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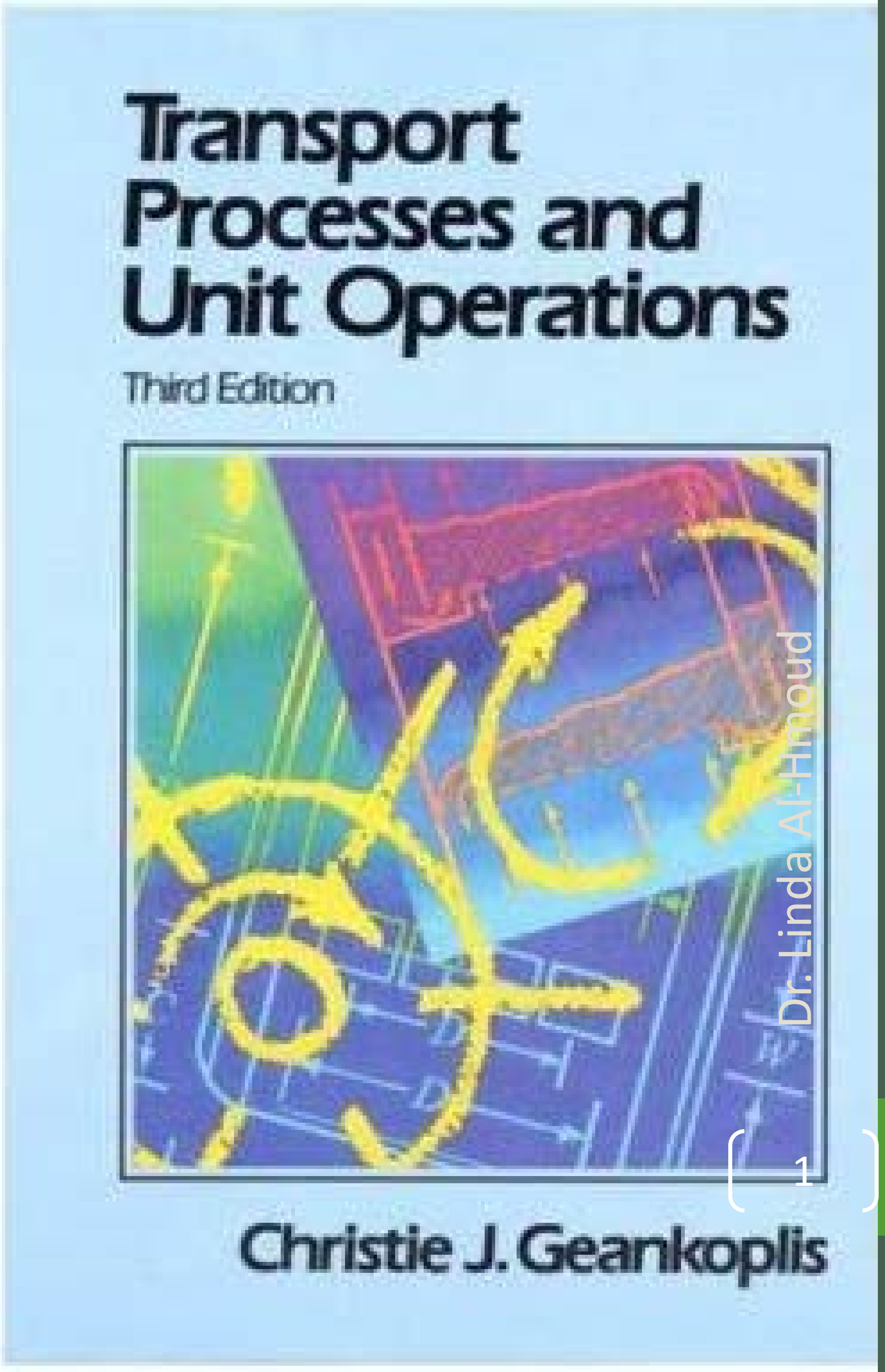
School of Engineering
Chemical Engineering Department

(0915341)
TRANSPORT PHENOMENA

Dr. Linda Al-Hmoud



What Are The Transport Phenomena?

The subject of transport phenomena includes three closely related topics:

Fluid Dynamics

involves the transport of **momentum**

Heat Transfer

deals with the transport of **energy**

Mass Transfer

concerned with the transport of **mass** of various chemical species

What Are The Transport Phenomena?

- These three transport phenomena should, at the introductory level, be studied together for the following reasons:
  - They frequently occur simultaneously in industrial, biological, agricultural, and meteorological problems; in fact, the occurrence of any one transport process by itself is the exception rather than the rule.
  - The **basic equations** that describe the three transport phenomena are closely related. The similarity of the equations under simple conditions is the basis for solving problems "by analogy."
  - The **mathematical tools** needed for describing these phenomena are very similar. Although it is not the aim of this book to teach mathematics, the student will be required to review various mathematical topics as the development unfolds. Learning how to use mathematics may be a very valuable by-product of studying transport phenomena.
  - The **molecular mechanisms** underlying the various transport phenomena are very closely related. All materials are made up of molecules, and the same molecular motions and interactions are responsible for viscosity, thermal conductivity, and diffusion.

Three Levels at which Transport Phenomena can be Studied

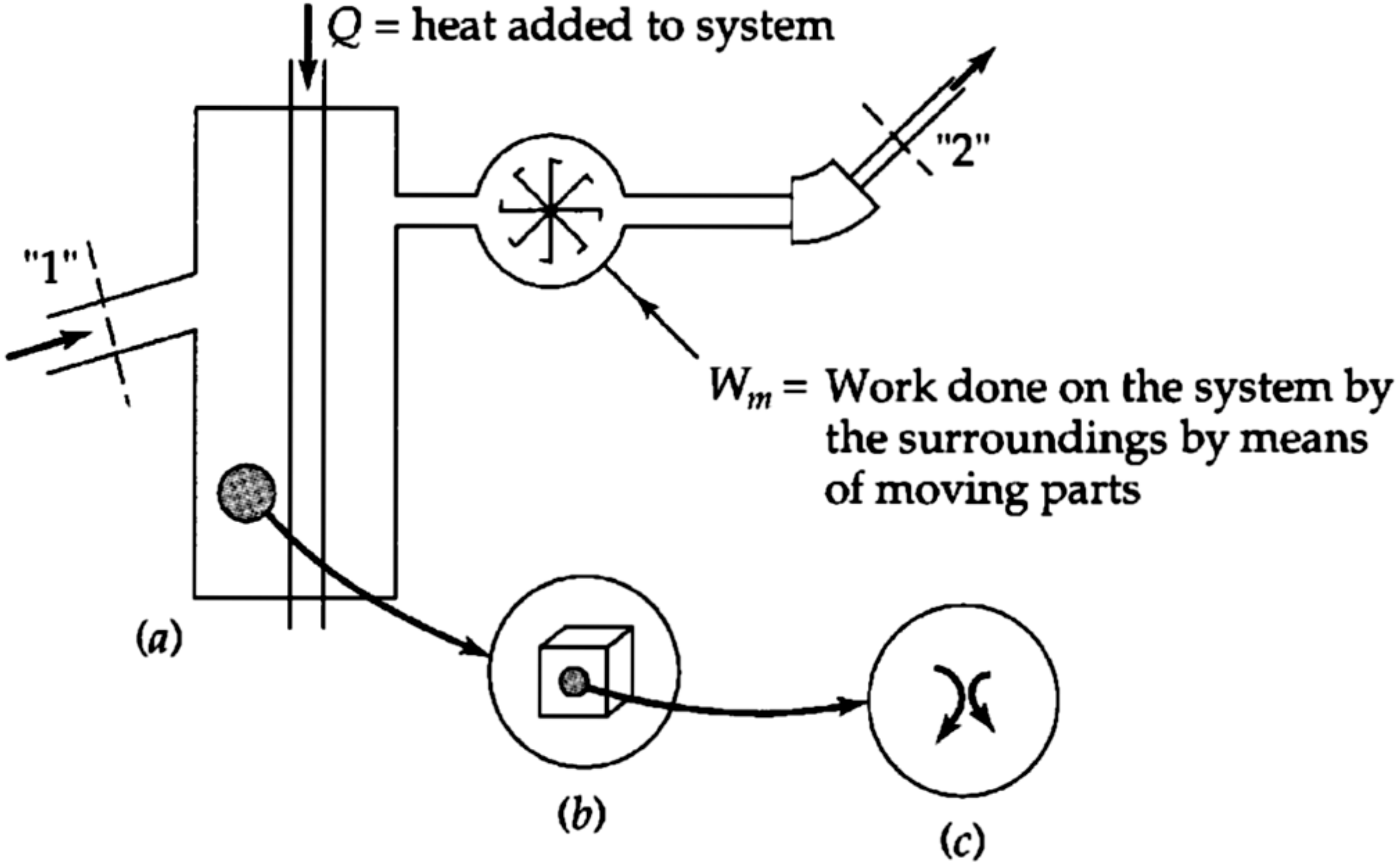
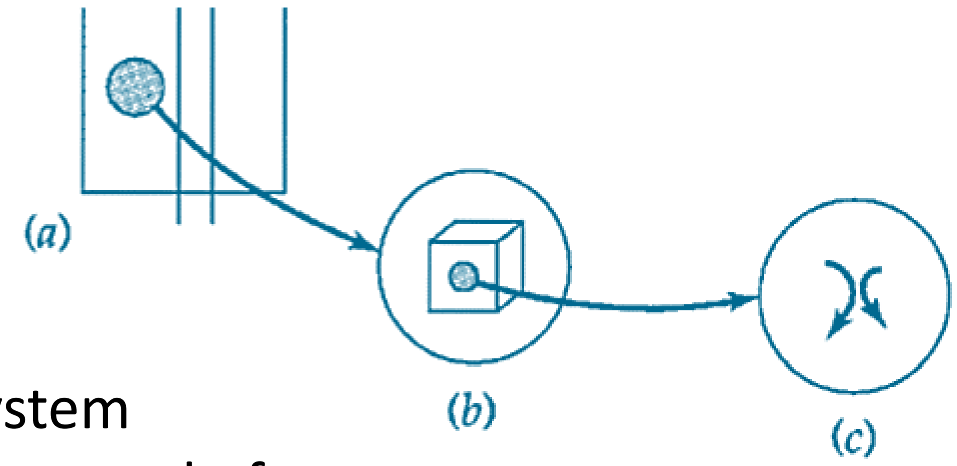


Fig. 0.2-1 (*a*) A macroscopic flow system containing N_2 and O_2; (*b*) a microscopic region within the macroscopic system containing N_2 and O_2, which are in a state of flow; (*c*) a collision between a molecule of N_2 and a molecule of O_2.

Three Levels at which Transport Phenomena can be Studied

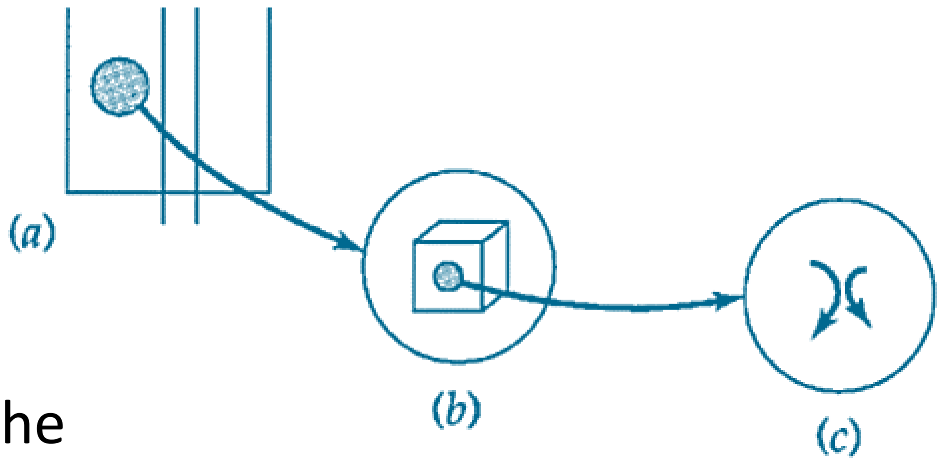
At the Macroscopic level (a)



- We write down a set of equations called the "**macroscopic balances**," which describe how the mass, momentum, energy, and angular momentum in the system change because of the introduction and removal of these entities via the entering and leaving streams, and because of various other inputs to the system from the surroundings. *No attempt is made to understand all the details of the system.*
- In studying an engineering or biological system it is a good idea to start with this macroscopic description in order to make a global assessment of the problem; in some instances, it is only this overall view that is needed.

Three Levels at which Transport Phenomena can be Studied

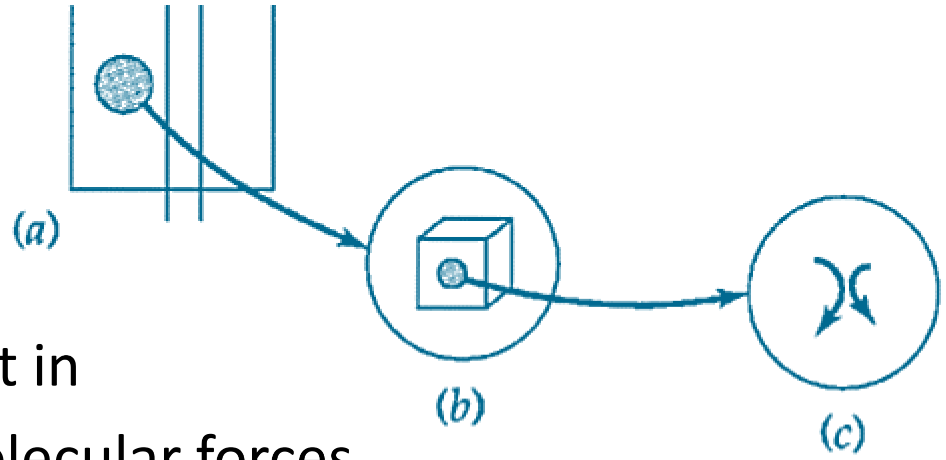
At the Microscopic level (b)



- We examine what is happening to the fluid mixture in a small region within the equipment.
- We write down a set of equations called the "**equations of change**," which describe how the mass, momentum, energy, and angular momentum change within this small region.
- The **aim here** is to get information about velocity, temperature, pressure, and concentration profiles within the system.
- This more detailed information may be required for the understanding of some processes.

Three Levels at which Transport Phenomena can be Studied

At the Molecular level (c)



- We seek a fundamental understanding of the mechanisms of mass, momentum, energy, and angular momentum transport in terms of molecular structure and intermolecular forces.
- Generally, this is the realm of the theoretical physicist or physical chemist, but occasionally engineers and applied scientists have to get involved at this level.
- This is particularly true if the processes being studied involve complex molecules, extreme ranges of temperature and pressure, or chemically reacting systems.

Relationship between Transport Phenomena

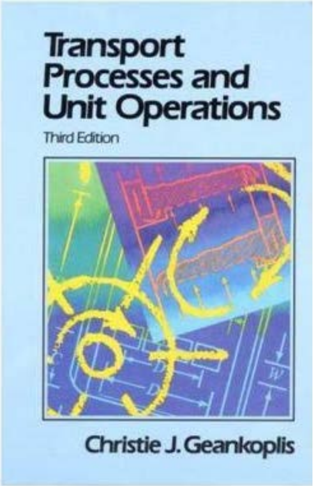
Type of transport	Momentum	Energy	Mass
Transport by molecular motion	1 Viscosity and the stress (momentum flux) tensor	9 Thermal conductivity and the heat-flux vector	17 Diffusivity and the mass-flux vectors
Transport in one dimension (shell-balance methods)	2 Shell momentum balances and velocity distributions	10 Shell energy balances and temperature distributions	18 Shell mass balances and concentration distributions
Transport in arbitrary continua (use of general transport equations)	3 Equations of change and their use [isothermal]	11 Equations of change and their use [nonisothermal]	19 Equations of change and their use [mixtures]
Transport with two independent variables (special methods)	4 Momentum transport with two independent variables	12 Energy transport with two independent variables	20 Mass transport with two independent variables

Relationship between Transport Phenomena

Type of transport	Momentum	Energy	Mass
Transport in turbulent flow, and eddy transport properties	5 Turbulent momentum transport; eddy viscosity	13 Turbulent energy transport; eddy thermal conductivity	21 Turbulent mass transport; eddy diffusivity
Transport across phase boundaries	6 Friction factors; use of empirical correlations	14 Heat-transfer coefficients; use of empirical correlations	22 Mass-transfer coefficients; use of empirical correlations
Transport in large systems, such as pieces of equipment or parts thereof	7 Macroscopic balances [isothermal]	15 Macroscopic balances [nonisothermal]	23 Macroscopic balances [mixtures]
Transport by other mechanisms	8 Momentum transport in polymeric liquids	16 Energy transport by radiation	24 Mass transport in multi-component systems; cross effects

Principles of Momentum Transfer and Overall Balances

Chapter 2



Introduction

- The flow and behavior of fluids is important in many of the unit operations in process engineering.

- **Fluid:** a substance that does not permanently resist distortion and, hence, will change its shape.
  - Gases
  - Liquids
  - Vapors

Have the Characteristics of Fluids

Introduction

- In process industries
  - Materials in fluid form
    - Stored
    - Handled
    - Pumped
    - Processed

It is necessary to become familiar with

- *the principles governing the flow of fluids*
- *the equipment used*

Typical Fluids

- *Water*
- *Air*
- *CO_2*
- *Oil*
- *Slurries*
- *Thick Syrups*

Types of Fluids

Incompressible Fluid

- Inappreciably affected by changes in pressure.
- Most liquids are incompressible.

Compressible Fluid

- Gases are considered to be compressible fluids.

If gases are subjected to small percentage changes in pressure and temperature, their <u>density</u> changes will be small

↓

they can be considered to be <u>*incompressible*</u>

Introduction

- Like all physical matter, a fluid is composed of an extremely large number of molecules per unit volume.
- A theory such as the **kinetic theory of gases or statistical mechanics** treats the motions of molecules in terms of statistical groups and not in terms of individual molecules.
- **In engineering** we are mainly concerned with the ***bulk or macroscopic behavior of a fluid*** rather than the individual ***molecular or microscopic behavior***.

Momentum Transfer

- In **momentum transfer** we treat the fluid as a continuous distribution of matter, or a “**continuum**.”
  - Valid when the smallest volume of fluid contains a number of molecules large enough that a statistical average is meaningful and the macroscopic properties of the fluid (density, pressure, and so on) vary smoothly or continuously from point to point.

- **Momentum Transfer ⇔ Fluid Mechanics**
  - **Fluid Statics** — **fluids at rest**
  - **Fluid Dynamics** — **fluids in motion**

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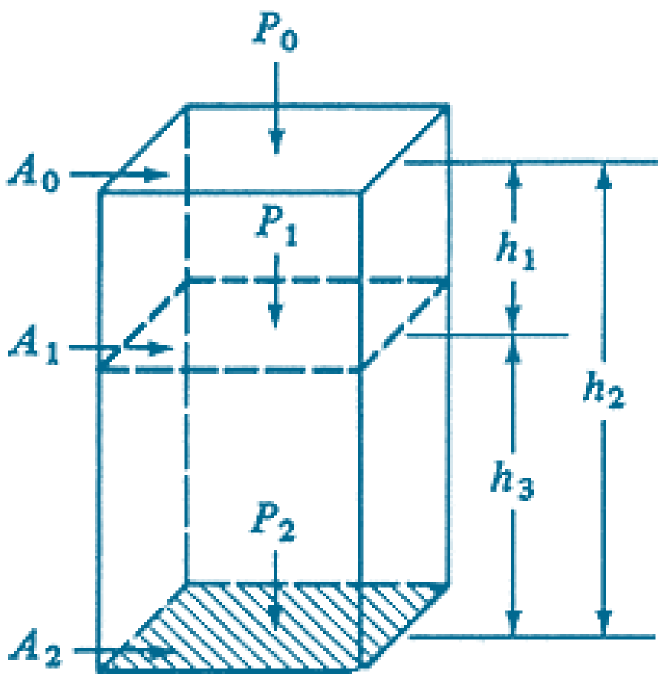
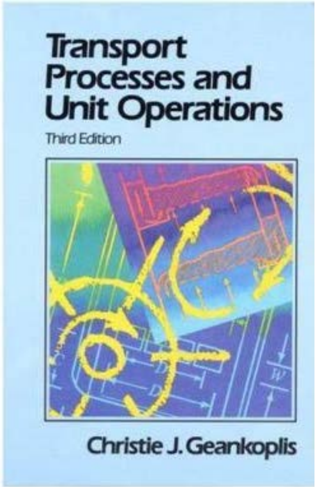
Fluid Statics



Section 2.3



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Force, Units, and Dimensions

- **Pressure**:
  - a surface force exerted by a fluid against the walls of its container.
  - Pressure exists at any point in a volume of a fluid.

$F = mg$ (SI units) $\quad F\ [=]\ N\ (kg \cdot m/s^2), m\ [=]\ kg, g = 9.80665\ m/s^2$

$F = \dfrac{mg}{g_c}$ (English units) $\quad F\ [=] lb_f, m\ [=] lb_m, g = 32.1740\ ft/s^2,$

$g_c = 32.1740\ lb_m \cdot ft/lb_f \cdot s^2$

1 Poundal = 1 $lb_m \cdot ft/s^2$ 1 Dyne = 1 $g \cdot cm/s^2$

EXAMPLE 2.2-1

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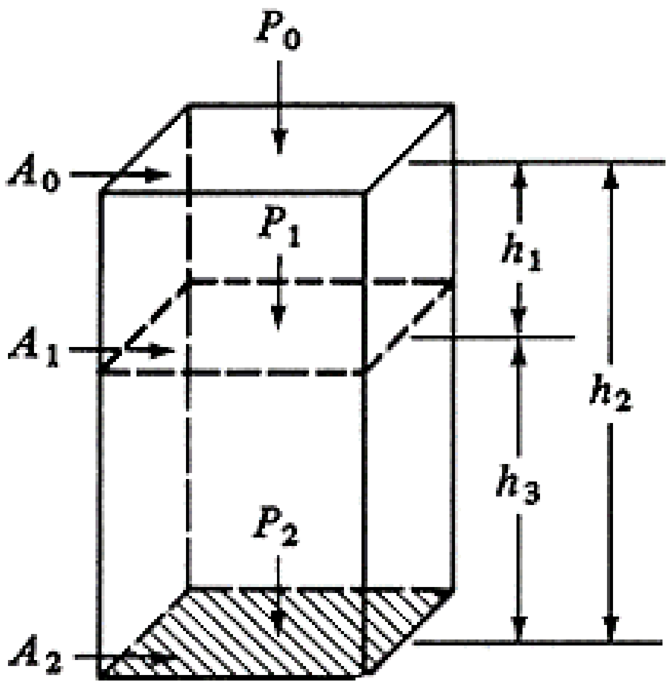
Pressure in a Fluid



$$\text{total kg fluid} = (h_2\ \text{m})(A\ \text{m}^2)\left(\rho \frac{\text{kg}}{\text{m}^3}\right) = h_2 A \rho\ \text{kg}$$

$$F = (h_2 A \rho\ \text{kg})(g\ \text{m/s}^2) = h_2 A \rho g \frac{\text{kg} \cdot \text{m}}{\text{s}^2}\ (\text{N})$$

$$P = \frac{F}{A} = (h_2 A \rho g)\frac{1}{A} = h_2 \rho g\ \text{N/m}^2 \quad \text{or} \quad \text{Pa}$$

- This is the pressure on A_2 due to the mass of the fluid above it.
- To get the **total pressure** P_2 on A_2, the pressure P_0 on the top of the fluid must be added:

$$P_2 = h_2 \rho g + P_0\ \text{N/m}^2 \quad \text{or} \quad \text{Pa}$$

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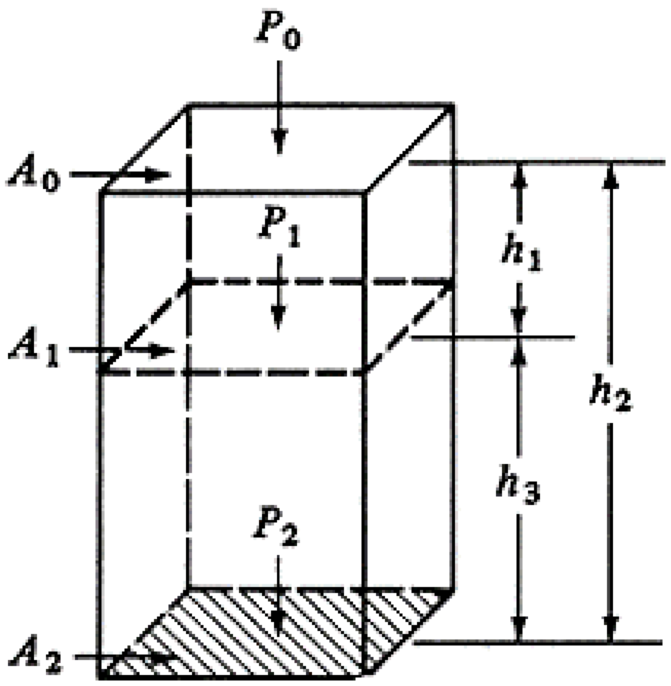
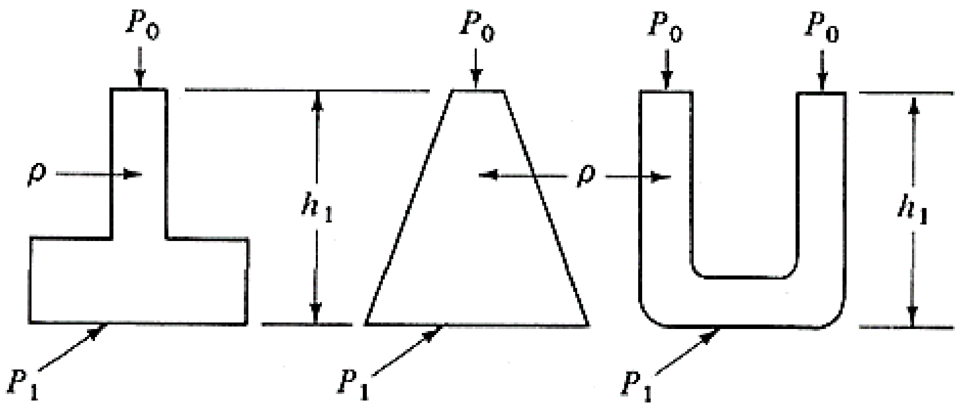
Pressure in a Fluid

- To calculate P_1, $P_1 = h_1 \rho g + P_0$
- The pressure difference between points 2 and 1 is

$$P_2 - P_1 = (h_2 \rho g + P_0) - (h_1 \rho g + P_0)$$
$$= (h_2 - h_1)\rho g \qquad \text{(SI units)}$$

$$P_2 - P_1 = (h_2 - h_1)\rho \frac{g}{g_c} \qquad \text{(English units)}$$





EXAMPLE 2.2-2

Head of a Fluid

- A common method of expressing pressures is in terms of head in m or feet of a **particular fluid**.
- This height or head in m or feet of the given fluid will exert the same pressure as the pressures it represents.

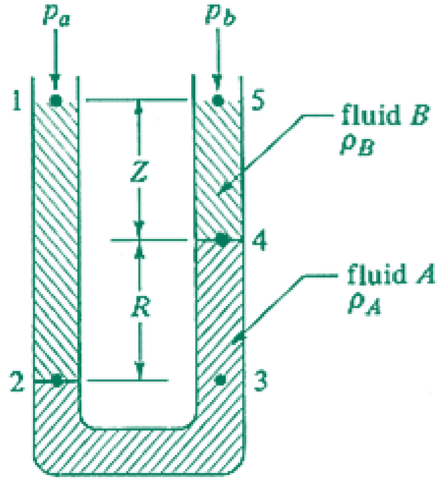
$$h(\text{head}) = \frac{p}{\rho g}\ \text{m} \qquad (\text{SI})$$

$$h = \frac{p g_c}{\rho g}\ \text{ft} \qquad (\text{English})$$

EXAMPLE 2.2-3

Pressure Measurement Devices

- **Simple U-tube manometer**



(a)

$$p_2 = p_a + (Z + R)\rho_B g \ \mathrm{N/m^2}$$

- P_3 must be equal to P_2 by the principles of hydrostatics: $p_3 = p_2$

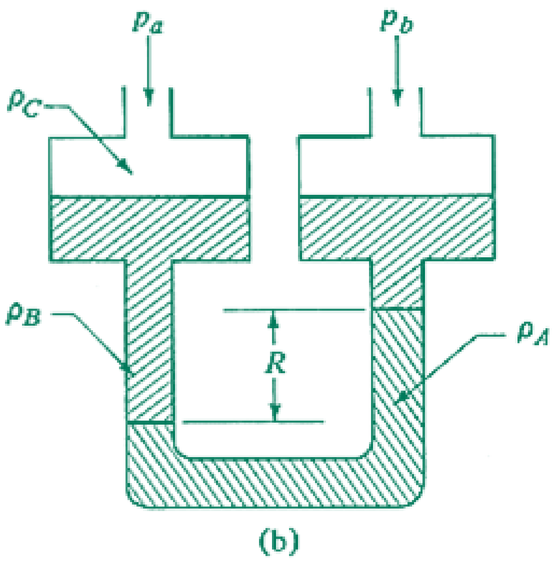
$$p_3 = p_b + Z\rho_B g + R\rho_A g$$

$$p_a + (Z + R)\rho_B g = p_b + Z\rho_B g + R\rho_A g$$

$$p_a - p_b = R(\rho_A - \rho_B)g$$

EXAMPLE 2.2-4

Pressure Measurement Devices



- **Two-fluid U tube**
  - a sensitive device for measuring small heads or pressure differences.

$$p_a - p_b = R(\rho_A - \rho_B)g \qquad \text{(SI)}$$

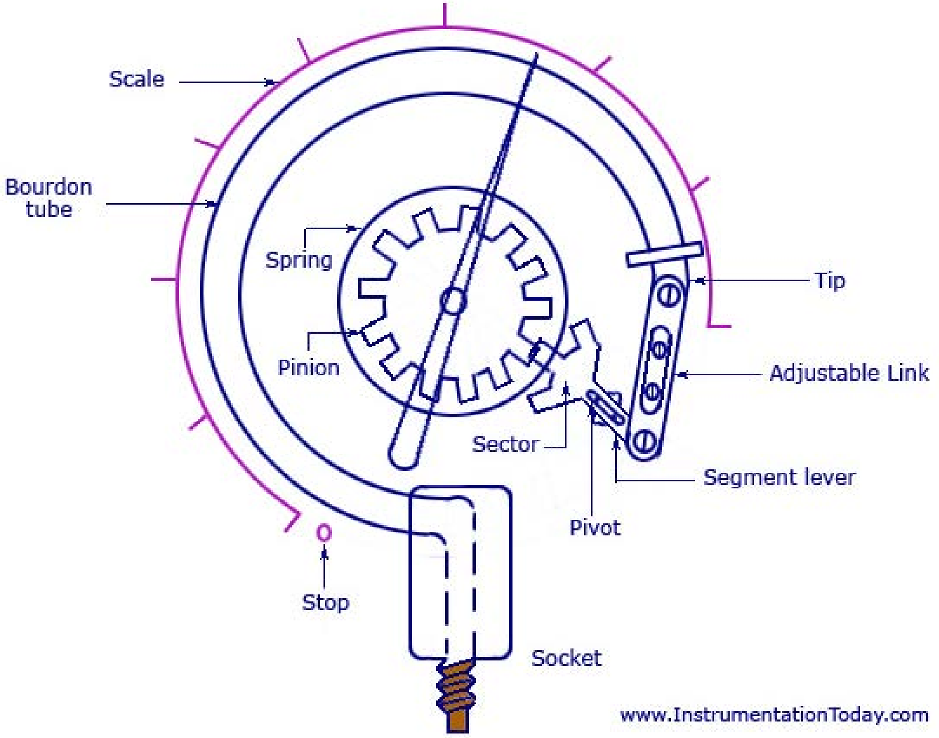
$$p_a - p_b = R(\rho_A - \rho_B)\frac{g}{g_c} \qquad \text{(English)}$$

  - If ρ_A and ρ_B are close to each other, the reading R is magnified.

