Boundary-Layer Flow

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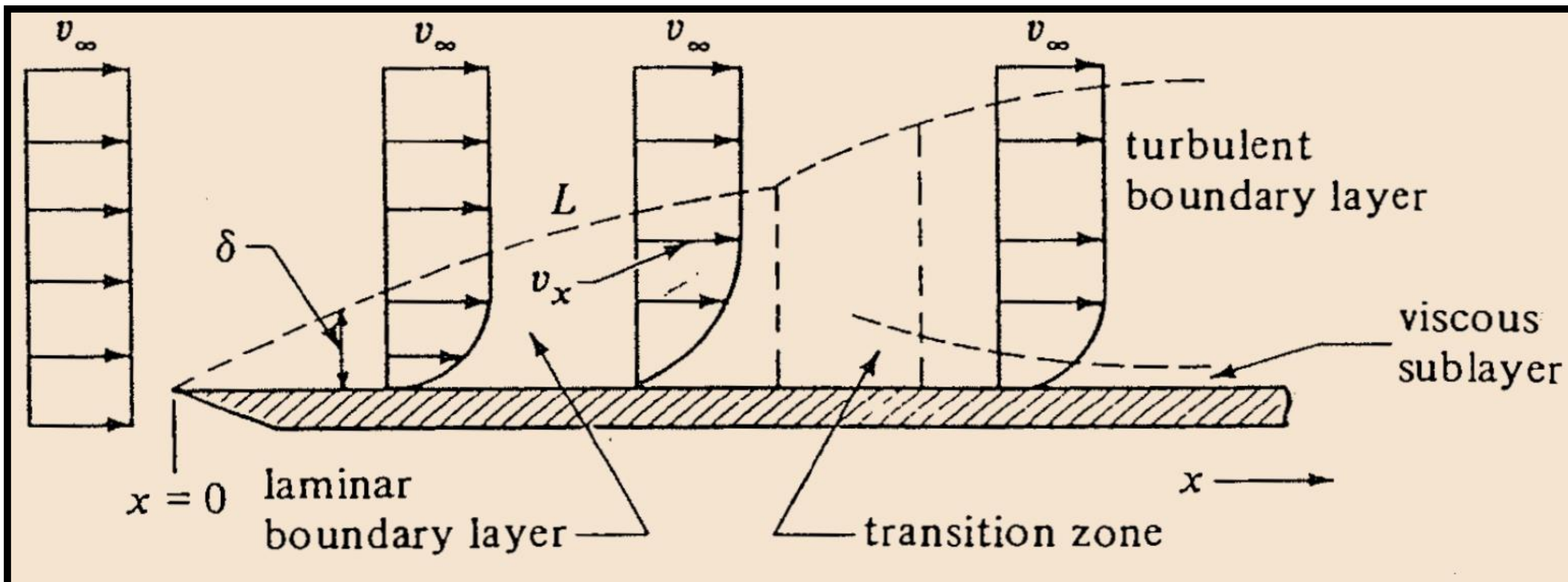
- In this section we are going to focus on the fluid flow around the solid objects. The region close to the solid object which is called the boundary layer will be considered in more detail.
- In general, the fluid motion in boundary-layer region near the solid is greatly affected by the solid surface.
- In the bulk of the fluid away from the boundary layer the flow can often be adequately described by the theory of ideal fluids with zero viscosity.
- However, in the thin boundary layer, viscosity is important. Since the region is thin, simplified solutions can be obtained for the boundary-layer region.

Fluid flow past a flat plate surface

- Assume a steady-state flow of fluid past a flat plate as shown in the following figure.
- A boundary-layer will be developed around the flat surface as shown on the given figure.



Notes

- The velocity of the upstream and at the leading edge x=0 is uniform across the entire fluid and its value is υ_∞.
- The velocity of the fluid at the wall is zero.
- The velocity v_x in the x direction increases as one goes farther from the plate.
- The velocity υ_x approaches asymptotically the velocity v_∞ of the bulk of the stream.
- The dashed line L is drawn so that the velocity at that point is 99% of the bulk velocity υ_∞.
- The layer or zone between the plate and the dashed line constitutes the boundary layer.
- When the flow is laminar, the thickness δ of the boundary layer increases with the $\sqrt{x}$ as we move in the x direction.

