



Transport Phenomena II

Lec 3: Steady-State Molecular Diffusion

Content

Diffusion in gases, Diffusion into an Infinite Standard Medium, Diffusion in liquids

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Content



- Steady State Mass Transfer
 - Introduction
 - Diffusion in gases
 - Diffusing of A through stagnant, non-diffusing B
 - Equimolar Counter Diffusion
 - Diffusion into an Infinite Standard Medium
 - Diffusion in liquids



Mass Transfer Equations



$$\frac{\partial N_{A,z}}{\partial x} = 0.0 \longrightarrow N_{A,z} = \text{Constant}$$

$$\frac{\partial (rN_{A,r})}{\partial r} = 0.0 \longrightarrow r N_A = \text{Constant}$$

$$\frac{\partial (r^2 N_{A,r})}{\partial r} = 0.0 \longrightarrow r^2 N_A = \text{Constant}$$

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General Flux Equation



For species A in binary mixture of A and B,

$$N_A = -c_T D_{AB} \frac{dx_A}{dz} + x_A (N_A + N_B) \quad (7d)$$

➤ Similarly for species B:

$$N_B = -c_T D_{AB} \frac{dx_B}{dz} + x_B (N_A + N_B) \quad (7e)$$

➤ The 1-D differential mass transfer equation

$$\frac{\partial N_{A,z}}{\partial z} + \frac{\partial C_A}{\partial t} = 0.0 \xrightarrow{\text{At steady state}} \frac{\partial N_{A,z}}{\partial z} = 0.0 \longrightarrow N_{A,z} = \text{constant}$$

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General Flux Equation



- Applying Eq. (7b) to the case of diffusion only in the z , direction, with N_A and N_B both constant (steady state) we can readily separate the variables and if D_{AB} is constant, it can be integrated

$$\int_{c_{A1}}^{c_{A2}} \frac{-dc_A}{N_A c_T - c_A(N_A + N_B)} = \frac{1}{c_T D_{AB}} \int_{z_1}^{z_2} dz$$

where 1 indicates the beginning of the diffusion path (c_A high) and 2 the end of the diffusion path (c_A low). Letting $z_2 - z_1 = z$, we get

$$\rightarrow N_A = \frac{N_A}{(N_A + N_B)} \frac{c_T D_{AB}}{z} \ln \left(\frac{\frac{N_A}{N_A + N_B} - \frac{c_{A2}}{c_T}}{\frac{N_A}{N_A + N_B} - \frac{c_{A1}}{c_T}} \right) \quad (1)$$

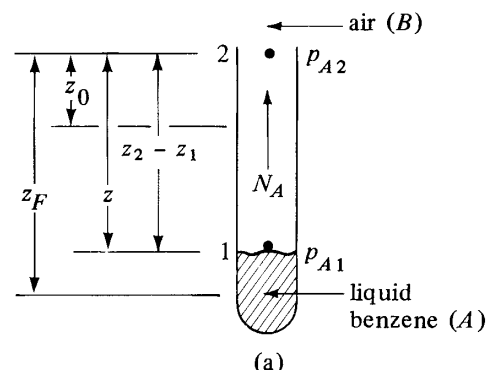
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Unimolecular Diffusion: Diffusing of A through stagnant, non-diffusing B



- One boundary at the end of the diffusion path is impermeable to component B, so it cannot pass through ($N_B = 0$)
 - Evaporation of a pure liquid (A) as benzene is at the bottom of a narrow tube.
 - Large amount of inert or non-diffusing air (B) is passed over the top.
 - Vapor A (benzene) diffuses through B (air) in the tube.
 - The boundary at the liquid surface (at point 1) is impermeable to B, since B is insoluble in liquid A.
- Hence, B cannot diffuse into or away from the surface.
- Therefore, $N_B = 0$
- At point 2 the partial pressure $P_{A2} = 0$, since a large volume of air is passing by.



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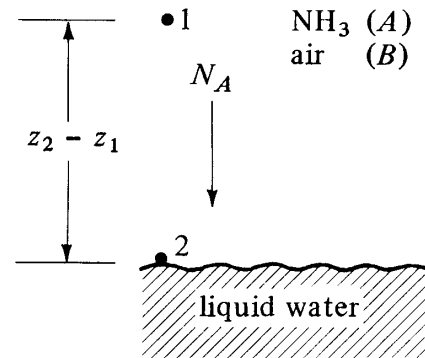


Diffusing of A through stagnant, non-diffusing B



➤ Absorption of NH_3 (A) vapor which is in air (B) by water.

