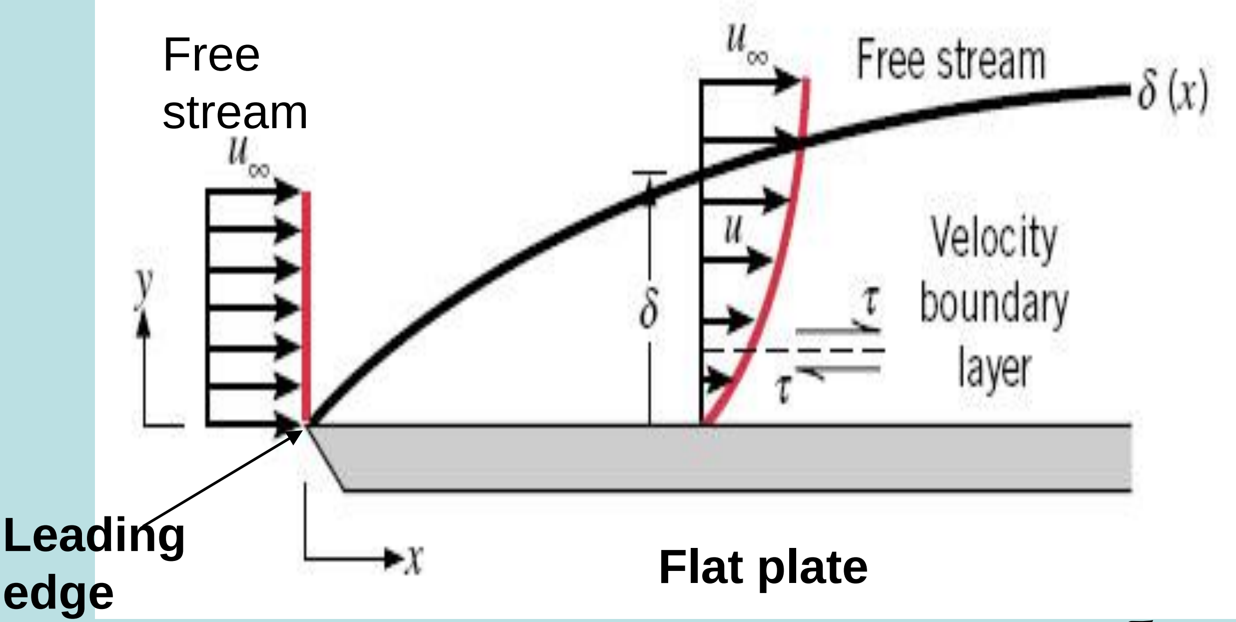
Convection Heat Transfer

Introduction & heat transfer coefficients

Convection Heat Transfer

- Bulk fluid motion (advection)
- Random motion of Fluid molecules (diffusion)

The velocity boundary layer



δ : Boundary layer thickness≡ the value of y for which u=0.99u$_\infty$

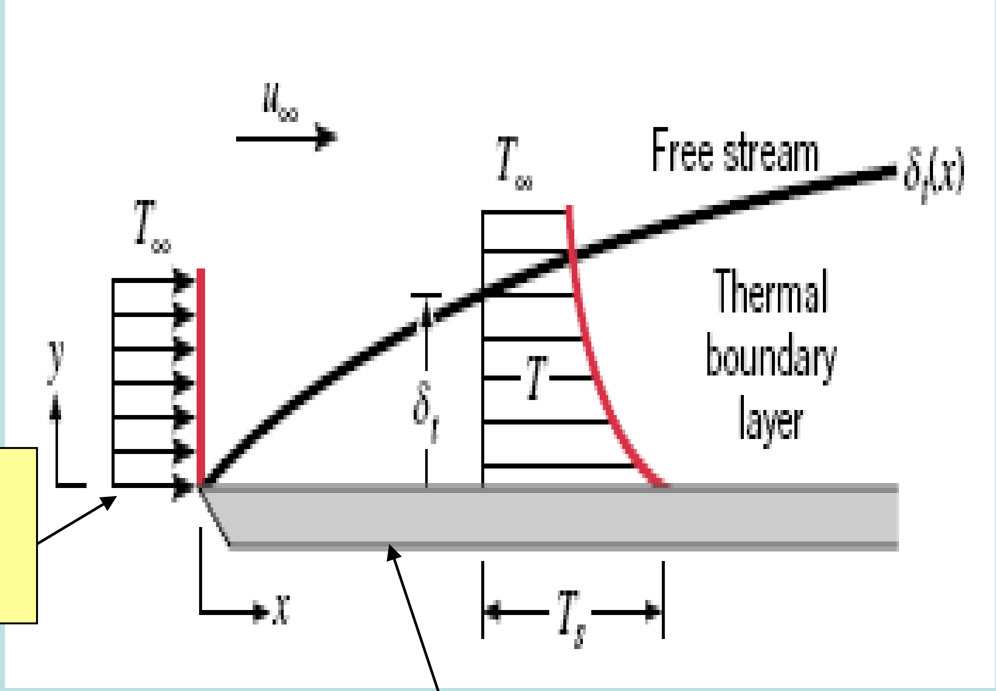
The local friction coefficient is $C_f = \dfrac{\tau_s}{\rho u_\infty^2 / 2}$

