Transport Phenomena II

Lec 6-Convective Mass Transfer

Content
Convection Mass Transfer, Mass Transfer Coefficient , Convective Mass Transfer Correlations

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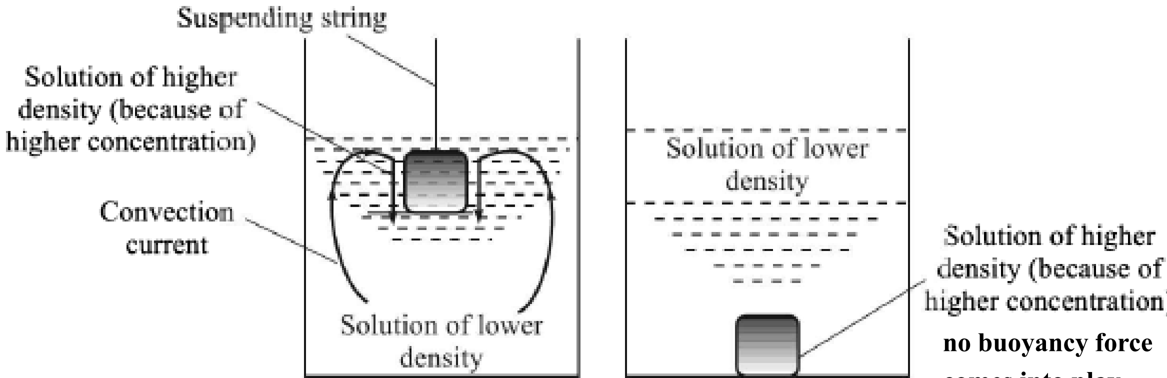
Content

- Convection Mass Transfer
- Mass transfer coefficient
- Convective Mass Transfer Correlations

Mass Transfer by Convection



no buoyancy force comes into play

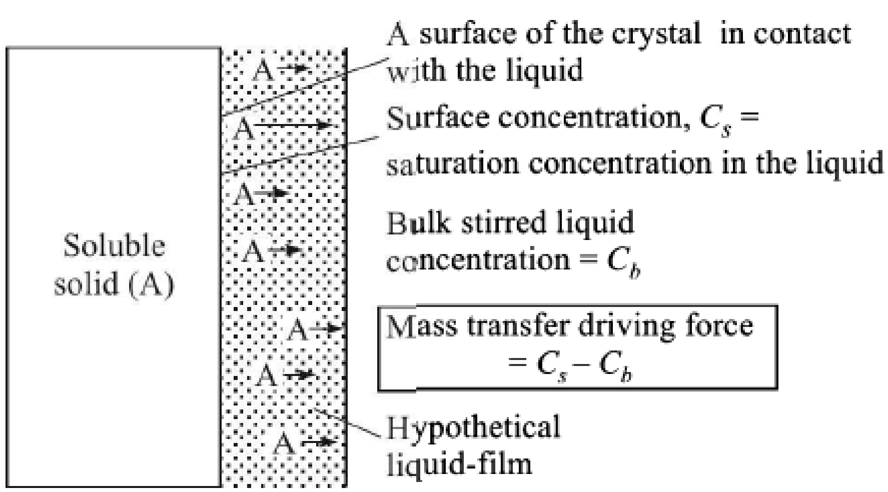
(a) The sugar cube is kept suspended in water; natural convection occurs.

(b) A small sugar cube rests at the bottom of a beaker; natural convection does not occur.

Similar to natural convection in heat transfer

Mass Transfer by Convection

- To have a fluid in convective flow usually requires the fluid to be flowing by another immiscible fluid or by a solid surface.



Visualization of the dissolution process of a solid.

Mass Transfer by Convection

- An example is a fluid flowing in a pipe; where part of the pipe wall is made by a slightly dissolving solid material such as benzoic acid.
  - The benzoic acid dissolves and is transported perpendicular to the main stream from the wall.
  - When a fluid is in turbulent flow and is flowing past a surface, the actual velocity of small particles of fluid cannot be described clearly as in laminar flow.
  - In turbulent motion there are no streamlines, but there are large eddies or "chunks" of fluid moving rapidly in seemingly random fashion.
- When a solute A is dissolving from a solid surface there is a high concentration of this solute in the fluid at the surface, and its concentration, in general, decreases as the distance from the wall increases.

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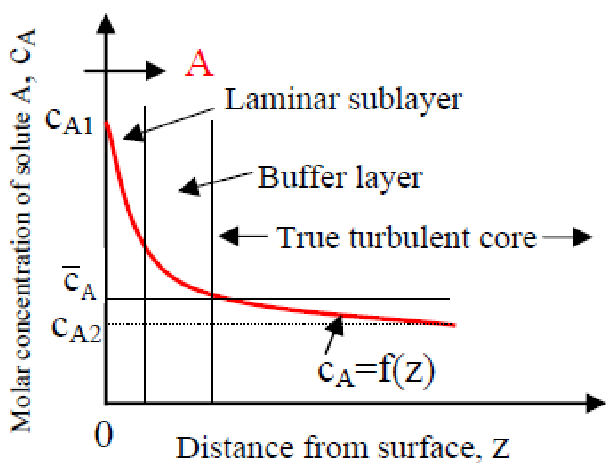
Mass Transfer by Convection



A typical plot for the mass transfer of a dissolving solid from a surface to a turbulent fluid in a conduit

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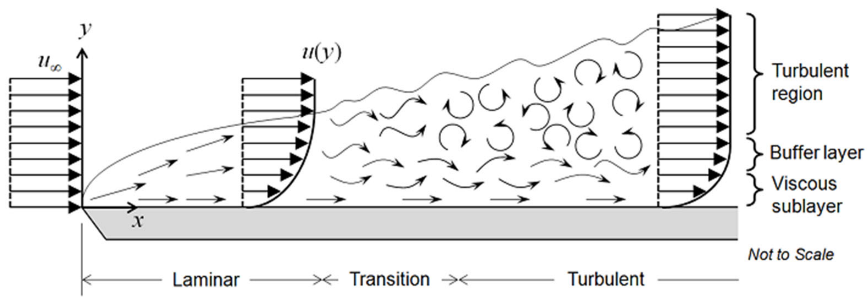
Mass Transfer by Convection

- Three regions of mass transfer can be visualized:

  i. In the first, which is adjacent to the surface, a thin viscous sub-layer film is present. Most of the mass transfer occurs by molecular diffusion.

  ii. The transition or buffer region is adjacent to the first region. Some eddies are present and the mass transfer is the sum of turbulent and molecular diffusion.

  iii. In the *turbulent region* adjacent to the buffer region, most of the transfer is by turbulent diffusion, with a small amount by molecular diffusion



Mass Transfer by Convection

- Mass transfer by convection involves the transport of material between a boundary surface (such as solid or liquid surface) and a moving fluid (in forced convection motion) or between two relatively immiscible, moving fluids.
- Convection mass transfer is strongly influenced by the flow field
- There are two different cases of convective mass transfer:

  i. Mass transfer takes place only in a single phase either to or from a phase boundary, as in sublimation of naphthalene (solid form) into the moving air.

  ii. Mass transfer takes place in the two contacting phases as in extraction and absorption.

- Mass-transfer problems involving flowing fluids are often solved using mass-transfer coefficients, which are analogous to heat-transfer coefficients.

Mass Transfer Coefficient

The mass transfer coefficient is defined on the following 'phenomenological basis'.

