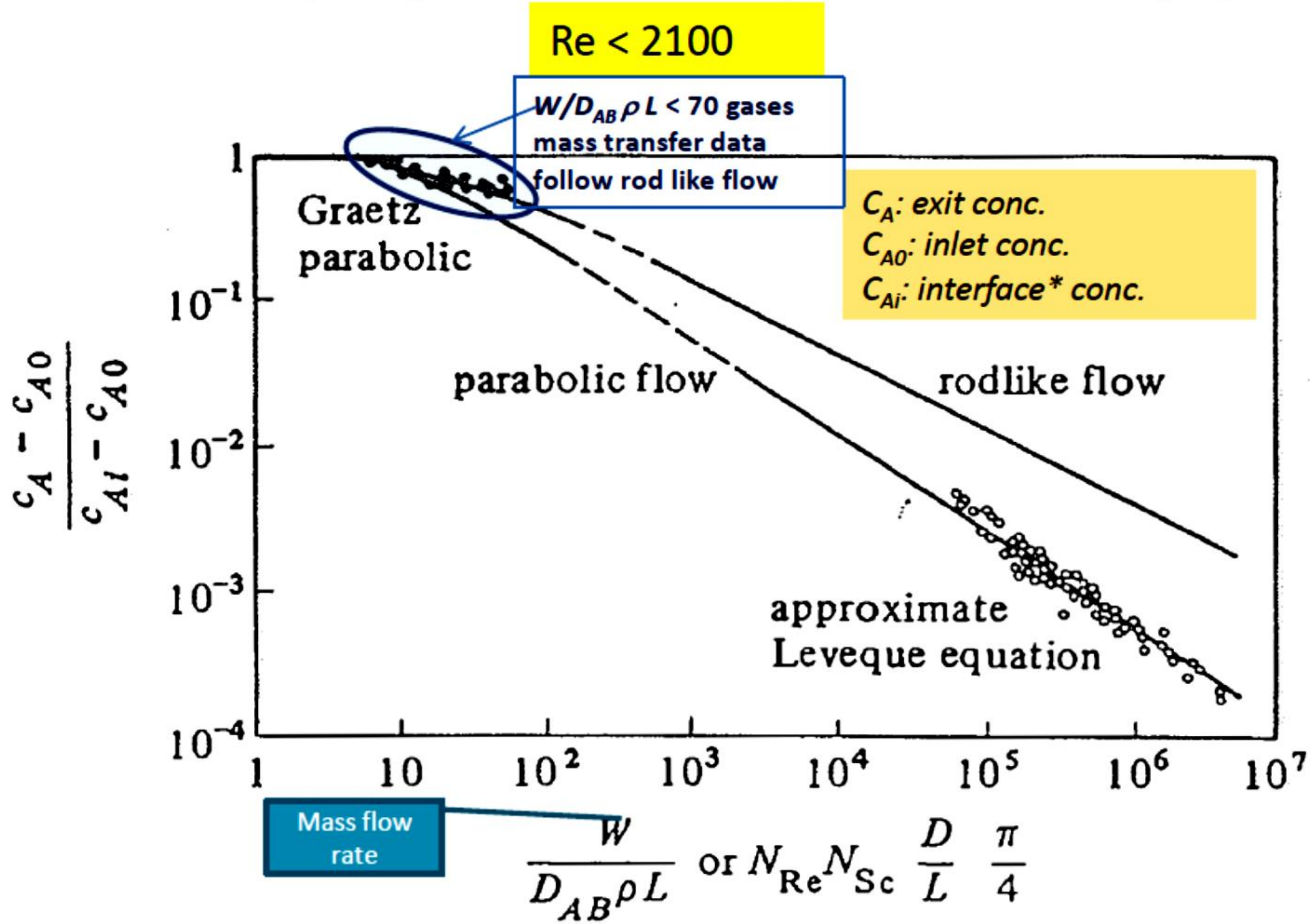
Correlations of mass transfer coefficients

For different geometries & systems, tubes, spheres, fixed bed, and suspensions

Mass transfer for laminar flow inside pipes.



