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TRANSPORT PHENOMENA I

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Lec "2"

2.3 :- General molecular transport eq. for momentum, heat and mass transfer.

* each molecule have a given quantity of property associated with it

* differance of property concentration from one region to an adjacent region ⟷ net transport of the property occurs.

"مثلاً" عندي نقطة ساخنة ونقطة باردة ـ عندي فرق بال heat ـه بيصير عندي انتقال
للـ heat من الـ high للـ low .

⇒ Dilute fluid (gases) :-

* molecule are relatively far apart.
* rate of transport of property should be relatively fast.
* few molecules are present to block the transport or interact.

⇒ Dense fluid (Liquids) :-

* molecule are relatively close together.
* transport or diffusion proceeds more slowly.

⇒ Solids :-

* molecule are relatively even more close packed than in Liquid.
* molecule migration is even more restricted. مقيدة

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* Rate of transfer process = $\frac{\text{driving Force}}{\text{resistance}}$

حارح يتم الترانسفير إلا إذا كان في قوة تتغلب على المقاومة

$$\boxed{\psi_z = -\delta \frac{d\Gamma}{dz}}$$

⇒ ψ_z = flux of property = amount o property being trasfered per unit time per cross sectinal area ⊥ to the z diriction of flow [=] amount of property / $s.m^2$

⇒ δ = proportionaly constant = diffusivity [=] m^2/s

⇒ Γ = Concentration of the property [=] amount of property / m^3

⇒ z = distance in the diriction of flow [=] m

* At steady state, ψ_z is constant

$$\psi_z \int_{z_1}^{z_2} dz = -\delta \int_{\Gamma_1}^{\Gamma_2} d\Gamma$$

⇒ $\boxed{\psi_z = \frac{\delta(\Gamma_1 - \Gamma_2)}{z_2 - z_1}}$ at steady state

Five Apple

مع بعض والتعويض بالإشارة

* For unsteady state :- Gen + cons

الـ acc هو تغير الـ prop مع "الزمن دائماً"

$$\text{In} - \text{Out} + \text{Gen} = \text{Acc}$$

unit area

$$(\psi_z)\cdot 1 \qquad (\psi_{(z+\Delta z)})\cdot 1 \qquad R(\Delta z \cdot 1) \qquad \frac{\partial \Gamma}{\partial t}(\Delta z \cdot 1)$$

$$\Rightarrow \psi_z + R(\Delta z) = \psi_{(z+\Delta z)} + \frac{\partial \Gamma}{\partial t}(\Delta z)$$

divided by Δz , Δz go to zero

$$\left\{ \Rightarrow \frac{\partial \Gamma}{\partial t} + \frac{\partial \psi_z}{\partial z} = R \quad \Rightarrow \frac{\partial \Gamma}{\partial t} - \delta \frac{\partial^2 \Gamma}{\partial z^2} = R \right\}$$

* IF no generation is present $\Rightarrow \frac{\partial \Gamma}{\partial t} = \delta \frac{\partial^2 \Gamma}{\partial z^2}$

$\Rightarrow$ Kinatic theory page 42 in text book.

* Introduction to molecular transport $\left\{ \psi_z = -\delta \frac{d\Gamma}{dz} \right\}$

momentum transport	heat transport	mass transport
* Newton's law	* Fourier's law	* Fick's law
$\tau_{zx} = -\nu \frac{d(v_x \rho)}{dz}$	$\frac{q_z}{A} = -\alpha \frac{d(\rho c_p T)}{dz}$	$J_{Az} = -D_{AB} \frac{dC_A}{dz}$

τ_{zx}: mom Flux

ν: mom viscosity

Ex 2.3-1

lec "3"

2.4 :- Viscosity of fluid

* when a fluid is flowing throw a closed channle there are 2 types of flow may occur (depending on the velocity of this fluid) :-

1) Laminar flow :-
   * At low velocity.
   * Flow without mixing.
   * like layers.
   * no cross currents ⊥ to the dirichion of flow nor eddies or swirls of fluid.

2) Turbulent flow :-
   * At higher velocity.
   * Eddies are formed.
   * Leads to lateral mixing.

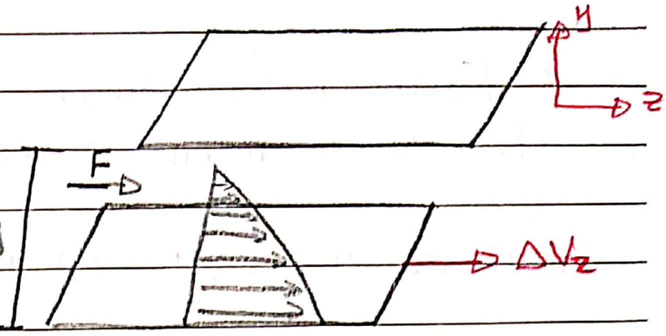
* Viscosity :- A property that lead to force risist the movement of layer in the fluid. stress $= \frac{F}{A}$

⇒ Solid
* elastic deformation $\propto$ to the applied stress يعود لحالته الطبيعية

⇒ Fluid استطيع التفريق بين الصلب والسائل عند تعرضها الـ stress
* Continuos deformation ⇒ لا يعود لحالته الأصلية
* stress ↑ ⇒ velocity ↑

* كلما زادت القوة بين الـ molecule في الفلويد كلما زادت الـ viscosity
ويصير اسم الفلويد ⇐ viscosit fluid

⇒ Laminar flow



* Viscous drag

Viscous Force

لأنه بعمل سحب

* من الأسفل للأعلى السرعة بتقل

$V_{top\,plate} = zero$ *

* exp ⇒ $\frac{F}{A} = -\mu \frac{\Delta V_z}{\Delta y}$

μ → Viscosity of the fluid [=] pa.s [=] Kg/m.s

$$\tau_{yz} = -\mu \frac{dV_z}{dy}$$

τ_{yz} → Shear stress (Force / unit area) [=] N/m^2 [=] pa

(ex 2.4-1)

* 1 cp = 0.01 g/cm.s = 1×10^{-3} Kg/m.s = 1×10^{-3} pa.s

* Momentum transfer in a fluid

⇒ shear stress (τ_{yz}) ≡ Flux of z directed momentum in y direction ≡ rate of flow momentum / unit area

⇒ momentem = mass X velocity [=] Kg.m/s

⇒ $\tau_{yz} = \frac{Kg.m/s}{m^2.s} = \frac{mom}{m^2.s}$

→ Faster moving layer send some of molecules into the slower moving layer

بأدي التصادم بين الجزيئات السريعة والبطيئة إلى تسريع الجزيئات البطيئة
في ال z-dirichion

mom transfer from → high velocity region to → low velocity region

* Viscosities on Newtonian Fluids

- Newtonian Fluids :- Fluid that follow Newton's Law of viscosity $\left(\tau_{zx} = -\nu \frac{d(V_x \rho)}{dz}\right)$

- In Newtonian Fluid there is a linear relation between shear stress τ_{yz} and velocity gradiant $\frac{dv_z}{dy}$ rate of shear stress

اتجاه العمودي على ال اتجاه الحركة واتجاه انتقال ال momentum

اتجاه الحركة

→ For non-newtonian → not linear.

⇒ Gases are newtonian fluids :-

* temp ↑ → Viscosity ↑
* to 1000 pa → Viscosity independent of pressure
* At higher pressure → Viscosity ↑

⇒ In Liquids

* temp ↑ → viscosity ↓
* Viscosity is not affected by pressure because Liquid is not Compressible

Tables in Appendix

Lec "4" Types of fluid and Reynolds number

* Reynolds number ⟷ dimensionless

				turbelence
1) Density	ρ	Kg/m^3	↑
2) Average velocity	$V = \frac{\dot{V}}{A}$ ($\dot{V}$: volumetric flow rate)	m/s	↑
3) Viscosity	μ	pa.s	↓
4) Tube diameter	D	m	↑

$$R_e = \frac{\rho V D}{\mu}$$

⇒ Reynolds no. determined by the ratio of Kinatic or inertial force to the viscous force.

$$\frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho V^2}{\frac{\mu V}{D}} = \frac{\rho V D}{\mu} = Re$$

* For a straight circular pipe :-

$Re \rightarrow < 2100 \equiv$ Laminar flow

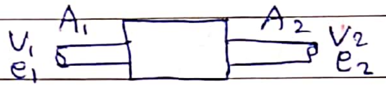
$Re \rightarrow 2100 \sim 4000 \equiv$ Transition region

$Re \rightarrow > 4000 \equiv$ Turbulent flow

* Overall mass balance and Continuity eq.