Rate of mass transfer $\propto$ Concentration driving force (i.e. the difference in concentration)
Rate of mass transfer $\propto$ Area of contact between the phases

$$n_A = k'_c\, A\left(C_{As} - C_A\right) \qquad \text{Or} \qquad N_A = k'_c\left(C_{As} - C_A\right)$$

- The molar flux N_A is measured relative to a set of axes fixed in space.
- The driving force is the difference between the concentration at the phase boundary, C_{AS} (a solid surface or a fluid interface) and the concentration at some arbitrarily defined point in the fluid medium, C_A (or bulk fluid concentration, C_{Ab})
- The convective mass transfer coefficient k'_c(defined based on concentration driving force in mol/time area-driving force) is a function of geometry of the system and the velocity and properties of the fluid similar to the heat transfer coefficient, h.
- The inverse of the mass transfer represent the mass transfer resistance

Mass Transfer Coefficient

$\Rightarrow$ Mass transfer coefficient, $k'_c = \dfrac{N_A}{\Delta C_A} = \dfrac{\text{molar flux}}{\text{concentration driving force}}$

- Using the following relationships between concentrations and partial pressures:

$$C_{A1} = p_{A1} / RT; \qquad C_{A2} = p_{A2} / RT$$

Then

$$N_A = k'_c\ (p_{A1} - p_{A2}) / RT = k_G\ (p_{A1} - p_{A2})$$

where $k_G = k'_c / RT$

k_G is a gas-phase mass-transfer coefficient based on a partial-pressure driving force.

Mass Transfer Coefficient

- Considering different driving force representations, the mass transfer coefficient for the transport of species A between two locations within a fluid can be also defined from the following relations:

Flux equations for equimolar counterdiffusion

Gases: $N_A = k'_c(c_{A1} - c_{A2}) = k'_G(p_{A1} - p_{A2}) = k'_y(y_{A1} - y_{A2})$

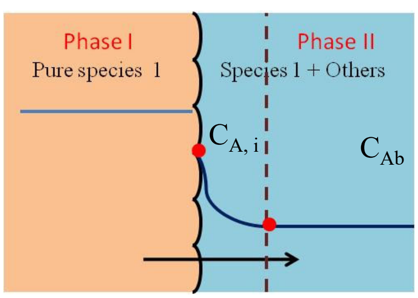
Liquids: $N_A = k'_c(c_{A1} - c_{A2}) = k'_L(c_{A1} - c_{A2}) = k'_x(x_{A1} - x_{A2})$

Flux equations for A diffusing through stagnant, nondiffusing B

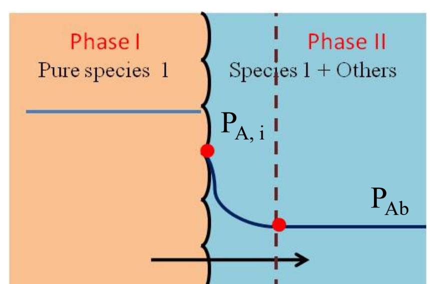
Gases: $N_A = k_c(c_{A1} - c_{A2}) = k_G(p_{A1} - p_{A2}) = k_y(y_{A1} - y_{A2})$

Liquids: $N_A = k_c(c_{A1} - c_{A2}) = k_L(c_{A1} - c_{A2}) = k_x(x_{A1} - x_{A2})$

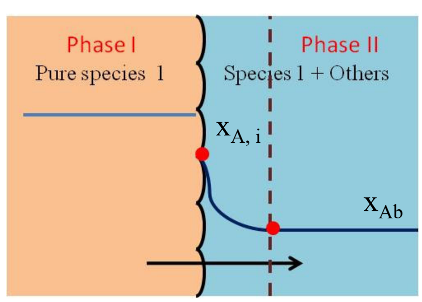
Mass Transfer Coefficient



$$N_A = k_c\left(C_{A,i} - C_{Ab}\right)$$



$$N_A = k_G\left(P_{A,i} - P_{Ab}\right)$$



$$N_A = k_x\left(x_{A,i} - x_{Ab}\right)$$

Mass Transfer Coefficient

➢ In these equations, N $_A$ is the molar flux of species A and the mass transfer coefficient k has different subscript and different units depending on the units of the driving force used in the expression

Conversions between mass-transfer coefficients

Gases:

$$k'_c c = k'_c \frac{P}{RT} = k_c \frac{p_{BM}}{RT} = k'_G P = k_G p_{BM} = k_y y_{BM} = k'_y = k_c y_{BM} c = k_G y_{BM} P$$

Liquids:

$$k'_c c = k'_L c = k_L x_{BM} c = k'_L \rho / M = k'_x = k_x x_{BM}$$

(where ρ is density of liquid and M is molecular weight)

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Mass Transfer Coefficient

Units of mass-transfer coefficients

	SI Units		*Cgs Units*	*English Units*
k_c, k_L, k'_c, k'_L	m/s		cm/s	ft/h
k_x, k_y, k'_x, k'_y	$\frac{\text{kg mol}}{\text{s} \cdot \text{m}^2 \cdot \text{mol frac}}$		$\frac{\text{g mol}}{\text{s} \cdot \text{cm}^2 \cdot \text{mol frac}}$	$\frac{\text{lb mol}}{\text{h} \cdot \text{ft}^2 \cdot \text{mol frac}}$
k_G, k'_G	$\frac{\text{kg mol}}{\text{s} \cdot \text{m}^2 \cdot \text{Pa}}$ (preferred)	$\frac{\text{kg mol}}{\text{s} \cdot \text{m}^2 \cdot \text{atm}}$	$\frac{\text{g mol}}{\text{s} \cdot \text{cm}^2 \cdot \text{atm}}$	$\frac{\text{lb mol}}{\text{h} \cdot \text{ft}^2 \cdot \text{atm}}$

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Notes

Diffusing of A through stagnant, non-diffusing B

$$N_A = \frac{c_T D_{AB}}{z_2 - z_1} \frac{(x_{A1} - x_{A2})}{x_{B,lm}} \Longrightarrow k_x = \frac{c_T D_{AB}}{z_2 - z_1} \frac{1}{x_{B,lm}}$$

Equimolar counter diffusion

$$N_A = \frac{D_{AB}}{z_2 - z_1}(C_{A1} - C_{A2}) \Longrightarrow k'_c = \frac{D_{AB}}{z_2 - z_1}$$

$$N_A = \frac{D_{AB}}{(z_2 - z_1) x_{BM}} \left(\frac{\rho}{M}\right)_{av} (x_{A1} - x_{A2}) \Longrightarrow k'_x = \frac{D_{AB}}{(z_2 - z_1) x_{BM}} \left(\frac{\rho}{M}\right)_{av}$$

Different types of mass transfer coefficients

Different types of mass transfer coefficients

Diffusion of A through non-diffusing B		*Equimolar counterdiffusion of A and B*		*Unit of the mass transfer coefficient*
Flux, N_A	*Mass transfer coefficient*	*Flux, N_A*	*Mass transfer coefficient*	
Gas-phase mass transfer				
$k_G(p_{A1} - p_{A2})$	$k_G = \frac{D_{AB}P}{RT\delta p_{BM}}$	$k'_G(p_{A1} - p_{A2})$	$k'_G = \frac{D_{AB}}{\delta RT}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta p_A)}$
$k_y(y_{A1} - y_{A2})$	$k_y = \frac{D_{AB}P^2}{RT\delta p_{BM}}$	$k'_y(y_{A1} - y_{A2})$	$k'_y = \frac{D_{AB}P}{\delta RT}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta y_A)}$
$k_c(C_{A1} - C_{A2})$	$k_c = \frac{D_{AB}P}{\delta p_{BM}}$	$k'_c(C_{A1} - C_{A2})$	$k'_c = \frac{D_{AB}}{\delta}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta C_A)}$
Liquid-phase mass transfer.				
$k_L(C_{A1} - C_{A2})$	$k_L = \frac{D_{AB}}{\delta x_{BM}}$	$k'_L(C_{A1} - C_{A2})$	$k'_L = \frac{D_{AB}}{\delta}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta C_A)}$
$k_x(x_{A1} - x_{A2})$	$k_x = \frac{CD_{AB}}{\delta x_{BM}}$	$k'_x(x_{A1} - x_{A2})$	$k'_x = \frac{CD_{AB}}{\delta}$	$\frac{\text{mol}}{(\text{time})(\text{area})(\Delta x_A)}$

