

Transport 2

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Mass transfer Mechanism :

طريقان لنقل الكتلة :

- 1- Molecular Diffusion (انتشار جزيئي)
- 2- Eddy Diffusion - Convection - (الحمل الجاري)
 - a. Natural
 - b. Forced

Mass transfer can occur within :

- Same phase in a fluid mixture
- Across a phase boundary

يحدث الانتقال للمادة داخل أو بين المراحل أو عبر الحدود (boundary)

* Molecular transport is **always present** in mass transfer (الانتقال الجزيئي دائماً موجود بيننا نقل المادة)

Fluxes always contain a molecular term plus a convection term.

Fluxes = molecular + convection

لقيم تتوسع بالجمع. ☺

طريقان لنقل الكتلة - مزج :

- 1- Diffusion : the net transport of substances in a stationary solid or in a stagnant fluid or in a fluid which are moving only in laminar flow, due to a concentration gradient. It may also be present even in highly developed turbulent flow but near the solid surface.

حيث شوائبها يمكن من طريقه لنقل المادة (الانتشار)
 والـ diffusion يصير من المادة الصلبة او السوائل
 سواء الـ (Stagnant) حيث الراكد او الـ (moving)
 المتحركة حلا في شرط للسوائل المتحركة يكون laminar
 واذا صار الـ turbulent يكون على (solid surface)
 طبقاً لحد الانتقال يصير نادراً على
 الـ (Concentration gradient) اختلاف التراكيز

2- Advection : the net transport of substances
 by the moving fluid, and so it can not
 happen in solids. It does not include transport
 of substances by simple diffusion.
 هو ان يمكن عن طريقه نقل بـ شئ عن طريق السائل المتحرك
 ولا يمكن ان يحدث في المواد الصلبة بذاته نسبة انه
 لا يمكن نقل المواد عن طريقه (diffu)

3. Convection The net transport of substance
 caused by both advective transport and
 diffusive transport in fluid.

Convection = Advection + ~~conve~~ Diffusion



Molecular Diffusion

- Caused by random microscopic movement of individual molecules in gas/liquid/solid as a result of thermal motion.
- Extremely slow.
- Occurs in solids and fluids that are stagnant or in laminar flow.
- Mass transfer under turbulent-flow but across an interface or near solid surface, the conditions near surface can be assumed laminar.
- Mathematically described by Fick's law:

$$J_{AZ}^* = -D_{AB} \frac{dc_A}{dz}$$



- يصير بسبب الحركة العشوائية (molecules) في غاز - سائل - صلب
نتيجة الحركة الكيميائية (thermal motion)

- يصير في fluid (سائل، غاز) أو الصلب سواء Stagnant
- laminar flow

- زي ما حكيينا انه يصير في turbulent flow بس فيه !!

اما عند interface أو near solid surface وفي هاهي الحالة

تقدر اعتبره (laminar)

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Convective Mass Transfer

- Caused by random **macroscopic** fluid **bulk** motion (dynamic characteristics).
- *Orders of magnitude* **greater** than molecular diffusion.
- Involves transport of materials at the interface between moving fluids (liquid-gas) or at interface between a moving fluid and a solid surface (liquid-solid, gas-solid).
- Mathematically described in a **manner analogous to Newton's law** :

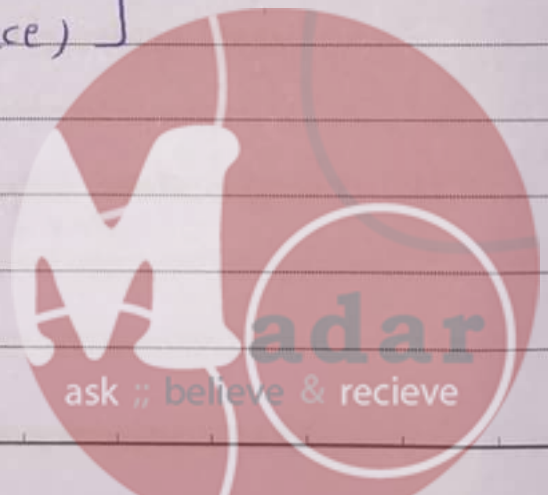


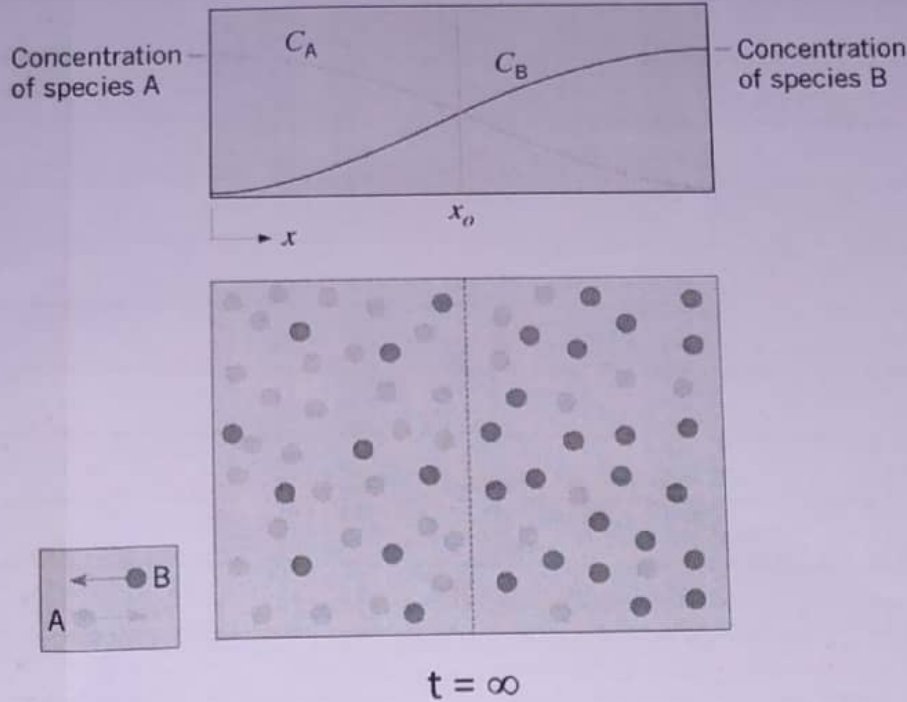
$$N_{AZ} = k_C \Delta C_A$$

- هاد النوع صيال اكثر للحوائص الديناميكية وبرهوا الحركة هون
عشوائية بيبي هون ننتج انه (macroscopic) عكس ال
Diffusion كان (micro)

Orders of magnitude < diffusion < الحجم اكبر من ال (diff) (interface) بين (moving fluid)

- نقل المواد } (interface) بين ال (moving fluid) & solid surface
liq - solid gas - solid





- علاهون في حادثين A و B نفس (P و T)
- اتجاه الـ diffusion يكون في اتجاه نقصان التركيز
- يعني A رح تنقل للسيس اما B للسيار
- فضل ينقل من التركيز الا على المنخفض لحد ما ييسر عندي
- تقاربي وليس يصير تركيز الحادثين A , B فوحده
- هون بوقت الـ diffusion
- A and B (approached)



Diffusion Kind :

Ordinary → Due to **concentration** (Chemical potential) gradient

Thermal → Due to **temperature** gradient
(Clusius-Dickel column)

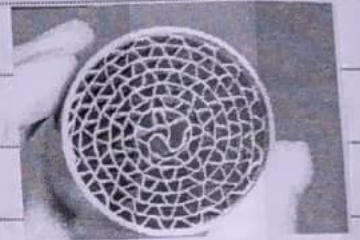
pressure → Due to **pressure** gradient
(centrifuges)

Forced → Due to **unequal external Forces** acting on the chemical species eg ions or charged particles in an electrical field
(Nernst-Planck equation)

Importance of mass Transfer (Applications)

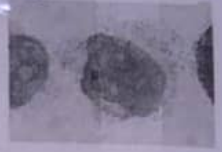
• **Industrial (ex:)**

1. Dispersion of flue gases
2. pollutant sequestration and mitigation
3. catalysis
4. Distillation / absorption / extraction / adsorption / SCF
5. Air conditioning



• Daily life (نشاطات الحياة اليومية)

1. Inhaling / exhaling / oxygen absorption in the lungs
2. Drug release and absorption
3. Sugar dissolution in tea or coffee
4. Moth balls (naphthalene)
5. Sweating



* Typically 50 to 90 percent of capital investment in chemical plant is for **Separations equipment**

← مجرد معلومات عامة للرأس مال المستثمر في مصنع كيميائي

Mass and Molar Notation

Concentration

- mass concentration ρ_A : is the mass of species A per unit volume of solution
- molar concentration $C_A = \rho_A / M_A$ number of moles of A
unit volume of solution

Fractions (النسب)

mass Fractions $W_A = \rho_A / \rho$

mole Fraction $X_A = C_A / C$

$\rho = \sum \rho_A$ total mass of all species / unit volume of sol.

$C = \sum C_A$ total # of moles of

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Table 24.1 Concentrations in a binary mixture of A and B

Molar Liquid Concentrations	Molar Gas Concentrations	Mass Concentrations (Gas or Liquid)
$c = n/V$	$c = n/V = P/RT$	$\rho = m/V$
$c_A = n_A/V$	$c_A = n_A/V = P_A/RT$	$\rho_A = m_A/V$
$c_B = n_B/V$	$c_B = n_B/V = P_B/RT$	$\rho_B = m_B/V$
$x_A = c_A/c$	$y_A = c_A/c = P_A/P$	$w_A = \rho_A/\rho$
$x_B = c_B/c$	$y_B = c_B/c = P_B/P$	$w_B = \rho_B/\rho$
$c = c_A + c_B$	$c = c_A + c_B, P = P_A + P_B$	$\rho = \rho_A + \rho_B$
$1 = x_A + x_B$	$1 = y_A + y_B$	$1 = w_A + w_B$

هباد الكيدل
فيه كل قاتون
ماتون احصاة
مواد

liquid
or
gas

هباد ار Nolar

قواسم كندل

gas

or liquid

به لبادي

اضمت

mass concentration

ماتيه القواسم

للتين

لستة لبادي

liquid

في عبا (V)

ماتيه القواسم

$PV = RT$

Ex 1. A gas mixture from a hydrocarbon reforming process contain 50% H_2 , 40% CO_2 and 10% Methane (CH_4) by volume at 400°C (673 K) and (1.5) atm total system pressure.

Determine the Molar Concentration^① and mass Fraction^② of each species in the mixture as well as the density of mixture^③.

Soln:

بدي اريح طاي واحط ريزون لكي عصفراو مركب مثلاً
 let $A = H_2$ $B = CO_2$ $C = CH_4$
 وانابدي افترضه انه عم يتعامل مع (ideal gas)

① molar concentration .

هذا جدول 24.1 دطلع القانون

$$C = \frac{P}{RT}$$

$$C = \frac{1.5 \text{ atm}}{(0.08206 \text{ m}^3 \cdot \text{atm} / \text{kg mol} \cdot \text{K}) (673 \text{ K})} = 2.72 \times 10^{-2} \text{ kg mol/m}^3$$

بدي اعرف في (ideal gas) تركيب الحجم المولي (volume percent) بكافيتي ان (mole percent) فشان صحت انابدي استعمل هذا القانون

لكل مادة $C_x = y_x C$ $\Leftarrow$ C طلعناها y_x

صبي النسبة

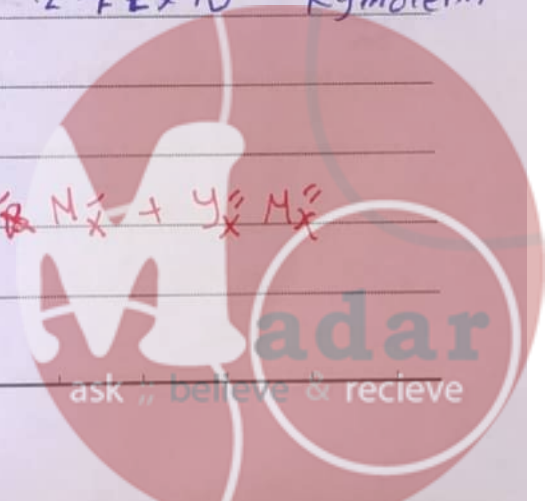
$$C_A = y_A C \rightarrow 0.50 \times 2.72 \times 10^{-2} \text{ kg mol/m}^3 = 1.36 \times 10^{-2} \text{ kg mole/m}^3$$

$$C_B = y_B C \rightarrow 0.40 \times 2.72 \times 10^{-2} = 1.10 \times 10^{-2} \text{ kg mole/m}^3$$

$$C_C = y_C C \rightarrow 0.10 \times 2.72 \times 10^{-2} = 2.72 \times 10^{-3} \text{ kg mole/m}^3$$

② mass Fraction

$$w_x = \frac{y_x M_x}{y_x M_x + y_B M_B + y_C M_C}$$



$$\text{So, } w_A = \frac{y_A M_A}{y_A M_A + y_B M_B + y_C M_C}$$

$$w_A = \frac{(0.50)(2)}{(0.50)(2) + (0.40)(44) + (0.10)(16)} \\ = 0.0495 \frac{\text{g H}_2}{\text{total g}}$$

$$w_B = \frac{(0.40)(44)}{(0.50)(2) + (0.40)(44) + (0.10)(16)} \\ = 0.871 \frac{\text{g CO}_2}{\text{total g}}$$

$$w_C = \frac{(0.10)(16)}{(0.50)(2) + (0.40)(44) + (0.10)(16)} \\ = 0.0793 \frac{\text{g Methan}}{\text{total g}}$$

③ density

$$\rho = \rho_A + \rho_B + \rho_C$$

$$= c_A M_A + c_B M_B + c_C M_C$$

$$= 1.36 \times 10^{-2} \frac{\text{kg mol}}{\text{m}^3} \times \frac{2 \text{ g}}{\text{g mol}} + 1.10 \times 10^{-2} \frac{\text{kg mol}}{\text{m}^3}$$

$$\times \frac{44 \text{ g}}{\text{g mol}} + 2.72 \times 10^{-3} \frac{\text{kg mol}}{\text{m}^3} \times \frac{16 \text{ g}}{\text{g mol}}$$

$$= 0.555 \frac{\text{kg}}{\text{m}^3}$$



Ex. 2 A waste water stream is contaminated with 200 mg/L of dissolved trichloroethylene (TCE) at 20°C, which is below its solubility limit in water. What are the molar concentration (in SI unit) and the mole fraction of TCE in the wastewater, assuming a dilute solution? At 20°C, the mass transfer of liquid water is 998.2 kg/m³ (Appendix 1). The molecular weight of the TCE is 131.4 g/gmole and the molecular weight of water is 18 g/gmole (18.1 kg/kg mole).

Soln :

let A = TCE (solute) , B = water (solvent)

The molar concentration of TCE in the wastewater C_A?

PA (mass concentration) بدي احسها في خلال الـ

$$\textcircled{1} \quad C_A = \frac{P_A}{M_A} = \frac{200 \text{ mg A/L} \times 1 \text{ g} \times 1 \text{ kg/mole} \times 1000 \text{ L}}{131.4 \text{ g/g mole} \times 1000 \text{ mg} \times 1000 \text{ g/mole} \times \text{m}^3}$$

$$= 1.52 \times 10^{-3} \text{ kg mole/m}^3$$

(A solute) في الـ TCE dilute
in (B solvent) solution

$$C \approx C_B = \frac{P_B}{M_B} = \frac{998.2 \text{ kg/m}^3}{18 \text{ kg/kg mole}}$$

ليست احسن تركيز B لانه اجمالي التركيز = 55.5 kg mole/m³
بديل لتركيز الـ (Solvent) كونه الاكثر كمية خدي

$$\textcircled{2} \quad \text{Mole Fraction } X_A = \frac{C_A}{C} = \frac{1.52 \times 10^{-3} \text{ kg mole/m}^3}{55.5 \text{ kg mole/m}^3}$$

$$= 2.74 \times 10^{-5}$$



Notation For velocities

* In diffusing mixture, the various chemical species are moving at different velocity (velocities).

سوف نستخدم

المختلط الموجود عندي فيه مواد هيا المواد بتكون عندي بسرعات مختلفة بس . يعني مثلا عندي وعاء فيه A, B, C سرعة A عندي B عندي C كل مادة اياها سرعة خاصة فيها .

- $V_\alpha \rightarrow$ the velocity of species α relative to stationary coordinates

هذه سرعة α species عندي بالنسبة ل إحداثيات ثابتة

- Diffusion velocity : the difference between the species velocity and mixture velocity V .

