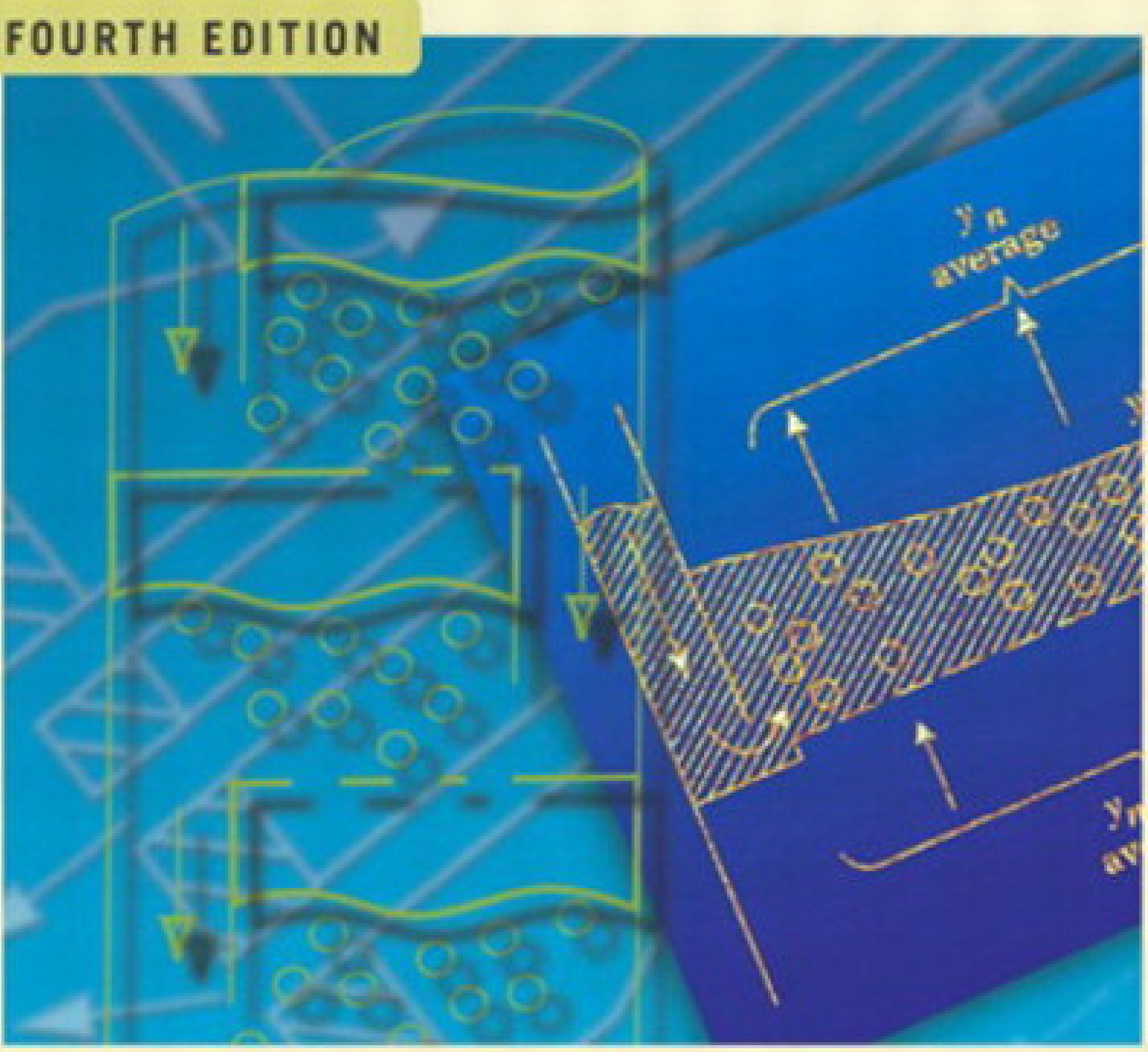
Transport Processes and Separation Process Principles

(INCLUDES UNIT OPERATIONS)

FOURTH EDITION



CHRISTIE JOHN GEANKOPLIS

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Chapter 9. Drying of Process Materials

INTRODUCTION AND METHODS OF DRYING

Purposes of Drying

The discussions of drying in this chapter are concerned with the removal of water from process materials and other substances. The term “drying” is also used to refer to removal of other organic liquids, such as benzene or organic solvents, from solids. Many of the types of equipment and calculation methods discussed for removal of water can also be used for removal of organic liquids.

Drying generally means removal of relatively small amounts of water from material. Evaporation refers to removal of relatively large amounts of water from material. In evaporation the water is removed as vapor at its boiling point. In drying the water is usually removed as a vapor by air.

In some cases water may be removed mechanically from solid materials by means of presses, centrifuging, and other methods. This is cheaper than drying by thermal means for removal of water, which will be discussed here. The moisture content of the final dried product varies depending upon the type of product. Dried salt contains about 0.5% water, coal about 4%, and many food products about 5%. Drying is usually the final processing step before packaging and makes many materials, such as soap powders and dyestuffs, more suitable for handling.

Drying or dehydration of biological materials, especially foods, is used as a preservation technique. Microorganisms that cause food spoilage and decay cannot grow and multiply in the absence of water. Also, many enzymes that cause chemical changes in food and other biological materials cannot function without water. When the water content is reduced below about 10 wt %, the microorganisms are not active. However, it is usually necessary to lower the moisture content below 5 wt % in foods to preserve flavor and nutrition. Dried foods can be stored for extended periods of time.

Some biological materials and pharmaceuticals, which may not be heated for ordinary drying, may be freeze-dried, as discussed in Section 9.11. Also, in Section 9.12, sterilization of foods and other biological materials is discussed, which is another method often employed to preserve such materials.

General Methods of Drying

Drying methods and processes can be classified in several different ways. Drying processes can be classified as *batch,* where the material is inserted into the drying equipment and drying proceeds for a given period of time, or as *continuous,* where the material is continuously added to the dryer and dried material is continuously removed.

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Drying processes can also be categorized according to the physical conditions used to add heat and remove water vapor: (1) in the first category, heat is added by direct contact with heated air at atmospheric pressure, and the water vapor formed is removed by the air; (2) in vacuum drying, the evaporation of water proceeds more rapidly at low pressures, and the heat is added indirectly by contact with a metal wall or by radiation (low temperatures can also be used under vacuum for certain materials that may discolor or decompose at higher temperatures); and (3) in freeze-drying, water is sublimed from the frozen material.

EQUIPMENT FOR DRYING

Tray Dryer

In *tray dryers,* which are also called shelf, cabinet, or compartment dryers, the material, which may be a lumpy solid or a pasty solid, is spread uniformly on a metal tray to a depth of 10 to 100 mm. Such a typical tray dryer, shown in Fig. 9.2-1, contains removable trays loaded in a cabinet.

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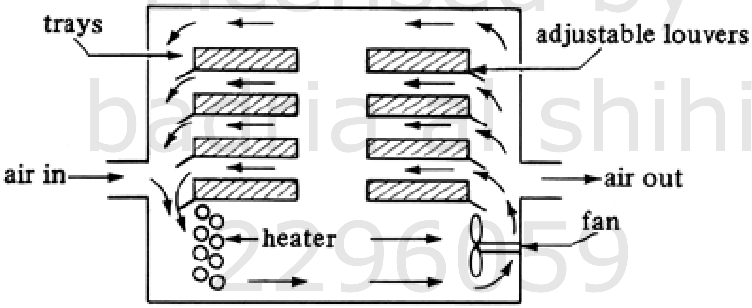

Figure 9.2-1. Tray or shelf dryer.

Steam-heated air is recirculated by a fan over and parallel to the surface of the trays. Electrical heat is also used, especially for low heating loads. About 10 to 20% of the air passing over the trays is fresh air, the remainder being recirculated air.

After drying, the cabinet is opened and the trays are replaced with a new batch of trays. A modification of this type is the tray–truck type, where trays are loaded on trucks which are pushed into the dryer. This saves considerable time, since the trucks can be loaded and unloaded outside the dryer.

In the case of granular materials, the material can be loaded on screens which form the bottom of each tray. Then, in this through-circulation dryer, heated air passes through the permeable bed, yielding shorter drying times because of the greater surface area exposed to the air.

Vacuum-Shelf Indirect Dryers

Vacuum-shelf dryers are indirectly heated batch dryers similar to tray dryers. Such a dryer consists of a cabinet made of cast-iron or steel plates with tightly fitted doors so that it can be operated under vacuum. Hollow shelves of steel are fastened permanently inside the chamber and are connected in parallel to inlet and outlet steam headers. The trays containing the solids to be dried rest upon the hollow shelves. The heat is conducted through the metal walls and aided by radiation from the shelf above. For low-temperature operation, circulating warm water is used instead of steam for furnishing the heat to vaporize the moisture. The vapors usually pass to a condenser.

These dryers are used to dry expensive or temperature-sensitive or easily oxidizable materials. They are useful for handling materials with toxic or valuable solvents.

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Continuous Tunnel Dryers

Continuous tunnel dryers are often batch truck or tray compartments operated in series, as shown in Fig. 9.2-2a. The solids are placed on trays or on trucks which move continuously through a tunnel with hot gases passing over the surface of each tray. The hot air flow can be countercurrent, co-current, or a combination. Many foods are dried in this way.

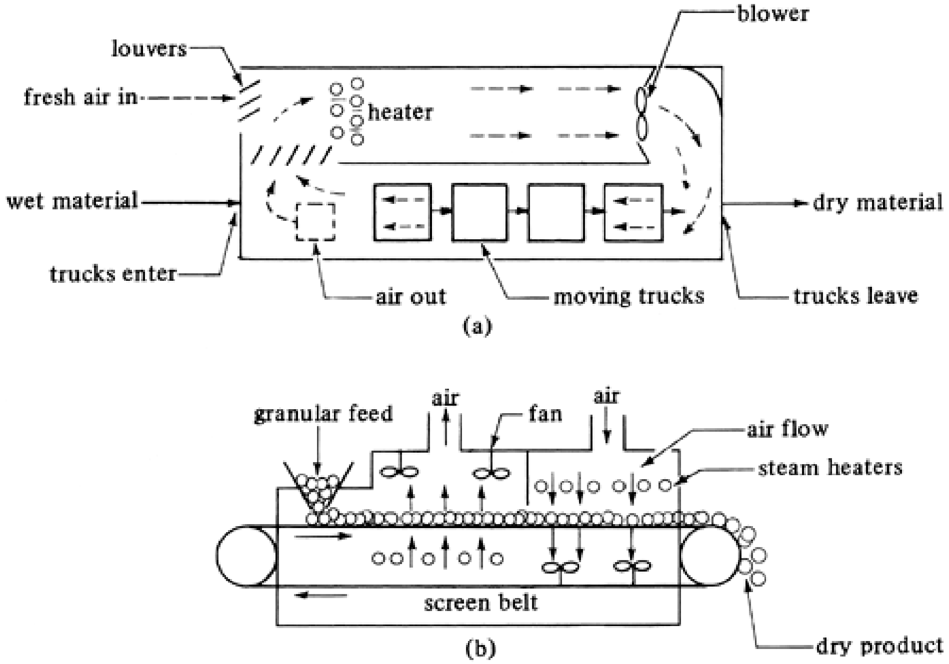

Figure 9.2-2. Continuous tunnel dryers: (a) tunnel dryer trucks with countercurrent air flow, (b) through-circulation screen conveyor dryer.

When granular particles of solids are to be dried, perforated or screen-belt continuous conveyors are often used, as in Fig. 9.2-2b. The wet granular solids are conveyed as a layer 25 to about 150 mm deep on a screen or perforated apron while heated air is blown upward through the bed or downward. The dryer consists of several sections in series, each with a fan and heating coils. A portion of the air is exhausted to the atmosphere by a fan. In some cases pasty materials can be preformed into cylinders and placed on the bed to be dried.

Rotary Dryers

A *rotary dryer* consists of a hollow cylinder which is rotated and usually slightly inclined toward the outlet. The wet granular solids are fed at the high end as shown in Fig. 9.2-3 and move through the shell as it rotates. The heating shown is by direct contact with hot gases in countercurrent flow. In some cases the heating is by indirect contact through the heated wall of the cylinder.

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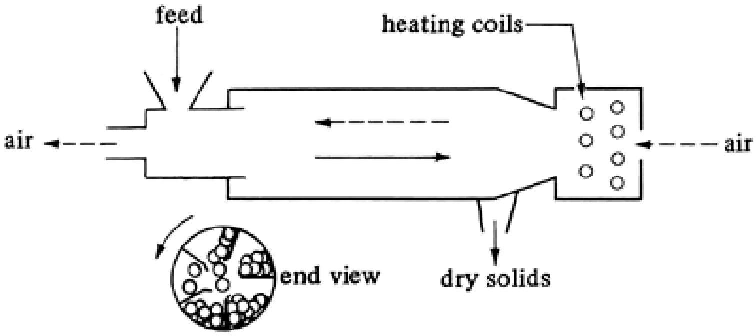

Figure 9.2.3. Schematic drawing of a direct-heat rotary dryer.

The granular particles move forward slowly a short distance before they are showered downward through the hot gases as shown. Many other variations of this rotary dryer are available; these are discussed elsewhere (P1).

Drum Dryers

A *drum dryer* consists of a heated metal roll, shown in Fig. 9.2-4, on the outside of which a thin layer of liquid or slurry is evaporated to dryness. The final dry solid is scraped off the roll, which is revolving slowly.

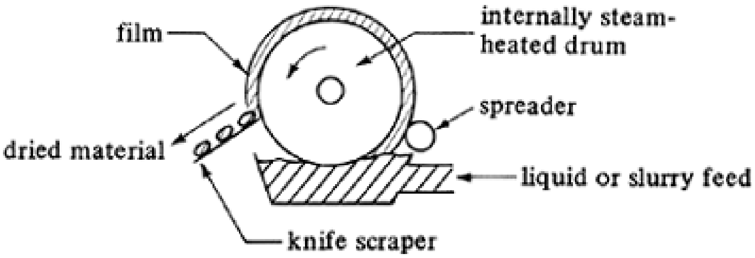

Figure 9.2.4. Rotary-drum dryer.

Drum dryers are suitable for handling slurries or pastes of solids in fine suspension and for solutions. The drum functions partly as an evaporator as well as a dryer. Other variations of the single-drum type are twin rotating drums with dip feeding or with top feeding to the two drums. Potato slurry is dried using drum dryers, to give potato flakes.

Spray Dryers

In a *spray dryer* a liquid or slurry solution is sprayed into a hot gas stream in the form of a mist of fine droplets. The water is rapidly vaporized from the droplets, leaving particles of dry solid which are separated from the gas stream. The flow of gas and liquid in the spray chamber may be countercurrent, cocurrent, or a combination.

The fine droplets are formed from the liquid feed by spray nozzles or high-speed rotating spray disks inside a cylindrical chamber, as shown in Fig. 9.2-5. It is necessary to ensure that the droplets or wet particles of solid do not strike and stick to solid surfaces before drying has taken place; hence, large chambers are used. The dried solids leave at the bottom of the chamber through a screw conveyor. The exhaust gases flow through a cyclone separator to remove any fines. The particles produced are usually light and quite porous. Dried milk powder is made from spray-drying milk.

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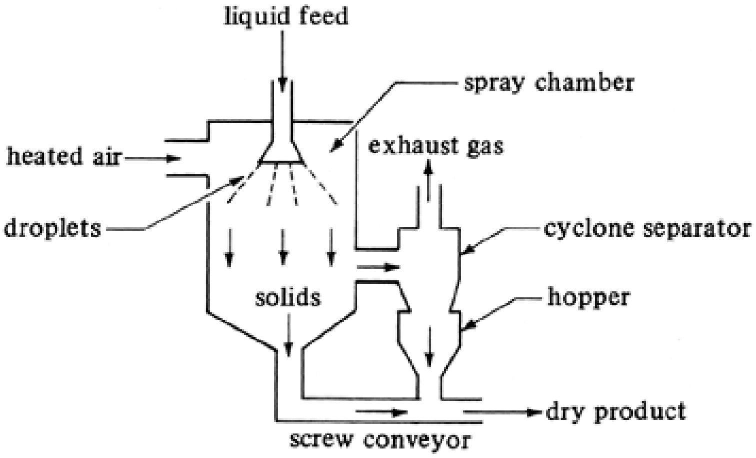

Figure 9.2-5. Process flow diagram of spray-drying apparatus.

Drying of Crops and Grains

In the drying of grain from a harvest, the grain contains about 30 to 35% moisture, and for safe storage for about 1 year should be dried to about 13 wt % moisture (H1). A typical continuous-flow dryer is shown in Fig. 9.2-6. In the drying bin the thickness of the layer of grain, through which the hot air passes, is 0.5 m or less. Unheated air in the bottom section cools the dry grain before it leaves. Other types of crop dryers and storage bins are described by Hall (H1).

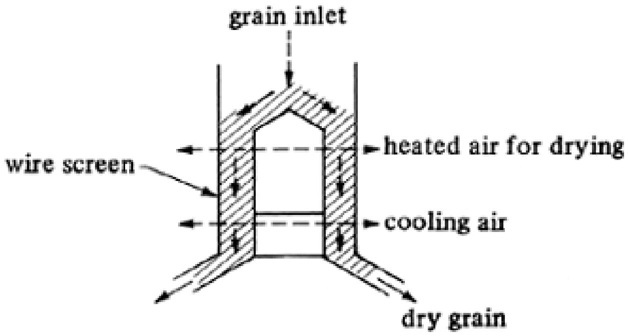

Figure 9.2-6. Vertical continuous-flow grain dryer.

VAPOR PRESSURE OF WATER AND HUMIDITY

Vapor Pressure of Water

Introduction

In a number of the separation processes and transport processes, it is necessary to make calculations involving the properties of mixtures of water vapor and air. These calculations involve knowledge of the concentration of water vapor in air under various conditions of temperature and pressure, the thermal properties of these mixtures, and the changes occurring when these mixtures are brought into contact with water or with wet solids in drying.

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Humidification involves the transfer of water from the liquid phase into a gaseous mixture of air and water vapor. *Dehumidification* involves the reverse transfer, whereby water vapor is transferred from the vapor state to the liquid state. Humidification and dehumidification can also refer to vapor mixtures of materials such as benzene, but most practical applications occur with water. To better understand humidity, it is first necessary to discuss the vapor pressure of water.

Vapor pressure of water and physical states

Pure water can exist in three different physical states: solid ice, liquid, and vapor. The physical state in which it exists depends on the pressure and temperature.

Figure 9.3-1 illustrates the various physical states of water and the pressure–temperature relationships at equilibrium. In Fig. 9.3-1 the regions of the solid, liquid, and vapor states are shown. Along the line *AB*, the phases liquid and vapor coexist. Along line *AC*, the phases ice and liquid coexist. Along line *AD*, ice and vapor coexist. If ice at point (1) is heated at constant pressure, the temperature rises and the physical condition is represented as moving horizontally. As the horizontal line crosses *AC*, the solid melts, and on crossing *AB* the liquid vaporizes. Moving from point (3) to (4), ice sublimes (vaporizes) to a vapor without becoming a liquid.

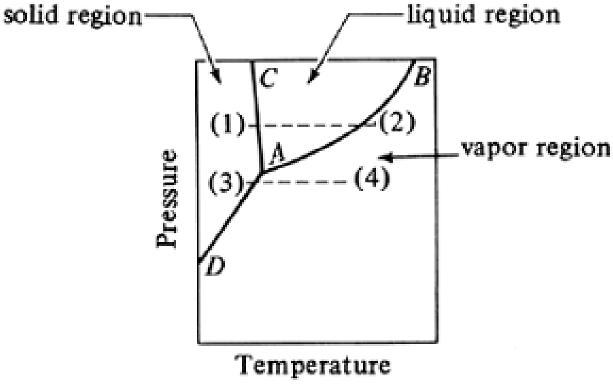

Figure 9.3-1. Phase diagram for water.

Liquid and vapor coexist in equilibrium along the line *AB*, which is the vapor-pressure line for water. Boiling occurs when the vapor pressure of the water is equal to the total pressure above the water surface. For example, at 100°C (212°F) the vapor pressure of water is 101.3 kPa (1.0 atm), and therefore it will boil at 1 atm pressure. At 65.6°C (150°F), from the steam tables in Appendix A.2, the vapor pressure of water is 25.7 kPa (3.72 psia). Hence, at 25.7 kPa and 65.6°C, water will boil.

If a pan of water is held at 65.6°C in a room at 101.3 kPa abs pressure, the vapor pressure of water will again be 25.7 kPa. This illustrates an important property of the vapor pressure of water, which is not influenced by the presence of an inert gas such as air; that is, the vapor pressure of water is essentially independent of the total pressure of the system.

Humidity and Humidity Chart

Definition of humidity

The humidity *H* of an air–water vapor mixture is defined as the kg of water vapor contained in 1 kg of dry air. The humidity so defined depends only on the partial pressure p_A of water vapor in the air and on the total pressure *P* (assumed throughout this chapter to be 101.325 kPa, 1.0 atm abs, or 760 mm Hg). Using the molecular weight of water (*A*) as 18.02 and of air as 28.97, the humidity *H* in kg H_2O/kg dry air, or in English units as lb H_2O/lb dry air, is as follows:

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Equation 9.3-1.

$$H\frac{\text{kg H}_2\text{O}}{\text{kg dry air}} = \frac{p_A}{P - p_A}\frac{\text{kg mol H}_2\text{O}}{\text{kg mol air}} \times \frac{18.02\text{ kg H}_2\text{O}}{\text{kg mol H}_2\text{O}} \times \frac{1}{28.97\text{ kg air/kg mol air}}$$

$$H = \frac{18.02}{28.97}\frac{p_A}{P - p_A}$$

Saturated air is air in which the water vapor is in equilibrium with liquid water at the given conditions of pressure and temperature. In this mixture the partial pressure of the water vapor in the air–water mixture is equal to the vapor pressure p_{AS} of pure water at the given temperature. Hence, the saturation humidity H_S is

Equation 9.3-2.

$$H_S = \frac{18.02}{28.97}\frac{p_{AS}}{P - p_{AS}}$$

Percentage humidity

The percentage humidity H_P is defined as 100 times the actual humidity H of the air divided by the humidity H_S if the air were saturated at the same temperature and pressure:

Equation 9.3-3.

$$H_P = 100\frac{H}{H_S}$$

Percentage relative humidity

The amount of saturation of an air–water vapor mixture is also given as percentage relative humidity H_R using partial pressures:

Equation 9.3-4.

$$H_R = 100\frac{p_A}{p_{AS}}$$

Note that $H_R \neq H_P$, since H_P expressed in partial pressures by combining Eqs. (9.3-1), (9.3-2), and (9.3-3) is

Equation 9.3-5.

$$H_P = 100\frac{H}{H_S} = (100)\frac{18.02}{28.97}\frac{p_A}{P - p_A}\Bigg/\frac{18.02}{28.97}\frac{p_{AS}}{P - p_{AS}} = \frac{p_A}{p_{AS}}\frac{P - p_{AS}}{P - p_A}(100)$$

This, of course, is not the same as Eq. (9.3-4).

EXAMPLE 9.3-1. Humidity from Vapor-Pressure Data

The air in a room is at 26.7°C (80°F) and a pressure of 101.325 kPa and contains water vapor with a partial pressure p_A = 2.76 kPa. Calculate the following:

a. Humidity, H.
b. Saturation humidity, H_S, and percentage humidity, H_P.
c. Percentage relative humidity, H_R.

