Transport Processes and Separation Process Principles

(INCLUDES UNIT OPERATIONS)

FOURTH EDITION

CHRISTIE JOHN GEANKOPLIS

Table of Contents

Chapter 10. Stage and Continuous Gas–Liquid Separation Processes

TYPES OF SEPARATION PROCESSES AND METHODS

Introduction

Many chemical process materials and biological substances occur as mixtures of different components in the gas, liquid, or solid phase. In order to separate or remove one or more of the components from its original mixture, it must be contacted with another phase. The two phases are brought into more or less intimate contact with each other so that a solute or solutes can diffuse from one to the other. The two bulk phases are usually only somewhat miscible in each other. The two-phase pair can be gas–liquid, gas–solid, liquid–liquid, or liquid–solid. During the contact of the two phases the components of the original mixture redistribute themselves between the two phases. The phases are then separated by simple physical methods. By choosing the proper conditions and phases, one phase is enriched while the other is depleted in one or more components.

Types of Separation Processes

Absorption

When the two contacting phases are a gas and a liquid, this operation is called *absorption.* A solute *A* or several solutes are absorbed from the gas phase into the liquid phase in absorption. This process involves molecular and turbulent diffusion or mass transfer of solute *A* through a stagnant, nondiffusing gas *B* into a stagnant liquid *C*. An example is absorption of ammonia *A* from air *B* by the liquid water *C*. Usually, the exit ammonia–water solution is distilled to recover relatively pure ammonia.

Another example is absorbing SO_2 from the flue gases by absorption in alkaline solutions. In the hydrogenation of edible oils in the food industry, hydrogen gas is bubbled into oil and absorbed. The hydrogen in solution then reacts with the oil in the presence of a catalyst. The reverse of absorption is called *stripping* or *desorption,* and the same theories and basic principles hold. An example is the steam stripping of nonvolatile oils, in which the steam contacts the oil and small amounts of volatile components of the oil pass out with the steam.

When the gas is pure air and the liquid is pure water, the process is called *humidification. Dehumidification* involves removal of water vapor from air.

Distillation

In the *distillation* process, a volatile vapor phase and a liquid phase that vaporizes are involved. An example is distillation of an ethanol–water solution, where the vapor contains a concentration of ethanol greater than in the liquid. Another example is distillation of an ammonia–water solution to produce a vapor richer in ammonia. In the distillation of crude petroleum, various fractions, such as gasoline, kerosene, and heating oils, are distilled off.

Liquid–liquid extraction

When the two phases are liquids, where a solute or solutes are removed from one liquid phase to another liquid phase, the process is called *liquid–liquid extraction.* One example is extraction of acetic acid from a water solution by isopropyl ether. In the pharmaceutical industry, antibiotics in an aqueous fermentation solution are sometimes removed by extraction with an organic solvent.

Leaching

If a fluid is being used to extract a solute from a solid, the process is called *leaching.* Sometimes this process is also called *extraction.* Examples are leaching copper from solid ores by sulfuric acid and leaching vegetable oils from solid soybeans by organic solvents such as hexane. Vegetable oils are also leached from other biological products, such as peanuts, rape seeds, and sunflower seeds. Soluble sucrose is leached by water extraction from sugar cane and beets.

Membrane processing

Separation of molecules by the use of membranes is a relatively new separation process and is becoming more important. The relatively thin, solid membrane controls the rate of movement of molecules between two phases. It is used to remove salt from water, to purify gases, in food processing, and so on.

Crystallization

Solute components soluble in a solution can be removed from a solution by adjusting the conditions, such as temperature or concentration, so that the solubility of one or more of the components is exceeded and they *crystallize* out as a solid phase. Examples of this separation process are crystallization of sugar from solution and crystallization of metal salts in the processing of metal ore solutions.

Adsorption

In an *adsorption* process, one or more components of a liquid or gas stream are adsorbed on the surface or in the pores of a solid adsorbent and a separation is obtained. Examples include removal of organic compounds from polluted water, separation of paraffins from aromatics, and removal of solvents from air.

Ion exchange

In an *ion-exchange* process, certain ions are removed by an ion-exchange solid. This separation process closely resembles adsorption.

Processing Methods

Several methods of processing are used in the separations discussed above. The two phases, such as gas and liquid, or liquid and liquid, can be mixed together in a vessel and then separated. This is a *single-stage process.* Often the phases are mixed in one stage, separated, and then brought into contact again in a *multiple-stage process.* These two methods can be carried out batchwise or continuously. In still another general method, the two phases can be contacted continuously in a packed tower.

In this chapter, humidification and absorption will be considered; in Chapter 11, distillation; in Chapter 12, adsorption, ion exchange, liquid–liquid extraction, leaching, and crystallization; in Chapter 13, membrane processes; and in Chapter 14, mechanical–physical separations. In these processes the equilibrium relations between the two phases being considered must be known. This is discussed for gas–liquid systems in Section 10.2 and for the other systems in Chapters 11, 12, and 13.

EQUILIBRIUM RELATIONS BETWEEN PHASES

Phase Rule and Equilibrium

In order to predict the concentration of a solute in each of two phases in equilibrium, experimental equilibrium data must be available. If the two phases are not at equilibrium, the rate of mass transfer is proportional to the driving force, which is the departure from equilibrium. In all cases involving equilibria, two phases are involved, such as gas–liquid or liquid–liquid. The important variables affecting the equilibrium of a solute are temperature, pressure, and concentration.

The equilibrium between two phases in a given situation is restricted by the phase rule:

Equation 10.2-1.

$$F = C - P + 2$$

where P is the number of phases at equilibrium, C the number of total components in the two phases when no chemical reactions are occurring, and F the number of variants or degrees of freedom of the system. For example, for the gas–liquid system of CO_2–air–water, there are two phases and three components (considering air as one inert component). Then, by Eq. (10.2-1),

$$F = C - P + 2 = 3 - 2 + 2 = 3$$

This means that there are 3 degrees of freedom. If the total pressure and temperature are set, only one variable is left that can be set arbitrarily. If the mole fraction composition x_A of CO_2 (A) in the liquid phase is set, the mole fraction composition y_A or pressure p_A in the gas phase is automatically determined.

The phase rule does not tell us the partial pressure p_A in equilibrium with the selected x_A. The value of p_A must be determined experimentally. The two phases can, of course, be gas–liquid, liquid–solid, and so on. For example, the equilibrium distribution of acetic acid between a water phase and an isopropyl ether phase has been determined experimentally for various conditions.

Gas–Liquid Equilibrium

Gas–liquid equilibrium data

To illustrate the obtaining of experimental gas–liquid equilibrium data, the system SO_2–air–water will be considered. An amount of gaseous SO_2, air, and water are put in a closed container and shaken repeatedly at a given temperature until equilibrium is reached. Samples of the gas and liquid are analyzed to determine the partial pressure p_A in atm of SO_2 (A) in the gas and mole fraction x_A in the liquid. Figure 10.2-1 shows a plot of data from Appendix A.3 of the partial pressure p_A of SO_2 in the vapor in equilibrium with the mole fraction x_A of SO_2 in the liquid at 293 K (20°C).

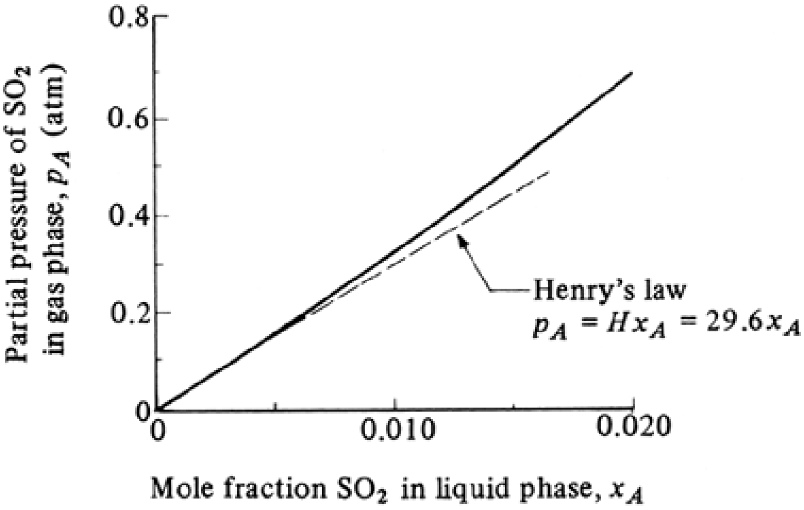

Figure 10.2-1. Equilibrium plot for SO_2–water system at 293 K (20°C).

Henry's law

Often the equilibrium relation between p_A in the gas phase and x_A can be expressed by a straight-line Henry's law equation at low concentrations:

Equation 10.2-2.

$$p_A = Hx_A$$

where H is the Henry's law constant in atm/mole fraction for the given system. If both sides of Eq. (10.2-2) are divided by total pressure P in atm,

Equation 10.2-3.

$$y_A = H'x_A$$

where H' is the Henry's law constant in mole frac gas/mole frac liquid and is equal to H/P. Note that H' depends on total pressure, whereas H does not.

In Fig. 10.2-1 the data follow Henry's law up to a concentration x_A of about 0.005, where $H = 29.6$ atm/mol frac. In general, up to a total pressure of about 5×10^5 Pa (5 atm) the value of H is independent of P. Data for some common gases with water are given in Appendix A.3.

EXAMPLE 10.2-1. Dissolved Oxygen Concentration in Water

What will be the concentration of oxygen dissolved in water at 298 K when the solution is in equilibrium with air at 1 atm total pressure? The Henry's law constant is 4.38×10^4 atm/mol fraction.

Solution: The partial pressure p_A of oxygen (A) in air is 0.21 atm. Using Eq. (10.2-2),

$$0.21 = Hx_A = 4.38 \times 10^4 x_A$$

Solving, $x_A = 4.80 \times 10^{-6}$ mol fraction. This means that 4.80×10^{-6} mol O_2 is dissolved in 1.0 mol water plus oxygen, or 0.000853 part O_2/100 parts water.

SINGLE AND MULTIPLE EQUILIBRIUM CONTACT STAGES

Single-Stage Equilibrium Contact

In many operations of the chemical and other process industries, the transfer of mass from one phase to another occurs, usually accompanied by a separation of the components of the mixture, since one component will be transferred to a larger extent than will another component.

A single-stage process can be defined as one in which two different phases are brought into intimate contact with each other and then are separated. During the time of contact, intimate mixing occurs and the various components diffuse and redistribute themselves between the two phases. If mixing time is long enough, the components are essentially at equilibrium in the two phases after separation and the process is considered a single equilibrium stage.

A single equilibrium stage can be represented as in Fig. 10.3-1. The two entering phases, L_0 and V_2, of known amounts and compositions, enter the stage; mixing and equilibration occur; and the two exit streams, L_1 and V_1, leave in equilibrium with each other. Making a total mass balance,

Equation 10.3-1.

$$L_0 + V_2 = L_1 + V_1 = M$$

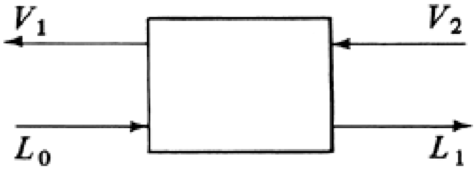

Figure 10.3-1. Single-stage equilibrium process.

where L is kg (lb$_m$), V is kg, and M is total kg.

Assuming that three components, A, B and C, are present in the streams and making a balance on A and C,

Equation 10.3-2.

$$L_0 x_{A0} + V_2 y_{A2} = L_1 x_{A1} + V_1 y_{A1} = M x_{AM}$$

Equation 10.3-3.

$$L_0 x_{C0} + V_2 y_{C2} = L_1 x_{C1} + V_1 y_{C1} = M x_{CM}$$

An equation for B is not needed since $x_A + x_B + x_C = 1.0$. The mass fraction of A in the L stream is x_A and is y_A in the V stream. The mass fraction of A in the M stream is x_{AM}.

To solve the three equations, the equilibrium relations between the components must be known. In Section 10.3B, this will be done for a gas–liquid system and in Chapter 11 for a vapor–liquid system. Note that Eqs. (10.3-1)–(10.3-3) can also be written using mole units, with L and V having units of moles and x_A and y_A units of mole fraction.

Single-Stage Equilibrium Contact for Gas–Liquid System

In the usual gas–liquid system the solute *A* is in the gas phase *V*, along with inert air *B*, and in the liquid phase *L*, along with inert water *C*. Assuming that air is essentially insoluble in the water phase and that water does not vaporize to the gas phase, the gas phase is a binary *A*–*B* and the liquid phase is a binary *A*–*C*. Using moles and mole fraction units, Eq. (10.3-1) holds for a single-stage process for the total material balance. Since component *A* is the only component that redistributes between the two phases, a balance on *A* can be written as follows:

Equation 10.3-4.

$$L'\left(\frac{x_{A0}}{1-x_{A0}}\right)+V'\left(\frac{y_{A2}}{1-y_{A2}}\right)=L'\left(\frac{x_{A1}}{1-x_{A1}}\right)+V'\left(\frac{y_{A1}}{1-y_{A1}}\right)$$

where L' is moles inert water *C* and V' is moles inert air *B*. Both L' and V' are constant and usually known.

To solve Eq. (10.3-4), the relation between y_{A1} and x_{A1} in equilibrium is given by Henry's law:

Equation 10.3-5.

$$y_{A1} = H'x_{A1}$$

If the solution is not dilute, equilibrium data in the form of a plot of p_A or y_A versus x_A must be available, as in Fig. 10.2-1.

EXAMPLE 10.3-1. Equilibrium Stage Contact for CO_2–Air–Water

A gas mixture at 1.0 atm pressure abs containing air and CO_2 is contacted in a single-stage mixer continuously with pure water at 293 K. The two exit gas and liquid streams reach equilibrium. The inlet gas flow rate is 100 kg mol/h, with a mole fraction of CO_2 of $y_{A2} = 0.20$. The liquid flow rate entering is 300 kg mol water/h. Calculate the amounts and compositions of the two outlet phases. Assume that water does not vaporize to the gas phase.

***Solution*:** The flow diagram is the same as given in Fig. 10.3-1. The inert water flow is $L' = L_0 = 300$ kg mol/h. The inert air flow V' is obtained from Eq. (10.3-6):

Equation 10.3-6.

$$V' = V(1 - y_A)$$

Hence, the inert air flow is $V' = V_2(1 - y_{A2}) = 100(1 - 0.20) = 80$ kg mol/h. Substituting into Eq. (10.3-4) to make a balance on CO_2 (*A*),

Equation 10.3-7.

$$300\left(\frac{0}{1-0}\right)+80\left(\frac{0.20}{1-0.20}\right)=300\left(\frac{x_{A1}}{1-x_{A1}}\right)+80\left(\frac{y_{A1}}{1-y_{A1}}\right)$$

At 293 K, the Henry's law constant from Appendix A.3 is $H = 0.142 \times 10^4$ atm/mol frac. Then $H' = H/P = 0.142 \times 10^4/1.0 = 0.142 \times 10^4$ mol frac gas/mol frac liquid. Substituting into Eq. (10.3-5),

Equation 10.3-8.

$$y_{A1} = 0.142 \times 10^4 x_{A1}$$

Substituting Eq. (10.3-8) into (10.3-7) and solving, $x_{A1} = 1.41 \times 10^{-4}$ and $y_{A1} = 0.20$. To calculate the total flow rates leaving,

$$L_1 = \frac{L'}{1 - x_{A1}} = \frac{300}{1 - 1.41 \times 10^{-4}} = 300 \text{ kg mol/h}$$

$$V_1 = \frac{V'}{1 - y_{A1}} = \frac{80}{1 - 0.20} = 100 \text{ kg mol/h}$$

In this case, since the liquid solution is so dilute, $L_0 \cong L_1$.

Countercurrent Multiple-Contact Stages

Derivation of general equation

In Section 10.3A we used single-stage contact to transfer the solute *A* between the *V* and *L* phases. In order to transfer more solute from, say, the V_1 stream, the single-stage contact can be repeated by again contacting the V_1 stream leaving the first stage with fresh L_0. This can be repeated using multiple stages. However, this is wasteful of the L_0 stream and gives a dilute product in the outlet L_1 streams. To conserve use of the L_0 stream and to get a more concentrated product, countercurrent multiple-stage contacting is generally used. This is somewhat similar to countercurrent heat transfer in a heat exchanger, where the outlet heated stream approaches more closely the temperature of the inlet hot stream.

The process flow diagram for a countercurrent stage process is shown in Fig. 10.3-2. The inlet *L* stream is L_0 and the inlet *V* stream is V_{N+1} instead of V_2 as for a single-stage in Fig. 10.3-1. The outlet product streams are V_1 and L_N and the total number of stages is *N*. The component *A* is being exchanged between the *V* and *L* streams. The *V* stream is composed mainly of component *B* and the *L* stream of component *C*. Components *B* and *C* may or may not be somewhat miscible in each other. The two-phase system can be gas–liquid, vapor–liquid, liquid–liquid, or other.

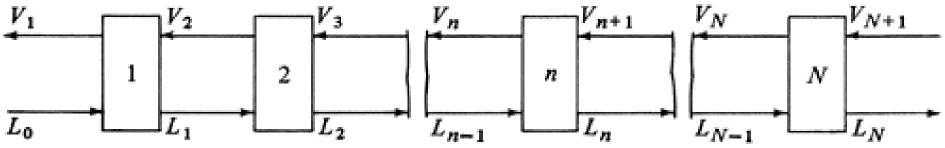

Figure 10.3-2. Countercurrent multiple-stage process.

Making a total overall balance on all stages,

Equation 10.3-9.

$$L_0 + V_{N+1} = L_N + V_1 = M$$

where V_{N+1} is mol/h entering L_N is mol/h leaving the process, and *M* is the total flow. Note in Fig. 10.3-2 that any two streams leaving a stage are in equilibrium with each other. For example, in stage *n*, V_n and L_n are in equilibrium. For an overall component balance on *A*, *B*, or *C*,

Equation 10.3-10.

$$L_0 x_0 + V_{N+1} y_{N+1} = L_N x_N + V_1 y_1 = M x_M$$

where *x* and *y* are mole fractions. Flows in kg/h (lb_m/h) and mass fraction can also be used in these equations.

Making a total balance over the first *n* stages,

Equation 10.3-11.

$$L_0 + V_{n+1} = L_n + V_1$$

Making a component balance over the first n stages,

Equation 10.3-12.

$$L_0 x_0 + V_{n+1} y_{n+1} = L_n x_n + V_1 y_1$$

Solving for y_{n+1} in Eq. (10.3-12),

Equation 10.3-13.

$$y_{n+1} = \frac{L_n x_n}{V_{n+1}} + \frac{V_1 y_1 - L_0 x_0}{V_{n+1}}$$

This is an important material-balance equation, often called an *operating line.* It relates the concentration y_{n+1} in the V stream with x_n in the L stream passing it. The terms V_1, y_1, L_0, and x_0 are constant and are usually known or can be determined from Eqs. (10.3-9)–(10.3-12).

Countercurrent contact with immiscible streams

An important case in which the solute A is being transferred occurs when the solvent stream V contains components A and B with no C and the solvent stream L contains A and C with no B. The two streams L and V are immiscible in each other, with only A being transferred. When Eq. (10.3-13) is plotted on an x-y plot (x_A and y_A of component A) such as Fig. 10.3-3, it is often curved, since the slope L_n/V_{n+1} of the operating line varies if the L and V streams vary from stage to stage.

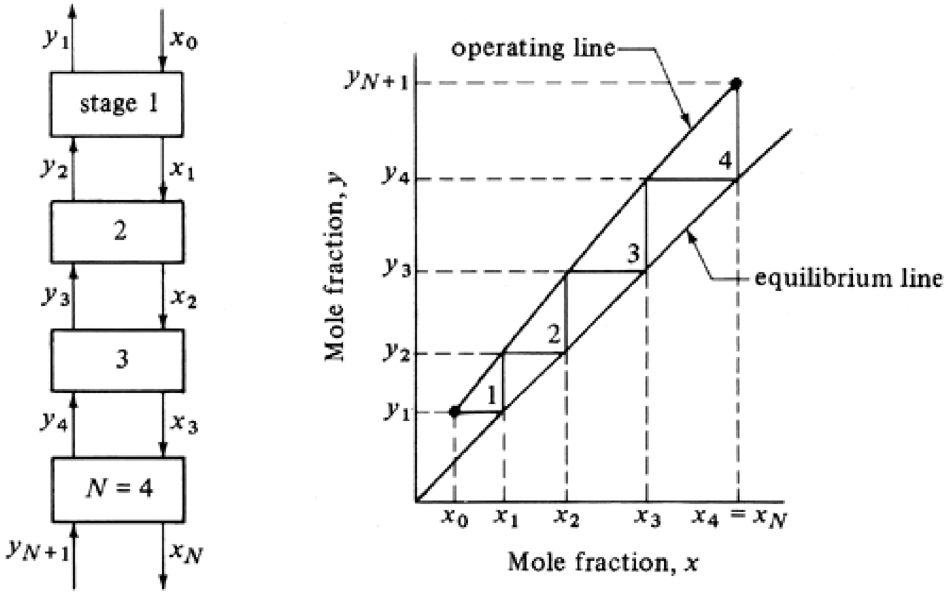

Figure 10.3-3. Number of stages in a countercurrent multiple-stage contact process.

In Fig. 10.3-3 the equilibrium line that relates the compositions of two streams leaving a stage in equilibrium with each other is plotted. To determine the number of ideal stages required to bring about a given separation or reduction of the concentration of A from y_{N+1} to y_1, the calculation is often done graphically. Starting at stage 1, y_1 and x_0 are on the operating line, Eq. (10.3-13), plotted in the figure. The vapor y_1 leaving is in equilibrium with the leaving x_1 and both compositions are on the equilibrium line. Then y_2 and x_1 are on the operating line and y_2 is in equilibrium with x_2, and so on. Each stage is represented by a step drawn on Fig. 10.3-3. The steps are continued on the graph until y_{N+1} is reached. Alternatively, we can start at y_{N+1} and draw the steps going to y_1.

If the streams *L* and *V* are dilute in component *A*, the streams are approximately constant and the slope L_n/V_{n+1} of Eq. (10.3-13) is nearly constant. Hence, the operating line is essentially a straight line on an *x*-*y* plot. In distillation, where only components *A* and *B* are present, Eq. (10.3-13) also holds for the operating line; this will be covered in Chapter 11. Cases where *A, B*, and *C* are appreciably soluble in each other often occur in liquid–liquid extraction and will be discussed in Chapter 12.

Example 10.3-2. Absorption of Acetone in a Countercurrent Stage Tower

It is desired to absorb 90% of the acetone in a gas containing 1.0 mol % acetone in air in a countercurrent stage tower. The total inlet gas flow to the tower is 30.0 kg mol/h, and the total inlet pure water flow to be used to absorb the acetone is 90 kg mol H_2O/h. The process is to operate isothermally at 300 K and a total pressure of 101.3 kPa. The equilibrium relation for the acetone (*A*) in the gas–liquid is $y_A = 2.53x_A$. Determine the number of theoretical stages required for this separation.

Solution: The process flow diagram is similar to Fig. 10.3-3. Given values are $y_{AN+1} = 0.01$, $x_{A0} = 0$, $V_{N+1} =$ 30.0 kg mol/h, and $L_0 = 90.0$ kg mol/h. Making an acetone material balance,

$$\text{amount of entering acetone} = y_{AN+1}V_{N+1} = 0.01(30.0) = 0.30 \text{ kg mol/h}$$

$$\text{entering air} = (1 - y_{AN+1})V_{N+1} = (1 - 0.01)(30.0)$$

$$= 29.7 \text{ kg mol air/h}$$

$$\text{acetone leaving in } V_1 = 0.10(0.30) = 0.030 \text{ kg mol/h}$$

$$\text{acetone leaving in } L_N = 0.90(0.30) = 0.27 \text{ kg mol/h}$$

$$V_1 = 29.7 + 0.03 = 29.73 \text{ kg mol air + acetone/h}$$

$$y_{A1} = \frac{0.030}{29.73} = 0.00101$$

$$L_N = 90.0 + 0.27 = 90.27 \text{ kg mol water + acetone/h}$$

$$x_{AN} = \frac{0.27}{90.27} = 0.00300$$

Since the flow of liquid varies only slightly from $L_0 = 90.0$ at the inlet to $L_N = 90.27$ at the outlet and *V* from 30.0 to 29.73, the slope L_n/V_{n+1} of the operating line in Eq. (10.3-13) is essentially constant. This line is plotted in Fig. 10.3-4 together with the equilibrium relation $y_A = 2.53x_A$. Starting at point y_{A1}, x_{A0}, the stages are drawn as shown. About 5.2 theoretical stages are required.

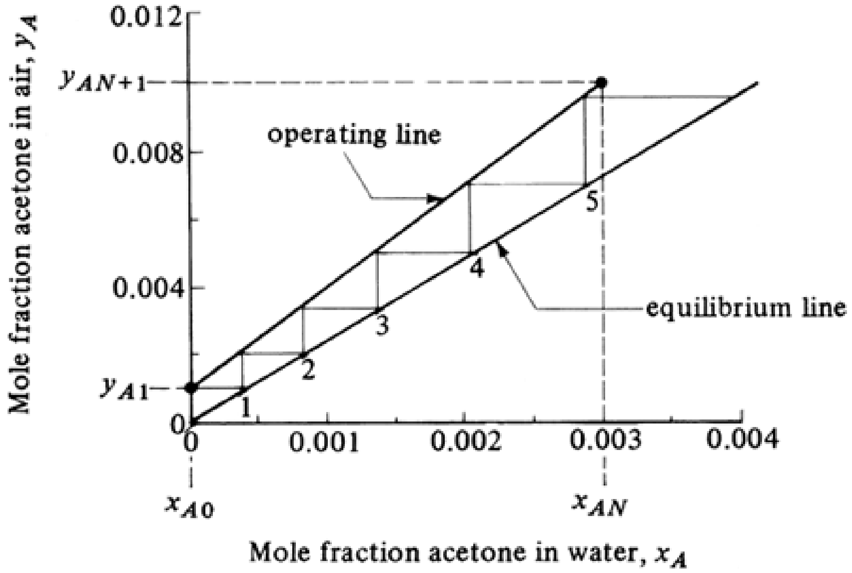

Figure 10.3-4. Theoretical stages for countercurrent absorption in Example 10.3-2.

Analytical Equations for Countercurrent Stage Contact

When the flow rates *V* and *L* in a countercurrent process are essentially constant, the operating-line equation (10.3-13) becomes straight. If the equilibrium line is also a straight line over the concentration range, simplified analytical expressions can be derived for the number of equilibrium stages in a countercurrent stage process.

Referring again to Fig. 10.3-2, Eq. (10.3-14) is an overall component balance on component *A*:

Equation 10.3-14.

$$L_0 x_0 + V_{N+1} y_{N+1} = L_N x_N + V_1 y_1$$

Rearranging,

Equation 10.3-15.

$$L_N x_N - V_{N+1} y_{N+1} = L_0 x_0 - V_1 y_1$$

Making a component balance for *A* on the first *n* stages,

Equation 10.3-16.

$$L_0 x_0 + V_{n+1} y_{n+1} = L_n x_n + V_1 y_1$$

Rearranging,

Equation 10.3-17.

$$L_0 x_0 - V_1 y_1 = L_n x_n - V_{n+1} y_{n+1}$$

Equating Eq. (10.3-15) to (10.3-17),

Equation 10.3-18.

$$L_n x_n - V_{n+1} y_{n+1} = L_N x_N - V_{N+1} y_{N+1}$$

Since the molar flows are constant, $L_n = L_N$ = constant = L and $V_{n+1} = V_{N+1}$ = constant = V. Then Eq. (10.3-18) becomes

Equation 10.3-19.

$$L(x_n - x_N) = V(y_{n+1} - y_{N+1})$$

Since y_{n+1} and x_{n+1} are in equilibrium and the equilibrium line is straight, $y_{n+1} = mx_{n+1}$. Also, $y_{N+1} = mx_{N+1}$. Substituting mx_{n+1} for y_{n+1} and calling $A = L/mV$, Eq. (10.3-19) becomes

Equation 10.3-20.

$$x_{n+1} - Ax_n = \frac{y_{N+1}}{m} - Ax_N$$

where A is an absorption factor and is constant.

All factors on the right-hand side of Eq. (10.3-20) are constant. This equation is a linear first-order difference equation and can be solved by the calculus of finite-difference methods (G1, M1). The final derived equations are as follows.

