Unsteady-State Diffusion

Unsteady-State diffusion in Various Geometries

- Analytical method
- Chart method
- The techniques of solution are similar to transient or unsteady-state heat transfer by conduction.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial c_A}{\partial t} = D_{AB} \frac{\partial^2 c_A}{\partial x^2}$$

- We are going to derive the solution for unsteady-state diffusion in the x direction for a plate of thickness $2x_1$

Conditions of the problem

- The initial profile of the concentration in the plate at t = 0 is uniform at $c = c_0$ for all x values, as shown in Fig. 1.
- At time t = 0, the concentration of the fluid in the environment outside is suddenly changed to c_1.
- For a very high mass-transfer coefficient outside the surface, resistance will be negligible and the concentration at the surface will be equal to that in the fluid, which is c_1.

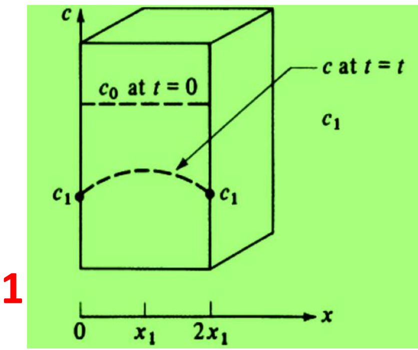


Fig. 1

Diffusion in a flat plate with negligible surface resistance "Analytical method"

- **One-dimensional Unsteady-state Diffusion**

$$\frac{\partial c_A}{\partial t} = D_{AB} \frac{\partial^2 c_A}{\partial x^2} \quad \text{.......................(A)}$$

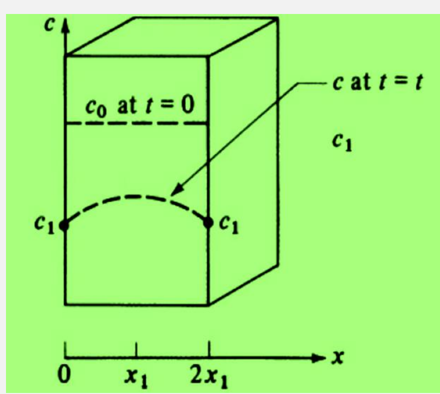
Define

$$Y = \frac{c_1 - c}{c_1 - c_0}$$

$$\frac{\partial Y}{\partial t} = D \frac{\partial^2 Y}{\partial x^2}$$

Unsteady-state diffusion in a flat plate with negligible surface resistance.

For a very high mass-transfer coefficient outside the surface, resistance will be negligible and the concentration at the surface will be equal to that in the fluid which is c_1.



The initial and boundary conditions are

$$c = c_0, \quad t = 0, \quad x = x, \quad Y = \frac{c_1 - c_0}{c_1 - c_0} = 1$$

$$c = c_1, \quad t = t, \quad x = 0, \quad Y = \frac{c_1 - c_1}{c_1 - c_0} = 0$$

$$c = c_1, \quad t = t, \quad x = 2x_1, \quad Y = \frac{c_1 - c_1}{c_1 - c_0} = 0$$

<u>Note</u>: The solution of Eq. (A) is an infinite Fourier series and is identical to the solution of the conduction 1-D heat transfer.

The solution to the previous equation is:

$$Y = \frac{c_1 - c}{c_1 - c_0} = \frac{4}{\pi}\left[\frac{1}{1}\exp\left(-\frac{1^2\pi^2 X}{4}\right)\sin\frac{1\pi x}{2x_1} + \frac{1}{3}\exp\left(-\frac{3^2\pi^2 X}{4}\right)\sin\frac{3\pi x}{2x_1} + \frac{1}{5}\exp\left(-\frac{5^2\pi^2 X}{4}\right)\sin\frac{5\pi x}{2x_1} + \cdots\right]$$

where,

$X = Dt/x_1^2$, dimensionless

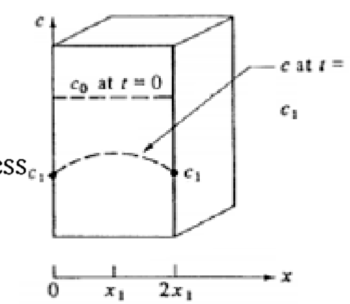
c = concentration at point x and time t in slab

$Y = (c_1 - c)/(c_1 - c_0)$, dimensionless

fraction of unaccomplished change, dimensionless

$1 - Y = (c - c_0)/(c_1 - c_0)$

= fraction of change



The above solution is time consuming!
<u>Charts</u> for various geometries are available and usually used.