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Solution: From the steam tables at 26.7°C, the vapor pressure of water is p_{AS} = 3.50 kPa (0.507 psia). Also, p_A = 2.76 kPa and P = 101.3 kPa (14.7 psia). For part (a), using Eq. (9.3-1),

$$H = \frac{18.02}{28.97}\frac{p_A}{P - p_A} = \frac{18.02(2.76)}{28.97(101.3 - 2.76)} = 0.01742 \text{ kg H}_2\text{O/kg air}$$

For part (b), using Eq. (9.3-2), the saturation humidity is

$$H_S = \frac{18.02}{28.97}\frac{p_{AS}}{P - p_{AS}} = \frac{18.02(3.50)}{28.97(101.3 - 3.50)} = 0.02226 \text{ kg H}_2\text{O/kg air}$$

The percentage humidity, from Eq. (9.3-3), is

$$H_P = 100\frac{H}{H_S} = \frac{100(0.01742)}{0.02226} = 78.3\%$$

For part (c), from Eq. (9.3-4), the percentage relative humidity is

$$H_R = 100\frac{p_A}{p_{AS}} = \frac{100(2.76)}{3.50} = 78.9\%$$

Dew point of an air–water vapor mixture

The temperature at which a given mixture of air and water vapor would be saturated is called the *dew-point temperature* or simply the *dew point.* For example, at 26.7°C (80°F), the saturation vapor pressure of water is p_{AS} = 3.50 kPa (0.507 psia). Hence, the dew point of a mixture containing water vapor having a partial pressure of 3.50 kPa is 26.7°C. If an air–water vapor mixture is at 37.8°C (often called the dry bulb temperature, since this is the actual temperature a dry thermometer bulb would indicate in this mixture) and contains water vapor of p_A = 3.50 kPa, the mixture would not be saturated. On cooling to 26.7°C, the air would be saturated, that is, at the dew point. On further cooling, some water vapor would condense, since the partial pressure cannot be greater than the saturation vapor pressure.

Humid heat of an air–water vapor mixture

The humid heat c_S is the amount of heat in J (or kJ) required to raise the temperature of 1 kg of dry air plus the water vapor present by 1 K or 1°C. The heat capacity of air and water vapor can be assumed constant over the temperature ranges usually encountered at 1.005 kJ/kg dry air · K and 1.88 kJ/kg water vapor · K, respectively. Hence, for SI and English units,

Equation 9.3-6.

$$c_S \text{ kJ/kg dry air} \cdot \text{K} = 1.005 + 1.88H \qquad \text{(SI)}$$

$$c_S \text{ btu/lb}_\text{m} \text{ dry air} \cdot {}^\circ\text{F} = 0.24 + 0.45H \qquad \text{(English)}$$

[In some cases c_S will be given as $(1.005 + 1.88H)10^3$ J/kg · K.]

Humid volume of an air–water vapor mixture

The humid volume v_H is the total volume in m^3 of 1 kg of dry air plus the vapor it contains at 101.325 kPa (1.0 atm) abs pressure and the given gas temperature. Using the ideal gas law,

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Equation 9.3-7.

$$v_H\ \text{m}^3/\text{kg dry air} = \frac{22.41}{273} T\,\text{K} \left(\frac{1}{28.97} + \frac{1}{18.02}H\right)$$

$$= (2.83 \times 10^{-3} + 4.56 \times 10^{-3}\, H)T\ \text{K}$$

$$v_H\ \text{ft}^3/\text{lb}_\text{m}\ \text{dry air} = \frac{359}{492} T\,°\text{R} \left(\frac{1}{28.97} + \frac{1}{18.02}H\right)$$

$$= (0.0252 + 0.0405H)T\,°\text{R}$$

For a saturated air–water vapor mixture, $H = H_S$, and v_H is the saturated volume.

Total enthalpy of an air–water vapor mixture

The total enthalpy of 1 kg of air plus its water vapor is H_y J/kg or kJ/kg dry air. If T_0 is the datum temperature chosen for both components, the total enthalpy is the sensible heat of the air–water vapor mixture plus the latent heat λ_0 in J/kg or kJ/kg water vapor of the water vapor at T_0. Note that $(T - T_0)$°C = $(T - T_0)$ K and that this enthalpy is referred to liquid water.

Equation 9.3-8.

$$H_y\ \text{kJ/kg dry air} = c_S(T - T_0) + H\lambda_0 = (1.005 + 1.88H)(T - T_0\,°\text{C}) + H\lambda_0$$

$$H_y\ \text{btu/lb}_\text{m}\ \text{dry air} = (0.24 + 0.45H)(T - T_0\,°\text{F}) + H\lambda_0$$

If the total enthalpy is referred to a base temperature T_0 of 0°C (32°F), the equation for H_y becomes

Equation 9.3-9.

$$H_y\ \text{kJ/kg dry air} = (1.005 + 1.88H)\,(T\,°\text{C} - 0) + 2501.4H \qquad \text{(SI)}$$

$$H_y\ \text{btu/lb}_\text{m}\ \text{dry air} = (0.24 + 0.45H)\,(T\,°\text{F} - 32) + 1075.4H \qquad \text{(English)}$$

Humidity chart of air–water vapor mixtures

A convenient chart of the properties of air–water vapor mixtures at 1.0 atm abs pressure is the humidity chart in Fig. 9.3-2. In this figure the humidity H is plotted versus the actual temperature of the air–water vapor mixture (dry bulb temperature).

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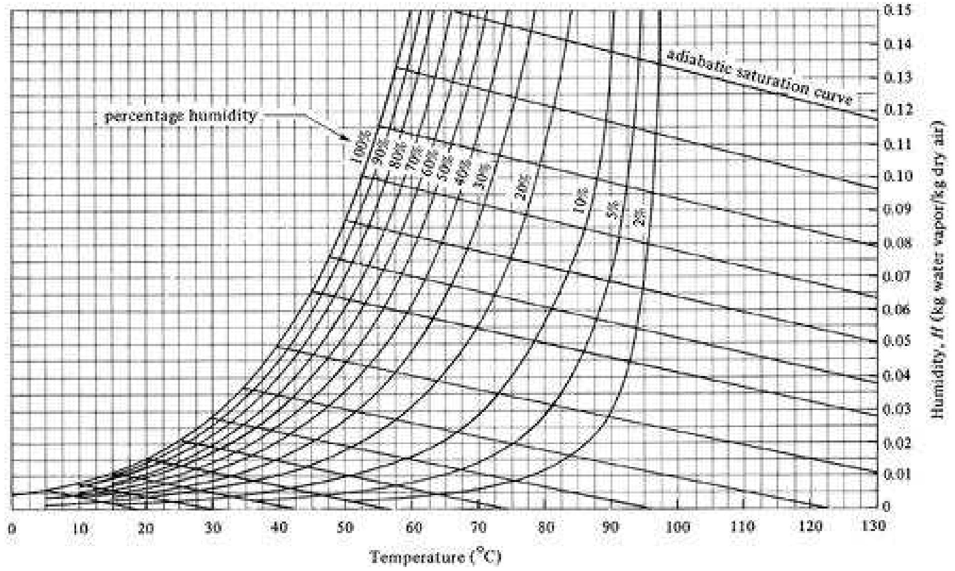

Figure 9.3-2. Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)

The curve marked 100% running upward to the right gives the saturation humidity H_S as a function of temperature. In Example 9.3-1, for 26.7°C, H_S was calculated as 0.02226 kg H_2O/kg air. Plotting this point for 26.7°C (80°F) and $H_S = 0.02226$ on Fig. 9.3-2, it falls on the 100% saturated line.

Any point below the saturation line represents unsaturated air–water vapor mixtures. The curved lines below the 100% saturation line and running upward to the right represent unsaturated mixtures of definite percentage humidity H_P. Going downward vertically from the saturation line at a given temperature, the line between 100% saturation and zero humidity H (the bottom horizontal line) is divided evenly into 10 increments of 10% each.

All the percentage humidity lines H_P mentioned and the saturation humidity line H_S can be calculated from the data for vapor pressure of water.

EXAMPLE 9.3-2. Use of Humidity Chart

Air entering a dryer has a temperature (dry bulb temperature) of 60°C (140°F) and a dew point of 26.7°C (80°F). Using the humidity chart, determine the actual humidity H, percentage humidity H_P, humid heat c_S, and humid volume v_H in SI and English units.

Solution: The dew point of 26.7°C is the temperature when the given mixture is at 100% saturation. Starting at 26.7°C (Fig. 9.3-2), and drawing a vertical line until it intersects the line for 100% humidity, a humidity of $H = 0.0225$ kg H_2O/kg dry air is read off the plot. This is the actual humidity of the air at 60°C. Stated in another way, if air at 60°C and having a humidity $H = 0.0225$ is cooled, its dew point will be 26.7°C. In English units, $H = 0.0225$ lb H_2O/lb dry air.

Locating this point where $H = 0.0225$ and $t = 60°C$ on the chart, the percentage humidity H_P is found to be 14%, by linear interpolation vertically between the 10 and 20% lines. The humid heat for $H = 0.0225$ is, from Eq. (9.3-6),

$$
\begin{aligned}
c_S &= 1.005 + 1.88(0.0225)\\
&= 1.047 \text{ kJ/kg dry air} \cdot \text{K} \qquad \text{or} \qquad 1.047 \times 10^3 \text{ J/kg} \cdot \text{K}\\
c_S &= 0.24 + 0.45(0.0225)\\
&= 0.250 \text{ btu/lb}_\text{m} \text{ dry air} \cdot {}^\circ\text{F} \qquad \text{(English)}
\end{aligned}
$$

The humid volume at 60°C (140°F), from Eq. (9.3-7), is

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$$v_H = (2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.0225)(60 + 273)$$
$$= 0.977 \text{ m}^3/\text{kg dry air}$$

In English units,

$$v_H = (0.0252 + 0.0405 \times 0.0225)(460 + 140) = 15.67 \text{ ft}^3/\text{lb}_\text{m} \text{ dry air}$$

Adiabatic Saturation Temperatures

Consider the process shown in Fig. 9.3-3, where the entering gas of air–water vapor mixture is contacted with a spray of liquid water. The gas leaves having a different humidity and temperature and the process is adiabatic. The water is recirculated, with some makeup water added.

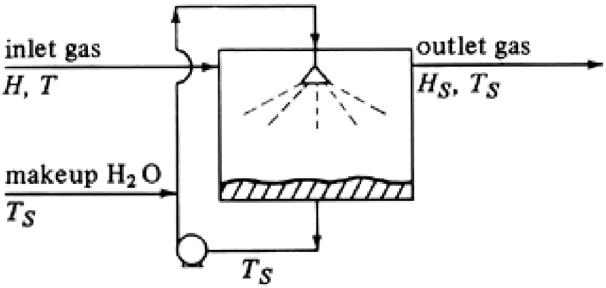

Figure 9.3-3. Adiabatic air–water vapor saturator.

The temperature of the water being recirculated reaches a steady-state temperature called the *adiabatic saturation temperature,* T_S. If the entering gas at temperature T having a humidity of H is not saturated, T_S will be lower than T. If the contact between the entering gas and the spray of droplets is enough to bring the gas and liquid to equilibrium, the leaving air is saturated at T_S, having a humidity H_S.

Writing an enthalpy balance (heat balance) over the process, a datum of T_S is used. The enthalpy of the makeup H_2O is then zero. This means that the total enthalpy of the entering gas mixture = enthalpy of the leaving gas mixture, or, using Eq. (9.3-8),

Equation 9.3-10.

$$c_S(T - T_S) + H\lambda_S = c_S(T_S - T_S) + H_S\lambda_S$$

Or, rearranging, and using Eq. (9.3-6) for c_S,

Equation 9.3-11.

$$\frac{H - H_S}{T - T_S} = -\frac{c_S}{\lambda_S} = \frac{1.005 + 1.88H}{\lambda_S} \qquad \text{(SI)}$$

$$\frac{H - H_S}{T - T_S} = \frac{0.24 + 0.45H}{\lambda_S} \qquad \text{(English)}$$

Equation (9.3-11) is the equation of an adiabatic humidification curve when plotted on Fig. 9.3-2, which passes through the point H_S and T_S on the 100% saturation curve and other points of H and T. These series of lines, running upward to the left, are called *adiabatic humidification lines* or *adiabatic saturation lines.* Since c_S contains the term H, the adiabatic lines are not quite straight when plotted on the humidity chart.

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If a given gas mixture at T_1 and H_1 is contacted for a sufficiently long time in an adiabatic saturator, it will leave saturated at H_{S1} and T_{S1}. The values of H_{S1} and T_{S1} are determined by following the adiabatic saturation line going through point T_1, H_1 until it intersects the 100% saturation line. If contact is not sufficient, the leaving mixture will be at a percentage saturation less than 100% but on the same line.

EXAMPLE 9.3-3. Adiabatic Saturation of Air

An air stream at 87.8°C having a humidity $H = 0.030$ kg H_2O/kg dry air is contacted in an adiabatic saturator with water. It is cooled and humidified to 90% saturation.

a. What are the final values of H and T?

b. For 100% saturation, what would be the values of H and T?

Solution: For part (a), the point $H = 0.030$ and $T = 87.8$°C is located on the humidity chart. The adiabatic saturation curve through this point is followed upward to the left until it intersects the 90% line at 42.5°C and $H = 0.0500$ kg H_2O/kg dry air.

For part (b), the same line is followed to 100% saturation, where $T = 40.5$°C and $H = 0.0505$ kg H_2O/kg dry air.

Wet Bulb Temperature

The adiabatic saturation temperature is the steady-state temperature attained when a large amount of water is contacted by the entering gas. The *wet bulb temperature* is the steady-state nonequilibrium temperature reached when a small amount of water is contacted under adiabatic conditions by a continuous stream of gas. Since the amount of liquid is small, the temperature and humidity of the gas are not changed, contrary to the case of adiabatic saturation, where the temperature and humidity of the gas are changed.

The method used to measure the wet bulb temperature is illustrated in Fig. 9.3-4, where a thermometer is covered by a wick or cloth. The wick is kept wet by water and is immersed in a flowing stream of air–water vapor having a temperature of T (dry bulb temperature) and humidity H. At steady state, water is evaporating to the gas stream. The wick and water are cooled to T_W and stay at this constant temperature. The latent heat of evaporation is exactly balanced by the convective heat flowing from the gas stream at T to the wick at a lower temperature T_W.

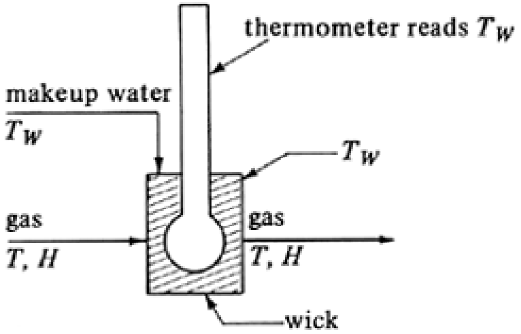

Figure 9.3-4. Measurement of wet bulb temperature.

A heat balance on the wick can be made. The datum temperature is taken at T_W. The amount of heat lost by vaporization, neglecting the small sensible heat change of the vaporized liquid and radiation, is

Equation 9.3-12.

$$q = M_A N_A \lambda_W A$$

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where q is kW (kJ/s), M_A is the molecular weight of water, N_A is kg mol H_2O evaporating/s · m^2, A is surface area m^2, and λ_W is the latent heat of vaporization at T_W in kJ/kg H_2O. In English units, q is btu/h, N_A is lb mol/h · ft^2, and λ_W is btu/lb_m H_2O. The flux N_A is

Equation 9.3-13.

$$N_A = \frac{k'_y}{x_{BM}}(y_W - y) = k_y(y_W - y)$$

where K'_y is the mass-transfer coefficient in kg mol/s · m^2 · mol frac, x_{BM} is the log mean inert mole fraction of the air, y_W is the mole fraction of water vapor in the gas at the surface, and y is the mole fraction in the gas. For a dilute mixture $x_{BM} \cong 1.0$ and $k'_y \cong k_y$. The relation between H and y is

Equation 9.3-14.

$$y = \frac{H/M_A}{1/M_B + H/M_A}$$

where M_B is the molecular weight of air and M_A the molecular weight of H_2O. Since H is small, as an approximation,

Equation 9.3-15.

$$y \cong \frac{HM_B}{M_A}$$

Substituting Eq. (9.3-15) into (9.3-13) and then substituting the resultant into Eq. (9.3-12),

Equation 9.3-16.

$$q = M_B k_y \lambda_W (H_W - H)A$$

The rate of convective heat transfer from the gas stream at T to the wick at T_W is

Equation 9.3-17.

$$q = h(T - T_W)A$$

where h is the heat-transfer coefficient in kW/m^2 · K (btu/h · ft^2 · °F).

Equating Eq. (9.3-16) to (9.3-17) and rearranging,

Equation 9.3-18.

$$\frac{H - H_W}{T - T_W} = -\frac{h/M_B k_y}{\lambda_W}$$

Experimental data on the value of $h/M_B k_y$, called the *psychrometric ratio,* show that for water vapor–air mixtures, the value is approximately 0.96–1.005. Since this value is close to the value of c_S in Eq. (9.3-11), approximately 1.005, Eqs. (9.3-18) and (9.3-11) are almost the same. This means that the adiabatic saturation lines can also be used for wet bulb lines with reasonable accuracy. (Note that this is only true for water vapor and not for other vapors, such as benzene.) Hence, the wet bulb determination is often used to measure the humidity of an air–water vapor mixture.

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EXAMPLE 9.3-4. Wet Bulb Temperature and Humidity

A water vapor–air mixture having a dry bulb temperature of $T = 60°C$ is passed over a wet bulb, as shown in Fig. 9.3-4, and the wet bulb temperature obtained is $T_W = 29.5°C$. What is the humidity of the mixture?

Solution: The wet bulb temperature of 29.5°C can be assumed to be the same as the adiabatic saturation temperature T_S, as discussed. Following the adiabatic saturation curve of 29.5°C until it reaches the dry bulb temperature of 60°C, the humidity is $H = 0.0135$ kg H_2O/kg dry air.

EQUILIBRIUM MOISTURE CONTENT OF MATERIALS

Introduction

As in other transfer processes, such as mass transfer, the process of drying of materials must be approached from the viewpoint of the equilibrium relationships together with the rate relationships. In most of the drying apparatus discussed in Section 9.2, material is dried in contact with an air–water vapor mixture. The equilibrium relationships between the air–water vapor and the solid material will be discussed in this section.

An important variable in the drying of materials is the humidity of the air in contact with a solid of given moisture content. Suppose that a wet solid containing moisture is brought into contact with a stream of air having a constant humidity H and temperature. A large excess of air is used, so its conditions remain constant. Eventually, after exposure of the solid sufficiently long for equilibrium to be reached, the solid will attain a definite moisture content. This is known as the *equilibrium moisture content* of the material under the specified humidity and temperature of the air. The moisture content is usually expressed on a dry basis as kg of water per kg of moisture-free (bone-dry) solid or kg H_2O/100 kg dry solid; in English units as lb H_2O/100 lb dry solid.

For some solids the value of the equilibrium moisture content depends on the direction from which equilibrium is approached. A different value for the equilibrium moisture content is obtained according to whether a wet sample is allowed to dry by desorption or a dry sample adsorbs moisture by adsorption. For drying calculations it is the desorption equilibrium that is the larger value and is of particular interest.

Experimental Data of Equilibrium Moisture Content for Inorganic and Biological Materials

Typical data for various materials

If the material contains more moisture than its equilibrium value in contact with a gas of a given humidity and temperature, it will dry until it reaches its equilibrium value. If the material contains less moisture than its equilibrium value, it will adsorb water until it reaches its equilibrium value. For air having 0% humidity, the equilibrium moisture value of all materials is zero.

The equilibrium moisture content varies greatly with the type of material for any given percent relative humidity, as shown in Fig. 9.4-1 for some typical materials at room temperature. Nonporous insoluble solids tend to have equilibrium moisture contents which are quite low, as shown for glass wool and kaolin. Certain spongy, cellular materials of organic and biological origin generally show large equilibrium moisture contents. Examples of these in Fig. 9.4-1 are wool, leather, and wood.

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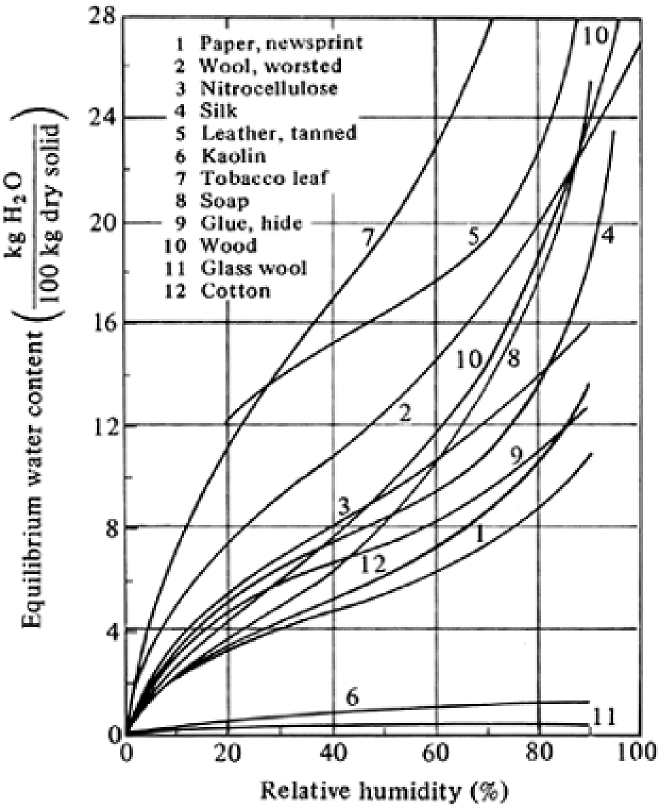

Figure 9.4-1. Typical equilibrium moisture contents of some solids at approximately 298 K (25°C). [From National Research Council, International Critical Tables, Vol. II. New York: McGraw-Hill Book Company, 1929. Reproduced with permission of the National Academy of Sciences.]

Typical food materials

In Fig. 9-4.2 the equilibrium moisture contents for some typical food materials are plotted versus percent relative humidity. These biological materials also show large values for equilibrium moisture content. Data in this figure and in Fig. 9.4-1 for biological materials show that at high percent relative humidities of about 60 to 80%, the equilibrium moisture content increases very rapidly with increases in relative humidity.

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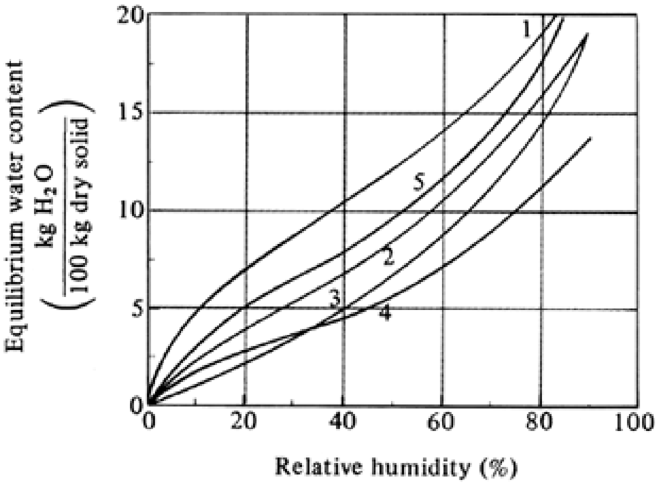

Figure 9.4-2. Typical equilibrium moisture contents of some food materials at approximately 298 K (25°C): (1) macaroni, (2) flour, (3) bread, (4) crackers, (5) egg albumin. [Curve (5) from ref. (E1). Curves (1) to (4) from National Research Council, International Critical Tables, Vol. II. New York: McGraw-Hill Book Company, 1929. Reproduced with permission of the National Academy of Sciences.]

In general, at low relative humidities the equilibrium moisture content is greatest for food materials high in protein, starch, or other high-molecular-weight polymers and lower for food materials high in soluble solids. Crystalline salts and sugars as well as fats generally adsorb small amounts of water.

Effect of temperature

The equilibrium moisture content of a solid decreases somewhat with an increase in temperature. For example, for raw cotton at a relative humidity of 50%, the equilibrium moisture content decreased from 7.3 kg H_2O/100 kg dry solid at 37.8°C (311 K) to about 5.3 at 93.3°C (366.5 K), a decrease of about 25%. Often, for moderate temperature ranges, the equilibrium moisture content will be assumed constant when experimental data are not available at different temperatures.

