Conservation Equations

Differential equations of the conservation

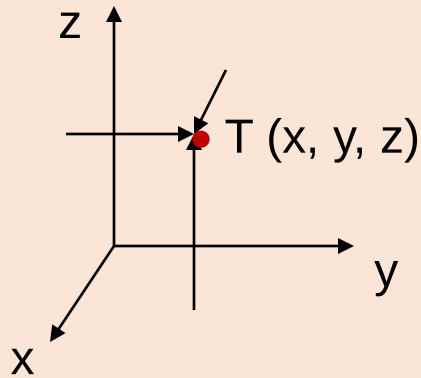
i. Differential equations of the conservation of mass (equation of continuity)

ii. Differential equations of the conservation of momentum

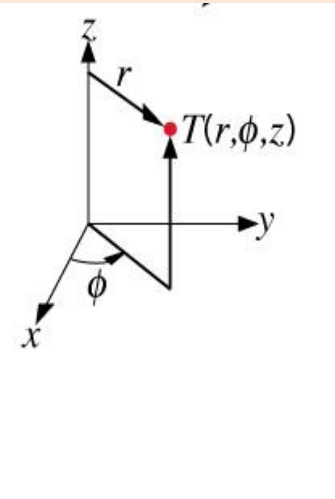
The conservation equations are usually called equations of change because they show the variations of properties with time and position.

Coordinate systems

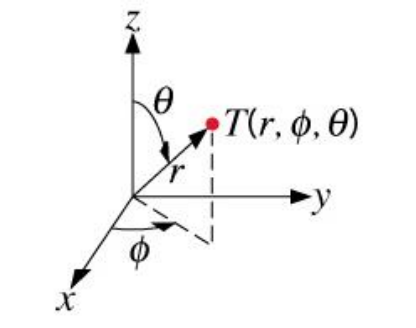
Coordinate systems



Cartesian coordinates: T (x, y, z)



Cylindrical Coordinates: $T(r, \phi, z)$



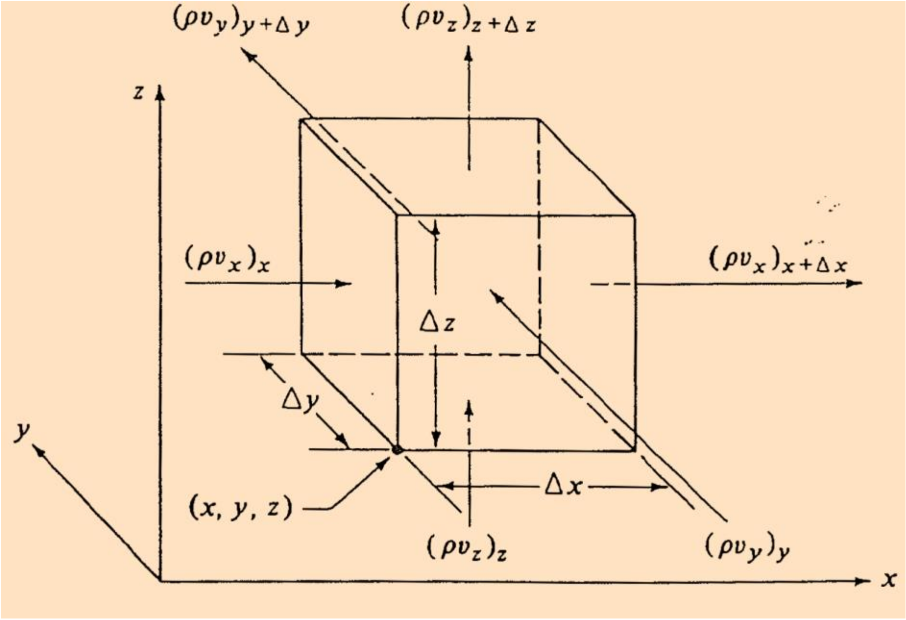
Spherical Coordinates: $T(r, \phi, \theta)$

Differential Equation of Continuity

- Consider the following differential volume element $\Delta x\ \Delta y\ \Delta z$ which is fixed in space. The mass balance for a pure fluid that is flowing through this element is:

$$\text{(rate of mass in)} - \text{(rate of mass out)} = \text{(rate of mass accumulation)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

- Let ρ is the density or conc. Of fluid kg/m^3
- For x-dir. Mass entering the face at x is $(\rho v_x)_x\ \Delta y\ \Delta z$ kg/s and leaving at $x + \Delta x$ is $(\rho v_x)_{x+\Delta x}\ \Delta y\ \Delta z$. And so to other directions.
- Note (ρv_x) means the mass flux in $kg/s.m^2$.



The rate of mass accumulation in the volume $\Delta x \ \Delta y \ \Delta z$ is

$$\text{rate of mass accumulation} = \Delta x \, \Delta y \, \Delta z \, \frac{\partial \rho}{\partial t} \quad \text{.................................... (2)}$$

After substituting previous expressions in eq.(1) and dividing on the volume $\Delta x \ \Delta y \ \Delta z$, we obtain

$$\frac{[(\rho v_x)_x - (\rho v_x)_{x+\Delta x}]}{\Delta x} + \frac{[(\rho v_y)_y - (\rho v_y)_{y+\Delta y}]}{\Delta y} + \frac{[(\rho v_z)_z - (\rho v_z)_{z+\Delta z}]}{\Delta z} = \frac{\partial \rho}{\partial t} \quad \text{.................................... (3)}$$

Taking the limit as Δx, Δy, and Δz approach zero, we obtain the equation of continuity or conservation of mass for a pure fluid.