For transfer of solute A from phase L to V (stripping),

Equation 10.3-21.

$$\frac{x_0 - x_N}{x_0 - (y_{N+1}/m)} = \frac{(1/A)^{N+1} - (1/A)}{(1/A)^{N+1} - 1}$$

Equation 10.3-22.

$$N = \frac{\ln\left[\frac{x_0 - (y_{N+1}/m)}{x_N - (y_{N+1}/m)}(1 - A) + A\right]}{\ln(1/A)}$$

When $A = 1$,

Equation 10.3-23.

$$N = \frac{x_0 - x_N}{x_N - (y_{N+1}/m)}$$

For transfer of solute A from phase V to L (absorption),

Equation 10.3-24.

$$\frac{y_{N+1} - y_1}{y_{N+1} - mx_0} = \frac{A^{N+1} - A}{A^{N+1} - 1}$$

Equation 10.3-25.

$$N = \frac{\ln\left[\frac{y_{N+1} - mx_0}{y_1 - mx_0}\left(1 - \frac{1}{A}\right) + \frac{1}{A}\right]}{\ln A}$$

When $A = 1$,

Equation 10.3-26.

$$N = \frac{y_{N+1} - y_1}{y_1 - mx_0}$$

The term A is often called the *absorption factor* and S the *stripping factor,* where $S = 1/A$. These equations can be used with any consistent set of units such as mass flow and mass fraction or molar flow and mole fraction. Such series of equations are often called *Kremser equations* and are convenient to use.

If the equilibrium line is not straight but curved somewhat, the slope will vary and, hence, m and $A = L/mV$ will vary. For absorption (referring to Fig. 10.3-3) at the concentrated end, the slope m_N or tangent of the equilibrium line at the concentrations y_N, x_N leaving this bottom stage N is used. This m_N is at point x_N on the equilibrium line. For the top or dilute end at stage 1 of the tower, the slope of the equilibrium line m_1 at the concentrations y_1, x_1 leaving this stage is employed. This m_1 is at point y_1 on the equilibrium line. Then $A_N = L_N/m_N V_{N+1}$ and $A_1 = L_0/m_1 V_1$. The geometric average is used, where $A = \sqrt{A_N A_1}$ (P1, T3). Also, the dilute m_1 is used in Eqs. (10.3-24)–(10.3-26).

For stripping at the top or concentrated stage, the slope m_1 or tangent to the equilibrium line at the concentrations y_1, x_1 leaving this stage is used. This m_1 is at point y_1 on the equilibrium line. At the bottom stage or dilute end of the tower, the slope m_N of the equilibrium line at the points y_N, x_N is used. This m_N is at x_N on the equilibrium line. Then, $A_N = L_N/m_N V_{N+1}$, $A_1 = L_0/m_1 V_1$, and $A = \sqrt{A_N A_1}$. Again the dilute m_N is used in Eqs. (10.3-21)–(10.3-23). Sometimes only the values of A and m at the dilute end are used since more of the stages are in this region.

Example 10.3-3. Number of Stages by Analytical Equation.

Repeat Example 10.3-2 but use the Kremser analytical equations for countercurrent stage processes.

Solution: At one end of the process at stage 1, $V_1 = 29.73$ kg mol/h, $y_{A1} = 0.00101$, $L_0 = 90.0$, and $x_{A0} = 0$. Also, the equilibrium relation is $y_A = 2.53x_A$ where $m = 2.53$. Then,

$$A_1 = \frac{L}{mV} = \frac{L_0}{mV_1} = \frac{90.0}{2.53 \times 29.73} = 1.20$$

At stage N, $V_{N+1} = 30.0$, $y_{AN+1} = 0.01$, $L_N = 90.27$, and $x_{AN} = 0.00300$.

$$A_N = \frac{L_N}{mV_{N+1}} = \frac{90.27}{2.53 \times 30.0} = 1.19$$

The geometric average $A = \sqrt{A_1 A_N} = \sqrt{1.20 \times 1.19} = 1.195$

The acetone solute is transferred from the V to the L phase (absorption). Substituting into Eq. (10.3-25),

$$N = \frac{\ln\left[\frac{0.01 - 2.53(0)}{0.00101 - 2.53(0)}\left(1 - \frac{1}{1.195}\right) + \frac{1}{1.195}\right]}{\ln(1.195)} = 5.04 \text{ stages}$$

This compares closely with 5.2 stages obtained using the graphical method.

MASS TRANSFER BETWEEN PHASES

Introduction and Equilibrium Relations

Introduction to interphase mass transfer

In Chapter 7 we considered mass transfer from a fluid phase to another phase, which was most often a solid phase. The solute *A* was usually transferred from the fluid phase by convective mass transfer and through the solid by diffusion. In the present section we shall be concerned with the mass transfer of solute *A* from one fluid phase by convection and then through a second fluid phase by convection. For example, the solute may diffuse through a gas phase and then diffuse through and be absorbed in an adjacent and immiscible liquid phase. This occurs in the case of absorption of ammonia from air by water.

The two phases are in direct contact with each other, such as in a packed, tray, or spray-type tower, and the interfacial area between the phases is usually not well defined. In two-phase mass transfer, a concentration gradient will exist in each phase, causing mass transfer to occur. At the interface between the two fluid phases, equilibrium exists in most cases.

Equilibrium relations

Even when mass transfer is occurring, equilibrium relations are important to determine concentration profiles for predicting rates of mass transfer. In Section 10.2 the equilibrium relation in a gas–liquid system and Henry's law were discussed. In Section 7.1C a discussion covered equilibrium distribution coefficients between two phases. These equilibrium relations will be used in discussion of mass transfer between phases in this section.

Concentration Profiles in Interphase Mass Transfer

In the majority of mass-transfer systems, two phases, which are essentially immiscible in each other, are present together with an interface between these two phases. Assuming solute *A* is diffusing from the bulk gas phase *G* to the liquid phase *L*, it must pass through phase *G*, through the interface, and then into phase *L* in series. A concentration gradient must exist to cause this mass transfer through the resistances in each phase, as shown in Fig. 10.4-1. The average or bulk concentration of *A* in the gas phase in mole fraction units is y_{AG}, where $y_{AG} = p_A/P$, and in the bulk liquid phase in mole fraction units it is x_{AL}.

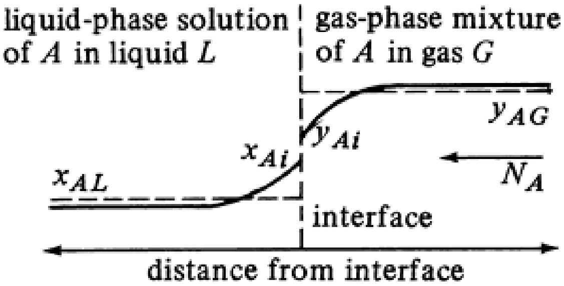

Figure 10.4-1. Concentration profile of solute A diffusing through two phases.

The concentration in the bulk gas phase y_{AG} decreases to y_{Ai} at the interface. The liquid concentration starts at x_{Ai} at the interface and falls to x_{AL}. At the interface, since there would be no resistance to transfer across this interface, y_{Ai} and x_{Ai} are in equilibrium and are related by the equilibrium distribution relation

Equation 10.4-1.

$$y_{Ai} = f(x_{Ai})$$

where y_{Ai} is a function of x_{Ai}. They are related by an equilibrium plot such as Fig. 10.1-1. If the system follows Henry's law, $y_A P$ or p_A and x_A are related by Eq. (10.2-2) at the interface.

Experimentally, the resistance at the interface has been shown to be negligible for most cases of mass transfer where chemical reactions do not occur, such as absorption of common gases from air to water and extraction of organic solutes from one phase to another. However, there are some exceptions. Certain surface-active compounds may concentrate at the interface and cause an "interfacial resistance" that slows down the diffusion of solute molecules. Theories for predicting when interfacial resistance may occur are often obscure and unreliable.

Mass Transfer Using Film Mass-Transfer Coefficients and Interface Concentrations

Equimolar counterdiffusion

For equimolar counterdiffusion the concentrations of Fig. 10.4-1 can be plotted on an *x*-*y* diagram as in Fig. 10.4-2. Point *P* represents the bulk phase compositions x_{AG} and x_{AL} of the two phases and point *M* the interface concentrations y_{Ai} and x_{Ai}. For *A* diffusing from the gas to liquid and *B* in equimolar counterdiffusion from liquid to gas,

Equation 10.4-2.

$$N_A = k'_y(y_{AG} - y_{Ai}) = k'_x(x_{Ai} - x_{AL})$$

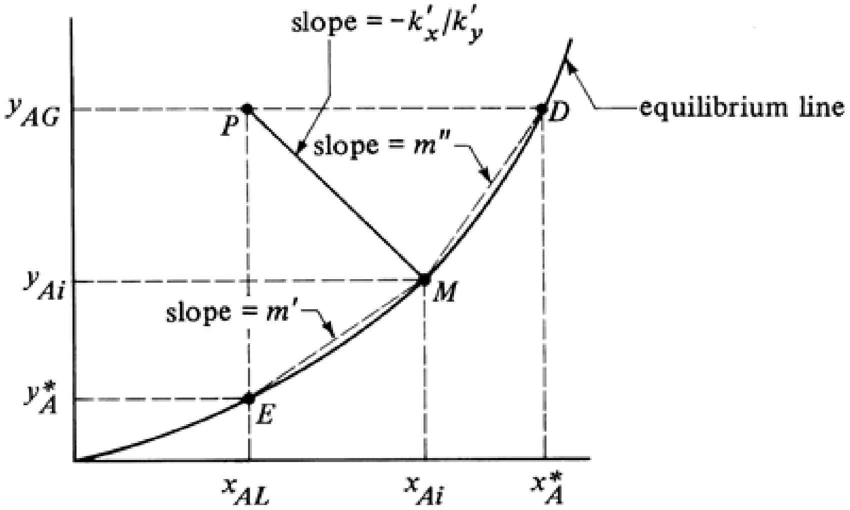

Figure 10.4-2. Concentration driving forces and interface concentrations in interphase mass transfer (equimolar counterdiffusion).

where k'_y is the gas-phase mass-transfer coefficient in kg mol/s · m^2 · mol frac (g mol/s · cm^2 · mol frac, lb mol/h · ft^2 · mol frac) and k'_x the liquid-phase mass-transfer coefficient in kg mol/s · m^2 · mol frac (g mol/s · cm^2 · mol frac, lb mol/h · ft^2 · mol frac). Rearranging Eq. (10.4-2),

Equation 10.4-3.

$$-\frac{k'_x}{k'_y} = \frac{y_{AG} - y_{Ai}}{x_{AL} - x_{Ai}}$$

The driving force in the gas phase is ($y_{AG} - y_{Ai}$) and in the liquid phase it is ($x_{Ai} - x_{AL}$). The slope of the line *PM* is $-k'_x/k'_y$ This means that if the two film coefficients k'_x and k'_y are known, the interface compositions can be determined by drawing line *PM* with a slope $-k'_x/k'_y$ intersecting the equilibrium line.

The bulk-phase concentrations y_{AG} and x_{AL} can be determined by simply sampling the mixed bulk gas phase and sampling the mixed bulk liquid phase. The interface concentrations are determined by Eq. (10.4-3).

Diffusion of A through stagnant or nondiffusing B

For the common case of *A* diffusing through a stagnant gas phase and then through a stagnant liquid phase, the concentrations are shown in Fig. 10.4-3, where *P* again represents bulk-phase compositions and *M* interface compositions. The equations for *A* diffusing through a stagnant gas and then through a stagnant liquid are

Equation 10.4-4.

$$N_A = k_y(y_{AG} - y_{Ai}) = k_x(x_{Ai} - x_{AL})$$

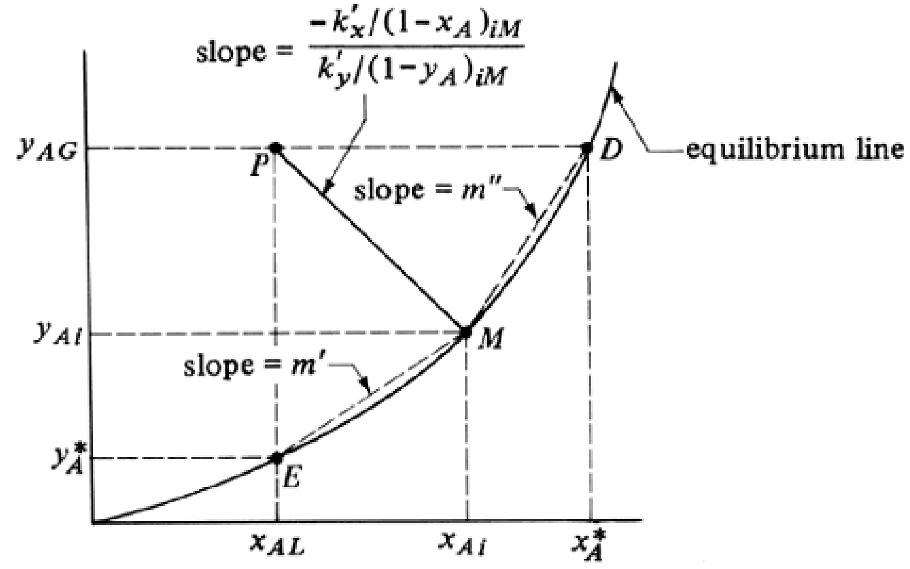

Figure 10.4-3. Concentration driving forces and interface concentrations in interphase mass transfer (A diffusing through stagnant B).

Now,

Equation 10.4-5.

$$k_y = \frac{k'_y}{(1 - y_A)_{iM}} \qquad k_x = \frac{k'_x}{(1 - x_A)_{iM}}$$

where

Equation 10.4-6.

$$(1 - y_A)_{iM} = \frac{(1 - y_{Ai}) - (1 - y_{AG})}{\ln[(1 - y_{Ai})/(1 - y_{AG})]}$$

Equation 10.4-7.

$$(1 - x_A)_{iM} = \frac{(1 - x_{AL}) - (1 - x_{Ai})}{\ln[(1 - x_{AL})/(1 - x_{Ai})]}$$

Then,

Equation 10.4-8.

$$N_A = \frac{k'_y}{(1 - y_A)_{iM}}(y_{AG} - y_{Ai}) = \frac{k'_x}{(1 - x_A)_{iM}}(x_{Ai} - x_{AL})$$

Note that $(1 - y_A)_{iM}$ is the same as y_{BM} of Eq. (7.2-11) but is written for the interface, and $(1 - x_A)_{iM}$ is the same as x_{BM} of Eq. (7.2-11). Using Eq. (10.4-8) and rearranging,

Equation 10.4-9.

$$\frac{-k'_x/(1 - x_A)_{iM}}{k'_y/(1 - y_A)_{iM}} = \frac{y_{AG} - y_{Ai}}{x_{AL} - x_{Ai}}$$

The slope of the line *PM* in Fig. 10.4-3 to obtain the interface compositions is given by the left-hand side of Eq. (10.4-9). This differs from the slope of Eq. (10.4-3) for equimolar counterdiffusion by the terms $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$. When *A* is diffusing through stagnant *B* and the solutions are dilute, $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$ are close to 1.

A trial-and-error method is needed to use Eq. (10.4-9) to get the slope, since the left-hand side contains the terms y_{Ai} and x_{Ai} that are being sought. For the first trial $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$ are assumed to be 1.0, and Eq. (10.4-9) is used to get the slope and y_{Ai} and x_{Ai} values. Then for the second trial, these values of y_{Ai} and x_{Ai} are used to calculate a new slope to get new values of y_{Ai} and x_{Ai}. This is repeated until the interface compositions do not change. Three trials are usually sufficient.

EXAMPLE 10.4-1. Interface Compositions in Interphase Mass Transfer

The solute *A* is being absorbed from a gas mixture of *A* and *B* in a wetted-wall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower the bulk gas concentration $y_{AG} = 0.380$ mol fraction and the bulk liquid concentration is $x_{AL} = 0.100$. The tower is operating at 298 K and 1.013×10^5 Pa and the equilibrium data are as follows:

x_A	y_A	x_A	y_A
0	0	0.20	0.131
0.05	0.022	0.25	0.187
0.10	0.052	0.30	0.265
0.15	0.087	0.35	0.385

The solute *A* diffuses through stagnant *B* in the gas phase and then through a nondiffusing liquid.

Using correlations for dilute solutions in wetted-wall towers, the film mass-transfer coefficient for *A* in the gas phase is predicted as $k_y = 1.465 \times 10^{-3}$ kg mol A/s · m^2 · mol frac (1.08 lb mol/h · ft^2 · mol frac) and for the liquid phase as $k_x = 1.967 \times 10^{-3}$ kg mol A/s · m^2 · mol frac (1.45 lb mol/h · ft^2 · mol frac). Calculate the interface concentrations y_{Ai} and x_{Ai} and the flux N_A.

***Solution*:** Since the correlations are for dilute solutions, $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$ are approximately 1.0 and the coefficients are the same as k'_y and k'_x. The equilibrium data are plotted in Fig. 10.4-4. Point *P* is plotted at $y_{AG} = 0.380$ and $x_{AL} = 0.100$. For the first trial, $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$ are assumed as 1.0 and the slope of line *PM* is, from Eq. (10.4-9),

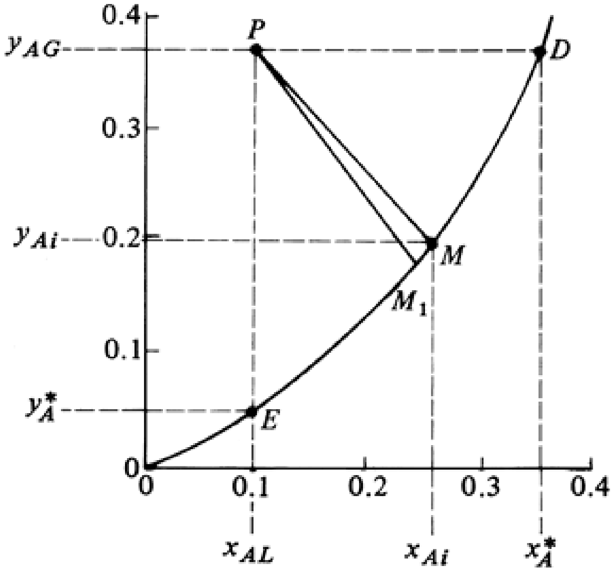

Figure 10.4-4. Location of interface concentrations for Example 10.4-1.

$$-\frac{k'_x/(1-x_A)_{iM}}{k'_y/(1-y_A)_{iM}} = -\frac{1.967\times10^{-3}/1.0}{1.465\times10^{-3}/1.0} = -1.342$$

A line through point P with a slope of −1.342 is plotted in Fig. 10.4-4 intersecting the equilibrium line at M_1, where $y_{Ai} = 0.183$ and $x_{Ai} = 0.247$.

For the second trial we use y_{Ai} and x_{Ai} from the first trial to calculate the new slope. Substituting into Eqs. (10.4-6) and (10.4-7),

$$(1-y_A)_{iM} = \frac{(1-y_{Ai})-(1-y_{AG})}{\ln[(1-y_{Ai})/(1-y_{AG})]}$$

$$= \frac{(1-0.183)-(1-0.380)}{\ln[(1-0.183)/(1-0.380)]} = 0.715$$

$$(1-x_A)_{iM} = \frac{(1-x_{AL})-(1-x_{Ai})}{\ln[(1-x_{AL})/(1-x_{Ai})]}$$

$$= \frac{(1-0.100)-(1-0.247)}{\ln[(1-0.100)/(1-0.247)]} = 0.825$$

Substituting into Eq. (10.4-9) to obtain the new slope,

$$-\frac{k'_x/(1-x_A)_{iM}}{k'_y/(1-y_A)_{iM}} = -\frac{1.967\times10^{-3}/0.825}{1.465\times10^{-3}/0.715} = -1.163$$

A line through point P with a slope of −1.163 is plotted and intersects the equilibrium line at M, where $y_{Ai} = 0.197$ and $x_{Ai} = 0.257$.

Using these new values for the third trial, the following values are calculated:

$$(1 - y_A)_{iM} = \frac{(1 - 0.197) - (1 - 0.380)}{\ln[(1 - 0.197)/(1 - 0.380)]} = 0.709$$

$$(1 - x_A)_{iM} = \frac{(1 - 0.100) - (1 - 0.257)}{\ln[(1 - 0.100)/(1 - 0.257)]} = 0.820$$

$$-\frac{k'_x/(1 - x_A)_{iM}}{k'_y/(1 - y_A)_{iM}} = -\frac{1.967 \times 10^{-3}/0.820}{1.465 \times 10^{-3}/0.709} = -1.160$$

This slope of −1.160 is essentially the same as the slope of −1.163 for the second trial. Hence, the final values are $y_{Ai} = 0.197$ and $x_{Ai} = 0.257$ and are shown as point *M*.

To calculate the flux, Eq. (10.4-8) is used:

$$N_A = \frac{k'_y}{(1 - y_A)_{iM}}(y_{AG} - y_{Ai}) = \frac{1.465 \times 10^{-3}}{0.709}(0.380 - 0.197)$$
$$= 3.78 \times 10^{-4}\ \text{kg mol/s} \cdot \text{m}^2$$

$$N_A = \frac{1.08}{0.709}(0.380 - 0.197) = 0.2785\ \text{lb mol/h} \cdot \text{ft}^2$$

$$N_A = \frac{k'_x}{(1 - x_A)_{iM}}(x_{Ai} - x_{AL}) = \frac{1.967 \times 10^{-3}}{0.820}(0.257 - 0.100)$$
$$= 3.78 \times 10^{-4}\ \text{kg mol/s} \cdot \text{m}^2$$

Note that the flux N_A through each phase is the same as in the other phase, which should be the case at steady state.

Overall Mass-Transfer Coefficients and Driving Forces

Introduction

Film or single-phase mass-transfer coefficients k'_y and k'_x or k_y and k_x are often difficult to measure experimentally, except in certain experiments designed so that the concentration difference across one phase is small and can be neglected. As a result, overall mass-transfer coefficients K'_y and k'_x are measured based on the gas phase or liquid phase. This method is used in heat transfer, where overall heat-transfer coefficients are measured based on inside or outside areas instead of film coefficients.

The overall mass transfer K'_y is defined as

Equation 10.4-10.

$$N_A = K'_y(y_{AG} - y^*_A)$$

where K'_y is based on the overall gas-phase driving force in kg mol/s · m^2 · mol frac, and y^*_A is the value that would be in equilibrium with x_{AL}, as shown in Fig. 10.4-2. Also, k'_x is defined as

Equation 10.4-11.

$$N_A = K'_x(x^*_A - x_{AL})$$

where k'_x is based on the overall liquid-phase driving force in kg mol/s · m^2 · mol frac and x^*_A is the value that would be in equilibrium with y_{AG}.

Equimolar counterdiffusion and/or diffusion in dilute solutions

Equation (10.4-2) holds for equimolar counterdiffusion, or, when the solutions are dilute, Eqs. (10.4-8) and (10.4-2) are identical:

Equation 10.4-2.

$$N_A = k'_y(y_{AG} - y_{Ai}) = k'_x(x_{Ai} - x_{AL})$$

From Fig. 10.4-2,

Equation 10.4-12.

$$y_{AG} - y^*_A = (y_{AG} - y_{Ai}) + (y_{Ai} - y^*_A)$$

Between the points E and M the slope m' can be given as

Equation 10.4-13.

$$m' = \frac{y_{Ai} - y^*_A}{x_{Ai} - x_{AL}}$$

Solving Eq. (10.4-13) for (y_{Ai} – y^*_A) and substituting into Eq. (10.4-12),

Equation 10.4-14.

$$y_{AG} - y^*_A = (y_{AG} - y_{Ai}) + m'(x_{Ai} - x_{AL})$$

Then, on substituting Eqs. (10.4-10) and (10.4-2) into (10.4-14) and canceling out N_A,

Equation 10.4-15.

$$\frac{1}{K'_y} = \frac{1}{k'_y} + \frac{m'}{k'_x}$$

The left-hand side of Eq. (10.4-15) is the total resistance based on the overall gas driving force and equals the gas film resistance $1/k'_y$ plus the liquid film resistance m'/k'_x.

In a similar manner, from Fig. 10.4-2,

Equation 10.4-16.

$$x^*_A - x_{AL} = (x^*_A - x_{Ai}) + (x_{Ai} - x_{AL})$$

Equation 10.4-17.

$$m'' = \frac{y_{AG} - y_{Ai}}{x^*_A - x_{Ai}}$$

Proceeding as before,

Equation 10.4-18.

$$\frac{1}{K'_x} = \frac{1}{m''k'_y} + \frac{1}{k'_x}$$

Several special cases of Eqs. (10.4-15) and (10.4-18) will now be discussed. The numerical values of k'_x and k'_y are very roughly similar. The values of the slopes m' and m'' are very important. If m' is quite small, so that the equilibrium curve in Fig. 10.4-2 is almost horizontal, a small value of y_A in the gas will give a large value of x_A in equilibrium in the liquid. The gas solute A is then very soluble in the liquid phase, and hence the term m'/k'_x in Eq. (10.4-15) is very small. Then,

Equation 10.4-19.

$$\frac{1}{K'_y} \cong \frac{1}{k'_y}$$

and the major resistance is in the gas phase, or the "gas phase is controlling." The point M has moved down very close to E, so that

Equation 10.4-20.

$$y_{AG} - y^*_A \cong y_{AG} - y_{Ai}$$

Similarly, when m'' is very large, the solute A is very insoluble in the liquid, $1/(m''k'_y)$ becomes small, and

Equation 10.4-21.

$$\frac{1}{K'_x} \cong \frac{1}{k'_x}$$

The "liquid phase is controlling" and $x_{Ai} \cong x^*_A$. Systems for absorption of oxygen or CO_2 from air by water are similar to Eq. (10.4-21).

Diffusion of A through stagnant or nondiffusing B

For the case of A diffusing through nondiffusing B, Eqs. (10.4-8) and (10.4-14) hold and Fig. 10.4-3 is used:

Equation 10.4-8.

$$N_A = \frac{k'_y}{(1-y_A)_{iM}}(y_{AG} - y_{Ai}) = \frac{k'_x}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})$$

Equation 10.4-14.

$$y_{AG} - y^*_A = (y_{AG} - y_{Ai}) + m'(x_{Ai} - x_{AL})$$

We must, however, define the equations for the flux using overall coefficients as follows:

Equation 10.4-22.

$$N_A = \left[\frac{K'_y}{(1-y_A)_{*M}}\right](y_{AG} - y^*_A) = \left[\frac{K'_x}{(1-x_A)_{*M}}\right](x^*_A - x_{AL})$$

The bracketed terms are often written as follows:

Equation 10.4-23.

$$K_y = \frac{K'_y}{(1 - y_A)_{*M}} \qquad K_x = \frac{K'_x}{(1 - x_A)_{*M}}$$

where K_y is the overall gas mass-transfer coefficient for *A* diffusing through stagnant *B* and K_x the overall liquid mass-transfer coefficient. These two coefficients are concentration-dependent. Substituting Eqs. (10.4-8) and (10.4-22) into (10.4-14), we obtain

Equation 10.4-24.

$$\frac{1}{K'_y/(1 - y_A)_{*M}} = \frac{1}{k'_y/(1 - y_A)_{iM}} + \frac{m'}{k'_x/(1 - x_A)_{iM}}$$

where

Equation 10.4-25.

$$(1 - y_A)_{*M} = \frac{(1 - y_A^*) - (1 - y_{AG})}{\ln[(1 - y_A^*)/(1 - y_{AG})]}$$

Similarly, for k'_x,

Equation 10.4-26.

$$\frac{1}{K'_x/(1 - x_A)_{*M}} = \frac{1}{m''k'_y/(1 - y_A)_{iM}} + \frac{1}{k'_x/(1 - x_A)_{iM}}$$

where

Equation 10.4-27.

$$(1 - x_A)_{*M} = \frac{(1 - x_{AL}) - (1 - x_A^*)}{\ln[(1 - x_{AL})/(1 - x_A^*)]}$$

It should be noted that the relations derived here also hold for any two-phase system, where *y* stands for one phase and *x* for the other phase. For example, for the extraction of the solute acetic acid (*A*) from water (*y* phase) by isopropyl ether (*x* phase), the same relations will hold.

EXAMPLE 10.4-2. Overall Mass-Transfer Coefficients from Film Coefficients

Using the same data as in Example 10.4-1, calculate the overall mass-transfer coefficient K'_y, the flux, and the percent resistance in the gas and liquid films. Do this for the case of *A* diffusing through stagnant *B*.

