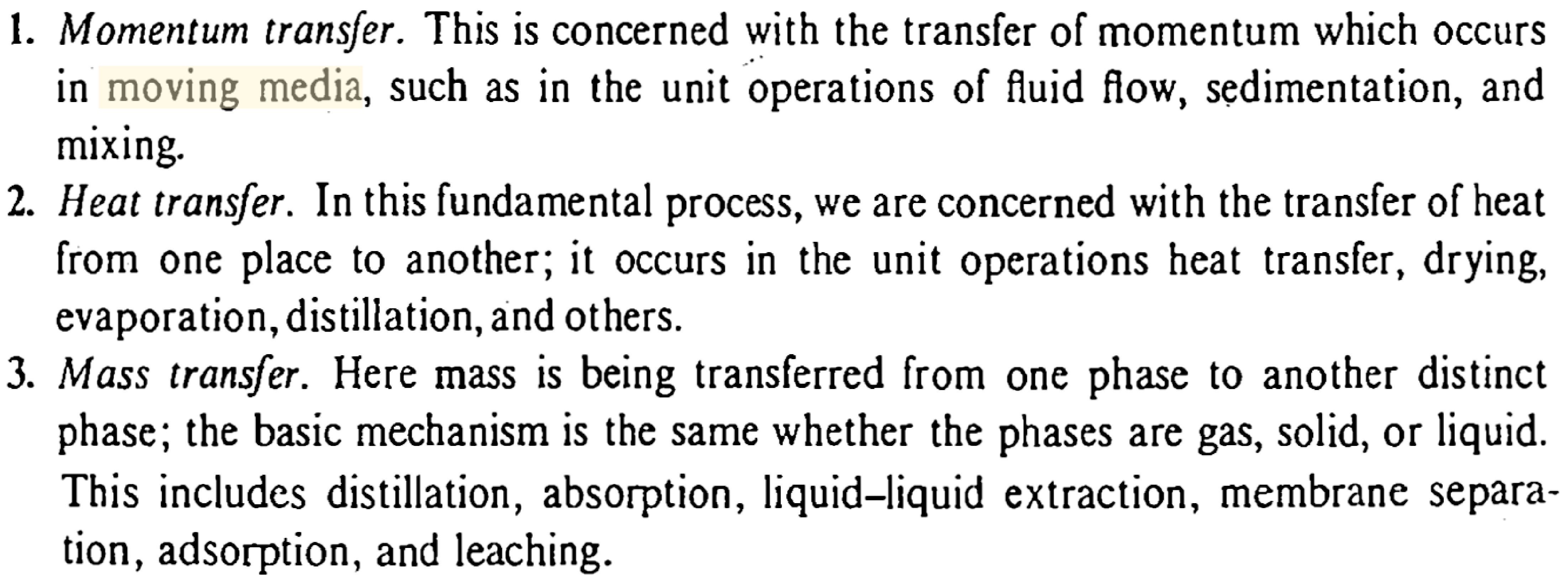
Transport Processes 1

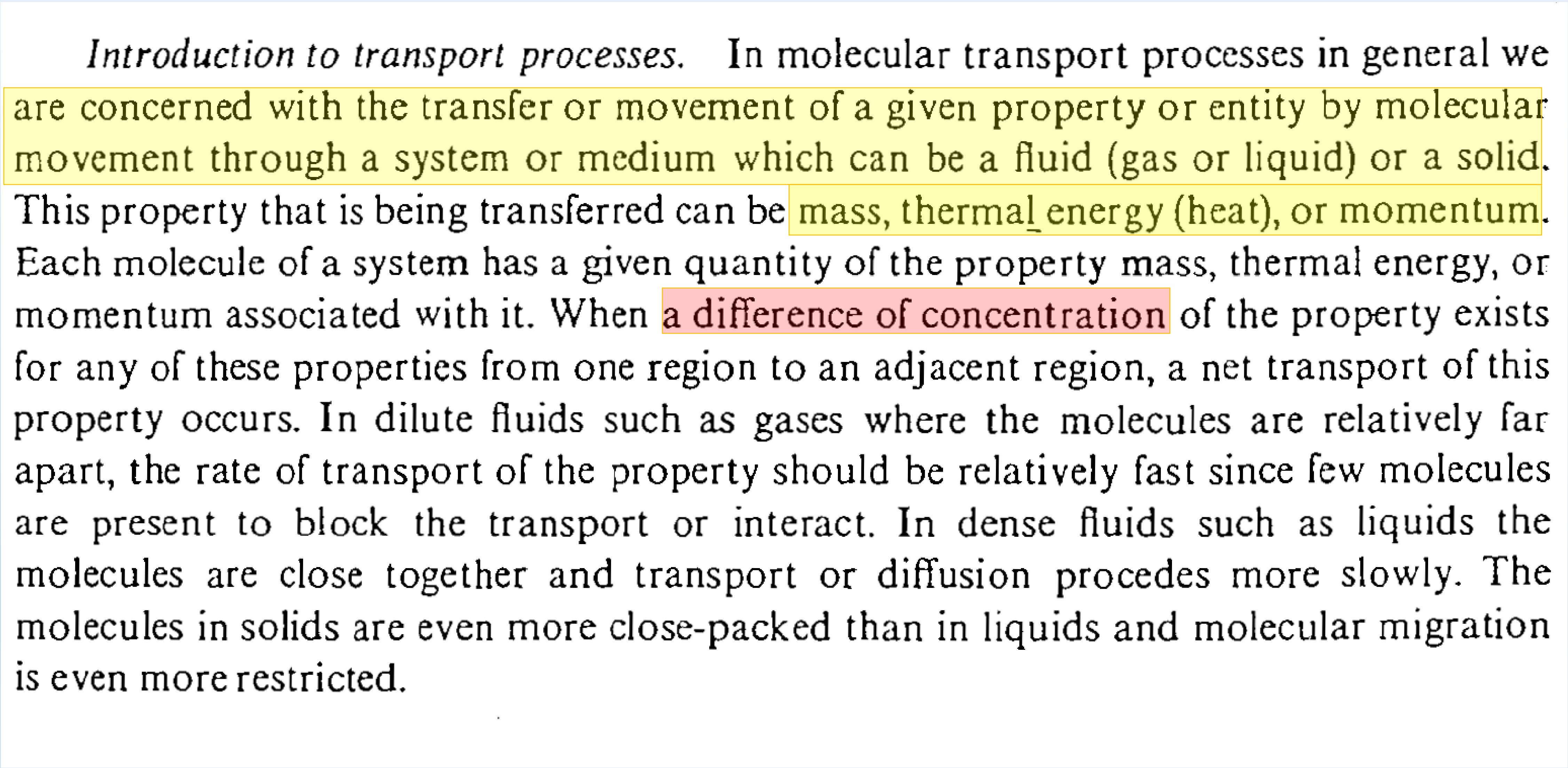
- Momentum Transfer
- Heat Transfer

Fundamental Transport Processes



1. *Momentum transfer.* This is concerned with the transfer of momentum which occurs in moving media, such as in the unit operations of fluid flow, sedimentation, and mixing.
2. *Heat transfer.* In this fundamental process, we are concerned with the transfer of heat from one place to another; it occurs in the unit operations heat transfer, drying, evaporation, distillation, and others.
3. *Mass transfer.* Here mass is being transferred from one phase to another distinct phase; the basic mechanism is the same whether the phases are gas, solid, or liquid. This includes distillation, absorption, liquid–liquid extraction, membrane separation, adsorption, and leaching.