The surface shear stress is evaluated at the wall surface $\tau_s = \mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0}$

The Thermal Boundary Layer

- Thermal B.L is created as a temperature difference is found between surface and a fluid.
- The region of the fluid in which temperature gradients develop is the thermal B.L.

Fluid of Uniform temp



Isothermal plate

Thermal B.L. thickness δ_t is the value y for the ratio $[(T_s-T)/(T_s-T_\infty)]=0.99$

The local heat flux at any distance x from the leading edge is obtained by applying Fourier's law to the fluid at y=0.

$$q_s'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

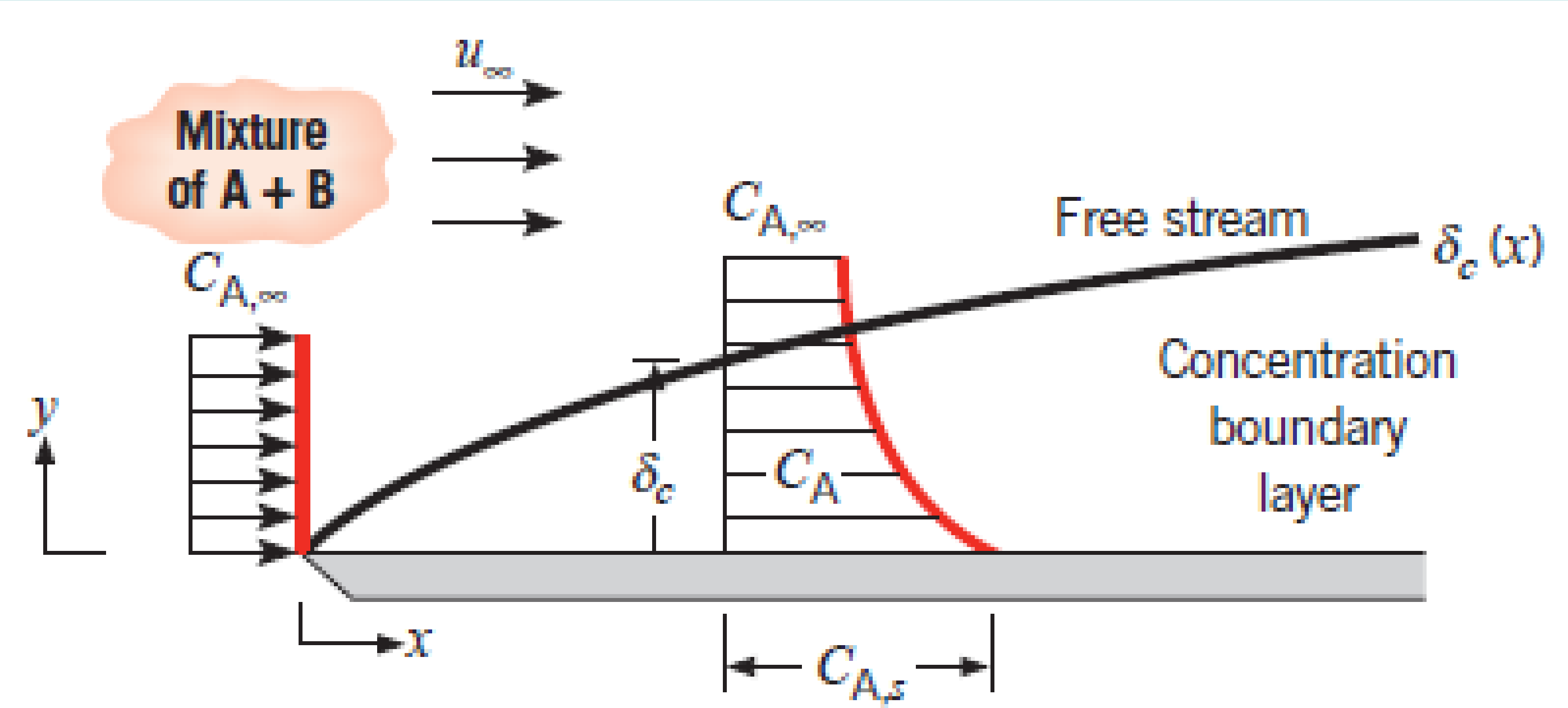
We can also apply Newton's law of cooling

