Transport Phenomena II

Lec 8: Concept of continuous contacting equipment

Content

Introduction, Equations for Packed Columns

Prof. Zayed Al-Hamamre

Chemical Engineering Department | University of Jordan | Amman 11942, Jordan
Tel. +962 6 535 5000 | 22888

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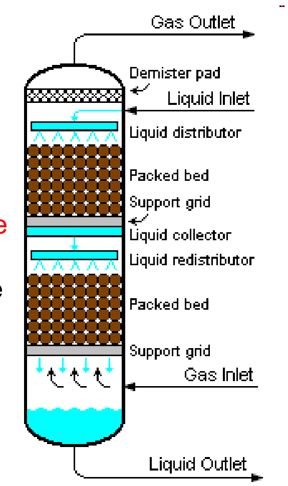
- The Operating Line for Packed Towers
- Calculating the height of packing in packed columns

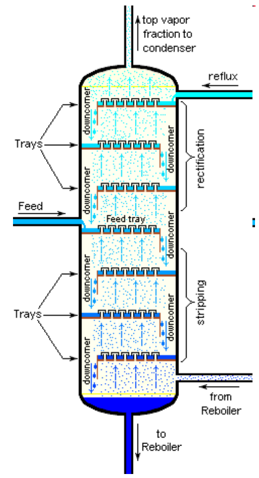
Introduction

- In continuous contacting equipment, the up-flowing gas remains in contact with down-flowing liquid throughout the packing, at every point of the tower.
- Therefore, packed tower is known as “continuous differential contact equipment It is different from the stage-wise contact equipment .

Continuous differential column

Requirement: Calculate the height of packing in a packed column where the separation process takes place.

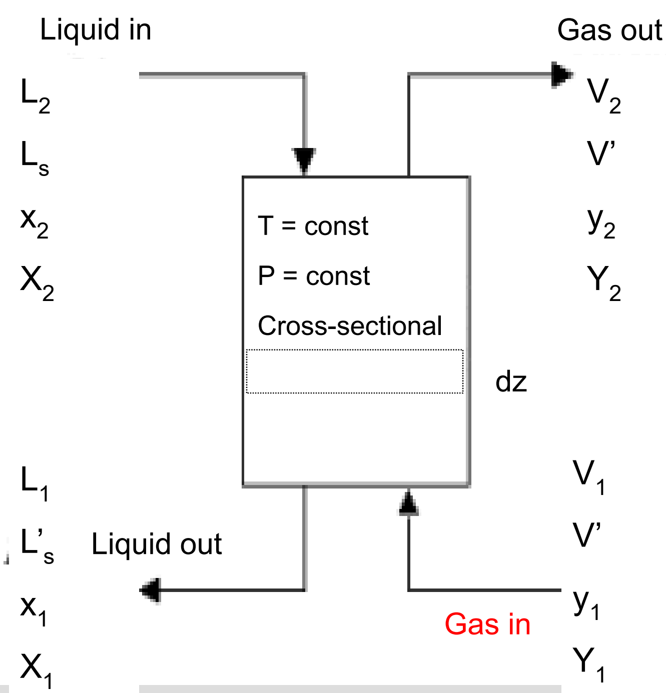




Stage wise contact is there in a plate column

Requirement: Calculate the number of plates required for the desired separation.

Mass Balances and the Operating Line for Packed Towers



V molar total gas flow rate (carrier gas + pollutant)

V’ molar inert carrier gas flow rate

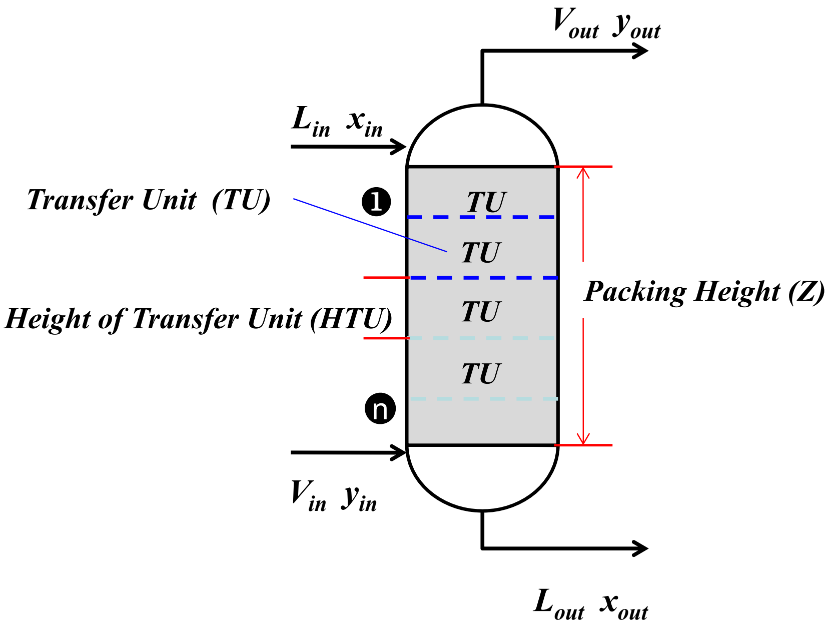
L molar total solvent flow rate (solvent + absorbed pollutant)

L’ molar solvent flow rate

x is the liquid mole fraction of pollutant, y is the gas phase mole fraction of the pollutants, X is the liquid phase mole ratio and

Y is the gas phase mole ratio

Packing Height (Z)



Packing Height (Z) = height of transfer unit (HTU) × number of transfer units (NTU)

Operating Line for Packed Absorption Towers

- A packed tower with countercurrent flow

- Mass balance around the cross-section A-A:

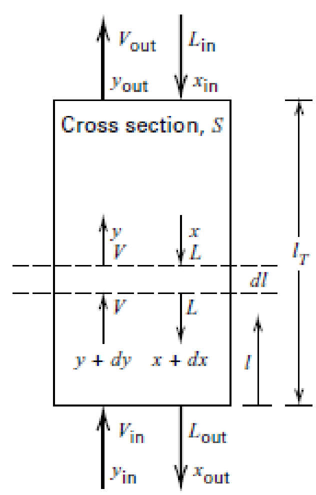
$$Vy_A + Lx_A = V(y_A - dy_A) + L(x_A + dx_A)$$

Therefore,

$$V\,dy_A = L\,dx_A$$

However, the flow rates do not remain constant through the column:

$$d(Vy_A) = d(Lx_A)$$



➢ Integrate from the top of the column down to the cross-section A-A:

$$Vy_A - V_1 y_{A1} = Lx_A - L_1 x_{A1} \quad \text{or} \quad x_{in}L_{in} + yV_l = xL_l + y_{out}V_{out} \qquad (3)$$

Operating Line for Packed Absorption Towers

- Rewrite the flow rates on a solute-free basis (use B):

$$V' = V(1-y_A) = V_1(1-y_{A1})$$

$$L' = L(1-x_A) = L_1(1-x_{A1})$$

- Rewrite Eq. (3):

$$\left(\frac{y_A}{1-y_A}\right)V' + \left(\frac{x_{A1}}{1-x_{A1}}\right)L' = \left(\frac{y_{A1}}{1-y_{A1}}\right)V' + \left(\frac{x_A}{1-x_A}\right)L' \qquad (4)$$

$$\Rightarrow Y_{n+1} = \frac{L'}{V'}X_n + \left(Y_1 - \frac{L'}{V'}X_o\right) \qquad (5)$$