General Molecular Transport Equation



Introduction to transport processes. In molecular transport processes in general we are concerned with the transfer or movement of a given property or entity by molecular movement through a system or medium which can be a fluid (gas or liquid) or a solid. This property that is being transferred can be mass, thermal energy (heat), or momentum. Each molecule of a system has a given quantity of the property mass, thermal energy, or momentum associated with it. When a difference of concentration of the property exists for any of these properties from one region to an adjacent region, a net transport of this property occurs. In dilute fluids such as gases where the molecules are relatively far apart, the rate of transport of the property should be relatively fast since few molecules are present to block the transport or interact. In dense fluids such as liquids the molecules are close together and transport or diffusion procedes more slowly. The molecules in solids are even more close-packed than in liquids and molecular migration is even more restricted.

General Molecular Transport Equation

- All the three transport processes; momentum, heat, mass, has the same general type of transport equation:

$$\text{rate of a transfer process} = \frac{\text{driving force}}{\text{resistance}} \qquad (1)$$

- Therefore, for any transfer process to proceed, we need a driving force to overcome a resistance to a transport process (similar to Ohm’s law in electricity { current, I = voltage /Resistance= V/R}.
- In general, we can formulize eq. 1 by writing an eq. that describe the molecular transport or diffusion of a property as follows

$$\psi_z = -\delta \frac{d\Gamma}{dz} \qquad (2)$$

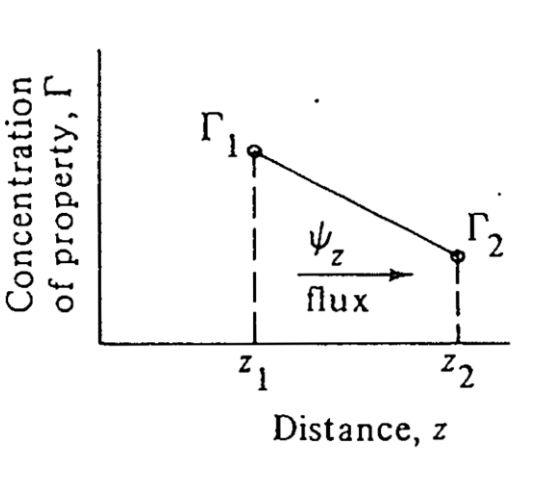
- Where, ψ_z is the property flux, amount of property per unit time per unit area perpendicular to the z direction of flow, δ is a proportionality constant called diffusivity in m^2/s, Γ is concentration of the property in amount of property $/m^3$, z is the distance in the direction of flow in m.

For Steady state process

- The flux ψ_z *is constant* . Thus eq. 2 on rearrangement and integration

$$\psi_z \int_{z_1}^{z_2} dz = -\delta \int_{\Gamma_1}^{\Gamma_2} d\Gamma$$

$$\psi_z = \frac{\delta(\Gamma_1 - \Gamma_2)}{z_2 - z_1} \qquad (3)$$



- Fig. 1 shows a plot of the conc., Γ,

versus z. It is a straight line since

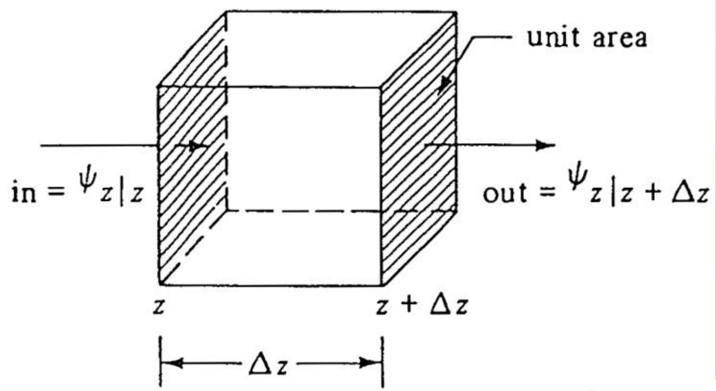
the flux is in the direction 1 to 2 of decreasing concentration, the slope $d\Gamma/dz$ is negative and the negative sign in Eq. (2) gives a positive flux in the direction 1 to 2.

For unsteady state process

- Here, ψ_z *is not constant and must be calculated over all the system. To do this, apply material balance on the system. Consider for the time being z-direction*

$$\begin{pmatrix}\text{rate of}\\ \text{property in}\end{pmatrix} + \begin{pmatrix}\text{rate of generation}\\ \text{of property}\end{pmatrix}$$

$$= \begin{pmatrix}\text{rate of}\\ \text{property out}\end{pmatrix} + \begin{pmatrix}\text{rate of accum-}\\ \text{ulation of property}\end{pmatrix}$$



unsteady-state general property balance

Unit area (1x1)

$$(\psi_{z|z}) \cdot 1 + R(\Delta z \cdot 1) = (\psi_{z|z+\Delta z}) \cdot 1 + \frac{\partial \Gamma}{\partial t}(\Delta z \cdot 1)$$

gen per unit vol

Using differential control volume dz, dy, dx

dy = dx = 1

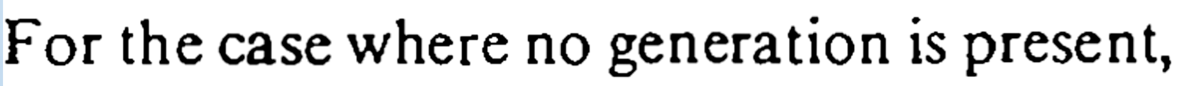
$$\psi_z + R\partial z = \psi_z + \frac{d\psi_z}{dz}\partial z + \frac{\partial \Gamma}{\partial t}\partial z$$

Dividing by ∂z

$$\therefore \quad \frac{d\psi_z}{dz} + \frac{\partial \Gamma}{\partial t} = R \qquad (4)$$