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] = -(\nabla \cdot \rho \mathbf{v}) \quad \text{.................................... (4)}$$

The vector notation on the right side of Eq. (4) comes from the fact that $\mathbf{v}$ is a vector.

Eq. (4) tells us how density ρ changes with time at a fixed point resulting from the changes in the mass velocity vector $\rho\mathbf{v}$.

- Eq. (4) can be expanded to give another form like this

$$\frac{\partial \rho}{\partial t} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) - \left(v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z}\right)$$

.... (5)

- Rearranging eq. (5) yields

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

.... (6)

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = -\rho(\nabla \cdot \mathbf{v})$$

.... (7)

- Or

$$\frac{D\rho}{Dt} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = -\rho(\nabla \cdot \mathbf{v})$$

.... (8)

Equation of continuity for constant density

- This situation often meets when the fluid is compressible where the density ρ is constant.
- Therefore, the term Dρ/Dt in eq. (8) is zero; hence, eq. (8) becomes at steady or unsteady state with ρ = constant.

$$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

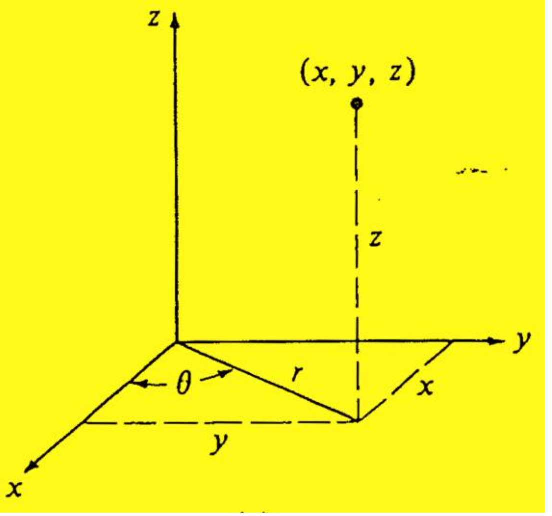
Relation between Cylindrical or system and Cartesian system

The relations between rectangular x, y, z and cylindrical r, θ, z coordinates are

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

$$r = +\sqrt{x^2+y^2} \qquad \theta = \tan^{-1}\frac{y}{x}$$

……………………………… (10)



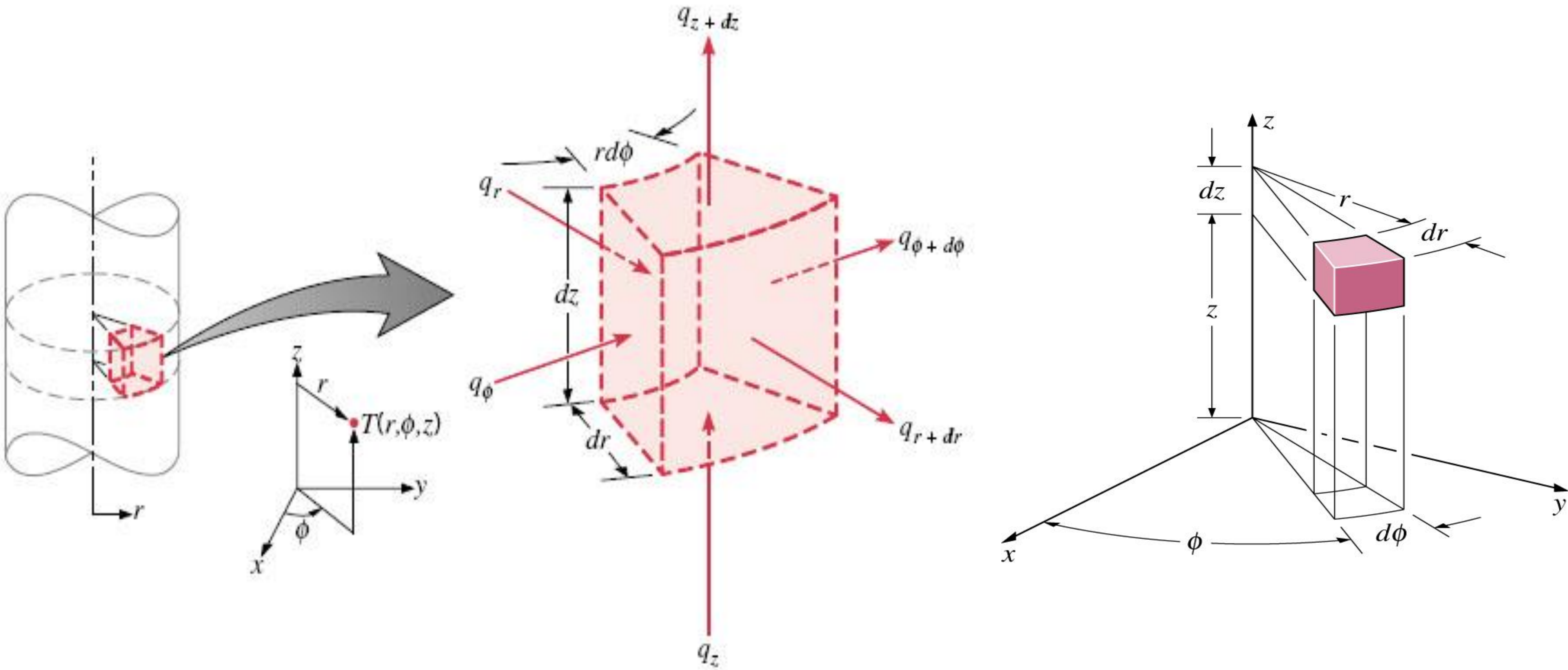
Using relations of eq.(10), the continuity eq. (4) for cylindrical coordinates becomes