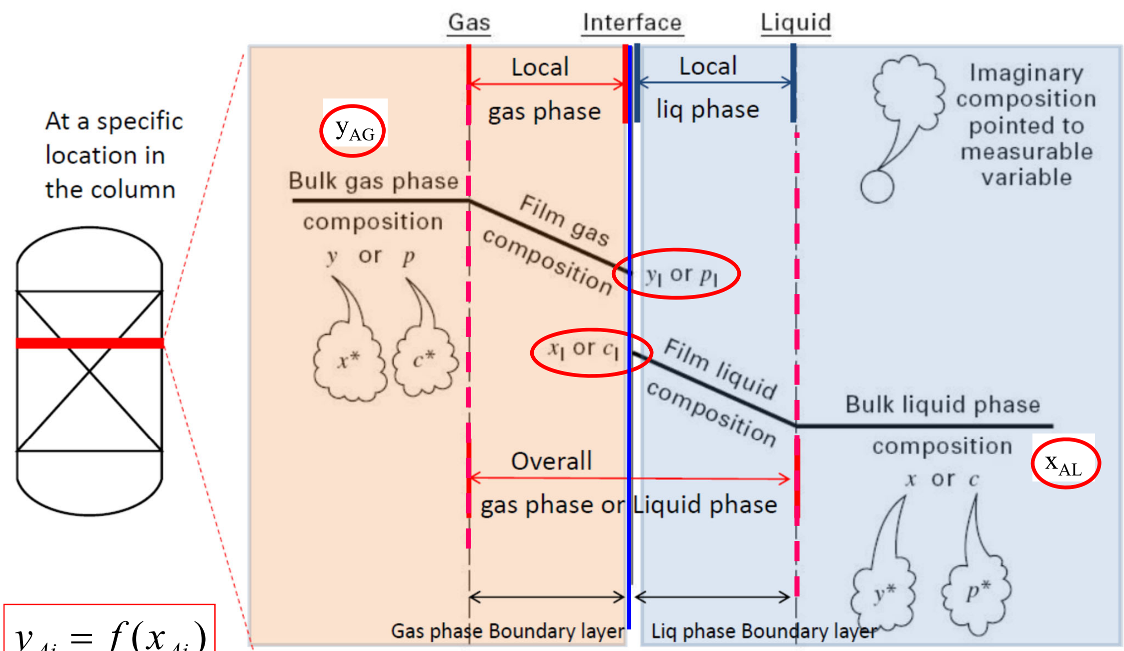
Eq. (5): Operating line equation

Operating Line for Packed Absorption Towers

For dilute solution: $(1-y_A) \approx 1.0, \ (1-x_{A1}) \approx 1.0, \ etc.$

$\Rightarrow$ $V' \approx V \qquad L' \approx L$

Two Film Theory: Review



$$y_{Ai} = f(x_{Ai})$$

Bulk concentration in the bulk liquid phase

Two Film Theory Applied at Steady-State

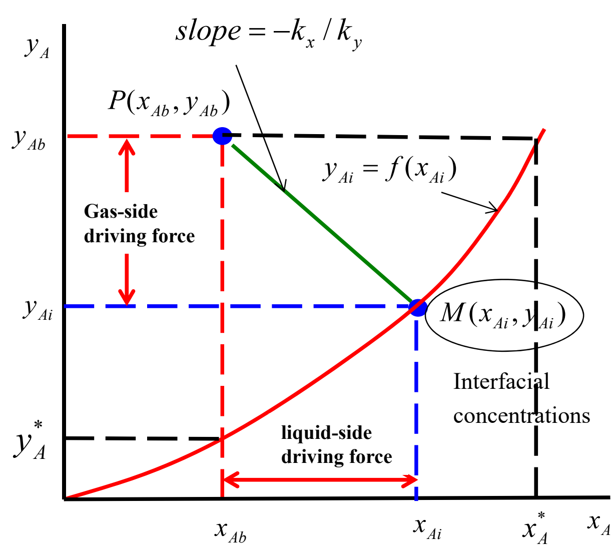
- Consider steady-state mass transfer of A from a gas, across an interface, and into a liquid.

$$N_A = k_y(y_{Ab} - y_{Ai}) = k_x(x_{Ai} - x_{Ab})$$

Gas-phase flux to the interface — Liquid-phase flux from the interface

- Using the above equation, We may write

$$\Rightarrow \quad -\frac{k_x}{k_y} = \frac{y_{Ab} - y_{Ai}}{x_{Ab} - x_{Ai}}$$



Two Film Theory: Review

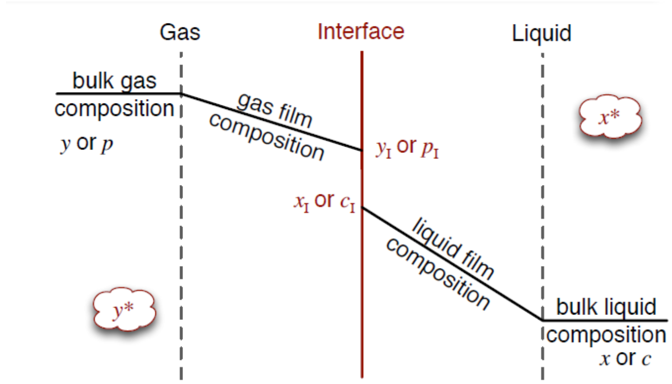
$$N_A = k_y a(y - y_I) = k_x a(x_I - x)$$

$$y = y_I - \frac{k_x a}{k_y a}(x - x_I)$$

relative resistance of mass transfer between the two phases

a = surface area per unit volume of packing

$$= \frac{\textit{interfacial area}}{\textit{unit volume}}$$



Overall mass transfer coefficient approach:

$$N_A = K_x a(x_A^* - x_A) = K_y a(y_A - y_A^*)$$

$$\frac{1}{K_y a} = \frac{1}{k_y a} + \frac{m}{k_x a}$$

$$\frac{1}{K_x a} = \frac{1}{k_x a} + \frac{1}{m k_y a}$$

Two Film Theory: Review

Where

y_A^* = equilibrium mole fraction of the solute in the vapor corresponding to the mole fraction x_A in the liquid

x_A^* = equilibrium mole fraction of the solute in the liquid corresponding to the mole fraction y_A in the vapor

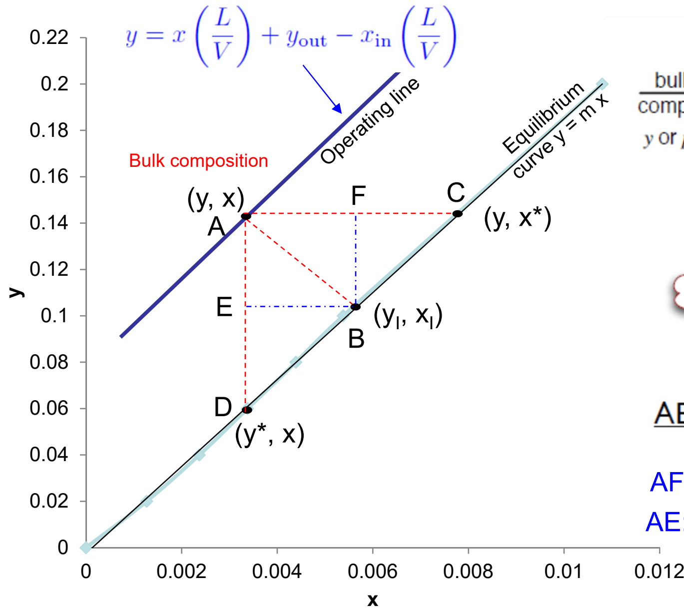
$K_y a,\ K_x a$: volumetric mass transfer coefficients

