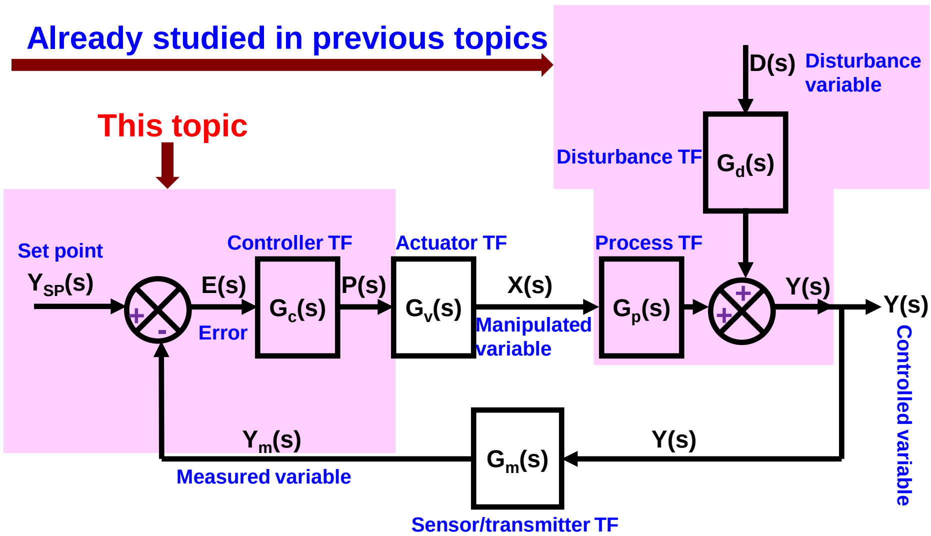
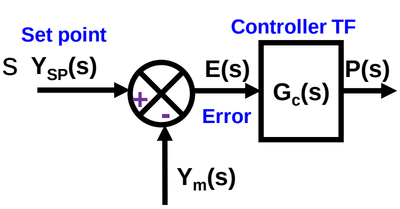
Process Controllers



"Standard block diagram of feedback controller with one disturbance"

Process Controllers



- Originally input and output signals are time varying:
  - **Controlled variable set point:**
    - **Servo control:**
      - Set point variable: $y_{SP}(t)$
      - Set point deviated variable: $Y_{SP}(t) = y_{SP}(t) - \bar{y}_{SP}$

        $L(Y_{SP}(t)) = Y_{SP}(s)$
    - **Regulatory control**:
      - Constant set point: $y_{SP}(t) = \bar{y}_{SP}$
      - Set point deviated variable: $Y_{SP}(t) = \bar{y}_{SP} - \bar{y}_{SP} = 0 \xrightarrow{L} Y_{SP}(s) = 0$

Note that:
Small letter : actual variable
Capital letter: deviated variable

Process Controllers

- Measured controlled variable: $y_m(t)$
  - Measured controlled variable (deviated value):

$$Y_m(t) = y_m(t) - \bar{y}_{SP}$$

$$L(Y_m(t)) = Y_m(s)$$

- Controller output variable: $p(t)$
  - Controller output variable (deviated value):

$$P(t) = p(t) - \bar{p}$$

$$L(P(t)) = P(s)$$

Where $\bar{p}$ is called Bias or nominal value of controller output signal which is usually adjusted, during calibration, to be half of the span of the range of the controller signal output.

Process Controllers

- **Definition of Span and Zero:**
  - Span: magnitude of the range of the output signal.
  - Zero: lower limit of the output signal

- According to Instrumental Society of America **(ISA)**, Controller output signal has the following standard ranges:

➢ Pneumatic signal: 3 – 15 psi → Zero = 3 psi ; Span=12 psi
➢ Electrical Signal: 4 - 20 mA → Zero = 4 mA ; Span=16 mA
➢ Electrical Signal: 0–10 VDC →Zero = 0 VDC ; Span=10 VDC

- Error in controlled variable: $e(t) = y_{sp}(t) - y_m(t)$ Watch units!

