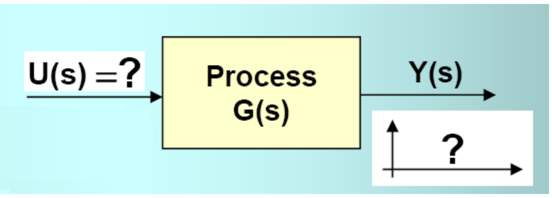
Process dynamic behavior

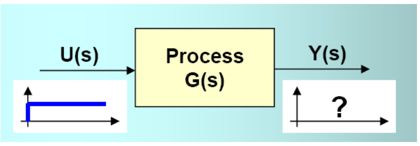
- ➢ In analyzing process dynamic and process control systems, it is important to know how the process responds to changes in the process inputs **U(s)**.
- ➢ A number of standard types of input changes are widely used for two reasons:
  1. They are representative of the types of changes that occur in plants.
  2. They are easy to analyze mathematically.



Process dynamic behavior

➢Standard types of input changes :

1. Step Input



A sudden change in a process input variable can be approximated by a step change of magnitude, *M*:

$$u_s(t)=\begin{cases}0 & t<0\\ M & t\geq 0\end{cases} \quad ; \quad U_s(s)=M/s$$

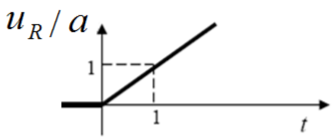
- The step change occurs at an arbitrary time denoted as *t* = 0.
- *Special Case:* If ***M* = 1**, we have a "**unit step change**". We give it the symbol, ***S*(*t*)**.
- ***Example of a step change***: A reactor feedstock is suddenly switched from one supply to another, causing sudden changes in feed concentration.

Process dynamic behavior

➢Standard types of input changes :

2. Ramp input

The process input variable increases linearly with time a rate of change , ***a*** :



$$u_R(t)=\begin{cases}0 & t<0\\ at & t\ge 0\end{cases} \qquad ; \qquad U_R(s)=a/s^2$$

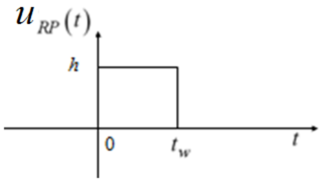
Example of ramp changes*:* Ramp setpoint to a new value; feed composition; heat exchanger fouling; catalyst activity.

Process dynamic behavior

➢Standard types of input changes :

3. Rectangular Pulse

It represents a brief sudden change in the input process input variable:



$$u_{RP}(t)=\begin{cases}0 & \text{for } \; t<0\\ h & \text{for } \; 0\le t<t_w\\ 0 & \text{for } \; t\ge t_w\end{cases}$$

$$\mathrm{U}_{\mathrm{RP}}(s)=\frac{h}{s}\left[1-e^{-t_w s}\right]$$

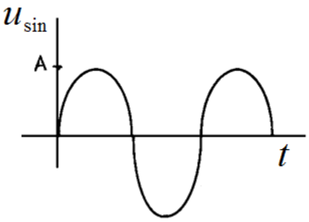
Example of rectangular pulse changes*:* Reactor feed is shut off for one hour; the fuel gas supply to a furnace is briefly interrupted.

Process dynamic behavior

➢**Standard types of input changes :**

4. Sinusoidal disturbance (Frequency Response)

Processes are also subject to periodic, or cyclic disturbances. They can be approximated by a sinusoidal disturbance:



$$u_{\sin}(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ A\sin(\omega t) & \text{for} \quad t \geq 0 \end{cases} \qquad U_{\sin}(s) = \frac{A\omega}{s^2 + \omega^2}$$

where: A = amplitude, ω = angular frequency

***Example of sinusoidal changes**:* 24 hour variations in cooling water temperature; electrical noise.

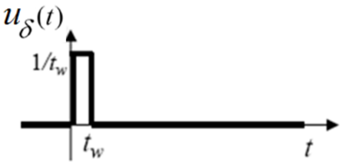
Process dynamic behavior

➢**Standard types of input changes :**

5. Impulse input

It represents a short, transient disturbance. It is the limit of a rectangular pulse for $t_w \rightarrow 0$ and $h = 1/t_w$:

$$u_\delta(t) = \lim_{t_w \to 0} \begin{cases} 0 & \text{for } t > t_w \\ 1/t_w & \text{for } t_w \geq t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



$$U_\delta(s) = 1$$

***Example of impulse changes**:* Electrical noise spike in a thermo-couple reading; Injection of a tracer dye.

Process dynamic behavior

➢ Dynamic behavior of 1st –order systems:

- First-order linear ODE (assume all deviation variables):

$$\tau\frac{dy(t)}{dt}=-y(t)+Ku(t)\xrightarrow{\mathbf{L}}(\tau s+1)Y(s)=KU(s)$$

- **Transfer function:** $\frac{Y(s)}{U(s)}=\frac{K}{(\tau s+1)}$

Gain

Time constant

(Space time)

Step change input: $U(s)=A/s$

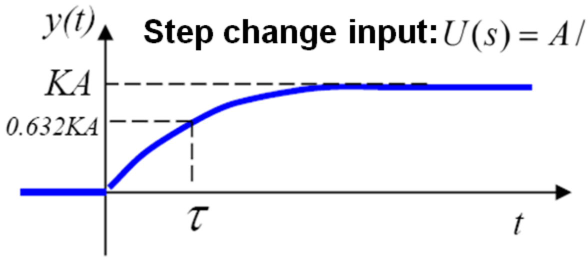
$$Y(s)=\frac{KA}{s(\tau s+1)}\xrightarrow{\mathbf{L}^{-1}}y(t)=KA(1-e^{-t/\tau})$$