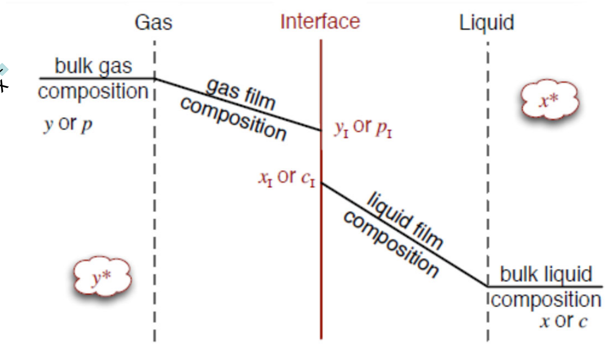
$$k'_y a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad k'_x a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad \text{(SI)}$$

$$K'_y a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad K'_x a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad \text{(SI)}$$

$$k'_y a = \frac{\text{lb mol}}{\text{h} \cdot \text{ft}^3 \text{ packing} \cdot \text{mol frac}} \quad k'_x a = \frac{\text{lb mol}}{\text{h} \cdot \text{ft}^3 \text{ packing} \cdot \text{mol frac}} \quad \text{(English)}$$

Two Film Theory: Review





AB line: $y = y_I - \frac{k_x a}{k_y a}(x - x_I)$

AF: Deriving force in the liquid side

AE: Deriving force in the gas side

$$N_A = K_x(x_A^* - x_{A_b}) = K_y(y_{A_b} - y_A^*)$$

$$x_A^* = \frac{y_{A_b}}{m} \qquad y_A^* = x_{A_b} m$$

Volume-Based Mass-Transfer Coefficients

- If A is the absorption tower cross-sectional area, and Z the packing height, then AZ is the tower packing volume.
  Defining A_i as the total interfacial area:

$$A_i = aAz$$

or in differential form:

$$dA_i = aAdz$$

- For constant mass flux, with units (moles/unit time) = (moles/unit time·area)(area):

$$d\overline{N_A} = N_A\, dA_i = K_y(y_A - y_A^*)aAdz$$

$$\Rightarrow d\overline{N_A} = K_y a(y_A - y_A^*)Adz = K_x a(x_A - x_A^*)Adz$$

Determining Height of Packing in the Tower: the HTU Method

- For the gas phase, the differential rate of mass transfer of component A is equal to the differential rate of change of the mass of A in the incoming gas stream in a height dz.

$$d\,\overline{N_A} = d(Vy_A) = d(Lx_A)$$

- Rewrite the flow rate on a solute-free basis: $V' = V(1-y)$

where $V = V'$ is a constant

then,
$$d(Vy_A) = V'd\left(\frac{y_A}{1-y_A}\right) = V'\frac{dy_A}{(1-y_A)^2} = V\frac{dy_A}{(1-y_A)}$$

since
$$V = \frac{V'}{(1-y_A)} \Rightarrow d(Vy_A) = V\frac{dy_A}{(1-y_A)}$$

- Therefore,

$$V\frac{dy}{(1-y_A)} = K_y a(y_A - y_A^*)Adz \qquad (8)$$

Determining Height of Packing in the Tower: the HTU Method

- Dropping the subscripts G and integrating, the final equations using overall coefficients are

$$Z = \int_{y_{out}}^{y_{in}} \left(\frac{V}{K_y aA}\right)\frac{dy_A}{(1-y_A)(y_A-y_A^*)}$$

$$(1-y_A)_{*LM} = \frac{(1-y_A)-(1-y_A^*)}{\ln\left[\frac{(1-y_A)}{(1-y_A^*)}\right]}$$

$$Z = \int_{y_{out}}^{y_{in}} \underbrace{\left(\frac{V}{K_y aA(1-y_A)_{*LM}}\right)}_{HTU_{OG}} \underbrace{\frac{(1-y_A)_{*LM}\,dy_A}{(1-y_A)(y_A-y_A^*)}}_{NTU_{OG}}$$

HTU_{OG} NTU_{OG}

Determining Height of Packing in the Tower: the HTU Method

- The term H_{OG} is called the <u>overall **H**eight of a **T**ransfer **U**nit</u> (*HTU*) based on the gas phase.
- Experimental data show that the *HTU* varies less with *V* than with $K_y a$.
- The smaller the *HTU*, the more efficient is the contacting.
- The term N_{OG} is called the <u>overall **N**umber of **T**ransfer **U**nits</u> (*NTU*) based on the gas phase.
- It represents the overall change in solute mole fraction divided by the average mole fraction driving force.
- The larger the *NTU*, the greater is the extent of contacting required.

Determining Height of Packing in the Tower: the HTU Method

But $K_y' = K_y(1-y_A)_{LM}$

$$\Rightarrow \quad Z = \int_{y_{out}}^{y_{in}} \left(\frac{V}{K_y' aA}\right) \frac{(1-y_A)_{*LM}}{(1-y_A)(y_A - y_A^*)} dy_A$$

- Graphical integration of right hand side of the equation is performed to find NTU_{OG}
- For dilute system (i.e. when the mole fractions y and x in the gas and liquid streams are less than about 0.10, i.e., 10%.)

$$Z = \left(\frac{V(1-y_A)_{*LM}}{K_y' aA(1-y_A)}\right) \int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^*)} dy_A$$

- Also , for dilute solutions: $(1-y_A) \approx (1-y_A)_{*LM} \approx 1.0$ and $K_y' = K_y$

$$\Rightarrow \quad Z = \left(\frac{V}{K'_y aA}\right)\int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^*)} dy_A$$

- The ratio of flow rate to mass transfer has been designated as the height of a transfer unit (HTU), or, for the gas phase, H_{OG}

$$H_{OG} = \left(\frac{V}{K'_y aA}\right)$$

- Therefore, H_{OG} has been defined in such a way that it remains constant through the absorption column.

Determining Height of Packing in the Tower: the HTU Method

- The number of overall mass transfer units (NTU), or for the gas phase, N_{OG} has been defined:

$$N_{OG} = \int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^*)} dy_A$$

- The height of the column may now be calculated from:

$$z = H_{OG} N_{OG}$$

Determining Height of Packing in the Tower: the HTU Method

- In similar manner, the analysis can be performed based on the liquid phase which would be useful in stripping calculations to give

$$\Rightarrow \quad Z = \left(\frac{L}{K_x' aA}\right) \int_{xin}^{xout} \frac{(1-x_A)_{*LM}}{(1-x_A)(x_A^* - x_A)} dx_A$$

Hence, $H_{OL} = \left(\frac{L}{K_x' aA}\right)$ and $N_{OL} = \int_{xin}^{xout} \frac{(1-x_A)_{*LM}}{(1-x_A)(x_A - x_A^*)} dx_A$

$$\Rightarrow \quad z = H_{OL} N_{OL}$$

- For dilute solutions: $(1-x_A) \approx (1-x_A)_{LM} \approx 1.0$ and $K_x' = K_x$

