*** Use the data and conditions from Example 4.13-2 and calculate the fin efficiency and rate of heat loss from the following different fin materials:

a. Carbon steel (k = 44 W/m · K).

b. Stainless steel (k = 17.9 W/m · K).

A74: **Ans.** (a) η_f = 0.66, q = 111.1 W

4.13-3. ***Heat Loss from Longitudinal Fin.*** A longitudinal aluminum fin as shown in Fig. 4.13-3a (k = 230 W/m · K) is attached to a copper tube having an outside radius of 0.04 m. The length of the fin is 0.080 m and the thickness is 3 mm. The tube base is held at 450 K and the external surrounding air at 300 K has a convective coefficient of 25 W/m^2 · K. Calculate the fin efficiency and the heat loss from the fin per 1.0 m of length.

4.13-4. ***Heat Transfer in Finned Tube Exchanger.*** Air at an average temperature of 50°C is being heated by flowing outside a steel tube (k = 45.1 W/m · K) having an inside diameter of 35 mm and a wall thickness of 3 mm. The outside of the tube is covered with 16 longitudinal steel fins with a length L = 13 mm and a thickness of t = 1.0 mm. Condensing steam inside at 120°C has a coefficient of 7000 W/m^2 · K. The outside coefficient of the air has been estimated as 30 W/m^2 · K. Neglecting fouling factors and using a tube 1.0 m long, calculate the overall heat-transfer coefficient U_i based on the inside area A_i.

4.14-1. ***Dimensional Analysis for Natural Convection.*** Repeat the dimensional analysis for natural convection heat transfer to a vertical plate as given in Section 4.14. However, do as follows:

a. Carry out all the detailed steps solving for all the exponents in the π's.

b. Repeat, but in this case select the four variables L, μ, c_p, and g to be common to all the dimensionless groups.

4.14-2. ***Dimensional Analysis for Unsteady-State Conduction.*** For unsteady-state conduction in a solid, the following variables are involved: ρ, c_p, L (dimension of solid), t, k, and z (location in solid). Determine the dimensionless groups relating the variables.

A78:

Ans. $$\pi_1 = \frac{kt}{\rho c_p L^2}, \pi_2 = \frac{z}{L}$$

4.15-1. ***Temperatures in a Semi-Infinite Plate.*** A semi-infinite plate is similar to that in Fig. 4.15-2. At the surfaces $x = 0$ and $x = L$, the temperature is held constant at 200 K. At the surface $y = 0$, the temperature is held at 400 K. If $L = 1.0$ m, calculate the temperature at the point y = 0.5 m and $x = 0.5$ m at steady state.

4.15-2. ***Heat Conduction in a Two-Dimensional Solid.*** For two-dimensional heat conduction as given in Example 4.15-1, derive the equation to calculate the total heat loss from the chamber per unit length using the nodes at the outside. There should be eight paths for one-fourth of the chamber. Substitute the actual temperatures into the equation and obtain the heat loss.

Ans. $q = 3426$ W

4.15-3. ***Steady-State Heat Loss from a Rectangular Duct.*** A chamber that is in the shape of a long, hollow, rectangular duct has outside dimensions of 3 × 4 m and inside dimensions of 1 × 2 m. The walls are 1 m thick. The inside surface temperature is constant at 800 K and the outside constant at 200 K. The $k = 1.4$ W/m · K. Calculate the steady-state heat loss per unit m of length of duct. Use a grid size of $\Delta x = \Delta y = 0.5$ m. Also, use the outside nodes to calculate the total heat conduction. Use a spreadsheet for the calculation.

A80: **Ans.** $q = 7428$ W

4.15-4. ***Two-Dimensional Heat Conduction and Different Boundary Conditions.*** A very long, solid piece of material 1 by 1 m square has its top face maintained at a constant temperature of 1000 K and its left face at 200 K. The bottom face and right face are exposed to an environment at 200 K and have a convection coefficient of $h = 10$ W/m^2 · K. The $k = 10$ W/m · K. Use a grid size of $\Delta x = \Delta y = \frac{1}{3}$ m and calculate the steady-state temperatures of the various nodes.

4.15-5. ***Nodal Point at Exterior Corner Between Insulated Surfaces.*** Derive the finite-difference equation for the case for the nodal point $T_{n,m}$ at an exterior corner between insulated surfaces. The diagram is similar to Fig. 4.15-6c except that the two boundaries are insulated.

A82:

Ans. $$\bar{q}_{n,m} = \tfrac{1}{2}(T_{n-1,m} + T_{n,m-1}) - T_{n,m}$$

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