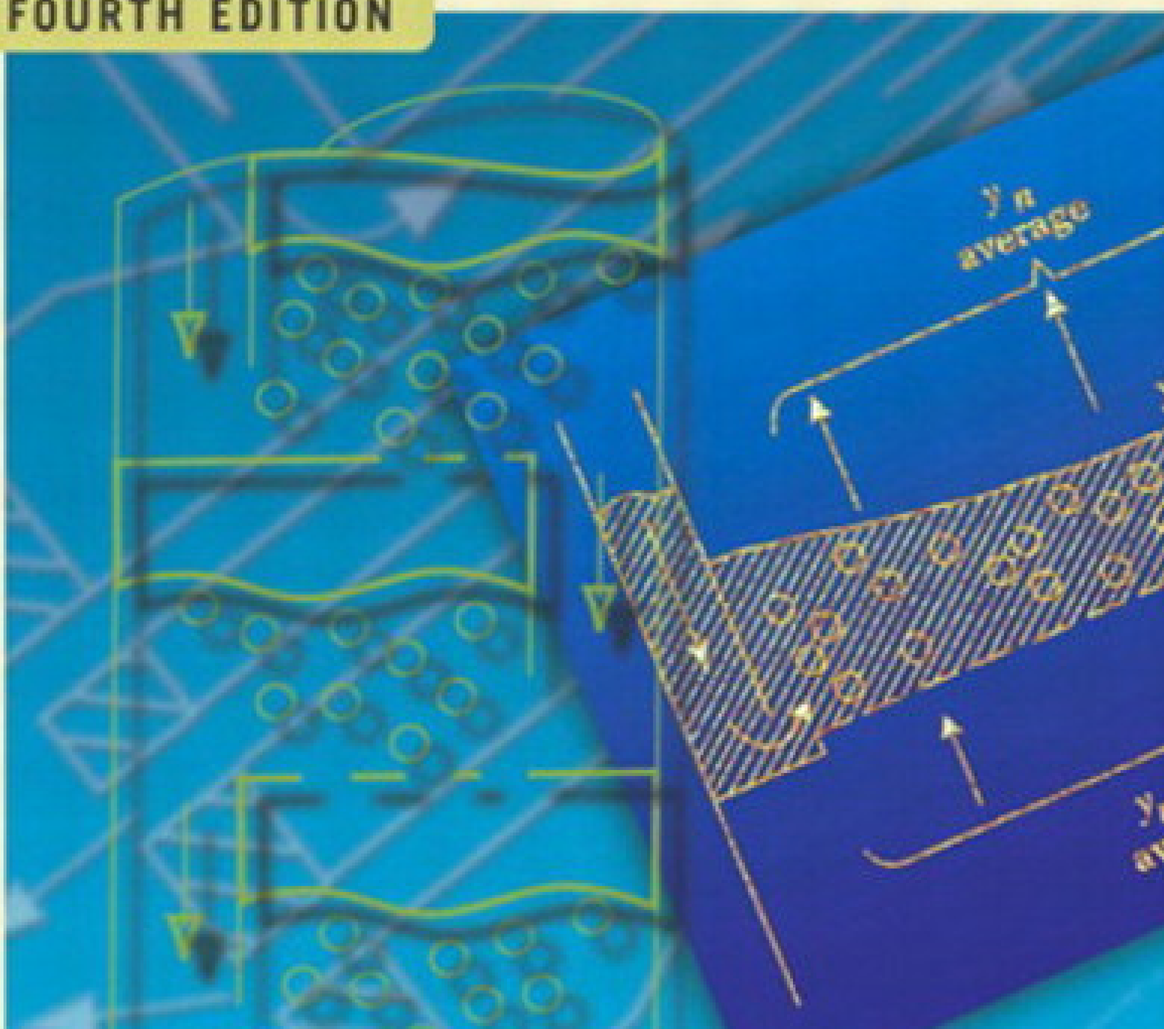
# Transport Processes and Separation Process Principles

(INCLUDES UNIT OPERATIONS)

FOURTH EDITION




CHRISTIE JOHN GEANKOPLIS

# Table of Contents

# Chapter 14. Mechanical–Physical Separation Processes

## INTRODUCTION AND CLASSIFICATION OF MECHANICAL–PHYSICAL SEPARATION PROCESSES

### Introduction

In Chapters 10 and 11, gas–liquid and vapor–liquid separation processes were considered. These separation processes depended on molecules diffusing or vaporizing from one distinct phase to another phase. In Chapter 12, liquid–liquid separation processes were discussed, in which the two liquid phases are quite different chemically, leading to separation on a molecular scale according to physical–chemical properties. Also in Chapter 12, we considered liquid–solid leaching, ion-exchange, and adsorption separation processes. Again, differences in the physical–chemical properties of the molecules lead to separation on a molecular scale. In Chapter 13, we discussed membrane separation processes, where the separation also depends on physical–chemical properties.

All the separation processes considered so far have been based upon physical–chemical differences in the molecules themselves and on mass transfer of the molecules. In this way individual molecules are separated into two phases because of these molecular differences. In the present chapter, a group of separation processes will be considered where the separation is not accomplished on a molecular scale nor is it due to the differences among the various molecules. The separation will be accomplished using mechanical–physical forces and not molecular or chemical forces and diffusion. These mechanical–physical forces will be acting on particles, liquids, or mixtures of particles and liquids themselves and not necessarily on the individual molecules.

These mechanical–physical forces include gravitational and centrifugal, actual mechanical, and kinetic forces arising from flow. Particles and/or fluid streams are separated because of the different effects produced on them by these forces.

### Classification of Mechanical–Physical Separation Processes

These mechanical–physical separation processes are considered under the following classifications:

#### Filtration

The general problem of the separation of solid particles from liquids can be solved by using a wide variety of methods, depending on the type of solids, the proportion of solid to liquid in the mixture, viscosity of the solution, and other factors. In filtration, a pressure difference is set up that causes the fluid to flow through small holes in a screen or cloth which block the passage of the large solid particles; these, in turn, build up on the cloth as a porous cake.

**Settling and sedimentation**

In settling and sedimentation, the particles are separated from the fluid by gravitational forces acting on particles of various sizes and densities.

**Centrifugal settling and sedimentation**

In centrifugal separations, the particles are separated from the fluid by centrifugal forces acting on particles of various sizes and densities. Two general types of separation processes are used. In the first type of process, centrifugal settling or sedimentation occurs.

**Centrifugal filtration**

In the second type of centrifugal separation process, centrifugal filtration occurs, which is similar to ordinary filtration, where a bed or cake of solids builds up on a screen, but centrifugal force is used to cause the flow instead of a pressure difference.

**Mechanical size reduction and separation**

In mechanical size reduction, the solid particles are broken mechanically into smaller particles and separated according to size.



# FILTRATION IN SOLID–LIQUID SEPARATION

## Introduction

In filtration, suspended solid particles in a fluid of liquid or gas are physically or mechanically removed by using a porous medium that retains the particles as a separate phase or cake and passes the clear filtrate. Commercial filtrations cover a very wide range of applications. The fluid can be a gas or a liquid. The suspended solid particles can be very fine (in the micrometer range) or much larger, very rigid or plastic particles, spherical or very irregular in shape, aggregates of particles or individual particles. The valuable product may be the clear filtrate from the filtration or the solid cake. In some cases, complete removal of the solid particles is required, in other cases only partial removal.

The feed or slurry solution may carry a heavy load of solid particles or a very small amount. When the concentration is very low, the filters can operate for very long periods of time before the filter needs cleaning. Because of the wide diversity of filtration problems, a multitude of types of filters have been developed.

Industrial filtration equipment differs from laboratory filtration equipment only in the amount of material handled and in the necessity for low-cost operation. A typical laboratory filtration apparatus is shown in Fig. 14.2-1, which is similar to a Büchner funnel. The liquid is made to flow through the filter cloth or paper by a vacuum on the exit end. The slurry consists of the liquid and the suspended particles. The passage of the particles is blocked by the small openings in the pores of the filter cloth. A support with relatively large holes is used to hold the filter cloth. The solid particles build up in the form of a porous filter cake as the filtration proceeds. This cake itself also acts as a filter for the suspended particles. As the cake builds up, resistance to flow also increases.

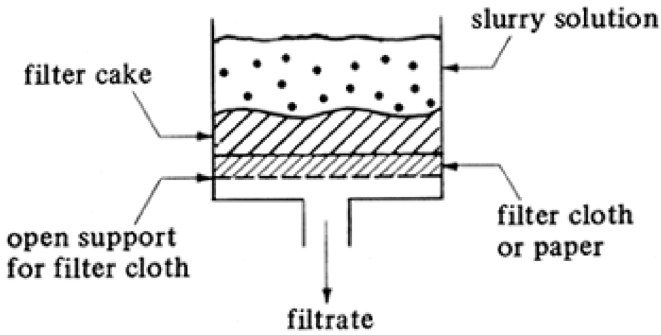



*Figure 14.2-1. Simple laboratory filtration apparatus.*

In section 14.2 the ordinary type of filtration will be considered, where a pressure difference is used to force the liquid through the filter cloth and the filter cake that builds up.

In Section 14.4E, centrifugal filtration will be discussed, where centrifugal force is used instead of a pressure difference. In many filtration applications, ordinary filters and centrifugal filters are often competitive and either type can be used.

## Types of Filtration Equipment

### Classification of filters

There are a number of ways to classify types of filtration equipment; unfortunately, it is not possible to devise a single classification scheme that includes all types of filters. In one classification, filters are classified according to whether the desired product is the filter cake or the clarified filtrate or outlet liquid. In either case, the slurry can have a relatively large percentage of solids so that a cake is formed, or have just a trace of suspended particles.

Filters can also be classified by operating cycle. Filters can be operated as batch, where the cake is removed after a run, or continuous, where the cake is removed continuously. In another classification, filters can be of the gravity type, where the liquid simply flows by means of a hydrostatic head, or pressure or vacuum can be used to increase the flow rates. An important method of classification depends upon the mechanical arrangement of the filter media. The filter cloth can be in a series arrangement as flat plates in an enclosure, as individual leaves dipped in the slurry, or on rotating-type rolls in the slurry. In the following sections only the most important types of filters will be described. For more details, see references (B1, P1).

### Bed filters

The simplest type of filter is the bed filter shown schematically in Fig. 14.2-2. This type is useful mainly in cases where relatively small amounts of solids are to be removed from large amounts of water in clarifying the liquid. Often the bottom layer is composed of coarse pieces of gravel resting on a perforated or slotted plate. Above the gravel is fine sand, which acts as the actual filter medium. Water is introduced at the top onto a baffle which spreads the water out. The clarified liquid is drawn out at the bottom.

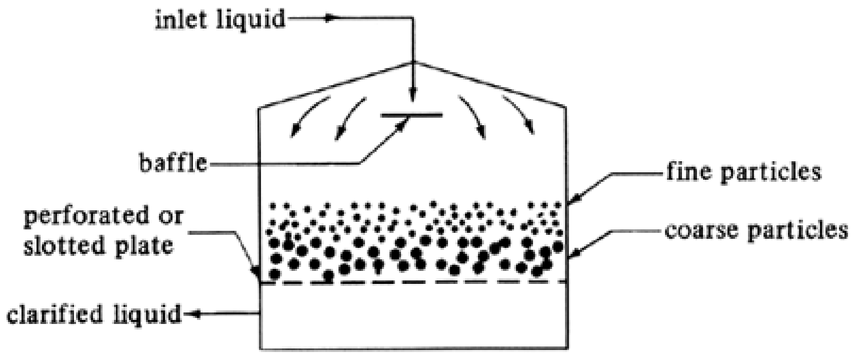



*Figure 14.2-2. Bed filter of solid particles.*

The filtration continues until the precipitate of filtered particles has clogged the sand so that the flow rate drops. Then the flow is stopped, and water is introduced in the reverse direction so that it flows upward, backwashing the bed and carrying the precipitated solid away. This apparatus can only be used on precipitates that do not adhere strongly to the sand and can be easily removed by backwashing. Open-tank filters are used in filtering municipal water supplies.

**Plate-and-frame filter presses**

One of the important types of filters is the plate-and-frame filter press, which is shown diagrammatically in Fig. 14.2-3a. These filters consist of plates and frames assembled alternately with a filter cloth over each side of the plates. The plates have channels cut in them so that clear filtrate liquid can drain down along each plate. The feed slurry is pumped into the press and flows through the duct into each of the open frames so that slurry fills the frames. The filtrate flows through the filter cloth and the solids build up as a cake on the frame side of the cloth. The filtrate flows between the filter cloth and the face of the plate through the channels to the outlet.

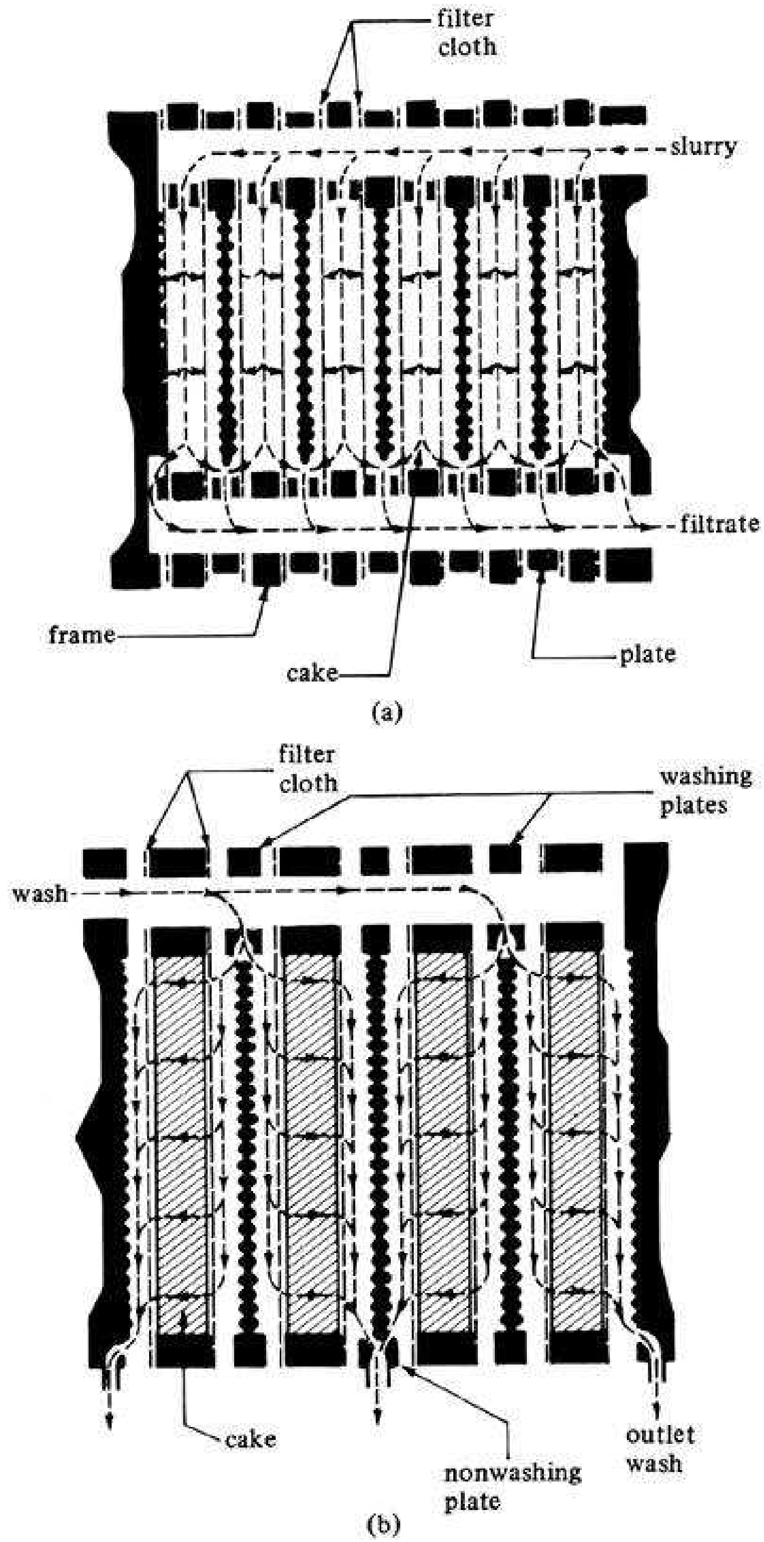



*Figure 14.2-3. Diagrams of plate-and-frame filter presses: (a) filtration of slurry with closed delivery, (b) through washing in a press with open delivery.*

The filtration proceeds until the frames are completely filled with solids. In Fig. 14.2-3a all the discharge outlets go to a common header. In many cases the filter press will have a separate discharge to the open for each frame. Then visual inspection can be made to see if the filtrate is running clear. If one is running cloudy because of a break in the filter cloth or other factors, it can be shut off separately. When the frames are completely full, the frames and plates are separated and the cake removed. Then the filter is reassembled and the cycle is repeated.

If the cake is to be washed, the cake is left in the plates and through-washing is performed, as shown in Fig. 14.2-3b. In this press a separate channel is provided for the wash-water inlet. The wash water enters the inlet, which has ports opening behind the filter cloths at every other plate of the filter press. The wash water then flows through the filter cloth, through the entire cake (not half the cake as in filtration), through the filter cloth at the other side of the frames, and out the discharge channel. It should be noted that there are two kinds of plates in Fig. 14.2-3b: those having ducts to admit wash water behind the filter cloth, alternating with plates without such ducts.

The plate-and-frame presses suffer from the disadvantages common to batch processes. The cost of labor for removing the cakes and reassembling plus the cost of fixed charges for downtime can be an appreciable part of the total operating cost. Some newer types of plate-and-frame presses have duplicate sets of frames mounted on a rotating shaft. Half of theframes are in use while the others are being cleaned, saving downtime and labor costs. Other advances in automation have been applied to these types of filters.

Filter presses are used in batch processes but cannot be employed for high-throughput processes. They are simple to operate, very versatile and flexible in operation, and can be used at high pressures, when necessary, if viscous solutions are being used or the filter cake has a high resistance.

**Leaf filters**

The filter press is useful for many purposes but is not economical for handling large quantities of sludge or for efficient washing with a small amount of wash water. The wash water often channels in the cake and large volumes of wash water may be needed. The leaf filter shown in Fig. 14.2-4 was developed for larger volumes of slurry and more efficient washing. Each leaf is a hollow wire framework covered by a sack of filter cloth.

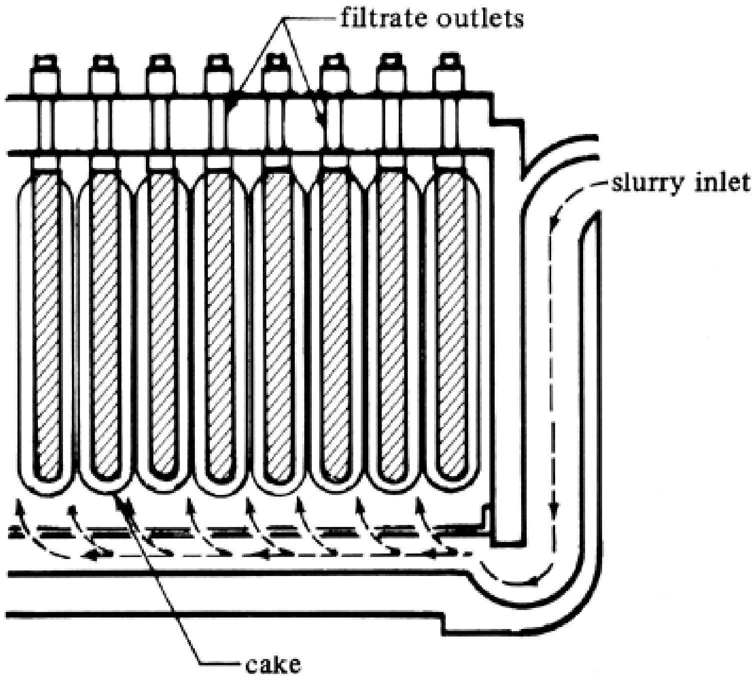



*Figure 14.2-4. Leaf filter.*

A number of these leaves are hung in parallel in a closed tank. The slurry enters the tank and is forced under pressure through the filter cloth, where the cake deposits on the outside of the leaf. The filtrate flows inside the hollow framework and out a header. The wash liquid follows the same path as the slurry. Hence, the washing is more efficient than the through-washing in plate-and-frame filter presses. To remove the cake, the shell is opened. Sometimes air is blown in the reverse direction into the leaves to help in dislodging the cake. If the solids are not wanted, water jets can be used to simply wash away the cakes without opening the filter.

Leaf filters also suffer from the disadvantages of batch operation. They can be automated for the filtering, washing, and cleaning cycle. However, they are still cyclical and are used for batch processes and relatively modest throughput processes.