$$q_s'' = h(T_s - T_\infty)$$

combing the previous two equations

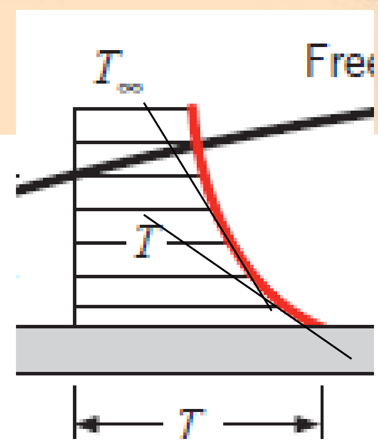
$$h = \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty} \quad \text{....................(1)}$$

Species concentration boundary layer development on a flat plate.

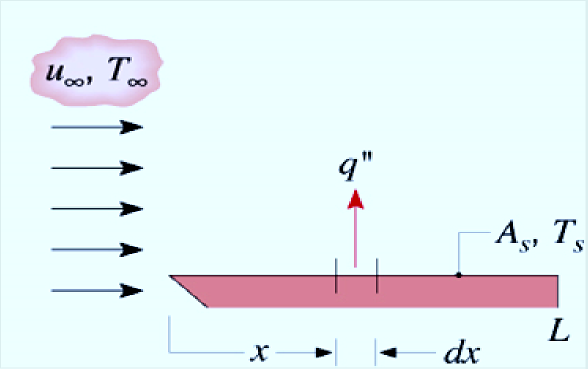


Vapor (species A) in gas (species B). **!!!** **H_2O in air over the surface of ocean**

Hence, conditions in the thermal boundary layer, which strongly influence the wall temperature gradient $\partial T/\partial y|_{y=0}$, determine the rate of heat transfer across the boundary layer. Since $(T_s - T_\infty)$ is a constant, independent of x, while δ_t increases with increasing x, temperature gradients in the boundary layer must decrease with increasing x. Accordingly, the magnitude of $\partial T/\partial y|_{y=0}$ decreases with increasing x, and it follows that q_s'' and h decrease with increasing x.



The relation between local and average convection coefficients



$$q = \int_{A_s} q'' dA_s \qquad \text{........(2)}$$

$$q = (T_s - T_\infty)\int_{A_s} h dA_s \qquad \text{....................(3)}$$

Defining an *average convection coefficient* $\bar{h}$ for the entire surface, the total heat transfer rate may also be expressed as

$$\boxed{q = \bar{h} A_s (T_s - T_\infty)} \qquad \text{....................(4)}$$

Equating Equations (3) and (4), it follows that the average and local convection coefficients are related by an expression of the form

$$\boxed{\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s} \qquad \text{....................(5)}$$

