

# Separation processes I



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# Mass Transfer: Separation processes Operations

## Introduction

A large number of unit operations are used in the separation of a substance into its component parts. They involve a change in composition of mixtures and solutions. The changes are mostly in nature.

Separations involve a non-spontaneous process because it reverses the process of mixing. Separation decreases the entropy of the system, and a separating agent (energy, mass or membrane) must be used to carry out the process.

Separation of mixtures may be mechanical in nature: filtration, sedimentation, etc.

Separation of solutions involves mass transfer operations characterized by the transfer of a substance through another on a molecular scale. These operations are interphase mass transfer processes involving the creation of a second phase by the addition of an energy-separating agent (ESA) or by a mass separating agent (MSA).

\* why the separations process be non-spontaneous ?!  
because it reverse the process of mixing.

\* عملیات جداسازی را نمی توان به صورت 100% انجام داد

\* Separating agent ( جداساز )

mass separating agent  
(MSA)

در جداسازی با استفاده از حلال  
مثلاً: جداسازی الکل از آب با استفاده از بنزین

energy agent  
(ESA)

در جداسازی با استفاده از انرژی  
مثلاً: جداسازی آب از الکل با استفاده از حرارت

adar  
ask, believe & recieve



## # The main separation techniques are:

### > Phase creation

Most common industrial technique. A second phase is created which is immiscible with the feed. This is done by increasing or decreasing the temperature or pressure on the feed.

\* تتم عملية الفصل عن طريق خلق طور جديد غير قابل للمختلج مع mixture feed

### > Phase addition

Involves the introduction of a solvent to the system which selectively solvates one component of the feed.

\* إضافة مذيب آخر إلى mixture ليحلل أحد مكونات الخليط في فصله عن بقية المكونات

### > Barrier separation

Increasingly important. Barrier selectively transports one species relative to another across the membrane.

\* يتم الانتقال من phase إلى phase بواسطة membrane بشكل انتقائي بين 2 phases

### > Solid agent separation

Solid particles are added to the feed which are either inert and carry substances using separation; or react with the feed to cause separation.

\* يتم إضافة مواد صلبة مثل activated carbon إلى الخليط

### > Separation by force field

Common in laboratory and for specialized separations and involves the use of electric and magnetic fields to preferentially move one species over another.

adsorption (adsorption) لبعض الغازات المسالة على سطح وديور الغاز السقي

قوة كهربائية أو مغناطيسية & density

\* ex : phase addition:

mixture  $\Rightarrow$  water + Acetone

يتم فصل Acetone عن الماء بإضافة مذيب آخر وهو (Toluene) حيث تكون ذائبيتان في الماء.

\* ex : Barrier separation:

تجسيم (إكل) في بطلعوا دم من جسم الإنسان وبعثروا في فلاتر، يكون الدم من هجوت ومنه الجهد الإضافي للفلاتر يوجد سائل آخر (ماء + ترابيزة هجوت من الدم)، تنقل السموم من الدم عبر الفلتر إلى السائل (ماء + دم).  
\* الهون من إضافة بعض الأملاح مع الماء حتى لا تنقل الدم إلى السائل لانه الهون تنقية الدم من السموم فقط.



## Types of Mass Transfer separation Operations

The type depends on the way different phases are contacted:

### I. Direct Contact

- Phases are immiscible or partially miscible
- Most widely used and most important
- Depends on the fact that the contacted phases, consisting of several components, are initially not at equilibrium. The system tends to reach equilibrium as a result of a slow process of diffusion between the two phases.

\* phases التي على اتصال Direct contact في البداية تكونا بمرحلة التوازن ، يكون تركيز المواد في phase 1 قبل التوازن في phase 2 وهذا يؤدي قوة دافعة لنقل المادة ، وبعد زوال دفع إلى مرحلة equilibrium في systems في الطبيعة تميل لنضع في مرحلة equilibrium .





- Depending on the state of the phases, the following interfaces may be distinguished:

**Gas/Gas:** Practically not realized (almost all gases are completely soluble).  
غازات نادرة  
مذابة لغازات ذائبة  
بعضها

**Gas/Liquid:** - Distillation: all components are present in different proportions in the two phases

- Gas Absorption (or desorption): one or more components distribute between the two phases.  
تتم مع طوري عكسي لغاز بسائل مناسب  
لهدي لغازات تذوب أو تخرج في  
السائل وبالتالي يوصف إذائته من تحديد

- Humidification (or dehumidification): one phase is pure and the other consists of two or more components

**Gas/solid:** - Fractional sublimation (vaporization of solid solutions (not used practically).

\* sublimation  $\Rightarrow$  gas  $\rightarrow$  solid  $\Rightarrow$  تسامي

من المذابات على انتقال  $H_2O$  من الحالة الغازية إلى

الحالة الصلبة كمثل طيقات جود بعض الأشياء.

عملية تفرز حيث يتم فيها إطلالة مادة مذابة سابقاً في سائل  
قدرة هذه يكتسب الجزيء طاقة كافية للتحرك خارج تشبيك الطاقات المحيطة التي تبقى في السطح  
\* desorption



مثال : جفاف المربى  
↓

- **Drying**: not all components are present in the two phases.
- **Adsorption (or desorption)**: Diffusion takes place from gas phase to solid phase or vice versa.

حالة صلبة محببة مغطاة بـ microscope يظهر جزيئاتها على سطح خشن في Gas phase

**Liquid / Liquid**: solvent extraction  $\Rightarrow$  exp  $\Rightarrow$  toluene & water + acetone

**Liquid/Solid**: Crystallization  $\Rightarrow$  بلورة  $\Rightarrow$  مثال : خلل عشيق يملح حبيب بعد غليته (بنو) يبدأ الملع تيلمر ويترسب.

- **Leaching**

مثال : اختزال الماء لمادة  $\Rightarrow$  coefficient الموجودة جزيئات المكونة  
الشيء " " الشيء

**Solid/Solid**: very slow rate of diffusion within solid phases.

يستخدم في صناعات البقايا عن طريق (imperfection) هذه الشوائب  
بإتات البقايا diffusion ببقائه صلب.



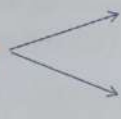
## II) Indirect contact:

- \* phases are separated by a membrane

محتوي على حركات انتقال المادة من phase 1 إلى phase 2



- \* membrane



porous (molecular sieves)

nonporous



محتوي على membrane (gel) تدفق المادة فيه وتسمى diffusion في gel ثم تنتقل إلى phase 2

- \* substances move between phases by diffusion through the membrane
- \* porous membranes are selective - they permit the diffusion of certain molecules.
- \* non porous membranes preferentially dissolve the solute or the solute and solvent which then diffuses through the membrane - the process is called permeation or dialysis.





- these operations involve different phases:

- **Gas/Gas**
  - effusion-microporous membrane
  - permeation - non-porous membrane.
- **Gas/Liquid** - permeation processes (non-porous membranes)
- **Liquid | Liquid**: Dialysis - membrane permeable to solvent and dissolved substance.
- **Electro dialysis** - electromotive force is used to assist the diffusion process.
- **Reverse osmosis (R/O)** - desalination of water.

الخاصية البرسوزية في انتقال المادة من التركيز العالي إلى التركيز المنخفض  
 ولكن هذه العملية عكس الخاصية البرسوزية (reverse) حيث تنقل المادة  
 من التركيز المنخفض وهذا يتم عن طريق التأثير بضغط عالي جداً



مثال: إذا كان لدينا خليط من oil و water في cylinder وبعدها في cylinder آخر  
 III) **Direct contact of miscible phases:** <sup>الذي يحدث في oil و water</sup> <sup>الذي يحدث في oil و water</sup> <sup>الذي يحدث في oil و water</sup>  
 separation <sup>الذي يحدث في oil و water</sup>

- **Thermal diffusion** - concentration difference is created by a temperature gradient
- **Centrifugation** - separation occurs as a result of slightly different forces acting on various molecules of different masses. <sup>الذي يحدث في oil و water</sup>

#### IV) Surface Concentration:

<sup>الذي يحدث في oil و water</sup>  
**Foam separation:** some substances when dissolved in a liquid in contact with a gas are found to concentrate on the free surface. By forming foam on the surface of the liquid, the surface area between the gas and the liquid is increased and the solute is concentrated in the foam and can be collected at the surface.

تم اختراع بعض المواد الذائبة في oil و water ، عند عمل contact بين oil و gas ، تتكون هذه المواد على سطح البحر ، عند تطبيق تقنية رغوة على سطح oil ، وتزداد مساحة سطح بين oil و gas ، وتتركز هذه المواد في الرغوة ويمكن جمعها على السطح.



## 2. Phase Equilibrium

توازن الطور  $\Rightarrow$  net transfer = 0

### 2.1 Review of relations for gaseous mixtures:

#### Partial Pressure: ( $P_i$ )

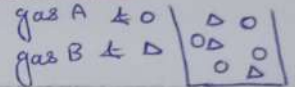
In a mixture of different gases the molecules of each gas are distributed throughout the available space and their motion contributes to the total pressure of the system.

The partial pressure of a component gas that is present in the mixture is the pressure that would be exerted by that component if it were present alone in the same volume and at the same temperature as the mixture.

Partial pressure

الضغط الجزئي

pressure force



(الضغط الجزئي)

For an ideal gas:

$$P_i = n_i \frac{RT}{V}$$

V : total volume

Partial pressure of i

#### Pure component volume: $V_i$

The pure component volume of a component gas that is present in a mixture of gases is the volume that would be occupied by that component if it were present alone at the same temperature and pressure as the mixture.

For an ideal Gas  $V_i = n_i \frac{RT}{P}$  ; P : total pressure

#### Relation between Partial pressure, pure component volume and mole fraction:

$$\frac{P_i}{V_i} = \frac{n_i(RT/V)}{n_i(RT/P)} \Rightarrow \frac{P_i}{P} = \frac{V_i}{V} \Rightarrow \frac{P_i}{P} = \frac{V_i}{V}$$

$$\Rightarrow \frac{P_i}{P} = \frac{V_i}{V} = \frac{n_i}{n} = y_i$$

$$\frac{P_i}{P} = \frac{n_i(RT/V)}{n(RT/V)} \Rightarrow \frac{P_i}{P} = \frac{n_i}{n}$$





### Dalton's Law of Partial Pressures:

The total pressure exerted by a gaseous mixture is equal to the sum of the partial pressures of the components present in the mixture:

$$P = \sum P_i$$

### Amagats Law:

$$P_{\text{tot}} = n_1 \frac{RT}{V} + n_2 \frac{RT}{V} + \dots = \frac{RT}{V} (n_1 + n_2 + \dots) = \frac{RT}{V} n_{\text{tot}}$$

The total volume occupied by a gaseous mixture is equal to the sum of the pure component volumes.

$$V = \sum V_i$$

**Vapor pressure:**

A vapor is a gas below its critical point. Vapor pressure is the pressure at which the liquid and vapor phases of a pure substance may exist at equilibrium at a certain temperature.

**Change of vapor pressure with temperature:**

\* Super heated vapor = gas

\* vapor  $\Rightarrow$   $\downarrow$   
below  $T_c$

**Antoine Equation:** (predictions of vapor pressures)

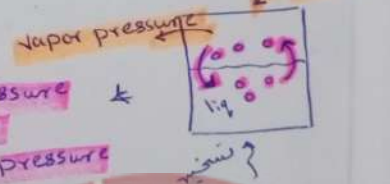
$\ln(P) = -\frac{A}{T+C} + B$   
 range of temperature  $\rightarrow$   $T$   
 absolute  $P$   
 absolute temperature  $\rightarrow$   $T$   
 $\rightarrow$  sub. vap.  $\rightarrow$   $P$

A, B and C are constants specific for each substance. T is the absolute temperature.

**Change of vapor pressure with pressure:**

Under normal conditions it is negligible.

where  $P_{\text{ext}}$  is the external pressure



\* الجزيئات الحرة في السطح تمتلك kinetic energy التي تمكنها من السطح وتتحول إلى vapor  
 \* عند التوازن (equilibrium) معدل البخر (التحول إلى vapor) = معدل التكثيف (التحول إلى liquid) ، إذا كان الفرق  
 بين  $P$  و  $P_{\text{vapor}}$  لا يساوي صفرًا (يعني يوجد فرق بين pure vapor و  $P_{\text{vapor}}$ ) ، فإن vapor حصة من الغازات  
 (saturated vapor) vapor pressure

### Boiling point:

It is the temperature at which the vapor pressure of a pure liquid becomes equal to the external pressure.

### Saturation:

A gaseous mixture in contact with a pure liquid is said to be saturated when the partial pressure of the liquid vapor in the mixture is equal to the vapor pressure of the liquid at a certain temperature. The vapor and liquid are at equilibrium.

### Phase Rule:

For a nonchemically reactive system at heterogeneous equilibrium.

$$F = C - P + 2$$

C = Number of components (chemical species)

P = Number of phases

F = Number of degrees of freedom (variance)

= Number of intensive properties (independent variables) that must be specified to completely fix the state of the system [In this course the intensive properties are T, P and concentration].

Ex:

$$F = 1 - 1 + 2 \\ = 2$$

For a pure gas  $F = 2 \Rightarrow$  T and P can be changed by small amounts without changing the state of the gas [no change in phase].

$$F = 1 - 3 + 2 \\ = 0$$

At Triple Point  $P = 3 \Rightarrow F = 0$

It is therefore impossible to have solid / liquid / vapor at equilibrium at conditions other than the triple point.



## 2.2 Vapor Liquid Equilibrium

### 2.2.1 Binary Vapor Liquid Equilibrium

**System:** vapor - Liquid mixture of components A and B.

$F = 2 - 2 + 2 = 2$  **Phase Rule:**  $C = 2$   $P = 2 \Rightarrow F = 2$

**Independent variables:** T, P and the concentration of one of the components in the vapor (y) and in the liquid (x).

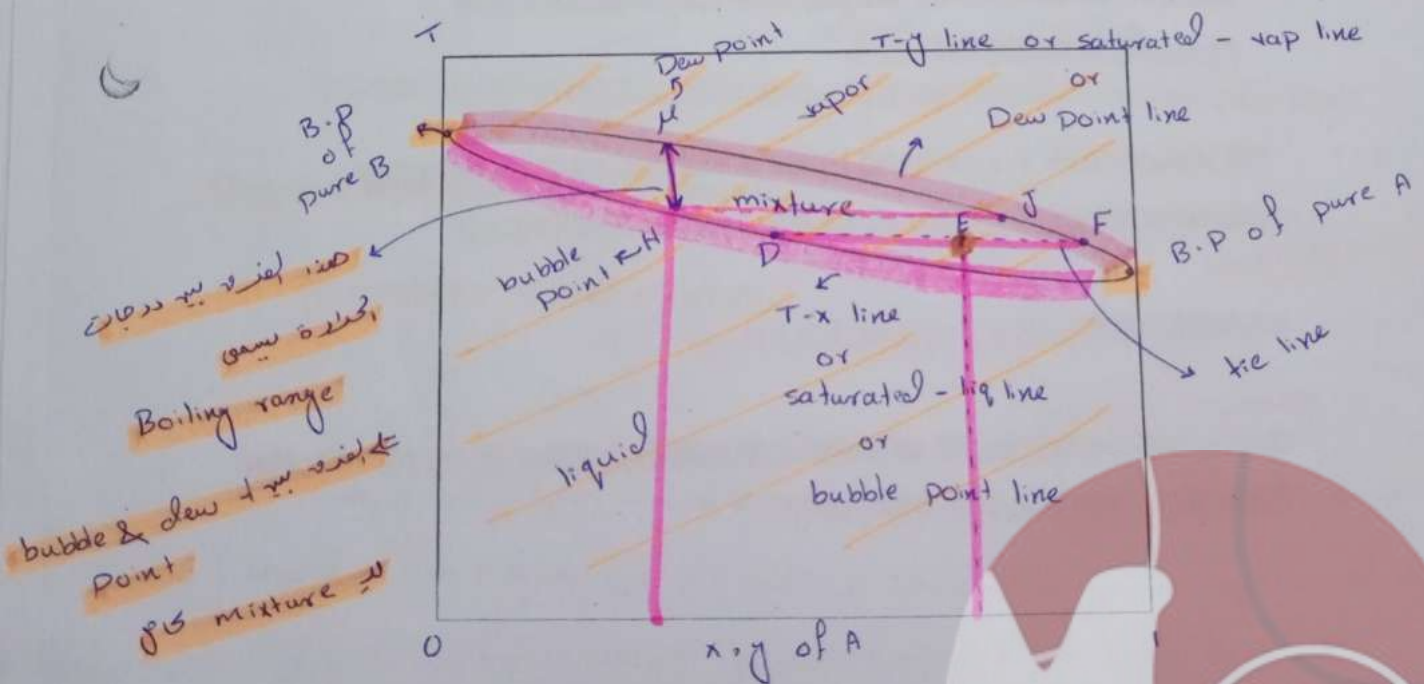
#### 2.2.1.1 Phase equilibrium diagrams:

- Usually pressure is specified (fixed)  $\Rightarrow$  one independent variable remains to be fixed.
- At constant pressure four isobaric phase equilibrium diagrams involving the variables T, x and y can be constructed:

• T-x • T-y • T-x-y • x-y

#### Constant Pressure Equilibrium:

• T-x • T-y • T-x-y diagrams:





- Liquid and vapor mixtures at equilibrium are at the same temperature and pressure and a horizontal line called a tie line joins their respective compositions.
- A mixture between two equilibrium curves such as E is a two phase mixture consisting of a liquid with composition at D and a vapor with composition at F.

The relative amounts of equilibrium phases are given by the lever rule

$$\frac{\text{Moles } D}{\text{Moles } F} = \frac{\overline{EF}}{\overline{DE}}$$

- Heating a liquid mixture composition (G):

**Closed system with constant pressure.** If the liquid is heated, the first bubble of vapor forms at (H) with vapor composition at (J) richer in the more volatile component (mvc). As more mixture is vaporized more vapor forms at the expense of liquid but the entire mass composition is still the same. The last drop of liquid mixture vaporizes at M and has composition at N. The boiling range of the mixture is HM.

**Open system:** If a solution is boiled in an open vessel, the vapor is richer in the more volatile component. As vaporization proceeds the liquid becomes leaner. The temperature and composition of saturated liquid move along HN as vaporization (distillation) proceeds.

**x vs. y diagram:**



### 2.2.1.2 Relative volatility

The greater the distance between the equilibrium curve and the diagonal, the greater the difference in VL composition and the easier the separation.

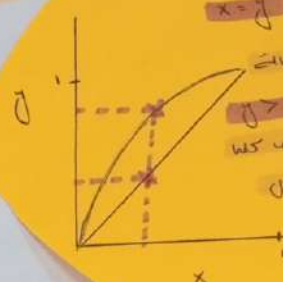
Relative volatility is a measure of the difference between vapor and liquid composition at equilibrium. It is called separation factor.

$$\alpha = \frac{\frac{y_A}{y_B}}{\frac{x_A}{x_B}} = \frac{\text{(moles A / moles B) in vapor}}{\text{(moles A / moles B) in liquid}}$$

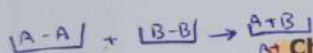
$$= \frac{y_A / (1 - y_A)}{x_A / (1 - x_A)}$$

$$= \frac{y_A (1 - x_A)}{x_A (1 - y_A)}$$

$x = y \leftarrow \alpha = 1$  no separation  
 $\alpha > 1$  separable mixture



### 2.2.1.3 VLE for ideal solutions:



intermolecular forces between A-B  
 ↓ mixing  
 ↓ gives mixing low intermolecular forces between (A-A) & (B-B)

#### Characteristics of ideal solutions:

- Average inter molecular forces in the solution are unchanged upon mixing.
- No heat will be absorbed or released upon mixing.  $\Delta H = 0$
- Volume of solution varies linearly with composition [i.e. Total solution volume equals the sum of pure component volumes].  $V_{mix} = V_A + V_B$
- The total vapor pressure of solution varies linearly with composition.  $\Rightarrow$  Raoult's Law

Ideal solutions do not exist in reality. However, many solutions approach ideality. For example: adjacent members of homologous series of organic compounds. e.g. benzene in toluene.

#### Raoult's Law:

When the gas mixture in equilibrium with an ideal liquid solution also follows ideal gas law, the partial pressure of a solute gas is linearly proportional to its

$$P_i \propto x_i$$

at equilibrium

|           |                                 |
|-----------|---------------------------------|
| $y_i$ A+B | $\Rightarrow$ if ideal gas      |
| $P_i$ vap |                                 |
| $x_i$ A+B | $\Rightarrow$ if ideal solution |
| $P_i$ liq |                                 |

Raoult's law



mole fraction in the liquid phase. The constant of proportionality being the vapor pressure of the pure species at the system temperature.

$$P_i = P_i^s x_i$$

↑  
saturated vapor pressure of i

Equilibrium partial pressure    vapor pressure    mole fraction in liquid

It is seen that the solubility of a certain gas in an ideal solution is independent of the nature of the solvent.

$$\Rightarrow P_i = P_i^s x_i \Rightarrow x_i = \frac{P_i}{P_i^s} \Rightarrow \text{solubility of gas in solvent}$$

For a binary mixture we have:

$$\begin{aligned} P_A &= P_A^s x_A & P_B &= P_B^s (1 - x_A) \\ \text{Dalton's law} \Rightarrow P_i &= P_A + P_B \\ &= P_A^s x_A + P_B^s (1 - x_A) \end{aligned}$$

$$P_i = (P_A^s - P_B^s) x_A + P_B^s \Rightarrow \text{straight line equation}$$

↑                      ↑  
slope                      intercept

At constant temperature these are linear since  $P_A^s$  and  $P_B^s$  are constant.

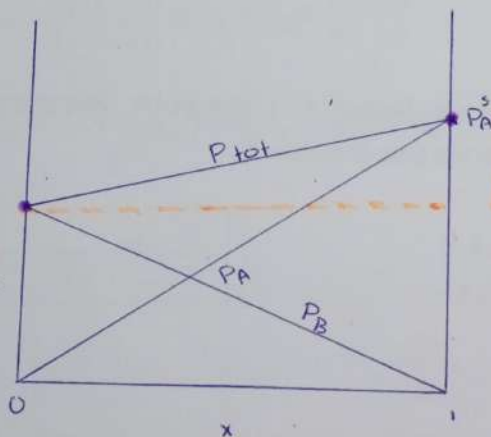
\* slope of  $P_{tot}$  =  $\frac{\Delta y}{\Delta x}$

$$\Delta y = P_A^s - P_B^s$$

$$\Delta x = 1 - 0$$

\* slope of  $P_{tot}$  =  $\frac{P_A^s - P_B^s}{1}$

\* intercept of  $P_{tot}$  =  $P_B^s$



$P_A$  is linear proportional with  $x$   
at  $x_A = 1 \Rightarrow P_A = P_A^s$

$P_B$  is linear proportional with  $x$   
at  $x_B = 1 \Rightarrow P_B = P_B^s$

$$P_{tot} = P_A + P_B$$

\*  $P_A^s$  &  $P_B^s \Rightarrow \text{constant} \Rightarrow$

Antoine equation can be used to calculate

For ideal solutions it is possible to calculate the vapor liquid equilibria from vapor pressure data of pure substances.





For Binary mixture: At a given temperature:  $T$  &  $P$

$$x_A + x_B = 1.0 \quad ; \quad y_A + y_B = 1.0$$

$$P_t = (P_A^s - P_B^s) x_A + P_B^s$$

$$x_A = \frac{P_t - P_B^s}{P_A^s - P_B^s}$$

$$x_A = \frac{(P_t - P_B^s)}{(P_A^s - P_B^s)}$$

$$P_t \Rightarrow$$

Relative volatility:

$$\alpha_{AB} = \frac{y_A (1 - x_A)}{x_A (1 - y_A)}$$

$$= \frac{P_A^s x_A (1 - x_A)}{P_t x_A \frac{P_B^s}{P_t} (1 - x_A)}$$

$\therefore$

$$\alpha_{AB} = \frac{P_A^s}{P_B^s}$$

$$y_A = \frac{P_A}{P_t} \Rightarrow$$

$$y_A = \left( \frac{P_A^s}{P_t} \right) x_A$$

$$P_A = P_A^s x_A \Rightarrow \text{Raoult's law}$$

$$y_A = \frac{P_A x_A}{P_t} = \left( \frac{P_A^s}{P_t} \right) x_A$$

\* range of temperature

B & A's w/ rili qaw mib



For Binary mixture: At a given temperature:  $T$  &  $P$

$$x_A + x_B = 1.0$$

$$y_A + y_B = 1.0$$

$$P_t = (P_A^s - P_B^s) x_A + P_B^s$$

$$x_A = \frac{P_t - P_B^s}{P_A^s - P_B^s}$$

$$x_A = \frac{(P_t - P_B^s)}{(P_A^s - P_B^s)}$$

$$P_t >$$

Relative volatility:

$$\alpha_{AB} = \frac{y_A (1 - x_A)}{x_A (1 - y_A)}$$

$$y_A = \left( \frac{P_A^s}{P_t} \right) x_A$$

$$P_A = P_A^s x_A \Rightarrow \text{Raoult's law}$$

$$y_A = \frac{P_A x_A}{P_t} = \left( \frac{P_A^s}{P_t} \right) x_A$$

\* range of temperature

B و A سب سے زیادہ زیادہ

more volatile  $\Rightarrow$   $P_A^s > P_B^s$

$$1 - y_A = y_B = \frac{P_B^s}{P_{tot}} \cdot x_B \Rightarrow \frac{P_A^s x_A}{P_t x_A} \cdot \frac{(1 - x_A)}{P_t (1 - x_A)}$$

$$\alpha_{AB} = \frac{y_A / y_B}{x_A / x_B} \Rightarrow \therefore$$

$$\alpha_{AB} = \frac{P_A^s}{P_B^s}$$

$$= \frac{(P_A^s x_A / P_t)}{(P_B^s x_B / P_t)} = \frac{P_A^s}{P_B^s}$$

$x_A / x_B$

ex. Compute the vapor liquid equilibria at constant pressure of 1 std atm for mixtures of n-heptane and n-octane.