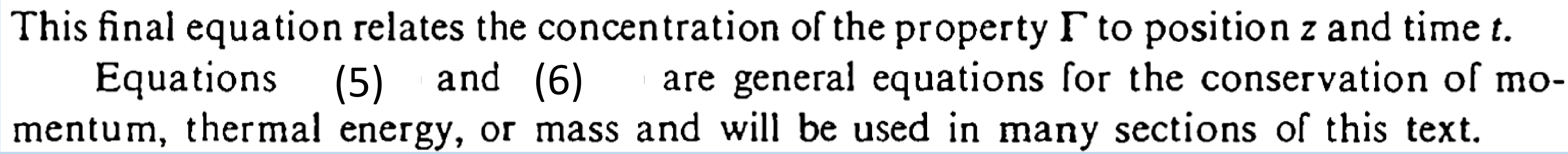
- Substituting ψ_z eq. 2, we obtain

$$\frac{\partial\Gamma}{\partial t} - \delta\frac{\partial^2\Gamma}{\partial z^2} = R \qquad (5)$$



For the case where no generation is present,

$$\frac{\partial\Gamma}{\partial t} = \delta\frac{\partial^2\Gamma}{\partial z^2} \qquad (6)$$



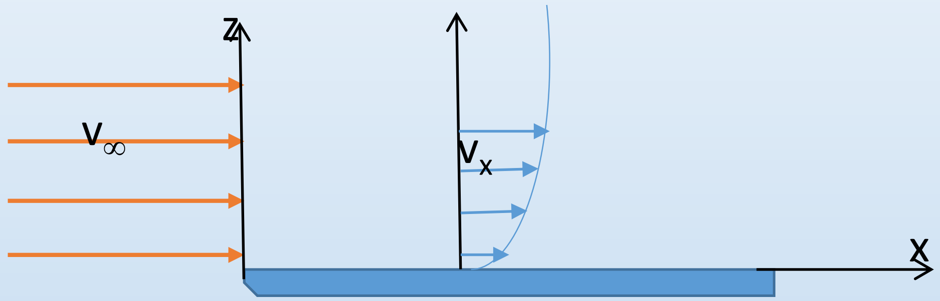
This final equation relates the concentration of the property Γ to position z and time t.

Equations (5) and (6) are general equations for the conservation of momentum, thermal energy, or mass and will be used in many sections of this text.

Molecular transport

1. Momentum Transport and Newton’s Law

- The velocity v_x in x direction decreases as the fluid approaches the surface (with z direction).
- The fluid has x directed momentum and its value is $v_x \cdot \rho$ (momentum/m^3), where momentum has units kg .m/s.



Fluid is flowing in x-direction parallel to a solid surface

- Using the kinetic theory of gases (translation, vibration, collision …etc. are all could be used to lead momentum, energy, mass transfer from high to low levels; high velocity to low velocity, high temp. to low temp., high conc. to low conc. and so on), we can explain how the transport takes place.
- In general, each individual molecule has a property of motion in all directions, hence there are fluxes in all directions. Therefore if there is a conc. gradient of the property, there will be a net flux of this property from high to low conc.

- Return back to figure, due to random diffusion of molecules, there is an exchange of molecules in the z direction, equal number moving in each direction (+ z and – z) between the faster moving layer of molecules and the slower adjacent layer. Hence the x-directed momentum has been transferred in the z-direction from the faster to the slower moving layer.
- The eq. for this transport of momentum is similar to eq. 2 and is Newton's Law of viscosity:

$$\tau_{zx} = -\nu \frac{d(v_x \rho)}{dz} \qquad (7)$$

where τ_{zx} is flux of x-directed momentum in the z direction, $(\mathrm{kg \cdot m/s})/\mathrm{s \cdot m^2}$; ν is μ/ρ, the momentum diffusivity in $\mathrm{m^2/s}$; z is the direction of transport or diffusion in m; ρ is the density in $\mathrm{kg/m^3}$; and μ is the viscosity in $\mathrm{kg/m \cdot s}$.

2. Heat transport and Fourier’s Law

- Fourier’s Law for molecular transport of heat or heat conduction in a fluid or solid can be written as (for cons. density and heat capacity)

$$\frac{q_z}{A} = -\alpha \frac{d(\rho c_p T)}{dz} \qquad (8)$$

where q_z/A is the heat flux in $\mathrm{J/s \cdot m^2}$, α is the thermal diffusivity in $\mathrm{m^2/s}$, and $\rho c_p T$ is the concentration of heat or thermal energy in $\mathrm{J/m^3}$.

3. Mass transport and Fick’s Law

- Fick’s law for molecular transport of mass in a fluid or solid (for cons. total concentration) is

$$J^*_{Az} = -D_{AB} \frac{dc_A}{dz} \qquad (9)$$

where J^*_{Az} is the flux of A in kg mol $A/\mathrm{s \cdot m^2}$, D_{AB} is the molecular diffusivity of the molecule A in B in $\mathrm{m^2/s}$, and c_A is the concentration of A in kg mol $A/\mathrm{m^3}$.

The one-dimension transport equations in their analogous form

Transport	**Flux**	**Flux units (SI)**	**Diffusivity**	**Gradient of concentration of property**	**Eq. No. and Law**
General	ψ_z		δ	$\partial\Gamma/\partial z$	(2)
Heat	q_z/A	J m^{-2} s^{-1}	α	$\partial(\rho c_p T)/\partial z$	(8) Fourier's law
Mass	J^*_{Az}/A	Kmol m^{-2} s^{-1}	D	$\partial C_A/\partial z$	(9) Fick's Law
Momentum	τ_{zx}	kg m^{-1} s^{-2} or N m^{-2} or (kg m/s)/m^2 s	ν	$\partial(\upsilon_x\rho)/\partial z$	(7) Newton's Law of viscosity

Viscosity of fluids & Newton's Law of viscosity

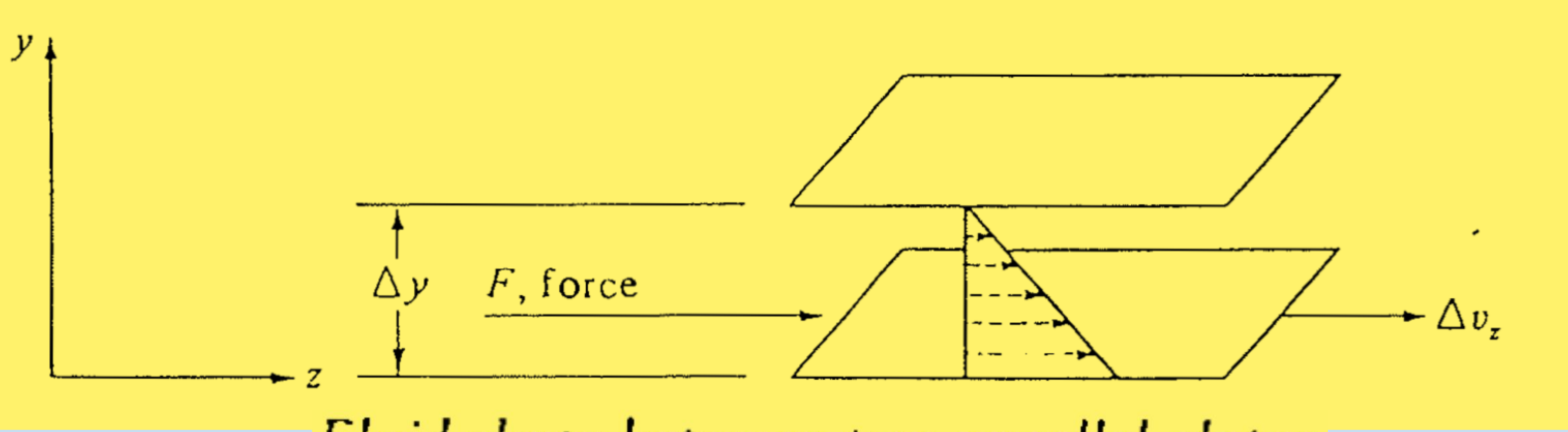
- Comparison between solid (assume elastic solid) and a fluid when they were subjected by applied stress.