#### Continuous rotary filters

Plate-and-frame filters suffer from the disadvantages common to all batch processes and cannot be used for large-capacity processes. A number of continuous-type filters are available, as discussed below.

a. ***Continuous rotary vacuum-drum filter.*** This filter, shown in Fig. 14.2-5, filters, washes, and discharges the cake in a continuous, repeating sequence. The drum is covered with a suitable filtering medium. The drum rotates and an automatic valve in the center serves to activate the filtering, drying, washing, and cake-discharge functions in the cycle. The filtrate leaves through the axle of the filter.

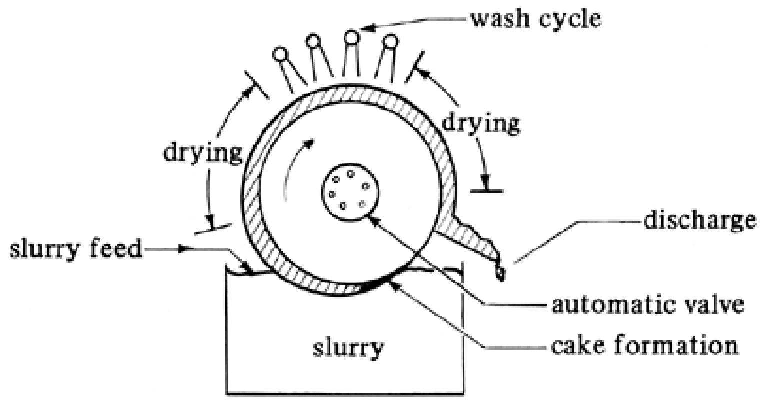



*Figure 14.2-5. Schematic of continuous rotary-drum filter.*

The automatic valve provides separate outlets for the filtrate and the wash liquid. Also, if needed, a connection for compressed-air blowback just before discharge can be used to help in cake removal by the knife scraper. The maximum pressure differential for the vacuum filter is only 1 atm. Hence, this type is not suitable for viscous liquids or for liquids that must be enclosed. If the drum is enclosed in a shell, pressures above atmospheric can be used; however, the cost of a pressure-type filter is about two times that of a vacuum-type rotary-drum filter (P2).

Modern, high-capacity processes use continuous filters. The important advantages are that the filters are continuous and automatic and labor costs are relatively low. However, the capital cost is relatively high.

b. ***Continuous rotary disk filter.*** This filter consists of concentric vertical disks mounted on a horizontal rotating shaft. The filter operates on the same principle as the vacuum rotary-drum filter. Each disk is hollow and covered with a filter cloth and is partly submerged in the slurry. The cake is washed, dried, and scraped off when the disk is in the upper half of its rotation. Washing is less efficient than with the rotating-drum type.

c. ***Continuous rotary horizontal filter.*** This type is a vacuum filter with the rotating annular filtering surface divided into sectors. As the horizontal filter rotates, it successively receives slurry, is washed, is dried, and the cake is scraped off. The washing efficiency is better than with the rotary-disk filter. This filter is widely used in ore-extraction processes, pulp washing, and other large-capacity processes.

## Filter Media and Filter Aids

### Filter media

The filter medium for industrial filtration must fulfill a number of requirements. First and foremost, it must remove the solids to be filtered from the slurry and give a clear filtrate. Also, the pores should not become plugged so that the rate of filtration becomes too slow. The filter medium must allow the filter cake to be removed easily and cleanly. Obviously, it must have sufficient strength not to tear and must be chemically resistant to the solutions used.

Some widely used filter media are twill or duckweave heavy cloth, other types of woven heavy cloth, woolen cloth, glass cloth, paper, felted pads of cellulose, metal cloth, nylon cloth, dacron cloth, and other synthetic cloths. The ragged fibers of natural materials are more effective in removing fine particles than the smooth plastic or metal fibers. Sometimes the filtrate may come through somewhat cloudy at first before the first layers of particles, which help filter the subsequent slurry, are deposited. This filtrate can be recycled for refiltration.

#### Filter aids

Certain filter aids may be used to aid filtration. These are often incompressible diatomaceous earth or kieselguhr, which is composed primarily of silica. Also used are wood cellulose and other inert porous solids.

These filter aids can be used in a number of ways. They can be used as a precoat before the slurry is filtered. This will prevent gelatinous-type solids from plugging the filter medium and also give a clearer filtrate. They can also be added to the slurry before filtration. This increases the porosity of the cake and reduces resistance of the cake during filtration. In a rotary filter, the filter aid may be applied as a precoat; subsequently, thin slices of this layer are sliced off with the cake.

The use of filter aids is usually limited to cases where the cake is discarded or where the precipitate can be separated chemically from the filter aid.

### Basic Theory of Filtration

#### Pressure drop of fluid through filter cake

Figure 14.2-6 is a section through a filter cake and filter medium at a definite time *t* s from the start of the flow of filtrate. At this time the thickness of the cake is *L* m (ft). The filter cross-sectional area is *A* m$^2$ (ft$^2$), and the linear velocity of the filtrate in the *L* direction is *v* m/s (ft/s) based on the filter area *A* m$^2$.

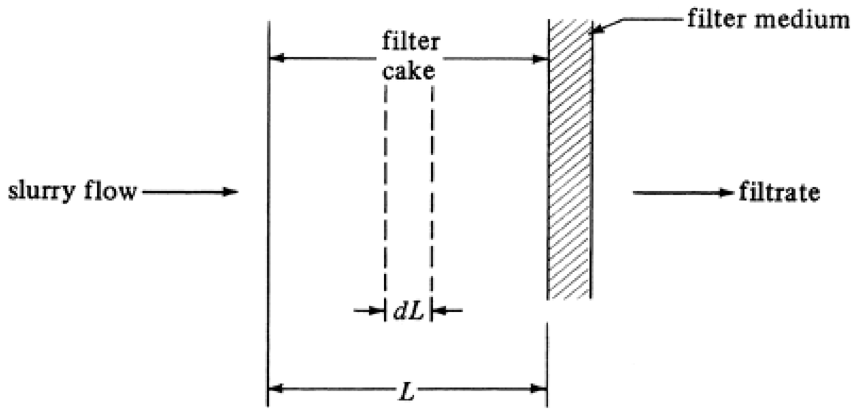



*Figure 14.2-6. Section through a filter cake.*

The flow of the filtrate through the packed bed of cake can be described by an equation similar to Poiseuille's law, assuming laminar flow occurs in the filter channels. Equation (2.10-2) gives Poiseuille's equation for laminar flow in a straight tube, which can be written

*Equation 14.2-1.*

$$-\frac{\Delta p}{L} = \frac{32\mu v}{D^2} \qquad \text{(SI)}$$

$$-\frac{\Delta p}{L} = \frac{32\mu v}{g_c D^2} \qquad \text{(English)}$$

where $\Delta p$ is pressure drop in N/m$^2$ (lb$_f$/ft$^2$), v is open-tube velocity in m/s (ft/s), *D* is diameter in m (ft), *L* is length in m (ft), μ is viscosity in Pa · s or kg/m · s (lb$_m$/ft · s), and $g_c$ is 32.174 lb$_m$ · ft/lb$_f$ · s$^2$.

For laminar flow in a packed bed of particles, the *Carman–Kozeny relation,* which is similar to Eq. (14.2-1) and to the Blake–Kozeny equation (3.1-17), has been shown to apply to filtration:

*Equation 14.2-2.*

$$-\frac{\Delta p_c}{L} = \frac{k_1 \mu v (1-\varepsilon)^2 S_0^2}{\varepsilon^3}$$

where $k_1$ is a constant and equals 4.17 for random particles of definite size and shape, $\mu$ is viscosity of filtrate in Pa · s ($lb_m$/ft · s), $v$ is linear velocity based on filter area in m/s (ft/s), $\varepsilon$ is void fraction or porosity of cake, $L$ is thickness of cake in m (ft), $S_0$ is specific surface area of particle in $m^2$ ($ft^2$) of particle area per $m^3$ ($ft^3$) volume of solid particle, and $\Delta p_c$ is pressure drop in the cake in N/$m^2$ ($lb_f$/$ft^2$). For English units, the right-hand side of Eq. (14.2-2) is divided by $g_c$. The linear velocity is based on the empty cross-sectional area and is

*Equation 14.2-3.*

$$v = \frac{dV/dt}{A}$$

where $A$ is filter area in $m^2$ ($ft^2$) and $V$ is total $m^3$ ($ft^3$) of filtrate collected up to time $t$ s. The cake thickness $L$ may be related to the volume of filtrate $V$ by a material balance. If $c_S$ is kg solids/$m^3$ ($lb_m/ft^3$) of filtrate, a material balance gives

*Equation 14.2-4.*

$$LA(1-\varepsilon)\rho_p = c_S(V + \varepsilon LA)$$

where $\rho_p$ is density of solid particles in the cake in kg/$m^3$ ($lb_m/ft^3$) solid. The final term of Eq. (14.2-4) is the volume of filtrate held in the cake. This is usually small and will beneglected.

Substituting Eq. (14.2-3) into (14.2-2) and using Eq. (14.2-4) to eliminate $L$, we obtain the final equation as

*Equation 14.2-5.*

$$\frac{dV}{A\,dt} = \frac{-\Delta p_c}{\dfrac{k_1(1-\varepsilon)S_0^2}{\rho_p \varepsilon^3}\dfrac{\mu c_S V}{A}} = \frac{-\Delta p_c}{\alpha \dfrac{\mu c_S V}{A}}$$

where α is the specific cake resistance in m/kg(ft/$lb_m$), defined as

*Equation 14.2-6.*

$$\alpha = \frac{k_1(1-\varepsilon)S_0^2}{\rho_p \varepsilon^3}$$

For the filter-medium resistance, we can write, by analogy with Eq. (14.2-5),

*Equation 14.2-7.*

$$\frac{dV}{A\,dt} = \frac{-\Delta p_f}{\mu R_m}$$

where $R_m$ is the resistance of the filter medium to filtrate flow in $m^{-1}$ ($ft^{-1}$) and $\Delta p_f$ is the pressure drop. When $R_m$ is treated as an empirical constant, it includes the resistance to flow of the piping leads to and from the filter and the filter medium resistance.

Since the resistances of the cake and the filter medium are in series, Eqs. (14.2-5) and (14.2-7) can be combined, and become

*Equation 14.2-8.*

$$\frac{dV}{A\ dt} = \frac{-\Delta p}{\mu\left(\frac{\alpha c_S V}{A} + R_m\right)}$$

where $\Delta p = \Delta p_c + \Delta p_f$. Sometimes Eq. (14.2-8) is modified as follows:

*Equation 14.2-9.*

$$\frac{dV}{A\ dt} = \frac{-\Delta p}{\frac{\mu \alpha c_S}{A}(V + V_e)}$$

where $V_e$ is a volume of filtrate necessary to build up a fictitious filter cake whose resistance is equal to $R_m$.

The volume of filtrate $V$ can also be related to $W$, the kg of accumulated dry cake solids, as follows:

*Equation 14.2-10.*

$$W = c_S V = \frac{\rho c_x}{1 - m c_x} V$$

where $c_x$ is mass fraction of solids in the slurry, $m$ is mass ratio of wet cake to dry cake, and $\rho$ is density of filtrate in kg/m$^3$ (lb$_m$/ft$^3$).

**Specific cake resistance**

From Eq. (14.2-6) we see that the specific cake resistance is a function of void fraction $\varepsilon$ and $S_0$. It is also a function of pressure, since pressure can affect $\varepsilon$. By conducting constant-pressure experiments at various pressure drops, the variation of $\alpha$ with $-\Delta p$ can be determined.

Alternatively, compression–permeability experiments can be performed. A filter cake at a low pressure drop and atm pressure is built up by gravity filtering in a cylinder with a porous bottom. A piston is loaded on top and the cake compressed to a given pressure. Then filtrate is fed to the cake and $\alpha$ is determined by a differential form of Eq. (14.2-9). This is then repeated for other compression pressures (G1).

If $\alpha$ is independent of $-\Delta p$, the sludge is incompressible. Usually, $\alpha$ increases with $-\Delta p$, since most cakes are somewhat compressible. An empirical equation often used is

*Equation 14.2-11.*

$$\alpha = \alpha_0(-\Delta p)^s$$

where $\alpha_0$ and $s$ are empirical constants. The compressibility constant $s$ is zero for incompressible sludges or cakes. The constant $s$ usually falls between 0.1 to 0.8. Sometimes the following is used:

*Equation 14.2-12.*

$$\alpha = \alpha_0'[1 + \beta(-\Delta p)^{s'}]$$

where $\alpha_0'$, $\beta$, and $s'$ are empirical constants. Experimental data for various sludges are given by Grace (G1).

The data obtained from filtration experiments often do not have a high degree of reproducibility. The state of agglomeration of the particles in the slurry can vary and have an effect on the specific cake resistance.

## Filtration Equations for Constant-Pressure Filtration

### Basic equations for filtration rate in batch process

Often a filtration is done under conditions of constant pressure. Equation (14.2-8) can be inverted and rearranged to give

*Equation 14.2-13.*

$$\frac{dt}{dV} = \frac{\mu \alpha c_S}{A^2(-\Delta p)}V + \frac{\mu}{A(-\Delta p)}R_m = K_p V + B$$

where $K_p$ is in s/m$^6$ (s/ft$^6$) and $B$ in s/m$^3$ (s/ft$^3$):

*Equation 14.2-14.*

$$K_p = \frac{\mu \alpha c_S}{A^2(-\Delta p)} \qquad \text{(SI)}$$

$$K_p = \frac{\mu \alpha c_S}{A^2(-\Delta p)g_c} \qquad \text{(English)}$$

*Equation 14.2-15.*

$$B = \frac{\mu R_m}{A(-\Delta p)} \qquad \text{(SI)}$$

$$B = \frac{\mu R_m}{A(-\Delta p)g_c} \qquad \text{(English)}$$

For constant pressure, constant $\alpha$, and incompressible cake, $V$ and $t$ are the only variables in Eq. (14.2-13). Integrating to obtain the time of filtration in $t$ s,

*Equation 14.2-16.*

$$\int_0^t dt = \int_0^V (K_p V + B)\,dV$$

*Equation 14.2-17.*

$$t = \frac{K_p}{2}V^2 + BV$$

Dividing by $V$

*Equation 14.2-18.*

$$\frac{t}{V} = \frac{K_p V}{2} + B$$

where $V$ is total volume of filtrate in m$^3$ (ft$^3$) collected to $t$ s.

To evaluate Eq. (14.2-17) it is necessary to know $\alpha$ and $R_m$. This can be done by using Eq. (14.2-18). Data for $V$ collected at different times $t$ are obtained. Then the experimental data are plotted as $t/V$ versus $V$, as in Fig. 14.2-7 Often, the first point on the graph does not fall on the line and is omitted. The slope of the line is $K_p/2$ and the intercept $B$. Then, using Eqs. (14.2-14) and (14.2-15), values of $\alpha$ and $R_m$ can be determined.

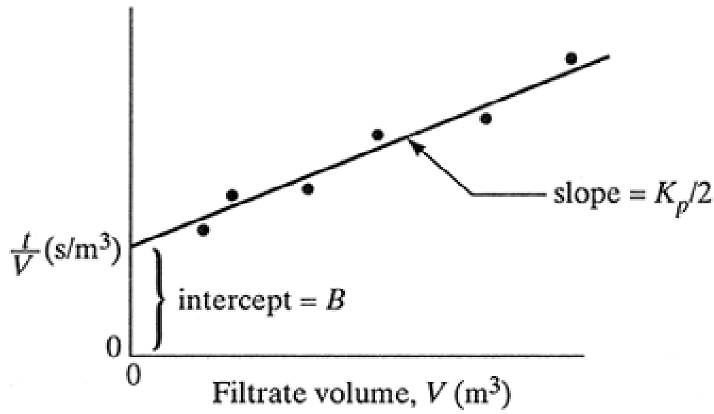



*Figure 14.2-7. Determination of constants in a constant-pressure filtration run.*

## EXAMPLE 14.2-1. Evaluation of Filtration Constants for Constant-Pressure Filtration

Data for the laboratory filtration of $CaCO_3$ slurry in water at 298.2 K (25°C are reported as follows at a constant pressure ($-\Delta p$) of 338 kN/m$^2$ (7060 lb$_f$/ft$^2$) (R1, R2, M1). The filter area of the plate-and-frame press was $A$ = 0.0439 m$^2$ (0.473 ft$^2$) and the slurry concentration was $c_s$ = 23.47 kg/m$^3$ (1.465 lb$_m$/ft$^3$). Calculate the constants $\alpha$ and $R_m$ from the experimental data given, where $t$ is time in s and $V$ is filtrate volume collected in m$^3$.

| $t$ | $V$ | $t$ | $V$ | $t$ | $V$ |
|---|---|---|---|---|---|
| 4.4 | $0.498 \times 10^{-3}$ | 34.7 | $2.498 \times 10^{-3}$ | 73.6 | $4.004 \times 10^{-3}$ |
| 9.5 | $1.000 \times 10^{-3}$ | 46.1 | $3.002 \times 10^{-3}$ | 89.4 | $4.502 \times 10^{-3}$ |
| 16.3 | $1.501 \times 10^{-3}$ | 59.0 | $3.506 \times 10^{-3}$ | 107.3 | $5.009 \times 10^{-3}$ |
| 24.6 | $2.000 \times 10^{-3}$ | | | | |

***Solution.*** First, the data are calculated as $t/V$ and tabulated in Table 14.2-1. The data are plotted as $t/V$ versus $V$ in Fig. 14.2-8 and the intercept is determined as $B$ = 6400 s/m$^3$ (181 s/ft$^3$) and the slope as $K_p/2$ = 3.00 × 10$^6$ s/m$^6$. Hence, $K_p$ = 6.00 × 10$^6$ s/m$^6$ (4820 s/ft$^6$).

*Table 14.2-1. Values of t/V for Example 14.2-1 (t =s, V = m$^3$)*

| $t$ | $V \times \sigma 10^3$ | $(t/V) \times 10^{-3}$ |
|---|---|---|
| **0** | **0** | |
| **4.4** | **0.498** | **8.84** |
| **9.5** | **1.000** | **9.50** |
| **16.3** | **1.501** | **10.86** |
| **24.6** | **2.000** | **12.30** |
| **34.7** | **2.498** | **13.89** |
| **46.1** | **3.002** | **15.36** |
| **59.0** | **3.506** | **16.83** |
| **73.6** | **4.004** | **18.38** |
| **89.4** | **4.502** | **19.86** |
| **107.3** | **5.009** | **21.42** |

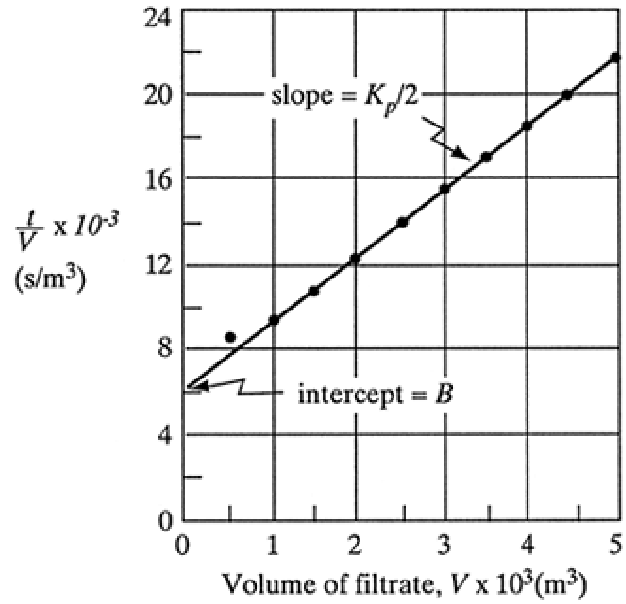



*Figure 14.2-8. Determination of constants for Example 14.2-1.*

At 298.2 K the viscosity of water is $8.937 \times 10^{-4}$ Pa ∊· s = $8.937 \times 10^{-4}$ kg/ms. Substituting known values into Eq. (14.2-14) and solving,

$$K_p = 6.00 \times 10^6 = \frac{\mu \alpha c_S}{A^2(-\Delta p)} = \frac{(8.937 \times 10^{-4})(\alpha)(23.47)}{(0.0439)^2(338 \times 10^3)}$$

$$\alpha = 1.863 \times 10^{11}\,\text{m/kg}\,(2.77 \times 10^{11}\,\text{ft/lb}_\text{m})$$

Substituting into Eq. (14.2-15) and solving,

$$B = 6400 = \frac{\mu R_m}{A(-\Delta p)} = \frac{(8.937 \times 10^{-4})(R_m)}{0.0439(338 \times 10^3)}$$

$$R_m = 10.63 \times 10^{10}\,\text{m}^{-1}\,(3.24 \times 10^{10}\,\text{ft}^{-1})$$

## EXAMPLE 14.2-2. Time Required to Perform a Filtration

The same slurry used in Example 14.2-1 is to be filtered in a plate-and-frame press having 20 frames and 0.873 m² (9.4 ft²) area per frame. The same pressure will be used in constant-pressure filtration. Assuming the same filter-cake properties and filter cloth, calculate the time to recover 3.37 m³ (119 ft³) filtrate.