- $y(\tau)=KA(1-e^{-\tau/\tau})\approx 0.632KA$
- $KA(1-e^{-t/\tau})\geq 0.99KA\Rightarrow t\approx 4.6\tau$ (Settling time=4$\tau\sim 5\tau$)
- $y'(0)=KAe^{-t/\tau}/\tau\Big|_{t=0}=KA/\tau\neq 0$ (Nonzero initial slope)

Process dynamic behavior

➢ Dynamic behavior of 1st –order systems:

Step change input: $U(s)=A/s$

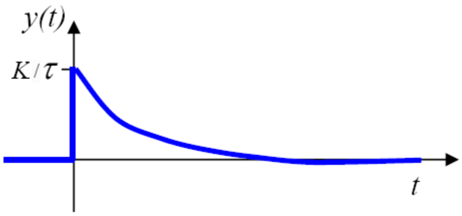


- **Impulse Input:** $U(s)=1$

$$Y(s)=\frac{K}{(\tau s+1)}$$

$$\Big\downarrow\ \mathbf{L}^{-1}$$

$$y(t)=\frac{K}{\tau}e^{-t/\tau}$$



Process dynamic behavior

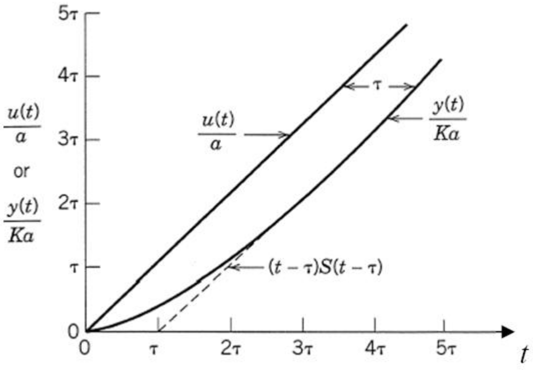
➢ Dynamic behavior of 1st –order systems:

- **Ramp input:** $U(s)=a/s^2$ $\quad u(t)=at$

$$Y(s)=\frac{Ka}{s^2(\tau s+1)}\xrightarrow{\mathbf{L}^{-1}} y(t)=Ka\tau e^{-t/\tau}+Ka(t-\tau)$$

$$\frac{y(t)}{Ka}=\tau e^{-t/\tau}+(t-\tau)$$

$$\frac{u(t)}{a}=t$$



Process dynamic behavior

➢ Dynamic behavior of 1st –order systems:

- **Sinusoidal input:** $U(s)=\mathbf{L}\left[A\sin\omega t\right]=A\omega/(s^2+\omega^2)$

$$Y(s)=\frac{K}{\tau s+1}\cdot\frac{A\omega}{s^2+\omega^2}=\frac{\alpha_0}{\tau s+1}+\frac{\alpha_1 s}{s^2+\omega^2}+\frac{\alpha_2}{s^2+\omega^2}$$

By partial fraction decomposition:

$$\alpha_0=\frac{\omega KA\tau^2}{\omega^2\tau^2+1}\quad;\quad \alpha_1=\frac{-\omega KA\tau}{\omega^2\tau^2+1}\quad;\quad \alpha_2=\frac{\omega KA}{\omega^2\tau^2+1}$$

$$Y(s)=\frac{\omega KA}{\omega^2\tau^2+1}\left[\frac{\tau^2}{\tau s+1}-\frac{\tau s}{s^2+\omega^2}+\frac{1}{s^2+\omega^2}\right]$$

$$\Big\downarrow \mathbf{L}^{-1}$$

$$y(t)=\frac{KA}{\omega^2\tau^2+1}(\omega\tau e^{-t/\tau}-\omega\tau\cos\omega t+\sin\omega t)$$

Process dynamic behavior

- **Dynamic behavior of 1st –order systems:**

Ultimate sinusoidal response $(t \to \infty)$

$$y_\infty(t) = \lim_{t\to\infty} \frac{KA}{\omega^2\tau^2+1}\left(\cancelto{0}{\omega\tau e^{-t/\tau}} - \omega\tau\cos\omega t + \sin\omega t\right)$$

$$= \frac{KA}{\omega^2\tau^2+1}(-\omega\tau\cos\omega t + \sin\omega t)$$

$$= \underbrace{\frac{KA}{\sqrt{\omega^2\tau^2+1}}}_{\textbf{Amplitude}} \sin(\omega t + \underbrace{\phi}_{\textbf{Phase angle}}) \qquad (\phi = -\tan^{-1}\omega\tau)$$

– For large t, $y(t)$ is also sinusoidal

Process dynamic behavior

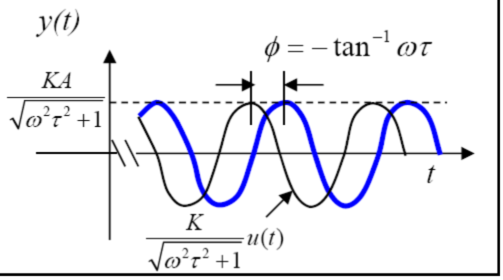
- **Dynamic behavior of 1st –order systems:**

– The output has the same period of oscillation as the input.

– But the amplitude is attenuated and the phase is shifted.

Normalized Amplitude Ratio (AR_N) $= \dfrac{1}{\sqrt{\omega^2\tau^2+1}} < 1$ **Phase angle** $= -\tan^{-1}\omega\tau$

– High frequency input will be attenuated more and phase is shifted more.