- Solid deforms by an amount proportional to the applied stress.
- Solid resists the stress.
- This resistance gives rise to the shear forces.

- Fluid also deforms but will continue to deform (to flow at a velocity that increases with increasing stress).
- Fluid resists the applied stress. Here **viscosity** is the property of a fluid which gives rise to forces that resist the movement of adjacent layers in the fluid.
- These forces (between molecules) are called **viscous forces.**

Viscosity of fluids

- Assume a fluid is contained between two infinite parallel plate.



Fluid shear between two parallel plates.

- The above figure shows that the bottom plate moves parallel to the top one at constant velocity Δv_z due to the applied force F.
- The liquid slides as layers over each other. The bottom layer (adjacent to the bottom plate) has a velocity as Δv_z whilst the layer's velocity decreases as we going up to the top plate with y direction.

- It has been found experimentally for many fluids that the force F in newtons is directly proportional to the velocity Δv_z in m/s, to the area A in m^2 of the plate used, and inversely proportional to the distance Δy in m. Or, as given by Newton's law of viscosity when the flow is laminar,

$$\frac{F}{A} = -\mu \frac{\Delta v_z}{\Delta y} \qquad (10)$$

- Where μ is the proportionality constant called the viscosity of the fluid. If Δy approach to zero then use the definition of derivative

$$\tau_{yz} = -\mu \frac{dv_z}{dy} \qquad (11)$$

where $\tau_{yz} = F/A$ and is the shear stress

- **Note:**

The units of viscosity in the cgs system are g/cm · s, called *poise* or centipoise (cp). In the SI system, viscosity is given in Pa · s (N · s/m^2 or kg/m · s).

$$1 \text{ cp} = 1 \times 10^{-3} \text{ kg/m} \cdot \text{s} = 1 \times 10^{-3} \text{ Pa} \cdot \text{s} = 1 \times 10^{-3} \text{ N} \cdot \text{s/m}^2 \text{ (SI)}$$

$$1 \text{ cp} = 0.01 \text{ poise} = 0.01 \text{ g/cm} \cdot \text{s}$$

$$1 \text{ cp} = 6.7197 \times 10^{-4} \text{ lb}_m/\text{ft} \cdot \text{s}$$

Question

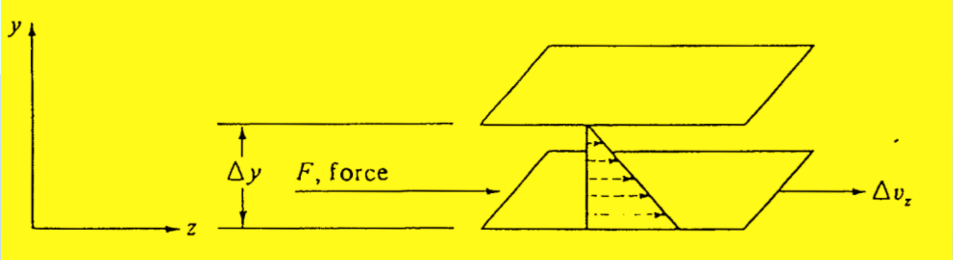
- What is the difference between kinematic viscosity and dynamic viscosity?
- $\nu = \mu / \rho$ where ν is the kinematic viscosity and μ the dynamic viscosity.

Example

- Referring to Figure a, the distance between plates is $\Delta y = 0.5$ cm, $\Delta v_z = 10$ cm/s, and the fluid is ethyl alcohol at 273 K having a viscosity of 1.77 cp (0.0177 g/cm · s).

  Calculate the shear stress τ_{yz} and the velocity gradient or shear rate dv_z/dy using cgs units.

Figure a



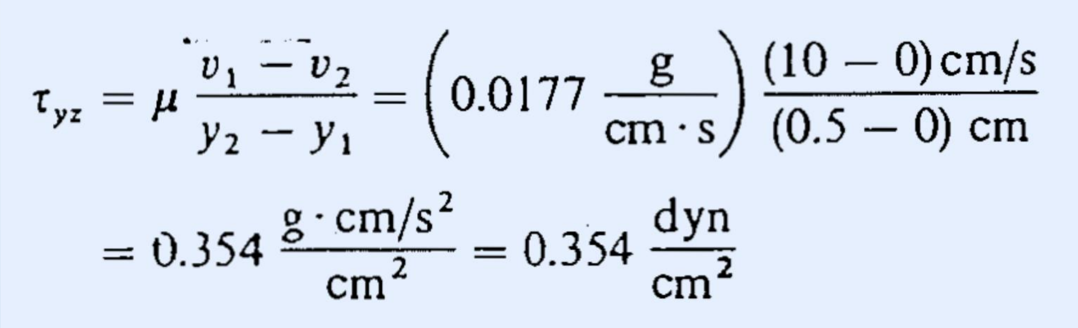
Solution

Integrate eq. 11, assuming the bottom plate is point 1.

$$\tau_{yz} \int_{y_1=0}^{y_2=0.5} dy = -\mu \int_{v_1=10}^{v_2=0} dv_z$$

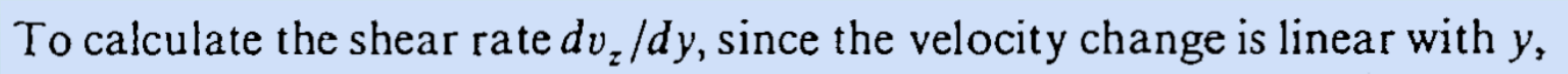
$$\tau_{yz} = \mu \frac{v_1 - v_2}{y_2 - y_1}$$