***Solution:*** In Example 14.2-1, the area $A$ = 0.0439 m², $K_p$ = $6.00 \times 10^6$ s/m⁶, and $B$ = 6400 s/m³. Since the α and $R_m$ will be the same as before, $K_p$ can be corrected. From Eq. (14.2-14), $K_p$ is proportional to $1/A^2$. The new area is $A$ = 0.873(20) = 17.46 m² (188 ft²). The new $K_p$ is

$$K_p = 6.00 \times 10^6(0.0439/17.46)^2 = 37.93\,\text{s/m}^6\,(0.03042\,\text{s/ft}^6)$$

The new $B$ is proportional to $1/A$ from Eq. (14.2-15):

$$B = (6400)\frac{0.0439}{17.46} = 16.10\,\text{s/m}^3\,(0.456\,\text{s/ft}^3)$$

Substituting into Eq. (14.2-17),

$$t = \frac{K_p}{2}V^2 + BV = \frac{37.93}{2}(3.37)^2 + (16.10)(3.37) = 269.7 \text{ s}$$

Using English units,

$$t = \frac{K_p}{2}V^2 + BV = \frac{0.03042}{2}(119)^2 + (0.456)(119) = 269.7 \text{ s}$$

**Equations for washing of filter cakes and total cycle time**

The washing of a cake after the filtration cycle has been completed takes place by displacement of the filtrate and by diffusion. The amount of wash liquid should be sufficient to give the desired washing effect. To calculate washing rates, it is assumed that the conditions during washing are the same as those that existed at the end of the filtration. It is assumed that the cake structure is not affected when wash liquid replaces the slurry liquid in the cake.

In filters where the wash liquid follows a flow path similar to that during filtration, as in leaf filters, the final filtering rate gives the predicted washing rate. For constant-pressure filtration, using the same pressure in washing as in filtering, the final filtering rate is the reciprocal of Eq. (14.2-13):

*Equation 14.2-19.*

$$\left(\frac{dV}{dt}\right)_f = \frac{1}{K_p V_f + B}$$

where $(dV/dt)_f$ = rate of washing in $m^3/s$ ($ft^3/s$) and $V_f$ is the total volume of filtrate for the entire period at the end of filtration in $m^3$ ($ft^3$).

For plate-and-frame filter presses, the wash liquid travels through a cake twice as thick and an area only half as large as in filtering, so the predicted washing rate is one-fourth of the final filtration rate:

*Equation 14.2-20.*

$$\left(\frac{dV}{dt}\right)_f = \frac{1}{4}\frac{1}{K_p V_f + B}$$

In actual experience the washing rate may be less than predicted because of cake consolidation, channeling, and formation of cracks. Washing rates in a small plate-and-frame filter were found to be from 70 to 92% of that predicted (M1).

After washing is completed, additional time is needed to remove the cake, clean the filter, and reassemble the filter. The total filter-cycle time is the sum of the filtration time, plus the washing time, plus the cleaning time.

## EXAMPLE 14.2-3. Rate of Washing and Total Filter-Cycle Time

At the end of the filtration cycle in Example 14.2-2, a total filtrate volume of 3.37 $m^3$ is collected in a total time of 269.7 s. The cake is to be washed by through-washing in the plate-and-frame press using a volume of wash water equal to 10% of the filtrate volume. Calculate the time of washing and the total filter-cycle time if cleaning the filter takes 20 min.

***Solution*.** For this filter, Eq. (14.2-20) holds. Substituting $K_p$ = 37.93 $s/m^6$, $B$ = 16.10 $s/m^3$, and $V_f$ = 3.37 $m^3$, the washing rate is as follows:

$$\left(\frac{dV}{dt}\right)_f = \frac{1}{4}\frac{1}{(37.93)(3.37) + 16.10} = 1.737 \times 10^{-3} \text{ m}^3/\text{s } (0.0613 \text{ ft}^3/\text{s})$$

The time of washing is then as follows for 0.10(3.37), or 0.337 $m^3$ of wash water:

$$t = \frac{0.337\ \text{m}^3}{1.737 \times 10^{-3}\ \text{m}^3/\text{s}} = 194.0\ \text{s}$$

The total filtration cycle is

$$\frac{269.7}{60} + \frac{194.0}{60} + 20 = 27.73\ \text{min}$$

**Equations for continuous filtration**

In a filter that is continuous in operation, such as a rotary-drum vacuum type, the feed, filtrate, and cake move at steady, continuous rates. In a rotary drum the pressure drop is held constant for the filtration. The cake formation involves a continual change in conditions. In continuous filtration, the resistance of the filter medium is generally negligible compared with the cake resistance. So in Eq. (14.2-13), $B = 0$.

Integrating Eq. (14.2-13), with $B = 0$,

*Equation 14.2-21.*

$$\int_0^t dt = K_p \int_0^V V\,dV$$

*Equation 14.2-22.*

$$t = K_p \frac{V^2}{2}$$

where $t$ is the time required for formation of the cake. In a rotary-drum filter, the filter time $t$ is less than the total cycle time $t_c$ by

*Equation 14.2-23.*

$$t = f t_c$$

where $f$ is the fraction of the cycle used for cake formation. In the rotary drum, $f$ is the fraction submergence of the drum surface in the slurry.

Next, substituting Eq. (14.2-14) and Eq. (14.2-23) into (14.2-22) and rearranging,

*Equation 14.2-24.*

$$\text{flow rate} = \frac{V}{A t_c} = \left[\frac{2f(-\Delta p)}{t_c \mu \alpha c_S}\right]^{1/2}$$

If the specific cake resistance varies with pressure, the constants in Eq. (14.2-11) are needed to predict the value of $\alpha$ to be used in Eq. (14.2-24). Experimental verification of Eq. (14.2-24) shows that the flow rate varies inversely with the square root of the viscosity and the cycle time (N1).

When short cycle times are used in continuous filtration and/or the filter medium resistance is relatively large, the filter resistance term $B$ must be included, and Eq. (14.2-13) becomes

*Equation 14.2-25.*

$$t = f t_c = K_p \frac{V^2}{2} + BV$$

Then Eq. (14.2-25) becomes

*Equation 14.2-26.*

$$\text{flow rate} = \frac{V}{At_c} = \frac{-R_m/t_c + [R_m^2/t_c^2 + 2c_S\alpha(-\Delta p)f/(\mu t_c)]^{1/2}}{\alpha c_S}$$

## EXAMPLE 14.2-4. Filtration in a Continuous Rotary-Drum Filter

A rotary-vacuum-drum filter having a 33% submergence of the drum in the slurry is to be used to filter a $CaCO_3$ slurry as given in Example 14.2-1 using a pressure drop of 67.0 kPa. The solids concentration in the slurry is $c_x$ = 0.191 kg solid/kg slurry and the filter cake is such that the kg wet cake/kg dry cake = $m$ = 2.0. The density and viscosity of the filtrate can be assumed as those of water at 298.2 K. Calculate the filter area needed to filter 0.778 kg slurry/s. The filter-cycle time is 250 s. The specific cake resistance can be represented by $\alpha = (4.37 \times 10^9)(-\Delta p)^{0.3}$, where $-\Delta p$ is in Pa and $\alpha$ in m/kg.

***Solution***: From Appendix A.2 for water, $\rho = 996.9$ kg/m$^3$, $\mu = 0.8937 \times 10^{-3}$ Pa · s. From Eq. (14.2-10),

$$c_S = \frac{\rho c_x}{1 - mc_x} = \frac{996.9(0.191)}{1-(2.0)(0.191)} = 308.1 \text{ kg solids/m}^3 \text{ filtrate}$$

Solving for $\alpha$, $\alpha = (4.37 \times 10^9)(67.0 \times 10^3)^{0.3} = 1.225 \times 10^{11}$ m/kg. To calculate the flow rate of the filtrate,

$$\begin{aligned}\frac{V}{t_c} &= 0.778(c_x)/(c_s)\\ &= \left(0.778\frac{\text{kg slurry}}{\text{s}}\right)\left(0.191\frac{\text{kg solid}}{\text{kg slurry}}\right)\Big/\left(\frac{1}{308.1 \text{ kg solid/m}^3 \text{ filtrate}}\right)\\ &= 4.823 \times 10^{-4} \text{ m}^3 \text{ filtrate/s}\end{aligned}$$

Substituting into Eq. (14.2-24), neglecting and setting $B = 0$, and solving,

$$\frac{V}{At_c} = \frac{4.823 \times 10^{-4}}{A} = \left[\frac{2(0.33)(67.0 \times 10^3)}{250(0.8937 \times 10^{-3})(1.225 \times 10^{11})(308.1)}\right]^{1/2}$$

Hence, $A = 6.60$ m$^2$.

### Filtration Equations for Constant-Rate Filtration

In some cases filtration runs are made under conditions of constant rate rather than constant pressure. This occurs if the slurry is fed to the filter by a positive-displacement pump. Equation (14.2-8) can be rearranged to give the following for a constant rate ($dV/dt$) m$^3$/s:

*Equation 14.2-27.*

$$-\Delta p = \left(\frac{\mu\alpha c_S}{A^2}\frac{dV}{dt}\right)V + \left(\frac{\mu R_m}{A}\frac{dV}{dt}\right) = K_V V + C$$

where

*Equation 14.2-28.*

$$K_V = \left(\frac{\mu \alpha c_S}{A^2}\frac{dV}{dt}\right) \qquad \text{(SI)}$$

$$K_V = \left(\frac{\mu \alpha c_S}{A^2 g_c}\frac{dV}{dt}\right) \qquad \text{(English)}$$

*Equation 14.2-29.*

$$C = \left(\frac{\mu R_m}{A}\frac{dV}{dt}\right)\text{(SI)}$$

$$C = \left(\frac{\mu R_m}{A g_c}\frac{dV}{dt}\right)\text{(English)}$$

$K_V$ is in N/m$^5$ (lb$_f$/ft$^5$) and $C$ is in N/m$^2$ (lb$_f$/ft$^2$).

Assuming that the cake is incompressible, $K_V$ and $C$ are constants characteristic of the slurry, cake, rate of filtrate flow, and so on. Hence, a plot of pressure, $-\Delta p$, versus the total volume of filtrate collected, $V$, gives a straight line for a constant rate $dV/dt$. The slope of the line is $K_V$ and the intercept is $C$. The pressure increases as the cake thickness increases and the volume of filtrate collected increases.

The equations can also be rearranged in terms of $-\Delta p$ and time $t$ as variables. At any moment during the filtration, the total volume $V$ is related to the rate and total time $t$ as follows:

*Equation 14.2-30.*

$$V = t\,\frac{dV}{dt}$$

Substituting Eq. (14.2-30) into Eq. (14.2-27),

*Equation 14.2-31.*

$$-\Delta p = \left[\frac{\mu \alpha c_S}{A^2}\left(\frac{dV}{dt}\right)^2\right] t + \left(\frac{\mu R_m}{A}\frac{dV}{dt}\right)$$

For the case where the specific cake resistance $\alpha$ is not constant but varies as in Eq. (14.2-11), this can be substituted for $\alpha$ in Eq. (14.2-27) to obtain a final equation.

# SETTLING AND SEDIMENTATION IN PARTICLE–FLUID SEPARATION

## Introduction

In filtration, the solid particles are removed from the slurry by forcing the fluid through a filter medium, which blocks the passage of the solid particles and allows the filtrate to pass through. In settling and sedimentation, the particles are separated from the fluid by gravitational forces acting on the particles.

Applications of settling and sedimentation include removal of solids from liquid sewage wastes, settling of crystals from the mother liquor, separation of liquid–liquid mixture from a solvent-extraction stage in a settler, settling of solid food particles from a liquid food, and settling of a slurry from a soybean leaching process. The particles can be solid particles or liquid drops. The fluid can be a liquid or gas and it may be at rest or in motion.

In some processes of settling and sedimentation, the purpose is to remove the particles from the fluid stream so that the fluid is free of particle contaminants. In other processes, the particles are recovered as the product, as in recovery of the dispersed phase in liquid–liquid extraction. In some cases the particles are suspended in fluids so that the particles can be separated into fractions differing in size or in density.

When a particle is at a sufficient distance from the walls of the container and from other particles so that its fall is not affected by them, the process is called *free settling.* Interference is less than 1% if the ratio of the particle diameter to the container diameter is less than 1:200 or if the particle concentration is less than 0.2 vol % in the solution. When the particles are crowded, they settle at a lower rate and the process is called *hindered settling.* The separation of a dilute slurry or suspension by gravity settling into a clear fluid and a slurry of higher solids content is called *sedimentation.*

## Theory of Particle Movement Through a Fluid

### Derivation of basic equations for rigid spheres

Whenever a particle is moving through a fluid, a number of forces will be acting on the particle. First, a density difference is needed between the particle and the fluid. An external force of gravity is needed to impart motion to the particle. If the densities of the fluid and particle are equal, the buoyant force on the particle will counterbalance the external force and the particle will not move relative to the fluid.

For a rigid particle moving in a fluid, there are three forces acting on the body: gravity acting downward, buoyant force acting upward, and resistance or drag force acting in opposite direction to the particle motion.

We will consider a particle of mass $m$ kg falling at a velocity $v$ m/s relative to the fluid. The density of the solid particle is $\rho_p$ kg/m$^3$ solid and that of the liquid is $\rho$ kg/m$^3$ liquid. The buoyant force $F_b$ in N on the particle is

*Equation 14.3-1.*

$$F_b = \frac{m\rho g}{\rho_p} = V_p \rho g$$

where $m/\rho_p$ is the volume $V_p$ in m$^3$ of the particle and $g$ is the gravitational acceleration in m/s$^2$.

The gravitation or external force $F_g$ in N on the particle is

*Equation 14.3-2.*

$$F_g = mg$$

The drag force $F_D$ on a body in N may be derived from the fact that, as in flow of fluids, the drag force or frictional resistance is proportional to the velocity head $v^2/2$ of the fluid displaced by the moving body. This must be multiplied by the density of the fluid and by a significant area $A$, such as the projected area of the particle. This was defined previously in Eq. (3.1-1):

*Equation 14.3-3.*

$$F_D = C_D \frac{v^2}{2} \rho A$$

where the drag coefficient $C_D$ is the proportionality constant and is dimensionless.

The resultant force on the body is then $F_g - F_b - F_D$. This resultant force must equal the force due to acceleration:

*Equation 14.3-4.*

$$m\frac{dv}{dt} = F_g - F_b - F_D$$

Substituting Eqs. (14.3-1)–(14.3-3) into (14.3-4),

*Equation 14.3-5.*

$$m\frac{dv}{dt} = mg - \frac{m\rho g}{\rho_p} - \frac{C_D v^2 \rho A}{2}$$

If we start from the moment the body is released from its position of rest, the falling of the body consists of two periods: the period of accelerated fall and the period of constant-velocity fall. The initial acceleration period is usually very short, on the order of a tenth of a second or so. Hence, the period of constant-velocity fall is the important one. The velocity is called the *free settling velocity* or *terminal velocity* $v_t$.

To solve for the terminal velocity in Eq. (14.3-5), $dv/dt = 0$ and the equation becomes

*Equation 14.3-6.*

$$v_t = \sqrt{\frac{2g(\rho_p - \rho)m}{A\rho_p C_D \rho}}$$

For spherical particles $m = \pi D_p^3 \rho_p/6$ and $A = \pi D_p^2/4$. Substituting these into Eq. (14.3-6), we obtain, for spherical particles,

*Equation 14.3-7.*

$$v_t = \sqrt{\frac{4(\rho_p - \rho)gD_p}{3C_D\rho}}$$

where $v_t$ is m/s (ft/s), $\rho$ is kg/m$^3$ (lb$_m$/ft$^3$), $g$ is 9.80665 m/s$^2$ (32.174 ft/s$^2$), and $D_p$ is m (ft).

**Drag coefficient for rigid spheres**

The drag coefficient for rigid spheres has been shown to be a function of the Reynolds number $D_p v\rho/\mu$ of the sphere and is shown in Fig. 14.3-1. In the laminar-ow region, called the Stokes' law region for $N_{Re} < 1$, as discussed in Section 3.1B, the drag coefficient is

*Equation 14.3-8.*

$$C_D = \frac{24}{D_p v\rho/\mu} = \frac{24}{N_{Re}}$$

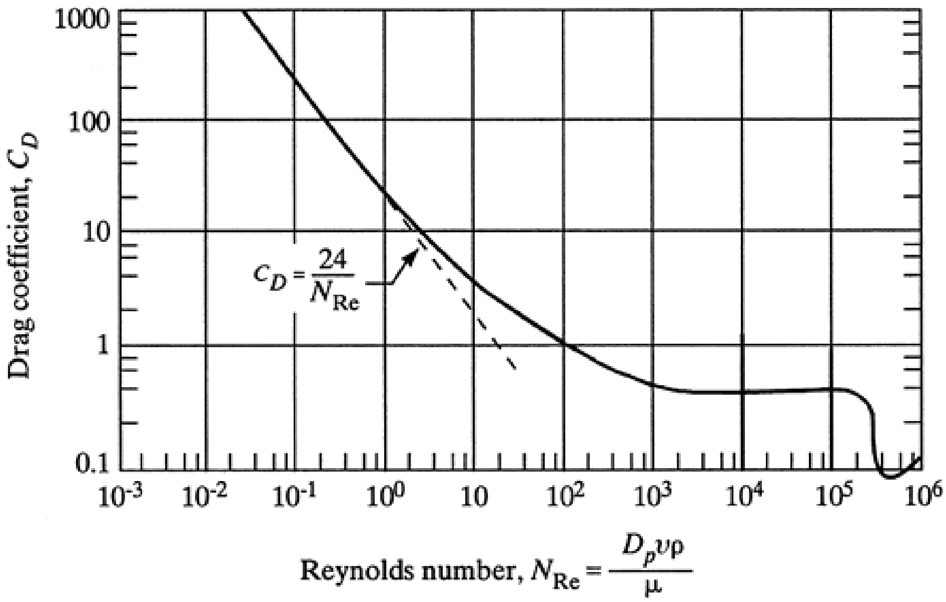



*Figure 14.3-1. Drag coefficient for a rigid sphere.*

where μ is the viscosity of the liquid in Pa · s or kg/m · s ($lb_m$/ft s). Substituting this into Eq. (14.3-7) for laminar flow,

*Equation 14.3-9.*

$$v_t = \frac{g D_p^2 (\rho_p - \rho)}{18\mu}$$

For other shapes of particles, drag coefficients will differ from those given in Fig. 14.3-1, and data are given in Fig. 3.1-2 and elsewhere (B2, L2, P1). In the turbulent Newton's law region above a Reynolds number of about 1000 to 2.0 × $10^5$, the drag coefficient is approximately constant at $C_D$ = 0.44.

Solution of Eq. (14.3-7) is by trial and error when the particle diameter is known and the terminal velocity is to be obtained, because $C_D$ also depends upon the velocity $v_t$.

If the particles are quite small, Brownian motion is present. *Brownian motion* is the random motion imparted to the particle by collisions between the molecules of the fluid surrounding the particle and the particle. This movement of the particles in random directions tends to suppress the effect of gravity, so settling of the particles may occur more slowly or not at all. At particle sizes of a few micrometers, the Brownian effect becomes appreciable, and at sizes of less than 0.1 μm, the effect predominates. For very small particles, application of centrifugal force helps reduce the effect of Brownian motion.

## EXAMPLE 14.3-1. Settling Velocity of Oil Droplets

Oil droplets having a diameter of 20 μm (0.020 mm) are to be settled from air at temperature of 37.8°C (311 K) and 101.3 kPa pressure. The density of the oil is 900 kg/$m^3$. Calculate the terminal settling velocity of the droplets.

***Solution:*** The various knowns are $D_p = 2.0 \times 10^{-5}$ m and $\rho_p$ = 900 kg/$m^3$. From Appendix A.3, for air at 37.8°C, $\rho$ = 1.137 kg/$m^3$, μ = 1.90 × $10^{-5}$ Pa · s. The droplet will be assumed to be a rigid sphere.