Solution:

We need to establish  $t$ - $x$ , $y$  diagram at 1 atm in the temperature range between boiling points of the pure components.

$$\text{At 1 atm } BP_{n-C_7} = 98.4^\circ\text{C}$$

$$BP_{n-C_8} = 125.6^\circ\text{C}$$

$$x_A = \frac{P_t - P_B^s}{P_A^s - P_B^s}$$

$$y_A = \frac{P_A^s}{P_t} x_A$$

Antoine equation

Sample calculation

At  $110^\circ\text{C}$

$$P_A^s = 1050 \text{ mm Hg}$$

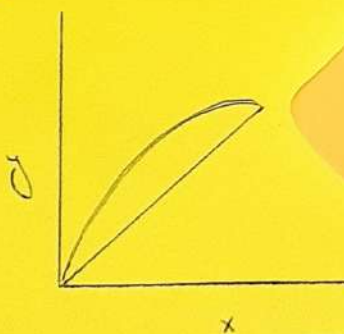
$$P_B^s = 484 \text{ mm Hg}$$

$$P_t = 760 \text{ mm Hg}$$

$T(x-y)$   $x, y$  سے متعلق

کل نقطہ  $(x, y)$  سے متعلق

$T(x-y)$   $T$ -line سے متعلق





$$x = \frac{760 - 484}{1050 - 484} = 0.487$$

$$y = \frac{1050}{760} \times 0.487 = 0.674$$

$\alpha_{AB}$ : at 110 °C

$$\alpha_{AB} = \frac{P_A^s}{P_B^s} = \frac{1050}{484} = 2.17$$

| Temp. °C | $P_A^s$ mm Hg | $P_B^s$ mm Hg | $X_A$ | $Y_A$ | $\alpha_{AB}$ |
|----------|---------------|---------------|-------|-------|---------------|
| 98.4     |               |               |       |       | 2.28          |
| 105.0    |               |               |       |       | 2.25          |
| 110.0    | 1050          | 484           | .487  | .674  | 2.17          |
| 115.0    |               |               |       |       | 2.14          |
| 120.0    |               |               |       |       | 2.08          |
| 125.0    |               |               |       |       | 2.02          |

Note:  $\alpha$  for ideal solutions does not vary considerably. An average value of  $\alpha$  can be calculated and used.

geometric Average  $\rightarrow$   $\sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$

$$\text{geometric average} = \sqrt[n]{\prod_{i=1}^n x_i}$$

$$= \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n}$$

$$\alpha_{AB} = \frac{y_A (1 - x_A)}{x_A (1 - y_A)} \Rightarrow y_A = \alpha_{AB} x_A \frac{(1 - y_A)}{(1 - x_A)}$$

$\Downarrow$

$$y_A [(1 - x_A) + \alpha_{AB} x_A] = \alpha_{AB} x_A \Leftarrow y_A (1 - x_A) = \alpha_{AB} x_A (1 - y_A)$$

$\Downarrow$

$$y_A = \frac{\alpha_{AB} x_A}{1 + x_A (\alpha_{AB} - 1)}$$

$\alpha_{AB}$  هو متوسط  $x_A$  و  $y_A$   $\rightarrow$   $\alpha_{AB}$  هو متوسط  $x_A$  و  $y_A$

في  $x_A$  و  $y_A$  من 0 إلى 1  $\rightarrow$   $\alpha_{AB}$  هو متوسط  $x_A$  و  $y_A$

$\alpha_{AB}$  هو متوسط  $x_A$  و  $y_A$

Notes:

1. Raoult's Law can generally be applied to a component whose mole fraction in the liquid phase approaches unity or to solutions of components quite similar in chemical nature.
2. Equilibrium constant or an equilibrium distribution coefficient defined.

\* Ideal solution:

الحلويات له solution يكونا  $\rightarrow$   $\alpha_{AB}$  هو متوسط  $x_A$  و  $y_A$

- shape
- size
- chemical interaction

$$\text{at } x_A = 0 \Rightarrow y_A = 0$$

$$y_A = \frac{\alpha_{AB} x_A}{1 + x_A (\alpha_{AB} - 1)}$$

\* at  $x_A = 1$

$$y_A = \frac{\alpha_{AB}}{1 + \alpha_{AB} - 1} = 1$$

$$\therefore y_A = 1$$



$$P_i = P_i^s \cdot x_i$$

$$P_i = y_i \cdot P_t$$

$\Downarrow$

distribution coefficient  $\leftarrow K_{i, \text{Raoult}} = \frac{y_i}{x_i} = \frac{P_i^s}{P_t}$

or  
equilibrium constant

In general VLE data may be represented by:

$$y_i = k_i x_i$$

$$K_i = f(T, P, \text{composition})$$

It can be seen that

في حالت بكون فيه  $k > 1$   
تثبت يعني لا يفصل ولا شيء

$k > 1$   
volatile  $x_i < y_i$   
 $k = 1$   
 $x_i = y_i$   
 $k < 1$   
less volatile  $x_i > y_i$

$$\alpha_{ij} = \frac{K_i}{K_j} = \frac{\frac{y_i}{x_i}}{\frac{y_j}{x_j}} = \frac{y_i}{y_j} \cdot \frac{x_j}{x_i} = \frac{y_i (1 - x_i)}{x_i (1 - y_i)}$$

$\alpha_{ij} > 1$   $\alpha_{ij} = 1$   $\alpha_{ij} < 1$   
separation

#### 2.2.1.4 Actual solutions:

**Miscible binary mixtures:**  $\Rightarrow$  2 phases  $\Rightarrow$  في ذاتية كاطت ولا يفصل

Few binary miscible systems obey Raoult's law throughout the range of concentrations. Most systems deviate from Raoult's law to a lesser extent.

# For real mixtures  $P_i \neq P_i^s \cdot x_i$   
# but instead  $P_i = \gamma_i P_i^s x_i$

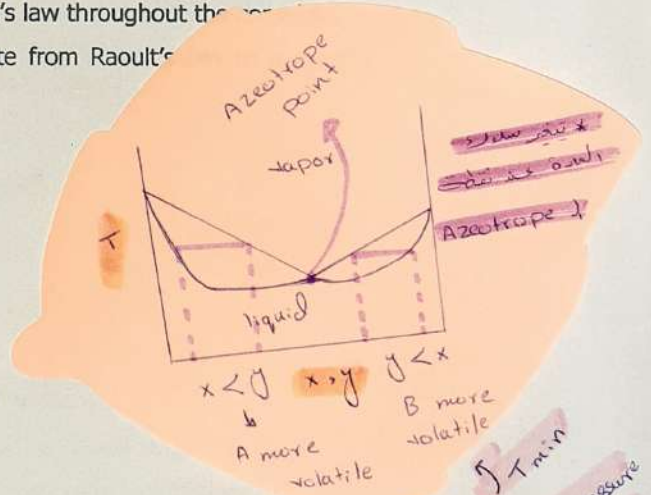
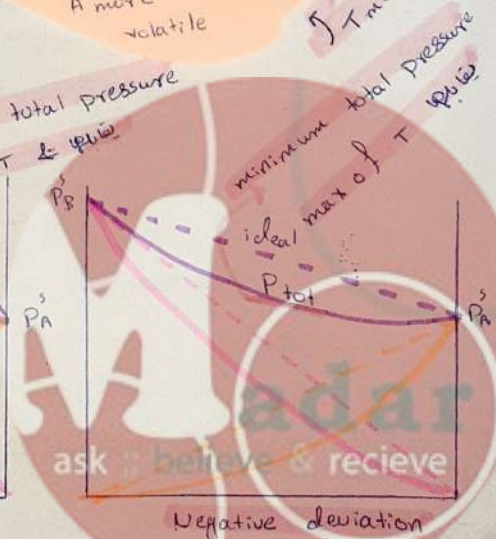
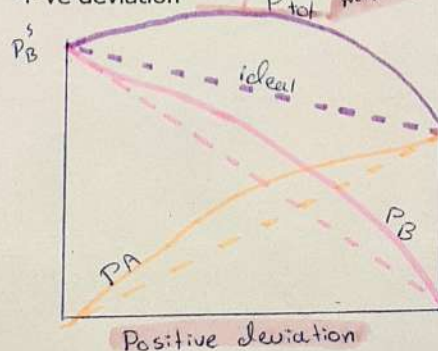
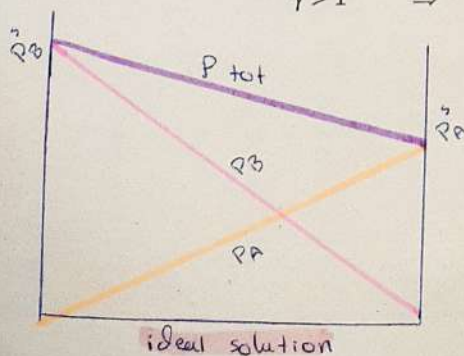
$\gamma_i$  = activity coefficient

$$= \frac{\text{real eqm partial pressure}}{\text{ideal eqm partial pressure}} = \frac{P_i \text{ real}}{P_i^s \cdot x_i}$$

therefore  $\gamma$  is a measure of the deviation from ideality

$\gamma = 1 \Rightarrow$  ideal solution

$\gamma > 1 \Rightarrow$  + ve deviation





$\gamma < 1 \Rightarrow$  - ve deviation

All miscible pairs can be classified into three general types:

**Type I:** systems whose total vapor pressure is intermediate between those of pure components.

Ex.  $\text{CCl}_4$  – cyclohexane  
 $\text{CCl}_4$  – benzene  
Benzene – toluene  
Water – methyl alcohol.

**Type II:** systems exhibiting a **maximum** in the total vapor pressure curve. They exhibit positive deviation from ideality and they form **minimum boiling point** mixtures.

Ex. Carbon disulfide – acetone, benzene – cyclohexane, water – ethyl alcohol.

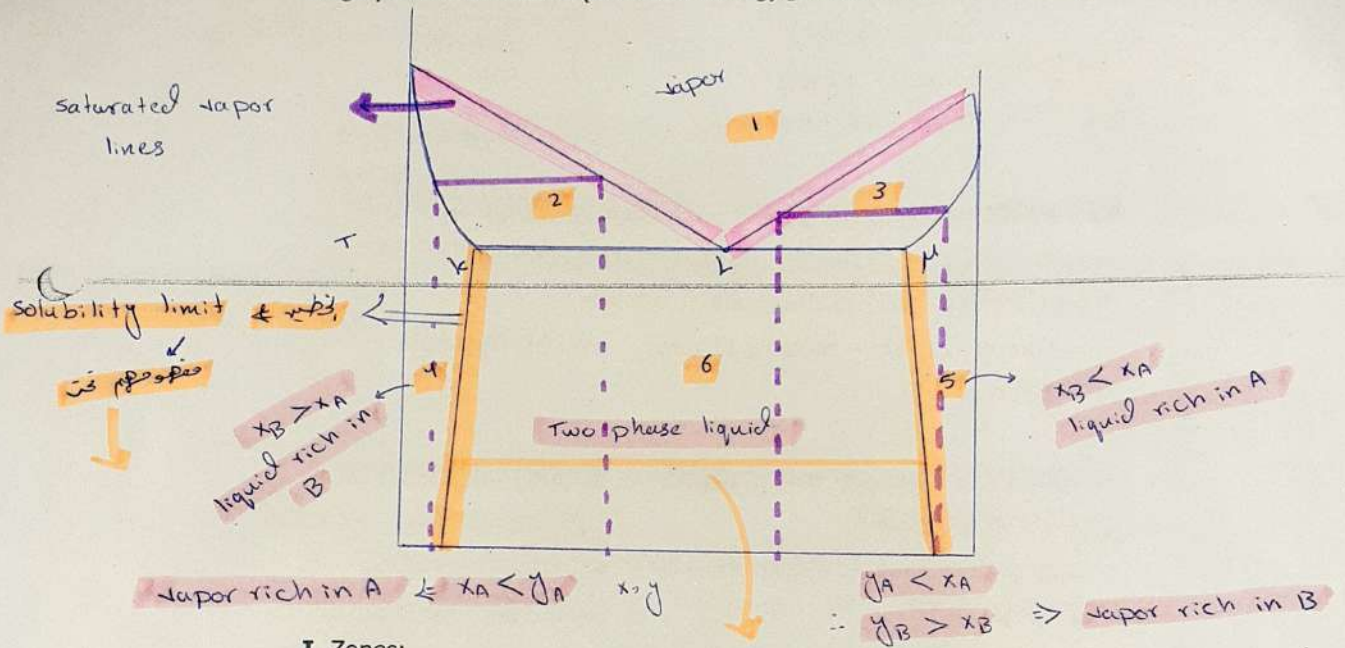
**Type III:** systems exhibiting a minimum in the total vapor pressure curve. They exhibit negative deviation from ideality and they **form maximum boiling point** mixtures.





## Partially Miscible Binary Mixtures:

Some systems do not dissolve completely in all proportions. They exhibit large positive deviations (minimum boiling) [Forces of repulsion  $\rightarrow 2\phi$ ]



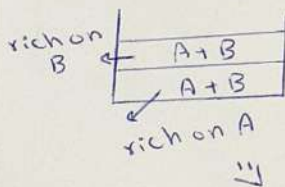
### I. Zones:

1. all vapor
2. vapor and liquid in equilibrium with vapor richer in A
3. vapor and liquid in equilibrium with vapor richer in B
4. Liquid solution richer in B (solution of A in B)
5. Liquid solution richer in A (solution of B in A)

is constant at given T

6. Two-phase mixtures. Each phase represents a solution of one component in the other. At constant temperature the composition of each phase is constant; however the proportions of phases are different.

II. For homogeneous liquids (zones 4 and 5), the vapor-liquid equilibrium phenomena are normal.



\* solubility limit =

ما قبل نقطة يكون في ذاتية حالة  
للحالة بعض الحالات  
one phase  
two phase

soluble A in B

soluble B in A

solvent rich

ask :: believe & recieve



مختلطة، وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq) وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq) وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq)

III. Two phase mixtures within the range KM will boil at the temperature of line KM, and they all give rise to the same vapor of composition L in equilibrium with two liquids at K and M which are also in equilibrium with each other.

IV. A two-phase mixture of average composition L which produces a vapor of the same composition is sometimes called a **heteroazeotrope**.

V. Phase rule for systems at temperature KM:

$$C = 2 \quad P = 3 \quad F = C - P + 2 \Rightarrow F = 1$$

∴ If pressure is set, everything else is specified.

مختلطة، وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq) وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq) وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq)

independently

properties of each

phase not affect

on other phase

مختلطة، وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq) وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq) وبنفس الطريقة تكون درجة حرارة الغليان (vap - liq)

### IMMISCIBLE BINARY MIXTURES (heterogeneous solutions)

Immiscible liquids are mutually insoluble – addition of one liquid to the other does not affect the properties of either liquid. Each will behave independently and will exert its own vapor pressure corresponding to the pure liquid at a given temperature.

$$P_t = P_A^s + P_B^s$$

The mixture will boil at a certain temperature when the sum of the vapor pressures equals the external pressure. The vapor composition is:

$$y_A = \frac{P_A^s}{P_t} \rightarrow \text{partial pressure of A}$$

\* boiling point  $\Rightarrow$  when:

$$P_t = P_{\text{external}}$$

Immiscible  $\Leftarrow$  A & B liq. + liq.

(miscible)  $\Leftarrow$  A & B vap. + liq.

$$P_B^s + P_A^s \rightarrow \text{mixture} \rightarrow \text{vapor}$$

ask & recieve





### Notes:

1. The boiling point of any mixture of two immiscible liquids is less than the boiling point of either of the two liquids.

نقطة الغليان لأي خليط من سائلين غير قابلين للامتزاج أقل من نقطة الغليان لأي من السائلين.

2. Boiling point of all possible mixtures of two immiscible liquids is the same (no change in vapor pressure with composition).  $\Rightarrow$

$$P_t = P_A^s + P_B^s$$

function on T

3. At any boiling point, the vapor composition remains constant and is independent of overall liquid composition.

$$P \propto \frac{1}{T}$$

$$y_i = \frac{P_i^s}{P_t}$$

ليكون التركيب البخاري ثابتاً في نقطة الغليان

### 2.2.2 VLE for multi-component systems: $n > 2$

- Graphical presentation of more than three component systems is extremely complex. or it's impossible

- Many of the multi-component systems of industrial importance

can be considered nearly ideal solutions. Raoult's law can be applied and vapor liquid equilibrium data can be calculated from pure component data.

equilibrium data can be calculated from binary

For multi-component systems, equilibrium data is represented by means of

nts.

nts.

For ideal solutions at moderate pressures,  $k_i$  is independent of composition. It depends only on temperature and total pressure.

At equilibrium:

Dalton's law

$$k_i = \frac{y_i}{x_i} = \frac{P_i/P_t}{P_i/P_i^s} = \frac{P_i^s}{P_t}$$

depends on  $P_{tot}$  &  $T$  composition

Raoult's law

It is also customary to choose a convenient reference component (in many cases heaviest component) to which data can be referred:

# De priester chart  $\rightarrow$  مخطط دى بريستر

reference component  $\rightarrow$  مكون مرجعي

T

adar ask :: believe & recieve



At equilibrium:

$$\alpha_{ij} = \frac{(y_i/x_i)}{(y_j/x_j)} = \frac{k_i}{k_j} \rightarrow \text{reference}$$

$$\alpha_{ij} = \frac{P_i^s/P_t}{P_j^s/P_t} = \frac{P_i^s}{P_j^s}$$

VLE For a given vapor-liquid system at equilibrium:

$$\alpha_{ij} = \frac{y_i/x_j}{x_i/x_j}$$

$$\alpha_{ij} = \frac{y_i/x_i}{y_j/x_j}$$

$$y_i = \frac{\alpha_{ij} x_i}{(x_j/y_j)}$$

$$x_i = \frac{y_i/\alpha_{ij}}{(y_j/x_j)}$$

$$\sum_{i=1}^n y_i = \sum_{i=1}^n \frac{\alpha_{ij} x_i}{(x_j/y_j)} = 1$$

\* درجه غلیظ mixture یکنواخت

$$\sum x_i = \sum \frac{y_i/\alpha_{ij}}{y_j/x_j} = 1.0$$

$$\left(\frac{x_j}{y_j}\right) = \sum_{i=1}^n \alpha_{ij} x_i$$

مقدار غلیظ mixture یکنواخت

مقدار غلیظ

$$\frac{y_j}{x_j} = \frac{1}{\sum \alpha_{ij} x_i}$$

$$\left(\frac{y_j}{x_j}\right) = \sum y_i/\alpha_{ij}$$

substitute for  $\frac{y_j}{x_j} = \frac{y_i}{\alpha_{ij} x_i}$

$$\# y_i = \frac{\alpha_{ij} x_i}{\sum \alpha_{ij} x_i}$$

\* مقدار غلیظ mixture یکنواخت

از این معادله می توانیم

تعیین نقطه جوش و نقطه جوش را

useful for Bubble point calculation

مثلاً  $\epsilon_j = 1$  و  $\epsilon_j$  نزدیک به 1

در محاسبات Bubble point  $T \geq x_i$

در محاسبات Bubble point  $T \geq x_i$

در محاسبات Bubble point  $T \geq x_i$

در محاسبات Bubble point  $T \geq x_i$

useful for Dew point calculation

\* مقدار غلیظ mixture یکنواخت

ask :: believe & recieve



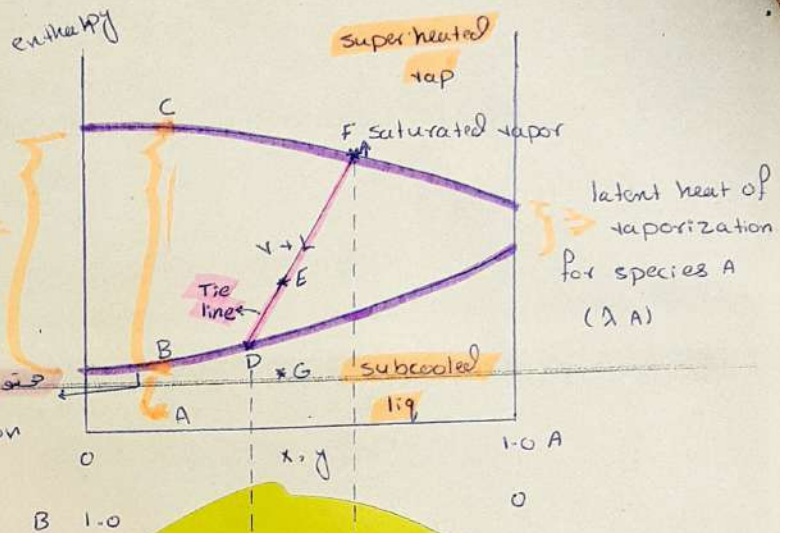
### 2.2.3 Enthalpy concentration diagrams:

Binary VLE data can also be represented by enthalpy concentration diagrams [useful in showing energy changes as well as composition changes].

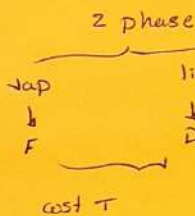
$\lambda_B > \lambda_A$   
more volatile & A than B

latent heat of vaporization for species B ( $\lambda_B$ )

enthalpy of saturated liquid as function of composition ( $x$  &  $y$ )



enthalpy of mixture



components:

solution:

$H_L$  = sensible heat and heat of mixing

$$H_L = C_L (t_L - t_0) M_{av} + \Delta H_S$$

$C_L$ : heat capacity of solution. For ideal solution it is the weighted average of the pure components heat capacities. (energy / mass. deg.). (average heat capacity of the solution)

$M_{av}$ : average molecular weight.

$t_L$ : liquid temperature (for a saturated liquid it is the bubble point).

$T_0$ : reference temperature.

$\Delta H_S$  = heat of solution at  $T_0$  for the given concentration.

enthalpy of heating mixture A to bubble point B +

latent heat of vaporization of mixture

absolute value of energy is 0 because the energy is a relative quantity

ask :: believe & recieve



### Saturated vapor:

We can assume pure liquids are heated separately to the dew point  $t_v$ , each vaporized at this point and vapors are mixed.

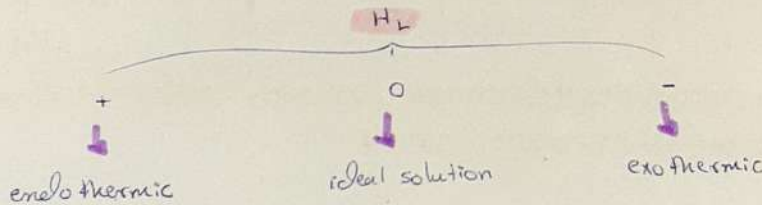
$H_v = \text{Enthalpy of Vap A} + \text{Enthalpy of Vap B}$   
 $H_v = y [C_{L,A} (t_v - t_0) M_A + \lambda_A M_A] + (1-y) [C_{L,B} (t_v - t_0) M_B + \lambda_B M_B]$

$\uparrow$   $\uparrow$   
 $\tau$  of vapor      mol fraction of B  
 mol fraction of the more volatile compound

$\lambda$ : latent heat of vaporization of pure substances at  $t_v$  [energy/mass]  
 $C_L$ : heat capacity of pure liquid energy/mass deg.

### ⇒ heat of mixing ( $H_L$ ) ⇒

هو المحتوى الحراري الذي يتم تحديده أو استنتاجه من قاعدة عند الخلط ، عندما يتم بيع مادة مع مواد أخرى ، فإن المحتوى الحراري الناتج يكون ناتج عن التفاعلات الجذبية بين المواد .



### ⇒ heat of solution ( $\Delta H_s$ ) ⇒

التغير في المحتوى الحراري الناتج من إذابة مادة صلبة (مذابة) في سائل (مذيب) (المحلول) (المذاب)



## 2.3 Gas – Liquid systems, Henry's Law

**System:** Gaseous mixture A+B

A to be absorbed by liquid S (solvent)  
B carrier gas.

**Thermodynamics variables:**  $P, T, x_A, x_B, x_g, y_A, y_B, y_s$

solvent (s) & gas mixture & A sol. & B carrier gas.

**Degrees of freedom:**  $C = 3 \quad P = 2 \Rightarrow F = 3 - 2 + 2 = 3$

Therefore if three variables are specified, all other variables are determined. If B is insoluble in S, and S has negligible vapor pressure, then the remaining variables are:  $P, T, x_A$  and  $y_A$ .

$x_B = 0$

$y_s = 0$

$y_B = 1 - y_A$

$x_s = 1 - x_A$

gas absorption & was in water & atmosphere.

solvent & gas mixture & B carrier gas.