- The water surface is impermeable to the air, since air is only very slightly soluble in water,
- Air has a source but no sink. Hence, it will not diffuse,
- The flux of B to a stationary observer appears to be zero, i.e. $(N_B = 0)$.
- But NH_3 (A) has a source (the bulk of air) can dissolve in water (sink)



(b)

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Diffusing of A through stagnant, non-diffusing B



$$N_A = -c_T D_{AB} \frac{dx_A}{dz} + x_A (N_A + \cancel{N_B})$$

$$\rightarrow N_A = -c_T D_{AB} \frac{dx_A}{dz} + x_A N_A$$

Rearranging

$$N_A = \frac{-c_T D_{AB}}{1 - x_A} \frac{dx_A}{dz} \quad (2)$$

➤ Eq. (16) may be integrated between the two boundary conditions:

$$\text{at } z = z_1 \quad x_A = x_{A1}$$

$$\text{And at } z = z_2 \quad x_A = x_{A2}$$

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Diffusing of A through stagnant, non-diffusing B



- Assuming the diffusivity (D_{AB}) is to be independent of concentration, and realizing that, for diffusion through constant area at steady state, N_A is constant along the diffusion path, by integration we get

$$N_A \int_{z_1}^{z_2} dz = c_T D_{AB} \int_{x_{A1}}^{x_{A2}} \frac{-dx_A}{1-x_A}$$

$$\rightarrow N_A = \frac{c_T D_{AB}}{z_2 - z_1} \ln \left(\frac{1-x_{A2}}{1-x_{A1}} \right) \quad (3a)$$

Or

$$N_A = \frac{c_T D_{AB}}{z_2 - z_1} \ln \left(\frac{c_T - c_{A2}}{c_T - c_{A1}} \right)$$

- Thus, the mole-fraction variation as a function of z is

$$x_A = 1 - (1 - x_{A1}) \exp \left(\frac{N_A (z - z_1)}{c_T D_{AB}} \right) \quad (3b)$$

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Diffusing of A through stagnant, non-diffusing B



- The log mean average concentration of component B is defined as

$$x_{B,lm} = \frac{x_{B2} - x_{B1}}{\ln \left(\frac{x_{B2}}{x_{B1}} \right)} \quad (4)$$

since $x_B = 1 - x_A$

$$\rightarrow x_{B,lm} = \frac{(1-x_{A2}) - (1-x_{A1})}{\ln \left(\frac{(1-x_{A2})}{(1-x_{A1})} \right)} = \frac{x_{A1} - x_{A2}}{\ln \left(\frac{(1-x_{A2})}{(1-x_{A1})} \right)} \quad (5a)$$

Substituting from Eq. (5a) in Eq. (3a),

$$N_A = \frac{c_T D_{AB}}{z_2 - z_1} \frac{(x_{A1} - x_{A2})}{x_{B,lm}} \quad (6a)$$

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Diffusing of A through stagnant, non-diffusing B



- For ideal gas and using $x_A = c_A/c_T$, $p_A = c_A RT$ and $P = c_T RT$, equation (17) can be converted to the following:

$$N_A = \frac{P D_{AB}}{RT(z_2 - z_1)} \ln \left(\frac{P - p_{A2}}{P - p_{A1}} \right) \quad (7)$$

Introduce the log mean value of inert B as follows:

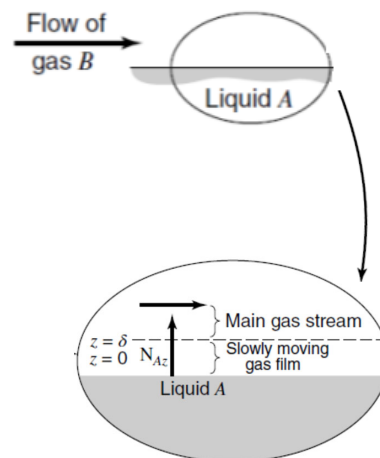
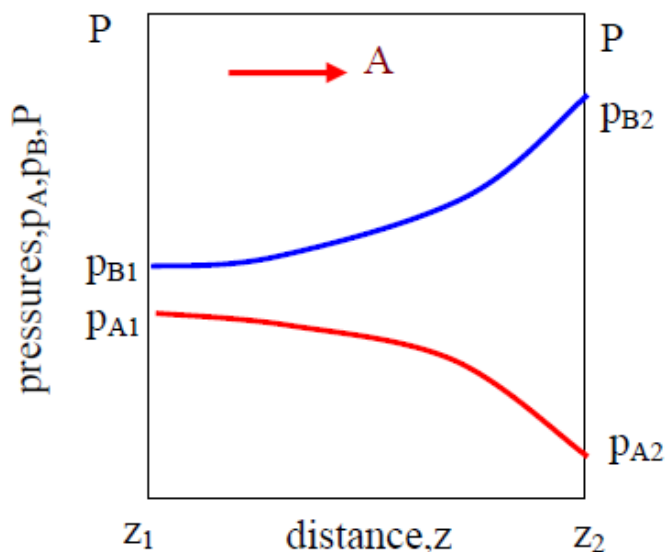
$$p_{B,lm} = \frac{(p_{B2} - p_{B1})}{\ln \left(\frac{p_{B2}}{p_{B1}} \right)} = \frac{(P - p_{A2}) - (P - p_{A1})}{\ln \left(\frac{(P - p_{A2})}{(P - p_{A1})} \right)} \quad (5b)$$

$$\rightarrow N_A = \frac{P D_{AB}}{RT(z_2 - z_1)} \frac{(p_{A1} - p_{A2})}{p_{B,lm}} \quad (6b)$$