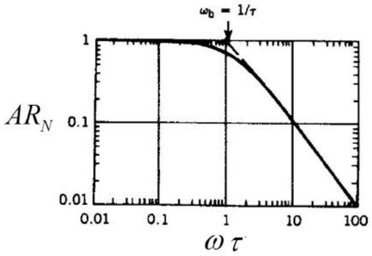
Process dynamic behavior

➢ Dynamic behavior of 1st –order systems:

- **Pode plot for 1st order system:**
  - AR plot asymptote

$$AR_N(\omega \to 0) = \lim_{\omega \to 0} \frac{1}{\sqrt{\omega^2\tau^2+1}} = 1$$

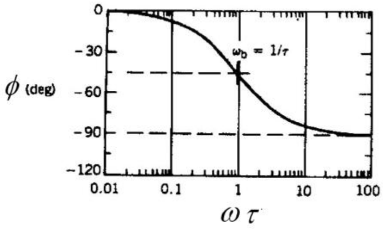
$$AR_N(\omega \to \infty) = \lim_{\omega \to \infty} \frac{1}{\sqrt{\omega^2\tau^2+1}} = 0$$



  - Phase angle plot asymptote:

$$\phi(\omega \to 0) = -\lim_{\omega \to 0} \tan^{-1}\omega\tau = 0^{\circ}$$

$$\phi(\omega \to \infty) = -\lim_{\omega \to \infty} \tan^{-1}\omega\tau = -90^{\circ}$$



Process dynamic behavior

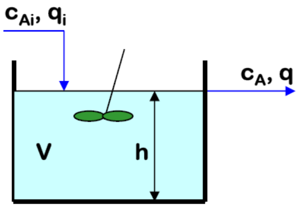
➢ Examples of 1st –order processes:

- **Continuous Stirred Tank (Isothermal):**

$$V\frac{dc_A}{dt} = qc_{Ai} - qc_A$$

$$\frac{C_A(s)}{C_{Ai}(s)} = \frac{q}{Vs+q} = \frac{1}{(V/q)s+1}$$

(c_{Ai} and c_A are deviated variables)

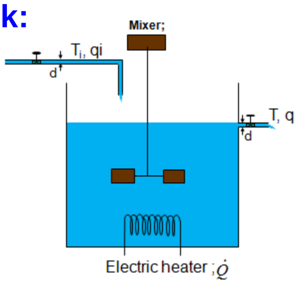


- **Non-isothermal with constant liquid volume, heat capacity and density and neglecting shaft work:**

$$\tau\frac{dT}{dt} = T_i - T + \frac{\dot{Q}}{C\rho q}$$

$$\frac{T(s)}{T_i(s)} = \frac{1}{\tau s+1}; \frac{T(s)}{\dot{Q}(s)} = \frac{1/C\rho q}{\tau s+1}$$

($T_i, T,$ and $\dot{Q}$ are deviated variables)



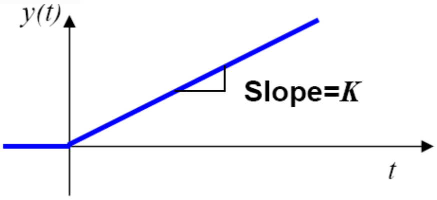
Process dynamic behavior

- **Dynamic behavior of 1st –order (integrating systems):**

- $$\frac{dy(t)}{dt} = Ku(t) \xrightarrow{\mathbf{L}} sY(s) = KU(s)$$
- **Transfer Function:** $$\frac{Y(s)}{U(s)} = \frac{K}{s}$$
- **Unit step change response:**

With $U(s) = 1/s$,

$$Y(s) = \frac{K}{s^2} \xrightarrow{\mathbf{L}^{-1}} y(t) = Kt$$



- The output is an **integration of input**.
- Impulse response is a step function output.
- Integrating system is non self-regulating system.
- Steady-state gain is not defined for integrating system.

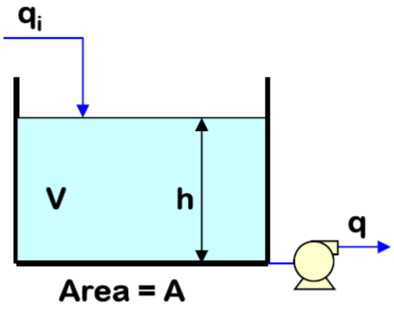
Process dynamic behavior

- **Example of 1st –order integrating processes:**
  - **Storage tank with constant outlet flow:**
    - Outlet flow is pumped out by a constant-speed, constant-volume pump.
    - Outlet flow is not a function of head.

$$A\frac{dh}{dt} = q_i - q$$

$$\frac{H(s)}{Q_i(s)} = \frac{1}{As}$$

$$\frac{H(s)}{Q(s)} = -\frac{1}{As}$$



($q_i, q,$ and h **are deviated variables)**

Process dynamic behavior

➢ Dynamic behavior of 2nd–order systems:

- Composed of one 2nd-order system: $U(s) \rightarrow \boxed{G(s)} \rightarrow Y(s)$
- Or it is composed of two 1st-order subsystems (G_1 and G_2):

$$U(s) \longrightarrow \boxed{G_1(s)} \xrightarrow{Y_1(s)} \boxed{G_2(s)} \longrightarrow Y(s)$$