- Substituting the given data

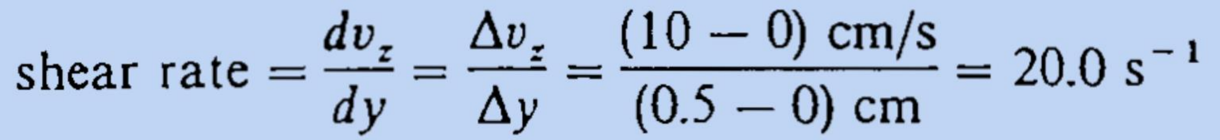


$$\tau_{yz} = \mu \frac{v_1 - v_2}{y_2 - y_1} = \left(0.0177 \frac{\mathrm{g}}{\mathrm{cm \cdot s}}\right) \frac{(10 - 0)\,\mathrm{cm/s}}{(0.5 - 0)\ \mathrm{cm}}$$

$$= 0.354 \frac{\mathrm{g \cdot cm/s^2}}{\mathrm{cm^2}} = 0.354 \frac{\mathrm{dyn}}{\mathrm{cm^2}}$$



To calculate the shear rate dv_z/dy, since the velocity change is linear with y,



$$\text{shear rate} = \frac{dv_z}{dy} = \frac{\Delta v_z}{\Delta y} = \frac{(10 - 0)\ \mathrm{cm/s}}{(0.5 - 0)\ \mathrm{cm}} = 20.0\ \mathrm{s^{-1}}$$

Momentum Transfer in a Fluid

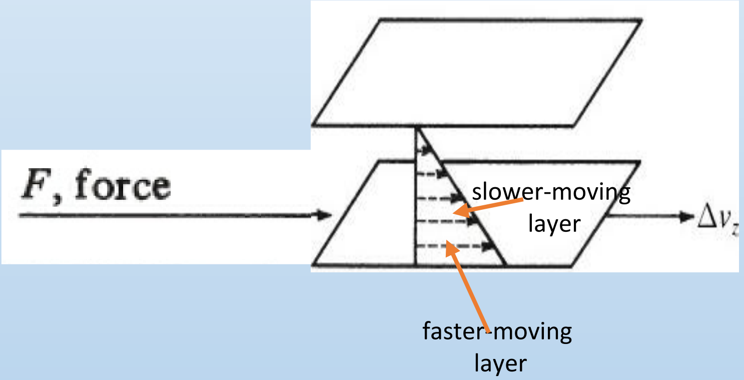
- The shear stress τ_{yz} in Eqs. (10)–(11) can also be interpreted as a flux of z-directed momentum in the y direction, which is the rate of momentum per unit area.
- The units of momentum are mass times velocity in kg · m/s. Thus, the shear stress can also be written as

$$\tau_{yz} = \frac{\mathrm{kg \cdot m/s}}{\mathrm{m^2 \cdot s}} = \frac{\mathrm{momentum}}{\mathrm{m^2 \cdot s}}$$

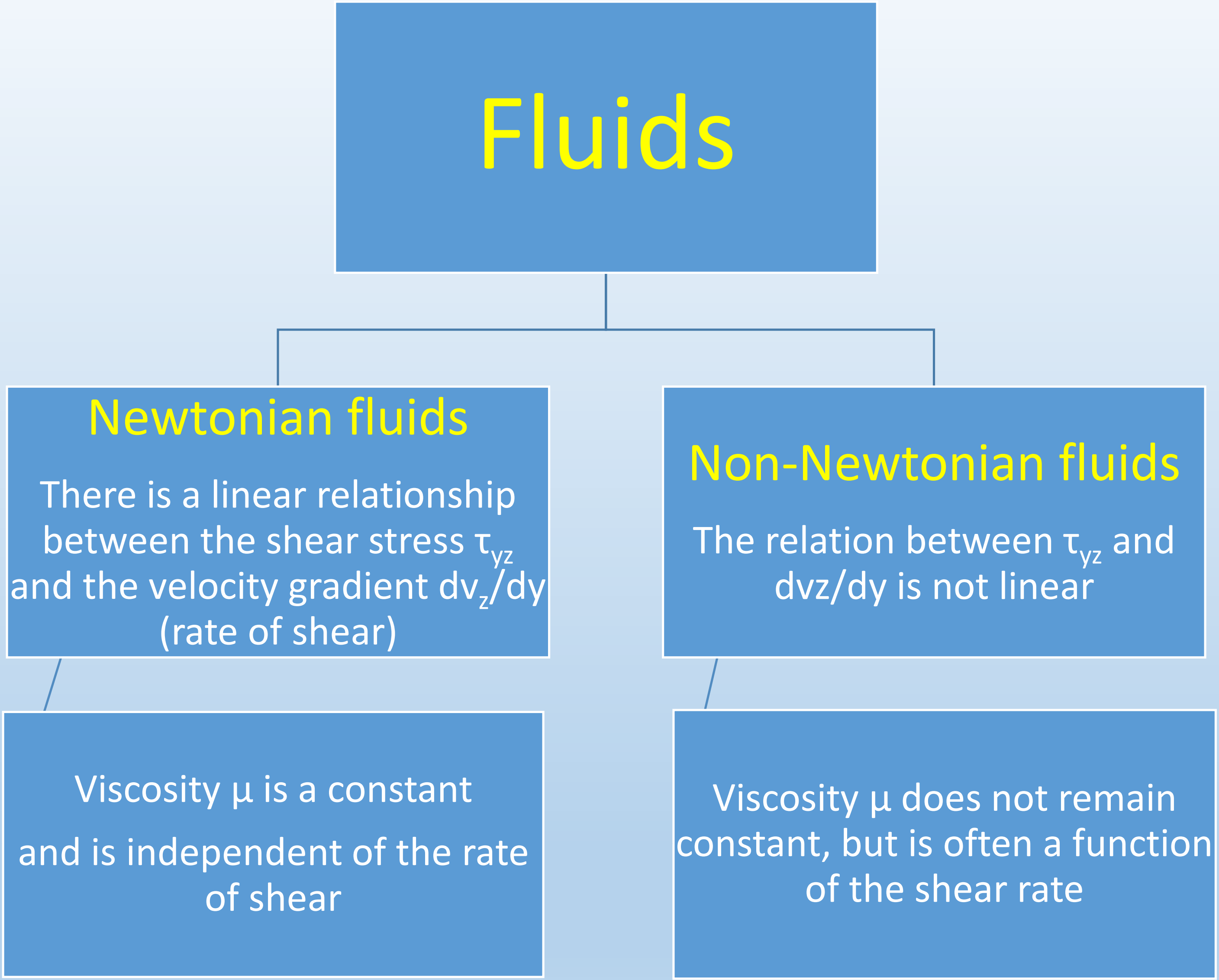
- This gives the amount of momentum transferred per unit area per second.