$$\frac{\partial\rho}{\partial t} = -\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] = -(\nabla\cdot\rho\mathbf{v})$$

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial\theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

……………………………… (11)

See next slides for derivations

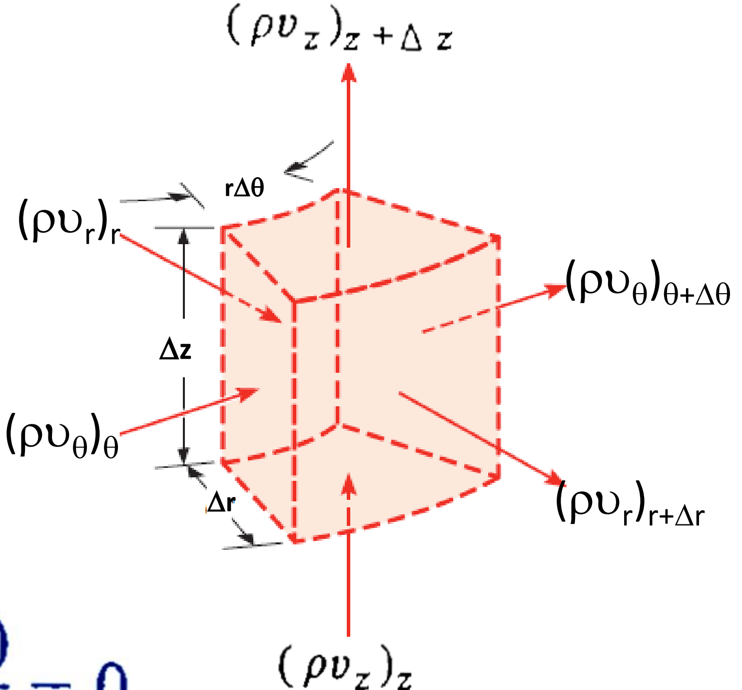


Differential control volume, $dr \cdot r\,d\phi \cdot dz$, for conduction analysis in cylindrical coordinates (r, ϕ, z).

Cylindrical Coordinates

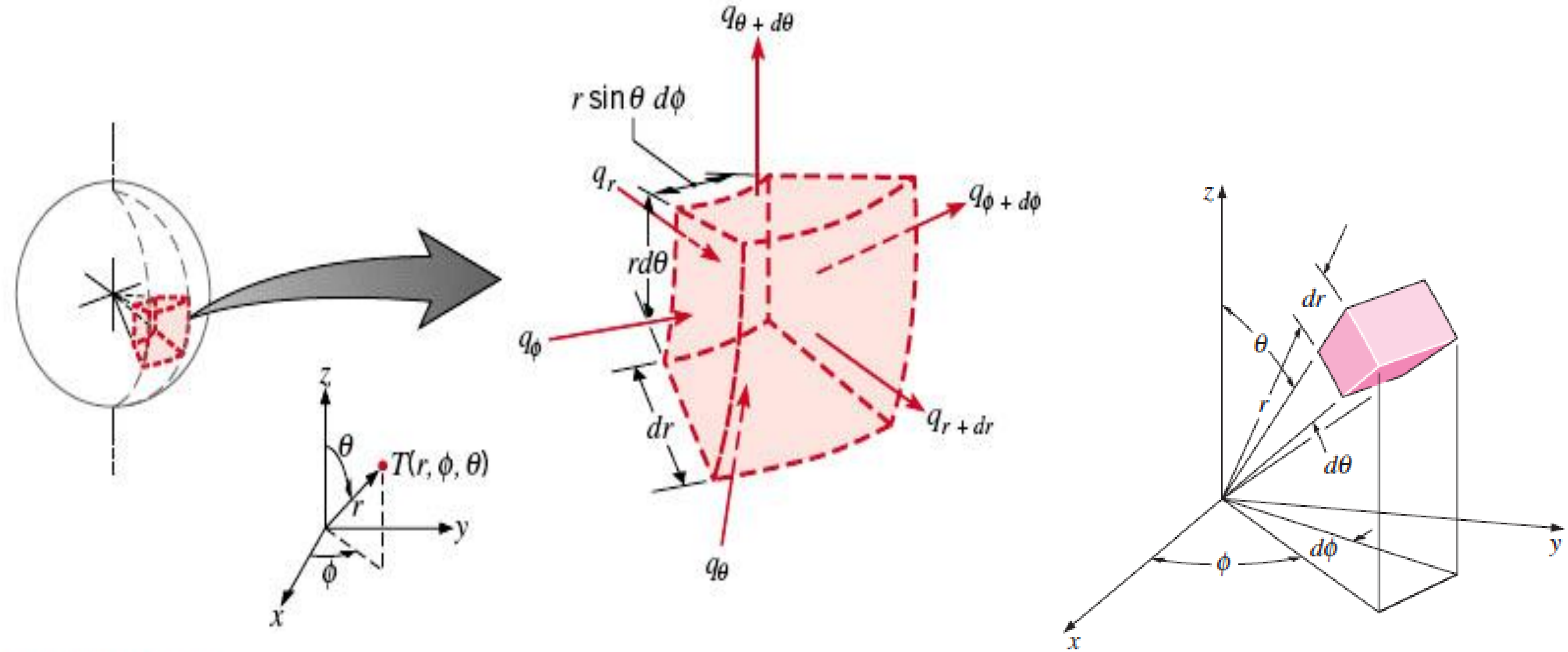
The continuity eq. in cylindrical coordinates can be derived from a mass balance performed on cylindrical element. The result is