- Simple mass balances

in = out + acc



- At steady state

in = out

$$m \rightarrow \; m = \rho_1 \underbrace{v_1 A_1}_{V_1} = \rho_2 \underbrace{v_2 A_2}_{V_2}$$

See ex 2.6-1

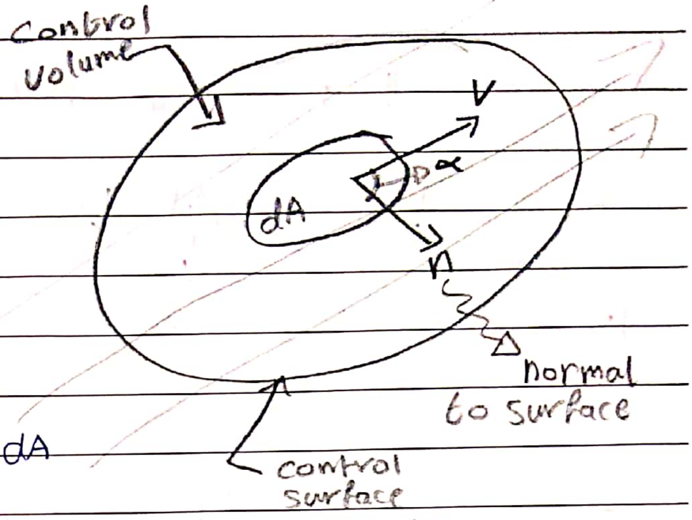
* Control Volume for balance

rate of mass output from control volum − rate of mass input to control volume + rate of mass accum in control volume

= rate of mass Generation in control volume

* ---- acc.. $= \frac{d}{dt}\iiint_V \rho \, dv$

$= \frac{dM}{dt}$



* net mass eflux from control volume $- \iint_A v \rho \cos\alpha \, dA$

at in $\alpha = 180$ $\cos = -1$

at out $\alpha = 0$ $\cos = 1$

$\Rightarrow \iint_A v\rho\cos\alpha \, dA = \iint_{A_2} v\rho\cos\alpha_2 \, dA + \iint_{A_1} v\rho\cos\alpha_1 \, dA$

$= V_2 \rho_2 A_2 - V_1 \rho_1 A_1$

m $\Rightarrow$ $V_2 \rho_2 A_2 - V_1 \rho_1 A_1 + \frac{dM}{dt} = 0$ → Generation

In generation $\Rightarrow$ $m_{i_2} - m_{i_1} + \frac{dM_i}{dt} = R_i$ → gen

note: if velocity is not constant :-

$\rightarrow V_{av} = \frac{1}{A} \iint_A v \, dA$

lec "5"

* Over all momentum balance

- $P = M V \quad [=] \; kg.m/s$

P → linear momentum (victor quantity)

M → mass

V → velocity

- $\sum F = \frac{dP}{dt} \quad [=] \; N$

Gen

- $\boxed{\text{Sum of force}}$ on control volum = rate of mom out − rate in + acc

→ momentum is not conserved because it is generated by external forces on the system

إذا ما عندي قوة خارجية بكون conserved

* For a small element of area dA on a control surface

- rate of momentum eflux $= v(\rho V)(dA \cos\alpha)$

$= \rho v \underbrace{(v \cdot n)}_{\cos\alpha} dA$

→ net mom eflux from control volume $= \iint_A v(\rho v)\cos\alpha \, dA = \iint_A \rho v(v \cdot n)\, dA$

→ rate of accumulation of mom control volume $= \frac{\partial}{\partial t} \iiint_V \rho v \, dV$

$$\Rightarrow \sum F = \iint_A \rho v (v.n)\, dA + \frac{\partial}{\partial t} \iiint_V \rho v\, dV$$

$$\left.\begin{aligned}
&\Rightarrow \sum F_x = \iint_A \rho v\, v_x \cos\alpha\, dA + \frac{\partial}{\partial t} \iiint_V \rho v_x\, dV \\
&\Rightarrow \sum F_y = \iint_A \rho v\, v_y \cos\alpha\, dA + \frac{\partial}{\partial t} \iiint_V \rho v_y\, dV \\
&\Rightarrow \sum F_z = \iint_A \rho v\, v_z \cos\alpha\, dA + \frac{\partial}{\partial t} \iiint_V \rho v_z\, dV
\end{aligned}\right\} \text{vector eq.}$$

* The Force term, $\sum F_x$ is composed of the sum of several forces :-

1) Body Force (F_{xg}) :- Caused by gravity $\{F_{xg} = Mg_x$
   - if it horizontal $\rightarrow F_{xg} =$ zero

2) pressure Force (F_{xp}) :- Caused by the pressure acting on the surface of the fluid.
   - $\perp$ to the surface
   - If gage pressure is used $\Rightarrow$ we can ignored the integral of the constant, pressure over the entire outer surface.
     external

3) Friction Force (F_{xs}) :- Shear or friction force, done on the fluid and solid walls, may be negligible.

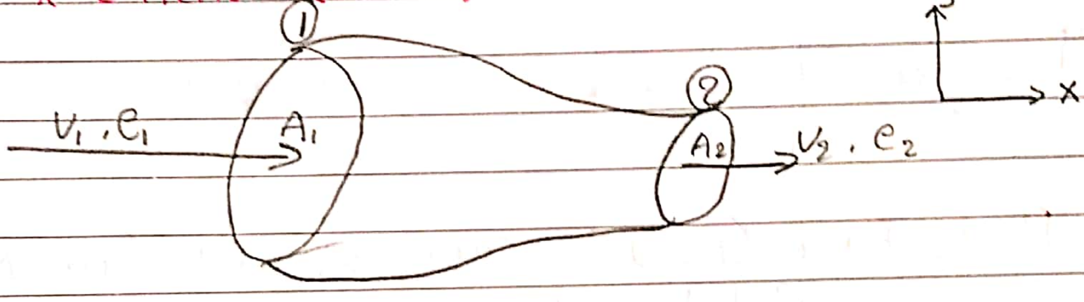
4) Solid Surface force (R_x) :- Cause where the control surface cuts through a solid (Force extered by the solid on the fluid).

$$\rightarrow \sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x$$

(similar eq. for y, z)

$$= \iint_A \rho v v_x \cos\alpha \, dA + \frac{\partial}{\partial t} \iiint_V \rho \, v_x \, dV$$

* For a fluid flowing at steady state in the control volume in the x-diriction $(v = v_x)$



$$\Rightarrow \sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x = \iint_A v_x \, \rho \, v_x \cos\alpha \, dA$$

$$\{ \cos\alpha = \pm 1 \quad , \quad \rho A = m / V_{av} \}$$

التربيع للسرعة مو للافريج

$$\rightarrow F_{xg} + F_{xp} + F_{xs} + R_x = \frac{m \, (V_{x_2}^2)_{av}}{V_{x_2 av}} - \frac{m \, (V_{x_1}^2)_{av}}{V_{x_1 av}}$$

$$* \quad (V_x^2)_{av} = \frac{1}{A} \iint_A V_x^2 \, dA$$

$$\rightarrow \frac{(V_x^2)_{av}}{V_{x\,av}} = \frac{V_{x\,av}}{\beta} \quad , \quad \beta \rightarrow \begin{cases} 0.95 - 0.99 & \text{for turbulent flow} \\ 0.75 & \text{for laminar flow} \end{cases}$$

(ex 2.8-1)

$\rightarrow F_{xp} = P_1 A_1 - P_2 A_2$, F_{xs} will be neg , $F_{xg} = 0$

gravity is acting only in y dir

Flat velocity ≡ turbilant

⟹ $R_x = mV_2 - mV_1 + P_2A_2 - P_1A_1$

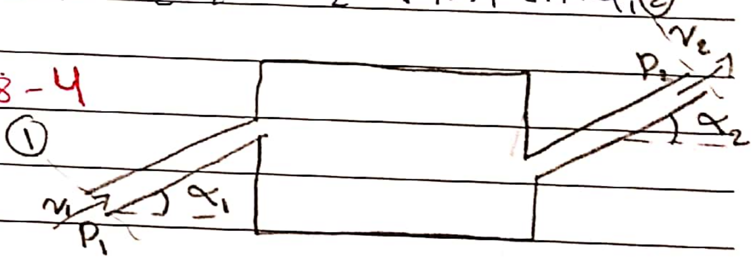
- R_x is the force exerted by the solid on fluid
- the force of the fluid on solid is $(-R_x)$

* overall momentum balance in 2D

$$R_x = mV_2 \cos\alpha_2 - mV_1 \cos\alpha_1 + P_2A_2 \cos\alpha_2 - P_1A_1 \cos\alpha_1$$

$$R_y = mV_2 \sin\alpha_2 - mV_1 \sin\alpha_1 + P_2A_2 \sin\alpha_2 - P_1A_1 \sin\alpha_1$$

See ex 2.8-3, 2.8-4



* Overall momentum balance for free jet striking a fixed vane

(2 pic in page 77)

⟶ For smooth flat vane (b)

* $m_2 = \frac{m_1}{2}(1 + \cos\alpha_2)$
* $m_3 = \frac{m_1}{2}(1 - \cos\alpha_2)$

⟹ the resultant force exerted by the p:-

resultant force $= R = m_1 V_1 \sin\alpha_2$

OR ⟹ $R_x = m_2V_2\cos\alpha_2 - m_1V_1\cos\alpha_1 + m_3V_3(-\cos\alpha_2)$

$R_y = m_2V_2\sin\alpha_2 - m_1V_1\sin\alpha_1 + m_3V_3(-\sin\alpha_2)$

* $V_1 = V_2 = V_3$ (no energy loss) ⟹ $R = \sqrt{R_x^2 + R_y^2}$

See P.2.8-3

Lec "6"

2.9 :- Shell momentum balance and velocity profile In laminar flow.