Unsteady-State Diffusion in Various Geometries- Chart Method

- *Convection and boundary conditions at the surface.*

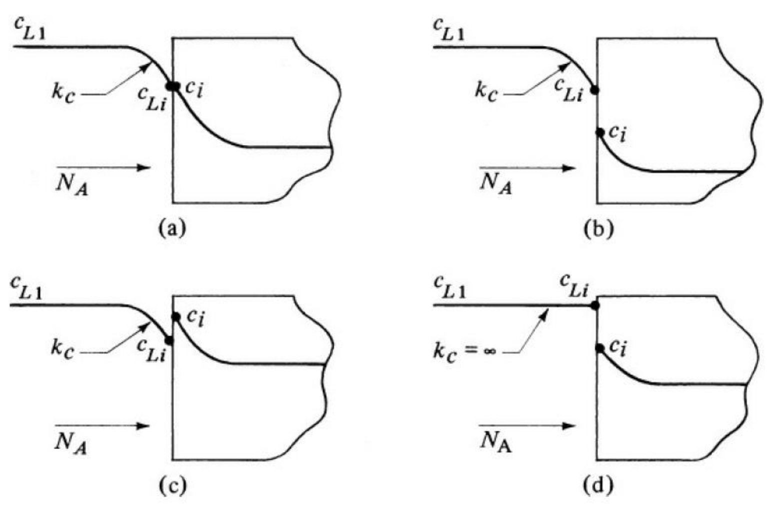
In many cases when a fluid is outside the solid, convective mass transfer is occurring at the surface. A convective mass-transfer coefficient k_c, similar to a convective heat-transfer coefficient, is defined as follows:

$$N_A = k_c(c_{L1} - c_{Li})$$

where k_c is a mass-transfer coefficient in m/s, c_{L1} is the bulk fluid concentration in kg mol A/m^3, and c_{Li} is the concentration in the fluid just adjacent to the surface of the solid. The coefficient k_c is an empirical coefficient and will be discussed more fully in **Next lectures.**

Different Conditions of mass transfer coefficients.

Case a where a mass-transfer coefficient is present at the boundary is shown. The concentration drop across the fluid is $c_{L1} - c_{Li}$. The concentration in the solid c_i at the surface is in equilibrium with c_{Li}.

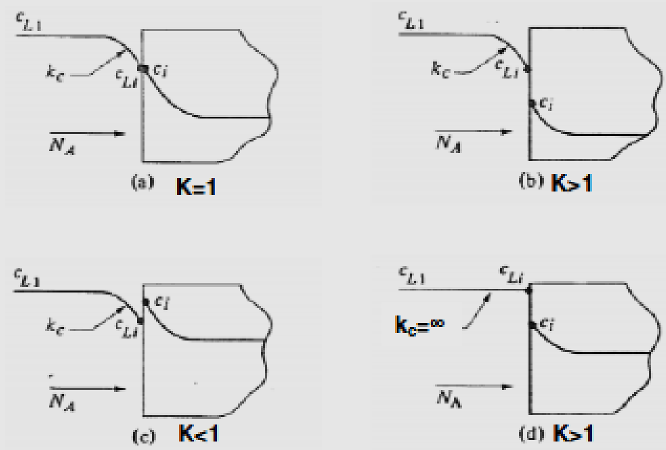


Note: for case **a**, the concentration c_{Li} in the liquid adjacent to the solid and c_i in the solid at the surface are in equilibrium and are equal.

Unlike heat transfer, where the temperatures are equal, the concentrations here are in equilibrium and are related by $K = \frac{c_{Li}}{c_i}$ where K is the equilibrium distribution coefficient (similar to henry's Law coefficient for gas and liquid). K for case a = 1.