- $$\tau^2 \frac{d^2y(t)}{dt^2} + 2\zeta\tau \frac{dy(t)}{dt} + y(t) = Ku(t)$$

$$\xrightarrow{\mathbf{L}} (\tau^2 s^2 + 2\zeta\tau s + 1)Y(s) = KU(s)$$

- **Transfer Function:**

$$\frac{Y(s)}{U(s)} = \frac{K}{(\tau^2 s^2 + 2\zeta\tau s + 1)}$$

Gain

Time constant

Damping Coefficient

Process dynamic behavior

➢ Dynamic behavior of 2nd–order systems:

- **Unit step response**
- **Roots of the denominator of TF:**
  - **Real part of roots should be negative for stability:** $\zeta \geq 0$
  - **Two distinct real roots (** $\zeta > 1$ **): overdamped (no oscillation)**
  - **Double root (** $\zeta = 1$ **): critically damped (no oscillation)**
  - **Complex roots (** $0 \leq \zeta < 1$ **): underdamped (oscillation)**

- **Case I** ($\zeta > 1$)

$$Y(s) = \frac{K}{s(\tau^2 s^2 + 2\zeta\tau s + 1)} = \frac{K}{s(\tau_1 s + 1)(\tau_2 s + 1)} \xrightarrow{\mathbf{L}^{-1}} \boxed{y(t) = K\left(1 - \frac{\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}}{(\tau_1 - \tau_2)}\right)}$$

$$\tau = \sqrt{\tau_1\tau_2};\ \zeta = \frac{\tau_1 + \tau_2}{2\sqrt{\tau_1\tau_2}}$$

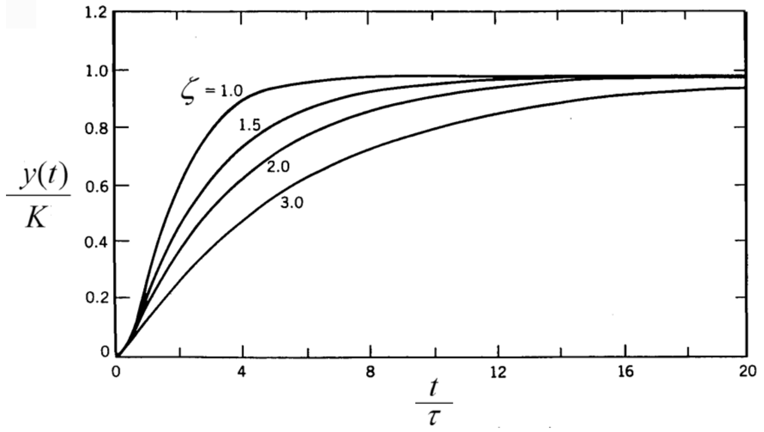
- **Case II** ($\zeta = 1$)

$$Y(s) = \frac{K}{s(\tau^2 s^2 + 2\tau s + 1)} = \frac{K}{s(\tau s + 1)^2} \xrightarrow{\mathbf{L}^{-1}} \boxed{y(t) = K\left[1 - (1 + t/\tau)e^{-t/\tau}\right]}$$

Process dynamic behavior

➢ Dynamic behavior of 2nd–order systems:

- **Unit step response:** **with** $U(s)=1/s$



“Step response of critically-damped and overdamped 2nd-order processes”

Process dynamic behavior

➢ Dynamic behavior of 2nd–order systems:

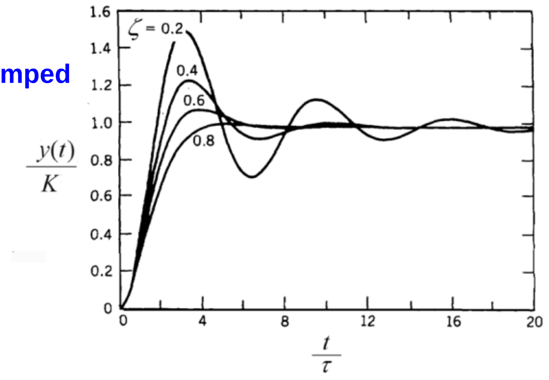
- **Step response:**

- **Case III** $(0 \le \zeta < 1)$

Natural frequency

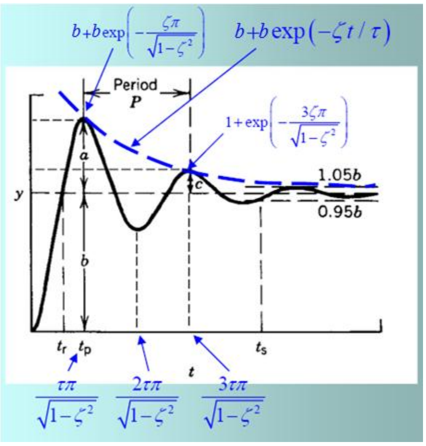
$$Y(s)=\frac{K}{s(\tau^2 s^2+2\zeta\tau s+1)} \xrightarrow{\mathbf{L}^{-1}} \boxed{y(t)=K\left[1-e^{-\zeta t/\tau}\left\{\cos\alpha t+\frac{\zeta}{\alpha\tau}\sin\alpha t\right\}\right] \quad (\alpha=\frac{\sqrt{1-\zeta^2}}{\tau})}$$

“Step response of underdamped 2nd-order processes”