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Diffusing of A through stagnant, non-diffusing B



Film model for mass transfer of component A into a moving gas stream

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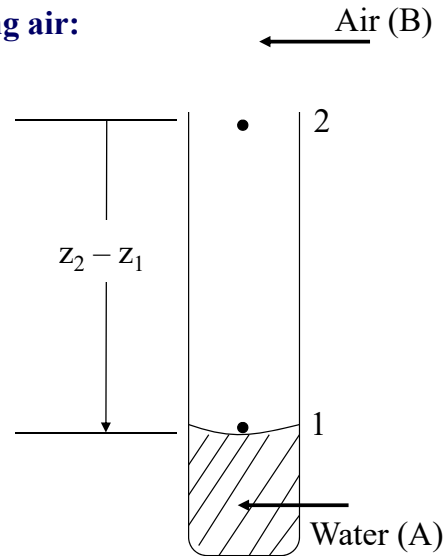


Example



Diffusion of water through stagnant, non-diffusing air:

Water in the bottom of a narrow metal tube is held at a constant temperature of 293 K. The total pressure of air (assumed to be dry) is 1 atm and the temperature is 293 K. Water evaporates and diffuses through the air in the tube, and the diffusion path is 0.1524 m long. Calculate the rate of evaporation at steady state. The diffusivity of water vapour at 1 atm and 293 K is $0.250 \times 10^{-4} \text{ m}^2/\text{s}$. Assume that the vapour pressure of water at 293 K is 0.0231 atm.



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Example cont.



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Example cont.



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Example



Oxygen is diffusing in a mixture of oxygen-nitrogen at 1 std atm, 25°C. Concentration of oxygen at planes 2 mm apart are 10 and 20 volume % respectively. Nitrogen is non-diffusing. Calculate the flux of oxygen. Diffusivity of oxygen in nitrogen = $1.89 \times 10^{-5} \text{ m}^2/\text{sec}$.

Let us denote oxygen as A and nitrogen as B

Given:

$$D_{AB} = 1.89 \times 10^{-5} \text{ m}^2/\text{sec}$$

$$P_t = 1 \text{ atm} = 1.01325 \times 10^5 \text{ N/m}^2$$

$$T = 25^\circ\text{C} = 273 + 25 = 298 \text{ K}$$

$$z = 2 \text{ mm} = 0.002 \text{ m}$$

$$P_{A1} = 0.2 \times 1 = 0.2 \text{ atm (From Ideal gas law and additive pressure rule)}$$

$$P_{A2} = 0.1 \times 1 = 0.1 \text{ atm}$$

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Example cont.

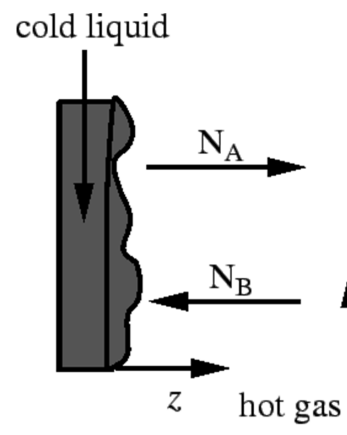


Equimolar Counter Diffusion



- A physical situation which is encountered in the distillation of two constituents whose molar latent heats of vaporization are essentially equal, stipulates that the flux of one gaseous component is equal to but acting in the opposite direction from the other gaseous component; that is, $N_A = -N_B$.
- The molar flux N_A , for a binary system at constant temperature and pressure is described by

$$N_A = -D_{AB} \frac{dC_A}{dz} + x_A (N_A + N_B) \quad (7b)$$



Equimolar Counter Diffusion



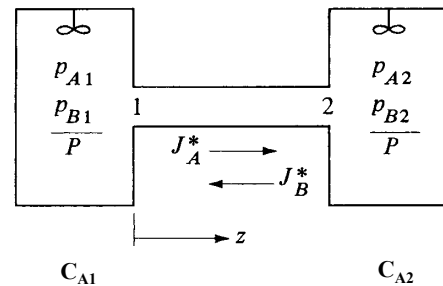
with the substitution of $N_B = -N_A$, the above equation becomes

$$N_A = -D_{AB} \frac{dC_A}{dz} \quad (1b)$$

For steady state, the above diffusion Equation may be integrated, using the boundary conditions:

$$\begin{aligned} \text{at } z = z_1 \quad C_A &= C_{A1} \\ \text{and } z = z_2 \quad C_A &= C_{A2} \end{aligned}$$