At present, theoretical understanding of the structure of solids and surface phenomena does not enable us to predict the variation of equilibrium moisture content of various materials from first principles. However, by using models such as those used for adsorption isotherms of multilayers of molecules and others, attempts have been made to correlate experimental data. Henderson (H2) gives an empirical relationship between equilibrium moisture content and percent relative humidity for some agricultural materials. In general, empirical relationships are not available for most materials, and equilibrium moisture contents must be determined experimentally. Also, equilibrium moisture relationships often vary from sample to sample of the same kind of material.

Bound and Unbound Water in Solids

In Fig. 9.4-1, if the equilibrium moisture content of a given material is continued to its intersection with the 100% humidity line, the moisture is called *bound water.* This water in the solid exerts a vapor pressure less than that of liquid water at the same temperature. If such a material contains more water than indicated by intersection with the 100% humidity line, it can still exert a vapor pressure only as high as that of ordinary water at the same temperature. This excess moisture content is called *unbound water,* and it is held primarily in the voids of the solid. Substances containing bound water are often called *hygroscopic materials.*

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As an example, consider curve 10 for wood in Fig. 9.4-1. This intersects the curve for 100% humidity at about 30 kg H_2O/100 kg dry solid. Any sample of wood containing less than 30 kg H_2O/100 kg dry solid contains only bound water. If the wood sample contained 34 kg H_2O/100 kg dry solid, 4 kg H_2O would be unbound and 30 kg H_2O bound per 100 kg dry solid.

The bound water in a substance may exist under several different conditions. Moisture in cell or fiber walls may have solids dissolved in it and have a lower vapor pressure. Liquid water in capillaries of very small diameter will exert a lowered vapor pressure because of the concave curvature of the surface. Water in natural organic materials is in chemical and physical–chemical combination.

Free and Equilibrium Moisture of a Substance

Free moisture content in a sample is the moisture above the equilibrium moisture content. This free moisture is the moisture that can be removed by drying under the given percent relative humidity. For example, in Fig. 9.4-1 silk has an equilibrium moisture content of 8.5 kg H_2O/100 kg dry material in contact with air of 50% relative humidity and 25°C. If a sample contains 10 kg H_2O/100 kg dry material, only 10.0 − 8.5, or 1.5, kg H_2O/100 kg dry material is removable by drying; this is the free moisture of the sample under these drying conditions.

In many texts and references, the moisture content is given as percent moisture on a dry basis. This is exactly the same as the kg H_2O/100 kg dry material multiplied by 100.

RATE-OF-DRYING CURVES

Introduction and Experimental Methods

Introduction

In the drying of various types of process materials from one moisture content to another, it is usually desired to estimate the size of dryer needed, the various operating conditions of humidity and temperature for the air used, and the time needed to perform the amount of drying required. As discussed in Section 9.4, equilibrium moisture contents of various materials cannot be predicted and must be determined experimentally. Similarly, since our knowledge of the basic mechanisms of rates of drying is quite incomplete, it is necessary in most cases to obtain some experimental measurements of drying rates.

Experimental determination of rate of drying

To experimentally determine the rate of drying for a given material, a sample is usually placed on a tray. If it is a solid material it should fill the tray so that only the top surface is exposed to the drying air stream. By suspending the tray from a balance in a cabinet or duct through which the air is flowing, the loss in weight of moisture during drying can be determined at different intervals without interrupting the operation.

In doing batch-drying experiments, certain precautions should be observed to obtain usable data under conditions that closely resemble those to be used in the large-scale operations. The sample should not be too small in weight and should be supported in a tray or frame similar to the large-scale one. The ratio of drying to nondrying surface (insulated surface) and the bed depth should be similar. The velocity, humidity, temperature, and direction of the air should be the same and constant to simulate drying under constant drying conditions.

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Rate of Drying Curves for Constant-Drying Conditions

Conversion of data to rate-of-drying curve

Data obtained from a batch-drying experiment are usually obtained as *W* total weight of the wet solid (dry solid plus moisture) at different times *t* hours in the drying period. These data can be converted to rate-of-drying data in the following ways. First, the data are recalculated. If *W* is the weight of the wet solid in kg total water plus dry solid and W_S is the weight of the dry solid in kg,

Equation 9.5-1.

$$X_t = \frac{W - W_S}{W_S} \frac{\text{kg total water}}{\text{kg dry solid}} \left(\frac{\text{lb total water}}{\text{lb dry solid}} \right)$$

For the given constant drying conditions, the equilibrium moisture content X^* kg equilibrium moisture/kg dry solid is determined. Then the free moisture content *X* in kg free water/kg dry solid is calculated for each value of X_t:

Equation 9.5-2.

$$X = X_t - X^*$$

Using the data calculated from Eq. (9.5-2), a plot of free moisture content *X* versus time *t* in h is made, as in Fig. 9.5-1a. To obtain the rate-of-drying curve from this plot, the slopes of the tangents drawn to the curve in Fig. 9.5-1a can be measured, which give values of dX/dt at given values of *t*. The rate *R* is calculated for each point by

Equation 9.5-3.

$$R = -\frac{L_S}{A} \frac{dX}{dt}$$

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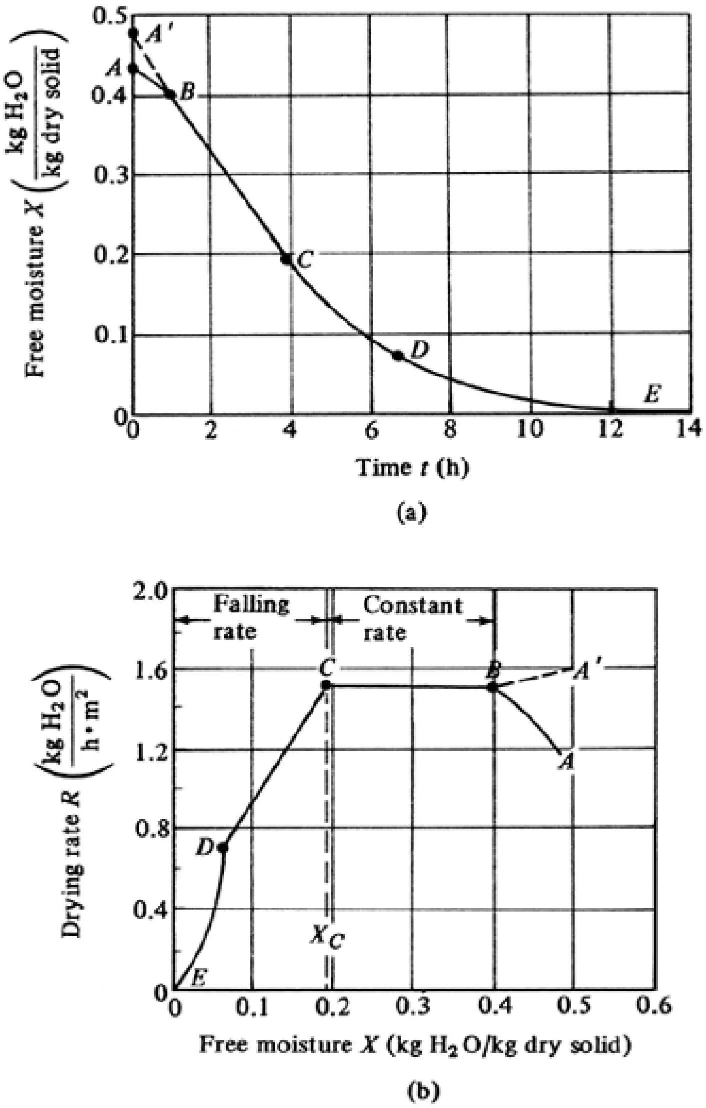

Figure 9.5-1. Typical drying-rate curve for constant drying conditions: (a) plot of data as free moisture versus time, (b) rate of drying curve as rate versus free moisture content.

where R is drying rate in kg H_2O/h · m^2, L_S kg of dry solid used, and A exposed surface area for drying in m^2. In English units, R is lb_m H_2O/h · ft^2, L_S is lb_m dry solid, and A is ft^2. For obtaining R from Fig. 9.5-1a, a value of L_S/A of 21.5 kg/m^2 was used. The drying-rate curve is then obtained by plotting R versus the moisture content, as in Fig. 9.5-1b.

Another method for obtaining the rate-of-drying curve is to first calculate the weight loss ΔX for a Δt time. For example, if $X_1 = 0.350$ at a time $t_1 = 1.68$ h and $x_2 = 0.325$ at a time $t_2 = 2.04$ h, $\Delta X/\Delta t = (0.350 - 0.325)/(2.04 - 1.68)$. Then, using Eq. (9.5-4) and $L_S/A = 21.5$,

$$R = -\frac{L_S}{A}\frac{\Delta X}{\Delta t} = 21.5\left(\frac{0.350 - 0.325}{2.04 - 1.68}\right) = 1.493$$

This rate R is the average over the period 1.68 to 2.04 h and should be plotted at the average concentration $X = (0.350 + 0.325)/2 = 0.338$.

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Plot of rate-of-drying curve

In Fig. 9.5-1b the rate-of-drying curve for constant-drying conditions is shown. At zero time the initial free moisture content is shown at point *A*. In the beginning the solid is usually at a colder temperature than its ultimate temperature, and the evaporation rate will increase. Eventually, at point *B*, the surface temperature rises to its equilibrium value. Alternatively, if the solid is quite hot to start with, the rate may start at point A'.This initial unsteady-state adjustment period is usually quite short and it is often ignored in the analysis of times of drying.

From point *B* to point *C* in Fig. 9.5-1a the line is straight, and hence the slope and rate are constant during this period. This *constant-rate-of-drying period* is shown as line *BC* in Fig. 9.5-1b.

At point *C* on both plots, the drying rate starts to decrease in the *falling-rate period* until it reaches point *D*. In this first falling-rate period, the rate shown as line *CD* in Fig. 9.5-1b is often linear.

At point *D* the rate of drying falls even more rapidly, until it reaches point *E*, where the equilibrium moisture content is X^* and $X = X^* - X^* = 0$. In some materials being dried, the region *CD* may be missing completely, or it may constitute all of the falling-rate period.

Drying in the Constant-Rate Period

Drying of different solids under different constant conditions of drying will often give curves of different shapes in the falling-rate period, but in general the two major portions of the drying-rate curve —constant-rate period and falling-rate period—are present.

In the constant-rate drying period, the surface of the solid is initially very wet and a continuous film of water exists on the drying surface. This water is entirely unbound water and it acts as if the solid were not present. The rate of evaporation under the given air conditions is independent of the solid and is essentially the same as the rate from a free liquid surface. Increased roughness of the solid surface, however, may lead to higher rates than from a flat surface.

If the solid is porous, most of the water evaporated in the constant-rate period is supplied from the interior of the solid. This period continues only as long as the water is supplied to the surface as fast as it is evaporated. Evaporation during this period is similar to that in determining the wet bulb temperature, and in the absence of heat transfer by radiation or conduction, the surface temperature is approximately the same as the wet bulb temperature.

Drying in the Falling-Rate Period

Point *C* in Fig. 9.5-1b is at the *critical free moisture content* X_C. At this point there is insufficient water on the surface to maintain a continuous film of water. The entire surface is no longer wetted, and the wetted area continually decreases in this first falling-rate period until the surface is completely dry, at point *D* in Fig. 9.5-1b.

The second falling-rate period begins at point *D* when the surface is completely dry. The plane of evaporation slowly recedes from the surface. Heat for the evaporation is transferred through the solid to the zone of vaporization. Vaporized water moves through the solid into the air stream.

In some cases no sharp discontinuity occurs at point *D*, and the change from partially wetted to completely dry conditions at the surface is so gradual that no distinct change is detectable.

The amount of moisture removed in the falling-rate period may be relatively small, but the time required may be long. This can be seen in Fig. 9.5-1. The period *BC* for constant-rate drying lasts for about 3.0 h and reduces *X* from 0.40 to about 0.19, a reduction of 0.21 kg H_2O/kg dry solid. The falling-rate period *CE* lasts about 9.0 h and reduces *X* only from 0.19 to 0.

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Moisture Movements in Solids During Drying in the Falling-Rate Period

When drying occurs by evaporation of moisture from the exposed surface of a solid, moisture must move from the depths of the solid to the surface. The mechanisms of this movement affect the drying during the constant-rate and falling-rate periods. Some of the theories advanced to explain the various types of falling-rate curves will be briefly reviewed.

Liquid diffusion theory

According to this theory, diffusion of liquid moisture occurs when there is a concentration difference between the depths of the solid and the surface. This method of transport of moisture is usually found in nonporous solids where single-phase solutions are formed with the moisture, such as in paste, soap, gelatin, and glue. It is also found in drying the last portions of moisture from clay, flour, wood, leather, paper, starches, and textiles. In drying many food materials, the movement of water in the falling-rate period also occurs by diffusion.

The shapes of the moisture-distribution curves in the solid at given times are qualitatively consistent with the unsteady-state diffusion equations given in Chapter 7. The moisture diffusivity D_{AB} usually decreases with decreased moisture content, so that the diffusivities are usually average values over the range of concentrations used. Materials drying in this way are usually said to be drying by diffusion, although the actual mechanisms may be quite complicated. Since the rate of evaporation from the surface is quite fast, that is, the resistance is quite low, compared to the diffusion rate through the solid in the falling-rate period, the moisture content at the surface is at the equilibrium value.

The shape of a diffusion-controlled curve in the falling-rate period is similar to Fig. 9.5-2a. If the initial constant-rate drying is quite high, the first falling-rate period of unsaturated surface evaporation may not appear. If the constant-rate drying is quite low, the period of unsaturated surface evaporation is usually present in region *CD* in Fig. 9.5-1b and the diffusion-controlled curve is in region *DE*. Equations for calculating drying in this period where diffusion controls are given in Section 9.9. Also, Problem 7.1-4 for the drying of clay and Problem 7.1-6 for the drying of wood using diffusion theory are given in the Chapter 7 Problems.

Capillary movement in porous solids

When granular and porous solids such as clays, sand, soil, paint pigments, and minerals are being dried, unbound or free moisture moves through the capillaries and voids of the solids by capillary action, not by diffusion. This mechanism, involving surface tension, is similar to the movement of oil in a lamp wick.

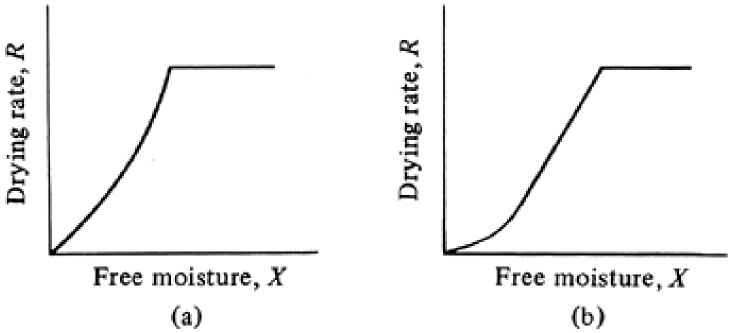

Figure 9.5-2. Typical drying-rate curves: (a) diffusion-controlled falling-rate period, (b) capillary-controlled falling-rate period in a fine porous solid.

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A porous solid contains interconnecting pores and channels of varying pore sizes. As water is evaporated, a meniscus of liquid water is formed across each pore in the depths of the solid. This sets up capillary forces by the interfacial tension between the water and the solid. These capillary forces provide the driving force for moving water through the pores to the surface. Small pores develop greater forces than do large pores.

At the beginning of the falling-rate period at point *C* in Fig. 9.5-1b, the water is being brought to the surface by capillary action, but the surface layer of water starts to recede below the surface. Air rushes in to fill the voids. As the water is continuously removed, a point is reached where there is insufficient water left to maintain continuous films across the pores, and the rate of drying suddenly decreases at the start of the second falling-rate period at point *D*. Then the rate of diffusion of water vapor in the pores and rate of conduction of heat in the solid may become the main factors in drying.

In fine pores in solids, the rate-of-drying curve in the second falling-rate period may conform to the diffusion law; the curve is concave upward, as shown in Fig. 9.5-2b. For very porous solids, such as a bed of sand, where the pores are large, the rate-of-drying curve in the second falling-rate period is often straight, and hence the diffusion equations do not apply.

Effect of shrinkage

A factor often greatly affecting the drying rate is the shrinkage of the solid as moisture is removed. Rigid solids do not shrink appreciably, but colloidal and fibrous materials such as vegetables and other foodstuffs do undergo shrinkage. The most serious effect is that there may be developed a hard layer on the surface which is impervious to the flow of liquid or vapor moisture and slows the drying rate; examples are clay and soap. In many foodstuffs, if drying occurs at too high a temperature, a layer of closely packed, shrunken cells, which are sealed together, forms at the surface. This presents a barrier to moisture migration and is known as *case hardening*. Another effect of shrinkage is to cause the material to warp and change its structure. This can happen in drying wood.

Sometimes, to decrease these effects of shrinkage, it is desirable to dry with moist air. This decreases the rate of drying so that the effects of shrinkage on warping or hardening at the surface are greatly reduced.

CALCULATION METHODS FOR CONSTANT-RATE DRYING PERIOD

Method Using Experimental Drying Curve

Introduction

Probably the most important factor in drying calculations is the length of time required to dry a material from a given initial free moisture content X_1 to a final moisture content X_2. For drying in the constant-rate period, we can estimate the time needed by using experimental batch-drying curves or by using predicted mass- and heat-transfer coefficients.

Method using drying curve

To estimate the time of drying for a given batch of material, the best method is based on actual experimental data obtained under conditions where the feed material, relative exposed surface area, gas velocity, temperature, and humidity are essentially the same as in the final drier. Then the time required for the constant-rate period can be determined directly from the drying curve of free moisture content versus time.

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EXAMPLE 9.6-1. Time of Drying from Drying Curve

A solid whose drying curve is represented by Fig. 9.5-1a is to be dried from a free moisture content X_1 = 0.38 kg H_2O/kg dry solid to X_2 = 0.25 kg H_2O/kg dry solid. Estimate the time required.

Solution: From Fig. 9.5-1a for X_1 = 0.38, t_1 is read off as 1.28 h. For X_2 = 0.25, t_2 = 3.08 h. Hence, the time required is

$$t = t_2 - t_1 = 3.08 - 1.28 = 1.80 \text{ h.}$$

Method using rate-of-drying curve for constant-rate period

Instead of using the drying curve, the rate-of-drying curve can be used. The drying rate R is defined by Eq. (9.5-3) as

Equation 9.5-3.

$$R = -\frac{L_S}{A}\frac{dX}{dt}$$

This can be rearranged and integrated over the time interval for drying from X_1 at t_1 = 0 to X_2 at t_2 = t:

Equation 9.6-1.

$$t = \int_{t_1=0}^{t_2=t} dt = \frac{L_S}{A}\int_{X_2}^{X_1}\frac{dX}{R}$$

If the drying takes place within the constant-rate period, so that both X_1 and X_2 are greater than the critical moisture content X_C, then R = constant = R_C. Integrating Eq. (9.6-1) for the constant-rate period,

Equation 9.6-2.

$$t = \frac{L_S}{AR_C}(X_1 - X_2)$$

EXAMPLE 9.6-2. Drying Time from Rate-of-Drying Curve

Repeat Example 9.6-1 but use Eq. (9.6-2) and Fig. 9.5-1b.

Solution: As stated previously, a value of 21.5 for L_S/A was used to prepare Fig. 9.5-1b from 9.5-1a. From Fig. 9.5-1b, R_C = 1.51 kg $H_2O/h \cdot m^2$. Substituting into Eq. (9.6-2),

$$t = \frac{L_S}{AR_C}(X_1 - X_2) = \frac{21.5}{1.51}(0.38 - 0.25) = 1.85 \text{ h}$$

This is close to the value of 1.80 h in Example 9.6-1.

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Method Using Predicted Transfer Coefficients for Constant-Rate Period

Introduction

In the constant-rate period of drying, the surfaces of the grains of solid in contact with the drying air flow remain completely wetted. As stated previously, the rate of evaporation of moisture under a given set of air conditions is independent of the type of solid and is essentially the same as the rate of evaporation from a free liquid surface under the same conditions. However, surface roughness may increase the rate of evaporation.

During this constant-rate period, the solid is so wet that the water acts as if the solid were not there. The water evaporated from the surface is supplied from the interior of the solid. The rate of evaporation from a porous material occurs by the same mechanism as that occurring at a wet bulb thermometer, which is essentially constant-rate drying.

Equations for predicting constant-rate drying

Drying of a material occurs by mass transfer of water vapor from the saturated surface of the material through an air film to the bulk gas phase or environment. The rate of moisture movement within the solid is sufficient to keep the surface saturated. The rate of removal of the water vapor (drying) is controlled by the rate of heat transfer to the evaporating surface, which furnishes the latent heat of evaporation for the liquid. At steady state, the rate of mass transfer balances the rate of heat transfer.

To derive the equation for drying, we neglect heat transfer by radiation to the solid surface and also assume no heat transfer by conduction from metal pans or surfaces. Conduction and radiation will be considered in Section 9.8. Assuming only heat transfer to the solid surface by convection from the hot gas to the surface of the solid and mass transfer from the surface to the hot gas (Fig. 9.6-1), we can write equations which are the same as those for deriving the wet bulb temperature T_W in Eq. (9.3-18).

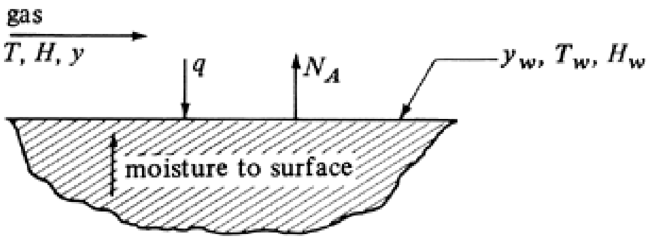

Figure 9.6-1. Heat and mass transfer in constant-rate drying.

The rate of convective heat transfer q in W (J/s, btu/h) from the gas at T°C (°F) to the surface of the solid at T_W°C, where $(T - T_W)$°C = $(T - T_W)$ K is

Equation 9.6-3.

$$q = h(T - T_W)A$$

where h is the heat-transfer coefficient in W/m^2 · K (btu/h · ft^2 · °F) and A is the exposed drying area in m^2 (ft^2). The equation of the flux of water vapor from the surface is the same as Eq. (9.3-13) and is

Equation 9.6-4.

$$N_A = k_y(y_W - y)$$

Using the approximation from Eq. (9.3-15) and substituting into Eq. (9.6-4),

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Equation 9.6-5.

$$N_A = k_y \frac{M_B}{M_A}(H_W - H)$$

The amount of heat needed to vaporize N_A kg mol/s · m^2 (lb mol/h · ft^2) water, neglecting the small sensible heat changes, is the same as Eq. (9.3-12):

Equation 9.6-6.

$$q = M_A N_A \lambda_W A$$

where λ_W is the latent heat at T_W in J/kg (btu/lb$_m$).

Equating Eqs. (9.6-3) and (9.6-6) and substituting Eq. (9.6-5) for N_A,

Equation 9.6-7.

$$R_C = \frac{q}{A\lambda_W} = \frac{h(T - T_W)}{\lambda_W} = k_y M_B(H_W - H)$$

Equation (9.6-7) is identical to Eq. (9.3-18) for the wet bulb temperature. Hence, in the absence of heat transfer by conduction and radiation, the temperature of the solid is at the wet bulb temperature of the air during the constant-rate drying period. Thus, the rate of drying R_C can be calculated using the heat-transfer equation $h(T - T_W)/\lambda_W$ or the mass-transfer equation $k_y M_B(H_W - H)$. However, it has been found more reliable to use the heat-transfer equation (9.6-8), since an error in determining the interface temperature T_W at the surface affects the driving force $(T - T_W)$ much less than it affects $(H_W - H)$.