Note: If the mass transfer at the surface is not negligible, then the following conditions may present:

Summary for all cases



Interface conditions for mass transfer.

The concentration in the liquid is obtained from a relation similar to Henry's law for gases:

$$c_{Li} = Kc_i$$

This K is called the equilibrium distribution coefficient

Charts for Diffusion

Charts for diffusion in various geometries. The various heat-transfer charts for unsteady-state conduction can be used for unsteady-state diffusion and are as follows.

1. Semiinfinite solid, Fig. 5.3-3.
2. Flat plate, Figs. 5.3-5 and 5.3-6.
3. Long cylinder, Figs. 5.3-7 and 5.3-8.
4. Sphere, Figs. 5.3-9 and 5.3-10.
5. Average concentrations, zero convective resistance, Fig. 5.3-13.

See Geankoplis text Book for these charts

Unsteady-state heal conducted in a semi-infinite solid with Surface convection

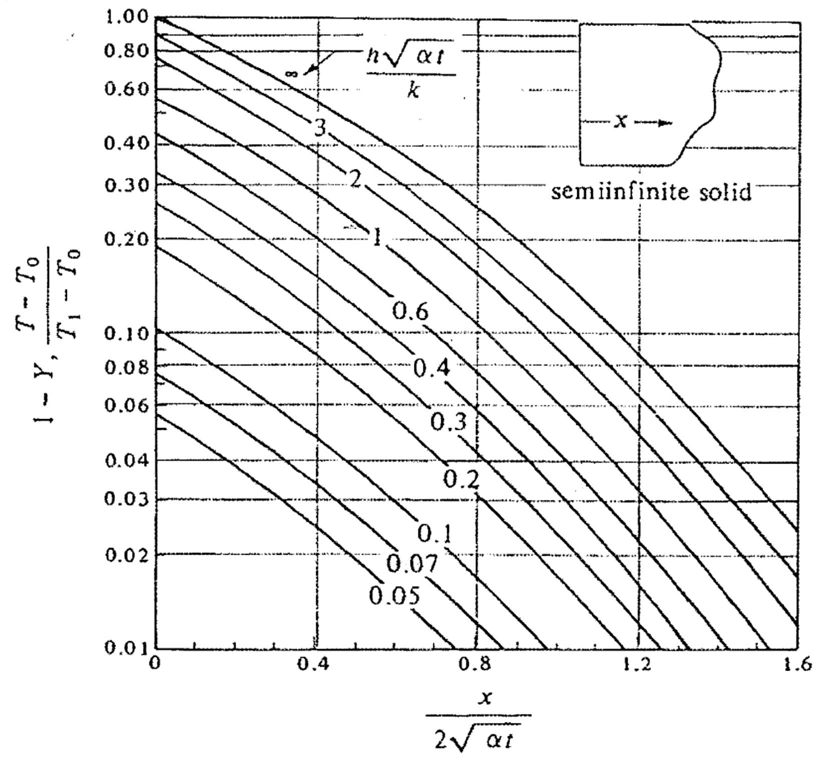
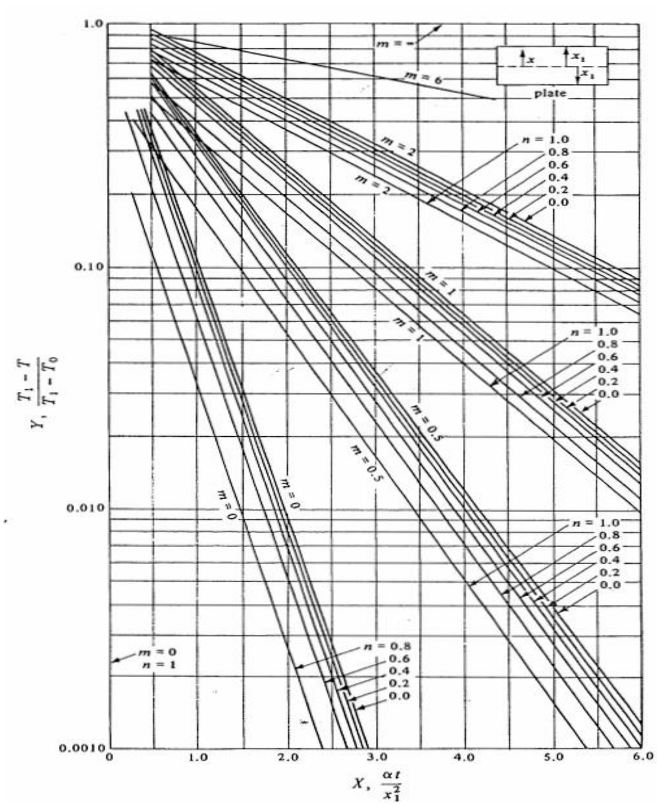