EXAMPLE 2.2-5

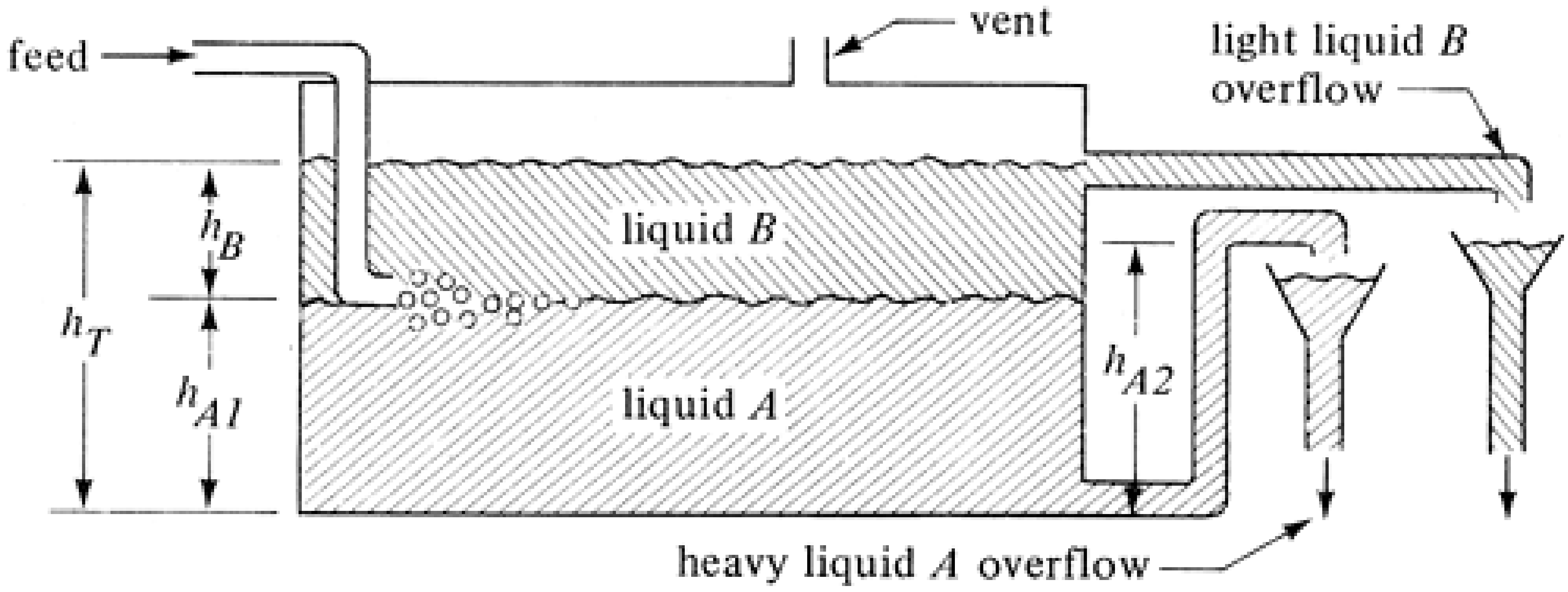
Pressure Measurement Devices

- **Bourdon pressure gage**

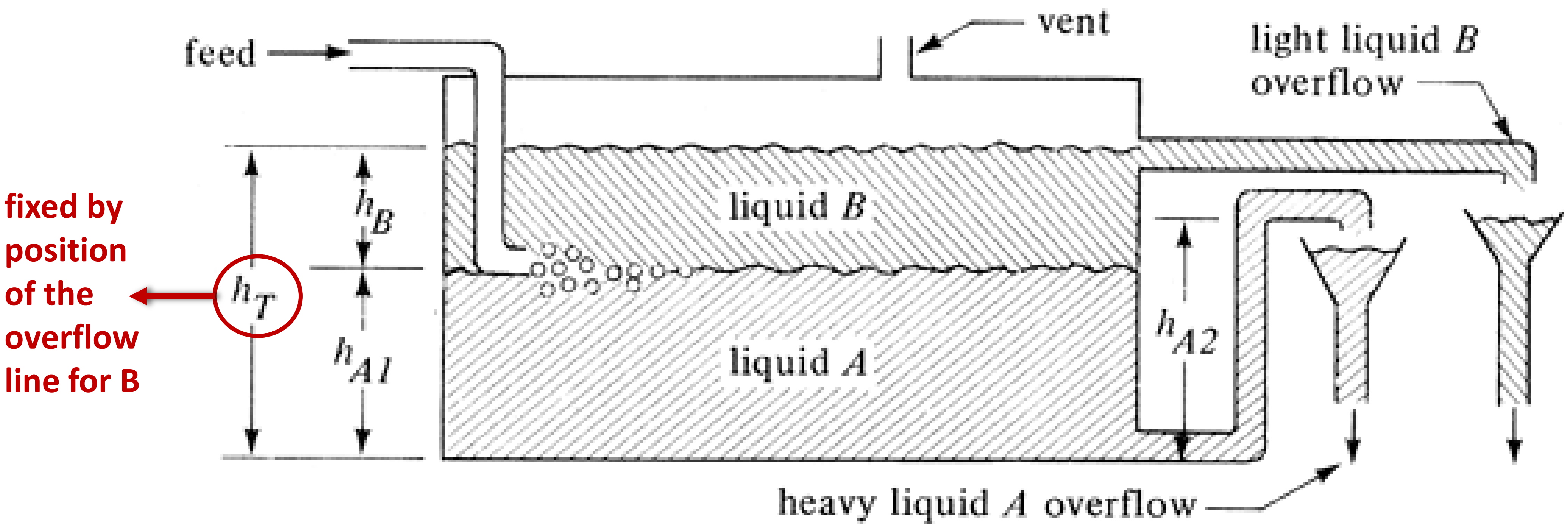


Pressure Measurement Devices

- **Gravity separator for two immiscible liquids**



Gravity separator for two immiscible liquids



- **A hydrostatic balance gives**

$$h_B \rho_B g + h_{A1} \rho_A g = h_{A2} \rho_A g \quad \longrightarrow \quad h_{A1} = \frac{h_{A2} - h_T \rho_B / \rho_A}{1 - \rho_B / \rho_A}$$

  - **The position of the interface, h_{A1} depends on the ratio of the densities of the two liquids and on the elevations h_{A2} and h_T.**
  - **Usually, h_{A2} is movable so that the interface level can be adjusted.**

Momentum Transport

Part One

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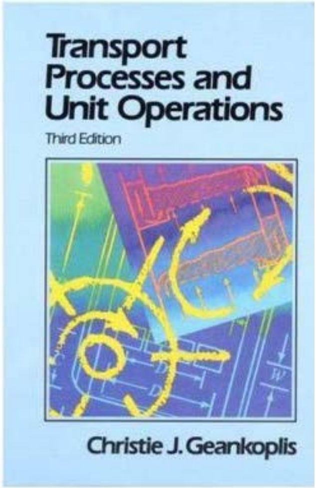
General Molecular Transport Equation for Momentum, Heat, and Mass Transfer

Section 2.3



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Introduction to Transport Processes

- In molecular transport processes in general ⇔ the transfer (movement) of a given property by molecular movement through a system [fluid (gas or liquid) or solid].
  - **Property**:

mass	heat	momentum

  - Each molecule of a system has a given quantity of the property associated with it.
  - Difference of property concentration from one region to an adjacent region ⇔ net transport of this property occurs.

Dilute fluids (gases
- molecules are relatively far apart
- rate of transport of property should be relatively fast
- few molecules are present to block the transport or interact.

Dense fluids (liquids)
- molecules are close together
- transport or diffusion proceeds more slowly.

Solids
- molecules are even more close-packed than in liquids
- molecular migration is even more restricted.

General Molecular Transport Equation

$$\text{rate of a transfer process} = \frac{\text{driving force}}{\text{resistance}}$$

we need a driving force to overcome a resistance in order to transport a property

- **Ohm's law in electricity**

$$\text{rate of flow of electricity} = \frac{\text{voltage drop (driving force)}}{\text{resistance}}$$

General Molecular Transport Equation

$$\text{rate of a transfer process} = \frac{\text{driving force}}{\text{resistance}}$$

$$\psi_z = -\delta \frac{d\Gamma}{dz}$$

- ψ_z = flux of property = amount of property being transferred per unit time per unit cross-sectional area perpendicular to the *z* direction of flow [=] amount of property/s.m^2
- δ = proportionality constant = diffusivity [=] m^2/s
- Γ = concentration of the property [=] amount of property/m^3
- z = distance in the direction of flow [=] m

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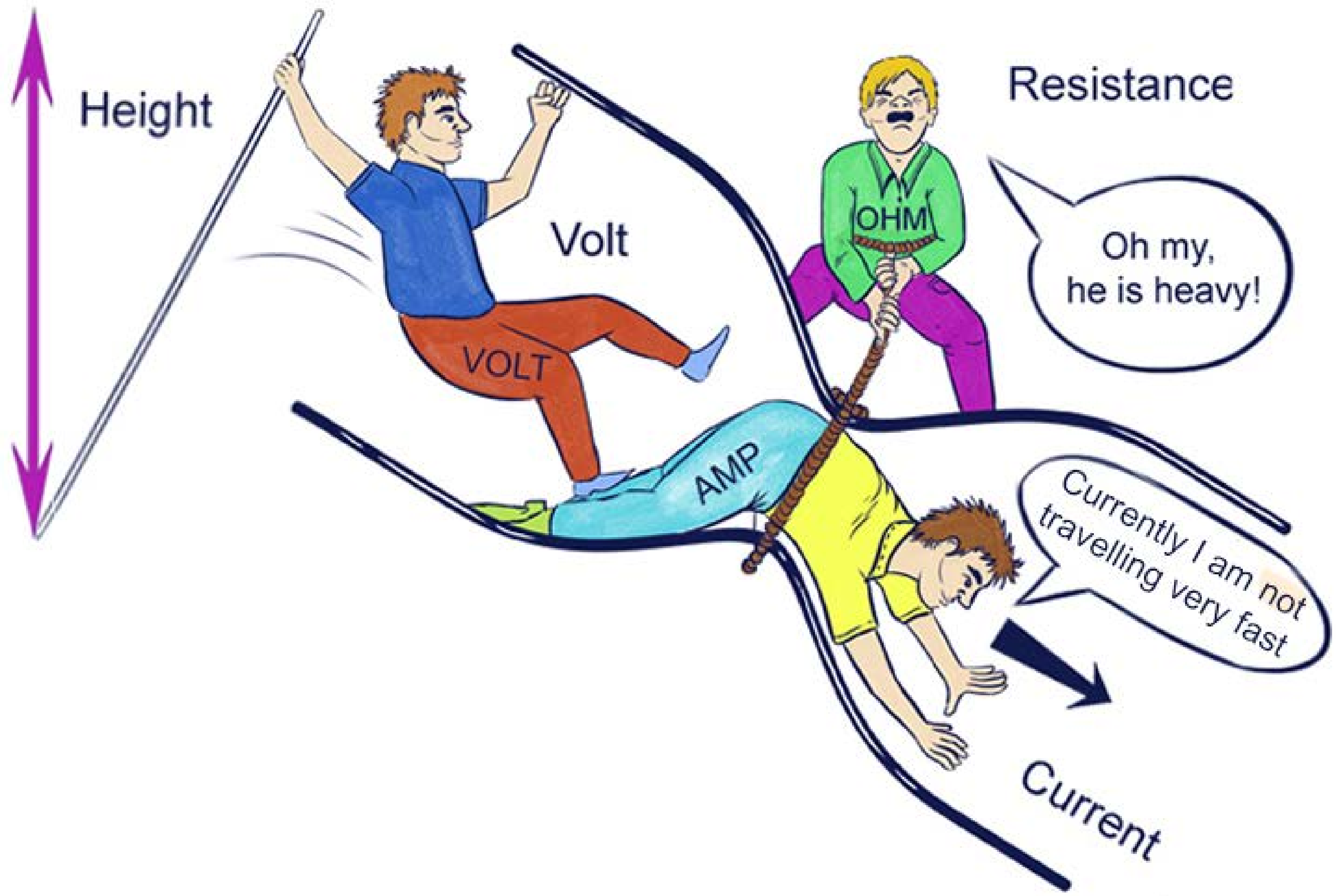
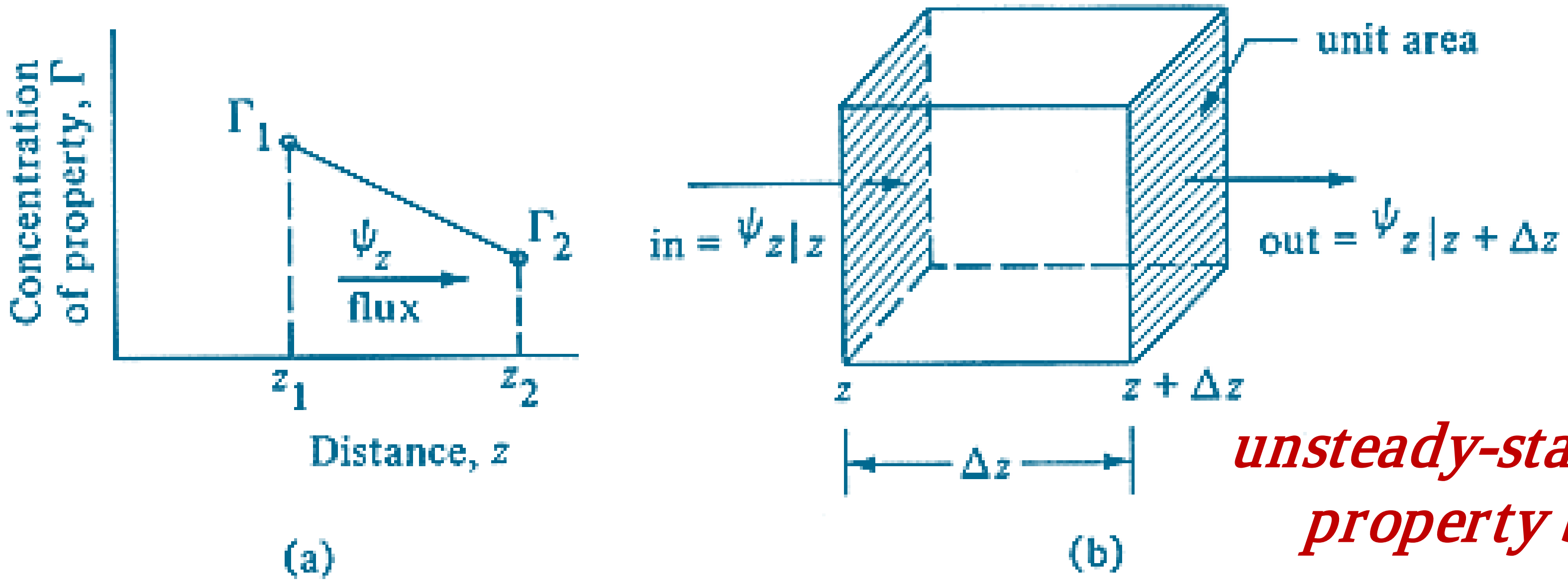


General Molecular Transport Equation

- At steady state, ψ_z is constant

$$\psi_z \int_{z_1}^{z_2} dz = -\delta \int_{\Gamma_1}^{\Gamma_2} d\Gamma \quad \Rightarrow \quad \psi_z = \frac{\delta(\Gamma_1 - \Gamma_2)}{z_2 - z_1}$$



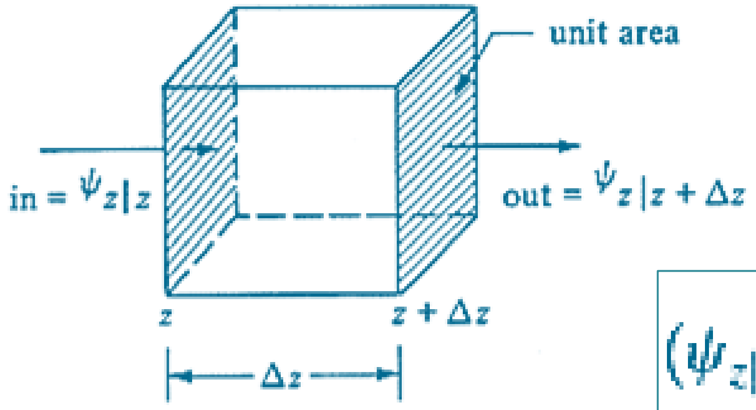
unsteady-state general property balance

***EXAMPLE* 2.3-1**

General Property Balance for Unsteady State

Rate of Property In − Rate of Property Out + Rate of Generation of Property = Rate of Accumulation of Property

$$(\psi_{z|z})\cdot 1 \qquad (\psi_{z|z+\Delta z})\cdot 1 \qquad R\,(\Delta z\cdot 1) \qquad \frac{\partial \Gamma}{\partial t}(\Delta z\cdot 1)$$



$$(\psi_{z|z})\cdot 1 + R(\Delta z\cdot 1) = (\psi_{z|z+\Delta z})\cdot 1 + \frac{\partial \Gamma}{\partial t}(\Delta z\cdot 1)$$

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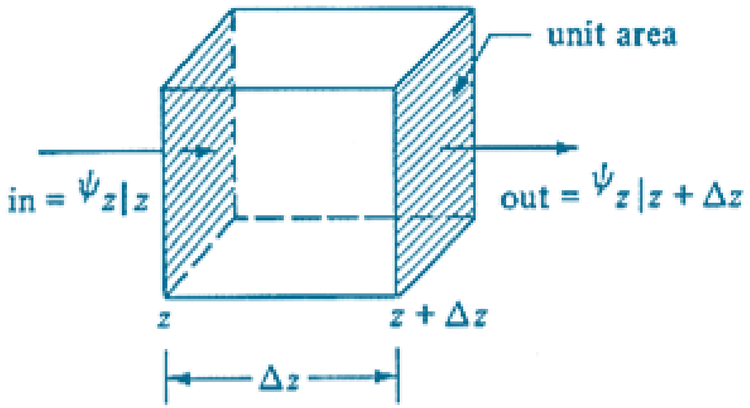
General Property Balance for Unsteady State

$$(\psi_{z|z})\cdot 1 + R(\Delta z\cdot 1) = (\psi_{z|z+\Delta z})\cdot 1 + \frac{\partial \Gamma}{\partial t}(\Delta z\cdot 1)$$

Dividing by Δz and letting Δz go to zero, $$\frac{\partial \Gamma}{\partial t} + \frac{\partial \psi_z}{\partial z} = R$$

Substituting for $\psi_z = -\delta \frac{d\Gamma}{dz}$

$$\frac{\partial \Gamma}{\partial t} - \delta \frac{\partial^2 \Gamma}{\partial z^2} = R$$



For the case where no generation is present,

$$\frac{\partial \Gamma}{\partial t} = \delta \frac{\partial^2 \Gamma}{\partial z^2}$$

General Equations for the Conservation of Momentum, Heat, or Mass

Introduction to Molecular Transport

Kinetic Theory of Gases

- Because of their kinetic energy the molecules are in rapid random movement, often colliding with each other
- Molecular transport / molecular diffusion of a property (momentum, heat, or mass) occurs in a fluid because of these random movements of individual molecules.
- Each individual molecule moves randomly in all directions and there are fluxes in all directions.
- If there is a concentration gradient of the property, there will be a net flux of the property from high to low concentration.
- This occurs because equal numbers of molecules diffuse in each direction between the high-concentration and low concentration regions.

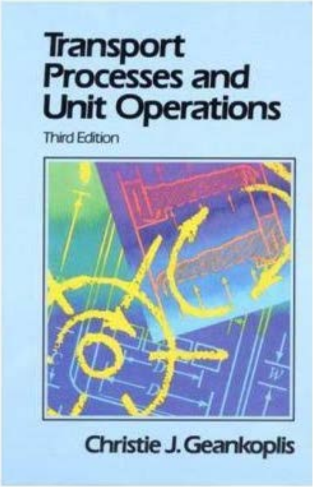
Introduction to Molecular Transport

$$\psi_z = -\delta \frac{d\Gamma}{dz}$$

- **Momentum transport**
  - Newton's law
  - $$\tau_{zx} = -\nu \frac{d(v_x \rho)}{dz}$$
- **Heat transport**
  - Fourier's law
  - $$\frac{q_z}{A} = -\alpha \frac{d(\rho c_p T)}{dz}$$
- **Mass transport**
  - Fick's law
  - $$J^*_{Az} = -D_{AB} \frac{dc_A}{dz}$$

Viscosity of Fluids

Section 2.4



Viscosity of Fluids

- Newton's Law & Viscosity
  - When a fluid is flowing through a closed channel such as a pipe or between two flat plates, either of two types of flow may occur, depending on the velocity of this fluid:

Laminar Flow

- **At low velocities, the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards.**
- **There are no cross currents perpendicular to the direction of flow, nor eddies or swirls of fluid.**

Turbulent Flow

- **At higher velocities eddies form, which leads to lateral mixing.**

Laminar Flow

$$stress = \frac{force}{area}$$

Viscosity

- a property of a fluid which gives rise to forces that resist the relative movement of adjacent layers in the fluid.

These viscous forces arise from forces existing between the molecules in the fluid and are of similar character as the shear forces in solids.

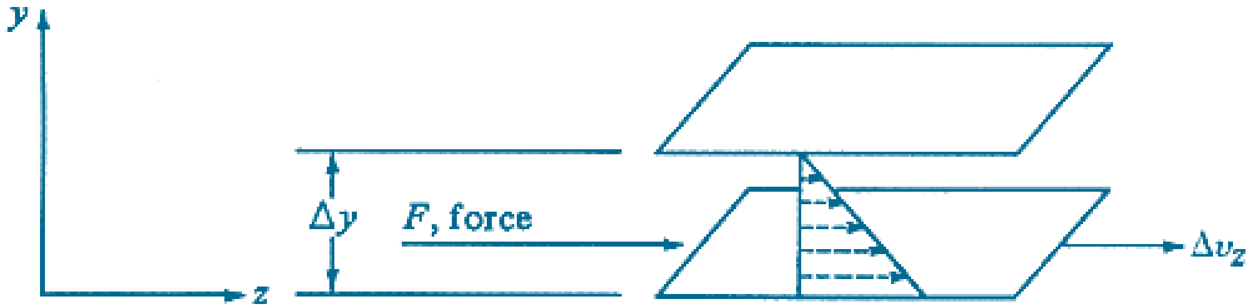
A fluid can be distinguished from a solid by its behavior when subjected to a stress/applied force.

An elastic solid deforms by an amount proportional to the applied stress.

A fluid when subjected to a similar applied stress will continue to deform, i.e., to flow at a velocity that increases with increasing stress.

A fluid exhibits resistance to this stress.

Laminar Flow



- A fluid is contained between two infinite (very long and very wide) parallel plates.
- The bottom plate is moving parallel to the top plate and at a constant velocity Δv_z **m/s** faster relative to the top plate because of a steady force F **newtons** being applied.
  - This force is called the **viscous drag**, and it arises from the viscous forces in the fluid.
- The plates are Δy **m** apart. Each layer of liquid moves in the z direction.
  - The layer immediately adjacent to the bottom plate is carried along at the velocity of this plate. The layer just above is at a slightly slower velocity, each layer moving at a slower velocity as we go up in the y direction. This velocity profile is linear, with y direction.
- An analogy to a fluid is a deck of playing cards!

Laminar Flow

- It has been found experimentally for many fluids that

$$\frac{F}{A} = -\mu \frac{\Delta v_z}{\Delta y}$$

μ = proportionality constant = ***viscosity*** of the fluid [=] $\mathrm{Pa \cdot s}$ [=] $\mathrm{kg/m \cdot s}$

- Let $\Delta y \rightarrow 0$,

$$\tau_{yz} = -\mu \frac{dv_z}{dy}$$

$\tau_{yz} = F/A$ = ***shear stress*** = force per unit area [=] $\mathrm{N/m^2}$ [=] Pa

- In cgs system, viscosity [=] $\mathrm{g/cm \cdot s}$ == **centipoise (cp)**

$$1\ \mathrm{cp} = 1 \times 10^{-3}\ \mathrm{kg/m \cdot s} = 1 \times 10^{-3}\ \mathrm{Pa \cdot s} = 1 \times 10^{-3}\ \mathrm{N \cdot s/m^2} \qquad \mathrm{(SI)}$$

$$1\ \mathrm{cp} = 0.01\ \mathrm{poise} = 0.01\ \mathrm{g/cm \cdot s}$$

$$1\ \mathrm{cp} = 6.7197 \times 10^{-4}\ \mathrm{lb_m/ft \cdot s}$$

***EXAMPLE* 2.4-1**

EXAMPLE 2.4-1. Calculation of Shear Stress in a Liquid

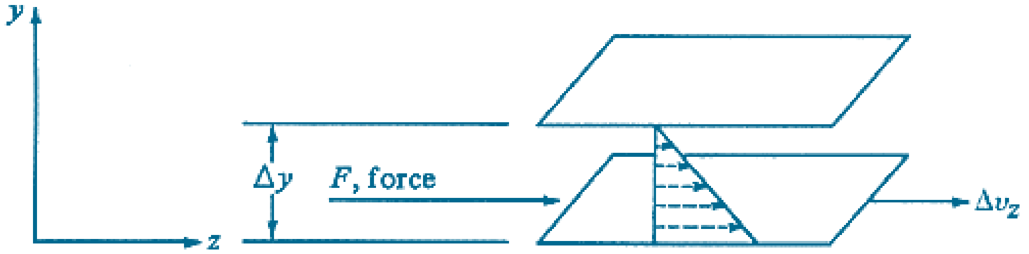
Referring to Fig. 2.4-1, the distance between plates is $\Delta y = 0.5$ cm, $\Delta v_z = 10$ cm/s, and the fluid is ethyl alcohol at 273 K having a viscosity of 1.77 cp (0.0177 g/cm · s).

(a) Calculate the shear stress τ_{yz} and the velocity gradient or shear rate dv_z/dy using cgs units.

(b) Repeat, using lb force, s, and ft units (English units).

(c) Repeat, using SI units.

$$\tau_{yz} = \frac{F}{A} = -\mu \frac{dv_z}{dy}$$



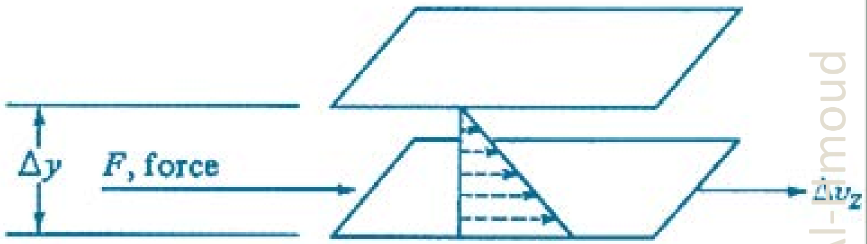
$1 \text{ cp} = 1 \times 10^{-3} \text{ kg/m} \cdot \text{s} = 1 \times 10^{-3} \text{ Pa} \cdot \text{s} = 1 \times 10^{-3} \text{ N} \cdot \text{s/m}^2$

$1 \text{ cp} = 0.01 \text{ poise} = 0.01 \text{ g/cm} \cdot \text{s} = 6.7197 \times 10^{-4} \text{ lb}_m/\text{ft} \cdot \text{s}$

Momentum Transfer in a Fluid

- Shear stress (τ_{yz}) ⇔ a flux of z-directed momentum in the y direction = the rate of flow of momentum per unit area.
- The units of momentum are mass $\times$ velocity [=] kg· m/s.
- The shear stress can be written as the amount of momentum transferred per second per unit area:

$$\tau_{yz} = \frac{kg \cdot m/s}{m^2 \cdot s} = \frac{\text{momentum}}{m^2 \cdot s}$$



- Random motions of molecules in the faster-moving layer send some of molecules into the slower-moving layer, where they collide with the slower-moving molecules and tend to speed them up or increase their momentum in the z direction.
- Molecules in the slower layer tend to retard those in the faster layer.
- This exchange of molecules between layers produces **a transfer or flux of z-directed momentum from high-velocity to low-velocity layers**.
- The negative sign indicates that momentum is transferred down the gradient from high- to low-velocity regions.