The solution is trial and error since the velocity is unknown. Hence, $C_D$ cannot be directly determined. The Reynolds number is as follows:

*Equation 14.3-10.*

$$N_{\mathrm{Re}} = \frac{D_p v_t \rho}{\mu} = \frac{(2.0 \times 10^{-5})(v_t)(1.137)}{1.90 \times 10^{-5}} = 1.197 v_t$$

For the first trial, assume that $v_t$ = 0.305 m/s. Then $N_{\mathrm{Re}}$ = 1.197(0.305) = 0.365. Substituting into Eq. (14.3-7) and solving for $C_D$,

*Equation 14.3-11.*

$$v_t = \sqrt{\frac{4(\rho_p - \rho) g D_p}{3 C_D \rho}} = \sqrt{\frac{4(900 - 1.137)(9.8066)(2.0 \times 10^{-5})}{(3) C_D (1.137)}}$$

$$C_D = \frac{0.2067}{v_t^2}$$

Using $v_t$ = 0.305 m/s, $C_D$ = 0.2067/(0.305)$^2$ = 2.22.

Assuming that $v_t$ = 0.0305 m/s, $N_{\mathrm{Re}}$ = 0.0365 from Eq. (14.3-10) and $C_D$ = 222 from Eq. (14.3-11). For the third trial, assuming that $v_t$ = 0.00305 m/s, $N_{\mathrm{Re}}$ = 0.00365 and $C_D$ = 22 200. These three values calculated for $N_{\mathrm{Re}}$ and $C_D$ are plotted on a graph similar to Fig. 14.3-1 and shown in Fig. 14.3-2. It can be shown that the line through these points is a straight line. The intersection of this line and the drag-coefficient correlation line is the solution to the problem at $N_{\mathrm{Re}}$ = 0.012. The velocity can be calculated from the Reynolds number in Eq. (14.3-10):

$$N_{\mathrm{Re}} = 0.012 = 1.197 v_t$$

$$v_t = 0.0100 \text{ m/s } (0.0328 \text{ ft/s})$$

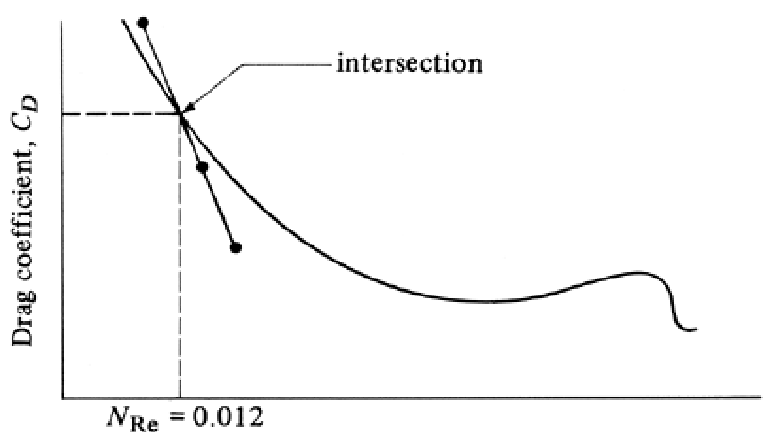



*Figure 14.3-2. Solution of Example 14.3-1 for settling velocity of a particle.*

The particle is in the Reynolds number range less than 1, which is the laminar Stokes' law region. Alternatively, the velocity can be calculated by substituting into Eq. (14.3-9):

$$v_t = \frac{9.8066(2.0 \times 10^{-5})^2(900 - 1.137)}{18(1.90 \times 10^{-5})} = 0.0103 \text{ m/s}$$

Note that Eq. (14.3-9) could not be used until it was determined that the particle fall was in the laminar region.

For particles that are rigid but nonspherical, the drag depends upon the shape of the particle and the orientation of the particle with respect to its motion. Correlations of drag coefficients for particles of different shapes are given in a number of references (B2, C1, P1).

#### Drag coefficients for nonrigid spheres

When particles are nonrigid, internal circulation inside the particle and particle deformation can occur. Both of these effects affect the drag coefficient and terminal velocity. Drag coefficients for air bubbles rising in water are given in Perry and Green (P1), and for a Reynolds number less than about 50, the curve is the same as for rigid spheres in water.

For liquid drops in gases, the same drag relationship as for solid spherical particles is obtained up to a Reynolds number of about 100 (H1). Large drops will deform with an increase in drag. Small liquid drops in immiscible liquids behave like rigid spheres and the drag coefficient curve follows that for rigid spheres up to a Reynolds number of about 10. Above this and up to a Reynolds number of 500, the terminal velocity is greater than that for solids because of internal circulation in the drop.

### Hindered Settling

For many cases of settling, a large number of particles are present, and the surrounding particles interfere with the motion of individual particles. The velocity gradients surrounding each particle are affected by the close presence of other particles. The particles settling in the liquid displace the liquid, and an appreciable upward velocity of the liquid is generated. Hence, the velocity of the liquid is appreciably greater with respect to the particle than with respect to the apparatus itself.

For such hindered flow, the settling velocity is less than would be calculated from Eq. (14.3-9) for Stokes' law. The true drag force is greater in the suspension because of the interference of the other particles. This higher effective viscosity of the mixture $\mu_m$ is equal to the actual viscosity of the liquid itself, $\mu$, divided by an empirical correction factor, $\psi_p$, which depends upon $\varepsilon$, the volume fraction of the slurry mixture occupied by the liquid (S1):

*Equation 14.3-12.*

$$\mu_m = \frac{\mu}{\psi_p}$$

where $\psi_p$ is dimensionless and is as follows (S1):

*Equation 14.3-13.*

$$\psi_p = \frac{1}{10^{1.82(1-\varepsilon)}}$$

The density of the fluid phase effectively becomes the bulk density of the slurry $\rho_m$, which is as follows:

*Equation 14.3-14.*

$$\rho_m = \varepsilon\rho + (1 - \varepsilon)\rho_p$$

where $\rho_m$ is density of slurry in kg solid + liquid/m$^3$. The density difference is now

*Equation 14.3-15.*

$$\rho_p - \rho_m = \rho_p - [\varepsilon\rho + (1 - \varepsilon)\rho_p] = \varepsilon(\rho_p - \rho)$$

The settling velocity, $v_t$ with respect to the apparatus is ε times the velocity calculated by Stokes' law.

Substituting mixture properties of $\mu_m$ from Eq. (14.3-12) for $\mu$ in Eq. (14.3-9), ($\rho_p - \rho_m$) from Eq. (14.3-15) for ($\rho_p - \rho$), and multiplying the result by ε for the relative-velocity effect, Eq. (14.3-9) becomes, for laminar settling,

*Equation 14.3-16.*

$$v_t = \frac{gD_p^2(\rho_p - \rho)}{18\mu}(\varepsilon^2\psi_p)$$

This is the velocity calculated from Eq. (14.3-9), multiplied by the correction factor ($\varepsilon^2\psi_p$).

The Reynolds number is then based on the velocity relative to the fluid and is

*Equation 14.3-17.*

$$N_{Re} = \frac{D_p v_t \rho_m}{\mu_m\,\varepsilon} = \frac{D_p^3\, g(\rho_p - \rho)\rho_m\, \varepsilon\psi_p^2}{18\mu^2}$$

When the Reynolds number is less than 1, the settling is in the Stokes' law range. For Reynolds numbers above 1.0, see (P1). The effect of concentration is greater for nonspherical particles and angular particles (S1).

## EXAMPLE 14.3-2. Hindered Settling of Glass Spheres

Calculate the settling velocity of glass spheres having a diameter of $1.554 \times 10^{-4}$ m ($5.10 \times 10^{-4}$ ft) in water at 293.2 K (20°C). The slurry contains 60 wt % solids. The density of the glass spheres is $\rho_p$ = 2467 kg/m$^3$ (154 lb$_m$/ft$^3$).

***Solution.*** Density of water $\rho$ = 998 kg/m$^3$ (62.3 lb$_m$/ft$^3$), and viscosity of water μ = $1.005 \times 10^{-3}$ Pa · s ($6.72 \times 10^{-4}$ lb$_m$/ft · s). To calculate the volume fraction $\varepsilon$ of the liquid,

$$\varepsilon = \frac{40/998}{40/998 + 60/2467} = 0.622$$

The bulk density of the slurry $\rho_m$ according to Eq. (14.3-14) is

$$\rho_m = \varepsilon\rho + (1 - \varepsilon)\rho_p = 0.622(998) + (1 - 0.622)(2467)$$
$$= 1553 \text{ kg/m}^3 \text{ (96.9 lb}_m\text{/ft}^3)$$

Substituting into Eq. (14.3-13),

$$\psi_p = \frac{1}{10^{1.82(1-\varepsilon)}} = \frac{1}{10^{1.82(1-0.622)}} = 0.205$$

Substituting into Eq. (14.3-16), using SI and English units,

$$v_t = \frac{9.807(1.554 \times 10^{-4})^2(2467 - 998)(0.622^2 \times 0.205)}{18(1.005 \times 10^{-3})}$$
$$= 1.525 \times 10^{-3} \text{ m/s}$$
$$v_t = \frac{32.174(5.1 \times 10^{-4})^2(154 - 62.3)(0.622^2 \times 0.205)}{18(6.72 \times 10^{-4})}$$
$$= 5.03 \times 10^{-3} \text{ ft/s}$$

The Reynolds number is obtained by substituting into Eq. (14.3-17):

$$N_{\text{Re}} = \frac{D_p v_t \rho_m}{\mu_m \varepsilon} = \frac{D_p v_t \rho_m}{(\mu/\psi_p)\varepsilon} = \frac{(1.554 \times 10^{-4})(1.525 \times 10^{-3})1553}{(1.005 \times 10^{-3}/0.205)0.622}$$

$$= 0.121$$

Hence, the settling is in the laminar range.

## Wall Effect on Free Settling

When the diameter $D_p$ of the particle becomes appreciable with respect to the diameter $D_W$ of the container in which the settling is occurring, a retarding effect known as the *wall effect* is exerted on the particle. The terminal settling velocity is reduced. In the case of settling in the Stokes' law regime, the computed terminal velocity can be multiplied by the following to allow for the wall effect (Z1) for $D_p/D_W < 0.05$:

*Equation 14.3-18.*

$$k_W = \frac{1}{1 + 2.1(D_p/D_W)}$$

For a completely turbulent regime, the correction factor is

*Equation 14.3-19.*

$$k'_W = \frac{1 - (D_p/D_W)^2}{[1 + (D_p/D_W)^4]^{1/2}}$$

## Differential Settling and Separation of Solids in Classification

### Sink-and-float methods

Devices for the separation of solid particles into several fractions based upon their rates of flow or settling through fluids are known as *classifiers.* There are several separation methods for accomplishing this, namely, sink-and-float and differential settling. In the *sink-and-float method,* a liquid is used whose density is intermediate between that of the heavy or high-density material and that of the light-density material. In this liquid, the heavy particles will not float but settle out from the medium, while the light particles will float.

This method is independent of the sizes of the particles and depends only upon the relative densities of the two materials. This means liquids used must have densities greater than water, since most solids have high densities. Unfortunately, few such liquids exist that are cheap and noncorrosive. As a result, pseudoliquids are used, consisting of a suspension in water of very fine solid materials with high specific gravities, such as galena (specific gravity = 7.5) and magnetite (specific gravity = 5.17).

Hindered settling is used and the bulk density of the medium can be varied widely by varying the amount of the fine solid materials in the medium. Common applications of this technique are concentrating ore materials and cleaning coal. The fine solid materials in the medium are so small in diameter that their settling velocity is negligible, giving a relatively stable suspension.

**Differential settling methods**

The separation of solid particles into several size fractions based upon their settling velocities in a particular medium is called *differential settling* or *classification.* The density of the medium is less than that of either of the two substances to be separated.

In differential settling, both light and heavy materials settle through the medium. A disadvantage of this method if the light and heavy materials both have a range of particle sizes is that the smaller heavy particles settle at the same terminal velocity as the larger light particles.

Suppose that we consider two different materials: heavy-density material *A* (such as galena, with a specific gravity $\rho_A$ = 7.5) and light-density material *B* (such as quartz, with a specific gravity $\rho_B$ = 2.65). The terminal settling velocity of components *A* and *B*, from Eq. (14.3-7), can be written

*Equation 14.3-20.*

$$v_{tA} = \left[\frac{4(\rho_{pA} - \rho)gD_{pA}}{3C_{DA}\rho}\right]^{1/2}$$

*Equation 14.3-21.*

$$v_{tB} = \left[\frac{4(\rho_{pB} - \rho)gD_{pB}}{3C_{DB}\rho}\right]^{1/2}$$

For particles of equal settling velocities, $v_{tA} = v_{tB}$ and we obtain, by equating Eq. (14.3-20) to (14.3-21), canceling terms, and squaring both sides,

*Equation 14.3-22.*

$$\frac{(\rho_{pA} - \rho)D_{pA}}{\rho C_{DA}} = \frac{(\rho_{pB} - \rho)D_{pB}}{\rho C_{DB}}$$

or

*Equation 14.3-23.*

$$\frac{D_{pA}}{D_{pB}} = \frac{\rho_{pB} - \rho}{\rho_{pA} - \rho}\frac{C_{DA}}{C_{DB}}$$

For particles that are essentially spheres at very high Reynolds numbers in the turbulent Newton's law region, $C_D$ is constant and $C_{DA} = C_{DB}$, giving

*Equation 14.3-24.*

$$\frac{D_{pA}}{D_{pB}} = \left(\frac{\rho_{pB} - \rho}{\rho_{pA} - \rho}\right)^{1.0}$$

For laminar Stokes' law settling,

*Equation 14.3-25.*

$$C_{DA} = \frac{24\mu}{D_{pA}v_{tA}\rho} \qquad C_{DB} = \frac{24\mu}{D_{pB}v_{tB}\rho}$$

Substituting Eq. (14.3-25) into (14.3-23) and rearranging for Stokes' law settling, where $v_{tA} = v_{tB}$,

*Equation 14.3-26.*

$$\frac{D_{pA}}{D_{pB}} = \left(\frac{\rho_{pB} - \rho}{\rho_{pA} - \rho}\right)^{0.5}$$

For transition flow between laminar and turbulent flow,

*Equation 14.3-27.*

$$\frac{D_{pA}}{D_{pB}} = \left(\frac{\rho_{pB} - \rho}{\rho_{pA} - \rho}\right)^{n} \quad \text{where } \tfrac{1}{2} < n < 1$$

For particles settling in the turbulent range, Eq. (14.3-24) holds for equal settling velocities. For particles where $D_{pA} = D_{pB}$ and settling is in the turbulent Newton's law region, combining Eqs. (14.3-20) and (14.3-21),

*Equation 14.3-28.*

$$\frac{v_{tA}}{v_{tB}} = \left(\frac{\rho_{pA} - \rho}{\rho_{pB} - \rho}\right)^{1/2}$$

If both *A* and *B* particles are settling in the same medium, then Eqs. (14.3-24) and (14.3-28) can be used to make the plots given in Fig. 14.3-3 for the relation of velocity to diameter for *A* and *B*.

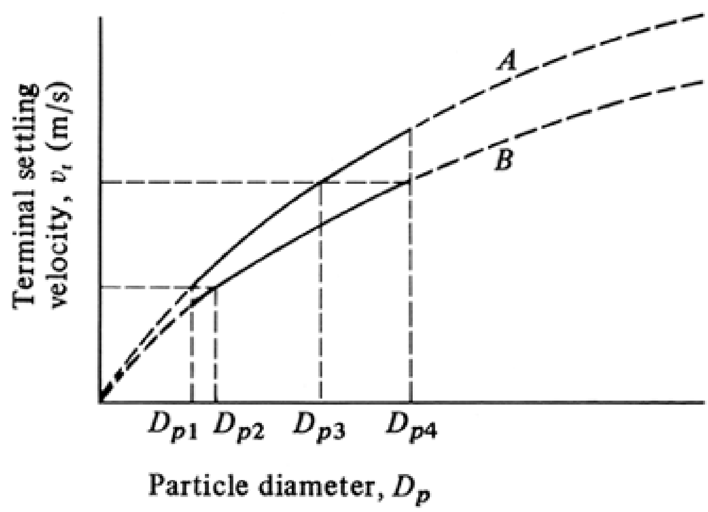



*Figure 14.3-3. Settling and separation of two materials A and B in Newton's law region.*

First, we consider a mixture of particles of materials *A* and *B* with a size range of $D_{p1}$ to $D_{p4}$ for both types of material. In the size range $D_{p1}$ to $D_{p2}$ in Fig. 14.3-3, a pure fraction of substance *B* can be obtained, since no particles of *A* settle as slowly. In the size range $D_{p3}$ to $D_{p4}$, a pure fraction of *A* can be obtained, since no *B* particles settle as fast as the *A* particles in this size range. In the size range $D_{p1}$ to $D_{p3}$, *A* particles settle as rapidly as *B* particles in the size range $D_{p2}$ to $D_{p4}$, forming a mixed fraction of *A* and *B*.

Increasing the density $\rho$ of the medium in Eq. (14.3-24), the numerator becomes smaller proportionately faster than the denominator, and the spread between $D_{pA}$ and $D_{pB}$ is increased. Somewhat similar curves are obtained in the Stokes' law region.

## EXAMPLE 14.3-3. Separation of a Mixture of Silica and Galena

A mixture of silica ($B$) and galena ($A$) solid particles having a size range of $5.21 \times 10^{-6}$ m to $2.50 \times 10^{-5}$ m is to be separated by hydraulic classification using free settling conditions in water at 293.2 K (B1). The specific gravity of silica is 2.65 and that of galena is 7.5. Calculate the size range of the various fractions obtained in the settling. If the settling is in the laminar region, the drag coefficients will be reasonably close to that for spheres.

***Solution:*** The particle-size range is $D_p = 5.21 \times 10^{-6}$ m to $D_p = 2.50 \times 10^{-5}$ m. Densities are $\rho_{pA} = 7.5(1000) = 7500$ kg/m$^3$, $\rho_{pB} = 2.65(1000) = 2650$ kg/m$^3$, $\rho = 998$ kg/m$^3$ for water at 293.2 K (20°C). The water viscosity $\mu = 1.005 \times 10^{-3}$ Pa · s $= 1.005 \times 10^{-3}$ kg/ms.

Assuming Stokes' law settling, Eq. (14.3-9) becomes as follows:

*Equation 14.3-29.*

$$v_{tA} = \frac{gD_{pA}^2(\rho_{pA} - \rho)}{18\mu}$$

The largest Reynolds number occurs for the largest particle and the biggest density, where $D_{pA} = 2.50 \times 10^{-5}$ m and $\rho_{pA} = 7500$. Substituting into Eq. (14.3-29),

$$v_{tA} = \frac{9.807(2.50 \times 10^{-5})^2(7500 - 998)}{18(1.005 \times 10^{-3})} = 2.203 \times 10^{-3} \text{ m/s}$$

Substituting into the Reynolds number equation,

*Equation 14.3-30.*

$$N_{\text{Re}} = \frac{D_{pA} v_{tA} \rho}{\mu}$$

$$= \frac{(2.50 \times 10^{-5})(2.203 \times 10^{-3})998}{1.005 \times 10^{-3}} = 0.0547$$

Hence, the settling is in the Stokes' law region.

Referring to Fig. 14.3-3 and using the same nomenclature, the largest size is $D_{p4} = 2.50 \times 10^{-5}$ m. The smallest size is $D_{p1} = 5.21 \times 10^{-6}$ m. The pure fraction of $A$ consists of $D_{pA4} = 2.50 \times 10^{-5}$ m to $D_{pA3}$. The particles, having diameters $D_{pA3}$ and $D_{pB4}$, are related by having equal settling velocities in Eq. (14.3-26). Substituting $D_{pB4} = 2.50 \times 10^{-5}$ m into Eq. (14.3-26) and solving,

$$\frac{D_{pA3}}{2.50 \times 10^{-5}} = \left(\frac{2650 - 998}{7500 - 998}\right)^{1/2}$$

$$D_{pA3} = 1.260 \times 10^{-5} \text{ m}$$

The size range of pure $B$ fraction is $D_{pB2}$ to $D_{pB1} = 5.21 \times 10^{-6}$ m. The diameter $D_{pB2}$ is related to $D_{pA1} = 5.21 \times 10^{-6}$ by Eq. (14.3-26) at equal settling velocities:

$$\frac{5.21 \times 10^{-6}}{D_{pB2}} = \left(\frac{2650 - 998}{7500 - 998}\right)^{1/2}$$

$$D_{pB2} = 1.033 \times 10^{-5} \text{ m}$$

The three fractions recovered are as follows:

1. The size range of the first fraction of pure $A$ (galena) is as follows:

$$D_{pA3} = 1.260 \times 10^{-5}\ \text{m} \quad \text{to} \quad D_{pA4} = 2.50 \times 10^{-5}\ \text{m}$$

2. The mixed-fraction size range is as follows:

$$D_{pB2} = 1.033 \times 10^{-5}\ \text{m} \quad \text{to} \quad D_{pB4} = 2.50 \times 10^{-5}\ \text{m}$$

$$D_{pA1} = 5.21 \times 10^{-6}\ \text{m} \quad \text{to} \quad D_{pA3} = 1.260 \times 10^{-5}\ \text{m}$$

3. The size range of the third fraction of pure *B* (silica) is as follows:

$$D_{pB1} = 5.21 \times 10^{-6}\ \text{m} \quad \text{to} \quad D_{pB2} = 1.033 \times 10^{-5}\ \text{m}$$

## Sedimentation and Thickening

### Mechanisms of sedimentation

When a dilute slurry is settled by gravity into a clear fluid and a slurry of higher solids concentration, the process is called *sedimentation* or sometimes *thickening.* To illustrate the method for determining settling velocities and the mechanisms of settling, a batch settling test is carried out by placing a uniform concentration of the slurry in a graduated cylinder. At the start, as shown in Fig. 14.3-4a, all the particles settle by free settling in suspension zone *B*. The particles in zone *B* settle at a uniform rate at the start, and a clear liquid zone *A* appears in Fig. 14.3-4b. The height *z* drops at a constant rate. Also, zone D begins to appear, which contains the settled particles at the bottom. Zone C is a transition layer whose solids content varies from that in zone B to that in zone D. After further settling, zones B and C disappear, as shown in Fig. 14.3-4c. Then compression first appears; this moment is called the critical point. During compression, liquid is expelled upward from zone D and the thickness of zone D decreases.