Figure 5.3-3.

One-dimensional Unsteady-state Diffusion _ Chart Method

- **For a flat plate:** *Figure 5.3.5*



*Relation Between Mass- and Heat-Transfer Parameters for Unsteady-State Diffusion**

Table A

Heat Transfer	Mass Transfer: $K = c_L/c = 1.0$	Mass Transfer: $K = c_L/c \neq 1.0$
$Y, \frac{T_1 - T}{T_1 - T_0}$	$\frac{c_1 - c}{c_1 - c_0}$	$\frac{c_1/K - c}{c_1/K - c_0}$
$1 - Y, \frac{T - T_0}{T_1 - T_0}$	$\frac{c - c_0}{c_1 - c_0}$	$\frac{c - c_0}{c_1/K - c_0}$
$X, \frac{\alpha t}{x_1^2}$	$\frac{D_{AB} t}{x_1^2}$	$\frac{D_{AB} t}{x_1^2}$
$\frac{x}{2\sqrt{\alpha t}}$	$\frac{x}{2\sqrt{D_{AB} t}}$	$\frac{x}{2\sqrt{D_{AB} t}}$
$m, \frac{k}{h x_1}$	$\frac{D_{AB}}{k_c x_1}$	$\frac{D_{AB}}{K k_c x_1}$
$\frac{h}{k}\sqrt{\alpha t}$	$\frac{k_c}{D_{AB}}\sqrt{D_{AB} t}$	$\frac{K k_c}{D_{AB}}\sqrt{D_{AB} t}$
$n, \frac{x}{x_1}$	$\frac{x}{x_1}$	$\frac{x}{x_1}$

* x is the distance from the center of the slab, cylinder, or sphere; for a semiinfinite slab, x is the distance from the surface. c_0 is the original uniform concentration in the solid, c_1 the concentration in the fluid outside the slab, and c the concentration in the solid at position x and time t.

For a flat plate

Chart for determining concentration at the center of a large flat plate for unsteady state diffusion