$$N_A \int_{z_1}^{z_2} dz = -D_{AB} \int_{C_{A1}}^{C_{A2}} dC_A$$



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Equimolar Counter Diffusion

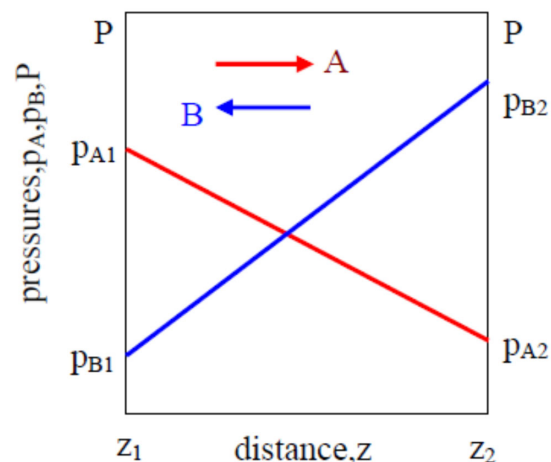


$$\Rightarrow N_A = \frac{D_{AB}}{z_2 - z_1} (C_{A1} - C_{A2})$$

➤ For ideal gases

$$C_A = \frac{n_A}{V} = \frac{p_A}{RT}$$

$$\Rightarrow N_A = \frac{D_{AB}}{RT(z_2 - z_1)} (P_{A1} - P_{A2})$$



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Equimolar Counter Diffusion



- Concentration profile in these equimolar counter diffusion may be obtained from,

$$\frac{d}{dz} (N_A) = 0 \quad (\text{Since } N_A \text{ is constant over the diffusion path}).$$

Hence
$$\frac{d}{dz} \left(-D_{AB} \frac{dC_A}{dz} \right) = 0 \quad \longrightarrow \quad \frac{d^2 C_A}{dz^2} = 0. \quad (8)$$

- This equation may be solved using the boundary conditions to give

$$\frac{C_A - C_{A1}}{C_{A1} - C_{A2}} = \frac{z - z_1}{z_1 - z_2} \quad (9)$$

- Equation, (25) indicates a linear concentration profile for equimolar counter diffusion

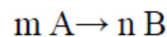
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Other Relations Between N_A and N_B



- **The Relationship between N_A and N_B is Fixed by Reaction Stoichiometry**



$$\frac{N_A}{m} = - \frac{N_B}{n} \quad \longrightarrow \quad N_B = - (n/m) N_A$$

- **The Relationship between N^A and N^B is Given by the Latent Heats of Vaporization**

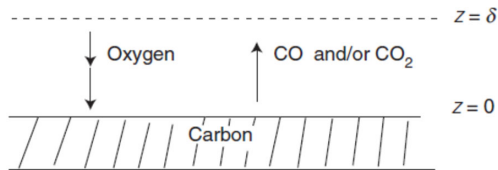
- In the rectification of a binary liquid solution, mass transfer takes place both in liquid and vapor phases.
- The more volatile component transfers from liquid to vapor and the less volatile from vapor to liquid

$$\lambda_A N_A = - \lambda_B N_B, \quad \longrightarrow \quad N_B = - (\lambda_A / \lambda_B) N_A$$

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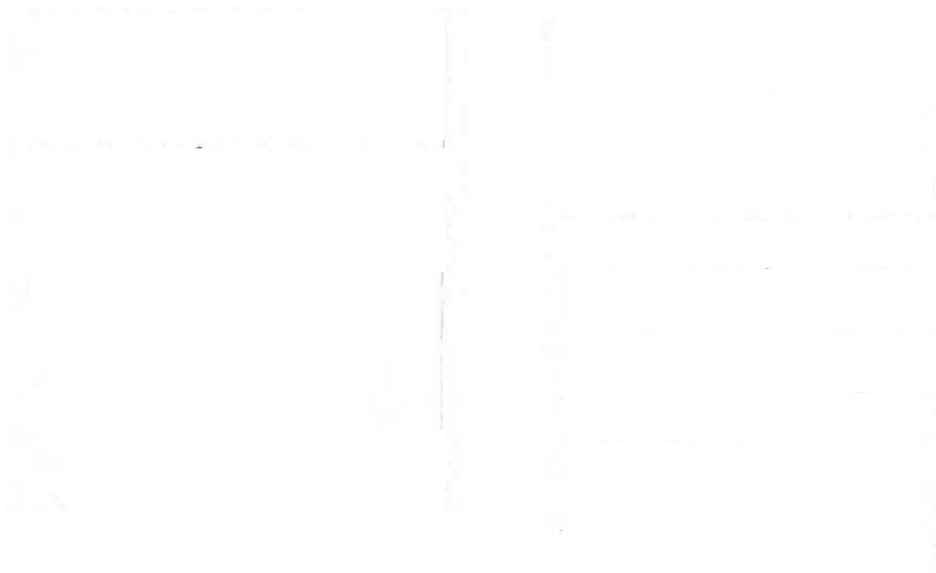
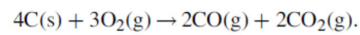


26.25 In a combustion chamber, oxygen diffuses through air to the carbon surface where it reacts to make CO and/or CO₂. The mole fraction of oxygen at $z=0$ is 0.21. The reaction at the surface may be assumed to be instantaneous. No reaction occurs in the gas film.