Solution: From Fig. 10.4-4, $y_A^* = 0.052$, which is in equilibrium with the bulk liquid $x_{AL} = 0.10$. Also, $y_{AG} = 0.380$. The slope of chord *m'* between *E* and *M* from Eq. (10.4-13) is, for $y_{Ai} = 0.197$ and $x_{Ai} = 0.257$,

$$m' = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_{AL}} = \frac{0.197 - 0.052}{0.257 - 0.100} = 0.923$$

From Example 10.4-1,

$$\frac{k'_y}{(1 - y_A)_{iM}} = \frac{1.465 \times 10^{-3}}{0.709} \qquad \frac{k'_x}{(1 - x_A)_{iM}} = \frac{1.967 \times 10^{-3}}{0.820}$$

Using Eq. (10.4-25),

$$(1 - y_A)_{*M} = \frac{(1 - y_A^*) - (1 - y_{AG})}{\ln[(1 - y_A^*)/(1 - y_{AG})]}$$

$$= \frac{(1 - 0.052) - (1 - 0.380)}{\ln[(1 - 0.052)/(1 - 0.380)]} = 0.733$$

Then, using Eq. (10.4-24),

$$\frac{1}{K'_y/0.773} = \frac{1}{1.465 \times 10^{-3}/0.709} + \frac{0.923}{1.967 \times 10^{-3}/0.820}$$

$$= 484.0 + 384.8 = 868.8$$

Solving, K'_y = 8.90 × 10^{-4}. The percent resistance in the gas film is (484.0/868.8)100 = 55.7% and in the liquid film 44.3%. The flux is as follows, using Eq. (10.4-22):

$$N_A = \frac{K'_y}{(1 - y_A)_{*M}}(y_{AG} - y_A^*) = \frac{8.90 \times 10^{-4}}{0.773}(0.380 - 0.052)$$

$$= 3.78 \times 10^{-4} \text{ kg mol/s} \cdot \text{m}^2$$

This, of course, is the same flux value as was calculated in Example 10.4-1 using the film equations.

Discussion of overall coefficients

If the two-phase system is such that the major resistance is in the gas phase, as in Eq. (10.4-19), then to increase the overall rate of mass transfer, efforts should be centered on increasing the gas-phase turbulence, not the liquid-phase turbulence. For a two-phase system where the liquid film resistance is controlling, turbulence should be increased in this phase to increase rates of mass transfer.

To design mass-transfer equipment, the overall mass-transfer coefficient is synthesized from the individual film coefficients, as discussed in this section.

Generally, when the major resistance to mass transfer is in the gas phase, the overall mass transfer coefficient K'_y or the film coefficient k'_y is used. An example would be absorption of ammonia from air to water. When the major resistance is in the liquid phase, as in absorption of oxygen from air by water, k'_x or k'_x is used.

CONTINUOUS HUMIDIFICATION PROCESSES

Introduction and Types of Equipment for Humidification

Introduction to gas–liquid contactors

When a relatively warm liquid is brought into direct contact with gas that is unsaturated, some of the liquid is vaporized. The liquid temperature will drop mainly because of the latent heat of evaporation. This direct contact of a gas with a pure liquid occurs most often in contacting air with water. This is done for the following purposes: humidifying air for control of the moisture content of air in drying or air conditioning; dehumidifying air, where cold water condenses some water vapor from warm air; and water cooling, where evaporation of water to the air cools warm water.

In Chapter 9 the fundamentals of humidity and adiabatic humidification were discussed. In this section the performance and design of continuous air–water contactors is considered. The emphasis is on cooling of water, since this is the most important type of process in the process industries. There are many cases in industry in which warm water is discharged from heat exchangers and condensers when it would be more economical to cool and reuse it than to discard it.

Towers for water cooling

In a typical water-cooling tower, warm water flows countercurrent to an air stream. Typically, the warm water enters the top of a packed tower and cascades down through the packing, leaving at the bottom. Air enters at the bottom of the tower and flows upward through the descending water. The tower packing often consists of slats of plastic or of a packed bed. The water is distributed by troughs and overflows to cascade over slat gratings or packing that provides large interfacial areas of contact between the water and air in the form of droplets and films of water. The flow of air upward through the tower can be induced by the buoyancy of the warm air in the tower (natural draft) or by the action of a fan. Detailed descriptions of towers are given in other texts (B1, T1).

The water cannot be cooled below the wet bulb temperature. The driving force for the evaporation of the water is approximately the vapor pressure of the water less the vapor pressure it would have at the wet bulb temperature. The water can be cooled only to the wet bulb temperature, and in practice it is cooled to about 3 K or more above this. Only a small amount of water is lost by evaporation in cooling water. Since the latent heat of vaporization of water is about 2300 kJ/kg, a typical change of about 8 K in water temperature corresponds to an evaporation loss of about 1.5%. Hence, the total flow of water is usually assumed to be constant in calculations of tower size.

In humidification and dehumidification, intimate contact between the gas phase and liquid phase is needed for large rates of mass transfer and heat transfer. The gas-phase resistance controls the rate of transfer. Spray or packed towers are used to give large interfacial areas and to promote turbulence in the gas phase.

Theory and Calculations for Water-Cooling Towers

Temperature and concentration profiles at interface

In Fig. 10.5-1 the temperature profile and concentration profile in terms of humidity are shown at the water–gas interface. Water vapor diffuses from the interface to the bulk gas phase with a driving force in the gas phase of $(H_i - H_G)$ kg H_2O/kg dry air. There is no driving force for mass transfer in the liquid phase, since water is a pure liquid. The temperature driving force is $T_L - T_i$ in the liquid phase and $T_i - T_G$ K or °C in the gas phase. Sensible heat flows from the bulk liquid to the interface in the liquid. Sensible heat also flows from the interface to the gas phase. Latent heat also leaves the interface in the water vapor, diffusing to the gas phase. The sensible heat flow from the liquid to the interface equals the sensible heat flow in the gas plus the latent heat flow in the gas.

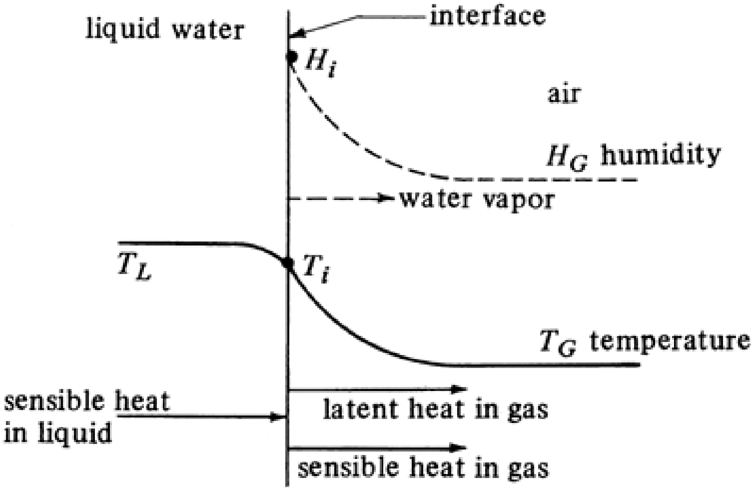

Figure 10.5-1. Temperature and concentration profiles in upper part of cooling tower.

The conditions in Fig. 10.5-1 occur at the upper part of the cooling tower. In the lower part of the cooling tower, the temperature of the bulk water is higher than the wet bulb temperature of the air but may be below the dry bulb temperature. Then the direction of the sensible heat flow in Fig. 10.5-1 is reversed.

Rate equations for heat and mass transfer

We shall consider a packed water-cooling tower with air flowing upward and water countercurrently downward in the tower. The total interfacial area between the air and water phases is unknown, since the surface area of the packing is not equal to the interfacial area between the water droplets and the air. Hence, we define a quantity *a*, defined as m^2 of interfacial area per m^3 volume of packed section, or m^2/m^3. This is combined with the gas-phase mass-transfer coefficient k_G in kg mol/s · m^2 Pa or kg mol/s · m^2 · atm to give a volumetric coefficient k_Ga in kg mol/s · m^3 volumePa or kg mol/s · m^3 · atm (lb mol/h · ft^3 · atm).

The process is carried out adiabatically; the various streams and conditions are shown in Fig. 10.5-2, where

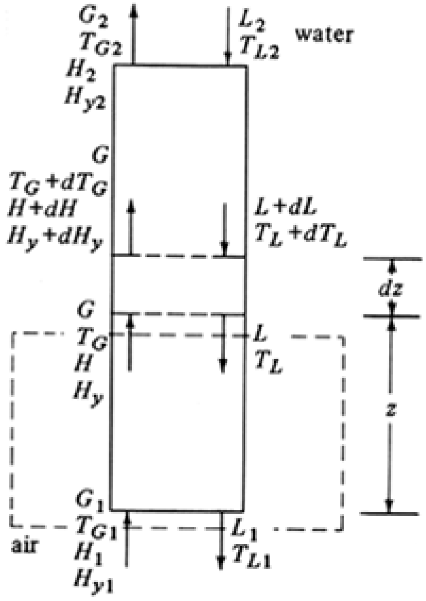

Figure 10.5-2. Continuous countercurrent adiabatic water cooling.

L = water flow, kg water/s · m^2 (lb$_m$/h · ft^2)

T_L = temperature of water, °C or K (°F)

G = dry air flow, kg/s · m^2 (lb$_m$/h · ft^2)

T_G = temperature of air, °C or K (°F)

H = humidity of air, kg water/kg dry air (lb water/lb dry air)

H_y = enthalpy of air–water vapor mixture, J/kg dry air (btu/lb$_m$ dry air)

The enthalpy H_y as given in Eq. (9.3-8) is

Equation 9.3-8.

$$H_y = c_S(T - T_0) + H\lambda_0 = (1.005 + 1.88H)10^3(T - 0) + 2.501 \times 10^6 H \qquad \text{(SI)}$$

$$H_y = c_S(T - T_0) + H\lambda_0 = (0.24 + 0.45H)(T - 32) + 1075.4H \qquad \text{(English)}$$

The base temperature selected is 0°C or 273 K (32°F). Note that $(T - T_0)$°C = $(T - T_0)$ K.

Making a total heat balance for the dashed-line box shown in Fig. 10.5-2, an operating line is obtained:

Equation 10.5-1.

$$G(H_y - H_{y1}) = Lc_L(T_L - T_{L1})$$

This assumes that L is essentially constant, since only a small amount is evaporated. The heat capacity c_L of the liquid is assumed constant at 4.187×10^3 J/kg · K (1.00 btu/lb$_m$ · °F). When plotted on a chart of H_y versus T_L, Eq. (10.5-1) is a straight line with a slope of Lc_L/G. Making an overall heat balance over both ends of the tower,

Equation 10.5-2.

$$G(H_{y2} - H_{y1}) = Lc_L(T_{L2} - T_{L1})$$

Again, making a heat balance for the *dz* column height and neglecting sensible-heat terms compared to the latent heat,

Equation 10.5-3.

$$Lc_L\, dT_L = G\, dH_y$$

The total sensible heat transfer from the bulk liquid to the interface is (refer to Fig. 10.5-1)

Equation 10.5-4.

$$Lc_L\, dT_L = G\, dH_y = h_L a\, dz(T_L - T_i)$$

where h_La is the liquid-phase volumetric heat-transfer coefficient in $W/m^3 \cdot K$ ($btu/h \cdot ft^3 \cdot °F$) and T_i is the interface temperature.

For adiabatic mass transfer the rate of heat transfer due to the latent heat in the water vapor being transferred can be obtained from Eq. (9.3-16) by rearranging and using a volumetric basis:

Equation 10.5-5.

$$\frac{q_\lambda}{A} = M_B k_G a P \lambda_0 (H_i - H_G)\, dz$$

where $q\lambda/A$ is in W/m^2 ($btu/h \cdot ft^2$), M_B = molecular weight of air, k_Ga is a volumetric mass-transfer coefficient in the gas in kg mol/s · m^3 · Pa, P = atm pressure in Pa, λ_0 is the latent heat of water in J/kg water, H_i is the humidity of the gas at the interface in kg water/kg dry air, and H_G is the humidity of the gas in the bulk gas phase in kg water/kg dry air. The rate of sensible heat transfer in the gas is

Equation 10.5-6.

$$\frac{q_S}{A} = h_G a (T_i - T_G)\, dz$$

where q_S/A is in W/m^2 and $h_G\, a$ is a volumetric heat-transfer coefficient in the gas in $W/m^3 \cdot K$.

Now from Fig. 10.5-1, Eq. (10.5-4) must equal the sum of Eqs. (10.5-5) and (10.5-6):

Equation 10.5-7.

$$G\, dH_y = M_B k_G a P \lambda_0 (H_i - H_G) dz + h_G a (T_i - T_G)\, dz$$

Equation (9.3-18) states that

Equation 10.5-8.

$$\frac{h_G a}{M_B k_y a} \cong c_S$$

Substituting Pk_Ga for k_ya,

Equation 10.5-9.

$$\frac{h_G a}{M_B P k_G a} \cong c_S$$

Substituting Eq. (10.5-9) into Eq. (10.5-7) and rearranging,

Equation 10.5-10.

$$G d\,H_y = M_B k_G a P\; dz[(c_S T_i + \lambda_0 H_i) - (c_S T_G + \lambda_0 H_G)]$$

Adding and subtracting $c_S T_0$ inside the brackets,

Equation 10.5-11.

$$G\; dH_y = M_B k_G a P\; dz\{c_S(T_i - T_0) + H_i \lambda_0 - [c_S(T_G - T_0) + H_G \lambda_0]\}$$

The terms inside the braces are ($H_{yi} - H_y$), and Eq. (10.5-11) becomes

Equation 10.5-12.

$$G\; dH_y = M_B k_G a P\; dz(H_{yi} - H_y)$$

Integrating, the final equation to use for calculating the tower height is

Equation 10.5-13.

$$\int_0^z dz = z = \frac{G}{M_B k_G a P} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y}$$

If Eq. (10.5-4) is equated to Eq. (10.5-12) and the result rearranged,

Equation 10.5-14.

$$-\frac{h_L a}{k_G a M_B P} = \frac{H_{yi} - H_y}{T_i - T_L}$$

Design of Water-Cooling Tower Using Film Mass-Transfer Coefficients

The tower design is done using the following steps:

1. The enthalpy of saturated air H_{yi} is plotted versus T_i on an H-versus-T plot as shown in Fig. 10.5-3. This enthalpy is calculated by means of Eq. (9.3-8) using the saturation humidity from the humidity chart for a given temperature, with 0°C (273 K) as a base temperature. Calculated values are tabulated in Table 10.5-1.

Table 10.5-1. Enthalpies of Saturated Air–Water Vapor Mixtures (0°C Base Temperature)

T_L		H_y		T_L		H_y	
°F	°C	btu/lb_m dry air	J/kg dry air	°F	°C	btu/lb_m dry air	J/kg dry air
60	15.6	18.78	43.68×10^3	100	37.8	63.7	148.2×10^3
80	26.7	36.1	84.0×10^3	105	40.6	74.0	172.1×10^3
85	29.4	41.8	97.2×10^3	110	43.3	84.8	197.2×10^3
90	32.2	48.2	112.1×10^3	115	46.1	96.5	224.5×10^3
95	35.0	55.4	128.9×10^3	140	60.0	198.4	461.5×10^3

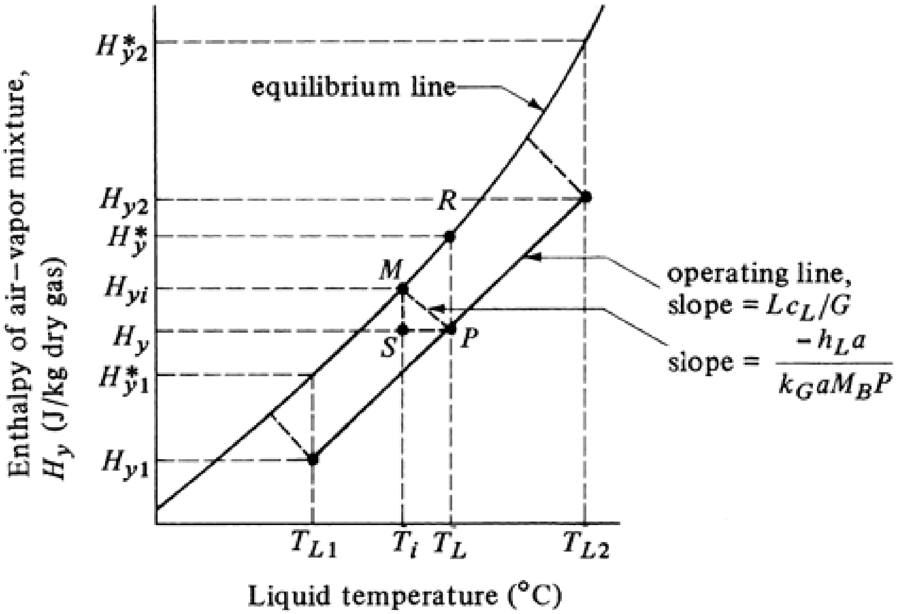

Figure 10.5-3. Temperature enthalpy diagram and operating line for water-cooling tower.

2. Knowing the entering air conditions T_{G1} and H_1, the enthalpy of this air H_{y1} is calculated from Eq. (9.3-8). The point H_{y1} and T_{L1} (desired leaving water temperature) is plotted in Fig. 10.5-3 as one point on the operating line. The operating line is plotted with a slope Lc_L/G and ends at point T_{L2}, which is the entering water temperature. This gives H_{y2}. Alternatively, H_{y2} can be calculated from Eq. (10.5-2).
3. Knowing h_La and k_Ga, lines with a slope of $-h_La/k_GaM_BP$ are plotted as shown in Fig. 10.5-3. From Eq. (10.5-14), point P represents H_y and T_L on the operating line, and point M represents H_{yi} and T_i, the interface conditions. Hence, line MS or $H_{yi} - H_y$ represents the driving force in Eq. (10.5-13).
4. The driving force $H_{yi} - H_y$ is computed for various values of T_L between T_{L1} and T_{L2}. Then the function $1/(H_{yi} - H_y)$ is integrated from H_{y1} to H_{y2} by numerical or graphical integration to obtain the value of the integral in Eq. (10.5-13). Finally, the height z is calculated from Eq. (10.5-13).

Design of Water-Cooling Tower Using Overall Mass-Transfer Coefficients

Often, only an overall mass-transfer coefficient K_Ga in kg mol/s · m^3 · Pa or kg mol/s · m^3 · atm is available, and Eq. (10.5-13) becomes

Equation 10.5-15.

$$z = \frac{G}{M_B k_G a P} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H^*_y - H_y}$$

The value of H^*_y is determined by going vertically from the value of H_y at point P up to the equilibrium line to give H^*_y at point R, as shown in Fig. 10.5-3. In many cases the experimental film coefficients k_Ga and h_La are not available. The few experimental data available indicate that h_La is quite large; the slope of the lines $-h_La/(k_GaM_BP)$ in Eq. (10.5-14) would be very large and the value of H_{yi} would approach that of H^*_y in Fig. 10.5-3.

The tower design using the overall mass-transfer coefficient is done using the following steps:

1. The enthalpy–temperature data from Table 10.5-1 are plotted as shown in Fig. 10.5-3.
2. The operating line is calculated as in steps 1 and 2 for the film coefficients and plotted in Fig. 10.5-3.
3. In Fig. 10.5-3 point *P* represents H_y and T_L on the operating line and point *R* represents H_y^* on the equilibrium line. Hence, the vertical line *RP* or $H_y^* - H_y$ represents the driving force in Eq. (10.5-15).
4. The driving force $H_y^* - H_y$ is computed for various values of T_L between T_{L1} and T_{L2}. Then the function $1/(H_y^* - H_y)$ is integrated from H_{y1} to H_{y2} by numerical or graphical methods to obtain the value of the integral in Eq. (10.5-15). Finally, the height *z* is obtained from Eq. (10.5-15).

If experimental cooling data from an actual run in a cooling tower with known height *z* are available, then, using Eq. (10.5-15), the experimental value of K_Ga can be obtained.

EXAMPLE 10.5-1. Design of Water-Cooling Tower Using Film Coefficients

A packed countercurrent water-cooling tower using a gas flow rate of $G = 1.356$ kg dry air/s · m^2 and a water flow rate of $L = 1.356$ kg water/s · m^2 is to cool the water from $T_{L2} = 43.3$°C (110°F) to $T_{L1} = 29.4$°C (85°F). The entering air at 29.4°C has a wet bulb temperature of 23.9°C. The mass-transfer coefficient k_Ga is estimated as 1.207×10^{-7} kg mol/s · m^3 · Pa and h_La/k_GaM_BP as 4.187×10^4 J/kg · K (10.0 btu/lb$_m$ · °F). Calculate the height of packed tower *z*. The tower operates at a pressure of 1.013×10^5 Pa.

Solution: Following the steps outlined, the enthalpies from the saturated air–water vapor mixtures from Table 10.5-1 are plotted in Fig. 10.5-4. The inlet air at $T_{G1} = 29.4$°C has a wet bulb temperature of 23.9°C. The humidity from the humidity chart is $H_1 = 0.0165$ kg H_2O/kg dry air. Substituting into Eq. (9.3-8), noting that (29.4 − 0)°C = (29.4 − 0) K,

$$H_{y1} = (1.005 + 1.88 \times 0.0165)10^3(29.4 - 0) + 2.501 \times 10^6(0.0165)$$

$$= 7.17 \times 10^3 \text{ J/kg}$$

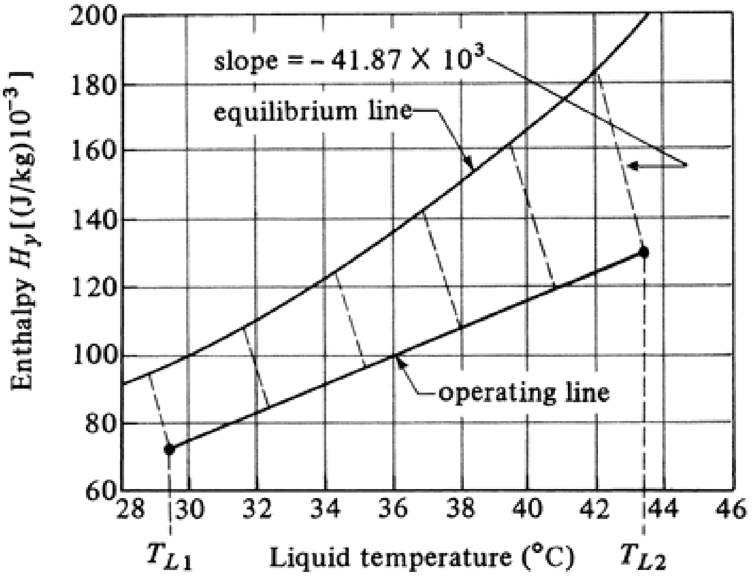

Figure 10.5-4. Graphical solution of Example 10.5-1.

The point $H_{y1} = 71.7 \times 10^3$ and $T_{L1} = 29.4$°C is plotted. Then substituting into Eq. (10.5-2) and solving,

$$1.356(H_{y2} - 71.7 \times 10^3) = 1.356(4.187 \times 10^3)(43.3 - 29.4)$$

$H_{y2} = 129.9 \times 10^3$ J/kg dry air (55.8 btu/lb$_m$). The point $H_{y2} = 129.9 \times 10^3$ and $T_{L2} = 43.3$°C is also plotted, giving the operating line. Lines with slope $-h_L a/k_G a M_B P = -41.87 \times 10^3$ J/kg · K are plotted, giving H_{yi} and H_y values, which are tabulated in Table 10.5-2 along with derived values as shown. Values of the function $1/(H_{yi} - H_y)$ are used with numerical integration between the values $H_{y1} = 71.7 \times 10^3$ to $H_{y2} = 129.9 \times 10^3$ to obtain the integral

Table 10.5-2. Enthalpy Values for Solution to Example 10.5-1 (enthalpy in J/kg dry air)

H_{yi}	H_y	$H_{yi} - H_y$	$1/(H_{yi} - H_y)$
94.4×10^3	71.7×10^3	22.7×10^3	4.41×10^{-5}
108.4×10^3	83.5×10^3	24.9×10^3	4.02×10^{-5}
124.4×10^3	94.9×10^3	29.5×10^3	3.39×10^{-5}
141.8×10^3	106.5×10^3	35.3×10^3	2.83×10^{-5}
162.1×10^3	118.4×10^3	43.7×10^3	2.29×10^{-5}
184.7×10^3	129.9×10^3	54.8×10^3	1.82×10^{-5}

$$\int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y} = 1.82$$

Substituting into Eq. (10.5-13),

$$z = \frac{G}{M_B k_G a P} \int \frac{dH_y}{H_{y_i} - H_y} = \frac{1.356}{29(1.207 \times 10^{-7})(1.013 \times 10^5)}(1.82)$$

$$= 6.98 \text{ m } (22.9 \text{ ft})$$

Minimum Value of Air Flow

Often the air flow G is not fixed but must be set for the design of the cooling tower. As shown in Fig. 10.5-5, for a minimum value of G, the operating line MN is drawn through the point H_{y1} and T_{L1} with a slope that touches the equilibrium line at T_{L2}, point N. If the equilibrium line is quite curved, line MN could become tangent to the equilibrium line at a point farther down the equilibrium line than point N. For the actual tower, a value of G greater than G_{min} must be used. Often, a value of G equal to 1.3 to 1.5 times G_{min} is used.

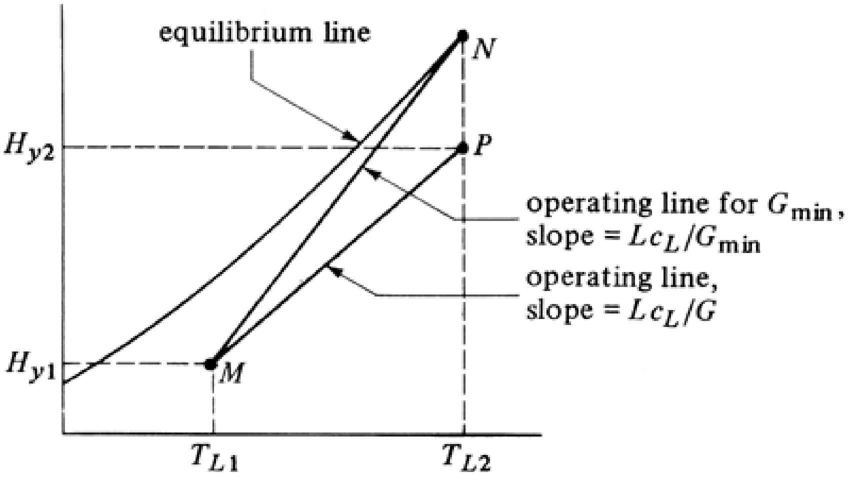

Figure 10.5-5. Operating-line construction for minimum gas flow.

Design of Water-Cooling Tower Using Height of a Transfer Unit

Often another form of the film mass-transfer coefficient is used in Eq. (10.5-13):

Equation 10.5-16.

$$z = H_G \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y}$$

Equation 10.5-17.

$$H_G = \frac{G}{M_B k_G a P}$$

where H_G is the height of a gas enthalpy transfer unit in m, and the integral term is called the number of transfer units. The term H_G is often used since it is less dependent upon flow rates than $k_G a$.

In many cases another form of the overall mass-transfer coefficient $K_G a$ in kg mol/s · m^3· Pa or kg mol/s · m^3 · atm is used, and Eq. (10.5-15) becomes

Equation 10.5-18.

$$z = \frac{G}{M_B K_G a P} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_y^* - H_y} = H_{OG} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_y^* - H_y}$$

where H_{OG} is the height of an overall gas enthalpy transfer unit in m. The value of H_y^* is determined by going vertically from the value of H_y up to the equilibrium line, as shown in Fig. 10.5-3. This method should be used only when the equilibrium line is almost straight over the range used. However, the H_{OG} is often used even if the equilibrium line is somewhat curved because of the lack of film mass-transfer-coefficient data.