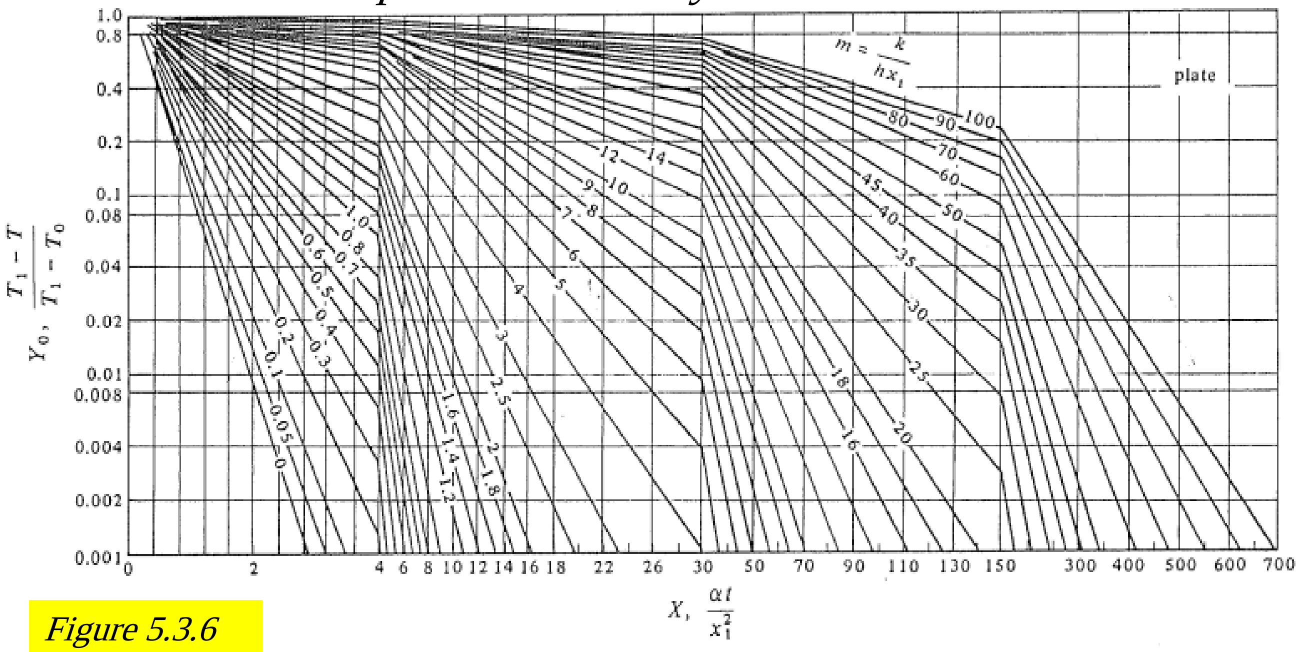


Figure 5.3.6

One-dimensional Unsteady-state Diffusion

For a long cylinder

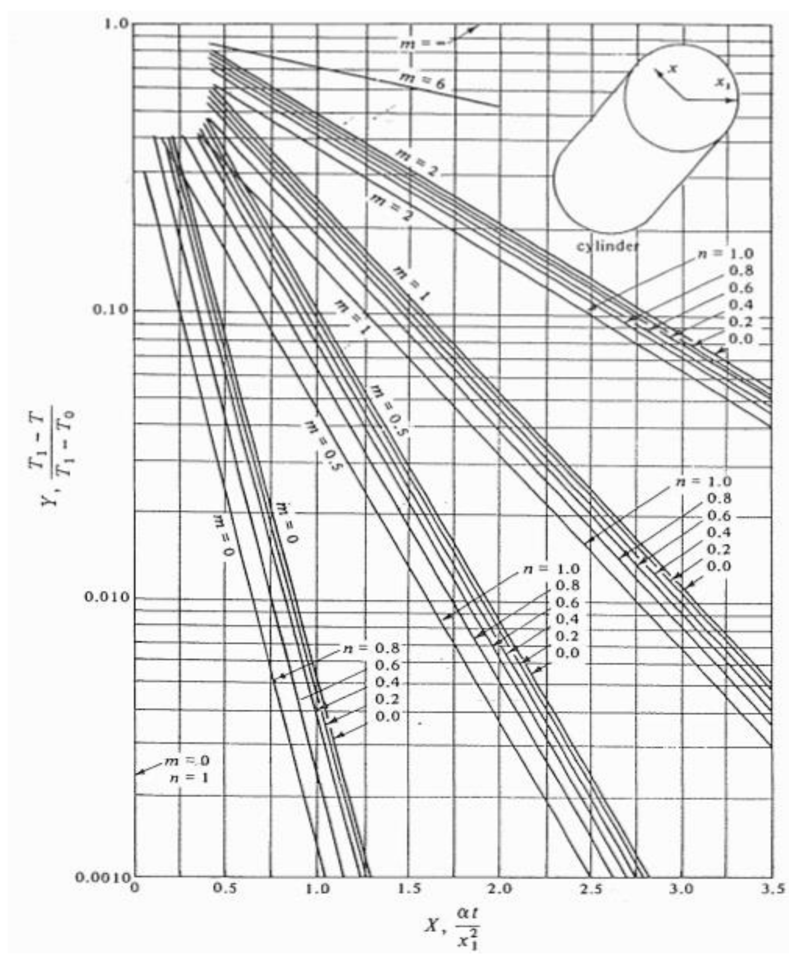


