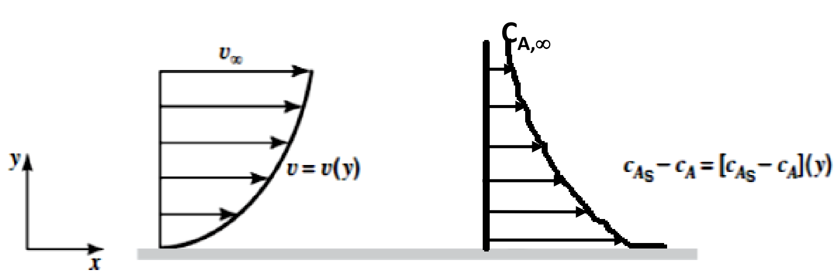
Convection mass transfer

Dimensional analysis

Convection mass transfer

- Consider the mass transfer of solute A from a solid to a fluid flowing past the surface of the solid. The concentration profile is depicted in the Figure below. For such a case, the mass transfer between the surface and the fluid may be written as

$$N_A = k_c(c_{A,s} - c_{A,\infty}) \quad (1)$$



Since the mass transfer at the surface is by molecular diffusion, the mass transfer may also be described by

$$N_A = -D_{AB} \left. \frac{dc_A}{dy} \right|_{y=0}$$

When the boundary concentration, $C_{A,s}$, is constant, this equation simplifies to

$$N_A = -D_{AB} \left. \frac{d(c_A - c_{A,s})}{dy} \right|_{y=0} \qquad (2)$$

Equations (1) and (2) may be equated, as they define the same flux of component A leaving the surface and entering the fluid. This gives the relation

$$k_c(c_{A,s} - c_{A,\infty}) = -D_{AB} \left. \frac{d}{dy}(c_A - c_{A,s}) \right|_{y=0}$$

which may be rearranged into the following form:

$$\frac{k_c}{D_{AB}} = \frac{-d(c_A - c_{A,s})/dy|_{y=0}}{(c_{A,s} - c_{A,\infty})} \qquad (3)$$

Multiplying both sides of equation (3) by a significant length, L, we obtain the following dimensionless expression:

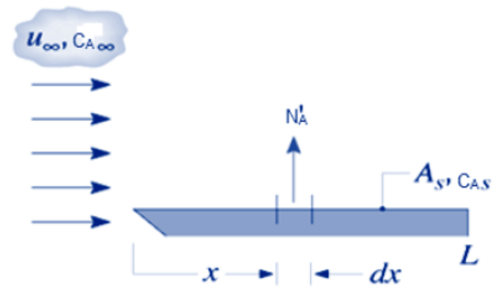
$$\frac{k_c L}{D_{AB}} = \frac{-d(c_A - c_{A,s})/dy|_{y=0}}{(c_{A,s} - c_{A,\infty})/L} \qquad (4)$$

Note

- The right-hand side of equation (4) is the ratio of the concentration gradient at the surface to an overall- or reference-concentration gradient. Accordingly, it may be considered a ratio of the molecular mass-transport resistance to the convective mass transfer resistance of the fluid.
- This ratio is referred to as the ***Sherwood number, Sh***

The relation between local and average convection coefficients

$$N_A = \int_{A_s} N'_A \, dA_s$$



$$N_A = (C_{As} - C_{A\infty}) \int_{A_s} k_c \, dA_s$$

$$N_A = \bar{k}_c \, A_s (C_{As} - C_{A\infty})$$

combining the two previous equations yield

$$\bar{k}_c = \frac{1}{A_s} \int_{A_s} k_c \, dA_s$$

since the width is constant, so the previous eq. can be written as

$$\bar{k}_c = \frac{1}{L} \int_{A_s} k_c \, dx$$

Example

Determine the Schmidt number for methanol in air at 298 K and 1.013 x 10^5 Pa and in liquid water at 298 K.

