madarju.com

Summary

Transport Phenomena 2

Dr. Zayed Al-Hamamre

By. Maisaa Alsaoud

Madar
ask ;; believe & recieve

Lec "1"

$$\Rightarrow \underset{\text{diffusion}}{J^*_{Az}} = -\overset{\frac{Kmol(A+B)}{m^3}}{C_T}\ \overset{(m^2/s)}{D_{AB}}\ \frac{dx_A}{dz_{om}} = -D_{AB}\frac{dC_A}{dz}$$

$\bar{J}/A \Rightarrow$ molar flux $(K\text{-mol }A/(m^2\text{-}s))$

Const. $T \Rightarrow$ isothermal

Const. $P \Rightarrow$ isobaric

for gases $\Rightarrow$ $C_T = \frac{n}{V} = \frac{P}{RT}$

$\Rightarrow$ Equimolar counter diffusion in gases

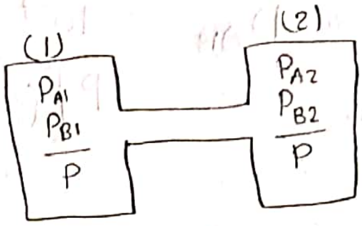
P_T , C_T $\Rightarrow$ Constant

steady state

$$J^*_{AZ} = -J^*_{BZ} \qquad (D_{AB} = D_{BA})$$

مقدار الزيادة في A بقابلة نقصان في B $\Rightarrow$ $dC_A = -dC_B$

* أنا بعرض اتجاه بالبداية إذا طلعت الاشارة موجبة ـــ الاتجاه صح
إذا السالبة ،، العكس



$R = 8314.3 \ \frac{m^3 \cdot Pa}{Kg\text{-mol}\cdot K}$

$= 82.057 \ \frac{Cm^3 \cdot atm}{g\text{-mol}\cdot K}$

التركيز على
الوحدات
مهم جداً

Slide 40

$\Rightarrow$ Convection mass transfer

$$\overset{\frac{K\text{-mol}}{m^2 \cdot s}}{\ } \leftarrow N_{AZ} = \underset{m/s}{K_c}\ \Delta C_A \rightarrow \frac{K\text{-mol}}{m^3}$$

$N_{AZ} = \frac{\bar{N}}{A}$

$\Rightarrow$ General flux equation

* Velocity $\Rightarrow$ $V_{A\text{-diffusion}} = \dfrac{J^*_A\ (mol/(m^2 \cdot s))}{C_A\ (mol/m^3)}$ $\qquad J^* = V\,C_A$

avg molar velocity $\rightarrow$ $V_{bulk} = \dfrac{\sum_{i=1}^{n} C_i V_i}{C_T}$ $\quad \Rightarrow \quad V_i = V_{bulk} + V_{A\text{-diffusion}}$

For binary mix $\Rightarrow$ $V_{bulk} = \dfrac{C_A V_A + C_B V_B}{C_T}$

$$N_A = C_A V_A$$
$$N_B = C_B V_B$$

$$V_{bulk} = \frac{N_A + N_B}{C_T}$$

$\rightarrow$ general flux eq. $\Rightarrow$

$$N_A = \underbrace{J^*_A}_{-D_{AB}\frac{dC_A}{dz}} + \underbrace{C_A \frac{(N_A + N_B)}{C_T}}_{x_A}$$

$C = \frac{P}{RT}$

$$N_A = -\frac{D_{AB}}{RT}\frac{dP_A}{dz} + \frac{P_A}{P_T}(N_A + N_B)$$

if the mixture is diluted in A

$$\Rightarrow N_A = J^*_A = -D_{AB}\frac{dC_A}{dz}$$

at Low $P \Rightarrow D_{AB} \uparrow$ with temp

$D_{AB} \downarrow$ with pressure

$\Rightarrow$ Diffusivity (L^2/t)

gases $\rightarrow$ $5\times10^{-6} \rightarrow 1\times10^{-5}$ m^2/s

Liquids $\rightarrow$ $10^{-6} \rightarrow 10^{-9}$ m^2/s

Solids $\rightarrow$ $5\times10^{-4} \rightarrow 1\times10^{-10}$ m^2/s

* For gases $\rightarrow$ binary mixture

$$D_{AB} = \frac{10^{-7} \, T^{1.75} \left(\frac{1}{M_A}+\frac{1}{M_B}\right)^{1/2}}{P\left[\left(\Sigma V_A\right)^{1/3}+\left(\Sigma V_B\right)^{1/3}\right]^2}$$