Temperature and Humidity of Air Stream in Tower

The procedures outlined above do not yield any information on the changes in temperature and humidity of the air–water vapor stream through the tower. If this information is of interest, a graphical method by Mickley (M2) is available. The equation used for the graphical method is derived by first setting Eq. (10.5-6) equal to $Gc_S dT_G$ and then combining it with Eqs. (10.5-12) and (10.5-9) to yield Eq. (10.5-19):

Equation 10.5-19.

$$\frac{dH_y}{dT_G} = \frac{H_{yi} - H_y}{T_i - T_G}$$

Dehumidification Tower

For the cooling or humidification tower discussed above, the operating line lies below the equilibrium line, and water is cooled and air humidified. In a dehumidification tower, cool water is used to reduce the humidity and temperature of the air that enters. In this case the operating line is above the equilibrium line. Similar calculation methods are used (T1).

ABSORPTION IN PLATE AND PACKED TOWERS

Introduction to Absorption

As discussed briefly in Section 10.1B, absorption is a mass-transfer process in which a vapor solute *A* in a gas mixture is absorbed by means of a liquid in which the solute is more or less soluble. The gas mixture consists mainly of an inert gas and the solute. The liquid also is primarily immiscible in the gas phase; that is, its vaporization into the gas phase is relatively slight. A typical example is absorption of the solute ammonia from an air–ammonia mixture by water. Subsequently, the solute is recovered from the solution by distillation. In the reverse process of desorption or stripping, the same principles and equations hold.

Equilibrium relations for gas–liquid systems in absorption were discussed in Section 10.2, and such data are needed for design of absorption towers. Some data are tabulated in Appendix A.3. Other, more extensive data are available in Perry and Green (P1, P2).

Equipment for Absorption and Distillation

Various types of tray (plate) towers for absorption and distillation

In order to efficiently bring the vapor and liquid into contact in absorption and distillation, tray towers of the following types are often used.

1. ***Sieve tray.*** The sieve tray shown in Fig. 10.6-1a is very common. Essentially the same tray is used in distillation and gas absorption. In the sieve tray, vapor bubbles up through simple holes in the tray through the flowing liquid. Hole sizes range from 3 to 12 mm in diameter, with 5 mm a common size. The vapor area of the holes varies between 5 to 15% of the tray area. The liquid is held on the tray surface and prevented from flowing down through the holes by the kinetic energy of the gas or vapor. The depth of liquid on the tray is maintained by an overflow, outlet weir. The overflow liquid flows into the downspout to the next tray below.

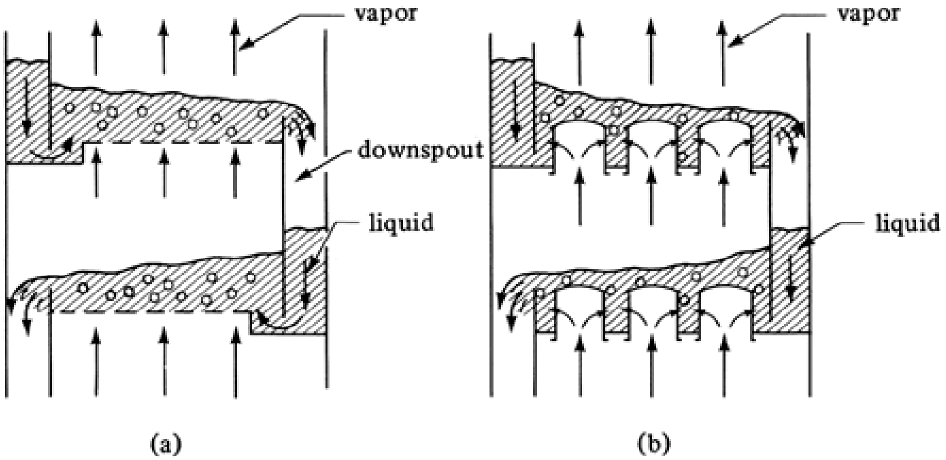

Figure 10.6-1. Tray contacting devices: (a) detail of sieve-tray tower, (b) detail of valve-tray tower.

2. ***Valve tray.*** A modification of the sieve tray is the valve tray shown in Fig. 10.6-1b. This consists of an opening in the tray and a lift-valve cover with guides to keep the cover properly positioned over the opening. This provides a variable open area which is varied by the vapor flow inhibiting leakage of liquid down the opening at low vapor rates. Hence, this type of tray can operate over a greater range of flow rates than the sieve tray, with a cost of only about 20% more than a sieve tray. The valve tray is being increasingly used today (S5).
3. ***Bubble-cap tray.*** Bubble-cap trays have been used for over 100 years, but since 1950 they have been generally superseded by sieve-type or valve trays because of their cost, which is almost double that of sieve-type trays. In the bubble tray, the vapor or gas rises through the opening in the tray into the bubble caps. Then the gas flows through slots in the periphery of each cap and bubbles upward through the flowing liquid. Details and design procedures for many of these and other types of trays are given elsewhere (B2, P2, T1). Efficiencies for the different types of trays are discussed in Section 11.5.

Structured packing for absorption and distillation

Structured packing has become competitive with conventional tray towers, especially in tower revamps where increased capacity and/or efficiency is desired (K1, L2). A typical corrugated-sheet packing is shown in Fig. 10.6-2 (F1). The thin corrugated-metal sheets are formed in a triangular cross-section, as shown in Fig. 10.6-2a. The vapor flow goes upward through the triangular channels, which are set at a 45° angle with the vertical. The sheets are arranged so the liquid flows downward in the opposite direction and spreads over the surfaces, as in a wetted-wall tower.

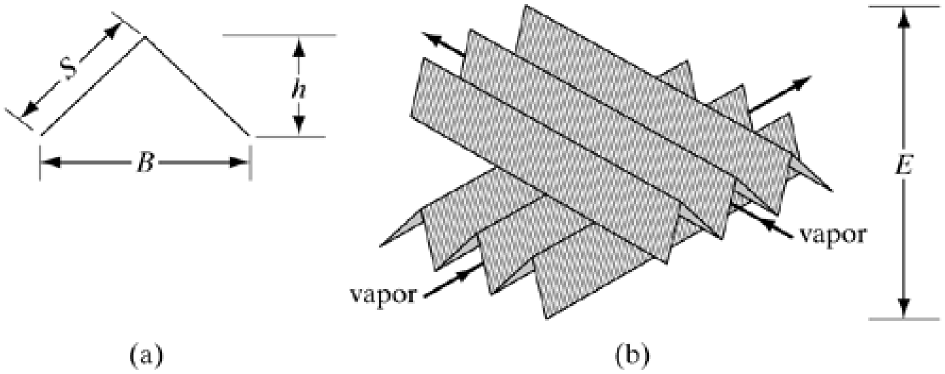

Figure 10.6-2. Typical corrugated structured packing: (a) triangular cross section of flow channel; (b) flow-channel arrangement, with vapor flowing upward, indicated by arrows, and liquid downward. [From J. R. Fair and J. L. Bravo, Chem. Eng. Progr., 86, (Jan.), 19 (1990). With permission.]

The corrugated sheets are assembled into an element whose height *E*, as shown in Fig. 10.6-2b, is about 20 to 30 cm tall (8–12 in.). A shorter height tends to increase the liquid and vapor spread in the horizontal plane. Each adjacent element is rotated 90° in the horizontal plane with respect to the layer below in order to spread the liquid and vapor uniformly in all radial planes. The ratios of *B/h* in Fig. 10.6-2a are in the range of 2/1 to 4/1. The size ranges of the triangle are typically: for *B*, 2.4–4.0 cm, for *S*, 1.7–2.6 cm, and for *h*, 1.2–1.8 cm (K1). Smaller sizes of the triangle mean that more sheets are present, giving a greater surface area. This results in a higher efficiency but smaller openings. However, the increased resistance to gas flow gives a lower capacity and a greater sensitivity to plugging (K1).

The open void fraction typically varies from 0.91 to 0.96 and the specific surface area from 165 to 330 m^2/m^3 volume (50 to 100 ft^2/ft^3). In many cases the packing sheet contains perforations or holes about 2–4 mm ID spaced 0.5–1.5 cm apart to help wet both the upper and lower sides of the sheet.

Packed towers for absorption and distillation

Packed towers are used for continuous countercurrent contacting of gas and liquid in absorption as well as for vapor–liquid contacting in distillation. The tower in Fig. 10.6-3 consists of a cylindrical column containing a gas inlet and distributing space at the bottom, a liquid inlet and distributing device at the top, a gas outlet at the top, a liquid outlet at the bottom, and a packing or filling in the tower. The gas enters the distributing space below the packed section and rises upward through the openings or interstices in the packing and contacts the descending liquid flowing through the same openings. A large area of intimate contact between the liquid and gas is provided by the packing.

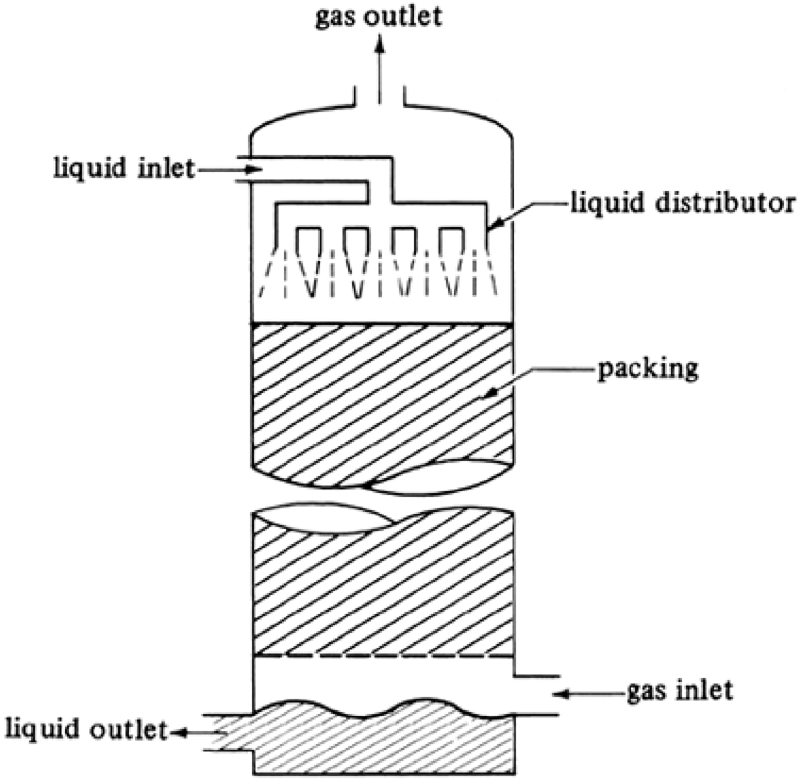

Figure 10.6-3. Packed tower flows and characteristics for absorption.

Types of random packing for absorption and distillation

Many different types of tower packings have been developed and a number are used quite often. Common types of packing which are dumped at random in the tower are shown in Fig. 10.6-4. Such packings and other commercial packings are available in sizes of 3 mm to about 75 mm. Most of the tower packings are made of materials such as clay, porcelain, metal, or plastic. High void spaces of 65–95% are characteristic of good packings. The packings permit relatively large volumes of liquid to pass countercurrent to the gas flow through the openings with relatively low pressure drops for the gas. These same types of packings are also used in vapor–liquid separation processes of distillation.

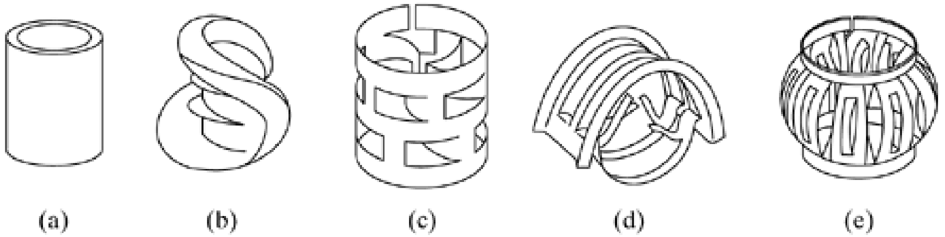

Figure 10.6-4. Typical random or dumped tower packings: (a) Raschig ring; (b) Berl saddle; (c) Pall ring; (d) Intalox metal, IMTP; (e) Jaeger Metal Tri-Pack.

Ceramic Raschig rings and Berl saddles shown in Figs. 10.6-4a and b are older types of random packing and are seldom used now (K1). Pall rings (second-generation packing) shown in Fig. 10.6-4c, are made of plastic or metal; they are much more efficient and are still used now. They have porosities or void spaces of 0.90–0.96 and areas of 100–200 m^2/m^3 (30–60 ft^2/ft^3). The latest or third-generation packings are the Intalox metal type, shown in Fig. 10.6-4d, which is a combination of the Berl saddle and the Pall ring, and the Metal Tri-Pack, shown in Fig. 10.6-4e, which is a Pall ring in spherical shape. Porosities range from 0.95 to 0.98. Many other types of new packings are available. These third-generation packings are only slightly more efficient than the Pall rings.

Stacked packings having sizes of 75 mm or so and larger are also used. The packing is stacked vertically, with open channels running uninterruptedly through the bed. The advantage of the lower pressure drop of the gas is offset in part by the poorer gas–liquid contact in stacked packings. Typical stacked packings are wood grids, drip-point grids, spiral partition rings, and others.

Pressure Drop and Flooding in Packed Towers

In a given packed tower with a given type and size of packing and with a definite flow of liquid, there is an upper limit to the rate of gas flow, called the *flooding velocity.* Above this gas velocity the tower cannot operate. At low gas velocities the liquid flows downward through the packing, essentially uninfluenced by the upward gas flow. As the gas flow rate is increased at low gas velocities, the pressure drop is proportional to the flow rate to the 1.8 power. At a gas flow rate called the *loading point,* the gas starts to hinder the liquid downflow, and local accumulations or pools of liquid start to appear in the packing. The pressure drop of the gas starts to rise at a faster rate. As the gas flow rate is increased, the liquid holdup or accumulation increases. At the flooding point, the liquid can no longer flow down through the packing and is blown out with the gas.

In an actual, operating tower, the gas velocity is well below flooding. The optimum economic gas velocity is about one-half or more of the flooding velocity. It depends upon a balance of economic factors including equipment cost, pressure drop, and processing variables. Pressure drop in the packing is an important consideration in design of a tower and is covered in detail below.

Pressure drop in random packings

Empirical correlations for various random packings based on experimental data are used to predict the pressure drop in the gas flow. The original correlation by Eckert (K1) correlated the gas and liquid flow rates and properties with pressure drop. The latest version has been replotted by Strigle (K1, S4) and is shown in Fig. 10.6-5. The line for $\Delta P = 2.0$ in. H_2O/ft has been extrapolated. The ordinate (capacity parameter) is $v_G[\rho_G/(\rho_L - \rho_G)]^{0.5} F_p^{0.7} v^{0.05}$ and the abscissa (flow parameter) is $(G_L/G_G)(\rho_G/\rho_L)^{0.5}$, where v_G is superficial gas velocity in ft/s, ρ_G is gas density in lb_m/ft^3, $v_G = G_G/\rho_G$, ρ_L is liquid density in lb_m/ft^3, F_p is a packing factor in ft^{-1}, v is kinematic viscosity $\mu_L/(\rho_L/62.4)$ in centstokes, μ_L is liquid viscosity in cp, G_L is liquid mass velocity in $lb_m/(s \cdot ft^2)$, and G_G is gas mass velocity in $lb_m/(s \cdot ft^2)$.Note that this capacity parameter is not dimensionless and that only these units should be used. This correlation predicts pressure drops to an accuracy of ±11% (L2).

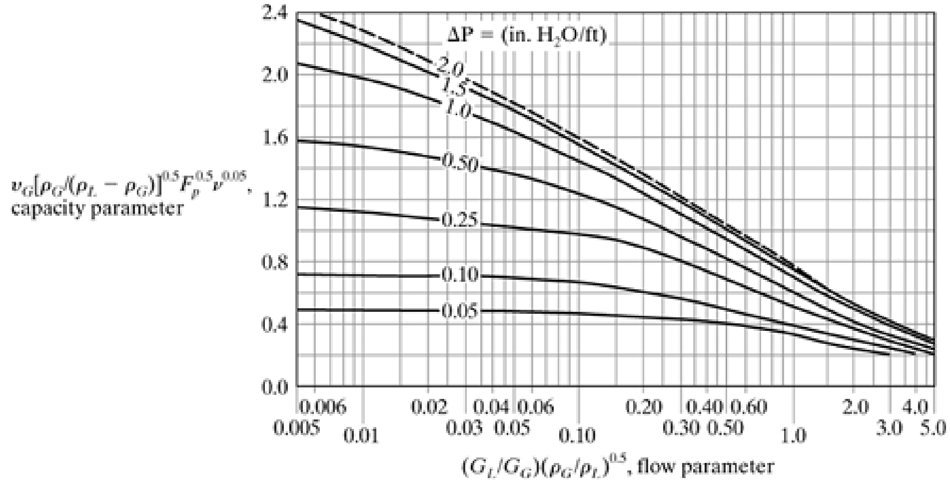

Figure 10.6-5. Pressure-drop correlation for random packings by Strigle. (From R. F. Strigle, Jr., Random Packings and Packed Towers, Houston: Gulf Publishing Company, 1987. With permission from Elsevier Science.)

The packing factor F_p is almost inversely proportional to packing size. This packing factor F_p is determined empirically for each size and type of packing, and some data are given in Table 10.6-1. A very extensive list of values of F_p is given by Kister (K1).

Table 10.6-1. Packing Factors for Random and Structured Packing

Type	*Material*	*Nominal size, in.*	*Void fraction, ε*	***Surface area, a, ft^2/ft^3 (m^2/m^3)***	***Packing factor, F_p, ft^{-1} (m^{-1})***	***Relative mass-transfer coefficient, f_p***
Random Packing						
Raschig Rings	**Ceramic**	**1/2**	**0.64**	**111 (364)**	**580 (1900)**	**1.52**
		1	**0.74**	**58 (190)**	**179 (587)**	**1.20**
		1 1/2	**0.73**	**37 (121)**	**95 (312)**	**1.00**
		2	**0.74**	**28 (92)**	**65 (213)**	**0.85**
Berl Saddles	**Ceramic**	**1/2**	**0.62**	**142 (466)**	**240 (787)**	**1.58**
		1	**0.68**	**76 (249)**	**110 (361)**	**1.36**
		2		**32 (105)**	**45 (148)**	
Pall Rings	**Metal**	**1**	**0.94**	**63 (207)**	**56 (184)**	**1.61**
		1 1/2	**0.95**	**39 (128)**	**40 (131)**	**1.34**
		2	**0.96**	**31 (102)**	**27 (89)**	**1.14**
Metal Intalox (IMTP)	**Metal**	**1**	**0.97**	**70 (230)**	**41 (134)**	**1.78**

Data from Ref. (K1, L2, P2, S4). The relative mass-transfer coefficient, f_p, is discussed in Section 10.8B.

Type		*Material*	*Nominal size, in.*	*Void fraction, ε*	***Surface area, a, ft^2/ft^3 (m^2/m^3)***		***Packing factor, F_p, ft^{-1}***	(m^{-1})	***Relative mass-transfer coefficient, f_p***
			2	0.98	30	(98)	18	(59)	1.27
Nor-Pac		Plastic	1	0.92	55	(180)	25	(82)	
			2	0.94	31	(102)	12	(39)	
Hy-Pak		Metal	1	0.96	54	(177)	45	(148)	1.51
			2	0.97	29	(95)	26	(85)	1.07
		Plastic	1	0.92	55	(180)	25	(82)	
			2	0.94	31	(102)	12	(39)	
Structured Packing									
Mellapak	250Y	Metal		0.95	76	(249)	20	(66)	
	500Y				152	(499)	34	(112)	
Flexipac	2			0.93	68	(223)	22	(72)	
	4			0.98			6	(20)	
Gempak	2A			0.93	67	(220)	16	(52)	
	4A			0.91	138	(452)	32	(105)	
Norton Intalox	2T			0.97	65	(213)	17	(56)	1.98
	3T			0.97	54	(177)	13	(43)	1.94
Montz	B300				91	(299)	33	(108)	
Sulzer	CY	Wire Mesh		0.85	213	(700)	70	(230)	
	BX			0.90	150	(492)	21	(69)	

Data from Ref. (K1, L2, P2, S4). The relative mass-transfer coefficient, f_p, is discussed in Section 10.8B.

Pressure drop in structured packings

An empirical correlation for structured packings is given in Fig. 10.6-6 by Kister and Gill (K1). They modified the Eckert correlation for random packings to better fit only the structured-packing data. An extrapolated line for $\Delta P = 0.05$ in. H_2O/ft and for $\Delta P = 2.0$ has been added. The packing factors F_p to be used for structured packing are those given in Table 10.6-1 and references (K1, L2, P2). The units on the ordinate and abscissa of Fig. 10.6-6 are the same as those for Fig. 10.6-5.

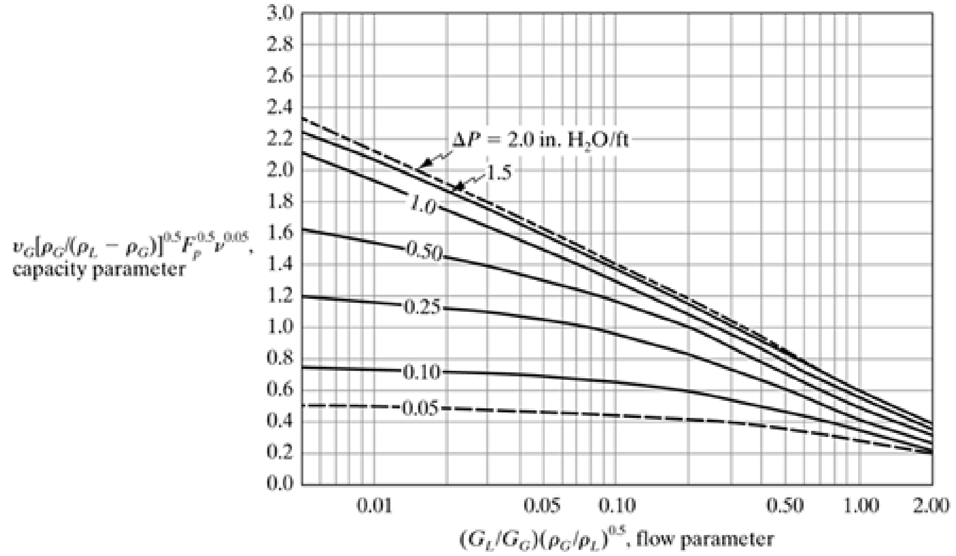

Figure 10.6-6. Pressure-drop correlation for structured packings by Kister and Gill (K2). (From H. Z. Kister, Distillation Design, New York: McGraw-Hill Book Company, 1992. With permission.)

Flooding pressure drop in packed and structured packings

It is important for proper design to be able to predict the flooding pressure drop in towers and, hence, the limiting flow rates at flooding. Figures 10.6-5 and 10.6-6 do not predict flooding conditions. Kister and Gill (K2) have developed an empirical equation to predict the limiting pressure drop at flooding. This equation is

Equation 10.6-1.

$$\Delta P_{flood} = 0.115F_p^{0.7}$$

where ΔP_{flood} is in in. H_2O/ft height of packing and F_p is the packing factor in ft^{-1} given in Table 10.6-1 for random or structured packing. To convert from English to SI units, 1.00 in. H_2O/ft height = 83.33 mm H_2O/m height of packing. This can be used for packing factors from 9 up to 60. It predicts all of the data for flooding within ±15% and most for ±10%. At a packing factor of 60 or higher, Eq. (10.6-1) should not be used; instead, the pressure drop at flooding can be taken as 2.00 in. H_2O/ft (166.7 mm H_2O/m).

The following procedure can be used to determine the limiting flow rates and the tower diameter.

1. First, a suitable random packing or structured packing is selected, giving an F_p value.
2. A suitable liquid-to-gas ratio G_L/G_G is selected along with the total gas flow rate.
3. The pressure drop at flooding is calculated using Eq. (10.6-1), or if F_p is 60 or over, the $\Delta P_{flooding}$ is taken as 2.0 in./ft packing height.
4. Then the flow parameter is calculated, and using the pressure drop at flooding and either Fig. 10.6-5 or 10.6-6, the capacity parameter is read off the plot.
5. Using the capacity parameter, the value of G_G is obtained, which is the maximum value at flooding.
6. Using a suitable % of the flooding value of G_G for design, a new G_G and G_L are obtained. The pressure drop can also be obtained from Figure 10.6-5 or 10.6-6.
7. Knowing the total gas flow rate and G_G, the tower cross-sectional area and ID can be calculated.

Approximate design factors to use

In using random packing, the radio of tower diameter to packing size should be 10/1 or greater. This is to ensure good liquid and gas distribution. For every 3 m (10 ft) height of packing, a liquid redistribution should be used to prevent channeling of liquid to the sides. Random-packed towers are generally used only for diameters of 1.0 m (3.3 ft) or less. Tray towers less than 0.6 m (2 ft) in diameter are usually not used because of cleaning and access problems.

The start of loading in packed towers is usually at about 65–70% of the flooding velocity (L2). For absorption, the tower should be designed using about 50–70% of the gas flooding velocity, with the high value used at high flow parameters. For atmospheric-pressure distillation, values of 70–80% can be used (S5). For distillation and structured packing, 80% of flooding is often used in design (L3). For tray towers, see Section 11.5.

EXAMPLE 10.6-1. Pressure Drop and Tower Diameter for Ammonia Absorption

Ammonia is being absorbed in a tower using pure water at 25°C and 1.0 atm abs pressure. The feed rate is 1440 lb_m/h (653.2 kg/h) and contains 3.0 mol % ammonia in air. The process design specifies a liquid-to-gas mass flow rate ratio G_L/G_G of 2/1 and the use of 1-in. metal Pall rings.

a. Calculate the pressure drop in the packing and gas mass velocity at flooding. Using 50% of the flooding velocity, calculate the pressure drop, gas and liquid flows, and tower diameter.
b. Repeat (a) above but use Mellapak 250Y structured packing.

Solution: The gas and liquid flows in the bottom of the tower are the largest, so the tower will be sized for these flows. Assume that approximately all of the ammonia is absorbed. The gas average mol wt is 28.97(0.97) + 17.0(0.03) = 28.61. The weight fraction of ammonia is 0.03(17)/(28.61) = 0.01783.

$$\rho_G = \left(\frac{492}{460 + 77}\right)\frac{(28.64)}{(359)} = 0.07309\ \text{lb}_\text{m}/\text{ft}^3$$

Assuming the water is dilute, from Appendix A.2-4, the water viscosity μ = 0.8937 cp. From A.2-3, the water density is 0.99708 gm/cm^3. Then, ρ_L = 0.99708(62.43) = 62.25 $\text{lb}_\text{m}/\text{ft}^3$. Also, $\nu = \mu/\rho$ = 0.8937/0.99708 = 0.8963 centistokes.

From Table 10.6-1, for 1-in. Pall rings, F_p = 56 ft^{-1}. Using Eq. 10.6-1, $\Delta P_{flood} = 0.115 F_p^{0.7} = 0.115(56)^{0.7}$ = 1.925 in. H_2O/ft packing height. The flow parameter for Fig. 10.6-5 is

$$(G_L/G_G)(\rho_G/\rho_L)^{0.5} = (2.0)(0.07309/62.25)^{0.5} = 0.06853$$

Using Fig. 10.6-5, for a flow parameter of 0.06853 (abscissa) and a pressure drop of 1.925 in./ft at flooding, a capacity parameter (ordinate) of 1.7 is read off the plot. Then, substituting into the capacity parameter equation and solving for v_G,

$$1.7 = v_G[\rho_G/(\rho_L - \rho_G)]^{0.5}\ F_p^{0.5}\nu^{0.05}$$

$$= v_G\left[\frac{0.07309}{(62.25 - 0.07309)}\right]^{0.5}(56)^{0.5}(0.8963)^{0.05}$$

v_G = 6.663 ft/s. Then $G_G = v_G\rho_G$ = 6.663(0.07309) = 0.4870 $\text{lb}_\text{m}/(\text{s}\cdot\text{ft}^2)$ at flooding. Using 50% of the flooding velocity for design, G_G = 0.5(0.4870) = 0.2435 $\text{lb}_\text{m}/(\text{s}\cdot\text{ft}^2)$ [1.189 kg/(s · m^2)]. Also, the liquid flow rate G_L = 2.0(0.2435) = 0.4870 $\text{lb}_\text{m}/(\text{s}\cdot\text{ft}^2)$ [2.378 kg/(s · m^2)].

To calculate the tower pressure drop at 50% of flooding, G_G = 0.2435 and G_L = 0.4870, the new capacity parameter is 0.5(1.7) = 0.85. Using this value of 0.85 and the same flow parameter, 0.06853, a value of 0.18 in. water/ft is obtained from Fig. 10.6-5.