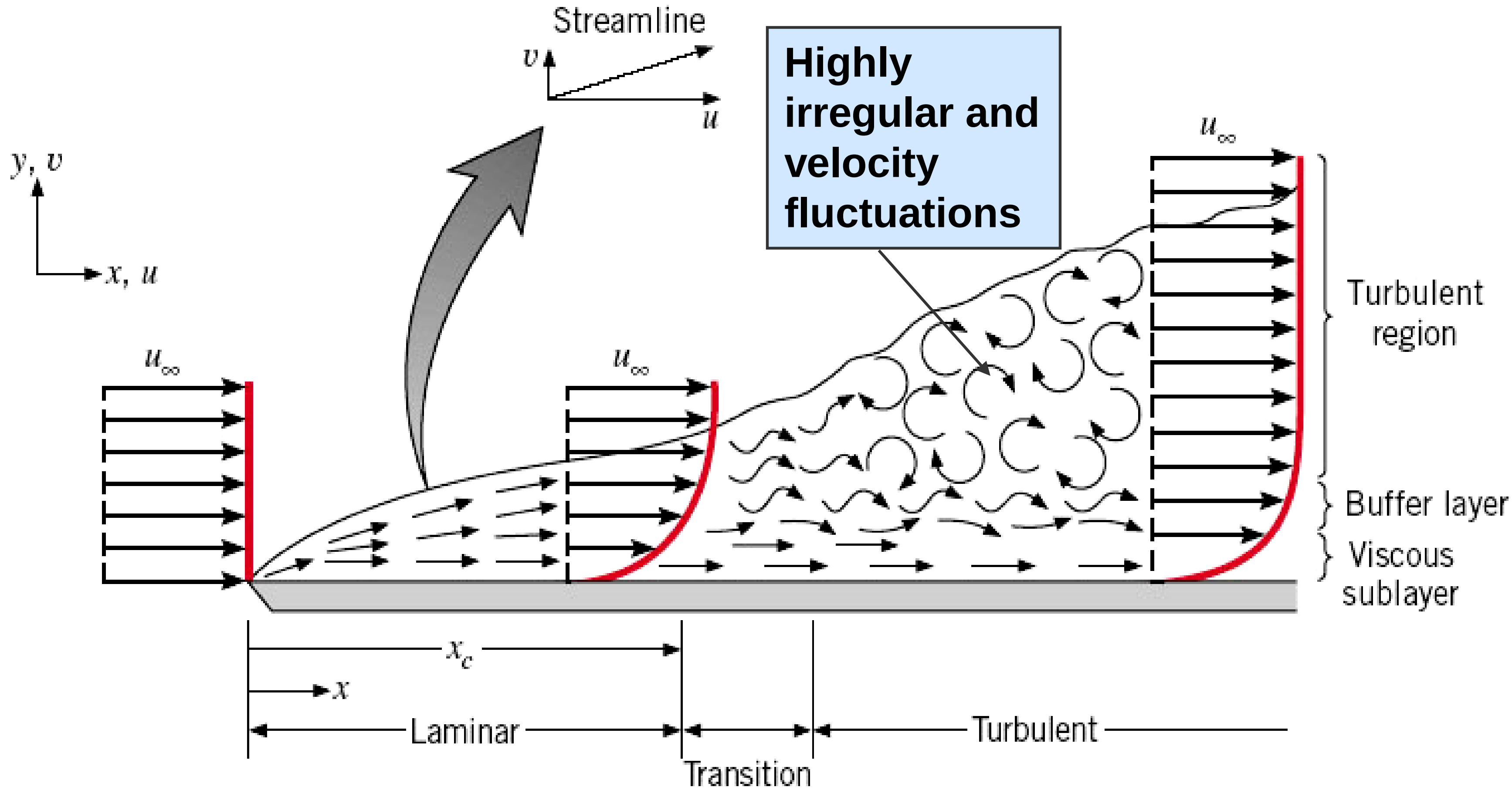
Note that for the special case of flow over a flat plate as in above fig. , h varies only with the distance x from the leading edge and Equation **(5)** reduces to

$$\bar{h} = \frac{1}{L}\int_0^L h dx$$

Summary of the Boundary layers

	Velocity B.L **$\delta(x)$**	Thermal B.L **$\delta_t(x)$**
Characterized by	velocity gradients and shear stress	temperature gradients and heat transfer
Engineering applications	Surface friction	Convection heat transfer
Key parameter	Friction coeff. 'C_f'	Convection heat transfer coeff. 'h'

Laminar and turbulent Flow



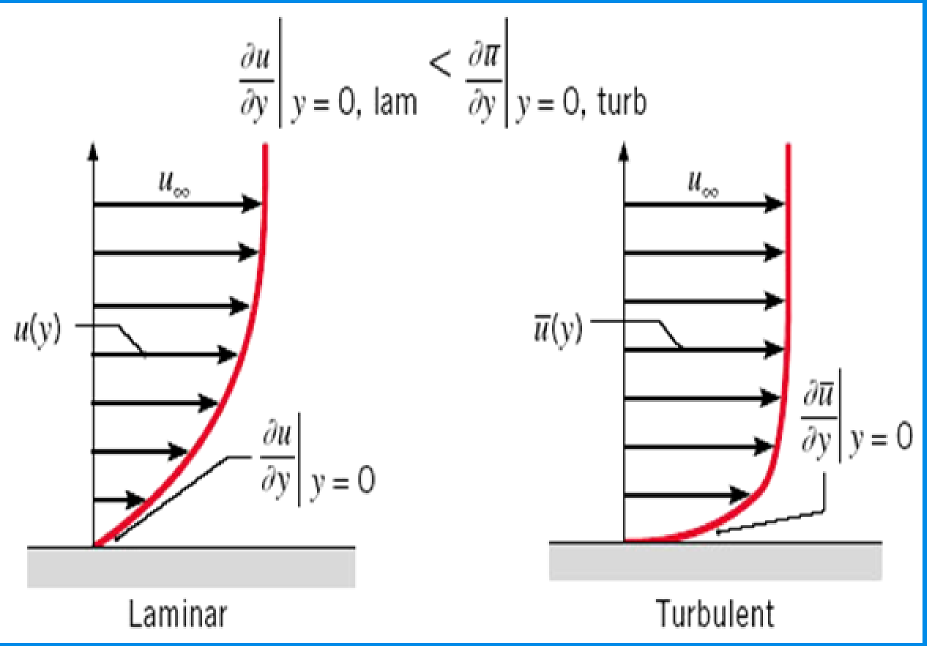
Velocity boundary layer development on a flat plate.