$$\Rightarrow \quad N_{OL} = \int_{xin}^{Xout} \frac{1}{(x_A^* - x_A)} dx_A$$

Summary

Table 6.7 Alternative Mass-Transfer Coefficient Groupings

	Height of a Transfer Unit, HTU			Number of Transfer Units, NTU		
Driving Force	Symbol	EM Diffusion or Dilute UM Diffusion	UM Diffusion	Symbol	EM Diffusion[a] or Dilute UM Diffusion	UM Diffusion
1. $(y - y^*)$	H_{OG}	$\frac{V}{K_y a S}$	$\frac{V}{K_y' a (1-y)_{LM} S}$	N_{OG}	$\int \frac{dy}{(y-y^*)}$	$\int \frac{(1-y)_{LM} dy}{(1-y)(y-y^*)}$
2. $(p - p^*)$	H_{OG}	$\frac{V}{K_G a P S}$	$\frac{V}{K_G' a (1-y)_{LM} P S}$	N_{OG}	$\int \frac{dp}{(p-p^*)}$	$\int \frac{(P-p)_{LM} dp}{(P-p)(p-p^*)}$
3. $(Y - Y^*)$	H_{OG}	$\frac{V'}{K_Y a S}$	$\frac{V'}{K_Y a S}$	N_{OG}	$\int \frac{dY}{(Y-Y^*)}$	$\int \frac{dY}{(Y-Y^*)}$
4. $(y - y_I)$	H_G	$\frac{V}{k_y a S}$	$\frac{V}{k_y' a (1-y)_{LM} S}$	N_G	$\int \frac{dy}{(y-y_I)}$	$\int \frac{(1-y)_{LM} dy}{(1-y)(y-y_I)}$
5. $(p - p_I)$	H_G	$\frac{V}{k_p a P S}$	$\frac{V}{k_p' a (P-p)_{LM} S}$	N_G	$\int \frac{dp}{(p-p_I)}$	$\int \frac{(P-p)_{LM} dp}{(P-p)(p-p_I)}$
6. $(x^* - x)$	H_{OL}	$\frac{L}{K_x a S}$	$\frac{L}{K_x' a (1-x)_{LM} S}$	N_{OL}	$\int \frac{dx}{(x^*-x)}$	$\int \frac{(1-x)_{LM} dx}{(1-x)(x^*-x)}$
7. $(c^* - c)$	H_{OL}	$\frac{L}{K_L a (\rho_L/M_L) S}$	$\frac{L}{K_L' a (\rho_L/M_L - c)_{LM} S}$	N_{OL}	$\int \frac{dc}{(c^*-c)}$	$\int \frac{(\rho_L/M_L - c)_{LM} dx}{(\rho_L/M_L - c)(c^*-c)}$
8. $(X^* - X)$	H_{OL}	$\frac{L'}{K_X a S}$	$\frac{L'}{K_X a S}$	N_{OL}	$\int \frac{dX}{(X^*-X)}$	$\int \frac{dX}{(X^*-X)}$
9. $(x_I - x)$	H_L	$\frac{L}{k_x a S}$	$\frac{L}{k_x' a (1-x)_{LM} S}$	N_L	$\int \frac{dx}{(x_I-x)}$	$\int \frac{(1-x)_{LM} dx}{(1-x)(x_I-x)}$
10. $(c_I - c)$	H_L	$\frac{L}{k_L a (\rho_L/M_L) S}$	$\frac{L}{k_L' a (\rho_L/M_L - c)_{LM} S}$	N_L	$\int \frac{dc}{(c_I-c)}$	$\int \frac{(\rho_L/M_L - c)_{LM} dc}{(\rho_L/M_L - c)(c_I-c)}$

[a]The substitution $K_y = K_y' y_{B_{LM}}$ or its equivalent can be made.

STEP-BY-STEP PROCEDURE

(1) For a particular gas-liquid system, draw equilibrium curve on *X*-*Y* plane.

(2) Draw operating line in *X*-*Y* plane (*PQ*) using material balance Equation. Lower terminal *Q* (X_2, Y_2) and upper terminal *P* (X_1, Y_1) are placed in *x*-*y* plane.

Overall mass balance Equation for the absorption tower is as follows:

$$L'\left(\frac{x_{A0}}{1-x_{A0}}\right)+V'\left(\frac{y_{A2}}{1-y_{A2}}\right)=L'\left(\frac{x_{A1}}{1-x_{A1}}\right)+V'\left(\frac{y_{A1}}{1-y_{A1}}\right)$$

If liquid mass flow rate, L' is not known, minimum liquid mass flow rate $L'_{\min}$, is to be determined. L' is generally 1.2 to 2 times the $L'_{\min}$.

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STEP-BY-STEP PROCEDURE

In the figure, lower terminal of absorption tower is represented by *Q* (X_2, Y_2); i.e., bottom of the tower. Operating line is *PQ*. If liquid rate is decreased, slope of operating line (L'/V') also decreases and operating line shifts from *PQ* to $P'Q$, when touches equilibrium line. This operating line is tangent to equilibrium line.

The driving force for absorption is zero at P' and is called "PINCH POINT".

(3) A point *A* (*x*, *y*) is taken on the operating line. From the known value of k_x and k_y or $k_x\bar{a}$ and $k_y\bar{a}$, a line is drawn with slope of k_x/k_y to equilibrium line, $B(x_i,y_i)$. Line *AB* is called "TIE LINE" and x_i and y_i are known for a set of values of *x* and *y*.

(4) Step (3) is repeated for other points in the operating line to get several (x_i,y_i) sets for $y_1 \geq y \geq y_2$.

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STEP-BY-STEP PROCEDURE

(5) Calculate flow rate of gas $V = V'/(1 - y_A)$ at each point

(6) Calculate height of the packing graphically or numerically.

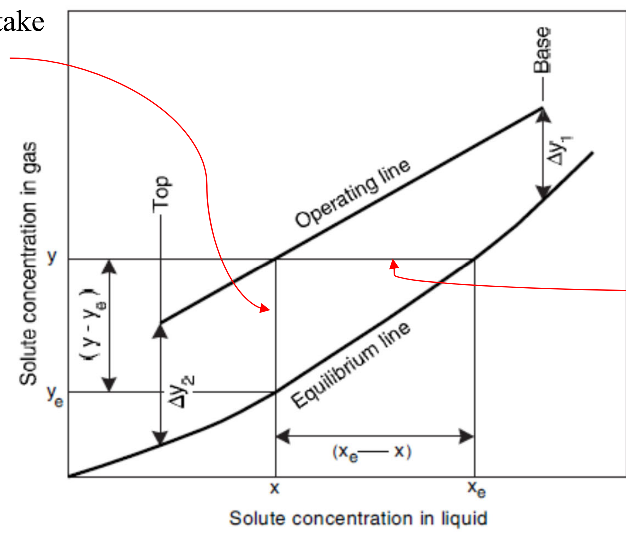
$$Z = \left(\frac{V}{K'_y aA}\right) \int_{y_{out}}^{y_{in}} \frac{(1 - y_A)_{*LM}}{(1 - y_A)(y_A - y_A^*)} dy_A$$