To specify the Regime

- Use Reynolds number
- Reynolds number is defined as

$$\mathrm{Re} = \frac{v_{\infty} \rho x}{\mu} \qquad \text{...(54)}$$

- Where x is the distance downstream from the leading edge.

- Re ranges

Re	$< 2 \times 10^5$	$2 \times 10^5 - 3 \times 10^6$	$> 3 \times 10^6$
Regime	Laminar	Transition	turbulent

For calculations assume $\mathrm{Re} \geq 2 \times 10^5$ is Turbulent

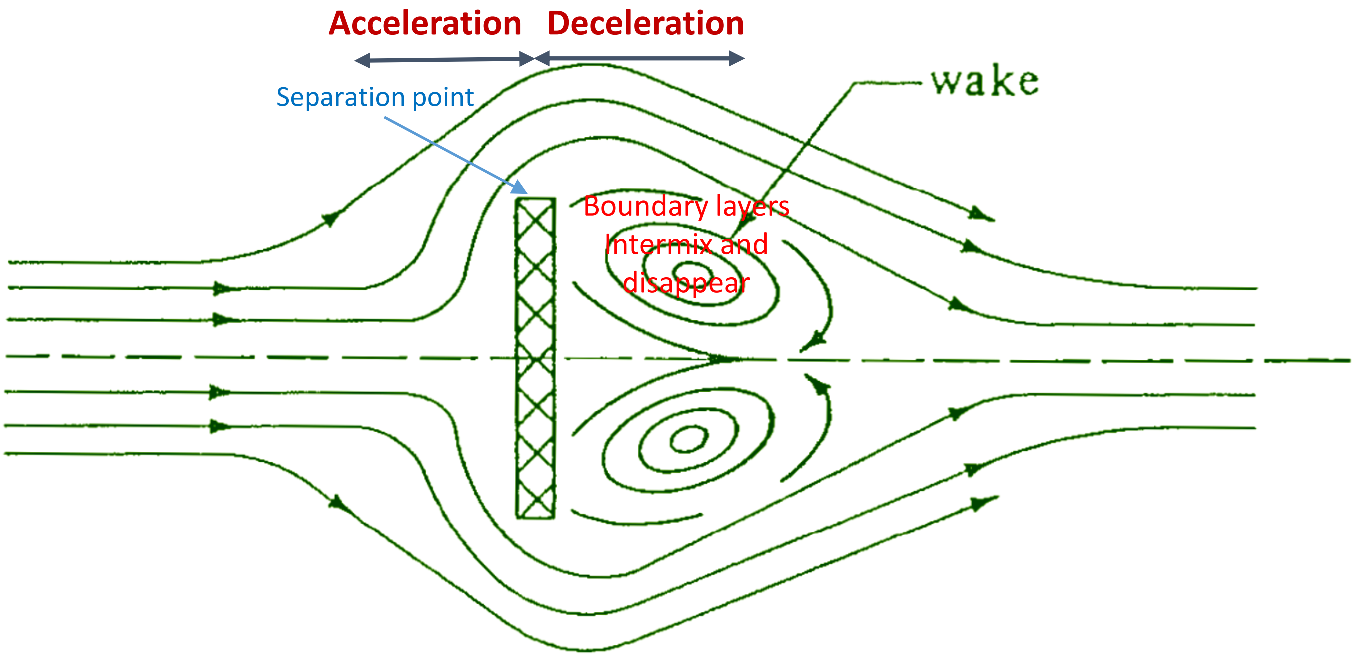
Notes

- For turbulent, a thin viscous sublayer persists next to the plate.

- Drag arises due to viscous shear {called Skin or wall friction}. This is only for flat plate.
- What about spherical shapes and other shapes ~ “form drag” due change in pressure & directions see previous lectures types of drags.
- Total drag = Skin + form

Boundary Layer separation and formation of Wakes

- Consider a flat plate perpendicular to the fluid flow as shown in the following figure.



- A growth of boundary layer takes place as discussed before.