Process dynamic behavior

- **Performance characteristics of the step response of underdamped process:**



- **Rise time (t_r)**

$$t_r = \tau(n\pi - \cos^{-1}\zeta)/\sqrt{1-\zeta^2} \quad (n=1)$$

- **Time to 1st peak (t_p)**

$$t_p = \tau\pi/\sqrt{1-\zeta^2}$$

- **Settling time (t_s)**

$$t_s \approx -\tau/\zeta \ln(0.05)$$

- **Overshoot (OS)**

$$OS = a/b = \exp\left(-\pi\zeta/\sqrt{1-\zeta^2}\right)$$

- **Decay ratio (DR): a function of damping coefficient only!**

$$DR = c/a = (OS)^2 = \exp\left(-2\pi\zeta/\sqrt{1-\zeta^2}\right)$$

- **Period of oscillation (P)** $P = 2\pi\tau/\sqrt{1-\zeta^2}$

Process dynamic behavior

- **1st order vs. 2nd order (overdamped)**

- **Initial slope of step response**

$$\text{1st order: } y'(0) = \lim_{s\to\infty}\left\{s^2Y(s)\right\} = \lim_{s\to\infty}\frac{KAs}{\tau s+1} = \frac{KA}{\tau} \neq 0$$

$$\text{2nd order: } y'(0) = \lim_{s\to\infty}\left\{s^2Y(s)\right\} = \lim_{s\to\infty}\frac{KAs}{\tau^2s^2+2\zeta\tau s+1} = 0$$

- **Shape of the curve (Convexity)**

$$\text{1st order: } y''(t) = -(KA/\tau^2)e^{-t/\tau} < 0 \;\; (\text{For } K>0) \;\Rightarrow\; \text{No inflection}$$

$$\text{2nd order: } y''(t) = -\frac{KA}{\tau_1-\tau_2}\left(\frac{e^{-t/\tau_1}}{\tau_1} - \frac{e^{-t/\tau_2}}{\tau_2}\right)$$

$$(+ \rightarrow - \text{ as } t\uparrow) \;\Rightarrow\; \text{Inflection}$$

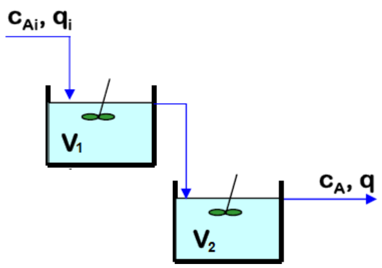
Process dynamic behavior

- **Some remarks on underdamped processes:**
  - Many examples can be found in mechanical and electrical system.
  - Among chemical processes, open-loop underdamped process is quite rare.
  - However, when the processes are controlled, the responses are usually underdamped.
  - Depending on the controller tuning, the shape of response will be decided.
  - Slight overshoot results short rise time and often more desirable.
  - Excessive overshoot may results long-lasting oscillation.

Process dynamic behavior

➢ **Examples of 2nd –order processes:**

- **Non interacting storage tanks (Constant flow rates, and constant liquid level) :**
  - The first tank affects the second tank but second tank does not affect the first tank. This is called “non-interacting”.
  - **Transfer Function:**



$$\frac{C_A(s)}{C_{Ai}(s)} = \frac{1}{((V_1/q)s+1)((V_2/q)s+1)}$$

$$= \frac{1}{(\ \tau_1 s+1\ )(\ \tau_2 s+1\ )}$$

$$= \frac{1}{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1} = \frac{1}{\tau^2 s^2+2\zeta\tau s+1}$$

$$\tau = \sqrt{\tau_1\tau_2}$$

$$\zeta = \frac{\tau_1+\tau_2}{2\sqrt{\tau_1\tau_2}}$$

Process dynamic behavior

- **Examples of 2nd –order processes:**
  - **Non interacting storage tanks with constant flow rates:**
    - Case I. $V_1 = V_2$: $\Rightarrow \tau_1 = \tau_2 = \tau \Rightarrow \zeta = \dfrac{\tau_1+\tau_2}{2\sqrt{\tau_1\tau_2}} = \dfrac{\tau+\tau}{2\sqrt{\tau\tau}} = \dfrac{2\tau}{2\tau} = 1 \Rightarrow$

      $\Downarrow$

      "Critically damped response"
    - Case II. $V_1 \neq V_2$:

      always $\zeta \leq 1 \Rightarrow$ "**Overdamped response"**
    - **Remember that (c_{Ai} and c_A) are deviated concentrations.**

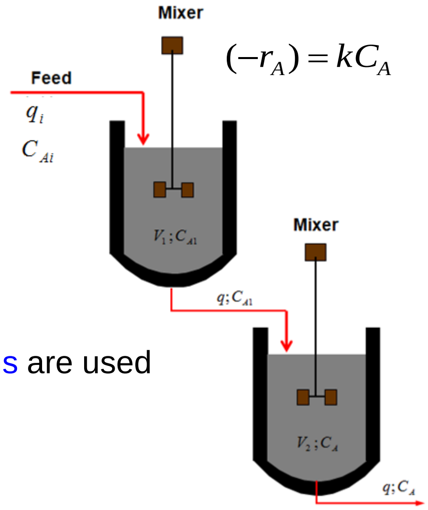
Process dynamic behavior

- **Examples of 2nd –order processes:**
  - **Non interacting two CSTRs with constant flow rates and first-order elementary reaction:**

$$\tau_1 \frac{dC_{A1}}{dt} + (1+k\tau_1)C_{A1} = C_{Ai}$$

$$\tau_2 \frac{dC_A}{dt} + (1+k\tau_2)C_A = C_{A1}$$

$$q = q_i; \tau_1 = V_1/q; \tau_2 = V_2/q;$$



$$(-r_A) = kC_A$$

Note that deviated concentrations are used in the above equations.

Process dynamic behavior

- Take L.T:
- Transfer functions:

$$\frac{C_{A1}}{C_{Ai}}=\frac{1}{\tau_1 s+(1+k\tau_1)}=\frac{1/(1+k\tau_1)}{\frac{\tau_1}{(1+k\tau_1)}s+1}=\frac{K_1}{z_1 s+1}$$

$$\frac{C_A}{C_{A1}}=\frac{1}{\tau_2 s+(1+k\tau_2)}=\frac{1/(1+k\tau_2)}{\frac{\tau_2}{(1+k\tau_2)}s+1}=\frac{K_2}{z_2 s+1}$$

$$\frac{C_A}{C_{Ai}}=\frac{K_1K_2}{(z_1s+1)(z_2s+1)}=\frac{\mathrm{K}}{(z_1s+1)(z_2s+1)}$$

$$=\frac{\mathrm{K}}{z_1z_2s^2+(z_1+z_2)s+1}=\frac{\mathrm{K}}{\tau^2s^2+2\xi\tau s+1}$$

Where $K_1=1/(1+k\tau_1);K_2=1/(1+k\tau_2);\mathrm{K}=K_1K_2$

$$z_1=\frac{\tau_1}{(1+k\tau_1)};z_2=\frac{\tau_2}{(1+k\tau_2)}$$

Process dynamic behavior

- Case I. $V_1 = V_2$:

$$\Rightarrow \tau_1=\tau_2$$

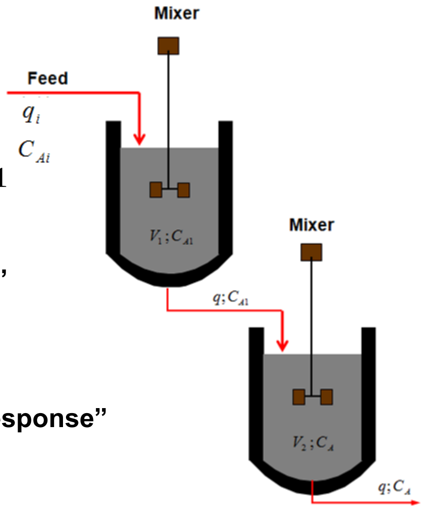
$$\Rightarrow z_1=z_2=z\Rightarrow\zeta=\frac{z_1+z_2}{2\sqrt{z_1z_2}}=\frac{z+z}{2\sqrt{zz}}=1$$

$$\Downarrow$$

“Critically damped response”

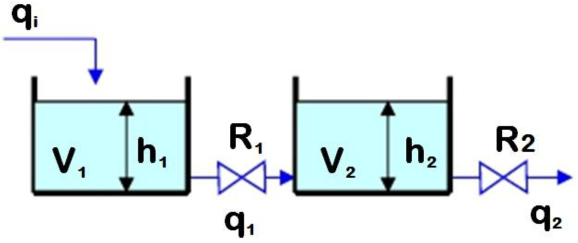
- Case II. $V_1 \neq V_2$:

always $\zeta\leq 1\Rightarrow$ **“Overdamped response”**



Process dynamic behavior

- **Interacting two storage tanks :**
  - Many chemical processes exhibit interacting nature.



- **MBs (deviated variables):** $A_1\frac{dh_1}{dt}=q_i-q_1$ $\qquad A_2\frac{dh_2}{dt}=q_1-q_2$
- **MEBs and after linearizing the resulting flow rate equations:**

$$q_1=\frac{1}{R_1}(h_1-h_2) \qquad q_2=\frac{1}{R_2}h_2$$

Where R_1 *is the resistance to flow rate* q_1 *and* R_2 *is the resistance to flow rate* q_2 *.*

Process dynamic behavior

- **Interacting two storage tanks :**

$$A_1\frac{dh_1}{dt}=q_i-\frac{1}{R_1}(h_1-h_2) \qquad A_2\frac{dh_2}{dt}=\frac{1}{R_1}(h_1-h_2)-\frac{1}{R_2}h_2$$

$$A_1R_1sH_1(s)+H_1(s)-H_2(s)=R_1Q_i(s)$$

$$\frac{A_2R_1R_2}{R_1+R_2}sH_2(s)+H_2(s)=\frac{R_2}{R_1+R_2}H_1(s)\Rightarrow\frac{H_2(s)}{H_1(s)}=\frac{R_2/(R_1+R_2)}{A_2R_1R_2/(R_1+R_2)s+1}$$

$$\frac{H_2(s)}{Q_i(s)}=\frac{R_2}{A_1A_2R_1R_2s^2+(A_1R_1+A_2R_2+A_1R_2)s+1}$$

$$\frac{H_2(s)}{Q_i(s)}=\frac{R_2}{\tau^2s^2+2\zeta\tau s+1} \quad \text{where } \tau=\sqrt{A_1A_2R_1R_2}\,,\ \zeta=\frac{(A_1R_1+A_2R_2+A_1R_2)}{2\sqrt{A_1A_2R_1R_2}}$$

$\zeta>1$ (overdamped)

Process dynamic behavior

➢Poles and Zeros:

$$G(s)=\frac{N(s)}{D(s)}=\frac{K(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + 1)}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + 1)}$$

- Poles (D(s)=0):

- Where a transfer function cannot be defined.
- Roots of the denominator of the transfer function.
- Determine modes of the response.
- Decide the stability.

- Zeros (N(s)=0)

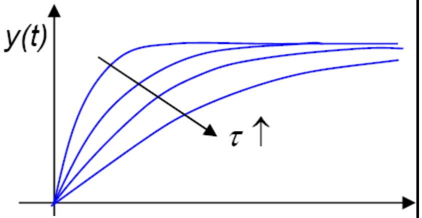
– Where a transfer function becomes zero.
– Roots of the numerator of the transfer function
– Decide weightings for each mode of response
– Decide the size of overshoot or inverse response

- Zeros and poles can be real or complex.