Figure 5.3.7

For a long cylinder {Center point}

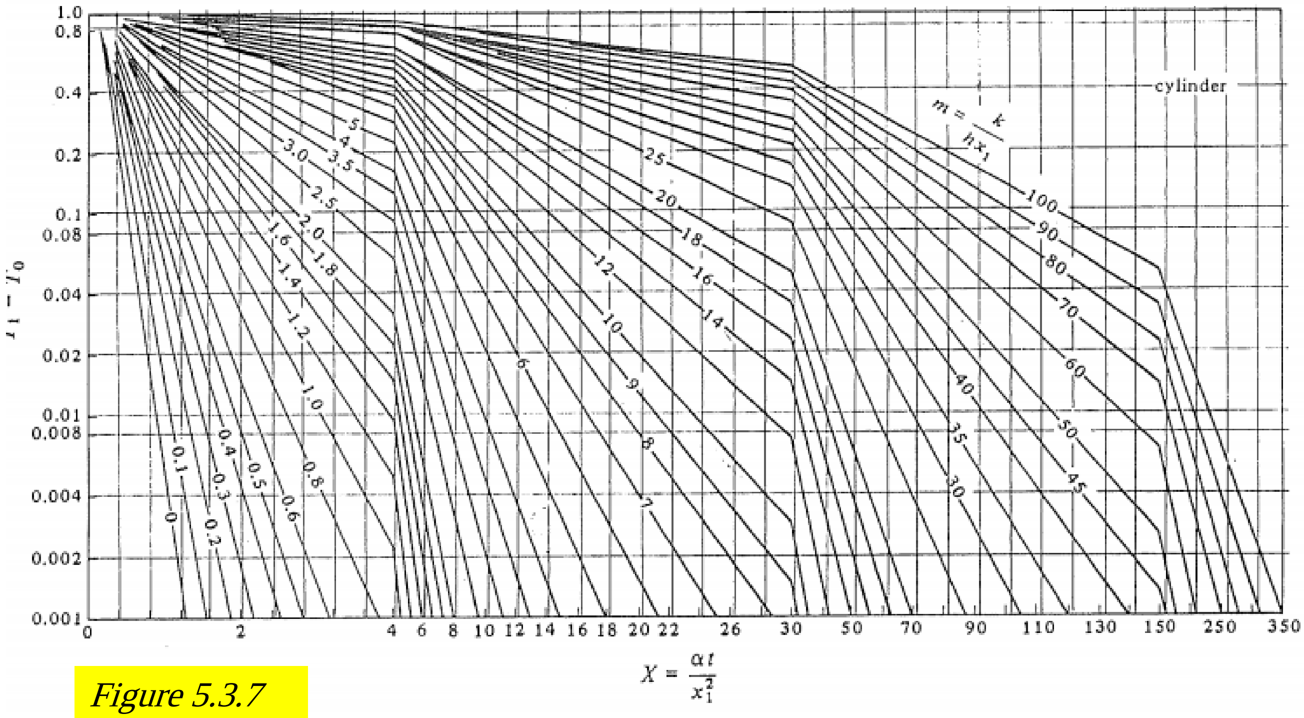
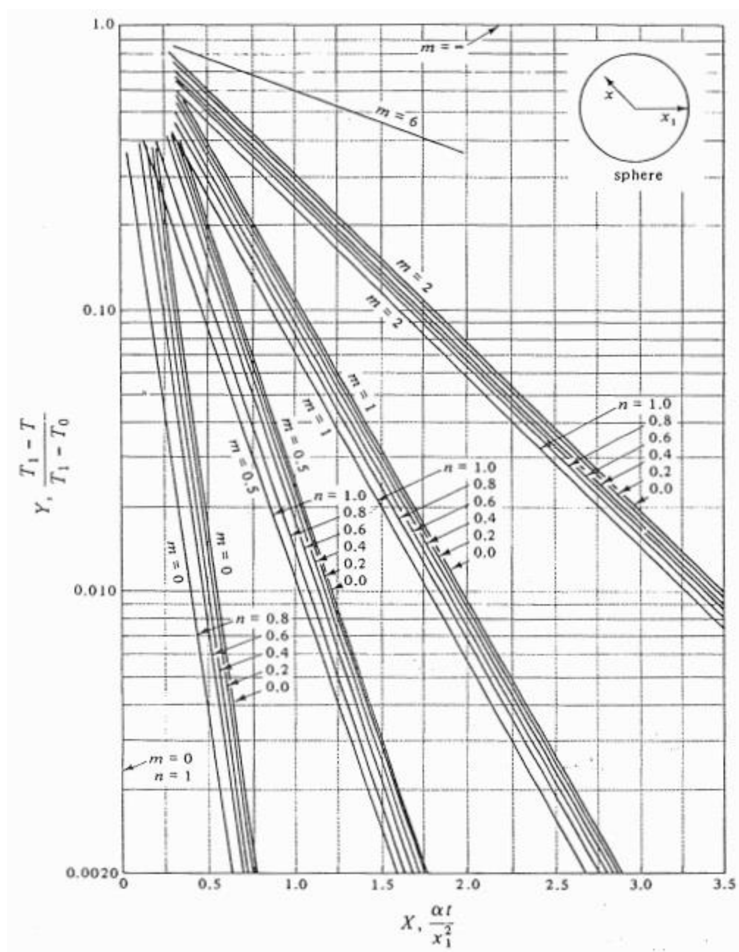