- overall momentum balance does not tell about the details of what happens inside the control volume.

- Small control volume will be analyzed and then sharnk to differential size.

* Shell momentum balance on a fluid flowing in a Circular tube. (pic in page 78)

- horizontal siction of pipe
- the fluid is incompressible Newtonian fluid
- the fluid is flowing in one-dimensional, steady state and Laminar flow

* the flow is fully developed :→
- It isn't influenced by entrance effects.
- the velocity profile dose not vary along the axis of flow in the x-direction.

→ At steady state

sum of force acting on control volum = rate of mom out control volume − rate of mom into control volume

* pressure force $= PA|_x - PA|_{x+\Delta x} = p(2\pi r\Delta r)|_x - p(2\pi r\Delta r)|_{x+\Delta x}$

* Shear force $= (\tau_{rx} 2\pi r\Delta x)|_{r+\Delta r} - (\tau_{rx} 2\pi r\Delta x)|_r$ = net rate of mom efflux

$\Rightarrow$ Flow is fully developed $\rightarrow$ net convective mom flux across the annular surface at x and $x+\Delta x$ is zero and the terms are indep of $x \Rightarrow V_x|_x = V_x|_{x+\Delta x}$

$$\Rightarrow p(2\pi r\Delta r)|_x - p(2\pi r\Delta r)|_{x+\Delta x} = (\tau_{rx} 2\pi r\Delta x)|_{r+\Delta r} - (\tau_{rx} 2\pi r\Delta x)|_r$$

$$\frac{r(P|_x - P|_{x+\Delta x})}{\Delta x} = \frac{(r\tau_{rx})|_{r+\Delta r} - (r\tau_{rx})|_r}{\Delta r}$$

* In fully developed flow $\Rightarrow \frac{\Delta P}{\Delta x}$ = constant $= \frac{\Delta P}{L}$ $L \rightarrow$ pipe Length

$$\Rightarrow \frac{d(r\tau_{rx})}{dr} = r\left(\frac{\Delta P}{\Delta L}\right)$$, Sepating and integrating

$$\tau_{rx} = \left(\frac{\Delta P}{L}\right)\frac{r}{2} + \frac{C_1}{r}$$ at $r=0$, $C_1 = 0$

$$\Rightarrow \tau_{rx} = \frac{\Delta P\, r}{2L} \Rightarrow \tau_{rx} = \left(\frac{P_o - P_L}{2L}\right) r$$ Linear relation with r

* the max value when $R = r$ at the wall

$$\Rightarrow \tau_{rx} = -\mu \frac{dV_x}{dr} = \left(\frac{P_o - P_L}{2L}\right) r$$ separte and integrate $V_x = 0$ at $r = R$

$$\Rightarrow V_x = \frac{P_o - P_L}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2\right]$$ {velocity distribution is parabolic}

* at $r=0$ (center) $\rightarrow$ max velocity $\rightarrow$ shear $= 0$
* at walls $\rightarrow$ Velocity = zero

$$\Rightarrow V_{x\,av} = \frac{1}{A}\iint_A V_x\,dA = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R V_x\,r\,dr\,d\theta = \int_0^R V_x\,2\pi r\,dr$$

$$\Rightarrow \boxed{V_{x\,av} = \frac{(P_o - P_L)\,R^2}{8\mu L} = \frac{(P_o - P_L)\,D^2}{32\mu L}}$$

* max velocity $\rightarrow r = 0$

$$\boxed{V_{x\,max} = \frac{P_o - P_L}{4\mu L}\,R^2}$$

$$\boxed{V_{x\,av} = \frac{V_{x\,max}}{2}}$$

* Shell momentum balance for Falling Film

(pic in page 81)

* we use Newtonian fluid, Laminar flow.

$$\rightarrow \text{net efflux} = \underbrace{L\overset{\text{العمق}}{W}}_{A}(\tau_{xz})\Big|_{x+\Delta x} - LW(\tau_{xz})\Big|_{x}$$

$$\rightarrow \left(V_z\Big|_{z=0} = V_z\Big|_{z=L}\right) \rightarrow \text{net efflux} = \text{zero}$$

الفرق بين أعلى نقطة وأخفض نقطة

$$\text{net efflux} = \Delta x W V_z (\underbrace{\rho V_z}_{\dot m})\Big|_{z=L} - \Delta x W V_z(\rho V_z)\Big|_{z=0} = 0$$

* gravity force $= \underbrace{\Delta x W L}_{V}(\rho g)$

$$\Rightarrow \Delta x W L(\rho g) = LW(\tau_{xz})\Big|_{x+\Delta x} - LW(\tau_{xz})\Big|_{x} + 0$$

$$\frac{\tau_{xz}|_{x+\Delta x} - \tau_{xz}|_{x}}{\Delta x} = \rho g$$

$$\frac{d}{dx}\tau_{xz} = \rho g$$, integrate

then → $$\boxed{\tau_{xz} = \rho g x}$$

→ $$\tau_{xz} = -\mu \frac{dv_x}{dx} = \rho g x$$

→ $$\frac{dv_x}{dx} = -\left(\frac{\rho g}{\mu}\right) x$$, integrate

→ $$\boxed{V_x = \frac{\rho g \delta^2}{2\mu}\left[1-\left(\frac{x}{\delta}\right)^2\right]}$$

→ $$\boxed{V_{z\,max} = \frac{\rho g \delta^2}{2\mu}}$$

→ $$\boxed{V_{z\,av} = \frac{\rho g \delta^2}{3\mu}}$$

→ $$\boxed{V_{z\,av} = \frac{2}{3} V_{z\,max}}$$

volumetric flow rate → $$\boxed{q = \frac{\rho g \delta^3 W}{3\mu}}$$ m^3/s

* Γ :- mass rate of flow / unit width of walls ($kg/s.m$)

→ $$\boxed{\Gamma = \rho\delta V_{z\,av}}$$

→ $$\boxed{N_{Re} = \frac{4\Gamma}{\mu} = \frac{4\rho\delta V_{z\,av}}{\mu}}$$

$N_{Re} < 1200$ → Laminar

see P. 2.9-1

Chapter "3"

3.6: Differential equation of continuity

→ **Overall balances:**

الكم دقيقة كنه سابقا

* allow to solve many elementary problemes on fluid flow.
* done on a control volume
* don't require knowledge of what happen inside the finite control volume.

التفاصيل الداخلية

* if we need greater details about what happen inside ---
  * use a differential element for a control volume.
  * differential balances in a single phase boundary using the boundary conditions.

→ Diff momentem balance eq. is based on Newton's second law

* allowes to determine the way of velocity varies with position and time.
* allows to determin the pressure drop in laminar flow.

→ The eq. of mom bal can be used for turbelent flow (with certain modification). → مع بعض التعديلات

→ Often these conservation eq. are called (eq. of change)
→ because they describe the variations in the properties of the fluid with respect to position and time. تغير

* Types of time derivations and Vector Notation

التغير ليس في الزمن فقط

1. Partial time derivative : the local change of fluid prop with time at a fixed point x, y and z.

ex ⇒ $\frac{\partial e}{\partial t}$ = partial time derivative of density e.

2. Total time derivative :

$$\frac{de}{dt} = \frac{\partial e}{\partial t} + \frac{\partial e}{\partial x}\frac{dx}{dt} + \frac{\partial e}{\partial y}\frac{dy}{dt} + \frac{\partial e}{\partial z}\frac{dz}{dt}$$

⇒ the density is a fun of (t) and of the velocity components (dx/dt, dy/dt and dz/dt) at which the observer is moving.

3. Substantial time derivation : derivative that follows the motion.

$$\frac{D\rho}{Dt} = \frac{\partial e}{\partial t} + v_x\frac{\partial e}{\partial x} + v_y\frac{\partial e}{\partial y} + v_z\frac{\partial e}{\partial z} + (\vec{v} \cdot \nabla e)$$

4. Scalars : quantity have magnitude but no direction
Such as : Concentration, temp, Length, Volume and time.

5. Vectors : have magnitude and direction. such as : velocity, force, momentum, acceleration. → يكتب الرمز وفوقه

⇒ $\vec{B} = iB_x + jB_y + kB_z$

$r\vec{B} = \vec{B}r$

$(\vec{B} \cdot \vec{C}) = (\vec{C} \cdot \vec{B})$

$(\vec{B} \cdot \vec{C})\vec{D} \neq \vec{B}(\vec{C} \cdot \vec{D})$

$(\vec{B} \cdot \vec{C}) = BC \cos\phi_{BC}$ → الزاوية بين B و C

6. Differential Operations with scalar and vectors :-

→ the gradiant or "grad" of the scalar field is

$$\nabla e = i\frac{\partial e}{\partial x} + j\frac{\partial e}{\partial y} + k\frac{\partial e}{\partial z}$$