10^{-3}, T in K

D_{AB}: m^2/s, cm^2/s

ΣV_A: From Table in slide 61

$$\Rightarrow D_{AB}(T_2, P_2) = D_{AB}(T_1, P_1)\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)^{1.75}$$

for more than 2 comp.

$$\rightarrow D_{1-mix} = \frac{1}{\frac{x_2'}{D_{1-2}} + \frac{x_3'}{D_{1-3}} + \cdots + \frac{x_n'}{D_{1-n}}}$$

$$x_2' = \frac{x_2}{x_2 + x_3 + \cdots + x_n} \quad \text{without comp. 1}$$

* For Liquid $\rightarrow$

large molecule $\Rightarrow$ $$D_{AB} = \frac{9.96\times10^{-12}\, T}{\mu \, V_A^{1/3}}$$

$V_A \rightarrow$ حجم مولي

small molecule $\Rightarrow$ $$D_{AB} = \frac{7.4\times10^{-8}\,(\Phi M_B)^{1/2}\, T}{\mu_B \, V_A^{0.6}}$$

Lec "2"

* The eq. of Continuity for component A :-

in rectan. for comp. A
$$\frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Ay}}{\partial y} + \frac{\partial N_{Az}}{\partial z} + \frac{\partial C_A}{\partial t} - r_A = 0$$

→ if → no bulk flow :- $N_A = J_A^*$

no chemical rxn :- $r_A = 0$

$$\Rightarrow \frac{\partial C_A}{\partial t} = D_{AB}\left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2}\right)$$ Fick's Second Law

⇒ @ steady state ⇒ $\frac{\partial C_A}{\partial t} = 0$

if $\frac{\partial N_A}{\partial x} = 0 \Rightarrow (N_A = \text{constant}$ @ bulk flow) velocity profile (Linear)

For Cylindrical ⇒ $$\frac{\partial C_A}{\partial t} = \left[\frac{1}{r}\frac{\partial}{\partial r}(r N_{A,r}) + \frac{1}{r}\frac{\partial N_{A,\theta}}{\partial \theta} + \frac{\partial N_{A,z}}{\partial z}\right] = R_A$$

with no bulk flow (no fluid motion) in r dirихtion and no chem. rxn

$$\rightarrow \frac{\partial C_A}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(r N_{A,r})$$ $J_A^* = N_{A,r} = -D_{AB}\frac{dC_A}{dr}$

$$\frac{\partial C_A}{\partial t} = \frac{D_{AB}}{r}\frac{\partial}{\partial r}\left(r \frac{\partial C_A}{\partial r}\right)$$

For spherical ⇒ $$\frac{\partial C_A}{\partial t} + \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 N_{A,r}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(N_{A,\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial N_{A,\phi}}{\partial \phi}\right] = R_A$$

$$\rightarrow \frac{\partial C_A}{\partial t} = \frac{1}{r^2}\frac{\partial (r^2 N_{A,r})}{\partial r}$$

$$= \frac{D_{AB}}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial C_A}{\partial r}\right)$$

ex
at symmetry
$z=0 \rightarrow \frac{\partial C_A}{\partial z} = 0$

In interface
$N_A = K_c \, \Delta C_A$

Lec "3"

* rectangular → $\partial N_{Az}/\partial x = 0 \Rightarrow N_{Az} = \text{Const.}$
  cylinrical → $\partial (r N_{Az})/\partial r = 0 \Rightarrow r N_{Az} = \text{Const.}$
  spherical → $\partial (r^2 N_{Ar})/\partial r = 0 \Rightarrow r^2 N_{Ar} = \text{Const.}$

* general flux in binary mixture (A and B)

$$\Rightarrow N_A = -C_T D_{AB} \frac{dX_A}{dz} + X_A (N_A + N_B)$$

no bulk flow $\Rightarrow N_A = J_A^* = -C_T D_{AB} \frac{dX_A}{dz}$

* 1-D , no chem. rxn , steady state $\Rightarrow \frac{\partial C_A}{\partial t} = 0 \Rightarrow \frac{\partial N_{A,z}}{\partial z} = 0 \Rightarrow N_{A,z} = \text{Cons}$

→ يعني مافي تغير لل N_A طول ال diff. path (z)

* Diffusion of A through stagnant , non-diffusing B

→ B dose not dissolve in A

point 1 in the surfase of A
" 2 " air

→ $C_{A,2} = 0$, $P_{A,2} = 0$

→ $N_B = 0$

in ideal gas volume% = mol%

$$\Rightarrow N_A = -C_T D_{AB} \frac{dX_A}{dz} + X_A (N_A + \cancelto{0}{N_B})$$