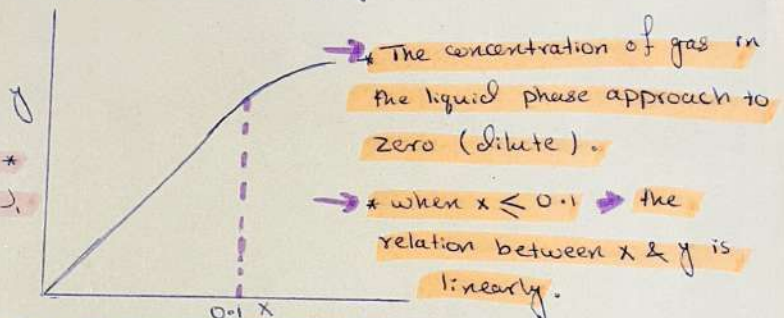
**vap - liq equilb:**

2 phase

**gas - liq equilb:**

2 phase

$x_B = 0 \rightarrow$  liquid



### Henry's Law:

$x \ll 0.1$

$$P_A = k_h \cdot x_A$$

Partial pressure of A      Henry's law constant      mole fraction of A in liquid

almost all liquids will deviate above mole fractions of 0.1.

Equilibrium constant:

$$K_A = \frac{y_A}{x_A} = \frac{k_h}{P_i}$$





For Binary mixture: At a given temperature:  $2P$

$$x_A + x_B = 1.0$$

$$y_A + y_B = 1.0$$

$$P_t = (P_A^S - P_B^S) x_A + P_B^S$$

$$X_A = \frac{P_t - P_B^s}{P_A^s - P_B^s}$$

$$X_A = \frac{(P_t - P_B^S)}{(P_A^S - P_B^S)}$$

$$\Rightarrow P_A \neq P_B$$

$$y_A = \frac{P_A}{P_t}$$

$$y_A = \left( \frac{P_A^S}{P_t} \right) x_A$$

$$P_A = P_A^s \times A \Rightarrow \text{Raoult's law}$$

$$Y_A = \frac{P_A^s X_A}{P_t} = \left( \frac{P_A^s}{P_t} \right) X_A$$

\* range of temperature

B و A بسوی رابطه مستقیم می باشد

$$1 - y_A = y_B = \frac{P_B^s}{P_{\text{tot}}} \cdot x_B \leq \frac{P_A^s x_A}{P_i x_A} \frac{P_B^s}{P} (1 - x_A)$$

$$\alpha_{AB} = \frac{y_A / y_B}{x_A / x_B} \quad \leftarrow \therefore$$

$$\alpha_{AB} = \frac{P_A^S}{P_B^S}$$

$$= \frac{(P_A^s \times x_A / P_t)}{(P_B^s \times x_B / P_t)} = \frac{P_A^s}{P_B^s}$$

ex. Compute the vapor liquid equilibria at constant pressure of 1 std atm for mixtures of n-heptane and n-octane.

**Solution:**

We need to establish  $t-x,y$  diagram at 1 atm in the temperature range between boiling points of the pure components.

At 1 atm  $BP_{\text{n-C}_7} = 98.4^\circ\text{C}$

$$BP_{\text{тн-C}_8} = 125.6^\circ\text{C}$$

$$X_A = \frac{P_i - P_B^S}{P_A^S - P_B^S}$$

$$y_A = \frac{P_A^s}{P_1} x_A$$

### Sample calculation

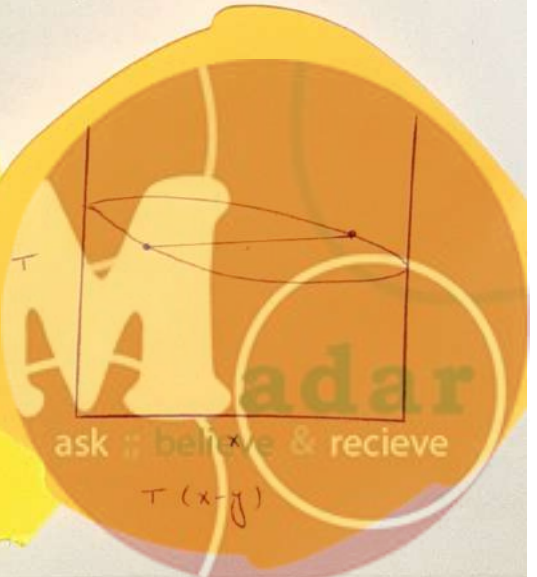
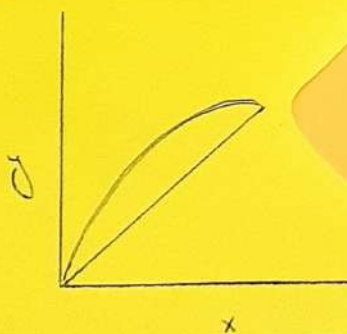
At 110 °C

$$P_A^S = 1050 \text{ mm Hg}$$
$$P_B^S = 484 \text{ mm Hg}$$
 $P_t = 760 \text{ mm Hg}$ 

فرضه  $x, y$  فضاى  $T(x-y)$

كل نقطة  $P$  ممتدة  $(x, y)$  تعتبر

$T(x-y)$  is  $\perp$  Tie line





$$x = \frac{760 - 484}{1050 - 484} = 0.487$$

$$y = \frac{1050}{760} \times 0.487 = 0.674$$

$\alpha_{AB}$ : at 110 °C

$$\alpha_{AB} = \frac{P_A^s}{P_B^s} = \frac{1050}{484} = 2.17$$

| Temp. °C | $P_A^s$ mm Hg | $P_B^s$ mm Hg | $X_A$ | $Y_A$ | $\alpha_{AB}$ |
|----------|---------------|---------------|-------|-------|---------------|
| 98.4     |               |               |       |       | 2.28          |
| 105.0    |               |               |       |       | 2.25          |
| 110.0    | 1050          | 484           | .487  | .674  | 2.17          |
| 115.0    |               |               |       |       | 2.14          |
| 120.0    |               |               |       |       | 2.08          |
| 125.0    |               |               |       |       | 2.02          |

Note:  $\alpha$  for ideal solutions does not vary considerably. An average value of  $\alpha$  can be calculated and used.

geometric Average  $\rightarrow$  متوسط هندسي \*

$$\text{geometric average} = \sqrt[n]{\prod_{i=1}^n x_i}$$

$$= \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdots x_n}$$

$$\alpha_{AB} = \frac{y_A (1 - x_A)}{x_A (1 - y_A)} \Rightarrow y_A = \alpha_{AB} x_A \frac{(1 - y_A)}{(1 - x_A)}$$

$\Downarrow$

$$y_A [(1 - x_A) + \alpha_{AB} x_A] = \alpha_{AB} x_A \Leftarrow y_A (1 - x_A) = \alpha_{AB} x_A (1 - y_A)$$

$\Downarrow$

$$y_A = \frac{\alpha_{AB} x_A}{1 + x_A (\alpha_{AB} - 1)}$$

$\alpha_{AB}$  هو متوسط  $x_A$  و  $y_A$   $\rightarrow$  متوسط هندسي \*

في حالة  $\alpha_{AB}$  قيمته (10 - 1)  $\rightarrow$   $x_A$  و  $y_A$  قريبين

$\rightarrow$   $y_A$  قريب من  $x_A$

Notes:

1. Raoult's Law can generally be applied to a component whose mole fraction in the liquid phase approaches unity or to solutions of components quite similar in chemical nature.
2. Equilibrium constant or an equilibrium distribution coefficient defined.

ideal soln!  $\rightarrow$  مثالي

\* Ideal solution:

:  $\rightarrow$  solution له شكل و حجم و تفاعل كيميائي

- shape
- size
- chemical interaction

$$\text{at } x_A = 0$$

$$\Rightarrow y_A = 0$$

$$y_A = \frac{\alpha_{AB} x_A}{1 + x_A (\alpha_{AB} - 1)}$$

\* at  $x_A = 1$

$$y_A = \frac{\alpha_{AB}}{1 + \alpha_{AB} - 1} = 1$$

$$\therefore y_A = 1$$



$$P_i = P_i^s \cdot x_i$$

$$P_i = y_i \cdot P_t$$

$\Downarrow$

distribution coefficient  $\leftarrow K_{i, \text{Raoult}} = \frac{y_i}{x_i} = \frac{P_i^s}{P_t}$

or  
equilibrium constant

In general VLE data may be represented by:

$$y_i = k_i x_i$$

$$K_i = f(T, P, \text{composition})$$

It can be seen that

في حالت بكون ضغط  $k$   
 ثابت يعني لا يتغير  $k$   
 شي

$k > 1$   
volatile المادة  
 $x_i < y_i$   
 $k = 1$   
 $x_i = y_i$   
 $k < 1$   
less volatile المادة  
 $x_i > y_i$

$$\alpha_{ij} = \frac{K_i}{K_j} = \frac{\frac{y_i}{x_i}}{\frac{y_j}{x_j}} = \frac{y_i}{y_j} \cdot \frac{x_j}{x_i} = \frac{y_i (1 - x_i)}{x_i (1 - y_i)}$$

$\alpha_{ij} > 1$  separation  
 $\alpha_{ij} = 1$   
 $\alpha_{ij} < 1$

#### 2.2.1.4 Actual solutions:

**Miscible binary mixtures:**  $\Rightarrow$  2 phases يكون في ذاتية كاطت

Few binary miscible systems obey Raoult's law throughout the range of concentrations. Most systems deviate from Raoult's law to a lesser extent.

# For real mixtures  $P_i \neq P_i^s \cdot x_i$   
# but instead  $P_i = \gamma_i P_i^s x_i$

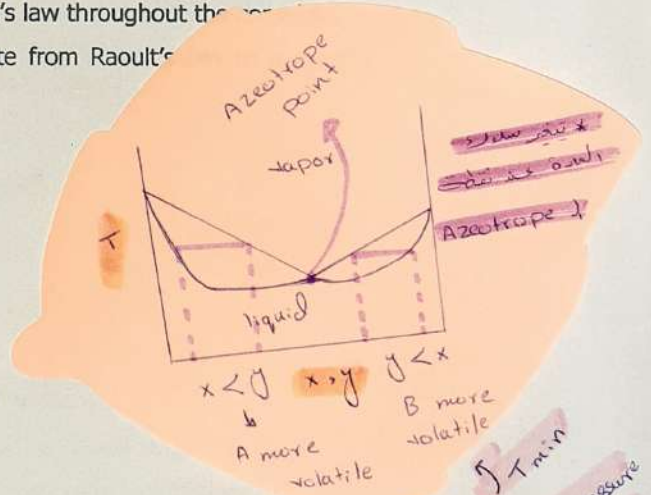
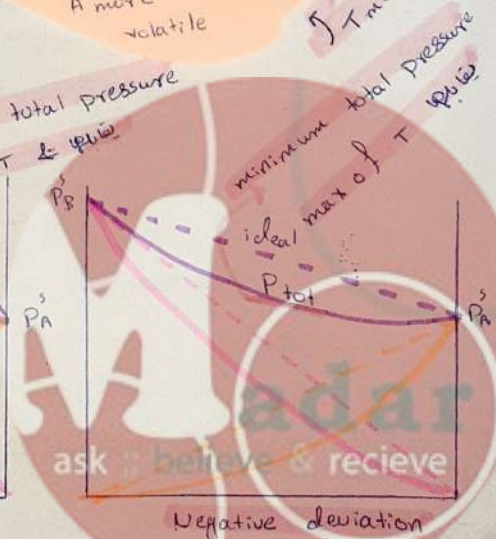
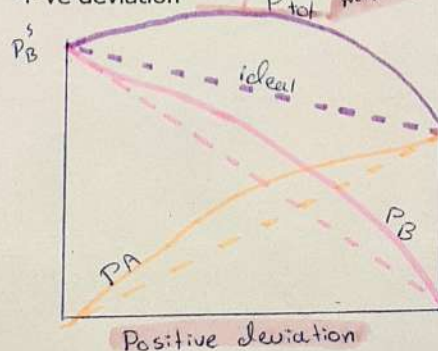
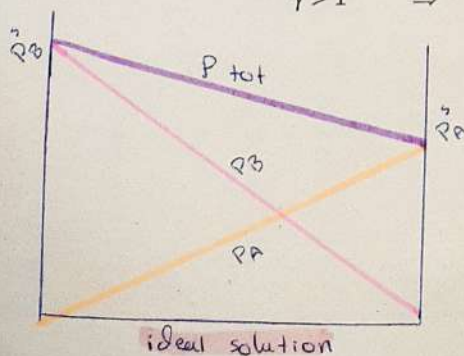
$\gamma_i$  = activity coefficient

$$= \frac{\text{real eqm partial pressure}}{\text{ideal eqm partial pressure}} = \frac{P_i \text{ real}}{P_i^s \cdot x_i}$$

therefore  $\gamma$  is a measure of the deviation from ideality

$\gamma = 1 \Rightarrow$  ideal solution

$\gamma > 1 \Rightarrow$  + ve deviation





$\gamma < 1 \Rightarrow$  - ve deviation

All miscible pairs can be classified into three general types:

**Type I:** systems whose total vapor pressure is intermediate between those of pure components.

Ex.  $\text{CCl}_4$  – cyclohexane  
 $\text{CCl}_4$  – benzene  
Benzene – toluene  
Water – methyl alcohol.

**Type II:** systems exhibiting a **maximum** in the total vapor pressure curve. They exhibit positive deviation from ideality and they form **minimum boiling point** mixtures.

Ex. Carbon disulfide – acetone, benzene – cyclohexane, water – ethyl alcohol.

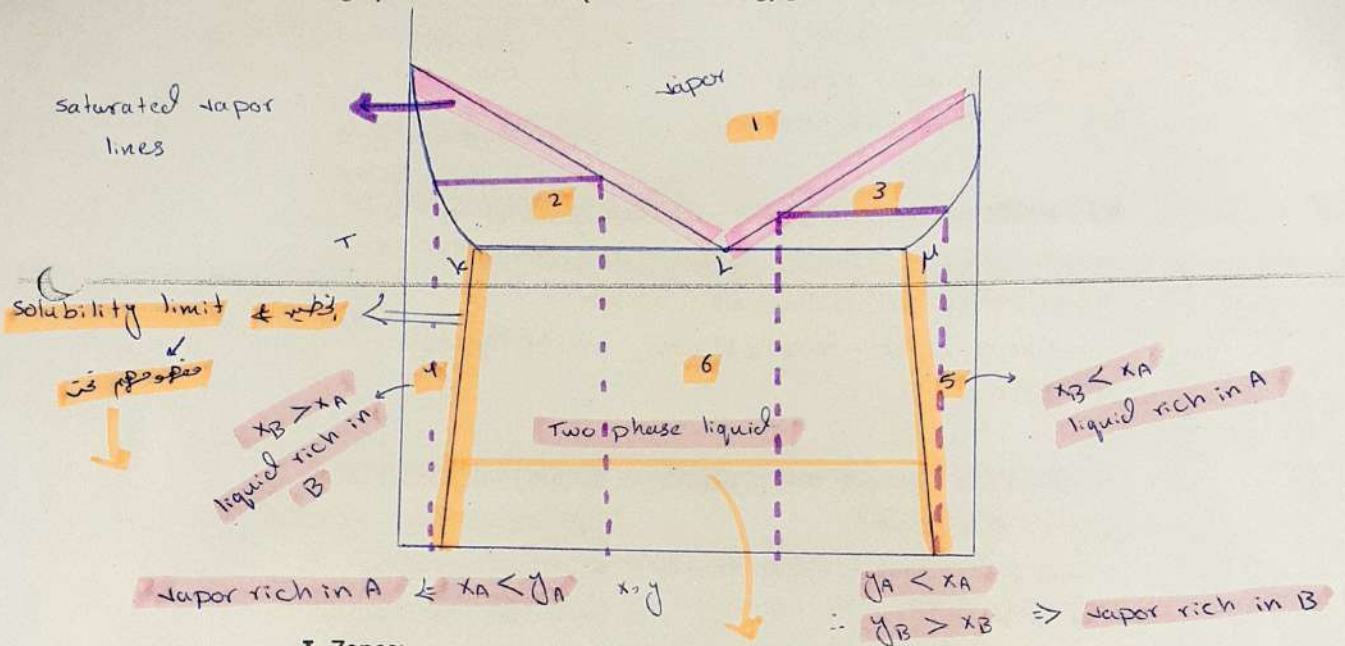
**Type III:** systems exhibiting a minimum in the total vapor pressure curve. They exhibit negative deviation from ideality and they **form maximum boiling point** mixtures.





## Partially Miscible Binary Mixtures:

Some systems do not dissolve completely in all proportions. They exhibit large positive deviations (minimum boiling) [Forces of repulsion  $\rightarrow 2\phi$ ]



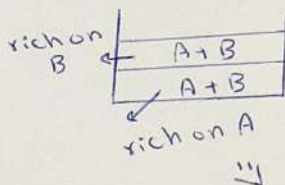
### I. Zones:

1. all vapor
2. vapor and liquid in equilibrium with vapor richer in A
3. vapor and liquid in equilibrium with vapor richer in B
4. Liquid solution richer in B (solution of A in B)
5. Liquid solution richer in A (solution of B in A)

Tie line  $\Rightarrow$  the equilibrium composition is constant at given T

6. Two-phase mixtures. Each phase represents a solution of one component in the other. At constant temperature the composition of each phase is constant; however the proportions of phases are different.

II. For homogeneous liquids (zones 4 and 5), the vapor-liquid equilibrium phenomena are normal.



\* solubility limit =

ما قبل نقطة يكون في ذاتية حالت  
للحالة بعض في شكلها  
one phase  
عند نقطة يكون في حالتها  
two phase

soluble A in B

soluble B in A

solvent rich

ask :: believe & recieve



مختلطة mixture، وبقول ل.ق. (vap - liq) وبعدها درجة الحرارة تكون درجة خلط mixture

III. Two phase mixtures within the range KM will boil at the temperature of line KM, and they all give rise to the same vapor of composition L in equilibrium with two liquids at K and M which are also in equilibrium with each other.

IV. A two-phase mixture of average composition L which produces a vapor of the same composition is sometimes called a **heteroazeotrope**.

V. Phase rule for systems at temperature KM:

$$C = 2 \quad P = 3 \quad F = C - P + 2 \Rightarrow F = 1$$

∴ If pressure is set, everything else is specified.

تختلف باختلاف الضغط ودرجة الحرارة

independently

properties of each

phase not affect

on other phase

مختلطة

### IMMISCIBLE BINARY MIXTURES (heterogeneous solutions)

Immiscible liquids are mutually insoluble – addition of one liquid to the other does not affect the properties of either liquid. Each will behave independently and will exert its own vapor pressure corresponding to the pure liquid at a given temperature.

$$P_t = P_A^s + P_B^s$$

The mixture will boil at a certain temperature when the sum of the vapor pressures equals the external pressure. The vapor composition is:

$$y_A = \frac{P_A^s}{P_t} \rightarrow \text{partial pressure of A}$$

\* boiling point  $\Rightarrow$  when:

$$P_t = P_{\text{external}}$$

Immiscible  $\Leftarrow$  A & B liq. + liq.

(المختلطة بالجوهر)  $\Leftarrow$  miscible  $\Leftarrow$  A & B vap + liq.

$P_B^s + P_A^s$  mixture  $\rightarrow$  vapor

المختلطة = مختلطة





### Notes:

1. The boiling point of any mixture of two immiscible liquids is less than the boiling point of either of the two liquids.

نقطة الغليان لأي خليط من سائلين غير قابلين للامتزاج أقل من نقطة غليان أحدهما.

2. Boiling point of all possible mixtures of two immiscible liquids is the same (no change in vapor pressure with composition).  $\Rightarrow$

$$P_t = P_A^s + P_B^s$$

$$P \propto \frac{1}{T}$$

3. At any boiling point, the vapor composition remains constant and is independent of overall liquid composition.

function on T

$$y_i = \frac{P_i^s}{P_t}$$

ليس له علاقة بتركيب السائل

### 2.2.2 VLE for multi-component systems: $n > 2$

- Graphical presentation of more than three component systems is extremely complex. or it's impossible

- Many of the multi-component systems of industrial importance

can be considered nearly ideal solutions. Raoult's law can be applied and vapor liquid equilibrium data can be calculated from pure component data.

equilibrium data can be calculated from binary

For multi-component systems, equilibrium data is represented by means of

nts.

nts.

For ideal solutions at moderate pressures,  $k_i$  is independent of composition. It depends only on temperature and total pressure.

At equilibrium:

Dalton's law

$$k_i = \frac{y_i}{x_i} = \frac{P_i/P_t}{P_i/P_i^s} = \frac{P_i^s}{P_t}$$

$$\rightarrow$$

depend on

$P_{tot}$  &

$T$

$\rightarrow$

composition

$\rightarrow$

composition

$\rightarrow$

composition

$\rightarrow$

composition

Raoult's law

It is also customary to choose a convenient reference component (in many cases heaviest component) to which data can be referred:

# De priester chart  $\rightarrow$

نوع من كارت

reference

في المثال السابق

T

T

T

T

T

T

T

T

T

T

adar

ask :: believe & recieve



At equilibrium:

$$\alpha_{ij} = \frac{(y_i/x_i)}{(y_j/x_j)} = \frac{k_i}{k_j} \rightarrow \text{reference}$$

$$\alpha_{ij} = \frac{P_i^s/P_t}{P_j^s/P_t} = \frac{P_i^s}{P_j^s}$$

VLE For a given vapor-liquid system at equilibrium:

$$\alpha_{ij} = \frac{y_i/x_j}{x_i/x_j}$$

$$\alpha_{ij} = \frac{y_i/x_i}{y_j/x_j}$$

$$y_i = \frac{\alpha_{ij} x_i}{(x_j/y_j)}$$

$$x_i = \frac{y_i/\alpha_{ij}}{(y_j/x_j)}$$

$$\sum_{i=1}^n y_i = \sum_{i=1}^n \frac{\alpha_{ij} x_i}{(x_j/y_j)} = 1$$

\* درجه غلیظ mixture را می توانیم

$$\sum x_i = \sum \frac{y_i/\alpha_{ij}}{y_j/x_j} = 1.0$$

$$\left( \frac{x_j}{y_j} \right) = \sum_{i=1}^n \alpha_{ij} x_i$$

مقدار غلیظ mixture را می توانیم

مقدار غلیظ mixture را می توانیم

$$\frac{y_j}{x_j} = \frac{1}{\sum \alpha_{ij} x_i}$$

$$\left( \frac{y_j}{x_j} \right) = \sum y_i/\alpha_{ij}$$

substitute for  $\frac{y_j}{x_j} = \frac{y_i}{\alpha_{ij} x_i}$

$$\# y_i = \frac{\alpha_{ij} x_i}{\sum \alpha_{ij} x_i}$$

\* مقدار غلیظ mixture را می توانیم

از این معادله می توانیم

بسیار T به T با boiling point & boiling point

جمله به هم می آید

useful for Bubble

point calculation

مثال:  $\epsilon_j = 1$

از این معادله می توانیم

\* مقدار غلیظ mixture را می توانیم

در محاسبات Bubble point

T و x

مقدار

در محاسبات Bubble point

T bubble

مثال:  $\epsilon_j = 1$

useful for Dew  
point calculation

\* مقدار غلیظ mixture را می توانیم



ask :: believe & recieve



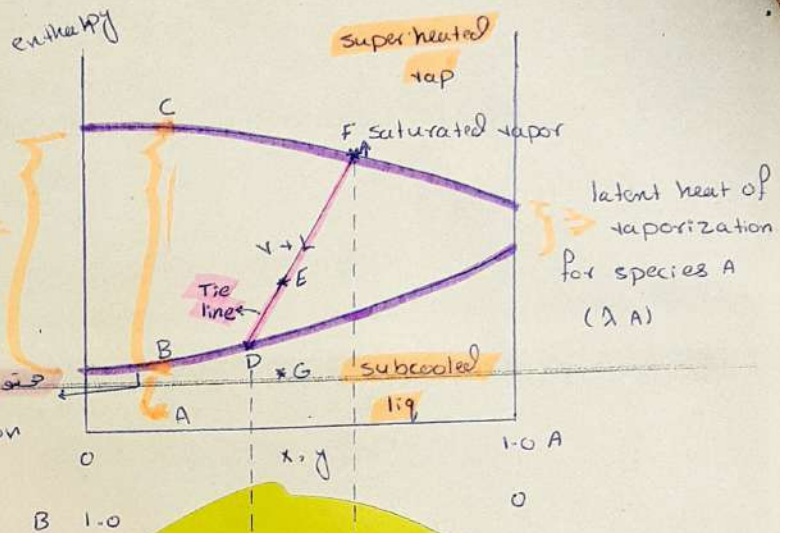
## 2.2.3 Enthalpy concentration diagrams:

Binary VLE data can also be represented by enthalpy concentration diagrams [useful in showing energy changes as well as composition changes].

$\lambda_B > \lambda_A$   
more volatile & A than B

latent heat of vaporization for species B ( $\lambda_B$ )

saturated liquid, enthalpy is as function of composition ( $x$  &  $y$ )



of as function of composition ( $x$  &  $y$ )



solution:

$H_L$  = sensible heat and heat of mixing

$$H_L = C_L (t_L - t_0) M_{av} + \Delta H_S$$

$C_L$ : heat capacity of solution. For ideal solution it is the weighted average of the pure components heat capacities. (energy / mass. deg.). (average heat capacity of the solution)

$M_{av}$ : average molecular weight.