→ the divergence or "div" of a vector $\underline{V}$ is

$$(\nabla \cdot V) = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

→ the laplacian of a scalar field is

المشتقة الثانية $$\nabla^2 e = \frac{\partial^2 e}{\partial x^2} + \frac{\partial^2 e}{\partial y^2} + \frac{\partial^2 e}{\partial z^2}$$

→ $\nabla r s = r \nabla s + s \nabla r$

$$(\nabla \cdot s\vec{V}) = (\nabla s \cdot \vec{V}) + s(\nabla \cdot \vec{V})$$

$$V \cdot \nabla s = V_x\frac{\partial s}{\partial x} + V_y\frac{\partial s}{\partial y} + V_z\frac{\partial s}{\partial z}$$

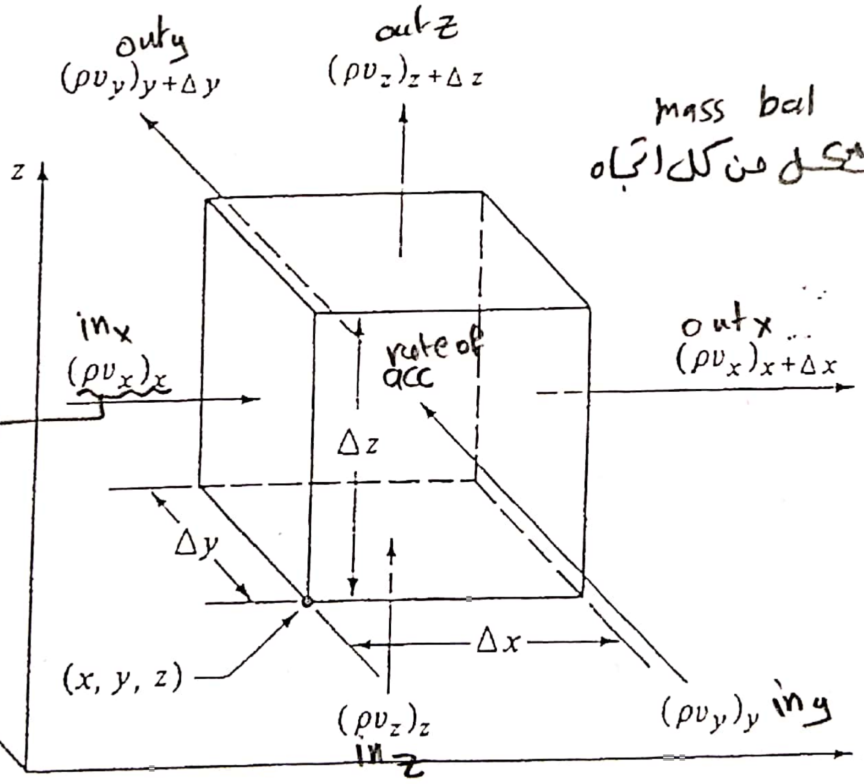
3.6C Differential Equation of Continuity

1. *Derivation of equation of continuity.* A mass balance will be made for a pure fluid flowing through a stationary volume element $\Delta x \, \Delta y \, \Delta z$ which is fixed in space as in Fig. 3.6-2. The mass balance for the fluid with a concentration of ρ kg/m^3 is

$$\text{(rate of mass in)} - \text{(rate of mass out)} = \text{(rate of mass accumulation)} \tag{3.6-17}$$

In the x direction the rate of mass entering the face at x having an area of $\Delta y \, \Delta z$ m^2 is $(\rho v_x)_x \, \Delta y \, \Delta z$ kg/s and that leaving at $x + \Delta x$ is $(\rho v_x)_{x+\Delta x} \, \Delta y \, \Delta z$. The term (ρv_x) is a mass flux in kg/s · m^2. Mass entering and that leaving in the y and the z directions are also shown in Fig. 3.6-2.

بنا نعمل mass bal
على هاد الشكل من كل اتجاه



نضرب كل متجه بالمساحة
حتى أحصل على الكتلة
$m = \rho v \underline{A}$

FIGURE 3.6-2. *Mass balance for a pure fluid flowing through a fixed volume $\Delta x \, \Delta y \, \Delta z$ in space.*

The rate of mass accumulation in the volume $\Delta x\ \Delta y\ \Delta z$ is

$$\text{rate of mass accumulation} = \underbrace{\Delta x\ \Delta y\ \Delta z}_{\text{Volume}}\frac{\partial \rho}{\partial t} \tag{3.6-18}$$

Substituting all these expressions into Eq. (3.6-17) and dividing both sides by $\Delta x\ \Delta y\ \Delta z$,

$$\frac{[\overset{\text{in}}{(\rho v_x)_x} - \overset{\text{out}}{(\rho v_x)_{x+\Delta x}}]}{\Delta x} + \frac{[(\rho v_y)_y - (\rho v_y)_{y+\Delta y}]}{\Delta y} + \frac{[(\rho v_z)_z - (\rho v_z)_{z+\Delta z}]}{\Delta z} = \frac{\partial \rho}{\partial t} \tag{3.6-19}$$

Taking the limit as Δx, Δy, and Δz approach zero, we obtain the equation of continuity or conservation of mass for a pure fluid.

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] = -(\nabla \cdot \rho \mathbf{v}) \tag{3.6-20}$$

(The vector notation on the right side of Eq. (3.6-20) comes from the fact that $\mathbf{v}$ is a vector.) Equation (3.6-20) tells us how density ρ changes with time at a fixed point resulting from the changes in the mass velocity vector $\rho \mathbf{v}$.

We can convert Eq. (3.6-20) into another form by carrying out the actual partial differentiation.

من التعريفات السابقة

$$\frac{\partial \rho}{\partial t} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) - \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z}\right) \tag{3.6-21}$$

Rearranging Eq. (3.6-21),

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) \tag{3.6-22}$$

The left-hand side of Eq. (3.6-22) is the same as the substantial derivative in Eq. (3.6-2). Hence, Eq. (3.6-22) becomes

$$\frac{D\rho}{Dt} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = -\rho(\nabla \cdot \mathbf{v}) \tag{3.6-23}$$

2 (*Equation of continuity for constant density.*) Often in engineering with liquids that are relatively incompressible, the density ρ is essentially constant. Then ρ remains constant for a fluid element as it moves along a path following the fluid motion, or $D\rho/Dt = 0$. Hence, Eq. (3.6-23) becomes for a fluid of constant density at steady or unsteady state,

$$\Rightarrow\ (\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{3.6-24}$$

At steady state, $\partial \rho/\partial t = 0$ in Eq. (3.6-22).

EXAMPLE 3.6-1. Flow over a Flat Plate

An incompressible fluid flows past one side of a flat plate. The flow in the x direction is parallel to the flat plate. At the leading edge of the plate the flow is uniform at the free stream velocity v_{x0}. There is no velocity in the z direction. The y direction is the perpendicular distance from the plate. Analyze this case using the equation of continuity.

Solution: For this case where ρ is constant, Eq. (3.6-24) holds.

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \tag{3.6-24}$$

Since there is no velocity in the z direction, we obtain

$$\frac{\partial v_x}{\partial x} = -\frac{\partial v_y}{\partial y} \tag{3.6-25}$$

At a given small value of y close to the plate, the value of v_x must decrease from its free stream velocity v_{x0} as it passes the leading edge in the x direction because of fluid friction. Hence, $\partial v_x/\partial x$ is negative. Then from Eq. (3.6-25), $\partial v_y/\partial y$ is positive and there is a component of velocity away from the plate.

3\. *Continuity equation in cylindrical and spherical coordinates.* It is often convenient to use cylindrical coordinates to solve the equation of continuity if fluid is flowing in a cylinder. The coordinate system as related to rectangular coordinates is shown in Fig. 3.6-3a. The relations between rectangular x, y, z and cylindrical r, θ, z coordinates are

$$\boxed{x = r\cos\theta} \qquad \boxed{y = r\sin\theta} \qquad \boxed{z = z}$$
$$\boxed{r = +\sqrt{x^2 + y^2}} \qquad \boxed{\theta = \tan^{-1}\frac{y}{x}} \tag{3.6-26}$$

Using the relations from Eq. (3.6-26) with Eq. (3.6-20), the equation of continuity in cylindrical coordinates is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0 \tag{3.6-27}$$

For spherical coordinates the variables r, θ, and ϕ are related to x, y, z by the following as shown in Fig. 3.6-3b.

$$\boxed{x = r\sin\theta\cos\phi} \qquad \boxed{y = r\sin\theta\sin\phi} \qquad \boxed{z = r\cos\theta}$$
$$\boxed{r = +\sqrt{x^2 + y^2 + z^2}} \qquad \boxed{\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}} \qquad \boxed{\phi = \tan^{-1}\frac{y}{x}} \tag{3.6-28}$$

The equation of continuity in spherical coordinates becomes

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\phi)}{\partial \phi} = 0 \tag{3.6-29}$$