Summary of the turbulent layer

Regions	Transport by	Velocity profile
1. Laminar sublayer	diffusion	Nearly linear
2. Buffer layer	diffusion + turbulent mixing	Not linear
3. Turbulent zone	Turbulent mixing	Not linear

Comparison of Laminar and turbulent velocity B.L. profiles for the same free stream velocity

The turbulent velocity profile is relatively flat due to mixing that occurs within the buffer and turbulent layers giving rise to large velocity gradients within the viscous sublayer.



$$\mathrm{Re}_x = \frac{\rho u_\infty x}{\mu}$$

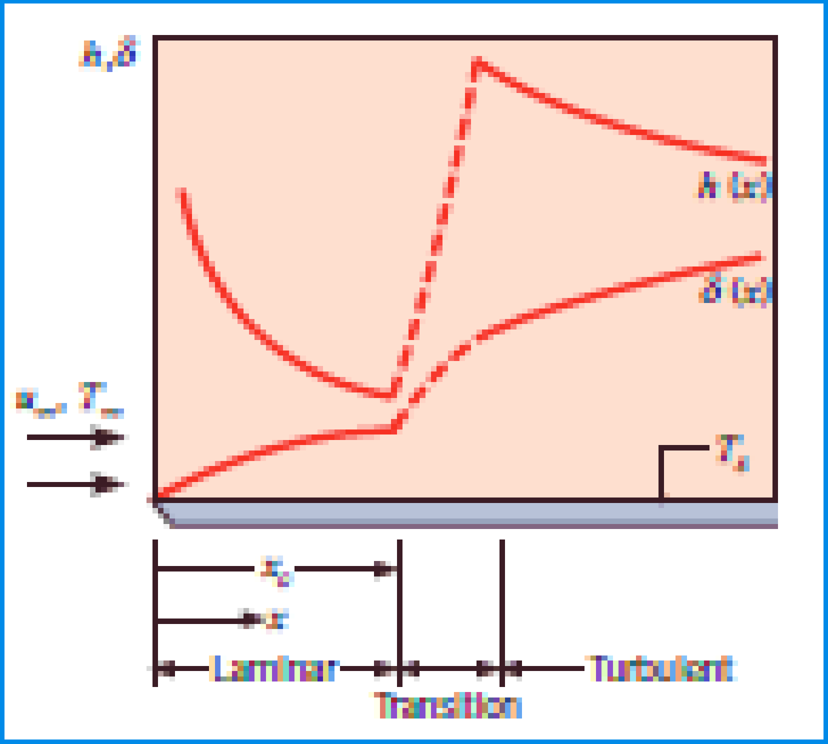
Transition begins at a critical distance, x_c defined by

$$\mathrm{Re}_{x,c} = \frac{\rho u_\infty x_c}{\mu} = 5x10^5$$

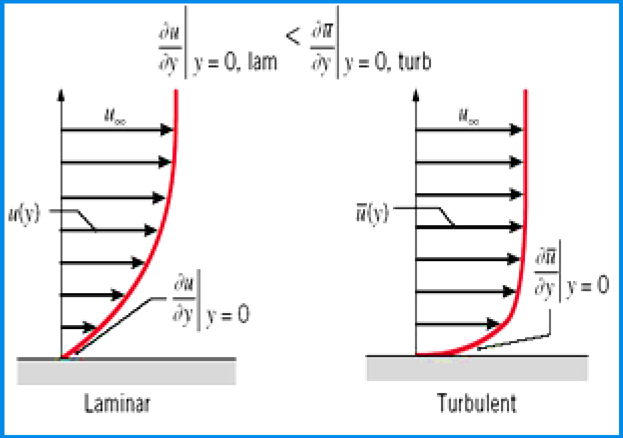
$$\tau_s\big|_{\mathrm{turb}} > \tau_s\big|_{\mathrm{lam}}$$