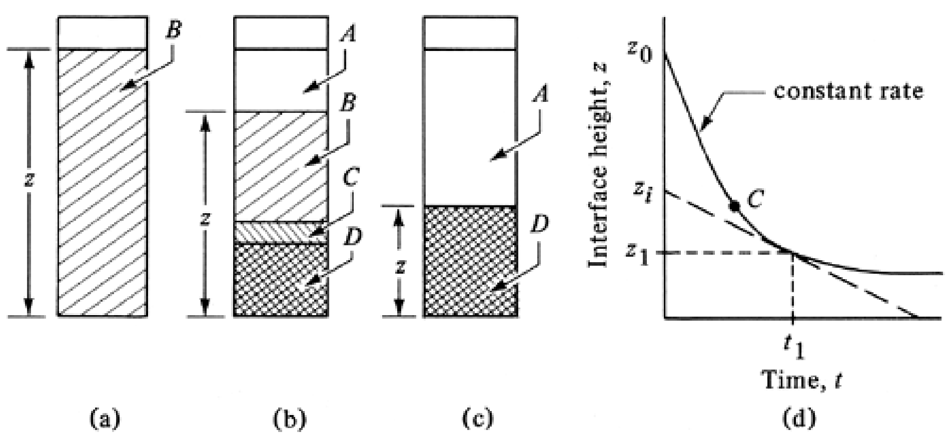



*Figure 14.3-4. Batch sedimentation results: (a) original uniform suspension, (b) zones of settling after a given time, (c) compression of zone D after zones B and C disappear, (d) clear liquid interface height z versus time of settling.*

### Determination of settling velocity

In Fig. 14.3-4d the height *z* of the clear-liquid interface is plotted versus time. As shown, the velocity of settling, which is the slope of the line, is constant at first. The critical point is shown at point *C*. Since sludges vary greatly in their settling rates, experimental rates for each sludge are necessary. Kynch (K1) and Talmage and Fitch (T1) describe a method for predicting thickener sizes from the batch settling test.

The settling velocity $v$ is determined by drawing a tangent to the curve in Fig. 14.3-4d at a given time $t_1$, with slope $-dz/dt = v_1$. At this point the height is $z_1$, and $z_i$ is the intercept of the tangent to the curve. Then,

*Equation 14.3-31.*

$$v_1 = \frac{z_i - z_1}{t_1 - 0}$$

The concentration $c_1$ is, therefore, the average concentration of the suspension if $z_i$ is the height of this slurry. This is calculated by

*Equation 14.3-32.*

$$c_1 z_i = c_0 z_0 \quad \text{or} \quad c_1 = \left(\frac{z_0}{z_i}\right) c_0$$

where $c_0$ is the original slurry concentration in kg/m$^3$ at $z_0$ height and $t = 0$. This is repeated for other times, and a plot of settling velocity versus concentration is made. Further details of this and other methods of designing the thickener are given elsewhere (C1, F1, F2, T1, P1). These and other methods in the literature are highly empirical and care should be exercised in their use.

## Equipment for Settling and Sedimentation

### Simple gravity settling tank

In Fig. 14.3-5a a simple gravity settler is shown for removing by settling a dispersed liquid phase from another phase. The velocity horizontally to the right must be slow enough to allow time for the smallest droplets to rise from the bottom to the interface or from the top down to the interface and coalesce.

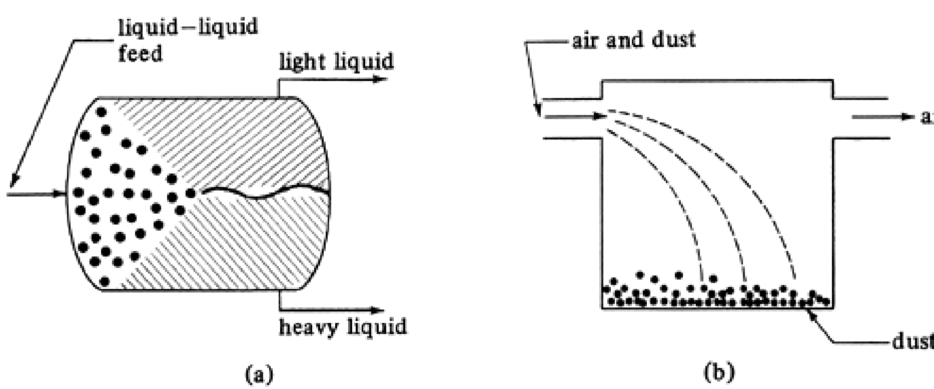



*Figure 14.3-5. Gravity settling tanks: (a) settler for liquid–liquid dispersion, (b) dust-settling chambers.*

In Fig. 14.3-5b a gravity settling chamber is shown schematically. Dust-laden air enters at one end of a large, boxlike chamber. Particles settle toward the floor at their terminal settling velocities. The air must remain in the chamber a sufficient length of time (residence time) so that the particles reach the floor of the chamber. Knowing the throughput of the air stream through the chamber and the chamber size, the residence time of the air in the chamber can be calculated. The vertical height of the chamber must be small enough that this height, divided by the settling velocity, gives a time less than the residence time of the air.

**Equipment for classification**

The simplest type of classifier is one in which a large tank is subdivided into several sections, as shown in Fig. 14.3-6. A liquid slurry feed enters the tank containing a size range of solid particles. The larger, faster-settling particles settle to the bottom close to the entrance and the slower-settling particles settle to the bottom close to the exit. The linear velocity of the entering feed decreases as a result of the enlargement of the cross-sectional area at the entrance. The vertical baffles in the tank allow for the collection of several fractions. The settling-velocity equations derived in this section hold.

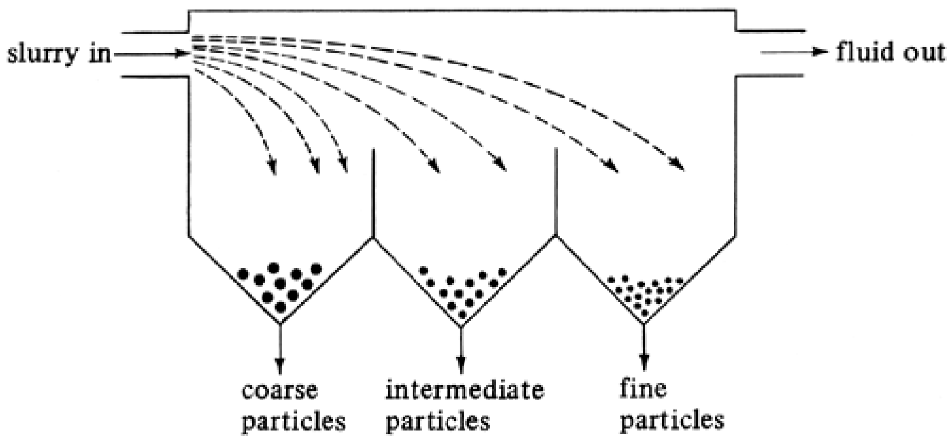



*Figure 14.3-6. Simple gravity settling classifier.*

**Spitzkasten classifier**

Another type of gravity settling chamber is the *Spitzkasten,* shown in Fig. 14.3-7, which consists of a series of conical vessels of increasing diameter in the direction of flow. The slurry enters the first vessel, where the largest and fastest-settling particles are separated. The overflow goes to the next vessel, where another separation occurs. This continues in the succeeding vessel or vessels. In each vessel the velocity of upflowing inlet water is controlled to give the desired size range for each vessel.

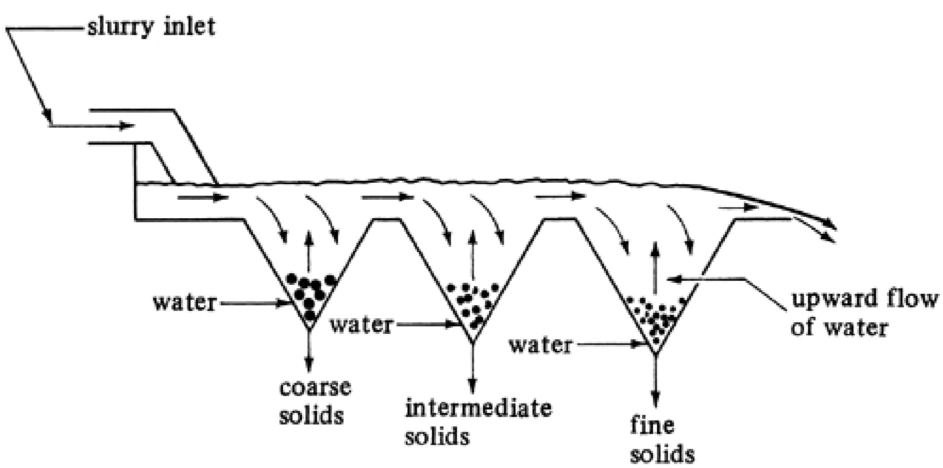



*Figure 14.3-7. Spitzkasten gravity settling chamber.*

**Sedimentation thickener**

The separation of a dilute slurry by gravity settling into a clear fluid and a slurry of higher solids concentration is called *sedimentation.* Industrially, sedimentation operations are often carried out continuously in equipment called *thickeners.* A continuous thickener with a slowly revolving rake for removing the sludge or thickened slurry is shown in Fig. 14.3-8.

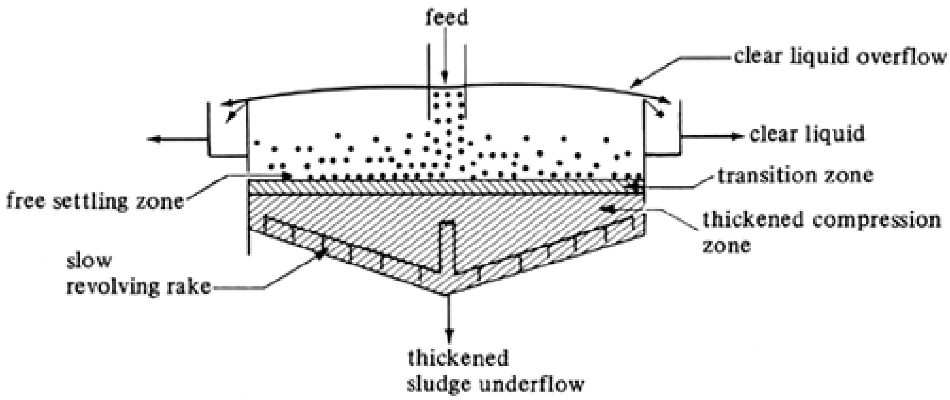



*Figure 14.3-8. Continuous thickener.*

The slurry in Fig. 14.3-8 is fed at the center of the tank several feet below the surface of the liquid. Around the top edge of the tank is a clear-liquid-overflow outlet. The rake serves to scrape the sludge toward the center of the bottom for removal. This gentle stirring aids in removing water from the sludge.

In the thickener the entering slurry spreads radially through the cross section of the thickener and the liquid flows upward and out the overflow. The solids settle in the upper zone by free settling. Below this dilute settling zone is the transition zone, in which the concentration of solids increases rapidly, and then the compression zone. A clear overflow can be obtained if the upward velocity of the fluid in the dilute zone is less than the minimal terminal settling velocity of the solids in this zone.

The settling rates are quite slow in the thickened zone, which consists of a compression of the solids with liquid being forced upward through the solids. This is an extreme case of hindered settling. Equation (14.3-16) may be used to estimate the settling velocities, but the results can be in considerable error because of agglomeration of particles. As a result, laboratory settling or sedimentation data must be used in the design of a thickener, as discussed previously in Section 14.3F.

# CENTRIFUGAL SEPARATION PROCESSES

## Introduction

**Centrifugal settling or sedimentation**

In Section 14.3 were discussed the processing methods of settling and sedimentation, where particles are separated from a fluid by gravitational forces acting on the particles. The particles were solid, gas, or liquid and the fluid was a liquid or a gas. In the present section we discuss settling or separation of particles from a fluid by centrifugal forces acting on the particles.

Use of centrifuges increases the forces on particles manyfold. Hence, particles that will not settle readily or at all in gravity settlers can often be separated from fluids by centrifugal force. The high settling force means that practical rates of settling can be obtained with much smaller particles than in gravity settlers. These high centrifugal forces do not change the relative settling velocities of small particles, but these forces do overcome the disturbing effects of Brownian motion and free convection currents.

Sometimes gravity separation may be too slow because of the closeness of the densities of the particles and the fluid, or because of association forces holding the components together, as in emulsions. An example in the dairy industry is the separation of cream from whole milk, giving skim milk. Gravity separation takes hours, while centrifugal separation is accomplished in minutes in a cream separator. Centrifugal settling or separation is employed in many food industries, such as breweries, vegetable-oil processing, fish-protein-concentrate processing, fruit juice processing to remove cellular materials, and so on. Centrifugal separation is also used in drying crystals and for separating emulsions into their constituent liquids or solid–liquid. The principles of centrifugal sedimentation are discussed in Sections 14.4B and 14.4C.

#### Centrifugal filtration

Centrifuges are also used in centrifugal filtration, where a centrifugal force is used instead of a pressure difference to cause the flow of slurry in a filter where a cake of solids builds up on a screen. The cake of granular solids from the slurry is deposited on a filter medium held in a rotating basket, washed, and then spun "dry." Centrifuges and ordinary filters are competitive in most solid–liquid separation problems. The principles of centrifugal filtration are discussed in Section 14.4E.

### Forces Developed in Centrifugal Separation

#### Introduction

Centrifugal separators make use of the common principle that an object whirled about an axis or center point at a constant radial distance from the point is acted on by a force. The object being whirled about an axis is constantly changing direction and is thus accelerating, even though the rotational speed is constant. This centripetal force acts in a direction toward the center of rotation.

If the object being rotated is a cylindrical container, the contents of fluid and solids exert an equal and opposite force, called *centrifugal force,* outward to the walls of the container. This is the force that causes settling or sedimentation of particles through a layer of liquid or filtration of a liquid through a bed of filter cake held inside a perforated rotating chamber.

In Fig. 14.4-1a a cylindrical bowl is shown rotating, with a slurry feed of solid particles and liquid being admitted at the center. The feed enters and is immediately thrown outward to the walls of the container, as in Fig. 14.4-1b. The liquid and solids are now acted upon by the vertical gravitational force and the horizontal centrifugal force. The centrifugal force is usually so large that the force of gravity may be neglected. The liquid layer then assumes the equilibrium position, with the surface almost vertical. The particles settle horizontally outward and press against the vertical bowl wall.

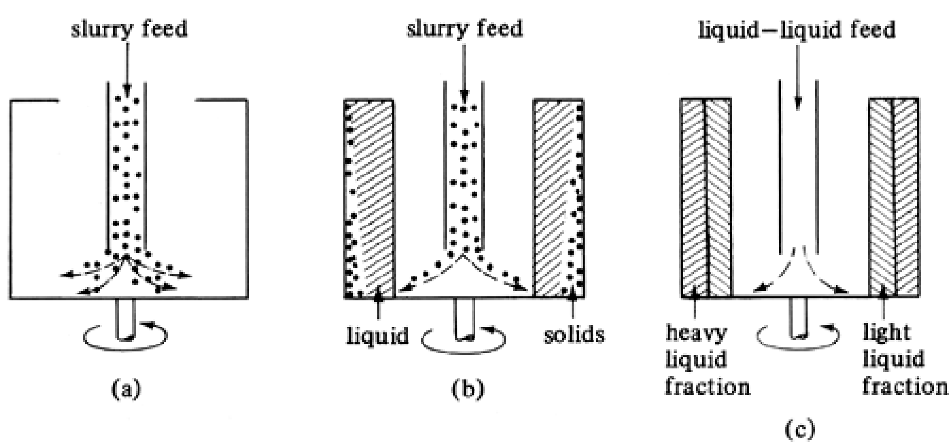



*Figure 14.4-1. Sketch of centrifugal separation: (a) initial slurry feed entering, (b) settling of solids from a liquid, (c) separation of two liquid fractions.*

In Fig. 14.4-1c two liquids having different densities are being separated by the centrifuge. The denser fluid will occupy the outer periphery, since the centrifugal force on it is greater.

**Equations for centrifugal force**

In circular motion the acceleration due to the centrifugal force is

*Equation 14.4-1.*

$$a_e = r\omega^2$$

where $a_e$ is the acceleration from a centrifugal force in m/s$^2$ (ft/s$^2$), $r$ is radial distance from the center of rotation in m (ft), and $\omega$ is angular velocity in rad/s.

The centrifugal force $F_c$ in N (lb$_f$) acting on the particle is given by

*Equation 14.4-2.*

$$F_c = ma_e = mr\omega^2 \qquad \text{(SI)}$$

$$F_c = \frac{mr\omega^2}{g_c} \qquad \text{(English)}$$

where $g_c$ = 32.174 lb$_m$ · ft/lb$_f$s$^2$.

Since $\omega = v/r$, where $v$ is the tangential velocity of the particle in m/s (ft/s),

*Equation 14.4-3.*

$$F_c = mr\left(\frac{v}{r}\right)^2 = \frac{mv^2}{r}$$

Often rotational speeds are given as $N$ rev/min and

*Equation 14.4-4.*

$$\omega = \frac{2\pi N}{60}$$

*Equation 14.4-5.*

$$N = \frac{60v}{2\pi r}$$

Substituting Eq. (14.4-4) into Eq. (14.4-2),

*Equation 14.4-6.*

$$F_c \text{ newton} = mr\left(\frac{2\pi N}{60}\right)^2 = 0.01097mrN^2 \qquad \text{(SI)}$$

$$F_c \text{ lb}_f = \frac{mr}{g_c}\left(\frac{2\pi N}{60}\right)^2 = 0.000341mrN^2 \qquad \text{(English)}$$

By Eq. (14.3-2), the gravitational force on a particle is

*Equation 14.3-2.*

$$F_g = mg$$

where $g$ is the acceleration of gravity and is 9.80665 m/s$^2$. In terms of gravitational force, the centrifugal force is as follows, by combining Eqs. (14.3-2), (14.4-2), and (14.4-3):

*Equation 14.4-7.*

$$\frac{F_c}{F_g} = \frac{r\omega^2}{g} = \frac{v^2}{rg} = \frac{r}{g}\left(\frac{2\pi N}{60}\right)^2 = 0.001118 rN^2 \qquad \text{(SI)}$$

$$\frac{F_c}{F_g} = 0.000341 rN^2 \qquad \text{(English)}$$

Hence, the force developed in a centrifuge is $r\omega^2/g$ or $v^2/rg$ times as large as the gravitational force. This is often expressed as equivalent to so many $g$ forces.

### EXAMPLE 14.4-1. Force in a Centrifuge

A centrifuge having a radius of the bowl of 0.1016 m (0.333 ft) is rotating at $N$ = 1000 rev/min.

a. Calculate the centrifugal force developed in terms of gravity forces.
b. Compare this force to that for a bowl with a radius of 0.2032 m rotating at the same rev/min.

***Solution:*** For part (a), $r$ = 0.1016 m and $N$ = 1000. Substituting into Eq. (14.4-7),

$$\frac{F_c}{F_g} = 0.001118 rN^2 = 0.001118(0.1016)(1000)^2 \qquad \text{(SI)}$$

$$= 113.6 \text{ gravities or } g\text{'s}$$

$$\frac{F_c}{F_g} = 0.000341(0.333)(1000)^2 = 113.6 \qquad \text{(English)}$$

For part (b), $r$ = 0.2032 m. Substituting into Eq. (14.4-7),

$$\frac{F_c}{F_g} = 0.001118(0.2032)(1000)^2 = 227.2 \text{ gravities or } g\text{'s}$$

## Equations for Rates of Settling in Centrifuges

### General equation for settling

If a centrifuge is used for sedimentation (removal of particles by settling), a particle of a given size can be removed from the liquid in the bowl if there is sufficient residence time of the particle in the bowl for the particle to reach the wall. For a particle moving radially at its terminal settling velocity, the diameter of the smallest particle which can be removed can be calculated.