Δz ثابتة يعني بيأ لهيك $N_A = \text{Const.}$ نقدر أجيب ⇒ قيمة X_A عند أي $\underline{z}$

$$N_A = \frac{-C_T D_{AB}}{1 - X_A} \frac{dX_A}{dz} \qquad \begin{Bmatrix} z = z_1 & X_A = X_{A1} \\ z = z_2 & X_A = X_{A2} \end{Bmatrix}$$

$$\rightarrow N_A = \frac{C_T D_{AB}}{z_2 - z_1} \ln\left[\frac{1 - X_{A2}}{1 - X_{A1}}\right] = \frac{D_{AB}}{z_2 - z_1} \ln\left[\frac{C_T - C_{A2}}{C_T - C_{A1}}\right]$$

$$\rightarrow N_A = \frac{C_T D_{AB}}{z_2 - z_1} \frac{(X_{A1} - X_{A2})}{X_{B,Lm}}$$

$$X_{B,Lm} = \frac{X_{A1} - X_{A2}}{\ln\left(\frac{1 - X_{A2}}{1 - X_{A1}}\right)}$$

$$X_{B,Lm} = \frac{X_{B2} - X_{B1}}{\ln(X_{B2}/X_{B1})}$$

$\Rightarrow$ diluted [A] $\Rightarrow X_{B,Lm} = 1$

$$\rightarrow N_A = \frac{P D_{AB}}{RT(z_2 - z_1)} \frac{(P_{A1} - P_{A2})}{P_{B,Lm}}$$

$$P_{B,Lm} = \frac{(P - P_{A2}) - (P - P_{A1})}{\ln\left[\frac{P - P_{A2}}{P - P_{A1}}\right]} = \frac{P_{A1} - P_{A2}}{\ln\left[\frac{P - P_{A2}}{P - P_{A1}}\right]}$$

$$\Rightarrow X_{A_2} = 1 - (1 - X_{A_1}) \exp\left[\frac{N_A (z_2 - z_1)}{C_T D_{AB}}\right]$$

* Equimolar Counter Diffusion

$$N_A = J_A = -D_{AB} \frac{dC_A}{dz} \qquad \left\{ \begin{array}{ll} z = z_1 & C_A = C_{A1} \\ z = z_2 & C_A = C_{A2} \end{array} \right\}$$

$$\Rightarrow N_A = D_{AB} \frac{C_{A1} - C_{A2}}{z_2 - z_1} = \frac{D_{AB}}{RT (z_2 - z_1)} (P_{A1} - P_{A2})$$

$N_A \rightarrow$ constant $\Rightarrow$ $$\frac{C_A - C_{A1}}{C_{A1} - C_{A2}} = \frac{z - z_1}{z_1 - z_2}$$

ال rxn سريع $\Rightarrow$ instaneous
بامتدادي بالعلايات لأنّا ال rxn سريع
تركيز ال O_2 عند نقطة 2 = 0

* Other Relations between N_A and N_B

$$mA \longrightarrow nB$$

$$\frac{N_A}{m} = \frac{-N_B}{n}$$

* The relation between N_A, N_B is given by the latent heats of vap. as:-

$$\lambda_A N_A = -\lambda_B N_B \Rightarrow N_B = -\frac{\lambda_A}{\lambda_B} N_A$$

* Pseudo steady state diffusion

$$\Rightarrow N_A = \frac{C_T D_{AB}}{z} \frac{(X_{A1} - X_{A2})}{X_{B,lm}}$$

$$\boxed{N_A = \frac{e_{A,L}}{M_A} \frac{dz}{dt}}$$

$$\Rightarrow \frac{e_{A,L}}{M_A} \frac{dz}{dt} = \frac{C_T D_{AB}}{z} \frac{(X_{A1} - X_{A2})}{X_{B,Lm}}$$

{integrate}

rect.

$$\Rightarrow t = \frac{e_{A,L} X_{B,Lm} / M_A}{C_T D_{AB} (X_{A1} - X_{A2})} \left[\frac{z_t^2 - z_{t_0}^2}{2}\right]$$

لحساب التغير قبل وبعد التبخر

$z_{t_0} \rightarrow z_t$

$$\Rightarrow D_{AB} = \frac{e_{A,L} X_{B,Lm} (z_t^2 - z_{t_0}^2)}{2t C_T M_A (X_{A1} - X_{A2})}$$