Equation 9.6-8.

$$R_C \text{ kg H}_2\text{O/h} \cdot \text{m}^2 = \frac{h}{\lambda_W}(T - T_W {}^\circ\text{C})(3600) \qquad \text{(SI)}$$

$$R_C \text{ lb}_\text{m} \text{ H}_2\text{O/h} \cdot \text{ft}^2 = \frac{h}{\lambda_W}(T - T_W\ {}^\circ\text{F}) \qquad \text{(English)}$$

To predict R_C in Eq. (9.6-8), the heat-transfer coefficient must be known. For the case where the air is flowing parallel to the drying surface, Eq. (4.6-3) can be used for air. However, because the shape of the leading edge of the drying surface causes more turbulence, the following can be used for an air temperature of 45–150°C and a mass velocity G of 2450–29 300 kg/h · m^2 (500–6000 lb$_m$/h · ft^2) or a velocity of 0.61–7.6 m/s (2–25 ft/s):

Equation 9.6-9.

$$h = 0.0204G^{0.8} \qquad \text{(SI)}$$

$$h = 0.0128G^{0.8} \qquad \text{(English)}$$

where in SI units G is $v\rho$ kg/h · m^2 and h is W/m^2 · K. In English units, G is in lb$_m$/h · ft^2 and h in btu/h · ft^2 · °F. When air flows perpendicular to the surface for a G of 3900–19 500 kg/h · m^2 or a velocity of 0.9–4.6 m/s (3–15 ft/s),

Equation 9.6-10.

$$h = 1.17G^{0.37} \qquad \text{(SI)}$$

$$h = 0.37G^{0.37} \qquad \text{(English)}$$

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Equations (9.6-8)–(9.6-10) can be used to estimate the rate of drying during the constant-rate period. However, when possible, experimental measurements of the drying rate are preferred.

To estimate the time of drying during the constant-rate period, substituting Eq. (9.6-7) into (9.6-2),

Equation 9.6-11.

$$t = \frac{L_S \lambda_W (X_1 - X_2)}{Ah(T - T_W)} = \frac{L_S (X_1 - X_2)}{A k_y M_B (H_W - H)}$$

EXAMPLE 9.6-3. Prediction of Constant-Rate Drying

An insoluble wet granular material is dried in a pan 0.457 × 0.457 m (1.5 × 1.5 ft) and 25.4 mm deep. The material is 25.4 mm deep in the pan, and the sides and bottom can be considered to be insulated. Heat transfer is by convection from an air stream flowing parallel to the surface at a velocity of 6.1 m/s (20 ft/s). The air is at 65.6°C (150°F) and has a humidity of 0.010 kg H_2O/kg dry air. Estimate the rate of drying for the constant-rate period using SI and English units.

***Solution*:** For a humidity $H = 0.010$ and dry bulb temperature of 65.6°C, using the humidity chart, Fig. 9.3-2, the wet bulb temperature T_W is found to be 28.9°C (84°F) and $H_W = 0.026$ by following the adiabatic saturation line (the same as the wet bulb line) to the saturated humidity. Using Eq. (9.3-7) to calculate the humid volume,

$$\begin{aligned} v_H &= (2.83 \times 10^{-3} + 4.56 \times 10^{-3} H)T \\ &= (2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.01)(273 + 65.6) \\ &= 0.974 \text{ m}^3/\text{kg dry air} \end{aligned}$$

The density for 1.0 kg dry air + 0.010 kg H_2O is

$$\rho = \frac{1.0 + 0.010}{0.974} = 1.037 \text{ kg/m}^3 \ (0.0647 \text{ lb}_m/\text{ft}^3)$$

The mass velocity G is

$$G = v\rho = 6.1(3600)(1.037) = 22\,770 \text{ kg/h} \cdot \text{m}^2$$

$$G = v\rho = 20(3600)(0.0647) = 4660 \text{ lb}_m/\text{h} \cdot \text{ft}^2$$

Using Eq. (9.6-9),

$$h = 0.0204 G^{0.8} = 0.0204(22\,770)^{0.8} = 62.45 \text{ W/m}^2 \cdot \text{K}$$

$$h = 0.0128 G^{0.8} = 0.0128(4660)^{0.8} = 11.01 \text{ btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

At $T_W = 28.9$°C (84°F), $\lambda_W = 2433$ kJ/kg (1046 btu/lb_m) from steam tables.

Substituting into Eq. (9.6-8) and noting that (65.6 − 28.9)°C = (65.6 − 28.9) K,

$$\begin{aligned} R_C &= \frac{h}{\lambda_W}(T - T_W)(3600) = \frac{62.45}{2433 \times 1000}(65.6 - 28.9)(3600) \\ &= 3.39 \text{ kg/h} \cdot \text{m}^2 \end{aligned}$$

$$R_C = \frac{11.01}{1046}(150 - 84) = 0.695 \text{ lb}_m/\text{h} \cdot \text{ft}^2$$

The total evaporation rate for a surface area of 0.457 × 0.457 m^2 is

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$$\text{total rate} = R_C A = 3.39(0.457 \times 0.457) = 0.708 \text{ kg H}_2\text{O/h}$$

$$= 0.695(1.5 \times 1.5) = 1.564 \text{ lb}_\text{m} \text{ H}_2\text{O/h}$$

Effect of Process Variables on Constant-Rate Period

As stated previously, experimental measurements of the drying rate are usually preferred over using the equations for prediction. However, these equations are quite helpful in predicting the effect of changing the drying-process variables when limited experimental data are available.

Effect of air velocity

When conduction and radiation heat transfer are not present, the rate R_C of drying in the constant-rate region is proportional to h and hence to $G^{0.8}$ as given by Eq. (9.6-9) for air flow parallel to the surface. The effect of gas velocity is less important when radiation and conduction are present.

Effect of gas humidity

If the gas humidity H is decreased for a given T of the gas, then from the humidity chart the wet bulb temperature T_W will decrease. Then, using Eq. (9.6-7), R_C will increase. For example, if the original conditions are R_{C1}, T_1, T_{W1}, H_1, and H_{W1}, then if H_1 is changed to H_2 and H_{W1} is changed to H_{W2}, R_{C2} becomes

Equation 9.6-12.

$$R_{C2} = R_{C1} \frac{T - T_{W2}}{T - T_{W1}} \frac{\lambda_{W1}}{\lambda_{W2}} = R_{C1} \frac{H_{W2} - H_2}{H_{W1} - H_1}$$

However, since $\lambda_{W1} \cong \lambda_{W2}$,

Equation 9.6-13.

$$R_{C2} = R_{C1} \frac{T - T_{W2}}{T - T_{W1}} = R_{C1} \frac{H_{W2} - H_2}{H_{W1} - H_1}$$

Effect of gas temperature

If the gas temperature T is increased, T_W is also increased somewhat, but not as much as the increase in T. Hence, R_C increases as follows:

Equation 9.6-14.

$$R_{C2} = R_{C1} \frac{T_2 - T_{W2}}{T_1 - T_{W1}} = R_{C1} \frac{H_{W2} - H_2}{H_{W1} - H_1}$$

Effect of thickness of solid being dried

For heat transfer by convection only, the rate R_C is independent of the thickness x_1 of the solid. However, the time t for drying between fixed moisture contents X_1 and X_2 will be directly proportional to the thickness x_1. This is shown by Eq. (9.6-2), where increasing the thickness with a constant A will directly increase the amount of L_S kg dry solid.

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Experimental effect of process variables

Experimental data tend to bear out the conclusions reached on the effects of material thickness, humidity, air velocity, and $T - T_W$.

CALCULATION METHODS FOR FALLING-RATE DRYING PERIOD

Method Using Numerical Integration

In the falling-rate drying period, as shown in Fig. 9.5-1b, the rate of drying R is not constant but decreases when drying proceeds past the critical free moisture content X_C. When the free moisture content X is zero, the rate drops to zero.

The time of drying for any region between X_1 and X_2 has been given by Eq. (9.6-1):

Equation 9.6-1.

$$t = \frac{L_S}{A}\int_{X_2}^{X_1}\frac{dX}{R}$$

If the rate is constant, Eq. (9.6-1) can be integrated to give Eq. (9.6-2). However, in the falling-rate period, R varies. For any shape of falling-rate drying curve, Eq. (9.6-1) can be integrated by plotting $1/R$ versus X and determining the area under the curve using graphical integration or numerical integration with a spreadsheet. (See Section 1.8 for methods of numerical integration.)

EXAMPLE 9.7-1. Numerical Integration in Falling-Rate Period

A batch of wet solid whose drying-rate curve is represented by Fig. 9.5-1b is to be dried from a free moisture content of $X_1 = 0.38$ kg H_2O/kg dry solid to $X_2 = 0.04$ kg H_2O/kg dry solid. The weight of the dry solid is $L_S = 399$ kg dry solid and $A = 18.58$ m^2 of top drying surface. Calculate the time for drying. Note that $L_S/A = 399/18.58 = 21.5$ kg/m^2.

***Solution*:** From Fig. 9.5-1b, the critical free moisture content is $X_C = 0.195$ kg H_2O/kg dry solid. Hence, the drying occurs in the constant-rate and falling-rate periods.

For the constant-rate period, $X_1 = 0.38$ and $X_2 = X_C = 0.195$. From Fig. 9.5-1b, $R_C = 1.51$ kg H_2O/hm^2. Substituting into Eq. (9.6-2),

$$t = \frac{L_S}{AR_C}(X_1 - X_2) = \frac{399(0.38 - 0.195)}{(18.58)(1.51)} = 2.63\ \text{h}$$

For the falling-rate period, reading values of R for various values of X from Fig. 9.5-1b, the following table is prepared:

X	R	$1/R$	X	R	$1/R$
0.195	1.51	0.663	0.065	0.71	1.41
0.150	1.21	0.826	0.050	0.37	2.70
0.100	0.90	1.11	0.040	0.27	3.70

To determine this area by numerical integration using a spreadsheet, the calculations are given in the following table:

X	R	$1/R$	ΔX	$(1/R)_{av}$	$(\Delta X)(1/R)_{av}$
0.195	0.51	0.663	0.045	0.745	0.0335

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X	R	$1/R$	ΔX	$(1/R)_{av}$		$(\Delta X)(1/R)_{av}$
0.150	1.21	0.826	0.050	0.969		0.0485
0.100	0.90	1.111	0.035	1.260		0.0441
0.065	0.71	1.408	0.015	2.055		0.0308
0.050	0.37	2.702	0.010	3.203		0.0320
0.040	0.27	3.704		Total	=	0.1889

The area of the first rectangle is the average height (0.663 + 0.826)/2, or 0.745, times the width ΔX (0.195 − 0.150), or 0.045, giving 0.0335. Other values are similarly calculated, giving a total of 0.1889.

Substituting into Eq. (9.6-1),

$$t = \frac{L_S}{A} \int_{X_2}^{X_1} \frac{dX}{R} = \frac{399}{18.58}(0.1889) = 4.06 \text{ h}$$

The total time is 2.63 + 4.06 = 6.69 h.

Calculation Methods for Special Cases in Falling-Rate Region

In certain special cases in the falling-rate region, the equation for the time of drying, Eq. (9.6-1), can be integrated analytically.

Rate is a linear function of X

If both X_1 and X_2 are less than X_C and the rate R is linear in X over this region,

Equation 9.7-1.

$$R = aX + b$$

where a is the slope of the line and b is a constant. Differentiating Eq. (9.7-1) gives $dR = a\,dX$. Substituting this into Eq. (9.6-1),

Equation 9.7-2.

$$t = \frac{L_S}{aA} \int_{R_2}^{R_1} \frac{dR}{R} = \frac{L_S}{aA} \ln \frac{R_1}{R_2}$$

Since $R_1 = aX_1 + b$ and $R_2 = aX_2 + b$,

Equation 9.7-3.

$$a = \frac{R_1 - R_2}{X_1 - X_2}$$

Substituting Eq. (9.7-3) into (9.7-2),

Equation 9.7-4.

$$t = \frac{L_S(X_1 - X_2)}{A(R_1 - R_2)} \ln \frac{R_1}{R_2}$$

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Rate is a linear function through origin

In some cases a straight line from the critical moisture content passing through the origin adequately represents the whole falling-rate period. In Fig. 9.5-1b this would be a straight line from *C* to *E* at the origin. Often, for lack of more detailed data, this assumption is made. Then, for a straight line through the origin, where the rate of drying is directly proportional to the free moisture content,

Equation 9.7-5.

$$R = aX$$

Differentiating, $dX = dR/a$. Substituting into Eq. (9.6-1),

Equation 9.7-6.

$$t = \frac{L_S}{aA}\int_{R_2}^{R_1}\frac{dR}{R} = \frac{L_S}{aA}\ln\frac{R_1}{R_2}$$

The slope a of the line is R_C/X_C, and for $X_1 = X_C$ at $R_1 = R_C$,

Equation 9.7-7.

$$t = \frac{L_S X_C}{AR_C}\ln\frac{R_C}{R_2}$$

Noting also that $R_C/R_2 = X_C/X_2$,

Equation 9.7-8.

$$t = \frac{L_S X_C}{AR_C}\ln\frac{X_C}{X_2}$$

or

Equation 9.7-9.

$$R = R_C\frac{X}{X_C}$$

EXAMPLE 9.7-2. Approximation of Straight Line for Falling-Rate Period

Repeat Example 9.7-1, but as an approximation assume a straight line for the rate *R* versus *X* through the origin from point X_C to $X = 0$ for the falling-rate period.

Solution: $R_C = 1.51$ kg $H_2O/h \cdot m^2$ and $X_C = 0.195$. Drying in the falling-rate region is from X_C to $X_2 = 0.040$. Substituting into Eq. (9.7-8),

$$t = \frac{L_S X_C}{AR_C}\ln\frac{X_C}{X_2} = \frac{399(0.195)}{18.58(1.51)}\ln\frac{0.195}{0.040}$$

$$= 4.39 \text{ h}$$

This value of 4.39 h compares favorably with the value of 4.06 h obtained in Example 9.7-1 by numerical integration.

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COMBINED CONVECTION, RADIATION, AND CONDUCTION HEAT TRANSFER IN CONSTANT-RATE PERIOD

Introduction

In Section 9.6B an equation was derived for predicting the rate of drying in the constant-rate period. Equation (9.6-7) was derived assuming heat transfer to the solid by convection only from the surrounding air to the drying surface. Often the drying is done in an enclosure, where the enclosure surface radiates heat to the drying solid. Also, in some cases the solid may be resting on a metal tray, and heat transfer by conduction through the metal to the bottom of the solid may occur.

Derivation of Equation for Convection, Conduction, and Radiation

In Fig. 9.8-1 a solid material being dried by a stream of air is shown. The total rate of heat transfer to the drying surface is

Equation 9.8-1.

$$q = q_C + q_R + q_K$$

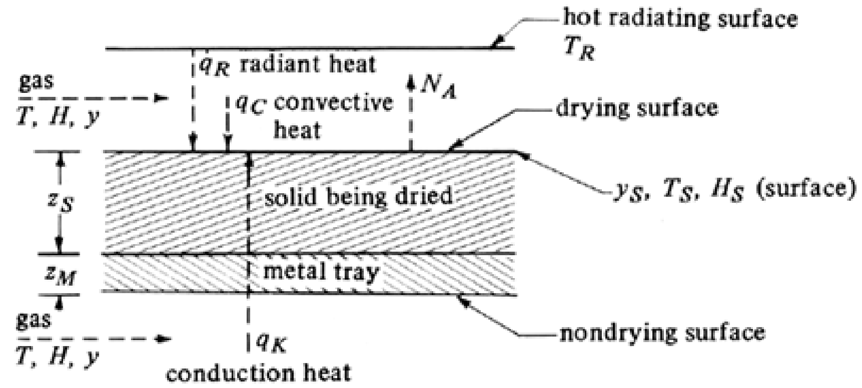

Figure 9.8-1. Heat and mass transfer in drying a solid from the top surface.

where q_C is the convective heat transfer from the gas at T°C to the solid surface at T_S°C in W (J/s), q_R is the radiant heat transfer from the surface at T_R to T_S in W (J/s), and q_K is the rate of heat transfer by conduction from the bottom in W. The rate of convective heat transfer is similar to Eq. (9.6-3) and is as follows, where $(T - T_S)$°C = $(T - T_S)$ K:

Equation 9.8-2.

$$q_C = h_C(T - T_S)A$$

where A is the exposed surface area in m^2. The radiant heat transfer is

Equation 9.8-3.

$$q_R = h_R(T_R - T_S)A$$

where h_R is the radiant-heat-transfer coefficient defined by Eq. (4.10-10):

Equation 4.10-10.

$$h_R = \varepsilon(5.676)\frac{\left(\frac{T_R}{100}\right)^4 - \left(\frac{T_S}{100}\right)^4}{T_R - T_S}$$

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Note that in Eq. (4.10-10) T_R and T_S are in K. For the heat transfer by conduction from the bottom, the heat transfer is first by convection from the gas to the metal plate, then by conduction through the metal, and finally by conduction through the solid. Radiation to the bottom of the tray is often small if the tray is placed above another tray, and it will be neglected here. Also, if the gas temperatures are not too high, radiation from the top surface to the tray will be small. Hence, the heat by radiation should not be overemphasized. The heat by conduction is

Equation 9.8-4.

$$q_K = U_K(T - T_S)A$$

Equation 9.8-5.

$$U_K = \frac{1}{1/h_C + z_M/k_M + z_S/k_S}$$

where z_M is the metal thickness in m, k_M the metal thermal conductivity in W/m · K, z_S the solid thickness in m, and k_S the solid thermal conductivity. The value of h_C in Eq. (9.8-4) is assumed to be the same as in Eq. (9.8-2).

The equation for the rate of mass transfer is similar to Eq. (9.6-5) and is

Equation 9.8-6.

$$N_A = k_y \frac{M_B}{M_A}(H_S - H)$$

Also, rewriting Eq. (9.6-6),

Equation 9.8-7.

$$q = M_A N_A \lambda_S A$$

Combining Eqs. (9.8-1), (9.8-2), (9.8-3), (9.8-4), (9.8-6), and (9.8-7),

Equation 9.8-8.

$$R_C = \frac{q}{A\lambda_S} = \frac{(h_C + U_K)(T - T_S) + h_R(T_R - T_S)}{\lambda_S} = k_y M_B(H_S - H)$$

This equation can be compared to Eq. (9.6-7), which gives the wet bulb temperature T_W when radiation and conduction are absent. Equation (9.8-8) gives surface temperature T_S greater than the wet bulb temperature T_W. Equation (9.8-8) must also intersect the saturated humidity line at T_S and H_S, and $T_S > T_W$ and $H_S > H_W$. The equation must be solved by trial and error.

To facilitate solution of Eq. (9.8-8), it can be rearranged (T1) in the following way:

Equation 9.8-9.

$$\frac{(H_S - H)\lambda_S}{h_C/k_y M_B} = \left(1 + \frac{U_K}{h_C}\right)(T - T_S) + \frac{h_R}{h_C}(T_R - T_S)$$

The ratio h_C/k_yM_B was shown in the wet bulb derivation of Eq. (9.3-18) to be approximately c_S in Eq. (9.3-6):

Equation 9.3-6.

$$c_S = (1.005 + 1.88H)10^3 \text{ J/kg} \cdot \text{K}$$

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EXAMPLE 9.8-1. Constant-Rate Drying When Radiation, Conduction, and Convection Are Present

An insoluble granular material wet with water is being dried in a pan 0.457 × 0.457 m and 25.4 mm deep. The material is 25.4 mm deep in the metal pan, which has a metal bottom of thickness z_M = 0.610 mm having a thermal conductivity k_M = 43.3 W/m · K. The thermal conductivity of the solid can be assumed as k_S = 0.865 W/m · K. Heat transfer is by convection from an air stream flowing parallel to the top drying surface and the bottom metal surface at a velocity of 6.1 m/s and having a temperature of 65.6°C and humidity H = 0.010 kg H_2O/kg dry air. The top surface also receives direct radiation from steam-heated pipes whose surface temperature T_R = 93.3°C. The emissivity of the solid is ε = 0.92. Estimate the rate of drying for the constant-rate period.

Solution: Some of the given values are as follows:

$$T = 65.6°\text{C}, \quad z_S = 0.0254\text{ m}, \quad k_M = 43.3, \quad k_S = 0.865$$

$$z_M = 0.00061\text{ m} \quad \varepsilon = 0.92, \quad H = 0.010$$

The velocity, temperature, and humidity of the air are the same as in Example 9.6-3 and the convective coefficient was predicted as h_C = 62.45 W/m^2 · K.

The solution of Eq. (9.8-9) is by trial and error. The temperature T_S will be above the wet bulb temperature of T_W = 28.9°C and will be estimated as T_S = 32.2°C. Then λ_S = 2424 kJ/kg from the steam tables. To predict h_R from Eq. (4.10-10) for ε = 0.92, T_l = 93.3 + 273.2 = 366.5 K, and T_2 = 32.2 + 273.2 = 305.4 K,

$$h_R = (0.92)(5.676)\frac{(366.5/100)^4 - (305.4/100)^4}{366.5 - 305.4} = 7.96\text{ W/m}^2\cdot\text{K}$$

Using Eq. (9.8-5),

$$U_K = \frac{1}{1/h_C + z_M/k_M + z_S/k_S} = \frac{1}{1/62.45 + 0.00061/43.3 + 0.0254/0.865}$$
$$= 22.04\text{ W/m}^2\cdot\text{K}$$

From Eq. (9.3-6),

$$c_S = (1.005 + 1.88H)10^3 = (1.005 + 1.88 \times 0.010)10^3$$
$$= 1.024 \times 10^3\text{ J/kg}\cdot\text{K}$$

This can be substituted for (h_C/k_yM_B) into Eq. (9.8-9). Also, substituting other knowns,

Equation 9.8-10.

$$\frac{(H_S - 0.01)\lambda_S}{1.024 \times 10^3} = (1 + 22.04/62.45)(65.6 - T_S) + (7.96/62.45)(93.3 - T_S)$$
$$= 1.353(65.6 - T_S) + 0.1275(93.3 - T_S)$$

For T_S assumed as 32.2°C, λ_S = 2424 × 10^3 J/kg. From the humidity chart for T_S = 32.2°C, the saturation humidity H_S = 0.031. Substituting into Eq. (9.8-10) and solving for T_S,

$$\frac{(0.031 - 0.010)(2424 \times 10^3)}{1.024 \times 10^3} = 1.353(65.6 - T_S) + 0.1275(93.3 - T_S)$$

$$T_S = 34.4°\text{C}$$

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For the second trial, assuming that $T_S = 32.5°\text{C}$, $\lambda_S = 2423 \times 10^3$, and H_S from the humidity chart at saturation is 0.032. Substituting into Eq. (9.8-10) while assuming that h_R does not change appreciably, a value of $T_S = 32.8°\text{C}$ is obtained. Hence, the final value is 32.8°C. This is 3.9°C greater than the wet bulb temperature of 28.9°C in Example 9.6-3, where radiation and conduction were absent.

Using Eq. (9.8-8),

$$\begin{aligned} R_C &= \frac{(h_C + U_K)(T - T_S) + h_R(T_R - T_S)}{\lambda_S}(3600) \\ &= \frac{(62.45 + 22.04)(65.6 - 32.8) + 7.96(93.3 - 32.8)}{2423 \times 10^3}(3600) \\ &= 4.83 \text{ kg/h} \cdot \text{m}^2 \end{aligned}$$

This compares favorably with 3.39 kg/h · m^2 for Example 9.6-3 for no radiation or conduction.

DRYING IN FALLING-RATE PERIOD BY DIFFUSION AND CAPILLARY FLOW

Introduction

In the falling-rate period, the surface of the solid being dried is no longer completely wetted, and the rate of drying steadily falls with time. In Section 9.7 empirical methods were used to predict the time of drying. In one method the actual rate-of-drying curve was numerically or graphically integrated to determine the time of drying.