Process dynamic behavior

- **Effects of Poles:**

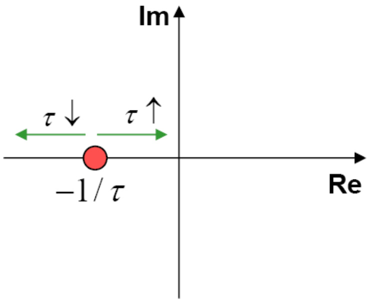
Example. Real pole from $D(s)=\tau s+1$



- One pole: $D(s)=\tau s+1=0$

$$\rightarrow s=-\frac{1}{\tau}$$

- Exponential Mode: $e^{-t/\tau}$



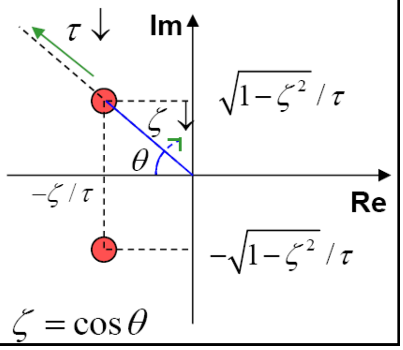
- If the pole is at the origin, it becomes “integrating pole”.
- Unstable response if the pole is in RHP, i.e. the response increases exponentially.
- Stable response if the pole is in LHP.

Process dynamic behavior

- **Effects of Poles:**

Example. Complex poles: $D(s)=(\tau^2 s^2+2\zeta\tau s+1)\quad(-1<\zeta<1)$

- Two poles: $D(s)=0 \rightarrow s=-\frac{\zeta}{\tau}\pm j\frac{\sqrt{1-\zeta^2}}{\tau}=-\alpha\pm j\beta$
- $|s|=\sqrt{\frac{\zeta^2+1-\zeta^2}{\tau^2}}=\frac{1}{\tau}$ (function of τ only)
- $\measuredangle s=\pm\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ (function of ζ only)



Process dynamic behavior

- **Effects of Poles:**

- Modes: $e^{-\alpha t\pm j\beta t}=e^{-\alpha t}(\cos\beta t\pm j\sin\beta t)$

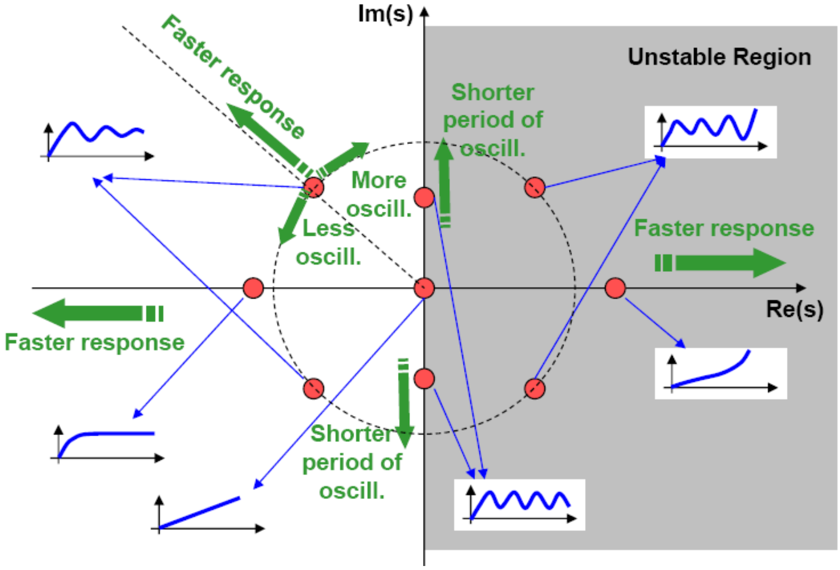
$$=e^{-\zeta t/\tau}(\cos\frac{\sqrt{1-\zeta^2}}{\tau}t\pm j\sin\frac{\sqrt{1-\zeta^2}}{\tau}t)$$

→ **For positive τ:**

○ If $\zeta<0$, the exponential part will grow as t increases: unstable
○ If $\zeta>0$, the exponential part will shrink as t increases: stable
○ If $\zeta=0$, the roots are pure imaginary: sustained oscillation.

Process dynamic behavior

- **Poles Locations:**



Process dynamic behavior

- **Effects of Zeros:**

$$G(s)=\frac{N(s)}{(s+p_1)\cdots(s+p_n)}=w_1\frac{1}{(s+p_1)}+\cdots+w_n\frac{1}{(s+p_n)}$$

- It is clear that the numerator (zeros) will change the weighting factors($w_1,\ldots w_n$).
- The effects on weighting factors are not always obvious.

Example. Lead-Lag module: $G(s)=\frac{N(s)}{D(s)}=\frac{K(\tau_a s+1)\xrightarrow{}\textbf{Lead}}{(\tau_1 s+1)\xrightarrow{}\textbf{Lag}}$

- For M step change input:

$$Y(s)=\frac{KM(\tau_a s+1)}{s(\tau_1 s+1)}=KM\left\{\frac{1}{s}+\frac{\tau_a-\tau_1}{\tau_1 s+1}\right\} \qquad y(t)=KM\left[1-\left(1-\frac{\tau_a}{\tau_1}\right)e^{-t/\tau_1}\right]$$

Process dynamic behavior

- **Effects of Zeros**

  - If: **(a)** $\tau_a > \tau_1 > 0$

    The lead dominates the lag.

