



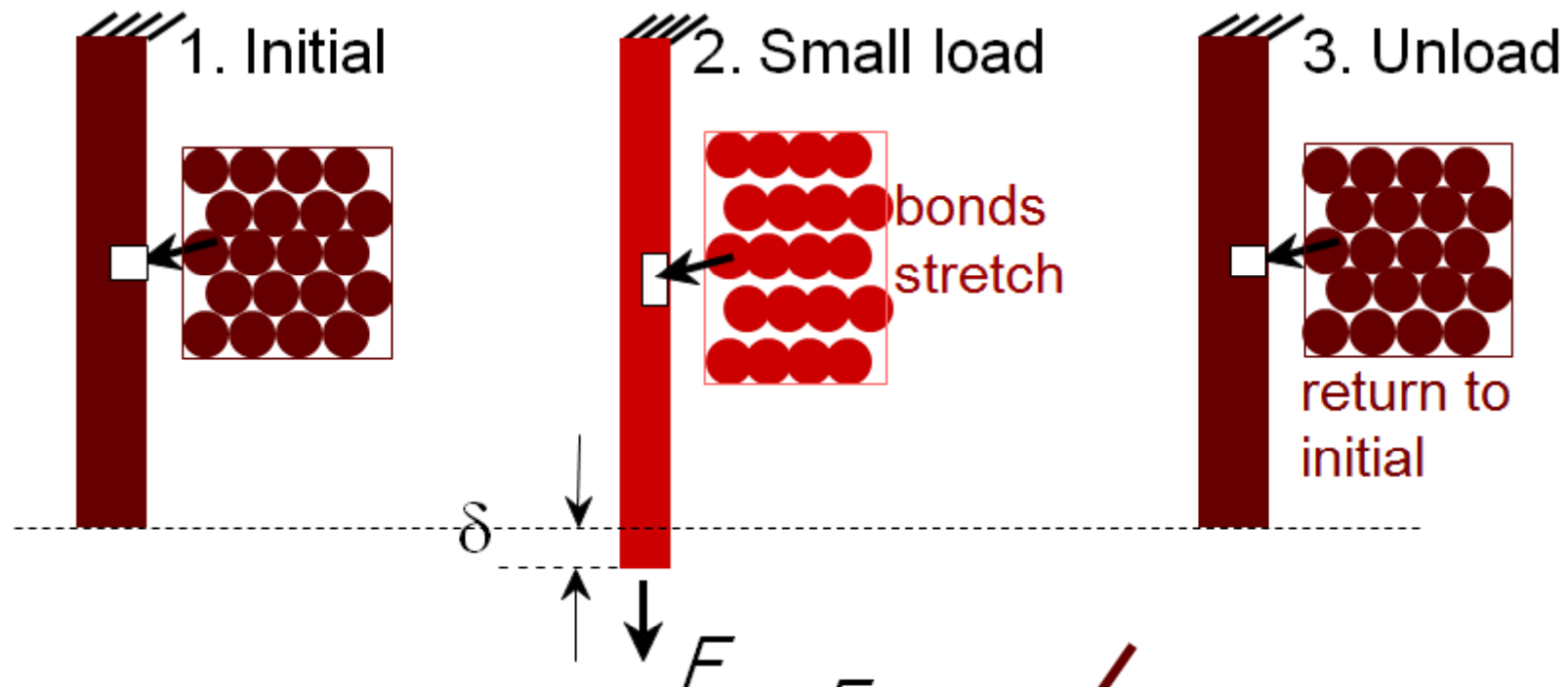
Chapter 5

Mechanical Properties

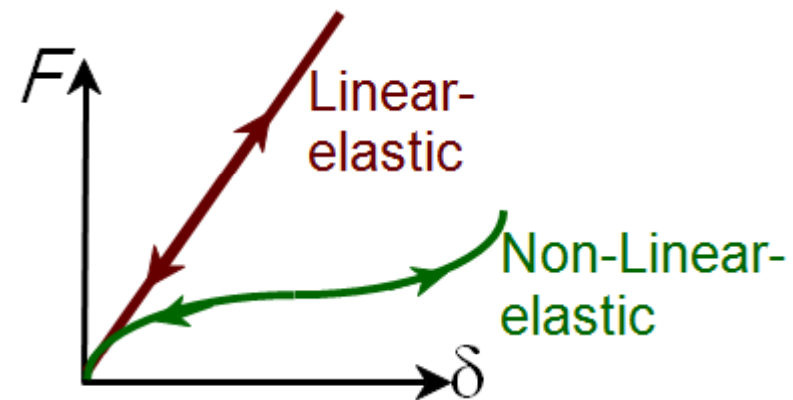
Outline

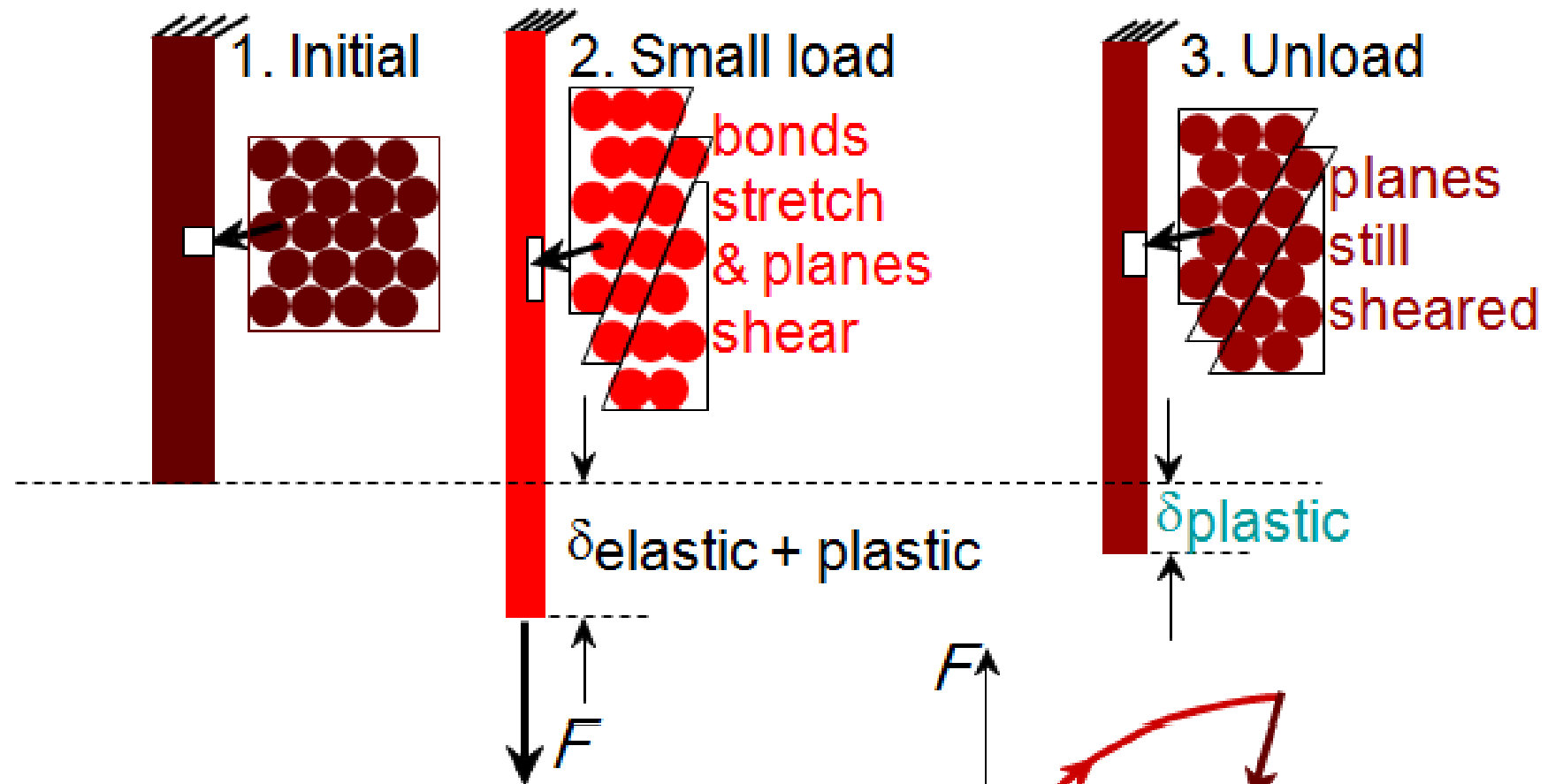
- **Stress and strain:** What are they and why are they used instead of load and deformation?
- **Elastic behavior:** When loads are small, how much deformation occurs? What materials deform least?
- **Plastic behavior:** At what point does permanent deformation occur? What materials are most resistant to permanent deformation?
- **Toughness and ductility:** What are they and how do we measure them?

Elastic Deformation

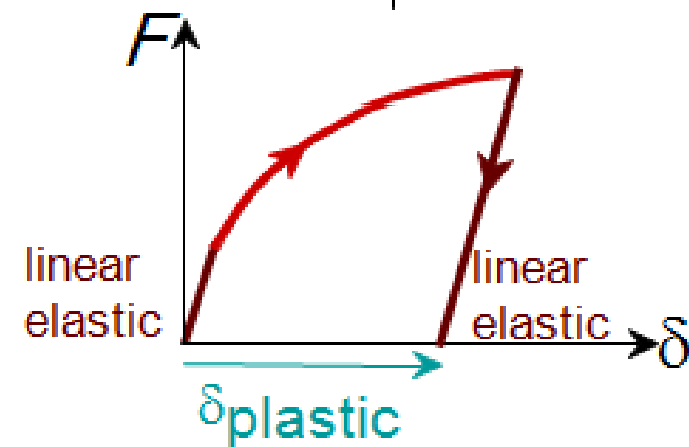


Elastic means **reversible**!