- At the trailing edge or rear edge of the flat plate, the boundary layers are present at the top and bottom sides of the plate.
- On leaving the plate, the boundary layers gradually intermix and disappear.
- At the edge of the plate, the momentum in the fluid prevents it from making the abrupt turn around the edge of the plate, and it separates from the plate.
- A zone of decelerated fluid is present behind the plate, and large eddies (vortices), called the *wake*, are formed in this area.
- The eddies consume large amounts of mechanical energy.
- This separation of boundary layers occurs when the change in velocity of the fluid flowing by an object is too large in direction or magnitude for the fluid to adhere to the surface.

Laminar Flow and Boundary-Layer Theory

- *Boundary-layer equations*
  - In case laminar flow, some terms in Navier-Stokes equation become negligible and can be eliminated.
  - the thickness of the boundary layer δ is taken as the distance away from the surface where the velocity is 99% of the free stream velocity υ_∞.
  - For two-dimensional laminar flow in the x and y directions of a fluid having a constant density, Eqs. 32 and 33

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)-\frac{\partial p}{\partial x}+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)-\frac{\partial p}{\partial y}+\rho g_y$$

- The previous eqs. After the following assumption:
  - Steady state flow conditions
  - Only x and y directions
  - Body forces is negligible as the boundary layer is thin i.e g_x and $g_y = 0$.

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) \quad \text{.................(55)}$$

$$v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right) \quad \text{................(56)}$$

- The continuity equation for two-dimensional flow becomes

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \quad \text{..................................(57)}$$

- In eq. 55 the term $\mu/\rho(\partial^2 v_x/\partial x^2)$ is negligible in comparison with the other terms in the equation. Also, it can be shown that all the terms containing v_y : and its derivatives are small. {See notes below}
- Hence the final two boundary layer equations to be solved are 57 and 58. {this means the continuity Eq. and the x-Navier-Stokes Eq. after approximations}

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\mu}{\rho}\frac{\partial^2 v_x}{\partial y^2} \quad \text{.....................(58)}$$

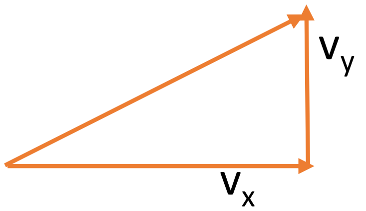
Notes: v_x horizontal component, v_y vertical component