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Viscosities of Newtonian Fluids

$$\tau_{yz} = -\nu \frac{d(v_z\rho)}{dy}$$

- ***Newtonian fluids***: fluids that follow Newton's law of viscosity
  - For a Newtonian fluid, there is a linear relation between shear stress τ_{yz} and velocity gradient $\frac{dv_z}{dy}$ (rate of shear).

    $\Rightarrow$ Viscosity μ is a constant and independent of the rate of shear.
- For ***non-Newtonian fluids***, the relation between τ_{yz} and $\frac{dv_z}{dy}$ is not linear $\Rightarrow$ Viscosity μ does not remain constant but is a function of shear rate.
  - Certain liquids do not obey this simple Newton's law. These are primarily pastes, slurries, high polymers, and emulsions.
- The science of the flow and deformation of fluids is often called ***rheology***.

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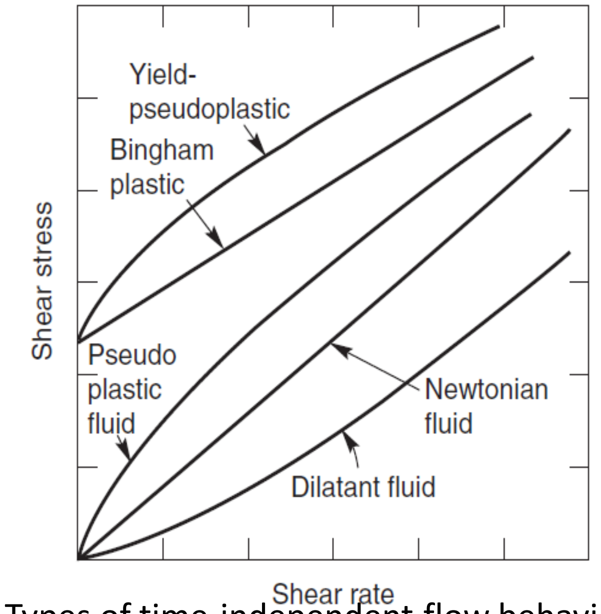
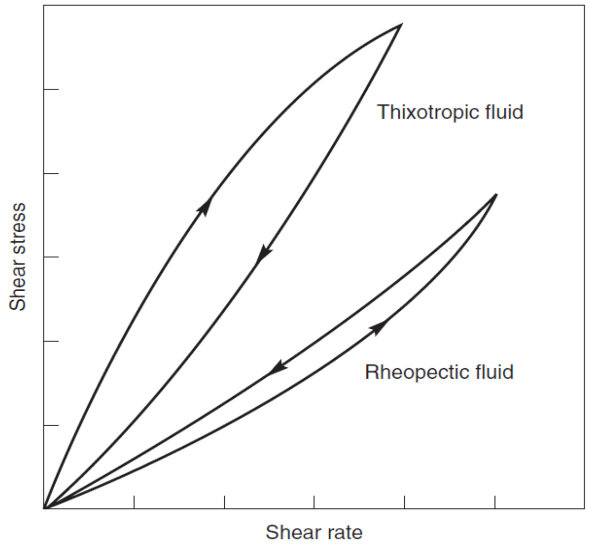
Non-Newtonian Fluids

- A fluid whose flow curve (shear stress versus shear rate) is nonlinear or does not pass through the origin,
  - The apparent viscosity, shear stress divided by shear rate, is not constant at a given temperature and pressure but is dependent on flow conditions:
    - Flow geometry
    - Shear rate
    - Sometimes even on the kinematic history of the fluid element under consideration.
- Apparent viscosity is the ratio of shear stress to shear rate, though the latter ratio is a function of the shear stress or shear rate and/or of time



Types of time-independent flow behavior



Shear stress–shear rate for time-dependent fluid behavior

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- fluids for which the rate of shear at any point is determined only by the value of the shear stress at that point at that instant; these fluids are variously known as ' time independent ' , ' purely viscous ' , ' inelastic ' or ' generalized Newtonian fluids ' (GNF);
- more complex fluids for which the relation between shear stress and shear rate depends, in addition, upon the duration of shearing and their kinematic history; they are called ' time-dependent fluids ';
- Substances exhibiting characteristics of both ideal fluids and elastic solids and showing partial elastic recovery, after deformation; these are categorized as ' visco-elastic fluids ' .

Viscosities of Newtonian Fluids

- **Gases** are Newtonian fluids.
  - Viscosity of gases **increases** with temperature and is approximately independent of pressure up to a pressure of about 1000 kPa.
  - At higher pressures, the viscosity of gases increases with increase in pressure.
  - For example, the viscosity of N_2 gas at 298 K approximately doubles in going from 100 kPa to about 5 x 10^4 kPa.
- In **liquids**, the viscosity **decreases** with increasing temperature.
  - Since liquids are essentially incompressible, the viscosity is not affected by pressure.

Viscosities of Newtonian Fluids

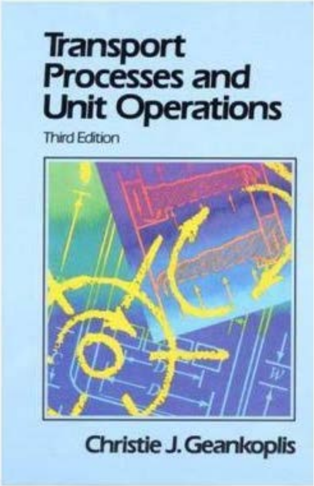
TABLE 2.4-1. *Viscosities of Some Gases and Liquids at 101.32 kPa Pressure*

Gases				Liquids			
Substance	Temp., K	Viscosity $(Pa \cdot s)10^3$ or $(kg/m \cdot s)\ 10^3$	Ref.	Substance	Temp., K	Viscosity $(Pa \cdot s)10^3$ or $(kg/m \cdot s)\ 10^3$	Ref.
Air	293	0.01813	N1	Water	293	1.0019	S1
CO_2	273	0.01370	R1		373	0.2821	S1
	373	0.01828	R1	Benzene	278	0.826	R1
CH_4	293	0.01089	R1				
				Glycerol	293	1069	L1
SO_2	373	0.01630	R1	Hg	293	1.55	R2
				Olive oil	303	84	E1

More complete tables of viscosities are given in Appendix A.2 (for water), Appendix A.3 (for inorganic and organic liquids and gases), and Appendix A.4 (for biological and food liquids).

Types of Fluid Flow and Reynolds Number

Section 2.5



Types of Fluid Flow

Laminar Flow

- When the velocity of flow is slow, the flow patterns are smooth.
- The layers of fluid seem to slide by one another without eddies or swirls being present
- Newton's law of viscosity holds

Turbulent Flow

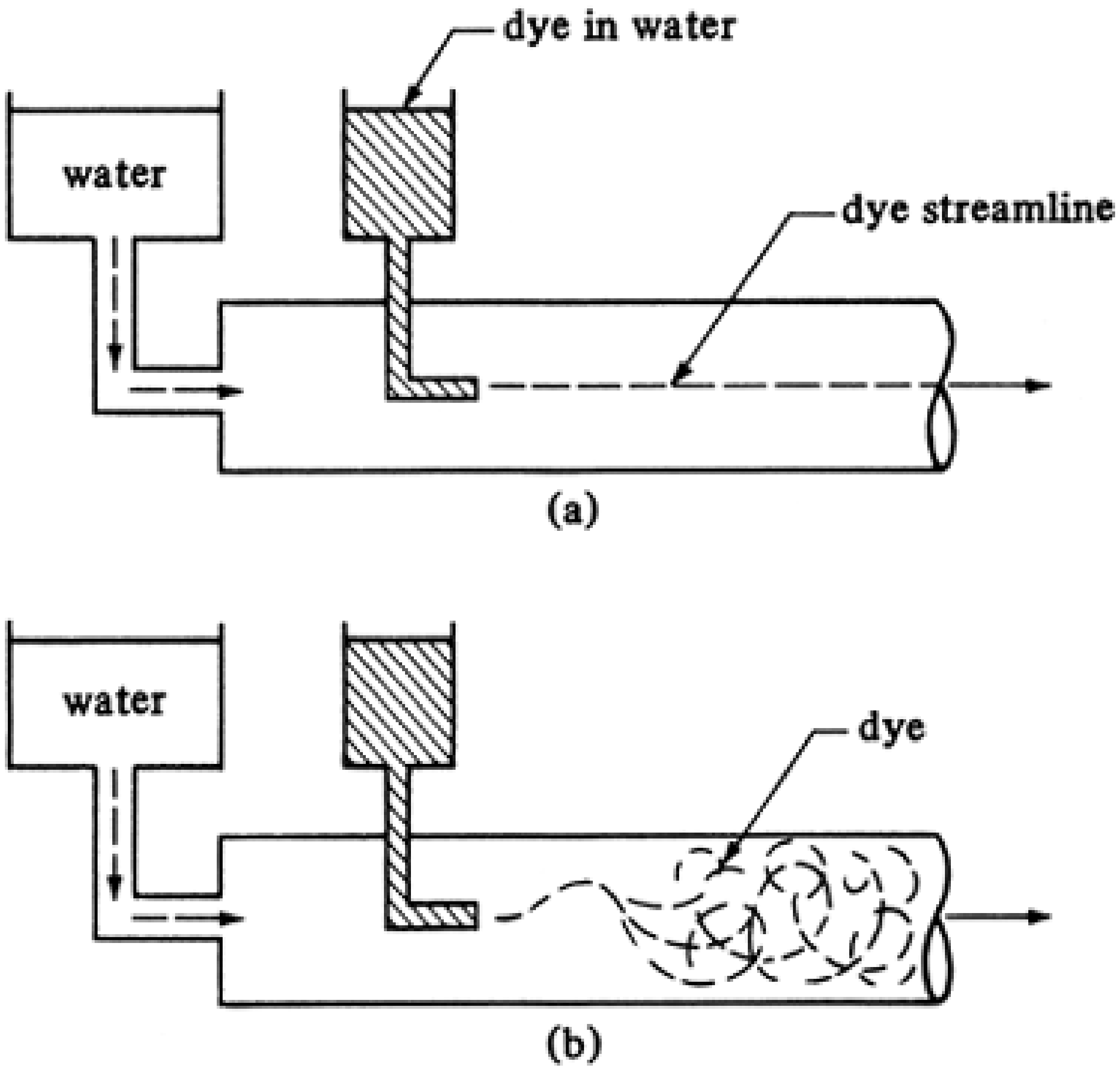
- When the velocity is quite high, an unstable pattern is observed in which eddies or small packets of fluid particles are present moving in all directions and at all angles to the normal line of flow.
- Eddies are present giving the fluid a fluctuating nature.

Reynold's Experiment

a) **Laminar / Viscous flow**

critical velocity

b) **Turbulent flow**



Reynold's Experiment

Reynolds Number $\leftrightarrow$ Dimensionless

Density	$\rho\ (\mathrm{kg/m^3})$	$\uparrow$
Average Velocity	$v\ (\mathrm{m/s}) = \dfrac{\dot{V}}{A}$	$\uparrow$
Viscosity	$\mu\ (\mathrm{Pa \cdot s})$	$\downarrow$
Tube Diameter	$D\ (\mathrm{m})$	$\uparrow$

$$Re = \frac{\rho v D}{\mu}$$

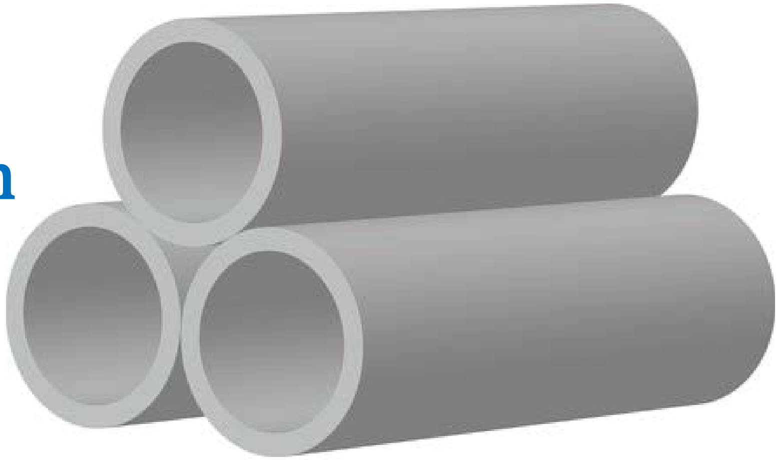
Reynolds Number $\leftrightarrow$ Dimensionless

- The instability of the flow that leads to disturbed / turbulent flow is determined by the ratio of the **kinetic or inertial forces** to the **viscous forces** in the fluid stream.
  - Inertial Forces $\propto \rho v^2$
  - Viscous Forces $\propto \mu v/D$

$$\frac{\text{Inertial Forces}}{\text{Viscous Forces}} \propto \frac{\rho v^2}{\frac{\mu v}{D}} = \frac{\rho v D}{\mu} = Re$$

- **For a straight circular pipe**

$$Re \begin{cases} \nearrow & < 2100 & \equiv \text{Laminar flow} \\ \rightarrow & 2100 \sim 4000 & \equiv \text{Transition region} \\ \searrow & > 4000 & \equiv \text{Turbulent flow} \end{cases}$$



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P2.5-1. Reynolds Number for Milk Flow

- Whole milk at 293 K having a density of 1030 kg/m^3 and viscosity of 2.12 cp is flowing at the rate of O.605 kg/s in a glass pipe having a diameter of 63.5 mm.
  a) Calculate the Reynolds number. Is this turbulent flow?

$1\ \text{cp} = 1 \times 10^{-3}\ \text{kg/m}\cdot\text{s}$

$$Re = \frac{\rho v D}{\mu}, \qquad v = \frac{\dot{V}}{A} = \frac{\dot{m}/\rho}{\frac{\pi}{4}D^2} = 0.185\ m/s$$

$$Re = \frac{\rho v D}{\mu} = \frac{1030 \times 0.185 \times 0.0635}{2.12 \times 10^{-3}} = 5707 == \textbf{turbulent}$$

  b) Calculate the flow rate needed in m^3/s for a Reynolds number of 2100 and the velocity in m/s.

$$v = \frac{Re \times \mu}{\rho D} = \frac{2100 \times 2.12 \times 10^{-3}}{1030 \times 0.0635} = 0.068\ m/s$$

$$\Rightarrow \dot{V} = vA = \frac{\pi}{4}D^2 v = \frac{\pi}{4}(0.0635)^2 \times 0.068 = 2.16 \times 10^{-4}\ m^3/s$$

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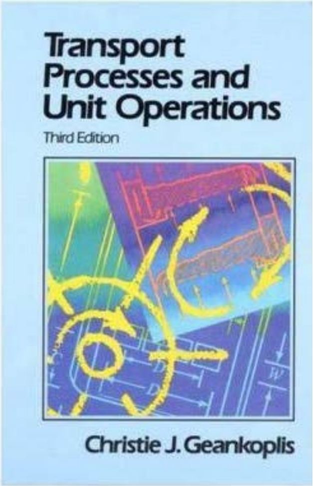
P2.3-2 Mass Balance for Flow of Sucrose Solution

Integration of General Property Equation for Steady State. Integrate the general property equation (2.3-11) for steady state and no generation between the points Γ_1 at z_1 and Γ_2 at z_2. The final equation should relate Γ to z.

$$\frac{\partial \Gamma}{\partial t} - \delta \frac{\partial^2 \Gamma}{\partial z^2} = R$$

Overall Mass Balance and Continuity Equation

Section 2.6



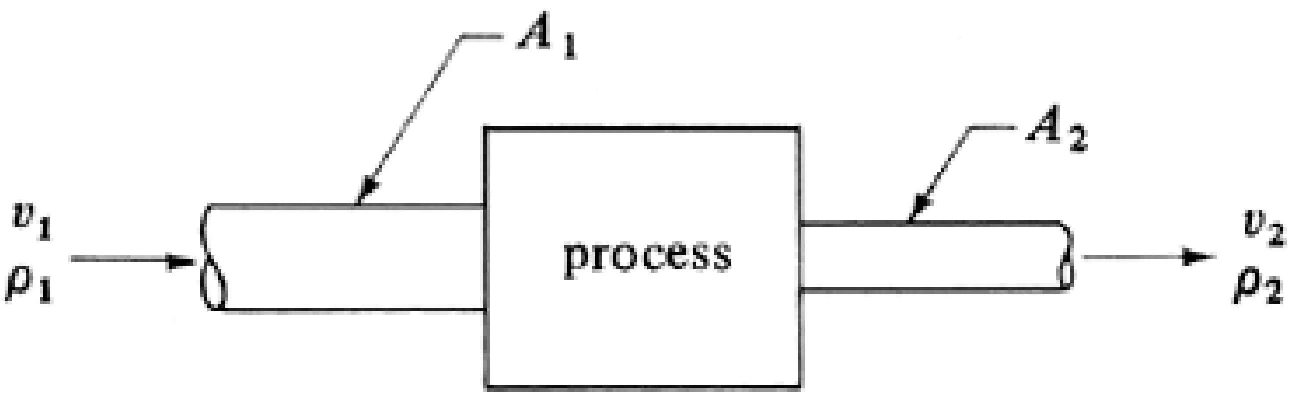
Introduction and Simple Mass Balances

- Simple mass / material balances:

$$\text{input} = \text{output} + \text{accumulation}$$

- At steady state:

$$\text{rate of input} = \text{rate of output}$$



$$m = \rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

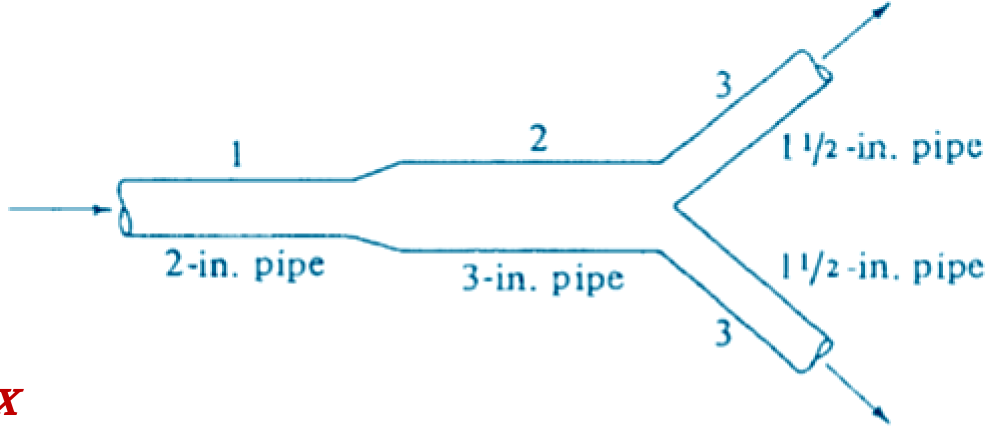
Often, $v\rho$ is expressed as $G = v\rho$, where G is *mass velocity* or *mass flux*.

EXAMPLE 2.6-1. Flow of Crude Oil and Mass Balance

A petroleum crude oil having a density of 892 kg/m^3 is flowing through the piping arrangement shown in Fig. 2.6-2 at a total rate of 1.388×10^{-3} m^3/s entering pipe 1.

The flow divides equally in each of pipes 3. The steel pipes are schedule 40 pipe (see Appendix A.5 for actual dimensions). Calculate the following using SI units.

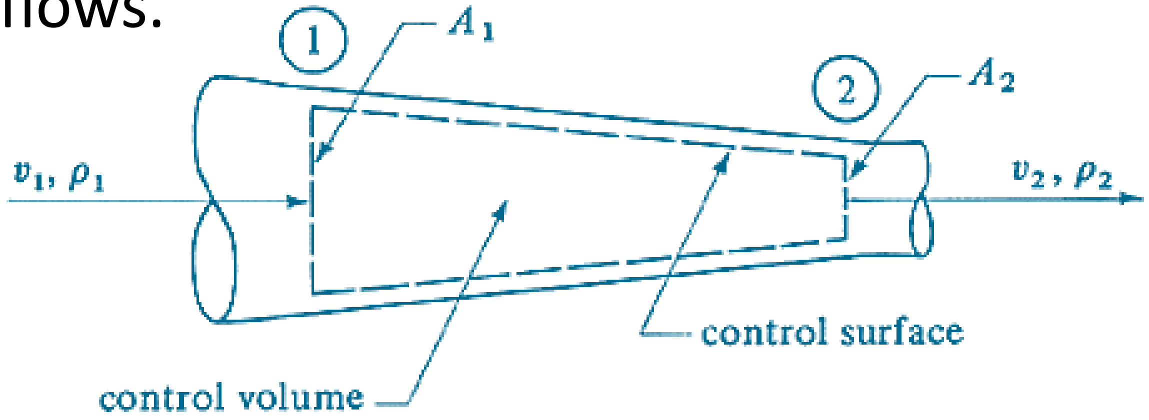
(a) The total mass flow rate m in pipe 1 and pipes 3.
(b) The average velocity v in 1 and 3.
(c) The mass velocity G in 1.



EXAMPLE 2.6-1. (pg. 51) (pg. 892 – Appendix A.5)

Control Volume for Balances

- Laws for the conservation of mass, energy, and momentum are all stated in terms of a **system**, and these laws give the interaction of a system with its **surroundings**.
  - A ***system*** is defined as a collection of fluid of fixed identity.
- In flow of fluids, individual particles are not easily identifiable.
  - attention is focused on a given **space** through which the fluid flows rather than to a given mass of fluid.
  - Used method: select a **control volume**, which is a region fixed in space through which the fluid flows.



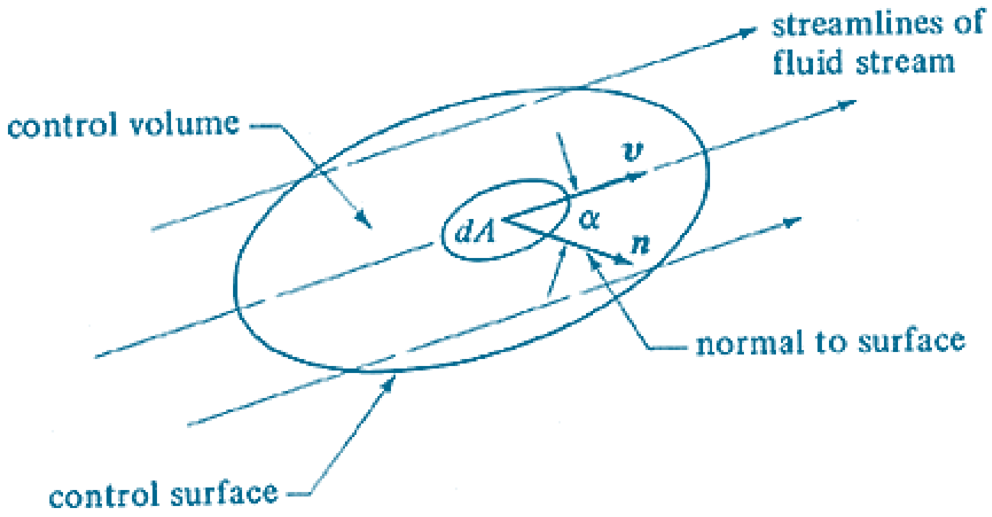
Overall Mass-Balance Equation

$$\begin{pmatrix}\text{rate of mass output}\\ \text{from control volume}\end{pmatrix} - \begin{pmatrix}\text{rate of mass input}\\ \text{to control volume}\end{pmatrix} + \begin{pmatrix}\text{rate of mass accumulation}\\ \text{in control volume}\end{pmatrix}$$

$$= \begin{pmatrix}\text{rate of mass generation}\\ \text{in control volume}\end{pmatrix}$$

$$\begin{pmatrix}\text{rate of mass}\\ \text{accumulation in}\\ \text{control volume}\end{pmatrix} = \frac{\partial}{\partial t}\iiint_V \rho\, dV = \frac{dM}{dt}$$

$$\begin{pmatrix}\text{net mass eflux}\\ \text{from control volume}\end{pmatrix} = \iint_A v\rho \cos\alpha\, dA$$



Overall Mass-Balance Equation

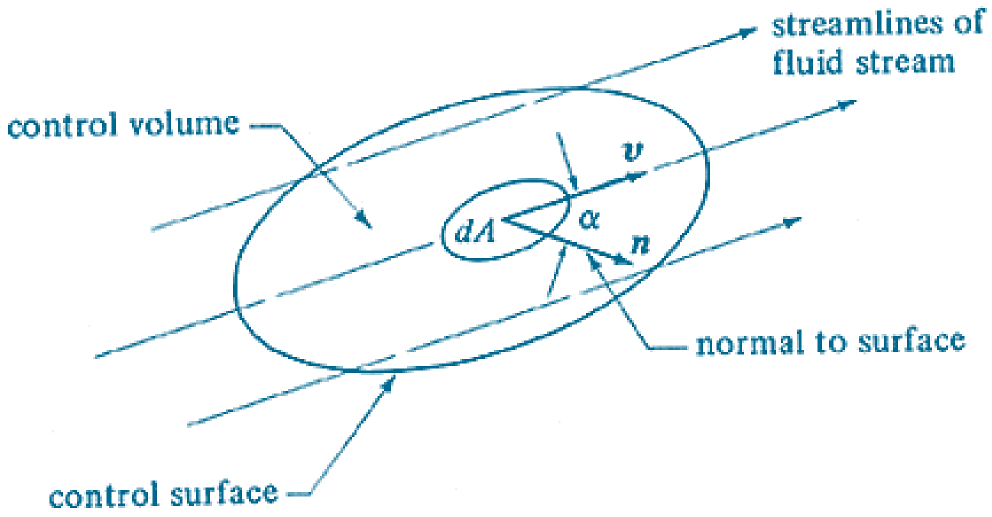
$$\iint_A v\rho \cos\alpha \, dA = \iint_{A_2} v\rho \cos\alpha_2 \, dA + \iint_{A_1} v\rho \cos\alpha_1 \, dA = \boldsymbol{v_2\,\rho_2\,A_2 - v_1\,\rho_1\,A_1}$$

- For a control volume where no mass is being generated:

$$\boldsymbol{v_2\,\rho_2\,A_2 - v_1\,\rho_1\,A_1} + \frac{dM}{dt} = 0$$

- And in general:

$$\boldsymbol{m_{i2} - m_{i1}} + \frac{dM_i}{dt} = R_i$$



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Average Velocity to Use in Overall Mass Balance

- If the velocity is not constant but varies across the surface area, an average or bulk velocity is defined by

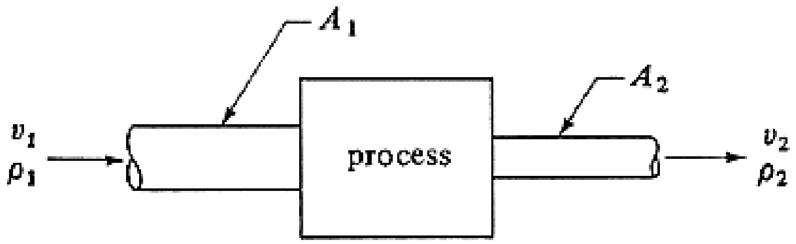
$$v_{av} = \frac{1}{A} \iint_{A} v dA$$

P2.6-2. Flow of Liquid in a Pipe and Mass Balance

- A hydrocarbon liquid enters a simple flow system shown in Fig.2.6-1 at an average velocity of 1.282 m/s, where $A_1 = 4.33 \times 10^{-3}$ m^2 and $\rho_1 = 902$ kg/m^3. The liquid is heated in the process and the exit density is 875 kg/m^3. The cross-sectional area at point 2 is 5.26×10^{-3} m^2. The process is steady state.

(a) Calculate the mass flow rate m at the entrance and exit.

(b) Calculate the average velocity v in 2 and the mass velocity G in 1

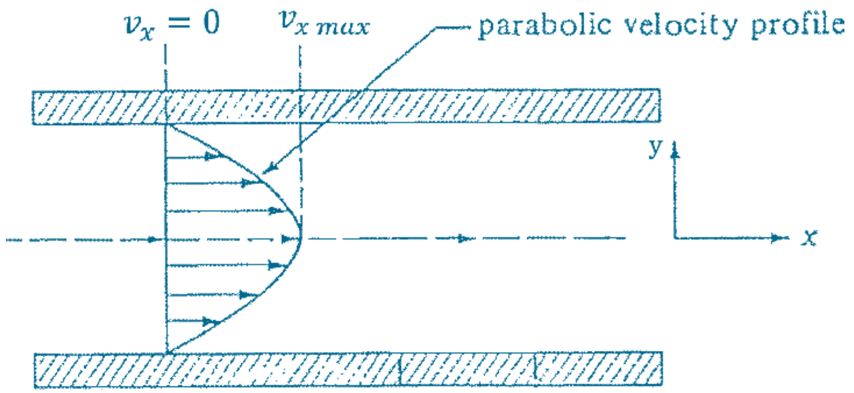


P2.6-4. Bulk Velocity for Flow Between Parallel Plates

Bulk Velocity for Flow Between Parallel Plates. A fluid flowing in laminar flow in the x direction between two parallel plates has a velocity profile given by the following.

$$v_x = v_{x\,max}\left[1 - \left(\frac{y}{y_0}\right)^2\right]$$

where $2y_0$ is the distance between the plates, y is the distance from the center line, and v_x is the velocity in the x direction at position y. Derive an equation relating v_{xav} (bulk or average velocity) with $v_{x\,max}$.