similar to the continuity eq. (4) but for cylindrical coordinates.



$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

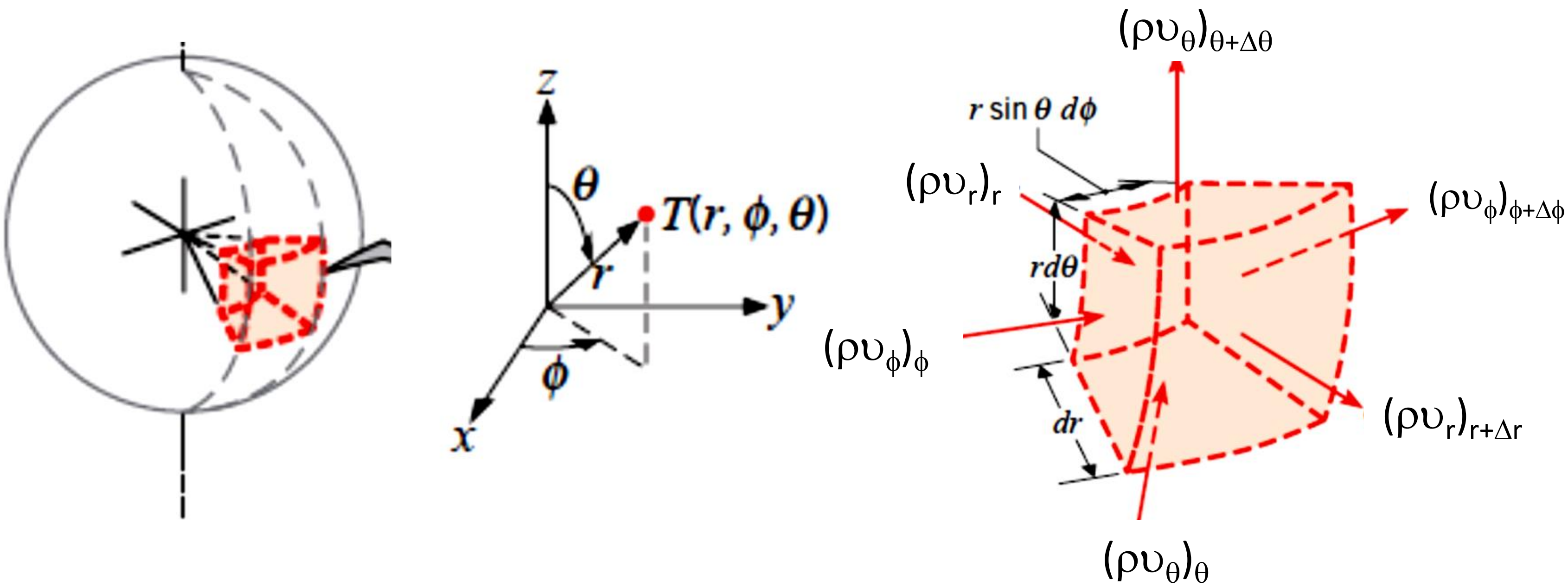
.... (11)



Differential control volume, $dr \cdot r \sin\theta \, d\phi \cdot r \, d\theta$, for conduction analysis in spherical coordinates (r, ϕ, θ).

Spherical Coordinate

Applying eq.1 to the spherical differential volume element, we obtain the continuity equation in spherical coordinates.