Comments on momentum transfer

- The random motions of the molecules in the faster-moving layer send some of the molecules into the slower-moving layer, where they collide with the slower-moving molecules and tend to speed them up or increase their momentum in the *z* direction.
- In the same fashion, molecules in the slower layer also tend to retard those in the faster layer.
- This exchange of molecules between layers produces a transfer or flux of *z*-directed momentum from high-velocity to low-velocity layers.
- The negative sign in Eq. (11) indicates that momentum is transferred down the gradient from high- to low-velocity regions. This is similar to the transfer of heat from high- to low temperature regions.



$$\tau_{yz} = -\mu \frac{dv_z}{dy}$$



Fluids

Newtonian fluids

There is a linear relationship between the shear stress τ_{yz} and the velocity gradient dv_z/dy (rate of shear)

Viscosity μ is a constant

and is independent of the rate of shear

Non-Newtonian fluids

The relation between τ_{yz} and dvz/dy is not linear

Viscosity μ does not remain constant, but is often a function of the shear rate

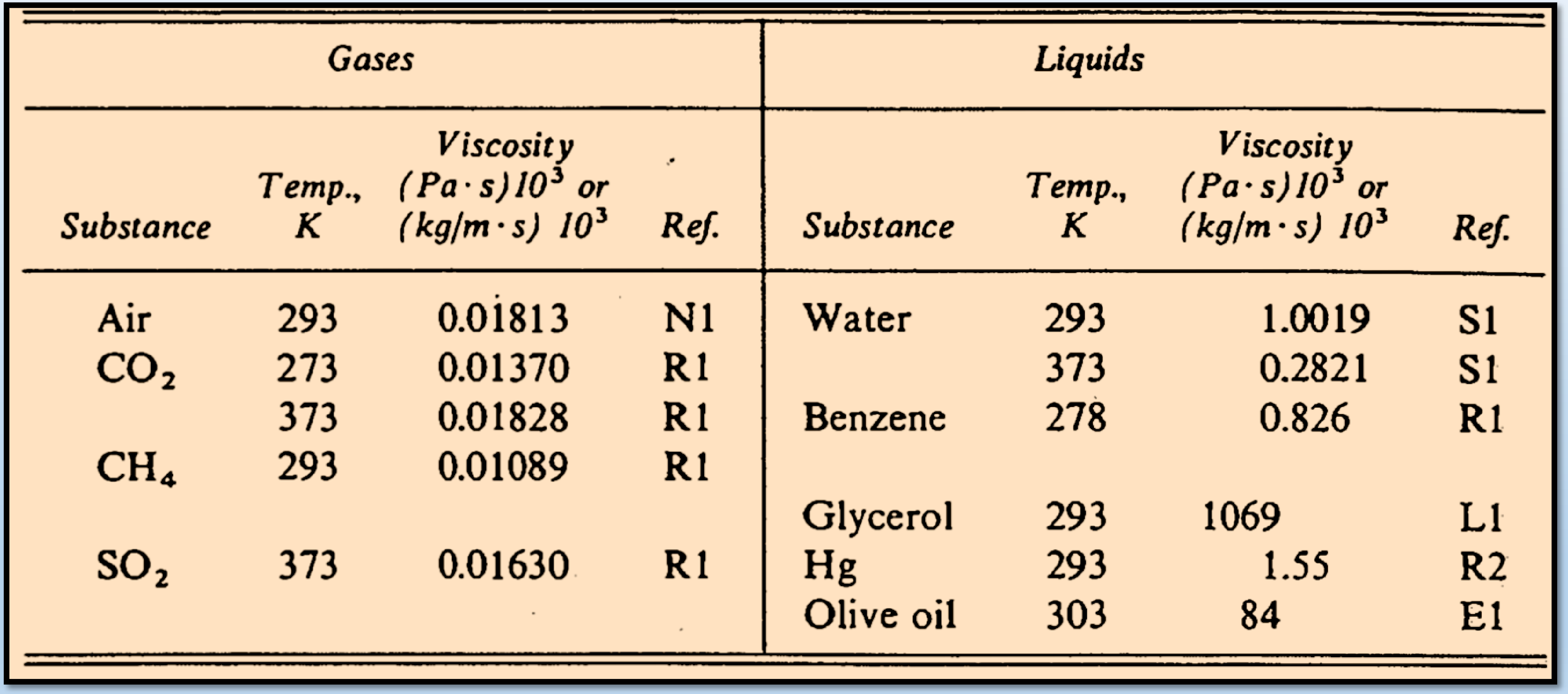
Note: Certain liquids—primarily pastes, slurries, high-molecular-weight polymers, and emulsions—do not obey Newton’s law

Viscosity of Newtonian fluids

- The viscosity of gases, which are Newtonian fluids, increases with increasing temperature and is approximately independent of pressure up to a pressure of about 1000 kPa. At higher pressures, the viscosity of gases increases with increase in pressure.
- For example, the viscosity of N_2 gas at 298 K approximately doubles when going from 100 kPa to about 5×10^4 kPa

- For liquids, the viscosity decreases with increasing temperature.
- Since liquids are essentially incompressible, their viscosities are usually not affected by pressure.
- The viscosities for gases are the lowest values and do not differ markedly from gas to gas. The viscosities for liquids are much greater.
- For details see text and appendices.

Viscosities of Some Gases and Liquids at 101.32 kPa Pressure



Gases				Liquids			
Substance	*Temp., K*	*Viscosity (Pa·s)10^3 or (kg/m·s) 10^3*	*Ref.*	*Substance*	*Temp., K*	*Viscosity (Pa·s)10^3 or (kg/m·s) 10^3*	*Ref.*
Air	293	0.01813	N1	Water	293	1.0019	S1
CO_2	273	0.01370	R1		373	0.2821	S1
	373	0.01828	R1	Benzene	278	0.826	R1
CH_4	293	0.01089	R1				
				Glycerol	293	1069	L1
SO_2	373	0.01630	R1	Hg	293	1.55	R2
				Olive oil	303	84	E1