Summary

At a given x take

$\Delta y = (y - y_e)$

$$\int_{y_{A\,out}}^{y_{A\,in}} \frac{dy}{(y_A - y_A^*)}$$



Gas absorption concentration relationships

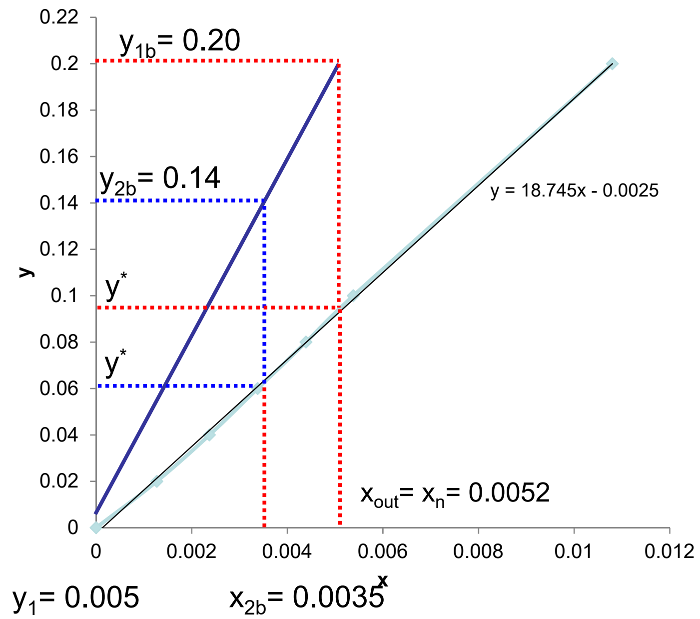
At a given y take

$\Delta x = (x_e - x)$

$$N_{OL} = \int_{Xin}^{Xout} \frac{1}{(x_A^* - x_A)} dx_A$$

Example

$$\Rightarrow y_{n+1} = 37.645 x_n + 0.005$$



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Summary

$$\int_{y_{A\,out}}^{y_{A\,in}} \frac{dy}{(y_A - y_A^*)}$$

y_A	y_A^*	$(y_A-y_A^*)$	$1/(y_A-y_A^*)$

- Draw y_A vs. $1/(y_A{}^* - y_A)$
- Then find area under the curve numerically

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Summary

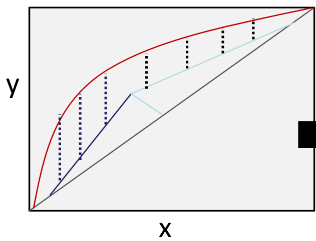
$$Z = \left(\frac{V}{K_y a A}\right) \int_{yout}^{yin} \frac{1}{(y_A - y_A^*)} dy_A$$

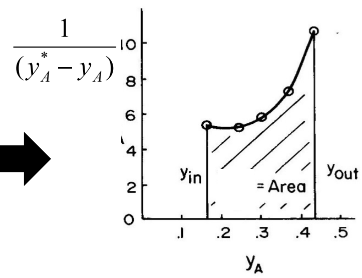
H_{OG}

Substitute values to calculate H_{OG}

Integration = N_{OG}

- N_{OG} is evaluated graphically by numerical integration using the equilibrium and operating lines.
- Draw $1/(y_A^* - y_A)$ (on y-axis) vs. y_A (on x-axis). Area under the curve is the value of integration.





Evaluate area under the curve by numerical integration

Area = N

Numerical Evaluation of Integrals

Trapezoidal rule (2-point):

$$\int_0^{X_1} f(x)dx = \frac{h}{2}[f(X_0) + f(X_1)]$$

$$h = X_1 - X_0$$

Simpson's one-third rule (3-point):

$$\int_0^{X_2} f(x)dx = \frac{h}{3}[f(X_0) + 4f(X_1) + f(X_2)]$$

$$h = \frac{X_2 - X_0}{2} \qquad X_1 = X_0 + h$$

Simpson's three-eights rule (4-point):

$$X_1 = X_0 + h \quad X_2 = X_0 + 2h$$

$$\int_0^{X_3} f(x)dx = \frac{3}{8}h[f(X_0) + 3f(X_1) + 3f(X_2) + f(X_3)]$$

$$h = \frac{X_3 - X_0}{3}$$

Simpson's five-point quadrature :

$$\int_0^{X_4} f(x)dx = \frac{h}{3}[f(X_0) + 4f(X_1) + 2f(X_2) + 4f(X_3) + f(X_4)]$$

$$h = \frac{X_4 - X_0}{4}$$

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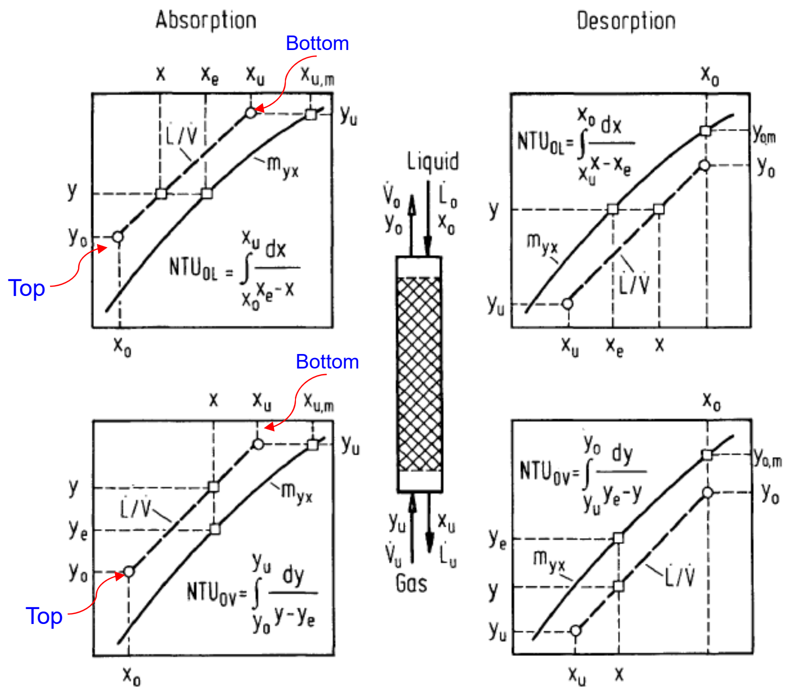
Summary



Summary

x, y . Mole fraction in liquid, gas or vapour

m_{yx} Slope of the equilibrium line ———

$\dot{L}/\dot{V}$ Molar liquid-to-gas ratio or slope of the operating line – – –

X, Y Load fraction of transfer component in liquid / gas

L, V Carrier stream of liquid / gas phase

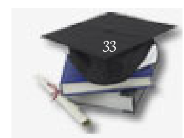
$$X_u = X_o + \frac{1}{L/V}(Y_u - Y_o) \qquad \frac{L}{V} = \frac{Y_o - Y_u}{X_o - X_u}, \qquad Y_o = Y_u + \frac{L}{V}(X_o - X_u)$$

Example

We wish to strip SO_2 from water using air at 20C. The inlet air is pure. The outlet water contains 0.0001 mole fraction SO_2, while the inlet water contains 0.0011 mole fraction SO_2. Operation is at 855 mmHg and L/V = 0.9×$(L/V)_{max}$. Assume H_{OL} = 2.76 feet and that the Henry's law constant is 22,500 mmHg/mole frac SO_2.

Calculate the packing height required.

$$Z = \left(\frac{L}{K_x' aA} \right) \int_{xin}^{xout} \frac{(1-x_A)_{LM}}{(1-x_A)(x_A^* - x_A)} dx_A$$



Example contd.