$t_L$ : liquid temperature (for a saturated liquid it is the bubble point).

$T_0$ : reference temperature.

$\Delta H_S$  = heat of solution at  $T_0$  for the given concentration.

enthalpy of heating mixture A to bubble point B +

latent heat of vaporization of mixture

absolute value of energy is 0 because the energy is a relative quantity

$$\mu_{av} = \sum \text{mol fraction} \cdot \mu_w$$

$$\mu_{av} = x_1 \mu_{w1} + x_2 \mu_{w2} \dots$$

A & B are solvent &

B & C are solvent &

reference state is 0

ask :: believe & recieve



### Saturated vapor:

We can assume pure liquids are heated separately to the dew point  $t_v$ , each vaporized at this point and vapors are mixed.

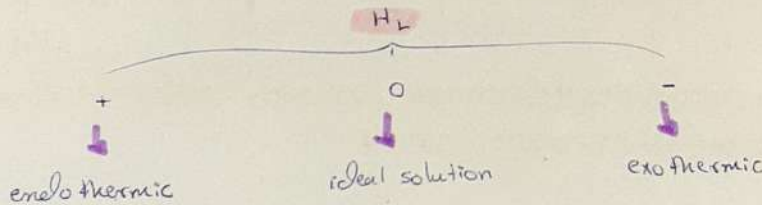
$H_v = \text{Enthalpy of Vap A} + \text{Enthalpy of Vap B}$   
 $H_v = y [C_{L,A} (t_v - t_0) M_A + \lambda_A M_A] + (1-y) [C_{L,B} (t_v - t_0) M_B + \lambda_B M_B]$

$\uparrow$   $\uparrow$   
 $\tau$  of vapor      mol fraction of B  
 mol fraction of the more volatile compound

$\lambda$ : latent heat of vaporization of pure substances at  $t_v$  [energy/mass]  
 $C_L$ : heat capacity of pure liquid energy/mass deg.

### ⇒ heat of mixing ( $H_L$ ) ⇒

هو المحتوى الحراري الذي يتم تحديده أو استنتاجه من قاعدة عند الخلط ، عندما يتم بيع مادة مع مواد أخرى ، فإن المحتوى الحراري الناتج يكون ناتج عن التفاعلات الجذبية بين المواد .



### ⇒ heat of solution ( $\Delta H_s$ ) ⇒

التغير في المحتوى الحراري الناتج من إذابة مادة صلبة (مذابة) في سائل (مذيب) (مذابة) (المذاب)





## 2.3 Gas – Liquid systems, Henry's Law

**System:** Gaseous mixture A+B

A to be absorbed by liquid S (solvent)  
B carrier gas.

**Thermodynamics variables:**  $P, T, x_A, x_B, x_g, y_A, y_B, y_s$

solvent (s) & gas mixture & A sol.

**Degrees of freedom:**  $C = 3 \quad P = 2 \Rightarrow F = 3 - 2 + 2 = 3$

Therefore if three variables are specified, all other variables are determined. If B is insoluble in S, and S has negligible vapor pressure, then the remaining variables are:  $P, T, x_A$  and  $y_A$

$x_B = 0$

$y_s = 0$

$x_s = 1 - x_A$

gas absorption & was in atmosphere

solvent & gas mixture

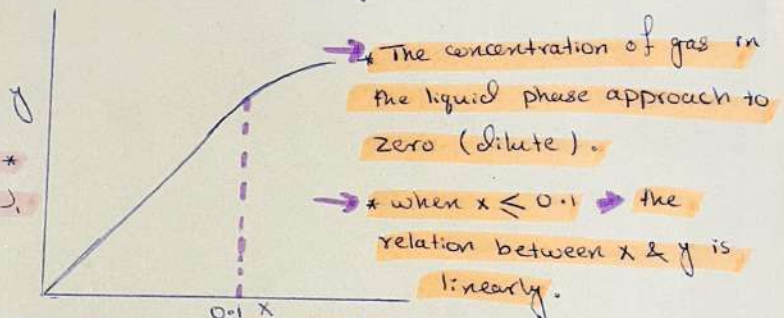
vap - liq equilb:

2 phase

gas - liq equilb:

2 phase

$x_B = 0 \rightarrow$  liquid



### Henry's Law:

$$P_A = k_h \cdot x_A$$

Partial pressure  
of A

Henry's  
law constant

mole fraction  
of A in liquid

almost all liquids will deviate above mole fractions of 0.1.

Equilibrium constant:

$$K_A = \frac{y_A}{x_A} = \frac{k_h}{P_i}$$





$y_i$  → mol fraction of solute in phase E

$$y_i = \frac{S_i}{E_s + S_i} = \frac{S_i}{E_i}$$

$m_i$  → mol fraction of solute in phase R

$$m_i = \frac{S_i^R}{R_s + S_i^R} = \frac{S_i^R}{R_i}$$

mole fraction

$$E_i = E_s + S_i$$

flow rate of solvent

flow rate of solute

If  $E_i$  is pure  $\rightarrow S_i = 0$

E and R and single solute transferring from phase R to Phase E

### Extracting Processes

the rate of transfer depend on equilibrium

at equilibrium  $\rightarrow$  net transfer = 0

If the feed is pure

$E_i$  = flow rate of solvent only

Flow rate of solvent + flow rate of solute

Flow rate of solvent

Solvent + solute

Counter

←

→

عكس بعينه

INPUT 1

II

$E_1 y_1$

$E_s y_1$

$R_1 x_1$

$R_s x_1$

Solvent

Solute

Contacting Device

B

$E E_s y Y$

OUTPUT 2

Solvent + solute

$E_2 y_2$

$E_s y_2$

$R_2 x_2$

$R_s x_2$

$E_1 < E_2$

solute in phase 1

انتقل من phase 1

ل phase 2

في phase E

في phase R

$R_1 > R_2$

solvent in phase 1

انتقل من phase 1

ل phase 2

في phase E

في phase R

$R R_s x X$

B'

Co current

$R_i, E_i$  : moles total material/time ;  $R_s, E_s$  :

$x_i, y_i$  : mole fractions of solute in stream

$i$  : stream number

### Solute Material Balance

Envelope I (overall)

$$\text{moles of solute in R phase} \quad \overbrace{R_1 x_1 + E_1 y_1}^{\text{IN}} = \overbrace{R_2 x_2 + E_2 y_2}^{\text{OUT}}$$

OR

$$R_1 x_1 - R_2 x_2 = E_2 y_2 - E_1 y_1$$

$$R_1 x_1 = R_s \frac{x_1}{1 - x_1} = R_s x_1$$

$$E_1 y_1 = E_s \frac{y_1}{1 - y_1} = E_s y_1$$

$$R_s (x_1 - x_2) = E_s (y_2 - y_1)$$

$$\frac{y_2 - y_1}{x_1 - x_2} = -\frac{R_s}{E_s} \quad \text{Straight line with slope} = -\frac{R_s}{E_s} \quad (\text{line PQ})$$

$$y_2 - y_1 = -\frac{R_s}{E_s} (x_1 - x_2)$$

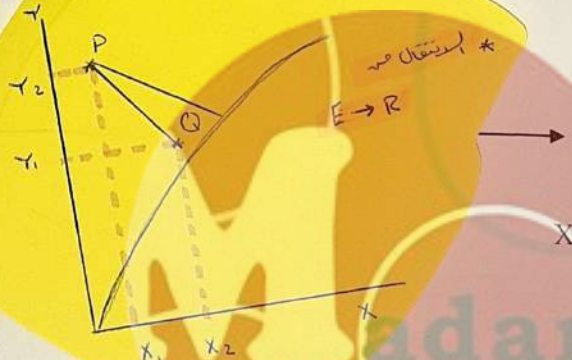
$X$  &  $Y$  mole Ratio

نسبة مولات solute في phase 1

نسبة مولات solvent في phase 1

$$Y = \frac{S}{E_s} = \frac{y E_i}{(1 - y) E_i} = \frac{y}{1 - y}$$

$$X = \frac{S_i^R}{R_s} = \frac{m R_i}{(1 - m) R_i} = \frac{m}{1 - m}$$



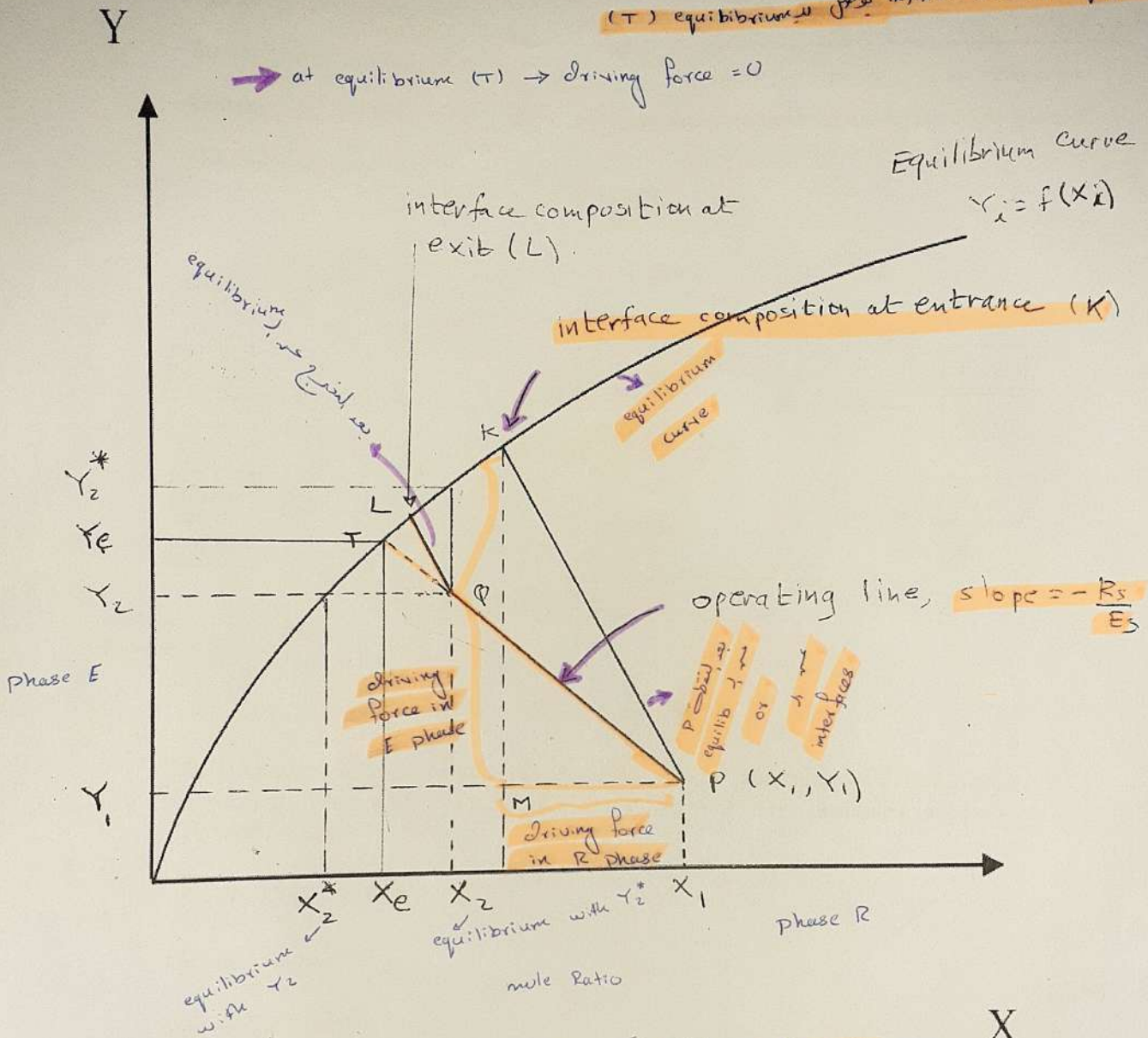
ask :: believe & recieve



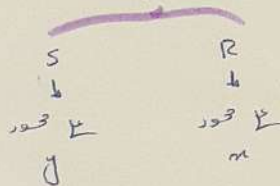
\* متى انتقلت T اختار لنا PCQ لو استمرت بعضه لنقل infinite time وكذا عندى

(T) equilibrium infinite size of equipment

→ at equilibrium (T) → driving force = 0



انتقلت P من ل إلى 2 phases وقتوى على ل



→ حار في انتقال لـ Solute من R → E حار في تقوية بتركيز لـ solute في R phase

→ من انتقلت X<sub>1</sub> → X<sub>2</sub> (بيننا لـ Phase E لاداد تركيز لـ solute من Y<sub>1</sub> → Y<sub>2</sub>)

→ انتقلت Q نفس المخرج وقتوى على 2 phases وذلك X<sub>1</sub> > X<sub>2</sub> بسبب انتقال

\* اختار في قيم X<sub>2</sub> > X<sub>1</sub> Driving force في R phase  
E phase

ask : believe & recieve

### Envelope II (General Balance)

On solute free basis

Straight line: slope

general + h  
PQ no xesi

#  $P_D(x_1 - x) = E_D(Y - Y_1)$  slope  
 $\hookrightarrow (x_1, Y_1)$  نقطة PQ  
 انظر فليجرب على بعضي ولكن هذا الذي  
 انظر PQ اني ما درست للمنتج

 $(V - Y_1)$ 

compositions of phases in equilibrium with distance from

Q represent inlet and exit concentrations

called Operating line

at entrance

position at interface

Forces in E-phase (KM) and R-phase (MP) at entrance conditions

line at exit

### Surface composition

$\Gamma$  represents equilibrium compositions ( $X_e, Y_e$ ) if the equipment were long enough. At this point the driving force is zero.

PQ →

### Representation using other units

قهرن ت curve

$R \triangleleft E$  v.u.

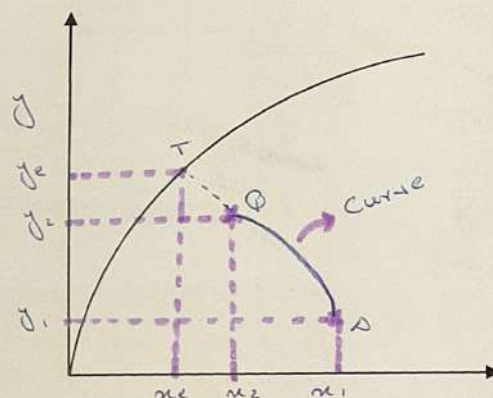
$$R_1 x_1 - R x = E y - E_1 y_1 \quad \text{Operating curve}$$
$$E y = R_1 x_1 + E_1 y_1 - R x$$

not constant

mole Ratio  $\rightarrow$  is  $y = \frac{R_1 x_1 + E_1 y_1}{E} - \frac{R}{E} x$  Operating Curve

تَغْيِرُوا تَغْيِيرَ الْمَوْقِعِ  $\rightarrow$  R & E

مفسرہ



➡ mole fraction

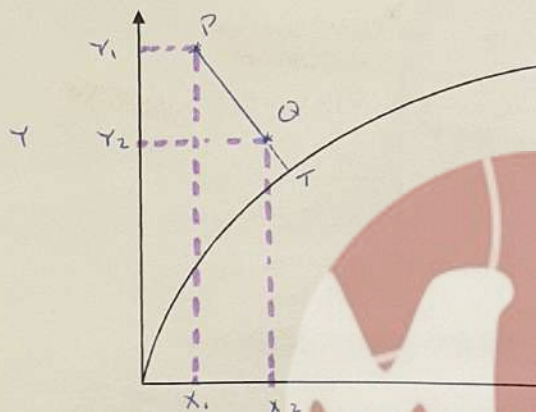
نو Ratio

Note: If

$$E_1 = E_2 = E$$
$$R_1 = R_2 = R$$

Straight line operating line in terms of mole fractions

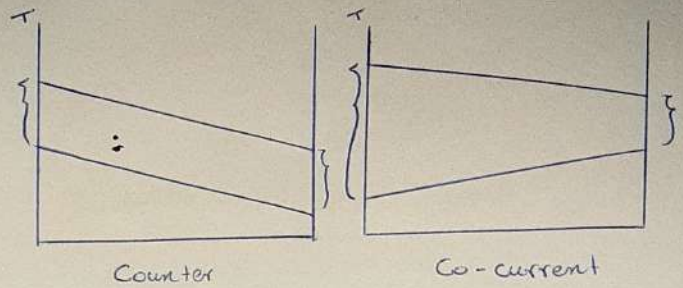
Solute transferring from E  $\rightarrow$  R





heat transfer معلومات من

driving force في counter-current  
في counter-current driving force تقريباً ثابتة

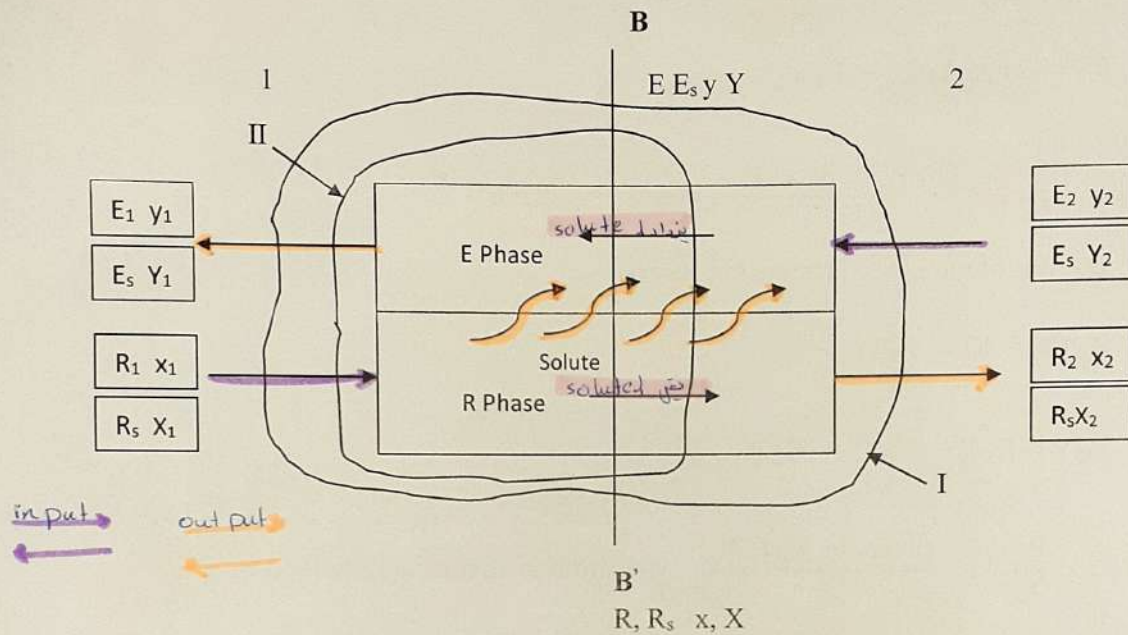


## Material Balances:

### Steady State Contacting Processes

System: two insoluble phases E and R and single solute transferring from phase R to Phase E

### Counter Current Process:



$R_i, E_i$  : moles total material/time ;  $R_s, E_s$  : moles non diffusing material/time (constant)  
 $x_i, y_i$  : mole fractions of solute in stream  $i$  ;  $X_i, Y_i$  : mole ratios of solute in stream  $i$   
 $i$  : stream number

Solute diffusing from  $R \rightarrow E$

Solute Material balance

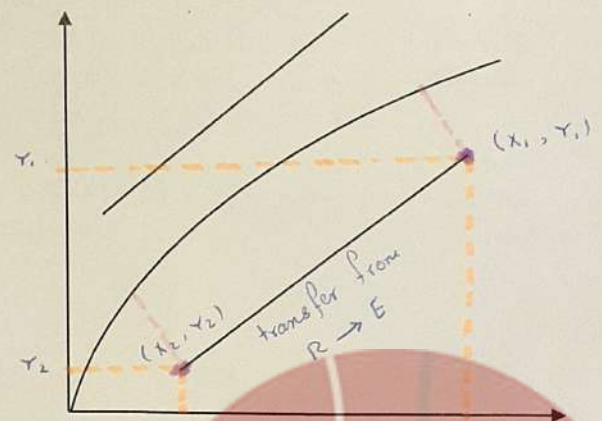
Envelop I (overall)

$$\underbrace{R_1 x_1 + E_2 y_2}_{\text{Input}} = \underbrace{R_2 x_2 + E_1 y_1}_{\text{Output}} \quad \text{mole fractions}$$

$$R_i x_i = R_s X_i \quad E_i y_i = E_s Y_i$$

$$R_s(X_1 - X_2) = E_s(Y_1 - Y_2) \quad \text{mole ratios}$$

This is a straight line equation with slope  $+\frac{R_s}{E_s}$



\* Driving force

equilibrium curve, line بعد

وبقي ما كان تقريباً ثابت

## Envelop II (General) operating lines

In terms of mole fractions:

$$\underbrace{R_1 x_1 + E y}_{\text{Input}} = \underbrace{R x + E_1 y_1}_{\text{Output}} \quad \text{mole fractions}$$

$E \& R \Rightarrow$  Input + Output  
or output

$$E y = R x + E_1 y_1 - R_1 x_1$$

$$y = \frac{R}{E} x + \frac{(E_1 y_1 - R_1 x_1)}{E}$$

operating curve

$\frac{R}{E}$  net cost  $\Rightarrow$

slope of line

In terms of mole ratios (solute free basis)

$$R_S (X_1 - X) = E_S (Y_1 - Y)$$

$$E_S Y = R_S X - R_S X_1 + E_S Y_1$$

$$Y = \frac{R_S}{E_S} X - \frac{(R_S X_1 - E_S Y_1)}{E_S}$$

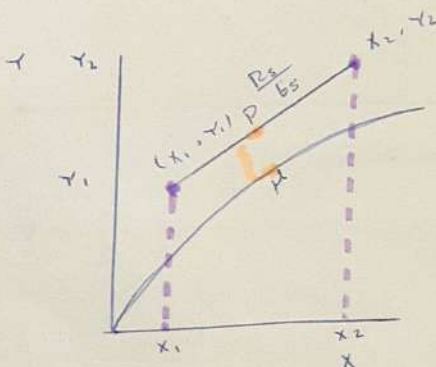
point & slope of overall line  
straight line operating line slope =  $+\frac{R_S}{E_S}$

In the case where the solute is passing from  $E \rightarrow R$ , the operating line will be above the equilibrium curve.

passing  
stream

- The operating line represents the material balance passing from point at one end to the point at the other end.
- A point on the operating line represents bulk concentrations of passing streams
- Lines such as PM indicate driving force

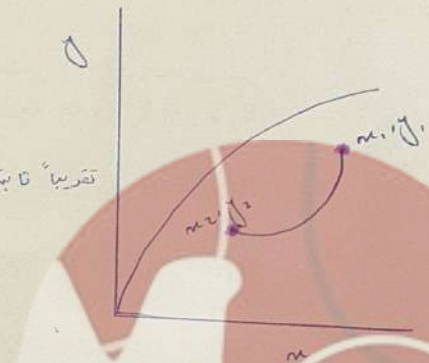
from solute material balance  $\Rightarrow$  net flow  $\Rightarrow$   $E y_1 - R x_1 = E_2 y_2 - R_2 x_2 = \text{constant}$



transfer from  $E \rightarrow R$

$P \mu \Rightarrow$  driving force  $\Rightarrow$

دریافت



In terms of mole fractions

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**Equilibrium Stages**  $\Rightarrow$  long time  $\rightarrow$  two phase  $\rightarrow$  gas

net transfer = 0  $\rightarrow$  no net transfer

It is a theoretical concept representing the contact of two phases for sufficient time until they reach equilibrium

The number of equilibrium stages represents the theoretical number of contacts required for a desired separation

The use of stage efficiencies (based on mass transfer rates) and the number of equilibrium stages can be used to determine the number of the actual stages required for the separation.