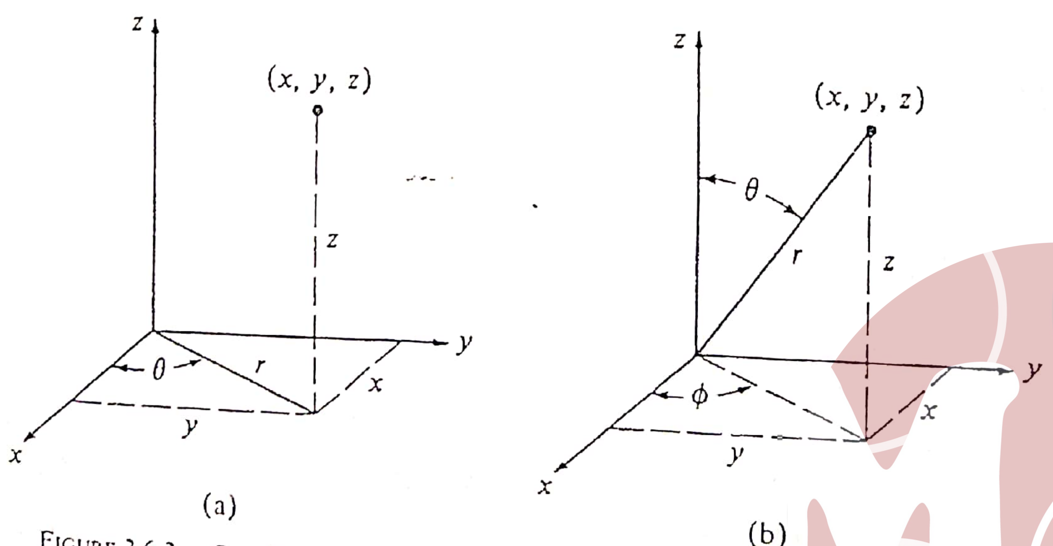


FIGURE 3.6-3. *Curvilinear coordinate systems: (a) cylindrical coordinates, (b) spherical coordinates.*

3.7: Differential equation of momentum Transfer

→ eq. of motion == eq. for the conservation of mom eq.

in − out + ΣF = acc

* Consider the x-component of each term:

– net convective x-momentum flow into the volume element ($\Delta x \Delta y \Delta z$) is:

$$\left[(\rho v_x v_x)_x - (\rho v_x v_x)_{x+\Delta x}\right]\Delta y \Delta z + \left[(\rho v_y v_x)_y - (\rho v_y v_x)_{y+\Delta y}\right]\Delta x \Delta z$$

$$+\left[(\rho v_z v_x)_z - (\rho v_z v_x)_{z+\Delta z}\right]\Delta x \Delta y$$

معناها تركيز الخاصية (الزخم هنا)

→ ρv_x ⇒ Concentration [=] mom/m^3

$\rho v_x v_x$ ⇒ mom flux [=] mom/$s.m^2$

(mom flux) * (Area) = Total mom

– net x-component of momentum by molecular transfer is:

$$\left[(\tau_{xx})_x - (\tau_{xx})_{x+\Delta x}\right]\Delta y \Delta z + \left[(\tau_{yx})_y - (\tau_{yx})_{y+\Delta y}\right]\Delta x \Delta z$$

$$+\left[(\tau_{zx})_z - (\tau_{zx})_{z+\Delta z}\right]\Delta x \Delta y$$

→ τ_{yx} = x direction shear stress on the y face

τ_{xx} = normal stress on the x face

– net fluid pressure force:-

$$\left[P_x - P_{x+\Delta x}\right]\Delta y \Delta z$$

- Gravitational force in the x-dirichion

$\Rightarrow \rho g_x \Delta x \Delta y \Delta z$

$\Rightarrow g_x$ = x component of the gravitational vector g

- Rate of Accumulation of x-momentum in the element is:

$\Delta x \Delta y \Delta z \dfrac{\partial(\rho V_x)}{\partial t}$ $\quad \underbrace{\rho V}_{m} = mom$

$\Rightarrow$ substituting, dividing by $\Delta x \Delta y \Delta z$, and taking the limit as $\Delta x, \Delta y, \Delta z \to 0$

$$\frac{\partial(\rho V_x)}{\partial t} = -\left[\frac{\partial(\rho V_x V_x)}{\partial x} + \frac{\partial(\rho V_y V_x)}{\partial y} + \frac{\partial(\rho V_z V_x)}{\partial z}\right] - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial P}{\partial x} + \rho g_x$$

$\Rightarrow$ x-component of the differential eq. of motion

مشتقة مزد
ونعوض

$\Rightarrow$ Using the eq. of continuity

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z}\right]$$

$\Rightarrow$ eq. of motion for the x, y and z components are obtained :-

We can use Eq. (3.6-20), which is the continuity equation, and Eq. (3.7-7) and obtain an equation of motion for the x component and also do the same for the y and z components as follows:

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = -\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right] + \rho g_x - \frac{\partial p}{\partial x} \tag{3.7-10}$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = -\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right] + \rho g_y - \frac{\partial p}{\partial x} \tag{3.7-11}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = -\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right] + \rho g_z - \frac{\partial p}{\partial x} \tag{3.7-12}$$

Adding vectorially, we obtain an equation of motion for a pure fluid.

$$\rho \frac{D\mathbf{v}}{Dt} = -(\nabla \cdot \boldsymbol{\tau}) - \nabla p + \rho \mathbf{g} \tag{3.7-13}$$

1. Shear-stress components for Newtonian fluids in rectangular coordinates

$$\tau_{xx} = -2\mu \frac{\partial v_x}{\partial x} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v}) \tag{3.7-14}$$

$$\tau_{yy} = -2\mu \frac{\partial v_y}{\partial y} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v}) \tag{3.7-15}$$

$$\tau_{zz} = -2\mu \frac{\partial v_z}{\partial z} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v}) \tag{3.7-16}$$

$$\tau_{xy} = \tau_{yx} = -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \tag{3.7-17}$$

$$\tau_{yz} = \tau_{zy} = -\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) \tag{3.7-18}$$

$$\tau_{zx} = \tau_{xz} = -\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right) \tag{3.7-19}$$

$$(\nabla \cdot \mathbf{v}) = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) \tag{3.7-20}$$

4. Equation of Motion for Newtonian fluids with varying density and viscosity After Eqs. (3.7-14)–(3.7-20) for shear-stress components are substituted into Eq. (3.7-10) for the x-component of momentum, we obtain the general (equation of motion for a Newtonian fluid with varying density and viscosity)

$$\rho \frac{Dv_x}{Dt} = \frac{\partial}{\partial x}\left[2\mu \frac{\partial v_x}{\partial x} - \frac{2}{3}\mu(\nabla \cdot \mathbf{v})\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)\right] - \frac{\partial p}{\partial x} + \rho g_x \tag{3.7-35}$$

(Similar equations are obtained for the y and z components of momentum.)

*eq. of continuity for constant density

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$$

1. Equation of motion in rectangular coordinates. For Newtonian fluids for constant ρ and μ for the x component, y component, and z component we obtain, respectively,

1: $x \to$

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) - \frac{\partial p}{\partial x} + \rho g_x \tag{3.7-36}$$

2: $y \to$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) - \frac{\partial p}{\partial y} + \rho g_y \tag{3.7-37}$$

3: $z \to$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) - \frac{\partial p}{\partial z} + \rho g_z \tag{3.7-38}$$

1, 2 and 3 Called the Navier - Strokes eq.s

Combining the three equations for the three components, we obtain

* Notes on section 3.8 :-

- ex 3.8-1 and 3.8-2

* لما أقرأ السؤال بدي أفصله كامل وأهتم بكل المعلومات

* Velocity distribution = change velocity with x , y and z

* steady state ≡ no accumilation (acc = change with time)

* Constant or varying (ρ and μ) ⇒ كي أعرف على اي معادلة أروح
ونختار المعادلة حسب اتجاه السرعة

* بعض الحدود بتنحذف من فهمنا لمضمون السؤال

* المحور الي بالعمق (مثلاً z) ~~بيعني~~ يعني بالمركز بكون عندي max velocity ومثلاً اتجاه السرعة بال x-dir

$$\frac{\partial v_x}{\partial z} = 0 \quad \text{and} \quad \frac{\partial^2 v_x}{\partial z^2} = 0$$

→ at width velocity is constant

* horizontal diriction → $g_x = 0$

* For integration :-
  - at center → max velocity → $\frac{\partial v}{\partial (dir)} = 0$
  - at wall → velocity = 0

-(ex 3.8-3 to 3.8-6)

* Incompressible ≡ constant density

* In cylindrical coordinates :-
  - we can use rectangular coordinates with (relations between rec and cyl in page 169)
  - OR we can use eq. of motion for cyl coor in page 174

* In ex 3.8-4 → max velocity determined by the ratio of r to r_1

* T (torque) $= \tau * A$

Chapter "4"

⇒ Principles of steady-State Heat Transfer

* Introduction:-

- The transfer of energy (heat) occurs in many chemical and other processes.

- Heat transfer often occurs in combination with other unit operations; such as:-
  - drying of lumber of food.
  - alcohol distillation.
  - burning of fuel.
  - evaporation.