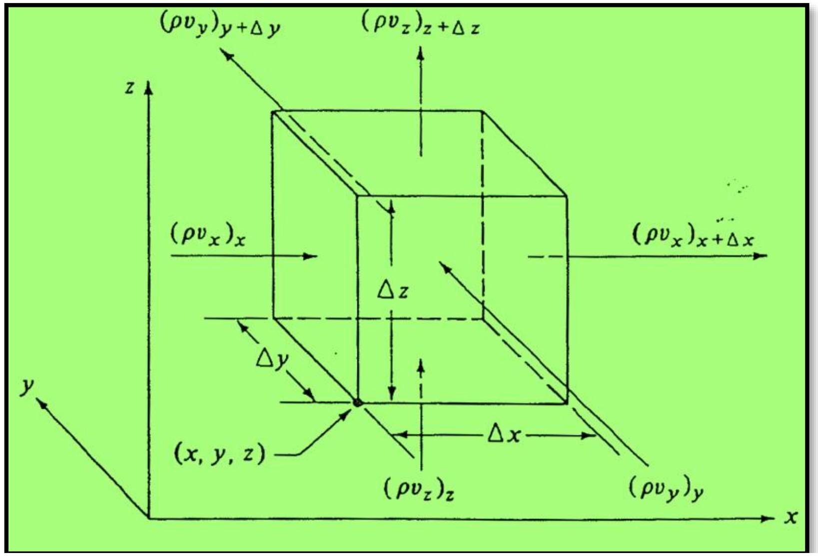
$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial(\rho v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r \sin\theta}\frac{\partial(\rho v_\phi)}{\partial \phi} = 0$$

……………………………… (12)

The momentum Equation or the Equation of Motion

Derivation of Equations of Momentum Transfer

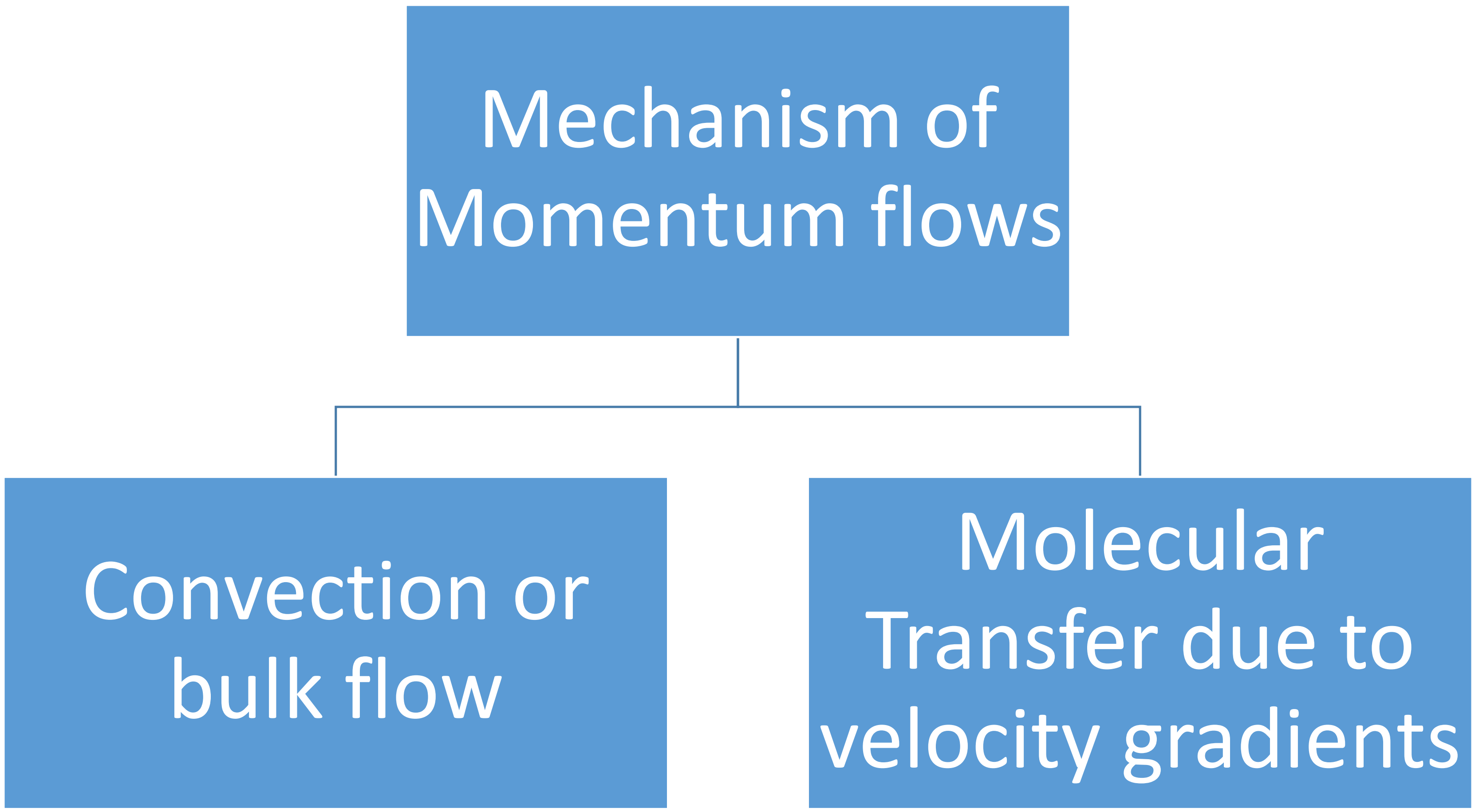
- A momentum balance written for a control volume Δx by Δy by Δz is



$$\begin{pmatrix}\text{rate of}\\ \text{momentum in}\end{pmatrix} - \begin{pmatrix}\text{rate of}\\ \text{momentum out}\end{pmatrix} + \begin{pmatrix}\text{sum of forces}\\ \text{acting on system}\end{pmatrix} = \begin{pmatrix}\text{rate of momentum}\\ \text{accumulation}\end{pmatrix}$$