(deviated error):

$$e(t) = y_{sp}(t) - y_m(t) = y_{sp}(t) - \bar{y}_{sp} + \bar{y}_{sp} - y_m(t) = Y_{sp}(t) - Y_m(t) = E(t)$$

$$\xrightarrow{L} E(s) = Y_{sp}(s) - Y_m(s)$$

Process Controllers

- Controller is the "brain" of the control loop. It decides "what to do" based on the error between the desired set point value (SP) of controlled variable and its measured value.

- **Basic types of controllers:**

  - **On-Off controller**.

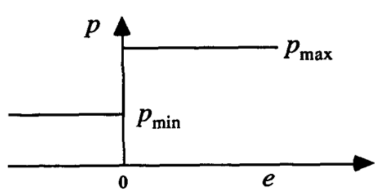
  - **Different modes of proportional (P), Integral (I) and derivative (D) Controllers**:
    - Proportional (P) controller
    - Proportional Integral (PI) controller
    - Proportional Derivative (PD) controller
    - Ideal Proportional Integral Derivative (PID) controller
    - Actual (PID) controller with filter.

Process Controllers

➢On-Off controllers:

- Synonyms: "two-position" or "bang-bang" controllers.
- Controller output has two possible values.
- Ideal case:

$$p(t) = \begin{cases} p_{max} & if \quad e > 0 \\ p_{min} & if \quad e < 0 \end{cases}$$

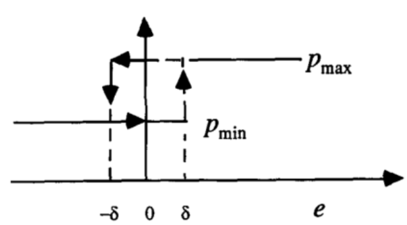


p_{max} is the "on" value
p_{min} is the "off" value.

- Practical case (dead band):

$$p(t) = \begin{cases} P_{max} & for \quad e > \delta \\ P_{min} & for \quad e < -\delta \end{cases}$$

δ = tolerance



Process Controllers

➢On-Off controllers:

- On-off Controllers is simple and cheap.
- Limited use in process control due to continuous cycling of controlled variable $\Rightarrow$ excessive wear on control valve.
- **Examples:**
  - Batch process control.
  - Residential heating and domestic refrigerators.
  - Solenoid in home heating unit
  - Sprinkler systems.

Process Controllers

➢Three mode PID controller:

1. Proportional (P) controller:

- In P controller, the **controller output signal** $p(t)$ varies linearly with error $e(t)$:

$$\Rightarrow p(t) = \bar{p} + K_c e(t)$$

Where K_c is called controller proportional gain.

- When error(offset) has zero value, the controller output signal reaches its steady state Bias value($\bar{p}$).

- **Action of controller**: as absolute error increases , the controller output signal must increase to give large change in manipulated variable, X(t).

Process Controllers

1. Proportional (P) controller:

- **Reverse or Direct action controller**:

$$\Rightarrow p(t) = \bar{p} + K_c e(t) = \bar{p} + K_c\left[y_{sp}(t) - y_m(t)\right]$$

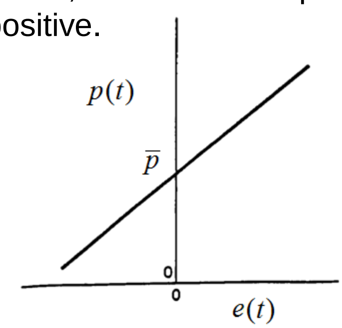
- **Direct acting controller (K_c <0):** choose **negative *K_c* *value*** *to* increase the controller output signal as the measured controlled variables increases above the set point.

- **Reverse acting controller (K_c>0):** choose **positive *K_c* *value*** *to* increase the controller output signal as the measured controlled variables decreases below the set point.