* Diffution into an Infinite standard medium (spherical particle) $(\text{Area})(N_A) = W_A = \text{const.}$

W_A → mol/time

→ $N_B = 0$ (air is stagnant (assume))

$$\Rightarrow \left(N_A = \frac{-C_T D_{AB}}{1 - X_A} \frac{dX_A}{dr} \right) * r^2 \qquad \{ r^2 N_A = r_o^2 N_{Ao} \}$$

$$X_{AS} = \frac{P_A}{P_T}$$

$$r_o^2 N_{Ao} = \frac{-r^2 C_T D_{AB}}{1 - X_A} \frac{dX_A}{dr} \qquad \begin{cases} r = r_o \\ r = \infty \end{cases} \qquad \begin{cases} X_A = X_{AS} \\ X_A = X_{A\infty} \end{cases}$$

→ at surface

→ " Large space (تركيز ثابت)

$$\text{integrate} \Rightarrow \left(N_{Ao} = \frac{C_T D_{AB}}{r_o} \ln \left[\frac{1 - X_{A\infty}}{1 - X_{AS}} \right] \right)$$

$$\rightarrow N_{Ao} = \frac{D_{AB} P}{R T r_o} \frac{(P_{A1} - P_{A2})}{P_{B,Lm}}$$

→ dilute gas phase P_{A1} is small ⇒ $P_{B,Ln} = P$

$$= \frac{2 D_{AB}}{D_1} (C_{A1} - C_{A2})$$

$$\Rightarrow \frac{\text{moles of water diffusing}}{\text{unit time}} = \frac{\text{moles of water leaving the droplet}}{\text{unit time}}$$

$$\underbrace{4\pi r_o^2 N_{Ao}}_{W_{Ao}} = \frac{-dn_A}{dt} = \frac{-d}{dt} \left(\frac{m_A}{M_A} \right) \qquad m = \rho V$$

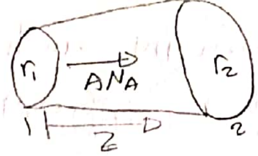
$$4\pi r_o^2 N_{Ao} = \frac{-d}{dt} \left[\frac{4}{3} \pi r_o^3 \frac{\rho_A}{M_A} \right]$$

$$\frac{C_T D_{AB}}{r_o} \ln \left[\frac{1 - X_{A\infty}}{1 - X_{AS}} \right] = -4\pi r_o^2 \frac{\rho_A}{M_A} \frac{dr}{dt} \qquad \{\text{integrate}\} \quad \begin{matrix} t = 0 \\ r_o = r_1 \end{matrix}$$

$$\text{for spherical shape} \Rightarrow t = \frac{\rho_A}{M_A} \frac{1}{2 C_T D_{AB}} \frac{r_1^2}{\ln \left[\frac{1 - X_{A\infty}}{1 - X_{AS}} \right]}$$

$$= \frac{\rho_A R T r_1^2}{2 M_A D_{AB} P} \frac{P_{B,Lm}}{(P_{A1} - P_{A2})}$$

* Diffution through varying cross sectional Area



$$\Rightarrow \frac{\partial (r^2 N_A)}{\partial r} = 0 \quad \Rightarrow r^2 N_A = \text{const.}$$

$$\Rightarrow \boxed{A_1 N_{A1} = A_2 N_{A2}}$$

$r \propto z$

$$r = \left(\frac{r_2 - r_1}{z_2 - z_1}\right) z + r_1$$

$N_B = 0 \Rightarrow$

$$N_A = \frac{-C_T D_{AB}}{1 - X_A} \frac{dX_A}{dz}$$

$$= \frac{-D_{AB}}{RT(1 - P_A/P)} \frac{dP_A}{dz} \qquad \begin{cases} z = z_1 & P_A = P_{A1} \\ z = z_2 & P_A = P_{A2} \end{cases}$$

integrate $\Rightarrow$

$$\bar{N}_A = \frac{\pi D_{AB} P}{RT} \frac{r_1}{z} \left[r_1 - (r_1 - r_2)\left(\frac{z}{L}\right)\right] \ln \frac{P}{P - P_A^{0}}$$