In another method an approximately straight line from the critical free moisture content to the origin at zero free moisture was assumed. Here the rate of drying was assumed to be a linear function of the free moisture content. The rate of drying R is defined by Eq. (9.5-3):

Equation 9.5-3.

$$R = -\frac{L_S}{A}\frac{dX}{dt}$$

When R is a linear function of X in the falling-rate period,

Equation 9.7-5.

$$R = aX$$

where a is a constant. Equating Eq. (9.7-5) to Eq. (9.5-3),

Equation 9.9-1.

$$R = -\frac{L_S}{A}\frac{dX}{dt} = aX$$

Rearranging,

Equation 9.9-2.

$$\frac{dX}{dt} = -\frac{aA}{L_S}X$$

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In many instances, however, as mentioned briefly in Section 9.5E, the rate of moisture movement in the falling-rate period is governed by the rate of internal movement of the liquid by liquid diffusion or by capillary movement. These two mechanisms of moisture movement will be considered in more detail and the theories related to experimental data in the falling-rate region.

Liquid Diffusion of Moisture in Drying

When liquid diffusion of moisture controls the rate of drying in the falling-rate period, the equations for diffusion described in Chapter 7 can be used. Using the concentrations as X kg free moisture/kg dry solid instead of kg mol moisture/m^3, Fick's second law for unsteady-state diffusion, Eq. (7.10-10), can be written as

Equation 9.9-3.

$$\frac{\partial X}{\partial t} = D_L \frac{\partial^2 X}{\partial x^2}$$

where D_L is the liquid diffusion coefficient in m^2/h and x is distance in the solid in m.

This type of diffusion is often characteristic of relatively slow drying in nongranular materials such as soap, gelatin, and glue, and in the later stages of drying of bound water in clay, wood, textiles, leather, paper, foods, starches, and other hydrophilic solids.

A major difficulty in analyzing diffusion drying data is that the initial moisture distribution is not uniform throughout the solid at the start if a drying period at constant rate precedes this falling-rate period. During diffusion-type drying, the resistance to mass transfer of water vapor from the surface is usually very small, and the diffusion in the solid controls the rate of drying. Then the moisture content at the surface is at the equilibrium value X^*. This means that the free moisture content X at the surface is essentially zero.

Assuming that the initial moisture distribution is uniform at $t = 0$, Eq. (9.9-3) may be integrated by the methods in Chapter 7 to give the following:

Equation 9.9-4.

$$\frac{X_t - X^*}{X_{t1} - X^*} = \frac{X}{X_1} = \frac{8}{\pi^2}\left[e^{-D_L t(\pi/2x_1)^2} + \frac{1}{9}e^{-9D_L t(\pi/2x_1)^2} + \frac{1}{25}e^{-25D_L t(\pi/2x_1)^2} + \cdots\right]$$

where X = average free moisture content at time t h, X_1 = initial free moisture content at $t = 0$, X^* = equilibrium free moisture content, $x_1 = \frac{1}{2}$ the thickness of the slab when drying occurs from the top and bottom parallel faces, and x_1 = total thickness of slab if drying occurs only from the top face.

Equation (9.9-4) assumes that D_L is constant, but D_L is rarely constant; it varies with moisture content, temperature, and humidity. For long drying times, only the first term in Eq. (9.9-4) is significant, and the equation becomes

Equation 9.9-5.

$$\frac{X}{X_1} = \frac{8}{\pi^2}\ e^{-D_L t(\pi/2x_1)^2}$$

Solving for the time of drying,

Equation 9.9-6.

$$t = \frac{4x_1^2}{\pi^2 D_L} \ln \frac{8X_1}{\pi^2 X}$$

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In this equation, if the diffusion mechanism starts at $X = X_C$, then $X_1 = X_C$. Differentiating Eq. (9.9-6) with respect to time and rearranging,

Equation 9.9-7.

$$\frac{dX}{dt} = -\frac{\pi^2 D_L X}{4x_1^2}$$

Multiplying both sides by $-L_S/A$,

Equation 9.9-8.

$$R = -\frac{L_S}{A}\frac{dX}{dt} = \frac{\pi^2 L_S D_L}{4x_1^2 A}X$$

Hence, Eqs. (9.9-7) and (9.9-8) state that when internal diffusion controls for long periods of time, the rate of drying is directly proportional to the free moisture X and the liquid diffusivity, and that the rate of drying is inversely proportional to the thickness squared. Or, stated as the time of drying between fixed moisture limits, the time varies directly as the square of the thickness. The drying rate should be independent of gas velocity and humidity.

EXAMPLE 9.9-1. Drying Slabs of Wood When Diffusion of Moisture Controls

The experimental average diffusion coefficient of moisture in a given wood is 2.97×10^{-6} m^2/h (3.20×10^{-5} ft^2/h). Large planks of wood 25.4 mm thick are dried from both sides by air having a humidity such that the equilibrium moisture content in the wood is $X^* = 0.04$ kg H_2O/kg dry wood. The wood is to be dried from a total average moisture content of $X_{t1} = 0.29$ to $X_t = 0.09$. Calculate the time needed.

Solution: The free moisture content $X_1 = X_{t1} - X^* = 0.29 - 0.04 = 0.25$, $X = X_t - X^* = 0.09 - 0.04 = 0.05$. The half-slab thickness $x_1 = 25.4/(2 \times 1000) = 0.0127$ m. Substituting into Eq. (9.9-6),

$$t = \frac{4x_1^2}{\pi^2 D_L}\ln\frac{8X_1}{\pi^2 X} = \frac{4(0.0127)^2}{\pi^2(2.97 \times 10^{-6})}\ln\frac{8 \times 0.25}{\pi^2 \times 0.05}$$
$$= 30.8 \text{ h}$$

Alternatively, Fig. 5.3-13 for the average concentration in a slab can be used. The ordinate $E_a = X/X_1 = 0.05/0.25 = 0.20$. Reading off the plot a value of $0.56 = D_L t/x_1^2$, substituting, and solving for t,

$$t = \frac{x_1^2(0.56)}{D_L} = \frac{(0.0127)^2(0.56)}{2.97 \times 10^{-6}} = 30.4 \text{ h}$$

Capillary Movement of Moisture in Drying

Water can flow from regions of high concentration to those of low concentration as a result of capillary action rather than by diffusion if the pore sizes of the granular materials are suitable.

The capillary theory (P1) assumes that a packed bed of nonporous spheres contains a void space between the spheres called *pores.* As water is evaporated, capillary forces are set up by the interfacial tension between the water and the solid. These forces provide the driving force for moving the water through the pores to the drying surface.

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A modified form of Poiseuille's equation for laminar flow can be used in conjunction with the capillary-force equation to derive an equation for the rate of drying when flow is by capillary movement. If the moisture movement follows the capillary-flow equations, the rate of drying R will vary linearly with X. Since the mechanism of evaporation during this period is the same as during the constant-rate period, the effects of the variables gas velocity, temperature of the gas, humidity of the gas, and so on, will be the same as for the constant-rate drying period.

The defining equation for the rate of drying is

Equation 9.5-3.

$$R = -\frac{L_S}{A}\frac{dX}{dt}$$

For the rate R varying linearly with X, given previously,

Equation 9.7-9.

$$R = R_C\frac{X}{X_C}$$

Equation 9.7-8.

$$t = \frac{L_S X_C}{A R_C}\ln\frac{X_C}{X_2}$$

We define t as the time when $X = X_2$ and

Equation 9.9-9.

$$L_S = x_1 A \rho_S$$

where ρ_S = solid density kg dry solid/m^3. Substituting Eq. (9.9-9) and $X = X_2$ into Eq. (9.7-8),

Equation 9.9-10.

$$t = \frac{x_1 \rho_S X_C}{R_C}\ln\frac{X_C}{X}$$

Substituting Eq. (9.6-7) for R_C,

Equation 9.9-11.

$$t = \frac{x_1 \rho_S \lambda_W X_C}{h(T - T_W)}\ln\frac{X_C}{X}$$

Hence, Eqs. (9.9-10) and (9.9-11) state that when capillary flow controls in the falling-rate period, the rate of drying is inversely proportional to the thickness. The time of drying between fixed moisture limits varies directly as the thickness and depends upon the gas velocity, temperature, and humidity.

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Comparison of Liquid Diffusion and Capillary Flow

To determine the mechanism of drying in the falling-rate period, the experimental data obtained for moisture content at various times using constant drying conditions are often analyzed as follows. The *unaccomplished moisture change,* defined as the ratio of free moisture present in the solid after drying for *t* hours to the total free moisture content present at the start of the falling-rate period, X/X_C, is plotted versus time on semilog paper. If a straight line is obtained, such as curve B in Fig. 9.9-1 using the upper scale for the abscissa, then either Eqs. (9.9-4)–(9.9-6) for diffusion or Eqs. (9.9-10) and (9.9-11) for capillary flow are applicable.

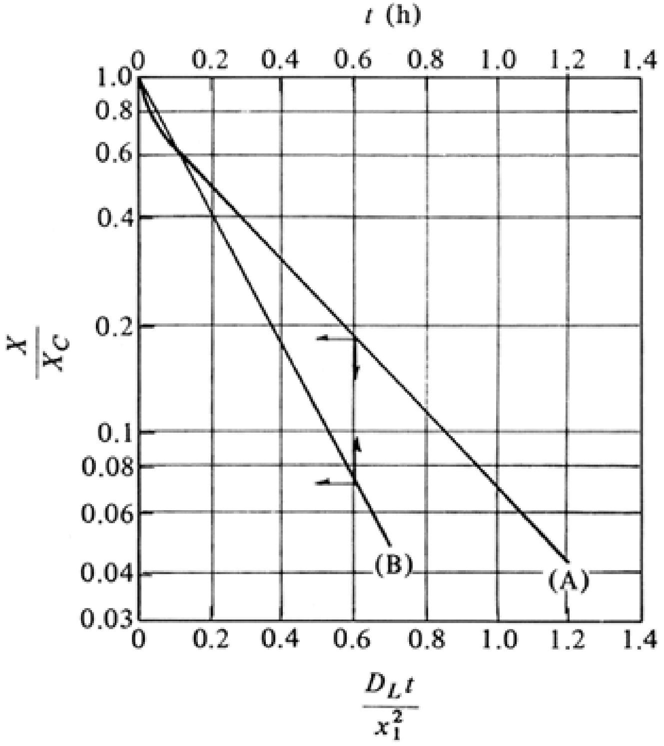

Figure 9.9-1. Plot of equations for falling-rate period: (A) Eq. (9.9-4) for moisture movement by diffusion, (B) Eq. (9.9-10) for moisture movement by capillary flow. (From R. H. Perry and C. H. Chilton, Chemical Engineers Handbook, 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.)

If the equations for capillary flow apply, the slope of the falling-rate drying line B in Fig. 9.9-1 is related to Eq. (9.9-10), which contains the constant drying rate R_C. The value of R_C is calculated from the measured slope of the line, which is $-R_C/x_1\rho_S X_C$, and if it agrees with the experimental value of R_C in the constant-drying period or the predicted value of R_C, the moisture movement is by capillary flow.

If the values of R_C do not agree, the moisture movement is by diffusion and the slope of line B in Fig. 9.9-1 from Eq. (9.9-6) should equal $-\pi^2 D_L/4x_1^2$. In actual practice, however, the diffusivity D_L is usually less at small moisture contents than at large moisture contents, and an average value of D_L is usually determined experimentally over the moisture range of interest. A plot of Eq. (9.9-4) is shown as line A, where $\ln(X/X_1)$ or $\ln(X/X_C)$ is plotted versus $D_L t/x_1^2$. This is the same plot as Fig. 5.3-13 for a slab and shows a curvature in the line for values of X/X_C between 1.0 and 0.6 and a straight line for $X/X_C < 0.6$.

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When the experimental data show that the movement of moisture follows the diffusion law, the average experimental diffusivities can be calculated as follows for different concentration ranges. A value of X/X_C is chosen at 0.4, for example. From an experimental plot similar to curve B, Fig. 9.9-1, the experimental value of t is obtained. From curve A at $X/X_C = 0.4$, the theoretical value of $(D_L t/x_1^2)_{\text{theor}}$ is obtained. Then, by substituting the known values of t and x_1 into Eq. (9.9-12), the experimental average value of D_L over the range $X/X_C = 1.0–0.4$ is obtained:

Equation 9.9-12.

$$D_L = \left(\frac{D_L t}{x_1^2}\right)_{\text{theor}} \frac{x_1^2}{t}$$

This is repeated for various values of X/X_C. Values of D_L obtained for $X/X_C > 0.6$ are in error because of the curvature of line A.

EXAMPLE 9.9-2. Diffusion Coefficient in the Tapioca Root

Tapioca flour is obtained from drying and then milling the tapioca root. Experimental data on drying thin slices of the tapioca root 3 mm thick on both sides in the falling-rate period under constant-drying conditions are tabulated below. The time $t = 0$ is the start of the falling-rate period.

X/X_C	t (*h*)	X/X_C	t (*h*)	X/X_C	t (*h*)
1.0	0	0.55	0.40	0.23	0.94
0.80	0.15	0.40	0.60	0.18	1.07
0.63	0.27	0.30	0.80		

It has been determined that the data do not follow the capillary-flow equations but appear to follow the diffusion equations. Plot the data as X/X_C versus t on semilog coordinates and determine the average diffusivity of the moisture up to a value of $X/X_C = 0.20$.

Solution: In Fig. 9.9-2 the data are plotted as X/X_C on the log scale versus t on a linear scale and a smooth curve is drawn through the data. At $X/X_C = 0.20$, a value of $t = 1.02$ h is read off the plot. The value of $x_1 = 3$ mm/2 = 1.5 mm for drying from both sides. From Fig. 9.9-1, line A, for $X/X_C = 0.20$, $(D_L t/x_1^2)_{\text{theor}} = 0.56$. Then substituting into Eq. (9.9-12),

$$D_L = (D_L t/x_1^2)_{\text{theor}} \frac{x_1^2}{t} = \frac{0.56(1.5/1000)^2}{1.02 \times 3600} = 3.44 \times 10^{-10} \text{ m}^2/\text{s}$$

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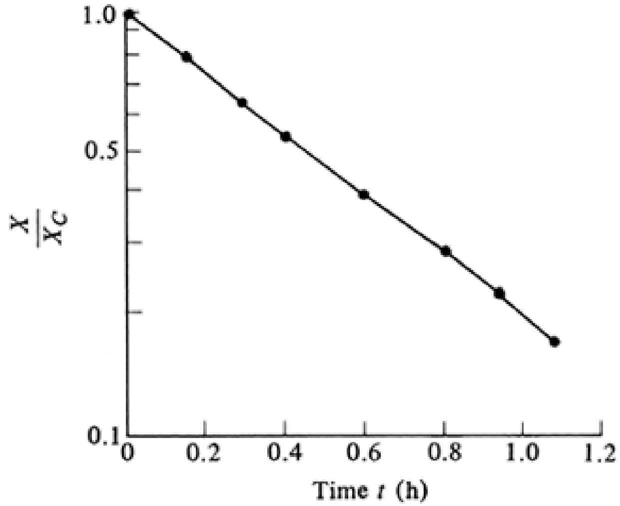

Figure 9.9-2. Plot of drying data for Example 9.9-2.

EQUATIONS FOR VARIOUS TYPES OF DRYERS

Through-Circulation Drying in Packed Beds

For through-circulation drying, where the drying gas passes upward or downward through a bed of wet granular solids, both a constant-rate period and a falling-rate period of drying may result. Often the granular solids are arranged on a screen so that the gas passes through the screen and through the open spaces or voids between the solid particles.

Derivation of equations

To derive the equations for this case, no heat losses will be assumed, so the system is adiabatic. The drying will be for unbound moisture in the wet granular solids. We shall consider a bed of uniform cross-sectional area A m^2, where a gas flow of G kg dry gas/h · m^2 cross section enters with a humidity of H_1. By a material balance on the gas at a given time, the gas leaves the bed with a humidity H_2. The amount of water removed from the bed by the gas is equal to the rate of drying at this time:

Equation 9.10-1.

$$R = G(H_2 - H_1)$$

where R = kg H_2O/h · m^2 cross section and G = kg dry air/h · m^2 cross section.

In Fig. 9.10-1 the gas enters at T_1 and H_1 and leaves at T_2 and H_2. Hence, the temperature T and humidity H both vary through the bed. Making a heat balance over the short section dz m of the bed,

Equation 9.10-2.

$$dq = -Gc_S A \ dT$$

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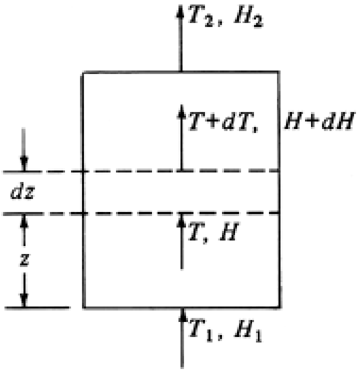

Figure 9.10-1. Heat and material balances in a through- circulation dryer in a packed bed.

where A = m^2 cross-sectional area, q is the heat-transfer rate in W (J/s), and c_S is the humid heat of the air–water vapor mixture in Eq. (9.3-6). Note that G in this equation is in kg/s · m^2. The heat-transfer equation gives

Equation 9.10-3.

$$dq = haA\ dz(T - T_W)$$

where T_W = wet bulb temperature of solid, h is the heat-transfer coefficient in W/m^2 · K, and a is m^2 surface area of solids/m^3 bed volume. Equating Eq. (9.10-2) to (9.10-3), rearranging, and integrating,

Equation 9.10-4.

$$\frac{ha}{Gc_S}\int_0^z dz = -\int_{T_1}^{T_2} \frac{dT}{T - T_W}$$

Equation 9.10-5.

$$\frac{haz}{Gc_S} = \ln \frac{T_1 - T_W}{T_2 - T_W}$$

where z = bed thickness = x_1 m.

For the constant-rate period of drying by air flowing parallel to a surface, Eq. (9.6-11) was derived:

Equation 9.6-11.

$$t = \frac{L_S \lambda_W (X_1 - X_2)}{Ah(T - T_W)} = \frac{L_S(X_1 - X_2)}{Ak_y M_B(H_W - H)}$$

Using Eq. (9.9-9) and the definition of a, we obtain

Equation 9.10-6.

$$\frac{L_S}{A} = \frac{\rho_S}{a}$$

Substituting Eq. (9.10-6) into (9.6-11) and setting $X_2 = X_C$ for drying to X_C, we obtain the equation for through-circulation drying in the constant-rate period:

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Equation 9.10-7.

$$t = \frac{\rho_S \lambda_W (X_1 - X_C)}{ah(T - T_W)} = \frac{\rho_S (X_1 - X_C)}{ak_y M_B (H_W - H)}$$

In a similar manner, Eq. (9.7-8) for the falling-rate period, which assumes that R is proportional to X, becomes, for through-circulation drying,

Equation 9.10-8.

$$t = \frac{\rho_S \lambda_W X_C \ln(X_C/X)}{ah(T - T_W)} = \frac{\rho_S X_C \ln(X_C/X)}{ak_y M_B (H_W - H)}$$

Both Eqs. (9.10-7) and (9.10-8), however, hold for only one point in the bed in Fig. 9.10-1, since the temperature T of the gas varies throughout the bed. Hence, in a manner similar to the derivation in heat transfer, a log mean temperature difference can be used as an approximation for the whole bed in place of $T - T_W$ in Eqs. (9.10-7) and (9.10-8):

Equation 9.10-9.

$$(T - T_W)_{LM} = \frac{(T_1 - T_W) - (T_2 - T_W)}{\ln[(T_1 - T_W)/(T_2 - T_W)]} = \frac{T_1 - T_2}{\ln[(T_1 - T_W)/(T_2 - T_W)]}$$

Substituting Eq. (9.10-5) for the denominator of Eq. (9.10-9) and substituting the value of T_2 from Eq. (9.10-5) into (9.10-9),

Equation 9.10-10.

$$(T - T_W)_{LM} = \frac{(T_1 - T_W)(1 - e^{-haz/Gc_S})}{haz/Gc_S}$$

Substituting Eq. (9.10-10) into (9.10-7) for the constant-rate period and setting $x_1 = z$,

Equation 9.10-11.

$$t = \frac{\rho_S \lambda_W x_1 (X_1 - X_C)}{Gc_S (T_1 - T_W)(1 - e^{-hax_1/Gc_S})}$$

Similarly, for the falling-rate period an approximate equation is obtained:

Equation 9.10-12.

$$t = \frac{\rho_S \lambda_W x_1 X_C \ \ln(X_C/X)}{Gc_S (T_1 - T_W)(1 - e^{-hax_1/Gc_S})}$$

A major difficulty with the use of Eq. (9.10-12) is that the critical moisture content is not easily estimated. Different forms of Eqs. (9.10-11) and (9.10-12) can also be derived, using humidity instead of temperature (T1).

Heat-transfer coefficients

For through-circulation drying, where the gases pass through a bed of wet granular solids, the following equations for estimating h for adiabatic evaporation of water can be used (G1, W1):

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Equation 9.10-13.

$$\left.\begin{aligned} h &= 0.151\frac{G_t^{0.59}}{D_p^{0.41}} \qquad \text{(SI)} \\ h &= 0.11\frac{G_t^{0.59}}{D_p^{0.41}} \qquad \text{(English)} \end{aligned}\right\} \frac{D_p G_t}{\mu} > 350$$

Equation 9.10-14.

$$\left.\begin{aligned} h &= 0.214\frac{G_t^{0.49}}{D_p^{0.51}} \qquad \text{(SI)} \\ h &= 0.15\frac{G_t^{0.49}}{D_p^{0.51}} \qquad \text{(English)} \end{aligned}\right\} \frac{D_p G_t}{\mu} < 350$$

where h is in $W/m^2 \cdot K$, D_p is diameter in m of a sphere having the same surface area as the particle in the bed, G_t is the total mass velocity entering the bed in $kg/h \cdot m^2$, and μ is viscosity in $kg/m \cdot h$. In English units, h is $btu/h \cdot ft^2 \cdot °F$, D_p is ft, G_t is $lb_m/h \cdot ft^2$, and μ is $lb_m/ft \cdot h$.

Geometry factors in a bed

To determine the value of a, m^2 surface area/m^3 of bed, in a packed bed for spherical particles having a diameter D_p m,

Equation 9.10-15.

$$a = \frac{6(1 - \varepsilon)}{D_p}$$

where ε is the void fraction in the bed. For cylindrical particles,

Equation 9.10-16.

$$a = \frac{4(1 - \varepsilon)(h + 0.5D_c)}{D_c h}$$

where D_c is diameter of cylinder in m and h is length of cylinder in m. The value of D_p to use in Eqs. (9.10-13) and (9.10-14) for a cylinder is the diameter of a sphere having the same surface area as the cylinder, as follows:

Equation 9.10-17.

$$D_p = (D_c h + 0.5D_c^2)^{1/2}$$

Equations for very fine particles

The equations derived for the constant- and falling-rate periods in packed beds hold for particles of about 3–19 mm in diameter in shallow beds about 10–65 mm thick (T1, M1). For very fine particles of 10–200 mesh (1.66–0.079 mm) and bed depth greater than 11 mm, the interfacial area a varies with the moisture content. Empirical expressions are available for estimating a and the mass-transfer coefficient (T1, A1).

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EXAMPLE 9.10-1. Through-Circulation Drying in a Bed

A granular paste material is extruded into cylinders with a diameter of 6.35 mm and length of 25.4 mm. The initial total moisture content X_{t1} = 1.0 kg H_2O/kg dry solid and the equilibrium moisture is X^* = 0.01. The density of the dry solid is 1602 kg/m^3 (100 lb_m/ft^3). The cylinders are packed on a screen to a depth of x_1 = 50.8 mm. The bulk density of the dry solid in the bed is ρ_S = 641 kg/m^3. The inlet air has a humidity H_1 = 0.04 kg H_2O/kg dry air and a temperature T_1 = 121.1°C. The gas superficial velocity is 0.811 m/s and the gas passes through the bed. The total critical moisture content is X_{tC} = 0.50. Calculate the total time to dry the solids to X_t = 0.10 kg H_2O/kg dry solid.