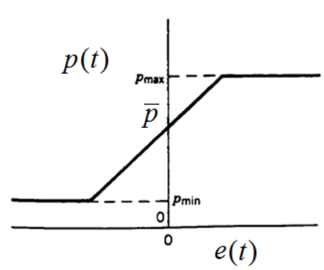
Process Controllers

1. Proportional (P) controller:

- Therefore, regardless of possible negative or positive errors, controller output signal, p(t) should be always positive.



“ideal behavior”



“actual behavior”

- As error changes, p(t) will change immediately (fast corrective action in very simple form).

Process Controllers

1. Proportional (P) controller:

- **Proportional band (PB):** $PB \equiv \frac{100}{K_c} \quad (\%)$

- Proportional band is defined only for dimensionless K_c.
- As will be shown later, the final error "offset" can be reduced by increasing the absolute value of K_c (reducing PB%).
- Introducing very high values of K_c will lead to oscillatory response or even unstable situation.

- Proportional Controller **Transfer Function** :

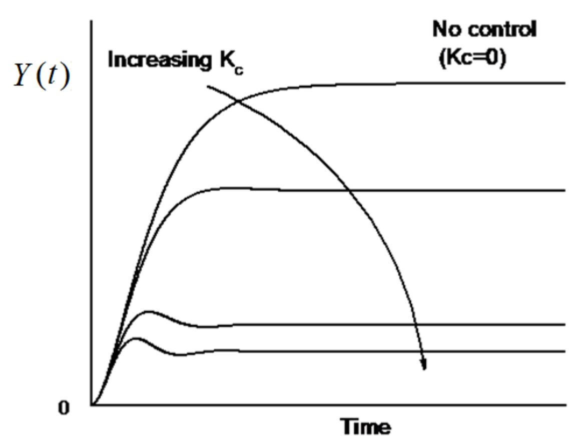
$$p(t) - \bar{p} = K_c e(t) \Rightarrow P(t) = K_c E(t) \xrightarrow{L} P(s) = K_c E(s)$$

$$\Rightarrow G_c(s) = \frac{P(s)}{E(s)} = K_c$$

Process Controllers

1. Proportional (P) controller:

- Effect of K_c on the controlled variable response under step change in disturbance variable:



Process Controllers

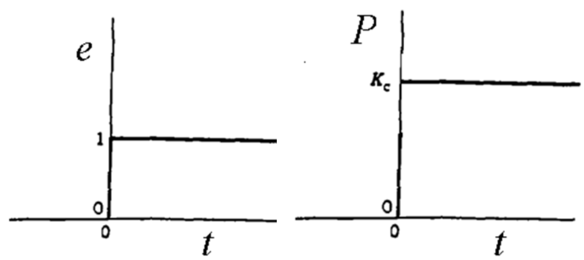
1. Proportional (P) controller:

- **Example.** How is the response of proportional controller to unit step change in e(t)?

$$e(t) = 1; E(s) = 1/s$$

$$\Rightarrow P(S) = G_c(s)E(s) = \frac{K_c}{s}$$

$$\xrightarrow{L^{-1}} \quad P(t) = K_c$$



- **Exercise.** How is the P controller response to ramp change in e(t)?

Process Controllers

2. Proportional Integral (PI) controller:

Synonyms: "reset", "floating control"

- **PI Controller signal output**:

$$p(t) = \overline{p} + \frac{K_c}{\tau_I}\int_0^t e(t^*)dt^* + K_c e(t)$$

$\tau_I \equiv$ reset time or integral time (adjustable parameter)

- The integral mode will change the bias value to eliminate the steady state error(offset).
- The action is not immediate until the integral becomes significant.
- The integral mode tends the system to be more oscillatory or even unstable.