Determine the rate of oxygen diffusion per hour through one square meter of area if

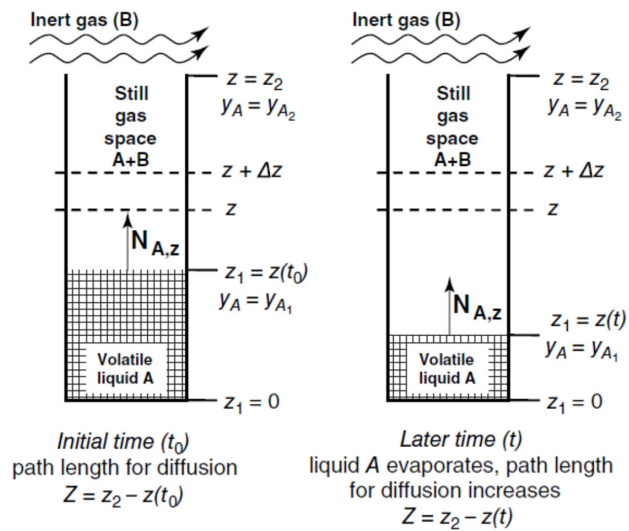
- only carbon monoxide, CO, is produced at the carbon surface;
- only carbon dioxide, CO₂, is produced at the carbon surface;
- the following instantaneous reaction occurs at the carbon surface:



Pseudo Steady State Diffusion Through A Stagnant Film



- In many mass transfer operations, one of the boundaries may move with time.
- If the length of the diffusion path changes a small amount over a long period of time, a pseudo steady state diffusion model may be used.
- When this condition exists, the equation of steady state diffusion through stagnant gas' can be used to find the flux.



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Pseudo Steady State Diffusion Through A Stagnant Film



- If the difference in the level of liquid A over the time interval considered is only a small fraction of the total diffusion path (level drops very slowly), and
- $t_0 - t$ is relatively long period of time, at any given instant in that period,
- The molar flux in the gas phase may be evaluated by

$$N_A = \frac{c_T D_{AB}}{Z} \frac{(x_{A1} - x_{A2})}{x_{B,lm}} \quad (6a)$$

Where $z = z_2 - z_1$ is the length of the diffusion path at time t

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Pseudo Steady State Diffusion Through A Stagnant Film



- The molar flux N_A is related to the amount of A leaving the liquid by

$$N_A = \frac{\rho_{A,L}}{M_A} \frac{dz}{dt} \quad (10)$$

where $\frac{\rho_{A,L}}{M_A}$ is the molar density of A in the liquid phase

Note: The concentration term ' C_A ' is replaced by average molar density

- Under Pseudo steady state conditions, equations (6a) & (10) can be equated to give

$$\frac{\rho_{A,L}}{M_A} \frac{dz}{dt} = \frac{c_T D_{AB}}{z} \frac{(x_{A1} - x_{A2})}{x_{B,lm}} \quad (11)$$

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Pseudo Steady State Diffusion Through A Stagnant Film



- Equation. (25) may be integrated from $t = 0$ to t and from $z = z_{t0}$ to $z = z_t$ as:

$$\int_{t=0}^t dt = \frac{\rho_{A,L} x_{B,lm} / M_A}{c_T D_{AB} (x_{A1} - x_{A2})} \int_{z_{t0}}^{z_t} z dz$$

$$\Rightarrow t = \frac{\rho_{A,L} x_{B,lm} / M_A}{c_T D_{AB} (x_{A1} - x_{A2})} \left(\frac{z_t^2 - z_{t0}^2}{2} \right) \quad (12)$$

- Eq. (26) used to experimentally determine the diffusivity D_{AB} as

$$D_{AB} = \frac{\rho_{A,L} x_{B,lm}}{M_A c_T (x_{A1} - x_{A2}) t} \left(\frac{z_t^2 - z_{t0}^2}{2} \right) \quad (13)$$

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Example



A vertical glass tube 3 mm in diameter is filled with liquid toluene to a depth of 20mm from the top opened. After 275 hrs at 39.4 °C and a total pressure of 760 mm Hg the level has dropped to 80 mm from the top. Calculate the value of diffusivity.

Data:

vapor pressure of toluene at 39.4°C = 7.64 kN / m²,

density of liquid toluene = 850 kg/m³

Molecular weight of toluene (C₆ H₆ CH₃)= 92

760 mm Hg = 101.3 kN/m²

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Example cont.



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Example cont.

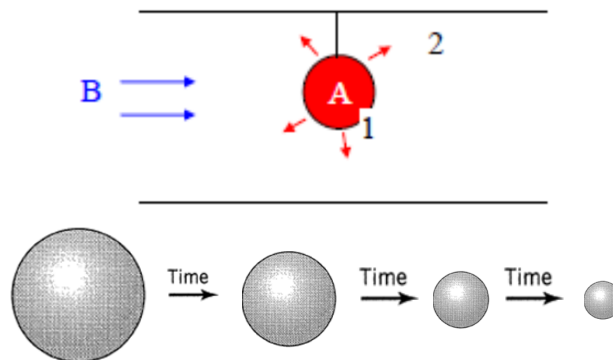


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Diffusion into an Infinite Standard Medium

(Vaporization of Liquid Drops or Sublimation of Solid Spheres)



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Diffusion into an Infinite Standard Medium



- Here we will discuss problems involving **diffusion from a spherical particle (either liquid or sublimable solid) into an infinite body of stagnant gas.**
- The purpose in doing this is to demonstrate how to set up differential equations that describe the diffusion in these processes.
- The solutions developed here for these problems actually represent a special case of the more common situation involving both molecular diffusion and convective mass transfer.
- As an example of such problems, we shall consider the evaporation of spherical droplet such as a raindrop or sublimation of naphthalene ball and the diffusion of nutrients to a spherical-like microorganism in a liquid.
- The vapor formed at the surface of the droplet is assumed to diffuse by molecular motions into the large body of stagnant gas that surrounds the droplet.