The tower cross-sectional area = (1440/3600 lb_m/s)(1/0.2435 $\text{lb}_m/(\text{s} \cdot \text{ft}^2)$) = 1.6427 ft^2 = $(\pi/4)D^2$. Solving, D = 1.446 ft (0.441 m). The amount of ammonia in the outlet water assuming all of the ammonia is absorbed is 0.01783(1440) = 25.68 lb. Since the liquid flow rate is 2 times the gas flow rate, the total liquid flow rate is 2.0(1440) = 2880 lb_m/hr. Hence, the flow rate of the pure inlet water is 2880 − 25.68 = 2858.3 lb_m/s.

For part (b), using Mellapak 250Y, F_p = 20 from Table 10.6-1. The $\Delta p_{flood} = 0.115 F_p^{0.7} = 0.115(20)^{0.7} = 0.936$ in. water/ft. The flow parameter of 0.06853 is the same. Using Fig. 10.6-6, a capacity parameter = 1.38 at flooding is obtained. Then,

$$1.38 = \frac{G_G}{\rho_G}\left[\frac{\rho_G}{\rho_L - \rho_G}\right]^{0.5} F_p^{0.5} \nu^{0.05}$$

$$= \frac{(G_G)}{(0.07309)}\left[\frac{0.07309}{(62.25 - 0.07309)}\right]^{0.5} (20)^{0.5}(0.8963)^{0.05}$$

Solving, G_G = 0.6615 $\text{lb}_m/(\text{h} \cdot \text{ft}^2)$. For 50% flood, G_G = 0.5(0.6615) = 0.3308 $\text{lb}_m/(\text{s} \cdot \text{ft}^2)$ [1.615 $\text{kg}/(\text{s} \cdot \text{m}^2)$]. Also, the liquid flow rate is G_L = 2(0.3308) = 0.6616 $\text{lb}_m/(\text{s} \cdot \text{ft}^2)$ [3.230 $\text{kg}/(\text{s} \cdot \text{m}^2)$]. The new capacity parameter is 0.5(1.38) = 0.69. Using this and Fig. 10.6-6, a pressure drop of 0.11 in. water/ft is obtained.

The tower cross-sectional area = (1440/3600)(1/0.3308) = 1.209 ft^2 = $(\pi/4)D^2$. Solving, D = 1.241 ft (0.3784 m). Note that the tower with structured packing uses about 25% less cross-sectional area.

Design of Plate Absorption Towers

Operating-line derivation

A plate (tray) absorption tower has the same process flow diagram as the countercurrent multiple-stage process in Fig. 10.3-2 and is shown as a vertical tray tower in Fig. 10.6-7. In the case of solute A diffusing through a stagnant gas (B) and then into a stagnant fluid, as in the absorption of acetone (A) from air (B) by water, the moles of inert or stagnant air and inert water remain constant throughout the entire tower. If the rates are V' kg mol inert air/s and L' kg mol inert solvent water/s or in kg mol inert/s · m^2 units (lb mol inert/h · ft^2), an overall material balance on component A in Fig. 10.6-7 is

Equation 10.6-2.

$$L'\left(\frac{x_0}{1 - x_0}\right) + V'\left(\frac{y_{N+1}}{1 - y_{N+1}}\right) = L'\left(\frac{x_N}{1 - x_N}\right) + V'\left(\frac{y_1}{1 - y_1}\right)$$

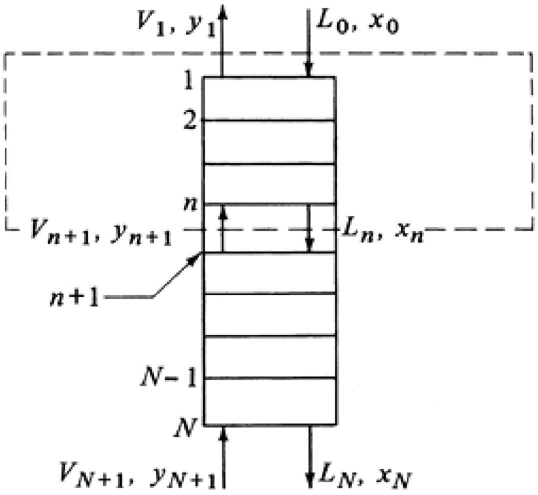

Figure 10.6-7. Material balance in an absorption tray tower.

A balance around the dashed-line box gives

Equation 10.6-3.

$$L'\left(\frac{x_0}{1-x_0}\right)+V'\left(\frac{y_{n+1}}{1-y_{n+1}}\right)=L'\left(\frac{x_n}{1-x_n}\right)+V'\left(\frac{y_1}{1-y_1}\right)$$

where x is the mole fraction A in the liquid, y the mole fraction of A in the gas, L_n the total moles liquid/s, and V_{n+1} the total moles gas/s. The total flows/s of liquid and gas vary throughout the tower.

Equation (10.6-3) is the material balance or operating line for the absorption tower and is similar to Eq. (10.3-13) for a countercurrent-stage process, except that the inert streams L' and V' are used instead of the total flow rates L and V. Equation (10.6-3) relates the concentration y_{n+1} in the gas stream with x_n in the liquid stream passing it. The terms V', L', x_0, and y_1 are constant and are usually known or can be determined.

Graphical determination of the number of trays

A plot of the operating-line equation (10.6-3) as y versus x will give a curved line. If x and y are very dilute, the denominators $1 - x$ and $1 - y$ will be close to 1.0, and the line will be approximately straight, with a slope $\cong L'/V'$. The number of theoretical trays is determined by simply stepping off the number of trays, as done in Fig. 10.3-3 for a countercurrent multiple-stage process.

EXAMPLE 10.6-2. Absorption of SO_2 in a Tray Tower

A tray tower is to be designed to absorb SO_2 from an air stream by using pure water at 293 K (68°F). The entering gas contains 20 mol % SO_2 and that leaving 2 mol % at a total pressure of 101.3 kPa. The inert air flow rate is 150 kg air/h · m^2, and the entering water flow rate is 6000 kg water/h · m^2. Assuming an overall tray efficiency of 25%, how many theoretical trays and actual trays are needed? Assume that the tower operates at 293 K (20°C).

***Solution*:** First calculating the molar flow rates,

$$V' = \frac{150}{29} = 5.18 \text{ kg mol inert air/h} \cdot \text{m}^2$$

$$L' = \frac{6000}{18.0} = 333 \text{ kg mol inert water/h} \cdot \text{m}^2$$

Referring to Fig. 10.6-7, $y_{N+1} = 0.20$, $y_1 = 0.02$, and $x_0 = 0$. Substituting into Eq. (10.6-2) and solving for x_N,

$$333\left(\frac{0}{1-0}\right) + 5.18\left(\frac{0.20}{1-0.20}\right) = 333\left(\frac{x_N}{1-x_N}\right) + 5.18\left(\frac{0.02}{1-0.02}\right)$$

$$x_N = 0.00355$$

Substituting into Eq. (10.6-3), using V' and L' as kg mol/h · m^2 instead of kg mol/s · m^2,

$$333\left(\frac{0}{1-0}\right) + 5.18\left(\frac{y_{n+1}}{1-y_{n+1}}\right) = 333\left(\frac{x_n}{1-x_n}\right) + 5.18\left(\frac{0.02}{1-0.02}\right)$$

In order to plot the operating line, several intermediate points will be calculated. Setting $y_{n+1} = 0.07$ and substituting into the operating equation,

$$0 + 5.18\left(\frac{0.07}{1-0.07}\right) = 333\left(\frac{x_n}{1-x_n}\right) + 5.18\left(\frac{0.02}{1-0.02}\right)$$

Hence, $x_n = 0.000855$. To calculate another intermediate point, we set $y_{n+1} = 0.13$, and x_n is calculated as 0.00201. The two end points and the two intermediate points on the operating line are plotted in Fig. 10.6-8, as are the equilibrium data from Appendix A.3. The operating line is somewhat curved.

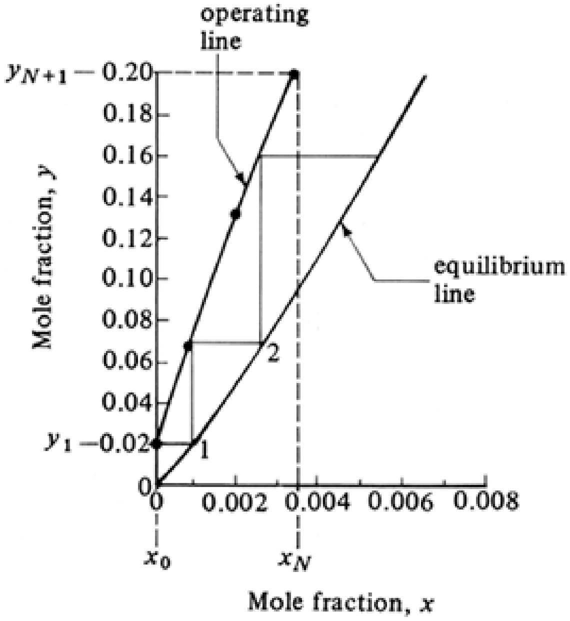

Figure 10.6-8. Theoretical number of trays for absorption of SO_2 in Example 10.6-2.

The number of theoretical trays is determined by stepping off the steps to give 2.4 theoretical trays. The actual number of trays is 2.4/0.25 = 9.6 trays.

Design of Packed Towers for Absorption

Operating-line derivation

For the case of solute *A* diffusing through a stagnant gas and then into a stagnant fluid, an overall material balance on component *A* in Fig. 10.6-9 for a packed tower is (note the notation change in Fig. 10.6-9 with inlets y_1 and x_2)

Equation 10.6-4.

$$L'\left(\frac{x_2}{1-x_2}\right)+V'\left(\frac{y_1}{1-y_1}\right)=L'\left(\frac{x_1}{1-x_1}\right)+V'\left(\frac{y_2}{1-y_2}\right)$$

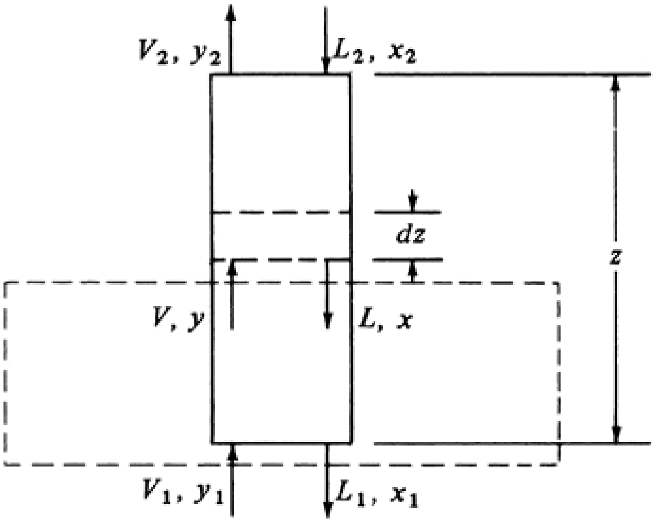

Figure 10.6-9. Material balance for a countercurrent packed absorption tower.

where L' is kg mol inert liquid/s or kg mol inert liquid/s · m^2, V' is kg mol inert gas/s or kg mol inert gas/s · m^2, and y_1 and x_1 are mole fractions *A* in gas and liquid, respectively. The flows L' and V' are constant throughout the tower, but the total flows *L* and *V* are not constant.

A balance around the dashed-line box in Fig. 10.6-9 gives the operating-line equation:

Equation 10.6-5.

$$L'\left(\frac{x}{1-x}\right)+V'\left(\frac{y_1}{1-y_1}\right)=L'\left(\frac{x_1}{1-x_1}\right)+V'\left(\frac{y}{1-y}\right)$$

This equation, when plotted on *yx* coordinates, will give a curved line, as shown in Fig. 10.6-10a. Equation (10.6-5) can also be written in terms of partial pressure p_1 of *A*, where $y_1/(1 - y_1) = p_1/(P - p_1)$, and so on. If *x* and *y* are very dilute, $(1 - x)$ and $(1 - y)$ can be taken as 1.0 and Eq. (10.6-5) becomes

Equation 10.6-6.

$$L'x + V'y_1 \cong L'x_1 + V'y$$

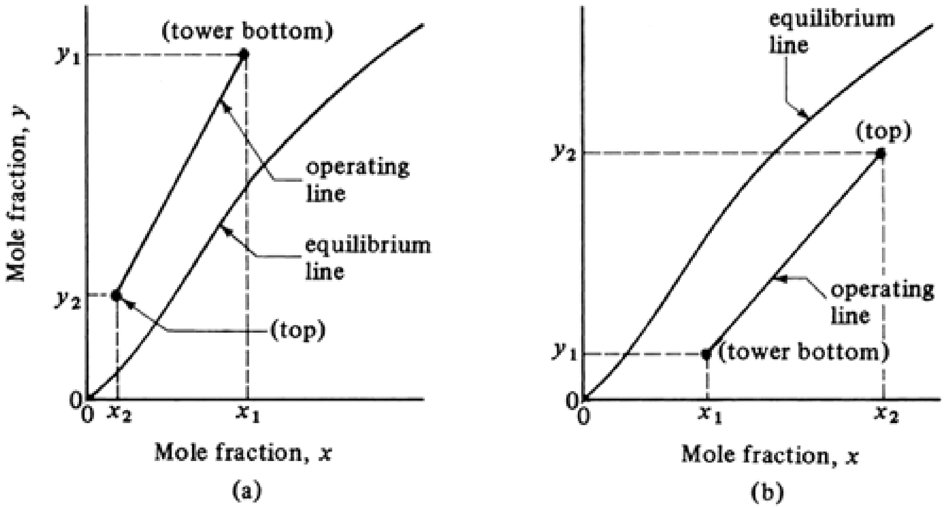

Figure 10.6-10. Location of operating lines: (a) for absorption of A from V to L stream, (b) for stripping of A from L to V stream.

This has a slope L'/V' and the operating line is essentially straight.

When the solute is being transferred from the L to the V stream, the process is called *stripping.* The operating line is below the equilibrium line, as shown in Fig. 10.6-10b.

Limiting and optimum L'/V' ratios.

In the absorption process, the inlet gas flow V_1 (Fig. 10.6-9) and its composition y_1 are generally set. The exit concentration y_2 is also usually set by the designer, and the concentration x_2 of the entering liquid is often fixed by process requirements. Hence, the amount of the entering liquid flow L_2 or L' is open to choice.

In Fig. 10.6-11a the flow V_1 and the concentrations y_2, x_2, and y_1 are set. When the operating line has a minimum slope and touches the equilibrium line at point P, the liquid flow L' is a minimum at $L'_{\min}$. The value of x_1 is a maximum at $x_{1\max}$ when L' is a minimum. At point P the driving forces $y - y^*$, $y - y_i$, $x^* - x$, and $x_i - x$ are all zero. To solve for $L'_{\min}$, the values y_1 and $x_{1\max}$ are substituted into the operating-line equation. In some cases, if the equilibrium line is curved concavely downward, the minimum value of L is reached by the operating line becoming tangent to the equilibrium line instead of intersecting it.

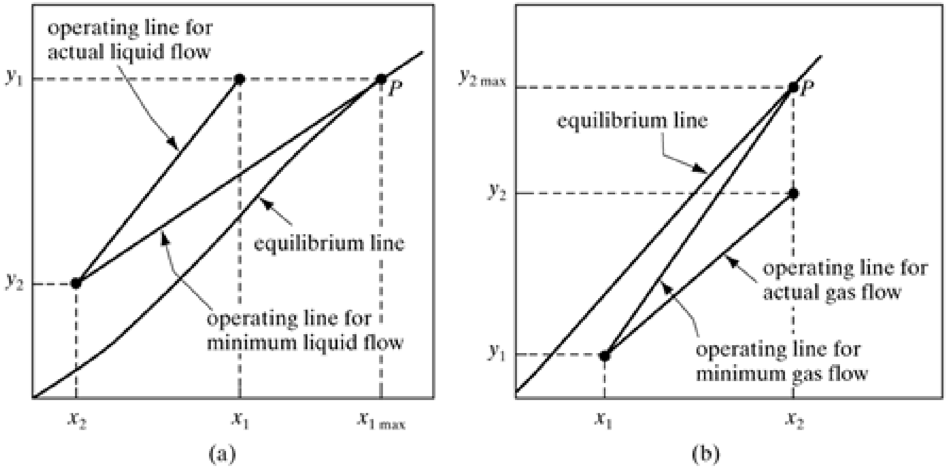

Figure 10.6-11. Operating line for limiting conditions: (a) absorption, (b) stripping.

The choice of the optimum L'/V' ratio to use in the design depends on an economic balance. In absorption, too high a value requires a large liquid flow and hence a large-diameter tower. The cost of recovering the solute from the liquid by distillation will be high. A small liquid flow results in a high tower, which is also costly. As an approximation, for absorption the optimum liquid flow rate can be taken as 1.2–1.5 times the limiting rate $L'_{\min}$, with 1.5 usually used (S6).

For stripping or transfer of solute from L to V, where the operating line has a maximum slope and touches the equilibrium line at point P in Fig. 10.6-11b, then the gas flow is at the minimum $V'_{\min}$ The value of y_2 is at a maximum at $y_{2\max}$. As in absorption, the optimum gas flow rate is taken at about 1.5 times $V'_{\min}$. In liquid extraction, covered later in Section 12.7, the same conditions for stripping hold for extracting solute from feed liquid L to solvent V.

Analytical equations for theoretical number of steps or trays

Analytical equations to calculate the number of theoretical trays N in an absorption tower are the same as Eqs. (10.3-22) and (10.3-25) for calculating the number of stages in countercurrent stage processes. These are as follows. (Note that the notation refers to Fig. 10.6-9 and not Fig. 10.3-3.)

For transfer of the solute from phase V to phase L (absorption),

Equation 10.6-7.

$$N = \frac{\ln\left[\left(\frac{y_1 - mx_2}{y_2 - mx_2}\right)(1 - 1/A) + 1/A\right]}{\ln A}$$

For transfer of the solute from phase L to phase V (stripping),

Equation 10.6-8.

$$N = \frac{\ln\left[\left(\frac{x_2 - y_1/m}{x_1 - y_1/m}\right)(1 - A) + A\right]}{\ln(1/A)}$$

where $A = L/mV$.

If the equilibrium line and/or operating line is slightly curved, m and $A = L/mV$ will vary. For absorption (referring to Fig. 10.6-9), at the bottom concentrated end tray, the slope m_1 or tangent at the point x_1 on the equilibrium line is used. At the dilute top tray, the slope m_2 of the equilibrium line at point y_2 is used. Then $A_1 = L_1/m_1V_1$, $A_2 = L_2/m_2V_2$, and $A = \sqrt{A_1A_2}$. Also, the dilute m_2 is used in Eq. (10.6-7). For stripping, at the top or concentrated stage, the slope m_2 or tangent at point y_2 on the equilibrium line is used. At the bottom or dilute end, the slope m_1 at point x_1 on the equilibrium line is used. Then, $A_1 = L_1/m_1V_1$, $A_2 = L_2/m_2V_2$, and $A = \sqrt{A_1A_2}$. Again, the dilute m_1 is used in Eq. (10.6-8).

EXAMPLE 10.6-3. Minimum Liquid Flow Rate and Analytical Determination of Number of Trays

A tray tower is absorbing ethyl alcohol from an inert gas stream using pure water at 303 K and 101.3 kPa. The inlet gas stream flow rate is 100.0 kg mol/h and it contains 2.2 mol % alcohol. It is desired to recover 90% of the alcohol. The equilibrium relationship is $y = mx = 0.68x$ for this dilute stream. Using 1.5 times the minimum liquid flow rate, determine the number of trays needed. Do this graphically and also using the analytical equations.

Solution: The given data are $y_1 = 0.022$, $x_2 = 0$, $V_1 = 100.0$ kg mol/h, $m = 0.68$. Then $V' = V_1(1 - y_1) = 100.0(1 - 0.022) = 97.8$ kg mol inert/h. Moles alcohol/h in V_1 are $100 - 97.8 = 2.20$. Removing 90%, moles/h in outlet gas $V_2 = 0.10(2.20) = 0.220$. $V_2 = V' + 0.220 = 97.8 + 0.22 = 98.02$. $y_2 = 0.22/98.02 = 0.002244$. The equilibrium line is plotted in Fig. 10.6-12 along with y_2, x_2, and y_1. The operating line for minimum liquid flow $L_{\min}$ is drawn from y_2, x_2 to point P, touching the equilibrium line where $x_{1\max} = y_1/m = 0.022/0.68 = 0.03235$. Substituting into the operating-line equation (10.6-4) and solving for $L_{\min}$,

$$L'\left(\frac{x_2}{1 - x_2}\right) + V'\left(\frac{y_1}{1 - y_1}\right) = L'\left(\frac{x_1}{1 - x_1}\right) + V'\left(\frac{y_2}{1 - y_2}\right)$$

$$L'_{\min}\left(\frac{0}{1 - 0}\right) + 97.8\left(\frac{0.022}{1 - 0.022}\right)$$

$$= L'_{\min}\left(\frac{0.03235}{1 - 0.03235}\right) + 97.8\left(\frac{0.002244}{1 - 0.002244}\right)$$

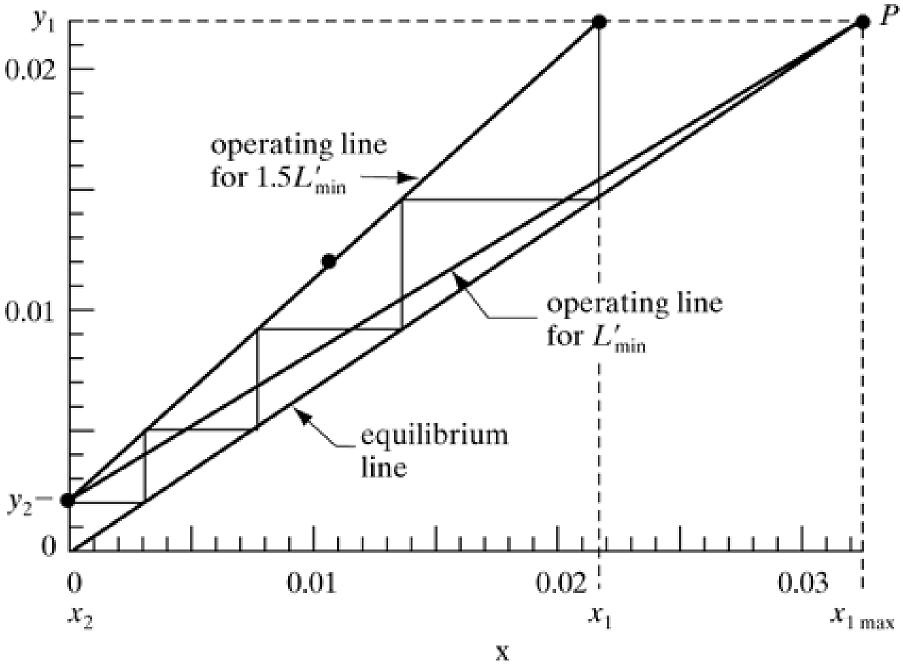

Figure 10.6-12. Operating line for minimum and actual liquid flow in Example 10.6-3.

$L'_{\min} = 59.24$ kg mol/h. Using $1.5L'_{\min}$, $L' = 1.5(59.24) = 88.86$. Using $L' = 88.86$ in Eq. (10.6-4) and solving for the outlet concentration, $x_1 = 0.02180$. The operating line is plotted as a straight line through the points y_2, x_2 and y_1, x_1 in Fig. 10.6-12. An intermediate point is calculated by setting $y = 0.012$ in Eq. (10.6-5) and solving for $x = 0.01078$. Plotting this point shows that the operating line is very linear. This occurs because the solutions are dilute.

The number of theoretical trays obtained by stepping them off is 4.0 trays. The total flow rates are $V_1 = 100.0$, $V_2 = V/(1 - y_2) = 97.8/(1 - 0.002244) = 98.02$, $L_2 = L' = 88.86$, and $L_1 = L'/(1 - x_1) = 88.86/(1 - 0.02180) = 90.84$. To calculate the number of trays analytically, $A_1 = L_1/mV_1 = 90.84/(0.68)(100) = 1.336$, $A_2 = L_2/mV_2 = 88.86/(0.68)(98.02) = 1.333$. Using the geometric average, $A = 1.335$. Using Eq. (10.6-7),

$$N = \frac{1}{\ln A} \ln\left[\left(\frac{y_1 - mx_2}{y_2 - mx_2}\right)(1 - 1/A) + 1/A\right]$$

$$= \frac{1}{\ln 1.335} \ln\left[\left(\frac{0.022 - 0}{0.002244 - 0}\right)(1 - 1/1.335) + 1/1.335\right]$$

N = 4.04 theoretical steps, which agrees closely with the 4.0 steps obtained graphically.

Film and overall mass-transfer coefficients in packed towers

As discussed in Section 10.5, it is very difficult to measure experimentally the interfacial area *A* m^2 between phases *L* and *V*. Also, it is difficult to measure the film coefficients k'_x and k'_y and the overall coefficients k'_x and K'_y. Usually, experimental measurements in a packed tower yield a volumetric mass-transfer coefficient that combines the interfacial area and mass-transfer coefficient.

Defining *a* as interfacial area in m^2 per m^3 volume of packed section, the volume of packing in a height *dz* m (Fig. 10.6-9) is *S dz*, and

Equation 10.6-9.

$$dA = aS\ dz$$

where *S* is m^2 cross-sectional area of tower. The volumetric film and overall mass-transfer coefficients are then defined as

$$k'_y a = \frac{\text{kg mol}}{\text{s}\cdot\text{m}^3\ \text{packing}\cdot\text{mol frac}} \qquad k'_x a = \frac{\text{kg mol}}{\text{s}\cdot\text{m}^3\ \text{packing}\cdot\text{mol frac}} \qquad \text{(SI)}$$

$$K'_y a = \frac{\text{kg mol}}{\text{s}\cdot\text{m}^3\ \text{packing}\cdot\text{mol frac}} \qquad K'_x a = \frac{\text{kg mol}}{\text{s}\cdot\text{m}^3\ \text{packing}\cdot\text{mol frac}} \qquad \text{(SI)}$$

$$k'_y a = \frac{\text{lb mol}}{\text{h}\cdot\text{ft}^3\ \text{packing}\cdot\text{mol frac}} \qquad k'_x a = \frac{\text{lb mol}}{\text{h}\cdot\text{ft}^3\ \text{packing}\cdot\text{mol frac}} \qquad \text{(English)}$$

Design method for packed towers using mass-transfer coefficients

For absorption of *A* from stagnant *B*, the operating-line equation (10.6-5) holds. For the differential height of tower *dz* in Fig. 10.6-9, the moles of *A* leaving *V* equal the moles entering *L*:

Equation 10.6-10.

$$d(Vy) = d(Lx)$$

where *V* = kg mol total gas/s, *L* = kg mol total liquid/s, and *d*(*Vy*) = *d*(*Lx*) = kg mol *A* transferred/s in height *dz* m. The kg mol *A* transferred/s from Eq. (10.6-10) must equal the kg mol *A* transferred/s from the mass-transfer equation for N_A. Equation (10.4-8) gives the flux N_A using the gas-film and liquid-film coefficients:

Equation 10.4-8.

$$N_A = \frac{k'_y}{(1-y_A)_{iM}}(y_{AG} - y_{Ai}) = \frac{k'_x}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})$$

where $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$ are defined by Eqs. (10.4-6) and (10.4-7). Multiplying the left-hand side of Eq. (10.4-8) by *dA* and the two right-side terms by *aS dz* from Eq. (10.6-9),

Equation 10.6-11.

$$N_A\ dA = \frac{k'_y a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\ dz = \frac{k'_x a}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})S\ dz$$

where $N_A\,dA$ = kg mol A transferred/s in height dz m (lb mol/h).