* Diffution in Liquid

مثال ← مكعب سكر في الماء

→ dispesion في كل الاتجاهات

→ diffution في اتجاه واحد

→ diffution ← إذا ذاب كلا لوحده

→ $\underbrace{\text{'' + advection}}_{=\text{ convection}}$ → mixing إذا علة

⇒ diffution in Liq slower than diff in gasses

لأنه المسافة بين الجزيئات بال Liq أقل, والتصادمات عالية فتتعيق حركة الانتشار

→ equimolar counter diff $(N_A = -N_B)$

⇒ all eq. between Liq and gases are same expect :-

$$\Rightarrow C_{AV} = \frac{\frac{\rho_1}{M_1} + \frac{\rho_2}{M_2}}{2} \qquad \cancel{C_T = \frac{P}{RT}} \qquad M_{AV} = X_{A1} M_A + X_{B1} M_B + \cdots$$

$$\rightarrow N_A = \frac{D_{AB}}{z} \left(\frac{\rho}{M}\right)_{avg} (X_{A1} - X_{A2})$$

$$N_A = \frac{D_{AB}}{(z_2 - z_1)\, X_{BM}} \left(\frac{\rho}{M}\right)_{av} (X_{A1} - X_{A2})$$

$$X_{BLm} = \frac{X_{A1} - X_{A2}}{\ln\left(\frac{1 - X_{A2}}{1 - X_{A1}}\right)}$$

Lec "4"

* Diffusion of solid occurs in 2 different ways :-

1) Following Fick's Law — ما بتعتمد على ال solid struc.

→ steady state diffusion through a solid slab (planner surface) → Constant cross sectional Area

$$N_A = -D_{AB}\frac{dC_A}{dz} + \cancel{\frac{C_A}{C_T}(N_A+N_B)} \to 0$$

$$\Rightarrow N_A = J^*_{Az} = -D_{AB}\frac{dC_A}{dz}$$

bulk flow مافي

$$\Rightarrow N_A = D_{AB}\frac{C_{A1}-C_{A2}}{z_2-z_1}$$

→ through a hollow cylindrical surface

$$W_A = -2\pi r L D_A \frac{dC_A}{dr}$$

* N_A مو constant بتتغير بتغير r

بس W_A constant

$$\Rightarrow W_A = N_A A$$

$$\rightarrow W_A = 2\pi L D_A \frac{(C_{A1}-C_{Ao})}{\ln(r_o/r_1)}$$

→ Conc. profile :-

$$\int_{C_{A1}}^{C_A(r)} dC_A = -\frac{W_A}{D_A 2\pi L}\int_{r_1}^{r}\frac{dr}{r}$$

$$C_A(r) = C_{A1} - \frac{(C_{A1}-C_{Ao})}{\ln(r_o/r_1)}\ln\left(\frac{r}{r_1}\right)$$

→ through a spherical Cavity

$$W_A = -4\pi r^2 D_A \frac{dC_A}{dr}$$

$$\rightarrow \cancel{W_A = 4\pi r^2 D_A \frac{(C_{A1} - C_{A0})}{r_0 - r_1}}$$

max mass transfer
→ when the diff. of conc = 0

→ Conc. profile.

$$\int_{C_{A1}}^{C_A(r)} dC_A = -\frac{W_A}{4\pi r^2 D_A} \int_{r_1}^{r} \frac{dr}{r^2}$$

$$C_A(r) = C_{A1} - 2\frac{(C_{A1} - C_{A0})}{r_0 - r_1}\left(\frac{1}{r_1} - \frac{1}{r}\right)$$

2) Dependent on the nature of the solid

→ diffusion of Liquids in Solids

* نفس الحالات السابقة ولكن الفرق

$$\hookrightarrow \left[D_{Aef} = \frac{\varepsilon}{K_t^2} D_{AB}\right] \rightarrow D_{AB} \text{ بدل}$$

$$\rightarrow N_A = \frac{\varepsilon}{K_t^2} D_{AB} \frac{C_{A1} - C_{A2}}{Z_2 - Z_1}$$

→ diffusion of gases in solids

$$* \ \lambda_A = \frac{3.2 \mu_A}{P}\left[\frac{RT}{2\pi M_A}\right]^{0.5}$$

$\lambda \rightarrow m$
$\mu \rightarrow Kg/(m.s)$
$P \rightarrow N/m^2$
$T \rightarrow K$
$R = 8314 \ J/(K\text{-mol}.K)$

$$* \ K_n = \frac{\lambda}{d_{pore}}$$

$\rightarrow Kn < 0.01$ $\qquad d_{pore} > \lambda$

pure molecular diffusion $\hookrightarrow$ follow Fick's Law $\rightarrow D_{AB}$ $\qquad$ A-B

$$\rightarrow N_A = -\frac{D_{AB} P}{RT} \frac{dy_A}{dz} + y_A (N_A + N_B)$$

$\rightarrow Kn > 10$ $\qquad \lambda > d_{pore}$

Pure Knudsen $\hookrightarrow D_{KA} = 4850\, d_{pore} \sqrt{\frac{T}{M_A}}$ $\qquad$ A-wall $\qquad$ conv. ما في

$$\rightarrow N_A = -D_{KA} \frac{dC_A}{dz} = \frac{D_{KA} P}{RT \Delta z} (y_{A1} - y_{A2})$$