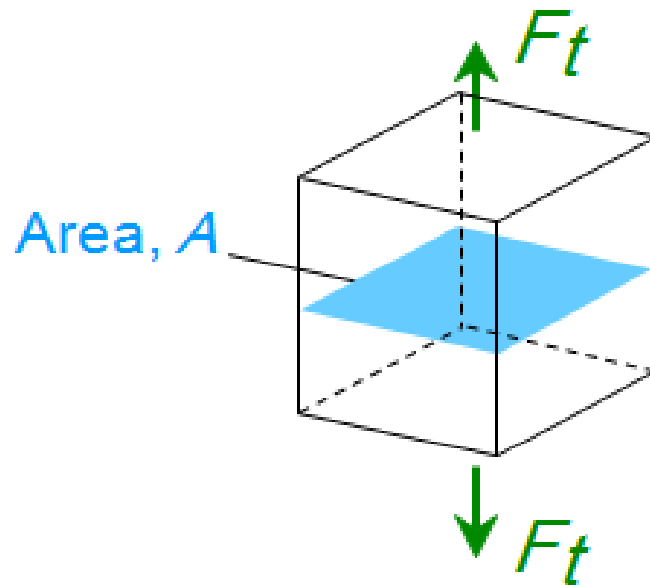


Plastic means **permanent!**



Engineering Stress

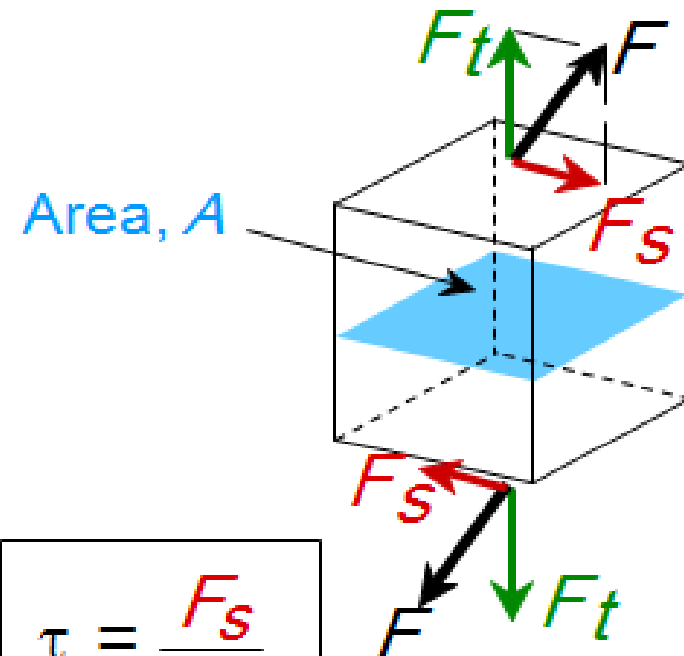
- Tensile stress, σ :



$$\sigma = \frac{F_t}{A_o} = \frac{\text{lb}_f}{\text{in}^2} \text{ or } \frac{\text{N}}{\text{m}^2}$$

original area
before loading

- Shear stress, τ :



$$\tau = \frac{F_s}{A_o}$$

∴ Stress has units:
N/m² or lb_f/in²

Common States of Stress

- **Simple tension:** cable

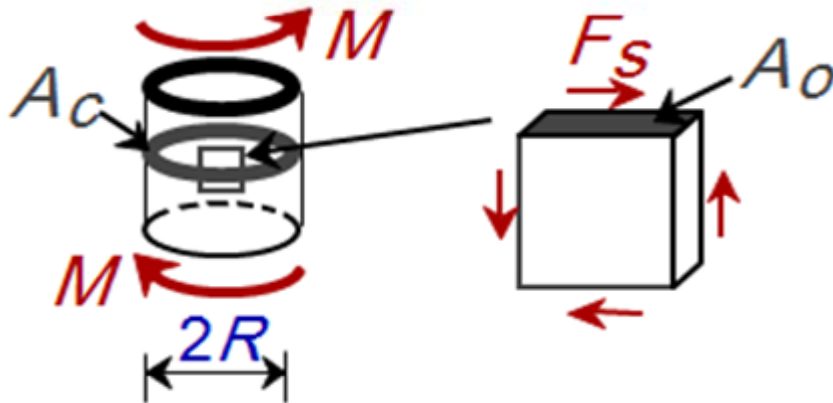


A_o = cross sectional area (when unloaded)

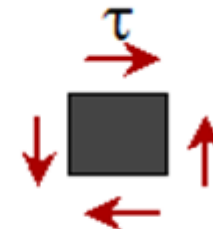
$$\sigma = \frac{F}{A_o}$$



- **Torsion** (a form of shear): drive shaft



$$\tau = \frac{F_s}{A_o}$$



Note: $\tau = M/A_c R$ here.

Other Common Stress States (1)

- **Simple** compression:



Balanced Rock, Arches National Park



Canyon Bridge, Los Alamos, NM

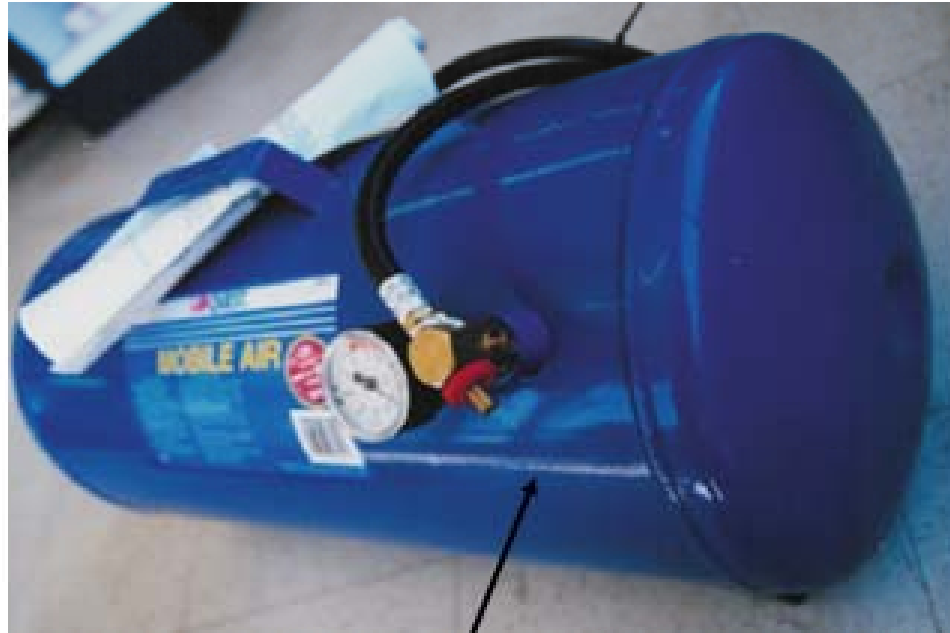
$$\sigma = \frac{F}{A_o}$$



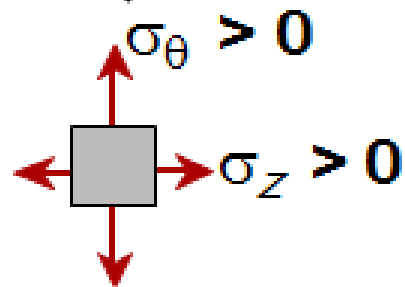
Note: compressive structure member ($\sigma < 0$ here).

Other Common Stress States (2)

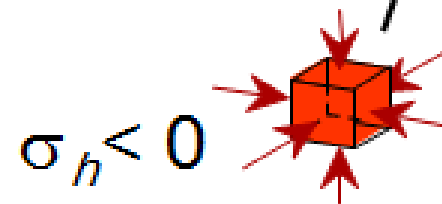
- **Bi-axial tension:**
- **Hydrostatic compression:**