In Fig. 14.4-2 a schematic of a tubular-bowl centrifuge is shown. The feed enters at the bottom, and it is assumed that all the liquid moves upward at a uniform velocity, carrying solid particles with it. The particle is assumed to be moving radially at its terminal settling velocity $v_t$. The trajectory or path of the particle is shown in Fig. 14.4-2. A particle of a given size is removed from the liquid if sufficient residence time is available for the particle to reach the wall of the bowl, where it is held. The length of the bowl is $b$ m.

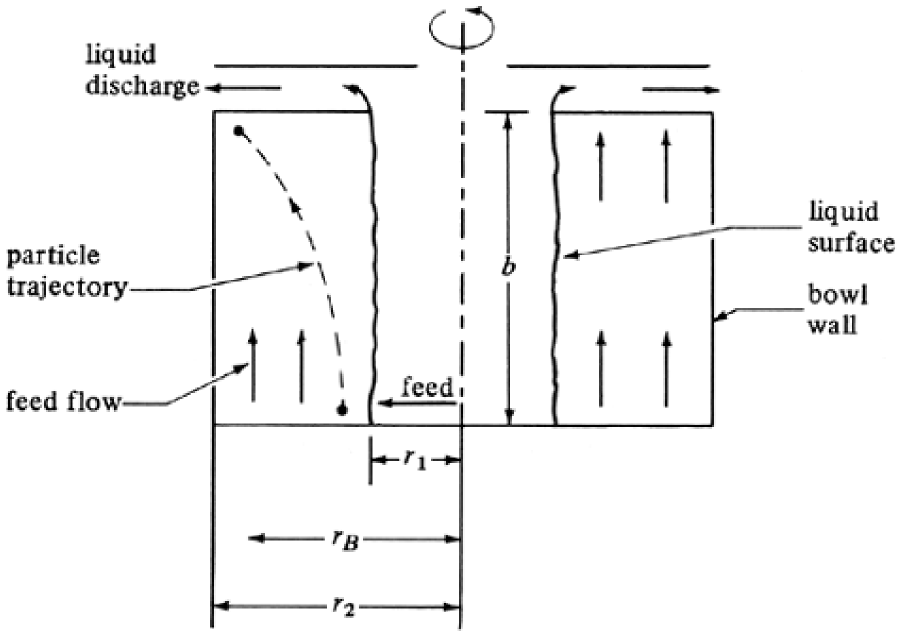



*Figure 14.4-2. Particle settling in sedimenting tubular-bowl centrifuge.*

At the end of the residence time of the particle in the fluid, the particle is at a distance $r_B$ m from the axis of rotation. If $r_B < r_2$, then the particle leaves the bowl with the fluid. If $r_B = r_2$, it is deposited on the wall of the bowl and effectively removed from the liquid.

For settling in the Stokes' law range, the terminal settling velocity at a radius $r$ is obtained by substituting Eq. (14.4-1) for the acceleration $g$ into Eq. (14.3-9):

*Equation 14.4-8.*

$$v_t = \frac{\omega^2 r D_p^2(\rho_p - \rho)}{18\mu}$$

where $v_t$ is settling velocity in the radial direction in m/s, $D_p$ is particle diameter in $m$, $\rho_p$ is particle density in kg/m$^3$, $\rho$ is liquid density in kg/m$^3$, and $\mu$ is liquid viscosity in Pa · s. If hindered settling occurs, the right-hand side of Eq. (14.4-8) is multiplied by the factor ($\varepsilon^2\psi_p$) given in Eq. (14.3-16).

Since $v_t = dr/dt$, then Eq. (14.4-8) becomes

*Equation 14.4-9.*

$$dt = \frac{18\mu}{\omega^2(\rho_p - \rho)D_p^2}\frac{dr}{r}$$

Integrating between the limits $r = r_1$ at $t = 0$ and $r = r_2$ at $t = t_T$,

*Equation 14.4-10.*

$$t_T = \frac{18\mu}{\omega^2(\rho_p - \rho)D_p^2}\ln\frac{r_2}{r_1}$$

The residence time $t_T$ is equal to the volume of liquid $V$ m$^3$ in the bowl divided by the feed volumetric flow rate $q$ in m$^3$/s. The volume $V = \pi b(r_2^2 - r_1^2)$. Substituting into Eq. (14.4-10) and solving for $q$,

*Equation 14.4-11.*

$$q = \frac{\omega^2(\rho_p - \rho)D_p^2}{18\mu \ln(r_2/r_1)}(V) = \frac{\omega^2(\rho_p - \rho)D_p^2}{18\mu \ln(r_2/r_1)}[\pi b(r_2^2 - r_1^2)]$$

Particles having diameters smaller than that calculated from Eq. (14.4-11) will not reach the wall of the bowl and will go out with the exit liquid. Larger particles will reach the wall and be removed from the liquid.

A cut point or critical diameter $D_{pc}$ can be defined as the diameter of a particle that reaches half the distance between $r_1$ and $r_2$. This particle moves a distance of half the liquid layer or $(r_2 - r_1)/2$ during the time this particle is in the centrifuge. The integration is then between $r = (r_1 + r_2)/2$ at $t = 0$ and $r = r_2$ at $t = t_T$. Then we obtain

*Equation 14.4-12.*

$$q_c = \frac{\omega^2(\rho_p - \rho)D_{pc}^2}{18\mu \ln[2r_2/(r_1 + r_2)]}(V) = \frac{\omega^2(\rho_p - \rho)D_{pc}}{18\mu \ln[2r_2/(r_1 + r_2)]}[\pi b(r_2^2 - r_1^2)]$$

At this flow rate $q_c$, particles with a diameter greater than $D_{pc}$ will predominantly settle to the wall and most smaller particles will remain in the liquid.

**Special case for settling**

For the special case where the thickness of the liquid layer is small compared to the radius, Eq. (14.4-8) can be written for a constant $r \cong r_2$ and $D_p = D_{pc}$ as follows:

*Equation 14.4-13.*

$$v_t = \frac{\omega^2 r_2 D_{pc}^2(\rho_p - \rho)}{18\mu}$$

The time of settling $t_T$ is then as follows for the critical $D_{pc}$ case:

*Equation 14.4-14.*

$$t_T = \frac{V}{q_c} = \frac{(r_2 - r_1)/2}{v_t}$$

Substituting Eq. (14.4-13) into (14.4-14) and rearranging,

*Equation 14.4-15.*

$$q_c = \frac{\omega^2 r_2 D_{pc}^2(\rho_p - \rho)V}{18\mu[(r_2 - r_1)/2]}$$

The volume $V$ can be expressed as

*Equation 14.4-16.*

$$V \cong 2\pi r_2(r_2 - r_1)b$$

Combining Eqs. (14.4-15) and (14.4-16),

*Equation 14.4-17.*

$$q_c = \frac{2\pi b r_2^2 \omega^2 D_{pc}^2(\rho_p - \rho)}{9\mu}$$

The analysis above is somewhat simplified. The pattern of flow of the fluid is actually more complicated. These equations can also be used for liquid–liquid systems where droplets of liquid migrate according to the equations and coalesce in the other liquid phase.

### EXAMPLE 14.4-2. Settling in a Centrifuge

A viscous solution containing particles with a density $\rho_p$ = 1461 kg/m$^3$ is to be clarified by centrifugation. The solution density $\rho$ = 801 kg/m$^3$ and its viscosity is 100 cp. The centrifuge has a bowl with $r_2$ = 0.02225 m, $r_1$ = 0.00716 m, and height $b$ = 0.1970 m. Calculate the critical particle diameter of the largest particles in the exit stream if $N$ = 23 000 rev/min and the flow rate $q$ = 0.002832 m$^3$/h.

***Solution***: Using Eq. (14.4-4),

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(23\,000)}{60} = 2410 \text{ rad/s}$$

The bowl volume $V$ is

$$V = \pi b(r_2^2 - r_1^2)$$
$$= \pi(0.1970)[(0.02225)^2 - (0.00716)^2] = 2.747 \times 10^{-4} \text{ m}^3$$

Viscosity μ = 100 × 10$^{-3}$ = 0.100 Pa · s = 0.100 kg/m · s. The flow rate $q_c$ is

$$q_c = \frac{0.002832}{3600} = 7.87 \times 10^{-7} \text{ m}^3\text{/s}$$

Substituting into Eq. (14.4-12) and solving for $D_{pc}$,

$$q_c = 7.87 \times 10^{-7}$$
$$= \frac{(2410)^2(1461 - 801)D_{pc}^2(2.747 \times 10^{-4})}{18(0.100)\ln[2 \times 0.02225/(0.00716 + 0.02225)]}$$
$$D_{pc} = 0.746 \times 10^{-6} \text{ m} \quad \text{or} \quad 0.746\ \mu\text{m}$$

Substituting into Eq. (14.4-13) to obtain $v_t$ and then calculating the Reynolds number, the settling is in the Stokes' law range.

**Sigma values and scale-up of centrifuges**

A useful physical characteristic of a tubular-bowl centrifuge can be derived by multiplying and dividing Eq. (14.4-12) by 2$g$ and then substituting Eq. (14.3-9) written for $D_{pc}$ into Eq. (14.4-12) to obtain

*Equation 14.4-18.*

$$q_c = 2\frac{(\rho_p - \rho)gD_{pc}^2}{18\mu}\,\frac{\omega^2 V}{2g\ln[2r_2/(r_1 + r_2)]} = 2v_t \cdot \Sigma$$

where $v_t$ is the terminal settling velocity of the particle in a gravitational field and

*Equation 14.4-19.*

$$\Sigma = \frac{\omega^2 V}{2g\ \ln[2r_2/(r_1 + r_2)]} = \frac{\omega^2[\pi b(r_2^2 - r_1^2)]}{2g\ \ln[2r_2/(r_1 + r_2)]}$$

where Σ is a physical characteristic of the centrifuge and not of the fluid–particle system being separated. Using Eq. (14.4-17) for the special case of settling for a thin layer,

*Equation 14.4-20.*

$$\Sigma = \frac{\omega^2 \pi b 2 r_2^2}{g}$$

The value of Σ is really the area in m$^2$ of a gravitational settler that will have the same sedimentation characteristics as the centrifuge for the same feed rate. To scale up from a laboratory test of $q_1$ and $\Sigma_1$ to $q_2$ (for $v_{t1} = v_{t2}$),

*Equation 14.4-21.*

$$\frac{q_1}{\Sigma_1} = \frac{q_2}{\Sigma_2}$$

This scale-up procedure is dependable for centrifuges of similar type and geometry and if the centrifugal forces are within a factor of 2 from each other. If different configurations are involved, efficiency factors $E$ should be used, where $q_1/\Sigma_1 E_1 = q_2/\Sigma_2 E_2$. These efficiencies must be determined experimentally, and values for different types of centrifuges are given elsewhere (F1, P1).

### Separation of liquids in a centrifuge

Liquid–liquid separations in which the liquids are immiscible but finely dispersed as in an emulsion are common operations in the food and other industries. An example is the dairy industry, in which the emulsion of milk is separated into skim milk and cream. In these liquid–liquid separations, the position of the outlet overflow weir in the centrifuge is very important, not only in controlling the volumetric holdup $V$ in the centrifuge but also in determining whether a separation is actually made.

In Fig. 14.4-3, a tubular-bowl centrifuge is shown in which the centrifuge is separating two liquid phases, one a heavy liquid with density $\rho_H$ kg/m$^3$ and the second a light liquid with density $\rho_L$. The distances shown are as follows: $r_1$ is radius to surface of light liquid layer, $r_2$ is radius to liquid–liquid interface, and $r_4$ is radius to surface of heavy liquid downstream.

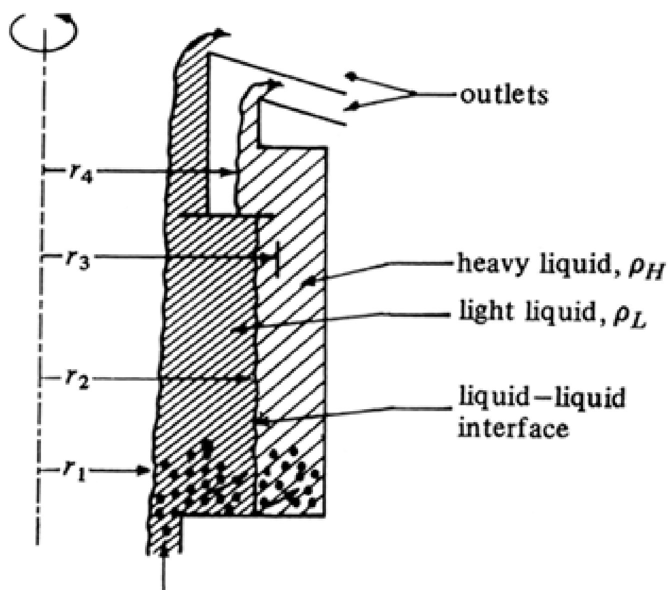



*Figure 14.4-3. Tubular bowl centrifuge for separating two liquid phases.*

To locate the interface, a balance must be made of the pressures in the two layers. The force on the fluid at distance *r* is, by Eq. (14.4-2),

*Equation 14.4-2.*

$$F_c = mr\omega^2$$

The differential force across a thickness *dr* is

*Equation 14.4-22.*

$$dF_c = dmr\omega^2$$

But,

*Equation 14.4-23.*

$$dm = [(2\pi rb)dr]\rho$$

where *b* is the height of the bowl in m and (2π*rb*) *dr* is the volume of fluid. Substituting Eq. (14.4-23) in (14.4-22) and dividing both sides by the area *A* = 2π*rb*,

*Equation 14.4-24.*

$$dP = \frac{dF_c}{A} = \omega^2 \rho r\, dr$$

where *P* is pressure in N/m$^2$ (lb$_f$/ft$^2$).

Integrating Eq. (14.4-24) between $r_1$ and $r_2$,

*Equation 14.4-25.*

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2)$$

Applying Eq. (14.4-25) to Fig. 14.4-3 and equating the pressure exerted by the light phase of thickness $r_2 - r_1$ to the pressure exerted by the heavy phase of thickness $r_2 - r_4$ at the liquid–liquid interface at $r_2$,

*Equation 14.4-26.*

$$\frac{\rho_H \omega^2}{2}(r_2^2 - r_4^2) = \frac{\rho_L \omega^2}{2}(r_2^2 - r_1^2)$$

Solving for $r_2^2$, the interface position,

*Equation 14.4-27.*

$$r_2^2 = \frac{\rho_H r_4^2 - \rho_L r_1^2}{\rho_H - \rho_L}$$

The interface at $r_2$ must be located at a radius smaller than $r_3$ in Fig. 14.4-3.

## EXAMPLE 14.4-3. Location of Interface in Centrifuge

In a vegetable-oil-refining process, an aqueous phase is being separated from the oil phase in a centrifuge. The density of the oil is 919.5 kg/m$^3$ and that of the aqueous phase is 980.3 kg/m$^3$. The radius $r_1$ for overflow of the light liquid has been set at 10.160 mm and the outlet for the heavy liquid at 10.414 mm. Calculate the location of the interface in the centrifuge.

*Solution*: The densities are $\rho_L$ = 919.5 and $\rho_H$ = 980.3 kg/m$^3$. Substituting into Eq. (14.4-27) and solving for $r_2$,

$$r_2^2 = \frac{980.3(10.414)^2 - 919.5(10.160)^2}{980.3 - 919.5}$$

$$r_2 = 13.75 \text{ mm}$$

## Centrifuge Equipment for Sedimentation

### Tubular centrifuge

A schematic of a *tubular-bowl centrifuge* is shown in Fig. 14.4-3. The bowl is tall and has a narrow diameter, 100–150 mm. Such centrifuges, known as *supercentrifuges,* develop a force about 13 000 times the force of gravity. Some narrow centrifuges, having a diameter of 75 mm and very high speeds of 60 000 or so rev/min, are known as *ultracentrifuges.* These supercentrifuges are often used to separate liquid–liquid emulsions.

### Disk bowl centrifuge

The *disk bowl centrifuge* shown in Fig. 14.4-4 is often used in liquid–liquid separations. The feed enters the actual compartment at the bottom and travels upward through vertically spaced feed holes, filling the spaces between the disks. The holes divide the vertical assembly into an inner section, where mostly light liquid is present, and an outer section, where mainly heavy liquid is present. This dividing line is similar to an interface in a tubular centrifuge.

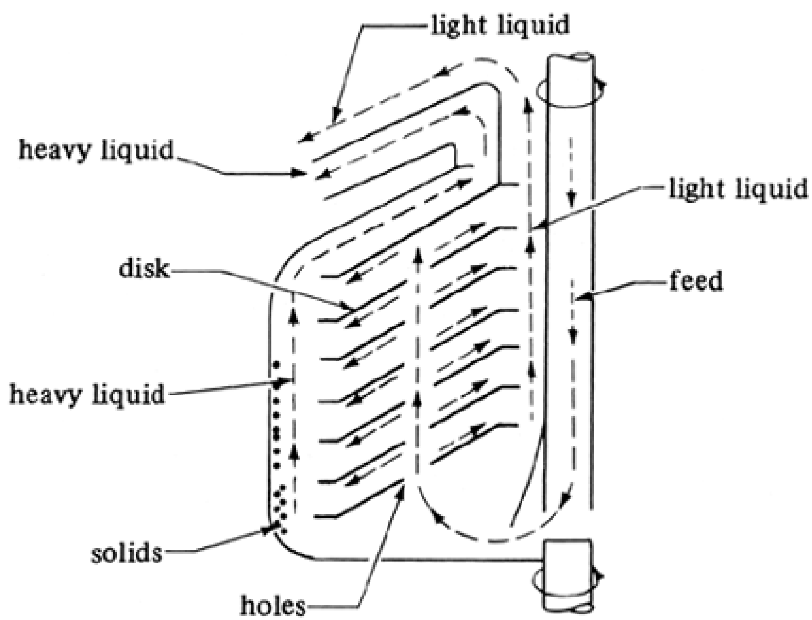



*Figure 14.4-4. Schematic of disk bowl centrifuge.*

The heavy liquid flows beneath the underside of a disk to the periphery of the bowl. The light liquid flows over the upper side of the disks and toward the inner outlet. Any small amount of heavy solids is thrown to the outer wall. Periodic cleaning is required to remove solids deposited. Disk bowl centrifuges are used in starch–gluten separation, concentration of rubber latex, and cream separation. Details are given elsewhere (P1, L1).

## Centrifugal Filtration

### Theory for centrifugal filtration

Theoretical prediction of filtration rates in centrifugal filters have not been too successful. The filtration in centrifuges is more complicated than for ordinary filtration using pressure differences, since the area for flow and driving force increase with distance from the axis and the specific cake resistance may change markedly. Centrifuges for filtering are generally selected by scale-up from tests on a similar-type laboratory centrifuge using the slurry to be processed.

The theory of constant-pressure filtration discussed in Section 14.2E can be modified and used where centrifugal force causes the flow instead of impressed pressure difference. The equation will be derived for the case where a cake has already been deposited, as shown in Fig. 14.4-5. he inside radius of the basket is $r_2$, $r_i$ is the inner radius of the face of the cake, and $r_1$ is the inner radius of the liquid surface. We will assume that the cake is nearly incompressible so that an average value of $\alpha$ can be used for the cake. Also, the flow is laminar. If we assume a thin cake in a large-diameter centrifuge, then the area $A$ for flow is approximately constant. The velocity of the liquid is

*Equation 14.4-28.*

$$v = \frac{q}{A} = \frac{dV}{A\,dt}$$

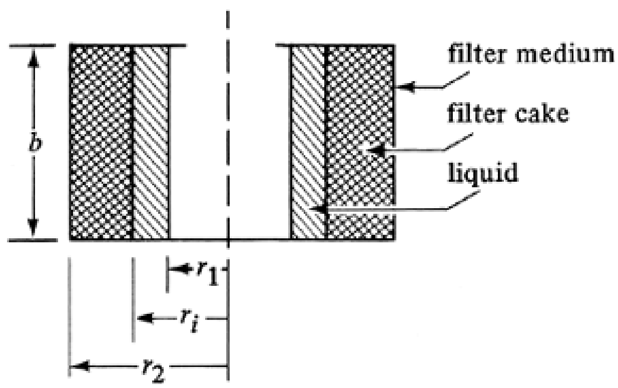



*Figure 14.4-5. Physical arrangement for centrifugal filtration.*

where $q$ is the filtrate flow rate in $m^3$/s and $v$ the velocity. Substituting Eq. (14.4-28) into (14.2-8),

*Equation 14.4-29.*

$$-\Delta p = q\mu\left(\frac{m_c\alpha}{A^2} + \frac{R_m}{A}\right)$$

where $m_c = c_S V$, mass of cake in kg deposited on the filter.