$v_y < v_x$

$\partial v_y/\partial x$ plus other v_y derivatives are very small

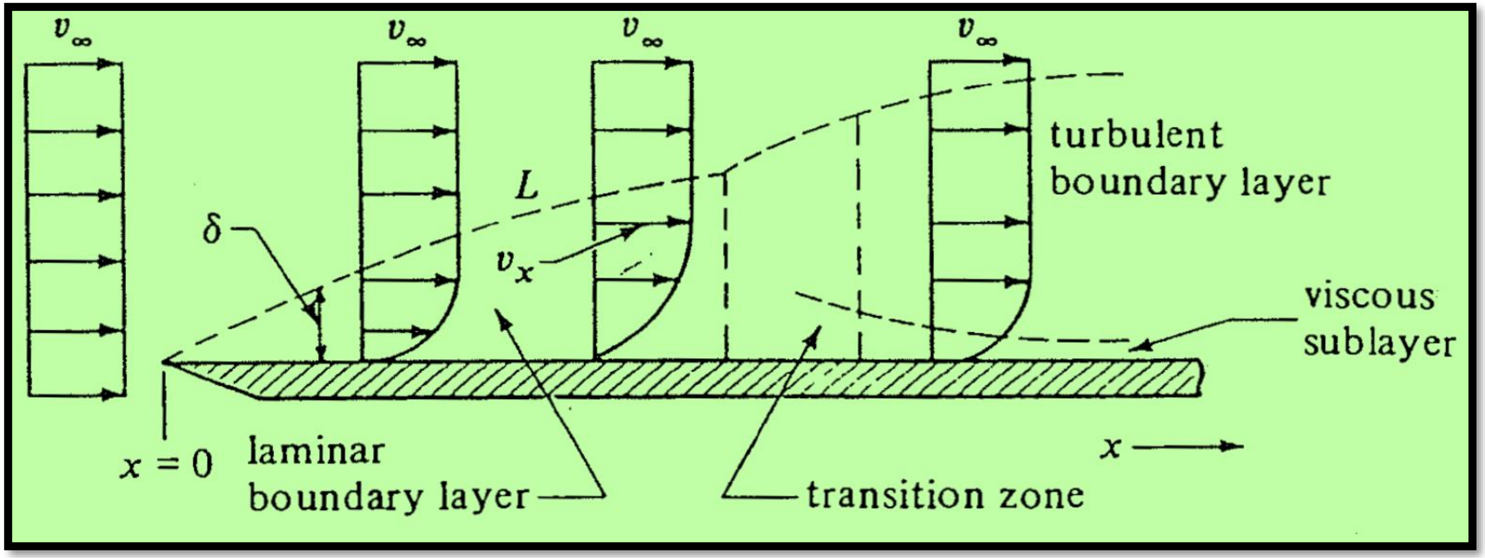
Or In general, eq. 56 is not important

X-direction component is important one i.e eq. 55 or 58



Solution for laminar boundary layer on a flat plate.

- For the below figure and for the laminar boundary layer we are looking to find some expressions for δ and drag force by solving the previous two partial differential equations. This is called the analytical method of solution. See below the details.



- We can do more simplification for Eq. 58 by assuming dp/dx =0 since the bulk velocity υ_∞ is constant.
- The final boundary-layer equations reduce to the equation of motion for the x direction and the continuity
- equation as follows:

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 v_x}{\partial y^2} \qquad \text{....................(59)}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \qquad \text{....................(60)}$$

- Boundary conditions are:

$\upsilon_x = \upsilon_y = 0$ at $y = 0$ (y is distance from plate)

$\upsilon_x = \upsilon_\infty$ at $y = \infty$

Solution

- The solution of this problem for laminar flow over a flat plate giving v_x and v_y as a function of x and y was first obtained by Blasius and later elaborated by Howarth
- In general, Blasius reduced the two equations to a single ordinary differential equation which is nonlinear.
- The equation could not be solved to give a definite form but a series solution was obtained.
- <u>The summary of Blasius solution is</u>:
- The thickness of the boundary-layer δ at $v_x \cong 0.99 v_\infty$ is

$$\delta = \frac{5.0x}{\sqrt{N_{Re,x}}} = 5.0\sqrt{\frac{\mu x}{\rho v_\infty}} \quad \text{......................(61)}$$

where $N_{Re,x} = xv_\infty \rho/\mu$. Hence, the thickness δ varies as $\sqrt{x}$.

- The drag in flow past a flat plate consists only of skin friction and is calculated from the shear stress at the surface at $y = 0$ for any x as follows.

$$\tau_0 = \mu \left(\frac{\partial v_x}{\partial y}\right)_{y=0} \qquad \text{.....................(62)}$$

- From the relation of v_x as a function of x and y obtained from the series solution eq. 62 becomes

$$\tau_0 = 0.332\mu v_\infty \sqrt{\frac{\rho v_\infty}{\mu x}} \qquad \text{.....................(63)}$$

- The total drag is given by the following for a plate of length L and width b:

$$F_D = b \int_0^L \tau_0 \, dx \qquad \text{.....................(64)}$$

- $$F_D = 0.664b\sqrt{\mu\rho v_\infty^3 L}$$ ....(65)
- The drag coefficient C_D related to the total drag on one side of the plate having an area $A = bL$ is defined as
  $$F_D = C_D \frac{v_\infty^2}{2} \rho A$$ ....(66)
- Substituting A value and eq. 65 in eq. 66
  $$C_D = 1.328\sqrt{\frac{\mu}{L v_\infty \rho}} = \frac{1.328}{N_{Re,L}^{1/2}}$$ ....(67)
- ***Notes:***
  a) The definition of C_D in eq. 67 is similar to Fanning friction factor *f* for pipes.
  b) The equation derived for C_D applies only to the laminar boundary layer for $N_{Re,L}$ less than about 5×10^5.