الفرق بين سرعة α species وسرعة الخليط

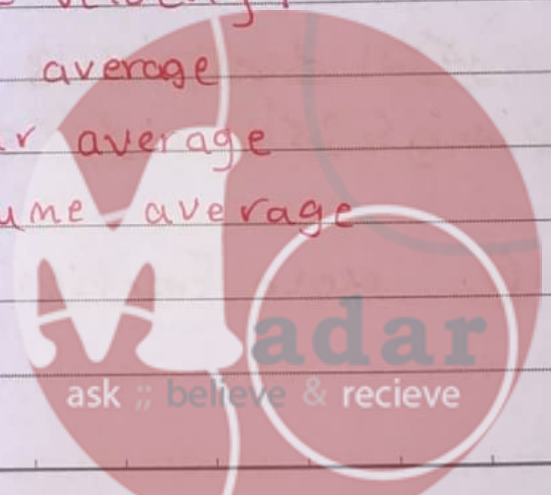
الفرق بين سرعة α species وسرعة الخليط (Mixture) - سرعة

- Diffusion velocity

(Mixture velocity)

- Mass average
- Molar average
- Volume average

- [ههنا ياتي بتقابل معاهم



Ex 2.1

Estimation of mass average,
Molar average velocity, volume
average velocity.

A gas mixture containing 65% NH_3 ,
8% N_2 , 24% H_2 , 3% Ar
is flowing through a pipe

2.5 mm in a diameter at a total pressure of 4.0 atm. The velocity of components are :

$$NH_3 = 0.03 \text{ m/s}$$

$$N_2 = 0.03 \text{ mls}$$

$$l_{t2} = 0.035 \text{ m/s}$$

$$Ar = 0.02 \text{ m/s}$$

مثال مع الحماية الحاسوبية

For NH_3

$y_i = 0.65$ Given

$$M_i = 3 \times 1 + 1 \times 14 = 17$$

$$v_i = 0.03 \text{ Given}$$

$$y_i v_i = 0.65 + 0.03 = 0.0195$$

$$y_i m_i = 0.65 * 17 = 11.05$$

اس

V $\left[\begin{array}{c} \text{قسمه اعلاى} \\ \text{وزن الصفه صاى} \end{array} \right]$
 V^*

name	i	yi	Mi	vi (m/s)	yivi (m/s)	yiMi	wi	wivi	vi-v	vi-v*	wi(vi-v*)	yi(vi-v)
H3	1	0.65	17	0.03	0.0195	11.05	0.73814	0.02214	0.00064	-0.0009	-0.00066	0.000417
	2	0.08	28	0.03	0.0024	2.24	0.14963	0.00449	0.00064	-0.0009	-0.00013	5.13E-05
	3	0.24	2	0.035	0.0084	0.48	0.03206	0.00112	0.00564	0.0041	0.000131	0.001354
	4	0.03	40	0.02	0.0006	1.2	0.08016	0.00160	-0.00936	-0.0109	-0.00087	-0.00028
	5	1	87		0.0309	14.97	1	0.02936	-0.00243	-0.00860	-0.00154	0.00154

$$v^* = \frac{\sum_{i=1}^N c_i v_i}{\sum_{i=1}^N c_i} = \frac{\sum_{i=1}^N c_i v_i}{\sum_{i=1}^N x_i v_i}$$

$$v = \frac{\sum_{i=1}^N \rho_i v_i}{\sum_{i=1}^N \rho_i} = \frac{\sum_{i=1}^N \rho_i v_i}{\sum_{i=1}^N \rho_i}$$

$v-v^*$

v^*-v

-0.00154

0.00154

$$V = \frac{\sum_{\alpha=1}^N \rho_{\alpha} V_{\alpha}}{\sum_{\alpha=1}^N \rho_{\alpha}} = \frac{\sum_{\alpha=1}^N \rho_{\alpha} V_{\alpha}}{\rho} = \sum_{\alpha=1}^N \omega_{\alpha} V_{\alpha}$$

*
mass avg velocity

*
molar avg velocity

$$V^* = \frac{\sum_{\alpha=1}^N c_{\alpha} V_{\alpha}}{\sum_{\alpha=1}^N c_{\alpha}} = \frac{\sum_{\alpha=1}^N c_{\alpha} V_{\alpha}}{C} = \sum_{\alpha=1}^N X_{\alpha} V_{\alpha}$$

Table 17.7-2 Notation for Velocities in Multicomponent Systems

Basic definitions:

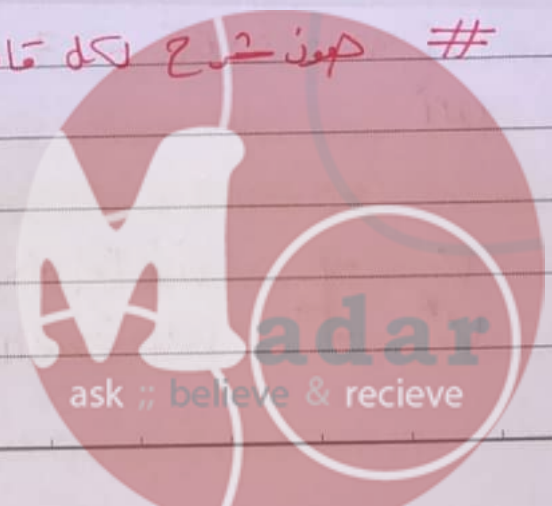
$\mathbf{v}_{\alpha}$	velocity of species α with respect to fixed coordinates
$\mathbf{v} = \sum_{\alpha=1}^N \omega_{\alpha} \mathbf{v}_{\alpha}$	mass average velocity
$\mathbf{v}^* = \sum_{\alpha=1}^N x_{\alpha} \mathbf{v}_{\alpha}$	molar average velocity

$\mathbf{v}_{\alpha} - \mathbf{v}$	diffusion velocity of species α with respect to the mass average velocity $\mathbf{v}$
$\mathbf{v}_{\alpha} - \mathbf{v}^*$	diffusion velocity of species α with respect to the molar average velocity $\mathbf{v}^*$

Additional relations:

$$\mathbf{v} - \mathbf{v}^* = \sum_{\alpha=1}^N \omega_{\alpha} (\mathbf{v}_{\alpha} - \mathbf{v}^*) \quad (F) \qquad \mathbf{v}^* - \mathbf{v} = \sum_{\alpha=1}^N x_{\alpha} (\mathbf{v}_{\alpha} - \mathbf{v})$$

هون شرح لكل قانون وكل رمز استخدميها بـ Ex 2.1



Convective Flux = (Quantity/volume) (Characteristic Velocity)

← انما هون رح انتقال فقط مع Diffusion و رح التعرف على convection

الفرق بين هذين النوعين :

- السائل هون ثابت تقريباً Diffusion →
- هون في velocity عم يتأثر عليه convection →

كيف هلا driving force في عني في mass transfer

Concentration ← اختلاف التراكيز هولي عبارة عن gradient

كلت عني mass transfer وطبعاً يتبدل من

high concentration → low concentration
ويقل سطر ال Mass transfer كلما ربح عني

equilibrium يعني التركز في كل مكان متساوي وقتها
بلغي (driving Force)

$$\text{Molecular Flux} = \frac{\text{Driving Force}}{\text{Resistance}}$$

$$= (\text{Diffusivity}) \times (\text{Gradient of quantity / volume})$$

$$= (\text{Diffusivity}) \times (\text{quantity / volume}) \times (\text{Characteristic length})$$

$$J_{Ay} = - \rho D_{AB} \frac{d\omega_A}{dy}$$

One-dimensional
Form of Fick's

One dimensional Form
of Fick's First law
of diffusion.

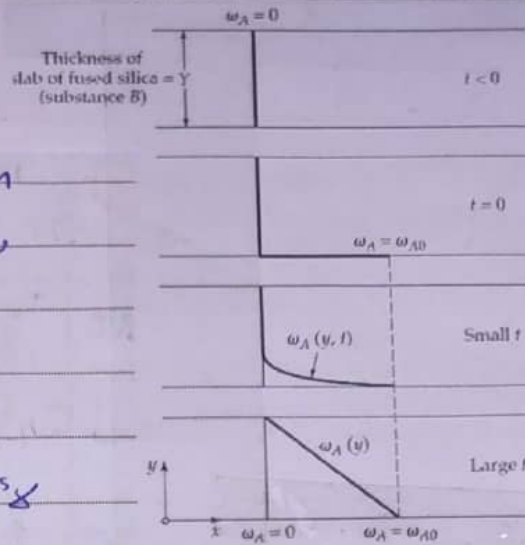


Fig. 17.3-1. Buildup to the steady-state concentration profile for the diffusion of helium (substance A) through fused silica (substance B). The symbol ω_A stands for the mass fraction of helium, and ω_{A0} is the solubility of helium in fused silica, expressed as the mass fraction. See Figs. 1.1-1 and 9.2-1 for analogous momentum and heat-transport situations.

هنا المعادلة اننا بقدرنا نستعملها
لاسي (binary) سواء كان
Solid, liq, gas
solution.

شرط ان J_{Ay} يتم تعريفه

بالنسبة لـ mass flux
mass avg velocity

الشرح :

اولا ان $\omega_A = 0$ هون لسا ما صار عندي Diffusion الحقة. $t < 0$
بعدين بصير في عندي Diffusion من قبل نتحل نقطة الجبر
وبصير تكبير في شوي وتنشأ من لسا ما صار عندي linear
لسا Curve هياي الحالة بعل حساب unsteady state وانا بخل حسابات
ورحل ينتشر لحد ما يصير linear زي اخر شكل هون
اننا بقدرنا نطبق Fick's law. عند t large



An empirical relation for molar flux, first postulated by Fick (accordingly often referred to as Fick's first law) can be written as follows for an isothermal, isobaric binary mixture of A and B in stagnant media or laminar flow regimes.

$$J_{Az}^* = -C_T D_{AB} \frac{dx_A}{dz}$$

(11)

Mixture
A + B

نصف اناء طبقاً لمعادلة فيك الأولى

طبقة سكونية أو جريان طبقة سكونية وحرارة متساوية

isothermal, isobaric

Stagnant أو جريان طبقة سكونية
Flow

نصف اناء طبقاً لمعادلة

J_A^*

is the molar flux of component A in the z direction relative to the molar-average velocity in kmol A/s.m² (fluxes of molecules)
تدفق المادة A باتجاه z بالنسبة لسرعة المتوسطة المولية

D_{AB}

is the molecular diffusivity of the molecule A in B in m²/s

انتشار A في B

C_T

is the total concentration in kmol (A + B) /m³.

التركيز الكلي A + B

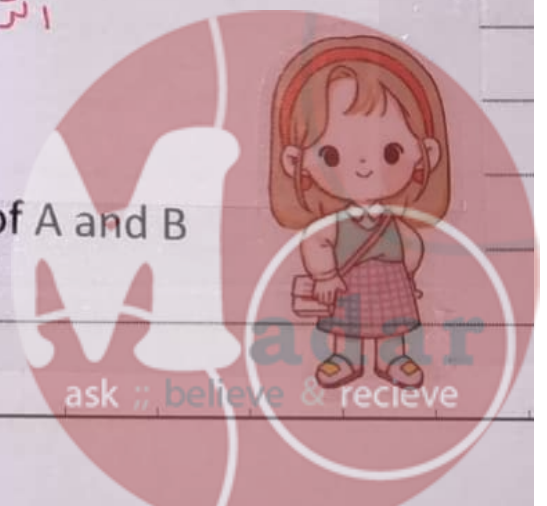
z

is the distance of diffusion in m

مسافة الانتشار

x_A

is mole fraction of A in the mixture of A and B



Transport Phenomena (Transport) : $\Delta \phi = \frac{J}{L} \cdot \Delta x$

Transport	Driving Force	Flux Equation	Phenomenological Coefficient	Flux Unit	Common Name
Mass	Concentration gradient	$J_m = -D \cdot \frac{dc}{dx}$	Diffusion coefficient $D [m^2/s]$	$\left[\frac{kg}{m^2 \cdot s} \right]$	Fick's law of diffusion
Energy/heat	Temperature gradient	$J_h = -k \cdot \frac{dT}{dx}$	Thermal conductivity $k [J/(s \cdot K \cdot m)]$	$\left[\frac{J}{m^2 \cdot s} \right]$	Fourier's law of heat conduction
Momentum	Velocity gradient	$J_n = -\mu \cdot \frac{dv}{dx}$	Dynamic viscosity $\mu [Pa \cdot s]$	$\left[\frac{kg \cdot (m/s)}{m^2 \cdot s} \right]$	Newton's law of viscosity
Volume	Pressure gradient	$J_v = -L_p \cdot \frac{dP}{dx}$	Permeability coefficient $L_p [m^2/(Pa \cdot s)]$	$\left[\frac{m^3}{m^2 \cdot s} \right]$	Darcy's law
Electrical	Voltage gradient	$J_e = -\sigma \cdot \frac{dE}{dx}$	Electrical conductance $\sigma [C^2/(s \cdot J \cdot m)]$	$\left[\frac{C}{m^2 \cdot s} \right]$	Ohm's law

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Ex. A mixture of He and N_2 gas is contained in a pipe at 298 K and 1 atm total pressure which is constant throughout. At one end of the pipe at point (1) the partial pressure P_{A_1} of He is 0.6 atm and at the other end 0.2 m $P_{A_2} = 0.2$ atm. Calculate the flux of He at steady state if D_{AB} of the He- N_2 mixture is $0.687 \times 10^{-4} \text{ m}^2/\text{s}$.

For steady state the flux $J_{Az}^{\#}$ is constant. Also D_{AB} for gas is constant, rearranging

$$J_{AZ}^* \int_{z_1}^{z_2} dz = -D_{AB} \int_{CA_1}^{CA_2} dC_A$$

(2) ---

$$J_{AZ}^* = \frac{D_{AB}(c_{A1} - c_{A2})}{z_2 - z_1}$$

From the perfect gas law:

$$P_A V = n_A RT, \quad C_{A1} = \frac{P_{A1}}{RT} = \frac{n_A}{V}$$

(3) ———

$$J_{A_z}^* = \frac{D_{AB}(p_{A1} - p_{A2})}{RT(z_2 - z_1)}$$

مبادی و معادلات دیفرانسیل

$$P_{A1} = 0.6 \text{ atm} = 0.6 * 1.01325 * 10^5$$
$$= 6.04 * 10^4 \text{ Pa}$$

$$P_{Ar} = 0.2 \text{ atm} = 2.027 \times 10^4 \text{ Pa}$$

using SI unit
ask :: believe & receive

نصف مادة رقم (3)

Soln :

$$J_{Az}^* = \frac{(0.687 \times 10^{-4}) (0.08 \times 10^4 - 2.027 \times 10^4)}{8314 (298) (0.20 - 0)}$$

$$= 5.63 \times 10^{-6} \text{ kg mol A / s.m}^2$$

بالانجليزي النظام pressure في (SI) unit

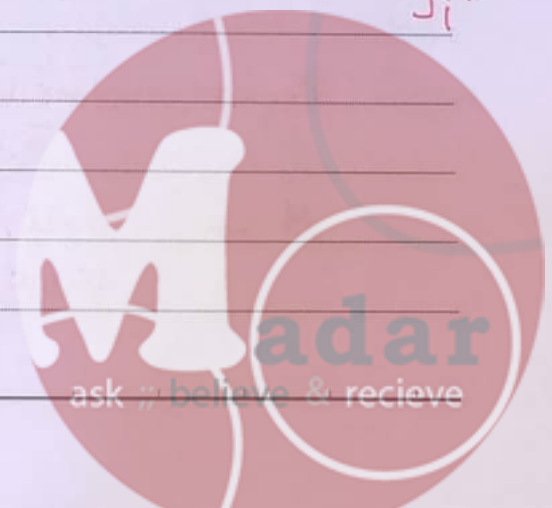
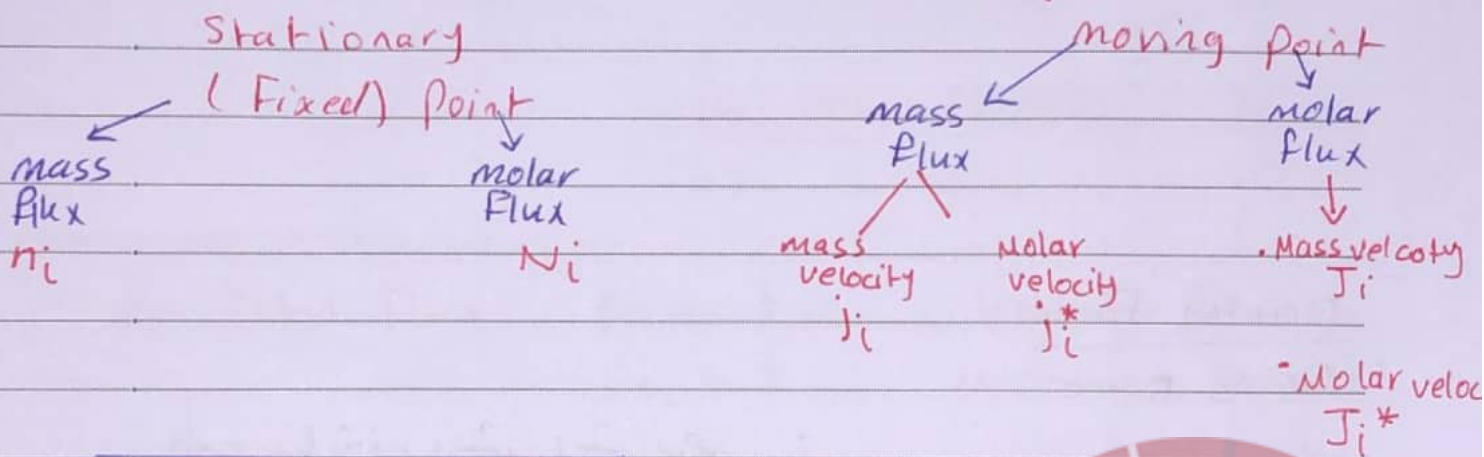
$$J_{Az}^+ = \frac{(0.687 \times 10^{-4}) (0.60 - 0.20)}{(82.06 \times 10^{-3}) (298) (0.20 - 0)}$$

$$5.63 \times 10^{-6} \text{ kg mol A / s.m}^2$$

بالعربي نفس الشيء لانهم بالانجليزي



Type of mass transfer fluxes





Diffusivities and Dimensionless Numbers

Dimensions of L^2/t

- Momentum diffusivity (kinematic viscosity)

$$\nu = \frac{\mu}{\rho}$$

- Thermal diffusivity

$$\alpha = \frac{k}{\rho \hat{C}_p}$$

- Mass diffusivity

$$D_{AB}$$

Dimensionless

- Prandtl number

$$Pr = \frac{\nu}{\alpha} = \frac{\hat{C}_p \mu}{k}$$

- Schmidt number

$$Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$

- Lewis number

$$Le = \frac{\alpha}{D_{AB}} = \frac{k}{\rho \hat{C}_p D_{AB}}$$

Driving Force

Dring. Force

كيفية تدفينا نعرفه اسهل اسهل فاعلم :