Notes

1. Similar to the laminar **velocity boundary layer**, the **thermal** boundary layer grows in increasing *x direction,*
2. *Temperature gradient in the fluid at y = 0* decrease in the x direction, and, from Equation (1), the heat transfer coefficient also decreases with increasing *x. See the adjacent figure.*

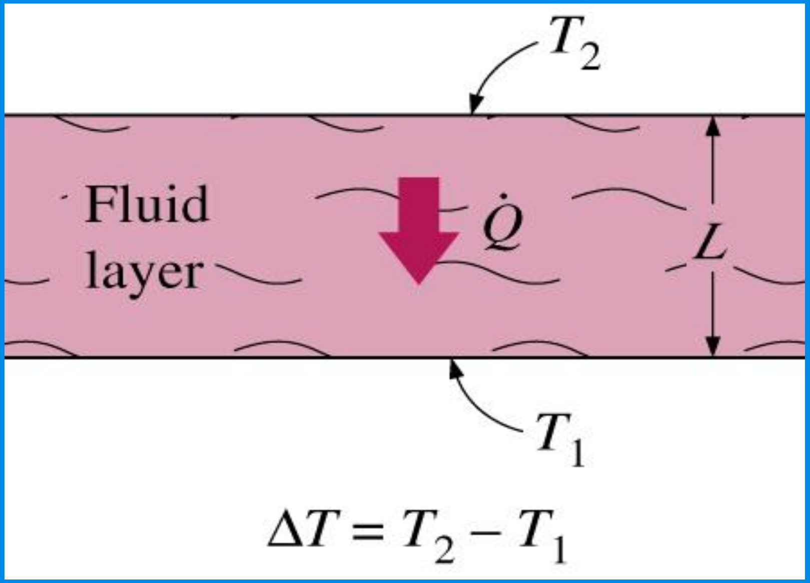


3. Just as it induces large velocity gradients at *y = 0 (Transition regime), as shown in the Figure, turbulent* mixing promotes large temperature gradient adjacent to the solid surface as well as a corresponding increase in the heat coefficient across the transition region. These effects are illustrated in the Figure for the velocity boundary layer thickness *and the local convection heat transfer coefficient h.*



Nusselt Number

Consider a fluid layer of thickness L and temp. difference $\Delta T = T_2 - T_1$. Heat transfer through the fluid layer will be by convection when the fluid involves some motion and by conduction when the fluid layer is motionless.



Heat flux

$$\dot{q}_{\text{conv}} = h\Delta T$$

and

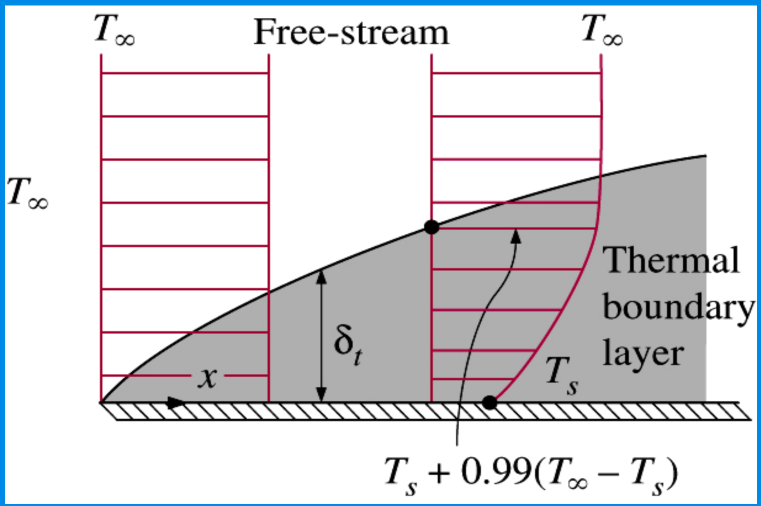
$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L}$$

Taking the ratio

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

The larger the Nusselt number, the more effective the convection. A Nusselt number of Nu 1 for a fluid layer represents heat transfer across the layer by pure conduction.

Boundary layer – temperature & Prandtl Number



<u>Definition</u>

thermal boundary layer : It is the flow region over the surface in which the temperature variation in the direction normal to the surface takes place.