MA = 0.5

Phase 1

fresh solvent

در این مرحله 0.4

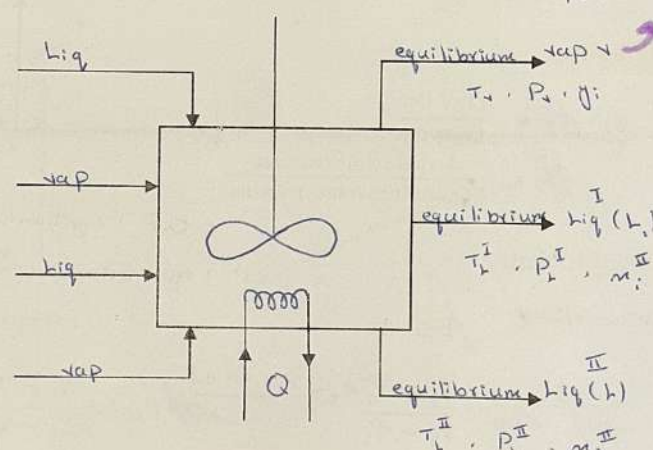
در این مرحله

product

fresh solvent

driving force

Incoming streams  
Not in equilibrium with each other



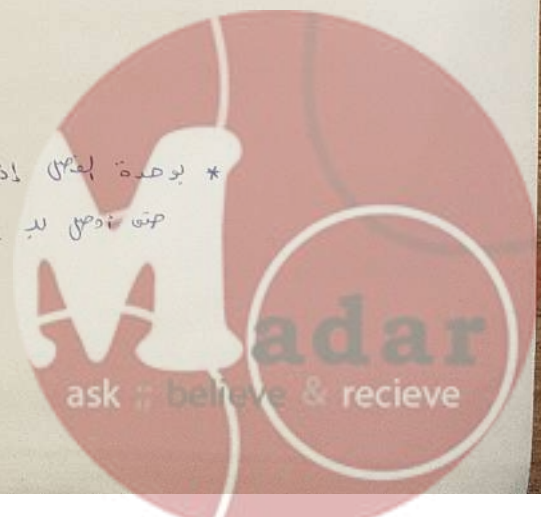
T/P equilibrium  $\Rightarrow$

T & P  $\rightarrow T_v = T_L^I = T_L^{II} \quad P_v = P_L^I = P_L^{II}$

VLE  $\rightarrow k_i^I = \frac{y_i}{n_i^I} \quad k_i^{II} = \frac{y_i}{n_i^{II}}$

LLE  $\rightarrow k_{D,i} = \frac{n_i^I}{n_i^{II}} \rightarrow$  Distribution coefficient

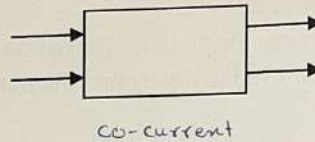
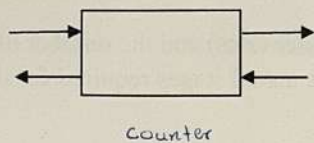
\* توجه (Note)  $\rightarrow$  در این حالت در equilibrium  $\rightarrow$  در این حالت در equilibrium



## # Single Stage

عملية قتراف

Effectively it is a concurrent operation. If the stage were ideal, the exit streams would be at equilibrium.



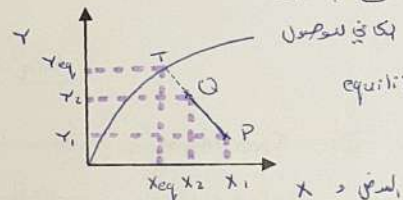
## ➡ Stage efficiency

جزئي

It is an expression of the fractional approach to equilibrium which a real stage produces.

## Possible definitions:

- # Line Qp
- # Line Tp
- #  $\frac{\text{Actual solute transfer}}{\text{Equilibrium solute transfer}}$



## Murphree stage efficiency

efficiency من المخرج في المرحلة

E-phase

$$E_{ME} = \frac{Y_2 - Y_1}{Y_2^* - Y_1}$$

فقط PA  
TP PA, PA

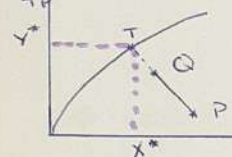
$$E_{MR} = \frac{X_1 - X_2}{X_1 - X_2^*} = \frac{PA}{TP}$$

$$Y_2^* = Y_{eq}$$

$$X_2^* = X_{eq}$$

$Y_2^*$  : in equilibrium with  $X_2$

$X_2^*$  : in equilibrium with  $Y_2$



R-phase efficiency

- This is an arbitrary definition since  $Y_2$  will never be greater than  $Y_e$  and  $X_2$  will never be lower than  $X_e$

- #  $E_{ME} \neq E_{MR}$

$$Y^* > Y_2$$

$$X^* < X_2$$

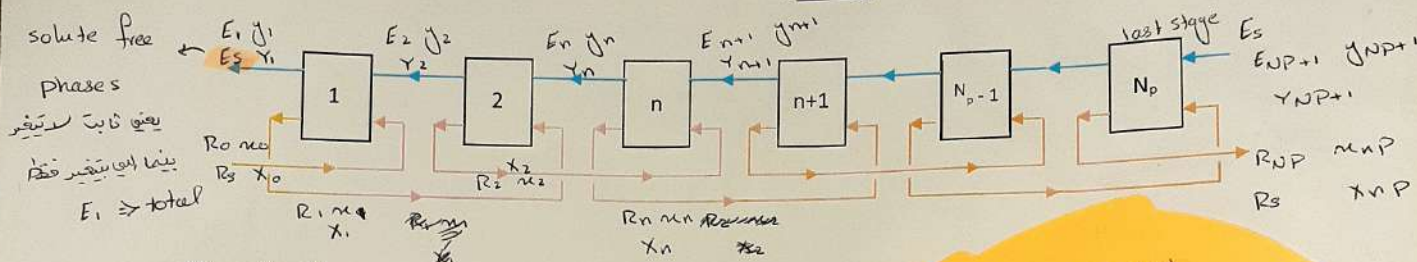
$Y^*$  : أكبر قيمة يمكن الحصول عليها  
بالقائ  $X^*$  أقل قيمة يمكن الحصول عليها.





## Counter Current Operations

- Most efficient arrangement [ least number of stages for a given separation].
- Each stage is identical in its action to a cocurrent process; however, the cascade (battery) has the characteristics of a counter current process.
- Passing streams are not in equilibrium  $\Rightarrow$  unit  $\nrightarrow$  equilibrium
- Streams leaving a stage are in equilibrium  $\Rightarrow$  equilibrium



### Operating Lines:

Operating line for any equilibrium stage (n)

INPUT  $(X_{n-1}, Y_{n+1})$  OUTPUT  $(X_n, Y_n)$

Mole fractions:  $R_{n-1} \cdot X_{n-1} + E_{n+1} \cdot Y_{n+1} = R_n \cdot X_n + E_n \cdot Y_n$

Mole ratios:  $R_s \cdot X_{n-1} + E_s \cdot Y_{n+1} = R_s \cdot X_n + E_s \cdot Y_n$

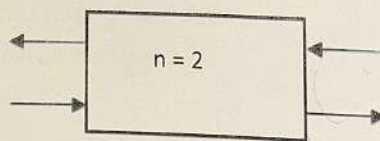
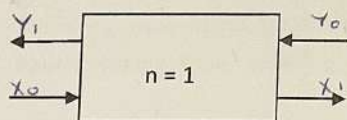
$$R_s(X_{n-1} - X_n) = E_s(Y_n - Y_{n+1})$$

$$Y_{n+1} = \frac{E_s Y_n - R_s X_{n-1} + R_s X_n}{E_s} \quad E_s Y_n - E_s Y_{n+1} = R_s X_{n-1} - R_s X_n$$

$$Y_{n+1} = \frac{R_s}{E_s} X_n + \frac{E_s Y_n - R_s X_{n-1}}{E_s} = -\frac{R_s}{E_s} X_{n-1} + \frac{R_s X_{n-1} + E_s Y_{n+1}}{E_s}$$

$\downarrow$   $\text{y-axis (slope)}$

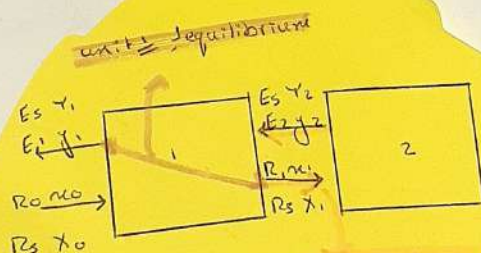
For example, take the first two stages:



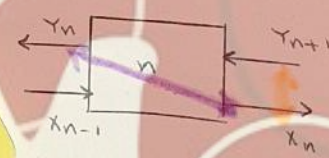
$$Y_2 = + \frac{R_s}{E_s} X_1 + \frac{-R_s X_0 + E_s Y_1}{E_s}$$

$$Y_3 = + \frac{R_s}{E_s} X_2 + \frac{-R_s X_1 + E_s Y_2}{E_s}$$

material balance  $\rightarrow$   
( $X_1, Y_2$ ) passing streams



passing streams  
operating line



equilibrium (2 phase)  
unit  $\rightarrow$  ask passing streams receive



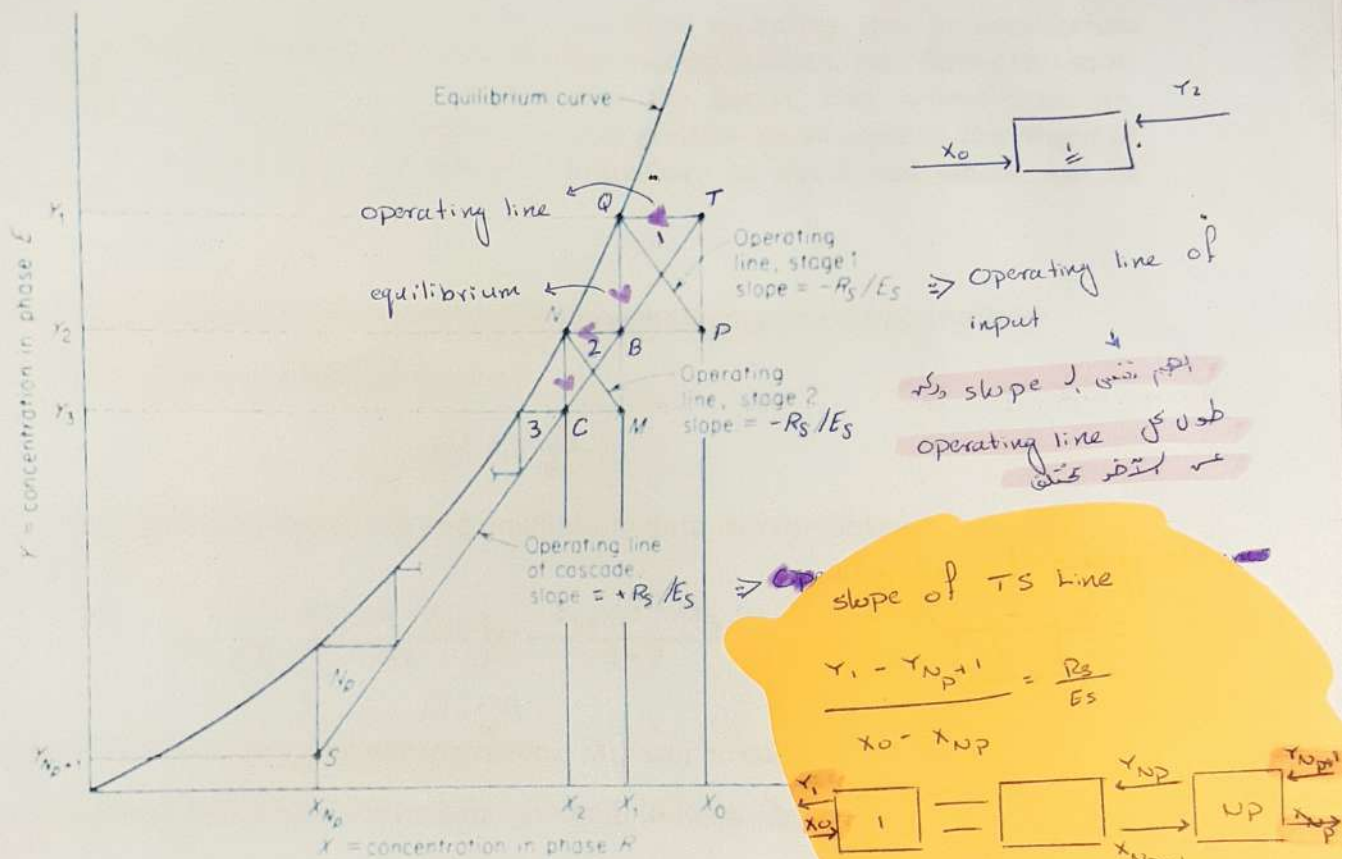


Figure 5.15 Countercurrent multistage cascade, solute transfer from ph operating line of cascade

### Notes:

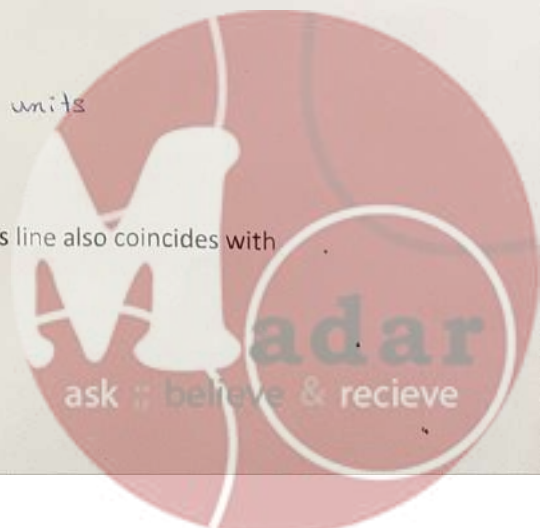
- All individual stages have parallel operating lines of slope  $= -\frac{R_s}{E_s}$
- Points  $x_n, y_n$  fall on equilibrium curve. They are exit streams from stage n [ Q , ... ]
- Points  $x_{n+1}, y_{n+1}$  these are inlet streams to stage n [ P , ... ]
- Points  $x_n, y_{n+1}$  are compositions of passing streams [ T, B, C ]. Lines like , have the same slope  $= +\frac{R_s}{E_s}$  and they all fall on the same line .

### Operating line for the cascade:

Entire cascade:  $R_s(X_0 - X_{N_p}) = E_s(Y_1 - Y_{N_p+1}) \Rightarrow$  for all units

For any stage n:  $R_s(X_0 - X_n) = E_s(Y_1 - Y_{n+1})$

$Y_{n+1} = +\frac{R_s}{E_s} X_n + \frac{E_s Y_1 - R_s X_0}{E_s}$  , Slope ; This line also coincides with



**Special Case:** The equilibrium curve is approximately line

For most cases, because of either a curved operating line or equilibrium curve, the relation between number of stages, compositions, and flow ratio must be determined graphically, as shown. For the *special case* where both are straight, however, with the equilibrium curve continuing straight to the origin of the distribution graph, an analytical solution can be developed which will be most useful.

If the equilibrium-curve slope is  $m = \frac{Y_{N+1}}{X_{N+1}}$  (straight line), and if an absorption factor (Extraction factor)  $A$  is defined as:

$$A = \frac{R_s}{mE_s},$$

then based on material balance and equilibrium data, we can obtain the following equation:

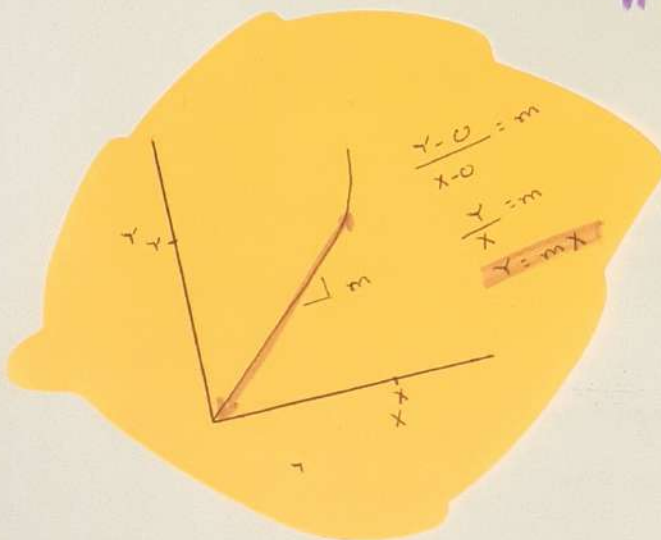
$$X_n = \left( X_0 - \frac{Y_{N+1}/m - AX_{N_2}}{1-A} \right) A^n + \frac{Y_{N+1}/m - AX_{N_2}}{1-A}$$

التركيز الطرفية

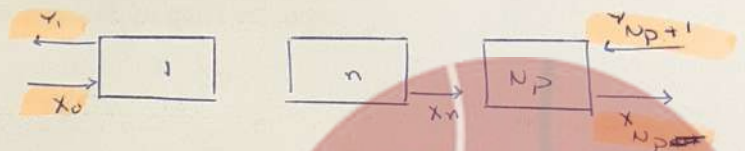
Therefore knowing **terminal concentrations**,  $X_n$  can be calculated.

In the opposite case where the transfer is from E to R, we define a stripping factor  $S$  as:

$$S = \frac{1}{A}$$



⇒ Special Case



تعتبر تركيزي طرفيت (تركيز الطرفية بقدر

Terminal Concentration (تركيز الطرفية)



## Kremser Equations: (Kremser-Brown-Souders)

In the case where  $n = N_p$ , then:

For transfer from R to E (stripping of R)

$A \neq 1$ :

$$A = \frac{R_s}{m E_s}$$

$$\frac{X_0 - X_{N_p}}{X_0 - Y_{N_p+1}/m} = \frac{(1/A)^{N_p+1} - 1/A}{(1/A)^{N_p+1} - 1}$$

$$N_p = \frac{\log \left[ \frac{X_0 - Y_{N_p+1}/m}{X_{N_p} - Y_{N_p+1}/m} (1 - A) + A \right]}{\log 1/A}$$

$A = 1$ :

$$\frac{X_0 - X_{N_p}}{X_0 - Y_{N_p+1}/m} = \frac{N_p}{N_p + 1}$$

$$N_p = \frac{X_0 - X_{N_p}}{X_{N_p} - Y_{N_p+1}/m}$$

For transfer from E to R (absorption into R). A similar treatment yields:

$A \neq 1$ :

$$\frac{Y_{N_p+1} - Y_1}{Y_{N_p+1} - mX_0} = \frac{A^{N_p+1} - A}{A^{N_p+1} - 1}$$

$$N_p = \frac{\log \left[ \frac{Y_{N_p+1} - mX_0}{Y_1 - mX_0} \left( 1 - \frac{1}{A} \right) + \frac{1}{A} \right]}{\log A}$$

$A = 1$ :

$$\frac{Y_{N_p+1} - Y_1}{Y_{N_p+1} - mX_0} = \frac{N_p}{N_p + 1}$$

$$N_p = \frac{Y_{N_p+1} - Y_1}{Y_1 - mX_0}$$

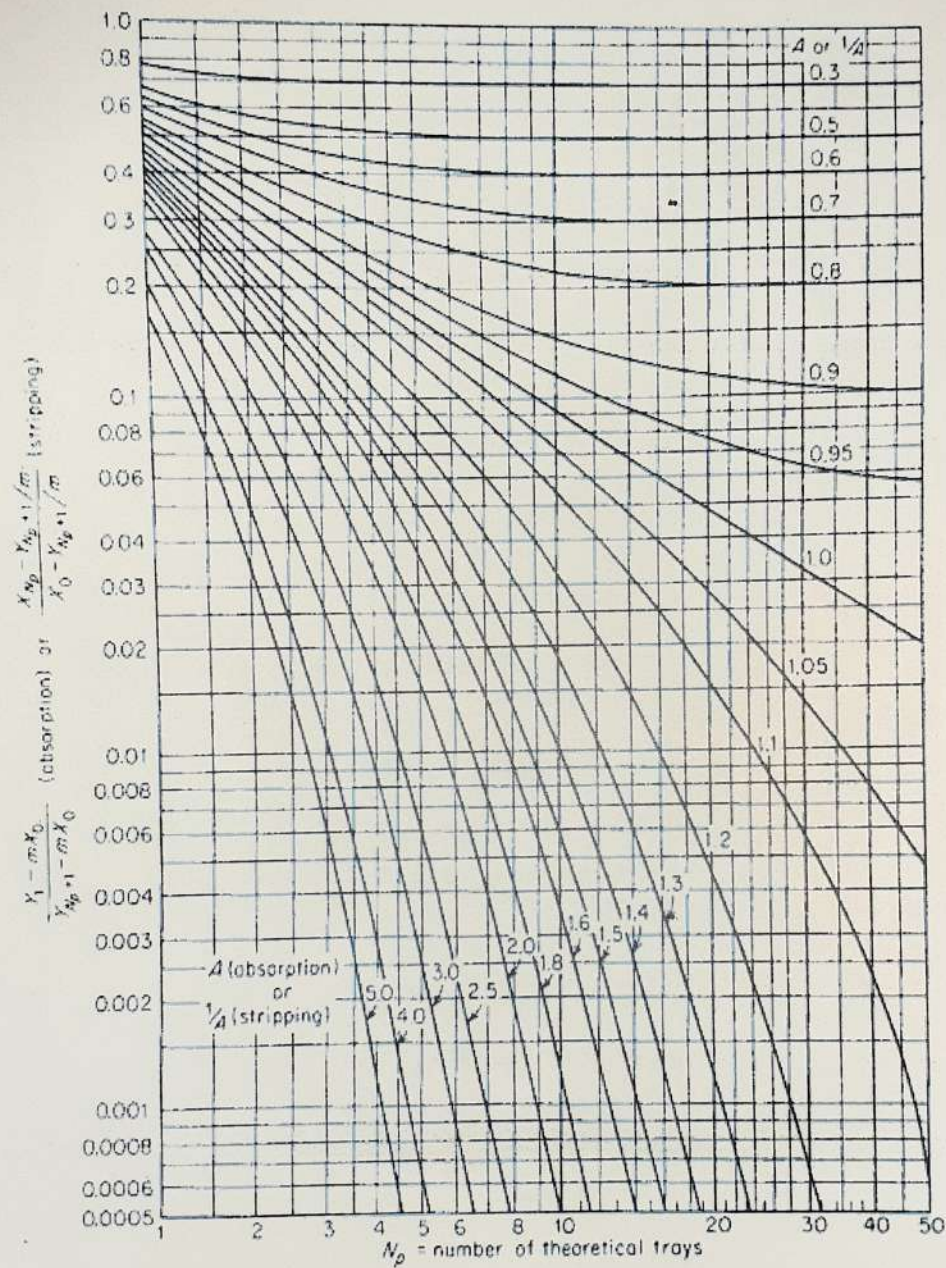
These are called the Kremser-Brown-Souders (or simply Kremser) equations, after those who derived them for gas absorption [7, 14] although apparently Turner [16] had used them earlier for leaching and solids washing. They are plotted in Fig. 5.16, which then becomes very convenient for quick solutions.

solid & liquid

leaching → من قاع السائل في الحبيبات

solid washing → من قاع الحبيبات في السائل





**Figure 5.16** Number of theoretical stages for countercurrent cascades, with Henry's law equilibrium and constant absorption or stripping factors. [After Hachmuth and Vance, *Chem. Eng. Prog.*, 48, 523, 570, 617 (1952).]





درجه غليظهم متفاوت، يعني نظامهم بيقي ادمج تقريباً، لانه ضغطهم باستخدام distillation مستحيل  
 Extraction يعني اي Extraction

Mixture of Water and p-dioxane having boiling points of  $100^{\circ}\text{C}$  and  $101.1^{\circ}\text{C}$  respectively at 1 atm which means that they cannot be separated by distillation. However, Benzene can be used to extract p-dioxane from Water at  $25^{\circ}\text{C}$ . It is known that Water and Benzene are mutually insoluble and therefore mass ratios are convenient to use. The equilibrium relationship has the form:

$$Y = K_D X$$

Y: mass ratio of p-dioxane in Benzene (mass p-dioxane/mass Benzene)

X: mass ratio of p-dioxane in Water (mass p-dioxane/mass Water)

lig - liq equilibrium

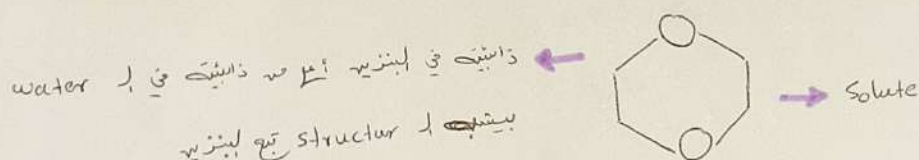
From equilibrium data it is known that the distribution coefficient ( $K_D$ ) varies between 1 to 1.4 over the concentration range of interest. For this problem assume  $K_D = 1.2$ .