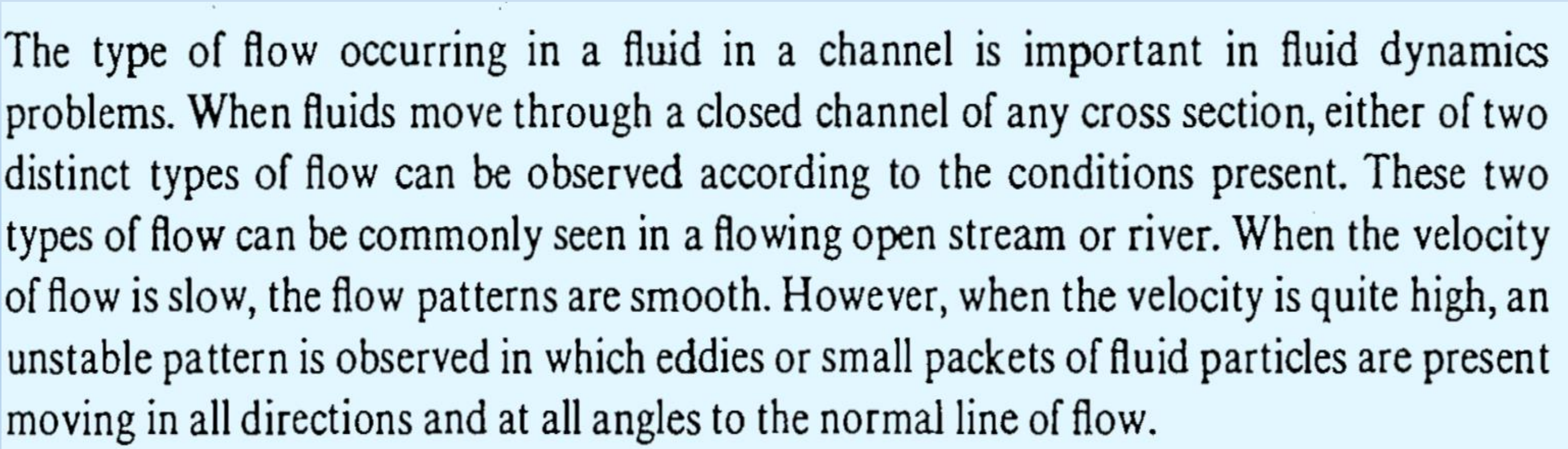
More complete tables of viscosities are given for water in Appendix A.2, for inorganic and organic liquids and gases in Appendix A.3, and for biological and food liquids in Appendix A.4. Extensive data are available in other references.

Types of fluid flow and Reynold's number

We have two types of fluid flow:

- Laminar flow
- Turbulent flow

- Reynold’s No. used to describe the regime or type of flow
- Regimes of flow

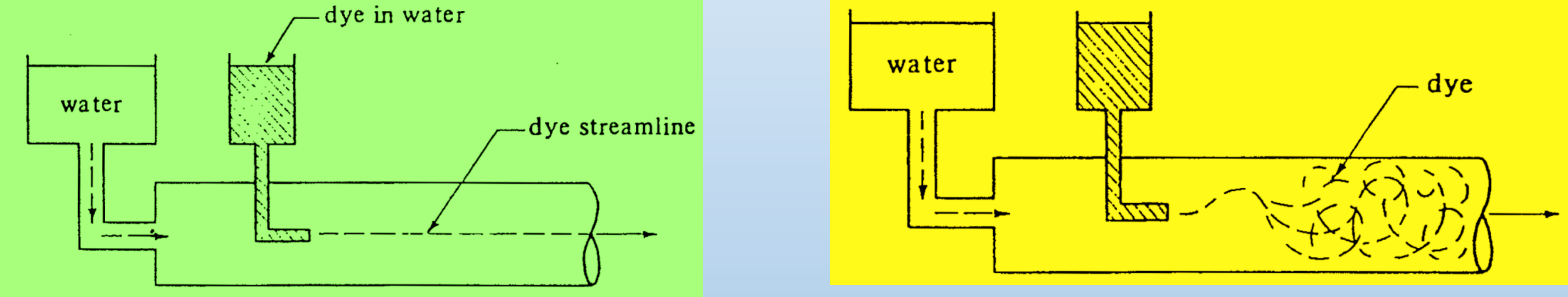


The type of flow occurring in a fluid in a channel is important in fluid dynamics problems. When fluids move through a closed channel of any cross section, either of two distinct types of flow can be observed according to the conditions present. These two types of flow can be commonly seen in a flowing open stream or river. When the velocity of flow is slow, the flow patterns are smooth. However, when the velocity is quite high, an unstable pattern is observed in which eddies or small packets of fluid particles are present moving in all directions and at all angles to the normal line of flow.

Types of fluid flow and Reynolds Number

- Laminar and turbulent flow

Reynolds' experiment for different types of flow

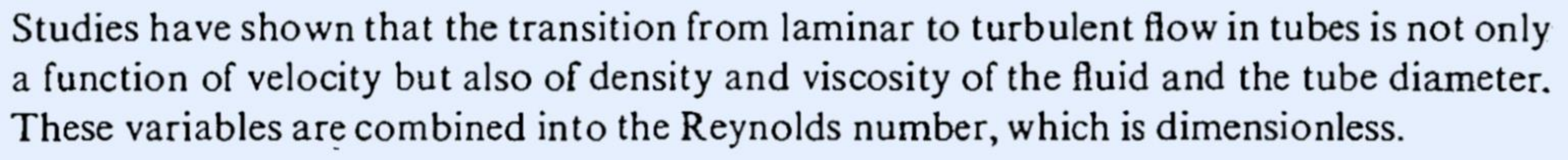


Laminar or viscous flow

turbulent flow

<u>Note</u>: The velocity at which the flow changes is known as the critical velocity.

Reynolds Number



Studies have shown that the transition from laminar to turbulent flow in tubes is not only a function of velocity but also of density and viscosity of the fluid and the tube diameter. These variables are combined into the Reynolds number, which is dimensionless.

$$N_{Re} = \frac{Dv\rho}{\mu} \quad Or \quad \mathrm{Re} = \frac{Dv\rho}{\mu} \equiv$$

Kinetic (Inertial) force/viscous force

$\equiv \rho v^2/(\mu v/D)$

Note:* *"for circular tube"

For Re < 2100 Laminar flow **for Re > 4000 Turbulent flow** **2100 < Re < 4000 transition regime**

Example 2

Water at 303 K is flowing at the rate of 10 gal/min in a pipe having an inside diameter (ID) of 2.067 in. Calculate the Reynolds number using both English units and SI units

Solution

From Appendix A.1, 7.481 gal = 1 ft^3. The flow rate is calculated as

$$\text{flow rate} = \left(10.0\frac{\text{gal}}{\text{min}}\right)\left(\frac{1\ \text{ft}^3}{7.481\ \text{gal}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= 0.0223\ \text{ft}^3/\text{s}$$

$$\text{pipe diameter, } D = \frac{2.067}{12} = 0.172\ \text{ft}$$

$$\text{cross-sectional area of pipe} = \frac{\pi D^2}{4} = \frac{\pi(0.172)^2}{4} = 0.0233\ \text{ft}^2$$

$$\text{velocity in pipe, } v = \left(0.0223\frac{\text{ft}^3}{s}\right)\left(\frac{1}{0.0233\ \text{ft}^2}\right) = 0.957\ \text{ft/s}$$