For a hydraulic head of $dz$ m, the pressure drop is

*Equation 14.4-30.*

$$dp = \rho g\,dz$$

In a centrifugal field, $g$ is replaced by $r\omega^2$ from Eq. (14.4-1) and $dz$ by $dr$. Then,

*Equation 14.4-31.*

$$dp = \rho r\omega^2\,dr$$

Integrating between $r_1$ and $r_2$,

*Equation 14.4-32.*

$$-\Delta p = \frac{\rho\omega^2(r_2^2 - r_1^2)}{2}$$

Combining Eqs. (14.4-29) and (14.4-32) and solving for $q$,

*Equation 14.4-33.*

$$q = \frac{\rho\omega^2(r_2^2 - r_1^2)}{2\mu\left(\dfrac{m_c\alpha}{A^2} + \dfrac{R_m}{A}\right)}$$

For the case where the flow area $A$ varies considerably with the radius, the following has been derived (G1):

*Equation 14.4-34.*

$$q = \frac{\rho\omega^2(r_2^2 - r_1^2)}{2\mu\left(\dfrac{m_c\alpha}{\overline{A}_L\overline{A}_a} + \dfrac{R_m}{A_2}\right)}$$

where $A_2 = 2\pi r_2 b$ (area of filter medium), $\overline{A}_L = 2\pi b(r_2 - r_i)/\ln(r_2/r_i)$ (logarithmic cake area), and $\overline{A}_a = (r_i + r_2)\pi b$ (arithmetic mean cake area). This equation holds for a cake of a given mass at a given time. It is not an integrated equation covering the whole filtration cycle.

**Equipment for centrifugal filtration**

In a centrifugal filter, slurry is fed continuously to a rotating basket which has a perforated wall and is covered with a filter cloth. The cake builds up on the surface of the filter medium to the desired thickness. hen, at the end of the filtration cycle, feed is stopped, and wash liquid is added or sprayed onto the cake. hen the wash liquid is stopped and the cake is spun as dry as possible. he motor is then shut off or slowed and the basked allowed to rotate while the solids are discharged by a scraper knife, so that the solids drop through an opening in the basket floor. Finally, the filter medium is rinsed clean to complete the cycle. Usually, the batch cycle is completely automated. Automatic batch centrifugals have basket sizes up to about 1.2 m in diameter and usually rotate below 4000 rpm.

Continuous centrifugal filters are available with capacities up to about 25 000 kg solids/h. Intermittently, the cake deposited on the filter medium is removed by being pushed toward the discharge end by a pusher, which then retreats, allowing the cake to build up once more. As the cake is being pushed, it passes through a wash region. The filtrate and wash liquid are kept separate by partitions in the collector. Details of different types of centrifugal filters are available (P1).

## Gas–Solid Cyclone Separators

### Introduction and equipment

For separation of small solid particles or mist from gases, the most widely used type of equipment is the cyclone separator, shown in Fig. 14.4-6. The cyclone consists of a vertical cylinder with a conical bottom. The gas–solid particle mixture enters in a tangential inlet near the top. This gas–solid mixture enters in a rotating motion, and the vortex formed develops centrifugal force, which throws the particles radially toward the wall.

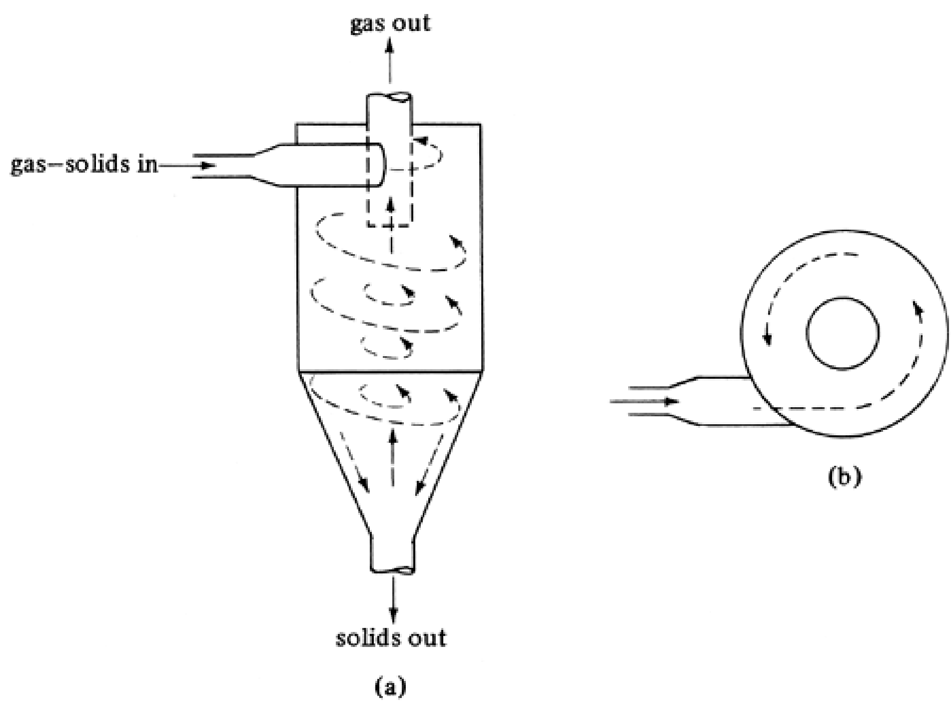



*Figure 14.4-6. Gas–solid cyclone separator: (a) side view, (b) top view.*

On entering, the air in the cyclone flows downward in a spiral or vortex adjacent to the wall. When the air reaches near the bottom of the cone, it spirals upward in a smaller spiral in the center of the cone and cylinder. Hence, a double vortex is present. The downward and upward spirals are in the same direction.

The particles are thrown toward the wall and fall downward, leaving out the bottom of the cone. A cyclone is a settling device in which the outward force on the particles at high tangential velocities is many times the force of gravity. Hence, cyclones accomplish much more effective separation than gravity settling chambers.

The centrifugal force in a cyclone ranges from about 5 times gravity in large, low-velocity units to 2500 times gravity in small, high-resistance units. These devices are used often in many applications, such as in spray-drying of foods, where the dried particles are removed by cyclones; in cleaning dust-laden air; and in removing mist droplets from gases. Cyclones offer one of the least expensive means of gas–particle separation. They are generally applicable in removing particles over 5 μm in diameter from gases. For particles over 200 μm in size, gravity settling chambers are often used. Wet-scrubber cyclones are sometimes used, where water is sprayed inside, helping to remove the solids.

### Theory for cyclone separators

It is assumed that particles on entering a cyclone quickly reach their terminal settling velocities. Particle sizes are usually so small that Stokes' law is considered valid. For centrifugal motion, the terminal radial velocity $v_{tR}$ is given by Eq. (14.4-8), with $v_{tR}$ being used for $v_t$:

*Equation 14.4-35.*

$$v_{tR} = \frac{\omega^2 r D_p^2(\rho_p - \rho)}{18\mu}$$

Since $\omega = v_{tan}/r$, where $v_{tan}$ is tangential velocity of the particle at radius $r$, Eq. (14.4-35) becomes

*Equation 14.4-36.*

$$v_{tR} = \frac{D_p^2 g(\rho_p - \rho)}{18\mu}\frac{v_{tan}^2}{gr} = v_t\frac{v_{tan}^2}{gr}$$

where $v_t$ is the gravitational terminal settling velocity $v_t$ in Eq. (14.3-9).

The higher the terminal velocity $v_t$, the greater the radial velocity $v_{tR}$ and the easier it should be to "settle" the particle at the walls. However, the evaluation of the radial velocity is difficult, since it is a function of gravitational terminal velocity, tangential velocity, and position radially and axially in the cyclone. Hence, the following empirical equation is often used (S2):

*Equation 14.4-37.*

$$v_{tR} = \frac{b_1 D_p^2(\rho_p - \rho)}{18\mu r^n}$$

where $b_1$ and $n$ are empirical constants.

### Efficiency of collection of cyclones

Smaller particles have smaller settling velocities according to Eq. (14.4-37) and do not have time to reach the wall to be collected. Hence, they leave with the exit air in a cyclone. Larger particles are more readily collected. The efficiency of separation for a given particle diameter is defined as the mass fraction of the size particles that are collected.

A typical collection-efficiency plot for a cyclone shows that the efficiency rises rapidly with particle size. The cut diameter $D_{pc}$ is the diameter for which one-half of the mass of the entering particles is retained.

# MECHANICAL SIZE REDUCTION

## Introduction

Many solid materials occur in sizes that are too large to be used and must be reduced. Often the solids are reduced in size so that the separation of various ingredients can be carried out. In general, the terms *crushing* and *grinding* are used to signify the subdividing of large solid particles into smaller particles.

In the food-processing industry, a large number of food products are subjected to size reduction. Roller mills are used to grind wheat and rye to flour and to grind corn. Soybeans are rolled, pressed, and ground to produce oil and flour. Hammer mills are often used to produce potato flour, tapioca, and other flours. Sugar is ground to a finer product.

Grinding operations are very extensive in the ore-processing and cement industries. Copper ores, nickel and cobalt ores, and iron ores, for example, are ground before chemical processing. Limestone, marble, gypsum, and dolomite are ground to use as fillers in paper, paint, and rubber. Raw materials for the cement industry, such as lime, alumina, and silica, are ground on a very large scale.

Solids may be reduced in size by a number of methods. *Compression* or *crushing* is generally used for reduction of hard solids to coarse sizes. *Impact* gives coarse, medium, or fine sizes. *Attrition* or *rubbing* yields fine products. *Cutting* is used to give definite sizes.

## Particle-Size Measurement

The feed-to-size reduction processes and the product are defined in terms of the particle-size distribution. One common way to plot particle sizes is to plot particle diameter (sieve opening in screen) in mm or μm versus the cumulative percent retained at that size. (Openings for various screen sizes are given in Appendix A.5.) Such a plot was given on arithmetic probability paper in Fig. 12.12-2.

Often the plot is made, instead, as the cumulative amount as percent smaller than the stated size versus particle size, as shown in Fig. 14.5-1a. In Fig. 14.5-1b the same data are plotted as a particle-distribution curve. The ordinate is obtained by taking the slopes of the 5-μm intervals of Fig. 14.5-1a and converting to percent by weight per μm. Complete particle-size analysis is necessary for most comparisons and calculations.

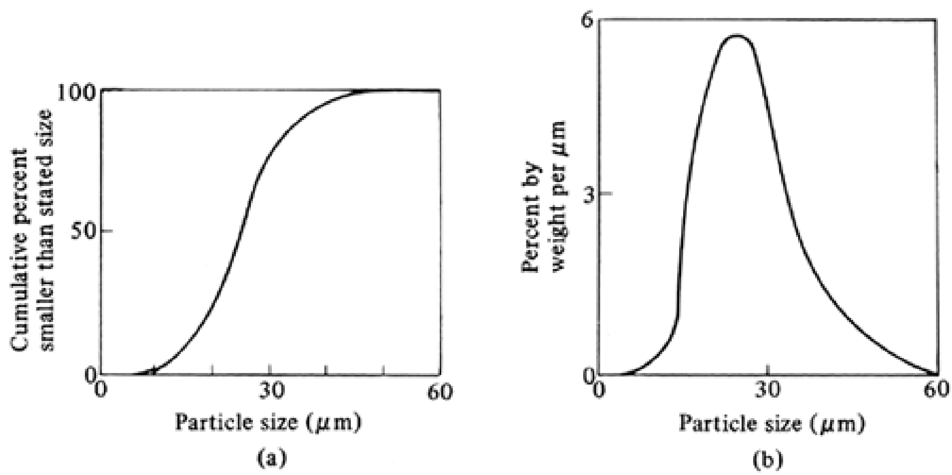



*Figure 14.5-1. Particle-size-distribution curves: (a) cumulative percent versus particle size, (b) percent by weight per μm versus particle size. (From R. H. Perry and C. H. Chilton, Chemical Engineers' Handbook, 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.)*

## Energy and Power Required in Size Reduction

### Introduction

In size reduction of solids, feed materials of solid are reduced to a smaller size by mechanical action. The materials are fractured. The particles of feed are first distorted and strained by the action of the size-reduction machine. This work to strain the particles is first stored temporarily in the solid as strain energy. As additional force is added to the stressed particles, the strain energy exceeds a certain level, and the material fractures into smaller pieces.

When the material fractures, new surface area is created. Each new unit area of surface requires a certain amount of energy. Some of the energy added is used to create the new surface, but a large portion of it appears as heat. The energy required for fracture is a complicated function of the type of material, size, hardness, and other factors.

The magnitude of the mechanical force applied; the duration; the type of force, such as compression, shear, and impact; and other factors affect the extent and efficiency of the size-reduction process. The important factors in the size-reduction process are the amount of energy or power used and the particle size and new surface formed.

**Power required in size reduction**

The various theories or laws proposed for predicting power requirements for size reduction of solids do not apply well in practice. The most important ones will be discussed briefly. Part of the problem with the theories is estimating the theoretical amount of energy required to fracture and create new surface area. Approximate calculations give actual efficiencies of about 0.1 to 2%.

The theories derived depend upon the assumption that the energy $E$ required to produce a change $dX$ in a particle of size $X$ is a power function of $X$:

*Equation 14.5-1.*

$$\frac{dE}{dX} = -\frac{C}{X^n}$$

where $X$ is size or diameter of particle in mm, and $n$ and $C$ are constants depending upon type and size of material and type of machine.

Rittinger proposed a law which states that the work in crushing is proportional to the new surface created. This leads to a value of $n = 2$ in Eq. (14.5-1), since area is proportional to length squared. Integrating Eq. (14.5-1),

*Equation 14.5-2.*

$$E = \frac{C}{n-1}\left(\frac{1}{X_2^{n-1}} - \frac{1}{X_1^{n-1}}\right)$$

where $X_1$ is mean diameter of feed and $X_2$ is mean diameter of product. Since $n = 2$ for Rittinger's equation, we obtain

*Equation 14.5-3.*

$$E = K_R\left(\frac{1}{X_2} - \frac{1}{X_1}\right)$$

where $E$ is work to reduce a unit mass of feed from $X_1$ to $X_2$ and $K_R$ is a constant. The law implies that the same amount of energy is needed to reduce a material from 100 mm to 50 mm as is needed to reduce the same material from 50 mm to 33.3 mm. It has been found experimentally that this law has some validity in grinding fine powders.

Kick assumed that the energy required to reduce a material in size was directly proportional to the size-reduction ratio. This implies $n = 1$ in Eq. (14.5-1), giving

*Equation 14.5-4.*

$$E = C \ln \frac{X_1}{X_2} = K_K \log \frac{X_1}{X_2}$$

where $K_K$ is a constant. This law implies that the same amount of energy is required to reduce a material from 100 mm to 50 mm as is needed to reduce the same material from 50 mm to 25 mm.

Recent data by Bond (B3) on correlating extensive experimental data suggest that the work required using a large-size feed is proportional to the square root of the surface/volume ratio of the product. This corresponds to $n = 1.5$ in Eq. (14.5-1), giving

*Equation 14.5-5.*

$$E = K_B \frac{1}{\sqrt{X_2}}$$

where $K_B$ is a constant. To use Eq. (14.4-5), Bond proposed a work index $E_i$ as the work in kW · h/ton required to reduce a unit weight from a very large size to 80% passing a 100-μm screen. Then the work $E$ is the gross work required to reduce a unit weight of feed with 80% passing a diameter $X_F$ μm to a product with 80% passing $X_P$ μm.

Bond's final equation, in terms of English units, is

*Equation 14.5-6.*

$$\frac{P}{T} = 1.46E_i\left(\frac{1}{\sqrt{D_P}} - \frac{1}{\sqrt{D_F}}\right)$$

where $P$ is hp, $T$ is feed rate in tons/min, $D_F$ is size of feed in ft, and $D_P$ is product size in ft. Typical values of $E_i$ for various types of materials are given in Perry and Green (P1) and by Bond (B3). Some typical values are bauxite ($E_i$ = 9.45), coal (11.37), potash salt (8.23), shale (16.4), and granite (14.39). These values should be multiplied by 1.34 for dry grinding.

### EXAMPLE 14.5-1. Power to Crush Iron Ore by Bond's Theory

It is desired to crush 10 ton/h of iron ore hematite. The size of the feed is such that 80% passes a 3-in. (76.2-mm) screen and 80% of the product is to pass a $\frac{1}{8}$ -in. (3.175-mm) screen. Calculate the gross power required. Use a work index $E_i$ for iron ore hematite of 12.68 (P1).

***Solution:*** The feed size is $D_F = \frac{3}{12}$ = 0.250 ft (76.2 mm) and the product size is $D_P = \frac{1}{8}/12$ = 0.0104 ft (3.175 mm). The feed rate is $T$ = 10/60 = 0.167 ton/min. Substituting into Eq. (14.5-6) and solving for $P$,

$$\frac{P}{0.167} = (1.46)(12.68)\left(\frac{1}{\sqrt{0.0104}} - \frac{1}{\sqrt{0.250}}\right)$$

$$P = 24.1 \text{ hp } (17.96 \text{ kW})$$

## Equipment for Size Reduction

### Introduction and classification

Size-reduction equipment may be classified according to the way the forces are applied as follows: between two surfaces, as in crushing and shearing; at one solid surface, as in impact; and by action of the surrounding medium, as in a colloid mill. A more practical classification is to divide the equipment into crushers, grinders, fine grinders, and cutters.

### Jaw crushers

Equipment for coarse reduction of large amounts of solids consists of slow-speed machines called *crushers.* Several types are in common use. In the first type, a jaw crusher, the material is fed between two heavy jaws or flat plates. As shown in the *Dodge crusher* in Fig. 14.5-2a, one jaw is fixed and the other reciprocating and movable on a pivot point at the bottom. The jaw swings back and forth, pivoting at the bottom of the V. The material is gradually worked down into a narrower space, being crushed as it moves.

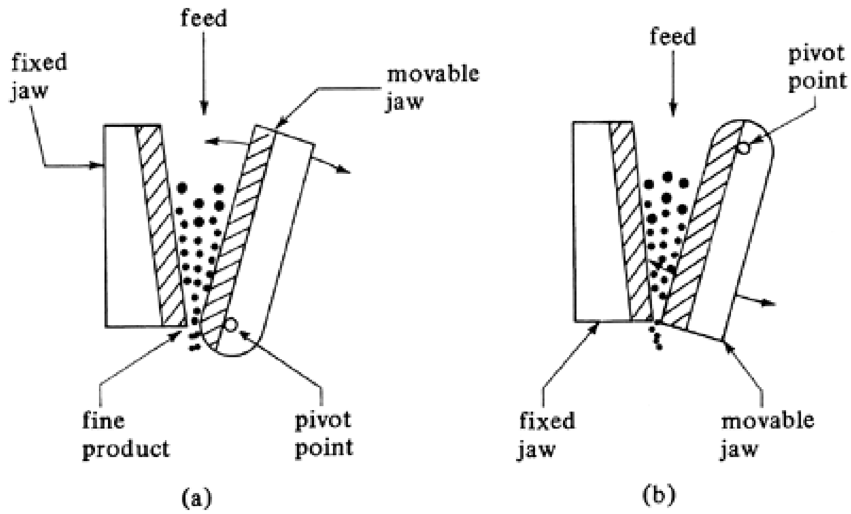



*Figure 14.5-2. Types of jaw crushers: (a) Dodge type, (b) Blake type.*

The *Blake crusher* in Fig. 14.5-2b is more commonly used, where the pivot point is at the top of the movable jaw. The reduction ratios average about 8:1 in the Blake crusher. Jaw crushers are used mainly for primary crushing of hard materials and are usually followed by other types of crushers.

**Gyratory crushers**

The *gyratory crusher* shown in Fig. 14.5-3a has to a large extent taken over in the field of large hard-ore and mineral crushing applications. Basically it is like a mortar-and-pestle crusher. The movable crushing head is shaped like an inverted truncated cone and is inside a truncated cone casing. The crushing head rotates eccentrically and the material being crushed is trapped between the outer fixed cone and the inner gyrating cone.