If 10,000 kg/hr of a 25% solution of p-dioxane in Water (R) is to be treated continuously with 15,000 kg/hr of pure Benzene (E), determine the effect of the number and arrangement of stages on the percent extraction of p-dioxane for the following situations:

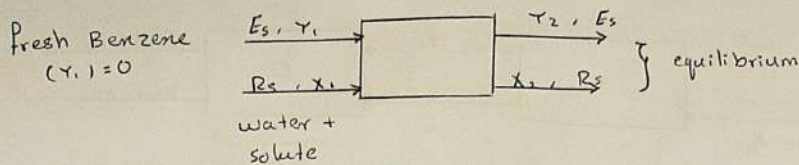
- Single equilibrium stage
- Cocurrent arrangement
- Cross current arrangement (2 stages)
- Countercurrent arrangement (2 stages)



$\Rightarrow$  P-dioxane : 1,4 dioxane  $\Rightarrow \text{C}_4\text{H}_8\text{O}_2$  it is an ether (R-O-R)



## a Single equilibrium stage



1 Definition:

$$\% \text{ extraction of P-dioxane} = \frac{\cancel{R_s} X_1 - \cancel{R_s} X_2}{\cancel{R_s} X_1} \times 100\% = \frac{X_1 - X_2}{X_1} \times 100\% = \frac{\text{Solute in} - \text{Solute out}}{\text{Solute in}} \times 100\%$$

$$\Rightarrow R_s X = \cancel{R_s} \frac{S}{\cancel{R_s}} = S \Rightarrow \text{solute } \downarrow \text{ mass}$$

Need  $X_1$  &  $X_2$

$$\underline{2} \quad X_1 = \frac{n_1 R}{(1-n_1)R} = \frac{\text{mass of solute}}{\text{mass of solvent}} = \frac{n_1}{1-n_1} = \frac{0.25}{1-0.25} = \frac{0.25}{0.75} = \underline{\underline{\frac{1}{3}}}$$

3  $X_2$ : from material balance on - P-dioxane

$$R_s X_1 + E_s Y_1 = R_s X_2 + E_s Y_2 \quad \dots \underline{1}$$

in = out  $\rightarrow$  for solute

$$\text{dissolve } Y_2 = k_D X_2 \quad \dots \underline{2}$$

$\neq$  solvent  $\neq$   $\neq$  solvent

$$R_s X_1 + E_s Y_1 = R_s X_2 + E_s k_D X_2$$

$$X_2 = \left( \frac{R_s X_1 + E_s Y_1}{R_s + E_s k_D} \right) \div \frac{R_s}{R_s}$$

$$X_2 = \frac{X_1 + \frac{E_s}{R_s} Y_1}{1 + \frac{E_s}{R_s} k_D} \quad \text{extraction factor (E)}$$

$Y_1 = 0 \rightarrow$  Pure Benzene

$$E = \frac{E_s}{R_s} k_D = \frac{15,000}{(1-0.25) \times 10,000} = \underline{\underline{2.4}}$$

$$E = \frac{E_s}{(1-n)R} k_D = \frac{15,000}{(1-0.25) \times 10,000} \times 1.2 = \underline{\underline{2.4}}$$

$$X_2 = \frac{X_1 + \frac{E_s}{R_s} Y_1}{1 + E} = \frac{1/3}{1 + 2.4} = \underline{\underline{0.0980}}$$

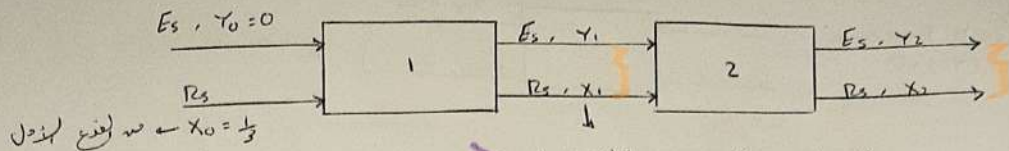
$$\% \text{ extraction} = \frac{X_1 - X_2}{X_1} \times 100\% = \frac{1/3 - 0.0980}{1/3} \times 100\% = \underline{\underline{70.6\%}}$$

$$Y_2 = X_2 k_D = 0.0980 \times 1.2 = \underline{\underline{0.1176}}$$





b Co-current  $\Rightarrow$  consider 2 stages



$\Rightarrow$  at equilibrium with each other

so  $X_1 = X_2$  &  $Y_1 = Y_2$

two streams  $(Y_1, X_1)$  &  $(Y_2, X_2)$

في كل مرحلة، يكونان في حالة اتزان

stage 2 في حالة اتزان

$$\% \text{ recovery} = \frac{X_0 - X_2}{X_0}$$

- From previous calculation

$$\frac{E_s}{R_s} = \frac{15,000}{7500} = 2 \quad \left. \vphantom{\frac{E_s}{R_s}} \right\} \text{ same for both stage}$$

$$E = \frac{E_s}{R_s} k_D = 2.4$$

\* For stage 1

$$X_1 = 0.098 \quad Y_1 = 0.1176$$

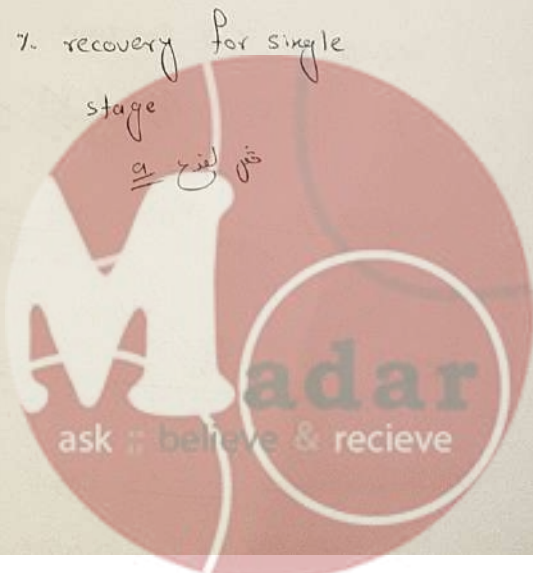
\* For stage 2

$$X_2 = \frac{X_1 + \frac{E_s}{R_s} Y_1}{1 + E} = \frac{0.098 + 2 \times 0.1176}{1 + 2.4} = 0.0980 = X_1$$

في كل مرحلة Co-current & في stage 2 يكونان في حالة اتزان

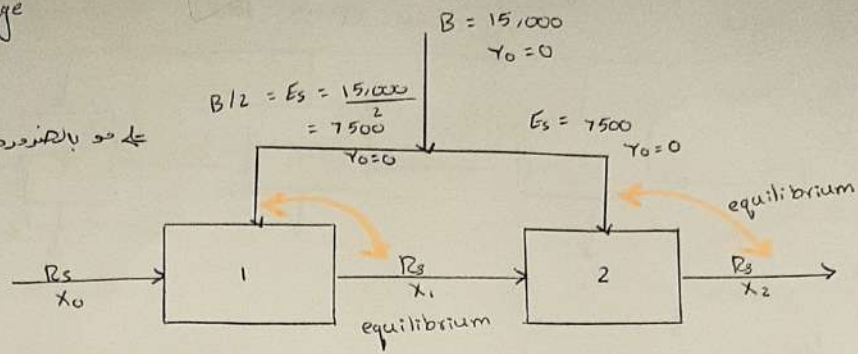
stage 1 & stage 2 في حالة اتزان

$$\% \text{ recovery} = \frac{0.333 - 0.098}{0.333} \times 100\% = 70.57\% \approx \% \text{ recovery for single stage}$$



## C Cross current 2 stage

مذيب في المذيبات المتبقية في المذيبات المتبقية



$$\% \text{ extraction} = \frac{X_0 - X_2}{X_0} \times 100\%$$

$$E = \frac{E_s}{R_s} K_D = \frac{7500}{7500} K_D = 1.2$$

$$X_1 = \frac{X_0 + 0}{1 + E} = \frac{1/3}{1 + 1.2} = 0.1515$$

$$X_2 = \frac{X_1 + 0}{1 + E} = \frac{0.1515}{1 + 1.2} = 0.0689 \Rightarrow$$

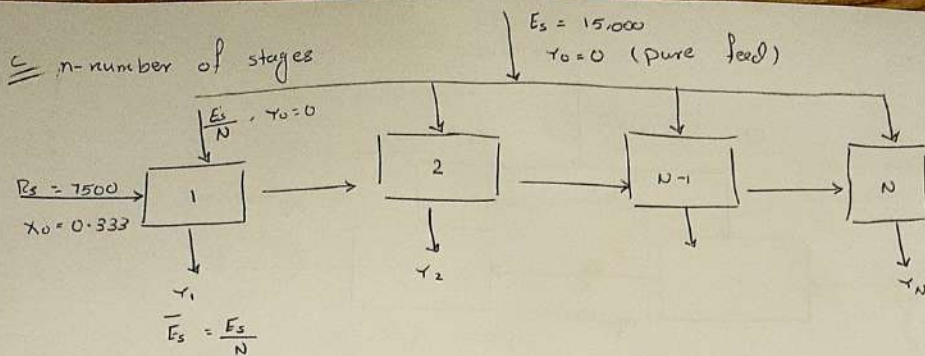
↓ no stage 2 is driving force

$$X_{out} = \frac{X_{in} + \frac{E_s}{R_s} Y_1}{1 + E} \rightarrow \text{general}$$

$$\% \text{ extraction} = \frac{0.333 - 0.0689}{0.333} = 79.34\%$$







$\Rightarrow \bar{E} = \frac{E_s}{N}$

Flow rate for each stage

$\bar{E} = \frac{(E_s/N)}{R_s} \cdot K_D = \frac{E}{N}$

extraction factor for each stage

$\Rightarrow E$  is effective extraction factor

% Recovery =  $\frac{X_0 - X_N}{X_0} \times 100\%$

$\Rightarrow X_1 = \frac{X_0 + \left(\frac{E_s}{N}\right) \cdot Y_0}{1 + \bar{E}} = \frac{X_0}{1 + \frac{E}{N}}$

$X_2 = \frac{X_1 + \frac{E_s}{N} \cdot Y_0}{1 + \bar{E}} = \frac{X_1}{1 + \bar{E}}$

$\Rightarrow X_2 = \frac{X_0}{1 + \frac{E}{N}} \cdot \frac{1}{1 + \frac{E}{N}} = \frac{X_0}{\left(1 + \frac{E}{N}\right)^2}$

$\Rightarrow X_N = \frac{X_0}{\left(1 + \frac{E}{N}\right)^N}$

# % Recovery =  $\frac{X_0 - X_N}{X_0} \times 100\%$

$= \frac{X_0 - \frac{X_0}{\left(1 + \frac{E}{N}\right)^N}}{X_0} \times 100\%$

$= 1 - \frac{1}{\left(1 + \frac{E}{N}\right)^N} \times 100\%$

% =  $\frac{\left(1 + \frac{E}{N}\right)^N - 1}{\left(1 + \frac{E}{N}\right)^N} \times 100\%$

From material balance of solute in stage 2

$R_s X_1 + \frac{E_s}{N} Y_0 = \frac{E_s}{N} Y_2 + R_s X_2$

$Y_2 = K_D X_2$

$R_s X_1 + \frac{E_s}{N} Y_0 = \frac{E_s}{N} K_D X_2 + R_s X_2$

$R_s X_1 + \frac{E_s}{N} Y_0 = X_2 \left( \frac{E_s}{N} K_D + R_s \right)$

$X_2 = \left( \frac{R_s X_1 + \frac{E_s}{N} Y_0}{\frac{E_s}{N} K_D + R_s} \right) \div R_s$

$X_2 = \frac{X_1 + \left(\frac{E_s}{N}\right) \cdot Y_0}{\bar{E} + 1}$

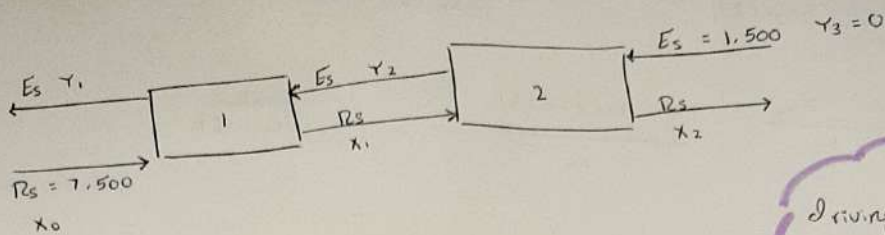
% Recovery independent on concentration

$\Rightarrow$  % Recovery depend on  $\bar{E}$

ask: extraction factor

$E \uparrow \Rightarrow \% \text{ Recovery} \uparrow$

## Counter current 2 stages



$$E = \frac{E_s}{R_s} k_D = 2.4 \text{ for each stage}$$

$$X_0 = 0.333$$

$$Y_3 = 0$$

Driving force  
cross  
Driving force

\* Stage  $\hat{=}$

$$R_s X_0 + E_s Y_2 = R_s X_1 + E_s Y_1$$

$$Y_1 = k_D X_1$$

$$R_s X_0 + E_s Y_2 = R_s X_1 + E_s k_D X_1$$

$$X_1 = \left( \frac{R_s X_0 + E_s Y_2}{R_s + E_s k_D} \right) \hat{=} \frac{R_s}{R_s}$$

$$X_1 = X_0 + \frac{E_s}{R_s} Y_2$$

$$X_2 = X_1 + \frac{E_s}{R_s} Y_3 = \frac{X_1}{1 + E}$$

در کمره اول،  
عوامل  $Y_2 = k_D X_2$

$$X_2 = \frac{X_0 + \frac{E_s}{R_s} k_D X_2}{(1 + E)^2}$$

$$X_2 = \frac{X_0 + E X_2}{(1 + E)^2} \Rightarrow X_2 = \frac{X_0 + E X_2}{1 + 2E + E^2}$$

$$X_2 = \frac{X_0}{1 + E + E^2} = \frac{0.333}{1 + 2.4 + (2.4)^2} = 0.036$$

$$\% \text{ Recovery} = \frac{0.333 - 0.036}{0.333} = 89.08\%$$





$$\lim_{N \rightarrow \infty} x_N = \frac{x_0}{\exp(E)} \Rightarrow x_\infty = \frac{x_0}{\exp(E)}$$

$$\% \text{ Recovery} = \frac{x_0 - x_\infty}{x_0} = \frac{x_0 - x_0 / \exp(E)}{x_0} = 1 - \frac{1}{\exp(E)}$$

\* ex:  $N = 5$

$$E = \frac{E_s}{R_s} \times K_D = \frac{15,000}{7,500} \times 1.2 = 2.4$$

$$\% \text{ recovery} = \left( 1 - \frac{1}{\left( 1 + \frac{E}{N} \right)^N} \right) \times 100\%$$

$$= \left[ 1 - \frac{1}{\left( 1 + \frac{2.4}{5} \right)^5} \right] \times 100\% = 85.92\%$$

نسبة استرجاع

stages 5

5 # of stages

100 # of stages  $\approx 5\%$

\* ex:  $N = 100$

$$x_{100} = \frac{0.333}{\left( 1 + \frac{2.4}{100} \right)^{100}} = 0.0311$$

$$\% \text{ Recovery} = \left( 1 - \frac{1}{\left( 1 + \frac{2.4}{100} \right)^{100}} \right) \times 100\% = 90.67\%$$

\* ex:  $N = \infty$

$$x_\infty = \frac{0.333}{\exp(2.4)} = 0.0302$$

$$\% \text{ Recovery} = 1 - \frac{1}{\exp(2.4)} = 90.93\%$$

(E) في % Recovery

extraction factor

solvent E في

$$E = \frac{E_s}{R_s} \times K_D$$

$$E_s \uparrow \quad E \uparrow$$

$1 < E$  في

ask believe & recieve

n-Number of stage  $\Rightarrow$  Counter

$$\# X_N = \frac{X_0}{\sum_{i=0}^N E^i}$$

ex:  $N=5$

$$X_5 = \frac{X_0}{1 + E + E^2 + E^3 + E^4 + E^5} = 0.0025$$

$$\% \text{ Recovery} = \frac{0.333 - 0.0025}{0.333} \times 100\% = 99.26\%$$

Limit  $N \rightarrow \infty$

$$\Rightarrow X_\infty = 0 \quad 1 \leq E \leq \infty \quad \Rightarrow \% \text{ Recovery} = 100\%$$

$$\Rightarrow X_\infty = (1-E) X_0 \quad E \leq 1 \quad \Rightarrow \% \text{ Recovery} = \frac{X_0 - (1-E) X_0}{X_0} = 1 - (1-E) = E$$

\* Summary: % Recovery

میزان بازیابی  
(0.5 : 0.5)

| # of stage | 1 atm stage | Co-current | Cross flow (X) | Counter current |
|------------|-------------|------------|----------------|-----------------|
| 1          | 70.6        | —          | —              | —               |
| 2          | 70.6        | 70.6       | 79.34          | 89.08           |
| 5          | 70.6        | 70.6       | 85.92          | 99.26           |
| $\infty$   | 70.6        | 70.6       | 90.93          |                 |

میزان بازیابی  
% Recovery

% recovery  $\rightarrow$  میزان بازیابی

$$\begin{cases} 100\% & 1 \leq E \leq \infty \\ E & \text{if } E \leq 1 \end{cases}$$

\* Note: for very small values of E (0.1 for example)

Cross flow (X)

Counter-current

$X_\infty$

$$\frac{X_0}{\exp(E)} = 0.301$$

$$(1-E) X_0 = 0.2997$$

% Recovery

9.55%

10.7%

در این حالت، بازیابی در جریان متقاطع بیشتر از جریان متضام است.

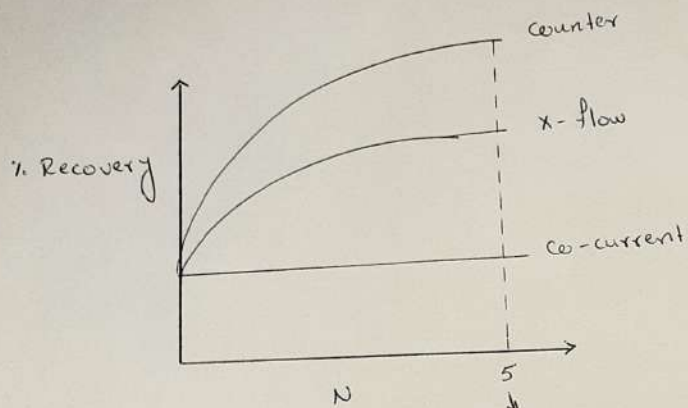
% Recovery کمتری است وقتی  $E \geq 1$  است.

ask : believe & recieve



- The difference is small

- Counter current is preferred since it is desirable to operate with  $E > 1$



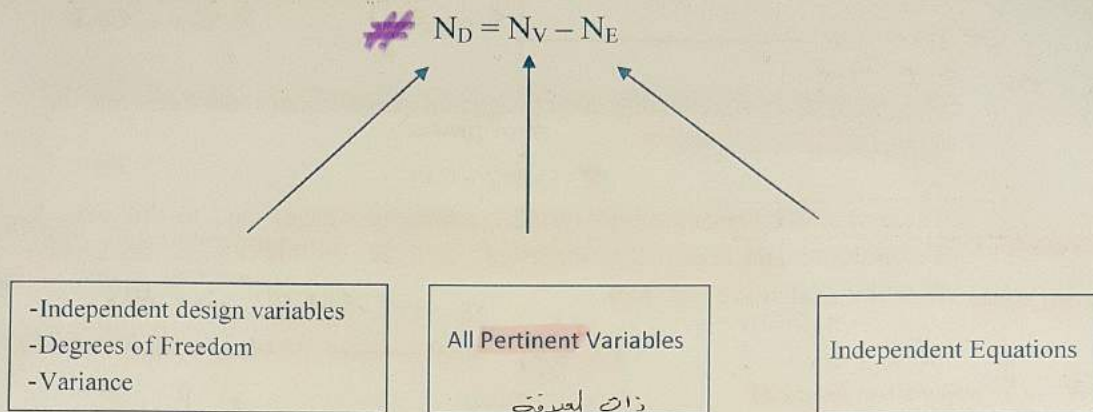
كفاءة رجوع 5 تقريبا ثابتة  
 $\propto \frac{1}{\omega_1}$



## Specification of Design Variables:

Solution of separation problems requires specification of a sufficient number of design variables in order to determine unknown (output) variables by solving an equal number of independent equations. The process will then be uniquely specified if the input variables are correctly specified.

The number of design variables reflects the number of degrees of freedom ( $N_D$ ) one has in correctly specifying a process. It is suggested that this number can be found as follows:



### Number of Variables $N_V$ :

The variables which can be counted in  $N_V$  are:

- ✓ Intensive: Variables  $T, P, x, \dots$  concentration (mole fraction, mass fraction, أو أي وحدة تركيز)
- ✓ Extensive: Flow rates,  $Q, \dots$
- ✓ Equipment Parameters: Number of Stages

Physical properties such as Enthalpy,  $K$  are not counted

### Number of Independent Equations $N_E$ :

Care must be taken in identification of independent equation

Types of Equations which can be used:

- ✓ Material Balances
- ✓ Energy Balances
- ✓ Phase Equilibrium Restrictions  $\Rightarrow$  Phases التي يكونوا متواجدين في unit
- ✓ Process Specifications  $T, P$
- ✓ Equipment configurations (counter, ...)

$$K_i = \frac{n_i}{y_i} \quad \text{equilibrium}$$

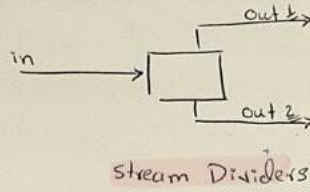




## Separation Equipment Consist of:

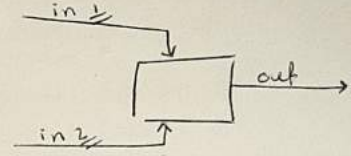
- ✓ Physically identifiable elements: Equilibrium stages, Condensers, Reboilers
- ✓ Stream Dividers
- ✓ Stream Mixers

### Examples:



Stream Dividers

$$E_{in} = E_{out}$$



Mixers

$$E_{in} = E_{out}$$

1- Stream Variables:  $E_{in} = E_{out}$

stream من صو  $P$  &  $T$  بقدر  $\Delta$   $sup$  heated,  $iq$ ,  $cap$  .....

For single phase stream containing  $C$  components; the phase rule states that the intensive variables are:

# of phases

$$C - \phi + 2 = C + 1$$

These are  $C-1$  concentrations (mole fractions or others), and to this we add Temperature and Pressure as intensive variables. To this number we add the flow rate (extensive variable).

$C+1$  stream  $\Delta$   $iq$ ,  $cap$  .....

intensive variables

$C+1 \Rightarrow \#$  of intensive variable  $N_v$

صو  $\Delta$   $iq$ ,  $cap$  .....

\*  $C-1 \Rightarrow$  Concentration

$\Delta$   $iq$ ,  $cap$  .....

$C$

$\Delta$   $iq$ ,  $cap$  .....

$C+1$

Intensive  $\Delta$   $iq$ ,  $cap$  .....

variable  $\Delta$   $iq$ ,  $cap$  .....

$P$  &  $T$

\* In counting the number of intensive variables, usually  $C-1$  concentrations specify all compositions since the remaining concentration is obtained by difference. However, we can specify all concentrations ( $C$ ) in counting the number of intensive variables and include in the list of equations the summation of concentrations constraints, such as the summation of mole fractions:

$$\sum_{i=1}^C \text{mole fractions} = 1$$

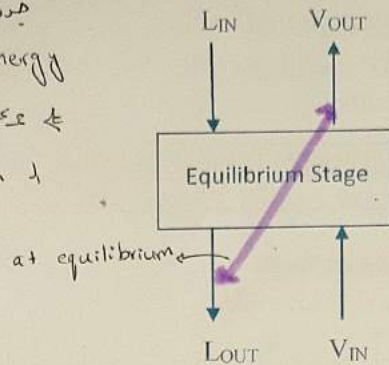
Therefore:

$$N_v = C + 3 \quad \text{Or} \quad N_v = C + 2 \quad \text{Or} \quad C + 2$$



## 2- Adiabatic Equilibrium Stage:

مردار 1 stage با تپش و در نتیجه  
 • energy  
 و در هر تپش و در نتیجه energy و در  
 1 stream در هر صورت مردار



\* open system  $\Rightarrow$  mass & energy  
 \* close system  $\Rightarrow$  تنقل ماده (energy) و در  
 لا تنقل mass  
 \* Isolated  $\Rightarrow$  mass & energy لا تنقل

NV:

The only variables are those associated with streams:

4 stream

$$N_V = 4(C+3) = 4C + 12 \quad \text{Or} \quad N_V = 4(C+2) = 4C + 8$$

$N_V = 4 N_V$  for single stream

NE:

int

C-1

C int

| Category                                                                         | Number of Equations |      |
|----------------------------------------------------------------------------------|---------------------|------|
| Equilibrium Restrictions:                                                        |                     |      |
| $P_{VOUT} = P_{LOUT}$                                                            | 1                   | 1    |
| $T_{VOUT} = T_{LOUT}$                                                            | 1                   | 1    |
| Phase Equilibrium<br>( $y_i$ ) <sub>VOUT</sub> = $k_i$ ( $x_i$ ) <sub>LOUT</sub> | C                   | C    |
| Component Balances                                                               | C-1                 | C-1  |
| Total Material Balance                                                           | 1                   | 1    |
| Adiabatic Enthalpy Balance                                                       | 1                   | 1    |
| Mole Fraction Constraints<br>(Sum of Mole Fractions)                             | 4                   | -    |
| NE =                                                                             | 2C+7                | 2C+3 |

$y_i = k_i x_i \Rightarrow$  کم غلظت بقدر آهن و  
 equilibrium  
 عدد لغزعات = عدد المكونات (C)

تپش و oil equil  
 $P_{VOUT} = P_{LOUT}$   
 $T_{VOUT} = T_{LOUT}$   
 concentration  
 كل stream تپش  
 component بالایی  
 كل stream تپش  
 $\sum_{i=1}^C x_i = 1$

Component material balance:

$$L_{IN}(x_i)_{L_{IN}} + V_{IN}(y_i)_{V_{IN}} = L_{OUT}(x_i)_{L_{OUT}} + V_{OUT}(y_i)_{V_{OUT}}$$

Total material balance:

$$L_{IN} + V_{IN} = L_{OUT} + V_{OUT}$$

Adiabatic enthalpy balance:

$$H_{L_{IN}} L_{IN} + H_{V_{IN}} V_{IN} = H_{L_{OUT}} L_{OUT} + H_{V_{OUT}} V_{OUT}$$

المعادلات التفاضلية كل comp

intensive

قابلیت تپش و لغزعات عدد هم 0

تپش و H با تپش و عدد بینظم  
 H at reference  
 تپش و  
 \* بقدر آهن و در عدد 1 stream  
 درجه حرارت تپش و در oil reference  
 و تپش و H عدد 0





Mole fraction constraints in any stream:

$$\sum_{i=1}^C \text{mole fractions} = 1$$

Design Variables,  $N_D$ :

|                   |                    |                   |
|-------------------|--------------------|-------------------|
| $N_D = N_V - N_E$ | $(4C+12) - (2C+7)$ | $(4C+8) - (2C+3)$ |
| $N_D =$           | $2C + 5$           | $2C + 5$          |

Therefore  $(2C+5)$  variables must be specified.