- The heat transfer occurs because of a temp difference driving force → (heat flows from high to low temp region)

* General thermal energy balance eq.

rate of heat in + rate of heat gen = rate of heat out + rate of heat accumulation

→ Assuming the rate of heat transfer occurs only by conduction, Fourier's law applies:

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

→ Unsteady state heat balance for the x-direction only on the control volume (pic in page 215)

$$q_x|_x + \underbrace{\dot{q}}_{\text{rate of heat gen per unit volume}}(\Delta x \cdot A) = q_x|_{x+\Delta x} + \rho c_p \frac{\partial T}{\partial t}(\Delta x \cdot A)$$

* At steady state :

$$q_x|_x = q_x|_{x+\Delta x}$$

in = out

→ Goal :→ to obtain expressions for the temp profile and heat flux.

* Basic Mecanisims of heat transfer :-

→ Conduction :-

- Solid, Liquids and gases
- transfer of energy of motion between adjacent molecules.

قريبين من بعض وينقلوا الهيت لبعض لتسبب حدوث الـ vibration

→ Convection :- 2 type :- 1) Forced Convection 2) Natural or Free

مثل المروحة بتجبر الهواء على التحرك

- transfer of heat by bulk transport and mixing of macroscopic elements of warmer portions with cooler portions of a gas or Liquid

زيادة الحرارة بتقلل الكثافة

فيتحرك الجزء السفلي للأعلى

→ Radiation :-

- no physical medium is needed for its propagation.
- through space by means of electromagnetic waves.

بتنتقل عبر الـ space أو الـ gas

examples :-

→ Conductive :-

1- heat transfer through walls of exchangers of a refrigerator.
2- heat treatment of steel forgings.
3 - Freezing of the ground during the winter

/→ Convection

1- loss of heat from a car radiator where the air is being circulated by a fan.

2- Cooking of foods in a vessel being stirred

3- Cooling of a hot cup of coffee by blowing over the surface

→ Radiation

1- transport of the heat to the earth from the sun.

2- Cooking of food when passed below red-hot electric heater.

3- heating of fluids in coils of tubing inside a combustion furnace.

✗ Fourier's Law of heat Conduction

In any property (mass or heat) ↓

rate of a transfer process = $\frac{\text{driving force}}{\text{resistance}}$

heat transfer rate in x-dirixion [=] W

$$\frac{q_x}{A} = -K \frac{dT}{dx}$$

dT → temperature [=] K

dx → distance [=] m

A: Cross-sectional area normal to diriction of heat flow [=] m^2

K: thermal conductivity [=] $W/m^2.K$

$\frac{q_x}{A}$ = heat flux [=] W/m^2 , $\frac{dT}{dx}$ = temp gradiant in the x-diriction

(−)K → if the heat flow is positive in a given dir → the temp decreases in this diriction.

flow of heat اتجاه ال

عكس اتجاه تغير ال temp

* For the case of steady state heat transfer through a flat wall of constant cross-sectional area (A) :-

$$\frac{q_x}{A}\int_{x_1}^{x_2} dx = -K\int_{T_1}^{T_2} dT$$

$$\rightarrow \left\{ \frac{q_x}{A}(x_2 - x_1) = -K(T_2 - T_1) \right\}$$

See ex. 4.1-1

* Thermal Conductivity :-

* Gases

- the molecules are in continuous random motion, colliding with one another and exchanging energy and momentum.

- if a molecule moves from high temp to lower temp → it transport K.E to this region and gives up this energy through collisions with lower energy molecules.

- Smaller molecules move faster, gases such as H_2 should have higher thermal conductivities.

table 4.1-1 page 218

Notes:-

1) Theories to predict K of gases are reasonably accurate $K \propto \sqrt{T}$

2) K is independent of pressure up to a few atmospheres.

3) At very low pressure (vacuum) → $K \simeq 0$

★ Liquids :-

- Mechanism :- higher energy molecules collide with lower energy molecules.

- most correlations to predict the thermal conductivities of Liq are emperical because adequate molecular theory of Liq is not avilable.

- K of Liq varies moderately with temp and often can be expressed as a linear variation :-

$$K = aT + b$$

- K of Liq is ~~ess~~ essentially independent of pressure.

⟹ Water has a high K compared to organic type Liq such as benzene.

★ Solids :-

- K of homog. solids varies quite widely.
- metallic solids of Cu and AL have very high K, and some insulating non metallic materials such as rock wool and corkboard have very low K

- Two mechanism :-
  - ★ In metallic solids :- الحرارة (مثل الكهرباء) يتم توصيلها بواسطة الالكترونات
  - ★ In all another solids :- الحرارة بنتقل عن طريق الاهتزازات بين الذرات

Convective

* when the fluid outside the solid surface is in forced or free convective motion, the rate of heat transfer from solid → fluid or vice versa, is expressed by:-

temp of solid surface [=] K

heat transfer rate [=] W

$$q = h A (T_w - T_f)$$

Convective heat transfer coefficient [=] $W/m^2.K$

Area [=] m^2

average or bulk temp of the fluid flowing by [=] K

→ The convective coefficient h is a function of:-

- system geometry.
- flow velocity.
- fluid properties.
- temp difference.

* In many case → empirical correlations are available to pridict this coefficient.

* when a fluid flows by a surface, there is a thin, almost stationary layer (film) of fluid adjacent to the wall presenting most of the resistance to heat transfer

→ h is often called (film coefficient)

Table 4.1-2 page 219

* Conduction heat transfer :-

(A) Through a flat Slab or wall   pic in page 220

$$\frac{q_x}{A} = -K \frac{dT}{dx} \quad , \text{A and K are constant}$$

$$\Rightarrow \frac{q}{A} = \frac{K}{x_2 - x_1}(T_1 - T_2) = \frac{K}{\Delta x}(T_1 - T_2)$$

* temp varies linearly with distance

$$q = \frac{T_1 - T_2}{\Delta x / KA} = \frac{T_1 - T_2}{R} = \frac{\text{driving force}}{\text{resistance}}$$

$$\rightarrow R = \frac{\Delta x}{KA} \; [=] \; K/W$$

* if $K = a + bT$

$$\frac{q}{A} = \frac{K_m}{\Delta x}(T_1 - T_2) \Rightarrow K_m = a + b\,\frac{T_1 + T_2}{2}$$

(B) Through a Hollow Cylinder   Pic page 221

$$\frac{q}{A} = -K \frac{dT}{dr} \quad , A = 2\pi r L \quad \text{and K is const}$$

$$\frac{q}{2\pi \cdot L}\int_{r_1}^{r_2} \frac{dr}{r} = -K\int_{T_1}^{T_2} dT \Rightarrow q = K \frac{2\pi L}{\ln(r_2/r_1)}(T_1 - T_2)$$

→ A isn't constant :-

$$A_{Lm} = \frac{A_2 - A_1}{\ln(A_2/A_1)} \rightarrow q = K A_{Lm} \frac{T_1 - T_2}{r_2 - r_1} = \frac{T_1 - T_2}{(r_2 - r_1)/(K A_{Lm})}$$

$$\rightarrow q = \frac{T_1 - T_2}{R} \rightarrow R = \frac{r_2 - r_1}{K A_{Lm}} = \frac{\ln(r_2/r_1)}{2\pi K L} \; [=] \; K/W$$

(c) Through a Hollow sphere

$$\frac{q}{A} = -K\frac{dT}{dr} \quad , A = 4\pi r^2 \text{ and } K \text{ is const}$$

$$\Rightarrow \frac{q}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -K\int_{T_1}^{T_2} dT$$

$$\Rightarrow q = \frac{4\pi K}{1/r_2 - 1/r_1}(T_1 - T_2)$$

$$\Rightarrow q = \frac{T_1 - T_2}{(1/r_2 - 1/r_1)/(4\pi K)} \quad , R = \frac{1/r_2 - 1/r_1}{4\pi K}$$

$$[=] \; K/W$$

See ex 4.2-1

☆ Conduction through solids In Series :-

Planes walls in Series — pic page 223

Multilayer cylinders — pic page 225

$$q = \frac{T_1 - T_4}{\Sigma R} = \frac{T_1 - T_4}{R_A + R_B + R_C}$$

$$R_j = \frac{\Delta x_i}{K_j A} \qquad R = \frac{r_2 - r_1}{K A_{Lm}}$$

★ Conduction Through matirials in parallel

$$* q_T = q_A + q_B$$

$$= \frac{T_1 - T_2}{\Delta X_A/(K_A A_A)} + \frac{T_3 - T_4}{\Delta X_B/(K_B A_B)}$$