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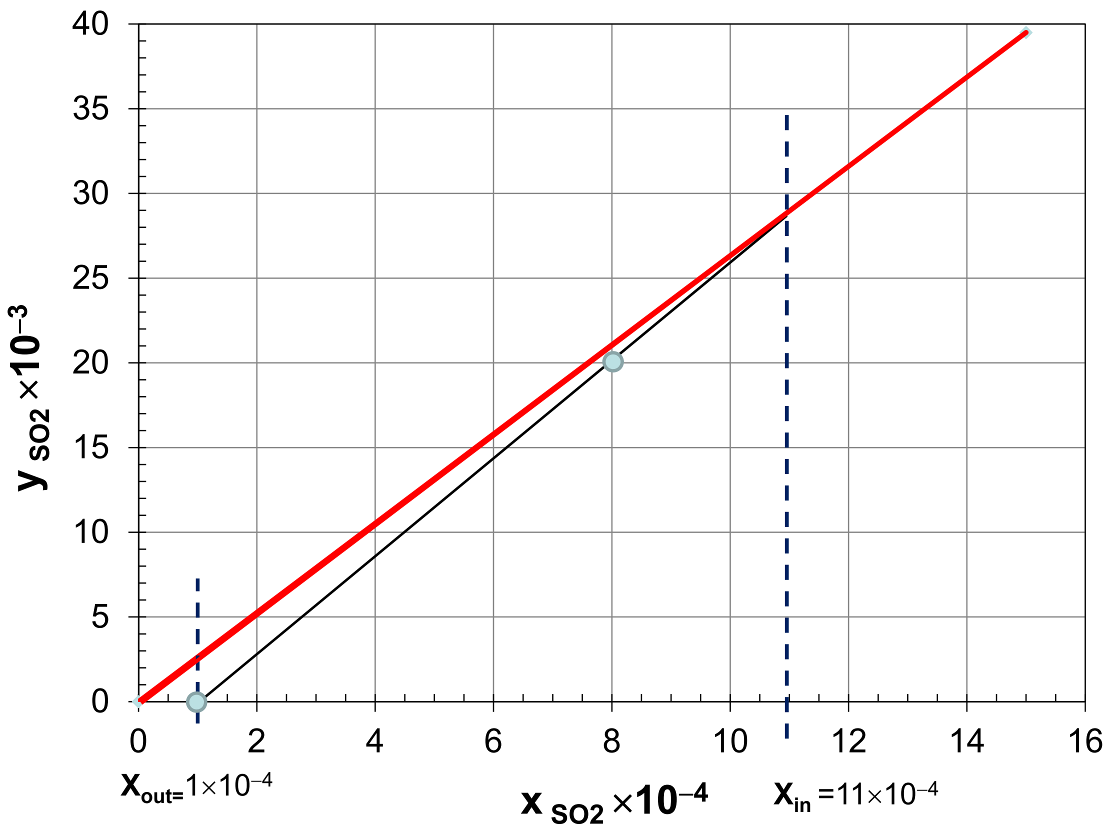

Example contd.

$(L/V) = 0.9\ (L/V)_{max}$

From pinch point and darwing, $(L/V)_{max}$ = slope= 29.29

$\Rightarrow (L/V) = 0.9 \times 29.29 = 26.36$

$y_{out} = 10\text{x}10^{-4}\ (L/V) = 10\text{x}10^{-4} \times 26.36$

$\Rightarrow$ **$y_{out} = 0.02636 = 26.36\times10^{-3}$**

Draw actual operating line

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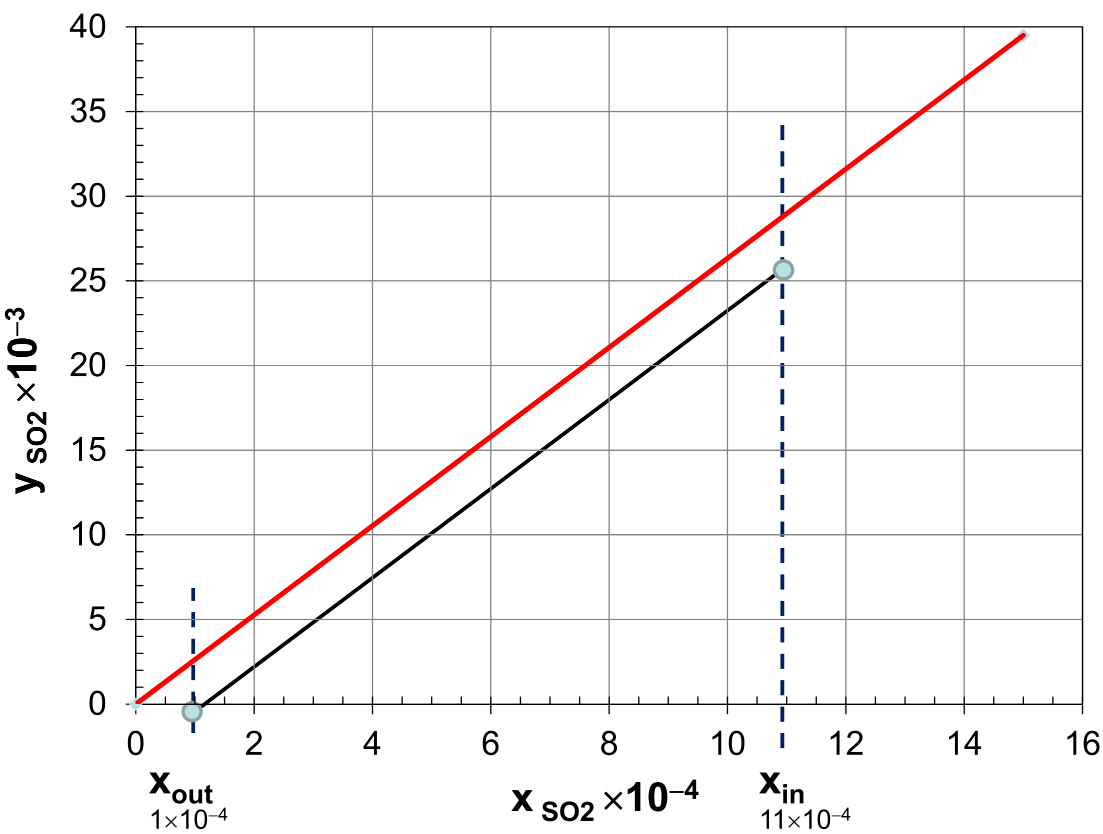
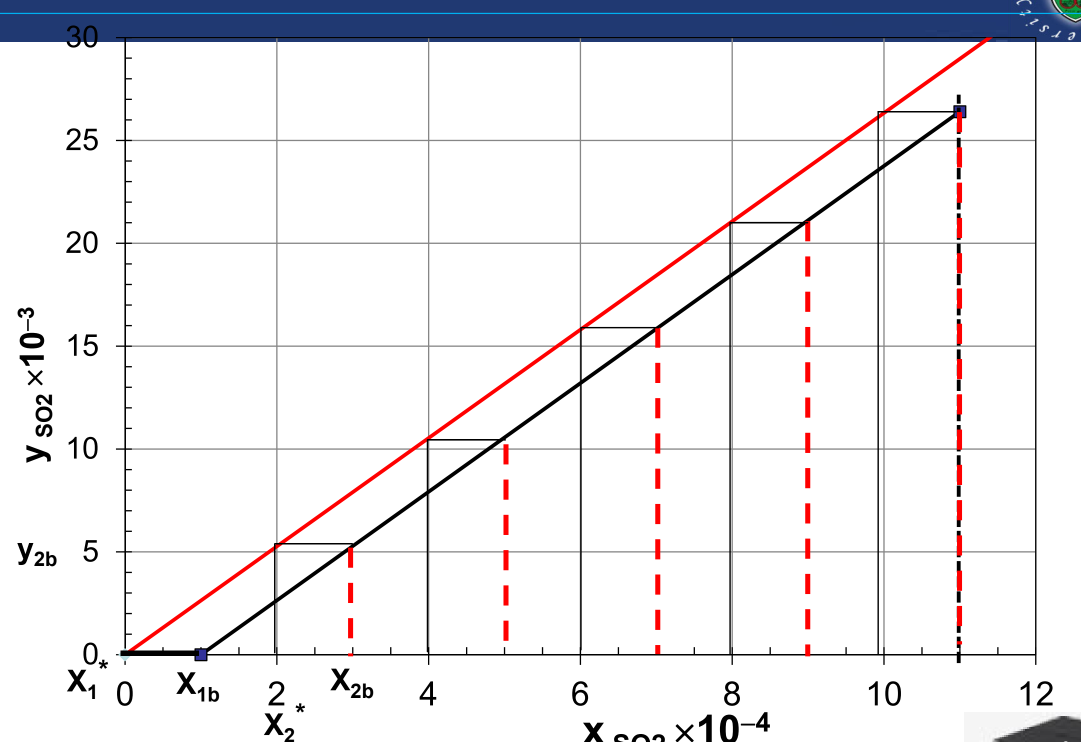

40
35
30
25
20
15
10
5
0
0
2
4
6
8
10
12
14
16
y SO2 ×10−3
x SO2 ×10−4
xout
1×10−4
xin
11×10−4


30
25
20
15
10
5
0
y2b
0
2
4
6
8
10
12
y SO2 ×10−3
x SO2 ×10−4
x1*
x1b
x2*
x2b

Example contd.

x	x*	1/(x-x*)
1.0E-4	0	10,000
3.0E-04	2.0E-04	10,000
5.0E-04	4.0E-04	10,000
7.0E-04	6.0E-04	10,000
9.0E-04	8.0E-04	10,000
1.1E-03	1.0E-03	10,000

Apply a graphical or numerical method for evaluating N_{OL}

$$\int_{x_{A\,out}=0.0001}^{x_{A\,in}=0.0011} \frac{dx}{(x_A - x_A^*)}$$

For example, we can use Simpson's rule.