Process Controllers

2. Proportional integral (PI) controller:

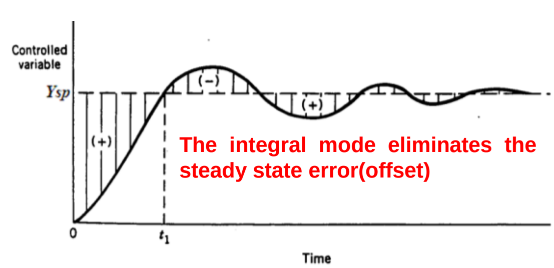
- **Reset rate:** $\tau_R = 1/\tau_I$
- Infinite integral time or zero reset rate → P controller
- **Advantages (Combined benefits)**
  – Fast action
  – Eliminate the offset
- **Disadvantages**
  – Oscillatory or unstable with integral control
  – Two parameters to tune (K_c and τ_I)

Process Controllers

2. Proportional integral (PI) controller:

➢ **Reset Time:**

- The bias value will be reset every τ_I by the amount of action taken by the P control. This is called “reset time”



Process Controllers

2. Proportional integral (PI) controller:

- **PI controller Transfer Function** :

$$p(t)-\bar{p}=\frac{K_c}{\tau_I}\int_0^t e(t^*)dt^* + K_c e(t)$$

$$\Rightarrow P(t)=\frac{K_c}{\tau_I}\int_0^t E(t^*)dt^* + K_c E(t)$$

$$\xrightarrow{L} P(s)=\frac{K_c}{\tau_I}\frac{E(s)}{s}+K_c E(s)=\left(\frac{K_c}{\tau_I s}+K_c\right)E(s)$$

$$\Rightarrow G_c(s)=\frac{P(s)}{E(s)}=\frac{K_c}{\tau_I s}+K_c=K_c\frac{\tau_I s+1}{\tau_I s}$$

Process Controllers

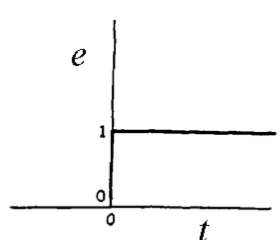
2. Proportional integral (PI) controller:

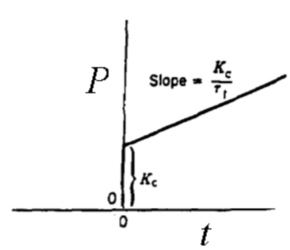
- **Example.** How is the response of PI controller to unit step change in e(t)?

$e(t)=1; E(s)=1/s$

$$\Rightarrow P(s)=G_c(s)E(s)=\frac{K_c}{s}\left(1+\frac{1}{\tau_I s}\right)=K_c\left(\frac{1}{s}+\frac{1}{\tau_I s^2}\right)$$

$$\xrightarrow{L^{-1}} P(t)=K_c\left(1+\frac{t}{\tau_I}\right)$$





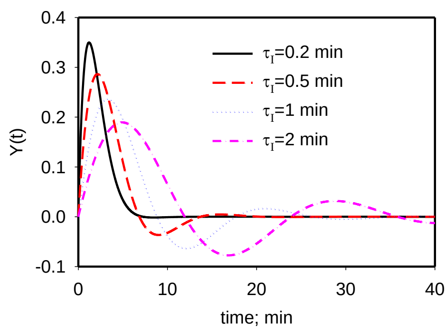
- **integral action** $\equiv K_c/\tau_I$

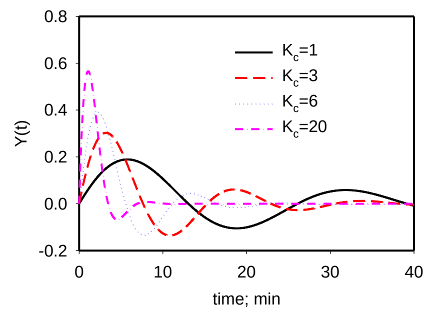
- **Exercise.** How is the PI controller response to ramp change in e(t)?

Process Controllers

2. Proportional integral (PI) controller:

- Effect of integral time τ_I and gain K_c of PI controller on the response of controlled deviated variable Y(t):





- As K_c increases or τ_I decreases : less offset, the response will be faster, more overshooting, less oscillation.