Solution: For the solid,

$$X_1 = X_{t1} - X^* = 1.00 - 0.01 = 0.99 \text{ kg } H_2O/\text{kg dry solid}$$

$$X_C = X_{tC} - X^* = 0.50 - 0.01 = 0.49$$

$$X = X_t - X^* = 0.10 - 0.01 = 0.09$$

For the gas, T_1 = 121.1°C and H_1 = 0.04 kg H_2O/kg dry air. The wet bulb temperature T_W = 47.2°C and H_W = 0.074. The solid temperature is at T_W if radiation and conduction are neglected. The density of the entering air at 121.1°C and 1 atm is as follows:

Equation 9.3-7.

$$\upsilon_H = (2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.04)(273 + 121.1)$$
$$= 1.187 \text{ m}^3/\text{kg dry air}$$

$$\rho = \frac{1.00 + 0.04}{1.187} = 0.876 \text{ kg dry air} + H_2O/\text{m}^3$$

The mass velocity of the dry air is

$$G = \upsilon\rho\left(\frac{1.0}{1.0 + 0.04}\right) = 0.811(3600)(0.876)\left(\frac{1}{1.04}\right) = 2459 \text{ kg dry air/h} \cdot \text{m}^2$$

Since the inlet H_1 = 0.040 and the outlet will be less than 0.074, an approximate average H of 0.05 will be used to calculate the total average mass velocity. The approximate average G_t is

$$G_t = 2459 + 2459(0.05) = 2582 \text{ kg air} + H_2O/\text{h} \cdot \text{m}^2$$

For the packed bed, the void fraction ε is calculated as follows for 1 m^3 of bed containing solids plus voids. A total of 641 kg dry solid is present. The density of the dry solid is 1602 kg dry solid/m^3 solid. The volume of the solids in 1 m^3 of bed is then 641/1602, or 0.40 m^3 solid. Hence, ε = 1 − 0.40 = 0.60. The solid cylinder length h = 0.0254 m. The diameter D_c = 0.00635 m. Substituting into Eq. (9.10-16),

$$a = \frac{4(1 - \varepsilon)(h + 0.5D_c)}{D_c h} = \frac{4(1 - 0.6)[0.0254 + 0.5(0.00635)]}{0.00635(0.0254)}$$
$$= 283.5 \text{ m}^2 \text{ surface area/m}^3 \text{ bed volume}$$

To calculate the diameter D_p of a sphere with the same area as the cylinder using Eq. (9.10-17),

$$D_p = (D_c h + 0.5D_c^2)^{1/2} = [0.00635 \times 0.0254 + 0.5(0.00635)^2]^{1/2}$$
$$= 0.0135 \text{ m}$$

The bed thickness x_1 = 50.8 mm = 0.0508 m.

To calculate the heat-transfer coefficient, the Reynolds number is first calculated. Assuming an approximate average air temperature of 93.3°C, the viscosity of air is μ = 2.15 × 10^{-5} kg/m s = 2.15 × 10^{-5}(3600) = 7.74 × 10^{-2} kg/m · h. The Reynolds number is

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$$N_{\text{Re}} = \frac{D_p G_t}{\mu} = \frac{0.0135(2582)}{7.74 \times 10^{-2}} = 450$$

Using Eq. (9.10-13),

$$h = 0.151 \frac{G_t^{0.59}}{D_p^{0.41}} = \frac{0.151(2582)^{0.59}}{(0.0135)^{0.41}} = 90.9 \text{ W/m}^2 \cdot \text{K}$$

For T_W = 47.2°C, λ_W = 2389 kJ/kg, or 2.389×10^6 J/kg (1027 btu/lb_m), from steam tables. The average humid heat, from Eq. (9.3-6), is

$$\begin{aligned} c_S &= 1.005 + 1.88H = 1.005 + 1.88(0.05) = 1.099 \text{ kJ/kg dry air} \cdot \text{K} \\ &= 1.099 \times 10^3 \text{ J/kg} \cdot \text{K} \end{aligned}$$

To calculate the time of drying for the constant-rate period using Eq. (9.10-11) and G = 2459/3600 = 0.6831 kg/s · m^2,

$$\begin{aligned} t &= \frac{\rho_S \lambda_W x_1 (X_1 - X_C)}{G c_S (T_1 - T_W)(1 - e^{-hax_1/Gc_S})} \\ &= \frac{641(2.389 \times 10^6)(0.0508)(0.99 - 0.49)}{(0.683)(1.099 \times 10^3)(121.1 - 47.2)[1 - e^{-(90.9 \times 283.5 \times 0.0508)/(0.683 \times 1.099 \times 10^3)}]} \\ &= 850 \text{ s} = 0.236 \text{ h} \end{aligned}$$

For the time of drying for the falling-rate period, using Eq. (9.10-12),

$$\begin{aligned} t &= \frac{\rho_S \lambda_W x_1 X_C \ \ln(X_C/X)}{G c_S (T_1 - T_W)(1 - e^{-hax_1/Gc_S})} \\ &= \frac{641(2.389 \times 10^6)(0.0508)(0.49)\ln(0.49/0.09)}{(0.6831)(1.099 \times 10^3)(121.1 - 47.2)[1 - e^{-(90.9 \times 283.5 \times 0.0508)/(0.683 \times 1.099 \times 10^3)}]} \\ &= 1412 \text{ s} = 0.392 \text{ h} \end{aligned}$$

$$\text{total time } t = 0.236 + 0.392 = 0.628 \text{ h}$$

Tray Drying with Varying Air Conditions

For drying in a compartment or tray dryer where the air passes in parallel flow over the surface of the tray, the air conditions do not remain constant. Heat and material balances similar to those for through circulation must be made to determine the exit-gas temperature and humidity.

In Fig. 9.10-2 air is shown passing over a tray. It enters having a temperature of T_1 and humidity H_1 and leaves at T_2 and H_2. The spacing between the trays is b m and dry air flow is G kg dry air/s m^2 cross-sectional area. Writing a heat balance over a length dL_t of tray for a section 1 m wide,

Equation 9.10-18.

$$dq = Gc_S(1 \times b)\ dT$$

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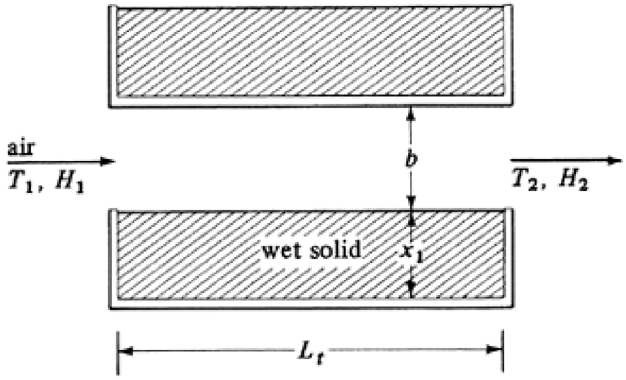

Figure 9.10-2. Heat and material balances in a tray dryer.

The heat-transfer equation is

Equation 9.10-19.

$$dq = h(1 \times dL_t)(T - T_W)$$

Rearranging and integrating,

Equation 9.10-20.

$$\frac{hL_t}{Gc_S b} = \ln \frac{T_1 - T_W}{T_2 - T_W}$$

Defining a log mean temperature difference similar to Eq. (9.10-10) and substituting into Eqs. (9.6-11) and (9.7-8), we obtain the following. For the constant-rate period,

Equation 9.10-21.

$$t = \frac{x_1 \rho_S L_t \lambda_W (X_1 - X_C)}{Gc_S b(T_1 - T_W)(1 - e^{-hL_t/Gc_S b})}$$

For the falling-rate period, an approximate equation is obtained:

Equation 9.10-22.

$$t = \frac{x_1 \rho_S L_t \lambda_W X_C \ \ln(X_C/X)}{Gc_S b(T_1 - T_W)(1 - e^{-hL_t/Gc_S b})}$$

Material and Heat Balances for Continuous Dryers

Simple heat and material balances

In Fig. 9.10-3 a flow diagram is given for a continuous-type dryer where the drying gas flows countercurrently to the solids flow. The solid enters at a rate of L_S kg dry solid/h, having a free moisture content X_1 and a temperature T_{S1}. It leaves at X_2 and T_{S2}. The gas enters at a rate G kg dry air/h, having a humidity H_2 kg H_2O/kg dry air and a temperature of T_{G2}. The gas leaves at T_{G1} and H_1.

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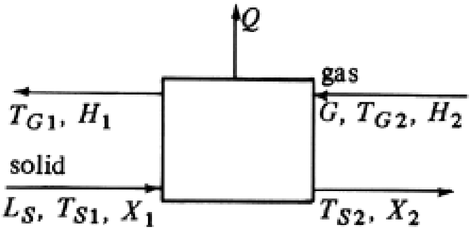

Figure 9.10-3. Process flow for a counter-current continuous dryer.

For a material balance on the moisture,

Equation 9.10-23.

$$GH_2 + L_S X_1 = GH_1 + L_S X_2$$

For a heat balance a datum of T_0°C is selected. A convenient temperature is 0°C (32°F). The enthalpy of the wet solid is composed of the enthalpy of the dry solid plus that of the liquid as free moisture. The heat of wetting is usually neglected. The enthalpy of the gas H'_G in kJ/kg dry air is

Equation 9.10-24.

$$H'_G = c_S(T_G - T_0) + H\lambda_0$$

where λ_0 is the latent heat of water at T_0°C, 2501 kJ/kg (1075.4 btu/lb_m) at 0°C, and c_S is the humid heat, given as kJ/kg dry air · K:

Equation 9.3-6.

$$c_S = 1.005 + 1.88H$$

The enthalpy of the wet solid H'_S in kJ/kg dry solid, where $(T_S - T_0)$°C = $(T_S - T_0)$ K, is

Equation 9.10-25.

$$H'_S = c_{pS}(T_S - T_0) + Xc_{pA}(T_S - T_0)$$

where c_{pS} is the heat capacity of the dry solid in kJ/kg dry solid · K and c_{pA} is the heat capacity of liquid moisture in kJ/kg H_2O · K. The heat of wetting or adsorption is neglected.

A heat balance on the dryer is

Equation 9.10-26.

$$GH'_{G2} + L_S H'_{S1} = GH'_{G1} + L_S H'_{S2} + Q$$

where Q is the heat loss in the dryer in kJ/h. For an adiabatic process Q = 0, and if heat is added, Q is negative.

EXAMPLE 9.10-2. Heat Balance on a Dryer

A continuous countercurrent dryer is being used to dry 453.6 kg dry solid/h containing 0.04 kg total moisture/kg dry solid to a value of 0.002 kg total moisture/kg dry solid. The granular solid enters at 26.7°C and is to be discharged at 62.8°C. The dry solid has a heat capacity of 1.465 kJ/kg K, which is assumed constant. Heating air enters at 93.3°C, having a humidity of 0.010 kg H_2O/kg dry air, and is to leave at 37.8°C. Calculate the air flow rate and the outlet humidity, assuming no heat losses in the dryer.

Solution: The flow diagram is given in Fig. 9.10-3. For the solid, L_S = 453.6 kg/h dry solid, c_{pS} = 1.465 kJ/kg dry solid · K, X_1 = 0.040 kg H_2O/kg dry solid, c_{pA} = 4.187 kJ/kg H_2OK, T_{S1} = 26.7°C, T_{S2} = 62.8°C, X_2 = 0.002. (Note that X values used are X_t values.) For the gas, T_{G2} = 93.3°C, H_2 = 0.010 kg H_2O/kg dry air, and T_{G1} = 37.8°C.

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Making a material balance on the moisture using Eq. (9.10-23),

Equation 9.10-27.

$$GH_2 + L_S X_1 = GH_1 + L_S X_2$$

$$G(0.010) + 453.6(0.040) = GH_1 + 453.6(0.002)$$

For the heat balance, the enthalpy of the entering gas at 93.3°C, using 0°C as a datum, is, by Eq. (9.10-24), ΔT°C = ΔTK and λ_0 = 2501 kJ/kg, from Table A.2-9,

$$\begin{aligned} H'_{G2} &= c_S(T_{G2} - T_0) + H_2\lambda_0 \\ &= [1.005 + 1.88(0.010)](93.3 - 0) + 0.010(2501) \\ &= 120.5 \text{ kJ/kg dry air} \end{aligned}$$

For the exit gas,

$$\begin{aligned} H'_{G1} &= c_S(T_{G1} - T_0) + H_1\lambda_0 \\ &= (1.005 + 1.88H_1)(37.8 - 0) + H_1(2501) = 37.99 + 2572H_1 \end{aligned}$$

For the entering solid, using Eq. (9.10-25),

$$\begin{aligned} H'_{S1} &= c_{pS}(T_{S1} - T_0) + X_1 c_{pA}(T_{S1} - T_0) \\ &= 1.465(26.7 - 0) + 0.040(4.187)(26.7 - 0) = 43.59 \text{ kJ/kg dry solid} \\ H'_{S2} &= c_{pS}(T_{S2} - T_0) + X_2 c_{pA}(T_{S2} - T_0) \\ &= 1.465(62.8 - 0) + 0.002(4.187)(62.8 - 0) = 92.53 \text{ kJ/kg} \end{aligned}$$

Substituting into Eq. (9.10-26) for the heat balance with $Q = 0$ for no heat loss,

Equation 9.10-28.

$$G(120.5) + 453.6(43.59) = G(37.99 + 2572H_1) + 453.6(92.53) + 0$$

Solving Eqs. (9.10-27) and (9.10-28) simultaneously,

$$G = 1166 \text{ kg dry air/h} \qquad H_1 = 0.0248 \text{ kg H}_2\text{O/kg dry air}$$

Air recirculation in dryers

In many dryers it is desired to control the wet bulb temperature at which the drying of the solid occurs. Also, since steam costs are often important in heating the drying air, recirculation of the drying air is sometimes used to reduce costs and control humidity. Part of the moist, hot air leaving the dryer is recirculated (recycled) and combined with the fresh air. This is shown in Fig. 9.10-4. Fresh air having a temperature T_{G1} and humidity H_1 is mixed with recirculated air at T_{G2} and H_2 to give air at T_{G3} and H_3. This mixture is heated to T_{G4} with $H_4 = H_3$. After drying, the air leaves at a lower temperature T_{G2} and a higher humidity H_2.

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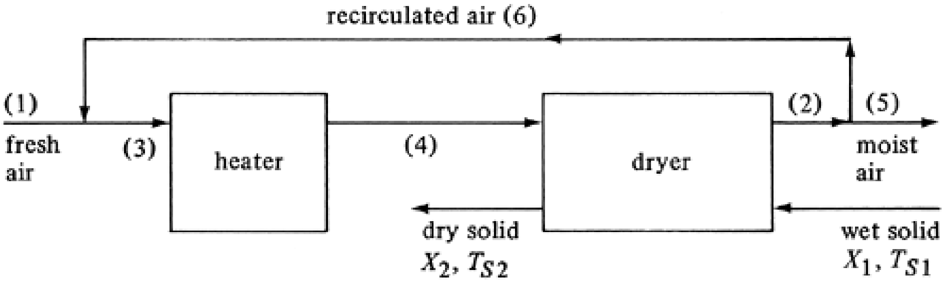

Figure 9.10-4. Process flow for air recirculation in drying.

The following material balances on the water can be made. For a water balance on the heater, noting that $H_6 = H_5 = H_2$,

Equation 9.10-29.

$$G_1H_1 + G_6H_2 = (G_1 + G_6)H_4$$

Making a water balance on the dryer,

Equation 9.10-30.

$$(G_1 + G_6)H_4 + L_SX_1 = (G_1 + G_6)H_2 + L_SC_2$$

In a similar manner heat balances can be made on the heater and dryer and on the overall system.

Continuous Countercurrent Drying

Introduction and temperature profiles

Drying continuously offers a number of advantages over batch-drying. Smaller sizes of equipment can often be used, and the product has a more uniform moisture content. In a continuous dryer the solid is moved through the dryer while in contact with a moving gas stream that may flow parallel or countercurrent to the solid. In countercurrent adiabatic operation, the entering hot gas contacts the leaving solid, which has been dried. In parallel adiabatic operation, the entering hot gas contacts the entering wet solid.

In Fig. 9.10-5 typical temperature profiles of the gas T_G and the solid T_S are shown for a continuous countercurrent dryer. In the preheat zone, the solid is heated up to the wet bulb or adiabatic saturation temperature. Little evaporation occurs here, and for low-temperature drying this zone is usually ignored. In the constant-rate zone, I, unbound and surface moisture are evaporated, and the temperature of the solid remains essentially constant at the adiabatic saturation temperature if heat is transferred by convection. The rate of drying would be constant here but the gas temperature is changing, as well as the humidity. The moisture content falls to the critical value X_C at the end of this period.

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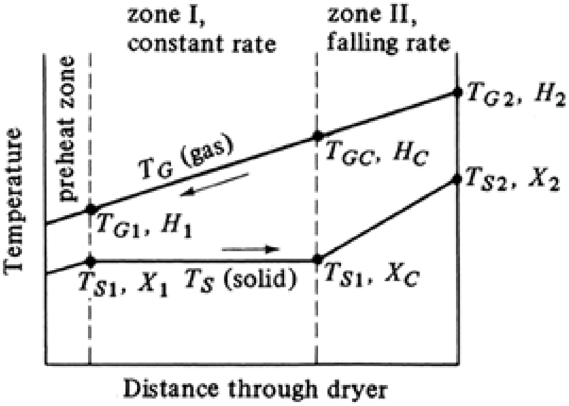

Figure 9.10-5. Temperature profiles for a continuous countercurrent dryer.

In zone II, unsaturated surface and bound moisture are evaporated and the solid is dried to its final value X_2. The humidity of the gas entering zone II is H_2 and it rises to H_C. The material-balance equation (9.10-23) may be used to calculate H_C as follows:

Equation 9.10-31.

$$L_S(X_C - X_2) = G(H_C - H_2)$$

where L_S is kg dry solid/h and G is kg dry gas/h.

Equation for constant-rate period

The rate of drying in the constant-rate region in zone I would be constant if it were not for the varying gas conditions. The rate of drying in this section is given by an equation similar to Eq. (9.6-7):

Equation 9.10-32.

$$R = k_y M_B(H_W - H) = \frac{h}{\lambda W}(T_G - T_W)$$

The time for drying is given by Eq. (9.6-1) using limits between X_1 and X_C:

Equation 9.10-33.

$$t = \left(\frac{L_S}{A}\right)\int_{X_C}^{X_1}\frac{dX}{R}$$

where A/L_S is the exposed drying surface m^2/kg dry solid. Substituting Eq. (9.10-32) into (9.10-33) and $(G/L_S)\ dH$ for dX,

Equation 9.10-34.

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right)\frac{1}{k_y M_B}\int_{H_C}^{H_1}\frac{dH}{H_W - H}$$

where G = kg dry air/h, L_S = kg dry solid/h, and A/L_S = m^2/kg dry solid. This can be integrated graphically or numerically.

For the case where T_W or H_W is constant for adiabatic drying, Eq. (9.10-34) can be integrated:

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Equation 9.10-35.

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right)\frac{1}{k_y M_B}\ \ln\frac{H_W - H_C}{H_W - H_1}$$

The above can be modified by use of a log mean humidity difference:

Equation 9.10-36.

$$\Delta H_{LM} = \frac{(H_W - H_C) - (H_W - H_1)}{\ln[(H_W - H_C)/(H_W - H_1)]} = \frac{H_1 - H_C}{\ln[(H_W - H_C)/(H_W - H_1)]}$$

Substituting Eq. (9.10-36) into (9.10-35), an alternative equation is obtained:

Equation 9.10-37.

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right)\frac{1}{k_y M_B}\frac{H_1 - H_C}{\Delta H_{LM}}$$

From Eq. (9.10-31), H_C can be calculated as follows:

Equation 9.10-38.

$$H_C = H_2 + \frac{L_S}{G}(X_C - X_2)$$

Equation for falling-rate period

For the situation where unsaturated surface drying occurs, H_W is constant for adiabatic drying, the rate of drying is directly dependent upon X as in Eq. (9.7-9), and Eq. (9.10-32) applies:

Equation 9.10-39.

$$R = R_C\frac{X}{X_C} = k_y M_B(H_W - H)\frac{X}{X_C}$$

Substituting Eq. (9.10-39) into (9.6-1),

Equation 9.10-40.

$$t = \left(\frac{L_S}{A}\right)\frac{X_C}{k_y M_B}\int_{X_2}^{X_C}\frac{dX}{(H_W - H)X}$$

Substituting $G\,dH/L_S$ for dX and $(H - H_2)G/L_S + X_2$ for X,

Equation 9.10-41.

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right)\frac{X_C}{k_y M_B}\int_{H_2}^{H_C}\frac{dH}{(H_W - H)[(H - H_2)G/L_S + X_2]}$$

Equation 9.10-42.

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right)\frac{X_C}{k_y M_B}\frac{1}{(H_W - H_2)G/L_S + X_2}\ \ln\frac{X_C(H_W - H_2)}{X_2(H_W - H_C)}$$

Again, to calculate H_C, Eq. (9.10-38) can be used.

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These equations for the two periods can also be derived using the last part of Eq. (9.10-32) and temperatures instead of humidities.

FREEZE-DRYING OF BIOLOGICAL MATERIALS

Introduction

Certain foodstuffs, pharmaceuticals, and biological materials, which may not be heated even to moderate temperatures in ordinary drying, may be freeze-dried. The substance to be dried is usually frozen by exposure to very cold air. In freeze-drying, the water is removed as a vapor by sublimation from the frozen material in a vacuum chamber. After the moisture sublimes to a vapor, it is removed by mechanical vacuum pumps or steam jet ejectors.

As a rule, freeze-drying produces the highest-quality food product obtainable by any drying method. A prominent factor is the structural rigidity afforded by the frozen substance when sublimation occurs. This prevents collapse of the remaining porous structure after drying. When water is added later, the rehydrated product retains much of its original structural form. Freeze-drying of biological and food materials also has the advantage of little loss of flavor and aroma. The low temperatures involved minimize the degrading reactions which normally occur in ordinary drying processes. However, freeze-drying is an expensive form of dehydration for foods because of the slow drying rate and the use of vacuum.

Since the vapor pressure of ice is very small, freeze-drying requires very low pressures or high vacuum. If the water were in a pure state, freeze-drying at or near 0°C (273 K) at a pressure of 4580 μm (4.58 mm Hg abs) could be performed. (See Appendix A.2 for the properties of ice.) However, since the water usually exists in a solution or a combined state, the material must be cooled below 0°C to keep the water in the solid phase. Most freeze-drying is done at −10°C (263 K) or lower at pressures of about 2000 μm or less.

Derivation of Equations for Freeze-Drying

In the freeze-drying process the original material is composed of a frozen core of material. As the ice sublimes, the plane of sublimation, which started at the outside surface, recedes, and a porous shell of material already dried remains. The heat for the latent heat of sublimation of 2838 kJ/kg (1220 btu/lb_m) ice is usually conducted inward through the layer of dried material. In some cases it is also conducted through the frozen layer from the rear. The vaporized water is transferred through the layer of dried material. Hence, heat and mass transfer are occurring simultaneously.

In Fig. 9.11-1 a material being freeze-dried is pictured. Heat by conduction, convection, and/or radiation from the gas phase reaches the dried surface and is then transferred by conduction to the ice layer. In some cases heat may also be conducted through the frozen material to reach the sublimation front or plane. The total drying time must be long enough that the final moisture content is below about 5 wt % to prevent degradation of the final material on storage. The maximum temperatures reached in the dried food and the frozen food must be low enough to keep degradation to a minimum.