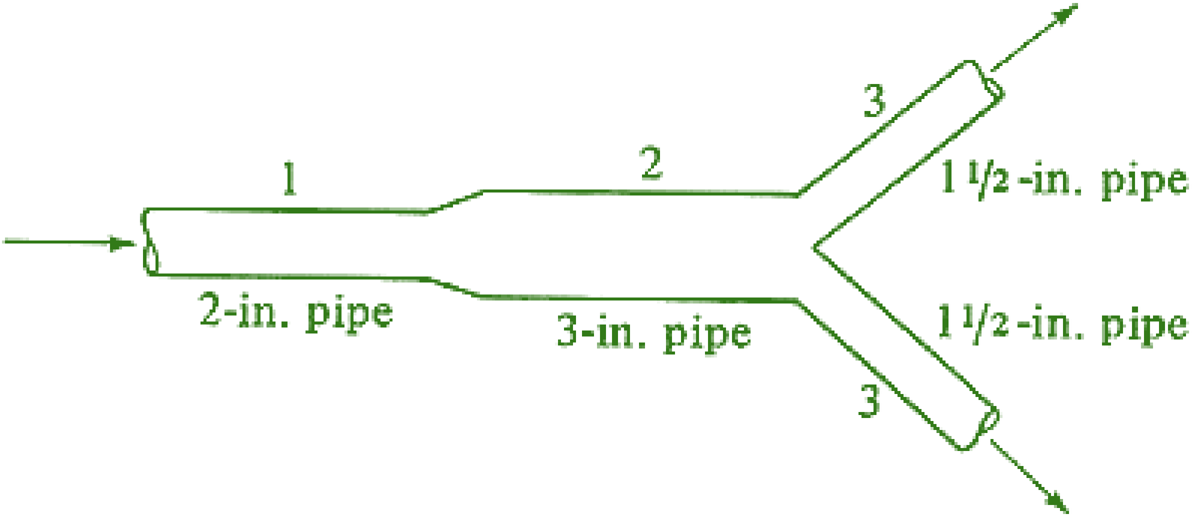
$$v_{av} = \frac{1}{A}\iint_A v\,dA$$

P2.6-7 Mass Balance for Flow of Sucrose Solution

Mass Balance for Flow of Sucrose Solution. A 20 wt % sucrose (sugar) solution having a density of 1074 kg/m^3 is flowing through the same piping system as Example 2.6-1 (Fig. 2.6-2). The flow rate entering pipe 1 is 1.892 m^3/h. The flow divides equally in each of pipes 3. Calculate the following:

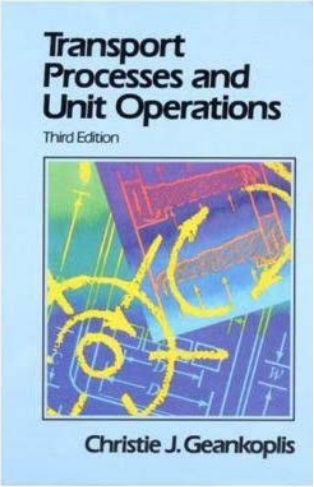
(a) The velocity in m/s in pipes 2 and 3.

(b) The mass velocity G kg/m$^2 \cdot$s in pipes 2 and 3.



Overall Momentum Balance

Section 2.8



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Overall Momentum Balance

- Momentum is a **vector** quantity, not like mass and energy.
- The total linear momentum vector $\vec{\boldsymbol{P}}$ of the total mass M of a moving fluid having a velocity of $\vec{\boldsymbol{v}}$ is

$$\vec{\boldsymbol{P}} = M\vec{\boldsymbol{v}} [=]\ kg.m/s$$

- ***Newton's second law***: The time rate of change of momentum of a system is equal to the summation of all forces acting on the system and takes place in the direction of the net force.

$$\sum \vec{\boldsymbol{F}} = \frac{d\vec{\boldsymbol{P}}}{dt}\ [=]\ N$$

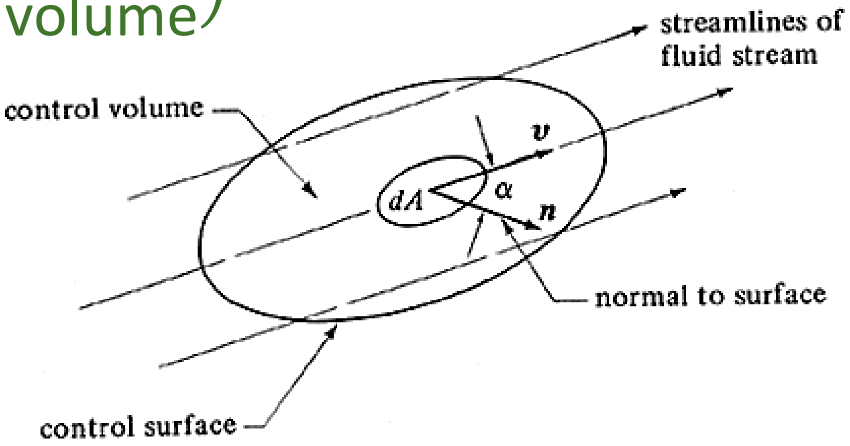
Overall Momentum Balance

- The equation for the conservation of momentum with respect to a control volume:

$$\begin{pmatrix}\text{sum of forces acting}\\ \text{on control volume}\end{pmatrix} = \begin{pmatrix}\text{rate of momentum out}\\ \text{of control volume}\end{pmatrix} - \begin{pmatrix}\text{rate of momentum}\\ \text{into control volume}\end{pmatrix} + \begin{pmatrix}\text{rate of accumulation of}\\ \text{momentum in control volume}\end{pmatrix}$$

The Generation Rate

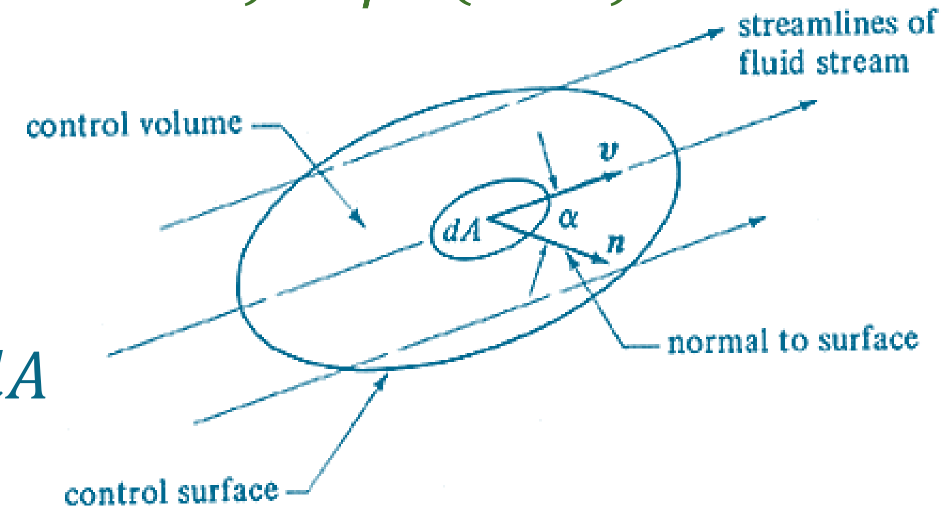
- Momentum is not conserved, since it is **generated by external forces** on the system.
  - If external forces are absent, momentum is conserved.



Overall Momentum Balance

- For a small element of area dA on the control surface,

$$\text{rate of momentum efflux} = \vec{\boldsymbol{v}}(\rho v)(dA\ \cos\alpha) = \rho\vec{\boldsymbol{v}}(\vec{\boldsymbol{v}}\cdot\boldsymbol{n})dA$$



$$\begin{pmatrix}\text{net momentum eflux}\\ \text{from control volume}\end{pmatrix} = \iint_A \vec{\boldsymbol{v}}(\rho v)\cos\alpha\, dA = \iint_A \rho\vec{\boldsymbol{v}}(\vec{\boldsymbol{v}}\cdot\vec{\boldsymbol{n}})\, dA$$

$$\begin{pmatrix}\text{rate of accumulation}\\ \text{of momentum in}\\ \text{control volume}\end{pmatrix} = \frac{\partial}{\partial t}\iiint_V \rho\vec{\boldsymbol{v}}dV$$

Overall Momentum Balance

- Overall linear momentum balance for a control volume:

$$\sum \boldsymbol{F} = \iint_A \rho \boldsymbol{v}\,(\boldsymbol{v}\cdot\boldsymbol{n})\,dA + \frac{\partial}{\partial t}\iiint_V \rho \boldsymbol{v}\,dV$$

which is a vector equation ➔

$$\sum F_x = \iint_A \rho v\, v_x\ \cos\alpha\, dA + \frac{\partial}{\partial t}\iiint_V \rho v_x dV$$

$$\sum F_y = \iint_A \rho v\, v_y\ \cos\alpha\, dA + \frac{\partial}{\partial t}\iiint_V \rho v_y dV$$

$$\sum F_z = \iint_A \rho v\, v_z\ \cos\alpha\, dA + \frac{\partial}{\partial t}\iiint_V \rho v_z dV$$

The Force term, $\sum F_x$

- The Force term, $\sum F_x$ is composed of the sum of several forces:
  1. **Body force, F_{xg}**, is the x-directed force caused by gravity acting on the total mass M in the control volume. $\boldsymbol{F_{xg} = Mg_x}$. F_{xg} = zero if the x direction is horizontal.
  2. **Pressure force, F_{xp}**, is the x-directed force caused by the pressure forces acting on the surface of the fluid system.
     When the control surface cuts through the fluid, the pressure is taken to be directed inward and perpendicular to the surface.
     In some cases, part of the control surface may be a solid, and this wall is included inside the control surface. Then there is a contribution to F_{xp} from the pressure on the outside of this wall, which is typically *atmospheric pressure*.
     If *gage pressure* is used, the integral of the constant external pressure over the entire outer surface can be automatically ignored.

The Force term, $\sum F_x$

- The Force term, $\sum F_x$ is composed of the sum of several forces:
  3. **Friction force:** When the fluid is flowing, an x-directed shear or friction force $\boldsymbol{F_{xs}}$, is present, which is exerted on the fluid by a solid wall when the control surface cuts between the fluid and the solid wall. In some or many cases this frictional force may be negligible compared to the other forces and is neglected.
  4. **Solid surface force:** In cases where the control surface cuts through a solid, there is present force $\boldsymbol{R_x}$, which is the x component of the resultant of the forces acting on the control volume at these points. This occurs in typical cases when the control volume includes a section of pipe and the fluid it contains. This is the force exerted by the solid surface on the fluid.

The Force term, $\sum F_x$

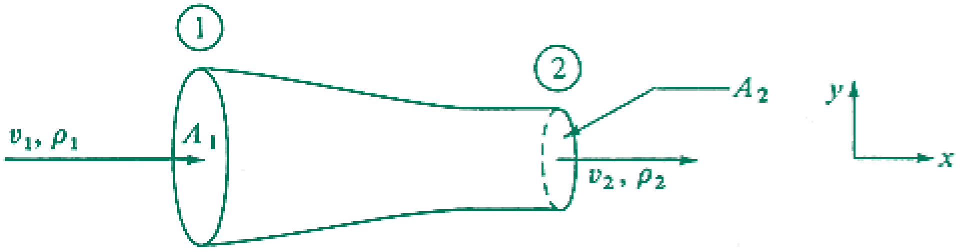
$$\sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x$$

$$= \iint_A \rho v \, v_x \, \cos\alpha \, dA + \frac{\partial}{\partial t} \iiint_V \rho v_x dV$$

Similar equations can be written for the y and z directions

Overall Momentum Balance in Flow System in One Direction

- For a fluid flowing at steady state in the control volume in the x direction, with $v = v_x$:



$$\sum F_x = F_{xg} + F_{xp} + F_{xs} + R_x = \iint_A v_x \, \rho v_x \, \cos\alpha \, dA$$

- Integrating with $\cos\alpha = \pm 1.0$ and $\rho A = m/v_{av}$

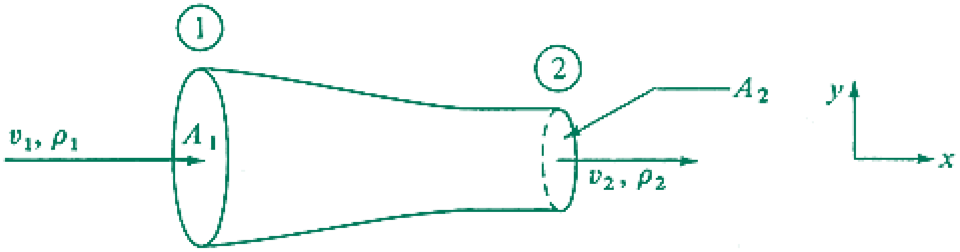
$$F_{xg} + F_{xp} + F_{xs} + R_x = m\frac{\left(v_{x_2}^2\right)_{av}}{v_{x_2\,av}} - m\frac{\left(v_{x_1}^2\right)_{av}}{v_{x_1\,av}}$$

Overall Momentum Balance in Flow System in One Direction

$$F_{xg} + F_{xp} + F_{xs} + R_x = m\frac{\left(v_{x_2}^2\right)_{av}}{v_{x_2\,av}} - m\frac{\left(v_{x_1}^2\right)_{av}}{v_{x_1\,av}}$$

If the velocity is not constant and varies across the surface area,

$$(v_x^2)_{av} = \frac{1}{A}\iint_A v_x^2\,dA$$



$$\frac{(v_x^2)_{av}}{v_{x\,av}} = \frac{v_{x\,av}}{\beta}, \beta = \begin{cases} 0.95 - 0.99 \text{ for turbulent flow} \\ 0.75 \text{ for laminar flow} \end{cases}$$ **(see *EXAMPLE 2.8–1*)**

$F_{xp} = p_1A_1 - p_2A_2$, F_{xs} will be neglected, $F_{xg} = 0$ (gravity is acting only in the y direction)

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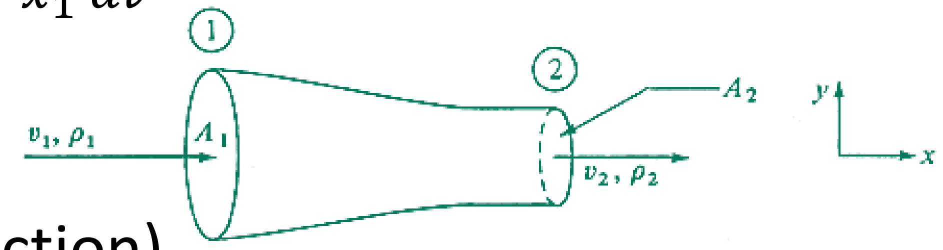
Overall Momentum Balance in Flow System in One Direction

$$F_{xg} + F_{xp} + F_{xs} + R_x = m\frac{(v_{x_2}^2)_{av}}{v_{x_2\,av}} - m\frac{(v_{x_1}^2)_{av}}{v_{x_1\,av}}$$



$F_{xp} = p_1A_1 - p_2A_2$, F_{xs} will be neglected

$F_{xg} = 0$ (gravity is acting only in the *y*-direction)

$$\frac{(v_x^2)_{av}}{v_{x\,av}} = \frac{v}{\beta}, \text{and setting } \beta = 1.0$$

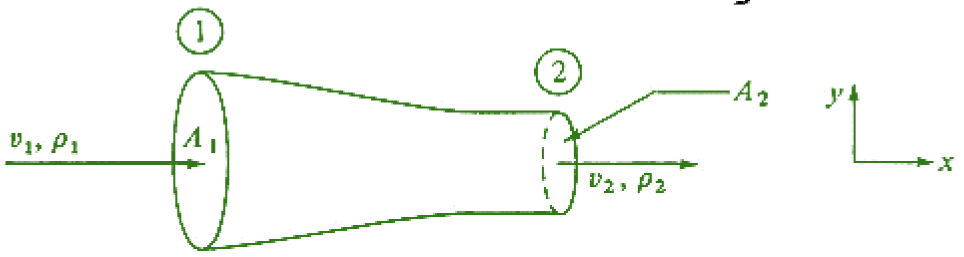
$$\boldsymbol{R_x = mv_2 - mv_1 + p_2A_2 - p_1A_1}$$

- **R_x is the force exerted by the solid on the fluid.**
- **The force of the fluid on the solid (reaction force) is the negative of this or $-R_x$.**

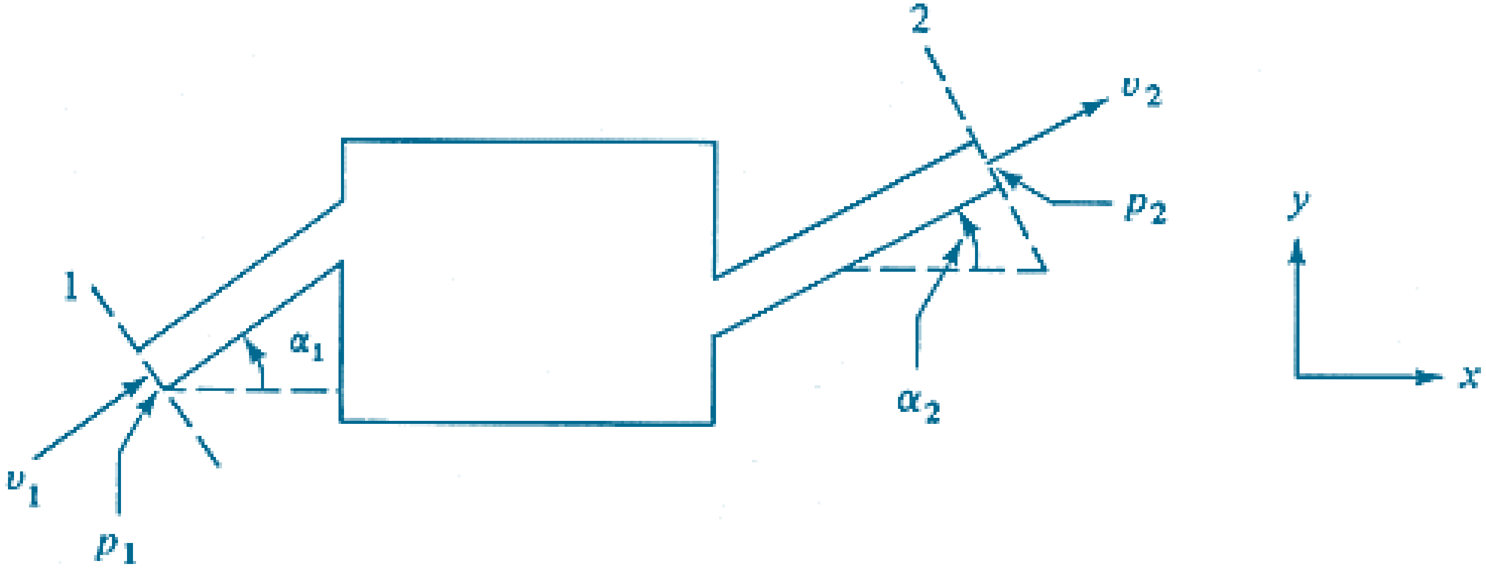
EXAMPLE 2.8-2. *Momentum Balance for Horizontal Nozzle*

Water is flowing at a rate of 0.03154 m^3/s through a horizontal nozzle shown in Fig. 2.8-1 and discharges to the atmosphere at point 2. The nozzle is attached at the upstream end at point 1 and frictional forces are considered negligible. The upstream ID is 0.0635 m and the downstream 0.0286 m. Calculate the resultant force on the nozzle. The density of the water is 1000 kg/m^3.

$$R_x = mv_2 - mv_1 - F_{xs} + p_2A_2 - p_1A_1$$



Overall Momentum Balance in Two Directions



$$R_x = mv_2 \cos\alpha_2 - mv_1 \cos\alpha_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1 \cos\alpha_1$$

$$R_y = mv_2 \sin\alpha_2 - mv_1 \sin\alpha_1 + p_2 A_2 \sin\alpha_2 - p_1 A_1 \sin\alpha_1 + m_t g$$

***EXAMPLE* 2.8-3. and 2.8-4.**

EXAMPLE 2.8-3. *Momentum Balance in a Pipe Bend*

Fluid is flowing at steady state through a reducing pipe bend, as shown in Fig. 2.8-3. Turbulent flow will be assumed with frictional forces negligible. The volumetric flow rate of the liquid and the pressure p_2 at point 2 are known as are the pipe diameters at both ends. Derive the equations to calculate the forces on the bend. Assume that the density ρ is constant.

$$R_x = mv_2 \cos\alpha_2 - mv_1 \cos\alpha_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1 \cos\alpha_1$$

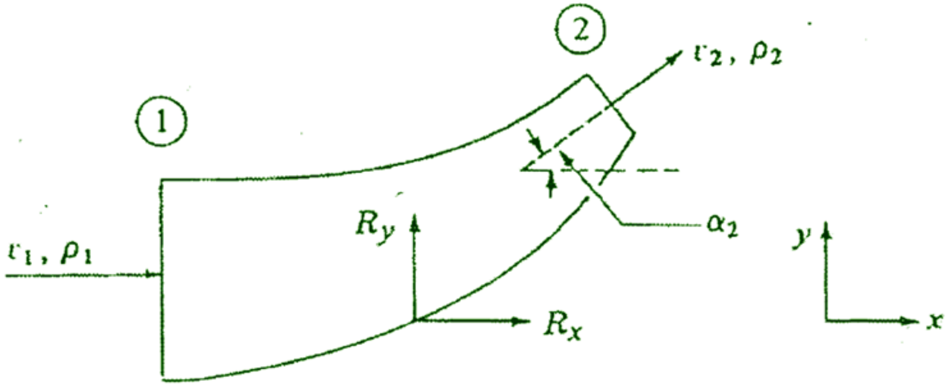
$$R_y = mv_2 \sin\alpha_2 - mv_1 \sin\alpha_1 + p_2 A_2 \sin\alpha_2 - p_1 A_1 \sin\alpha_1 + m_t g$$

$$\alpha_1 = 0,\ \cos\alpha_1 = 1, \sin\alpha_1 = 0$$

$$\Rightarrow R_x = mv_2 \cos\alpha_2 - mv_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1$$

$$R_y = mv_2 \sin\alpha_2 + p_2 A_2 \sin\alpha_2 + m_t g$$

$$|R| = \sqrt{R_x^2 + R_y^2},\ \ \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



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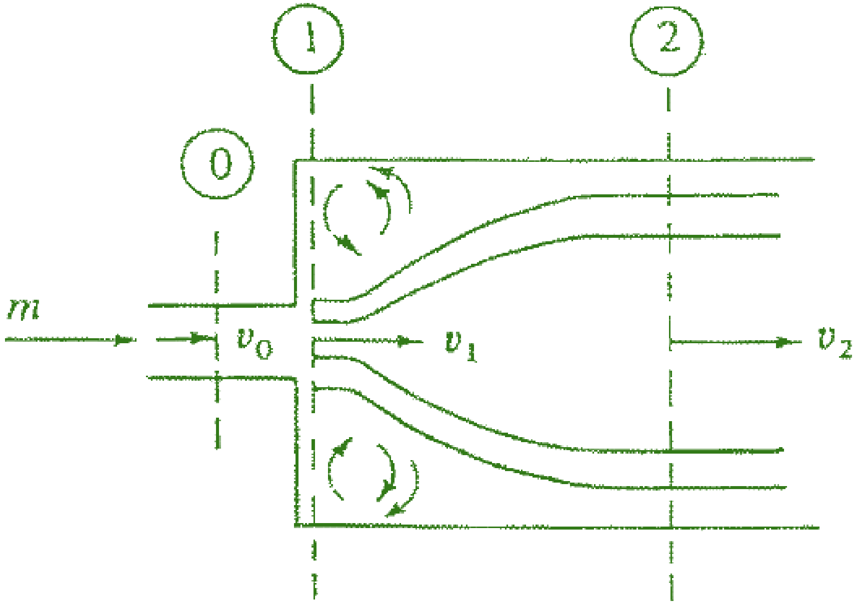
EXAMPLE 2.8-4. *Friction Loss in a Sudden Enlargement*

A mechanical-energy loss occurs when a fluid flows from a small pipe to a large pipe through an abrupt expansion, as shown in Fig. 2.8-4. Use the momentum balance and mechanical-energy balance to obtain an expression for the loss for a liquid. (*Hint*: Assume that $p_0 = p_1$ and $v_0 = v_1$. Make a mechanical-energy balance between points 0 and 2 and a momentum balance between points 1 and 2. It will be assumed that p_1 and p_2 are uniform over the cross-sectional area.)



1- Momentum balance between points 1 & 2

$$R_x = mv_2 - mv_1 + p_2A_2 - p_1A_1$$

- Control volume is selected so that it does not include the pipe wall ➔ R_x drops out
- And $A_2 = A_1, p_0 = p_1, v_1 = v_0$

➔ $p_2A_2 - p_1A_2 = mv_0 - mv_2$

Since $m = \rho A_0 v_0$ **and** $v_2 = (A_0/A_2)v_0$

➔ $\frac{p_2 - p_1}{\rho} = v_0^2\left(\frac{A_0}{A_2}\right)\left(1 - \frac{A_0}{A_2}\right)$

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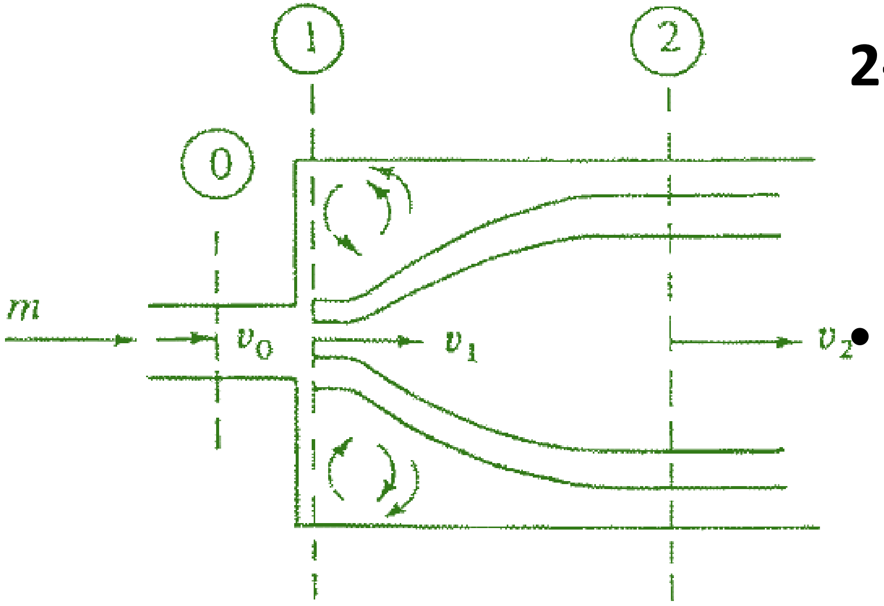
EXAMPLE 2.8-4. Friction Loss in a Sudden Enlargement

A mechanical-energy loss occurs when a fluid flows from a small pipe to a large pipe through an abrupt expansion, as shown in Fig. 2.8-4. Use the momentum balance and mechanical-energy balance to obtain an expression for the loss for a liquid. (*Hint*: Assume that $p_0 = p_1$ and $v_0 = v_1$. Make a mechanical-energy balance between points 0 and 2 and a momentum balance between points 1 and 2. It will be assumed that p_1 and p_2 are uniform over the cross-sectional area.)