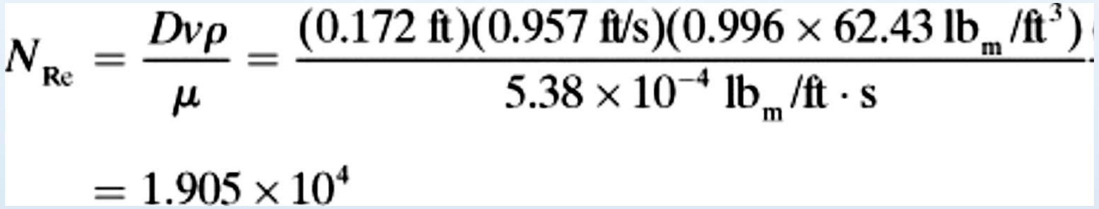
From Appendix A.2, for water at 303 K (30°C),

$$\text{density, } \rho = 0.996(62.43)\ \text{lb}_\text{m}/\text{ft}^3$$

$$\text{viscosity, } \mu = (0.8007\ \text{cp})\left(6.7197 \times 10^{-4}\ \frac{\text{lb}_\text{m}/\text{ft} \cdot \text{s}}{\text{cp}}\right)$$

$$= 5.38 \times 10^{24}\ \text{lb}_\text{m}/\text{ft} \cdot \text{s}$$

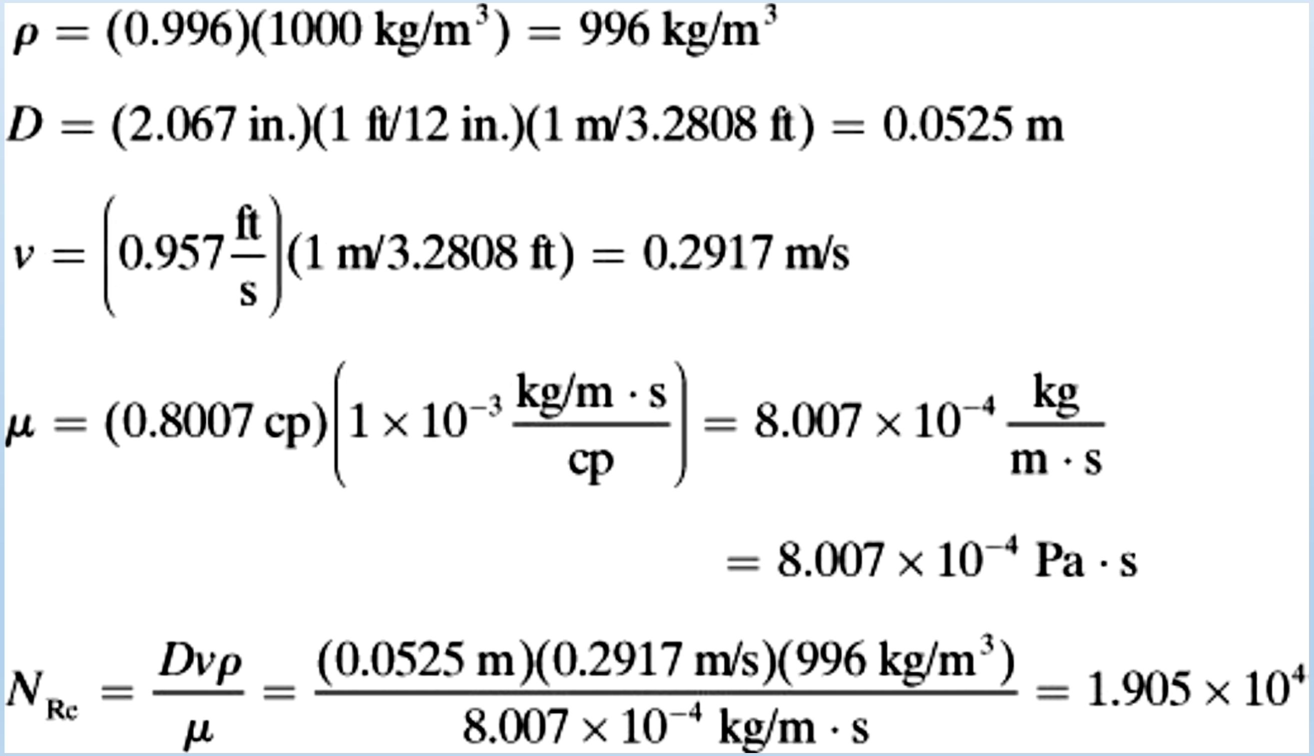
- Substituting in Reynolds Definition



$$N_{\mathrm{Re}} = \frac{Dv\rho}{\mu} = \frac{(0.172\ \mathrm{ft})(0.957\ \mathrm{ft/s})(0.996 \times 62.43\ \mathrm{lb_m/ft^3})}{5.38 \times 10^{-4}\ \mathrm{lb_m/ft \cdot s}}$$

$$= 1.905 \times 10^4$$

- Hence, the flow is turbulent. Using SI units,



$$\rho = (0.996)(1000\ \mathrm{kg/m^3}) = 996\ \mathrm{kg/m^3}$$

$$D = (2.067\ \mathrm{in.})(1\ \mathrm{ft/12\ in.})(1\ \mathrm{m/3.2808\ ft}) = 0.0525\ \mathrm{m}$$

$$v = \left(0.957 \frac{\mathrm{ft}}{\mathrm{s}}\right)(1\ \mathrm{m/3.2808\ ft}) = 0.2917\ \mathrm{m/s}$$

$$\mu = (0.8007\ \mathrm{cp})\left(1 \times 10^{-3} \frac{\mathrm{kg/m \cdot s}}{\mathrm{cp}}\right) = 8.007 \times 10^{-4} \frac{\mathrm{kg}}{\mathrm{m \cdot s}}$$

$$= 8.007 \times 10^{-4}\ \mathrm{Pa \cdot s}$$

$$N_{\mathrm{Re}} = \frac{Dv\rho}{\mu} = \frac{(0.0525\ \mathrm{m})(0.2917\ \mathrm{m/s})(996\ \mathrm{kg/m^3})}{8.007 \times 10^{-4}\ \mathrm{kg/m \cdot s}} = 1.905 \times 10^4$$

Summary

- we have described the two main types of fluid flows, *laminar flow* and *turbulent flow*, and how they are related to the fluid property of *viscosity*.
- We have shown how viscosity is related to the shear stress in a fluid and the shear rate at which it deforms. This relationship was observed to follow Newton’s law of viscosity

$$\tau_{yz} = -\mu \frac{dv_z}{dy} \qquad \text{(SI units)}$$

- The fluid property of *viscosity* was described in terms of macroscopic behavior (i.e., deformation and “resistance to flow”) as well on the molecular level in the exchange of momentum between adjacent molecules.
- It was shown that fluid viscosity is temperature dependent, and the relationship between temperature and viscosity varies if the fluid is a liquid or a gas.

- The differences between laminar flow and turbulent flow were also described based on Reynolds's original experiment.
- In order to help predict the flow regime, the Reynolds number was described as the ratio of inertial forces to viscous forces and defined as:

$$N_{\mathrm{Re}} = \frac{Dv\rho}{\mu}$$

- We also showed how the Reynolds number can be calculated from fluid and geometric properties, and how that value can be used to predict the flow behavior of the fluid.
- We have shown the different transport mechanisms plus the basic three laws for heat, mass and momentum.
- See the following Examples of transport.