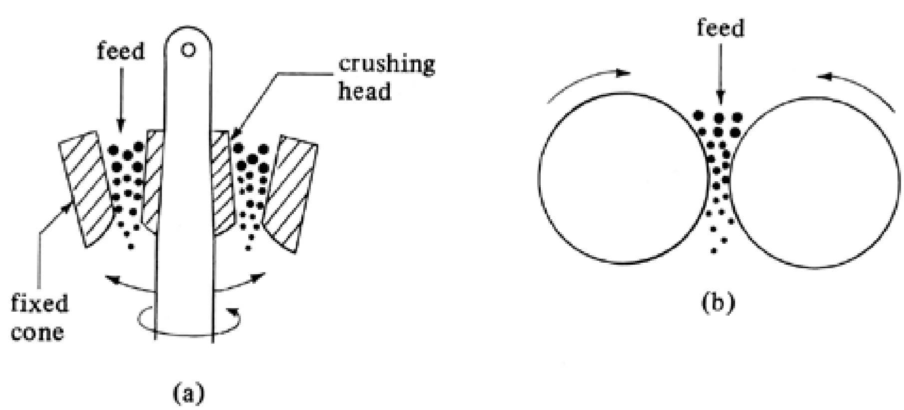



*Figure 14.5-3. Types of size-reduction equipment: (a) gyratory crusher, (b) roll crusher.*

**Roll crushers**

In Fig. 14.5-3b a typical smooth *roll crusher* is shown. The rolls are rotated toward each other at the same or different speeds. Wear of the rolls is a serious problem. The reduction ratio varies from about 4:1 to 2.5:1. Single rolls are often used, rotating against a fixed surface, and corrugated and toothed rolls are also used. Many food products that are not hard materials, such as flour, soybeans, and starch, are ground on rolls.

### Hammer mill grinders

Hammer mill devices are used to reduce intermediate-sized material to small sizes or powder. Often the product from jaw and gyratory crushers is the feed to the hammer mill. In the hammer mill a high-speed rotor turns inside a cylindrical casing. Sets of hammers are attached to pivot points at the outside of the rotor. The feed enters the top of the casing and the particles are broken as they fall through the cylinder. The material is broken by the impact of the hammers and pulverized into powder between the hammers and casing. The powder then passes through a grate or screen at the discharge end.

### Revolving grinding mills

For intermediate and fine reduction of materials, *revolving grinding mills* are often used. In such mills a cylindrical or conical shell rotating on a horizontal axis is charged with a grinding medium such as steel, flint, or porcelain balls, or with steel rods. The size reduction is effected by the tumbling of the balls or rods on the material between them. In the revolving mill, the grinding elements are carried up the side of the shell and fall on the particles underneath. These mills may operate wet or dry.

Equipment for very fine grinding is highly specialized. In some cases two flat disks are used, where one or both disks rotate and grind the material caught between the disks (P1).

## PROBLEMS

**14.2-1.** ***Constant-Pressure Filtration and Filtration Constants.*** Data for the filtration of $CaCO_3$ slurry in water at 298.2 K (25°C) are reported as follows (R1, R2, M1) at a constant pressure ($-\Delta p$) of 46.2 kN/m$^2$ (6.70 psia). The area of the plate-and-frame press was 0.0439 m$^2$ (0.473 ft$^2$) and the slurry concentration was 23.47 kg solid/m$^3$ filtrate. Calculate the constants $\alpha$ and $R_m$. Data are given as $t$ = time in s and $V$ = volume of filtrate collected in m$^3$.

| $V \times 10^3$ | $t$ | $V \times 10^3$ | $t$ | $V \times 10^3$ | $t$ |
|---|---|---|---|---|---|
| 0.5 | 17.3 | 1.5 | 72.0 | 2.5 | 152.0 |
| 1.0 | 41.3 | 2.0 | 108.3 | 3.0 | 201.7 |

**A1:** **Ans.** $\alpha = 1.106 \times 10^{11}$ m/kg ($1.65 \times 10^{11}$ ft/lb$_m$), $R_m = 6.40 \times 10^{10}$ m$^{-1}$ ($1.95 \times 10^{10}$ ft$^{-1}$)

**14.2-2.** ***Filtration Constants for Constants-Pressure Filtration.*** Data for constant-pressure filtration at 194.4 kN/m$^2$ are reported for the same slurry and press as in Problem 14.2-1 as follows, where $t$ is in s and $V$ in m$^3$:

| $V \times 10^3$ | $t$ | $V \times 10^3$ | $T$ | $V \times 10^3$ | $t$ |
|---|---|---|---|---|---|
| 0.5 | 6.3 | 2.5 | 51.7 | 4.5 | 134.0 |
| 1.0 | 14.0 | 3.0 | 69.0 | 5.0 | 160.0 |
| 1.5 | 24.2 | 3.5 | 88.8 | | |
| 2.0 | 37.0 | 4.0 | 110.0 | | |

**Q3:** Calculate the constants α and $R_m$.

**A3:** **Ans.** $\alpha = 1.61 \times 10^{11}$ m/kg

**14.2-3.** ***Compressibility of Filter Cake.*** Use the data for specific cake resistance $\alpha$ from Example 14.2-1 and Problems 14.2-1 and 14.2-2 and determine the compressibility constant $s$ in Eq. (14.2-11). Plot ln $\alpha$ versus ln($-\Delta p$) and determine the slope $s$.

14.2-4. ***Prediction of Filtration Time and Washing Time.*** The slurry used in Problem 14.2-1 is to be filtered in a plate-and-frame press having 30 frames and 0.873 $m^2$ area per frame. The same pressure, 46.2 $kN/m^2$, will be used in constant-pressure filtration. Assume the same filter-cake properties and filter cloth, and calculate the time to recover 2.26 $m^3$ of filtrate. At the end, using through-washing and 0.283 $m^3$ of wash water, calculate the time of washing and the total filter-cycle time if cleaning the press takes 30 min.

14.2-5. ***Constants in Constant-Pressure Filtration.*** McMillen and Webber (M2), using a filter press with an area of 0.0929 $m^2$, performed constant-pressure filtration at 34.5 kPa of a 13.9 wt % $CaCO_3$ solids in water slurry at 300 K. The mass ratio of wet cake to dry cake was 1.59. The dry-cake density was 1017 $kg/m^3$. The data obtained are as follows, where $W$ = kg filtrate and $t$ = time in s:

| $W$ | $t$ | $W$ | $t$ | $W$ | $t$ |
|---|---|---|---|---|---|
| 0.91 | 24 | 3.63 | 244 | 6.35 | 690 |
| 1.81 | 71 | 4.54 | 372 | 7.26 | 888 |
| 2.72 | 146 | 5.44 | 524 | 8.16 | 1188 |

Calculate the values of α and $R_m$.

14.2-6. ***Constant-Pressure Filtration and Washing in a Leaf Filter.*** An experimental filter press having an area of 0.0414 $m^2$ (R1) is used to filter an aqueous $BaCO_3$ slurry at a constant pressure of 267 kPa. The filtration equation obtained was

$$\frac{t}{V} = 10.25 \times 10^6 V + 3.4 \times 10^3$$

where $t$ is in s and $V$ in $m^3$.

a. If the same slurry and conditions are used in a leaf press having an area of 6.97 $m^2$, how long will it take to obtain 1.00 $m^3$ of filtrate?

b. After filtration, the cake is to be washed with 0.100 $m^3$ of water. Calculate the time of washing.

A7: **Ans.** (a) $t$ = 381.8 s

14.2-7. ***Constant-Rate Filtration of Incompressible Cake.*** The filtration equation for filtration at a constant pressure of 38.7 psia (266.8 kPa) is

$$\frac{t}{V} = 6.10 \times 10^{-5} V + 0.01$$

where $t$ is in s, $-\Delta p$ in psia, and $V$ in liters. The specific resistance of the cake is independent of pressure. If the filtration is run at a constant rate of 10 liters/s, how long will it take to reach 50 psia?

14.2-8. ***Effect of Filter-Medium Resistance on Continuous Rotary-Drum Filter.*** Repeat Example 14.2-4 for the continuous rotary-drum vacuum filter but do not neglect the constant $R_m$, which is the filter-medium resistance to flow. Compare with results of Example 14.2-4.

**A9:** **Ans.** $A = 7.78\ m^2$

**14.2-9.** ***Throughput in Continuous Rotary-Drum Filter.*** A rotary-drum filter having an area of 2.20 $m^2$ is to be used to filter the $CaCO_3$ slurry given in Example 14.2-4. The drum has a 28% submergence and the filter-cycle time is 300 s. A pressure drop of 62.0 $kN/m^2$ is to be used. Calculate the slurry feed rate in kg slurry/s for the following cases:

a. Neglect the filter-medium resistance.

b. Do not neglect the value of *B*.

**14.3-1.** ***Settling Velocity of a Coffee-Extract Particle.*** Solid spherical particles of coffee extract (F1) from a dryer having a diameter of 400 μm are falling through air at a temperature of 422 K. The density of the particles is 1030 $kg/m^3$. Calculate the terminal settling velocity and the distance of fall in 5 s. The pressure is 101.32 kPa.

**A11:** **Ans.** $v_t = 1.49$ m/s, 7.45 m fall

**14.3-2.** ***Terminal Settling Velocity of Dust Particles.*** Calculate the terminal settling velocity of dust particles having a diameter of 60 μm in air at 294.3 K and 101.32 kPa. The dust particles can be considered spherical, with a density of 1280 $kg/m^3$.

**A12:** **Ans.** $v_t = 0.1372$ m/s

**14.3-3.** ***Settling Velocity of Liquid Particles.*** Oil droplets having a diameter of 200 μm are settling from still air at 294.3 K and 101.32 kPa. The density of the oil is 900 $kg/m^3$. A settling chamber is 0.457 m high. Calculate the terminal settling velocity. How long will it take the particles to settle? (*Note:* If the Reynolds number is above about 100, the equations and form-drag correlation for rigid spheres cannot be used.)

**14.3-4.** ***Settling Velocity of Quartz Particles in Water.*** Solid quartz particles having a diameter of 1000 μm are settling from water at 294.3 K. The density of the spherical particles is 2650 $kg/m^3$. Calculate the terminal settling velocity of these particles.

**14.3-5.** ***Hindered Settling of Solid Particles.*** Solid spherical particles having a diameter of 0.090 mm and a solid density of 2002 $kg/m^3$ are settling in a solution of water at 26.7°C. The volume fraction of the solids in the water is 0.45. Calculate the settling velocity and the Reynolds number.

**14.3-6.** ***Settling of Quartz Particles in Hindered Settling.*** Particles of quartz having a diameter of 0.127 mm and a specific gravity of 2.65 are settling in water at 293.2 K. The volume fraction of the particles in the slurry mixture of quartz and water is 0.25. Calculate the hindered settling velocity and the Reynolds number.

**14.3-7.** ***Density Effect on Settling Velocity and Diameter.*** Calculate the terminal settling velocity of a glass sphere 0.080 mm in diameter having a density of 2469 $kg/m^3$ in air at 300 K and 101.32 kPa. Also calculate the diameter of a sphalerite sphere having a specific gravity of 4.00 with the same terminal settling velocity.

**14.3-8.** ***Differential Settling of Particles.*** Repeat Example 14.3-3 for particles having a size range of 1.27 × $10^{-2}$ mm to 5.08 × $10^{-2}$ mm. Calculate the size range of the various fractions obtained using free settling conditions. Also calculate the value of the largest Reynolds number occurring.

**14.3-9.** ***Separation by Settling.*** A mixture of galena and silica particles has a size range of 0.075–0.65 mm and is to be separated by a rising stream of water at 293.2 K. Use specific gravities from Example 14.3-3.

a. To obtain an uncontaminated product of galena, what velocity of water flow is needed and what is the size range of the pure product?

b. If another liquid, such as benzene, having a specific gravity of 0.85 and a viscosity of 6.50 × $10^{-4}$ Pa · s is used, what velocity is needed and what is the size range of the pure product?

**14.3-10.** ***Separation by Sink-and-Float Method.*** Quartz having a specific gravity of 2.65 and hematite having a specific gravity of 5.1 are present in a mixture of particles. It is desired to separate them by a sink-and-oat method using a suspension of fine particles of ferrosilicon having a specific gravity of 6.7 in water. At what consistency in vol % ferrosilicon solids in water should the medium be maintained for the separation?

**14.3-11.** ***Batch Settling and Sedimentation Velocities.*** A batch settling test on a slurry gave the following results, where the height $z$ in meters between the clear liquid and the suspended solids is given at time $t$ hours:

| $t$ (h) | $z$ (m) | $t$ (h) | $z$ (m) | $t$ (h) | $z$ (m) |
|---|---|---|---|---|---|
| 0 | 0.360 | 1.75 | 0.150 | 12.0 | 0.102 |
| 0.50 | 0.285 | 3.00 | 0.125 | 20.0 | 0.090 |
| 1.00 | 0.211 | 5.00 | 0.113 | | |

The original slurry concentration is 250 kg/m$^3$ of slurry. Determine the velocities of settling and concentrations and make a plot of velocity versus concentration.

**14.4-1.** ***Comparison of Forces in Centrifuges.*** Two centrifuges rotate at the same peripheral velocity of 53.34 m/s. The first bowl has a radius of $r_1$ = 76.2 mm and the second $r_2$ = 305 mm. Calculate the rev/min and the centrifugal forces developed in each bowl.

**A22:** **Ans.** $N_1$ = 6684 rev/min, $N_2$ = 1670 rev/min, 3806 $g$'s in bowl 1951 $g$'s in bowl 2

**14.4-2.** ***Forces in a Centrifuge.*** A centrifuge bowl is spinning at a constant 2000 rev/min. What radius bowl is needed for the following?

a. A force of 455 $g$'s.

b. A force four times that in part (a).

**A23:** **Ans.** (a) $r$ = 0.1017 m

**14.4-3.** ***Effect of Varying Centrifuge Dimensions and Speed.*** Repeat Example 14.4-2 but with the following changes:

a. Reduce the rev/min to 10 000 and double the outer-bowl radius $r_2$ to 0.0445 m, keeping $r_1$ = 0.00716 m.

b. Keep all variables as in Example 14.4-2 but double the throughput.

**A24:** **Ans.** (b) $D_p = 1.747 \times 10^{-6}$ m

**14.4-4.** ***Centrifuging to Remove Food Particles.*** A dilute slurry contains small solid food particles having a diameter of $5 \times 10^{-2}$ mm which are to be removed by centrifuging. The particle density is 1050 kg/m$^3$ and the solution density is 1000 kg/m$^3$. The viscosity of the liquid is $1.2 \times 10^{-3}$ Pa · s. A centrifuge at 3000 rev/min is to be used. The bowl dimensions are $b$ = 100.1 mm, $r_1$ = 5.00 mm, and $r_2$ = 30.0 mm. Calculate the expected flow rate in m$^3$/s just to remove these particles.

**14.4-5.** ***Effect of Oil Density on Interface Location.*** Repeat Example 14.4-3, but for the case where the vegetable-oil density has been decreased to 914.7 kg/m$^3$.

**14.4-6.** ***Interface in Cream Separator.*** A cream-separator centrifuge has an outlet discharge radius $r_1$ = 50.8 mm and outlet radius $r_4$ = 76.2 mm. The density of the skim milk is 1032 kg/m$^3$ and that of the cream is 865 kg/m$^3$ (E1). Calculate the radius of the interface neutral zone.

**A27:** **Ans.** $r_2$ = 150 mm

**14.4-7.** ***Scale-Up and Σ Values of Centrifuges.*** For the conditions given in Example 14.4-2, do as follows:

a. Calculate the Σ value.

b. A new centrifuge having the following dimensions is to be used: $r_2$ = 0.0445 m, $r_1$ = 0.01432 m, $b$ = 0.394 m, and $N$ = 26 000 rev/min. Calculate the new Σ value and scale up the flow rate using the same solution.

**A28:** **Ans.** (a) $\Sigma = 196.3$ m$^2$

**14.4-8.** ***Centrifugal Filtration Process.*** A batch centrifugal filter similar to Fig. 14.4-5 has a bowl height $b$ = 0.457 m and $r_2$ = 0.381 m and operates at 33.33 rev/s at 25.0°C. The filtrate is essentially water. At a given time in the cycle, the slurry and cake formed have the following properties: $c_S$ = 60.0 kg solids/m$^3$ filtrate, $\varepsilon$ = 0.82, $\rho_p$ = 2002 kg solids/m$^3$, cake thickness = 0.152 m, $\alpha = 6.38 \times 10^{10}$ m/kg, $R_m = 8.53 \times 10^{10}$ m$^{-1}$, $r_1$ = 0.2032 m. Calculate the rate of filtrate flow.

**A29:** **Ans.** $q = 6.11 \times 10^{-4}$ m$^3$/s

**14.5-1.** ***Change in Power Requirements in Crushing.*** In crushing a certain ore, the feed is such that 80% is less than 50.8 mm in size, and the product size is such that 80% is less than 6.35 mm. The power required is 89.5 kW. What will be the power required using the same feed so that 80% is less than 3.18 mm? Use the Bond equation. (*Hint:* The work index $E_i$ is unknown, but it can be determined using the original experimental data in terms of $T$. In the equation for the new size, the same unknowns appear. Dividing one equation by the other will eliminate these unknowns.)

**A30:** **Ans.** 146.7 kW

**14.5-2.** ***Crushing of Phosphate Rock.*** It is desired to crush 100 ton/h of phosphate rock from a feed size where 80% is less than 4 in. to a product where 80% is less than $\frac{1}{8}$ in. The work index is 10.13 (P1).

a. Calculate the power required.

b. Calculate the power required to crush the product further to where 80% is less than 1000 $\mu$m.

# REFERENCES

## Bibliography

[ch14biblio01entry01] (B1) W. L., Badger, and J. T Banchero,. *Introduction to Chemical Engineering*. New York: McGraw-Hill Book Company, 1955.

[ch14biblio01entry02] (B2) H. A. Becker, *Can. J. Chem. Eng.*, **37**, 85 (1951).

[ch14biblio01entry03] (B3) F. C. Bond, *Trans. A.I.M.E.*, **193**, 484 (1952).

[ch14biblio01entry04] (C1) J. M., Coulson, and J. F. Richardson, *Chemical Engineering*, Vol. 2, 3rd ed. New York: Pergamon Press, Inc., 1978.

[ch14biblio01entry05] (E1) R. L. Earle, *Unit Operations in Food Processing*. Oxford: Pergamon Press, Inc., 1966.

[ch14biblio01entry06] (F1) A. S. Foust,, et al. *Principles of Unit Operations*, 2nd ed. New York: John Wiley & Sons, Inc., 1980.

[ch14biblio01entry07] (F2) B. Fitch, *Ind. Eng. Chem.*, **58**(10), 18 (1966).

[ch14biblio01entry08] (G1) H. P. Grace, *Chem. Eng. Progr.*, **46**, 467 (1950); **49**, 303, 367, 427 (1953); *A.I.Ch.E. J.*, **2**, 307 (1956).

[ch14biblio01entry09] (H1) R. R., Hughes, and E. R. Gilliland, *Chem. Eng. Progr.*, **48**, 497 (1952).

[ch14biblio01entry10] (K1) G. J. Kynch, *Trans. Faraday Soc.*, **48**, 166 (1952).

[ch14biblio01entry11] (L1) M. G. Larian, *Fundamentals of Chemical Engineering Operations*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1958.

[ch14biblio01entry12] (L2) C. E., Lapple, and C. B. Shepherd, *Ind. Eng. Chem.*, **32**, 605 (1940).

[ch14biblio01entry13] (M1) W. L., McCabe, and J. C. Smith, *Unit Operations of Chemical Engineering*, 3rd ed. New York: McGraw-Hill Book Company, 1976.

[ch14biblio01entry14] (M2) E. L., McMillen, and H. A. Webber, *Trans. A.I.Ch.E.*, **34**, 213 (1938).

[ch14biblio01entry15] (N1) N., Nickolaus, and D. A. Dahlstrom, *Chem. Eng. Progr.*, **52**(3), 87M (1956).

[ch14biblio01entry16] (P1) R. H., Perry, and D. Green, *Perry's Chemical Engineers' Handbook*, 6th ed. New York: McGraw-Hill Book Company, 1984.

[ch14biblio01entry17] (P2) R. H., Perry, and C. H. Chilton, *Chemical Engineers' Handbook*, 5th ed. New York: McGraw-Hill Book Company, 1973.

[ch14biblio01entry18] (R1) B. F., Ruth, and L. L. Kempe, *Trans. A.I.Ch.E.*, **34**, 213 (1938).

[ch14biblio01entry19] (R2) B. F. Ruth, *Ind. Eng. Chem.*, **25**, 157 (1933).

[ch14biblio01entry20] (S1) H. H. Steinour, *Ind. Eng. Chem.*, **36**, 618, 840 (1944).

[ch14biblio01entry21] (S2) C. B., Shepherd, and C. E. Lapple, *Ind. Eng. Chem.*, **31**, 972 (1939); **32**, 1246 (1940).

[ch14biblio01entry22] (T1) W. P., Talmage, and E. B. Fitch, *Ind. Eng. Chem.*, **47**(1), 38 (1955).

[ch14biblio01entry23] (Z1) F. A., Zenz, and D. F. Othmer, *Fluidization and Fluid-Particle Systems*. New York: Reinhold Publishing Co., Inc., 1960.