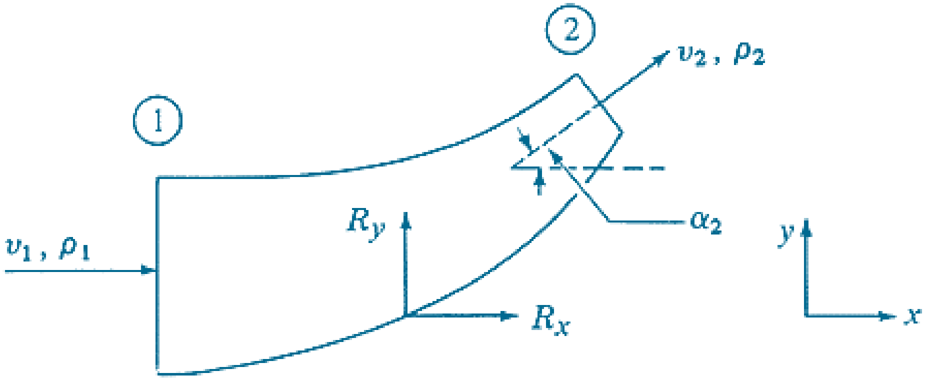
2- Mechanical energy balance between 0 & 2

$$\frac{v_0^2 - v_2^2}{2} - \sum F = \frac{p_2 - p_0}{\rho}$$

- Combining momentum and energy balance equations:

$$\sum F = \frac{v_0^2}{2}\left(1 - \frac{A_0}{A_2}\right)^2$$

P2.8-1. *Momentum Balance in a Reducing Bend*



Water is flowing at steady state through the reducing bend in Fig. 2.8-3. The angle α_2 = 90° (a right-angle bend). The pressure at point 2 is 1.0 atm abs. The flow rate is 0.020 m^3/s and the diameters at points 1 and 2 are 0.050 m and 0.030 m, respectively. Neglect frictional and gravitational forces. Calculate the resultant forces on the bend in newtons and lb-force. Use p = 1000 kg/m^3.

$$R_x = mv_2 \cos\alpha_2 - mv_1 \cos\alpha_1 + p_2A_2 \cos\alpha_2 - p_1A_1 \cos\alpha_1$$

$$R_y = mv_2 \sin\alpha_2 - mv_1 \sin\alpha_1 + p_2A_2 \sin\alpha_2 - p_1A_1 \sin\alpha_1 + m_t g$$

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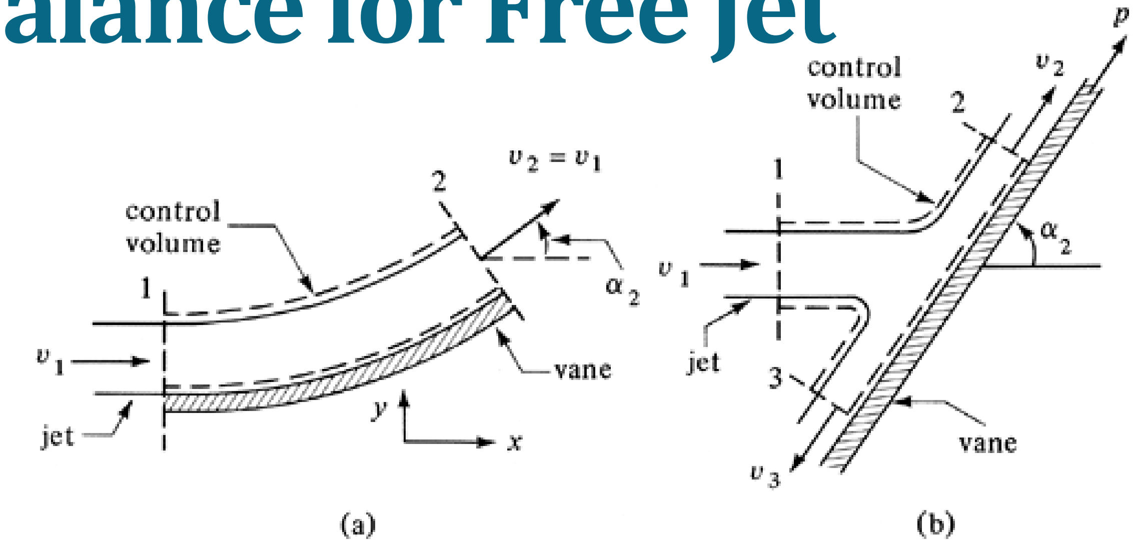
Overall Momentum Balance for Free Jet Striking a Fixed Vane



- For the curved vane (a):

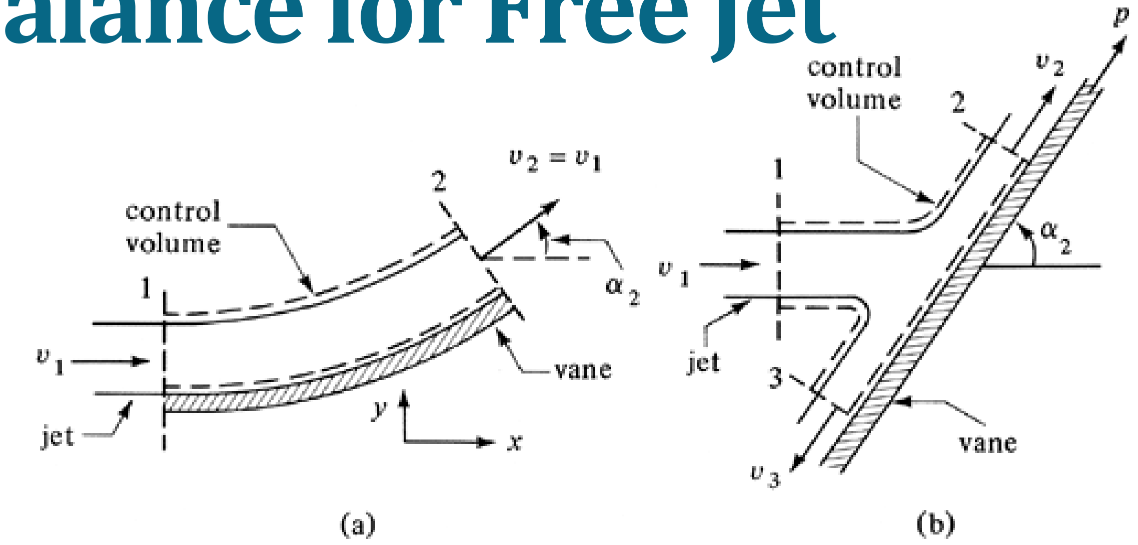
$$R_x = mv_1(\cos\alpha_2 - 1)$$

and neglecting the body force

$$R_y = mv_1 \sin\alpha_2$$

- Hence, R_x and R_y are the force components of the vane on the control volume fluid.
- The force components on the vane are $-R_x$ and $-R_y$.

Overall Momentum Balance for Free Jet Striking a Fixed Vane



- For smooth flat vane (b):

$$m_2 = \frac{m_1}{2}(1 + \cos\alpha_2)$$

$$m_3 = \frac{m_1}{2}(1 - \cos\alpha_2)$$

- The resultant force exerted by the plate on the fluid must be normal to it:

$$\text{resultant force} = R = m_1 v_1 \sin\alpha_2$$

OR

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R_x = m_2 v_2 \cos\alpha_2 - m_1 v_1 \cos\alpha_1 + m_3 v_3(-\cos\alpha_2)$$

$$R_y = m_2 v_2 \sin\alpha_2 - m_1 v_1 \sin\alpha_1 + m_3 v_3(-\sin\alpha_2)$$

$v_1 = v_2 = v_3$ (No energy loss)

P2.8-3. Force of Water Stream on a Wall

- Water at 298 K discharges from a nozzle and travels horizontally hitting a flat vertical wall. The nozzle has a diameter of 12 mm and the water leaves the nozzle with a flat velocity profile at a velocity of 6.0 m/s. Neglecting frictional resistance of the air on the jet, calculate the force in newtons on the wall.

$$m_2 = \frac{m_1}{2}(1 + \cos\alpha_2)$$

$$m_3 = \frac{m_1}{2}(1 - \cos\alpha_2)$$

$$\textbf{resultant force} = \boldsymbol{R = m_1 v_1 \sin\alpha_2}$$

OR

$$\boldsymbol{R = \sqrt{R_x^2 + R_y^2}}$$

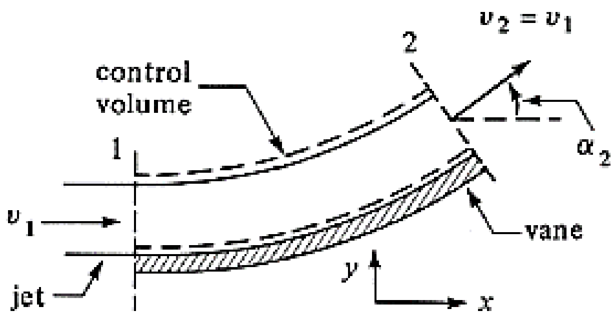
$$\boldsymbol{R_x = m_2 v_2 \cos\alpha_2 - m_1 v_1 \cos\alpha_1 + m_3 v_3(-\cos\alpha_2)}$$

$$\boldsymbol{R_y = m_2 v_2 \sin\alpha_2 - m_1 v_1 \sin\alpha_1 + m_3 v_3(-\sin\alpha_2)}$$

P2.8-6. Momentum Balance for Free Jet on a Curved, Fixed Vane.

A free jet having a velocity of 30.5 m/s and a diameter of 5.08×10^{-2} m is deflected by a curved, fixed vane as in Fig. 2.8-5a. However, the vane is curved downward at an angle of 60° instead of upward. Calculate the force of the jet on the vane. The density is 1000 kg/m^3.

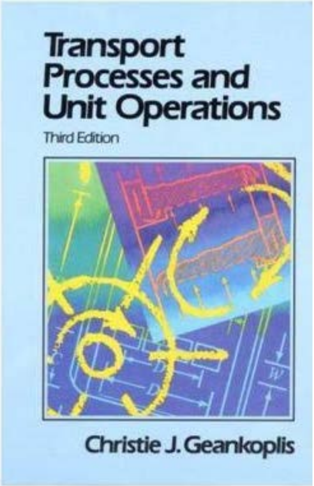
$$\boldsymbol{R_x = mv_1(\cos\alpha_2 - 1)} \qquad \boldsymbol{R_y = mv_1 \sin\alpha_2}$$



Ans. $-R_x$ = 942.8 N, $-R_y$ = 1633 N

Shell Momentum Balance and Velocity Profile in Laminar Flow

Section 2.9



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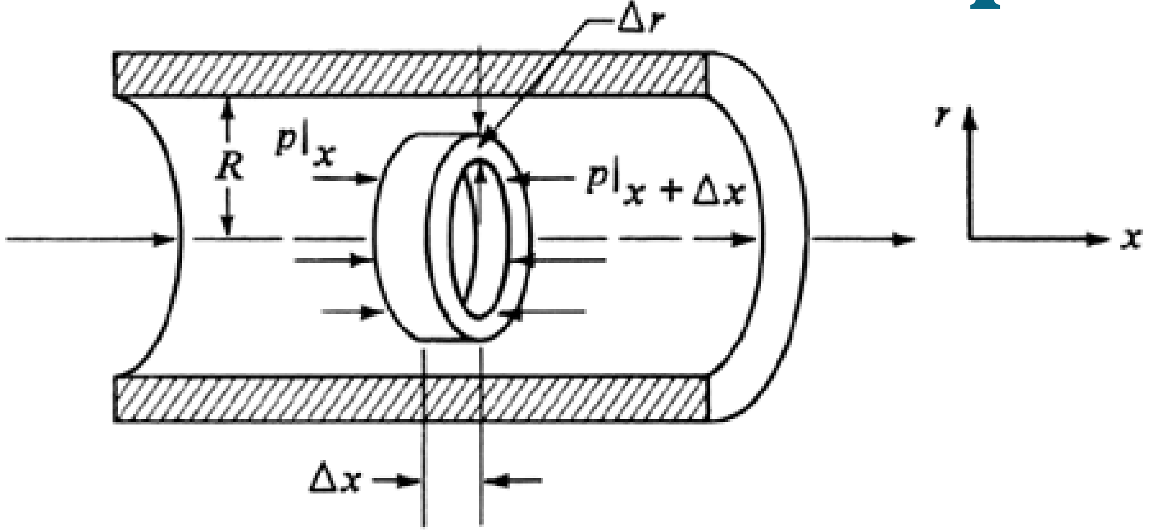
Shell Momentum Balance and Velocity Profile in Laminar Flow

- Overall momentum balance does not tell about the details of what happens inside the control volume.
- Here, a small control volume will be analyzed and then shrank to differential size.
  - Shell momentum balance using the momentum-balance concepts
  - using the definition of viscosity, an expression for the velocity profile inside the enclosure and the pressure drop will be obtained.

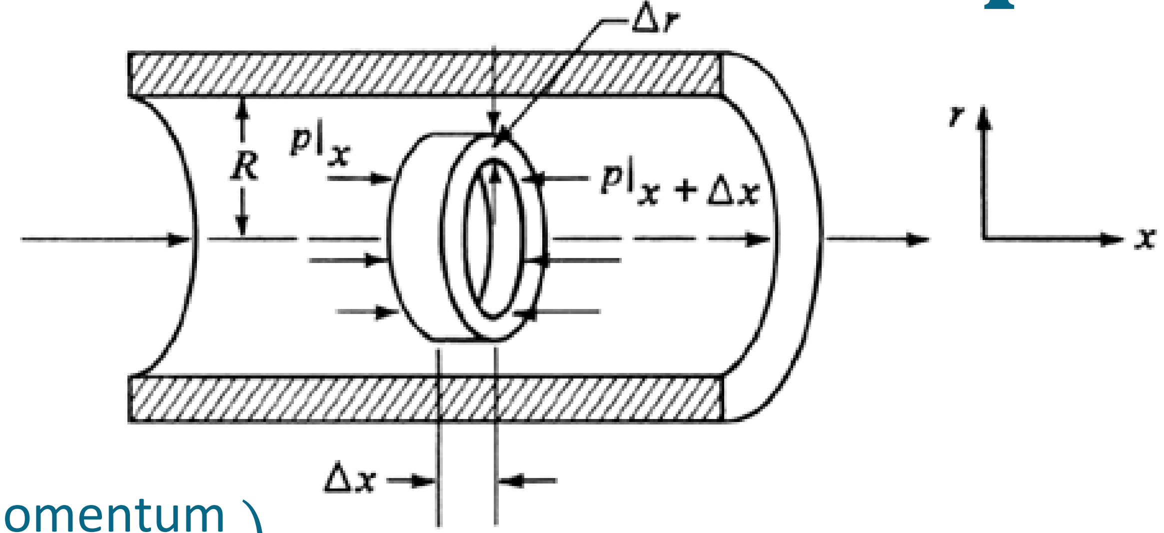
Shell Momentum Balance Inside a Pipe



- Engineers often deal with the flow of fluids inside a circular conduit or pipe.
- Analysis:
  - Horizontal section of pipe in which an incompressible Newtonian fluid is flowing in one-dimensional, steady-state, laminar flow.
  - **The flow is fully developed.**
    - It is not influenced by entrance effects.
    - the velocity profile does not vary along the axis of flow in the x direction.

Shell Momentum Balance Inside a Pipe

- At steady state the conservation of momentum becomes as follows:



$$\begin{pmatrix}\text{sum of forces acting}\\ \text{on control volume}\end{pmatrix}$$

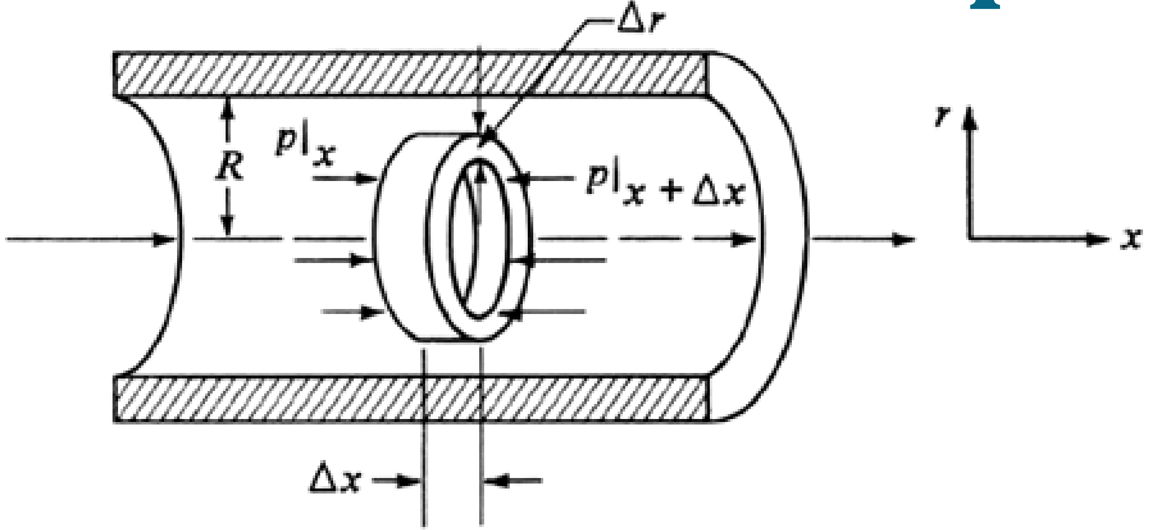
$$= \begin{pmatrix}\text{rate of momentum out}\\ \text{of control volume}\end{pmatrix} - \begin{pmatrix}\text{rate of momentum}\\ \text{into control volume}\end{pmatrix}$$

- Pressure force $= pA|_x - pA|_{x+\Delta x} = p(2\pi r\ \Delta r)|_x - p(2\pi r\ \Delta r)|_{x+\Delta x}$
- Shear force $= (\tau_{rx}\ 2\pi r\ \Delta x)|_{r+\Delta r} - (\tau_{rx}\ 2\pi r\ \Delta x)|_r$ = net rate of momentum efflux
- Flow is fully developed $\Rightarrow$ net convective momentum flux across the annular surface

  at x and $x + \Delta x$ is zero & the terms are independent of x $\Rightarrow v_x|_x = v_x|_{x+\Delta x}$

Shell Momentum Balance Inside a Pipe

- At steady state the conservation of momentum becomes as follows:



Pressure force + Shear force = 0

$$p(2\pi r\,\Delta r)|_x - p(2\pi r\,\Delta r)|_{x+\Delta x} = (\tau_{rx}\,2\pi r\,\Delta x)|_{r+\Delta r} - (\tau_{rx}\,2\pi r\,\Delta x)|_r$$

$$\frac{r\,(p|_x - p|_{x+\Delta x})}{\Delta x} = \frac{(r\tau_{rx})|_{r+\Delta r} - (r\tau_{rx})|_r}{\Delta r}$$

In fully developed flow, the pressure gradient ($\Delta p/\Delta x$) is constant = ($\Delta p/L$), where L = pipe length $\Rightarrow$

$$\frac{d(r\tau_{rx})}{dr} = r\left(\frac{\Delta p}{L}\right)$$

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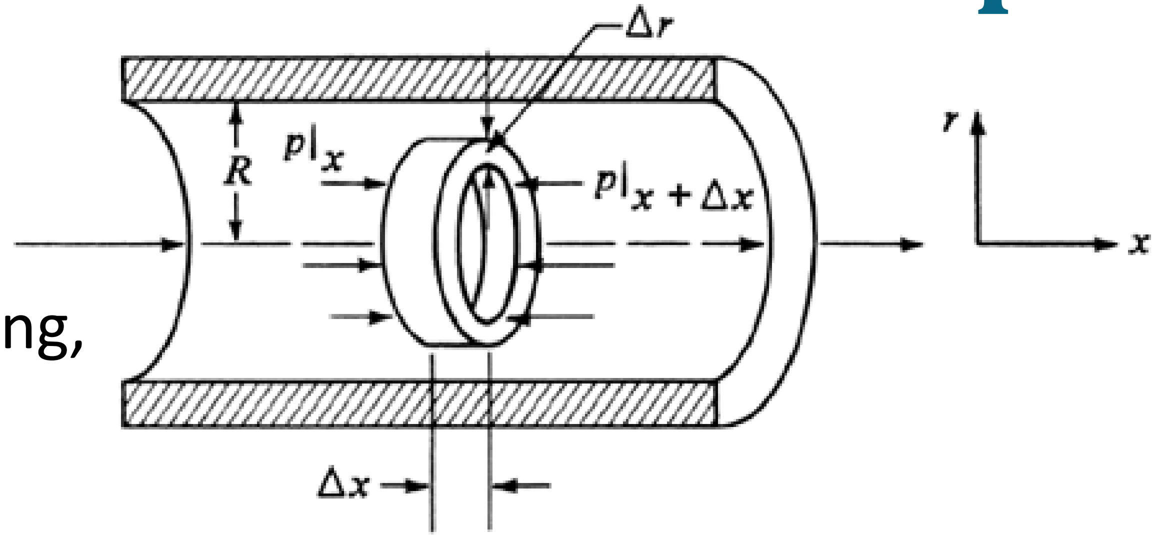
Shell Momentum Balance Inside a Pipe

$$\frac{d(r\tau_{rx})}{dr} = r\left(\frac{\Delta p}{L}\right)$$



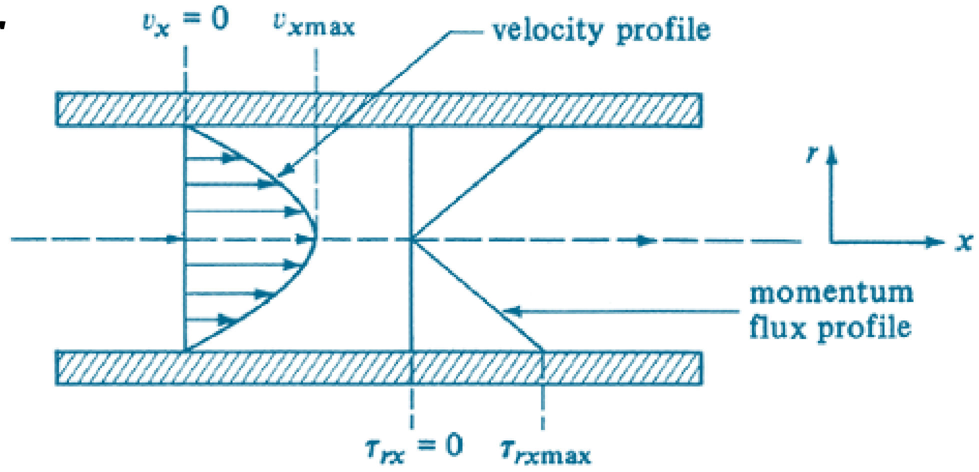
Separating variables and integrating,

$$\tau_{rx} = \left(\frac{\Delta p}{L}\right)\frac{r}{2} + \frac{C_1}{r}$$

$$\tau_{rx} \neq \infty \; at \; r = 0 \;\; \rightarrow C_1 = 0$$

$$\Rightarrow \tau_{rx} = \left(\frac{\Delta p}{2L}\right) r \Rightarrow \tau_{rx} = \left(\frac{p_0 - p_L}{2L}\right) r$$

$\Rightarrow$ momentum flux varies linearly with the radius, and the maximum value occurs at r = R at the wall



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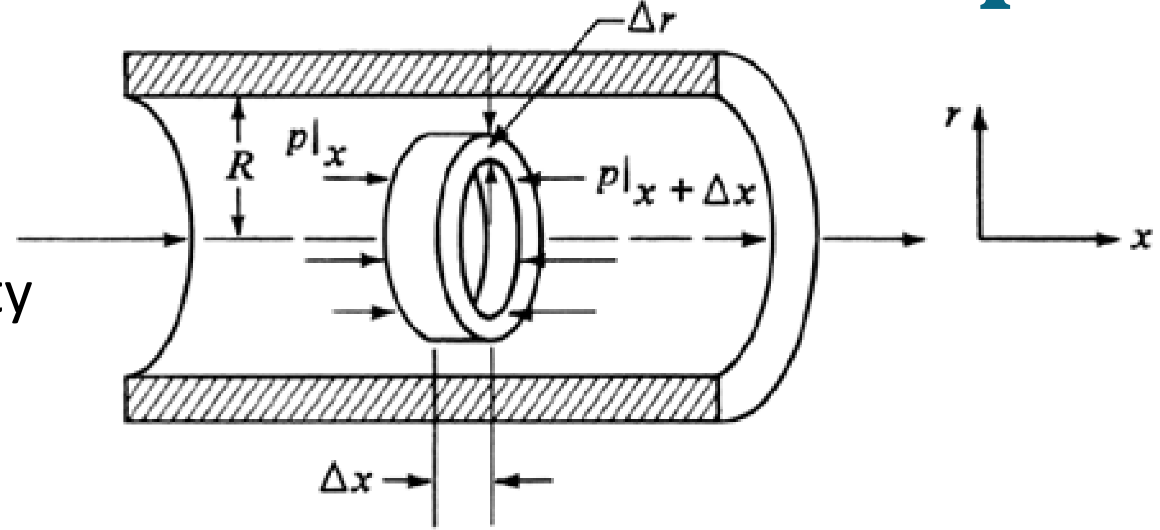
Shell Momentum Balance Inside a Pipe

$$\tau_{rx} = \left(\frac{p_0 - p_L}{2L}\right) r$$



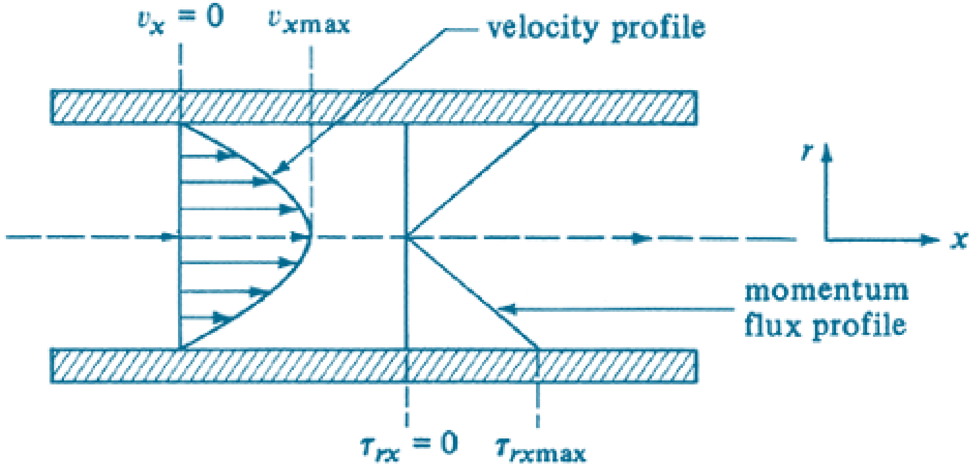
Substituting Newton's law of viscosity

$$\boldsymbol{\tau_{rx} = -\mu \frac{dv_x}{dr}} = \left(\frac{p_0 - p_L}{2L}\right) r$$

Integrating using the boundary condition that at the wall, $v_x = 0$ at $r = R$, we obtain the equation for the velocity distribution

$$v_x = \frac{p_0 - p_L}{4\mu L} R^2 \left[1 - \left(\frac{r}{R}\right)^2\right]$$

$\Rightarrow$ the velocity distribution is parabolic



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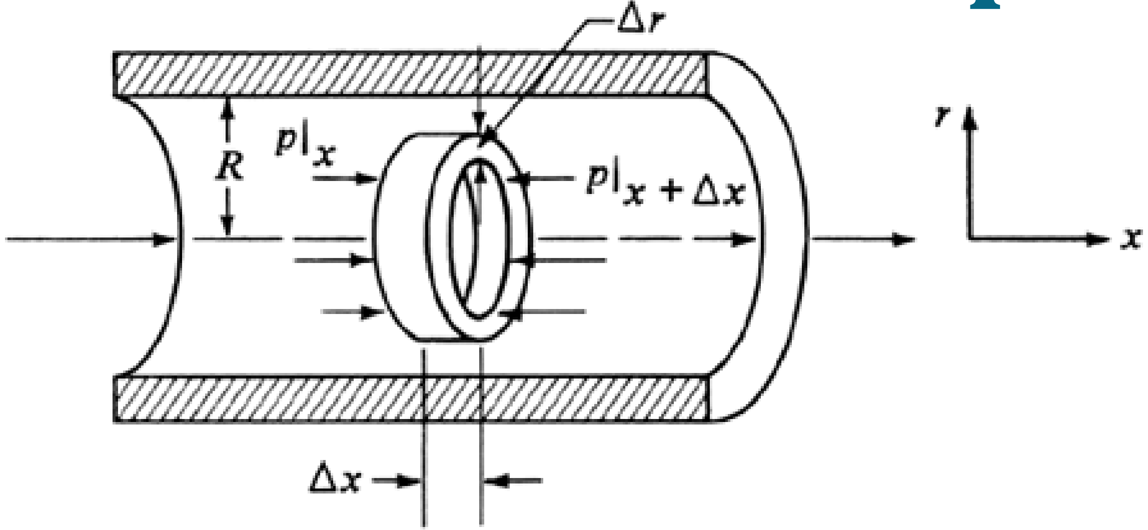
Shell Momentum Balance Inside a Pipe

$$v_x = \frac{p_0 - p_L}{4\mu L} R^2 \left[1 - \left(\frac{r}{R}\right)^2\right]$$



The average velocity $v_{x\ av}$ for a cross section is found by summing up all the velocities over the cross section and dividing by the cross-sectional area

$$v_{x\,\mathrm{av}} = \frac{1}{A}\iint_A v_x\, dA = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R v_x\, r\, dr\, d\theta = \frac{1}{\pi R^2}\int_0^R v_x\, 2\pi r\, dr$$

$$v_{x\,\mathrm{av}} = \frac{(p_0 - p_L)R^2}{8\mu L} = \frac{(p_0 - p_L)D^2}{32\mu L}$$

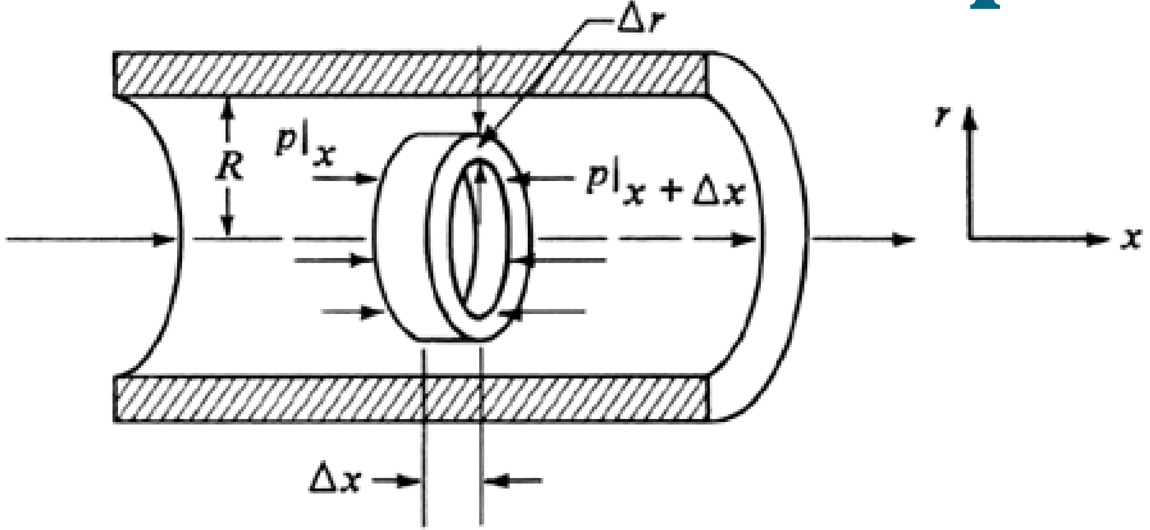
Shell Momentum Balance Inside a Pipe

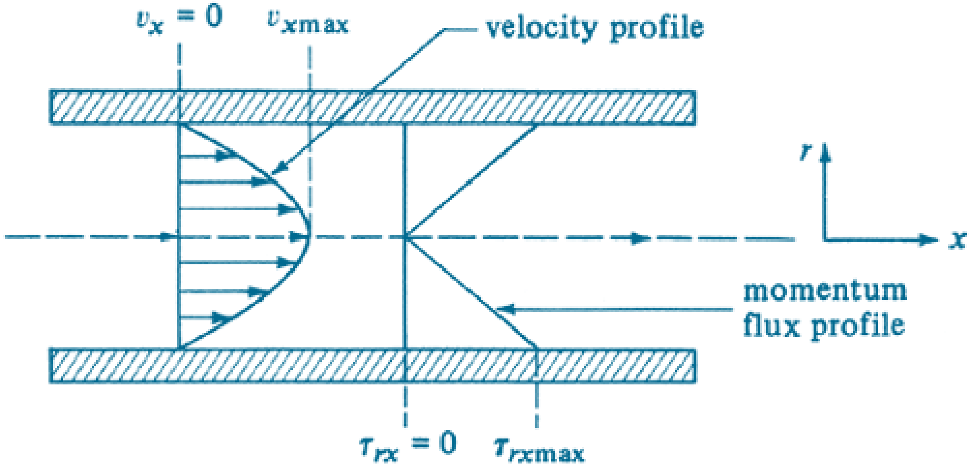
The maximum velocity for a pipe occurs at $r = 0$.