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + \ldots) \ +2(y_2 + y_4 + y_6 + \ldots) + y_n\right]$$

$$\Delta x = \frac{b-a}{n}$$

This gives us an easy way to remember Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[\text{FIRST} + 4(\text{sum of ODDs}) \ +2(\text{sum of EVENs}) + \text{LAST}\right]$$

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Example contd.

For the current problem

$$\int_{x_{A\,out}=0.0001}^{x_{A\,in}=0.0011} \frac{dx}{(x_A - x_A^*)}$$

$$f(X) = \frac{1}{(x - x^*)}$$

Substituting values from Table gives N_{OL}= 7.746.

Z = H_{OL}(given) × N_{OL}(calculated) = 2.76 × 7.746

⇒ **<u>Z = 21.38 ft</u>**

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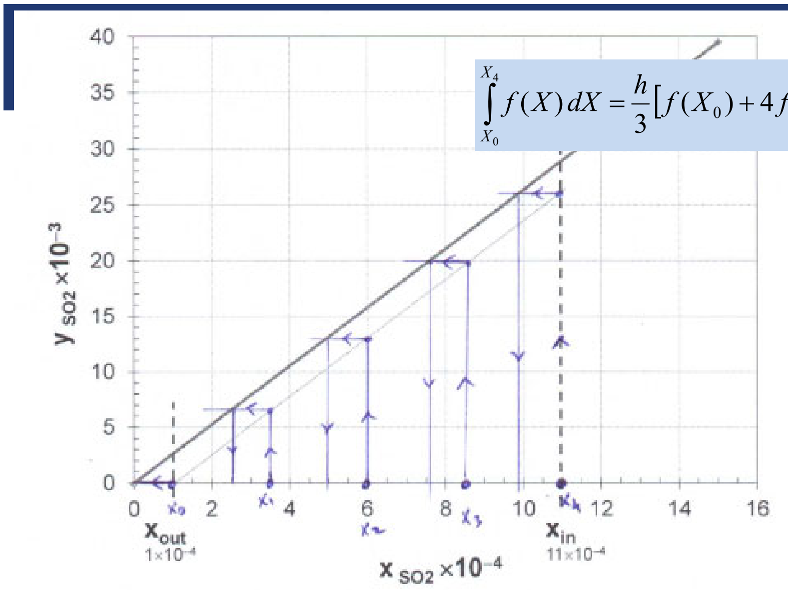
5-point method



$$\int_{X_0}^{X_4} f(X)\,dX = \frac{h}{3}\left[f(X_0)+4f(X_1)+2f(X_2)+4f(X_3)+f(X_4)\right]$$

$$h = \frac{(X_4 - X_0)}{4}$$

Pay attention to accuracy of drawing and obtaining data.

	x	x*	f(x) = 1/(x-x*)
0	1.0E-4	0	10,000
1	3.5×10^{-4}	2.5×10^{-4}	10,000
2	6×10^{-4}	5×10^{-4}	10,000
3	8.5×10^{-4}	7.5×10^{-4}	10,000
4	11E-4	10.0×10^{-4}	10,000

Analytical Solution: Dilute Solution Case

- Also, for dilute solutions, Henry's Law is usually a good choice for an equilibrium relationship.

$$y_{Ai} = m x_{Ai}$$

And $(1-y_A) \approx (1-y_A)_{LM} \approx 1.0$

Therefore,

$$N_{OG} = \int_{y_{out}}^{y_{in}} \frac{(1-y_A)_{LM}}{(1-y_A)(y_A - y_A^*)}\,dy_A = \int_{y_{out}}^{y_{in}} \frac{1}{(y_A - y_A^*)}\,dy_A$$

The operating line,

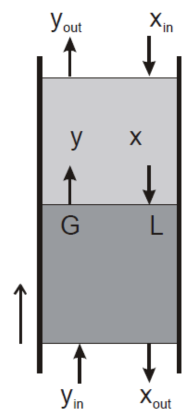
$$y = \frac{L}{G}x + y_{out} - \frac{L}{G}x_{in}$$

$$y_A = \frac{L}{G}(x_A - x_{in}) + y_{A1} = \frac{L}{mG}(mx_A - mx_{in}) + y_{out}$$

Analytical Solution: Dilute Solution Case

$$y - y^* = y - mx = y - m\left(\frac{G}{L}y - \frac{G}{L}y_{out} + x_{in}\right)$$

$$= \left(1 - \frac{mG}{L}\right)y + \frac{mG}{L}y_{\text{out}} - mx_{\text{in}}$$



$$\Rightarrow \quad n_{OG} = \int_{y_{out}}^{y_{in}} \frac{dy_A}{\left(1 - \frac{mG}{L}\right)y + \frac{mG}{L}y_{out} - mx_{in}}$$

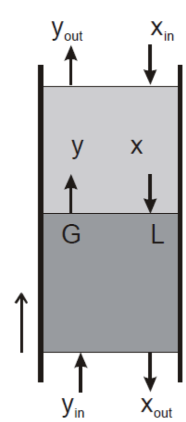
- Keep in mind that the absorption factor, A_b, can be defined as:

$$A_b = \frac{L}{mG}$$

Analytical Solution: Dilute Solution Case

or

$$n_{OG} = \frac{1}{1 - \frac{mG}{L}} \ln\left[\frac{\left(1 - \frac{mG}{L}\right)y_{in} + \frac{mG}{L}y_{out} - mx_{in}}{y_{out} - mx_{in}}\right]$$



Since $y^*_{out} = mx_{in}$, we have

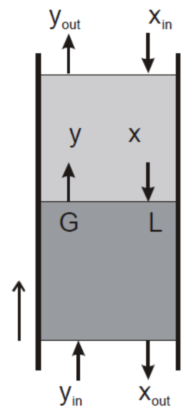
$$\Rightarrow \quad n_{OG} = \frac{1}{1 - \frac{mG}{L}} \ln\left[\left(1 - \frac{mG}{L}\right)\frac{y_{in} - y^*_{out}}{y_{out} - y^*_{out}} + \frac{mG}{L}\right]$$

Analytical Solution: Dilute Solution Case

We can follow a similar procedure to obtain the number of overall liquid transfer unit

$$n_{OL} = \int_{x_{in}}^{x_{out}} \frac{dx_A}{(x - x^*)}$$

$$n_{OL} = \frac{1}{1 - \frac{L}{mG}} \ln\left[\left(1 - \frac{L}{mG}\right)\frac{x_{in} - x^*_{out}}{x_{out} - x^*_{out}} + \frac{L}{mG}\right]$$



where $x^*_{out} = y_{in}/m$.

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Example

Subject: Absorption of SO_2 from air into water in an existing packed column.