$K \rightarrow$ مقياس انما ينقل حرارة

$C_p \rightarrow$ مقياس بقدر امتصاص حرارة

$\mu \rightarrow$ مقاومة المادة لـ Flow



$$\text{Total flux} = \text{Molecular flux} + \text{Convective flux}$$

$$\frac{\text{Convective flux}}{\text{Molecular flux}} \rightarrow \text{dimensionless}$$



$$= \frac{(\text{Characteristic length})(\text{characteristic velocity})}{\text{Diffusivity}}$$

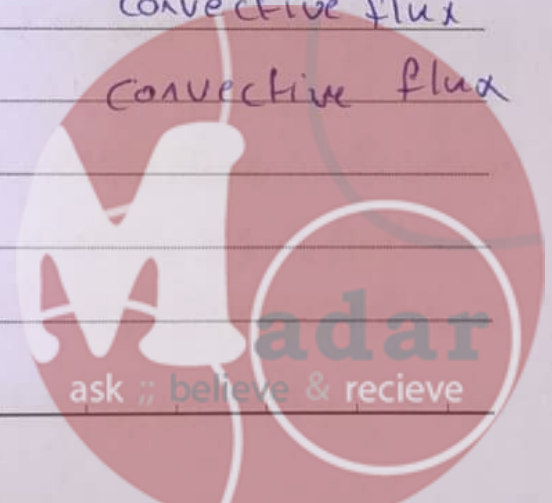
$$= Pe$$

$$Pe_{MT} = \frac{L_{ch} v_{ch}}{D_{AB}}$$

$$Pe_{HT} = \frac{L_{ch} v_{ch}}{\alpha}$$

$$Pe_{Mont} = \frac{L_{ch} v_{ch}}{r} = Re$$

#	Total flux	$Pe \ll 1$	Molecular flux
		$Pe \approx 1$	Molecular flux + convective flux
		$Pe \gg 1$	convective flux





Existence of Gradient

- * when a concentration gradient exists for one component of a binary Mixture in any direction, there must be a concentration difference for the other component of the mixture in the opposite direction.

سوف نرى هذا الحكي !
نَحْنُ نَحْضِرُ كَأَسَةِ فِي مِيقَا نَقْطَةِ حَبْرٍ عِنْدَ نَقْطَةِ اكْبَرِ التَّرْكِيزِ اعْلَى
فَأَمَّا نَحْنُ لِلْحَبْرِ وَاقِلٌ مَا يَكُونُ اللَّحْيُ بِالنَّاتِي الزِّيَادَةِ فِي تَرْكِيزِ مَادَّةِ
الْكِبَرِ بِرَافَقَةِ تَغَيُّرَاتٍ فِي مَادَّةِ ثَانِيَةٍ وَصِلَتْ .

- * As a result of these concentration differences, both components of the Mixture diffuse in in the opposite directions.

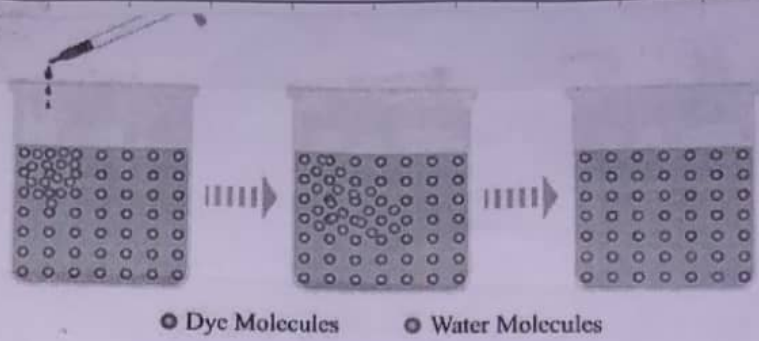
نَتَجَةُ لِاخْتِلَافِ التَّرَاكِيزِ رَجَحٌ يَنْسَرُّ كُلُّ خِلْطٍ فِي اتِّجَاهٍ
مُخَالِفٍ لِبَعْضٍ .

- * The rates of these diffusions are not equal in molar units, then the Mixture itself drifts in the direction of the component whose molar diffusional rate is greater

* نَتَذَكَّرُ لَمَّا كُنَّا نَلْعُو أَلْبَابَ وَصَارَ كُلُّ وَاحِدٍ يَتَحَرَّكُ بِحَسَبِ اتِّجَاهِ
حَرَكَةِ السَّبَابِ الْكُلِّيَّةِ كَانَتْ بِاتِّجَاهِ الْحَلِيلَةِ الْأَكْبَرِ (أَلْعُزْنَ الْأَكْثَرُ)
بِسَبَابَةِ الْخِلْطِ رَجَحٌ يَكُونُ بِاتِّجَاهِ مَادَّةِ
rate is greater

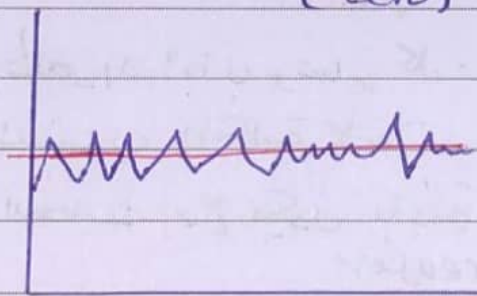
∴ So, it is obvious that total molar flux of each component for a fixed observer will be different than the diffusional fluxes of the components.

بِالْمُجْتَمِعِ التَّيَقُّنِ الْخَوَلِيِّ فَتَلَفَتْ لِكُلِّ مَادَّةٍ



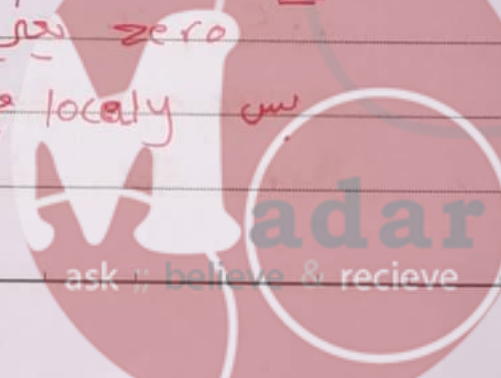
لكي نتوحي عندها الصورة انحنى فكر طبياً
 في عنا مادتين اول ما حصلنا اعادة
 A على المادة B تركيز اعادة
 A كان عالي وتركيز B قليل
 طبياً اعادة بلست تنتقل mass transfer
 هذا التركيز العالي للمخفف للمواد
 (equal) متساويين يعني التركيز في كل مكان
 ثابتة وصحت كقوة (Concentration gradient)
 نتوحيه Net Mass transfer
 = zero

طباً هو فعلاً = zero فلا لوبي احدى بكل فنية
 لا مرة بطرح مرة ينزل بس كـ Avg او net
 او ساوي (zero)



في حركة بسبب الحركة العشوائية
 لكن الخط هو ساوي
 Avg مساوي

zero يعني ما في حركة
 بس locally في حركة





Stefan - Maxwell Formalism

Early experimental investigations of molecular diffusion were **unable** to verify Fick's law of diffusion

كان من ناس علم تجارب و حمو theory لستيفان ماكسويل
و التجارب الباشية فاكنت بغير قانون (Fick)

* Attributed to mass is often transferred simultaneously by two possible means :

- as a result of the concentration differences as postulated by **Fick**
- and by **convection** induced by the **density differences** resulting from the concentration variation.

Stefan and Max
using the kinetic theory
of gases, showed that the
mass flux relative to a fixed
coordinate evolves as a result
of two contributions :

- the concentration gradient
- the bulk motion

Total
Mass transport = Mass transport
by diffusion

+ mass transport by bulk motion
of fluid

Stefan
(1872)



Maxwell
(1877)

ask :: believe & receive

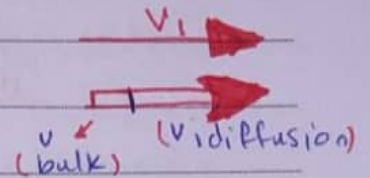


The Molar-Average Velocity and Relative velocity :

(bulk velocity

The molar average velocity (relative to stationary coordinates) for a multicomponent mixture is defined in terms of the molar concentration of all components by :

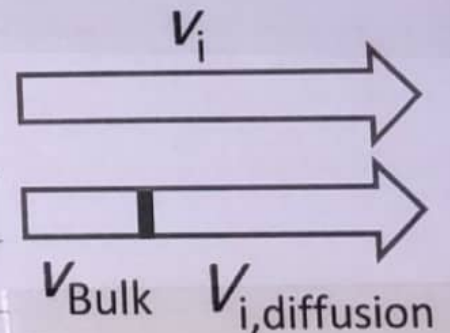
$$V = \frac{\sum_{i=1}^n c_i V_i}{\sum_{i=1}^N c_i} = \frac{\sum_{i=1}^n c_i V_i}{c} = \sum_{i=1}^n X_i V_i$$



لعبارة الحالة أنا بأحد الاتجاهات
لجميع المجموع وباتجاه الأعلى

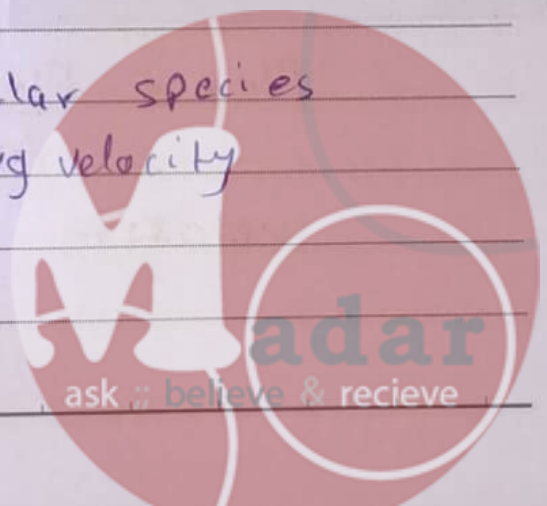
$V_i \rightarrow$ the velocity of each type of molecule in the specified direction.

$$V_{i,diffusion} = V_i - V_{Bulk}$$



diffusion velocity :

The velocity of particular species relative to the molar avg velocity





The Molar diffusive flux

- The Molar diffusive flux of A in B in a Binary Mixture is given by :

$$J_{Az}^* = -D_{AB} \frac{dc_A}{dz}$$

هنا المعادلة تستخدمها لحسابي الفلج
Molar diff flux

لحاجة معينة داخل مادة اخرى ففرضاً هنا
A و B . B داخل A تدفق
مستحالة الخيز (binary Mixture)

- The velocity of the diffusive flux of A in B (diffusion velocity of A) can Given by

$$V_{A,diffusion} (m/s) = \frac{J_A^* (mol/m^2.s)}{c_A (mol/m^3)}$$

$$\text{Flux} = \frac{\text{Flow}}{\text{Area}} = \frac{mol}{m^2.s}$$

$$\text{velocity} = \frac{mol / m^2.s}{mol / m^3} = m/s$$

adar
ask :: believe & recieve

(مقاييس سرعة الحسابات)

Bulk velocity and Total Molar Flux

The bulk velocity for a binary mixture is given by

$$V_{\text{Bulk}} (\text{m/s}) = \frac{(C_A V_A + C_B V_B) (\text{mol/m}^2 \cdot \text{s})}{C_T (\text{mol/m}^3)}$$

The total molar flux of components A and B relative to Fixed coordinates

$$N_A = C_A V_A$$

$$N_B = C_B V_B$$

The Bulk velocity is:

$$V_{\text{Bulk}} (\text{m/s}) = \frac{(N_A + N_B)}{C_T}$$

(نفس القارئ الأول بين يدي فوق حللنا)

هذا الـ diffusion يصير لا جسم أو المادة وهي سائلة -
Convection velocity يصير ا -

$$\text{Convection} = (\text{diffusion} + \text{advection})$$





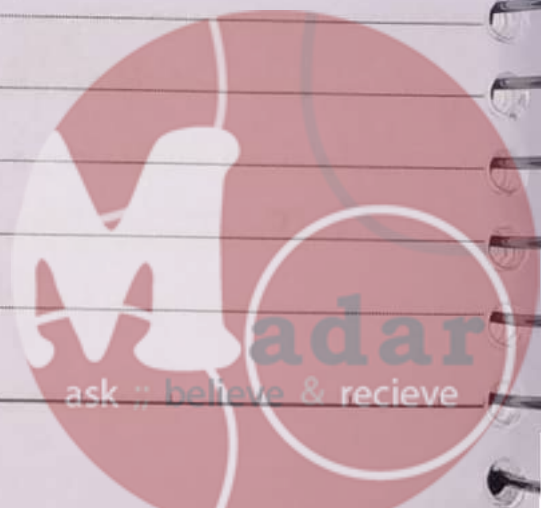
Total Molar Flux of Species and Net velocity

→ The total molar flux of species i (N_i) by convection (diff + advection) with respect to a stationary point is defined as the rate of transfer by unit area :

$$N_i (\text{mol/m}^2 \cdot \text{s}) = \frac{\bar{N}_i (\text{mol/s})}{A (\text{m}^2)}$$

∴ the velocity of the net flux of Air B (relative to stationary coordinates) given by :

$$V_A (\text{m/s}) = \frac{N_A (\text{mol/m}^2 \cdot \text{s})}{C_A (\text{mol/m}^3)}$$



General flux Equation

For species A, :

$$V_A = V_{A \text{ diffusion}} + V_{\text{Bulk}}$$

بضرب الطرفين بالمعادلة C_A

$$C_A V_A = C_A V_{A \text{ diff}} + C_A V_{\text{Bulk}}$$

$\downarrow$ Molecular Diffusion $\downarrow$ Bulk Flow Flux

$$C_A = \frac{P_A}{RT} \quad \text{For gass}$$

$$\therefore N_A = J_A^* + C_A \frac{(N_A + N_B)}{C_T}$$

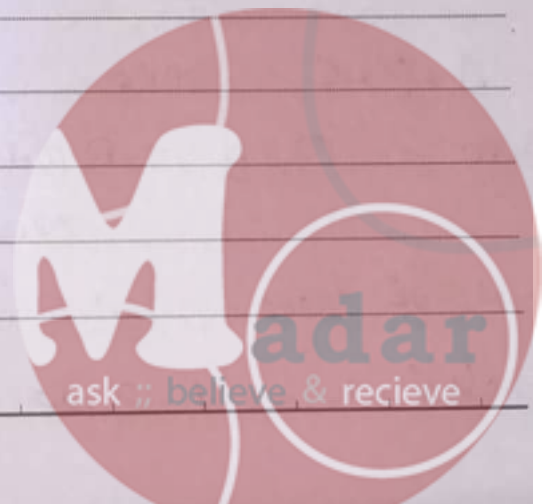
المعادلة

#

$$N_A = J_A^* + x_A (N_A + N_B)$$

$$\sum N_i = \sum J_i^* + \sum x_i N_T$$

هذه المعادلة بتبين علاقة D (diffusion coefficient)



Substituting J_A^* for the diffusive flux
From Fick's law

$$* N_A = -D_{AB} \frac{dc_A}{dz} + \frac{C_A}{C} (N_A + N_B)$$

Molecular
Diff flux of
A

Bulk flow
Flux $\propto f_A$

Total mass = by diff + by Bulk motion of fluid
Transported

Special case : $N_A \approx J_A^* = -D_{AB} \frac{dc_A}{dz}$

هنا نفس المعادلة بالفرق بين z term
سواءً $(N_A + N_B)$ يكون نفس
مolar zero = نفس الآخر