→ if $T_1 = T_3$ and $T_2 = T_4$

$$\Rightarrow q_T = \left(\frac{1}{R_A} + \frac{1}{R_B}\right)(T_1 - T_2)$$

⇒ Combined convection and conduction and overall coefficients pic 1 page 227 (wall)

$$q = h_i A (T_1 - T_2) = \frac{K_A A}{\Delta X_A}(T_2 - T_3) = h_o A (T_3 - T_4)$$

overall → $q = \dfrac{T_1 - T_4}{\dfrac{1}{h_i A} + \dfrac{\Delta X_A}{K_A A} + \dfrac{1}{h_o A}} = U A \Delta T_{overall}$

$$\frac{1}{U} = \frac{1}{h_i} + \frac{\Delta X_A}{K_A} + \frac{1}{h_o}$$

Pic 2 page 227 (cylindrical)

$$\rightarrow q = h_i A_i (T_1 - T_2) = \frac{K_A A_{A\,lm}}{r_o - r_i}(T_2 - T_3) = h_o A_o (T_3 - T_4)$$

$$q = \frac{T_1 - T_4}{\dfrac{1}{h_i A} + \dfrac{r_o - r_i}{K_A A_{A\,lm}} + \dfrac{1}{h_o A}} = \frac{T_1 - T_4}{\Sigma R} = U A \Delta T_{overall}$$

$$\Rightarrow q = U_i A_i (T_1 - T_4) = U_o A_o (T_1 - T_4)$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{(r_o - r_i) A_i}{K_A A_{ALm}} + \frac{A_i}{h_o A_o}$$

$$\frac{1}{U_o} = \frac{A_o}{h_i A_i} + \frac{(r_o - r_i) A_o}{K_A A_{ALm}} + \frac{1}{h_o}$$

* Conduction with Internal heat generation

ex: electric resistance heaters, nuclear fuel rods

* Heat Generation in plane wall
  - heat is conducted only in the one x-diriction.
  - The other walls are assumed to be insulated.
  - The volumitric rate of heat Generation is $\dot{q}$, W/m^3 and the thermal conductivity of the medium is K $W/m.K$

* The volumetric rate of heat gen is $\dot{q}$ (W/m^3) and the thermal conductivity of the medium is K $(W/m.K)$

* Energy balance without accumulation

$$\Rightarrow q_x|_x + \dot{q}(\Delta x . A) = q_x|_{x+\Delta x}$$

$$-\frac{dq_x}{dx} + \dot{q} A = 0$$

$\Rightarrow$ but $\frac{q_x}{A} = -K \frac{dT}{dx}$, so

$$\frac{d^2 T}{dx^2} + \frac{\dot{q}}{K} = 0 \quad \Rightarrow \text{integ}$$

$$\Rightarrow T = \frac{\dot{q} x^2}{2K} + C_1 x + C_2$$

* Boundary conditions:-

$X = L$ or $-L$, $T = T_w$

$X = 0$, $T = T_o$

$$\Rightarrow T = -\frac{\dot{q}}{2K} X^2 + T_o$$

$$T_o = \frac{\dot{q}}{2K} L^2 + T_w$$

* heat generation in Cylinder

$$T = \frac{\dot{q}}{4K} (R^2 - r^2) + T_o$$

$$T_o = \frac{q}{4K} R^2 + T_w$$

5.6:- Shell energy balance and Temp distribution in solid and laminar flow

* Energy balance is made over (a thin slab or shell) perpendiculer to the diriction of the heat flow.

⇓

* First order diff. eq. $\rightarrow$ heat flux distribution is obtained

⇓

* Subtutate Fourier's law of heat conduction into the heat Flux expression.

$\rightarrow$ First order diff eq. $\Rightarrow T = f$ (position)

* Determination of integration constant by using boundary conditions for temp or heat flux at the boundary surfaces.

☆ For shear stress systems, the law of the conservation of energy is :-

$$\begin{pmatrix}\text{rate of energy in} \\ \text{by convective transport}\end{pmatrix} - \begin{pmatrix}\text{rate of energy out} \\ \text{by convective transp:}\end{pmatrix}$$

$$+ \begin{pmatrix}\text{rate of energy in} \\ \text{by molecular}\end{pmatrix} - \begin{pmatrix}\text{rate of energy out} \\ \text{by molecular}\end{pmatrix}$$

$$+ \begin{pmatrix}\text{rate of external} \\ \text{work done on system}\end{pmatrix} - \begin{pmatrix}\text{rate of external} \\ \text{work done by system}\end{pmatrix}$$

= rate of acc of energy

⇒ Net convective energy transport

— energy transport by the bulk motion

$$\Delta y \Delta z \left[v_x \left(e u + \frac{e v^2}{2}\right)\right]_x - \Delta y \Delta x \left[v_x \left(e u + \frac{e v^2}{2}\right)\right]_{x+\Delta x}$$

⇒ Net moleculer energy transport

— Fourier's law of heat conduction

$$\Delta y \Delta z \left[(q_x)_x - (q_x)_{x+\Delta x}\right]$$

⇒ Net work done by system

* against the grav force

$$-e\ \Delta x \Delta y \Delta z\ (v_x g_x)$$

* against the static pressure (p)

$$\Delta y \Delta z \left[(p v_x)_{x+\Delta x} - (p v_x)_x\right]$$

* against the viscous force

$$(\Delta y \Delta z)\left[(\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z)_{x+\Delta x} - (\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z)_x\right]$$

⇒ writing similar eq in all 3 dim, sub these eq.s into the gen. balance eq., dividing by Δx, Δy and Δz and letting Δx, Δy and Δz → zero,

⇒ we obtain eq (5.6-8) page 366

⇒ Combain eq.(5.6-8) with the eq. of continuity, eq. of motion and expressing the internal energy in terms of fluid temp T and heat capacity C_p

$$\rho C_v \frac{DT}{Dt} = K \nabla^2 T - T\left(\frac{\partial P}{\partial T}\right)_e (\nabla \cdot V) + \mu \phi$$

→ $$K \nabla^2 T = K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

→ $\mu \phi$ = viscous dissipation term (general negligible expect :- where extermaly larg velocity gradiant exist)

* Common Types of Boundary Condition

a) The temp may be specified at a surface.

b) The heat flux normal to a surface may be given (this is equivalent to specifying the normal component of the temp gradiant).

c) At interfaces the continuity of temp and of the heat flux normal to the interface are requiered.

d) At a solid - fluid interface, the normal heat flux component may be related to the differance between the solid surface temp (T_o) and the "bulk" fluid temp (T_b) (Newton's Law of cooling):-

$$q = h(T_o - T_b)$$

→ Fluid at constant pressure :-

$$\rho C_p \frac{DT}{Dt} = K \nabla^2 T - \cancel{T \left(\frac{\partial P}{\partial T}\right)_e (\nabla \cdot v)}$$

Const press

* in rectangular Coordinates

$$\rho C_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right)$$

$$= K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

* in cylindrical coordinates

$$\rho C_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)$$

$$=K\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

* in spherical coordinates

$$\rho C_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{v_\theta}{r\sin\theta}\frac{\partial T}{\partial \phi}\right)$$

$$=K\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right]$$

→ Fluid at Constant Density

$$\rho C_p\frac{DT}{Dt}=K\nabla^2 T$$

→ Solid at Constant Density ($v=0$)

$$\rho C_p\frac{\partial T}{\partial t}=K\nabla^2 T$$

heat generation ⇒ $\rho C_p\dfrac{\partial T}{\partial t}=K\nabla^2 T+\dot{q}$

* Fourier's second law OR unsteady state heat Conduction :-

$$\alpha = \frac{k}{\rho C_p}$$

- rectangular coordinates

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right]$$

- cylindrical coordinates

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right]$$

- Spherical coordinates

$$\frac{\partial T}{\partial t} = \alpha \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right]$$

See ex 5.6-1 and 5.6-2

4.5 :- Forced Convection heat transfer inside pipes

- In most situations involving a liquid or gas in heat transfer, convective heat transfer usually occurs as well as condition

- In most industrial processes where heat transfer is occuring heat is being transformed from one fluid through a solid wall to a second fluid.

pic (4.5-1) page 237 shows :- temp profile for heat transfer by convection from hot flowing fluid to cold flowing fluid

$$\Rightarrow q = hA(T_w - T_f)$$

⇒ The coefficient (h) is a function of :-

1- system geometry
2- Type and velocity of flow
3- fluid physical properties
4- temp difference

* to correlate these data for heat transfer coff, dimensionless number such as :- the Reynolds and Prandtl number are used.

* Prandtl # = is the ratio of the shear component of diffusivity for mom (μ/ρ) to the diffusivity for heat ($k/\rho C_p$), and physically relates the relative thickness of the hydrodynamic layer and thermal boundary layer.

$$Pr = \frac{\mu/\rho}{k/\rho C_p} = \frac{C_p \mu}{k}$$

⇒ Values of Pr is in Appendix A.3, the range for:-

- gases ⇒ 0.5 to 1
- Liquid ⇒ 2 to 10^4

* The dimentionless Nusselt number (Nu)

$$Nu = \frac{hD}{K}$$

→ For flow inside a pipe ⇒ D is the diameter.

* heat transfer coefficient for Laminar flow inside a pipe

⇒ Types of flow: 1- Laminar flow $Re < 2100$

2- transition region $2100 < Re < 6000$

3- fully turbulent flow $Re > 6000$

→ For Laminar flow

$$Nu = \frac{h_a D}{K} = 1.86 \left(Re\, Pr \frac{D}{L}\right)^{1/3} \cdot \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

D: pipe diameter. L: pipe Length befor mixing occurs in the pipe.

μ_b: fluid viscosity at bulk average temp.

μ_w: viscosity at wall temp

K: thermal conductivity

h_a: average heat-transfer coefficient

⇒ * holds for $Re\,Pr\,\frac{D}{L} > 100$, and for > 10 with 20% error.