Given: Feed gas flow rate of 0.062 kmol/s containing 1.6 mol% SO_2. Absorbent is 2.2 kmol/s of pure water. Packed column is 1.5 m^2 in cross sectional area and packed with No. 2 plastic super Intalox saddles to a 3.5-m height. Exit gas contains an SO_2 mole fraction of 0.004. Operating pressure is 1 atm. At operating temperature, equilibrium curve for SO_2 is $y = Kx = 40x$

Assumptions: No stripping of water. No absorption of air.

Find: (a) L/L_{min}
(b) N_{OG} and N_t
(c) H_{OG} and HETP
(d) $K_G a$

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Example Contd

Example Contd

Example Contd

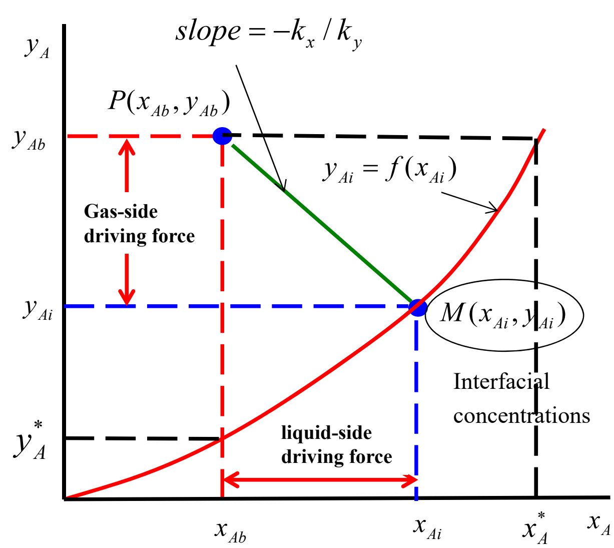
Interface composition in terms of the ratio of mass transfer coefficient

- Re-arranging

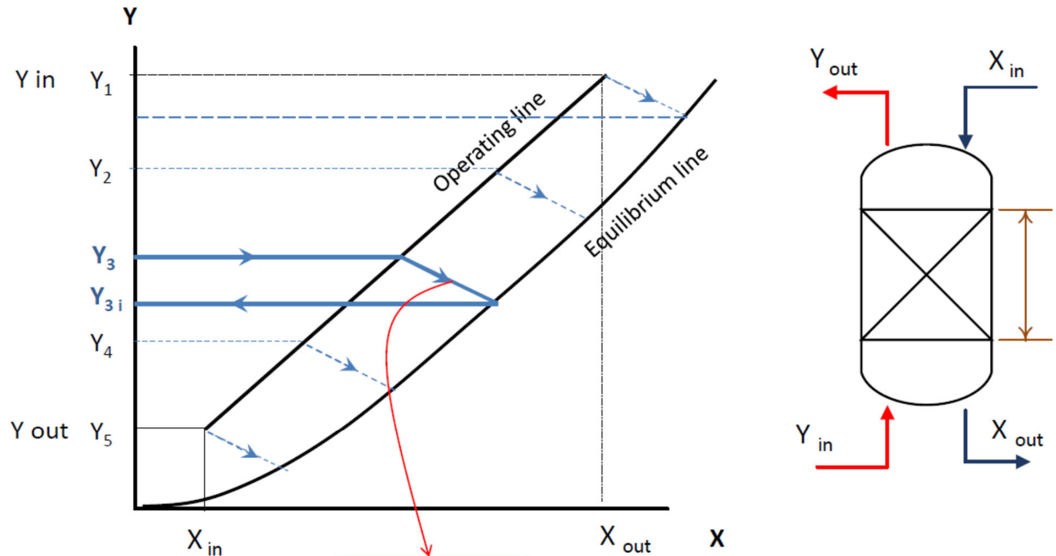
$$-\frac{k_x}{k_y} = \frac{y_{Ab} - y_{Ai}}{x_{Ab} - x_{Ai}}$$

- The mass transfer may now be written based on the overall mass-transfer coefficient

$$N_A = K_x(x_A^* - x_A) = K_y(y_A - y_A^*)$$



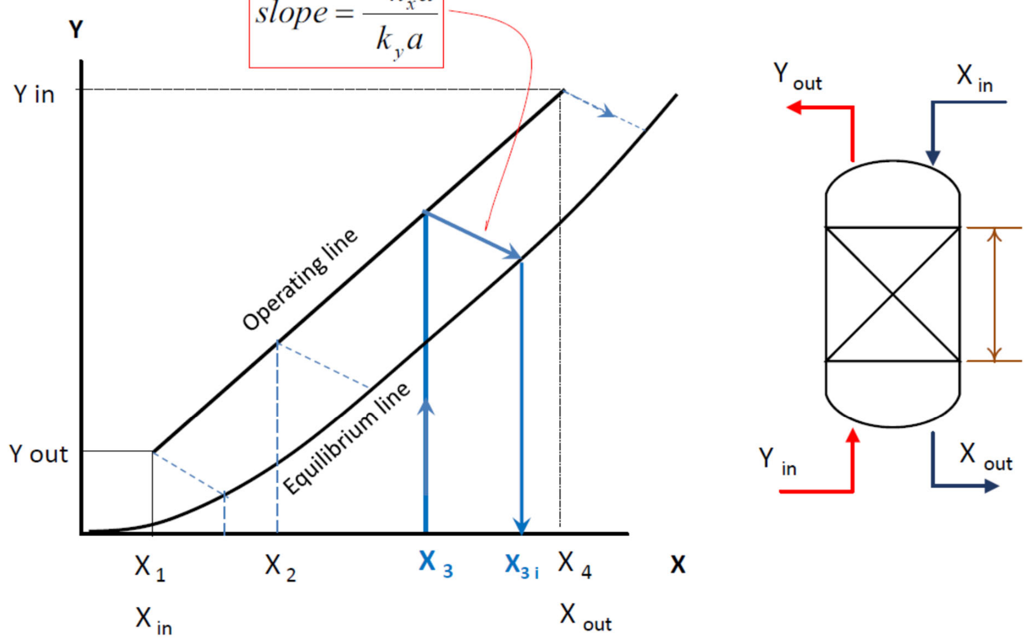
Counter-current Absorption (local gas phase)



$$slope = \frac{-k_x a}{k_y a}$$

Counter-current Absorption (local liquid phase)

$$slope = \frac{-k_x a}{k_y a}$$



Task 1

Experimental data have been obtained for air containing 1.6% by volume of SO_2 being scrubbed with pure water in a packed column of 1.5 m^2 in cross-sectional area and 3.5 m in packed height. Entering gas and liquid flow rates are 0.062 and 2.2 kmol/s, respectively. If the outlet mole fraction of SO_2 in the gas is 0.004 and column temperature is near ambient with K_{SO2} = 40, calculate the following:

a) The N_{OG} for absorption of SO_2

b) The H_{OG} in meters

c) The volumetric, overall mass-transfer coefficient, K_ya for SO_2 in kmol/m^3.s

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Task 2

A gaseous reactor effluent consisting of 2 mol% ethylene oxide in an inert gas is scrubbed with water at 30^oC and 20 atm. The total gas feed rate is 2500 lbmol/h, and the water rate entering the scrubber is 3500 lbmol/h. The column, with a diameter of 4 ft, is packed in two 12-ft-high sections with 1.5 in metal Pall rings. A liquid redistributer is located between the two packed sections. Under the operating conditions for the scrubber, the K-value (y = K x) for ethylene oxide is 0.85 and estimated values of k_ya and k_xa are 200 lbmol/h.ft^3 and 2643 lbmol/h.ft^3 , respectively. Calculate the following:

a) K_ya

b) H_{OG} and N_{OG}

c) Y_{out} and x_{out}

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