Process Controllers

3. Proportional Derivative (PD) controller:

- **PD Controller signal output**:

$$p(t) = \overline{p} + K_c \tau_D \frac{de(t)}{dt} + K_c e(t)$$

$\tau_D \equiv$ derivative or preact time (adjustable parameter)

- Zero derivative time $\rightarrow$ P controller
- Derivative mode is used to improve dynamic response of the controlled variable.
- Derivative mode it does NOT eliminate the offset.
- Two parameters to tune (K_c and τ_D)

Process Controllers

3. Proportional Derivative (PD) controller:

- PD controller **Transfer Function** :

$$p(t)-\bar{p}=K_c\tau_D\frac{de(t)}{dt}+K_c e(t)\Rightarrow P(t)=K_c\tau_D\frac{dE(t)}{dt}+K_c E(t)$$

$$\xrightarrow{L} P(s)=K_c\tau_D sE(s)+K_c E(s)\Rightarrow \boxed{G_c(s)=\frac{P(s)}{E(s)}=K_c\tau_D s+K_c}$$

- **Example.** How is the response of PD controller to unit step change in e(t)?

  $e(t)=1\Rightarrow\frac{de(t)}{dt}=0\rightarrow$ (Same as P controller)

- **Exercise.** How is the PD controller response to ramp change in e(t)?

Process Controllers

4. Proportional Integral Derivative (PID) controller:

- **PID controller signal output:**

$$p(t)=\bar{p}+\frac{K_c}{\tau_I}\int_0^t e(t^*)dt^*+K_c e(t)+K_c\tau_D\frac{de(t)}{dt}$$

- Infinite integral time and zero derivative time $\rightarrow$ P controller
- zero derivative time $\rightarrow$ PI controller
- Infinite integral time $\rightarrow$ PD controller

- **Advantages (Combined benefits)**
  – Fast action
  – Integral mode will eliminate the offset.
  – Derivative mode will make the process output to land on set point smoothly.
  – D mode tends to reduce the oscillation and enhance the stability.

Process Controllers

4. Proportional Integral Derivative (PID) controller:

- **Disadvantages**
  - If there is noise in the process variable, noise will be amplified by the derivative. If the measurement is noisy, use the measurement after smoothing out (filtering).
  - The derivative requires information on error in the future:

    impossible → use approximation of derivative
  - Three parameters to tune (K_c, τ_I, and τ_D). It is quite complicated for three tuning parameters.

- PID controller **Transfer Function** :

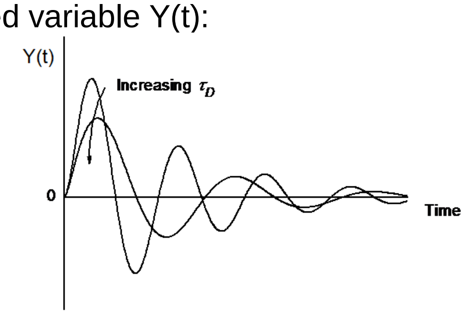
$$G_c(s) = \frac{P(s)}{E(s)} = K_c + \frac{K_c}{\tau_I s} + K_c \tau_D s = K_c \frac{\tau_I \tau_D s^2 + \tau_I s + 1}{\tau_I s}$$

Process Controllers

4. Proportional Integral Derivative (PID) controller:

- Effect of derivative time τ_D of PID controller on the response of controlled deviated variable Y(t):

• As τ_D increases: the response will be slower, less oscillatory, less overshooting (when there is no noise).



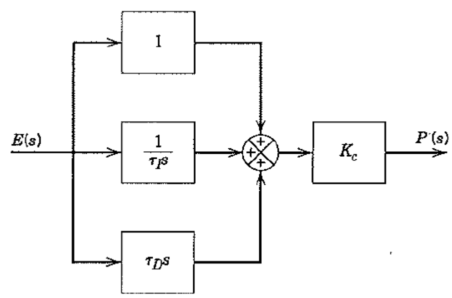
- **Example.** How is the response of PID controller to unit step change in e(t):

$$e(t) = 1 \Rightarrow \frac{de(t)}{dt} = 0 \rightarrow \text{(Same as PI controller)}$$

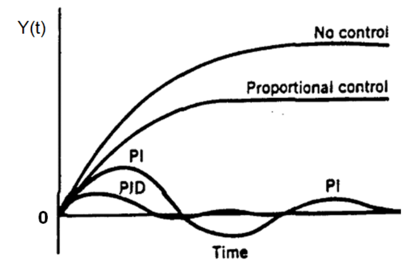
- **Exercise.** How is the PID controller response to ramp change in e(t)?