Equating Eq. (10.6-10) to (10.6-11) and using y_{AG} for the bulk gas phase and x_{AL} for the bulk liquid phase,

Equation 10.6-12.

$$d(Vy_{AG}) = \frac{k'_y a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\,dz$$

Equation 10.6-13.

$$d(Lx_{AL}) = \frac{k'_x a}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})S\,dz$$

Since $V' = V(1 - y_{AG})$ or $V = V'/(1 - y_{AG})$,

Equation 10.6-14.

$$d(Vy_{AG}) = d\left(\frac{V'}{(1-y_{AG})}y_{AG}\right) = V'\,d\left(\frac{y_{AG}}{1-y_{AG}}\right) = \frac{V'\,dy_{AG}}{(1-y_{AG})^2}$$

Substituting V for $V'/(1 - y_{AG})$ in Eq. (10.6-14) and then equating Eq. (10.6-14) to (10.6-12),

Equation 10.6-15.

$$\frac{V\,dy_{AG}}{1-y_{AG}} = \frac{k'_y a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\,dz$$

Repeating for Eq. (10.6-13), since $L = L'/(1 - x_{AL})$,

Equation 10.6-16.

$$\frac{L\,dx_{AL}}{1-x_{AL}} = \frac{k'_x a}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})S\,dz$$

Dropping the subscripts A, G, and L and integrating, the final equations are as follows using film coefficients:

Equation 10.6-17.

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{k'_y aS}{(1-y)_{iM}}(1-y)(y-y_i)}$$

Equation 10.6-18.

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{k'_x aS}{(1-x)_{iM}}(1-x)(x_i-x)}$$

In a similar manner, the final equations can be derived using overall coefficients:

Equation 10.6-19.

$$z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{K'_y aS}{(1-y)_{*M}}(1-y)(y-y^*)}$$

Equation 10.6-20.

$$z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{K'_x aS}{(1-x)_{*M}}(1-x)(x^*-x)}$$

In the general case, the equilibrium and the operating lines are usually curved, and $k'_x a$, $k'_y a$, $K'_y a$ and $K'_x a$ vary somewhat with total gas and liquid flows. Then Eqs. (10.6-17)–(10.6-20) must be integrated numerically or graphically. The methods for doing this for concentrated mixtures will be discussed in Section 10.7. Methods for dilute gases will be considered below.

Simplified Design Methods for Absorption of Dilute Gas Mixtures in Packed Towers

Since a considerable percentage of the absorption processes include absorption of a dilute gas *A*, these cases will be considered using a simplified design procedure.

The concentrations can be considered dilute for engineering design purposes when the mole fractions y and x in the gas and liquid streams are less than about 0.10, that is, 10%. The flows will vary by less than 10% and the mass-transfer coefficients by considerably less than this. As a result, the average values of the flows V and L and the mass-transfer coefficients at the top and bottom of the tower can be taken outside the integral. Likewise, the terms $(1 - y)_{iM}/(1 - y)$, $(1 - y)_{*M}/(1 - y)$, $(1 - x)_{iM}/(1 - x)$, and $(1 - x)_{*M}/(1 - x)$ can be taken outside and average values at the top and bottom of the tower used. (Often these terms are close to 1.0 and can be dropped out entirely.) Then Eqs. (10.6-17)–(10.6-20) become

Equation 10.6-21.

$$z = \left[\frac{V}{k'_y aS}\frac{(1-y)_{iM}}{1-y}\right]_{\text{av}} \int_{y_2}^{y_1} \frac{dy}{y-y_i}$$

Equation 10.6-22.

$$z = \left[\frac{L}{k'_x aS}\frac{(1-x)_{iM}}{1-x}\right]_{\text{av}} \int_{x_2}^{x_1} \frac{dx}{x_i-x}$$

Equation 10.6-23.

$$z = \left[\frac{V}{K'_y aS}\frac{(1-y)_{*M}}{1-y}\right]_{\text{av}} \int_{y_2}^{y_1} \frac{dy}{y-y^*}$$

Equation 10.6-24.

$$z = \left[\frac{L}{K'_x aS}\frac{(1-x)_{*M}}{1-x}\right]_{\text{av}} \int_{x_2}^{x_1} \frac{dx}{x^*-x}$$

Since the solutions are dilute, the operating line will be essentially straight. Assuming the equilibrium line is approximately straight over the range of concentrations used, ($y - y_i$) varies linearly with y and also with x:

Equation 10.6-25.

$$y - y_i = ky + b$$

where k and b are constants. Therefore, the integral of Eq. (10.6-21) can be integrated to give the following:

Equation 10.6-26.

$$\int_{y_2}^{y_1} \frac{dy}{y - y_i} = \frac{y_1 - y_2}{(y - y_i)_M}$$

where $(y - y_i)_M$ is the log mean driving force:

Equation 10.6-27.

$$(y - y_i)_M = \frac{(y_1 - y_{i1}) - (y_2 - y_{i2})}{\ln[(y_1 - y_{i1})/(y_2 - y_{i2})]}$$

Similarly, Eq. (10.6-23) gives

Equation 10.6-28.

$$(y - y^*)_M = \frac{(y_1 - y_1^*) - (y_2 - y_2^*)}{\ln[(y_1 - y_1^*)/(y_2 - y_2^*)]}$$

If the term $(1 - y)_{iM}/(1 - y)$ is considered 1.0, then by substituting Eq. (10.6-26) into (10.6-21) and doing the same for Eqs. (10.6-22)–(10.6-24), the final results are as follows:

Equation 10.6-29.

$$\frac{V}{S}(y_1 - y_2) = k'_y az(y - y_i)_M$$

Equation 10.6-30.

$$\frac{L}{S}(x_1 - x_2) = k'_x az(x_i - x)_M$$

Equation 10.6-31.

$$\frac{V}{S}(y_1 - y_2) = K'_y az(y - y^*)_M$$

Equation 10.6-32.

$$\frac{L}{S}(x_1 - x_2) = K'_x az(x^* - x)_M$$

where the left side is the kg mol absorbed/s · m^2 (lb mol/h · ft^2) by material balance and the right-hand side is the rate equation for mass transfer. The value of V is the average $(V_1 + V_2)/2$ and of L is $(L_1 + L_2)/2$.

Equations (10.6-29)–(10.6-32) can be used in slightly different ways. The general steps to follow are discussed below and shown in Fig. 10.6-13.

1. The operating-line equation (10.6-5) is plotted as in Fig. 10.6-13 as a straight line. Calculate V_1, V_2, and $V_{av} = (V_1 + V_2)/2$; also calculate L_1, L_2, and $L_{av} = (L_1 + L_2)/2$.
2. Average experimental values of the film coefficients $k'_y a$ and $k'_x a$ are available or may be obtained from empirical correlations. The interface compositions y_{i1} and x_{i1} at point y_1, x_1 in the tower are determined by plotting line P_1M_1, whose slope is calculated by Eq. (10.6-33):

   Equation 10.6-33.

   $$\text{slope} = -\frac{k'_x a/(1-x)_{iM}}{k'_y a/(1-y)_{iM}} = -\frac{k_x a}{k_y a}$$

   Equation 10.6-34.

   $$\text{slope} \cong -\frac{k'_x a/(1-x_1)}{k'_y a/(1-y_1)}$$

   If terms $(1 - x)_{iM}$ and $(1 - y)_{iM}$ are used, the procedure is trial and error, as in Example 10.4-1. However, since the solutions are dilute, the terms $(1 - x_1)$ and $(1 - y_1)$ can be used in Eq. (10.6-34) without trial and error and with only a small error in slope. If the coefficients $k_y a$ and $k_x a$ are available for the approximate concentration range, they can be used, since they include the terms $(1 - x)_{iM}$ and $(1 - y)_{iM}$. For line P_2M_2 at the other end of the tower, values of y_{i2} and x_{i2} are determined using Eq. (10.6-33) or (10.6-34) and y_2 and x_2.
3. If the overall coefficient $K'_y a$ is being used, y_1^* and y_2^* are determined as shown in Fig. 10.6-13. If $K'_x a$ is used, x_1^* and x_2^* are obtained.
4. Calculate the log mean driving force $(y - y_i)_M$ by Eq. (10.6-27) if $k'_y a$ is used. For $K'_y a$, $(y - y^*)_M$ is calculated by Eq. (10.6-28). Using the liquid coefficients, the appropriate driving forces are calculated.
5. Calculate the column height z m by substituting into the appropriate form of Eqs. (10.6-29)–(10.6-32).

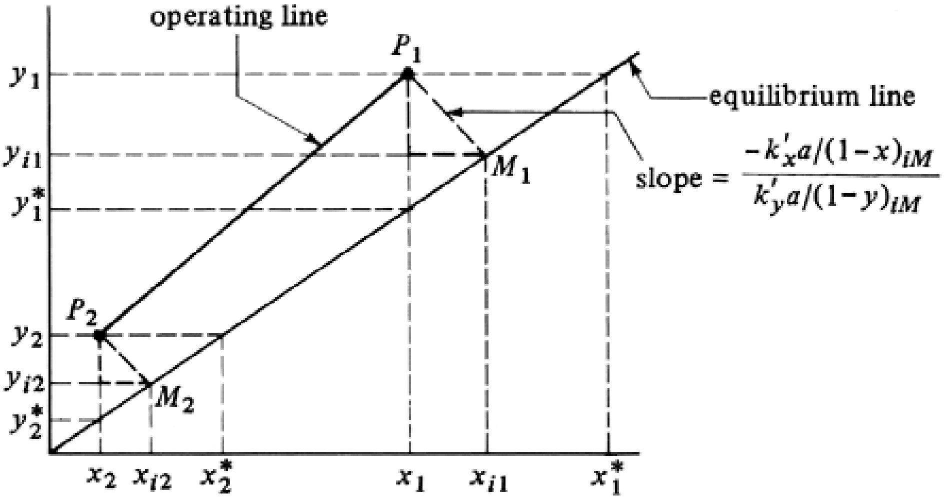

Figure 10.6-13. Operating-line and interface compositions in a packed tower for absorption of dilute gases.

EXAMPLE 10.6-4. Absorption of Acetone in a Packed Tower

Acetone is being absorbed by water in a packed tower having a cross-sectional area of 0.186 m^2 at 293 K and 101.32 kPa (1 atm). The inlet air contains 2.6 mol % acetone and outlet 0.5%. The gas flow is 13.65 kg mol inert air/h (30.1 lb mol/h). The pure water inlet flow is 45.36 kg mol water/h (100 lb mol/h). Film coefficients for the given flows in the tower are $k'_y a$ = 3.78×10^{-2} kg mol/s · m^3 · mol frac (8.50 lb mol/h · ft^3 · mol frac) and $k'_x a$ = 6.16×10^{-2} kg mol/s · m^3 · mol frac (13.85 lb mol/h · ft^3 · mol frac). Equilibrium data are given in Appendix A.3.

a. Calculate the tower height using. $k'_y a$.

b. Repeat, using. $k'_x a$.

c. Calculate $K'_y a$ and the tower height.

Solution: From Appendix A.3 for acetone–water and $x_A = 0.0333$ mol frac, $p_A = 30/760 = 0.0395$ atm or $y_A = 0.0395$ mol frac. Hence, the equilibrium line is $y_A = mx_A$ or $0.0395 = m(0.0333)$. Then, $y = 1.186x$. This equilibrium line is plotted in Fig. 10.6-14. The given data are $L' = 45.36$ kg mol/h, $V' = 13.65$ kg mol/h, $y_1 = 0.026$, $y_2 = 0.005$, and $x_2 = 0$.

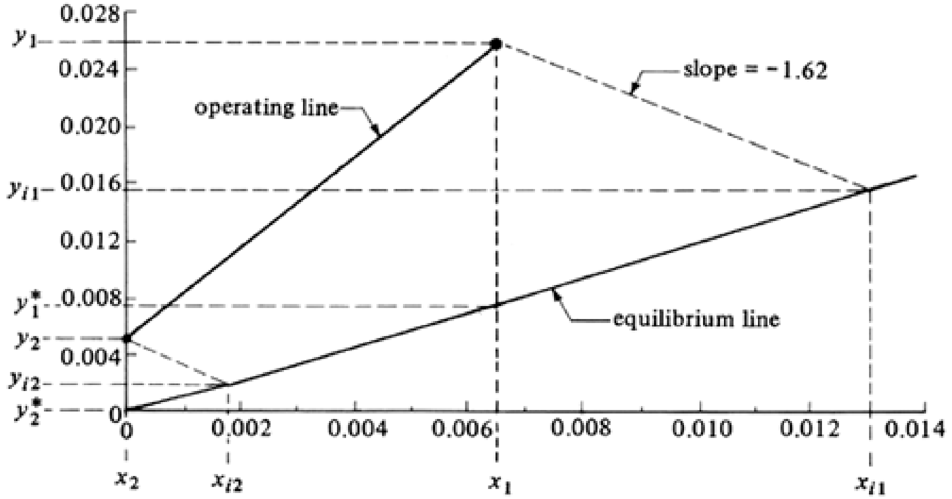

Figure 10.6-14. Location of interface compositions for Example 10.6-4.

Substituting into Eq. (10.6-4) for an overall material balance using flow rates as kg mol/h instead of kg mol/s,

$$45.36\left(\frac{0}{1-0}\right) + 13.65\left(\frac{0.026}{1-0.026}\right) = 45.36\left(\frac{x_1}{1-x_1}\right) + 13.65\left(\frac{0.005}{1-0.005}\right)$$

$$x_1 = 0.00648$$

The points y_1, x_1 and y_2, x_2 are plotted in Fig. 10.6-14 and a straight line is drawn for the operating line.

Using Eq. (10.6-34) the approximate slope at y_1, x_1 is

$$\text{slope} \cong -\frac{k'_x a/(1-x_1)}{k'_y a/(1-y_1)} = -\frac{6.16 \times 10^{-2}/(1-0.00648)}{3.78 \times 10^{-2}/(1-0.026)} = -1.60$$

Plotting this line through y_1, x_1, the line intersects the equilibrium line at $y_{i1} = 0.0154$ and $x_{i1} = 0.0130$. Also, y_1^* = 0.0077. Using Eq. (10.6-33) to calculate a more accurate slope, the preliminary values of y_{i1} and x_{i1} will be used in the trial-and-error solution. Substituting into Eq. (10.4-6),

$$(1 - y)_{iM} = \frac{(1 - y_{i1}) - (1 - y_1)}{\ln[(1 - y_{i1})/(1 - y_1)]}$$

$$= \frac{(1 - 0.0154) - (1 - 0.026)}{\ln[(1 - 0.0154)/(1 - 0.020)]} = 0.979$$

Using Eq. (10.4-7),

$$(1 - x)_{iM} = \frac{(1 - x_1) - (1 - x_{i1})}{\ln[(1 - x_1)/(1 - x_{i1})]}$$

$$= \frac{(1 - 0.00648) - (1 - 0.0130)}{\ln[(1 - 0.00648)/(1 - 0.0130)]} = 0.993$$

Substituting into Eq. (10.6-33),

$$\text{slope} = -\frac{k'_x a/(1 - x)_{iM}}{k'_y a/(1 - y)_{iM}} = -\frac{6.16 \times 10^{-2}/0.993}{3.78 \times 10^{-2}/0.929} = -1.61$$

Hence, the approximate slope and interface values are accurate enough.

For the slope at point y_2, x_2,

$$\text{slope} \cong -\frac{k'_x a/(1 - x_2)}{k'_y a/(1 - y_2)} = -\frac{6.16 \times 10^{-2}/(1 - 0)}{3.78 \times 10^{-2}/(1 - 0.005)} = -1.62$$

The slope changes little in the tower. Plotting this line, $y_{i2} = 0.0020$, $x_{i2} = 0.0018$, and $y_2^* = 0$.

Substituting into Eq. (10.6-27),

$$(y - y_i)_M = \frac{(y_1 - y_{i1}) - (y_2 - y_{i2})}{\ln[(y_1 - y_{i1})/(y_2 - y_{i2})]}$$

$$= \frac{(0.026 - 0.0154) - (0.005 - 0.0020)}{\ln[(0.026 - 0.0154)/(0.005 - 0.0020)]} = 0.00602$$

To calculate the total molar flow rates in kg mol/s,

$$V_1 = \frac{V'}{1 - y_1} = \frac{13.65/3600}{1 - 0.026} = 3.893 \times 10^{-3} \text{ kg mol/s}$$

$$V_2 = \frac{V'}{1 - y_2} = \frac{13.65/3600}{1 - 0.005} = 3.811 \times 10^{-3} \text{ kg mol/s}$$

$$V_{\text{av}} = \frac{V_1 + V_2}{2} = \frac{3.893 \times 10^{-3} + 3.811 \times 10^{-3}}{2} = 3.852 \times 10^{-3} \text{ kg mol/s}$$

$$L' \cong L_1 \cong L_2 \cong L_{\text{av}} = \frac{45.36}{3600} = 1.260 \times 10^{-2} \text{ kg mol/s}$$

For part (a), substituting into Eq. (10.6-29) and solving,

$$\frac{V_{av}}{S}(y_1 - y_2) = k'_y a z (y - y_i)_M$$

$$\frac{3.852 \times 10^{-3}}{0.186}(0.0260 - 0.005) = (3.78 \times 10^{-2}) z (0.00602)$$

$$z = 1.911 \text{ m } (6.27 \text{ ft})$$

For part (b), using an equation similar to Eq. (10.6-27),

$$(x_i - x)_M = \frac{(x_{i1} - x_1) - (x_{i2} - x_2)}{\ln[(x_{i1} - x_1)/(x_{i2} - x_2)]}$$

$$= \frac{(0.0130 - 0.00648) - (0.0018 - 0)}{\ln[(0.0130 - 0.00648)/(0.0018 - 0)]} = 0.00368$$

Substituting into Eq. (10.6-30) and solving,

$$\frac{1.260 \times 10^{-2}}{0.186}(0.00648 - 0) = (6.16 \times 10^{-2}) z (0.00368)$$

$$z = 1.936 \text{ m}$$

This agrees with part (a) quite closely.

For part (c), substituting into Eq. (10.4-25) for point y_1, x_1,

$$(1 - y)_{*M} = \frac{(1 - y_1^*) - (1 - y_1)}{\ln[(1 - y_1^*)/(1 - y_1)]} = \frac{(1 - 0.0077) - (1 - 0.026)}{\ln[(1 - 0.0077)/(1 - 0.026)]} = 0.983$$

The overall mass-transfer coefficient $K'_y a$ at point y_1, x_1 is calculated by substituting into Eq. (10.4-24):

$$\frac{1}{K'_y a/(1 - y)_{*M}} = \frac{1}{k'_y a/(1 - y)_{iM}} + \frac{m'}{k'_x a/(1 - x)_{iM}}$$

$$\frac{1}{K'_y a/0.983} = \frac{1}{3.78 \times 10^{-2}/0.979} + \frac{1.186}{6.16 \times 10^{-2}/0.993}$$

$$K'_y a = 2.183 \times 10^{-2} \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

Substituting into Eq. (10.6-28),

$$(y - y^*)_M = \frac{(y_1 - y_1^*) - (y_2 - y_2^*)}{\ln[(y_1 - y_1^*)/(y_2 - y_2^*)]}$$

$$= \frac{(0.0260 - 0.0077) - (0.0050 - 0)}{\ln[(0.0260 - 0.0077)/(0.0050 - 0)]} = 0.01025$$

Finally, substituting into Eq. (10.6-31),

$$\frac{3.852 \times 10^{-3}}{0.186}(0.0260 - 0.0050) = (2.183 \times 10^{-2})z(0.01025)$$

$$z = 1.944 \text{ m}$$

This is in agreement with parts (a) and (b).

Design of Packed Towers Using Transfer Units

Design for concentrated solutions

Another and in some ways more useful design method for packed towers is the use of the transfer-unit concept. For the most common case of *A* diffusing through stagnant and nondiffusing *B*, Eqs. (10.6-17)–(10.6-20) can be rewritten as

Equation 10.6-35.

$$z = H_G \int_{y_2}^{y_1} \frac{(1 - y)_{iM}\, dy}{(1 - y)(y - y_i)} = H_G N_G$$

Equation 10.6-36.

$$z = H_L \int_{x_2}^{x_1} \frac{(1 - x)_{iM}\, dx}{(1 - x)(x_i - x)} = H_L N_L$$

Equation 10.6-37.

$$z = H_{OG} \int_{y_2}^{y_1} \frac{(1 - y)_{*M}\, dy}{(1 - y)(y - y^*)} = H_{OG} N_{OG}$$

Equation 10.6-38.

$$z = H_{OL} \int_{x_2}^{x_1} \frac{(1 - x)_{*M}\, dx}{(1 - x)(x^* - x)} = H_{OL} N_{OL}$$

where

Equation 10.6-39.

$$H_G = \frac{V}{k'_y a S} = \frac{V}{k_y a (1 - y)_{iM} S}$$

Equation 10.6-40.

$$H_L = \frac{L}{k'_x a S} = \frac{L}{k_x a (1 - x)_{iM} S}$$

Equation 10.6-41.

$$H_{OG} = \frac{V}{K'_y a S} = \frac{V}{K_y a (1 - y)_{*M} S}$$

Equation 10.6-42.

$$H_{OL} = \frac{L}{K'_x aS} = \frac{L}{K_x a(1 - x)_{*M} S}$$

where $(1 - y)_{iM}$ is defined by Eq. (10.4-6), $(1 - x)_{iM}$ by Eq. (10.4-7), $(1 - y)_{*M}$ by Eq. (10.4-25), and $(1 - x)_{*M}$ by Eq. (10.4-27). The units of H are in m (ft). H_G is the height of a transfer unit based on the gas film. The values of the heights of transfer units are more constant than the mass-transfer coefficients. For example, $k'_y a$ is often proportional to $V^{0.7}$, so then $H_G \propto V^{1.0}/V^{0.7} \propto V^{0.3}$. The average values of the mass-transfer coefficients, $(1 - y)_{iM}$, $(1 - y)_{*M}$, $(1 - x)_{iM}$, and $(1 - x)_{*M}$, must be used in Eqs. (10.6-39)–(10.6-42).

The integrals on the right side of Eqs. (10.6-35)–(10.6-38) are the number of transfer units N_G, N_L, N_{OG}, and N_{OL}, respectively. The height of the packed tower is then

Equation 10.6-43.

$$z = H_G N_G = H_L N_L = H_{OG} N_{OG} = H_{OL} N_{OL}$$

These equations are basically no different from those using mass-transfer coefficients. One still needs $k'_y a$ and $k'_x a$ to determine interface concentrations. Disregarding $(1 - y)_{iM}/(1 - y)$, which is near 1.0 in Eq. (10.6-35), the greater the amount of absorption $(y_1 - y_2)$ or the smaller the driving force $(y - y_i)$, the larger the number of transfer units N_G and the taller the tower.

Design for dilute solutions

When the solutions are dilute, with concentrations below 10%, the terms $(1 - y)_{iM}/(1 - y)$, $(1 - x)_{iM}/(1 - x)$, $(1 - y)_{*M}/(1 - y)$, and $(1 - x)_{*M}/(1 - x)$ in Eqs. (10.6-35)–(10.6-38) can be taken outside the integral and average values used. Since these are quite close to 1.0, they can be dropped out. The equations then become

Equation 10.6-44.

$$z = H_G N_G = H_G \int_{y_2}^{y_1} \frac{dy}{y - y_i}$$

Equation 10.6-45.

$$z = H_L N_L = H_L \int_{x_2}^{x_1} \frac{dx}{x_i - x}$$

Equation 10.6-46.

$$z = H_{OG} N_{OG} = H_{OG} \int_{y_2}^{y_1} \frac{dy}{y - y^*}$$

Equation 10.6-47.

$$z = H_{OL} N_{OL} = H_{OL} \int_{x_2}^{x_1} \frac{dx}{x^* - x}$$

If both the operating and equilibrium lines are straight and dilute, the integrals in Eqs. (10.6-44)–(10.6-47) can be integrated, giving the number of transfer units:

Equation 10.6-48.

$$N_G = \frac{y_1 - y_2}{(y - y_i)_M}$$

Equation 10.6-49.

$$N_L = \frac{x_1 - x_2}{(x_i - x)_M}$$

Equation 10.6-50.

$$N_{OG} = \frac{(y_1 - y_2)}{(y - y^*)_M}$$

Equation 10.6-51.

$$N_{OL} = \frac{(x_1 - x_2)}{(x^* - x)_M}$$

where $(y - y_i)_M$ is defined in Eq. (10.6-27) and $(y - y^*)_M$ in (10.6-28).

When the major resistance to mass transfer is in the gas phase, as in absorption of acetone from air by water, the overall number of transfer units based on the gas phase N_{OG} or the film N_G should be used. When the major resistance is in the liquid phase, as in absorption of O_2 by water or stripping of a slightly soluble solute from water, N_{OL} or N_L should be employed. This was also discussed in detail in Section 10.4D. Often the film coefficients are not available, and then it is more convenient to use N_{OG} or N_{OL}.

By combining the operating line with the integrals in Eqs. (10.6-46) and (10.6-47), using the equilibrium-line equation $y = mx$, and letting $A = L/mV$, different forms of the equations for absorption with N_{OG} and for stripping with N_{OL} are obtained:

Equation 10.6-52.

$$N_{OG} = \frac{1}{(1 - 1/A)} \ln\left[(1 - 1/A)\left(\frac{y_1 - mx_2}{y_2 - mx_2}\right) + 1/A\right]$$

Equation 10.6-53.

$$N_{OL} = \frac{1}{(1 - A)} \ln\left[(1 - A)\left(\frac{x_2 - y_1/m}{x_1 - y_1/m}\right) + A\right]$$

The values of m and A to use in the above equations when operating and/or equilibrium lines are slightly curved are discussed in detail in Section 10.6E for use with Eqs. (10.6-7) and (10.6-8).

When the operating and equilibrium lines are straight and not parallel, the number of overall gas-transfer units N_{OG} for absorption in Eq. (10.6-52) is related to the number of theoretical trays or stages N in Eq. (10.6-7) by

Equation 10.6-54.

$$N_{OG} = \frac{N \ln A}{(1 - 1/A)}$$

The height of a theoretical tray or stage, HETP, in m is related to H_{OG} by

Equation 10.6-55.

$$\text{HETP} = H_{OG}\frac{\ln(1/A)}{(1-A)/A}$$

A detailed discussion of HETP is given in Section 11.5D. Note that if the operating and equilibrium lines are parallel ($A = 1.0$), then H_{OG} = HETP and $N_{OG} = N$. Equations (10.6-7) and (10.6-8) can be used to analytically calculate N, the number of theoretical steps.

EXAMPLE 10.6-5. Use of Transfer Units and Analytical Equations for Packed Tower

Repeat Example 10.6-4 using transfer units and height of a transfer unit as follows:

a. Use H_G and N_G to calculate tower height.
b. Use H_{OG} and N_{OG} to calculate tower height.
c. Use Eq. (10.6-52) to calculate N_{OG} and tower height.
d. Using the analytical equations, calculate HETP from Eq. (10.6-55), number of theoretical steps N from Eq. (10.6-7), and tower height.

Solution: From Example 10.6-4, $k'_y a = 3.78 \times 10^{-2}$ kg mol/s · m^2 · mol frac, $K'_y a = 2.183 \times 10^{-2}$ kg mol/s · m^2 · mol frac, average $V = 3.852 \times 10^{-3}$ kg mol/s, and $S = 0.186$ m^2.

For part (a), from Eq. (10.6-39),

$$H_G = \frac{V}{k'_y aS} = \frac{3.852 \times 10^{-3}}{(3.78 \times 10^{-2})(0.186)} = 0.548 \text{ m}$$

From Eq. (10.6-27) in Example 10.6-4, $(y - y_i)_{iM} = 0.00602$. Also, $y_1 = 0.026$ and $y_2 = 0.005$. Then, using Eq. (10.6-48),

$$N_G = \frac{y_1 - y_2}{(y - y_i)_{iM}} = \frac{0.026 - 0.005}{0.00602} = 3.488 \text{ transfer units}$$

Substituting in Eq. (10.6-44),

$$z = H_G N_G = (0.548)(3.488) = 1.911 \text{ m } (6.27 \text{ ft})$$

For part (b), using Eq. (10.6-41),

$$H_{OG} = \frac{V}{K'_y aS} = \frac{3.852 \times 10^{-3}}{(2.183 \times 10^{-2})(0.186)} = 0.949 \text{ m}$$

From Eq. (10.6-28) in Example 10.6-4, $(y - y^*)_{*M} = 0.01025$. Then using Eq. (10.6-50),

$$N_{OG} = \frac{(y_1 - y_2)}{(y - y^*)_{*M}} = \frac{(0.026 - 0.005)}{0.01025} = 2.049 \text{ transfer units}$$

From Eq. (10.6-46),

$$z = H_{OG} N_{OG} = (0.949)(2.049) = 1.945 \text{ m } (6.38 \text{ ft})$$

Note that the number of transfer units $N_{OG} = 2.049$ is not the same as $N_G = 3.488$.