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Diffusion into an Infinite Standard Medium



- Consider a water droplet of **spherical shape** with a radius r_0 , the flux of water vapor at any distance r from the center is given by

$$N_A = -c_T D_{AB} \frac{d x_A}{d r} + x_A (N_A + N_B)$$

Here $N_B = 0$ (since air is assumed to be stagnant), therefore,

$$N_A = -c_T D_{AB} \frac{d x_A}{d r} + x_A N_A$$

Rearranging

$$N_A = \frac{-c_T D_{AB}}{1 - x_A} \frac{d x_A}{d r} \quad (14)$$

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Diffusion into an Infinite Standard Medium



- The flux N_A is not constant, because of the spherical geometry; decreases as the distance from the center of sphere increases. But the molar flow rate at r and $r + \delta r$ are the same. This could be written as,

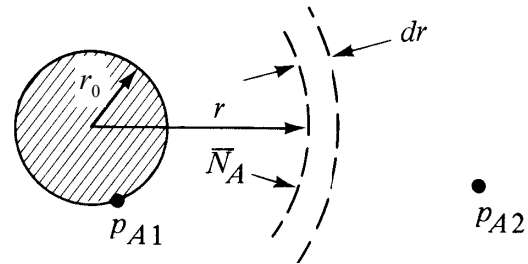
$$A N_A|_r = A N_A|_{r+\delta r} \quad (15)$$

where A = surface area of sphere at r or $r + \delta r$.

Substituting for $A = 4\pi r^2$ in the equation,

$$4\pi r^2 N_A|_{r+\delta r} - 4\pi r^2 N_A|_r = 0$$

$$\text{Or } \lim_{\delta r \rightarrow 0} \frac{r^2 N_A|_{r+\delta r} - r^2 N_A|_r}{\delta r} = 0$$



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Diffusion into an Infinite Standard Medium



$$\frac{d}{dr}(r^2 N_A) = 0$$

Or

- from the general equation and for radial symmetry in spherical coordinates **with no chemical reaction takes place**

$$\frac{\partial c_A}{\partial t} = \frac{1}{r^2} \frac{\partial (r^2 N_{A,r})}{\partial r}$$

At steady state

$$0 = \frac{1}{r^2} \frac{\partial (r^2 N_{A,r})}{\partial r} \quad \longrightarrow \quad \frac{d}{dr}(r^2 N_A) = 0$$

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Diffusion into an Infinite Standard Medium



Integrating

$$\rightarrow r^2 N_A = \text{constant}$$

hence

$$r^2 N_A = r_0^2 N_{A_0} \quad (16)$$

Substituting for N_A from equation (16),

$$\frac{-r^2 c_T D_{AB}}{1-x_A} \frac{dx_A}{dr} = r_0^2 N_{A_0}$$

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Diffusion into an Infinite Standard Medium



$$r_0^2 N_{A_0} \int \frac{dr}{r^2} = -c_T D_{AB} \int \frac{dx_A}{1-x_A} \quad (17)$$

Boundary condition : $r = r_0 \quad x_A = x_{AS}$

$r = \infty \quad x_A = x_{A\infty}$

$$\rightarrow r_0^2 N_{A_0} \left(-\frac{1}{r} \right)_{r_0}^{\infty} = \left[c_T D_{AB} \ln(1-x_A) \right]_{x_{AS}}^{x_{A\infty}}$$

Simplifying

$$\rightarrow N_{A_0} = \frac{c_T D_{AB}}{r_0} \ln \left(\frac{1-x_{A\infty}}{1-x_{AS}} \right) \quad (18)$$

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Diffusion into an Infinite Standard Medium



Using $c_T = \frac{P}{RT}$

$$\rightarrow N_{Ao} = \frac{D_{AB}P}{RT r_0} \frac{(p_{A1} - p_{A2})}{p_{B,lm}} \quad (19)$$

➤ If p_{A1} is small compared to P (a dilute gas phase), then $p_{B,lm} = P$ and again,

$$c_{A1} = \frac{P_{A1}}{RT}$$

$$\rightarrow N_{Ao} = \frac{2D_{AB}}{D_1} (c_{A1} - c_{A2}) \quad (20)$$

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Diffusion into an Infinite Standard Medium



➤ Time required for complete evaporation of the droplet may be evaluated from making mass balance.