* In laminar flow, ha depends strongly on heated length.

* the average (arithmetic mean) temp drop (ΔT_a) is used in the eq. to calculate the heat transfer rate q.

$$q = h_a A \Delta T_a = h_a A \frac{(T_w - T_{bi}) + (T_w - T_{bo})}{2}$$

⇒ T_{bi} = inlet bulk fluid temp
T_{bo} = outlet " " "
T_w = wall temp

⇒ For turbulent flow:-

$$Nu = \frac{h_L D}{K} = 0.027\, Re^{0.8}\, Pr^{1/3} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

⇒ holds for $0.7 < Pr < 16000$, $\frac{L}{D} > 60$

⇒ h_L = heat transfer coefficient based on the log mean driving force ΔT_{Lm}

⇒ All fluid property, expect μ_w, are evaluated at meat bulk temp (the mean of the inlet and outlet temp is used if they vary).

⇒ For air at 1 atm total pressure → the following simplified eq. holds for turbulent flow in pipes

$$h_L = 3.52 \frac{v^{0.8}}{D^{0.2}} \quad \text{[in SI unit]}$$

D [=] m , v [=] m/s , h_L [=] $W/m^2.s$

⇒ A simplified eq to use for water in temp range of $T = 4$ to 65 °C is :-

$$h_L = 1429 \left(1 + 0.0146\, T(°C)\right) \frac{v^{0.8}}{D^{0.2}} \quad \text{[in SI unit]}$$

⇒ A very simplifed eq for organic Liquids to use for approximations is :-

$$h_L = 423 \frac{v^{0.8}}{D^{0.2}} \quad \text{[in SI unit]}$$

See ex 4.5-1 page 240

→ For transtion region :-

See Figure (4.5-2) page 241

2.10:G

* Heat transfer coefficient for Noncircular Conduits

→ The equivalent Diameter (D_{eq}) must be used, which is defined as four times the hydraulic radius r_H

$$D_{eq} = 4\, r_H = 4\, \frac{\text{cross-sectional area of channel}}{\text{wetted perimeter of channel}}$$

→ For annular space with outside diameter D_2 and inside D_1 :-

$$D_{eq} = 4 r_H = 4\, \frac{\frac{\pi D_2^2}{4} - \frac{\pi D_1^2}{4}}{\pi D_1 + \pi D_2} = D_2 - D_1$$

→ For a rectangle with Liquid dipth (y) and width (b)

$$D_{eq} = \frac{4by}{b+2y}$$

See ex 4.5-2 . page 241

* Log mean temp difference and varying temp drop

⇒ at one point in the heat-transfer apparatus when the fluids are being heated or cooled,

$$q = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o)$$

⇒ As the fluids travel through the heat exchanger, they become heated or cooled and T_i and/or T_o vary.

→ $(T_i - T_o)$ or ΔT varies with position and some mean ΔT_m must be used over the whole apparatus.

* Two types of heat exchanger :- 1- Countercurrent flow
2- co-current

⟹ Countercurrent (ال flow عكس اتجاهين)

⟹ $q = UA\ \Delta T_m$

⟶ $\Delta T_{lm} = \dfrac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$

(نفس الاتجاه)

ΔT_{lm} should be used also for parallel (co-current) flow

See ex (4.5-4) p 245
(4.6-6) p 246

* Introduction to Radiation heat transfer

⟹ In (radiant) heat transfer the medium through which the heat is transferd usually is not heated

⟹ Radiation heat transfer is the transfer of heat by : electromagnatic radiation

⟹ Radiation often occurs in combination with conduction and convection.

⟹ Thermal Radiation :- is a form of electromagnatic radiation similar to X rays, light waves, gamma rays and so on, differing only in wavelength.

* Thermal Radiation :-

→ It obeys the same laws a light :
- travels in staright lines.
- can be transmitted through space and vacuum.

→ It is an important mode of heat transfer and is especially important where large temp differences occur, for example:
- in a furnace with boiler tubes.
- in radiant dryers
- in an oven baking food

* Mechanism of Radiant heat transfered :-

1) The thermal energy of a hot source, such as the wall of a furnace at T_1, is converted into the energy of electromagnatic radiation waves.

2) These waves travel through the intervening spase in straight lines and strike a cold object at T_2 such as a furnace tube containing water to be heated.

3) The electromagnatic waves that strike the body are absorbed by the body and converted back to thermal energy or heat.

⇒ Absorptivity and black Bodies :-

* When thermal radiation (like light waves) falls upon a body, part is absorbed by the body in the form of heat, part is reflected back into space and part may be actually transmitted through the body.

⇒ For most cases in process engineering, bodies are opaque to transmission, so this will be neglicted

⇒ Opaque Bodies :-

$$\underbrace{\alpha}_{\text{absorptivity or fraction absorbed}} + \underbrace{e}_{\text{reflectivity or fraction reflectied}} = 1$$

absorptivity or fraction absorbed

reflectivity or fraction reflectied

⇒ Black bodies :- one that absorbs all radiant energy and relects none. $e = 0$, $\alpha = 1$

* There are no perfect black bodies, but a close approximation to this is a small hole in a hollow body.

* Black bodies pic page 277

- the inside surface of the hollow body is blackened by charcoal.
- the radiation enters the hole and impinges on the rear wall;
  - part is absorbed there and part is reflected in all diriction.
  - The reflected rays impinges again, part is absorbed and the process continues.

- Hence, essentially all of the energy entering is absorbed and the area of the hole acts as a perfect black body.

- The surface of the inside walls are "rough" and rays are scattered in all direction, unlike a mirror where they are reflected at a definite angle.

- A black body absorbs all radiant energy falling on it and reflects none.

- Such a black body also emits radiation, depending on it's temp, and doesn't reflected any.

- The ratio of emissive power of a surface to that of a black body is called emissivity (ε) and it's (1) for a black body.

- Kirchhoff's law states that at the same temp T_1, α_1 and ε_1 of a given surface are the same or : $\alpha_1 = \varepsilon_1$

(holds for any black or non-black solid surface)

* Radiation From a Body and Emissivity

- Basic eq for heat transfer by radiation from a perfect black body with an emissivity $\varepsilon = 1$ is

heat flow [=] W $\leftarrow q = A \sigma T^4 \rightarrow$ temp of the black body

surface area of body [=] m^2

constant $= 5.676 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$

- For a body that is NOT a black body and $\varepsilon < 1$, the emissive power is reduced by ε, or :-

$$q = A \varepsilon \sigma T^4$$

- Gray Bodies
  - ☆ Substance that have $\varepsilon < 1$ when it is independent of the wavelength.
  - ☆ All real materials have $\varepsilon < 1$

See Table (4.10-1) page 278

* Radiation to a small Object from surroundings

- When we have the case of a small gray object of area $A_1 (m^2)$ at temp T_1 in a large enclosure at a higher temp T_2, there is a net radiation to the small object.

→ The small body emits an amount of radiation to the enclosure $= A_1 \varepsilon_1 \sigma T_1^4$.

→ ε_1 of this body is taken at T_1.

- The small body also absorbs energy from the surroundings at T_2 $= A_1 \alpha_{12} \sigma T_2^4$.

→ α_{12} = the absorptivity of body 1 for radiation from the enclosure at T_2.

→ The value of $\alpha_{12} \approx$ the emissivity of this body at T_2

- The net heat of absorpation is then, by the stefan-Boltzman equation :-

$$q = A_1 \varepsilon_1 \sigma T_1^4 - A_1 \alpha_{12} \sigma T_2^4 = A_1 \sigma (\varepsilon_1 T_1^4 - \alpha_{12} T_2^4)$$

- A further simplification is usually made for engineering purposes, by using one emissivity of the small body at temp T_2, ε.

$$q = A_1 \varepsilon \sigma (T_1^4 - T_2^4)$$

See ex (4.10-1)

* Combined Radiation and Convective heat transfer

- When radiation heat transfer occure from a surface it is usually accompanied by convective heat transfer unless the surface is in a vacuum.

- when radiating surface is at a uniform temp,
  - ⇒ we can calculated the heat transfer for natural or forced convection as befor.
  - ⇒ the radiation heat transfer is calculated by the stafan - Boltzman
  - ⇒ The total rate of heat transfer is the sum of convection + radiation

$$q = q_{conv} + q_{rad} = (h_c + h_r) A_1 (T_1 - T_2)$$

- To obtain an expression for h_r:

$$q_{rad} = h_r A_1 (T_1 - T_2) = A_1 \varepsilon \sigma (T_1^4 - T_2)^4$$

$$\Rightarrow h_r = \varepsilon (5.676) \frac{(T_1/100)^4 - (T_2/100)^4}{T_1 - T_2}$$

⇒ Figure (4.10-2).

* h_r given in english unit calculated with $\varepsilon = 1$

* To use value from the figure, the value obtained from this figure should be multiplied by (ε) to give the value of h_r

* If the air temp is not the same as T_2 of the enclosure, q_{conv} and q_{rad} must be calculated separately and not combined together as in ⇒ $q = q_{conv} + q_{rad}$

see ex (4.10-2)