For part (c), from Example 10.6-4, $m = 1.186$, and the average $V = 3.852 \times 10^{-3}$, $L = 1.260 \times 10^{-2}$, $y_1 = 0.026$, $y_2 = 0.005$, and $x_2 = 0$.

Then $A = L/mV = 1.260 \times 10^{-2}/(1.186 \times 3.852 \times 10^{-3}) = 2.758$. Substituting into Eq. (10.6-52),

$$N_{OG} = \frac{1}{1 - 1/2.758} \ln\left[(1 - 1/2.758)\left(\frac{0.026 - 1.186 \times 0}{0.005 - 1.186 \times 0}\right) + \frac{1}{2.758}\right]$$

$$= 2.043 \text{ transfer units}$$

This compares closely with the value of 2.049 in part (b). Also, $z = H_{OG}N_{OG} = (0.949)(2.043) = 1.939$ m.

For part (d), to calculate HETP, Eq. (10.6-55) is used:

$$\text{HETP} = H_{OG}\frac{\ln(1/A)}{(1 - A)/A} = (0.949)\frac{\ln(1/2.758)}{(1 - 2.758)/2.758} = 1.510 \text{ m}$$

Using Eq. (10.6-7) to calculate N,

$$N = \frac{1}{\ln A} \ln\left[\left(\frac{y_1 - mx_2}{y_2 - mx_2}\right)(1 - 1/A) + 1/A\right]$$

$$= \frac{1}{\ln 2.758} \ln\left[\left(\frac{0.026 - 0}{0.005 - 0}\right)(1 - 1/2.758) + 1/2.758\right]$$

$$N = 1.283 \text{ theoretical steps}$$

This compares with a value of 1.35 steps obtained graphically in Figure 10.6-14:

$$z = N(\text{HETP}) = 1.283(1.510 \text{ m}) = 1.938 \text{ m}$$

In order to relate H_{OG} to the film coefficients H_G and H_L, Eq. (10.4-24) is rewritten for dilute solutions:

Equation 10.6-56.

$$\frac{1}{K'_y a} = \frac{1}{k'_y a} + \frac{m}{k'_x a}$$

Then, substituting $H_G = V/k'_y aS$, $H_L = L/k'_x aS$, and $H_{OG} = V/K'_y a$ into Eq. (10.6-56) and canceling like terms,

Equation 10.6-57.

$$H_{OG} = H_G + (mV/L)H_L$$

Similarly, using Eq. (10.4-26),

Equation 10.6-58.

$$H_{OL} = H_L + (L/mV)H_G$$

Note that often in the literature, the terms H_y, H_x, H_{Oy}, and H_{Ox} are used instead of H_G, H_L, H_{OG}, and H_{OL}; they are identical to each other.

ABSORPTION OF CONCENTRATED MIXTURES IN PACKED TOWERS

In Section 10.6F simplified design methods were given for absorption of dilute gases in packed towers when the mole fractions in the gas and liquid streams were less than about 10%. Straight operating lines and approximately straight equilibrium lines are obtained. In concentrated gas mixtures the operating line and usually the equilibrium line will be substantially curved and $k'_x a$ and $k'_y a$ may vary with total flows. Then the design equations (10.6-17)–(10.6-20) must be integrated graphically or numerically:

Equation 10.6-17.

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{k'_y a S}{(1-y)_{iM}}(1-y)(y-y_i)}$$

Equation 10.6-18.

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{k'_x a S}{(1-x)_{iM}}(1-x)(x_i-x)}$$

Equation 10.6-19.

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{K'_y a S}{(1-y)_{*M}}(1-y)(y-y^*)}$$

Equation 10.6-20.

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{K'_x a S}{(1-x)_{*M}}(1-x)(x^*-x)}$$

The detailed general steps to follow are as follows:

1. The operating-line equation (10.6-5) and the equilibrium line are plotted.
2. The values of the film coefficients $k'_y a$ and $k'_x a$ are obtained from empirical equations. These two film coefficients are functions of G_y^n, kg total gas/s · m^2, and G_x^m, kg total liquid/s · m^2, where *n* and *m* are in the range 0.2–0.8. Using the operating-line-equation values, total *V* and *L* are calculated for different values of *y* and *x* in the tower and converted to G_y and G_x. Then values of $k'_y a$ and $k'_x a$ are calculated. If the variation between $k'_y a$ or $k'_x a$ at the top and bottom of the tower is small, an average value can be used.
3. Starting with the tower bottom at point $P_1(y_1, x_1)$, the interface compositions y_{i1}, x_{i1} are determined by plotting a line P_1M_1 with a slope calculated by Eq. (10.6-33). This line intersects the equilibrium line at the interface concentrations at point M_1:

Equation 10.6-33.

$$\text{slope} = -\frac{k'_x a/(1-x)_{iM}}{k'_y a/(1-y)_{iM}} = -\frac{k_x a}{k_y a}$$

where $(1 - y)_{iM}$ and $(1 - x)_{iM}$ are determined from Eqs. (10.4-6) and (10.4-7), respectively. This is trial and error. As a first trial, $(1 - x_1)$ can be used for $(1 - x)_{iM}$ and $(1 - y_1)$ for $(1 - y)_{iM}$. The values of y_{i1} and x_{i1} determined in the first trial are used in Eq. (10.6-33) for the second trial.

4. At point $P_2(y_2, x_2)$ determine a new slope using Eq. (10.6-33) and repeating step 3. Do this for several intermediate points in the tower. This slope may vary throughout the tower.
5. Using the values of y_i and x_i, calculate the values of $f(y)$ as follows:

Equation 10.7-1.

$$f(y) = \frac{V}{\dfrac{k'_y a S}{(1-y)_{iM}}(1-y)(y-y_i)}$$

Then numerically or graphically integrate Eq. (10.6-17) between y_2 and y_1 to obtain the tower height. If $k'_x a$ or other coefficients are used, the appropriate functions indicated in Eqs. (10.6-18) – (10.6-20) are used. If the stream is quite dilute, $(1 - y)_{iM}$ or $(1 - x)_{iM}$ can be assumed to be 1.0.

EXAMPLE 10.7-1. Design of an Absorption Tower with a Concentrated Gas Mixture

A tower packed with 25.4-mm ceramic rings is to be designed to absorb SO_2 from air by using pure water at 293 K and 1.013×10^5 Pa abs pressure. The entering gas contains 20 mol % SO_2 and that leaving 2 mol %. The inert air flow is 6.53×10^{-4} kg mol air/s and the inert water flow is 4.20×10^{-2} kg mol water/s. The tower cross-sectional area is 0.0929 m^2. For dilute SO_2, the film mass-transfer coefficients at 293 K are, for 25.4-mm (1-in.) rings (W1),

$$k'_y a = 0.0594 G_y^{0.7} G_x^{0.25} \qquad k'_x a = 0.152 G_x^{0.82}$$

where $k'_y a$ is kg mol/s · m^3 · mol frac, $k'_x a$ is kg mol/s · m^3mol frac, and G_x and G_y are kg total liquid or gas, respectively, per sec per m^2 tower cross section. Calculate the tower height.

***Solution*:** The given data are $V' = 6.53 \times 10^{-4}$ kg mol air/s (5.18 lb mol/h), $L' = 4.20 \times 10^{-2}$ kg mol/s (333 lb mol/h), $y_1 = 0.20$, $y_2 = 0.02$, and $x_2 = 0$.

Substituting into the overall material-balance equation (10.6-4),

$$L'\left(\frac{x_2}{1-x_2}\right) + V'\left(\frac{y_1}{1-y_1}\right) = L'\left(\frac{x_1}{1-x_1}\right) + V'\left(\frac{y_2}{1-y_2}\right)$$

$$4.20 \times 10^{-2}\left(\frac{0}{1-0}\right) + 6.53 \times 10^{-4}\left(\frac{0.2}{1-0.2}\right) = 4.20 \times 10^{-2}\left(\frac{x_1}{1-x_1}\right) + 6.53 \times 10^{-4}\left(\frac{0.02}{1-0.02}\right)$$

Solving, $x_1 = 0.00355$. The operating line Eq. (10.6-5) is

$$4.20 \times 10^{-2}\left(\frac{x}{1-x}\right) + 6.53 \times 10^{-4}\left(\frac{0.2}{1-0.2}\right) = 4.20 \times 10^{-2}\left(\frac{0.00355}{1-0.00355}\right) + 6.53 \times 10^{-4}\left(\frac{y}{1-y}\right)$$

Setting $y = 0.04$ in the operating-line equation above and solving for x, $x = 0.000332$. Selecting other values of y and solving for x, points on the operating line were calculated as shown in Table 10.7-1 and plotted in Fig. 10.7-1 together with the equilibrium data from Appendix A.3.

Table 10.7-1. Calculated Data for Example 10.7-1

Y	x	V	L	G_y	G_x	$k'_y a$	$k'_x a$	x_i	y_i	$1-y$	$(1-y)_{iM}$		$\dfrac{V}{\frac{k'_y aS}{(1-y)_{iM}}(1-y)(y-y_i)}$
0.02	0	6.66×10^{-4}	0.04200	0.2130	8.138	0.03398	0.848	0.0046	0.0090	0.980	0.985	0.0110	19.25
0.04	0.000332	6.80×10^{-4}	0.04201	0.2226	8.147	0.03504	0.849	0.0103	0.0235	0.960	0.968	0.0165	12.77
0.07	0.000855	7.02×10^{-4}	0.04203	0.2378	8.162	0.03673	0.850	0.0185	0.0476	0.930	0.941	0.0224	9.29
0.13	0.00201	7.51×10^{-4}	0.04208	0.2712	8.196	0.04032	0.853	0.0355	0.1015	0.870	0.885	0.0285	7.16
0.20	0.00355	8.16×10^{-4}	0.04215	0.3164	8.241	0.04496	0.857	0.0565	0.1685	0.800	0.816	0.0315	6.33

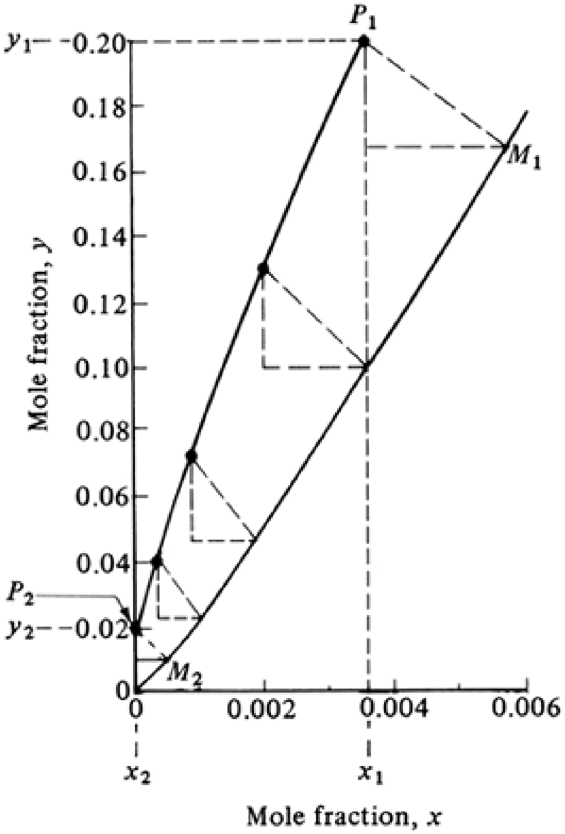

Figure 10.7-1. Operating line and interface compositions for Example 10.7-1.

The total molar flow V is calculated from $V = V'/(1 - y)$. At $y = 0.20$, $V = 6.53 \times 10^{-4}/(1 - 0.2) = 8.16 \times 10^{-4}$. Other values are calculated and tabulated in Table 10.7-1. The total mass flow G_y in kg/s · m^2 is equal to the mass flow of air plus SO$_2$ divided by the cross-sectional area:

$$G_y = \frac{6.53 \times 10^{-4}(29) \text{ kg air/s} + 6.53 \times 10^{-4}\left(\frac{y}{1-y}\right)(64.1) \text{ kg SO}_2\text{/s}}{0.0929 \text{ m}^2}$$

Setting $y = 0.20$,

$$G_y = \frac{6.53 \times 10^{-4}(29) + 6.53 \times 10^{-4}\left(\frac{0.2}{1-0.2}\right)64.1}{0.0929} = 0.3164 \text{ kg/s} \cdot \text{m}^2$$

Similarly, G_y is calculated for all points and tabulated. For the liquid flow, $L = L'/(1 - x)$. Also, for the total liquid mass flow rate,

$$G_x = \frac{4.20 \times 10^{-2}(18) + 4.20 \times 10^{-2}\left(\frac{x}{1-x}\right)64.1}{0.0929}$$

Calculated values of L and G_x for various values of x are tabulated in Table 10.7-1.

To calculate values of $k'_x a$, for $x = 0$, $G_x = 8.138$ and

$$k'_x a = 0.152 G_x^{0.82} = 0.152(8.138)^{0.82} = 0.848 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

The rest of these values are calculated and given in Table 10.7-1. For the value of $k'_y a$, for $y = 0.02$, $G_y = 0.2130$, $G_x = 8.138$, and

$$\begin{aligned} k'_y a = 0.0594 G_y^{0.7} G_x^{0.25} &= 0.0594(0.2130)^{0.7}(8.138)^{0.25} \\ &= 0.03398 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac} \end{aligned}$$

This and other calculated values of k'_y are tabulated. Note that these values vary considerably in the tower.

Next the interface compositions y_i and x_i must be determined for the given y and x values on the operating line. For the point $y_1 = 0.20$ and $x_1 = 0.00355$, we make a preliminary estimate of $(1 - y)_{iM} \cong 1 - y \cong 1 - 0.20 \cong 0.80$, together with the estimate of $(1 - x)_{iM} \cong 1 - x \cong 1 - 0.00355 \cong 0.996$. The slope of the line P_1M_1 by Eq. (10.6-33) is approximately

$$\text{slope} = -\frac{k'_x a/(1 - x)_{iM}}{k'_y a/(1 - y)_{iM}} = -\frac{0.857/(0.996)}{0.04496/(0.80)} = -15.3$$

Plotting this on Fig. 10.7-1, $y_i = 0.1688$ and $x_i = 0.00566$. Using these values for the second trial in Eqs. (10.4-6) and (10.4-7),

$$(1 - y)_{iM} = \frac{(1 - 0.1688) - (1 - 0.20)}{\ln[(1 - 0.1688)/(1 - 0.20)]} = 0.816$$

$$(1 - x)_{iM} = \frac{(1 - 0.00355) - (1 - 0.00566)}{\ln[(1 - 0.00355)/(1 - 0.00566)]} = 0.995$$

The new slope by Eq. (10.6-33) is $(-0.857/0.995)/(0.04496/0.816) = -15.6$. Plotting, $y_i = 0.1685$ and $x_i = 0.00565$. This is shown as point M_1. This calculation is repeated until point y_2, x_2 is reached. The slope of Eq. (10.6-33) increases markedly in going up the tower, being −24.6 at the top of the tower. The values of y_i and x_i are given in Table 10.7-1.

In order to integrate Eq. (10.6-17), values of $(1 - y)$, $(1 - y)_{iM}$, and $(y - y_i)$ are needed and are tabulated in Table 10.7-1. Then for $y = 0.20$, $f(y)$ is calculated from Eq. (10.7-1):

$$f(y) = \frac{V}{\dfrac{k'_y a S}{(1 - y)_{iM}}(1 - y)(y - y_i)} = \frac{8.16 \times 10^{-4}}{\dfrac{0.04496(0.0929)}{0.816}(0.800)(0.0315)} = 6.33$$

This is repeated for other values of y from y_2 to y_1. Then the function $f(y)$ is used to numerically integrate Eq. (10.6-17), giving $z = 1.588$ m.

ESTIMATION OF MASS-TRANSFER COEFFICIENTS FOR PACKED TOWERS

Experimental Determination of Film Coefficients

The individual film mass-transfer coefficients $k'_y a$ and $k'_x a$ depend generally upon Schmidt number, gas and liquid mass velocities, and the size and shape of the packing. The interactions among these factors are quite complex. Hence, the correlations for mass-transfer coefficients are highly empirical. The reliability of these correlations is not too satisfactory. Deviations of up to 25% are not uncommon. A major difficulty arises because it is an overall coefficient or resistance that is measured experimentally, which represents the two film resistances in series. To obtain the single-phase film coefficient, the experiment must be so arranged that the other film resistance is negligible or can be approximately calculated.

To measure the liquid film mass-transfer coefficient $k'_x a$, a system for absorption or desorption of very insoluble gases such as O_2 or CO_2 in water is used. The experiment gives $K'_x a$, which equals $k'_x a$ (which can give H_L), since the gas-phase resistance is negligible.

To measure the gas-phase film coefficient $k'_y a$, we desire to use a system such that the solute is very soluble in the liquid and the liquid-phase resistance is negligible. Most such systems, for example, NH_3–air–water, have a liquid-phase resistance of about 10%. By subtracting this known liquid-phase resistance (obtained by correcting $k'_x a$ data for absorption of CO_2 or O_2 to NH_3 data for $k'_x a$) from the overall resistance in Eq. (10.4-24), we obtain the coefficient $k'_y a$ or H_G. Details of these systems are discussed elsewhere (G1, S1, S2).

Correlations for Film Coefficients

Correlations for experimental coefficients can be expressed in terms of H_L and H_G or $k'_x a$ and $k'_y a$, which are related by Eqs. (10.6-39) and (10.6-40). For the first generation of packings, such as Raschig rings and Berl saddles, extensive correlations are available (T1). However, comprehensive data on the individual coefficients H_L and H_G for the newer packings, which have higher mass-transfer coefficients and capacity, are not generally available. These newer packings are more commonly used today.

However, as an alternative method for comparing the performance of different types and sizes of these newer random packings, the system CO_2–air–NaOH solution is often used (P2, S4). Air containing 1.0 mole CO_2 at 24°C (75°F) is absorbed in a packed tower using 1.0 N (4 wt %) NaOH solution (E2, E3, P2, S4). An overall coefficient $K_G a$ is measured.

In this system, the liquid film is controlling but the gas film resistance is not negligible. The fast chemical reaction between NaOH and CO_2 takes place close to the interface, which gives a steeper concentration gradient for CO_2 in the water film. Hence, the value of $K_G a$ is muc h larger than for absorption of CO_2 in water. Because of this, these experimental values are not used to predict the absorption for other systems in towers.

These experimental results, however, can be used to compare the performances of various packings. To do this, the ratio f_p of $K_G a$ for a given packing to that for $1\frac{1}{2}$-in. Raschig rings at a liquid velocity G_x of 5000 $\text{lb}_\text{m}/\text{h} \cdot \text{ft}^2$ (6.782 $\text{kg/s} \cdot \text{m}^2$) and G_y of 1000 $\text{lb}_\text{m}/\text{h} \cdot \text{ft}^2$ (1.356 $\text{kg/s} \cdot \text{m}^2$) is obtained; these are given in Table 10.6-1. The f_p value is a relative ratio of the total interfacial areas, since the reaction of CO_2 in NaOH solution takes place in the relatively static holdup pools and in the dynamic holdup. Some f_p data have been obtained at G_y = 500 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ instead of 1000 (E3). Eckert et al. (E2) showed that there is no effect of G_y in the range of 200–1000 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ on the overall K_Ga. This is expected where the liquid film resistance controls (S1). Values of f_p for various investigators agree within ±10% or less.

Predicting Mass-Transfer Film Coefficients

For estimating the performance $H_L(H_x)$ and $H_G(H_y)$ of a new packing, the values of f_p can be used to correct the experimental H_x values for oxygen absorption or desorption and the H_y value for NH_3 absorption with $1\frac{1}{2}$-in. Raschig rings. These values must also be corrected for Schmidt number, liquid viscosity, and flow rates.

Gas film coefficient H_y

Using the NH_3 absorption data corrected for the liquid film resistance of approximately 10%, H_G has been found to vary as G_y to an exponent between 0.3 and 0.4 (S1, T1) for values of G_y up to about 700 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ (0.949 $\text{kg/s} \cdot \text{m}^2$). A value of 0.35 is used. For liquid flows of G_x from 500 to 5000 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ (0.678–6.782 $\text{kg/s} \cdot \text{m}^2$), H_y varies as $G_x^{-0.4}$–$G_x^{-0.6}$, with the value of $G_x^{-0.5}$ used. Also, the value of H_y has been found to be proportional to $N_{\text{Sc}}^{0.5}$ of the gas phase. A value for H_y of 0.74 ft (0.226 m) is obtained from the correlation for $1\frac{1}{2}$-in. Raschig rings for the NH_3 system (S1) corrected for the small liquid film resistance of 10% at G_x = 5000 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ (6.782 $\text{kg/s} \cdot \text{m}^2$) and G_y = 500 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ (0.678 $\text{kg/s} \cdot \text{m}^2$). The value of G_y = 500 will be used instead of 1000, since there is no effect of G_y on f_p in this range. For the NH_3 system, N_{Sc} = 0.66 at 25°C. Then, for estimation of H_G for a new solute system and packing and flow rates of G_x and G_y using SI units,

Equation 10.8-1.

$$H_G = H_y = \left(\frac{0.226}{f_p}\right)\left(\frac{N_{\text{Sc}}}{0.660}\right)^{0.5}\left(\frac{G_x}{6.782}\right)^{-0.5}\left(\frac{G_y}{0.678}\right)^{0.35}$$

where f_p for the new packing is given in Table 10.6-1 and H_G is in m.

Liquid film coefficient H_x

For gas flow rates up to loading or about 50% of the flooding velocity, the effect of G_y on H_x is small and can be neglected (S1). Using the oxygen desorption data, H_x is proportional to the liquid $N_{\text{Sc}}^{0.5}$. The N_{Sc} = 372 at 25°C for O_2 in water and the viscosity μ is 0.8937×10^{-3} $\text{kg/m} \cdot \text{s}$. Data for different packings show that H_x is proportional to (G_x/μ) to the 0.22–0.35 exponent, with an average of $(G_x/\mu)^{0.3}$. A value of H_x = 1.17 ft (0.357 m), where G_x = 5000 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ is obtained from the correlation (S1) for the O_2 system and $1\frac{1}{2}$-in. Raschig rings. Then, to predict H_x for a new solute system and packing at velocities of G_x and G_y using SI units,

Equation 10.8-2.

$$H_L = H_x = \left(\frac{0.357}{f_p}\right)\left(\frac{N_{\text{Sc}}}{372}\right)^{0.5}\left(\frac{G_x/\mu}{6.782/0.8937 \times 10^{-3}}\right)^{0.3}$$

These equations can be used for values of G_y up to almost 1000 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ and G_x up to 5000 and remain below loading.

EXAMPLE 10.8-1. Prediction of Film Coefficients for CO_2 Absorption

Predict H_G, H_L, and H_{OL} for absorption of CO_2 from air by water in a dilute solution in a packed tower with $1\frac{1}{2}$-in. metal Pall rings at 303 K (30°C) and 101.32 kPa pressure. The flow rates are $G_x = 4.069$ kg/s · m^2 (3000 lb$_m$/h · ft^2) and $G_y = 0.5424$ kg/s · m^2 (400 lb$_m$/h · ft^2).

Solution: From Appendix A.3-18, for CO_2 at 1 atm, $p_A = 1.86 \times 10^3 x_A$. Hence, $y_A = p_A/1.0 = 1.86 \times 10^3 x_A$ (mole fraction units). Also, from Appendix A.3-3 for air at 303 K, $\mu = 1.866 \times 10^{-5}$ kg/m · s and the density $\rho = 1.166$ kg/m^3. The diffusivity for CO_2 at 276.2 K from Table 6.2-1 is 0.142×10^{-4} m^2/s. Correcting this to 303 K by Eq. (6.2-45), $D_{AB} = 0.142 \times 10^{-4} (303/276.2)^{1.75} = 0.1670 \times 10^{-4}$ m^2/s. Hence,

$$N_{Sc} = \frac{\mu}{\rho D} = \frac{1.866 \times 10^{-5}}{(1.166)(0.1670 \times 10^{-4})} = 0.958$$

From Table 10.6-1 the relative mass-transfer coefficient for $1\frac{1}{2}$-in. metal Pall rings compared to that for $1\frac{1}{2}$-in. Raschig rings is $f_p = 1.34$. Substituting into Eq. (10.8-1),

$$H_G = \frac{0.226}{1.34}\left(\frac{0.958}{0.660}\right)^{0.5}\left(\frac{4.069}{6.782}\right)^{-0.5}\left(\frac{0.5424}{0.678}\right)^{0.35} = 0.2426 \text{ m } (0.796 \text{ ft})$$

From Appendix A.2-4, the viscosity of water at 303 K is 0.8007×10^{-3} kg/m · s and the density $\rho = 995.68$ kg/m^3. At 298 K the viscosity of water is 0.8937×10^{-3} kg/s · m.

From Table 6.3-1, the D_{AB} of CO_2 in water is 2.00×10^{-9} m^2/s at 25°C. Using Eq. (6.3-9) to correct it to 303 K, $D_{AB} = (0.8937 \times 10^{-3}/0.8007 \times 10^{-3})(303/298)(2.00 \times 10^{-9}) = 2.270 \times 10^{-9}$ m^2/s. Then,

$$N_{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{0.8007 \times 10^{-3}}{995.68(2.270 \times 10^{-9})} = 354.3$$

Substituting into Eq. (10.8-2),

$$H_L = \left(\frac{0.357}{f_p}\right)\left(\frac{N_{Sc}}{372}\right)^{0.5}\left(\frac{G_x/\mu}{6.782/0.8937 \times 10^{-3}}\right)^{0.3}$$

$$H_L = \left(\frac{0.357}{1.34}\right)\left(\frac{354.3}{372}\right)^{0.5}\left(\frac{4.069/0.8007 \times 10^{-3}}{6.782/0.8937 \times 10^{-3}}\right)^{0.3} = 0.2306 \text{ m } (0.759 \text{ ft})$$

To calculate the value of H_{OL}, the molar flow rates are calculated, where for dilute air, $V = G_y/\text{MW} = 0.5424/28.97 = 0.01872$ kg mol/s · m^2, and for water, $L = G_x/\text{MW} = 4.069/18.0 = 0.2261$ kg mol/s · m^2. Substituting into Eq. (10.6-58),

$$\begin{aligned} H_{OL} &= H_{Ox} = H_L + (L/mV)H_G \\ &= 0.2306 + [(0.2261)/(1.86 \times 10^3 \times 0.01872)]0.2426 \\ &= 0.2306 + 0.001575 = 0.2322 \text{ m} \end{aligned}$$

The percent resistance in the gas film is 0.001575(100)/0.2322 = 0.68%. This shows that for a very insoluble gas, even though H_G is similar in value to H_L, the large value of m causes the gas-phase resistance to be very small. Hence, $H_{OL} \cong H_L$. For a soluble gas like NH_3, where $m = 1.20$ at 303 K compared to 1.86×10^3 for CO_2, the percent resistance in the gas film would be about 90%. (See Problem 10.8-1.) The different gas and liquid Schmidt numbers for NH_3 would have only a small effect on the results.

HEAT EFFECTS AND TEMPERATURE VARIATIONS IN ABSORPTION

Heat Effects in Absorption

The temperature in an absorption tower can vary from the top to the bottom of the tower when the inlet gas contains a relatively high concentration of solute. Solute absorption by the liquid yields a heat of solution which raises the liquid temperature. Also, if evaporation of the solvent to the gas phase occurs, the liquid temperature is cooled. The curvature of the equilibrium line depends upon the absorption of the solute, the heat-transfer rate between phases, and the evaporation or condensation of the liquid solvent.

Calculated temperature profiles may also have a maximum in the tower. This can occur when the entering gas temperature is considerably below the exit solvent temperature and/or the volatile solvent evaporates. Solvent evaporation cools the liquid near the tower bottom and a temperature maximum occurs. Detailed rigorous heat and mass-balance equations are needed for mass transfer of the solute, mass transfer of the solvent, heat transfer between the gas and liquid, and enthalpy balances. Equations and examples are given by Sherwood et al. (S1), Stockar and Wilke (S7), and Treybal (T2). Note that in actual towers, intercoolers are often used to keep the towers isothermal.

Simplified Design Method

A simplified adiabatic design method is sometimes used when the entering gas contains solvent vapor and/or solvent evaporation occurs. Heat of solution of the solute is also included in the design. Then, using an overall enthalpy balance, the exit liquid temperature is calculated. This rise in liquid temperature can then be used to adjust the equilibrium-line curvature. First, an overall enthalpy balance is made which includes sensible heat for inlet and outlet gas temperatures, heat of solution of the solute, heat of vaporization of the solvent, and sensible heat for the inlet and the unknown outlet liquid temperature. This outlet temperature is then obtained.