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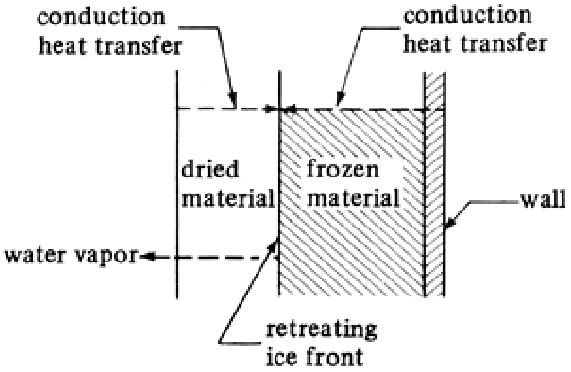

Figure 9.11-1. Heat and mass transfer in freeze drying.

The most widely used freeze-drying process is based upon the heat of sublimation being supplied from the surrounding gases to the sample surface. Then the heat is transferred by conduction through the dried material to the ice surface. A simplified model by Sandall et al. (S1) is shown in Fig. 9.11-2.

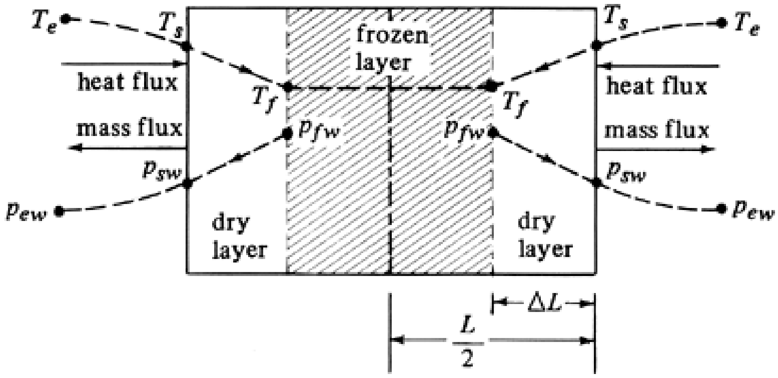

Figure 9.11-2. Model for uniformly retreating ice front in freeze drying.

The heat flux to the surface of the material in Fig. 9.11-2 occurs by convection and in the dry solid by conduction to the sublimation surface. The heat flux to the surface is equal to that conducted through the dry solid, assuming pseudo-steady state:

Equation 9.11-1.

$$q = h(T_e - T_s) = \frac{k}{\Delta L}(T_s - T_f)$$

where q is heat flux in W (J/s), h is external heat-transfer coefficient in W/m^2 · K, T_e is external temperature of the gas in °C, T_s is surface temperature of the dry solid in °C, T_f is temperature of the sublimation front or ice layer in °C, k is thermal conductivity of the dry solid in W/m · K, and ΔL is the thickness of the dry layer in m. Note that $(T_s - T_f)$°C = $(T_s - T_f)$ K.

In a similar manner, the mass flux of the water vapor from the sublimation front is

Equation 9.11-2.

$$N_A = \frac{D'}{RT\,\Delta L}(p_{fw} - p_{sw}) = k_g(p_{sw} - p_{ew})$$

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where N_A is flux of water vapor in kg mol/s · m^2, k_g is external mass-transfer coefficient in kg mol/s · m^2 · atm, p_{sw} is partial pressure of water vapor at the surface in atm, p_{ew} is partial pressure of water vapor in the external bulk gas phase in atm, T is the average temperature in the dry layer, D' is an average effective diffusivity in the dry layer in m^2/s, and p_{fw} is the partial pressure of water vapor in equilibrium with the sublimation ice front in atm.

Equation (9.11-1) can be rearranged to give

Equation 9.11-3.

$$q = \frac{1}{1/h + \Delta L/k}(T_e - T_f)$$

Also, Eq. (9.11-2) can be rearranged to give

Equation 9.11-4.

$$N_A = \frac{1}{1/k_g + RT\,\Delta L/D'}(p_{fw} - p_{ew})$$

The coefficients h and k_g are determined by the gas velocities and characteristics of the dryer and hence are constant. The values of T_e and p_{ew} are set by the external operating conditions. The values of k and D' are determined by the nature of the dried material.

The heat flux and mass flux at pseudo-steady state are related by

Equation 9.11-5.

$$q = \Delta H_s\, N_A$$

where ΔH_s is the latent heat of sublimation of ice in J/kg mol. Also, p_{fw} is uniquely determined by T_f, since it is the equilibrium vapor pressure of ice at that temperature; or

Equation 9.11-6.

$$p_{fw} = f(T_f)$$

Substituting Eqs. (9.11-3) and (9.11-4) into (9.11-5),

Equation 9.11-7.

$$\frac{1}{1/h + \Delta L/k}(T_e - T_f) = \Delta H_s\, \frac{1}{1/k_g + RT\,\Delta L/D'}(p_{fw} - p_{ew})$$

Also, substituting Eqs. (9.11-1) and (9.11-4) into (9.11-5),

Equation 9.11-8.

$$\frac{1}{\Delta L/k}(T_s - T_f) = \Delta H_s\, \frac{1}{1/k_g + RT\,\Delta L/D'}(p_{fw} - p_{ew})$$

As T_e and, hence, T_s are raised to increase the rate of drying, two limits may possibly be reached. First, the outer-surface temperature, T_s cannot be allowed to go too high because of thermal damage. Second, the temperature T_f must be kept well below the melting point. For the situation where $k/\Delta L$ is small compared to k_g and $D'/RT\Delta L$, the outer-surface temperature limit will be encountered first as T_s is raised. To further increase the drying rate, k must be raised. Hence, the process is considered to be heat-transfer-controlled. Most commercial freeze-drying processes are heat-transfer-controlled (K1).

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In order to solve the given equations, ΔL is related to x, the fraction of the original free moisture remaining:

Equation 9.11-9.

$$\Delta L = (1 - x)\frac{L}{2}$$

The rate of freeze-drying can be related to N_A by

Equation 9.11-10.

$$N_A = \frac{L}{2}\frac{1}{M_A V_S}\left(-\frac{dx}{dt}\right)$$

where M_A is molecular weight of water, V_S is the volume of solid material occupied by a unit kg of water initially ($V_S = 1/X_0\rho_S$), X_0 is initial free moisture content in kg H_2O/kg dry solid, and ρ_S is bulk density of dry solid in kg/m^3.

Combining Eqs. (9.11-3), (9.11-5), (9.11-9), and (9.11-10), we obtain, for heat transfer,

Equation 9.11-11.

$$\frac{L}{2}\frac{\Delta H_s}{M_A V_S}\left(-\frac{dx}{dt}\right) = \frac{1}{1/h + (1 - x)L/2k}(T_e - T_f)$$

Similarly, for mass transfer,

Equation 9.11-12.

$$\frac{L}{2}\frac{1}{M_A V_S}\left(-\frac{dx}{dt}\right) = \frac{1}{1/k_g + RT(1 - x)L/2D'}(p_{fw} - p_{ew})$$

Integrating Eq. (9.11-11) between the limits of $t = 0$ at $x_1 = 1.0$ and $t = t$ at $x_2 = x_2$, the equation for the time of drying to x_2 is as follows for h being very large (negligible external resistance):

Equation 9.11-13.

$$t = \frac{L^2}{4kV_S}\frac{\Delta H_s}{M_A}\frac{1}{T_e - T_f}\left(x_1 - x_2 - \frac{x_1^2}{2} + \frac{x_2^2}{2}\right)$$

where $\Delta H_s/M_A$ is heat of sublimation in J/kg H_2O. For $x_2 = 0$, the slab is completely dry.

Assuming that the physical properties and mass- and heat-transfer coefficients are known, Eq. (9.11-8) can be used to calculate the ice-sublimation temperature T_f when the environment temperature T_e and the environment partial pressure p_{ew} are set. Since h is very large, $T_e \cong T_S$. Then Eq. (9.11-8) can be solved for T_f, since T_f and p_{fw} are related by the equilibrium-vapor-pressure relation, Eq. (9.11-6). In Eq. (9.11-8) the value to use for T can be approximated by $(T_f + T_S)/2$.

The uniformly retreating ice-front model was tested by Sandall et al. (S1) against actual freeze-drying data. The model satisfactorily predicted the drying times for removal of 65–90% of the total initial water (S1, K1). The temperature T_f of the sublimation interface did remain essentially constant as assumed in the derivation. However, during removal of the last 10–35% of the water, the drying rate slowed markedly and the actual time was considerably greater than the predicted time for this period.

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The effective thermal conductivity *k* in the dried material has been found to vary significantly with the total pressure and with the type of gas present. Also, the type of material affects the value of *k* (S1, K1). The effective diffusivity *D′* of the dried material is a function of the structure of the material, Knudsen diffusivity, and molecular diffusivity (K1).

UNSTEADY-STATE THERMAL PROCESSING AND STERILIZATION OF BIOLOGICAL MATERIALS

Introduction

Materials of biological origin are usually not as stable as most inorganic and some organic materials. Hence, it is necessary to use certain processing methods to preserve biological materials, especially foods. Physical and chemical processing methods of preservation, such as drying, smoking, salting, chilling, freezing, and heating, are commonly used. Freezing and chilling of foods were discussed in Section 5.5 as methods of slowing the spoilage of biological materials. Also, in Section 9.11, freeze-drying of biological materials was discussed.

An important method is heat or thermal processing, whereby contaminating microorganisms that occur primarily on the outer surface of foods and cause spoilage and health problems are destroyed. This leads to longer storage times for food and other biological materials. A common method of preservation is to heat-seal cans of food. Likewise, thermal processing is used to sterilize aqueous fermentation media to be used in fermentation processes so that organisms that do not survive are unable to compete with the organism that is to be cultured.

The sterilization of food materials by heating destroys bacteria, yeast, molds, and so on, which cause spoilage, and also destroys pathogenic (disease-producing) organisms that may produce deadly toxins if not destroyed. The rate of destruction of microorganisms varies with the amount of heating and the type of organism. Some bacteria can exist in a vegetative growing form and in a dormant or spore form. The spore forms are much more resistant to heat. This mechanism of heat resistance is not clear.

For foods, it is necessary to kill essentially all the spores of *Clostridium botulinum,* which produces a toxin that is a deadly poison. Complete sterility with respect to this spore is the purpose of thermal processing. Since *Cl. botulinum* is so dangerous and often difficult to use, other spores, such as *Bacillus stearothermophilus,* which is a nonpathogenic organism of similar heat resistance, are often used for testing the heat-treating processes (A2, C1).

Temperature has a great effect on the growth rate of microorganisms, which have no temperature-regulating mechanism. Each organism has a certain optimal temperature range in which it grows best. If any microorganism is heated to a sufficiently high temperature for a sufficient time, it will be rendered sterile or killed.

The exact mechanism of thermal death of vegetative bacteria and spores is still somewhat uncertain. It is thought, however, to be due to the breakdown of enzymes, which are essential to the functioning of the living cell (B1).

Thermal Death-Rate Kinetics of Microorganisms

The destruction of microorganisms by heating means loss of viability and not destruction in the physical sense. If it is assumed that inactivation of a single enzyme in a cell will inactivate the cell, then in a suspension of organisms of a single species at a constant temperature, the death rate can be expressed as a first-order kinetic equation (A2). The rate of destruction (number dying per unit time) is proportional to the number of organisms:

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Equation 9.12-1.

$$\frac{dN}{dt} = -kN$$

where N is the number of viable organisms at a given time, t is time in min, and k is a reaction velocity constant in min^{-1}. The reaction velocity constant is a function of temperature and the type of microorganism.

After rearranging, Eq. (9.12-1) can be integrated as follows:

Equation 9.12-2.

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_{t=0}^{t} k\ dt$$

Equation 9.12-3.

$$\ln \frac{N_0}{N} = kt$$

where N_0 is the original number of organisms at $t = 0$ and N is the number at time t. Often N_0 is called the *contamination level* (original number of contaminating microbes before sterilization) and N the sterility level. Equation (9.12-3) can also be written as

Equation 9.12-4.

$$N = N_0\ e^{-kt}$$

Sometimes microbiologists use the term *decimal reduction time D,* which is the time in min during which the original number of viable microbes is reduced by $\frac{1}{10}$. Substituting into Eq. (9.12-4),

Equation 9.12-5.

$$\frac{N}{N_0} = \frac{1}{10} = e^{-kD}$$

Taking the $\log_{10}$ of both sides and solving for D,

Equation 9.12-6.

$$D = \frac{2.303}{k}$$

Combining Eqs. (9.12-3) and (9.12-6),

Equation 9.12-7.

$$t = D\ \log_{10} \frac{N_0}{N}$$

If the $\log_{10}(N/N_0)$ is plotted versus t, a straight line should result from Eq. (9.12-3). Experimental data bear this out for vegetative cells and approximately for spores. Data for the vegetative cell *E. coli* (A1) at constant temperature follow this logarithmic death-rate curve. Bacterial-spore plots sometimes deviate somewhat from the logarithmic rate of death, particularly during a short period immediately following exposure to heat. However, for thermal-processing purposes for use with spores such as *Cl. botulinum,* a logarithmic-type curve is used.

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To experimentally measure the microbial death rate, the spore or cell suspension in a solution is usually sealed in a capillary or test tube. A number of these tubes are then suddenly dipped into a hot bath for a given time. Then they are removed and immediately chilled. The number of viable organisms before and after exposure to the high temperature is then usually determined biologically by means of a plate count.

The effect of temperature on the reaction-rate constant *k* may be expressed by an Arrhenius-type equation:

Equation 9.12-8.

$$k = ae^{-E/RT}$$

where *a* = an empirical constant, *R* is the gas constant in kJ/g mol · K (cal/gmol · K), *T* is absolute temperature in K, and *E* is the activation energy in kJ/g mol (cal/g mol). The value of *E* is in the range 210 to about 418 kJ/g mol (50–100 kcal/g mol) for vegetative cells and spores (A2) and much lower for enzymes and vitamins.

Substituting Eq. (9.12-8) into (9.12-2) and integrating,

Equation 9.12-9.

$$\ln \frac{N_0}{N} = a \int_{t=0}^{t} e^{-E/RT}\, dt$$

At constant temperature *T*, Eq. (9.12-9) becomes (9.12-3). Since *k* is a function of temperature, the decimal reduction time *D*, which is related to *k* by Eq. (9.12-6), is also a function of temperature. Hence, *D* is often written as D_T to show that it is temperature-dependent.

Determination of Thermal Process Time for Sterilization

For canned foods, *Cl. botulinum* is the primary organism to be reduced in number (S2). It has been established that the minimum adequate heating process should reduce the number of spores by a factor of 10^{-12}. This means that, since *D* is the time required to reduce the original number by 10^{-1}, substituting $N/N_0 = 10^{-12}$ into Eq. (9.12-4) and solving for *t*,

Equation 9.12-10.

$$t = 12\frac{2.303}{k} = 12D$$

Thus the time *t* is equal to 12*D* (often called the *12D concept*). This time in Eq. (9.12-10) to reduce the number by 10^{-12} is called the *thermal death time.* Usually, the sterility level *N* is a number much less than one organism. These times do not represent complete sterilization but a mathematical concept which has been found empirically to give effective sterilization.

Experimental data for thermal death rates of *Cl. botulinum,* when plotted as the decimal reduction time D_T at a given *T* versus the temperature *T* in °F on a semilog plot, give essentially straight lines over the range of temperatures used in food sterilization (S2). A typical thermal destruction curve is shown in Fig. 9.12-1. Actually, by combining Eqs. (9.12-6) and (9.12-8), it can be shown that the plot of $\log_{10} D_T$ versus $1/T$ (*T* in degrees absolute) is a straight line, but over small ranges of temperature a straight line is obtained when $\log_{10} D_T$ is plotted versus *T*°F or °C.

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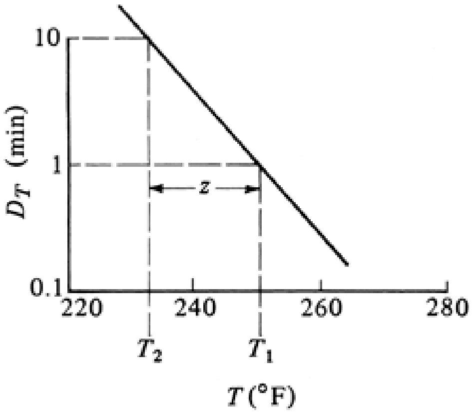

Figure 9.12-1. Thermal destruction curve: plot of decimal reduction time versus temperature.

In Fig. 9.12-1 the term z represents the temperature range in °F for a 10:1 change in D_T. Since the plot is a straight line, the equation can be represented as

Equation 9.12-11.

$$\log_{10} D_{T_2} - \log_{10} D_{T_1} = \frac{1}{z}(T_1 - T_2)$$

Letting T_1 = 250°F (121.1°C), which is the standard temperature against which thermal processes are compared, and calling $T_2 = T$, Eq. (9.12-11) becomes

Equation 9.12-12.

$$D_T = D_{250} \times 10^{(250-T)/z}$$

For the organism *Cl. botulinum* the experimental value of z = 18°F. This means that each increase in temperature of 18°F (10°C) will increase the death rate by a factor of 10. This compares with the factor of 2 for many chemical reactions for an 18°F increase in temperature.

Using Eq. (9.12-7),

Equation 9.12-7.

$$t = D_T \log_{10} \frac{N_0}{N}$$

Substituting T = 250°F (121.1°C) as the standard temperature into this equation and substituting F_0 for t,

Equation 9.12-13.

$$F_0 = D_{250} \log_{10} \frac{N_0}{N}$$

where the F_0 value of a process is the time t in min at 250°F that will produce the same degree of sterilization as the given process at its temperature T. Combining Eqs. (9.12-7), (9.12-12), and (9.12-13), the F_0 of the given process at temperature T is

Equation 9.12-14.

$$F_0 = t \cdot 10^{(T\,^\circ\text{C}-121.1)/(z\,^\circ\text{C})} \qquad \text{(SI)}$$

$$F_0 = t \cdot 10^{(T\,^\circ\text{F}-250)/(z\,^\circ\text{F})} \qquad \text{(English)}$$

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This is the F_0 value in min for the given thermal process at a given constant temperature T°F and a given time t in min. Values for F_0 and z for adequate sterilization of *Cl. botulinum* vary somewhat with the type of food. Data are tabulated by Stumbo (S2) and Charm (C2) for various foods and microorganisms.

The effects of different but successive sterilization processes in a given material are additive. Hence, for several different temperature stages T_1, T_2, and so on, each having different times t_1, t_2, . . . , the F_0 values for each stage are added to give the total F_0:

Equation 9.12-15.

$$F_0 = t_1 \cdot 10^{(T_1-250)/z} + t_2 \cdot 10^{(T_2-250)/z} + \cdots \qquad \text{(English)}$$

EXAMPLE 9.12-1. Sterilization of Cans of Food

Cans of a given food were heated in a retort for sterilization. The F_0 for *Cl. botulinum* in this type of food is 2.50 min and z = 18°F. The temperatures in the center of a can (the slowest-heating region) were measured and were approximately as follows, where the average temperature during each time period is listed: t_1 (0–20 min), T_1 = 160°F; t_2 (20–40 min), T_2 = 210°F; t_3 (40–73 min), T_3 = 230°F. Determine if this sterilization process is adequate. Use English and SI units.

***Solution*:** For the three time periods, the data are as follows:

$$\begin{aligned}
t_1 &= 20 - 0 = 20\text{ min}, & T_1 &= 160°\text{F } (71.1°\text{C}), & z &= 18°\text{F } (10°\text{C})\\
t_2 &= 40 - 20 = 20\text{ min}, & T_2 &= 210°\text{F } (98.9°\text{C})\\
t_3 &= 73 - 40 = 33\text{ min}, & T_3 &= 230°\text{F } (110°\text{C})
\end{aligned}$$

Substituting into Eq. (9.12-15) and solving, using English and SI units,

Equation 9.12-15.

$$\begin{aligned}
F_0 &= t_1 \cdot 10^{(T_1-250)/z} + t_2 \cdot 10^{(T_2-250)/z} + t_3 \cdot 10^{(T_3-250)/z}\\
&= (20)10^{(160-250)/18} + (20)10^{(210-250)/18} + (33)10^{(230-250)/18}\\
&= 0.0020 + 0.1199 + 2.555 = 2.68\text{ min} \qquad \text{(English)}\\
&= (20)10^{(71.1-121.1)/10} + (20)10^{(98.9-121.1)/10} + (33)10^{(110-121.1)/10}\\
&= 2.68\text{ min} \qquad \text{(SI)}
\end{aligned}$$

Hence, this thermal processing is adequate, since only 2.50 min is needed for complete sterilization. Note that the time period at 160°F (71.1°C) contributes an insignificant amount to the final F_0. The major contribution is at 230°F (110°C), which is the highest temperature.

In the general case when cans of food are being sterilized in a retort, the temperature is not constant for a given time period but varies continuously with time. Hence, Eq. (9.12-15) can be modified and written for a continuously varying temperature T by taking small time increments of dt min for each value of T and summing. The final equation is

Equation 9.12-16.

$$F_0 = \int_{t=0}^{t=t} 10^{(T°\text{F}-250)/(z°\text{F})}\, dt \qquad \text{(English)}$$

$$F_0 = \int_{t=0}^{t=t} 10^{(T°\text{C}-121.1)/(z°\text{C})}\, dt \qquad \text{(SI)}$$

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This equation can be used as follows. Suppose that the temperature of a process is varying continuously and that a graph or table of values of T versus t is known or can be calculated by means of the unsteady-state methods given in Chapter 5. Equation (9.12-16) can be graphically integrated by plotting values of $10^{(T-250)/z}$ versus t and taking the area under the curve. In most cases a numerical integration is used to determine F_0. (See Section 1.8 for methods of numerical integration.)

In many cases the temperature of a process that is varying continuously with time is determined experimentally by measuring the temperature in the slowest-heating region. In cans this is the center of the can. Methods given in Chapter 5 for unsteady-state heating of short, fat cylinders by conduction can be used to predict the center temperature of the can as a function of time. However, these predictions can be somewhat in error, since physical and thermal properties of foods are difficult to measure accurately and often can vary. Also, trapped air in the container and unknown convection effects can affect the accuracy of predictions.

EXAMPLE 9.12-2. Thermal Process Evaluation by Numerical Integration

In the sterilization of a canned purée, the temperature in the slowest-heating region (center) of the can was measured, giving the following time–temperature data for the heating and holding time. The cooling-time data will be neglected as a safety factor.

***t* (*min*)**	***T* (°*F*)**		***t* (*min*)**	***T* (°*F*)**	
0	80	(26.7°C)	40	225	(107.2°C)
15	165	(73.9)	50	230.5	(110.3)
25	201	(93.9)	64	235	(112.8)
30	212.5	(100.3)			

The F_0 value of *Cl. botulinum* is 2.45 min and z is 18°F. Calculate the F_0 value of the process above and determine if the sterilization is adequate.

Solution: In order to use Eq. (9.12-16), the values of $10(T-250)/z$ must be calculated for each time. For $t = 0$ min, $T = 80$°F, and $z = 18$°F,

$$10^{(T-250)/z} = 10^{(80-250)/18} = 3.6 \times 10^{-10}$$

For $t = 15$ min, $T = 165$°F,

$$10^{(165-250)/18} = 0.0000189$$

For $t = 25$ min, $T = 201$°F,

$$10^{(201-250)/18} = 0.00189$$

For $t = 30$ min,

$$10^{(201-250)/18} = 0.00825$$

For $t = 40$ min,

$$10^{(225-250)/18} = 0.0408$$

For $t = 50$ min,

$$10^{(230.5-250)/18} = 0.0825$$

For $t = 64$ min,

$$10^{(235-250)/18} = 0.1465$$

These calculated values are then used in Eq. (9.12-16) to perform a numerical integration to give $F_0 = 2.50$ min. The process value of 2.50 min is greater than the required value of 2.45 min, and the sterilization is adequate.