Example

Vaporizing A and Convective Mass Transfer

A large volume of pure gas B at 2 atm pressure is flowing over a surface from which pure A is vaporizing. The liquid A completely wets the surface, which is a blotting paper. Hence, the partial pressure of A at the surface is the vapor pressure of A at 298 K, which is 0.20 atm. The k'_y has been estimated to be 6.78x10^{-5} kg mol/s·m^2 . mol frac. Calculate N_A, the vaporization rate, and also the value of k_y and k_G.

Example contd.

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Example contd.

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Mass Transfer Coefficient

The most widely used approach for defining a mass-transfer coefficient:

- Consider the mass transfer of solute A from a solid to a fluid flowing past the surface of the solid. For such a case, the mass transfer between the solid surface and the fluid may be written as

$$N_A = k'_c \left(C_{As} - C_{A\infty}\right)$$

- Since the mass transfer at the surface is by molecular diffusion, the mass transfer may also described by

$$N_A = -D_{AB} \left.\frac{dC_A}{dy}\right|_{y=0}$$

- When the boundary concentration, C_{As} is constant,

$$N_A = -D_{AB} \left.\frac{d\left(C_A - C_{As}\right)}{dy}\right|_{y=0}$$

Mass Transfer Coefficient

$$k'_c \left(C_{As} - C_{A\infty}\right) = -D_{AB} \left.\frac{d}{dy}\left(C_A - C_{As}\right)\right|_{y=0}$$

- Rearranging and multiplying both sides by a characteristic length, L

$$\frac{k'_c L}{D_{AB}} = -\frac{d\left(C_A - C_{As}\right)/dy\,\big|_{y=0}}{\left(C_{AS} - C_{A\infty}\right)/L}$$

$$Sh = \text{Sherwood number} = \frac{k'_c L}{D_{AB}} = \frac{\text{Total mass transfer rate}}{\text{Mass transfer by molecular diffusion}}$$

Or it is the ratio of molecular mass-transport resistance to the convective mass-transport resistance of the fluid.

Hence

$$N_{Sh} = k'_c \frac{L}{D_{AB}} = k_c y_{BM} \frac{L}{D_{AB}} = \frac{k'_x}{c} \frac{L}{D_{AB}}$$

Significant Parameters in Convective Mass Transfer

- Dimensionless parameters are often used to correlate convective transfer data.
- In momentum transfer Reynolds number and friction factor play a major role.
- In the correlation of convective heat transfer data, Prandtl and Nusselt numbers are important.
- Some of the same parameters, along with some newly defined dimensionless numbers, will be useful in the correlation of convective mass-transfer data.
- The molecular diffusivities of the three transport process (momentum, heat and mass) have been defined as:

$$\text{Momentum diffusivity} \quad \nu = \frac{\mu}{\rho}$$

$$\text{Thermal diffusivity} \quad \alpha = \frac{k}{\rho C_p}$$

Significant Parameters in Convective Mass Transfer

and $$\text{Mass diffusivity } D_{AB}$$

- The ratio of the molecular diffusivity of momentum to the molecular diffusivity of heat (thermal diffusivity) is designated as the Prandtl Number

$$\frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \Pr = \frac{\nu}{\alpha} = \frac{C_p \mu}{K}$$

- The analogous number in mass transfer is Schmidt number given as

$$\frac{\text{Molecular Diffusivity of Momentum}}{\text{Molecular Diffusivity of Mass}} = Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$

Significant Parameters in Convective Mass Transfer

- The ratio of the molecular diffusivity of heat to the molecular diffusivity of mass is designated the Lewis Number, and is given by

$$\frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}} = Le = \frac{\alpha}{D_{AB}} = \frac{k}{\rho C_p D_{AB}}$$

- Lewis number is encountered in processes involving simultaneous convective transfer of mass and energy

Dimensionless Groups

- In order to obtain dimensionless group for convective mass transfer, consider mass transfer from a pipe wall to a fluid flowing inside the pipe at steady-state, where **A** shows the transferring component and **B** the fluid.
- Determination of the independent variables that affect the mass transfer coefficient constitutes the first step of the analysis

$$k'_c = f\left(D, D_{AB}, \rho, \mu, \bar{u}_x\right)$$

- The number of variables encountered is 6, and hence, the number of π groups required to give the relationship among the variables is 3,
- Using the Buckingham Pi Theorem

$$\Rightarrow \quad \frac{k'_c D}{D_{AB}} = \varphi\left(\frac{\rho D \bar{u}_x}{\mu}, \frac{\mu}{\rho D_{AB}}\right)$$

or $$Sh = \alpha \, Re^{\beta} \, Sc^{\gamma}$$

Dimensionless Groups

where

Sh = Sherwood number = kL/D_{AB}

Re_L = Reynolds number = $L\upsilon\rho/\mu$

Sc = Schmidt number = $\mu / (\rho D_{AB})$

k = overall mass transfer coefficient

L = length of sheet

D_{AB} = diffusivity of A in B

υ = velocity

μ = viscosity

ρ = density, and

μ/ρ = kinematic viscosity.

Γ is the mass rate of liquid flow per unit of film width in the x direction.

$$j_D = \frac{k_c}{v_\infty}(Sc)^{2/3}$$

J factor for mass transfer

$$\mathrm{St} = \mathrm{Sh} / \mathrm{Re\,Sc} = \frac{k_c}{v}$$

The Stanton number $= \dfrac{\text{Total mass transfer rate}}{\text{Inertia forces}}$

$$\mathrm{Pe} = \mathrm{Re\,Sc} = \frac{\ell v\rho}{\mu}\frac{\mu}{\rho D} = \frac{\ell v}{D}$$

Peclet number $= \dfrac{\text{Inertia forces}}{\text{Mass transfer by molecular diffusion}}$

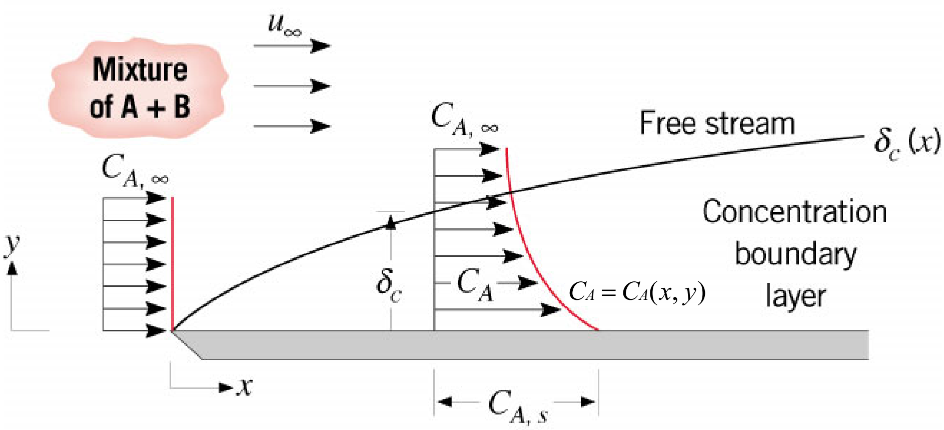
Convective Mass Transfer Correlations

- Extensive data have been obtained for the transfer of mass between a moving fluid and certain shapes, such as flat plates, spheres and cylinders.
- The techniques include sublimation of a solid, vapourization of a liquid into a moving stream of air and the dissolution of a solid into water.
- These data have been correlated in terms of dimensionless parameters and the equations obtained are used to estimate the mass transfer coefficients in other moving fluids and geometrically similar surfaces.