$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

Where μ=ρν,

Typical ranges of Prandtl numbers for common fluids

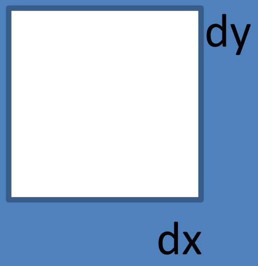
Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

Conservation of Energy equation within the thermal boundary layer

2-dimension equation

Control volume *dx. dy. 1*

In thermal boundary layer



Basic information

- $E_{in} - E_{out} = \Delta E_{sys}$
- Assume S.S. Flow $\Delta E_{sys} = 0$
- It is well known that energy can be transferred by heat, work and mass . Therefore, for S.S. Flow control volume

  $(E_{in} - E_{out})_{by\ heat} + (E_{in} - E_{out})_{by\ work} + (E_{in} - E_{out})_{by\ mass} = 0$ [1]
- Total energy of a flowing fluid per unit mass, e_{stream}

  $e_{stream} = h + KE + PE$

  where h : enthalpy = u + pv

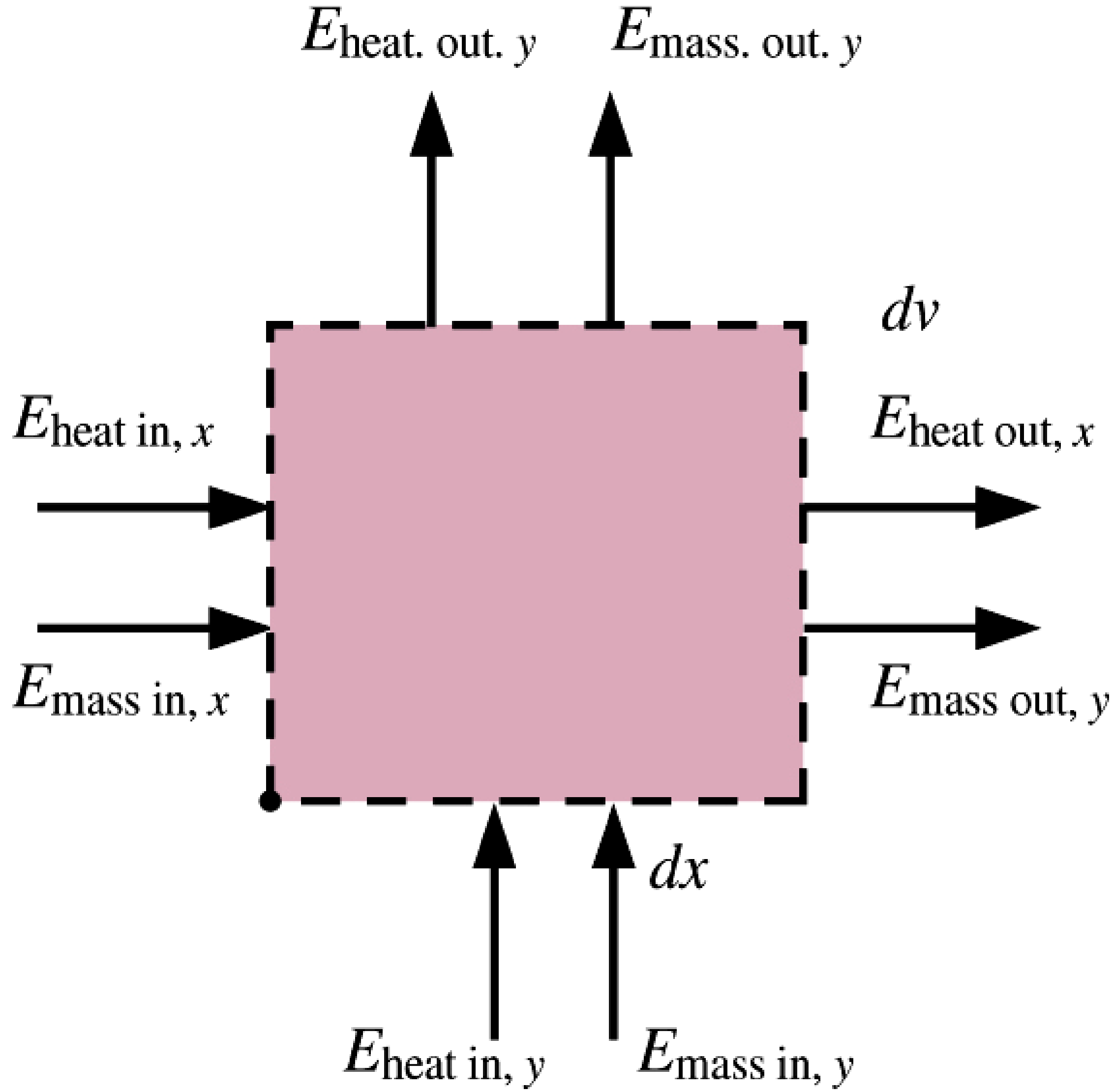
  internal energy

  Flow energy

Notes

- Ignore KE and PE because they are very small.
- Assume ρ , C_p , k and μ are constants.
- Hence, energy per unit mass becomes

  $e_{stream} = h = C_p T$
- Mass entering the control volume from left is $\rho u(dy.1)$.
- Now see the figure of energy transfer by heat and mass flow associated with the a differential control volume in the thermal boundary layer in steady state 2-dimensional flow.


$E_{\text{heat. out. }y}$
$E_{\text{mass. out. }y}$
dv
$E_{\text{heat in, }x}$
$E_{\text{heat out, }x}$
$E_{\text{mass in, }x}$
$E_{\text{mass out, }y}$
dx
$E_{\text{heat in, }y}$
$E_{\text{mass in, }y}$

Energy transfer to control volume in x-direction is

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}, x} = (\dot{m}e_{\text{stream}})_x - \left[(\dot{m}e_{\text{stream}})_x + \frac{\partial(\dot{m}e_{\text{stream}})_x}{\partial x} dx \right]$$

$$= -\frac{\partial[\rho u(dy \cdot 1)C_p T]}{\partial x} dx = -\rho C_p \left(u \frac{\partial T}{\partial x} + T \frac{\partial u}{\partial x} \right) dx\, dy$$

∴ Energy transfer to control volume in y-direction is

$$(E_{in} - E_{out})_{\text{by mass, y}} = -\rho C_p \left(v \frac{\partial T}{\partial y} + T \frac{\partial v}{\partial y}\right) dxdy$$

- Adding the previous two equations, we obtain the net rate of energy transfer by mass

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by mass}} = -\rho C_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy - \rho C_p\left(v\frac{\partial T}{\partial y} + T\frac{\partial v}{\partial y}\right)dx\,dy$$