Figure 5.3.7

One-dimensional Unsteady-state Diffusion

- For a sphere

Figure 5.3.9



For a sphere {Center point}

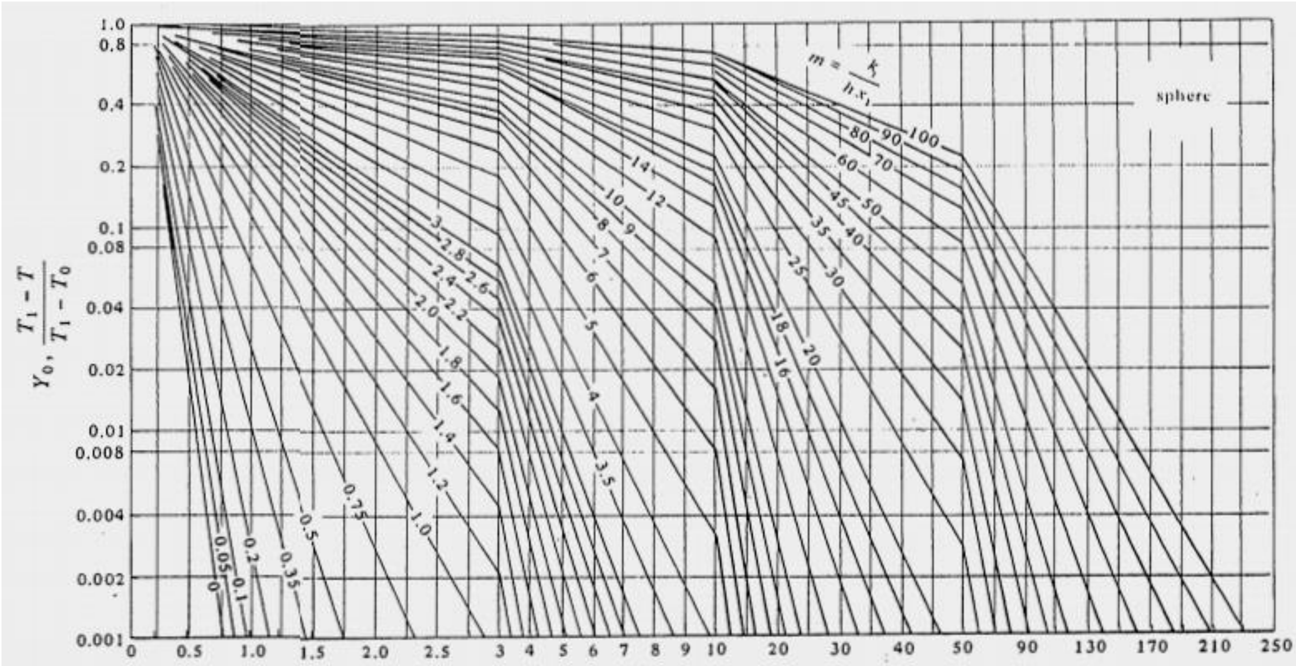


Figure 5.3.10