Flat Plate

- From the experimental measurements of rate of evaporation from a liquid surface or from the sublimation rate of a volatile solid surface into a controlled air-stream, several correlations are available.
- These correlation have been found to satisfy the equations obtained by theoretical analysis on boundary layers

Convective Mass Transfer Correlations: Flat Plate



thicknesses of the velocity and the concentration boundary layers depend upon the value of the Schmidt number, Sc (which is the ratio of the momentum diffusivity to the molecular diffusivity). If the Schmidt number is greater than unity, the thickness of the momentum boundary layer at any location on the plate is more than the concentration boundary layer. It can be shown that

$$\frac{\delta_\eta}{\delta_c} = Sc^{1/3}$$

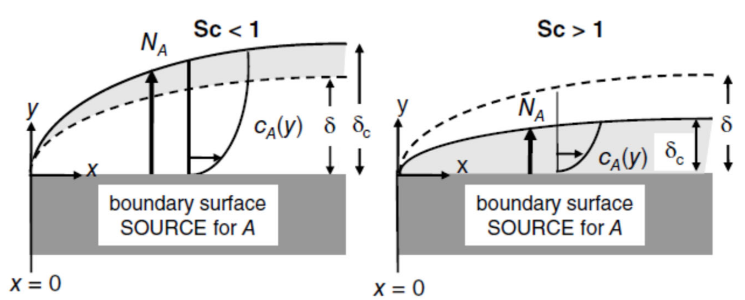
Convective Mass Transfer Correlations: Flat Plate

- For laminar flow over a flat plate, the thickness of the laminar hydrodynamic boundary layer is

$$\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$

- And thus, the concentration boundary layer thickness for laminar flow over a flat plate can be estimated by

$$\delta_c = 5\mathrm{Sc}^{1/3}\sqrt{\frac{\upsilon x}{\mathrm{v}_\infty}}$$



Convective Mass Transfer Correlations: Flat Plate

- If $Sc < 1$, then the concentration profile continues to develop beyond the hydrodynamic boundary layer.
- If $Sc = 1$, then the concentration and hydrodynamic boundary layers are of the same thickness.
- If $Sc > 1$, then the concentration boundary layer resides within the hydrodynamic boundary layer (as for convective mass transfer into flowing liquids).

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Flow Parallel to Flat Plates

2.	Unconfined flow parallel to flat plates‡	Transfer begins at leading edge $Re_x < 50\ 000$	$Sh_L = \frac{k_c L}{D_{AB}} = 0.664\ Re_L^{1/2}\ Sc^{1/3}$
		$Re_x = 5 \times 10^5$–3×10^7 $Sc = 0.7$–380	$Sh_L = \frac{k_c L}{D_{AB}} = 0.037\ Re_x^{0.8}\ Sc^{0.43} \left(\frac{Sc}{Sc_i}\right)^{0.25}$
		$Re_x = 2 \times 10^4$–5×10^5 $Sc = 0.7$–380	Between above and $Sh = 0.027\ Re_x\ Sc^{0.43} \left(\frac{Sc}{Sc_i}\right)^{0.25}$

- If the plate is long enough for the flow to be turbulent, but not long enough to disregard the laminar region, then

$$k_c = \frac{\int_0^L k_{c,x}(x)dx}{\int_0^L dx} = \frac{\int_0^{L_t} k_{c,\mathrm{lam}}(x)\,dx + \int_{L_t}^L k_{c,\mathrm{turb}}(x)\,dx}{L}$$

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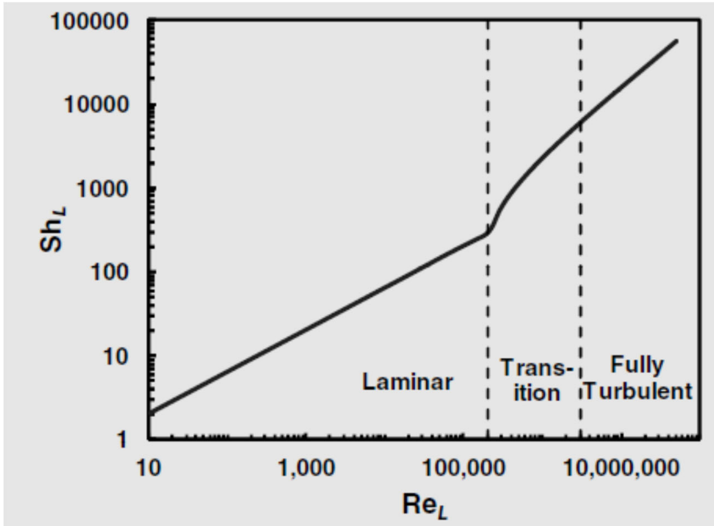
Flow Parallel to Flat Plates



Average Sherwood number (Sh_L) at Sc =1.0 for flow over a flat plate, showing transition from laminar to fully turbulent regimes,

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Flow Parallel to Flat Plates

- Then The average Sherwood over the entire plate is

$$\frac{k_c L}{D_{AB}} = \mathrm{Sh}_L = 0.664(\mathrm{Re}_t)^{1/2}\mathrm{Sc}^{1/3} + 0.0365\,\mathrm{Sc}^{1/3}\left[(\mathrm{Re}_L)^{4/5} - (\mathrm{Re}_t)^{4/5}\right]$$

- Experimental data for liquids are correlated within about ± 40% by the following for

$$J_D = 0.99 N_{Re,L}^{-0.5} \qquad 600 < \mathrm{Sc} < 50000$$

3. Confined gas flow parallel to a flat plate in a duct $\quad Re_e = 2600\text{–}22\,000 \quad j_D = 0.11\,Re_e^{-0.29}$

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Fluid motion	Range of conditions	Equation
1. Inside circular pipes	Re = 4000–60 000 Sc = 0.6–3000	$j_D = 0.023\ \mathrm{Re}^{-0.17}$ $\mathrm{Sh} = 0.023\ \mathrm{Re}^{0.83}\ \mathrm{Sc}^{1/3}$
	Re = 10 000 – 400 000 Sc > 100	$j_D = 0.0149\ \mathrm{Re}^{-0.12}$ $\mathrm{Sh} = 0.0149\ \mathrm{Re}^{0.88}\ \mathrm{Sc}^{1/3}$

For *laminar* flow of a fluid through a tube, with a Reynolds number range of $10 < \mathrm{Re} < 2000$, the appropriate mass-transfer correlation is

$$\mathrm{Sh} = 1.86\left(\frac{v_\infty D^2}{LD_{AB}}\right)^{1/3} = 1.86\left(\frac{D}{L}\frac{v_\infty D}{\nu}\cdot\frac{\nu}{D_{AB}}\right)^{1/3} = 1.86\left(\frac{D}{L}\mathrm{Re}\,\mathrm{Sc}\right)^{1/3} \qquad (30\text{-}19)$$

where L the length of the pipe and v_∞ is the bulk average velocity.

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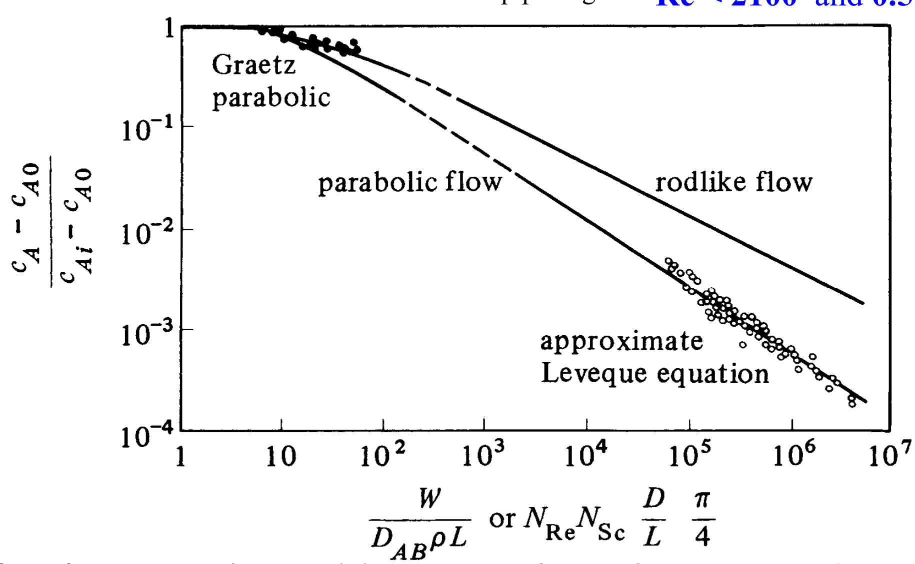
Figure 7.3-2: Mass transfer from the inner walls of a pipe to gases **Re < 2100** and **0.5 < Sc < 3.0**



c_A is the exit concentration, c_{A0} inlet concentration, and c_{Ai} concentration at the interface between the wall and the gas. The dimensionless abscissa is $W/D_{AB}\rho L$ or $N_{\mathrm{Re}}N_{\mathrm{Sc}}(D/L)(\pi/4)$, where W is flow in kg/s and L is length of mass-transfer section in m.

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Liquid film in Wetted Wall

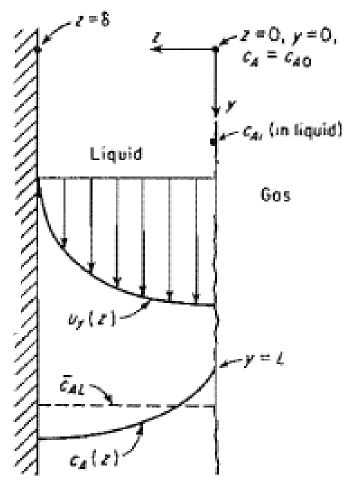
4. Liquid film in wetted-wall tower, transfer between liquid and gas

Re = 0–1200, $k_{L,\,av} = \left(\frac{6D_{AB}\Gamma}{\pi\rho\delta L}\right)^{1/2}$

$\text{Re} = 4\Gamma/\mu$ less than 100 $\quad \text{Sh}_{av} \approx 3.41$

$3{,}000 < \text{Re}' < 40{,}000;\ 0.5 < \text{Sc} < 3 \qquad j_D = 0.0328(\text{Re}')^{-0.23}$

$\text{Re} = \frac{4\Gamma}{\mu} = 1300\text{–}8300 \qquad \text{Sh} = (1.76\times10^{-5})\left(\frac{4\Gamma}{\mu}\right)^{1.506}\text{Sc}^{0.5}$



The film thickness is then

$$\delta = \left(\frac{3\bar{u}_y\mu}{\rho g}\right)^{1/2} = \left(\frac{3\mu\Gamma}{\rho^2 g}\right)^{1/3}$$

The Reynolds number of the liquid flowing down the tube is defined as

$$\text{Re}_L = \frac{4\Gamma}{\mu_L} = \frac{4w}{\pi D\mu_L}$$

where w is the mass flow rate of liquid, D is the inner diameter of the cylindrical column, and Γ is the mass flow rate of liquid per unit wetted perimeter of the column.

or the mass rate of liquid flow per unit of film width in the x direction.

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Past Single Sphere

For mass transfer into liquid streams, the equation of Brian and Hales[4]

$$\text{Sh} = \frac{k_L D}{D_{AB}} = \left(4 + 1.21\,\text{Pe}_{AB}^{2/3}\right)^{1/2} \qquad Pe_{AB} = \text{Re}\,Sc$$

correlates data where the mass-transfer Peclet number, Pe_{AB}, is less than 10,000. For Peclet numbers greater than 10,000, Levich[5] recommends the simpler relationship

$$\text{Sh} = \frac{k_L D}{D_{AB}} = 1.01\,\text{Pe}_{AB}^{1/3}$$

- Also, for liquid

$$N_{Sh} = 2 + 0.95 N_{Re}^{0.50} N_{Sc}^{1/3} \qquad 2 < \text{Re} < 2000$$

$$N_{Sh} = 0.347 N_{Re}^{0.62} N_{Sc}^{1/3} \qquad 2000 < \text{Re} < 17000$$

For mass transfer into gas streams, the Fröessling equation (evaporation and sublimation)

$$\text{Sh} = \frac{k_c D}{D_{AB}} = 2 + 0.552\,\text{Re}^{1/2}\,\text{Sc}^{1/3} \qquad 2 < \text{Re} < 48000,\ 0.6 < \text{Sc} < 2.7$$

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Past Single Sphere

- For drops of liquid falling through a gas, the mass transfer coefficient to and from drops may be approximately represented by

$$Sh = 1.13\ Re^{1/2}\ Sc^{1/2}$$

$d_p > 3.0$ cm

$500 < Re < 2000$

- Bubbles behave very much like drops, but their buoyancy and velocity of rise are much higher. Mass transfer within bubbles is relatively rapid since bubbles are filled with gas and molecular diffusion in gases is high. the following empirical equation is proposed by Johnson et al. (1969) gives a more reliable prediction

$$Sh = 1.13\, Pe \left(\frac{d_e}{0.45 + 0.2 d_e}\right)^{1/2}$$

$0.6 < d_p < 4.0$ cm,

$500 < Re < 2000$

Past Single Sphere

- Colombet et al. (2013) proposed the following relation that is valid for a spherical bubble whatever the value of Re and Pe,

$$Sh = 1 + \left[1 + \left(\frac{4}{3\pi}\right)^{2/3} (2\, Pe_{max})^{2/3}\right]^{3/4},$$

where Pe_{max} is the Péclet number based on the maximal velocity U_{max} of the liquid at the interface instead of the bubble velocity V_z, which is obtained from the correlation proposed by Legendre (2007),

$$\frac{U_{max}}{V_z} = \frac{1}{2}\,\frac{16 + 3.315 Re^{1/2} + 3Re}{16 + 3.315 Re^{1/2} + Re}.$$

- For very low Reynold's number, the Sherwood number should approach a value of 2.

$$\frac{k_c' D_p}{D_{AB}} = N_{Sh} = 2.0$$

Past Single Sphere

The following correlation of Steinberger and Treybal[8] is recommended when the transfer occurs in the presence of natural convection

$$\mathrm{Sh} = \mathrm{Sh}_o + 0.347(\mathrm{Re}\,\mathrm{Sc}^{1/2})^{0.62}$$

where Sh_o is dependent on Gr Sc

$$\mathrm{Sh}_o = 2 + 0.569(\mathrm{GrSc})^{0.25} \qquad \mathrm{Gr\,Sc} \leq 10^8$$

$$\mathrm{Sh}_o = 2 + 0.0254(\mathrm{GrSc})^{1/3}\,(\mathrm{Sc})^{0.244} \qquad \mathrm{Gr\,Sc} \geq 10^8$$

the Grashof number is defined as

$$\mathrm{Gr} = \frac{D^3 \rho g \,\Delta\rho}{\mu^2}$$

where density, ρ, and viscosity, μ, are taken at the bulk conditions of the flowing fluid, and $\Delta\rho$ is the positive density difference between the two phases in contact. The prediction for Sh is valid when $2 \leq \mathrm{Re} \leq 3 \times 10^4$ and $0.6 \leq \mathrm{Sc} \leq 3200$.

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Past Single Sphere

- for the mass-transfer coefficient associated with the transfer of a sparingly soluble gaseous solute A into solvent B by a swarm of gas bubbles in a natural convection process, the following correlation are used:

For gas bubble diameters (d_b) less than 2.5 mm, use

$$\mathrm{Sh} = \frac{k_L d_b}{D_{AB}} = 0.31\,\mathrm{Gr}^{1/3}\,\mathrm{Sc}^{1/3}$$

For bubble diameters greater or equal to 2.5 mm, use

$$\mathrm{Sh} = \frac{k_L d_b}{D_{AB}} = 0.42\,\mathrm{Gr}^{1/3}\,\mathrm{Sc}^{1/2}$$

In the above correlations, the Grashof number is defined as

$$\mathrm{Gr} = \frac{d_b^3 \rho_L g\,\Delta\rho}{\mu_L^2}$$

where $\Delta\rho$ is the difference of the density of the liquid and the density of the gas inside the bubble, with density (ρ_L) and viscosity (μ_L) determined at the bulk average properties of the liquid mixture. For dilute solutions, the fluid properties of the solvent approximate the fluid properties of the liquid mixture. The diffusion coefficient D_{AB} is with respect to dissolved gaseous solute A in solvent B.

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Perpendicular to Single Cylinders

5.	Perpendicular to single cylinders	Re = 400–25 000, Sc = 0.6–2.6	$\frac{k_G P (Sc)^{0.56}}{G_M} = \frac{k_c (Sc)^{0.56}}{v_\infty} = 0.281(\mathrm{Re}_D)^{-0.4}$
		Re′ = 0.1–10[5], Sc = 0.7–1500	$Sh = (0.35 + 0.34\,Re^{0.5} + 0.15 Re^{0.58}) Sc^{0.3}$

P is the system total pressure and G_M is the superficial molar velocity of the gas flowing normal to the cylinder in units of kg mol/m$^2 \cdot$ s. The Reynolds number for flow normal to a solid cylinder, Re_D, is defined as

$$\mathrm{Re}_D = \frac{\rho v_\infty D}{\mu}$$

where D is cylinder diameter, v_∞ is the fluid velocity normal to the solid cylinder, and ρ and μ for the gas stream evaluated at the film average temperature.

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Perpendicular to Single Cylinders

- Experimental data have been obtained for mass transfer from single cylinders when the flow is perpendicular to the cylinder. The cylinders are long and mass transfer to the ends of the cylinder is not considered.

$$J_D = 0.600(N_{Re})^{-0.487}$$

$$\left.\begin{array}{l}\textbf{For gases } 0.6 < Sc < 2.6 \\ \textbf{For liquid } 1000 < Sc < 3000\end{array}\right\} \; \mathbf{50 < Re < 50000}$$

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Example

A tube is coated on the inside with naphthalene and has an inside diameter of 20 mm and a length of 1.10 m. Air at 318 K and of 101.3 kPa flows through this pipe at a velocity of 0.8 m/s. Assuming that the pressure remains constant. Calculate the concentration of naphthalene in the exist air

Example Contd

Example

A large volume of pure water at 26.1°C is flowing parallel to a flat plate of solid benzoic acid, where $L = 0.244$ m in the direction of flow. The water velocity is 0.061 m/s. The solubility of benzoic acid in water is 0.02948 kg mol/m^3. The diffusivity of benzoic acid is 1.245×10^{-9} m^2/s. Calculate the mass-transfer coefficient k_L and the flux N_A.

Example Contd

Example Contd

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Example

Calculate the value of the mass-transfer coefficient and the flux for mass transfer from a sphere of napthalene to air at 45°C and 1 atm abs flowing at a velocity of 0.305 m/s. The diameter of the sphere is 25.4 mm. The diffusivity of naphthalene in air at 45°C is 6.92×10^{-6} m^2/s and the vapor pressure of solid naphthalene is 0.555 mm Hg. Use English and SI units.

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Example Contd

Example Contd

Mass Transfer to Small Particles

Mass transfer to small particles <0.6 mm

- The following equation has been shown to hold to predict mass-transfer coefficients from small gas bubbles such as oxygen or air to the liquid phase or from the liquid phase to the surface of small catalyst particles, microorganisms, other solids, or liquid drops

$$k'_L = \underbrace{\frac{2D_{AB}}{D_p}}_{\text{the molecular diffusion term}} + \underbrace{0.31 N_{Sc}^{-2/3}\left(\frac{\Delta\rho\mu_c g}{\rho_c^2}\right)^{1/3}}_{\text{free fall or rise of the sphere by gravitational forces}}$$

where D_{AB} is the diffusivity of the solute A in solution in m^2/s, D_p is the diameter of the gas bubble or the solid particle in m, μ_c is the viscosity of the solution in kg/m · s, $g = 9.80665\ m/s^2$, $\Delta\rho = (\rho_c - \rho_p)$ or $(\rho_p - \rho_c)$, ρ_c is the density of the continuous phase in kg/m^3, and ρ_p is the density of the gas or solid particle. The value of $\Delta\rho$ is always positive.

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Mass Transfer to Small Particles

Mass transfer to large gas bubbles > 2.5 mm.

- For large gas bubbles or liquid drops > 2.5 mm, the mass-transfer coefficient can be predicted by

$$k'_L = 0.42 N_{Sc}^{-0.5}\left(\frac{\Delta\rho\mu_c g}{\rho_c^2}\right)^{1/3}$$

- Large gas bubbles are produced when pure liquids are aerated in mixing vessels and sieve-plate columns

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Example

Calculate the maximum rate of absorption of O_2 in a fermenter from air bubbles at 1 atm abs pressure having diameters of 100 μm at 37°C into water having a zero concentration of dissolved O_2. The solubility of O_2 from air in water at 37°C is 2.26×10^{-7} g mol O_2/cm^3 liquid or 2.26×10^{-4} kg mol O_2/m^3. The diffusivity of O_2 in water at 37°C is 3.25×10^{-9} m^2/s. Agitation is used to produce the air bubbles.

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Example Contd

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Mass Transfer to and from Packed Beds

- Mass transfer to and from packed beds occurs often in processing operations, including
  - drying operations,
  - adsorption or desorption of gases or
  - liquids by solid particles such as charcoal, and
  - mass transfer of gases and liquids to catalyst particles.
- Using a packed bed a large amount of mass-transfer area can be contained in a relatively small volume.
- For packed and fluidized beds, the area of mass transfer is generally expressed in terms of specific interfacial area which is defined as the area per unit volume of packed bed. It can e expressed as: $a = \frac{6(1-\varepsilon)}{d_p}$

Mass Transfer to and from Packed Beds

Where ε is the porosity or void fraction and d_p is the particle diameter.