$\rightarrow 0.01 < Kn < 10$ $\qquad$ A-B , A-wall

$$\hookrightarrow \frac{1}{D_{Ae}} = \frac{1}{D_{AB}} + \frac{1}{D_{KA}}$$

random

$$D'_{Ae} = \varepsilon^2 D_{Ae}$$

* Relationship between the fluxes @ diffusion of gases in solid.

$\rightarrow$ closed system (equi. molar ---)

$$\rightarrow N_A = -N_B$$

$\rightarrow$ open system

$$\rightarrow N_A \sqrt{M_A} = -N_B \sqrt{M_B}$$

* Relating the conc. and Solubility

$\rightarrow C_A = S P_A / 22.414$ $\qquad$ P_A In atm

$$\rightarrow N_A = D_{AB} \frac{S (P_{A1} - P_{A2})}{22.414 (z_2 - z_1)}$$

STP @ 0°C , 1 atm

* permeability $\rightarrow P_M = D_{AB} S$

Lec "5"

1 → Transient diffusion in a semi-infinite medium

$$\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}$$

* @ $z=0 \rightarrow C_A = C_{AS}$, $t>0$

@ $z=\infty \rightarrow C_A = C_{Ao}$

$$\rightarrow \boxed{\frac{C_{AS} - C_A}{C_{AS} - C_{Ao}} = \text{erf}\left(\underbrace{\frac{z}{2\sqrt{D_{AB}\, t}}}_{\phi}\right)}$$

solution by {ϕ - erf(ϕ)} chart

* Total amount of species (A) transfered with time "t"

$$\rightarrow m_A(t) - m_{Ao} = A \int_0^t N_{Az}\Big|_{z=0} dt \qquad \boxed{N_{Az}\Big|_{z=0} = \sqrt{\frac{D_{AB}}{\pi t}}\,(C_{AS} - C_{Ao})}$$

$$= A \int_0^t \sqrt{\frac{D_{AB}}{\pi t}}\,(C_{AS} - C_{Ao})\, dt$$

$$\boxed{m_A(t) - m_{Ao} = A \sqrt{\frac{4 D_{AB}\, t}{\pi}}\,(C_{AS} - C_{Ao})}$$

* If, $t < \frac{L^2}{16 D_{AB}}$ <u>OR</u> $L \geq z^*$ $\qquad \boxed{z^* = 4\sqrt{D_{AB}\, t}}$

→ we can consider the system is semi-infinite.

* $\text{erf}(\phi) \simeq 1$ @ $\phi = 2 \rightarrow C_A = C_{Ao}$

2 → Transient diffusion in a finite - dimensional medium

(A) Medium Negligible surface Resistance : $m = 0$

$$* \; m = \frac{D_{AB}}{K_c X_1} = \frac{1}{Bi}$$

→ For a slab , $\frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}$

I.C and B.C

→ $t=0$, $C_A = C_{A0}$, $-L \leq z \leq L$

→ $z=0$, $\partial C_A / \partial z = 0$, $t>0$

→ $z = \pm L$, $C_A = C_{AS}$, $t > 0$

$* \; Y = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}}$, $F_{om} = \frac{D_{AB}\, t}{a^2}$ {3 (Y - Fom) chart for slab, cyl, sph}

(B) Un steady state diffution : $m > 0$, Bi → $0.1 < Bi < 40$

$$* \; N_A(L,t) = -D_{AB} \frac{\partial C_A}{\partial x}\Big|_{x=L} = K_c\,(C_{AS} - C_{A\infty})$$

$$* \; Y = \frac{C_{A\infty} - C_A}{C_{A\infty} - C_{A0}}$$

* 3 chart (X , Y)
for slab , cyl , sph
→ بحاجة أعرف 3 من 4
(X, Y , m , n)
المحاور — X, Y
المجروة كاملة — m
خط من المجروة — n

$$* \; X_D = \frac{D_{AB}\, t}{X_1^2}$$

$$* \; m = \frac{D_{AB}}{K_c X_1}$$

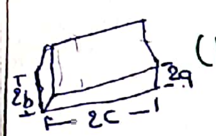
$* \; n = \frac{X}{X_1}$ for slab , $n = \frac{r}{R}$ for cyl , sph