Pressurized tank



Fish under water



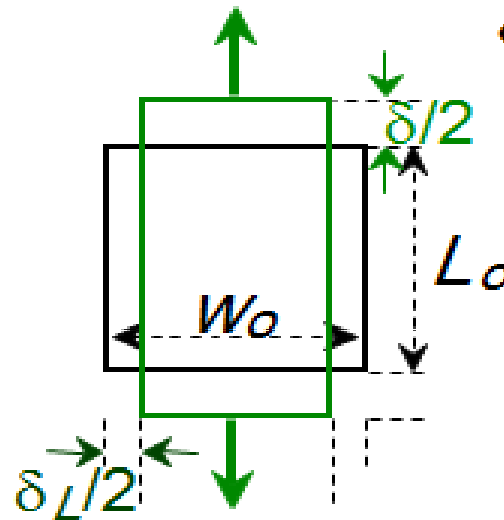
Engineering Strain

- **Tensile strain:**

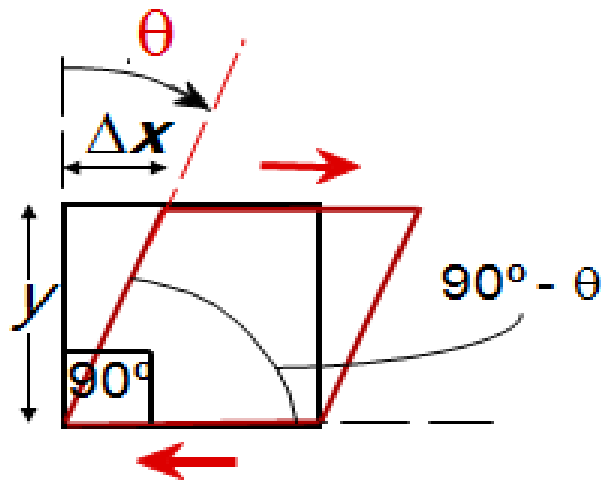
$$\varepsilon = \frac{\delta}{L_o}$$

- **Lateral strain:**

$$\varepsilon_L = \frac{-\delta_L}{W_o}$$



- **Shear strain:**

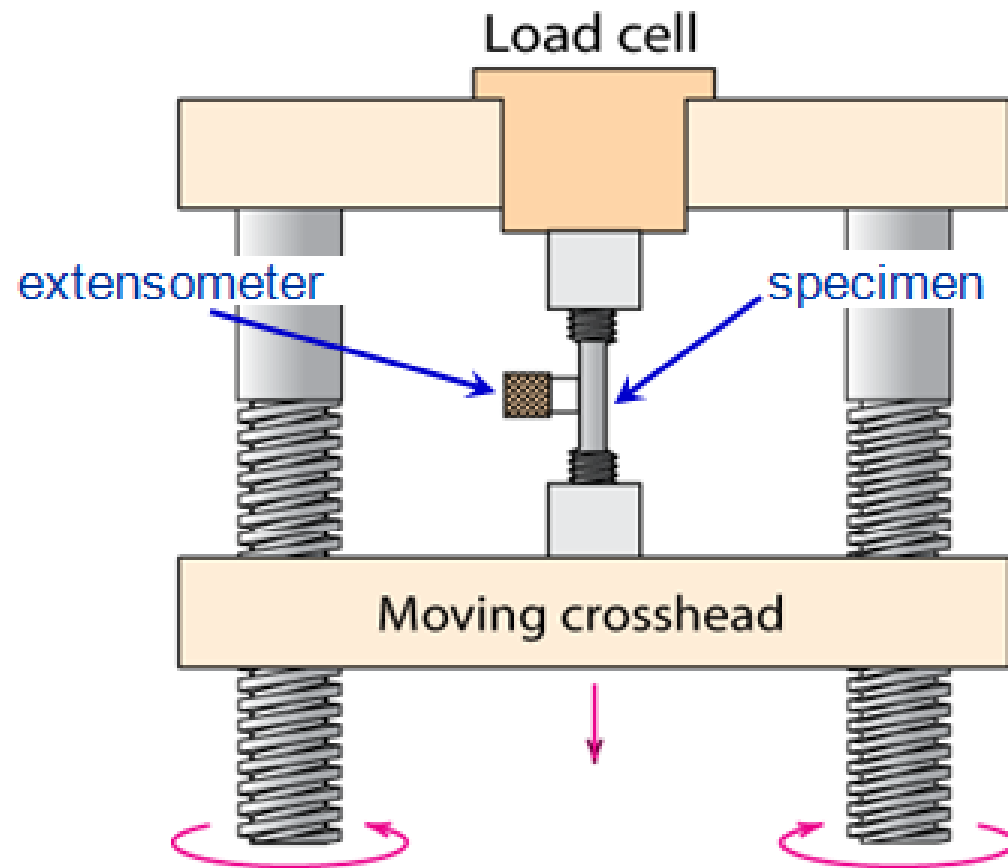


$$\gamma = \Delta x / y = \tan \theta$$

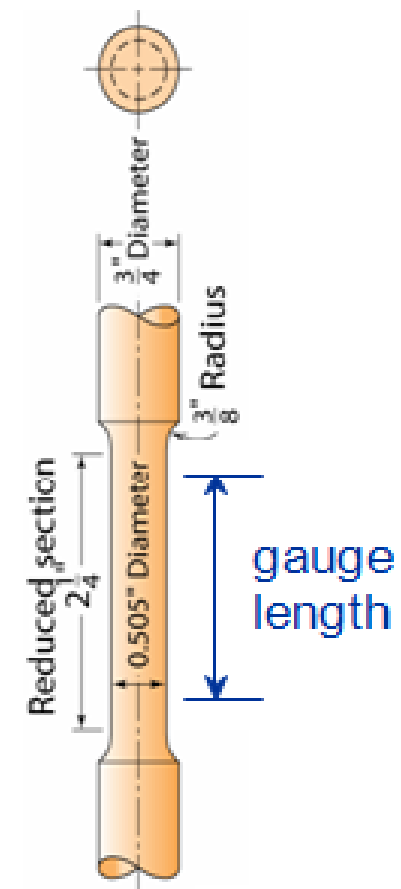
Strain is always dimensionless.

Stress-Strain Testing

- **Typical tensile test machine**



- **Typical tensile specimen**

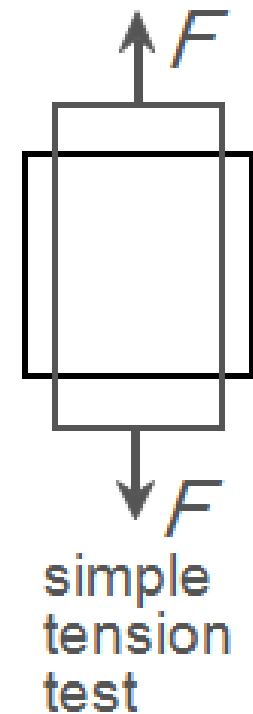
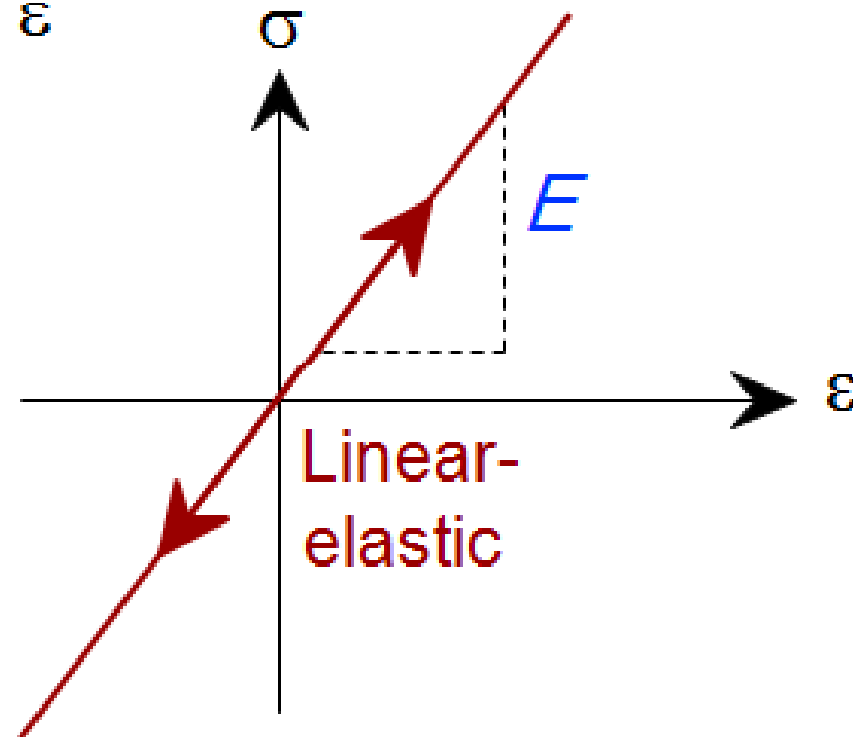


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Linear Elastic Properties

- **Modulus of Elasticity, E :**
(also known as Young's modulus)
- **Hooke's Law:**

$$\sigma = E \varepsilon$$



Poisson's ratio, ν

- **Poisson's ratio, ν :**

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$

metals: $\nu \sim 0.33$

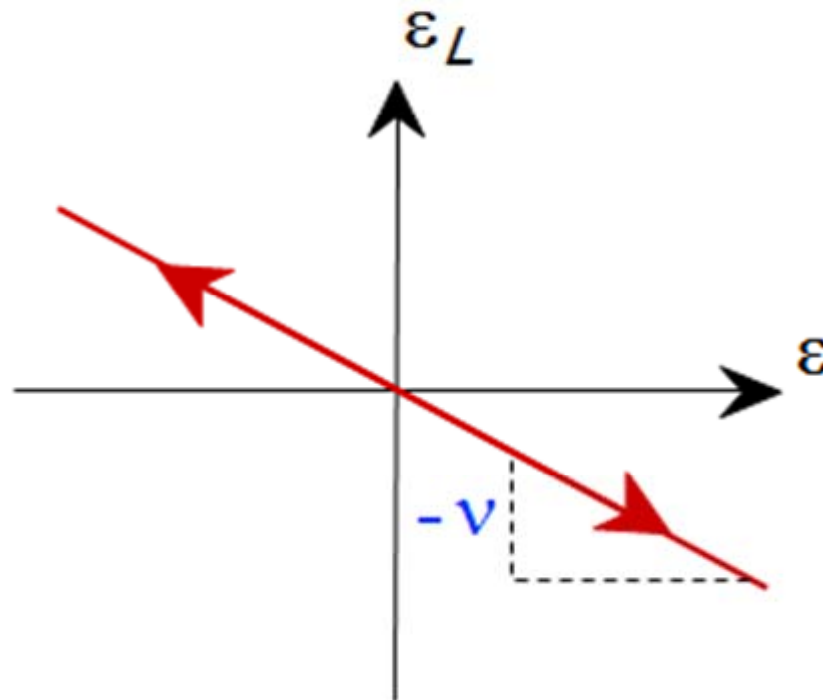
ceramics: $\nu \sim 0.25$

polymers: $\nu \sim 0.40$

Units:

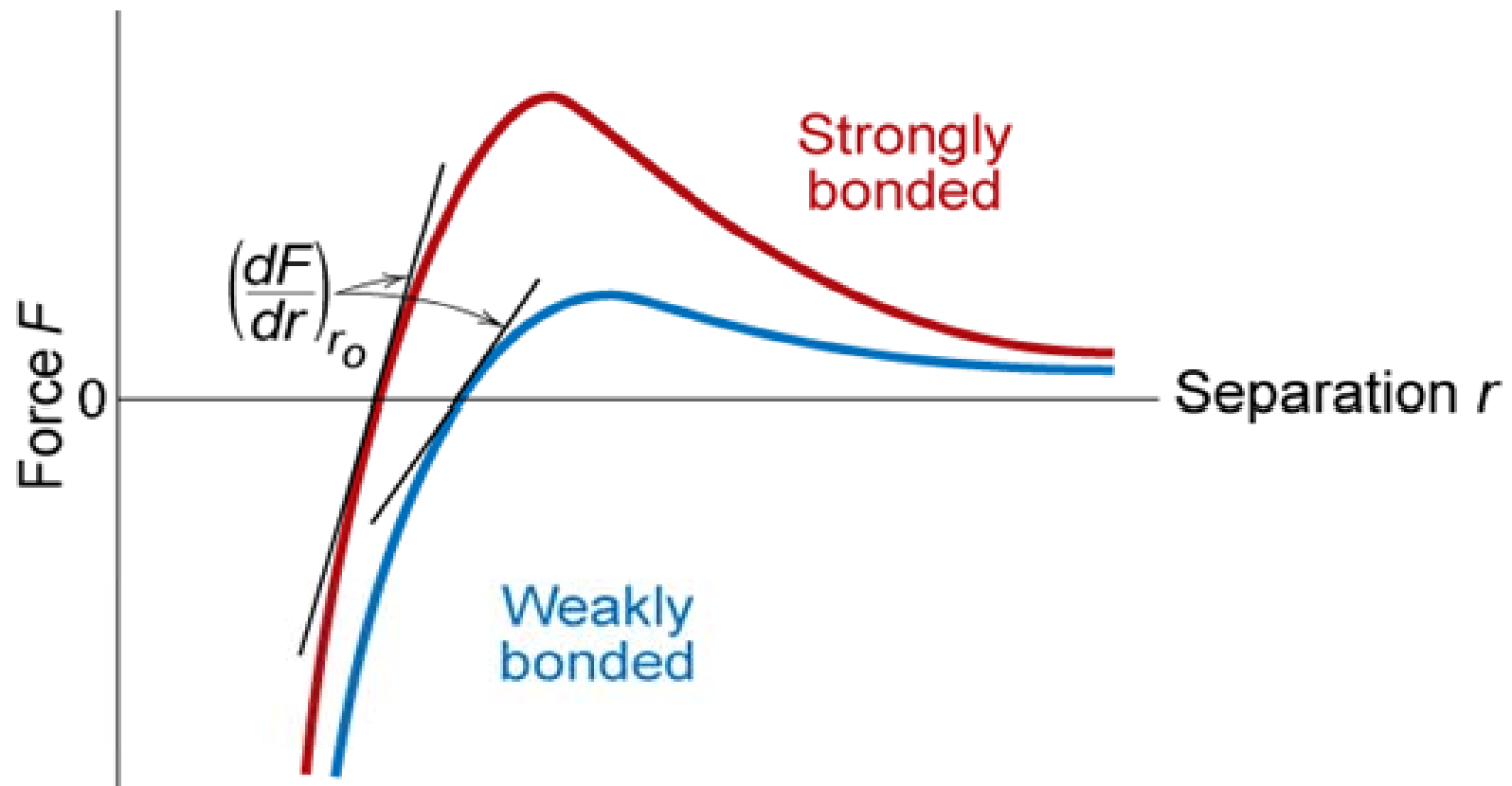
E : [GPa] or [psi]

ν : dimensionless



Mechanical Properties

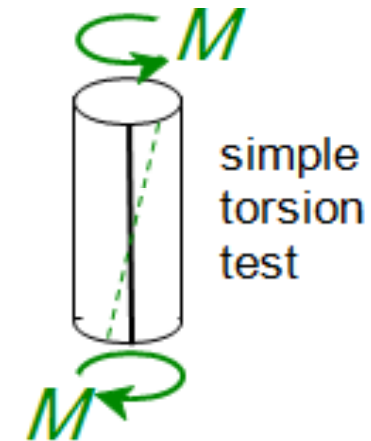
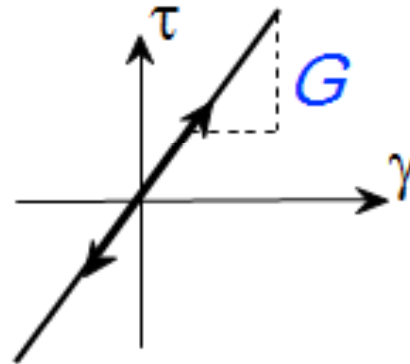
- Slope of stress strain plot (which is proportional to the elastic modulus) depends on bond strength of metal



Other Elastic Properties

- **Elastic Shear modulus, G :**

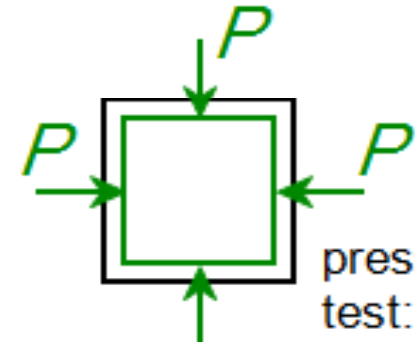
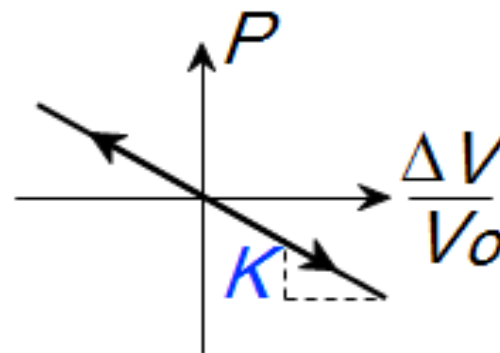
$$\tau = G \gamma$$



simple
torsion
test

- **Elastic Bulk modulus, K :**

$$P = -K \frac{\Delta V}{V_0}$$



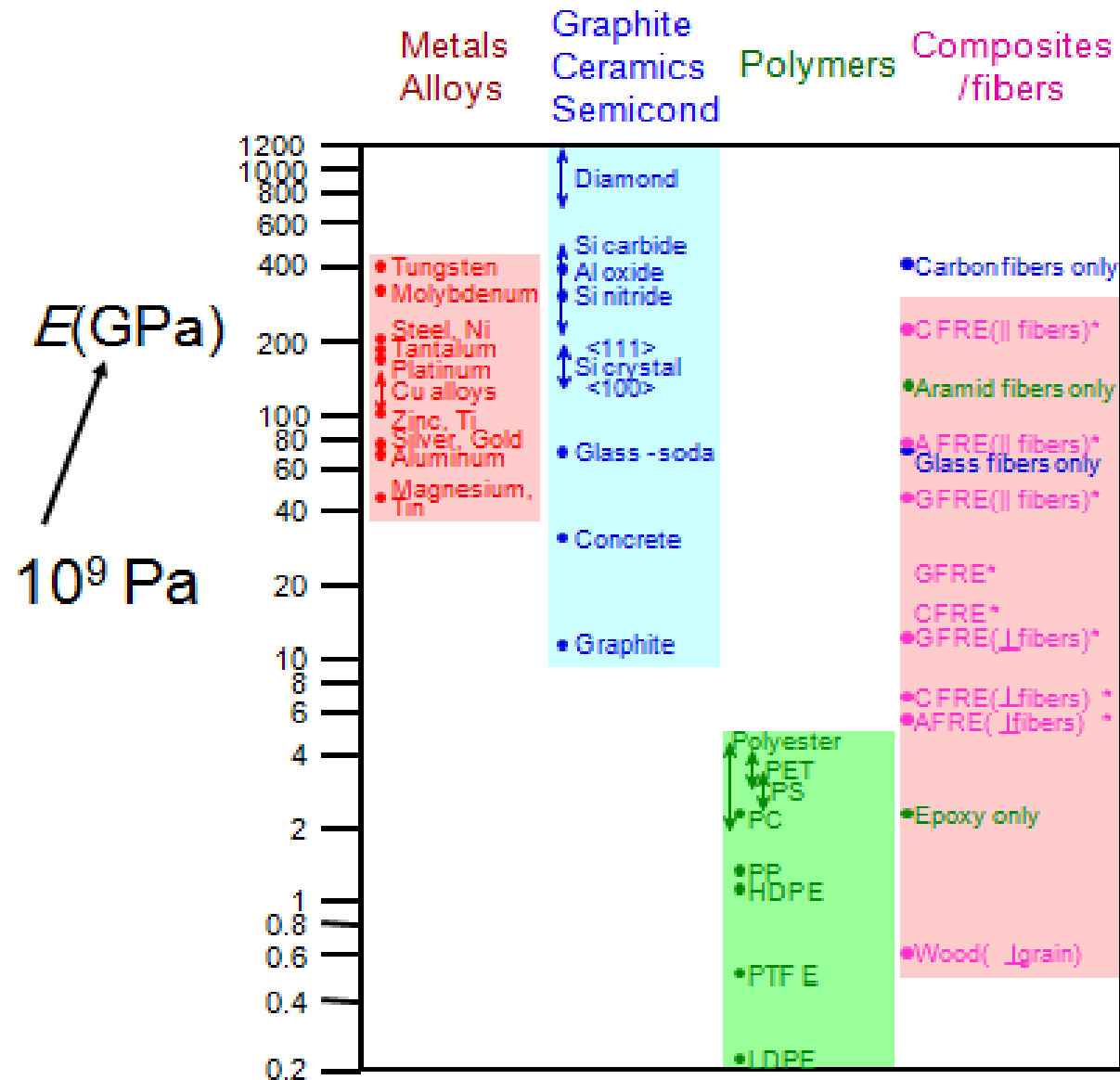
pressure
test: Init.
vol = V_0 .
Vol chg.
= ΔV

- **Special relations for isotropic materials:**

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

Young's Moduli: Comparison

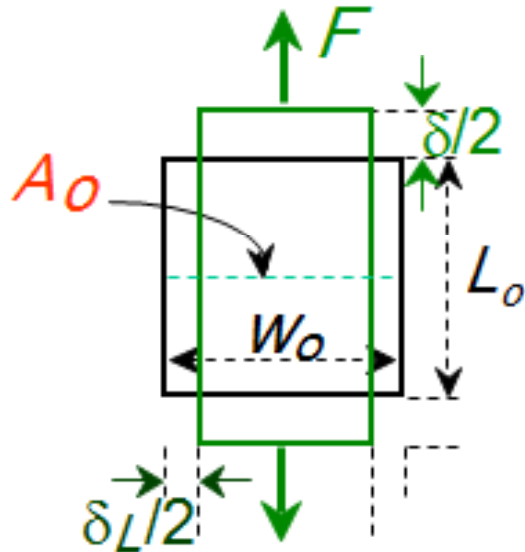


Useful Linear Elastic Relationships

- **Simple tension:**

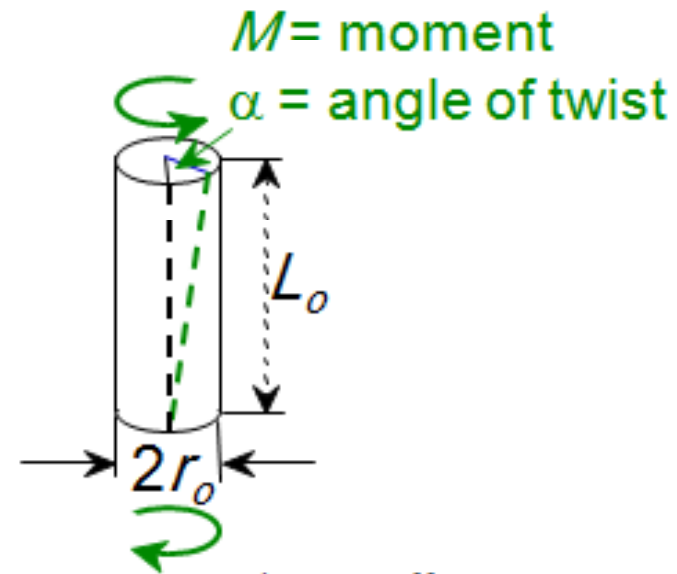
$$\delta = \frac{FL_o}{EA_o}$$

$$\delta_L = -\nu \frac{FW_o}{EA_o}$$



- **Simple torsion:**

$$\alpha = \frac{2ML_o}{\pi r_o^4 G}$$

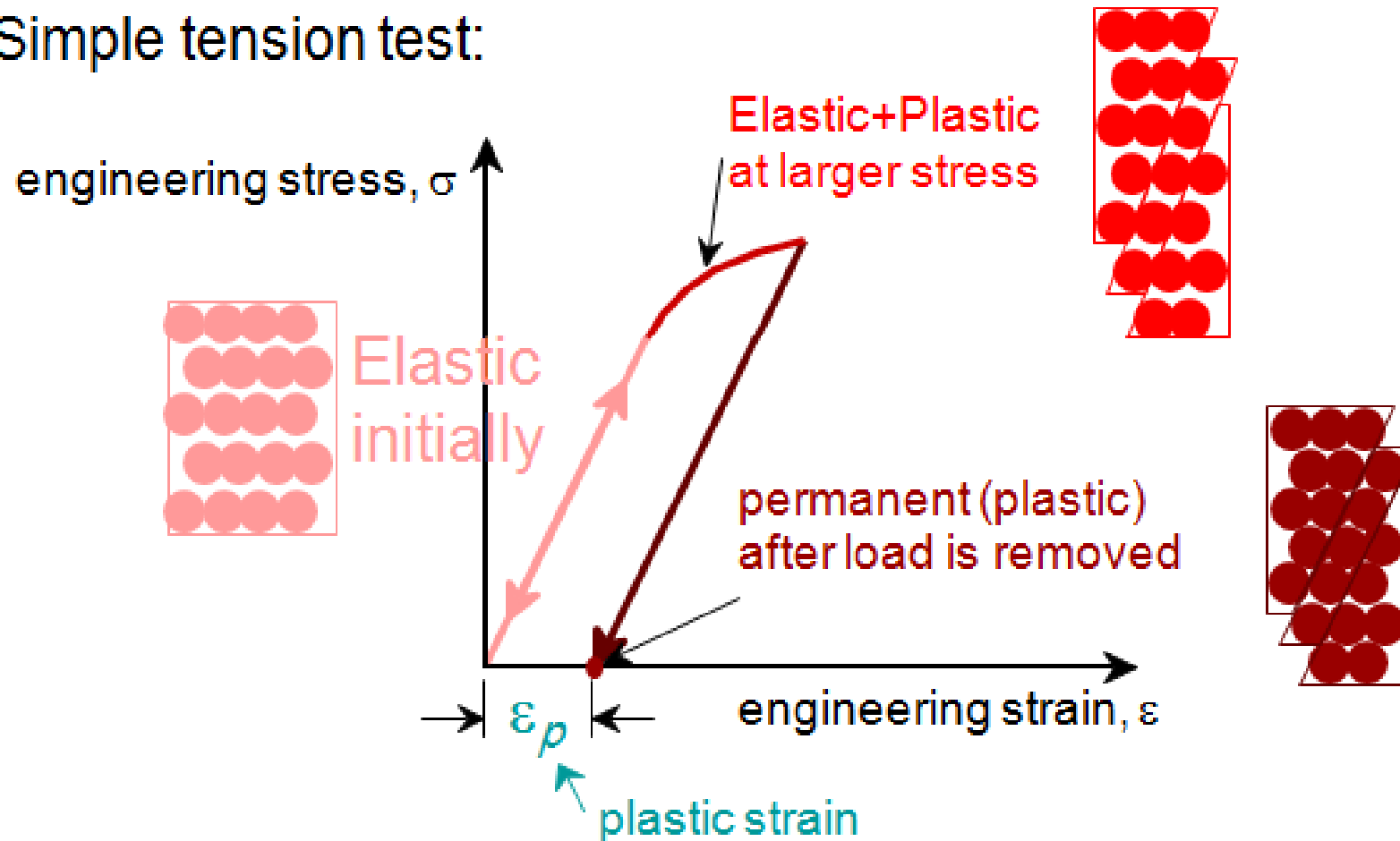


- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.

Plastic (Permanent) Deformation

(at lower temperatures, i.e. $T < T_{melt}/3$)

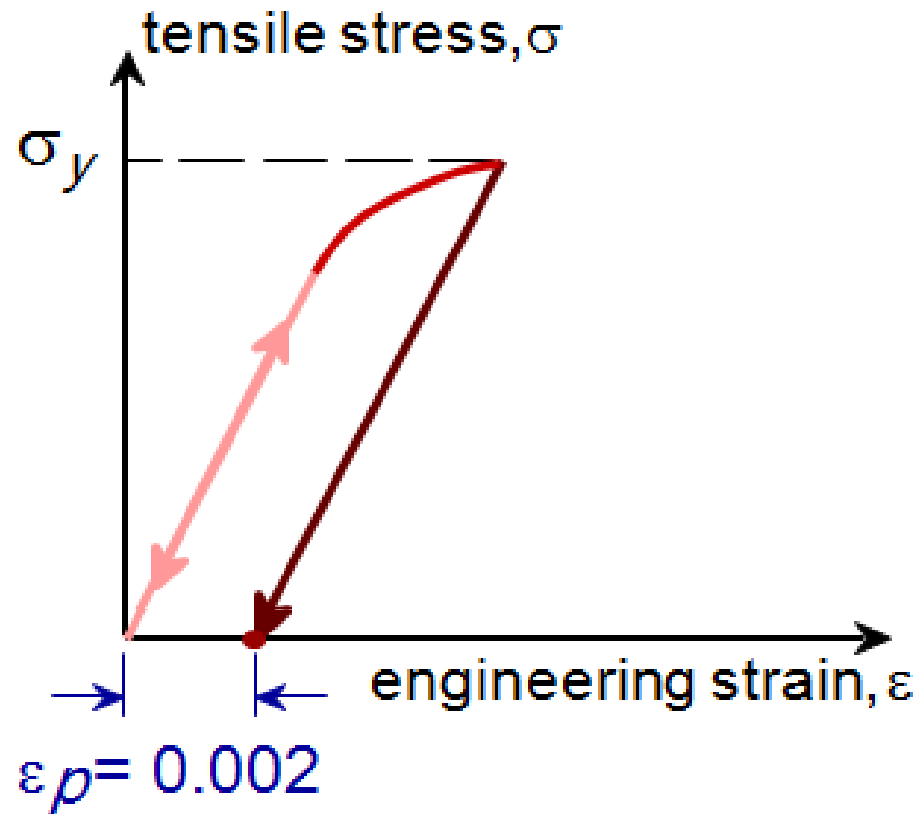
- Simple tension test:



Yield Strength, σ_y

- Stress at which *noticeable* plastic deformation has occurred.

when $\varepsilon_p = 0.002$



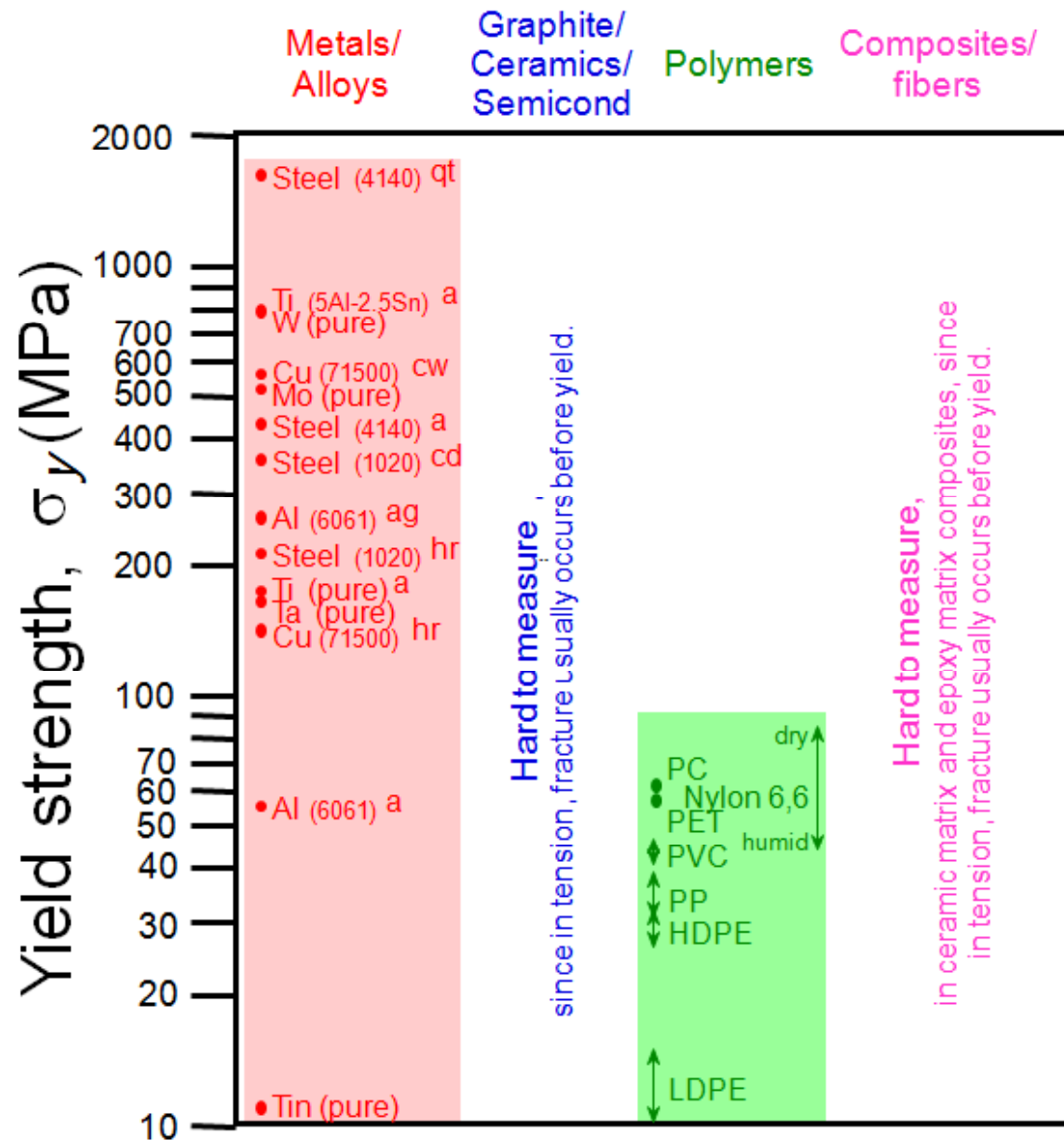
σ_y = yield strength

Note: for 2 inch sample

$$\varepsilon = 0.002 = \Delta z / z$$

$$\therefore \Delta z = 0.004 \text{ in}$$

Yield Strength : Comparison



Based on data in Table B4,
Callister 7e.

a = annealed

hr = hot rolled

ag = aged

cd = cold drawn

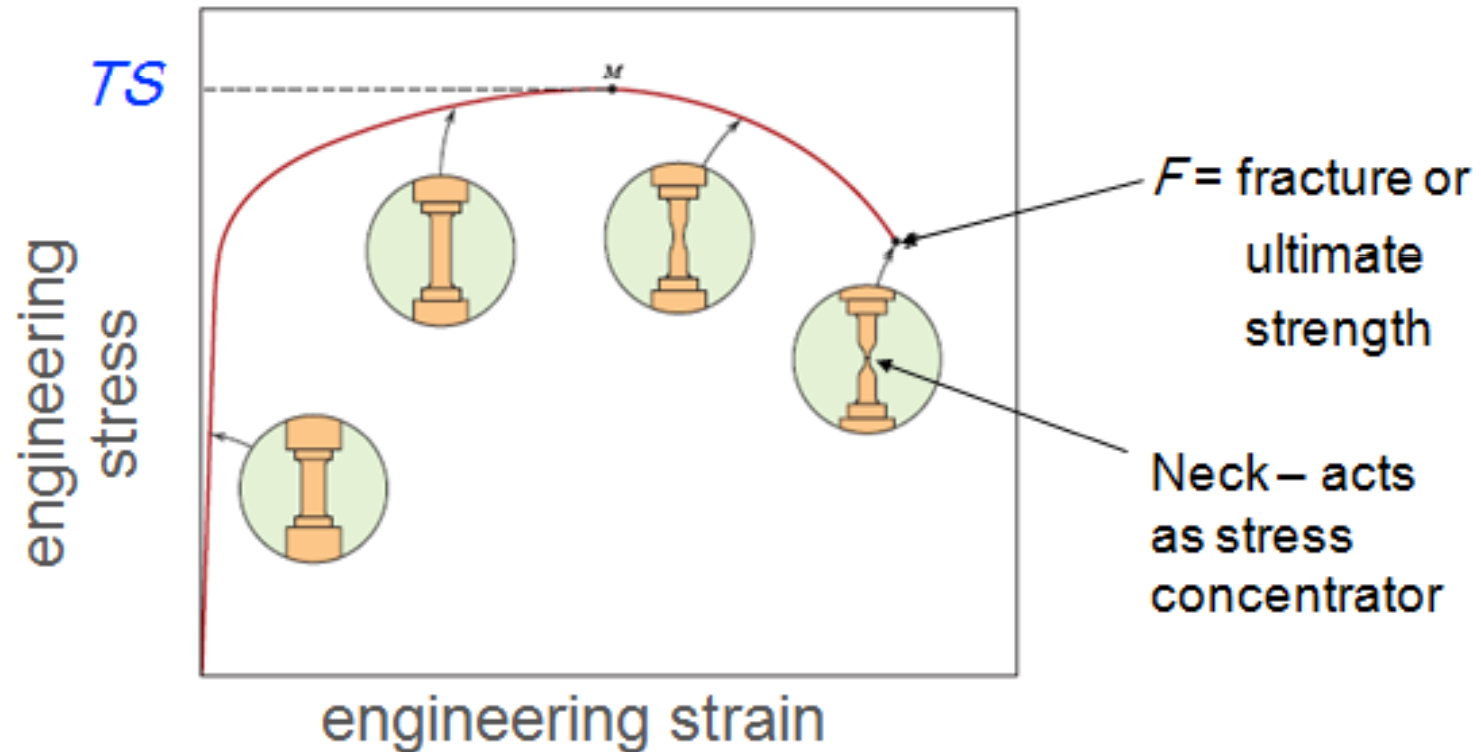
cw = cold worked

qt = quenched & tempered

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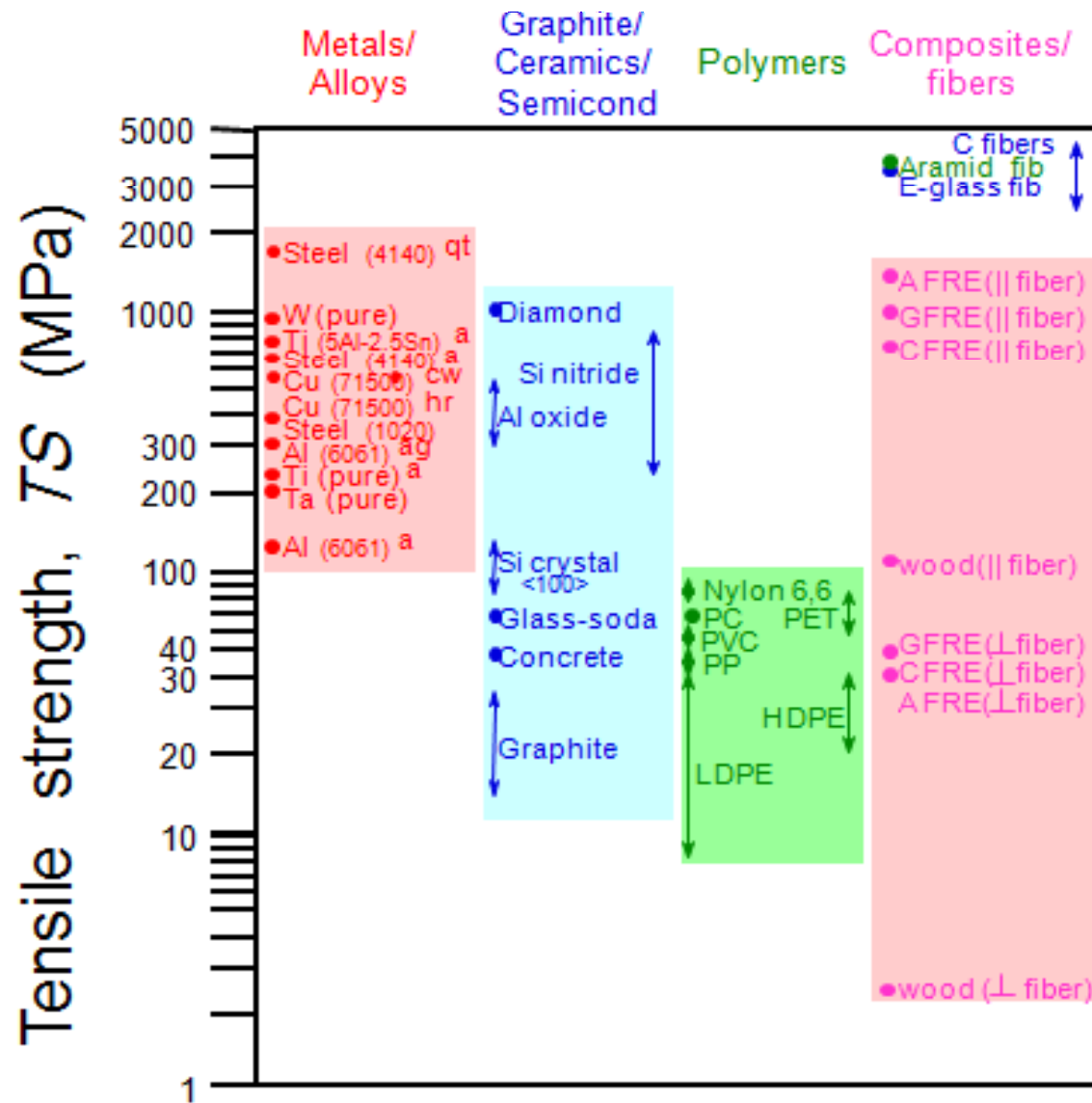
Tensile Strength, TS

- Maximum stress on engineering stress-strain curve.



- Metals:** occurs when noticeable necking starts.
- Polymers:** occurs when polymer backbone chains are aligned and about to break.

Tensile Strength : Comparison



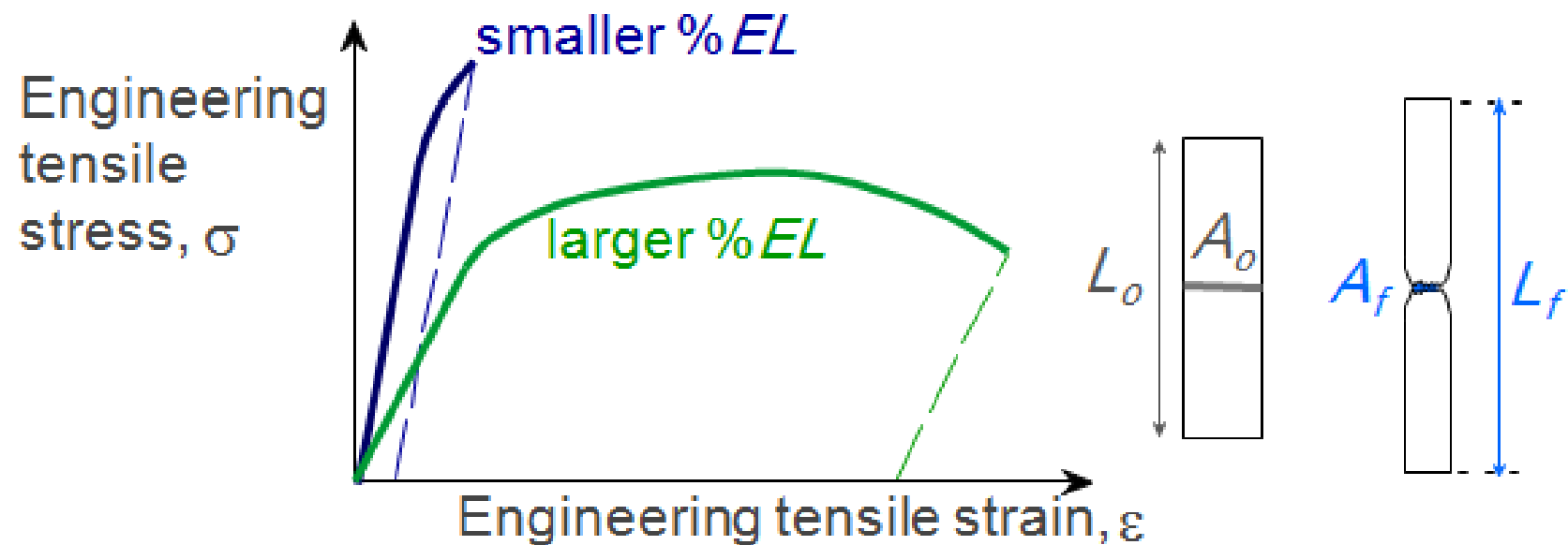
Room Temp. values

a = annealed
 hr = hot rolled
 ag = aged
 cd = cold drawn
 cw = cold worked
 qt = quenched & tempered
 AFRE, GFRE, & CFRE =
 aramid, glass, & carbon
 fiber-reinforced epoxy
 composites, with 60 vol%
 fibers.

Ductility

- Plastic tensile strain at failure:

$$\%EL = \frac{L_f - L_o}{L_o} \times 100$$

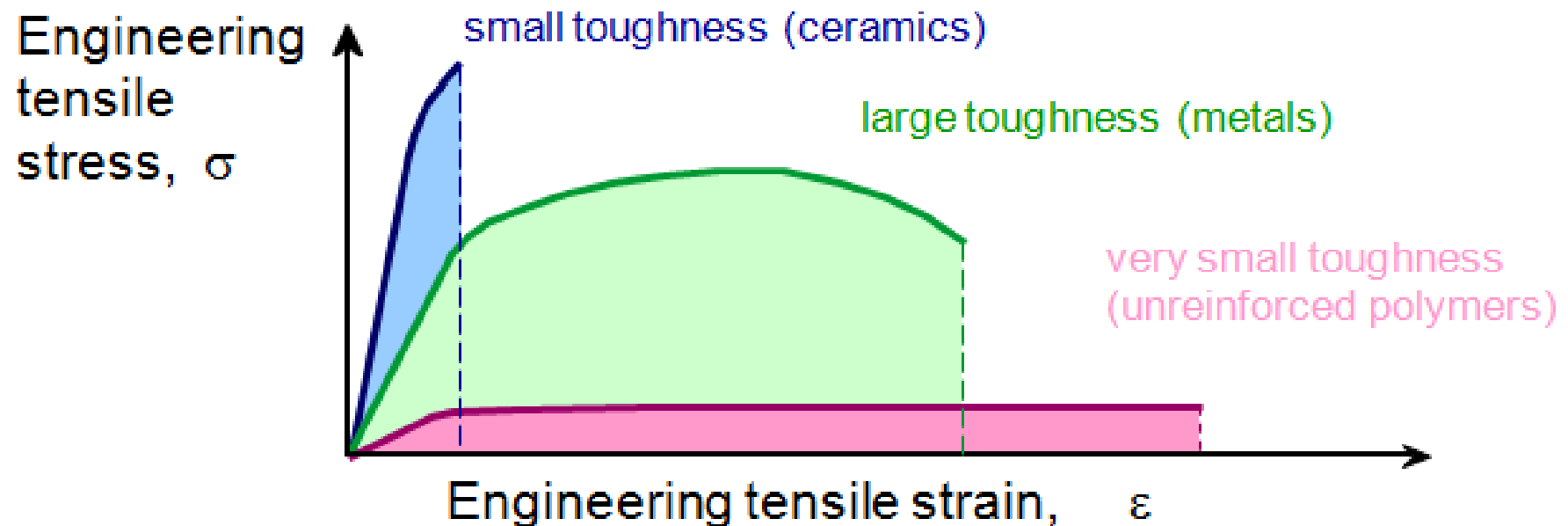


- Another ductility measure:

$$\%RA = \frac{A_o - A_f}{A_o} \times 100$$

Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.

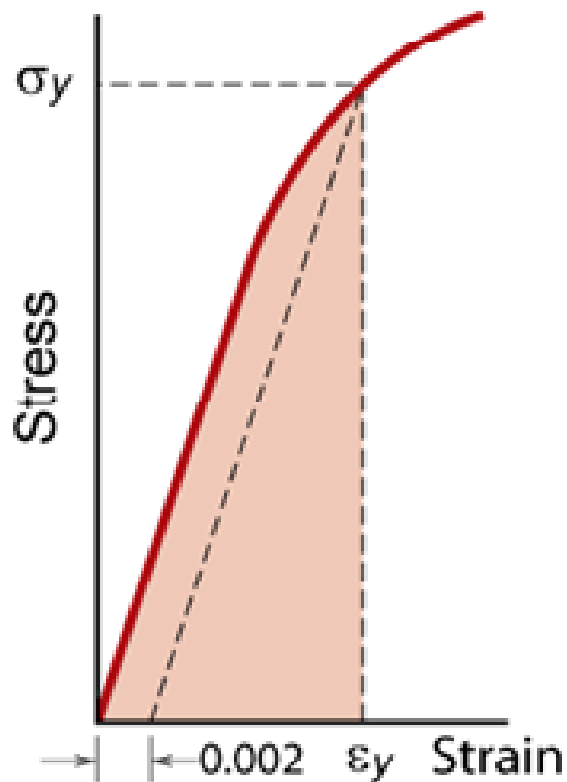


Brittle fracture: elastic energy

Ductile fracture: elastic + plastic energy

Resilience, U_r

- Ability of a material to store energy
 - Energy stored best in elastic region

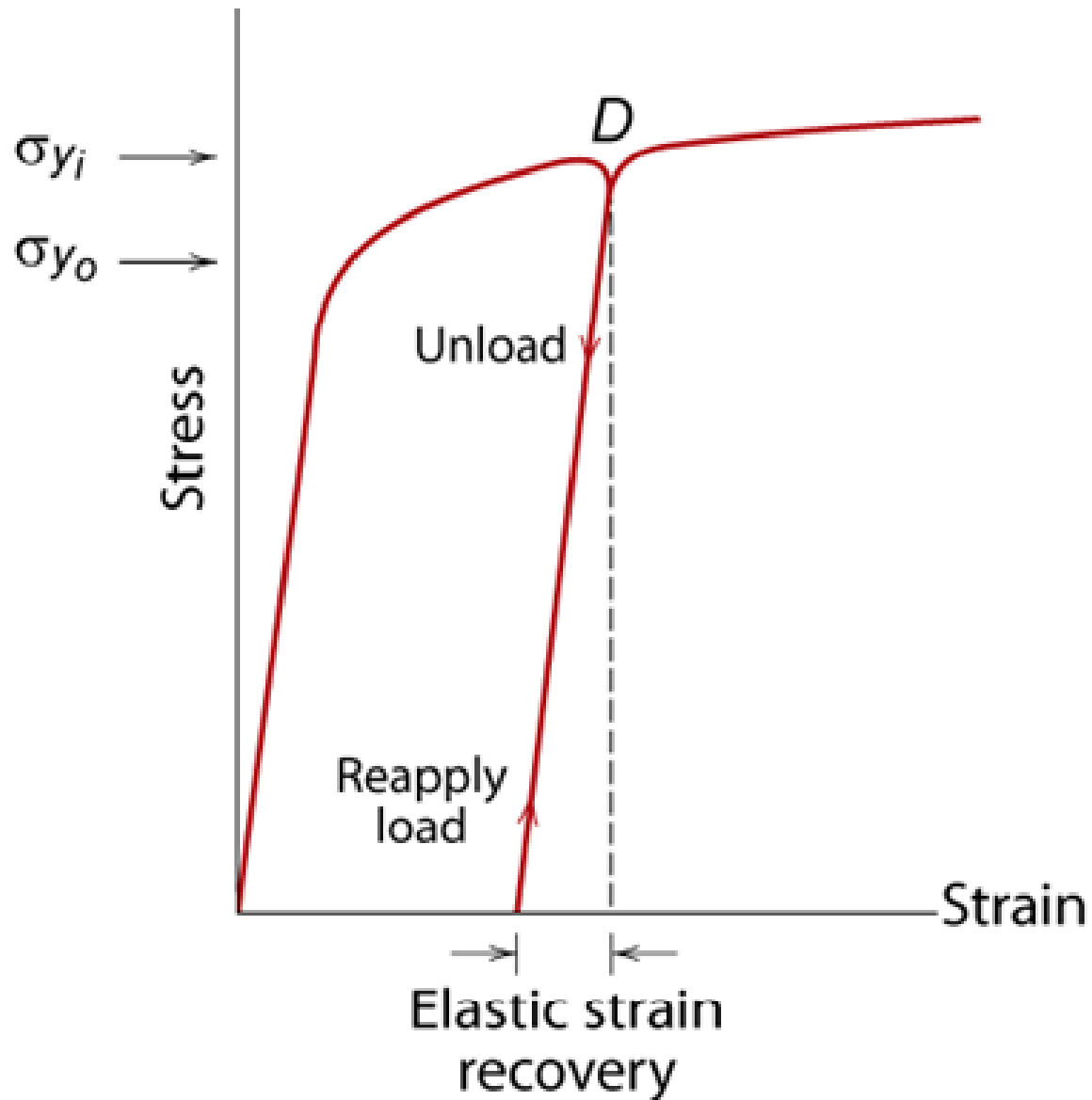


$$U_r = \int_0^{\epsilon_y} \sigma d\epsilon$$

If we assume a linear stress-strain curve this simplifies to

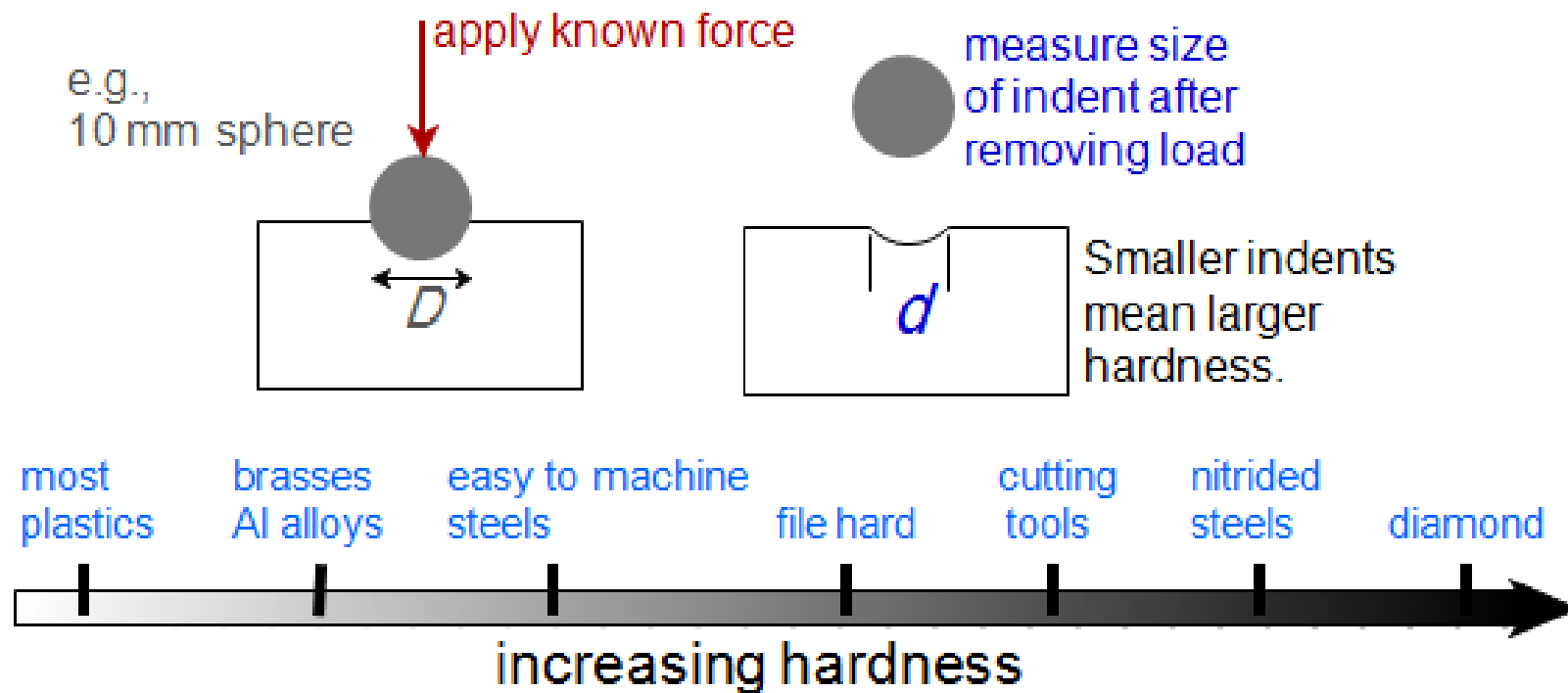
$$U_r \approx \frac{1}{2} \sigma_y \epsilon_y$$

Elastic Strain Recovery



Hardness

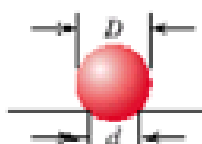
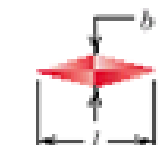
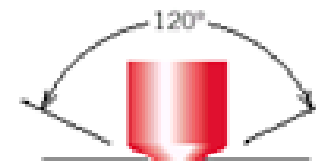
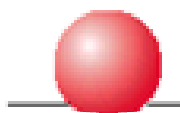
- Resistance to permanently indenting the surface.
- Large hardness means:
 - resistance to plastic deformation or cracking in compression.
 - better wear properties.



Hardness: Measurement

- Rockwell
 - No major sample damage
 - Each scale runs to 130 but only useful in range 20-100.
 - Minor load 10 kg
 - Major load 60 (A), 100 (B) & 150 (C) kg
 - A = diamond, B = 1/16 in. ball, C = diamond
- HB = Brinell Hardness
 - $TS(\text{psia}) = 500 \times \text{HB}$
 - $TS(\text{MPa}) = 3.45 \times \text{HB}$

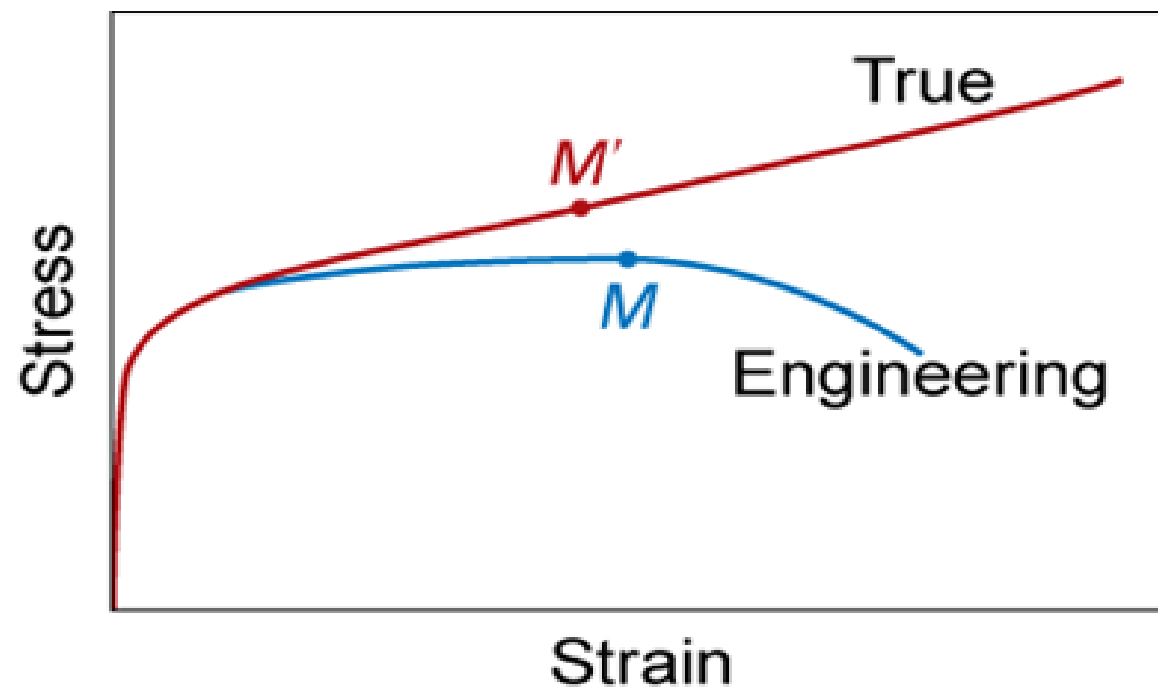
Hardness: Measurement

Test	Indenter	Shape of Indentation		Load	Formula for Hardness Number ^a
		Side View	Top View		
Brinell	10-mm sphere of steel or tungsten carbide			P	$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres	 	 	60 kg 100 kg 150 kg Rockwell 15 kg 30 kg 45 kg Superficial Rockwell	

True Stress & Strain

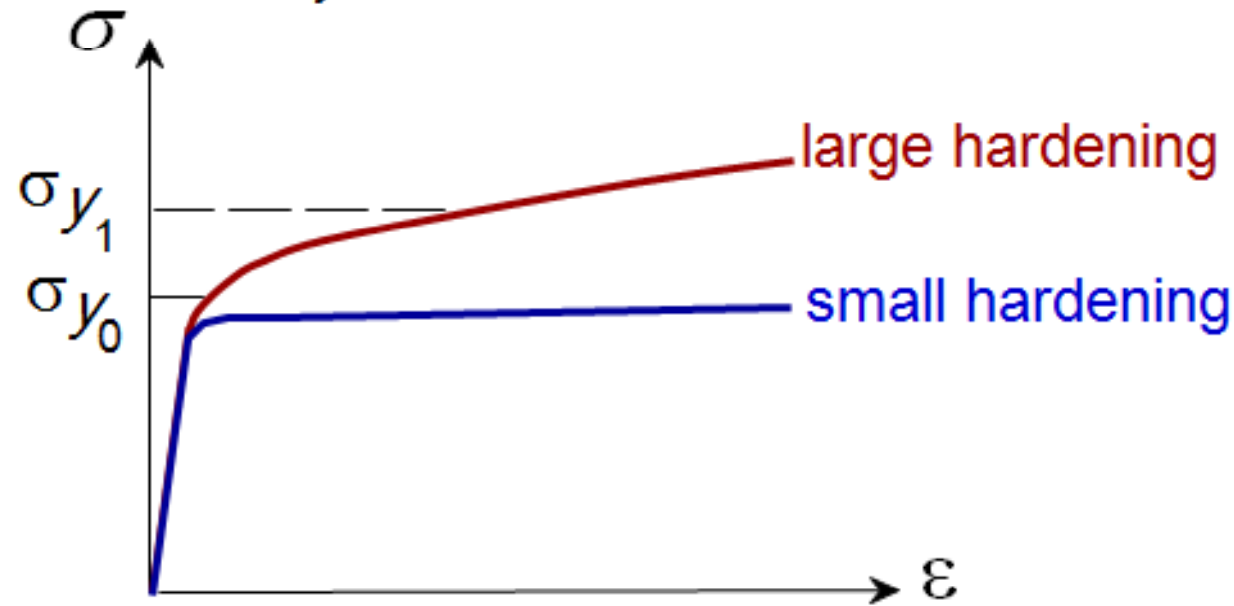
Note: S.A. changes when sample stretched

- True stress $\sigma_T = F/A_i$
 - True Strain $\epsilon_T = \ln(\ell_i/\ell_o)$
- $$\sigma_T = \sigma(1 + \epsilon)$$
$$\epsilon_T = \ln(1 + \epsilon)$$



Hardening

- An increase in σ_y due to plastic deformation.



- Curve fit to the stress-strain response:

$$\sigma_T = K(\epsilon_T)^n$$

Annotations for the equation:

- hardening exponent:
 $n = 0.15$ (some steels)
to $n = 0.5$ (some coppers)
- "true" stress (F/A)
- "true" strain: $\ln(L/L_0)$

Variability in Material Properties

- Elastic modulus is material property
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics

– Mean

$$\bar{x} = \frac{\sum_{n=1}^n x_n}{n}$$

– Standard Deviation

$$s = \left[\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \right]^{\frac{1}{2}}$$

where n is the number of data points

Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, N

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often N is between 1.2 and 4

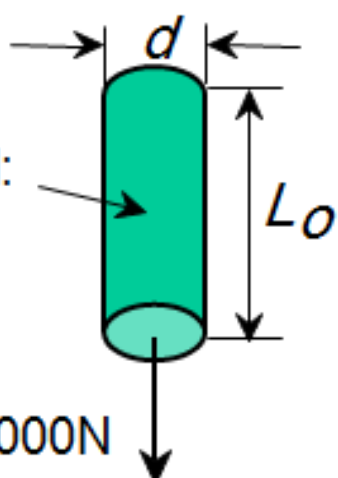
- Example: Calculate a diameter, d , to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$$\sigma_{working} = \frac{\sigma_y}{N}$$

1045 plain carbon steel:
 $\sigma_y = 310 \text{ MPa}$
 $TS = 565 \text{ MPa}$

$F = 220,000 \text{ N}$

$d = 0.067 \text{ m} = 6.7 \text{ cm}$



The diagram shows a vertical cylindrical rod of diameter d and length L_o . A downward arrow labeled $F = 220,000 \text{ N}$ is applied to the bottom of the rod. The rod is colored green.