Typical Set of Variables: Complete specification of the two incoming streams as well as stage pressure.

| Specified Variable                          | Number of Variables |
|---------------------------------------------|---------------------|
| Liquid IN mole fractions $(x_i)_{LIN}$      | C-1                 |
| $L_{IN}$ : Liquid IN flow rate              | 1                   |
| Vapor IN mole fractions $(y_i)_{VIN}$       | C-1                 |
| $V_{IN}$ : Vapor IN flow rate               | 1                   |
| Temperature and Pressure of $L_{IN}$        | 2                   |
| Temperature and Pressure of $V_{IN}$        | 2                   |
| Stage Pressure ( $P_{VOUT}$ OR $P_{LOUT}$ ) | 1                   |
| $N_D =$                                     | $(2C+5)$            |



Distillation بار اضافی مادی (mass) add mass to create a phase

**Distillation:** add energy to create a phases (مستطی)

- ✓ Separation is based on volatility difference
- ✓ Heat is supplied to create a vapour phase in equilibrium with a liquid
- ✓ All components in the feed are present in both phases in different proportions. The vapour phase is richer in mvc; while liquid phase is richer in lvc
- ✓ Separation is not isothermal but may be considered to be close to isothermal
- ✓ Process may be single stage (Flash vaporization or partial condensation), or continuous fractionation (packed towers or multi-tray columns), each offering a varying degree of separation.

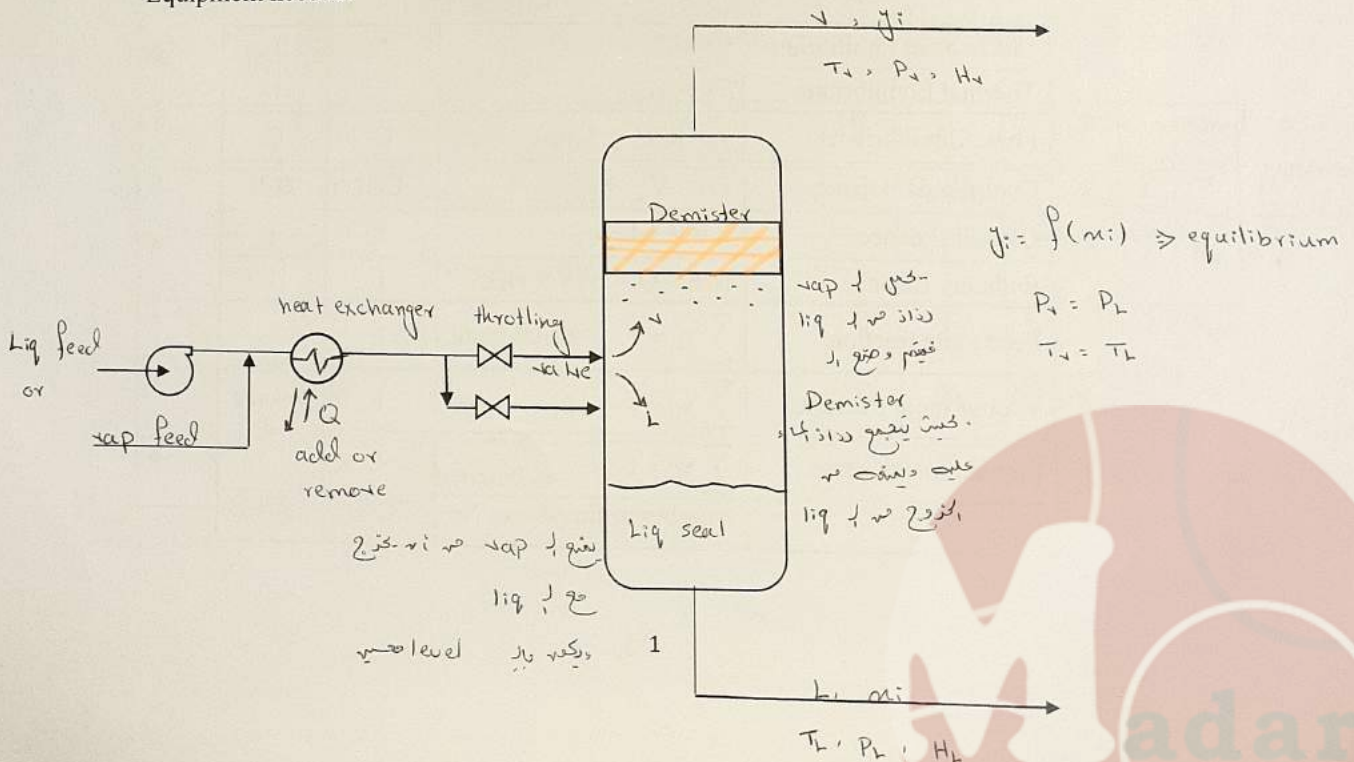
equi stage

### Equilibrium Flash Vaporization and Partial condensation

$0.95 < \alpha < 1.05$   
 به یحیی، اقلی در طریق،  
 Distillation، لایه ای volatility،  
 تقویناً قسمیات، قیاسی تطبیق، مادی  
 extraction، مادی

- Single equilibrium stage separation processes
- Unless relative volatilities are very large, the degree of separation is limited
- Useful for removal of light components from heavy ones (e.g. removal of  $N_2$  from crude oil)
- Flash vaporization: A liquid feed is partially vaporized to give a vapour richer in mvc in equilibrium with a liquid richer in lvc. The liquid may be flashed:
  - ❖ Adiabatically across a throttling valve (high pressure liquid is throttled into a flash drum).
  - ❖ Isothermally: throttling valve is not used (liquid under low pressure is vaporized in a heater then separated into two phases in a flash drum).
- Partial condensation: A vapour feed is cooled and partially condensed with phase separation taking place in a flash drum.
- Vapour and liquid leaving a flash drum are in equilibrium with each other
- Flash calculations can be used to determine the state of a stream of known composition, Temperature and Pressure.

Equipment needed:





### Operation: (Ex: adiabatic flash)

- Feed is pressurised and heated (if necessary)
- Feed is passed through a throttling valve or nozzle into a flash drum at a lower pressure
- Due to large drop in pressure, a fraction of the liquid vaporises
- Vapour is taken off overhead, while the liquid drains to the bottom of the drum
- Demister prevents liquid droplet from being entrained in the vapour

### Design Considerations

The design of an equipment involves

Stream variables calculations  
(chemical engineering)  $\Rightarrow n, T, P, \dots$

Size of equipment (last calculation)

Stream and process variables for flash or partial condensation,  $N_v$ :

composition of feed  
↑  
 $F, V, L, \bar{Z}, \bar{y}, \bar{x}, T_F, T_V, T_L, P_F, P_V, P_L$ , and  $Q$

3 stream  
↑  
 $\therefore N_v = 3(C+3) + 1$  OR  $N_v = 3(C+2) + 1$

T & P & flow rate  
↓  
Q (extensive property)

Number of Equations,  $N_E$ :

|                            |                             |      |             |
|----------------------------|-----------------------------|------|-------------|
| Mechanical Equilibrium     | $P_V = P_L$                 | 1    | 1           |
| Thermal Equilibrium        | $T_V = T_L$                 | 1    | 1           |
| Phase Equilibrium          | $y_i = K_i x_i$             | C    | C           |
| Component Balance          | $Fz_i = V y_i + L x_i$      | C-1  | C-1         |
| Overall Balance            | $F = V + L$                 | 1    | 1           |
| Enthalpy Balance           | $H_F F + Q = H_V V + H_L L$ | 1    | 1           |
| Feed mole fraction         | $\sum z_i = 1$              | 1    | Not Counted |
| Vapour mole fractions      | $\sum y_i = 1$              | 1    |             |
| Liquid mole fraction       | $\sum x_i = 1$              | 1    |             |
| Number of Equations, $N_E$ |                             | 2C+6 | 2C+3        |

$k$  is function on  $T$   
volatility  $\rightarrow$   $k$  is function of  $T$   
لا يتغير مع تغير  $T$   
 $k \approx \text{const}$



### Degrees of Freedom, $N_D$ :

|                   |                         |                         |
|-------------------|-------------------------|-------------------------|
| $N_D = N_V - N_E$ | $[3(C+3) + 1] - [2C+6]$ | $[3(C+2) + 1] - [2C+3]$ |
| $N_D$             | $C + 4$                 | $C + 4$                 |

Usually the designer has information about:

- Feed flow rate  $F$  1
- Feed Temperature  $T_F$  1
- Feed Pressure  $P_F$  1
- Feed Composition  $\vec{Z}$   $C-1$  since  $\sum z_i = 1$  is not needed

$(C+4) - (C+2) = 2$   $\frac{C+4}{C+2}$   
 $N_D = 2$

This means that two additional variables can be specified and the rest are calculated.

The choice of the specified variables controls the choice of the computational procedure.

The designer may be required to solve material balance equations and equilibrium equation then solve for energy balance (sequential solution procedure) or solve material, equilibrium and energy equation ( simultaneous solution).

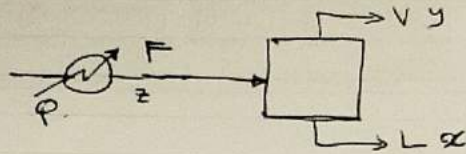
Examples:

| Case | Specified variables  | Type of Computational Procedure | Output Variables |            |     |                      |     |                      |
|------|----------------------|---------------------------------|------------------|------------|-----|----------------------|-----|----------------------|
| 1    | $T_V, P_V$           | Isothermal                      | $Q$              |            | $V$ | $\vec{y}$            | $L$ | $\vec{x}$            |
| 2    | $P_V$ and $Q = 0$    | Adiabatic                       |                  | $T_V$      | $V$ | $\vec{y}$            | $L$ | $\vec{x}$            |
| 3    | $L, P_V$             | Percent Liquid                  | $Q$              | $T_V$      | $V$ | $\vec{y}$            |     | $\vec{x}$            |
| 4    | $V, P_V$ (or $T_V$ ) | Percent Vapour                  | $Q$              | $T_V(P_V)$ |     | $\vec{y}$            | $L$ | $\vec{x}$            |
| 5    | $P_V, Q \neq 0$      | Non Adiabatic                   |                  | $T_V$      | $V$ | $\vec{y}$            | $L$ | $\vec{x}$            |
| 6    | $P_V, x_j$           | Liquid Purity                   | $Q$              | $T_V$      | $V$ | $\vec{y}$            | $L$ | $\vec{x}_{i \neq j}$ |
| 7    | $P_V, y_j$           | Vapour purity                   | $Q$              | $T_V$      | $V$ | $\vec{y}_{i \neq j}$ | $L$ | $\vec{x}$            |
| 8    | $y_j, x_k$           | Separation                      | $P_V, Q$         | $T_V$      | $V$ | $\vec{y}_{i \neq j}$ | $L$ | $\vec{x}_{i \neq k}$ |





## Binary Mixtures: Graphical Methods



**x-y diagram:** material balance with equilibrium

**Material Balances for MVC** & less volatile component by difference

$$zF = yV + Lx$$

Component:  $y = -\frac{L}{V}x + \frac{F}{V}z$        $\frac{y-z}{x-z} = -\frac{L}{V}$

Total:  $L = F - V$

$\psi = \frac{V}{F}$  (fraction vaporized)

$$y = -\frac{(F-V)}{V}x + \frac{F}{V}z$$

$$= -\frac{1-\psi}{\psi}x + \frac{z}{\psi}$$

\* when  $\psi = 0$

$y = -\frac{1}{0}x + \frac{z}{0} \Rightarrow \text{slope} = \infty \Rightarrow$  يعني لا ماسوي

\* when  $\psi = 1$

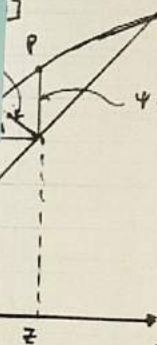
$y = -(0)x + z \Rightarrow \text{slope} = 0 \Rightarrow$  يعني خط افقي

st. line (operating line).

$$y = -\frac{(1-\psi)}{\psi}x + \frac{z}{\psi}$$

slope      intercept.

line also passes through the point  $z=x=y$   
 $\Rightarrow z=x=y$



• Bubble point  $\rightarrow$  Saturated liquid.

• M: non eqm stage product

• N: eqm stage product

• P: richest point of feed  $\psi = 0$

• T: leanest point of feed  $\psi = 1$

$$* y = -\frac{(1-\psi)}{\psi}x + \frac{z}{\psi}$$

$x = y = z$       45° line

$x = y$       45° line

$$x = -\frac{(1-\psi)}{\psi}x + \frac{z}{\psi}$$

$$(x + \frac{(1-\psi)}{\psi}x = \frac{z}{\psi}) \psi$$

$$\psi x + (1-\psi)x = z$$

$$x(1) = z \quad \#$$

$\psi = 1$  Dewpoint  
 $\rightarrow$  Saturated vapor

(sat liquid) bubble  $\psi = 0 = \frac{1}{F} \Rightarrow \frac{1}{F} = 0$

more rich in feed

(sat vapor) dew point  $\psi = 1 = \frac{1}{F} \Rightarrow \frac{1}{F} = 1$

leanest point of feed

energy & material balance with equilibrium

**Hyz diagram** (solving material and Energy Balances)

0 or - or +  $\frac{Q}{F}$

EB:  $H_F \cdot F + \frac{Q}{F} = H_V \cdot V + H_L \cdot L$

$$F \left( H_F + \frac{Q}{F} \right) = H_V \cdot V + H_L \cdot L$$

material with energy balance  $\Rightarrow (V+L) \left( H_F + \frac{Q}{F} \right) = H_V \cdot V + H_L \cdot L$

$$\Rightarrow [H_V - (H_F + \frac{Q}{F})]V = -[H_L - (H_F + \frac{Q}{F})] \cdot L$$

$$\Rightarrow \boxed{-\frac{L}{V} = \frac{H_V - (H_F + \frac{Q}{F})}{H_L - (H_F + \frac{Q}{F})} = \frac{y - z}{x - z}}$$

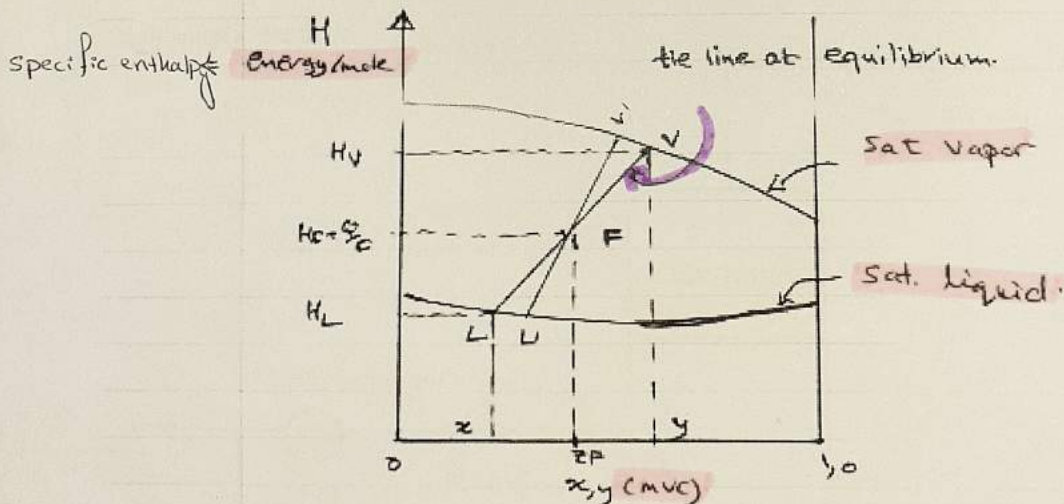
Enthalpy of feed after Heat exchanger

This is a straight line on the Hyz diagram with the following coordinates:

Point V  $\rightarrow (H_V, y)$

Point L  $\rightarrow (H_L, x)$

Point F  $\rightarrow \left[ \left( H_F + \frac{Q}{F} \right), z \right]$



L'V is also a tie line for an equilibrium process  
L'V' corresponds to a non equilibrium stage.



## Multicomponent Isothermal Flash and Partial Condensation Calculations:

Given

- ▲ Feed data ( $F, T_F, P_F, C-1$  values of  $\vec{z}$ )  
composition of feed
- ▲  $T_v$  (or  $T_L$ )
- ▲  $P_v$  (or  $P_L$ )

Required:  $V, L, Q, \vec{y}, \vec{x}, T_L$  (or  $T_v$ ),  $P_L$  (or  $P_v$ ),  $\vec{z}_c$  (remaining)

Remaining unknowns:  $2C+6$

➔ **Procedure:** The solution procedure is sequential.

**Step 1:**

solve equations containing one variable.

$$\begin{aligned} T_L &= T_v \\ P_L &= P_v \\ Z_k &= 1 - \sum_{\substack{i=1 \\ i \neq k}}^c Z_i \end{aligned} \quad c: \text{no. of components.}$$

**Step 2:** Solution of Material Balance and Eqm Relations

- ▲ Material balances are non linear → computational procedure is not straight forward and many solution strategies can be adopted
- ▲  $K_i$  values may be function of composition or independent of composition. This influences the solution strategy.

ملاحظة:  $K_i$  قد يكون  $T$  is const  $\Leftarrow$  composition  $\neq$

ملاحظة:  $K_i$  قد يكون case  $c$   $\rightarrow$   $K_i$  is const

$K_i$  is const  $\rightarrow$   $n$   $\neq$



Case 1:  $K_i$  values independent of composition.

component balance  $Fz_i = Vy_i + Lx_i$

total balance  $F = V + L$

$Fz_i = Vy_i + (F - V)x_i$

if  $K_i$  &  $z_i$  are known, solve for  $y_i$  or  $x_i$   
if  $y_i$  &  $K_i$  are known, solve for  $x_i$

Substitute  $y_i = K_i x_i$   
for  $y_i$  or  $x_i$   
solve for

$Fz_i = Vy_i + (F - V) \frac{y_i}{K_i}$

$y_i \left[ V + \frac{(F - V)}{K_i} \right] = Fz_i$

$y_i = \frac{Fz_i K_i}{VK_i + (F - V)}$

$y_i = \frac{z_i K_i}{\left(\frac{V}{F}\right)K_i + \left(1 - \frac{V}{F}\right)}$

$\psi = \frac{V}{F}$

$Fz_i = VK_i x_i + (F - V)x_i$

$x_i [VK_i + (F - V)] = Fz_i$

$x_i = \frac{Fz_i}{VK_i + (F - V)}$

$x_i = \frac{z_i}{\frac{V}{F}K_i + \left(1 - \frac{V}{F}\right)}$

$\psi = \frac{V}{F}$

if  $\psi$  is known, solve for  $y_i$  or  $x_i$

$\sum y_i = \sum x_i = 1$

if  $\psi$  is known, solve for  $y_i$  or  $x_i$   
sum = 1

Constraints  $\sum y_i = 1$

$\sum x_i = 1$

In iterative solutions

$\sum < 1$

reduce  $\psi$

$\sum > 1$

increase  $\psi$





One strategy consists of

- Finding  $K_i$ 's at  $T_v$
- Guessing values of  $\psi$  until  $\sum y_i = 1$  or  $\sum x_i = 1$

Another strategy:

The equations for  $y_i$  and  $x_i$  can be combined to give a non linear equation in  $\psi$  only.

$$\sum_{i=1}^C y_i - \sum_{i=1}^C x_i = 0$$

$$\sum_{i=1}^C \frac{K_i z_i}{1 + \psi(K_i - 1)} - \sum_{i=1}^C \frac{z_i}{1 + \psi(K_i - 1)} = 0$$

$$\sum_{i=1}^C \frac{z_i (K_i - 1)}{1 + \psi (K_i - 1)} = 0$$

This equation can be solved for  $\psi$ .

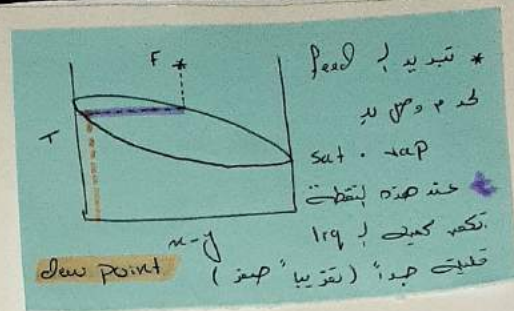
Step 3:

$$\bullet V = \psi F \Rightarrow L = F - V$$

$$\left. \begin{array}{l} \bullet x_i = \frac{z_i}{1 + \psi(K_i - 1)} \\ \bullet y_i = K_i x_i \end{array} \right\} \text{مقادير } \psi \text{ معروفه فـ } y_i \text{ و } x_i$$

$$Q = H_V \cdot V + H_L \cdot L - H_F \cdot F$$

Note: If  $T_F$  or/and  $P_F$  are not specified, then  $Q$  cannot be calculated. In this case eqm phase conditions are calculated only.



$n-j$   
 \* سَتِيْبُ اِ لْ فِئْدِ عَدَدٍ لِي سَات. ١: ٦  
 عَدَدٌ هُوَ اِتْمَعَتْ نَكْبَ كَمِيَدٍ اِ لْ صَافِ قَلِيَدٍ  
 بَدَأُ ( تَقْرِيْبًا صَوْر )

Joint Calculations: special case from Plush calculate

a two phase equilibrium mixture

A: As a first estimate

If all  $k_i$  values  $> 1 \Rightarrow$  exit phase above dew point (superheated vapor)

if all  $K$  values  $< 1 \Rightarrow$  exit phase below bubble point (subcooled liquid)

or  $k > 1$   $\Rightarrow$  Use of parameter  $\psi$  (more precise).  $\psi = \frac{1}{k}$

$$-\psi \in [0, 1] \quad f(\psi) = \sum \frac{z_i(1-k_i)}{1+\psi(k_i-1)} \quad \text{مع } x_i \text{ و } y_i \text{ مجموع}$$

\* when  $\psi = 0$   $\rightarrow$  bubble point

$$\sum z_i (1 - \kappa_i) = f(\psi)$$

$$P(\psi) = \sum z_i - \sum z_i k_i$$

$\Rightarrow$  bubble point  $\rightarrow$  this means the  
feed is liq.  $\Rightarrow z_i = m_i$

$$P(\psi) = 1 - \sum \frac{y_i}{n_i}$$

$$f(\psi) = 1 - \epsilon_j$$

- Bubble Point.  $\Rightarrow \psi = 0$

$$\psi=0 \Rightarrow f[\psi=0] = 1 - \sum_{y_i} z_i k_i$$

if  $f[\psi=0] > 0 \Rightarrow$  exit mixture below bubble  
 $1 - \sum y_i > 0$  vapor is not rich enough  $\sum y_i < 1$

If  $\delta[\psi=0]=0 \Rightarrow$  exit mixture at bubblepoint  $\epsilon y_i = 0$

$\therefore$  Bubble point criterion is

$$\sum_{i=1}^C \tilde{z}_i w_i = 1$$

with  $x_i = z_i$   
 $y_i = z_i k_i$  } In first bubble

Note:

$$y_i = \frac{\alpha_{ij} z_j}{\sum \alpha_{ki} z_k} \Rightarrow \sum \alpha_{kj} z_k = \frac{\alpha_{ij} z_j}{y_i} = \frac{\kappa_j}{\kappa_i} \frac{1}{\kappa_i} = \frac{1}{\kappa_i}$$

$$\therefore \sum \frac{K_k}{K_j} z_k = \frac{1}{K_j}$$

$j$ : reference component

heavy component

$$y_i = \frac{a_{ij} z_j}{\sum a_{kj} z_k}$$

$$\sum_k a_{kj} z_k = \frac{a_{ij} z_i}{y_i} = \frac{k_i}{k_j} \frac{y_i}{y_i} = \frac{k_i}{k_j} = \frac{1}{k_j}$$

at bubble  
 $n_i = 2$



$z_i = m_i$  @ bubble

ask :: believe &amp; receive point



### Dew Point:

$$f(\psi) = \sum \frac{z_i(1-k_i)}{1+\psi(k_i-1)} \quad \psi=1 \Rightarrow f[\psi=1] = \sum \frac{z_i}{k_i} - 1 \quad \sum z_i = 1$$

$$f(\psi) = \sum \frac{z_i(1-k_i)}{k_i}$$

if  $f[\psi=1] < 0$  exit stream above its dew point [super heated vapor  $K > 1$ ]

$$f(\psi) = \sum \frac{z_i}{k_i} - 1$$

if  $f[\psi=1] = 0$  exit stream is at its dew point

$$f(\psi) = \sum \frac{z_i}{k_i} - 1$$

at dew point  $z_i = y_i$

$\therefore$  Dew point criterion is

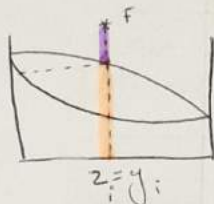
vapor feed

$$\sum \left( \frac{z_i}{k_i} \right) = 1 \quad \text{with} \quad \left. \begin{array}{l} y_i = z_i \\ x_i = \frac{z_i}{k_i} \end{array} \right\} \sum x_i = 1$$

$$= \sum \frac{y_i}{y_i/m_i} = \sum \frac{y_i}{m_i}$$

Note:

$$x_i = \frac{y_i/k_{ij}}{\sum y_k/k_{kj}} \Rightarrow \sum \frac{y_k}{x_{kj}} = \frac{y_i}{x_i} \frac{1}{\alpha_{ij}} = k_{ij}$$



a) dew point

$$\text{Assume } T \text{ find } \frac{y_k}{x_{kj}} \Rightarrow \sum \frac{y_k}{x_{kj}} = K_j$$

NOTE:

Bubble point and Dew point Calculations are used to determine saturation conditions for liquid and vapor streams respectively.