$$\frac{\text{Moles of water diffusing}}{\text{unit time}} = \frac{\text{moles of water leaving the droplet}}{\text{unit time}}$$

$$\begin{aligned} 4\pi r_0^2 N_{Ao} &= -\frac{dn_A}{dt} = -\frac{d}{dt} \left(\frac{m_A}{M_A} \right) \\ 4\pi r_0^2 N_{Ao} &= -\frac{d}{dt} \left(\frac{4}{3} \pi r_0^3 \frac{\rho_A}{M_A} \right) \\ &= -4\pi r_0^2 \frac{\rho_A}{M_A} \frac{dr_0}{dt} \end{aligned} \quad (21)$$

Substituting for N_{Ao} from equation (31),

$$\frac{c_T D_{AB}}{r_0} \ln \left(\frac{1 - x_A}{1 - x_{AS}} \right) = \frac{-\rho_A}{M_A} \frac{dr_0}{dt}$$

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Diffusion into an Infinite Standard Medium



Integrating this equation with Initial condition : at $t = 0, r_0 = r_1$

$$\int_0^t dt = \frac{-\rho_A}{M_A} \frac{1}{c_T D_{AB}} \frac{1}{\ln\left(\frac{1-x_{A\infty}}{1-x_{AS}}\right)} \int_{r_1}^0 r_0 dr_0$$

$$\rightarrow t = \frac{\rho_A}{M_A} \frac{1}{2c_T D_{AB}} \frac{r_1^2}{\ln\left(\frac{1-x_{A\infty}}{1-x_{AS}}\right)} \quad (22a)$$

$$\text{Or } t = \frac{\rho_A R T r_1^2}{2 M_A D_{AB} P} \frac{p_{B,lm}}{(p_{A1} - p_{A2})} \quad (22b)$$

- Equation (22) gives the total time t required for complete evaporation of spherical droplet of initial radius r_1 .

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Example



A sphere of naphthalene having a radius of 2.0 mm is suspended in a large volume of still air at 318 K and 1.101325×10^5 Pa (1 atm). The diffusivity of the naphthalene at 318 K is 6.92×10^{-6} m²/s. Calculate the rate of evaporation of naphthalene from the surface.

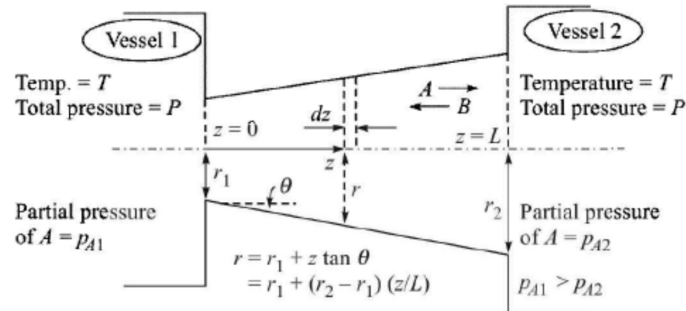
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Diffusion Through Varying Cross-sectional Area



- In the Figure component A is diffusion at steady state through a circular conduit which is tapered uniformly as shown.
- At point 1 the radius is r_1 and at point 2 it is r_2 .



Rate of input of A into the section at $z = (\pi r^2)N_A|_z$

Rate of output of A from the thin section at $z + \Delta z = (\pi r^2)N_A|_{z+\Delta z}$



Diffusion Through Varying Cross-sectional Area



- At steady state

$$(\pi r^2)N_A|_z - (\pi r^2)N_A|_{z+\Delta z} = 0$$

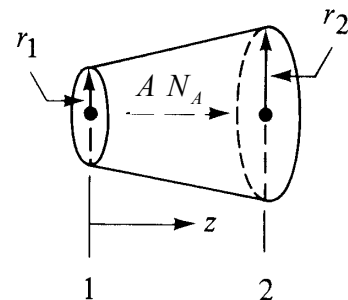
Dividing by Δz throughout and taking the limit $\Delta z \rightarrow 0$,

$$\lim_{\Delta z \rightarrow 0} \frac{(\pi r^2)N_A|_z - (\pi r^2)N_A|_{z+\Delta z}}{\Delta z} = 0$$

$$\Rightarrow -\frac{d}{dz}(\pi r^2 N_A) = 0 \quad \text{i.e.} \quad \pi r^2 N_A = \text{constant}$$

Or

$$A_1 N_{A1} = A_2 N_{A2} = \bar{N}_A$$



Diffusion Through Varying Cross-sectional Area



- At position z in the conduit, for A diffusing through stagnant, non-diffusing B , Eq. (16) can be applied

$$N_A = \frac{-c_T D_{AB}}{1-x_A} \frac{dx_A}{dz} \quad (6a)$$

Or

$$N_A = \frac{-D_{AB}}{RT(1-p_A/P)} \frac{dP_A}{dz} \quad (6b)$$

- The variable radius r can be related to position z in the path as follows:

$$r = \left(\frac{r_2 - r_1}{z_2 - z_1} \right) z + r_1$$

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Diffusion Through Varying Cross-sectional Area



- This value of r is then substituted into Eq. (6b) to eliminate r and the equation is integrated.