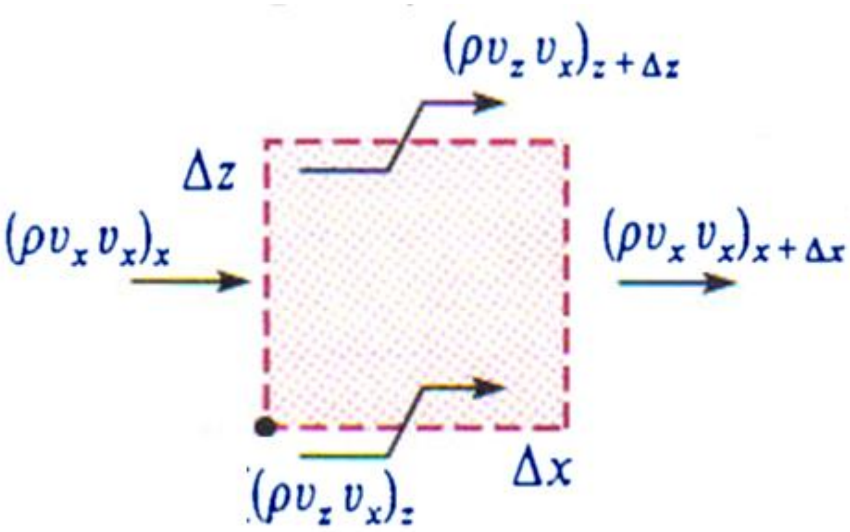
.... (13)

Note

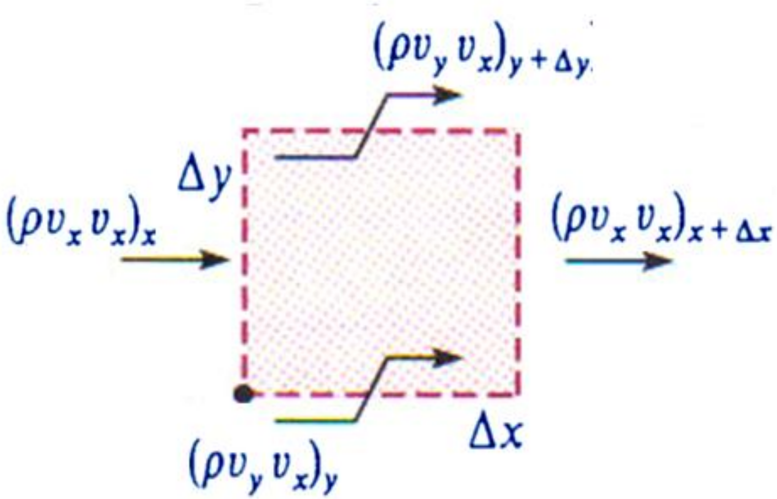

Mechanism of Momentum flows
Convection or bulk flow
Molecular Transfer due to velocity gradients

- It is known from Newton's Law that Eq. 13 can be written for a particular –x, y, or z. Eq. 13 is thus a vector eq. having three component equations.
- We are going to derive the x-directed momentum components and forces; similar derivations for y and z directions can be handled analogously.
- The rate at which the x component of momentum enters the face at x in the x direction by convection is $(\rho v_x v_x)_x \, \Delta y \, \Delta z$ and the rate at which it leaves at $x + \Delta x$ is $(\rho v_x v_x)_{x+\Delta x} \, \Delta y \, \Delta z$.
- The x component of momentum entering the face at y is $(\rho v_y v_x)_y \, \Delta x \, \Delta z$, and leaving at $y + \Delta y$ is $(\rho v_y v_x)_{y+\Delta y} \, \Delta x \, \Delta z$
- For the face at z we have $(\rho v_z v_x)_z \, \Delta x \, \Delta y$ entering, and at $z + \Delta z$ we have $(\rho v_z v_x)_{z+\Delta z} \, \Delta x \, \Delta y$ leaving.

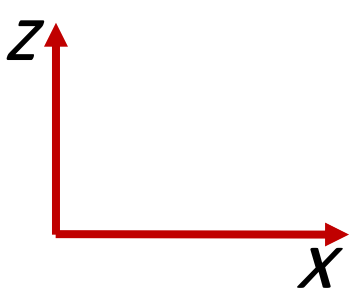
Note

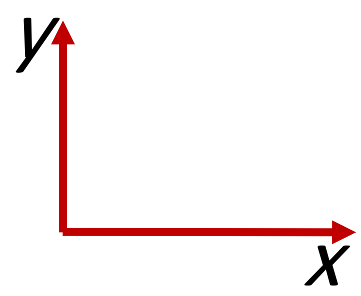


Side – front view



Top-bottom view





- Now, the net convective x-momentum flow into the volume element $\Delta x\ \Delta y\ \Delta z$ is

$$[(\rho v_x v_x)_x - (\rho v_x v_x)_{x+\Delta x}]\Delta y\ \Delta z + [(\rho v_y v_x)_y - (\rho v_y v_x)_{y+\Delta y}]\Delta x\ \Delta z + [(\rho v_z v_x)_z - (\rho v_z v_x)_{z+\Delta z}]\Delta x\ \Delta y \quad \text{.................................} \quad (14)$$