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Sterilization Methods Using Other Design Criteria

In types of thermal processing which are not necessarily involved with sterilization of foods, other types of design criteria are used. In foods the minimum adequate heat process should reduce the number of spores by a factor of 10^{-12}, that is, $N/N_0 = 10^{-12}$. However, in other batch-sterilization processes, such as in the sterilization of fermentation media, other criteria are often used. Often the equation for k, the reaction velocity constant for the specific organism to be used, is available:

Equation 9.12-8.

$$k = ae^{-E/RT}$$

Then Eq. (9.12-9) is written as

Equation 9.12-17.

$$\nabla = \ln \frac{N_0}{N} = a \int_{t=0}^{t} e^{-E/RT}\, dt = \int_{t=0}^{t} k\, dt$$

where ∇ is the design criterion. Usually, the contamination level N_0 is available and either the sterility level N or the time of sterilization at a given temperature is the unknown. In either case a numerical or graphical integration is used to solve the problem.

In sterilization of food in a container, the time required to render the material safe is calculated at the slowest-heating region of the container (usually the center). Other regions of the container are usually heated to higher temperatures and are overtreated. Hence, another method used is based on the probability of survival in the whole container. These details are given by others (C2, S2). In still another processing method, a short-time, continuous-flow process is used instead of a batch process in a container (B2).

Pasteurization

The term *pasteurization* is used today to apply to a mild heat treatment of foods that is less drastic than sterilization. It is used to kill organisms that are relatively low in thermal resistance compared to those which the more drastic sterilization processes are designed to eliminate. Pasteurization usually involves killing vegetative microorganisms and not heat-resistant spores.

The most common process is the pasteurization of milk to kill *Mycobacterium tuberculosis,* which is a non-spore-forming bacterium. This pasteurization does not sterilize the milk but kills the *M. tuberculosis* and reduces the other bacterial count sufficiently so that the milk can be stored if refrigerated.

For the pasteurization of such foods as milk, fruit juices, and beer, the same mathematical and numerical procedures covered for sterilization processes in this section are used to accomplish the degree of sterilization desired in pasteurization (B1, S2). The times involved are much shorter and the temperatures used in pasteurization are much lower. Generally, the F_0 value is given as 150°F (65.6°C) or a similar temperature rather than 250°F as in sterilization. Also, the concept of the z value is employed, in which a rise in temperature of z°F will increase the death rate by a factor of 10. An F_0 value written as F_{150}^{9} means the F value at 150°F with a z value of 9°F (S2).

In pasteurizing milk, batch and continuous processes are used. U.S. health regulations specify two equivalent sets of conditions; in one, the milk is held at 145°F (62.8°C) for 30 min, and in the other, at 161°F (71.7°C) for 15 s.

The general equations used for pasteurization are similar to sterilization and can be written as follows. Rewriting Eq. (9.12-13),

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Equation 9.12-18.

$$F^z_{T_1} = D_{T_1} \log_{10} \frac{N_0}{N}$$

Rewriting Eq. (9.12-14),

Equation 9.12-19.

$$F^z_{T_1} = t \cdot 10^{(T-T_1)/z}$$

where T_1 is the standard temperature being used, such as 150°F, z is the value of z in °F for a tenfold increase in death rate, and T is the temperature of the actual process.

EXAMPLE 9.12-3. Pasteurization of Milk

A typical F value given for the thermal processing of milk in a tubular heat exchanger is F^9_{150} = 9.0 min, and D_{150} = 0.6 min. Calculate the reduction in the number of viable cells under these conditions.

Solution: The z value is 9°F (5°C) and the temperature of the process is 150°F (65.6°C). Substituting into Eq. (9.12-18) and solving,

$$F^9_{150} = 9.0 = 0.6 \ \log_{10} \frac{N_0}{N}$$

$$\frac{N_0}{N} = \frac{10^{15}}{1}$$

This gives a reduction in the number of viable cells of 10^{15}.

Effects of Thermal Processing on Food Constituents

Thermal processing is used to cause the death of various undesirable microorganisms, but it also causes undesirable effects, such as the reduction of certain nutritional values. Ascorbic acid (vitamin C) and thiamin and riboflavin (vitamins B_1 and B_2) are partially destroyed by thermal processing. The reduction of these desirable constituents can also be given kinetic parameters such as F_0 and z values in the same way as for sterilization and pasteurization. Examples and data are given by Charm (C2).

These same kinetic methods of thermal death rates can also be applied to predict the time for detecting a flavor change in a food product. Dietrich et al. (D1) determined a curve for the number of days to detect a flavor change in frozen spinach versus temperature of storage. The data followed Eq. (9.12-8) and a first-order kinetic relation.

PROBLEMS

9.3-1. ***Humidity from Vapor Pressure.*** The air in a room is at 37.8°C and a total pressure of 101.3 kPa abs containing water vapor with a partial pressure p_A = 3.59 kPa. Calculate:

a. Humidity.
b. Saturation humidity and percentage humidity.
c. Percentage relative humidity.

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9.3-2. ***Percentage and Relative Humidity.*** The air in a room has a humidity H of 0.021 kg H_2O/kg dry air at 32.2°C and 101.3 kPa abs pressure. Calculate:

a. Percentage humidity H_P.

b. Percentage relative humidity H_R.

A2: **Ans.** (a) $H_P = 67.5\%$; (b) $H_R = 68.6\%$

9.3-3. ***Use of the Humidity Chart.*** The air entering a dryer has a temperature of 65.6°C (150°F) and dew point of 15.6°C (60°F). Using the humidity chart, determine the actual humidity and percentage humidity. Calculate the humid volume of this mixture and also calculate c_S using SI and English units.

A3: **Ans.** $H = 0.0113$ kg H_2O/kg dry air, $H_P = 5.3\%$, $c_S = 1.026$ kJ/kg · K (0.245 btu/lb_m · °F), $v_H = 0.976$ m^3 air + water vapor/kg dry air

9.3-4. ***Properties of Air to a Dryer.*** An air–water vapor mixture going to a drying process has a dry bulb temperature of 57.2°C and a humidity of 0.030 kg H_2O/kg dry air. Using the humidity chart and appropriate equations, determine the percentage humidity, saturation humidity at 57.2°C, dew point, humid heat, and humid volume.

9.3-5. ***Adiabatic Saturation Temperature.*** Air at 82.2°C having a humidity $H = 0.0655$ kg H_2O/kg dry air is contacted in an adiabatic saturator with water. It leaves at 80% saturation.

a. What are the final values of H and T°C?

b. For 100% saturation, what would be the values of H and T?

A5: **Ans.** (a) $H = 0.079$ kg H_2O/kg dry air, $T = 52.8$°C

9.3-6. ***Adiabatic Saturation of Air.*** Air enters an adiabatic saturator having a temperature of 76.7°C and a dew-point temperature of 40.6°C. It leaves the saturator 90% saturated. What are the final values of H and T°C?

9.3-7. ***Humidity from Wet and Dry Bulb Temperatures.*** An air–water vapor mixture has a dry bulb temperature of 65.6°C and a wet bulb temperature of 32.2°C. What is the humidity of the mixture?

A7: **Ans.** $H = 0.0175$ kg H_2O/kg dry air

9.3-8. ***Humidity and Wet Bulb Temperature.*** The humidity of an air–water vapor mixture is $H = 0.030$ kg H_2O/kg dry air. The dry bulb temperature of the mixture is 60°C. What is the wet bulb temperature?

9.3-9. ***Dehumidification of Air.*** Air having a dry bulb temperature of 37.8°C and a wet bulb temperature of 26.7°C is to be dried by first cooling to 15.6°C to condense water vapor and then heating to 23.9°C.

a. Calculate the initial humidity and percentage humidity.

b. Calculate the final humidity and percentage humidity. [*Hint*: Locate the initial point on the humidity chart. Then go horizontally (cooling) to the 100% saturation line. Follow this line to 15.6°C. Then go horizontally to the right to 23.9°C.]

A9: **Ans.** (b) $H = 0.0115$ kg H_2O/kg dry air, $H_P = 60\%$

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9.3-10. ***Cooling and Dehumidifying Air.*** Air entering an adiabatic cooling chamber has a temperature of 32.2°C and a percentage humidity of 65%. It is cooled by a cold water spray and saturated with water vapor in the chamber. After leaving, it is heated to 23.9°C. The final air has a percentage humidity of 40%.

a. What is the initial humidity of the air?

b. What is the final humidity after heating?

9.6-1. ***Time for Drying in Constant-Rate Period.*** A batch of wet solid was dried on a tray dryer using constant drying conditions and a thickness of material on the tray of 25.4 mm. Only the top surface was exposed. The drying rate during the constant-rate period was R = 2.05 kg $H_2O/h \cdot m^2$ (0.42 lb_m $H_2O/h \cdot ft^2$). The ratio L_S/A used was 24.4 kg dry solid/m^2 exposed surface (5.0 lb_m dry solid/ft^2). The initial free moisture was X_1 = 0.55 and the critical moisture content X_C = 0.22 kg free moisture/kg dry solid.

Calculate the time to dry a batch of this material from X_1 = 0.45 to X_2 = 0.30 using the same drying conditions but a thickness of 50.8 mm, with drying from the top and bottom surfaces. (*Hint*: First calculate L_S/A for this new case.)

A11: **Ans.** t = 1.785 h

9.6-2. ***Prediction of Effect of Process Variables on Drying Rate.*** Using the conditions in Example 9.6-3 for the constant-rate drying period, do as follows:

a. Predict the effect on R_C if the air velocity is only 3.05 m/s.

b. Predict the effect if the gas temperature is raised to 76.7°C and H remains the same.

c. Predict the effect on the time t for drying between moisture contents X_1 to X_2 if the thickness of material dried is 38.1 mm instead of 25.4 mm and the drying is still in the constant-rate period.

A12: **Ans.** (a) R_C = 1.947 kg $H_2O/h \cdot m^2$ (0.399 lb_m $H_2O/h \cdot ft^2$); (b) R_C = 4.21 kg $H_2O/h \cdot m^2$

9.6-3. ***Prediction in Constant-Rate Drying Region.*** A granular insoluble solid material wet with water is being dried in the constant-rate period in a pan 0.61 m × 0.61 m and the depth of material is 25.4 mm. The sides and bottom are insulated. Air flows parallel to the top drying surface at a velocity of 3.05 m/s and has a dry bulb temperature of 60°C and wet bulb temperature of 29.4°C. The pan contains 11.34 kg of dry solid having a free moisture content of 0.35 kg H_2O/kg dry solid, and the material is to be dried in the constant-rate period to 0.22 kg H_2O/kg dry solid.

a. Predict the drying rate and the time in hours needed.

b. Predict the time needed if the depth of material is increased to 44.5 mm.

9.6-4. ***Drying a Filter Cake in the Constant-Rate Region.*** A wet filter cake in a pan 1 ft × 1 ft square and 1 in. thick is dried on the top surface with air at a wet bulb temperature of 80°F and a dry bulb temperature of 120°F flowing parallel to the surface at a velocity of 2.5 ft/s.

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The dry density of the cake is 120 lb_m/ft^3 and the critical free moisture content is 0.09 lb H_2O/lb dry solid. How long will it take to dry the material from a free moisture content of 0.20 lb H_2O/lb dry material to the critical moisture content?

A14: **Ans.** $t = 13.3$ h

9.7-1. ***Numerical Integration for Drying in Falling-Rate Region.*** A wet solid is to be dried in a tray dryer under steady-state conditions from a free moisture content of $X_1 = 0.40$ kg H_2O/kg dry solid to $X_2 = 0.02$ kg H_2O/kg dry solid. The dry solid weight is 99.8 kg dry solid and the top surface area for drying is 4.645 m^2. The drying-rate curve can be represented by Fig. 9.5-1b.

a. Calculate the time for drying using numerical integration in the falling-rate period.

b. Repeat but use a straight line through the origin for the drying rate in the falling-rate period.

A15: **Ans.** (a) t(constant rate) = 2.91 h, t(falling rate) = 6.65 h, t(total) = 9.56 h

9.7-2. ***Drying Tests with a Foodstuff.*** In order to test the feasibility of drying a certain foodstuff, drying data were obtained in a tray dryer with air flow over the top exposed surface having an area of 0.186 m^2. The bone-dry sample weight was 3.765 kg dry solid. At equilibrium after a long period, the wet sample weight was 3.955 kg H_2O + solid. Hence, 3.955 − 3.765, or 0.190, kg of equilibrium moisture was present. The following sample weights versus time were obtained in the drying test:

Time (h)	*Weight (kg)*	*Time (h)*	*Weight (kg)*	*Time (h)*	*Weight (kg)*
0	4.944	2.2	4.554	7.0	4.019
0.4	4.885	3.0	4.404	9.0	3.978
0.8	4.808	4.2	4.241	12.0	3.955
1.4	4.699	5.0	4.150		

a. Calculate the free moisture content X kg H_2O/kg dry solid for each data point and plot X versus time. (*Hint:* For 0 h, 4.944 − 0.190 − 3.765 = 0.989 kg free moisture in 3.765 kg dry solid. Hence, $X = 0.989/3.765$.)

b. Measure the slopes, calculate the drying rates R in kg $H_2O/h \cdot m^2$, and plot R versus X.

c. Using this drying-rate curve, predict the total time to dry the sample from $X = 0.20$ to $X = 0.04$. Use numerical integration for the falling-rate period. What are the drying rate R_C in the constant-rate period and X_C?

A16: **Ans.** (c) $R_C = 0.996$ kg $H_2O/h \cdot m^2$, $X_C = 0.12$, $t = 4.1$ h (total)

9.7-3. ***Prediction of Drying Time.*** A material was dried in a tray-type batch dryer using constant-drying conditions. When the initial free moisture content was 0.28 kg free moisture/kg dry solid, 6.0 h was required to dry the material to a free moisture content of 0.08 kg free moisture/kg dry solid. The critical free moisture content is 0.14. Assuming a drying rate in the falling-rate region, where the rate is a

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straight line from the critical point to the origin, predict the time to dry a sample from a free moisture content of 0.33 to 0.04 kg free moisture/kg dry solid. (*Hint:* First use the analytical equations for the constant-rate and the linear falling-rate periods with the known total time of 6.0 h. Then use the same equations for the new conditions.)

9.8-1. ***Drying of Biological Material in Tray Dryer.*** A granular biological material wet with water is being dried in a pan 0.305 × 0.305 m and 38.1 mm deep. The material is 38.1 mm deep in the pan, which is insulated on the sides and bottom. Heat transfer is by convection from an air stream flowing parallel to the top surface at a velocity of 3.05 m/s, having a temperature of 65.6°C and humidity $H = 0.010$ kg H_2O/kg dry air. The top surface receives radiation from steam-heated pipes whose surface temperature $T_R = 93.3$°C. The emissivity of the solid is $\epsilon = 0.95$. It is desired to keep the surface temperature of the solid below 32.2°C so that decomposition will be kept low. Calculate the surface temperature and the rate of drying for the constant-rate period.

A18: **Ans.** $T_S = 31.3$°C, $R_C = 2.583$ kg H_2O/hm^2

9.8-2. ***Drying When Radiation, Conduction, and Convection Are Present.*** A material is granular and wet with water and is being dried in a layer 25.4 mm deep in a batch-tray dryer pan. The pan has a metal bottom having a thermal conductivity of $k_M = 43.3$ W/m · K and a thickness of 1.59 mm. The thermal conductivity of the solid is $k_S = 1.125$ W/m · K. The air flows parallel to the top exposed surface and the bottom metal at a velocity of 3.05 m/s and a temperature of 60°C and humidity $H = 0.010$ kg H_2O/kg dry solid. Direct radiation heat from steam pipes having a surface temperature of 104.4°C falls on the exposed top surface, whose emissivity is 0.94. Estimate the surface temperature and the drying rate for the constant-rate period.

9.9-1. ***Diffusion Drying in Wood.*** Repeat Example 9.9-1 using the physical properties given but with the following changes:

a. Calculate the time needed to dry the wood from a total moisture of 0.22 to 0.13. Use Fig. 5.3-13.

b. Calculate the time needed to dry planks of wood 12.7 mm thick from $X_{t1} = 0.29$ to $X_t = 0.09$. Compare with the time needed for 25.4 mm thickness.

A20: **Ans.** (b) $t = 7.60$ h (12.7 mm thick)

9.9-2. ***Diffusivity in Drying Tapioca Root.*** Using the data given in Example 9.9-2, determine the average diffusivity of the moisture up to a value of $X/X_C = 0.50$.

9.9-3. ***Diffusion Coefficient.*** Experimental drying data for a typical nonporous biological material obtained under constant drying conditions in the falling-rate region are tabulated below.

X/X_C	$t(h)$	X/X_C	$t(h)$
1.00	0	0.17	11.4
0.65	2.50	0.10	14.0
0.32	7.00	0.06	16.0

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Drying from one side occurs, with the material having a thickness of 10.1 mm. The data appear to follow the diffusion equation. Determine the average diffusivity over the range $X/X_C = 1.0–0.10$.

9.10-1. ***Drying a Bed of Solids by Through Circulation.*** Repeat Example 9.10-1 for drying of a packed bed of wet cylinders by through circulation of the drying air. Use the same conditions except that the air velocity is 0.381 m/s.

9.10-2. ***Derivation of Equation for Through-Circulation Drying.*** Different forms of Eqs. (9.10-11) and (9.10-12) can be derived using humidity and mass-transfer equations rather than temperature and heat-transfer equations. This can be done by writing a mass-balance equation similar to Eq. (9.10-2) for a heat balance and a mass-transfer equation similar to Eq. (9.10-3).

a. Derive the final equation for the time of drying in the constant-rate period using humidity and mass-transfer equations.

b. Repeat for the falling-rate period.

A24:

Ans.

$$\text{(a)}\ t = \frac{\rho_S x_1 (X_1 - X_C)}{G(H_W - H_1)(1 - e^{-k_y M_B a x_1 / G})}$$

9.10-3. ***Through-Circulation Drying in the Constant-Rate Period.*** Spherical wet catalyst pellets having a diameter of 12.7 mm are being dried in a through-circulation dryer. The pellets are in a bed 63.5 mm thick on a screen. The solids are being dried by air entering with a superficial velocity of 0.914 m/s at 82.2°C and having a humidity $H = 0.01$ kg H_2O/kg dry air. The dry solid density is determined as 1522 kg/m^3, and the void fraction in the bed is 0.35. The initial free moisture content is 0.90 kg H_2O/kg solid and the solids are to be dried to a free moisture content of 0.45, which is above the critical free moisture content. Calculate the time for drying in this constant-rate period.

9.10-4. ***Material and Heat Balances on a Continuous Dryer.*** Repeat Example 9.10-2, making heat and material balances, but with the following changes. The solid enters at 15.6°C and leaves at 60°C. The gas enters at 87.8°C and leaves at 32.2°C. Heat losses from the dryer are estimated as 2931 W.

9.10-5. ***Drying in a Continuous Tunnel Dryer.*** A rate of feed of 700 lb$_m$ dry solid/h containing a free moisture content of $X_1 = 0.4133$ lb H_2O/lb dry solid is to be dried to $X_2 = 0.0374$ lb H_2O/lb dry solid in a continuous-counterflow tunnel dryer. A flow of 13 280 lb$_m$ dry air/h enters at 203°F with an $H_2 = 0.0562$ lb H_2O/lb dry air. The stock enters at the wet bulb temperature of 119°F and remains essentially constant in temperature in the dryer. The saturation humidity at 119°F from the humidity chart is $H_W = 0.0786$ lb H_2O/lb dry air. The surface area available for drying is $(A/L_S) = 0.30$ ft^2/lb$_m$ dry solid.

A small-batch experiment was performed using approximately the same constant-drying conditions, air velocity, and temperature of the solid as in the continuous dryer. The equilibrium critical moisture content was found to be $X_C = 0.0959$ lb H_2O/lb dry solid, and the experimental value of $k_y M_B$ was found as 30.15 lb$_m$ air/h · ft^2. In the falling-rate period, the drying rate was directly proportional to X.

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For the continuous dryer, calculate the time in the dryer in the constant-rate zone and the falling-rate zone.

A27: **Ans.** H_C = 0.0593 lb H_2O/lb dry air, H_1 = 0.0760 lb H_2O/lb dry air, t = 4.20 h in the constant-rate zone, t = 0.47 h in the falling-rate zone

9.10-6. ***Air Recirculation in a Continuous Dryer.*** The wet feed material to a continuous dryer contains 50 wt % water on a wet basis and is dried to 27 wt % by countercurrent air flow. The dried product leaves at the rate of 907.2 kg/h. Fresh air to the system is at 25.6°C and has a humidity of H = 0.007 kg H_2O/kg dry air. The moist air leaves the dryer at 37.8°C and H = 0.020 and part of it is recirculated and mixed with the fresh air before entering a heater. The heated mixed air enters the dryer at 65.6°C and H = 0.010. The solid enters at 26.7°C and leaves at 26.7°C. Calculate the fresh-air flow, the percent air leaving the dryer that is recycled, the heat added in the heater, and the heat loss from the dryer.

A28: **Ans.** 32 094 kg fresh dry air/h, 23.08% recycled, 440.6 kW in heater

9.12-1. ***Sterilizing Canned Foods.*** In a sterilizing retort, cans of a given food were heated; the average temperature in the center of a can is approximately 98.9°C for the first 30 min. The average temperature for the next period is 110°C. If F_0 for the spore organism is 2.50 min and z = 10°C, calculate the time of heating at 110°C to make the process safe.

A29: **Ans.** 29.9 min

9.12-2. ***Temperature Effect on Decimal Reduction Time.*** Prove by combining Eqs. (9.12-6) and (9.12-8) that a plot of $\log_{10} D_T$ versus $1/T$ (T in degrees absolute) is a straight line.

9.12-3. ***Thermal Process Time for Pea Purée.*** For cans of pea purée, F_0 = 2.45 min and z = 9.94°C (C2). Neglecting heatup time, determine the process time for adequate sterilization at 112.8°C at the center of the can.

A31: **Ans.** t = 16.76 min

9.12-4. ***Process Time for Adequate Sterilization.*** The F_0 value for a given canned food is 2.80 min and z is 18°F (10°C). The center temperatures of a can of this food when heated in a retort were as follows for the time periods given: t_1 (0–10 min), T_1 = 140°F; t_2 (10–30 min), T_2 = 185°F; t_3 (30–50 min), T_3 = 220°F; t_4 (50–80 min), T_4 = 230°F; t_5 (80–100 min), T_5 = 190°F. Determine if adequate sterilization is obtained.

9.12-5. ***Process Time and Numerical Integration.*** The following time–temperature data were obtained for the heating, holding, and cooling of a canned food product in a retort, the temperature being measured in the center of the can:

t(*min*)	*T*(°*F*)	*t*(*min*)	*T*(°*F*)
0	110 (43.3°C)	80	232 (111.1)
20	165 (73.9)	90	225 (107.2)
40	205 (96.1)	100	160 (71.1)
60	228 (108.9)		

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The F_0 value used is 2.60 min and z is 18°F (10°C). Calculate the F_0 value for this process and determine if the thermal processing is adequate. Use SI and English units.

9.12-6. ***Sterility Level of Fermentation Medium.*** The aqueous medium in a fermentor is being sterilized and the time–temperature data obtained are as follows:

Time (min)	0	10	20	25	30	35
Temperature (°C)	100	110	120	120	110	100

The reaction velocity constant k in min^{-1} for the contaminating bacterial spores can be represented as (A1)

$$k = 7.94 \times 10^{38} e^{-(68.7 \times 10^3)/1.987T}$$

where T = K. The contamination level $N_0 = 1 \times 10^{12}$ spores. Calculate the sterility level N at the end and ∇.

9.12-7. ***Time for Pasteurization of Milk.*** Calculate the time in min at 62.8°C for pasteurization of milk. The F_0 value to be used at 65.6°C is 9.0 min. The z value is 5°C.

A35: **Ans.** t = 32.7 min

9.12-8. ***Reduction in Number of Viable Cells in Pasteurization.*** In a given pasteurization process the reduction in the number of viable cells used is 10^{15} and the F_0 value is 9.0 min. If the reduction is to be increased to 10^{16} because of increased contamination, what would be the new F_0 value?

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