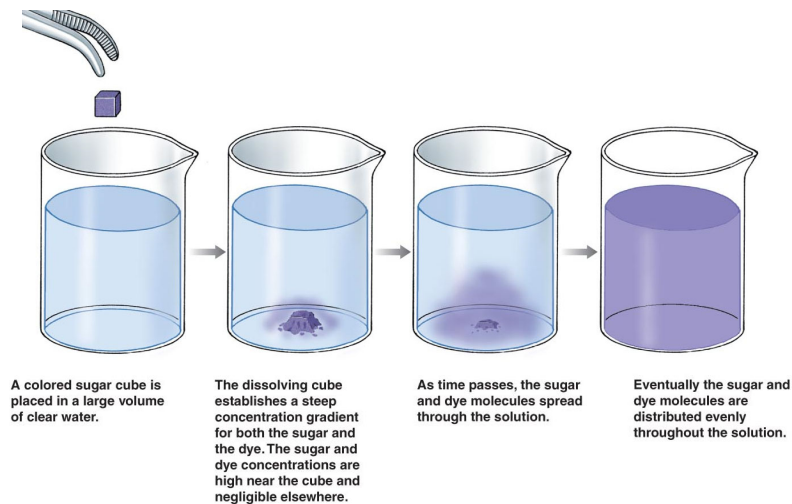
$$\frac{\bar{N}_A}{\pi} \int_{z_1}^{z_2} \frac{dz}{\left(\left(\frac{r_2 - r_1}{z_2 - z_1} \right) z + r_1 \right)^2} = \frac{-D_{AB}}{RT} \int_{P_{A1}}^{P_{A2}} \frac{dP_A}{(1-p_A/P)}$$

$$\Rightarrow \bar{N}_A = \frac{\pi D_{AB} P}{RT} \frac{r_1}{z} \left[r_1 - (r_1 - r_2) \left(\frac{z}{L} \right) \right] \ln \frac{P}{P - p_A^v}$$

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Diffusion in liquid



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Diffusion in liquid

- Diffusion of solutes in liquids is very important in many industrial processes, especially in such separation operations as liquid-liquid extraction or solvent extraction, gas absorption, and distillation.
- The rate of molecular diffusion in liquids is considerably slower than in gases. The molecules in a liquid are very close together compared to a gas.
- Hence, the molecules of the diffusing solute A will collide with molecules of liquid B more often and diffuse more slowly than in gases.
- Since the molecules in a liquid are packed together much more closely than in gases, the density and the resistance to diffusion in a liquid are much greater.
- Also, because of this closer spacing of the molecules, the attractive forces between molecules play an important role in diffusion.

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Diffusion in liquid



Equimolar Counter Diffusion

- Similar to the equation used for equimolar counter diffusion in gases, but the term c_T is replaced by ' c_{av} ' which is equal to the average molar density. With $N_A = -N_B$

$$N_A = \frac{D_{AB}}{z_2 - z_1} (C_{A1} - C_{A2}) = \frac{c_{av} D_{AB}}{z_2 - z_1} (x_{A1} - x_{A2})$$

where $c_{av} = \frac{\frac{\rho_1}{M_1} + \frac{\rho_2}{M_2}}{2}$

$$N_A = \frac{D_{AB}}{Z} \left(\frac{\rho}{M} \right)_{av} (x_{A1} - x_{A2}) \quad (2b)$$

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Diffusion in liquid



Diffusion of A through non diffusing B

- An example is a dilute solution of propionic acid (A) in a water (B) solution being contacted with toluene.
- Only the propionic acid (A) diffuses through the water phase, to the boundary, and then into the toluene phase.
- The toluene-water interface is a barrier to diffusion of B and $N_B = 0$
- Using equations (20a) and replacing C_T with C_{av} , then

$$\rightarrow N_A = \frac{D_{AB}}{(z_2 - z_1) x_{BM}} \left(\frac{\rho}{M} \right)_{av} (x_{A1} - x_{A2}) \quad (20b)$$

with $x_{B,lm} = \frac{x_{A1} - x_{A2}}{\ln \left(\frac{(1 - x_{A2})}{(1 - x_{A1})} \right)}$

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Example



An ethanol (A)-water (B) solution in the form of a stagnant film 2.0 mm thick at 293 K is in constant at one surface with an organic solvent in which ethanol is soluble and water is insoluble. Hence, $N_B = 0$. At point 1 the concentration of ethanol is 16.8 wt % and the solution density is $\rho_1 = 972.8 \text{ kg/m}^3$. At point 2 the concentration of ethanol is 6.8 wt % and $\rho_2 = 988.1 \text{ kg/m}^3$. The diffusivity of ethanol is $0.740 \times 10^{-9} \text{ m}^2/\text{s}$. Calculate the steady-state flux N_A .

The diffusivity of ethanol is $0.740 \times 10^{-9} \text{ m}^2/\text{s}$. The molecular weights of A and B are $M_A = 46.05$ and $M_B = 18.02$. for a wt % of 6.8, the mole fraction of ethanol (A) is as follows when using 100 kg solution:

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Example cont.



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Example cont.



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