The equilibrium line for the top entering liquid temperature is plotted, with the given slope at the dilute concentration. Next, an equilibrium line at this higher outlet liquid temperature is plotted. The outlet concentration x_1 is then located on this equilibrium line. Then, assuming a linear liquid temperature profile, the temperature of point x, which is halfway between the top and bottom concentrations, is obtained. This x value is then located on the equilibrium line for this new temperature. The final curved equilibrium line is plotted with the slope at the dilute end, through the x at the halfway point, and finally through the x_1 point. This method is illustrated in Example 10.9-1.

EXAMPLE 10.9-1. Heat Balance for Nonisothermal Absorption Tower

The gas feed at 20°C and 1 atm to a packed absorption tower contains 6.0 mol % of NH_3 (on a dry basis) in air. The inlet gas is saturated with water vapor. The absorbing pure liquid water enters at 25°C. The outlet gas contains 0.5% of NH_3 on a dry basis; it is assumed to be saturated with water vapor and leaves at 25°C. For a feed of 100 g mol dry gas, 190 g mol of pure water are to be used for absorption. The heat of solution for 1.0 g mol NH_3 gas absorbed in water is $\Delta H = -8310$ cal/g mol (S1). Equilibrium data (P1) are as follows for this system, where $y = mx$:

	20°C	25	30	35	40
x		m			
0.03077	0.778	1.005	1.266	1.578*	1.924

*Interpolated

Solution: For the inlet gas, where $y_1 = 0.06$, the moles $NH_3 = 0.06(100) = 6.0$ and moles air $= 0.94(100) = 94.0$. For the outlet gas, where $y_2 = 0.005$, the moles air $= 94.0$ and moles $NH_3 = 94.0\,[0.005/(1 - 0.005)] = 0.472$. The total is 94.472 moles. The moles NH_3 absorbed $= 6.0 - 0.472 = 5.528$. The outlet $x_1 = (5.528)/(190 + 5.528) = 0.0283$. From Appendix A.2-9, the latent heat of water at 25°C is (2442.3 J/g)(1/4.184 J/cal)(18.02 g/g mol) = 10 519 cal/g mol. The vapor pressure of water at 20°C = 2.346 kPa and at 25°C, 3.169 kPa.

The moles H_2O in the inlet gas at 20°C = (100.0)(2.346)/(101.325 − 2.346) = 2.370 moles vapor. In the outlet gas moles H_2O vapor = (94.472)(3.169)/(101.325 − 3.169) = 3.050 moles vapor. The moles of H_2O vaporized = 3.050 − 2.370 = 0.680.

An enthalpy balance is made for ammonia gas and air in and liquid water at 25°C. For air, from Appendix A. 3-3, c_p = (1.0048 J/g · K)(1/4.184 J/cal) (28.972g/g mol) = 6.957 cal/g mol · K. For water vapor, from the Appendix, Fig. A.3-1, c_p = 8.0 cal/g mol · K, and for NH_3 gas, c_p = 8.58 cal/g mol · K. The sensible heat for the entering gas at 20°C is as follows. For air, the sensible heat q = 94(6.957)(20 − 25) = −3270 cal. For water vapor, q = (2.370)(8.0)(20 − 25) = −95 cal. For NH_3 gas, q = (6.00)(8.58)(20 − 25) = −257 cal. The total sensible heat is −3270 − 95 − 257 = −3622 cal. The latent heat of water vapor entering q = 2.370(10 519) = 24 930 cal. The sensible heat of the entering liquid at 25°C is 0.

For the exit gas at 25°C, the sensible heat = 0. The latent heat of the water vapor = 3.050(10 519) = 32 083 cal. The heat of solution of NH_3 absorption = 5.528(−8310) = −45 938 cal. From Appendix A.2-11, assuming dilute liquid water, c_p = (1.000 cal/g · K)(18.02 g/g mol) = 18.02 cal/g mol · K. The total g moles outlet liquid = 190 + 5.528 − 0.68 = 194.85. The sensible heat q of the outlet liquid = (194.85)(18.02)(T_1 − 25) = 3511(T_1 − 25) cal, where T_1 is the unknown outlet temperature in °C. Equating the heat in to the heat out,

$$-3622 + 24\,930 = 0 + 32\,083 - 45\,938 + 3511(T_1 - 25)$$

Solving, (T_1 − 25) = 10.02°C, which is the temperature increase of the outlet liquid. The outlet T_1 = 35.02°C or 35°C.

In Fig. 10.9-1, a straight operating line is plotted since the solution is dilute. The equilibrium line for 25°C is also shown with a slope m = 1.005 at the top of the tower. At the tower bottom, at 35°C, the exit x_1 = 0.0283. The equilibrium line at 35°C is shown. The point x_1 is located on this 35°C line and on the curved equilibrium line for the tower. The assumed linear temperature profile is also shown. Selecting a value of x = (0 + 0.0283)/2 = 0.0142 with a temperature of (35 + 25)/2 or 30°C, the point x_1 = 0.0412 is plotted on the 30°C equilibrium line. The final curved equilibrium line is plotted with a slope at the origin of 1.005 at 25°C, through the point x = 0.0142 on the 30°C equilibrium line, and then through the point x_1 = 0.0283 on the 35°C equilibrium line. Since the equilibrium line is curved, the number of overall transfer units N_{Oy} can be obtained by numerical or graphical integration as before using Eq. (10.6-46).

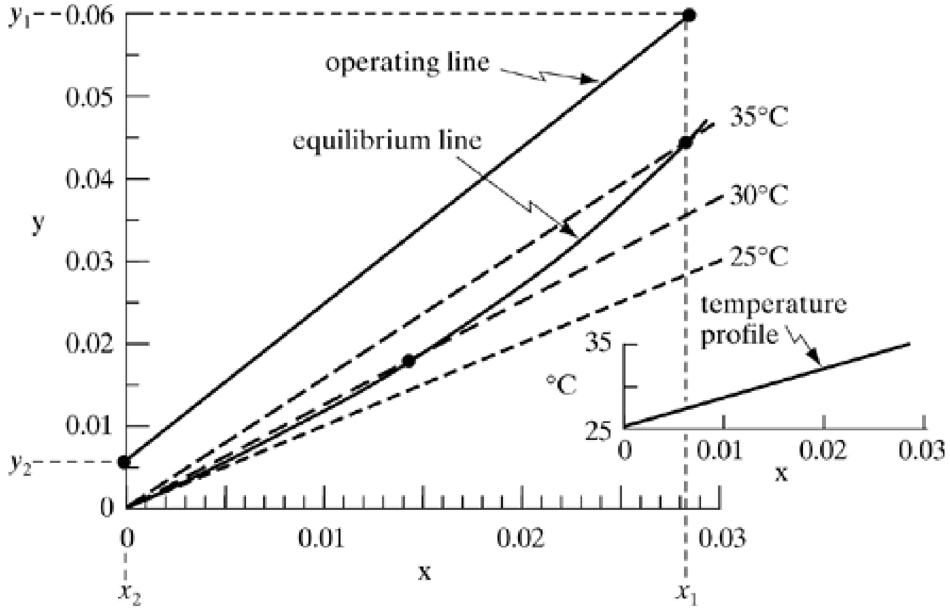

Figure 10.9-1. Operating line and curved equilibrium line for Example 10.9-1. The calculated linear temperature profile is also shown.

PROBLEMS

10.2-1. ***Equilibrium and Henry's Law Constant.*** The partial pressure of CO_2 in air is 1.333×10^4 Pa and the total pressure is 1.133×10^5 Pa. The gas phase is in equilibrium with a water solution at 303 K. What is the value of x_A for CO_2 in equilibrium in the solution? See Appendix A.3 for the Henry's law constant.

A1: **Ans.** $x_A = 7.07 \times 10^{-5}$ mol frac CO_2

10.2-2. ***Gas Solubility in Aqueous Solution.*** At 303 K the concentration of CO_2 in water is 0.90×10^{-4} kg CO_2/kg water. Using the Henry's law constant from Appendix A.3, what partial pressure of CO_2 must be kept in the gas to prevent the CO_2 from vaporizing from the aqueous solution?

A2: **Ans.** $p_A = 6.93 \times 10^3$ Pa (0.0684 atm)

10.2-3. ***Phase Rule for a Gas–Liquid System.*** For the system SO_2–air–water, the total pressure is set at 1 atm abs and the partial pressure of SO_2 in the vapor is set at 0.20 atm. Calculate the number of degrees of freedom, F. What variables are unspecified and hence can be arbitrarily set?

10.3-1. ***Equilibirum Stage Contact for Gas–Liquid System.*** A gas mixture at 2.026×10^5 Pa total pressure containing air and SO_2 is brought into contact in a single-stage equilibrium mixer with pure water at 293 K. The partial pressure of SO_2 in the original gas is 1.52×10^4 Pa. The inlet gas contains 5.70 total kg mol and the inlet water 2.20 total kg mol. The exit gas and liquid leaving are in equilibrium. Calculate the amounts and compositions of the outlet phases. Use equilibrium data from Fig. 10.2-1.

A4: **Ans.** $x_{A1} = 0.00495$, $y_{A1} = 0.0733$, $L_1 = 2.211$ kg mol, $V_1 = 5.69$ kg mol

10.3-2. ***Absorption in a Countercurrent Stage Tower.*** Repeat Example 10.3-2 using the same conditions but with the following change. Use a pure water flow to the tower of 108 kg mol H_2O/h, that is, 20% above the 90 used in Example 10.3-2. Determine the number of stages required graphically. Repeat, using the analytical Kremser equation.

10.3-3. ***Stripping Taint from Cream by Steam.*** Countercurrent stage stripping is to be used to remove a taint from cream. The taint is present in the original cream to the stripper at a concentration of 20 parts per million (ppm). For every 100 kg of cream entering per unit time, 50 kg of steam will be used for stripping. It is desired to reduce the concentration of the taint in the cream to 1 ppm. The equilibrium relation between the taint in the steam vapor and the liquid cream is $y_A = 10x_A$, where y_A is ppm of taint in the steam and x_A ppm in the cream (E1). Determine the number of theoretical stages needed. [*Hint:* In this case, for stripping from the liquid (L) stream to the vapor (V) stream, the operating line will be below the equilibrium line on the $y_A - x_A$ diagram. It is assumed that none of the steam condenses in the stripping. Use ppm in the material balances.]

A6: **Ans.** Number stages = 1.85 (stepping down starting from the concentrated end)

10.4-1. ***Overall Mass-Transfer Coefficient from Film Coefficients.*** Using the same data as in Example 10.4-1, calculate the overall mass-transfer coefficients K'_x and K_x, the flux, and the percent resistance in the gas film.

A7: **Ans.** K'_x = 1.173 × 10^{-3} kg mol/s · m^2 · mol frac, K_x = 1.519 × 10^{-3}, N_A = 3.78 × 10^{-4} kg mol/s · m^2, 36.7% resistance

10.4-2. ***Interface Concentrations and Overall Mass-Transfer Coefficients.*** Use the same equilibrium data and film coefficients k'_y and k'_x as in Example 10.4-1. However, use bulk concentrations of y_{AG} = 0.25 and x_{AL} = 0.05. Calculate the following:

a. Interface concentrations y_{Ai} and x_{Ai} and flux N_A.

b. Overall mass-transfer coefficients K'_y and K_y and flux N_A.

c. Overall mass-transfer coefficient K'_x and flux N_A.

10.5-1. ***Countercurrent Water-Cooling Tower.*** A forced-draft countercurrent water-cooling tower is to cool water from 43.3 to 26.7°C. The air enters the bottom of the tower at 23.9°C with a wet bulb temperature of 21.1°C. The value of H_G for the flow conditions is H_G = 0.533 m. The heat-transfer resistance in the liquid phase will be neglected; that is, h_L is very large. Hence, values of H_y^* should be used. Calculate the tower height needed if 1.5 times the minimum air rate is used.

10.5-2. ***Minimum Gas Rate and Height of Water-Cooling Tower.*** It is planned to cool water from 110°F to 85°F in a packed countercurrent water-cooling tower using entering air at 85°F with a wet bulb temperature of 75°F. The water flow is 2000 $lb_m/h \cdot ft^2$ and the air flow is 1400 lb_m air/h · ft^2. The overall mass-transfer coefficient is K_Ga = 6.90 lb mol/h · ft^3atm.

a. Calculate the minimum air rate that can be used.

b. Calculate the tower height needed if the air flow of 1400 lb_m air/h · ft^2 is used.

A10: **Ans.** (a) G_{min} = 935 lb_m air/h · ft^2 (4241 kg air/h · m^2); (b) z = 21.8 ft (6.64 m)

10.5-3. ***Design of Water-Cooling Tower.*** Recalculate Example 10.5-1, but calculate the minimum air rate and use 1.75 times the minimum air rate.

10.5-4. ***Effect of Changing Air Conditions on Cooling Tower.*** For the cooling tower in Example 10.5-1, to what temperature will the water be cooled if the entering air enters at 29.4°C but the wet bulb temperature is 26.7°C? The same gas and liquid flow rates are used. The water enters at 43.3°C, as before. (*Hint:* In this case T_{L1} is the unknown. The tower height is the same as in Example 10.5-1. The slope of the operating line is as before. The solution is trial and error. Assume a value of T_{L1} that is greater than 29.4°C. Do numerical or graphical integration to see if the same height is obtained.)

10.6-1. ***Amount of Absorption in a Tray Tower.*** An existing tower contains the equivalent of 3.0 theoretical trays and is being used to absorb SO_2 from air by pure water at 293 K and 1.013×10^5 Pa. The entering gas contains 20 mol % SO_2 and the inlet air flow rate is 150 kg inert air/h · m^2. The entering water rate is 6000 kg/h · m^2. Calculate the outlet composition of the gas. (*Hint:* This is a trial-and-error solution. Assume an outlet gas composition of, say, $y_1 = 0.01$. Plot the operating line and determine the number of theoretical trays needed. If this number is not 3.0 trays, assume another value of y_1, and so on.)

A13: **Ans.** $y_1 = 0.009$

10.6-2. ***Analytical Method for Number of Trays in Absorption.*** Use the analytical equations in Section 10.6 for countercurrent tray contact to calculate the number of theoretical trays needed for Example 10.6-3 using $1.3 L'_{\min}$.

10.6-3. ***Absorption of Ammonia in a Tray Tower.*** A tray tower is to be used to remove 99% of the ammonia from an entering air stream containing 6 mol % ammonia at 293 K and 1.013×10^5 Pa. The entering pure water flow rate is 188 kg H_2O/h · m^{2}and the inert air flow is 128 kg air/h · m^2. Calculate the number of theoretical trays needed. Use equilibrium data from Appendix A.3. For the dilute end of the tower, plot an expanded diagram to step off the number of trays more accurately.

A15: **Ans.** $y_1 = 0.000639$ (exit), $x_N = 0.0260$ (exit), 3.8 theoretical trays

10.6-4. ***Minimum Liquid Flow in a Packed Tower.*** The gas stream from a chemical reactor contains 25 mol % ammonia and the rest inert gases. The total flow is 181.4 kg mol/h to an absorption tower at 303 K and 1.013×10^5 Pa pressure, where water containing 0.005 mol frac ammonia is the scrubbing liquid. The outlet gas concentration is to be 2.0 mol % ammonia. What is the minimum flow $L'_{\min}$? Using 1.5 times the minimum, plot the equilibrium and operating lines.

A16: **Ans.** $L'_{\min} = 262.6$ kg mol/h

10.6-5. ***Steam Stripping and Number of Trays.*** A relatively nonvolatile hydrocarbon oil contains 4.0 mol % propane and is being stripped by direct superheated steam in a stripping tray tower to reduce the propane content to 0.2%. The temperature is held constant at 422 K by internal heating in the tower at 2.026×10^5 Pa pressure. A total of 11.42 kg mol of direct steam is used for 300 kg mol of total entering liquid. The vapor–liquid equilibria can be represented by $y = 25x$, where y is mole fraction propane in the steam and x is mole fraction propane in the oil. Steam can be considered as an inert gas and will not condense. Plot the operating and equilibrium lines and determine the number of theoretical trays needed.

A17: **Ans.** 5.6 theoretical trays (stepping down from the tower top)

10.6-6. ***Absorption of Ammonia in Packed Tower.*** A gas stream contains 4.0 mol % NH_3 and its ammonia content is reduced to 0.5 mol % in a packed absorption tower at 293 K and 1.013×10^5 Pa. The inlet pure water flow is 68.0 kg mol/h and the total inlet gas flow is 57.8

kg mol/h. The tower diameter is 0.747 m. The film mass-transfer coefficients are $k'_y a$ = 0.0739 kg mol/s · m^{3}mol frac and $k'_x a$ = 0.169 kg mol/s · m^{3}mol frac. Using the design methods for dilute gas mixtures, do as follows:

a. Calculate the tower height using $k'_y a$.

b. Calculate the tower height using $K'_y a$.

A18: **Ans.** (a) z = 2.362 m (7.75 ft)

10.6-7. ***Tower Height Using Overall Mass-Transfer Coefficient.*** Repeat Example 10.6-4, using the overall liquid mass-transfer coefficient $K'_x a$ to calculate the tower height.

10.6-8. ***Experimental Overall Mass-Transfer Coefficient.*** In a tower 0.254 m in diameter absorbing acetone from air at 293 K and 101.32 kPa using pure water, the following experimental data were obtained. Height of 25.4-mm Raschig rings = 4.88 m, V' = 3.30 kg mol air/h, y_1 = 0.01053 mol frac acetone, y_2 = 0.00072, L' = 9.03 kg mol water/h, x_1 = 0.00363 mol frac acetone. Calculate the experimental value of $K_y a$.

10.6-9. ***Conversion to Transfer-Unit Coefficients from Mass-Transfer Coefficients.*** Experimental data on absorption of dilute acetone in air by water at 80°F and 1 atm abs pressure in a packed tower with 25.4-mm Raschig rings were obtained. The inert gas flow was 95 lb$_m$ air/h · ft^2 and the pure water flow was 987 lb$_m$/h · ft^2. The experimental coefficients are $k_G a$ = 4.03 lb mol/h · ft^{3}atm and $k_L a$ = 16.6 lb mol/h · ft^3 · lb mol/h · ft^3. The equilibrium data can be expressed by $c_A = 1.37 p_A$, where c_A = lb mol/ft^3 and p_A = atm partial pressure of acetone.

a. Calculate the film height of transfer units H_G and H_L.

b. Calculate H_{OG}.

A21: **Ans.** H_{OG} = 0.957 ft (0.292 m)

10.6-10. ***Height of Tower Using Transfer Units.*** Repeat Example 10.6-4 but use transfer units and calculate H_L, N_L, and tower height.

10.6-11. ***Experimental Value of H_{OG}.*** Using the experimental data given in Problem 10.6-8, calculate the number of transfer units N_{OG} and the experimental value of H_{OG}.

A23: **Ans.** H_{OG} = 1.265 m

10.6-12. ***Pressure Drop and Tower Diameter.*** Use the same conditions as in Example 10.6-1 but with the following changes. The gas feed rate is 2000 lb$_m$/h and the design ratio of G_L/G_G is 2.2/1. Using 60% of flooding and 1 in Intalox packing, calculate the pressure drop, gas and liquid flows, and tower diameter.

A24: **Ans.** Δp = 0.250 in. water/ft packing, D = 1.452 ft (0.4425 m)

10.6-13. ***Minimum Liquid Flow Rate in Absorption.*** Using the data from Example 10.6-3, calculate the number of trays graphically and analytically for an operating flow rate of 1.3 times the minimum liquid flow rate.

A25: **Ans.** N = 5.34 steps (analytical)

10.6-14. ***Experimental Height of Transfer Unit and Analytical Equations.*** A packed tower 4.0 m tall is used to absorb ethyl alcohol from an inert gas by 90 kg mol/h of pure water at 303 K and 101.3 kPa. The total gas stream flow rate of 100 kg mol/h contains 2.0 mol % alcohol and the exit concentration is 0.20 mol %. The equilibrium relation is $y = mx = 0.68x$ for this dilute stream. Using the analytical equations, calculate the number of theoretical trays N, the number of transfer units N_{OG}, H_{OG}, and HETP.

A26: **Ans.** $H_{OG} = 0.860$ m (2.823 ft), $N = 4.016$ trays

10.7-1. ***Liquid Film Coefficients and Design of SO$_2$ Tower.*** Using the data for Example 10.7-1, calculate the height of the tower using Eq. (10.6-18), which is based on the liquid film mass-transfer coefficient . $k'_x a$ [*Note:* The interface values x_i have already been obtained.]

A27: **Ans.** $z = 1.586$ m

10.7-2. ***Design of SO$_2$ Tower Using Overall Coefficients.*** Using the data for Example 10.7-1, calculate the tower height using the overall mass-transfer coefficient . $K'_y a$ [*Hint:* Calculate $K'_y a$ at the top of the tower and at the bottom of the tower from the film coefficients. Then use a linear average of the two values for the design. Obtain the values of y^* from the operating- and equilibrium-line plot. Numerically or graphically integrate Eq. (10.6-19), keeping $K'_y a$ outside the integral.]

10.7-3. ***Height of Packed Tower Using Transfer Units.*** For Example 10.7-1, calculate the tower height using the H_G and the number of transfer units N_G. [*Hint:* Calculate H_G at the tower top using Eq. (10.6-39) and at the tower bottom. Use the linear average value for H_G. Calculate the number of transfer units N_G by numerical or graphical integration of the integral of Eq. (10.6-35). Then calculate the tower height.]

A29: **Ans.** $H_G = 0.2036$ m (average value)

10.7-4 ***Design of Absorption Tower Using Transfer Units.*** The gas SO_2 is being scrubbed from a gas mixture by pure water at 303 K and 1.013×10^5 Pa. The inlet gas contains 6.00 mol % SO_2 and the outlet 0.3 mol % SO_2. The tower cross-sectional area of packing is 0.426 m^2. The inlet gas flow is 13.65 kg mol inert air/h and the inlet water flow is 984 kg mol inert water/h. The mass-transfer coefficients are $H_L = 0.436$ m and $k_G a = 6.06 \times 10^{-7}$ kg mol/s · m^3 · Pa and are to be assumed constant in the tower for the given concentration range. Use equilibrium data from Appendix A.3. By numerical or graphical integration, determine N_G. Calculate the tower height. (*Note:* The equilibrium line is markedly curved, so numerical or graphical integration is necessary even for this dilute mixture.)

A30: **Ans.** $N_G = 8.47$ transfer units, $z = 1.311$ m

10.8-1. ***Correction of Film Coefficients for NH$_3$ Absorption.*** Use the same system and conditions given in Example 10.8-1, except that NH_3 is being absorbed instead of CO_2. The flow rates are the same. Predict H_G, H_L, H_{OG}, and percent resistance in the liquid phase.

A31: **Ans.** $H_{OG} = 0.2244$ m, % resistance = 9.4%

10.8-2.	***Prediction of Film Coefficients for Acetone Absorption.*** Predict H_G, H_L, and H_{OG} for absorption of acetone from air by water in a dilute aqueous solution using 2-in. Intalox metal IMTP packing at 20°C and 1 atm abs pressure. The flow rates are $G_x = 3.391$ kg/s · m^2 (2500 lb$_m$/h · ft^2) and $G_y = 0.678$ kg/s · m^2 (500 lb$_m$/h · ft^2). Use equilibrium data from Appendix A.3-21 and the diffusivity for acetone in water from Table 6.3-1. The diffusivity of acetone in air at 1 atm abs is 0.109×10^{-4} m^2/s at 0°C (P1).
A32:	**Ans.** $H_G = 0.3416$ m, $H_{OG} = 0.3921$ m
10.9-1.	***Nonisothermal Absorption Tower.*** Use the same operating conditions as in Example 10.9-1 for absorption of NH_3 except for the following change. The inlet gas temperature is 15°C and it is saturated with water vapor. Calculate the outlet water temperature T_1 and the overall number of transfer units N_{Oy} by numerical or graphical integration.
A33:	**Ans.** $T_1 = 32.05$°C

REFERENCES

Bibliography

[ch10biblio01entry01] (B1) W. L., Badger, and J. T. Banchero, *Introduction to Chemical Engineering*. New York: McGraw-Hill Book Company, 1955.

[ch10biblio01entry02] (B2) *Bubble Tray Design Manual*. American Institute of Chemical Engineers, New York, 1958.

[ch10biblio01entry03] (E1) R. L. Earle, *Unit Operations in Food Processing*. Oxford: Pergamon Press, Inc., 1966.

[ch10biblio01entry04] (E2) J. S., Eckert, E. H., Foote, and R. L. Huntington, *Chem. Eng. Progr.*, **54**(1), 70 (1958).

[ch10biblio01entry05] (E3) Eckert J. S., E. H., Foote, L. R., Rollison, and L. F. Walter, *Ind. Eng. Chem.*, **59**(2), 41 (1967).

[ch10biblio01entry06] (F1) J. R., Fair, and J. L. Bravo, *Chem. Eng. Progr.*, **86** (Jan.), 19 (1990).

[ch10biblio01entry07] (G1) C. J. Geankoplis, *Mass Transport Phenomena*. Columbus, Ohio: Ohio State University Bookstores, 1972.

[ch10biblio01entry08] (K1) H. Z. Kister, *Distillation Design*. New York: McGraw-Hill Book Company, 1992.

[ch10biblio01entry09] (K2) H. Z., Kister, and D. R. Gill, *Chem. Eng. Progr.*, **87** (Feb.), 32 (1991).

[ch10biblio01entry10] (L1) M. Leva, *Tower Packings and Packed Tower Design*, 2nd ed. Akron, Ohio: U.S. Stoneware, Inc., 1953.

[ch10biblio01entry11] (L2) E. E. Ludwig, *Applied Process Design for Chemical and Petroleum Plants*, Vol. 2, 3rd ed. Houston: Gulf Publishing Company, 1997.

[ch10biblio01entry12] (M1) H. S., Mickley, T K., Sherwood, and C. E. Reed, *Applied Mathematics in Chemical Engineering*, 2nd ed. New York: McGraw-Hill Book Company, 1957.

[ch10biblio01entry13] (M2) H. S. Mickley, *Chem. Eng. Progr.*, **45**, 739 (1949).

[ch10biblio01entry14] (P1) R. H., Perry, and D. Green, *Perry's Chemical Engineers' Handbook*, 6th ed. New York: McGraw-Hill Book Company, 1984.

[ch10biblio01entry15] (P2) R. H., Perry, and D. Green, *Perry's Chemical Engineers' Handbook*, 7th ed. New York: McGraw-Hill Book Company, 1997.

[ch10biblio01entry16] (S1) T. K., Sherwood, R. L., Pigford, and C. R. Wilkie, *Mass Transfer*. New York: McGraw-Hill Book Company, 1975.

[ch10biblio01entry17] (S2) H. L., Shulman, and coworkers. *A.I.Ch.E. J.*, **1**, 274 (1955); **9**, 479(1963).

[ch10biblio01entry18] (S3) T. K., Sherwood, and F. L.A. Holloway, *Trans. A.I.Ch.E.*, **30**, 39 (1940).

[ch10biblio01entry19] (S4) R. F., Strigle,, Jr. *Packed Tower Design and Applications: Random and Structured Packings*, 2nd ed. Houston: Gulf Publishing Company, 1994.

[ch10biblio01entry20] (S5) P. A. Schweitzer, *Handbook of Separation Techniques for Chemical Engineers*. New York: McGraw-Hill Book Company, 1997.

[ch10biblio01entry21] (S6) J. D., Seader, and E. J. Henley, *Separation Process Principles*. New York: John Wiley & Sons, Inc., 1998.

[ch10biblio01entry22] (S7) U., Stockar, and C. R. Wilke, *Ind. Eng. Chem. Fund.*, **16**(1), 88, 94 (1977).

[ch10biblio01entry23] (T1) R. E. Treybal, *Mass Transfer Operations*, 3rd ed. New York: McGraw-Hill Book Company, 1980.

[ch10biblio01entry24] (T2) R. E. Treybal, *Mass Transfer Operations*. New York: McGraw-Hill Book Company, 1955.

[ch10biblio01entry25] (T3) R. E. Treybal, *Liquid Extraction*, 2nd ed. New York: McGraw-Hill Book Company, 1963.

[ch10biblio01entry26] (W1) R. P., Whitney, and J. E. Vivian, *Chem. Eng. Progr.*, **45**, 323 (1949).