Data: At 298 K, the diffusivity of methanol in air

$$D_{\text{methanol-air}} P = 1.641 \text{ m}^2 \text{ Pa/s}$$

$\therefore$

$$D_{\text{methanol-air}} = \frac{1.641 \text{ m}^2 \text{ Pa/s}}{1.013 \times 10^5 \text{ Pa}} = 1.62 \times 10^{-5} \text{ m}^2/\text{s}$$

The kinematic viscosity of air is

$$\upsilon = 1.569 \times 10^{-5} \text{ m}^2/\text{s}.$$

- The Schmidt number of methanol in air is

$$Sc = \frac{v}{D_{AB}} = \frac{1.569 \times 10^{-5}\ m^2/s}{1.62 \times 10^{-5}\ m^2/s} = 0.968$$

- The Schmidt number of methanol in water

The diffusivity of methanol in water and the kinematic viscosity of water are, respectively:

$$D_{AB} = 1.738\ 10^{-9}\ m^2/s$$
$$v = 0.912 \times 10^{-6}\ m^2/s$$

$$Sc = \frac{v}{D_{AB}} = \frac{0.912 \times 10^{-6}\ m^2/s}{1.738 \times 10^{-9}\ m^2/s} = 525$$

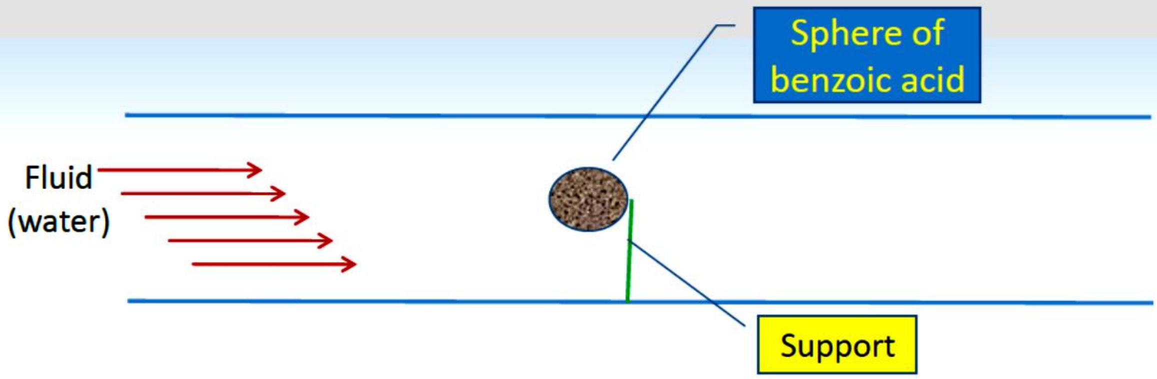
Methods to determine mass-transfer coefficients

In general, there are five methods of evaluating convective mass-transfer coefficients. They are:

- Experimental;
- dimensional analysis coupled with experiment;
- exact laminar boundary-layer analysis;
- approximate boundary-layer analysis;
- analogy between momentum, energy, and mass transfer

Experimental_ Example

In determining the mass-transfer coefficient to a sphere, Steele and Geankoplis used a solid sphere of benzoic acid held rigidly by a rear support in a pipe. Before the run the sphere was weighed. After flow of the fluid for a timed interval, the sphere was removed, dried, and weighed again to give the amount of mass transferred, which was small compared to the weight of the sphere. From the mass transferred and the area of the sphere, the flux N_A was calculated. Then the driving force $(c_{AS} - 0)$ was used to calculate k_L, where c_{AS} is the solubility and the water contained no benzoic acid.



DIMENSIONAL ANALYSIS OF CONVECTIVE MASS TRANSFER

Variable	Symbol	Dimensions
Tube diameter	D	L
Fluid density	ρ	M/L^3
Fluid viscosity	μ	M/Lt
Fluid velocity	v	L/t
Fluid diffusivity	D_{AB}	L^2/t
Mass-transfer coefficient	k_c	L/t

$$\mathrm{Sh} = f(\mathrm{Re}, \mathrm{Sc})$$

Analogous to the heat-transfer correlation $\mathrm{Nu} = f(\mathrm{Re}, \mathrm{Pr})$

Note: **The use of dimensional analysis is commonly used to predict the different dimensional groups which are very useful in correlating experimental mass transfer data.**

See the Text for details “Geankoplis”

EXACT ANALYSIS OF THE LAMINAR CONCENTRATION BOUNDARY LAYER

- Exactly, similar to what has been given in heat transfer course. See Incropera for details.
- It is simply known "Blasius an exact solution for the hydrodynamic, thermal and species concentration boundary layer for laminar flow parallel to a flat surface".
- The boundary-layer equations considered in the steady-state momentum transfer included the two-dimensional, incompressible continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- and the equation of motion in the x direction (momentum)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

- For the thermal boundary layer, the equation describing the energy transfer in a steady, incompressible, two-dimensional, isobaric flow with constant thermal diffusivity (Energy)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

- For the species concentration boundary layer, the equation describing the mass transfer in a steady, incompressible, two-dimensional, isobaric flow with constant mass diffusivity (species)

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

$$\left(v_x\frac{\partial c_A}{\partial x} + v_y\frac{\partial c_A}{\partial y} + \cancel{v_z\frac{\partial c_A}{\partial z}}\right) = D_{AB}\left(\cancel{\frac{\partial^2 c_A}{\partial x^2}} + \frac{\partial^2 c_A}{\partial y^2} + \cancel{\frac{\partial^2 c_A}{\partial z^2}}\right) + \cancel{R_A}$$

Summary

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Species: $$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

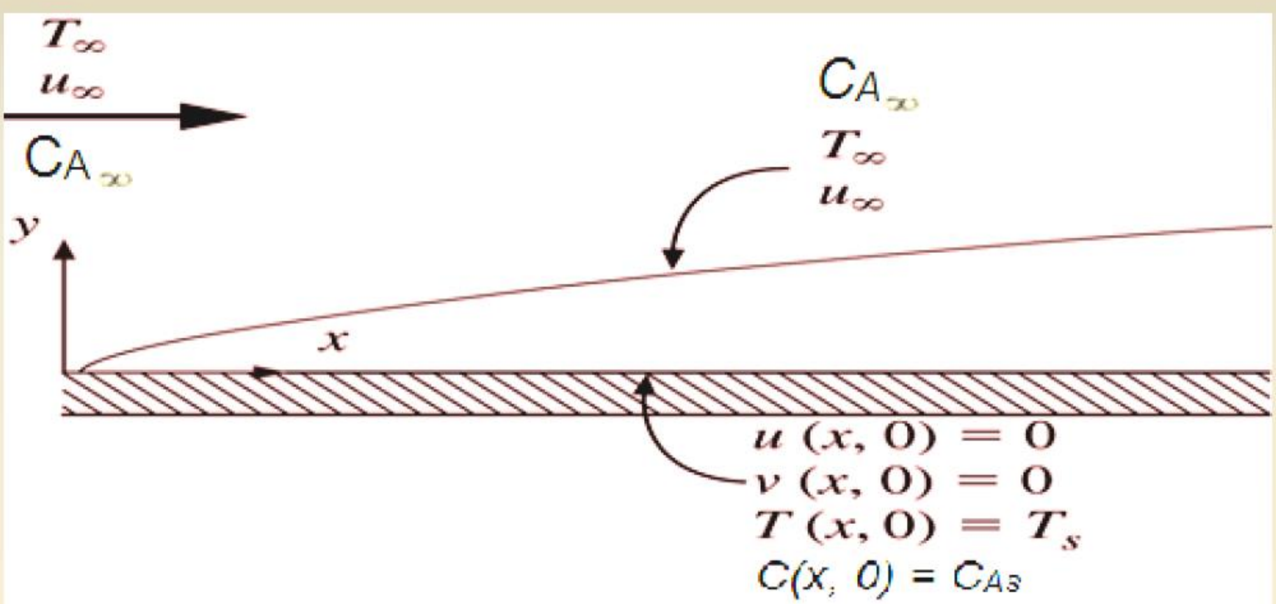
with the boundary conditions

At $x = 0$: $u(0, y) = u_\infty$, $T(0, y) = T_\infty$ $C(0,y) = C_{A\infty}$

At $y = 0$: $u(x, 0) = 0$, $v(x, 0) = 0$, $T(x, 0) = T_s$ $C(x, 0) = C_{As}$

As $y \rightarrow \infty$: $u(x, \infty) = u_\infty$, $T(x, \infty) = T_\infty$ $C(x, \infty) = C_{A\infty}$

Summary



With the foregoing simplification and approximations, the overall continuity equation and the x-momentum equation reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Also, the energy equation reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

And the species continuity equation becomes

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial y^2}$$

Boundary Layer Similarity Parameter

- Define the following dimensionless variables:

$$x^* = \frac{x}{L} \;,\; y^* = \frac{y}{L} \;,\; P^* = \frac{P_\infty}{\rho V^2}$$

$$u^* = \frac{u}{V} \;,\; v^* = \frac{v}{V}$$

$$T^* = \frac{T - T_s}{T_\infty - T_s} \;,\; C_A^* = \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}}$$

Where L is the characteristic length of the surface, and V is the velocity upstream of the surface.

- Using the above definitions, the velocity and temperature equations become as shown in the next table. Neglect viscous dissipation term.

Similarity Parameters and the dimensionless form of the B.L. Equations

TABLE 6.1 The boundary layer equations and their y-direction boundary conditions in nondimensional form

Boundary Layer	Conservation Equation	Boundary Conditions: Wall	Boundary Conditions: Free Stream	Similarity Parameter(s)
Velocity	$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$ (6.35)	$u^*(x^*,0)=0$, $v^*(x^*,0)=0$	$u^*(x^*,\infty)=\frac{u_\infty(x^*)}{V}$ (6.38)	$Re_L=\frac{VL}{\nu}$ (6.41)
Thermal	$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$ (6.36)	$T^*(x^*,0)=0$	$T^*(x^*,\infty)=1$ (6.39)	$Re_L, Pr=\frac{\nu}{\alpha}$ (6.42)
Concentration	$u^*\frac{\partial C_A^*}{\partial x^*}+v^*\frac{\partial C_A^*}{\partial y^*}=\frac{1}{Re_L Sc}\frac{\partial^2 C_A^*}{\partial y^{*2}}$ (6.37)	$C_A^*(x^*,0)=0$	$C_A^*(x^*,\infty)=1$ (6.40)	$Re_L, Sc=\frac{\nu}{D_{AB}}$ (6.43)

$$C_A^* \equiv \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}} \qquad u^*\frac{\partial C_A^*}{\partial x^*}+v^*\frac{\partial C_A^*}{\partial y^*}=\frac{1}{\mathrm{Re}_L\, Sc}\frac{\partial^2 C_A^*}{\partial y^{*2}}$$

- The concentration eq. suggests the following functional forms of solution

$$C_A^* = f\left(x^*, y^*, \mathrm{Re}_L, Sc, \frac{dp^*}{dx^*}\right) \qquad \text{(i)}$$

- Where the dependence on *dp*/dx** originates from the effect of the geometry on the fluid motion (u* and *v**),which, hence, affects the thermal conditions.

$$\therefore\ k_c = \frac{-D_{AB}\, \partial C_A / \partial y|_{y=0}}{C_{As} - C_{A\infty}}$$

$$k_c = -\frac{D_{AB}}{L}\frac{C_{A\infty} - C_{As}}{C_{As} - C_{A\infty}}\left.\frac{\partial C_A^*}{\partial y^*}\right|_{y^*=0} = \frac{D_{AB}}{L}\left.\frac{\partial C_A^*}{\partial y^*}\right|_{y^*=0} \qquad \text{(ii)}$$

$\therefore$ equation (ii) can be rearranged as

$$\frac{k_c L}{D_{AB}} = \left.\frac{\partial C_A^*}{\partial y^*}\right|_{y^*=0} \quad \text{this called Sherwood number}$$

$$\therefore \quad Sh = \frac{k_c L}{D_{AB}} = \left.\frac{\partial C_A^*}{\partial y^*}\right|_{y^*=0} \quad \text{dimensionless concetration gradient at surface}$$

$Sh \rightarrow$ local Sherwood number

For a prescribed geometry equqtion (i) becomes

$$Sh = f(x^*, \mathrm{Re}_L, Sc) \qquad \text{(iii)}$$

Equation (iii) shows that Sh is a function of x*, Re_L, and Sc. If this function is known, hence Sh can be computed for various fluids and for various values of V and L. Consequently, the coefficient k_c of mass transfer can be found from the computed value of Sh.

Boundary Layer analogies Heat and Mass transfer Analogy

- Definition:

  'If two or more processes are governed by dimensionless equations of the same form, the processes are said to be analogous'.

- The next table (6.3) shows the analogies between Heat and Mass transfer via eq.^{s} (6.47&6.51), (6.48&6.52), (6.49&6.53) and (6.50&6.54)

Summary of the functional relations and B.L. analogies

TABLE 6.3 Functional relations pertinent to the boundary layer analogies

Fluid Flow		Heat Transfer		Mass Transfer	
$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right)$	(6.44)	$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$	(6.47)	$C_A^* = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right)$	(6.51)
$C_f = \frac{2}{Re_L} \frac{\partial u^*}{\partial y^*}\bigg\vert_{y^*=0}$	(6.45)	$Nu = \frac{hL}{k} = +\frac{\partial T^*}{\partial y^*}\bigg\vert_{y^*=0}$	(6.48)	$Sh = \frac{k_c L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\bigg\vert_{y^*=0}$	(6.52)
$C_f = \frac{2}{Re_L} f(x^*, Re_L)$	(6.46)	$Nu = f(x^*, Re_L, Pr)$	(6.49)	$Sh = f(x^*, Re_L, Sc)$	(6.53)
		$\overline{Nu} = f(Re_L, Pr)$	(6.50)	$\overline{Sh} = f(Re_L, Sc)$	(6.54)

Conclusion

- If one has performed a set of heat experiments to find the functional form of equation 6.49, for example, the results may be used for the convective mass transfer involving the same geometry. This could be obtained by replacing Nu with Sh and Pr with Sc.
- In general, Nu and Sh are proportional to Pr^n and Sc^n , respectively.

Note:
Since the Pr and Sc dependence of Nu and Sh, respectively, is typically of the form Pr^n and Sc^n, where n is a positive exponent ($0.30 \le n \le 0.40$),

- Use the following analogy equations:

  Nu= $f(x^*, Re_L)$ Pr^n,   Sh= $f(x^*, Re_L)$ Sc^n

  in which case, with equivalent functions, $f(x^*, Re_L)$,

$$\frac{Nu}{Pr^n} = f\left(x^*, Re\right) = \frac{Sh}{Sc^n}$$

$$\frac{Nu}{Pr^n} = \frac{Sh}{Sc^n}$$

$$\frac{hL/k}{Pr^n} = \frac{k_c\, L/D_{AB}}{Sc^n}$$

Or

$$\frac{h}{k_c} = \frac{k}{D_{AB} Le^n} = \rho\, c_p\, Le^{1-n}$$

Note

For most engineering applications, assume a value of n=1/3

$$Le = \frac{\alpha}{D_{AB}} = \frac{\rho C_p}{k D_{AB}}$$

$$Sc = \frac{\nu}{D_{AB}}$$

Reynolds Analogy

- This analogy assumes the following:

  dp*/dx*=0 and Pr = Sc =1.

  and for a flat surface $u_\infty = V$

- Hence, the velocity, the thermal and the concentration Equations and boundary conditions become analogous and the functional form of the solutions for u*, T*, and C*, eqs. 6.44, 6.47, and 6.51 are equivalent.

TABLE 6.3 Functional relations pertinent to the boundary layer analogies

Fluid Flow		Heat Transfer		Mass Transfer	
$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right)$	(6.44)	$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$	(6.47)	$C_A^* = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right)$	(6.51)
$C_f = \frac{2}{Re_L} \frac{\partial u^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.45)	$Nu = \frac{hL}{k} = +\frac{\partial T^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.48)	$Sh = \frac{k_c L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\Big\vert_{y^*=0}$	(6.52)
$C_f = \frac{2}{Re_L} f(x^*, Re_L)$	(6.46)	$Nu = f(x^*, Re_L, Pr)$	(6.49)	$Sh = f(x^*, Re_L, Sc)$	(6.53)
		$\overline{Nu} = f(Re_L, Pr)$	(6.50)	$\overline{Sh} = f(Re_L, Sc)$	(6.54)

- From eqs. 6.45, 6.48 and 6.52 it follows that (see table 6.3)

$$C_f \frac{Re_L}{2} = Nu = Sh \qquad (6.66)$$

- Replacing Nu and Sh by the Stanton number, St, and the mass transfer Stanton number, St_m, respectively,

$$St = \frac{h}{\rho V c_p} = \frac{Nu}{Re\,Pr}$$

$$St_m = \frac{k_c}{V} = \frac{Sh}{Re\,Sc}$$

$$C_f \frac{Re_L}{2} = St\,Re = St_m\,Re \quad \{Note: Pr = Sc = 1\}$$

$$\therefore \quad \frac{C_f}{2} = St = St_m$$

- Eq. 6.66 may be expressed as

$$\frac{C_f}{2} = St = St_m \qquad Pr = Sc = 1$$

$$\text{and } dp^*/dx^* = 0$$

- The modified Reynolds, or Chilton-Colburn, analogies

$$\frac{C_f}{2} = St\,Pr^{2/3} = j_H \qquad 0.6 < Pr < 60$$

$$\frac{C_f}{2} = St_m\,Sc^{2/3} = j_m \qquad 0.6 < Sc < 3000$$

Note

- Applicable to laminar flow if $dp^*/dx^* \sim 0$.
- Generally applicable to turbulent flow without restriction on dp^*/dx^*.

Chilton-Colburn, analogies _ Summary

$$\frac{C_f}{2} = \text{St Pr}^{2/3} = \text{St}_{\text{mass}}\text{Sc}^{2/3}$$

$0.6 < \text{Pr} < 60$ and $0.6 < \text{Sc} < 3000$

$$\frac{\text{St}}{\text{St}_{\text{mass}}} = \left(\frac{\text{Sc}}{\text{Pr}}\right)^{2/3}$$

$$\frac{h_{\text{heat}}}{k_c} = \rho c_p \left(\frac{\text{Sc}}{\text{Pr}}\right)^{2/3} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} = \rho c_p \text{Le}^{2/3}$$

Chilton-Colburn Analogy

General:

$$k_c = \frac{h_{\text{heat}}}{\rho c_p}\left(\frac{D_{AB}}{\alpha}\right)^{2/3}$$

$$= \frac{1}{2} C_f V \left(\frac{D_{AB}}{\nu}\right)^{2/3}$$

Special case: $\nu = \alpha = D_{AB}$

$$k_c = \frac{h_{\text{heat}}}{\rho c_p} = \frac{1}{2} C_f V \quad \text{m/s}$$

This is k_c' use conversion table to get other units

When the friction or heat transfer coefficient is known, the mass transfer coefficient can be determined directly from the Chilton–Colburn analogy.

For air–water vapor mixtures:

$$h_{\text{heat}} \cong \rho c_p k_c \qquad \text{(air–water vapor mixtures)}$$

This relation is commonly used in air-conditioning applications.