طب سيجد نتائج مختلفة

$$(-D_{AB} + D_{BA}) C \frac{dy}{dx} = 0$$

$$A + B + C = \text{zero}$$

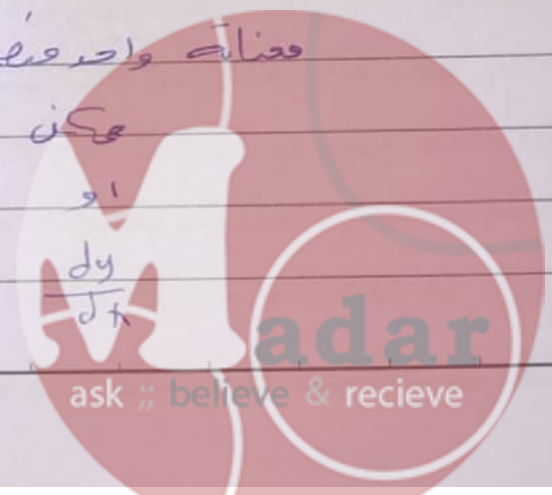
معادلة واحدة فقط = zero

$$\text{zero} = D_{AB} = D_{BA}$$

$$\text{zero} = C$$

zero = zero $\rightarrow$ معادلة واحدة

معادلة واحدة $\rightarrow$ معادلة واحدة



General Flux Equation [gases at low pressures]

$$C_A = \frac{n_A}{V} = \frac{P_A}{RT}$$

$$C_T = \frac{n_T}{V} = \frac{P}{RT}$$



∴ The general flux equation becomes :

$$N_A = - \frac{D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P} (N_A + N_B)$$

$$N_A = - \frac{D_{AB}}{RT} \frac{dP_A}{dz} + x_A (N_A + N_B)$$

##



Outline

- prelude
- Diffusivity For Gases
- Diffusivity For liquids
- Diffusivity For Solids



← D_{AB} diffusivity د تقارب د diffusivity
و شؤن لىقائىن ركل phase

(Diffusion coeff) D_{AB} بىكول عام سؤ وحدة
(S) $\text{Area} \rightarrow \text{Area}$ لىقائىن لىقائىن m^2/s , cm^2/s
SI unit

* D_{AB} Function in $T, P, \text{Composition}$
 $[D_{AB} = f(T, P, x)]$

* D_{AB} has the dimension L^2/t
indetical to the other transport properties :

kinematic viscosity $\gamma = \mu / \rho \rightarrow$ in momentum
thermal diffusivity $= \alpha = k / \rho c_p \rightarrow$ in heat



phase	$D_{AB} (\text{m}^2/\text{s})$
→ Gases	$5 \times 10^{-6} - 1 \times 10^{-5}$
→ liquids	$10^{-6} - 10^{-9}$
→ Gas - Solid	$10^{-12} - 10^{-14}$
Solid Gas Solid	$10^{-19} - 10^{-35}$

← سؤىن 10^{-35} ← دىقائىن (diffu) دىقائىن دىقائىن



Dependence of Diffusivity

→ Gases at low density

1 - Almost composition independent

at low $\Rightarrow$ (x) (composition) الغازات في الحالة منخفضة الكثافة

$P \Rightarrow$ ليس $\Rightarrow$ لا يتغير $\uparrow$ مع التركيزات تكون كبيرة $\Rightarrow$ في حالة غازات منخفضة الكثافة

at low (P) / low density

2 - increase with the temperature.

$\uparrow$ velocity $\uparrow$ KE $\uparrow$ temp $\uparrow$ سرعة $\uparrow$ الطاقة الحركية $\uparrow$ الحرارة

$\uparrow D_{AB}$ مع زيادة D_{AB} مع زيادة

3 - Vary inversely with pressure

الضغط عكسي $\Rightarrow$ التركيزات منخفضة

$\downarrow D_{AB}$ مع زيادة الضغط

4 - Inversely proportional to molecular weight

إذا زاد الوزن الجزيئي $\Rightarrow$ سرعة الحركة عكسي $\Rightarrow$ التركيزات منخفضة

→ liquids and solids

1 - strongly concentration dependent

2 - Increase with temperature

مع زيادة الحرارة $\Rightarrow$ التركيزات

* $\propto$ Pressure $\propto$ التركيزات

density $\propto$ الكثافة

* molecular weight $\propto$ الوزن الجزيئي

Dependence For Gas Phase

- pressure dependence for moderate ranges of pressures up to 25 atm:

$$D_{AB} \propto \frac{1}{P}$$

عكس التناسب مع الضغط

- Temperature dependence:

$$D_{AB} \propto T^{3/2 - 2}$$

قوة التناسب مع درجة الحرارة

$$\left(\frac{3}{2} - 2\right)$$

تأثير الـ Temp أكثر من تأثير الـ (P)

إذا طغت T الـ P_{AB} (4) (الضغط) حيز يزيد

الـ P يقل للنتيجة

عند العلاقات على 25 to 4 P

حيث المعنى حيث قليل وقليل على

$$D_{AB}(T, P) = D_{AB}(T_0, P_0) \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{P_0}{P}\right) \left(\frac{\Omega_P T_0}{\Omega(T)}\right)$$



ex (3.4) For the CO-C₂H₄ binary pair

with A = CO, B = ethylene (C₂H₄), the measured diffusion coefficient is $D_{AB} = 0.151 \text{ cm}^2/\text{sec}$ at $P_0 = 1.0 \text{ atm}$, $T_0 = 273 \text{ K}$. What will be the D_{AB} at 2.0 atm and $T = 350 \text{ K}$ given that ratio of the collision integral at the two conditions is 1.088?

$$D_{AB}(T_0, P_0) = 0.151 \frac{\text{cm}^2}{\text{s}}$$

$$P_0 = 1.0 \text{ atm}$$

$$P_1 = 2.0 \text{ atm}$$

$$T = 350 \text{ K}$$

$$T_0 = 273 \text{ K}$$

$$= (0.151) * \left(\frac{1.0}{2.0}\right) * \left(\frac{350}{273}\right)^{3/2} * \left(\frac{1.112}{1.022}\right)$$

$$= 0.119 \text{ cm}^2/\text{sec}$$

$$= 1.088$$

$$\text{ask :: } \frac{\Omega(T_0)}{\Omega(T)} = 1.088$$

$$\Omega(T)$$



Molecular Diffusion and Kinetic Theory of Gases :

← يعرفون الغازات تتحرك بسرعات عشوائية وفي اتجاهات مختلفة

← متجهين للجزيئات (diffusion) ← على حرارة اعلى من
الحرارة الصفرية

→ All particles undergo diffusion at temp above absolute zero

$$D_{AB} \propto \lambda v$$

$\lambda \rightarrow$ mean Free path

$v \rightarrow$ mean molecular velocity

mean Free path $\rightarrow$ the typical length that a particle travels for before it hits another particle

لو كان عن جزيئات تتبين وحدة ثابتة والثانية متحركة باتجاه
الثابتة الطاقة الثابتة ردة فعلها مع الطاقة المتحركة
من الاستجاب خرج تغير تغيرها لا نهالو اصبحت
خرج ثلثها كغير حركتها (KE) ياتي عندها

← كلما λ كانت اكبر ال diff D_{AB} , ح يكون اكبر

← كلما v كانت اسرع كلما D_{AB} صار اسرع



$\lambda \Rightarrow$ mean Free path is inversely proportional with (n) number of molecules

كلما كثرت الجزيئات تكون عاكسة كل ما كان (n) أكثر يعني λ ستكون أقل
أي إقبات ستكون قريبة من بعضها بسبب عدد (n)
Cross-sectional area $A = \pi d^2$

For this molecules

لذلك إذا زاد الحجم تآخى ال molec λ يقل λ يتآخى Area أكثر

$$\# \quad \lambda = \frac{1}{\sqrt{2}} \left(\frac{1}{nA} \right) = \frac{1}{\sqrt{2}} \left(\frac{1}{\pi n d^2} \right)$$

(ideal gas)

$$PV = nRT$$

$$n \propto \frac{P}{T} \rightarrow \lambda \propto \frac{T}{PA} \rightarrow \lambda \propto \sqrt{\frac{T}{M_{WT}}}$$

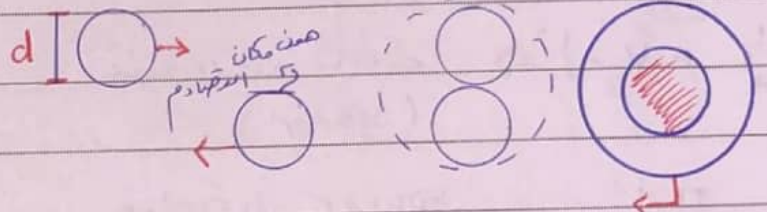
يرجع لمصنع $A = \pi d^2$ ريدنا نأخذ من (d) وهو قطر الجزيء

$d \Rightarrow$ collision diameter

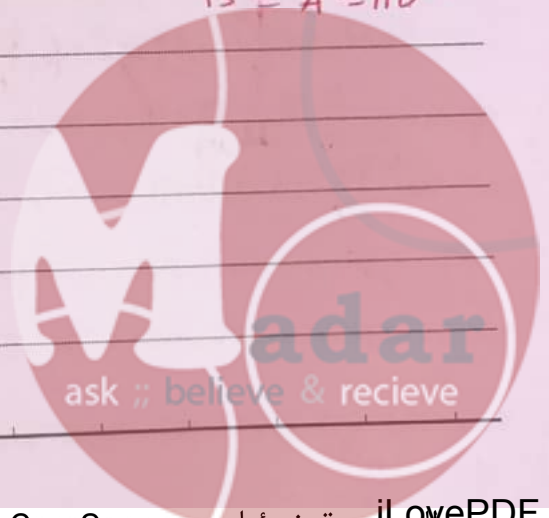
(the diameter of a

repulsive volume when

another molecule is approaching to the first one, it repulses without collision.)



The effective collision area is $A = \pi d^2$



Diffusion Coefficient From kinetic theory (Gilliland Equation)

$$D_{AB} \propto \sqrt{\frac{T}{M_A} + \frac{T}{M_B}} \cdot \frac{T}{P A_{avg}}$$

لا، عني فادش 8

$$D_{AB} = \frac{k T^{3/2}}{P A_{avg}} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}} = T^{1/2} + T$$

هنا عني (k) (constant) لا، عني فادش
Gilliland (1934) للعلماء انهم رطوة، راسبت مع

$$D_{AB} = 4.3 \times 10^{-9} \frac{T^{3/2}}{P (V_A^{1/3} + V_B^{1/3})^2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$$

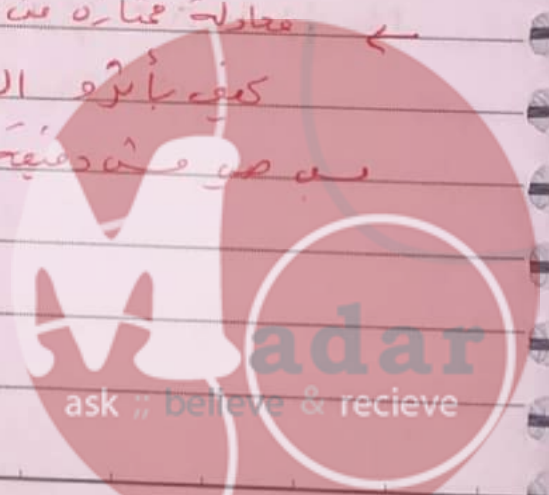
$\downarrow$ atm $\downarrow$ Molecular Volume At Normal Boiling ($m^3 / kg-mol$) $\downarrow$ Molar Mass ($kg / kg-mol$)

لا، A كيف ال $(V_A^{1/3} + V_B^{1/3})^2 \Rightarrow$ $\frac{4}{3} \pi r^3$ (sphere)

$$\pi d^2 \rightarrow \sqrt[3]{V}$$

مساحة ال sphere

(qualitively) معادلة متان من فادش كيف ال
 $\frac{P}{T} \rightarrow (D_{AB})$ ال variables
 M_{wt} مع فادش



(2) Statterry - Bird equation

دلالة (Volume) V في المعادلة
Polarity ← dielectric Constant ϵ $P_c \rightarrow T_c$

$$D_{AB} = a \left(\frac{T}{\sqrt{T_A T_B}} \right)^b \frac{(P_A P_B)^{1/3} (T_A T_B)^{5/12}}{P} \left[\frac{1}{M_A} + \frac{1}{M_B} \right]^{1/2}$$

where,

- Non polar gases $a = 2.745 \times 10^{-4}$ $D_{AB} = \text{cm}^2/\text{sec}$
Excluding H_2 & He $b = 1.823$ $P = \text{atm}$
 $T = K$

- H_2O non polar gas $a = 3.640 \times 10^{-4}$
 $b = 2.334$

في D_{AB} (V) $\leftarrow$
at low density $\leftarrow$

(3) Champ man - Enskog Equation (For ideal gas)

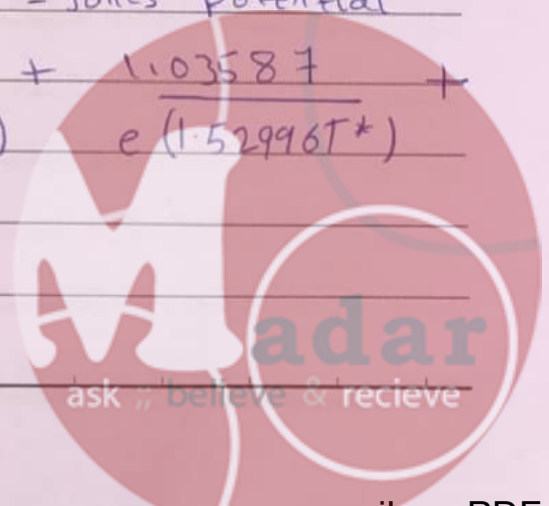
$$D_{AB} = 1.858 \times 10^{-7} T^{3/2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}} \rightarrow \text{kg/kg-mol} \quad C = \frac{P}{RT}$$

$m^2/\text{sec} \leftarrow$ $P \sigma_{AB}^2 \Omega_{DAB}$
 $\rightarrow$ Dimention less

Ω_{PAB} (collision integral) molecule $\leftarrow$
interaction potential $\leftarrow$

based on the Lennard-Jones Potential

$$\Omega_{PAB} = \frac{1.0603}{T^{*0.15610}} + \frac{0.19300}{e^{(0.47635 T^*)}} + \frac{1.03587}{e^{(1.52996 T^*)}} + \frac{1.76474}{e^{(3.89411 T^*)}}$$



القوة الجزيئية = molecular Force

في منطقة رح سياره

attraction و repulsion

8.8

بمسافات بعيدة

قريبه المسافة

6 (collision diameter)

Zero = molecules 2 molecular Force

$$\sigma_{AB} = \frac{\sigma_{AA} + \sigma_{BB}}{2}, \text{ (Collision diameter)}$$

$$\frac{\epsilon_{AB}}{k_B} = \left(\frac{\epsilon_{AA}}{k_B} \cdot \frac{\epsilon_{BB}}{k_B} \right)^{\frac{1}{2}}, \text{ well depth energy minimum}$$

$$T^* = \frac{k_B T}{\epsilon}, \text{ Reduced } T$$

(Dimension less)

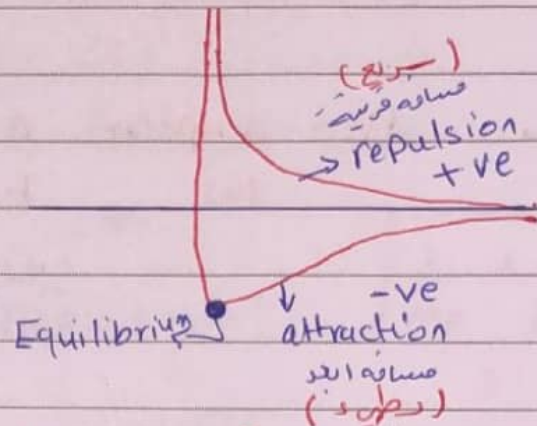


Table E.2
For ϵ, σ

$$U(r) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right]$$

repulsive attraction

* $\epsilon \rightarrow (e.v)$

* $\sigma \rightarrow A^\circ 10^{-10}$

(Lennard-Jones 12-6)
Potential

* $\frac{\epsilon}{k_B} (K)$

$$k_B = \frac{R}{N_{Avog}}$$

$$= 8.14$$

$$6.023 \times 10^{-23}$$

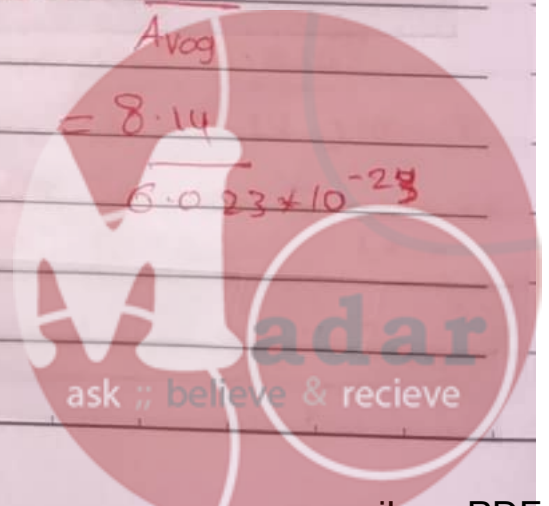


Table E.2 Collision Integrals for Use with the Lennard-Jones (6-12) Potential for the Prediction of Transport Properties of Gases at Low Densities^{a,b,c}

kT/ϵ or kT/ϵ_{AB}	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{\mathcal{D},AB}$ (for diffusivity)	kT/ϵ or kT/ϵ_{AB}	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{\mathcal{D},AB}$ (for diffusivity)
0.30	2.840	2.649	2.7	1.0691	0.9782
0.35	2.676	2.468	2.8	1.0583	0.9682
0.40	2.531	2.314	2.9	1.0482	0.9588
0.45	2.401	2.182	3.0	1.0388	0.9500
0.50	2.284	2.066	3.1	1.0300	0.9418
0.55	2.178	1.965	3.2	1.0217	0.9340
0.60	2.084	1.877	3.3	1.0139	0.9267
0.65	1.999	1.799	3.4	1.0066	0.9197
0.70	1.922	1.729	3.5	0.9996	0.9131
0.75	1.853	1.667	3.6	0.9931	0.9068
0.80	1.790	1.612	3.7	0.9868	0.9008
0.85	1.734	1.562	3.8	0.9809	0.8952
0.90	1.682	1.517	3.9	0.9753	0.8897
0.95	1.636	1.477	4.0	0.9699	0.8845
1.00	1.593	1.440	4.1	0.9647	0.8796
2.00	1.176	1.075	25.0	0.7198	0.6414
2.10	1.156	1.058	30.0	0.7010	0.6235
2.20	1.138	1.042	35.0	0.6854	0.6088
2.30	1.122	1.027	40.0	0.6723	0.5964
2.40	1.107	1.013	50.0	0.6510	0.5763
2.50	1.0933	1.0006	75.0	0.6140	0.5415
2.60	1.0807	0.9890	100.0	0.5887	0.5180





① عنوان صحت ← ربط ϵ , σ من الجدول

Ex. (Lennard-Jones) 12-6 Potential

Determine the inter molecular potential energy between two Argon (Ar) atoms separated by a distance of $4.0 \text{ \AA}$

$$k_B \rightarrow \text{Boltzmann's Const} = 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$$

$$N_A \rightarrow \text{Avogadro's Number} = 6.02 \times 10^{23} \frac{\text{Molecule}}{\text{mole}}$$

$$\text{From table } \epsilon / k_B = 122.4 \text{ K}^{-1}$$

$$\sigma = 3.432 \text{ \AA}$$

$$\epsilon = \frac{\epsilon}{k_B} \times k_B \times N_A$$

$$= 122.4 \times 1.38 \times 10^{-23} \times 6.022 \times 10^{23}$$

$$= 1017.6 \text{ J/mol}$$

$$= 1.0176 \text{ KJ/mol}$$

$$u = 4 \times 1.0176 \left[\left(\frac{3.432}{u} \right)^{12} - \left(\frac{3.432}{u} \right)^6 \right]$$

$\#$ Attractive energy $\leftarrow = -0.9761 \text{ KJ/mol}$



		Lennard-Jones parameters		
Substance	Molecular Weight <i>M</i>	σ (Å)	ϵ/k (K)	Ref.
Light elements:				
H ₂	2.016	2.915	38.0	<i>a</i>
He	4.003	2.576	10.2	<i>a</i>
Noble gases:				
Ne	20.180	2.789	35.7	<i>a</i>
Ar	39.948	3.432	122.4	<i>b</i>
Kr	83.80	3.675	170.0	<i>b</i>
Xe	131.29	4.009	234.7	<i>b</i>
Simple polyatomic gases:				
Air	28.964 ⁱ	3.617	97.0	<i>a</i>
N ₂	28.013	3.667	99.8	<i>b</i>
O ₂	31.999	3.433	113.	<i>a</i>
CO	28.010	3.590	110.	<i>a</i>
CO ₂	44.010	3.996	190.	<i>a</i>
NO	30.006	3.470	119.	<i>a</i>
N ₂ O	44.012	3.879	220.	<i>a</i>
SO ₂	64.065	4.026	363.	<i>c</i>
F ₂	37.997	3.653	112.	<i>a</i>
Cl ₂	70.905	4.115	357.	<i>a</i>
Br ₂	159.808	4.268	520.	<i>a</i>
I ₂	253.809	4.982	550.	<i>a</i>

Hydrocarbons:

CH ₄	16.04	3.780	154.	b
CH ₂ =CH	26.04	4.114	212.	d
CH ₂ =CH ₂	28.05	4.228	216.	b
C ₂ H ₆	30.07	4.388	232.	b
CH ₃ C≡CH	40.06	4.742	261.	d
CH ₃ CH=CH ₂	42.08	4.766	275.	b
C ₃ H ₈	44.10	4.934	273.	b
n-C ₄ H ₁₀	58.12	5.604	304.	b
i-C ₄ H ₁₀	58.12	5.393	295.	b
n-C ₅ H ₁₂	72.15	5.850	326.	b
i-C ₅ H ₁₂	72.15	5.812	327.	b
C(CH ₃) ₄	72.15	5.759	312.	b
n-C ₆ H ₁₄	86.18	6.264	342.	b
n-C ₇ H ₁₆	100.20	6.663	352.	b
n-C ₈ H ₁₈	114.23	7.035	361.	b
n-C ₉ H ₂₀	128.26	7.463	351.	b
Cyclohexane	84.16	6.143	313.	d
Benzene	78.11	5.443	387.	b
CH ₃ Cl	50.49	4.151	355.	c
CH ₂ Cl ₂	84.93	4.748	398.	c
CHCl ₃	119.38	5.389	340.	e
CCl ₄	153.82	5.947	323.	e
C ₂ N ₂	52.034	4.361	349.	e
COS	60.076	4.130	336.	e
CS ₂	76.143	4.483	467.	e
CCl ₂ F ₂	120.91	5.116	280.	b

BSL, p.864 for ϵ and σ .

ask :: believe & recieve

Ex 5.1-2 Use the Chapman-Enskog theory to estimate the diffusion of hydrogen in nitrogen at 21°C and 2 atm. The experimental value is 0.38 cm²/sec

$$\sigma_{AB} = \frac{2.915 + 3.667}{2} = 3.291 \text{ \AA} \quad \left| \begin{array}{l} \sigma_H \\ \sigma_N \end{array} \right.$$

$$\frac{\epsilon_{AB}}{K_B} = \left(\frac{\epsilon_{AA}}{K_B} \cdot \frac{\epsilon_{BB}}{K_B} \right)^{\frac{1}{2}} = (38 \times 99.8)^{\frac{1}{2}} = 61.6 \text{ K}$$

$$T^* = \frac{K_B T}{\epsilon} = \frac{294}{61.6} = 4.774$$

$$\Omega_{DAB} = \frac{1.06036}{(4.774)^{0.15610}} + \frac{0.19300}{e^{(0.47635 + 4.774)}} +$$

$$\frac{1.03587}{e^{(1.52996 + 4.774)}} + \frac{1.76474}{e^{(3.89411 + 4.774)}} = 0.85123$$

$$D_{AB} = 1.858 \times 10^{-7} \times 294.15^{3/2} \sqrt{\frac{1}{2.016} + \frac{1}{28.013}} \times 0.85123$$

$$= 3.707 \times 10^{-5} \text{ m}^2/\text{s}$$



Ex. Chapman and Enskog

Estimate the diffusion coefficient for the system $N_2 - CO_2$ at 590 K and 1 bar. The experimental value reported Ellis and Holser 1989 is $0.583 \text{ cm}^2/\text{s}$.

$$\begin{aligned} \sigma(CO_2) &= 3.941 \text{ \AA} \rightarrow 3.996 \text{ \AA} \\ \sigma(N_2) &= 3.798 \text{ \AA} \rightarrow 3.667 \text{ \AA} \end{aligned}$$

معدل التصادم (Table) $\rightarrow$ معدل التصادم

$$\begin{aligned} \epsilon(CO_2)/K &= 195.2 \text{ K} \rightarrow 190 \text{ K} \\ \epsilon(N_2)/K &= 71.4 \text{ K} \rightarrow 99.8 \text{ K} \end{aligned}$$

$$M(CO_2) = 44.0 \quad M(N_2) = 28.0$$

1- اولى خطوة: معدل التصادم، σ_{AB}

2- معدل التصادم، ϵ_{AB}

$$\sigma_{AB} = \frac{\sigma_A + \sigma_B}{2} \rightarrow \frac{3.941 + 3.798}{2} = 3.8695$$

arithmetic

معدل التصادم

$$\epsilon_{AB} = (\epsilon_A \cdot \epsilon_B)^{1/2} \rightarrow [(195.2)(71.4)]^{1/2} = 118 \text{ K}$$

Geometrical

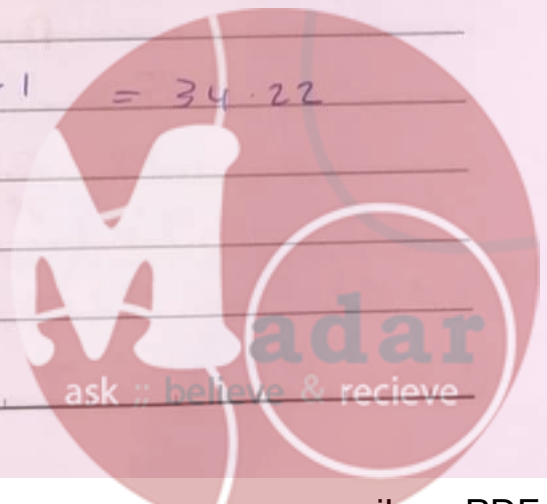
$$T^* = \frac{KT}{\epsilon} \quad \frac{590}{118} = 5.0$$

3- معدل التصادم، T^*

4- معدل التصادم، M_{AB}

$$\begin{aligned} M_{AB} &= 2 \left[\left(\frac{1}{M_A} \right) + \left(\frac{1}{M_B} \right) \right]^{-1} \\ &= 2 \left(\frac{1}{44.0} + \frac{1}{28.0} \right)^{-1} = 34.22 \end{aligned}$$

طبي الحساب
اختياري



$$\begin{aligned} \Omega D &= \frac{1.06036}{5.0^{0.15610}} + \frac{0.19300}{e(0.47635 \times 5.0)} + \frac{1.03587}{e(1.52996 \times 5)} \\ &+ \frac{1.76474}{e(3.89411 \times 5)} \end{aligned}$$

$$= 0.8247 + 0.17827 + 4.93218 \times 10^{-9} + 6.176 \times 10^{-9} = 0.8430$$

$$D_{AB} = \frac{1.858 \times 10^{-7} \times T^{3/2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}}{P \times 6^2 \Omega}$$

$$= \frac{1.858 \times 10^{-7} \times (590)^{3/2} \sqrt{\frac{1}{44} + \frac{1}{28}}}{1 \times (3.8695)^2 \times 0.842}$$

$$= \frac{2.6627 \times 10^{-3} \times 0.241}{1 \times (3.8695)^2 \times 0.842} = 0.52 \text{ cm}^2/\text{s}$$

٥

(4) Fuller et al Correlation

(use atomic diffusion volume)

$\Sigma v \rightarrow$ sum atomic diffusion volume

$$V = \Sigma n_i v_i$$

وفي الـ جدول

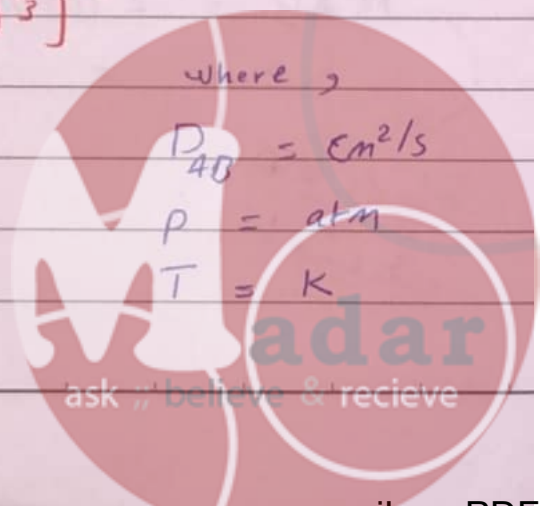
$$D_{AB} = \frac{10^{-3} \left(\frac{1}{M_A} + \frac{1}{M_B} \right)^{1/2} \times T^{1.75}}{P \left[(\Sigma v_A)^{1/3} + (\Sigma v_B)^{1/3} \right]^2}$$

where ,

$$D_{AB} = \text{cm}^2/\text{s}$$

$$P = \text{atm}$$

$$T = \text{K}$$



Atomic and Structural Diffusion-Volume Increments, v

C	16.5	Cl*	19.5
H	1.98	S*	17.0
O	5.48	Aromatic ring	-20.2
N*	5.69	Heterocyclic ring	-20.2

Diffusion Volumes for Simple Molecules, Σv

H ₂	7.07	CO	18.9
D ₂	6.70	CO ₂	26.9
He	2.88	N ₂ O	35.9
N ₂	17.9	NH ₃	14.9
O ₂	16.6	H ₂ O	12.7
Air	20.1	CCl ₂ F ₂ *	114.8
Ar	16.1	SF ₆ *	69.7
Kr	22.8	Cl ₂ *	37.7
Xe*	37.9	Br ₂ *	67.2
		SO ₂ *	41.1





Ex 5.1-2

Use Fuller's correlation to estimate the diffusion of hydrogen in nitrogen at 21°C and 2 atm. The experimental value is $0.38 \text{ cm}^2/\text{sec}$

أولاً نحدد قيم V و M لكل غاز

$$V_{\text{H}_2} = 7.07$$

$$V_{\text{N}_2} = 17.9$$

$$M_{\text{H}_2} = 2.02$$

$$M_{\text{N}_2} = 28.0$$

ثم نستخدم العلاقة

$$D = 10^{-3} T^{1.75} \left(\frac{1}{M_{\text{H}_2}} + \frac{1}{M_{\text{N}_2}} \right)^{1/2} \frac{1}{P [V_{\text{H}_2}^{1/3} + V_{\text{N}_2}^{1/3}]^2}$$

$$= 10^{-3} \times (21 + 273)^{1.75} \left(\frac{1}{2.02} + \frac{1}{28.0} \right)^{1/2} \frac{1}{2 [(7.07)^{1/3} + (17.9)^{1/3}]^2}$$

$$= 0.37 \text{ cm}^2/\text{sec}$$

Ex. Normal butanol ($\text{C}_4\text{H}_{10}\text{O}$) is diffusing through air at 1 atm abs. Using the Fuller et al. method estimate the diffusivity D_{AB} at 0°C .

$$V_{\text{C}} = 16.5$$

$$V_{\text{H}} = 1.98$$

$$V_{\text{O}} = 5.48$$

$$4(16.5) + 10(1.98) + 1(5.48) = 91.28$$

$$V_{\text{air}} = 20.1$$

العلاقة التي استخدمها (V) لمركب ما يكون

$$T = 273 \text{ K}$$

$$M_{\text{butanol}} = 74.1$$

$$M_{\text{air}} = 29$$

$$D_{AB} = 1.0 \times 10^{-7} (273)^{1.75} \left(\frac{1}{74.1} + \frac{1}{29} \right)^{1/2} \frac{1}{1.0 [(91.28)^{1/3} + (20.1)^{1/3}]^2}$$

$$= 7.73 \times 10^{-6} \text{ m}^2/\text{s}$$

ask :: believe & recieve

Polar Gases at low pressures

(Lennard-Jones - Stockmayer)



(nonpolar) ← attractive force by nature
(polar) ← attractive force by nature

! (Stockmayer)

* if one or both components of a gas mixture are polar often used.

① Brokaw Method : using a different set of collision diameters and well ② depth + the dipole ③ moment for the molecules concerned μ in (Debye)
لـ، لـ، لـ

$$S = 1.94 \times 10^3 \mu^2 \rightarrow \text{dipole moment (debyes)}$$

liquid $\leftarrow V_b T_b \rightarrow$ normal boiling point (K)

molar volume at

the normal boiling point
 cm^3/mol

$$\frac{\epsilon}{K_B} = 1.18 (1 + 1.3 S^2) T_b$$

$$\delta = \left[\frac{1.585 V_b}{1 + 1.3 S^2} \right]^{1/3}$$

[Geometric avg] مبروح و جداول رطوب و $\frac{\epsilon}{K_B}$ من جدول * من جدول

$$\sigma_{AB} = (\sigma_{AA} \cdot \sigma_{BB})^{1/2}$$

$$\frac{\epsilon_{AB}}{K} = \left(\frac{\epsilon_{AA}}{K} \cdot \frac{\epsilon_{BB}}{K} \right)^{1/2}$$

$$\epsilon_{AB} = (\epsilon_{AA} \cdot \epsilon_{BB})^{1/2}$$



$$\# \quad \Omega_D = \Omega + \frac{0.19 \delta_{AB}^2}{T^*}$$

$$\# \quad D_{AB} = 1.858 \times (10^{-7})^{3/2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}} \quad \frac{P \delta_{AB}^2 \Omega}{\text{cm}^2 \text{s}^{-1}}$$

Ex Modified Chapman - Enskog theory
for polar gases:

estimate the diffusion coefficient for a mixture
of methyl chloride MC (CH_3Cl) and
sulfur dioxide SD SO_2 at 1 bar and 323 K

	MC	SD
Dipol moment debyes	1.9	1.6
liquid Molar (V) at T_b cm ³ /mol	50.1	44.03
Normal Boiling temp (K)	248.95	263.13

$$\text{Soln: } \delta = \frac{1.94 \times 10^3 \text{ MP}^2}{VT}$$

$$\begin{aligned} \delta_{MC} &= \frac{1.94 \times 10^3 \times 1.9^2}{(50.1)(248.95)} \\ &= 0.56 \end{aligned}$$

$$\begin{aligned} \delta_{SD} &= \frac{1.94 \times 10^3 \times 1.6^2}{44.03 \times 263.13} \\ &= 0.43 \end{aligned}$$

$$\begin{aligned} \delta_{MC-SD} &= (0.56 + 0.43)^{1/2} \\ &= 0.49 \end{aligned}$$



$$\frac{\epsilon}{k_B} = 1.18 (1 + 1.3 \delta^2) T_b$$

$$\frac{\epsilon_{MC}}{k_B} = 1.18 (1 + 1.3 \times 0.56^2) (248.95) = 414 \text{ K}$$

$$\frac{\epsilon_{SD}}{k_B} = 1.18 (1 + 1.3 \times 0.43^2) (\cancel{248.95}) (263.13) = 385 \text{ K}$$

$$\frac{\epsilon_{MC-SD}}{k_B} = (414 + 385)^{1/2} = 399 \text{ K}$$

$$\delta = \left(\frac{1.585 V_b}{1 + 1.3 \delta^2} \right)^{1/3} \quad \delta_{MC} = \left[\frac{1.585 \times 50.1}{1 + 1.3 \times 0.56^2} \right]^{1/3} = 3.84 \text{ Å}^0$$

$$\delta_{SD} = \left[\frac{1.585 \times 44.03}{1 + 1.3 \times 0.43^2} \right]^{1/3} = 3.83 \text{ Å}^0$$

$$\delta_{MC-SD} = (3.84 + 3.83)^{1/2} = 3.84 \text{ Å}^0$$

$$T^* = \frac{k_B T}{\epsilon_{MC-SD}} = \frac{323}{399} = 0.810$$

$$\begin{aligned} \Omega_{MC-SD} &= \frac{1.06036}{0.810^{0.15610}} + \frac{0.19300}{e(0.47635 \times 0.810)} + \frac{1.03587}{e(1.52996 \times 0.810)} \\ &+ \frac{1.76474}{e(3.89411 \times 0.810)} = 1.60 \end{aligned}$$

$$\Omega_D = \Omega_{MC-SD} + \frac{0.195^2}{T^*} = 1.60 + 0.19 \times 0.49^2$$

$$= 1.66$$

ask, believe & recieve

$$\begin{aligned}
 D_{\text{Mc-SD}} &= 1.858 \times 10^{-7} \times T^{3/2} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}} \rho \delta^2 \Omega \\
 &= 1.858 \times 10^{-7} \times 323^{3/2} \times \sqrt{\frac{1}{50.04} + \frac{1}{66.06}} \\
 &\quad \times 1 \times 3.842 \times 1.66 \\
 &= 8.40 \times 10^{-6} \text{ m}^2/\text{s}
 \end{aligned}$$

Multi component Diffusion

!! (لا تتركوا كذا)



(qualitative point about Diffusivity in liquids)

- Diffusivity in liquids is about ten times lower than those in dilute gases

طبيعتا في الغازات أعلى من السوائل
لأنه الجزيئات أقرب في الغازات

- This lower value of diffusivity limit the overall rates for processes taking place in liquids e.g. reaction between two components.

• Diffusivity limits the rate of acid-base reaction

• Responsible for the rates of liq-liq (solvent) extraction

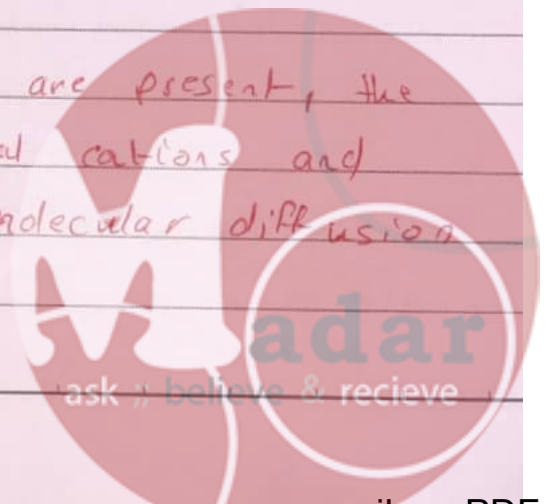
- Certain molecules diffuse as molecules, while others which are designated as electrolytes ionize in solutions and diffuse as ions

مثال جزء من جزيئات واثبات فيس في جزيئات في محال
في محال في شكل ايونات في محال

• Sodium chloride NaCl diffuses in water as ion Na^+ and Cl^-

• Though each ion has a different mobility, the electrical neutrality of the solution implies that ions must diffuse at the same rate accordingly it is possible to speak of a diffusion coefficient. For molecular electrolytes such as NaCl

• However, if several ions are present, the diffusion rates of the individual cations and anions must be considered and molecular diffusion coefficient have no meaning.



← الات يوصف العمليات تحتاج عنا تريع لل (diffusion)
كيف اعمل تريع (لا diffusion rates) من خلال
مواقع اما زيادة الحرارة او التحريك يساعد انه
الانتشار ريسر بكل مكان ويتوزع اسرع من اوسع
الطبعي ...

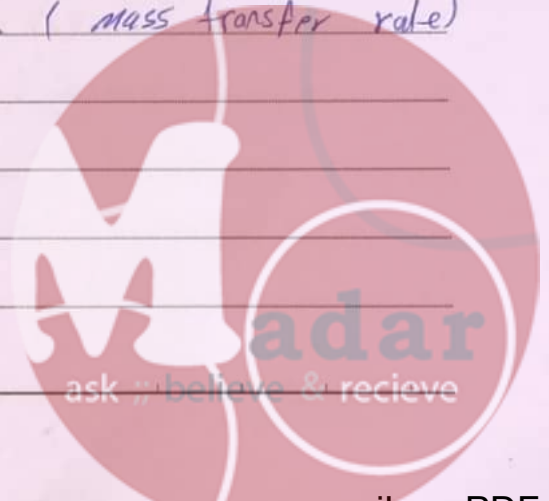
← الكمازات اللوحية (M) قلت او (diffusion)
وصادحتي طبيعي لان الكزيئات بتكون قريبة عي بعض
وصعب تحريكها .

→ Diffusivity varies inversely with viscosity
when the ratio of solute to solvent ratio
exceeds five.

• خلاخضاً انيدي اعل (reaction) (liq-liq) extraction
مينج ريسر او (reaction) على (interface) مثل زينة
موي حوا اعل عشان اشي ال (reaction) تحركهم بالتالي
رح ريسر عني (droplet) عني بالتالي المساحات
الطوية رح تزيد وريسر اعل من او (interface).
ولما ازيد المساحة صبي انه D_{AB} رح يزيد .
وبتزيد او (mass transfer rate)



ليس المساحة فابقدر نتحكم بحجم او drop بال عني
لحين فابقدر اتحكم ب Area يعني بقدرته اتحكم ب
(mass transfer rate)



→ Diffusivity varies inversely with viscosity when the ratio exceeds five.

→ In viscous materials diffusion becomes independent of viscosity.

$$\left(\frac{D_{AB} M}{T} \right)_{T_2} = \left(\frac{D_{AB} M}{T} \right)_{T_1} \rightarrow D_{AB}(T_2)$$

$$= D_{AB}(T_1) \frac{T_2}{T_1} \frac{M T_2}{M T_1} \rightarrow \quad (\text{عدلوهم})$$

Ex
مثال

$$D_{AB} \mid = 1 \times 10^{-9}$$

at 25°

$$M = 100 \text{ cp}$$

$$D_{AB} \text{ at } 100^\circ = ?$$

$$M = 10 \text{ cp}$$

$$D_{AB}(100) = 1 \times 10^{-9} \frac{(25 + 273)}{(100 + 273)} \frac{373}{298} \times \frac{10}{100}$$

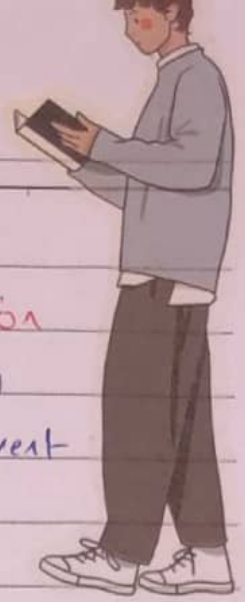
$$= 1.2 \times 10^{-10}$$

($M \ll \text{نقل الحرارة}$ For liq)



Diffusion Coefficient in liquids Stokes - Einstein Equation

- Based on hydrodynamics of creeping flow
- Applies for solute molecules larger than solvent
- Derived for infinitely dilute solution A in solvent B



diffusivity of solute A in solvent B D_{AB} = $\frac{RT}{6\pi\eta_B R_A}$ in K

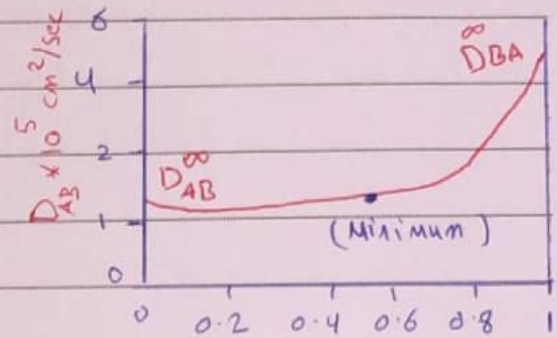
$R_A = \left(\frac{3V}{4\pi}\right)^{1/3}$ sphere in volume #

$R_A = \frac{6V}{\pi}$ radius of the solute molecule

η_B viscosity for solvent



Table (2.8)



not linear الحالة

composition as اعتاد as

in gas $D_{AB} = D_{BA}$

liquid لكن في

$$D_{AB} \neq D_{BA}$$

composition as اعتاد

(Minimum) 0.5 في هنا

و هو ا كثر في وجود

ب الغا ذرات

$D_{AB}^{\infty} \rightarrow \text{zero} = A$ تركيز A

المادة A خفيف في B

$D_{AB}^{\infty} \rightarrow$

Diffusion coefficient for solute A in solvent B

at an dilution for solute A

$D_{BA} \rightarrow$

zero = B تركيز B

المادة B خفيف في A

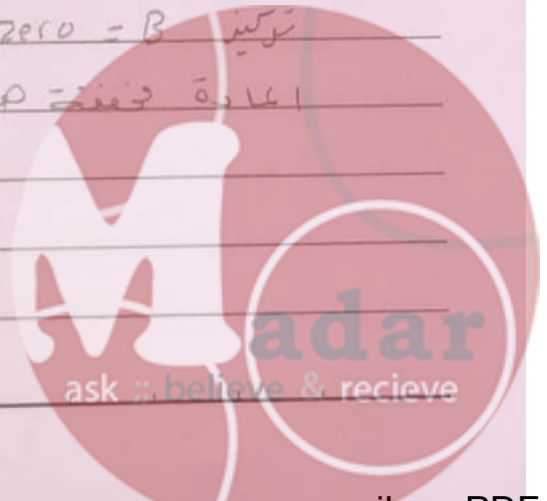


Table 2.8

Molecular and van der Waals Volumes of Some Liquids

Substance	$v_M (\text{\AA}^3)$	$v_w (\text{\AA}^3)$
Benzene	148.6	80.4
Cyclohexane	179.4	102.0
Ethylene glycol	92.4	60.8
Glycerol	121.3	88.5
Methanol	67.0	36.1
<i>n</i> -Pentane	192	96.4

Source: (Edward, 1970)



(Ex 2.11) Diffusion coefficient of lactalbumin, ovalbumin and bovine serum albumin (BSA) in 0.1 M phosphate buffered saline (PBS) solution at 298K are reported by Pluen et al. (1999) as 1.14×10^{-6} , 7.8×10^{-7} and $6.4 \times 10^{-7} \text{ cm}^2/\text{s}$ respectively. if the viscosity of the solvent is 0.8705 calculate the radius of these macro molecules.

$$R_A = \frac{kT}{6\pi\eta_B D_{AB}^\infty}$$

$$\eta_B = 0.8705$$

$$T = 298$$

$$k = 1.3806 \times 10^{-23}$$

$$D_{AB} \text{ (A)} = 1.14 \times 10^{-6}$$

$$D_{AB} \text{ (B)} = 7.8 \times 10^{-7}$$

$$D_{AB} \text{ (C)} = 6.4 \times 10^{-7}$$

lactalbumin :

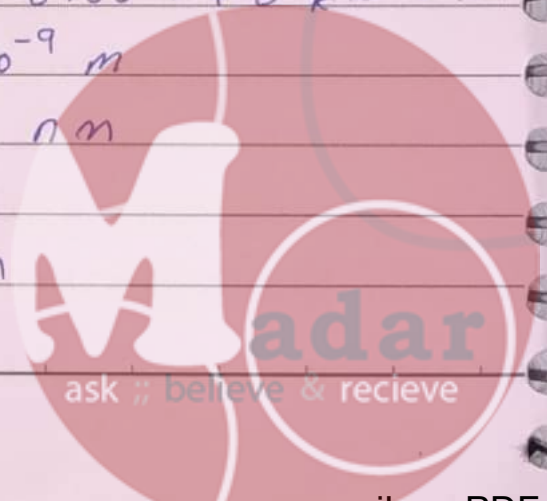
$$\begin{aligned} R_A &= \frac{1.3806 \times 10^{-23} \times 298}{6\pi \times 0.8705 \times 1.14 \times 10^{-6} \times 10^{-4}} \\ &= 2.199 \times 10^{-9} \text{ m} \\ &= 2.199 \text{ nm} \end{aligned}$$

ovalbumin :

$$\begin{aligned} R_A &= \frac{1.3806 \times 10^{-23} \times 298}{6\pi \times 0.8705 \times 7.8 \times 10^{-7} \times 10^{-4}} \\ &= 3.215 \times 10^{-9} \text{ m} \\ &= 3.215 \text{ nm} \end{aligned}$$

BSA :

$$= 3.918 \text{ nm}$$





- Empirical in nature
- For infinitely dilute solute in solvent
- based primarily on data with solvent viscosities 0.4 - 1.5 centipoise (Cp)
- More accurate than Stokes - Einstein equation, in particular when solute and solvent are similar in size.