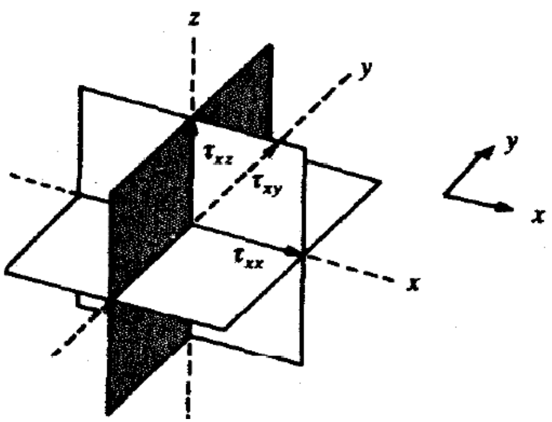
- Eq. 14 represents the mechanism of convection momentum flow due to bulk-fluid flow.
- Molecular mechanism: velocity gradients exist in the fluid due to forces exerted on the fluid (see viscosity definition).
- The rate at which the x component of momentum enters the face at x by molecular transfer is $(\tau_{xx})_x\ \Delta y\ \Delta z$, and the rate at which it leaves the surface at $x + \Delta x$ is $(\tau_{xx})_{x+\Delta x}\ \Delta y\ \Delta z$.
- Similar expressions can be written for the other faces.

- The net x component of momentum by molecular transfer is

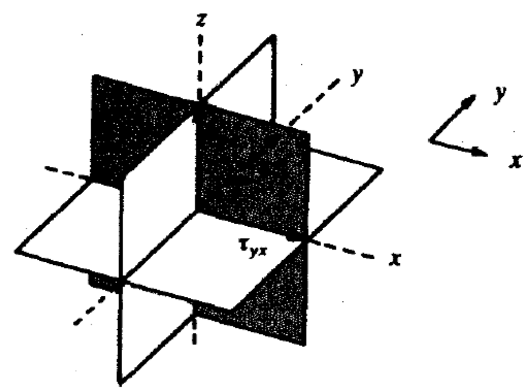
$$[(\tau_{xx})_x - (\tau_{xx})_{x+\Delta x}]\Delta y\,\Delta z + [(\tau_{yx})_y - (\tau_{yx})_{y+\Delta y}]\Delta x\,\Delta z + [(\tau_{zx})_z - (\tau_{zx})_{z+\Delta z}]\Delta x\,\Delta y \quad \text{.................................} \quad (15)$$

Note

These molecular fluxes of momentum may be considered as shear stresses and normal stresses. Hence, τ_{yx} is the x direction shear stress on the y face and τ_{zx} the shear stress on the z face. Also, τ_{xx} is the normal stress on the x face.



Stress tensor on the ***yz*** plane



Stress tensor on the ***xz*** plane

Forces acting on the fluid element

- There are a number of forces acting on the fluid element, such as
  1. Pressure force
  2. Gravity force
  3. Surface tension
  4. Magnetic effects
- In general, and in fluid mechanics it is common to consider only pressure and gravity forces.
- For x-direction these are

$$(p_x - p_{x+\Delta x})\Delta y\, \Delta z + \rho g_x\, \Delta x\, \Delta y\, \Delta z \quad \text{........................} \quad (16)$$

where g_x is the x component of the gravitational vector g.

- The rate of accumulation of x momentum in the element is

$$\Delta x \, \Delta y \, \Delta z \, \frac{\partial(\rho v_x)}{\partial t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(17)}$$

- Substituting equations 14-17 into eq.13 dividing on Δx Δy Δz and taking the limits as Δx, Δy, and Δz approach zero, we obtain

$$\frac{\partial(\rho v_x)}{\partial t} = -\left[\frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_y v_x)}{\partial y} + \frac{\partial(\rho v_z v_x)}{\partial z}\right] - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho g_x \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(18)}$$

- Eq. 18 is the x component of the equation of motion.

The y and z components of the differential equation of motion are, respectively,

$$\frac{\partial(\rho v_y)}{\partial t} = -\left[\frac{\partial(\rho v_x v_y)}{\partial x} + \frac{\partial(\rho v_y v_y)}{\partial y} + \frac{\partial(\rho v_z v_y)}{\partial z}\right] - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) - \frac{\partial p}{\partial y} + \rho g_y \qquad \text{.............................. (19)}$$

$$\frac{\partial(\rho v_z)}{\partial t} = -\left[\frac{\partial(\rho v_x v_z)}{\partial x} + \frac{\partial(\rho v_y v_z)}{\partial y} + \frac{\partial(\rho v_z v_z)}{\partial z}\right] - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) - \frac{\partial p}{\partial z} + \rho g_z \qquad \text{.............................. (20)}$$

Notes for next slide

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right]$$

Continuity eq.