$$= -\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dx\,dy \qquad [2]$$

$$\text{note } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \text{ from continuity equation}$$

- The net rate of heat conduction to volume element in x-direction is

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by heat}, x} = \dot{Q}_x - \left(\dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x} dx\right)$$

$$= -\frac{\partial}{\partial x}\left(-k(dy \cdot 1)\frac{\partial T}{\partial x}\right) dx = k\frac{\partial^2 T}{\partial x^2} dx\, dy$$

- The net rate of heat conduction to volume element in y-direction is

$$(E_{in} - E_{out})_{\text{by heat, y}} = k\frac{\partial^2 T}{\partial y^2} dxdy$$

- Adding the previous two equations, we obtain the net rate of energy transfer by heat

$$(\dot{E}_{in} - \dot{E}_{out})_{\text{by heat}} = k\frac{\partial^2 T}{\partial x^2}dx\,dy + k\frac{\partial^2 T}{\partial y^2}dx\,dy = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)dx\,dy \quad \textbf{[3]}$$

- Ignore the effect of the work done by body and surface forces, hence the energy eq. is obtains by substituting eqs. [2] and [3] into [1]

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad \textbf{[4]}$$

Notes

- When the viscous shear stresses are not negligible, add the term ($\mu\varphi$) to the previous equation [4] i.e.

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\varphi$$

- Where φ is the viscous dissipation function

$$\varphi = 2\left[(\frac{\partial u}{\partial x})^2+(\frac{\partial v}{\partial y})^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2$$

Equation [4] can be rearranged and noting that $\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$

$$\therefore \quad \rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\cancelto{0}{\frac{\partial^2 T}{\partial x^2}} + \frac{\partial^2 T}{\partial y^2}\right)$$

Energy equation becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Summary

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

with the boundary conditions

At $x = 0$:	$u(0, y) = u_\infty$,	$T(0, y) = T_\infty$
At $y = 0$:	$u(x, 0) = 0$,	$v(x, 0) = 0$, $T(x, 0) = T_s$
As $y \to \infty$:	$u(x, \infty) = u_\infty$,	$T(x, \infty) = T_\infty$