$$v_x = \frac{p_0 - p_L}{4\mu L} R^2 \left[1 - \left(\frac{r}{R}\right)^2\right]$$

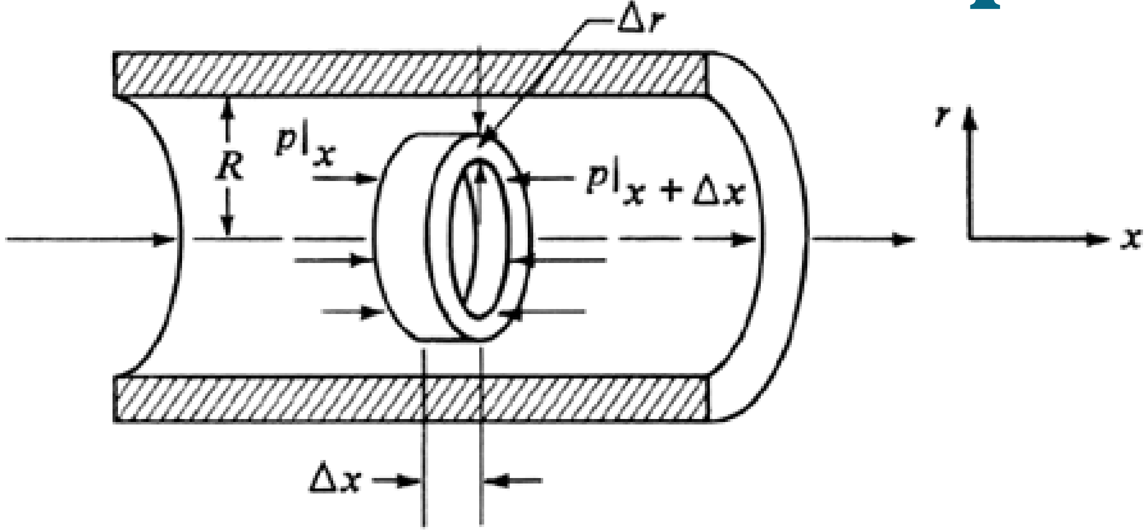
$$\Rightarrow v_{x\,max} = \frac{p_0 - p_L}{4\mu L} R^2$$

$$\boldsymbol{v_{x\,av} = \frac{v_{x\,max}}{2}}$$





Shell Momentum Balance Inside a Pipe



$$v_x = \frac{p_0 - p_L}{4\mu L} R^2 \left[1 - \left(\frac{r}{R}\right)^2\right]$$

The average velocity $v_{x\ av}$ for a cross section is found by summing up all the velocities over the cross section and dividing by the cross-sectional area

$$v_{x\,\mathrm{av}} = \frac{1}{A} \iint_A v_x \, dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R v_x \, r \, dr \, d\theta = \frac{1}{\pi R^2} \int_0^R v_x \, 2\pi r \, dr$$

$$v_{x\,\mathrm{av}} = \frac{(p_0 - p_L)R^2}{8\mu L} = \frac{(p_0 - p_L)D^2}{32\mu L}$$

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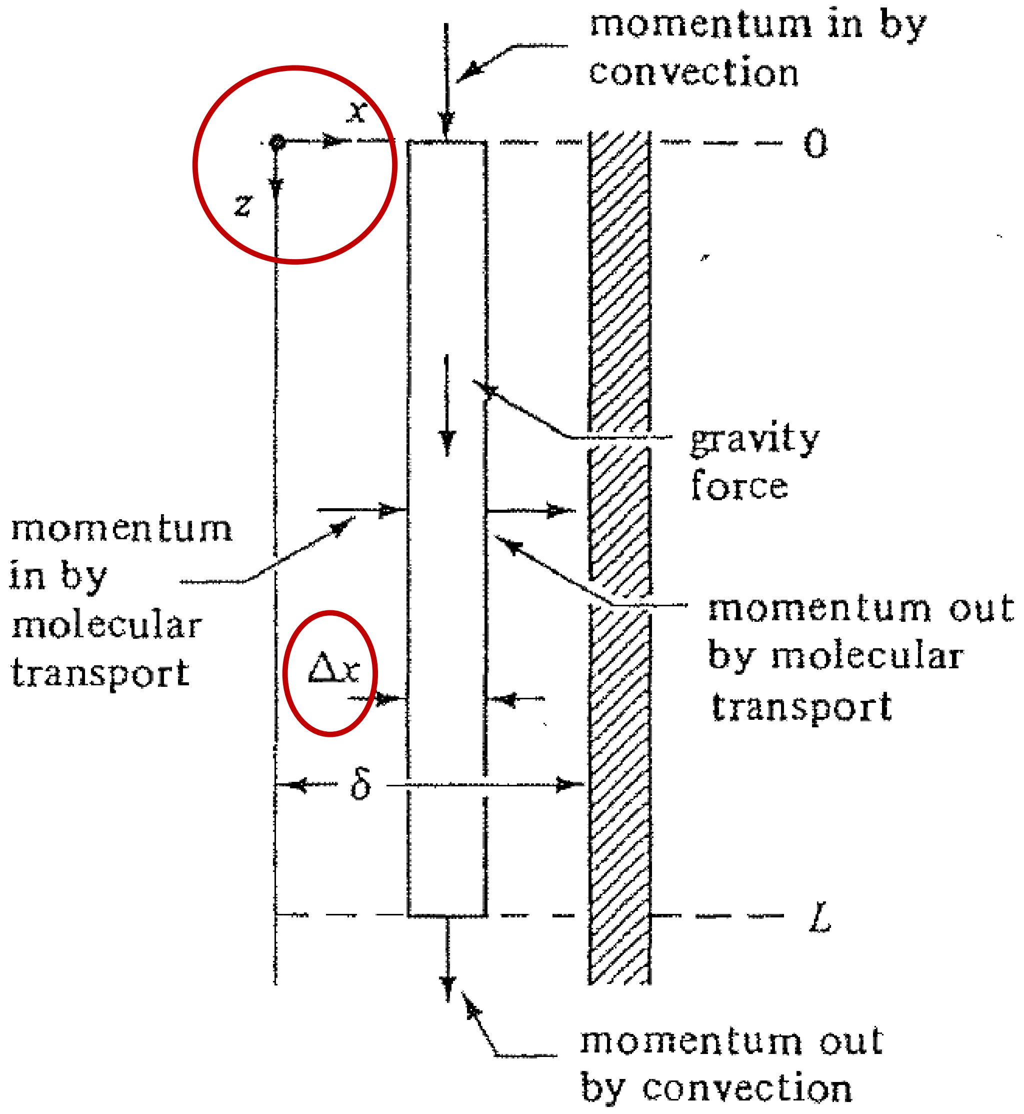
Shell Momentum Balance for Falling Film

- Falling films have been used to study various phenomena in mass transfer, coatings on surfaces, and so on.
- The control volume for the falling film is a shell of fluid having a thickness of Δx and a length of L in the vertical z direction;
  - sufficiently far from the entrance and exit regions so that the flow is not affected by these regions. $\Rightarrow$ the velocity $v_z(x)$ does not depend on position z.



Shell Momentum Balance for Falling Film

System: Δx thick, bounded in the z direction by the planes $z = 0$ and $z = L$, and extending a distance W in the y direction.

1. Momentum flux due to **molecular transport**:

$$\text{net efflux} = LW(\tau_{xz})|_{x+\Delta x} - LW(\tau_{xz})|_{x}$$

2. Net **convective** momentum flux:

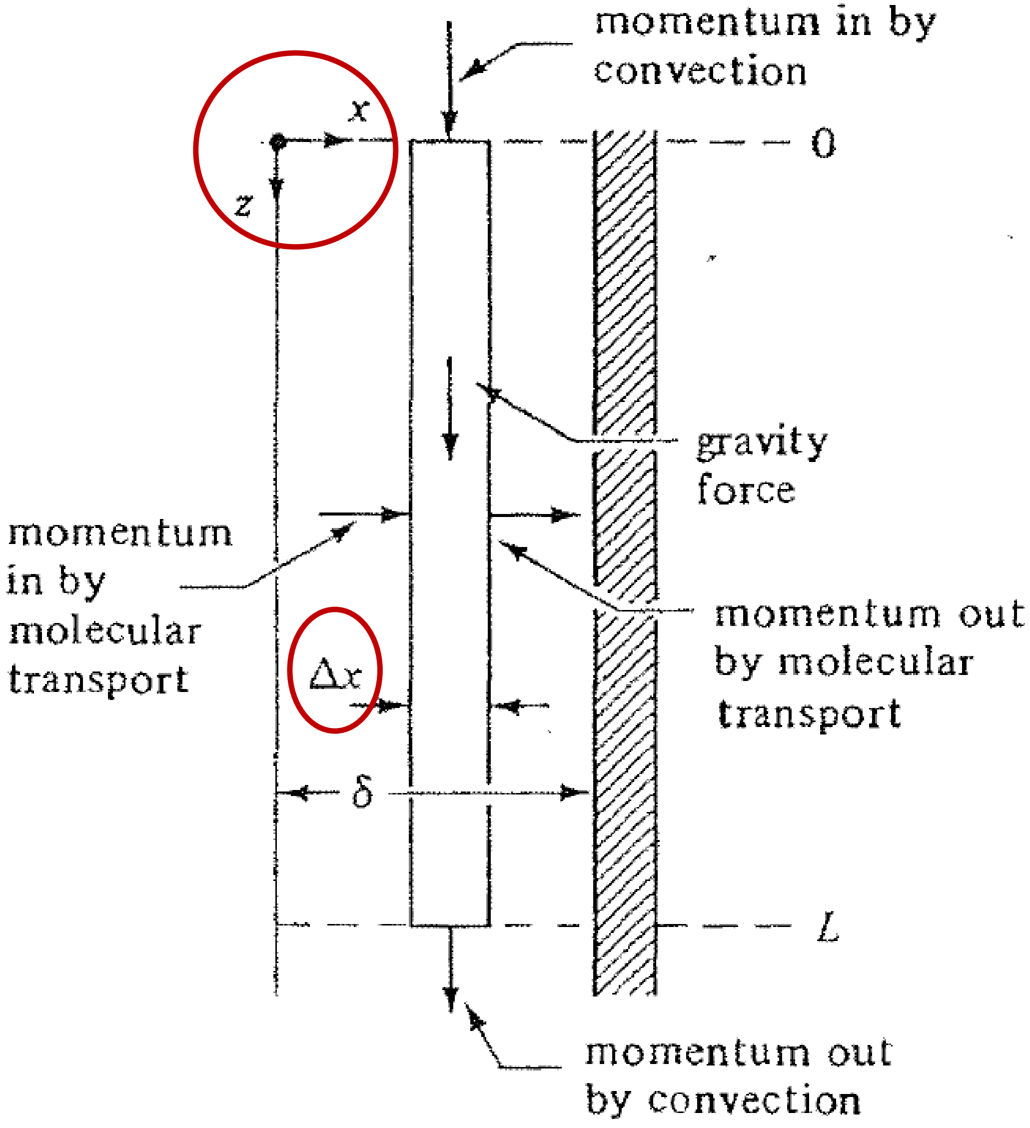
net efflux

$$= \Delta x W v_z(\rho v_z)|_{z=L} - \Delta x W v_z(\rho v_z)|_{z=0} = 0$$

equal

3. **Gravity** force acting on the fluid:

$$\text{gravity force} = \Delta x W L(\rho g)$$



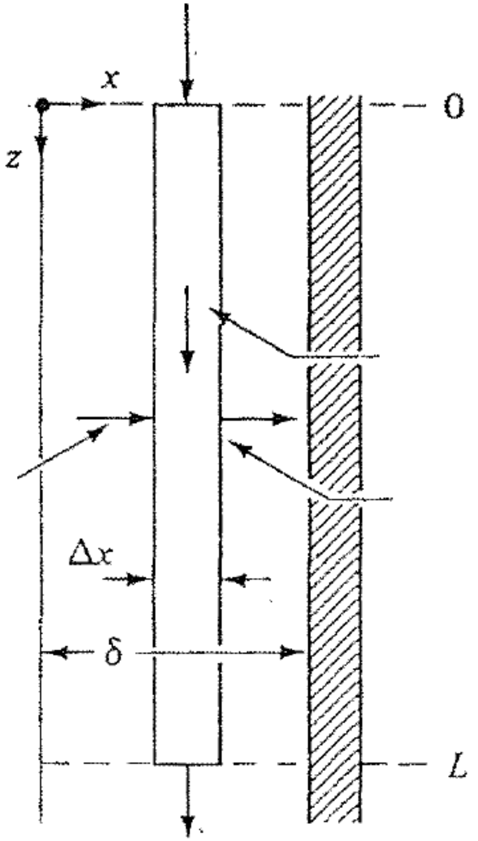
Shell Momentum Balance for Falling Film

$$\begin{pmatrix}\text{sum of forces acting}\\ \text{on control volume}\end{pmatrix} = \begin{pmatrix}\text{rate of momentum out}\\ \text{of control volume}\end{pmatrix} - \begin{pmatrix}\text{rate of momentum}\\ \text{into control volume}\end{pmatrix}$$

$$\Delta x W L(\rho g) = LW(\tau_{xz})|_{x+\Delta x} - LW(\tau_{xz})|_{x} + 0$$

Rearranging and letting $\Delta x \to 0$,

$$\frac{\tau_{xz}|_{x+\Delta x} - \tau_{xz}|_{x}}{\Delta x} = \rho g \quad \rightarrow \quad \frac{d\tau_{xz}}{dx} = \rho g$$



Boundary conditions:

at $x = 0$, $\tau_{xz} = 0$ at the free liquid surface,

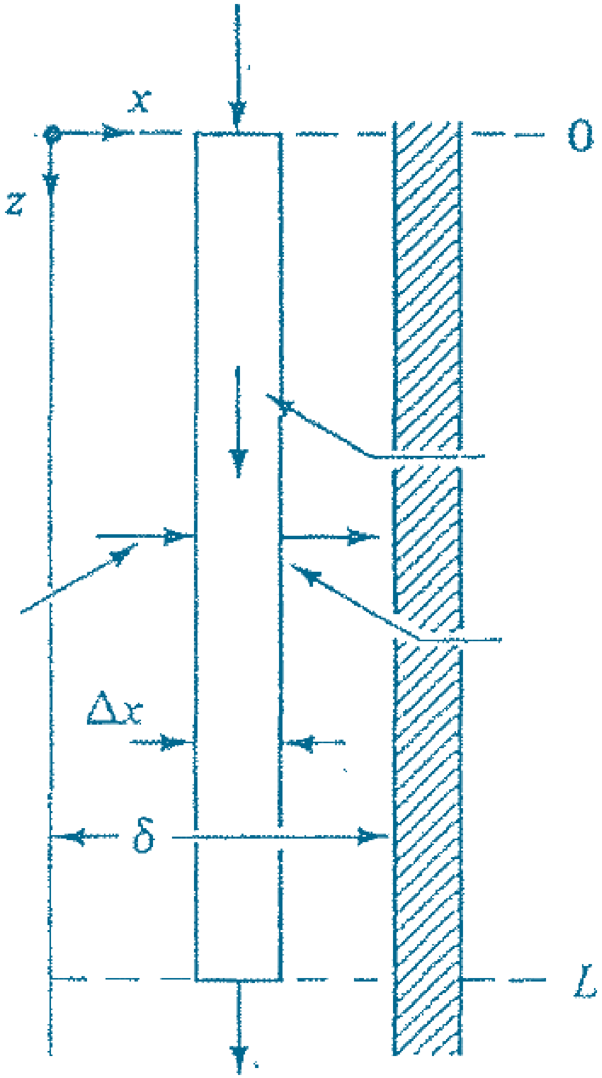
and at $x = x$, $\tau_{xz} = \tau_{xz} \quad v \quad \rightarrow \quad \tau_{xz} = \rho g x$

Shell Momentum Balance for Falling Film

- For a Newtonian fluid using Newton's law of viscosity,

$$\tau_{xz} = -\mu \frac{dv_z}{dx} \quad \text{and} \quad \boldsymbol{\tau_{xz} = \rho g x}$$

$$\frac{dv_z}{dx} = -\left(\frac{\rho g}{\mu}\right)x \xrightarrow{\textit{integrate}} v_z = -\left(\frac{\rho g}{2\mu}\right)x^2 + C_1$$



Boundary conditions:

at $x = \delta$, $\boldsymbol{v_x = 0}$

$$v_z = \frac{\rho g \delta^2}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

Shell Momentum Balance for Falling Film

$$v_z = \frac{\rho g \delta^2}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

- The maximum velocity occurs at $x = 0$

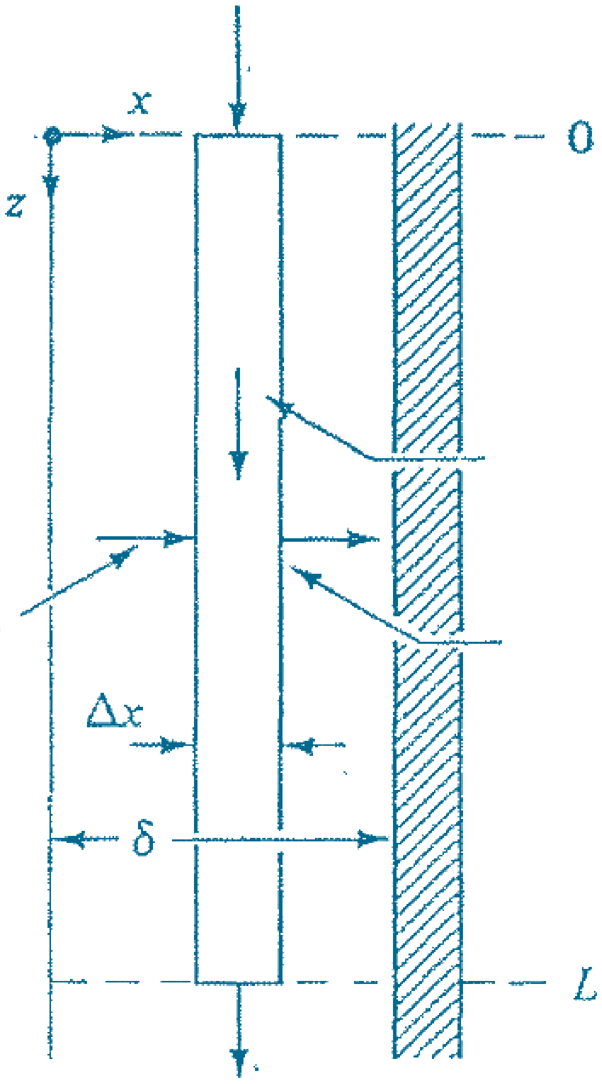
$$\Rightarrow \quad v_{z\,max} = \frac{\rho g \delta^2}{2\mu}$$

- Average velocity:

$$v_{z\,av} = \frac{1}{A}\iint_A v_z\, dA = \frac{1}{W\delta}\int_0^W\int_0^\delta v_z\, dx\, dy = \frac{W}{W\delta}\int_0^\delta v_z\, dx$$

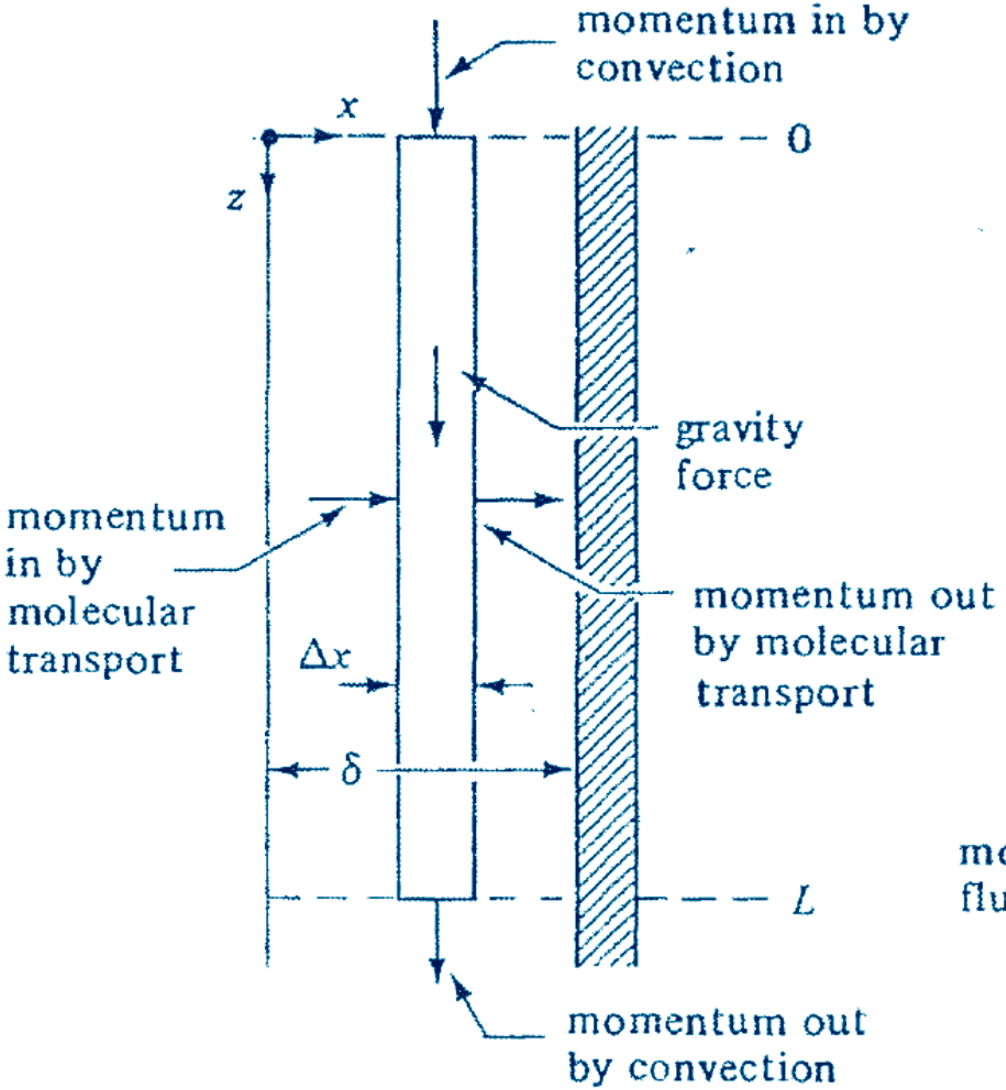
$$\Rightarrow \quad v_{z\,av} = \frac{\rho g \delta^2}{3\mu} = \frac{2}{3} v_{z\,max}$$

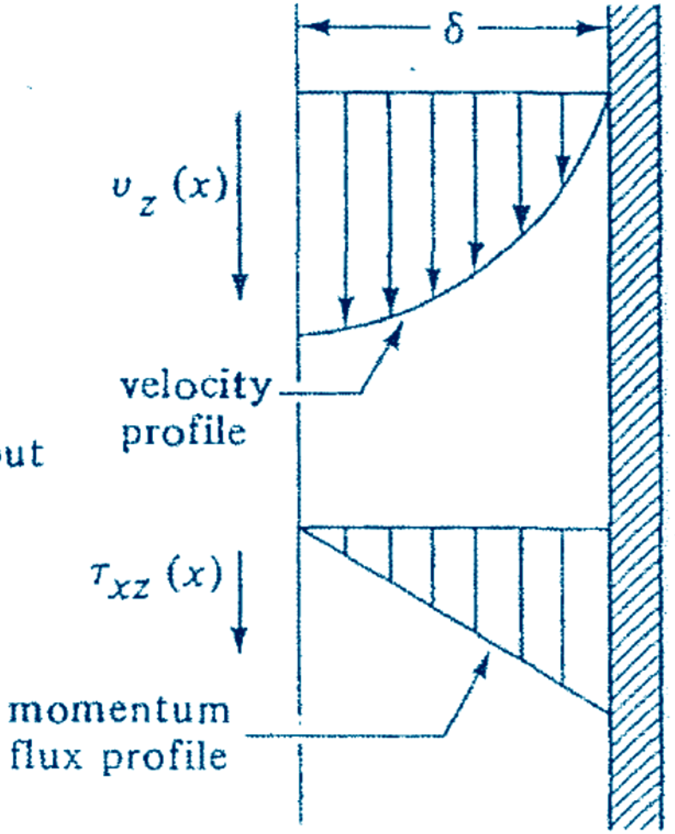
Laminar flow occurs for *Re* < 1200.



Shell Momentum Balance for Falling Film

$$v_z = \frac{\rho g \delta^2}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$





$$\tau_{xz} = \rho g x$$

2.9-1. ***Film of Water on Wetted-Wall Tower.*** Pure water at 20°C is flowing down a vertical wetted-wall column at a rate of 0.124 kg/s · m. Calculate the film thickness and the average velocity.

Ans. $\delta = 3.370 \times 10^{-4}$ m, $v_{z\,av} = 0.3687$ m/s

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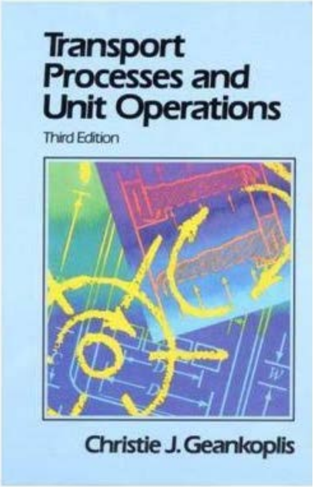
Differential Equations Of Continuity

Chapter 3
Section 3.6



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Differential Equations of Continuity

- Overall mass and momentum balances allowed us to solve many elementary problems on fluid flow.
  - balances done on *a control volume*
- Overall balances do not require knowledge of what goes on inside the finite control volume.
- To advance in studying these flow systems, must investigate in greater detail what goes on inside this finite control volume.
  - use a differential element for a control volume
  - differential balances in a single phase and integrate to the phase boundary using the boundary conditions.