- For a Reynolds number range of 10-10000 for gases in a packed bed of spheres

$$J_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069}$$

where

$N_{Re} = D_p v' \rho / \mu$,

The void fraction in a bed is ε, range from 0.3 to 0.5

D_p is diameter of the spheres

v' is the superficial mass average velocity in the empty tube without packing.

Mass Transfer to and from Packed Beds

7.	Through fixed beds of pellets§	Re″ = 90–4000 Sc = 0.6	$j_D = j_H = \frac{2.06}{\varepsilon}\text{Re}''^{-0.575}$
		Re″ = 5000–10 300 Sc = 0.6	$j_D = 0.95 j_H = \frac{20.4}{\varepsilon}\text{Re}''^{-0.815}$
		Re″ = 0.0016–55 Sc = 168–70 600	$j_D = \frac{1.09}{\varepsilon}\text{Re}''^{-2/3}$
		Re″ = 5–1500 Sc = 168–70 600	$j_D = \frac{0.250}{\varepsilon}\text{Re}''^{-0.31}$

† Average mass-transfer coefficients throughout, for constant solute concentrations at the phase surface. Generally, fluid properties are evaluated at the average conditions between the phase surface and the bulk fluid. The heat-mass-transfer analogy is valid throughout.

‡ Mass-transfer data for this case scatter badly but are reasonably well represented by setting $j_D = j_H$.

§ For fixed beds, the relation between ε and d_p is $a = 6(1-\varepsilon)/d_p$, where a is the specific solid surface, surface per volume of bed. For mixed sizes [58]

Gas-phase flow through a packed bed	$10 \le \text{Re}'' \le 2500$	$j_D = 1.17(\text{Re}'')^{-0.415}$
Liquid flow through a packed bed	$\text{Re}'' < 55$	$j_D = 1.09(\text{Re}'')^{-2/3}$
	$3 < \text{Re}'' < 10{,}000$	$\text{Sh} = 2 + 1.1(\text{Re})^{0.6}(\text{Sc})^{0.33}$

d = tube diameter; $\text{Re}_l = l v\rho/\mu$, l = characteristic length; $\text{Re}' = dv'\rho/\mu$, v' = gas velocity relative to the surface of the falling film; $\text{Re}'' = d_p v''\rho/\mu$, d_p = diameter of the sphere, v'' = superficial velocity of the fluid (i.e. velocity based on the bed cross-section).

Mass Transfer in Fluidized Beds of Spheres

- For fluidized beds of spheres and for gases and liquids

$$J_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069} \qquad \mathbf{10 < Re < 4000}$$

$$\varepsilon J_D = 1.1068\ N_{Re}^{-0.72} \qquad \mathbf{1 < Re < 10}$$

- For both gas and liquid packed and fluidized bed of spherical particle,

$$St_D Sc^{2/3} = \frac{0.010}{\varepsilon} + \frac{0.863/\varepsilon}{\text{Re}^{0.58} - 0.483} \qquad \text{where } 1 < \text{Re} < 2100$$

Mass Transfer to Packed Beds

➢ The total flux in a packed bed,

$$N_A A = A k_c \frac{(c_{Ai} - c_{A1}) - (c_{Ai} - c_{A2})}{\ln \dfrac{c_{Ai} - c_{A1}}{c_{Ai} - c_{A2}}}$$

where

c_{Ai} is the concentration at the surface of the solid, in kg mol/m^3

c_{A1} is the inlet bulk fluid concentration

c_{A2} is the outlet.

A is the total external surface area *in* m^2 $\quad A = aV_b$

a is the m^2 surface area/m^3 total volume of bed when the solids are spheres.

$$a = \frac{6(1 - \varepsilon)}{D_p}$$

V_b is total volume *in* m^3 of the bed (void plus solids),

Mass Transfer to Packed Beds

k_c is the mass transfer coefficient obtained from correlations

➢ Also, the material balance equation on the bulk stream is

$$N_A A = V(c_{A2} - c_{A1})$$

where V is volumetric flow rate of fluid entering in m^3/s.

Example

Pure water at 26.1°C flows at the rate of 5.514×10^{-7} m^3/s through a packed bed of benzoic acid spheres having a diameter of 6.375 mm. The total surface area of the spheres in the bed is 0.01198 m^2 and the void fraction is 0.436. The tower diameter is 0.0667 m. The solubility of benzoic acid in water is 2.948×10^{-2} kg mol/m^3.

(a) Predict the mass-transfer coefficient k_c. Compare with the experimental value of 4.665×10^{-6} m/s by Wilson and Geankoplis (W1).

(b) Using the experimental value of k_c, predict the outlet concentration of benzoic acid in the water.

Example Contd

Example Contd

Reynolds Analogy

- Reynolds postulated that the mechanisms for transfer of momentum, energy and mass are identical.
- Accordingly: $$\frac{k_c}{V_\infty} = \frac{h}{\rho V_\infty c_p} = \frac{C_f}{2}$$

Where: *h* is heat transfer coefficient, V_∞ is velocity of free stream

C_f is skin friction coefficient, $$C_f = \frac{\tau_0}{\rho v_\infty^2/2} = \frac{2\mu}{\rho v_\infty^2} \frac{\partial v_x}{\partial y}\bigg|_{y=0}$$

- This analogy is limited in application;
  - It is found to be accurate when <u>Prandtl and Schmidt numbers are equal to one.</u>
  - <u>This</u> is applicable for mass transfer by means of turbulent eddies in gases.
  - In this situation, we can estimate mass transfer coefficients from heat transfer coefficients or from friction factors.

Chilton – Colburn Analogy

- Chilton and Colburn, using experimental data, sought modifications to the Reynolds analogy that would not have the restrictions that Prandtl and Schmidt numbers must be equal to one
  - They defined for the *J* factor for mass transfer as

$$j_D = St_m (Sc)^{2/3}$$

Where St_m is the Stanton number for mass transfer $\quad St_m = \frac{Sh}{Re\, Sc} = \frac{k_c}{v_\infty}$

  - The analogous j factor for heat transfer as

$$j_H = St_H (Pr)^{2/3}$$

Where St_H is the Stanton number for heat transfer $\quad St_H = \frac{Nu}{Re\, Pr} = \frac{h}{\rho C_p V}$

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Chilton – Colburn Analogy

- Based on data collected in both laminar and turbulent flow regimes,

$$j_D = j_H = \frac{f}{2}$$

or

$$\frac{h}{\rho\, v_\infty c_p} (Pr)^{2/3} = \frac{k_c}{v_\infty} (Sc)^{2/3}$$

- This analogy is valid for gases and liquids within the range of $0.6 < Sc < 2500$ and $0.6 < Pr < 100$.

- The Chilton-Colburn analogy has been observed to hold for much different geometry for example, flow over flat plates, flow in pipes, and flow around cylinders.