.... (4)

Substitute

Expand this term

Expand this term

$$\frac{\partial(\rho v_x)}{\partial t} = -\left[\frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_y v_x)}{\partial y} + \frac{\partial(\rho v_z v_x)}{\partial z}\right]$$

$$-\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

.... (18)

Then you will get eq. (21)

- Combining continuity eq.(4) and eq. (18) we obtain an eq. of motion for the x component and we can also do the same for y and z components as follows:

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\left[\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right]+\rho g_x-\frac{\partial p}{\partial x}$$

.... (21)

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\left[\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{zy}}{\partial z}\right]+\rho g_y-\frac{\partial p}{\partial y}$$

.... (22)

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\left[\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}\right]+\rho g_z-\frac{\partial p}{\partial z}$$

.... (23)

Note

Adding vectorially, we obtain an equation of motion for a pure fluid.

$$\rho \frac{D\mathbf{v}}{Dt} = -(\nabla \cdot \boldsymbol{\tau}) - \nabla p + \rho \mathbf{g}$$

.... (24)

We should note that Eqs. (18) – (24) are valid for any continuous medium.

Equations of Motion for Newtonian fluids with varying density and viscosity

- Equations (18) to (24) are used to obtain velocity distributions.
- To use these eqs. you need expressions for stress in terms of velocity gradients and fluid properties.
- For Newtonian fluids the expressions for the stresses are (for rectangular coordinates)

$$\tau_{xx} = -2\mu \frac{\partial v_x}{\partial x} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v}) \quad \text{............ (25)}$$

$$\tau_{yy} = -2\mu \frac{\partial v_y}{\partial y} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v}) \quad \text{............ (26)}$$

$$\tau_{zz} = -2\mu \frac{\partial v_z}{\partial z} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v}) \quad \text{............ (27)}$$

$$\tau_{xy} = \tau_{yx} = -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \quad \text{............ (28)}$$

$$\tau_{yz} = \tau_{zy} = -\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) \quad \text{............ (29)}$$

$$\tau_{zx} = \tau_{xz} = -\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right) \quad \text{............ (30)}$$

$$(\nabla \cdot \mathbf{v}) = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) \quad \text{............ (31)}$$

$$\tau_{xx} = -\mu(\partial U_x/\partial x + \partial U_x/\partial x) = -2\mu(\partial U_x/\partial x)$$

$$\tau_{yx} = -\mu[(\partial U_x/\partial y) + (\partial U_y/\partial x)]$$

$$\tau_{xy} = -\mu[(\partial U_y/\partial x) + (\partial U_x/\partial y)]$$

Equation of Motion for Newtonian fluids with varying density and viscosity After

x-component of momentum

rectangular coordinates

- When eqs. (25) to (31) are combined with eq. (21), the result

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{z}}{\partial z}\right.$$

$$\rho\frac{Dv_x}{Dt} = \frac{\partial}{\partial x}\left[2\mu\frac{\partial v_x}{\partial x} - \frac{2}{3}\mu(\nabla\cdot\mathbf{v})\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)\right] - \frac{\partial p}{\partial x} + \rho g_x \quad \text{.....................} \quad (31)$$

Similar equations are obtained for the y and z components of momentum

Equations of Motion for Newtonian Fluids with Constant Density and Viscosity

- The equations above are seldom used in their complete forms
- When the density ρ and the viscosity μ are constant where $(\nabla \cdot \mathbf{v}) = 0$ the equations are simplified and we obtain the equations of motion for Newtonian fluids.
- These equations are also called the *Navier–Stokes equations.*
- *Equation of motion in rectangular coordinates.* ***for x-component***

Navier-Stokes equations

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

………………… (32)

Navier-Stokes equations

- We do the same for y and z components, respectively.

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) - \frac{\partial p}{\partial y} + \rho g_y$$

.... (33)

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) - \frac{\partial p}{\partial z} + \rho g_z$$

.... (34)

- Combining the three equations for the three components, we obtain

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{v}$$

.... (35)

Equation of motion in cylindrical coordinates..

- We do the same procedure as before for coordinate system.
- These equations are as follows for Newtonian fluids for constant ρ and μ for the r, θ, and z components, respectively.

Navier-Stokes equations

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r \quad \ldots\ldots\ldots\ldots (36)$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta \quad \ldots\ldots\ldots\ldots (37)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \ldots\ldots\ldots\ldots (38)$$

Recommended Method

- The continuity and Navier-Stokes Equations can be used to set up descriptive equations for various problems.
- It is possible to formulate an equation for each flow problem, but it is safer to begin with the general equations and cancel terms that do not apply. This is the recommended method in this course.
- The solution technique involves obtaining the descriptive equation for the system, and solving (if possible) subjected to the boundary conditions.
- For a majority of problems, especially in turbulent flow, the equations cannot be solved exactly, due to the presence of the nonlinear acceleration terms.

- A number of solutions exist for laminar-flow (also called viscous-flow) problems in which the acceleration terms vanish from the differential equation of motion.