Differential Equations of Continuity

- Differential-momentum-balance equation is based on Newton's second law
  - allows to determine the way velocity varies with position and time
  - allows to determine the pressure drop in laminar flow.
- The equation of momentum balance can be used for turbulent flow with certain modifications.
- Often these conservation equations are called *equations of change*, since they describe the variations in the properties of the fluid with respect to position and time:

Types of Time Derivatives & Vector Notation

1. ***Partial time derivative:*** the local change of fluid property with time at a fixed-point x, y, and z.
   - Example: $\frac{\partial \rho}{\partial t}$ = partial time derivative of density ρ.
2. ***Total time derivative.***

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}\frac{dx}{dt} + \frac{\partial \rho}{\partial y}\frac{dy}{dt} + \frac{\partial \rho}{\partial z}\frac{dz}{dt}$$

"the density is a function of t and of the velocity components dx/dt, dy/dt, and dz/dt at which the observer is moving"

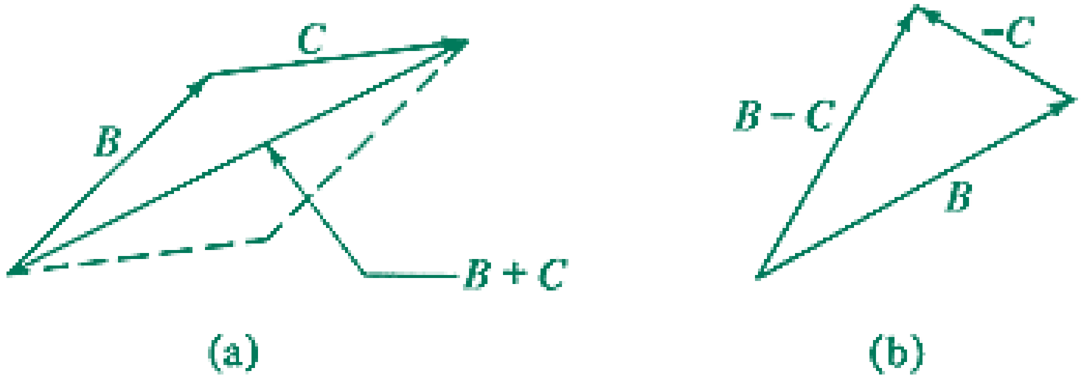
3. ***Substantial time derivative:*** derivative that follows the motion

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = \frac{\partial \rho}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla} \rho)$$

Types of Time Derivatives & Vector Notation

4. ***Scalars:*** quantities such as concentration, temperature, length, volume, time, and energy. They have magnitude but no direction.
5. ***Vectors.*** Velocity, force, momentum, and acceleration are considered vectors since they have magnitude and direction. They are written in boldface letters in textbooks, e.g., **v** for velocity.
   The vector $\boldsymbol{B}$ is represented by its three projections B_x, B_y, and B_z on the *x, y,* and *z* axes and

$$\mathbf{B} = \mathbf{i}B_x + \mathbf{j}B_y + \mathbf{k}B_z$$
$$r\mathbf{B} = \mathbf{B}r$$
$$(\mathbf{B} \cdot \mathbf{C}) = (\mathbf{C} \cdot \mathbf{B})$$
$$(\mathbf{B} \cdot \mathbf{C})\mathbf{D} \neq \mathbf{B}(\mathbf{C} \cdot \mathbf{D})$$
$$(\mathbf{B} \cdot \mathbf{C}) = BC \cos \phi_{BC}$$



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Types of Time Derivatives & Vector Notation

6. ***Differential operations with scalars and vectors.*** The gradient or "grad" of a scalar field is

$$\nabla\rho = \mathbf{i}\frac{\partial\rho}{\partial x} + \mathbf{j}\frac{\partial\rho}{\partial y} + \mathbf{k}\frac{\partial\rho}{\partial z}$$

The divergence or "div" of a vector **v** is

$$(\nabla\cdot\mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

The Laplacian of a scalar field is

$$\nabla^2\rho = \frac{\partial^2\rho}{\partial x^2} + \frac{\partial^2\rho}{\partial y^2} + \frac{\partial^2\rho}{\partial z^2}$$

Types of Time Derivatives & Vector Notation

- **Other useful operations:**

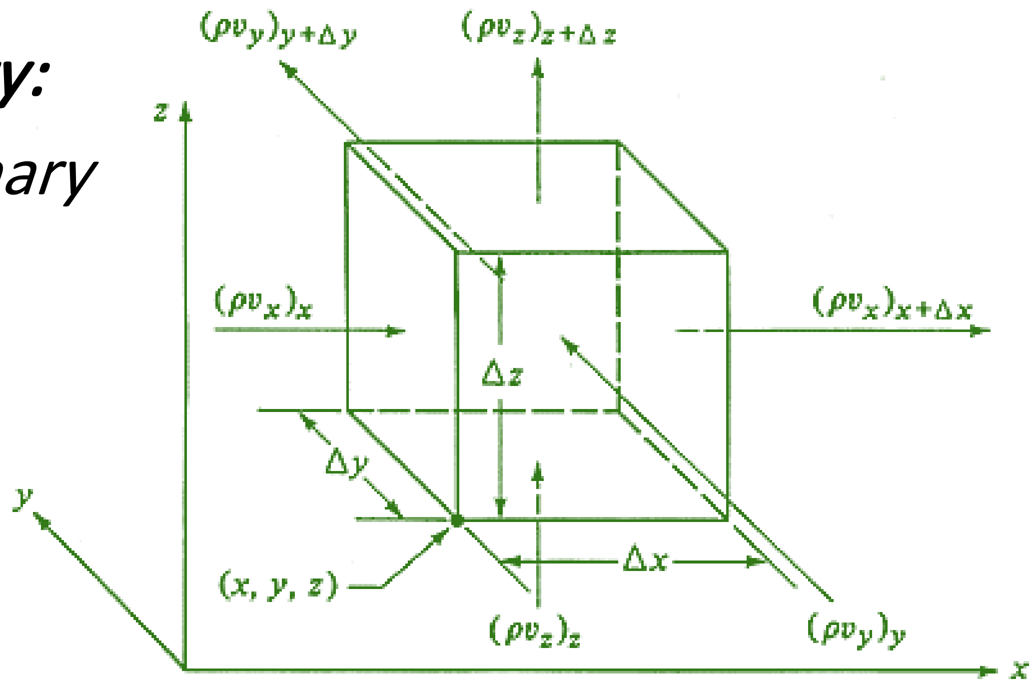
$$\nabla rs = r\nabla s + s\nabla r$$

$$(\nabla \cdot s\mathbf{v}) = (\nabla s \cdot \mathbf{v}) + s(\nabla \cdot \mathbf{v})$$

$$\mathbf{v} \cdot \nabla s = v_x \frac{\partial s}{\partial x} + v_y \frac{\partial s}{\partial y} + v_z \frac{\partial s}{\partial z}$$

Differential Equation of Continuity

- *Derivation of equation of continuity:*
  - *pure fluid flowing through stationary volume element*

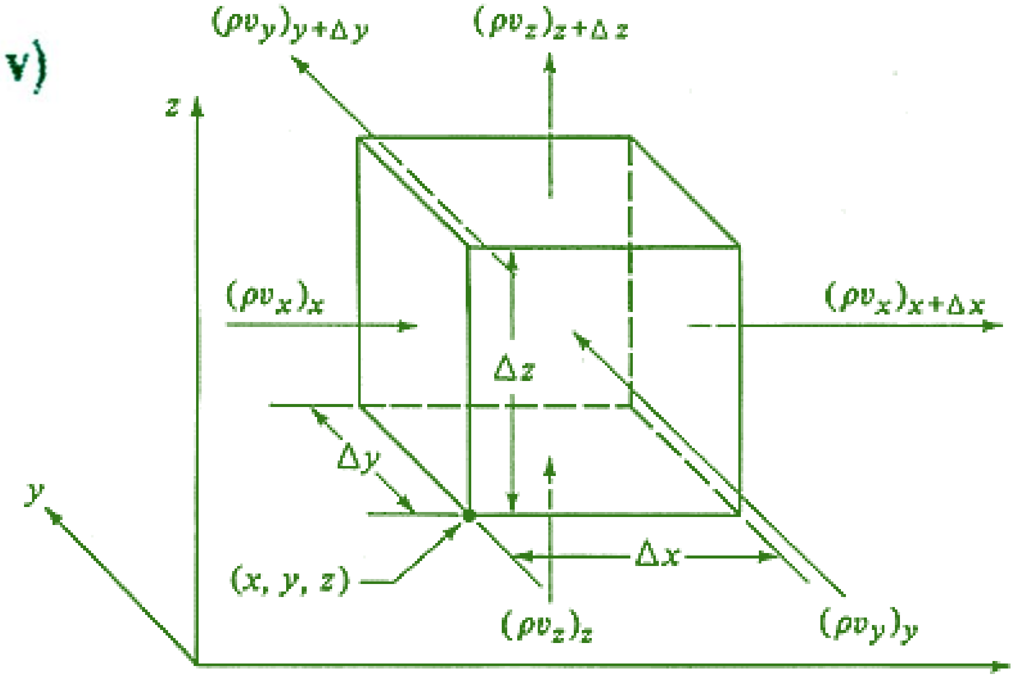


Differential Equation of Continuity

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] = -(\nabla \cdot \rho \mathbf{v})$$

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$\frac{D\rho}{Dt} = -\rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = -\rho(\nabla \cdot \mathbf{v})$$



For constant density:

$$(\nabla \cdot \mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

EXAMPLE **3.6-1.**

Continuity Equation in Cylindrical Coordinates

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

$$r = +\sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\frac{y}{x}$$

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial\theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

Continuity Equation in Spherical Coordinates

$$x = r\sin\theta\cos\phi \qquad y = r\sin\theta\sin\phi \qquad z = r\cos\theta$$

$$r = +\sqrt{x^2 + y^2 + z^2} \qquad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} \qquad \phi = \tan^{-1}\frac{y}{x}$$

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial(\rho r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\theta \sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(\rho v_\phi)}{\partial\phi} = 0$$

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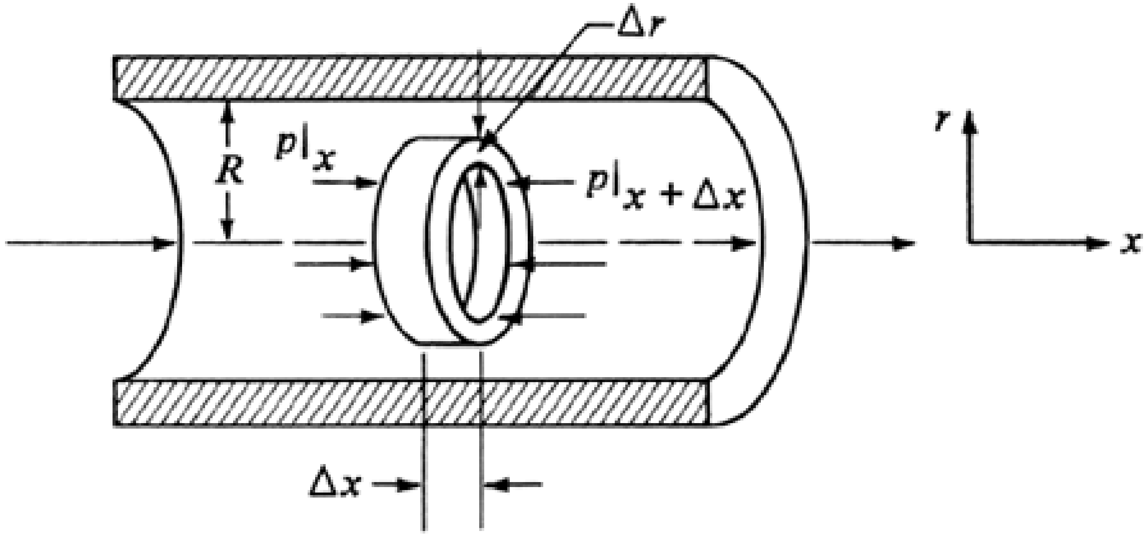
P3.6-1. Equation of Continuity in a Cylinder.

Fluid having a constant density ρ is flowing in the z direction through a circular pipe with axial symmetry. The radial direction is designated by r.

(a) Using a cylindrical shell balance with dimensions dr and dz, derive the equation of continuity for this system.

(b) Use the equation of continuity in cylindrical coordinates to derive the equation.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$



Δr
R
p|x
p|x + Δx
r
x
Δx

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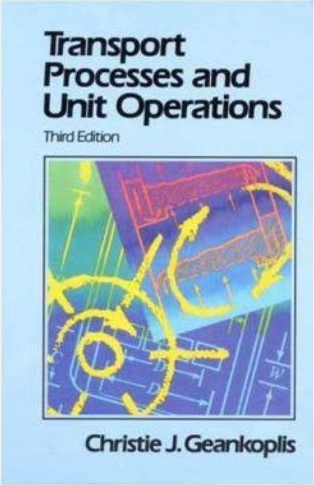
Differential Equations of Momentum Transfer or Motion

Section 3.7



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Differential Equation of Momentum Transfer

- *Equation of motion*

  == Equation for the conservation-of-momentum equation

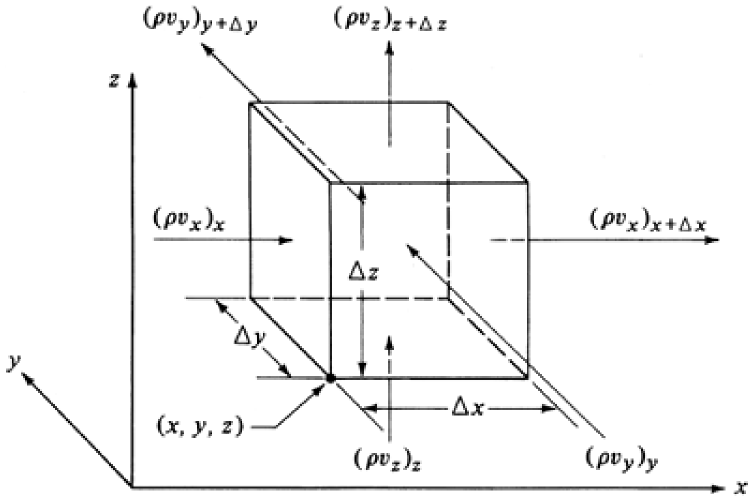
$$\begin{pmatrix}\text{rate of}\\ \text{momentum in}\end{pmatrix} - \begin{pmatrix}\text{rate of}\\ \text{momentum out}\end{pmatrix} + \begin{pmatrix}\text{sum of forces}\\ \text{acting on system}\end{pmatrix} = \begin{pmatrix}\text{rate of momentum}\\ \text{accumulation}\end{pmatrix}$$

- **Considering the x-component of each term:**
  - **Net Convective x-momentum flow into the volume element $\Delta x \Delta y \Delta z$ is**

$$[(\rho v_x v_x)_x - (\rho v_x v_x)_{x+\Delta x}]\Delta y \Delta z + \left[(\rho v_y v_x)_y - (\rho v_y v_x)_{y+\Delta y}\right]\Delta x \Delta z + [(\rho v_z v_x)_z - (\rho v_z v_x)_{z+\Delta z}]\Delta x \Delta y$$

$\rho v_x =$ concentration [=] momentum/m^3

$\rho v_x v_x =$ momentum flux [=] momentum/s·m^2



Differential Equation of Momentum Transfer

- **Considering the x-component of each term:**
  - **Net x-component of momentum by Molecular transfer is**

$$[(\tau_{xx})_x - (\tau_{xx})_{x+\Delta x}]\boldsymbol{\Delta y \Delta z} + \left[(\tau_{yx})_y - (\tau_{yx})_{y+\Delta y}\right]\boldsymbol{\Delta x \Delta z}$$

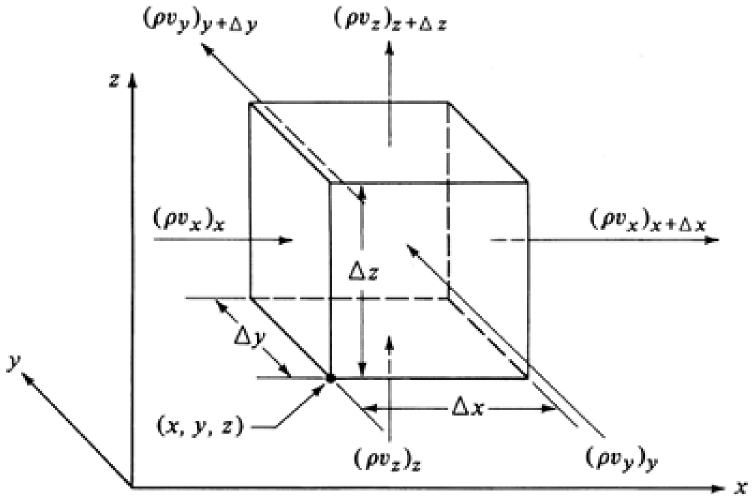
$$+[(\tau_{zx})_z - (\tau_{zx})_{z+\Delta z}]\boldsymbol{\Delta x \Delta y}$$

$\tau_{yx} =$ x direction shear stress on the y face

$\tau_{xx} =$ nomal stress on the x face

  - **net fluid pressure force:**

$$[p_x - p_{x+\Delta x}]\boldsymbol{\Delta y \Delta z}$$



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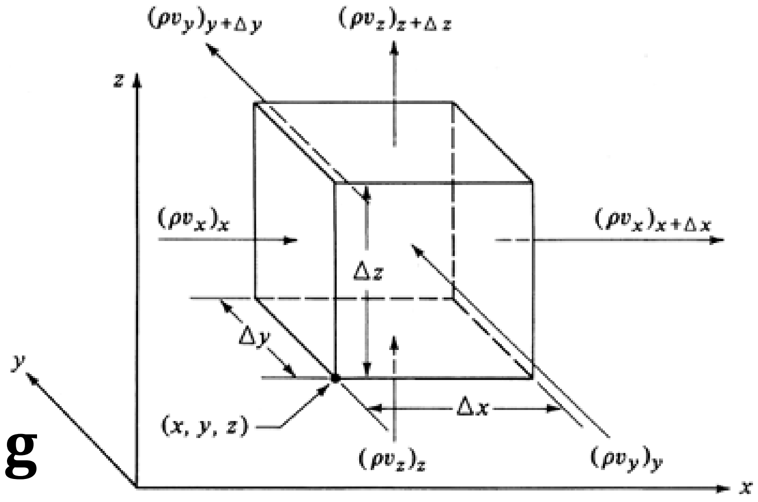
Differential Equation of Momentum Transfer



- **Considering the x-component of each term:**
  - **Gravitational force in the x direction is**

$$\rho g_x\, \Delta x \Delta y \Delta z$$

$$g_x = x\ component\ of\ the\ gravitational\ vector\ \mathbf{g}$$

  - **Rate of Accumulation of x momentum in the element is:**

$$\Delta x \Delta y \Delta z \frac{\partial(\rho v_x)}{\partial t}$$

- **Substituting, dividing by $\Delta x \Delta y \Delta z$, and taking the limit as $\Delta x, \Delta y, \Delta z \to 0$:**

$$\frac{\partial(\rho v_x)}{\partial t} = -\left[\frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_y v_x)}{\partial y} + \frac{\partial(\rho v_z v_x)}{\partial z}\right] - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

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Differential Equation of Momentum Transfer

- ***x***-component of the differential equation of motion.

$$\frac{\partial(\rho v_x)}{\partial t} = -\left[\frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_y v_x)}{\partial y} + \frac{\partial(\rho v_z v_x)}{\partial z}\right] - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

- **Using the equation of continuity:**

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right]$$

- **Equations of motion for the *x, y,* and *z* components are obtained:**

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) - \frac{\partial p}{\partial y} + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) - \frac{\partial p}{\partial z} + \rho g_z$$

Equations of Motion for Newtonian Fluids

Shear-stress components for Newtonian fluids

- in cylindrical coordinates
- In rectangular coordinates
- in spherical coordinates

$$\tau_{xx} = -2\mu \frac{\partial v_x}{\partial x} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v})$$

$$\tau_{yy} = -2\mu \frac{\partial v_y}{\partial y} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v})$$

$$\tau_{zz} = -2\mu \frac{\partial v_z}{\partial z} + \frac{2}{3}\mu(\nabla \cdot \mathbf{v})$$

$$\tau_{xy} = \tau_{yx} = -\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$\tau_{yz} = \tau_{zy} = -\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$$

$$\tau_{zx} = \tau_{xz} = -\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)$$

$$(\nabla \cdot \mathbf{v}) = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

Equations of Motion for Newtonian Fluids with Varying Density and Viscosity

- **For the x-component of momentum, the general equation of motion for a Newtonian fluid with varying density and viscosity is:**

$$\rho \frac{Dv_x}{Dt} = \frac{\partial}{\partial x}\left[2\mu \frac{\partial v_x}{\partial x} - \frac{2}{3}\mu(\nabla \cdot \mathbf{v})\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial x}\right)\right]$$

$$+ \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right)\right] - \frac{\partial p}{\partial x} + \rho g_x$$

- **Similar equations are obtained for the y and z components of momentum.**

Equations of Motion for Newtonian Fluids with Constant Density and Viscosity

$$\frac{\partial(\rho v_x)}{\partial t} = -\left[\frac{\partial(\rho v_x v_x)}{\partial x} + \frac{\partial(\rho v_y v_x)}{\partial y} + \frac{\partial(\rho v_z v_x)}{\partial z}\right] - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

- For constant viscosity fluid, Newton's law applies: $\tau_{yx} = -\mu \frac{dv_x}{dy}$

➔ *Equation of motion in rectangular coordinates:*

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

- **Similar equations are obtained for the y and z components.**

EXAMPLE 3.8-3. *Laminar Flow in a Circular Tub*

2. Equation of motion in cylindrical coordinates. These equations are as follows for Newtonian fluids for constant ρ and μ for the r, θ, and z components, respectively.

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r \qquad (3.7\text{-}40)$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rv_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta \qquad (3.7\text{-}41)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad (3.7\text{-}42)$$

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EXAMPLE 3.8-3. *Laminar Flow in a Circular Tube*

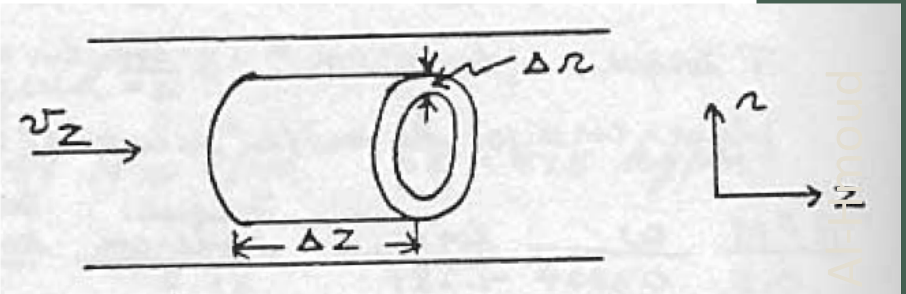
$$\frac{1}{\mu}\frac{dp}{dz} = \text{const} = \frac{d^2 v_z}{dr^2} + \frac{1}{r}\frac{dv_z}{dr} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

P3.6-1. *Equation of Continuity in a Cylinder.*

Fluid having a constant density ρ is flowing in the z direction through a circular pipe with axial symmetry. The radial direction is designated by r.

(a) Using a cylindrical shell balance with dimensions dr and dz, derive the equation of continuity for this system.

(b) Use the equation of continuity in cylindrical coordinates to derive the equation.



$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

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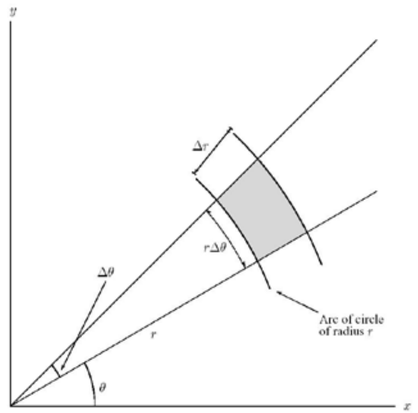
P3.8-1. *Average Velocity in a Circular Tube.*

Using Eq. (3.8-17) for the velocity in a circular tube as a function of radius *r*,

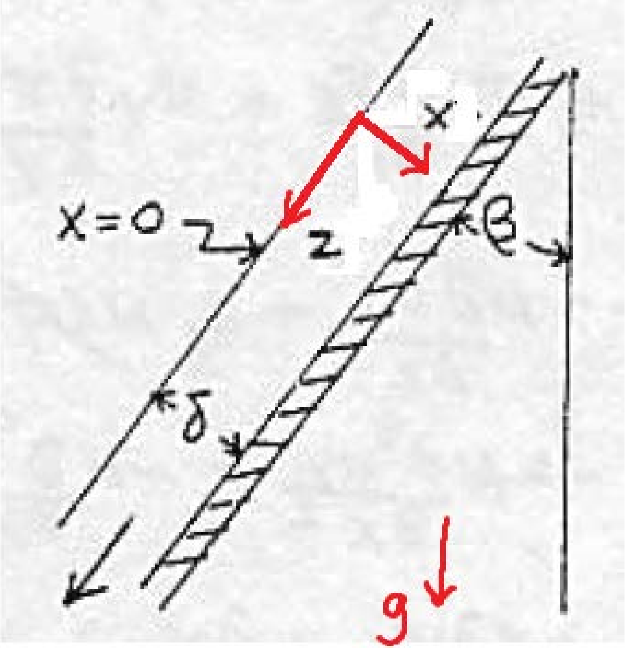
$$v_z = \frac{1}{4\mu}\frac{dp}{dz}(r^2 - r_0^2)$$

derive Eq. (3.8-19) for the average velocity.

$$v_{z\,av} = -\frac{r_0^2}{8\mu}\frac{dp}{dz}$$



P3.8-4. *Velocity Profile in Falling Film and Differential Momentum Balance.*



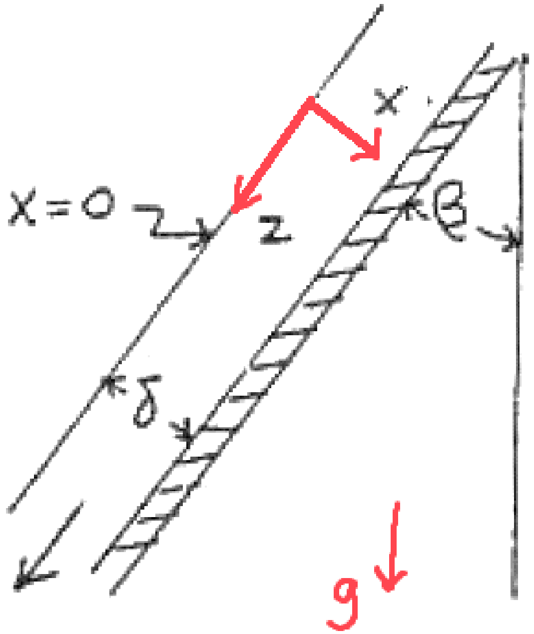
Newtonian liquid is flowing as a falling film on an inclined flat surface. The surface makes an angle of θ with the vertical. Assume that in this case the section being considered is sufficiently far from both ends that there are no end effects on the velocity profile. The thickness of the film is δ. The apparatus is similar to Fig. 2.9-3 but is not vertical. Do as follows.

(a) Derive the equation for the velocity profile of v_z as a function of x in this film using the differential momentum balance equation.

(b) What are the maximum velocity and the average velocity?

(c) What is the equation for the momentum flux distribution of τ_{xz}? [Hint: Can Eq. (3.7-19) be used here?]

P3.8-4. *Velocity Profile in Falling Film and Differential Momentum Balance.*



(a) Eq. (3.7-38) for z component

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}}_0 + v_x \cancel{\frac{\partial v_z}{\partial x}}_0 + v_y \cancel{\frac{\partial v_z}{\partial y}}_0 + v_z \cancel{\frac{\partial v_z}{\partial z}}_0\right)$$

$$= \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \cancel{\frac{\partial^2 v_z}{\partial y^2}}_0 + \cancel{\frac{\partial^2 v_z}{\partial z^2}}_0\right) - \cancel{\frac{\partial p}{\partial z}}_0 + \rho g_z$$

(1) $\mu \frac{\partial^2 v_z}{\partial x^2} = -\rho g_z$

But $g_z = g \cos\beta$

(2) $\mu \frac{\partial^2 v_z}{\partial x^2} = -\rho g \cos\beta$

Integrating (2),

(3) $\mu \frac{\partial v_z}{\partial x} = -\rho g \cos\beta\, x + C_1$ ← constant

B.C. $\frac{\partial v_z}{\partial x} = 0$, at $x = 0$

Hence, $C_1 = 0$

(4) $\mu \frac{\partial v_z}{\partial x} = -\rho g \cos\beta\, x + 0$

B.C. $v_z = 0$, at $x = \delta$

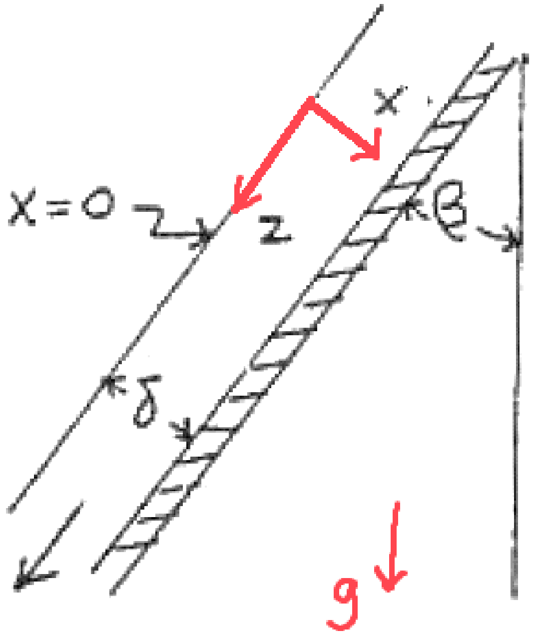
(5) $\mu v_z = -\rho g \cos\beta \frac{x^2}{2} + C_2 = 0$

Hence, $C_2 = \rho g \cos\beta \frac{\delta^2}{2}$

(6) $\mu v_z = -\rho g \cos\beta \frac{x^2}{2} + \rho g \cos\beta \frac{\delta^2}{2}$

Solving for v_z, $\boxed{v_z = \frac{\rho g \delta^2}{2\mu} \cos\beta \left(1 - \frac{x^2}{\delta^2}\right)}$

P3.8-4. *Velocity Profile in Falling Film and Differential Momentum Balance.*



(b) Maximum velocity at $x=0$

(7) $$\boxed{v_{z\,max} = \frac{\rho g \delta^2}{2\mu}\cos\beta}$$

Using Eq. (2.9-26) where W is distance in y direction

(8) $$v_{z\,av} = \frac{1}{A}\iint_A v_z\, dA = \frac{1}{W\delta}\int_0^W\int_0^\delta v_z\, dx\, dy = \frac{W}{W\delta}\int_0^\delta v_z\, dx$$

Substituting (6) into (8) and integrating, $\boxed{v_{z\,av} = \frac{\rho g \delta^2}{3\mu}\cos\beta}$

(c) Eq. (3.7-19) $$\tau_{xz} = -\mu\left(\frac{\partial v_z}{\partial x} + \underbrace{\cancel{\frac{\partial v_x}{\partial z}}}_{0}\right)$$

(9) $$\tau_{xz} = -\mu\frac{\partial v_z}{\partial x}$$

(10) Subst. (6) into (9)

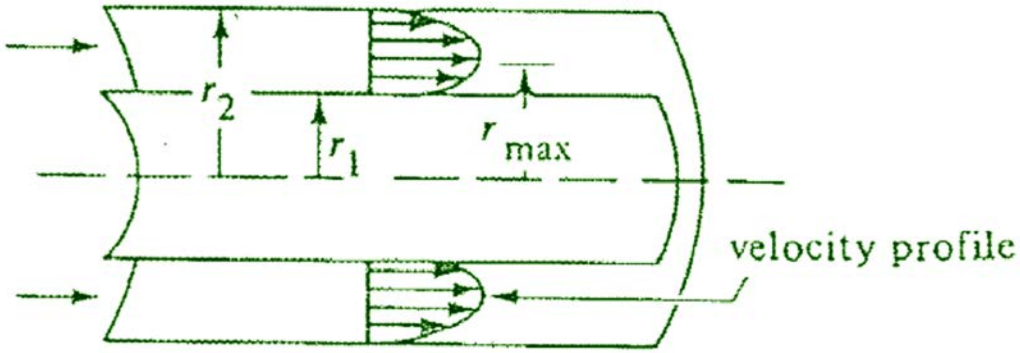
$$\tau_{xz} = -\mu\left(-\frac{\rho g x}{\mu}\cos\beta\right)$$

$$\boxed{\tau_{xz} = \rho g x \cos\beta}$$

P3.8-2. *Laminar Flow in a Cylindrical Annulus.*

Derive all the equations given in Example 3.8-4 showing all the steps. Also, derive the equation for the average velocity $v_{z,av}$. Finally, integrate to obtain the pressure drop from $z = 0$ for $p = p_0$ to $z = L$ for $p = p_L$.

Steady-state laminar flow inside the annulus between Two concentric horizontal pipes



$$\frac{r\,dv_z}{dr} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\left(\frac{r^2}{2} - \frac{r_{max}^2}{2}\right)$$

$$v_z = \left(\frac{1}{2\mu}\frac{dp}{dz}\right)\left(\frac{r^2}{2} - \frac{r_1^2}{2} - r_{max}^2 \ln\frac{r}{r_1}\right)$$

$$v_z = \left(\frac{1}{2\mu}\frac{dp}{dz}\right)\left(\frac{r^2}{2} - \frac{r_2^2}{2} - r^2_{max} \ln\frac{r}{r_2}\right)$$

$$r_{max} = \sqrt{\frac{1}{\ln(r_2/r_1)}(r_2^2 - r_1^2)/2}$$

$$v_{z\,av} = -\frac{1}{8\mu}\frac{dp}{dz}\left[r_2^2 + r_1^2 - \frac{r_2^2 - r_1^2}{\ln(r_2/r_1)}\right],$$

$$v_{z\,av} = \frac{p_0 - p_L}{8\mu L}\left[r_2^2 + r_1^2 - \frac{r_2^2 - r_1^2}{\ln(r_2/r_1)}\right]$$

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2. *Equation of motion in cylindrical coordinates.* These equations are as follows for Newtonian fluids for constant ρ and μ for the r, θ, and z components, respectively.

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r \qquad (3.7\text{-}40)$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta \qquad (3.7\text{-}41)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \qquad (3.7\text{-}42)$$

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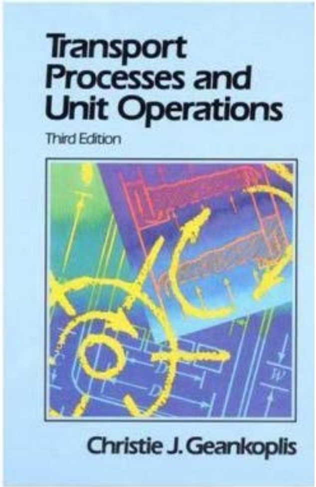
Boundary-Layer Flow and Turbulence

Section 3.10



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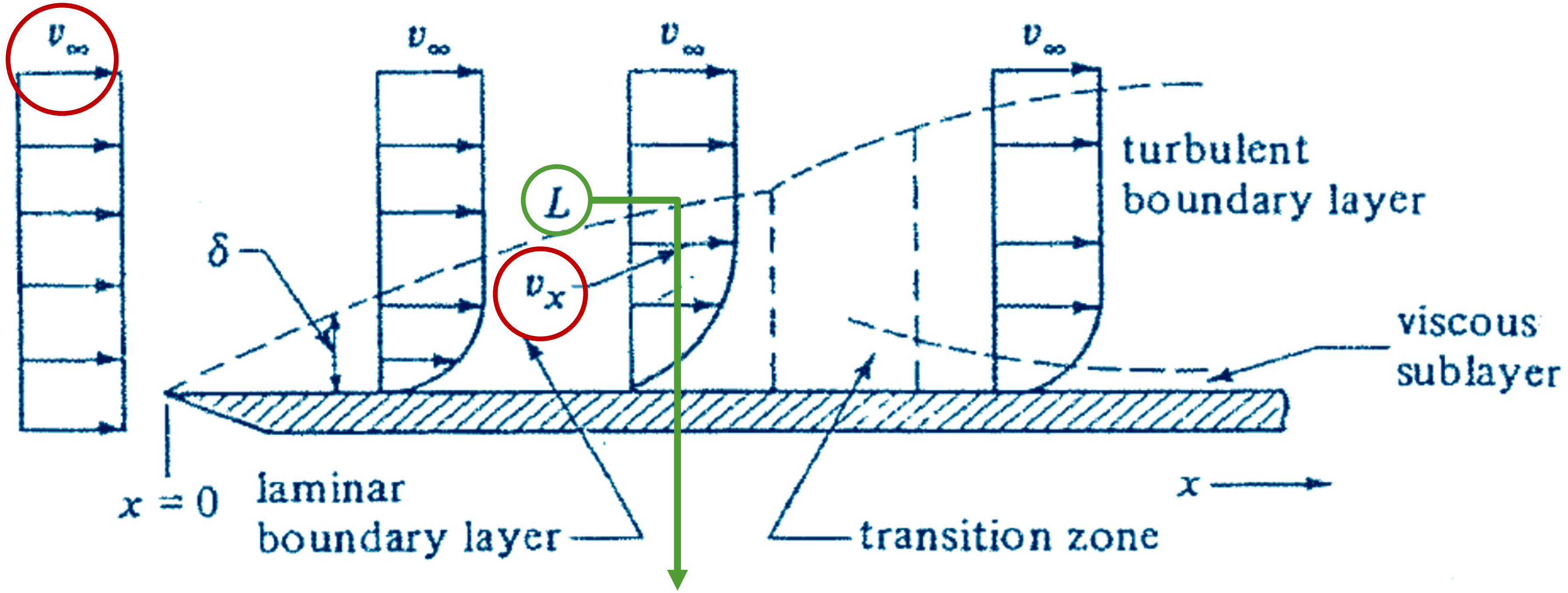
Boundary-Layer Theory

- **Boundary Layer:** the region close to the solid surface.
- In the boundary-layer region, the fluid motion is greatly affected by the solid surface.
- In the bulk of the fluid away from the boundary layer the flow can often be adequately described by the theory of ideal fluids with zero viscosity.
- However, in the thin boundary layer, viscosity is important.
- Since the region is thin, simplified solutions can be obtained for the boundary-layer region.
- Prandtl originally suggested this division of the problem into two parts, which has been used extensively in fluid dynamics.

Boundary Layer for Flow Past Flat Plate

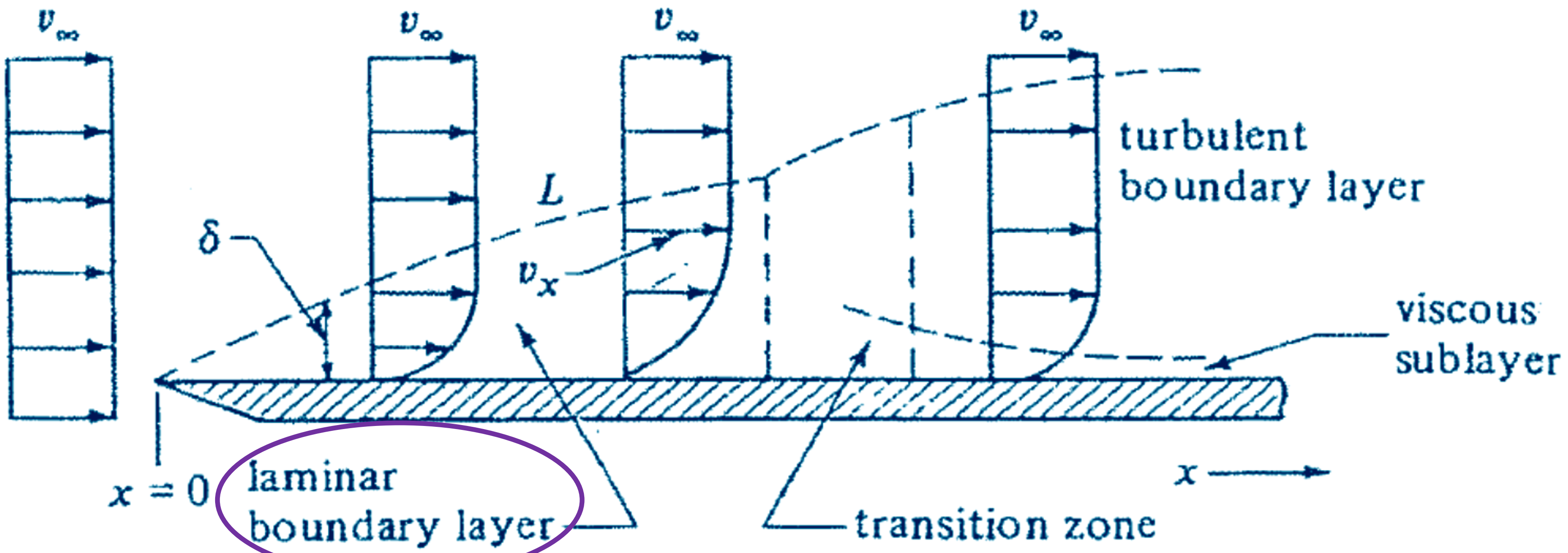
- Boundary-layer formation in the steady-state flow of a fluid past a flat plate.



At the points connected by the dashed line, L, the velocity is 99% of the bulk velocity v_∞

Boundary Layer for Flow Past Flat Plate

- Boundary-layer formation in the steady-state flow of a fluid past a flat plate.

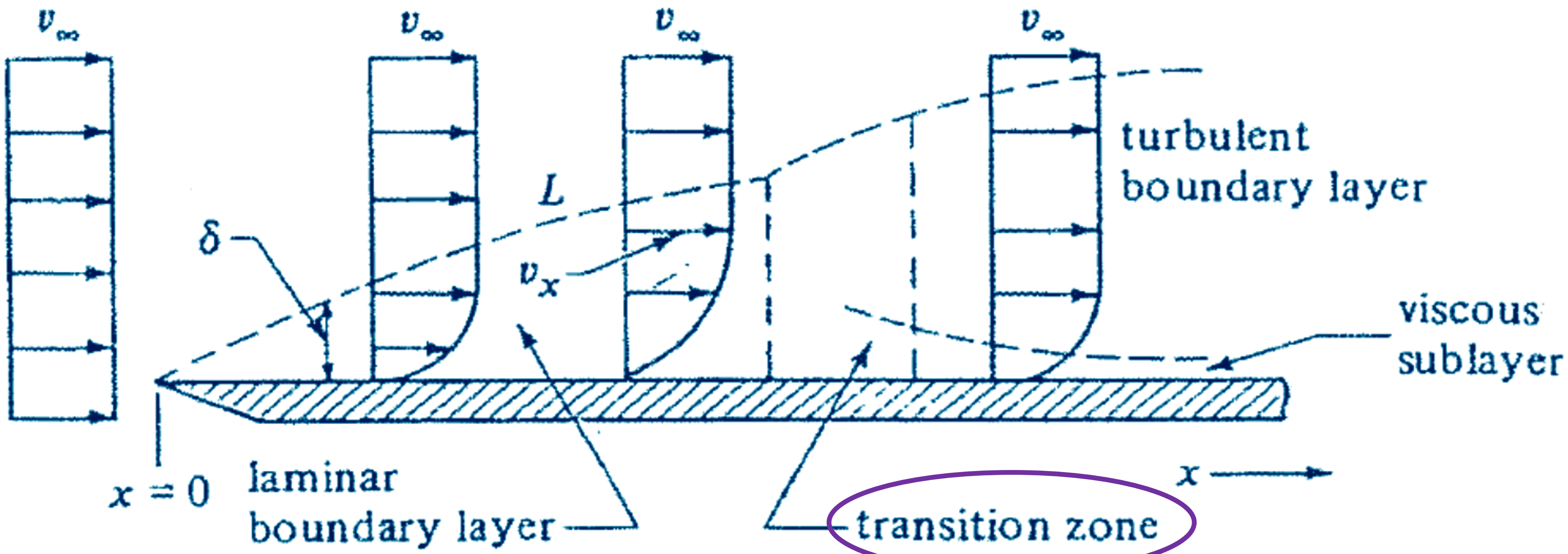


- When $Re_x = \frac{x v_\infty \rho}{\mu} < 5 \times 10^5$, the flow is laminar

the thickness δ of the boundary layer increases with the $\sqrt{x}$ as we move in the x direction

Boundary Layer for Flow Past Flat Plate

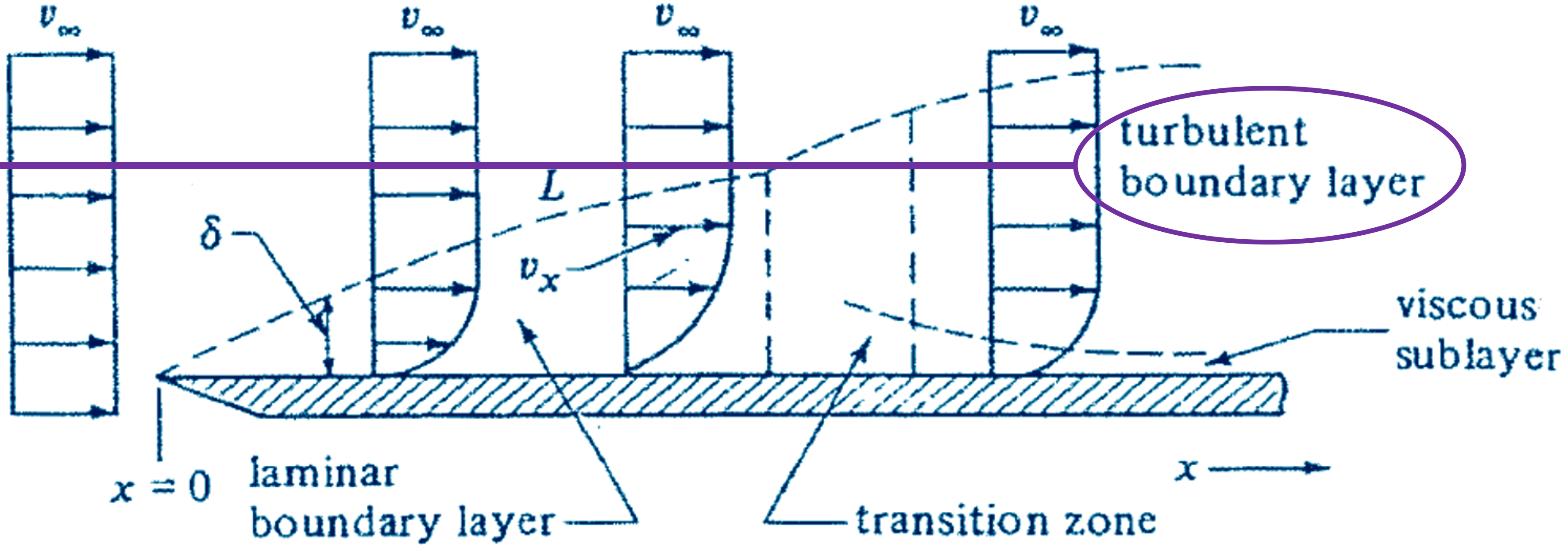
- Boundary-layer formation in the steady-state flow of a fluid past a flat plate.



- When $5 \times 10^5 < Re_x < 3 \times 10^6$, the flow is in the transition zone

Boundary Layer for Flow Past Flat Plate

- Boundary-layer formation in the steady-state flow of a fluid past a flat plate.



- When $Re_x > 3 \times 10^6$, the flow is turbulent

When the boundary layer is turbulent, a thin viscous sublayer persists next to the plate. The drag caused by the viscous shear in the boundary layers is called *skin friction:* the only drag present for flow past a flat plate.

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Laminar Flow and Boundary-Layer Theory

- **Boundary-layer equations:** When laminar flow is occurring in a boundary layer, certain terms in the Navier-Stokes equations become negligible and can be neglected.
- Thickness of boundary layer δ: the distance away from the surface where the velocity reaches 99% of the free stream velocity.
- The concept of a relatively thin boundary layer leads to some important simplifications of the Navier-Stokes equations.
- For two-dimensional laminar flow in the x and y directions of a fluid having a constant density, for flow at steady state, when the body forces g_x and g_y are neglected:

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \frac{\mu}{\rho}\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} = \frac{\mu}{\rho}\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

Laminar Flow and Boundary-Layer Theory

- The **continuity equation** for two-dimensional flow becomes

$$\boxed{\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0}$$

negligible in comparison with the other terms

Navier-Stokes equations

$$\begin{cases} v_x\dfrac{\partial v_x}{\partial x}+v_y\dfrac{\partial v_x}{\partial y}=\dfrac{\mu}{\rho}\left(\dfrac{\partial^2 v_x}{\partial x^2}+\dfrac{\partial^2 v_x}{\partial y^2}\right)-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x} \\[2ex] v_x\dfrac{\partial v_y}{\partial x}+v_y\dfrac{\partial v_y}{\partial y}=\dfrac{\mu}{\rho}\left(\dfrac{\partial^2 v_y}{\partial x^2}+\dfrac{\partial^2 v_y}{\partial y^2}\right)-\dfrac{1}{\rho}\dfrac{\partial p}{\partial y} \end{cases}$$

- All the terms containing v_y and its derivatives are small.

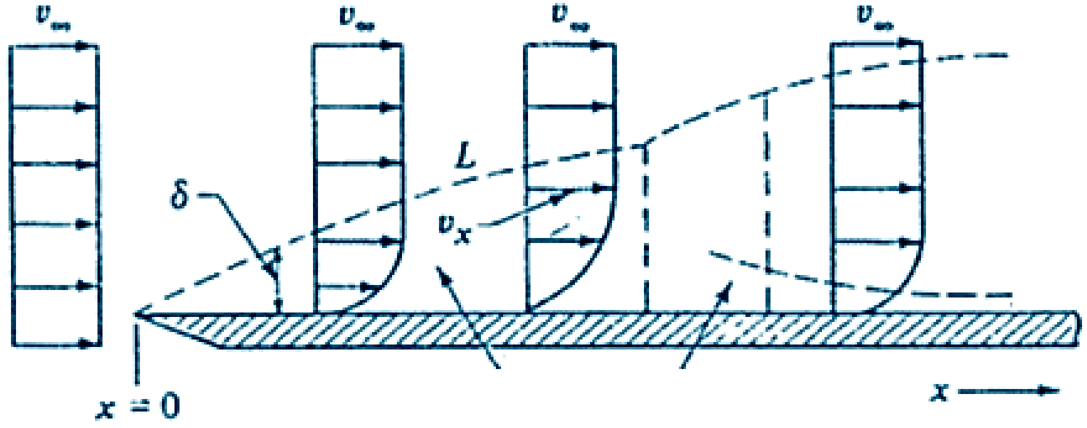
$$\Rightarrow \boxed{v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 v_x}{\partial y^2}-\frac{1}{\rho}\frac{\partial p}{\partial x}}$$

Solution for Laminar Boundary Layer on a Flat Plate

- Since v_∞ is constant, dp/dx is zero.
- The final boundary-layer equations reduce to

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 v_x}{\partial y^2}$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

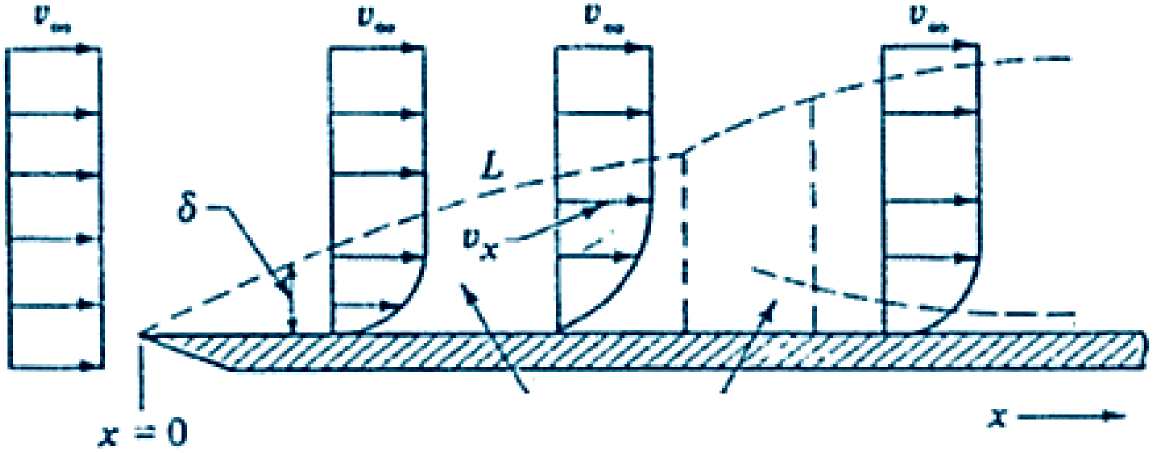


- **Boundary condition:**
  - $v_x = v_y = 0$ at $y = 0$,and $v_x = v_\infty$ at $y = \infty$
- Solution of this problem for laminar flow over a flat plate giving v_x and v_y as a function of x and y was first obtained by **Blasius** and later elaborated by **Howarth**.

Solution for Laminar Boundary Layer on a Flat Plate

- **Blasius** reduced the two equations to a single ordinary differential equation which is nonlinear.
- The equation could not be solved to give a closed form but a series solution was obtained.
- The results of Blasius work are as follows:
  - Boundary-layer thickness δ is given approximately by $\delta = \frac{5.0x}{\sqrt{Re_x}} = 5.0\sqrt{\frac{\mu x}{\rho v_\infty}}$
  - Hence δ varies as $\sqrt{x}$
  - Skin friction τ_0 is calculated from the shear stress at the surface at $y = 0$ for any x:

$$\tau_0 = 0.332\mu v_\infty\sqrt{\frac{\rho v_\infty}{\mu x}}$$



Solution for Laminar Boundary Layer on a Flat Plate

- Total drag for a plate of length L and width b is given by:

$$F_D = b\int_0^L \tau_0 dx$$

$$F_D = 0.664b\sqrt{\mu\rho v_\infty^3 L} = C_D \frac{v_\infty^2}{2}\rho A$$

where $C_D = 1.328\sqrt{\frac{\mu}{L\rho v_\infty}} = \frac{1.328}{\sqrt{Re_L}}$ (similar to Fanning friction factor f for pipes).

- This C_D equation applies only to the laminar boundary layer for $Re_L < 5 \times 10^5$.
- The results are valid only for positions where x is sufficiently far from the leading edge so that x or L is much greater than δ.
- Experimental results on the drag coefficient to a flat plate confirm the validity of the above equation.
- Boundary-layer flow past many other shapes has been successfully analyzed using similar methods.

P3.10-1

Laminar Boundary Layer on Flat Plate. Water at 20°C is flowing past a flat plate at 0.914 m/s. The plate is 0.305 m wide.

(a) Calculate the Reynolds number 0.305 m from the leading edge to determine if the flow is laminar.

(b) Calculate the boundary-layer thickness at $x = 0.152$ and $x = 0.305$ m from the leading edge.

(c) Calculate the total drag on the 0.305-m-long plate.

P3.10-2

Air Flow Past a Plate. Air at 294.3 K and 101.3 kPa is flowing past a flat plate at 6.1 m/s. Calculate the thickness of the boundary layer at a distance of 0.3 m from the leading edge and the total drag for a 0.3-m-wide plate.

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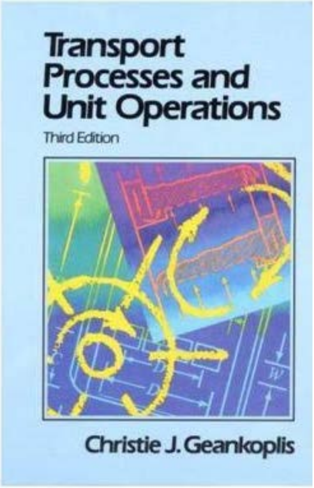
Boundary-Layer Flow and Turbulence in Heat Transfer

Section 5.7A



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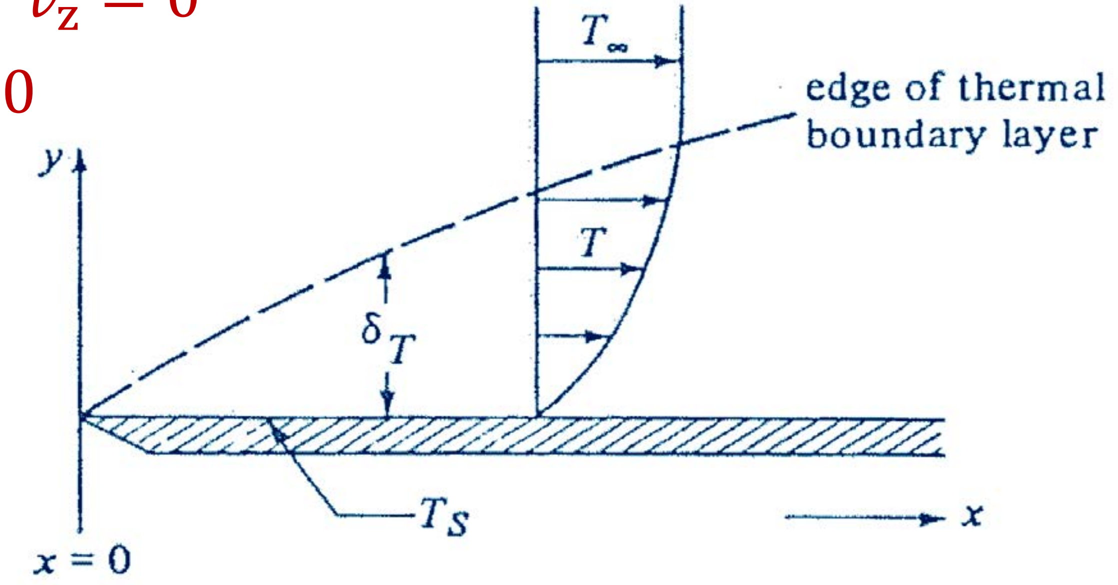
Laminar Flow and Boundary-Layer Theory in Heat Transfer

- Laminar Flow of Fluid past a Flat Plate and Thermal Boundary Layer
- T_∞ = temperature of fluid approaching the plate
- T_s = temperature of the plate at the surface.
- Starting with the differential energy balance:

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

- The flow is in x and y directions $\rightarrow$ $v_z = 0$
- The flow is at steady state $\rightarrow$ $\frac{\partial T}{\partial t} = 0$
- Conduction is neglected in the $\boldsymbol{x}$ and z directions $\rightarrow$ $\frac{\partial^2 T}{\partial x^2} = \frac{\partial^2 T}{\partial z^2} = 0$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2}$$



Laminar Flow and Boundary-Layer Theory in Heat Transfer

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \frac{\mathrm{k}}{\rho c_p} \frac{\partial^2 T}{\partial y^2}$$

- The simplified momentum balance equation used in the velocity boundary-layer derivation

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 v_x}{\partial y^2}$$

- The continuity equation used previously is

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

Laminar Flow and Boundary-Layer Theory in Heat Transfer

- **Boundary Conditions used by Blasius for solving the case of laminar boundary-layer flow:**

$$\frac{v_x}{v_\infty} = \frac{v_y}{v_\infty} = 0 \quad at \quad y = 0, \qquad \frac{v_x}{v_\infty} = 1 \quad at \quad y = \infty$$

$$\frac{v_x}{v_\infty} = 1 \qquad at \quad x = 0$$

- **Blasius solution can be applied similarly,**

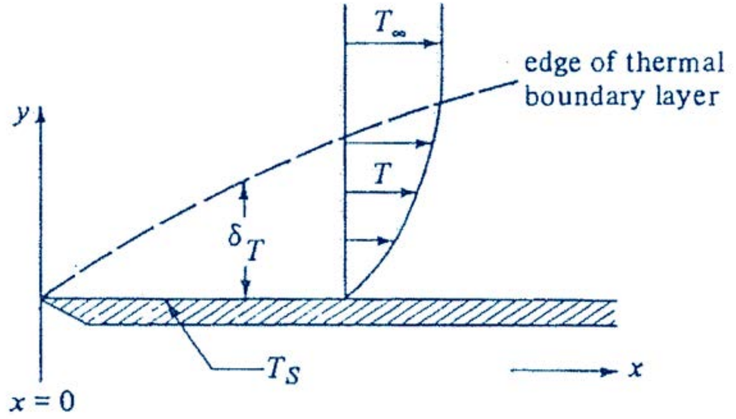
if $\frac{\mathrm{k}}{\rho c_p} = —$

Laminar Flow and Boundary-Layer Theory in Heat Transfer

- **The transfer of momentum and heat are directly analogous and the boundary-layer thickness δ for the velocity profile (hydrodynamic boundary layer) and the thermal boundary-layer thickness δ_T are equal.**

$$\left(\frac{\partial v_x}{\partial y}\right)_{y=0} = 0.332\frac{v_\infty}{x}\sqrt{Re_x}, \qquad Re_x = \frac{x v_\infty \rho}{\mu}$$

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = (T_\infty - T_s)\left(\frac{0.332}{x}\sqrt{Re_x}\right)$$



- **The convective equation can be related to the Fourier equation by the following:**

$$\frac{q_y}{A} = h_x(T_\infty - T_s) = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$\boxed{\frac{h_x\, x}{k} = Nu_x = 0.332\sqrt{Re_x}}$$

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Laminar Flow and Boundary-Layer Theory in Heat Transfer

- Pohlhausen showed that the relation between the hydrodynamic and thermal boundary layers for fluids with $\mathrm{P}r > 0.6$ is approximately:

$$\frac{\delta}{\delta_T} = Pr^{1/3}$$

- As a result, the equation for the local heat-transfer coefficient is

$$\frac{h_x\, x}{k} = Nu_x = 0.332 {Re_x}^{1/2} Pr^{1/3}$$

- The equation for the mean heat-transfer coefficient h from $x = 0$ to $x = L$ is for a plate of width b and area bL,

$$h = \frac{b}{A}\int_0^L h_x\, dx = 0.664\frac{k}{L}{Re_L}^{1/2} Pr^{1/3}$$

$$\frac{hL}{k} = 0.664 {Re_L}^{1/2} Pr^{1/3}$$

  - This laminar boundary layer on smooth plates holds for Re < $5\text{x}10^5$.
  - Fluid properties are evaluated at film temperature $T_f = (T_s + T_\infty)/2$.

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P5.7-1. Thermal and Hydrodynamic Boundary Layer Thicknesses

- Air at 294.3 K and 101.3 kPa with a free stream velocity of 12.2 $\mathrm{m/s}$ is flowing parallel to a smooth flat plate held at a surface temperature of 383 K. Do the following.

(a) At the critical $Re_L = 5 \times 10^5$, calculate the critical length $x = L$ of the plate, the thickness δ of the hydrodynamic boundary layer, and the thickness δ_T of the thermal boundary layer. Note that the Prandtl number is not 1.0.

(b) Calculate the average heat-transfer coefficient over the plate covered by the laminar boundary layer.

P5.7-2. Boundary Layer Thicknesses and Heat Transfer

- Air at 37.8 °C and 1 atm abs flows at a velocity of 3.05 m/s parallel to a flat plate held at 93.3 °C . The plate is 1 m wide. Calculate the following at a position 0.61 m from the leading edge.

(a) The thermal boundary layer thickness δ_T, and the hydrodynamic boundary layer thickness δ.

(b) Total heat-transfer from the plate.