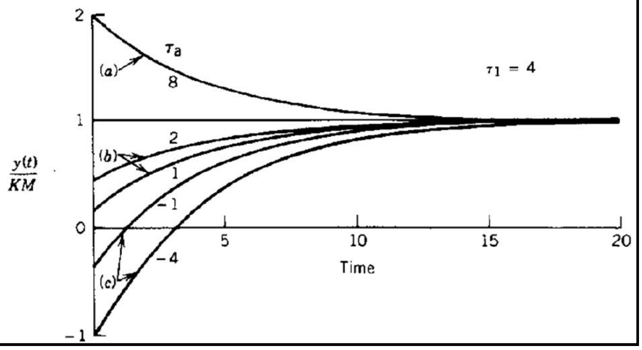
    (b) $0 \le \tau_a < \tau_1$

    The lag dominates the lead.

    (c) $0 > \tau_a$

    Inverse response

$$y(t) = KM\left[1-\left(1-\frac{\tau_a}{\tau_1}\right)e^{-t/\tau_1}\right]$$



Process dynamic behavior

- **Effects of Zeros**

Example. Overdamped 2nd-order+single zero system**:**

$$G(s) = \frac{N(s)}{D(s)} = \frac{K(\tau_a s+1)}{(\tau_1 s+1)(\tau_2 s+1)} \qquad (\text{assume } \tau_1 > \tau_2)$$

- For M step change input:

$$Y(s) = \frac{KM(\tau_a s+1)}{s(\tau_1 s+1)(\tau_2 s+1)} = KM\left\{\frac{1}{s}+\frac{\tau_1(\tau_a-\tau_1)}{\tau_1-\tau_2}\frac{1}{\tau_1 s+1}+\frac{\tau_2(\tau_a-\tau_2)}{\tau_2-\tau_1}\frac{1}{\tau_2 s+1}\right\}$$

$$y(t) = KM\left[1+\frac{\tau_a-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1}+\frac{\tau_a-\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}\right]$$

Process dynamic behavior

- **Effects of Zeros**

$$y(t) = KM\left[1 + \frac{\tau_a - \tau_1}{\tau_1 - \tau_2} e^{-t/\tau_1} + \frac{\tau_a - \tau_2}{\tau_2 - \tau_1} e^{-t/\tau_2}\right]$$

  - If: **(a)** $\tau_a > \tau_1 > 0$

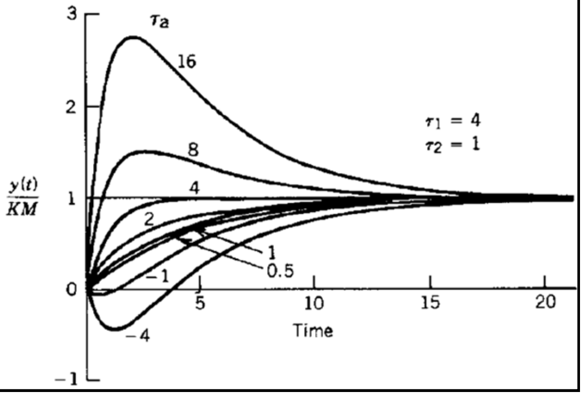
    The lead dominates the lags.

    (b) $0 < \tau_a \leq \tau_1$

    The lags dominate the lead.

    (c) $0 > \tau_a$

    Inverse response



Process dynamic behavior

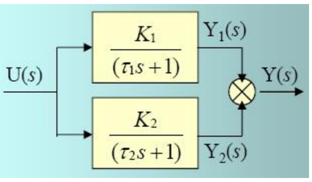
- **Effects of Zeros:**

$$G(s) = \frac{K(\tau_a s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} = \frac{K_1}{(\tau_1 s + 1)} + \frac{K_2}{(\tau_2 s + 1)}$$

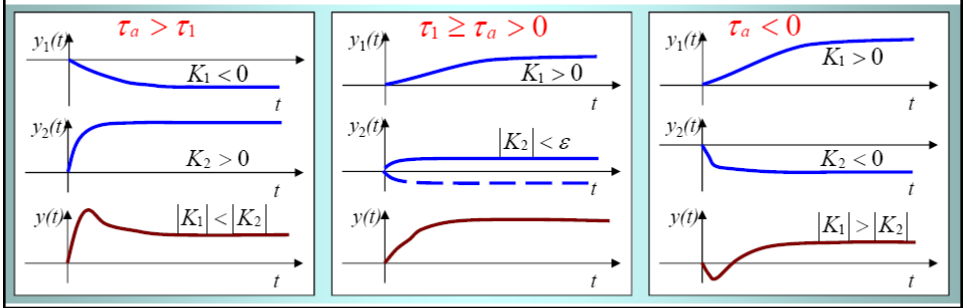
  - Another Analysis:

$$K_1 = \left.\frac{K(\tau_a s + 1)}{(\tau_2 s + 1)}\right|_{s=-1/\tau_1} = \frac{K(\tau_1 - \tau_a)}{(\tau_1 - \tau_2)}$$

$$K_2 = \left.\frac{K(\tau_a s + 1)}{(\tau_1 s + 1)}\right|_{s=-1/\tau_2} = \frac{K(\tau_a - \tau_2)}{(\tau_1 - \tau_2)}$$



- Since $\tau_1 > \tau_2$, 1 is slow dynamics and 2 is fast dynamics.



Process dynamic behavior

- **Zeros Locations:**