X_1 → one side $(X_1 = L)$
X_1 → both side $(X_1 = L/2)$

→ Average concentration profiles for unsteady state diffusion:- $Bi > 40$

$\overline{C_A} = C_{A\,avg}$

(A) for slab

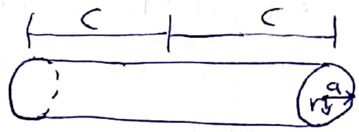


(1) 1 D $\quad \dfrac{C_{AS} - \overline{C_A}}{C_{AS} - C_{A0}} = f\left(\dfrac{D_{AB}t}{a^2}\right) = E_a$

(2) 2 D $\quad \dfrac{C_{AS} - \overline{C_A}}{C_{AS} - C_{A0}} = f\left(\dfrac{D_{AB}t}{a^2}\right) f\left(\dfrac{D_{AB}t}{b^2}\right) = E_a E_b$

(3) 3 D $\quad \dfrac{C_{AS} - \overline{C_A}}{C_{AS} - C_{A0}} = f\left(\dfrac{D_{AB}t}{a^2}\right) f\left(\dfrac{D_{AB}t}{b^2}\right) f\left(\dfrac{D_{AB}t}{c^2}\right) = E_a E_b E_c$

(B) for cylindrical



(1) 1 D $\quad = E_r$

(2) 2 D $\quad = E_r E_c$

(C) for sphere

(1) 1 D $\quad = f\left(\dfrac{D_{AB}t}{a^2}\right) = E_s$

→ 1 Chart for slab, cyl, sph $\left(E_{avg} - D_{AB}t/a^2, b^2, c^2\right)$

→ determine the conc. @

1) mid plane of Flat plate

2) Center line of a long cylinder.

3) center of sphere

⟹ $Y = \dfrac{C_{AS} - C_A}{C_{AS} - C_{Ao}}$

⟹ $m = \dfrac{1}{Bi} = \dfrac{D_{AB}}{K_c X_1}$

⟹ $F_{om} = X = \dfrac{D_{AB}\, t}{x_1^2}$

(4)

Lec "6"

→ mass transfer cofft.

1) in gas

(K_c', K_G', k_y') in equimolar , (K_c, K_G, K_y) in stagnant

$\rightarrow K_c' = K_c\, y_{BM}$

$\rightarrow K_G' = K_G\, y_{BM}$

$\rightarrow K_y' = K_y\, y_{BM}$

$\rightarrow K_G' = K_c'/RT$

$K_y' = K_c'\, C$

$K_y' = K_G'\, P$

$N_A =$

$= K_c' (C_{A1} - C_{A2})$

$= K_G' (P_{A1} - P_{A2})$

$= K_y' (y_{A1} - y_{A2})$

$= K_x' (X_{A1} - X_{A2})$

$= K_L' (C_{A1} - C_{A2})$

2) in Liquid

$\rightarrow K_x' = K_x\, X_{BM}$

$\rightarrow K_L' = K_L\, X_{BM}$

$\rightarrow K_c'\, C = K_L' \frac{\rho}{M}$

→ units ⇒ K_c, K_c', K_L, K_L' → m/s

K_x, K_x', K_y, K_y' → Kmol/(m².s.mol frac.)

K_G, K_G' → Kmol/(m².s.Pa)

$R = 8314 \frac{m^3 . Pa}{Kmol . K}$

dilute

$K_c' = K_c$

→ $Sh = \frac{K_c'\, L}{D_{AB}}$

→ $Re = \frac{D \rho v}{\mu}$

→ $Sc = \frac{\mu}{\rho D_{AB}}$

→ $j_D = \frac{K_c}{v_\infty} (Sc)^{2/3}$

→ $St = \frac{K_c}{v}$

→ $Gr = \frac{D^3 \rho g \Delta\rho}{\mu^2}$

* Mass transfer to :-

1) small partical < 0.6 mm

$$K_L' = \frac{2 D_{AB}}{D_p} + 0.31 (Sc)^{-2/3} \left(\frac{\Delta\rho\, \mu_c\, g}{\rho_c^2}\right)^{1/3}$$

2) Larg gas bubbles > 2.5 mm

$$K_L' = 0.42 (Sc)^{-0.5} \left(\frac{\Delta\rho\, \mu_c\, g}{\rho_c^2}\right)^{1/3}$$