Process Controllers

4. Proportional Integral Derivative (PID) controller:



"Block diagram of ideal PID controller"



"Response of controlled deviated variable Y(t) using different controllers"

- This PID controller is ideal noninteracting. In many cases it is **physically unrealizable.**
- Integral time must be equal or greater than the derivative time: $\tau_I \geq \tau_D$. Typically $\tau_I \approx 4\tau_D$.

Process Controllers

5. Actual Interacting PID controller:

- **Transfer function**:

$$G_c^*(s) = K_c^* \frac{(\tau_I^* s+1)}{\tau_I^* s} \frac{(\tau_D^* s+1)}{(\alpha \tau_D^* s+1)} \quad (0 < \alpha \ll 1)$$

Filtering effect

Where α is derivative filter parameter

- This controller is **physically realizable**.

Process Controllers

➢ Comparison between ideal and actual PID without filter:

- Actual PID without filter ($\alpha = 0$):

$$G_c^*(s) = K_c^* \frac{(\tau_D^* \tau_I^* s^2 + (\tau_I^* + \tau_D^*)s + 1)}{\tau_I^* s}$$

$$= \frac{K_c^*(\tau_I^* + \tau_D^*)}{\tau_I^*}\left(1 + \frac{1}{(\tau_I^* + \tau_D^*)}\frac{1}{s} + \frac{\tau_D^* \tau_I^*}{(\tau_I^* + \tau_D^*)} s\right)$$

- Ideal PID controller: $G_c(s) = K_c(1 + \frac{1}{\tau_I s} + \tau_D s)$

$\rightarrow$ Compare:

$$K_c = \frac{K_c^*(\tau_I^* + \tau_D^*)}{\tau_I^*}, \quad \tau_I = \tau_I^* + \tau_D^*, \quad \tau_D = \frac{\tau_D^* \tau_I^*}{(\tau_I^* + \tau_D^*)}$$

- In this form, $\tau_I \geq \tau_D$,is satisfied automatically.

Process Controllers

➢ Key characteristics of commercial PID controllers:

Controller Feature	Controller Parameter	Symbol	Units	Typical Range*
Proportional mode	*Controller gain*	K_c	Dimensionless [%/%, mA/mA]	0.1–100
	Proportional band	$PB = 100\%/K_c$	%	1–1000%
Integral mode	*Integral time* (or *reset time*)	τ_I	Time [min, s]	0.02–20 min 1–1000 s
	Reset rate	$1/\tau_I$	Repeats/time [min^{-1}, s^{-1}]	0.001–1 repeats/s 0.06–60 repeats/min
	Integral mode "gain"	K_I	Time^{-1} [min^{-1}, s^{-1}]	0.1–100
Derivative mode	*Derivative time*	τ_D	Time [min, s]	0.1–10 min. 5–500 s
	Derivative mode "gain"	K_D	Time [min, s]	0.1–100
	Derivative filter parameter	α	Dimensionless	0.05–0.2
Control interval (Digital controllers)		Δt	Time [s, min]	0.1 s–10 min

Process Controllers

- **Important remark:**
  - Basically, the controller gain, K_c, is dimensional quantity. Its physical unit is the ratio of the unit of controller output signal (mA, psig) to the unit of the controlled variable(^oC, m, Pa, gpm, mol/L, …etc).
  - K_c can be dimensionless quantity if the set point and measured controlled variable are transmitting to a signal as that of controller output: