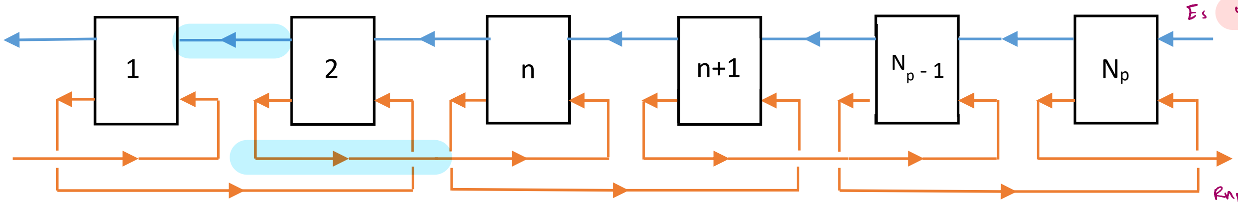
chapter 2
chapter 4 ⇐ Seader
Third edition

Treybal problem 5.3 ⇒ project. 27/12

Counter Current Operations

- Most efficient arrangement [least number of stages for a given separation].
- Each stage is identical in its action to a cocurrent process; however, the cascade (battery) has the characteristics of a counter current process.
- Passing streams are not in equilibrium
- Streams leaving a stage are in equilibrium

equilibrium → not in equilibrium



Operating Lines: "Material balance"

$$Y_n = \frac{R_s}{E_s} X_n + \frac{E_s Y_{n+1} - R_s X_{n-1}}{E_s}$$

Operating line for any equilibrium stage (n)

INPUT (X_{n-1}, Y_{n+1}) OUTPUT (X_n, Y_n)

Mole fractions: $R_{n-1}.x_{n-1} + E_{n+1}.y_{n+1} = R_n.x_n + E_n.y_n$

Mole ratios: $R_s.X_{n-1} + E_s.Y_{n+1} = R_s.X_n + E_s.Y_n$

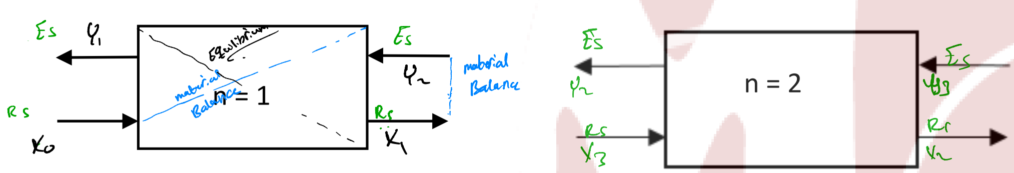
$$R_s(X_{n-1} - X_n) = E_s(Y_n - Y_{n+1})$$

$$E_s\, Y_n - E_s\, Y_{n+1} = R_s\, X_{n-1} - R_s\, X_n$$

$$\boxed{Y_n = + \frac{R_s}{E_s} X_n \pm \frac{R_s X_{n-1} + E_s Y_{n+1}}{E_s}}$$

intercept. ⇒ straight line, Slope ⇒ $-\frac{R_s}{E_s}$

For example, take the first two stages:



$$Y_1 = -\frac{R_S}{E_S} X_1 + \frac{R_S X_0 + E_S Y_2}{E_S} \qquad Y_2 = -\frac{R_S}{E_S} X_2 + \frac{R_S X_1 + E_S Y_3}{E_S}$$

$R_s X_0 + E_s Y_2 = E_s Y_1 + R_s X_1$

$E_s Y_2 = E_s Y_1 + R_s X_1 - R_s X_0$

$Y_2 = Y_1 + \frac{R_s}{E_s} X_1 - \frac{R_s}{E_s} X_0$

$Y_2 = \frac{R_s}{E_s} X_1 + Y_1 - \frac{R_s}{E_s} X_0$

$y_2 = \frac{R_s}{E_s} + \frac{Y_1 E_s - R_s X_0}{E_s}$

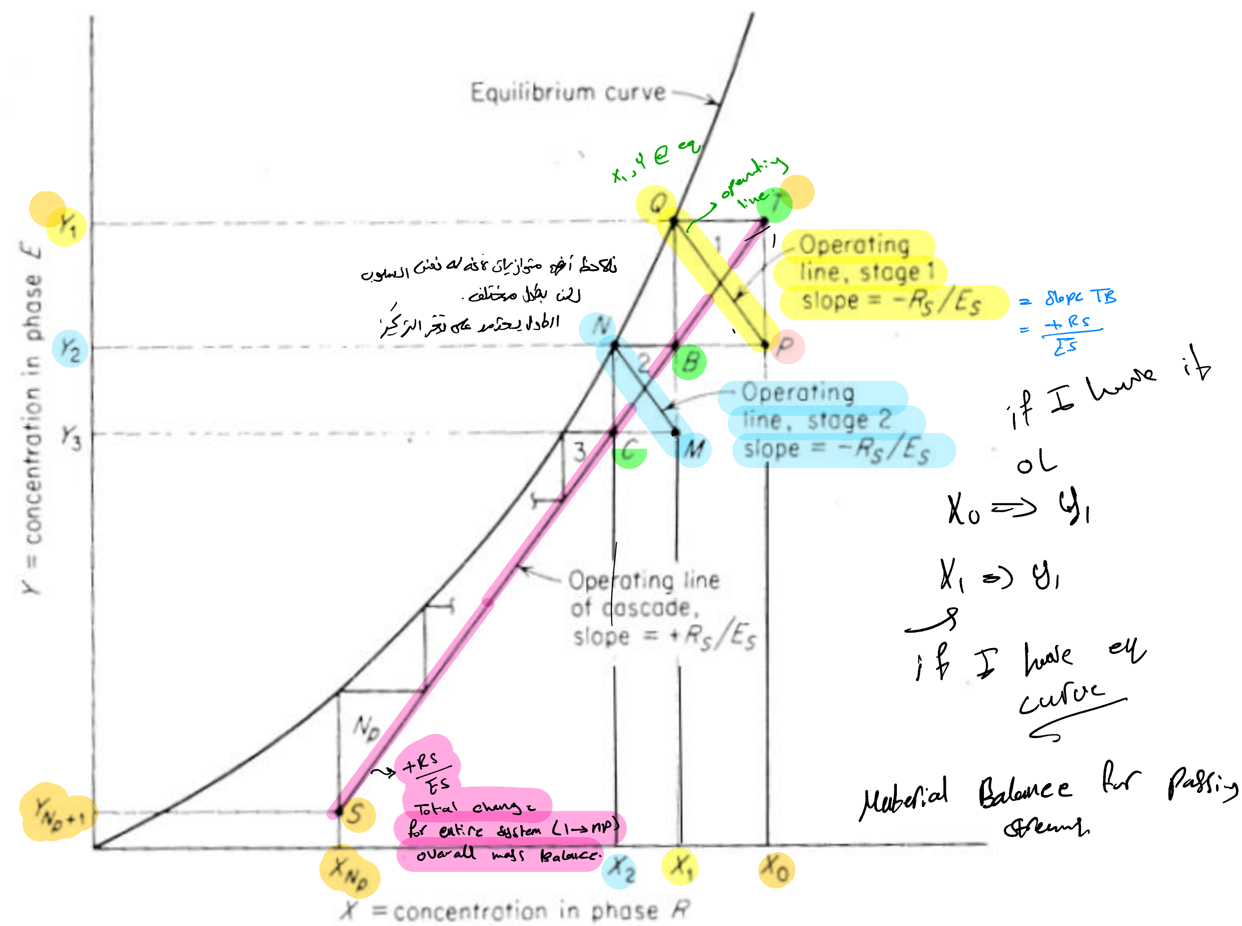


Figure 5.15 Countercurrent multistage cascade, solute transfer from phase R to phase E.

Notes:

- All individual stages have parallel operating lines of slope $= -\frac{R_S}{E_S}$
- Points (X_n, Y_n) fall on equilibrium curve. They are exit streams from stage n [Q, P, ……]
- Points (X_{n-1}, Y_{n+1}) these are inlet streams to stage n [P, M , …….]
- Points X_n, Y_{n+1} are compositions of passing streams [T, P, C, ___]. Lines like ,
  have the same slope TS $= +\frac{R_S}{E_S}$ and they all fall on the same line .

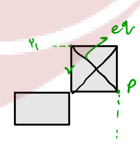
Operating line for the cascade:

Entire cascade: $R_S\left(X_o - X_{N_p}\right) = E_S\left(Y_1 - Y_{N_p+1}\right)$

For any stage n: $R_S(X_o - X_n) = E_S\,(Y_1 - Y_{n+1})$

$$Y_{n+1} = +\frac{R_S}{E_S}\,X_n + \frac{E_S\,Y_1 - R_S\,X_o}{E_S}$$

, Slope ; This line also coincides with .



Special Case:

=> if the operating line and equilibrium are in equilibrium => for dilluted mixtures] => analytical solutions.

For most cases, because of either a curved operating line or equilibrium curve, the relation between number of stages, compositions, and flow ratio must be determined graphically, as shown. For the *special* case where both are straight, however, with the equilibrium curve continuing straight to the origin of the distribution graph, an analytical solution can be developed which will be most useful.

graphically => plotting equilibrium and operating Then we start separation

خارجة من stage at equilibrium

If the equilibrium-curve slope is $m = \frac{Y_{n+1}}{X_{n+1}}$ (straight line), and if an absorption factor (Extraction factor) A is defined as:

$$\boxed{A = \frac{R_S}{mE_S},}$$

then based on material balance and equilibrium data, we can obtain the following equation:

for any stage

$$\boxed{X_n = \left(X_0 - \frac{Y_{N_p+1}/m - AX_{N_p}}{1 - A}\right)A^n + \frac{Y_{N_p+1}/m - AX_{N_p}}{1 - A}}$$

=> only valid for this case "Equilibrium curve => straight"

طرفيات

Therefore knowing terminal concentrations, X_n can be calculated.

molar fraction → X_0 → y_{np+}

In the opposite case where the transfer is from E to R, we define a stripping factor S as:

$$S = \frac{1}{A}$$ => اكثر خوة operating line

Kremser Equations: (Kremser-Brown-Souders)

$A = \frac{R_s}{m E_s}$

$m = \frac{y_{n+1}}{x_{n+1}}$

In the case where $n = N_p$, then:

For transfer from R to E (stripping of R)

$A \neq 1$:

$$\frac{X_0 - X_{N_p}}{X_0 - Y_{N_p+1}/m} = \frac{(1/A)^{N_p+1} - 1/A}{(1/A)^{N_p+1} - 1}$$

$$N_p = \frac{\log\left[\dfrac{X_0 - Y_{N_p+1}/m}{X_{N_p} - Y_{N_p+1}/m}(1 - A) + A\right]}{\log 1/A}$$

$$n_p = \frac{x_0 - \frac{y_{(n_p+1)}}{m}}{x_{n_p} - \frac{y_{n_p+1}}{m_1}}$$

$A = 1$:

$$\frac{X_0 - X_{N_p}}{X_0 - Y_{N_p+1}/m} = \frac{N_p}{N_p + 1}$$

$$N_p = \frac{X_0 - X_{N_p}}{X_{N_p} - Y_{N_p+1}/m}$$

#NP number of Equilibrium stages to reach a separation

For transfer from E to R (absorption into R). A similar treatment yields:

$A \neq 1$:

$$\frac{Y_{N_p+1} - Y_1}{Y_{N_p+1} - mX_0} = \frac{A^{N_p+1} - A}{A^{N_p+1} - 1}$$

$$N_p = \frac{\log\left[\dfrac{Y_{N_p+1} - mX_0}{Y_1 - mX_0}\left(1 - \dfrac{1}{A}\right) + \dfrac{1}{A}\right]}{\log A}$$

$A = 1$:

$$\frac{Y_{N_p+1} - Y_1}{Y_{N_p+1} - mX_0} = \frac{N_p}{N_p + 1}$$

$$N_p = \frac{\overbrace{Y_{N_p+1} - Y_1}^{\text{total}}}{\underbrace{Y_1 - mX_0}_{\text{total in stage 1}}}$$

These are called the Kremser-Brown-Souders (or simply Kremser) equations, after those who derived them for gas absorption [7, 14] although apparently Turner [16] had used them earlier for leaching and solids washing. They are plotted in Fig. 5.16, which then becomes very convenient for quick solutions.

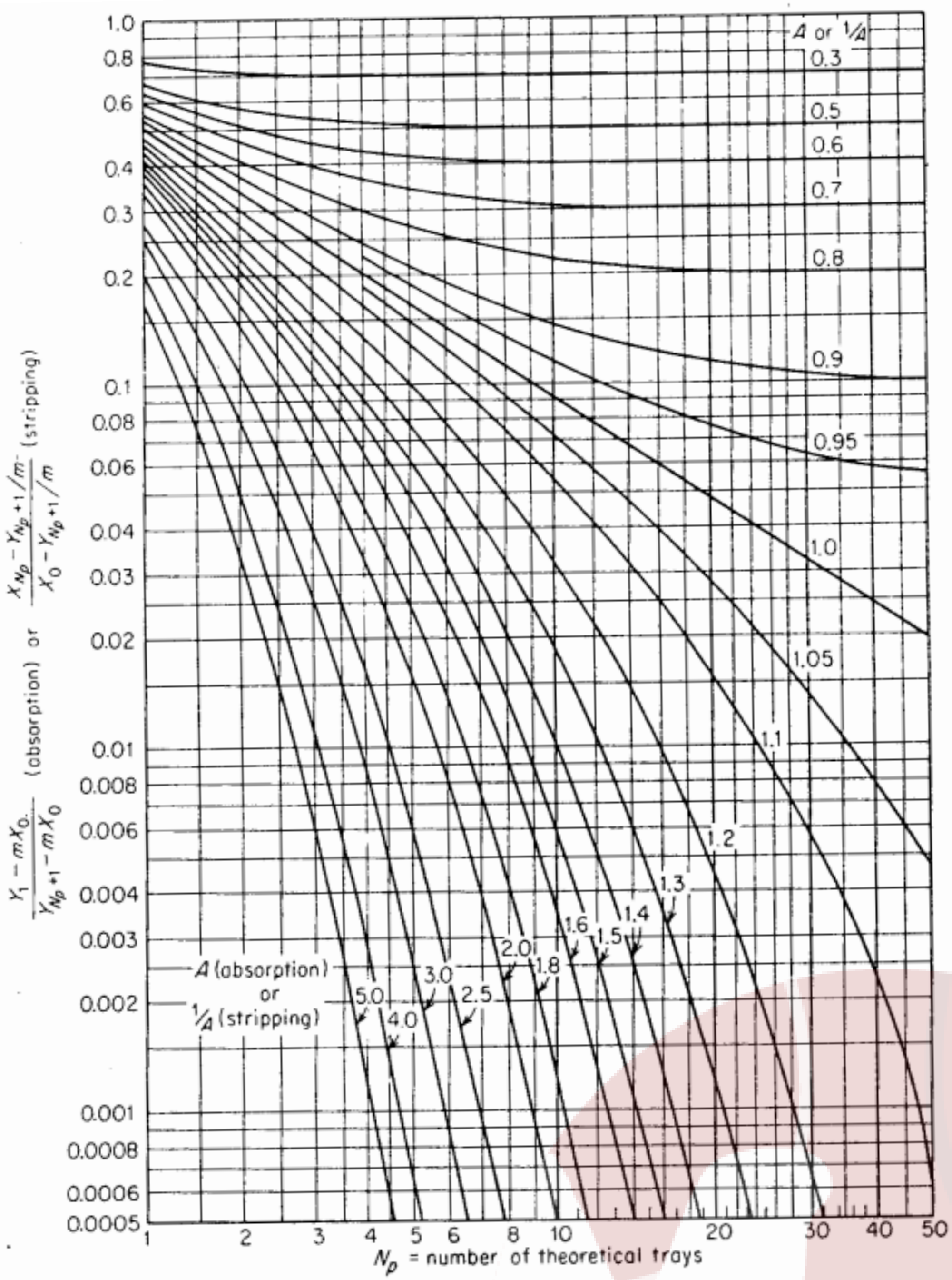


Figure 5.16 Number of theoretical stages for countercurrent cascades, with Henry's law equilibrium and constant absorption or stripping factors. [*After Hachmuth and Vance, Chem. Eng. Prog.*, **48**, *523, 570, 617 (1952).*]