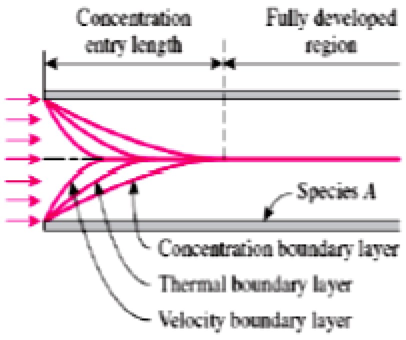
Data for diffusion in a fluid in streamline flow inside a pipe: filled circles, vaporization data ; open circls, dissolving-solids data

* *Interface means between wall and gas stream.*

Notes

- Experimental data of mass transfer of gases form follow the rodlike plot.
- Velocity profile is assumed fully developed to parabolic form at entrance.
- For liquids (of small values of D_{AB}),

data follow the parabolic flow line, which can be approximated by



The development of the velocity, thermal, and concentration boundary layers in internal flow.

$$\frac{c_A - c_{A0}}{c_{Ai} - c_{A0}} = 5.5\left(\frac{W}{D_{AB}\rho L}\right)^{-2/3}$$

$(W/D_{AB}\ \rho\ L) > 400$

Mass Transfer for turbulent flow inside pipes

- Re > 2100, for gases and liquids use

$$N_{Sh} = k'_c \frac{D}{D_{AB}} = \frac{k_c p_{BM}}{P} \frac{D}{D_{AB}} = 0.023\left(\frac{Dv\rho}{\mu}\right)^{0.83}\left(\frac{\mu}{\rho D_{AB}}\right)^{0.33}$$

$$0.6 < Sc > 3000$$

Sherwood number relations in mass convection for specified concentration at the surface corresponding to the Nusselt number relations in heat convection for specified surface temperature

Convective Heat Transfer	Convective Mass Transfer
1. *Forced Convection over a Flat Plate*	
(*a*) Laminar flow ($Re < 5 \times 10^5$)	
$Nu = 0.664\ Re_L^{0.5}\ Pr^{1/3}$, $Pr > 0.6$	$Sh = 0.664\ Re_L^{0.5}\ Sc^{1/3}$, $Sc > 0.5$
(*b*) Turbulent flow ($5 \times 10^5 < Re < 10^7$)	
$Nu = 0.037\ Re_L^{0.8}\ Pr^{1/3}$, $Pr > 0.6$	$Sh = 0.037\ Re_L^{0.8}\ Sc^{1/3}$, $Sc > 0.5$
2. *Fully Developed Flow in Smooth Circular Pipes*	
(*a*) Laminar flow ($Re < 2300$)	
$Nu = 3.66$	$Sh = 3.66$
(*b*) Turbulent flow ($Re > 10{,}000$)	
$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$, $0.7 < Pr < 160$	$Sh = 0.023\ Re^{0.8}\ Sc^{0.4}$, $0.7 < Sc$ 160

Example 1

A tube is coated on the inside with naphthalene and has an inside diameter of 20 mm and a length of 1.10 m. Air at 318 K and an average pressure of 101.3 kPa flows through this pipe at a velocity of 0.80 m/s. Assuming that the absolute pressure remains essentially constant, calculate the concentration of naphthalene in the exit air.

Solution

Properties: $D_{AB} = 6.92 \times 10^{-6}$ m^2/s and the vapor pressure $p_{Ai} = 74.0$ Pa or $c_{Ai} = p_{Ai}/RT = 74.0/(8314.3 \times 318) = 2.799 \times 10^{-5}$ kg mol/m^3. For air from Appendix A.3, $\mu = 1.932 \times 10^{-5}$ Pa·s, $\rho = 1.114$ kg/m^3. The Schmidt number is

$$N_{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{1.932 \times 10^{-5}}{1.114 \times 6.92 \times 10^{-6}} = 2.506$$

The Reynolds number is

$$N_{Re} = \frac{Dv\rho}{\mu} = \frac{0.020(0.80)(1.114)}{1.932 \times 10^{-5}} = 922.6$$

Hence, the flow is laminar. Then,

$$N_{Re} N_{Sc} \frac{D}{L} \frac{\pi}{4} = 922.6(2.506) \frac{0.020}{1.10} \frac{\pi}{4} = 33.02$$

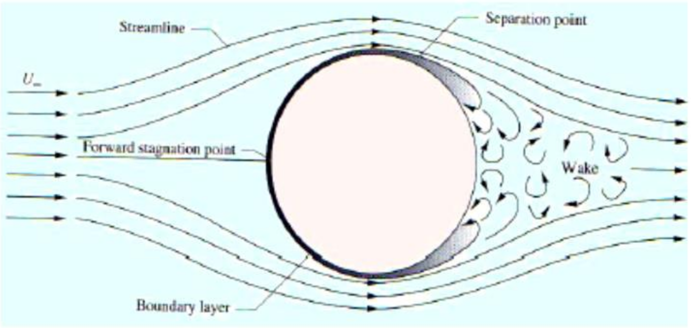
Using Fig. 7.3-2 and the rodlike flow line, $(c_A - c_{A0})/(c_{Ai} - c_{A0}) = 0.55$. Also, c_{A0}(inlet) $= 0$. Then, $(c_A - 0)/(2.799 \times 10^{-5} - 0) = 0.55$. Solving, c_A(exit concentration) $= 1.539 \times 10^{-5}$ kg mol/m^3.

Relations of Mass Transfer flow outside solid surface

- Mass transfer for flow past single spheres

For gases, $$N_{Sh} = 2 + 0.522 N_{Re}^{0.53} N_{Sc}^{1/3}$$

N_{Re}: 1-48,000



For liquids, N_{Re}: 2-2,000

$$N_{Sh} = 2 + 0.95 N_{Re}^{0.5} N_{Sc}^{1/3}$$

For liquids, N_{Re}: 2000 - 17000

$$N_{Sh} = 0.347 N_{Re}^{0.62} N_{Sc}^{1/3}$$

Mass Transfer to packed beds

Mass transfer to **packed beds of spherical particles**:

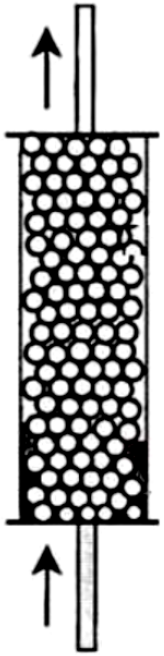
Applications: Drying, adsorption, mass transfer to catalysts, …

For gases, in a packed bed of sphere and N_{Re}: 10-10,000:

$$J_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069}$$

Where: $$J_D = \frac{k_c}{v'} (N_{Sc})^{2/3}$$

$$N_{Re} = \frac{D_p v' \rho}{\mu}$$



Superficial velocity

For liquids, N_{Re}: 0.0016-55, N_{SC}: 165-70,000:

$$J_D = \frac{1.09}{\varepsilon} N_{Re}^{-2/3}$$

For liquids, N_{Re}: 55-1500, N_{SC}: 165-10690:

$$J_D = \frac{0.25}{\varepsilon} N_{Re}^{-0.31}$$

Note

- **For non-spherical particles packed beds:**

The previous correlations of spherical packed beds can be used but with using surface diameter instead of spherical diameter.

LMCD
Log mean
conc. Diff.

V_b: total volume of the bed, m^3

A: total external surface area, m^2

a: A/V_b

$$N_A = k_c \frac{(c_{A1} - c_{Ai}) - (c_{A2} - c_{Ai})}{\ln \dfrac{(c_{A1} - c_{Ai})}{(c_{A2} - c_{Ai})}}$$

Mass transfer flux

C_{Ai}: concentration of A at the surface of the solid

C_{A1}: inlet bulk fluid concentration

C_{A2}: outlet bulk fluid concentration

Calculations method for Packed Beds

- Obtain k_c in m/s from the J_D.
- Knowing the total volume of the bed V_b, the total surface area, A, of the bed is calculated as follows:

$$A = a\,V_b$$

a Sp. surface area

Where $a = \frac{6(1-\varepsilon)}{D_p}$ *a is the m² surface area /m³ total volume of bed.*

- To calculate the mass-transfer rate, the log mean driving force at the inlet and outlet of the bed should be used

$$N_A A = Ak_c \frac{(c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})}{\ln \frac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}} \quad \text{....................(A)}$$

where the final term is the log mean driving force: c_{Ai} is the concentration at the surface of the solid, in kg mol/m^3; c_{A1} is the inlet bulk fluid concentration; and c_{A2} is the outlet. The material-balance equation on the bulk stream is

$$N_A A = V\left(c_{A2} - c_{A1}\right) \qquad \text{....................(B)}$$

where V is the volumetric flow rate of fluid entering in m^3/s.

Note

Equations (A) and (B) must both be satisfied. The use of these two equations is similar to the use of the log mean temperature difference and heat balance in heat exchangers. These two equations can also be used for a fluid flowing in a pipe or past a flat plate, where A is the pipe wall area or plate area.

Example 2

Pure water at 26.1°C flows at the rate of 5.514×10^{-7} m^3/s through a packed bed of benzoic acid spheres having diameters of 6.375 mm. The total surface area of the spheres in the bed is 0.01198 m^2 and the void fraction is 0.436. The tower diameter is 0.0667 m. The solubility of benzoic acid in water is 2.948×10^{-2} kg mol/m^3.

1. Predict the mass transfer coefficient k_c.
2. Predict the outlet concentration of benzoic acid in the water.

Solution

the physical properties of water will be used at 26.1°C

$$\mu = 0.8718 \times 10^{-3}\ \text{Pa}\cdot\text{s},$$

$$\rho = 996.7\ \text{kg/m}^3$$

At 25.0°C, $\mu = 0.8940 \times 10^{-3}\ \text{Pa}\cdot\text{s}$

$$D_{AB} = 1.21 \times 10^{-9}\ \text{m}^2/\text{s}.$$

Spheres diameter= 6.375 mm
Void fraction =0.436
Tower diameter=0.0667 m

Pure water
26.1 °C
5.514x10^{-7} m^3/s

To correct D_{AB} to 26.1°C , $D_{AB} \propto T/\mu$. Hence,

To correct D_{AB} to 26.1°C , $D_{AB} \propto T/\mu$. Hence,

$$D_{AB}(26.1^\circ\text{C}) = (1.21 \times 10^{-9})\left(\frac{299.1}{298}\right)\left(\frac{0.8940 \times 10^{-3}}{0.8718 \times 10^{-3}}\right)$$

$$= 1.254 \times 10^{-9}\ \text{m}^2/\text{s}$$

The tower cross-sectional area $= (\pi/4)(0.0667)^2 = 3.494 \times 10^{-3}$ m^2.

Then

The superficial velocity is

$$v' = (5.514 \times 10^{-7})/(3.494 \times 10^{-3}) = 1.578 \times 10^{-4} \text{ m/s.}$$

Then,

$$N_{Sc} = \frac{\mu}{\rho D_{AB}} = \frac{0.8718 \times 10^{-3}}{996.7(1.245 \times 10^{-9})} = 702.6$$

$$N_{Re} = \frac{Dv'\rho}{\mu} = \frac{0.006375(1.578 \times 10^{-4})(996.7)}{0.8718 \times 10^{-3}} = 1.150$$

$$J_D = \frac{1.09}{\varepsilon}(N_{Re})^{-2/3} = \frac{1.09}{0.436}(1.150)^{-2/3} = 2.277$$

Then,

$$J_D = \frac{k_c}{v'}(N_{Sc})^{2/3}$$

$$2.277 = \frac{k_c}{1.578 \times 10^{-4}}(702.6)^{2/3}$$

$$k_c = 4.447 \times 10^{-6} \text{ m/s}.$$

$$A k_c \frac{(c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})}{\ln \dfrac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}} = V(c_{A2} - c_{A1})$$

$$c_{Ai} = 2.948 \times 10^{-2}$$

$$c_{A1} = 0,$$

$$A = 0.01198,$$

$$V = 5.514 \times 10^{-7}$$ Vol flow rate m^3/s

$$\frac{0.01198(4.665 \times 10^{-6})(c_{A2} - 0)}{\ln \dfrac{2.948 \times 10^{-2} - 0}{2.948 \times 10^{-2} - c_{A2}}} = (5.514 \times 10^{-7})(c_{A2} - 0)$$

Solving, $c_{A2} = 2.842 \times 10^{-3}$ kg mol/m^3.

Mass transfer to suspensions of small particles

- Mass transfer to suspensions of small particles occurs in many applications, such as:
  - –liquid-phase hydrogenation, where hydrogen diffuses to small catalyst particles.
  - –Fermentation, where oxygen diffuses to small microorganisms.
  - –Mixing has small effect on mass transfer in such cases

Cases of mass transfer to suspensions of small particles

- Cases are based on the particle size range

1. Mass transfer to small particles <0.6 mm.

$$k'_L = \frac{2D_{AB}}{D_p} + 0.31 N_{Sc}^{-2/3} \left(\frac{\Delta\rho \mu_c g}{\rho_c^2} \right)^{1/3}$$

ρ_c is the density of the continuous phase

$\Delta\rho = (\rho_c - \rho_p)$

2. Mass transfer to large gas bubbles > 2.5 mm.

$$k'_L = 0.42 N_{Sc}^{-0.5} \left(\frac{\Delta\rho \mu_c g}{\rho_c^2} \right)^{1/3}$$

3. Mass transfer to particles in transition region. In mass transfer in the transition region between small and large bubbles in the size range 0.6 to 2.5 mm, the mass-transfer coefficient can be approximated by assuming that it increases linearly with bubble diameter,

4. Mass transfer to particles in highly turbulent mixers.

$$k'_L N_{Sc}^{2/3} = 0.13\left(\frac{(P/V)\mu_c}{\rho_c^2}\right)^{1/4}$$

where P/V is power input per unit volume