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Mass Transfer Coefficients for Tower Packing

Mass Transfer Coefficients in Various Commercial Packings

Packing	G, kg/m²s	L, kg/m²s	$k_c a$, s⁻¹
System: CO_2–aqueous NaOH ($k_c a$)			
1-in. Raschig ceramic	0.61–0.68	1.4–14	0.14–0.33
2-in. Raschig ceramic	0.61–0.68	1.4–14	0.09–0.31
1-in. Raschig metal	0.61–0.68	1.4–14	0.16–0.32
1-in. Pall plastic	0.61–0.68	1.4–27	0.16–0.33
2-in. Pall plastic	0.68	1.4–54	0.13–0.33
1-in. Pall metal	0.61–0.68	1.4–27	0.19–0.44
1-in. Intalox plastic	1.2	2.7–27	0.28–0.43
2-in. Intalox plastic	1.2	4.1–41	0.20–0.30
½-in. Intalox ceramic	0.61–0.68	1.4–14	0.30–0.51
1-in. Intalox ceramic	0.61–0.68	1.4–14	0.17–0.36
3-in. Intalox ceramic	1.2	1.4–54	0.04–0.22

Packing	G, kg/m²s	L, kg/m²s	$k_c a$ or $k_L a$, s⁻¹
System: NH_3–Water ($k_c a$)			
1-in. Raschig ceramic	0.54	0.68–6.1	1.3–5.2
2-in. Raschig ceramic	0.54	0.68–6.1	0.87–2.6
1-in. Berl ceramic	0.54	0.68–6.1	1.7–4.3
2-in. Berl ceramic	0.54	0.68–6.1	1.3–4.0
50-mm Pall plastic	0.45–2.5	4.2	2.0–7.0
System: CO_2–water ($k_L a$)			
50-mm Pall plastic	0.4–2.0	4.2	11
System: O_2–water ($k_L a$)			
1.5-in. Raschig ceramic	0.054–0.54	2.7	0.14–1.4

Mass Transfer Coefficients in Agitated Vessels

Mass Transfer Coefficients in Agitated Vessels

System	Correlation
Solid–liquid baffled vessel Re = $10^4 - 10^6$	$\frac{k_c d_v}{D} = 1.46\left(\frac{d_i^2 N\rho}{\mu}\right)^{0.65}(\mathrm{Sc})^{0.33}$
Solid–liquid unbaffled vessel Re = $10^2 - 10^5$	$\frac{k_c d_v}{D} = 0.402\left(\frac{d_i^2 N\rho}{\mu}\right)^{0.65}(\mathrm{Sc})^{0.33}$
Liquid–liquid	$\frac{k_c d_v}{D} = 0.052\left(\frac{d_i^2 N\rho}{\mu}\right)^{0.833}(\mathrm{Sc})^{0.5}$

- For the Sherwood number, the dimensional length to be used is the vessel diameter d_v.
- For the Reynolds number, the dimensional length is represented by the impeller diameter d_i and the dimensional velocity by the product $d_i N$, where N represents the number of revolutions per unit time. All other parameters are used in the same fashion as before; i.e., μ, ρ, and D are the viscosity, density, and diffusivity of the continuous phase.

Mass Transfer Coefficients for Flow around Simple Geometries

Mass Transfer Coefficients in Laminar Flow around Simple Geometries

Geometry	Correlation	Re
Flat plate	$St = 0.66(Re)^{-1/2}(Sc)^{-2/3}$	$<10^5$
Sphere	$Sh = 2.0 + 0.60\,(Re)^{1/2}(Sc)^{1/3}$	$<10^{-1}$
Cylinder	$Sh = 0.43 + 0.53\,(Re)^{1/2}(Sc)^{0.31}$	$1 < Re < 10$

Correlations for Mass Transfer Coefficients in Turbulent Flow

Range	Correlation $Sh = a\,Re^b\,Sc^c$
1. Flat plate	
$Re > 10^6$	$St = 0.036\,(Re)^{-0.2}\,(Sc)^{-0.67}$
2. Sphere	
Unlimited	$Sh = 2.0 + 0.60\,(Re)^{0.5}\,(Sc)^{0.33}$
3. Inside tubes	
$Re > 20{,}000$	$Sh = 0.026\,(Re)^{0.8}\,(Sc)^{0.33}$
4. Packed bed of spheres	
$Re > 50$	$St = 0.61\,(Re)^{-0.41}\,(Sc)^{-0.67}$

Summary

No.	Geometry	Equation	Application range	Charact. dimension	Type of fluid
1	Inside pipe flow	$Sh = 0.023\,Re^{0.83}\,Sc^{0.33}$ $Sh = 0.0149\,Re^{0.88}\,Sc^{0.33}$	4 000< Re< 60 000 0.6 <Sc <3 000 10 000< Re<4*10^5 Sc > 100	Pipe diameter	Fluid
2	Flow parallel to a flat plate	$J_D = 0.664\,Re_x^{-0.5}$ $Sh = 0.037\,Re_x^{0.8}\,Sc^{0.43}\left(\frac{Sc}{Sc_i}\right)^{0.25}$ $Sh = 0.027\,Re_x\,Sc^{0.43}\left(\frac{Sc}{Sc_i}\right)^{0.25}$	Re < 50 000 5*10^5<Re$_x$ <3*10^7 0.7<Sc <380 2*10^4<Re$_x$< 5*10^5 0.7<Sc <380	Length of plate	Fluid
3	Gas flow Parallel to a flat plate in confined duct	$J_D = 0.11\,Re^{-0.29}$	2 600<Re <22 000	Length of plate	Gas

Summary

4	Wetted-wall column	$Sh = 3.41$ $Sh = \left(\frac{3\delta}{2\pi h} Re\, Sc\right)^{0.5}$ $Sh = 1.76 \cdot 10^{-5} Re^{1.506} Sc^{0.5}$	$Re = \frac{4\Gamma}{\mu} < 100$ $100 < Re < 1200$ $1\,300 < Re < 8\,300$	Thickness of liquid film	Liquid
5	Flow Perpendicular To cylinder	$Sh = 0.281\, Re^{0.60} Sc^{0.44}$ $Sh = (0.35 + 0.34\, Re^{0.5} + 0.15\, Re^{0.58})\, Sc^{0.3}$	$400 < Re < 25\,000$, $0.6 < Sc < 2.6$ $0.1 < Re < 10^5$, $0.7 < Sc < 1\,500$	Diameter of cylinder	Gas Fluid
6	Flow past single sphere	$Sh = Sh_o + 0.347\,(Re.Sc^{0.5})^{0.62}$ $Sh_o = \begin{cases} 2 + 0.569(Gr\,Sc)^{0.25} & Gr\,Sc < 10^8 \\ 2 + 0.0254(Gr\,Sc)^{0.333}\,Sc^{0.244} & Gr\,Sc > 10^8 \end{cases}$	$1.8 < Re.Sc^{0.5} < 6 \cdot 10^5$ $0.6 < Sc < 3\,200$	Diameter of sphere	Fluid

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Summary

7	Packed column (*)	$J_D = \frac{2.06}{\varepsilon} Re^{-5.75}$ $J_D = \frac{20.4}{\varepsilon} Re^{-0.815}$ $J_D = \frac{1.09}{\varepsilon} Re^{-2/3}$ $J_D = \frac{0.250}{\varepsilon} Re^{-0.31}$	$90 < Re < 4\,000$, $Sc = 0.6$ $5\,000 < Re < 10\,300$, $Sc = 0.6$ $0.0016 < Re < 55$, $168 < Sc < 70\,600$ $5 < Re < 1\,500$, $168 < Sc < 70\,600$	Diameter of packing	Gas Gas Liquid Liquid

Grashof number, $Gr = \frac{g\ell^3 \Delta\rho}{\rho}\left(\frac{\rho}{\mu}\right)^2$; J_D- factor , $J_D = \frac{Sh}{Re\, Sc^{0.33}}$; ε : Void fraction

(*) In the Reynolds numbers superficial velocities (velocity based on empty column) are used.

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