Diffusivity $\leftarrow D_{AB} = 7.4 \times 10^{-8} \frac{(\Phi_B M_B)^{1/2} T}{M_B V_A^{0.6}} \rightarrow \text{in k}$

Solute A in solvent B

Φ_B Association Factor for solvent

M_B Molar Mass of solvent (g/mol)

V_A liquid molar volume of the solute at (nbp)

T normal boiling point

$\Phi_B \Rightarrow$ water - 2.6 H_2O

Methanol = 1.9 CH_3OH

Ethanol = 1.5 C_2H_5OH

benzene, ether, heptane,

other unassociated solvent = 1

table 3.3



Ex 5.2-1

Estimate the diffusion at 25°C For Oxygen dissolved in water using the Stokes-Einstein equation and the Wilke-Chang correlation compare your result with experimental value of $1.8 \times 10^{-5} \text{ cm}^2/\text{sec}$

1 Stokes-Einstein

$$R = \frac{1}{2} \sigma_1 = 1.73 \cdot 10^{-8} \text{ cm}$$

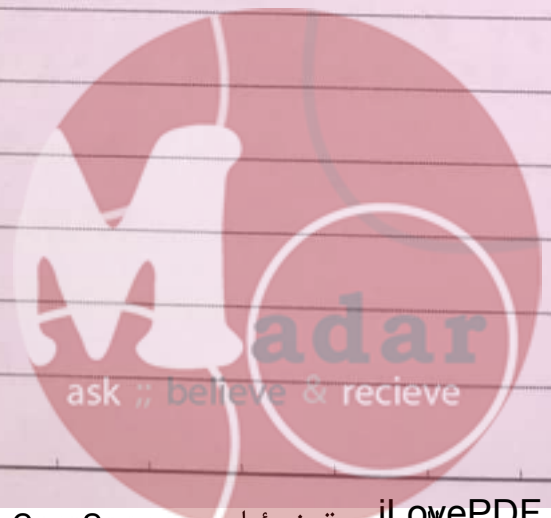
$$D = \frac{k_B T}{6 \pi \eta R} = \frac{1.38 \times 10^{-16} \text{ g cm}^2/\text{sec}^2 \text{ K} \times 298 \text{ K}}{6 \pi (0.01 \text{ g/cm sec}) 1.73 \times 10^{-8} \text{ cm}} \\ = 1.3 \times 10^{-5} \text{ cm}^2/\text{sec}$$

2 Wilke-Chang

$$D = 7.4 \times 10^{-8} (\Phi_{H_2O})^{1/2} + \frac{M_{H_2O}}{M_{O_2}} \sqrt{\frac{0.6}{0.2}}$$

$$= 7.4 \times 10^{-8} \times 2.8 \left[\frac{(18 \text{ cm}^3/\text{mol})}{1 \text{ Cp } (25 \text{ cm}^3/\text{mol})} \right]^{1/2} \times 298 \text{ K} \\ = 2.2 \times 10^{-5} \text{ cm}^2/\text{sec}$$

more accurate ✓



Ex (Wilke-Chang diffusivity for liquid)
 Use the Wilke-Chang correlation to estimate
 For ethylbenzene (A) diffusing into water (B)
 at 293 K. The viscosity of water at this
 temperature is 1.0 cP

(V) $\frac{M_w}{\rho}$ density $\rightarrow$ $\frac{M_w}{\rho}$

Ethylbenzene $C_6H_5C_2H_5$

$\rightarrow C_8H_{10}$

density = 0.761 g/cm^3 C_8H_{10}

$M_w = 106.17$

Association Factor For $H_2O = 2.6$

$$V_A = \frac{M_w}{\rho} = \frac{106.17}{0.761} = 139.5 \text{ cm}^3/\text{mol}$$

$$\begin{aligned} \frac{D}{AB} &= 7.4 \times 10^{-8} \frac{((2.6)(18.01))^{1/2} + 293}{1.0 \times (139.5)^{0.6}} \\ &= 0.77 \times 10^{-5} \text{ cm}^2/\text{sec} \end{aligned}$$





Tyn and Calus

000

Involves the ratio of surface tensions of the solute and solvent as

$$D_{AB} = 8.93 \times 10^{-8} \frac{V_B^{0.267}}{V_A^{0.433}} \frac{T}{\mu_B} \left(\frac{\sigma_B}{\sigma_A} \right)^{0.15}$$

$\sigma \rightarrow$ the surface tension in $\text{dyn/cm} = 10^{-3} \text{ N/m}$

1 $\approx \frac{\sigma_B}{\sigma_A}$ تقريباً متساوية لعل الحاد

$$D_{AB}^0 = 8.93 \times 10^{-8} \frac{V_B^{0.267}}{V_A^{0.433}} \frac{T \rightarrow K}{\mu_B \rightarrow \text{viscosity for solvent}}$$

Ex. use Tyn and Calus correlation for ethyl benzene (A) diffusing in water (B) at 293 K $\mu_B = 1.0 \text{ cp}$

(C8H10) -- (A) H_2O -- (B)
 $\rightarrow$ density $\approx 0.761 \text{ g/cm}^3$ $\rightarrow \mu = 1 \text{ cp}$
 $M_{wt} \approx 106.17$

$$V_B \approx 1811 = 18 \text{ cm}^3/\text{mol}$$

$$V_A \approx 106.17 / 0.761 = 139.5 \text{ cm}^3/\text{mol}$$

$$D_{AB} \approx 8.93 \times 10^{-8} \frac{(18)^{0.267}}{(139.5)^{0.433}} \times \frac{293}{1}$$

$$= 0.66 \times 10^{-5} \text{ cm}^2/\text{sec}$$

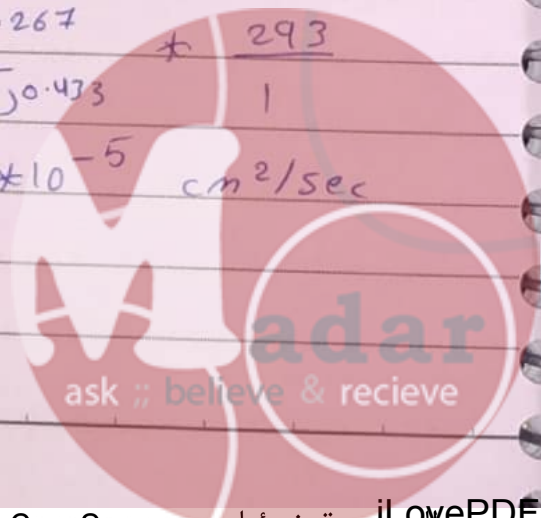


TABLE 11-5 Diffusion Coefficients in Liquids at Infinite Dilution

Solute A	Solvent B	T, K	Experimental $D_{AB}^\circ \times 10^5$, cm ² /s	Ref.**	Percent error*			
					Wilke and Chang	Tyn and Calus	Hayduk and Minhas	Nakanishi
Acetone	Chloroform	298	2.35	5	42	8.2	5.8	-8.8
		313	2.90		38	5.1	3.2	-11
Benzene	Benzene	288	2.51	10	1.3	-21	-22	-15
		328	4.25	20	-0.7	-23	-23	-17
288		2.20	10	47	12	8.3	-29	
298		2.13	20	29	7.4	4.5	4.4	
298		2.02	18	36	12	9.4	15	
298		2.13	18	37	9.3	6.9	-11	
298		2.09	4	28	-9.4	-10	-7.0	
298		1.96	17	0.4	0.1	0.1	-11	
298		1.38	4	28	0.6	0.8	-10	
281		1.45	22	-8.8	-6.4	-6.0	2.4	
298		2.09	20	-11	-6.9	-7.5	-3.5	
333		3.45		-8.1	-4.2	-5.0	-0.7	
288		2.25	10	-1.8	-5.2	-7.0	-39	
298		2.28	4	53	-4.2	-4.2	13	
298		2.10	3	-27	-16	-17	-6.2	
353		4.25		-20	-7.1	-8.9	3.6	
303		2.09	1	8.7	10	9.0	-8.5	
Naphthalene		281	1.19	22	-2.1	5.2	5.6	18
Toluene		298	1.85	20	0.1	4.1	3.5	10
1,2,4-Trichlorobenzene	281	1.34	22	-13	-8.5	-7.7	-0.8	
Vinyl chloride	281	1.77	22	8.7	0.1	-0.2	12	
Acetic Acid	Acetone	288	2.92	2	35	3.2	-3.2	4.2
		313	4.04	25	33	2.1	-3.7	3.1
298		2.62	4	13	-3.5	-8.2	-13	
298		3.77	4	56	5.5	0.1	21	

TABLE 11-5 Diffusion Coefficients in Liquids at Infinite Dilution (Continued)

Solute A	Solvent B	T, K	Experimental $D_{AB}^{\infty} \times 10^5$, cm ² /s	Ref. **	Percent error*			
					Wilke and Chang	Tyn and Calus	Hayduk and Minhas	Nakanishi
Nitrobenzene	Acetone	293	2.94	19	-2.4	8.7	3.0	-6.2
Water		298	4.56	16		5.6	3.7	-19
Bromobenzene	<i>n</i> -Hexane	281	2.60	25	16	17	12	26
Carbon tetrachloride		298	3.70	6	12	8.6	4.6	18
Dodecane		298	2.73	23	-17	8.1	-6.7	7.9
<i>n</i> -Hexane		298	4.21	15	-18	-9.0	-4.0	1.3
Methyl ethyl ketone		303	3.74	1	23	23	17	-0.2
Propane		298	4.87	7	3.4	0.6	20	1.3
Toluene		298	4.21	4	-9.0	-6.3	-11	-3.1
Allyl alcohol	Ethanol	293	0.98	9	22	9.7	14	30
Isoamyl alcohol		293	0.81	9	9.8	10	14	15
Benzene		298	1.81	14	-39	-3.0	2.8	-18
Iodine		298	1.32	4	2.4			12
Oxygen		303	2.64	11	-3.2	32	34	47
Pyridine		293	1.10	9	-1.0	-14	-9.1	3.3
Water		298	1.24	12		1.7	11	6.9
Carbon tetrachloride		298	1.50	14	-30	11	18	-6.7
Adipic acid	<i>n</i> -Butanol	303	0.40	1	1.1	16	29	8.0
Benzene		298	1.00	14	-52	5.3	20	-23
Butyric acid		303	0.51	1	1.5	7.4	19	2.5
<i>p</i> -Dichlorobenzene		298	0.82	14	-52	13	29	-22
Methanol		303	0.59	14	51	37	50	51
Oleic acid		303	0.25	1	-8.4	26	41	-4.4
Propane		298	1.57	2	-65	-23	-13	-47
Water		298	0.56	14		7.2	26	22
Benzene	<i>n</i> -Heptane	298	3.40	3	8.4	1.2	-1.2	13
		372	8.40		1.6	-5.0	-5.9	6.0
Acetic acid	Ethyl acetate	293	2.18	21	69	12	8.5	21
Acetone		293	3.18	21	3.2	-6.8	-9.6	-15
Ethyl benzoate		293	1.85	21	9.0	16	13	-3.9
Methyl ethyl ketone		303	2.93	1	14	8.1	4.8	-5.4
Nitrobenzene		293	2.25	21	10	6.4	4.2	-2.8
Water		298	3.20	12		16	17	-4.2
Methane	Water	275	0.85	26	10	-3.6	0.0	14
		333	3.55		15	0.7	-2.6	19
Carbon dioxide		298	2.00	24	1.6	-22	-13	-19
Propylene		298	1.44	24	-7.7	-13	-13	-6.2
Methanol		288	1.26	10	5.4	-8.7	-5.4	-9.6
Ethanol		288	1.00	9	5.3	-1.6	-2.7	-8.7
Allyl alcohol		288	0.90	9	5.5	0.5	-2.0	-7.4
Acetic acid		293	1.19	13	2.6	-5.0	-4.7	-24
Ethyl acetate		293	1.00	13	-10	-9.4	-16	-0.9
Aniline		293	0.92	13	-2.5	-5.9	-8.9	-10
Diethylamine		293	0.97	13	-8.6	-7.3	-15	-21
Pyridine		288	0.58	9	49	37	38	31
Ethylbenzene		293	0.81	26	-8.9	-0.2	-18	8.0
Methylcyclopentane		275	0.48	26	-2.5	0.3	-14	7.0
		293	0.85		-1.7	1.1	-9.0	7.8
		333	1.92		6.3	9.4	8.5	17
Vinyl chloride		298	1.34	8	3.6	-7.6	-3.3	4.3
		348	3.67		4.2	-7.1	3.0	4.9
ave. abs. % dev.					17	9	11	13

*Percent error = [(calc. - exp.)/exp.] × 100

**References: 1, Amourdam and Laddha (1967); 2, Bidlack and Anderson (1964); 3, Calus and Tyn (1973); 4, Chang and Wilke (1955); 5, Haluska and Colver (1971); 6, Hammond and Stokes (1955); 7, Hayduk, et al. (1973); 8, Hayduk and Laudie (1974); 9, Int. Critical Tables (1926); 10, Johnson and Babb (1956); 11, Krieger, et al. (1967); 12, Lees and Surram (1971); 13, Lewis (1955); 14, Lusi and Ratcliff (1971); 15, McCall and Douglas (1959); 16, Olander (1961); 17, Rao and Bennett (1971); 18, Ratcliff and Lusi (1971); 19, Reddy and Doraiswamy (1967); 20, Sanni and Hutchinson (1973); 21, Sitaraman, et al. (1963); 22, Stearn, et al. (1940); 23, Vadovic and Colver (1973); 24, Vivian and King (1964); 25, Wilke and Chang (1955); 26, Witherspoon and Bonoli (1969)

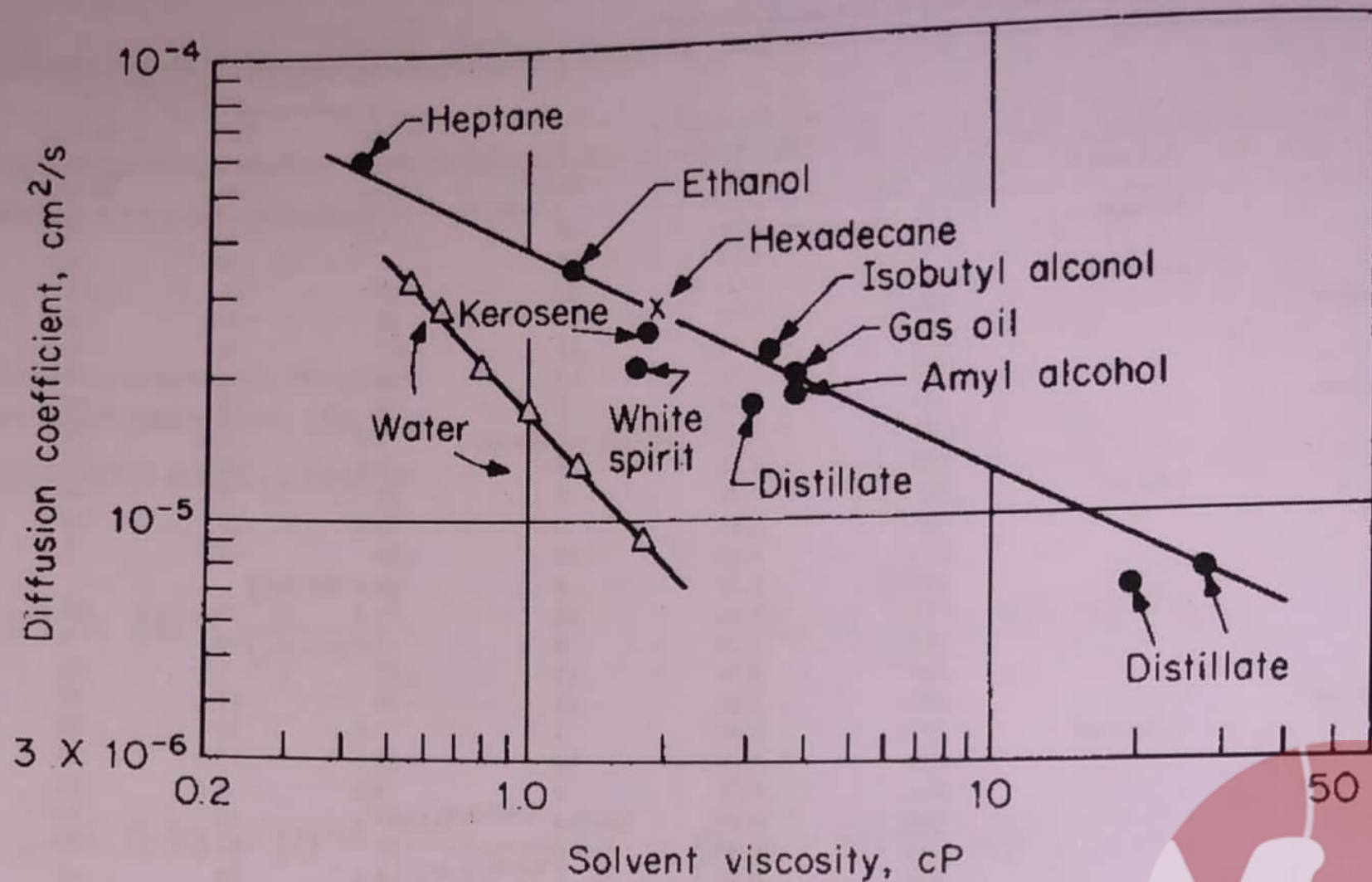
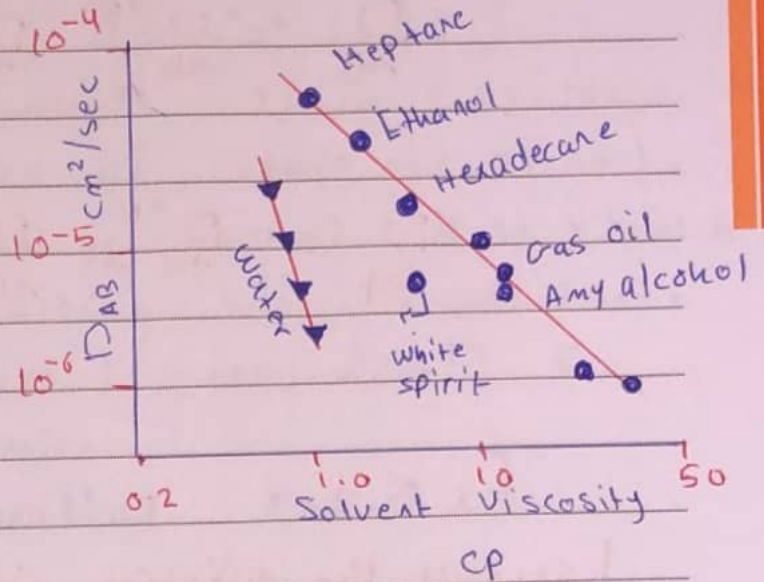


FIGURE 11-8 Diffusion coefficients of carbon dioxide in various solvents. ● Davies et al. (1967); × Hayduk and Cheng (1971); △ Himmelblau (1964)

Table 11.5

Figure (11-8)



العلاقة الخطية بين D_{AB} و μ (viscosity) في السوائل غير القطبية (non-polar) ←
 علاقة خطية بين D_{AB} و μ (viscosity) في السوائل القطبية (polar) ←
 (Water) استثناء

تفسير ← H_2O كجزيء قطبي لأن باقي الجزيئات polar (مواد عازلة) ← الجزيء (polar) قابلية (H_2O)