⊳ Total amount in the packed bed

$$* \; W = N_A A = A K_c \Delta C_{AM}$$

$$= A K_c \frac{(C_{Ai} - C_{A1}) - (C_{Ai} - C_{A2})}{\ln\left(\frac{C_{Ai} - C_{A1}}{C_{Ai} - C_{A2}}\right)}$$

$$* \; N_A A = V (C_{A2} - C_{A1})$$

$$N_A A = N_A A$$

"lec 7"

→ Y_{AG}, Y_{Ab} ⇒ bulk conc. in gas phase

X_{AL}, X_{Ab} ⇒ " " " Liq "

→ Y_{Ai} ⇒ conc. of gas @ interface

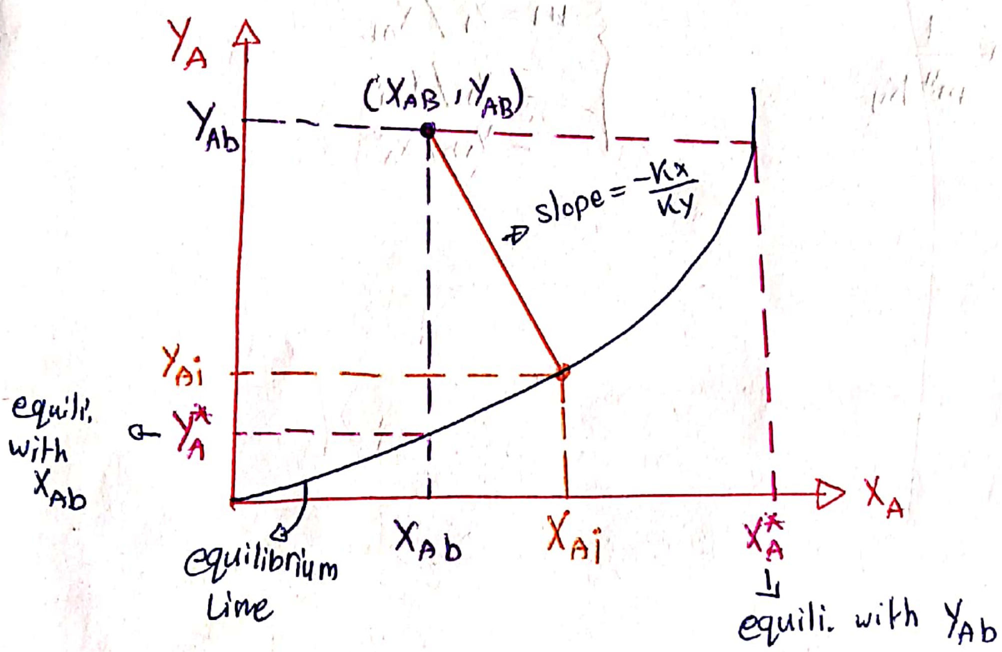
X_{Ai} ⇒ " " Liq " "

→ X_{Ai} and Y_{Ai} @ equilibrium

⇒ $P_{Ai} = H X_{Ai}$

→ $Y_{Ai} = H' X_{Ai}$ $(H' = H/P)$

→ Two film theory applied @ st. st.



$$-\frac{Kx}{Ky} = \frac{Y_{AB} - Y_{Ai}}{X_{Ab} - X_{Ai}}$$

dilute → $K_x' = K_x$, $K_y' = K_y$

→ Over all mass transfer coff.

→ $K_y = \frac{N_A}{Y_{Ab} - Y_A^*}$

capital → over all

→ $K_x = \frac{N_A}{X_A^* - X_{Ab}}$

small

$k_y = \frac{N_A}{Y_{Ab} - Y_{Ai}}$

$k_x = \frac{N_A}{X_{Ai} - X_{Ab}}$

→ non-Linear equilibrium.

→ $m' = \dfrac{y_{Ai} - y_A^*}{x_{Ai} - x_{Ab}}$

→ $m'' = \dfrac{y_{Ab} - y_{Ai}}{x_A^* - x_{Ai}}$

→ $\left(\dfrac{1}{K_y}\right) = \dfrac{1}{k_y} + \dfrac{m'}{k_x}$

over all mass transfer resistance

→ $\dfrac{1}{K_x} = \dfrac{1}{k_x} + \dfrac{1}{m'' k_y}$

* m' small → resistance in gas phase

m' Larg → " " Liq "

$m = y_A^* / x_{Ab}$

$= y_{Ai} / x_{Ai}$

$= y_{Ab} / x_A^*$