العلاقة بين D_{AB} و μ (viscosity) في السوائل القطبية (polar) ←
 تفسير ← (solvent)



Vigness's equation except for strongly associated binary mixtures

$$D_{AB} = D_{AB}^{x_B} D_{AB}^{x_A} \left[1 + \frac{\partial \ln \gamma_A}{\partial \ln x_A} \right]_{T, P}$$

$$D_{BA} = D_{AB}^{x_B} D_{AB}^{x_A} \left[1 + \frac{\partial \ln \gamma_B}{\partial \ln x_B} \right]_{T, P}$$

at constant c
 T, P

Ex 5.2-3 Diffusion in an acetone - water Mix

Estimate the diffusion coefficient in a 50-mole % mixture of acetone (1) and water (2)

This solution is highly nonideal so that

$\frac{\partial \ln \gamma}{\partial \ln c_1} = 0.69$ in pure acetone, the diffusion coefficient is $1.26 \times 10^{-5} \text{ cm}^2/\text{sec}$

in pure water it is $4.86 \times 10^{-5} \text{ cm}^2/\text{sec}$

$$D_0 = [D_0 x_1 = 1]^{x_1} + [D_0 x_2 = 1]^{x_2}$$

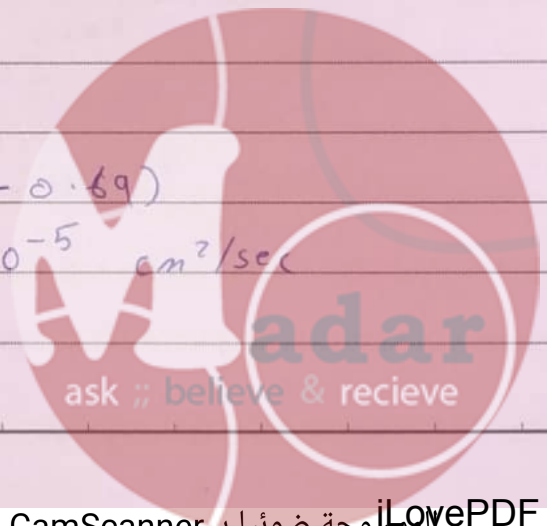
$$= (1.26 \times 10^{-5})^{0.5} + (4.86 \times 10^{-5})^{0.5}$$

$$= 2.43 \times 10^{-5} \text{ cm}^2/\text{sec}$$

$$D = D_0 \left(1 + \frac{\partial \ln \gamma_1}{\partial \ln c_1} \right)$$

$$= 2.43 \times 10^{-5} (1 - 0.69)$$

$$(D_{AB} \text{ at } 50\% \text{ mix}) = 0.75 \times 10^{-5} \text{ cm}^2/\text{sec}$$



(Diffusion in Electrolytes)

10.8

Wernst - Haskell equation for diffusion at infinite dilution

$$D_{AB} = \frac{RT}{F^2} \left[\frac{1/z_+ + 1/z_-}{1/\lambda_+^\infty + 1/\lambda_-^\infty} \right]$$

Faraday's constant = 96500 C/g-equiv

z_+ & z_- (valence of cation and anion)

R = universal gas constant 8.31 J/mol.K

λ_+ & λ_- - limiting ionic conductance
(A/cm²) (V/cm) g-equiv/cm³) For cation and anion



table 3.7

(effect of salt Normality on D_{AB})



Mass Transfer (905) Table 3.7 Limiting Ionic Conductances in Water at 25°C, in
(A/cm²)(V/cm)(g-equiv/cm³)

Anion	λ_-	Cation	λ_+
OH ⁻	197.6	H ⁺	349.8
Cl ⁻	76.3	Li ⁺	38.7
Br ⁻	78.3	Na ⁺	50.1
I ⁻	76.8	K ⁺	73.5
NO ₃ ⁻	71.4	NH ₄ ⁺	73.4
ClO ₄ ⁻	68.0	Ag ⁺	61.9
HCO ₃ ⁻	44.5	Tl ⁺	74.7
HCO ₂ ⁻	54.6	($\frac{1}{2}$)Mg ²⁺	53.1
CH ₃ CO ₂ ⁻	40.9	($\frac{1}{2}$)Ca ²⁺	59.5
ClCH ₂ CO ₂ ⁻	39.8	($\frac{1}{2}$)Sr ²⁺	50.5
CNCH ₂ CO ₂ ⁻	41.8	($\frac{1}{2}$)Ba ²⁺	63.6
CH ₃ CH ₂ CO ₂ ⁻	35.8	($\frac{1}{2}$)Cu ²⁺	54
CH ₃ (CH ₂) ₂ CO ₂ ⁻	32.6	($\frac{1}{2}$)Zn ²⁺	53
C ₆ H ₅ CO ₂ ⁻	32.3	($\frac{1}{3}$)La ³⁺	69.5
HC ₂ O ₄ ⁻	40.2	($\frac{1}{3}$)Co(NH ₃) ₆ ³⁺	102
($\frac{1}{2}$)C ₂ O ₄ ²⁻	74.2		
($\frac{1}{2}$)SO ₄ ²⁻	80		
($\frac{1}{3}$)Fe(CN) ₆ ³⁻	101		
($\frac{1}{4}$)Fe(CN) ₆ ⁴⁻	111		

Source: Poling, Prausnitz, and O'Connell [2].





يمكن ربط، شتافيا

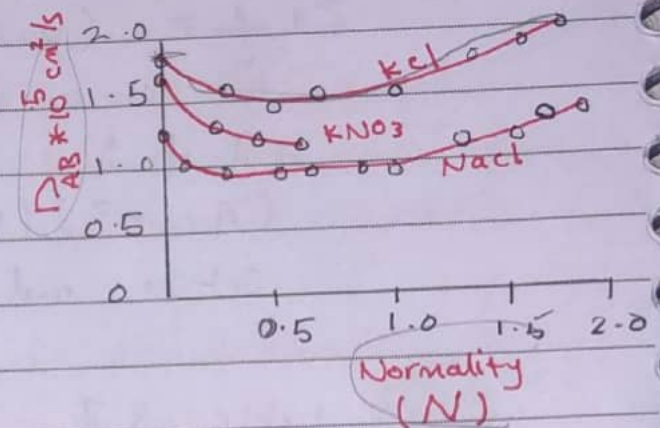
Use Gordan's equation up to 2N

$$D_{AB} = D_{AB}^0 \frac{\mu_B}{M} \left(\frac{\rho}{B} \bar{V}_B \right)^{-1} \left[1 + m \frac{d \ln \gamma_t}{d m} \right]$$

at ∞
dilution
= 1

كال ماعندي معلومات بالسؤال

Partial Molar volume
تصحیح لقانون



effect of concentration
on D_{AB} of electrolytes
in aqueous solution
at 18.5°C .

Normality & Molarity

بالكان الواحد 1:1

اما بالكان 2:1

Normality



Ex 11.11

Estimate the diffusion coefficient NaOH in a 2 N aqueous solution.

→ From data on densities of aqueous solution of (NaOH)

① 12 weight % $\rightarrow \frac{\text{mol}}{\text{L}} \quad \frac{\text{mass}}{\text{mass}}$

من خلال الجدول MW و solution اعتبره (H_2O)

② 3 N

③ تركيزه في aqueous solution حوالي (55.5)

إذا كان solution شوي dilute كثافته داخل solution = 1

$$55.5 = \frac{1000}{18}$$

④ $\gamma_{\pm}$ For $\text{NaOH} = 1950$ at 298 K

⑤ $m = 2 = 0.698$

⑥ value are plotted vs molality m the slope at $2m = 0.047$

$$m \frac{\partial \ln \gamma_{\pm}}{\partial m} = \frac{m}{\gamma_{\pm}} \frac{\partial \gamma_{\pm}}{\partial m} = \frac{2}{0.698} (0.047) = 0.135$$

The viscosities of water and 2N NaOH solution at 298 K = 0.894 and 1.42 cP



$$D_{AB}^0 = \frac{(2) (8.314) (298)}{(1150) (1) (1498) (96.500)^2}$$

$$= 2.12 \times 10^{-5} \text{ cm}^2/\text{s}$$

$$D_{AB} = (2.12 \times 10^{-5}) \frac{0.894}{1.42} \cdot \frac{55.5}{55.5} [1 + (2)(0.135)]$$

$$\bar{D}_{AB} \leftarrow = 1.70 \times 10^{-5} \text{ cm}^2/\text{s}$$

at ∞ dilution

at 288 K the viscosity of water = 1.144 cP
 so the D_{AB} at 288 K is

$$1.70 \times 10^{-5} \frac{288}{(334)(1.144)} = 1.28 \times 10^{-5} \text{ cm}^2/\text{s}$$

←
 جای ضربه بعد
 تصحیح

نزلت بعد التصحیح لأنه زاد الـ (ion)
 at dilution one (ion)

D_{AB} زاد وتركيزه يعني زاد الـ ion يعني قلت
 mean $\leftarrow \lambda$ لأنه قلت
 Free path



(Solid)

(Examples)

- 1- leaching of metal ores .
تلك الاشياء في الماء
- 2- Drying of timber and food .
الخشب ، والخبز ، الخ
- 3- adsorption of solutes .
- 4- Diffusion and catalytic reaction in Solid catalysts .
- 5- Separation of fluid by membranes صفاة
مائية
- 6- Treatment of metals at high temperature by gases .

Modes of diffusion in solid

Follow Fick's law : does not depend on the structure of the solid.

إذا كان يعتمد على البنية (structure) فإنه لا يتبع

(Fick's law)

→ an example is the diffusion of gasses into Polymeric membranes

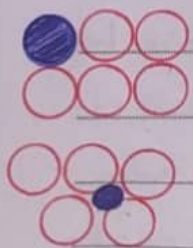
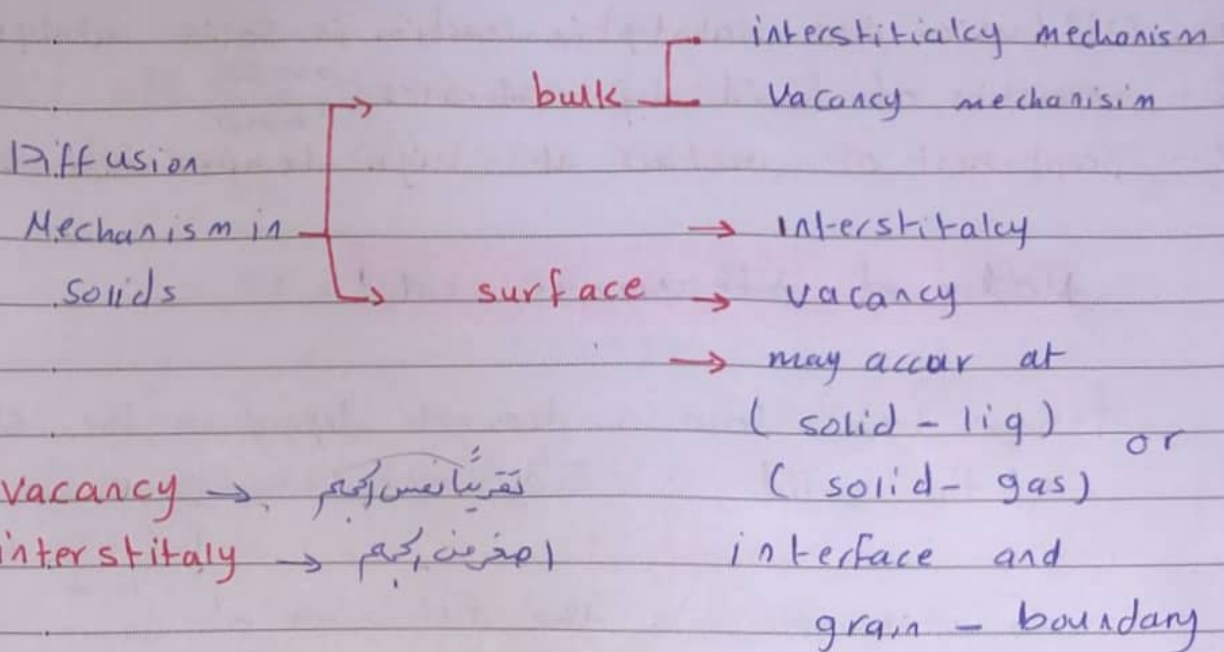
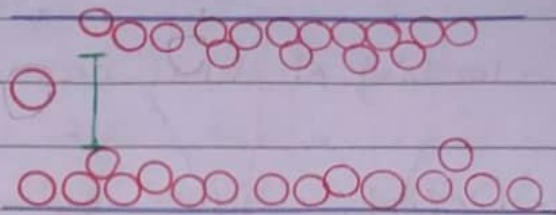
→ solubility of gas is described by Henry's law

هذه البنية الصلبة
لا تعتمد على البنية
البنية الصلبة
لا تعتمد على البنية
البنية الصلبة
لا تعتمد على البنية



Diffusion in porous solid : depends on the actual structure and void channels in the solid

← سائل سائل (gas) سائل
جزء من الغاز يذوب في السائل على سطح
وتتركيزه يصبح عالي مع الوقت (الراحة)
ينتقل إلى مساحة أكبر بالوقت، لا خسر



vacancy → تقريباً نفس الحجم
interstitially → اخر من الحجم

الـ Surface سائل (bulk)



Knudsen Diffusion

لے حوالی (50nm)

متوسط حساب

(mean free path)

MEAV

(Pore diameter

بسم الله الرحمن الرحيم

(low) حد

permeability

Porous

(2 ↑) ← (low pressure)

Pressure)

« الامم مسلمة بين ال حبيب والكارأهم من الامم مسلمة بين الخوفا
نفسها .

← پیتھ (pores) پکن حجم اقلین (50 nm)

Pore wall

→ ترکیب خون بند

d pore

تَرْكِزِ اَعْلَى عَائِلَتِی

$$d_{\text{pore}} \propto \left(\frac{\text{Solute}}{\text{Diameter}} \right)^{1/2}$$

(knudsen number) :

$$1.38 \times 10^{-26}$$

$k_n = \underline{2} \rightarrow$ mean free path

$$\lambda = k_B T$$

$d \rightarrow d$ pore

$$\sqrt{2\pi\sigma_{AB}^2 P}$$

(Kn) is used to identify :

1- $Kn < 0.001$ leads to the application of the no-slip boundary condition (continuum)

zero = 74% of molecules are

المرحلة 2، قبل (K1) و (Fick's)

$k_n = \frac{\lambda}{d}$ حلا انا کبیب (Kn) اذ اکان صغیر (d کبیر)

معادله انتقال (Fick's law)

(concentration gradient)

1) structural elements are divided into two main groups:

ask :: believe & recieve

Ex 3.3 Knudsen diffusion in Porous Solid

A Mix of O_2 (A) and N_2 (B) diffuses through the pores of a 2 mm-thick piece of (unglazed) porcelain at a total pressure of 0.1 atm and a temp of 293 K. The avg pore diameter is 0.1 μ m the porosity is 30.5% and the tortuosity is 4.39. Estimate the diffusion fluxes of both components when the mole fractions of O_2 are 80% and 20%. On either side of the porcelain, the collision diameters for oxygen and nitrogen ~~are~~ are 3.467 and 3.798 Å respectively.

$$\lambda_{AB} = \frac{(3.467 + 3.798)}{2} = 3.632 \text{ Å}$$

$$\lambda = \frac{KT}{\sqrt{2\pi} \sigma^2 p} = \frac{1.38 \times 10^{-23} (293)}{\sqrt{2\pi} (3.632 \times 10^{-10})^2 (0.10 \times 1.013 \times 10^5)} = 6.81 \times 10^{-7} \text{ m}$$

$$\lambda = 6810 \text{ Å}$$

$$D_{KA} = \frac{10^{-7}}{3} \left(\frac{8 + \frac{8314}{293}}{\pi \cdot 32} \right)^{1/2} = 1.47 \times 10^{-5} \text{ m}^2/\text{s} = 0.147 \text{ cm}^2/\text{s}$$

$$D_{KACPL} = \varepsilon \frac{D_{KA}}{L} = 0.305 \times 0.147$$

$$= \frac{4.39}{4.39} = 0.0102 \text{ cm}^2/\text{s}$$

$$Kn = \lambda/d$$

$$6810/1000 = 6.81 > 0.05$$

~~Knudsen~~

$$D_{KA} = \frac{d}{3} \left(\frac{8RT}{\pi M_i} \right)^{1/2}$$

2- $0.001 < Kn < 0.1$ leads to the application of the slip boundary condition
 (مستوى) zero $\neq$ molecule $\rightarrow$ surface $\leftarrow$

3- $Kn > 0.1$ leads to the inapplicability of the continuum assumption and most of the gas flow must be characterized using Statistical methods.

(molecular level) $\rightarrow$ مستوى جزيئي

effective diffusivity

$$D_{eff} = \frac{\varepsilon}{\tau} D_{AB}$$

$\rightarrow$ tortuosity (corrects for the path

Knudsen diffusivity

longer than linear diffusion path)

$$D_K = \frac{d}{3} \left[\frac{8RT}{\pi M_i} \right]^{1/2}$$

($T = 1.5 - 5$)