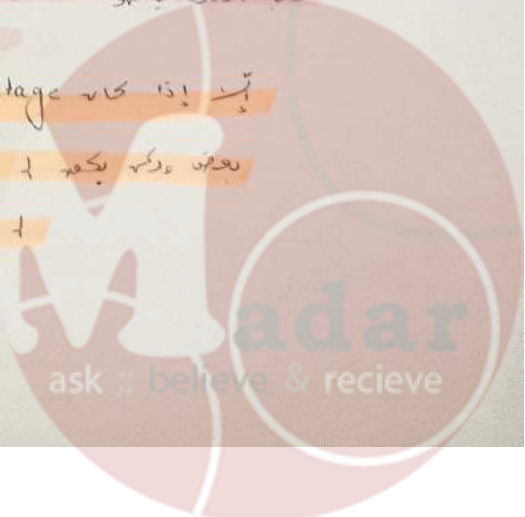
in VLE  $\begin{cases} \text{Vap is at its dew point (saturated)} \\ \text{Liq. is at its bubble point (saturated).} \end{cases}$

مخرج من عمود  $\rightarrow$  equilibrium with each other

equilibrium single stage vs is bi single stage vs is

dew point  $\rightarrow$  sat. vap  $\leftarrow$  vap  $\rightarrow$  sat. liq

bubble point  $\leftarrow$  sat. liq  $\leftarrow$  liq  $\rightarrow$



## Adiabatic Flash:

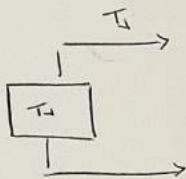
Given :  $\Delta$  Feed data

$\Delta P_v$

$\Delta Q = 0$

Reqd:  $v, L, y, x, T_v, T_L, P_L, Z_c$

Remaining unknowns  $2C+6$



Procedure: The solution procedure is simultaneous.

- $T_v$  is unknown  $\therefore K$  values cannot be obtained
- To find  $T_v$  - solve enthalpy balance.
- Since EB depends on composition and  $\psi$  it cannot be solved alone
- Net result  $\rightarrow$  Simultaneous soln.

comp. of E.B. making it simultaneous with  $\psi$  &  $\phi$  soln

One strategy:

Step 1: solve equations containing one variable

$$P_v = P_v$$

$$Z_c = 1 - \sum Z_i$$

Step 2: Estimate  $T_v \Rightarrow$  guess value of  $T_v$   $\rightarrow$  find B.P.  $P_i$  & mixture  $P_v$  B.P.

Step 3: use isothermal flash procedure at  $T_v, P_v$

- obtain  $K_i$  at  $T_v, P_v$
- solve for  $\psi$
- solve for  $y, x$
- solve for  $v, L$

$$T_v \rightarrow P_i \rightarrow P_v$$

Step 4: Calculate  $H_L, H_v$





After  $T_v$  is estimated step 5: Check  $T_v$  enthalpy balance,

From Enthalpy balance  $Q_{in}$  and M.B.  $L_{2F-v}$

$$Q = H_v V + H_L(F - V) - H_F \cdot F$$

$$\psi H_v + (1 - \psi) \cdot H_L - H_F = 0 = f(T_v)$$

$$f(T_v) = 0 \text{ at eqm.}$$

$$\text{Calculate } f(T_v) = \frac{\psi H_v - (1 - \psi) H_L - H_F}{1000} \stackrel{?}{=} 0$$

في هذه المرحلة  $H_L$  و  $H_v$  من 1000 و  $H_F$  من 1000  
نحتاج إلى  $T_v$  لتقدير  $H_L$  و  $H_v$

step 6: if  $f(T_v) \neq 0$  reestimate  $T_v$  and go back to step 3.

إذا  $T_v$  ليس هو  $f(T_v) \neq 0$ !

step 7: Stop and Print results.



\* procedure for doing isothermal flashing

\* ex:

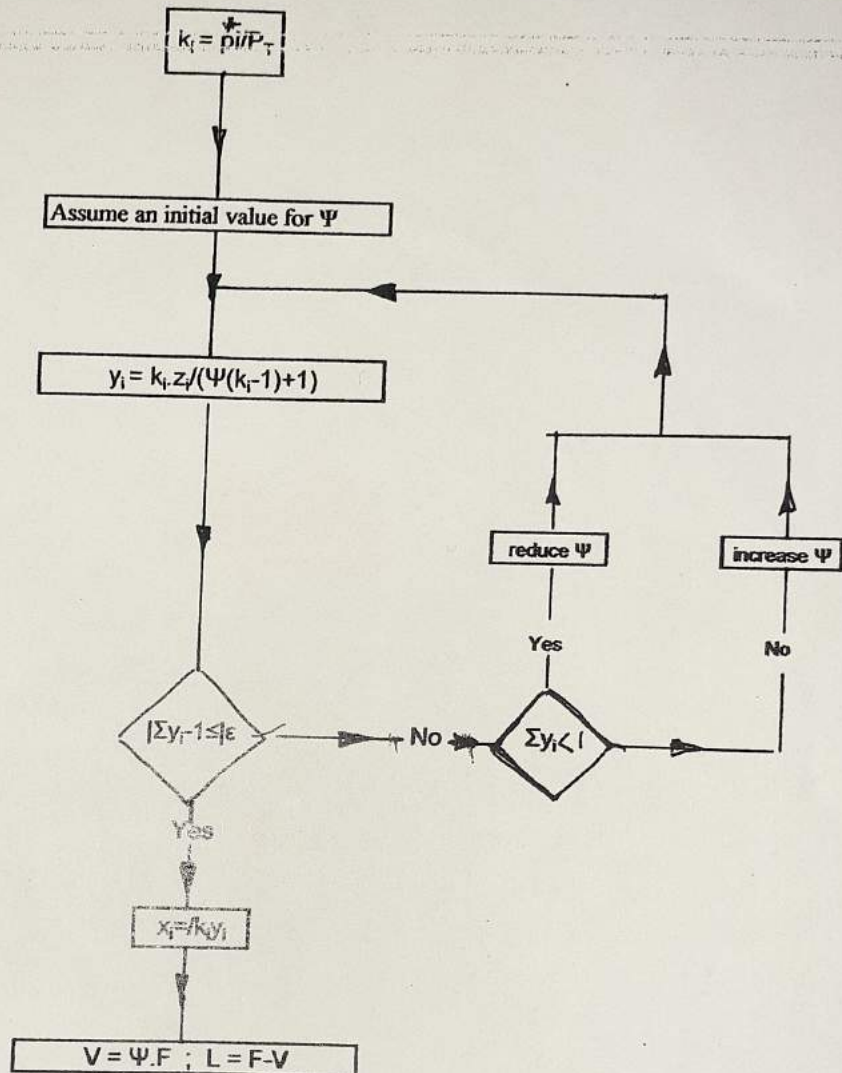
50% mole of Benzene (A)

25% mole of Toluene (B)

25% mole of O-xylene (C)

P = atm

T = 100°C isothermal



$P_i^*/P$

| Component | $P_i$<br>mmHg | $k_i$ | $z_i$ | $k_i z_i$ | $(k_i - 1)$ | $y_i = k_i z_i / (\Psi(k_i - 1) + 1)$ |            |               | $x_i = y_i / k_i$ |
|-----------|---------------|-------|-------|-----------|-------------|---------------------------------------|------------|---------------|-------------------|
|           |               |       |       |           |             | $\Psi = 0.5$                          | $\Psi = 1$ | $\Psi = 32.5$ |                   |
| A         | 1370          | 1.203 | 0.5   | 0.601     | 0.203       | 0.643                                 |            | 0.715         | 0.397             |
| B         | 550           | 0.724 | 0.25  | 0.181     | -0.276      | 0.210                                 |            | 0.199         | 0.274             |
| C         | 200           | 0.263 | 0.25  | 0.066     | -0.737      | 0.104                                 |            | 0.087         | 0.324             |

↓  
vapor pressure  
from Antoine  
equation  
at T = 100°C

$\sum z_i = 1$

$\sum y_i = 0.957$

$\sum y_i = 1.007$

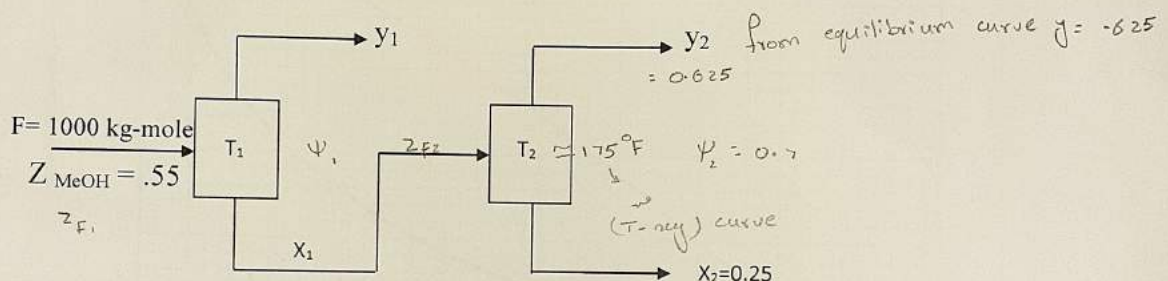
$\sum x_i = 1.000$

↓  
 $\Psi = 1$   
∴  $\Psi = 1$





Two flash distillation chambers are connected together as shown in the diagram. Both are at 1 atm pressure. The feed to the first drum is a binary mixture of methanol and water having 55 mole% methanol. Feed flow rate is 1000 kg-moles/hr. The second drum operates with a vapor to feed ratio of 0.7 and the liquid product is 25 mole % methanol. Equilibrium diagrams are given.



- What are  $y_1$ ,  $y_2$ ,  $x_1$ ,  $T_1$  and  $T_2$ ?
- What is the fraction vaporized in the first drum?

\* from material balance:

$$F \cdot Z_F = L \cdot x + V \cdot y$$

$$Z_F = \psi y + (1-\psi)x$$

equilibrium of  $z_F = x_1 = 0.512$   
curve

$y_1$  is

$$y_1 = 0.732$$

165  $\approx T_1$  is  $(T-xy)$  curve

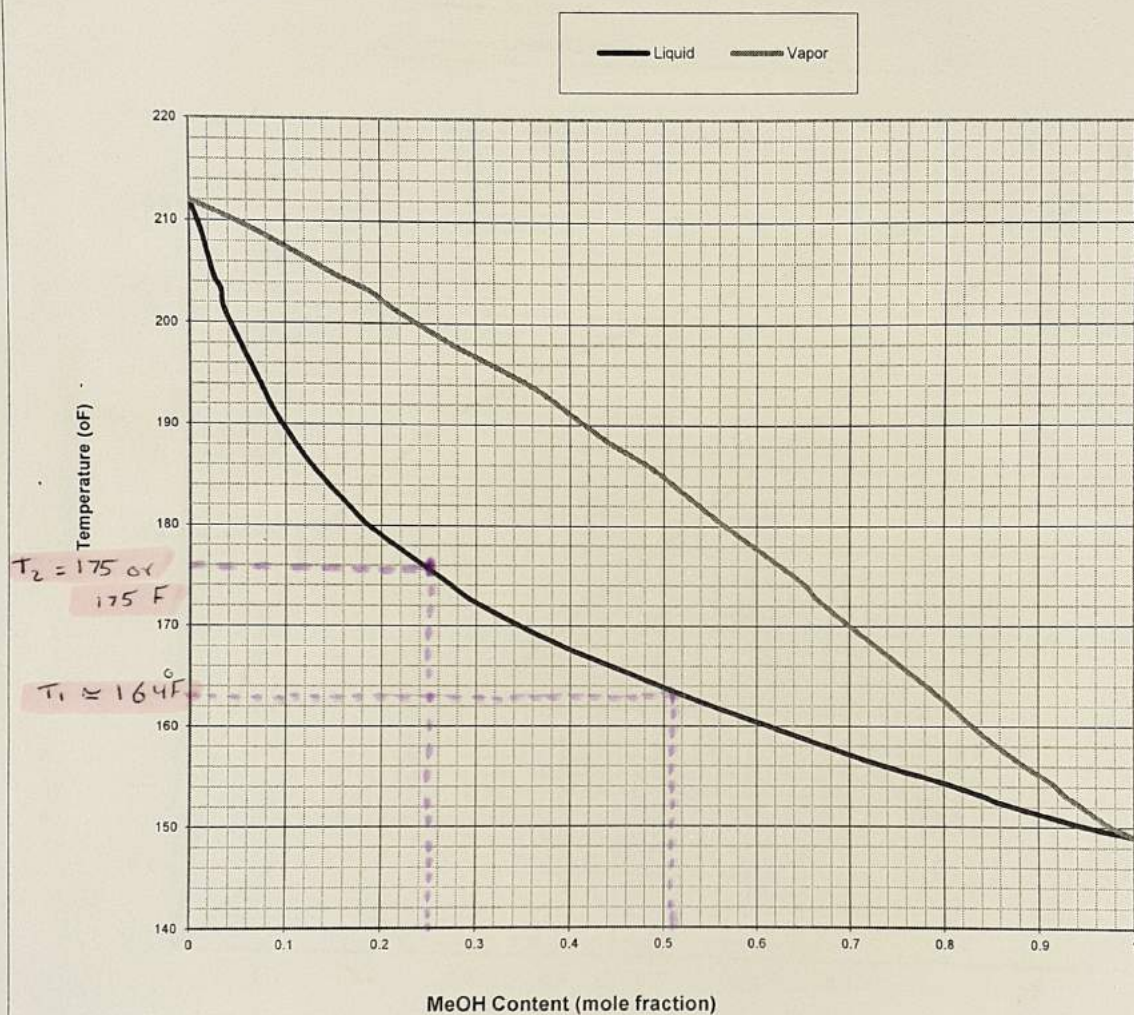
Operating line is  $\psi$  is

$$y = \frac{\psi}{1-\psi} x + \frac{Z_F}{1-\psi}$$

$$\psi = 1.47$$

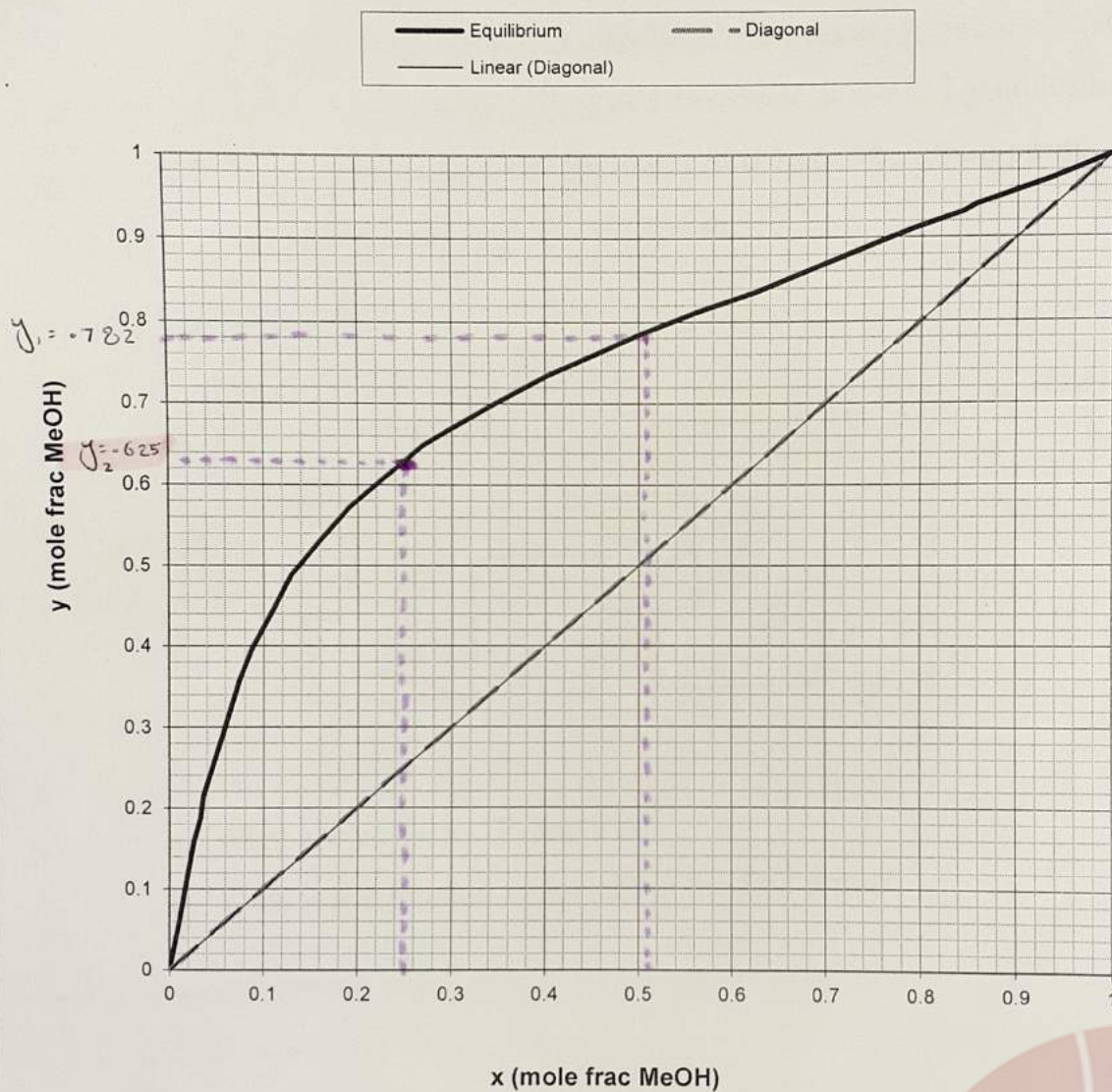


VLE for MeOH/H<sub>2</sub>O system @ 1 atm





VLE for MeOH/H<sub>2</sub>O system @ 1 atm



سفر: پائپ  
cylinder

\* اکثر انواع تجهیزات پائپ به شکل sphere or cylinder ←

## 2.9 Size Calculation

Once the vapor and liquid compositions and flow rates have been determined, the flash drum can be sized. This is an empirical procedure. We will discuss the specific procedure first for vertical flash drums (Figure 2-1) and then adjust the procedure for horizontal flash drums.

Step 1. Calculate the permissible vapor velocity,  $u_{perm}$

بزرگترین سرعت در پائپ مسدود →  $u_{perm} = K_{drum} \sqrt{\frac{\rho_L - \rho_v}{\rho_v}}$

(2-64)

$u_{perm}$  is the maximum permissible vapor velocity in feet per second at the maximum cross-sectional area.  $\rho_L$  and  $\rho_v$  are the liquid and vapor densities.  $K_{drum}$  is in ft/s.

$K_{drum}$  is an empirical constant that depends on the type of drum. For vertical drums the value has been correlated graphically by Watkins (1967) for 85% of flood with no demister. Approximately 5% liquid will be entrained with the vapor. Use of the same design with a demister will reduce entrainment to less than 1%. The demister traps small liquid droplets on fine wires and prevents them from exiting. The droplets then coalesce into larger droplets, which fall off the wire and through the rising vapor into the liquid pool at the bottom of the flash chamber. Blackwell (1984) fit Watkins' correlation to the equation

→  $K_{drum} = (\text{Const.}) \exp[A + B \ln F_{lv} + C(\ln F_{lv})^2 + D(\ln F_{lv})^3 + E(\ln F_{lv})^4]$

(2-65)

→ where  $F_{lv} = \frac{W_L}{W_v} \sqrt{\frac{\rho_v}{\rho_L}}$  and const = 1.0 ft/s,

with  $W_L$  and  $W_v$  being the liquid and vapor flow rates in weight units per hour (e.g., lb/h). The constants are (Blackwell, 1984):

$$A = -1.877478097$$

$$B = -0.8145804597$$

$$C = -0.1870744085$$

$$D = -0.0145228667$$

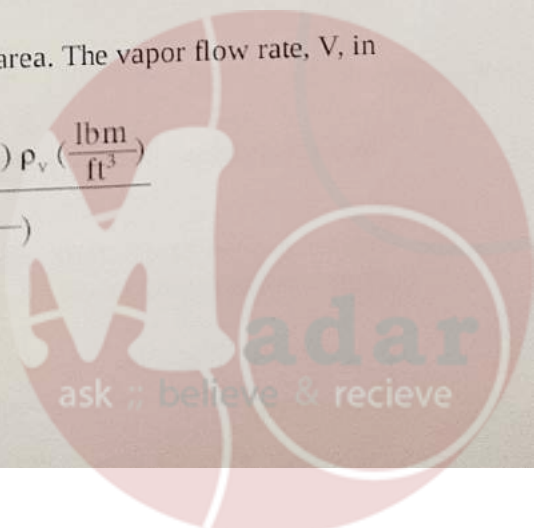
$$E = -0.0010148518$$

The resulting value for  $K_{drum}$  typically ranges from 0.1 to 0.35.

Step 2. Using the known vapor rate,  $V$ , convert  $u_{perm}$  into a horizontal area. The vapor flow rate,  $V$ , in lbmol/h is

$$V \left( \frac{\text{lbmol}}{\text{h}} \right) = \frac{u_{perm} \left( \frac{\text{ft}}{\text{s}} \right) \left( \frac{3600 \text{ s}}{\text{h}} \right) A_c (\text{ft}^2) \rho_v \left( \frac{\text{lbm}}{\text{ft}^3} \right)}{MW_{\text{vapor}} \left( \frac{\text{lbm}}{\text{lbmol}} \right)}$$

Solving for the cross-sectional area,





→ cross sectional area & Diameter

$$A_c = \frac{V(MW_v)}{u_{perm}(3600)\rho_v} \quad \text{mole flow rate/n}$$

\* بحسب  $D$  و ارتفاع  
بمقدار من قوتي  $D$  tank بشي  
vertical or horizontal

(2-66)

For a vertical drum, diameter  $D$  is

$$D = \sqrt{\frac{4A_c}{\pi}}$$

(2-67)

Usually, the diameter is increased to the next largest 6-in. increment.

Step 3. Set the length/diameter ratio either by rule of thumb or by the required liquid surge volume. For vertical flash drums, the rule of thumb is that  $h_{total}/D$  ranges from 3.0 to 5.0. The appropriate value of  $h_{total}/D$  within this range can be found by minimizing the total vessel weight (which minimizes cost).

Flash drums are often used as liquid surge tanks in addition to separating liquid and vapor. The design procedure for this case is discussed by Watkins (1967) for petrochemical applications. The height of the drum above the centerline of the feed nozzle,  $h_v$ , should be 36 in. plus one-half the diameter of the feed line (see Figure 2-14). The minimum of this distance is 48 in.

→ \* location of feed nozzle:  $h_v$

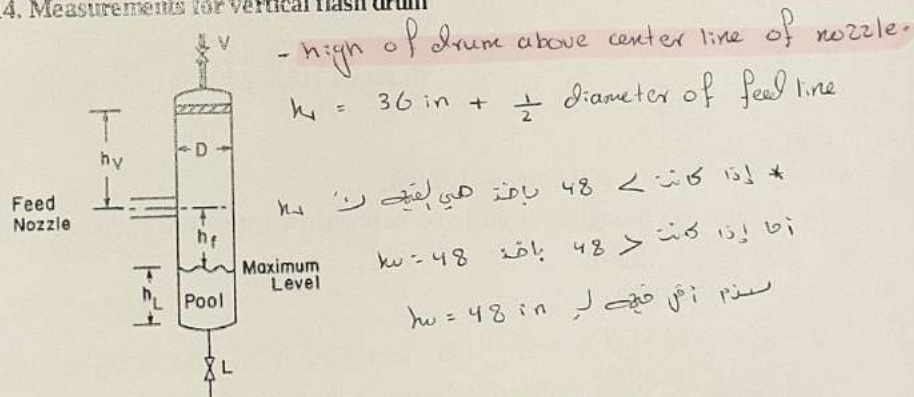
\* length / Diameter ratio

$$3 < \frac{L}{D_{drum}} < 5$$

# appropriate value in the range can be calculated by minimizing the total vessel weight.

weight of vessel ratio also \*

Figure 2-14. Measurements for vertical flash drum



The height of the center of the feed line above the maximum level of the liquid pool,  $h_f$ , should be 12 in. plus one-half the diameter of the feed line. The minimum distance for this free space is 18 in. The depth of the liquid pool,  $h_L$ , can be determined from the desired surge volume,  $V_{surge}$ .

$$h_L = \frac{V_{surge}}{\pi D^2/4}$$

Surge volume ⇒ ...

(2-68)

The geometry can now be checked, since

$$\frac{h_{total}}{D} = \frac{h_v + h_f + h_L}{D}$$

should be between 3 and 5. These procedures are illustrated in Example 2-4. If  $h_{total}/D < 3$ , a larger liquid surge volume should be allowed. If  $h_{total}/D > 5$ , a horizontal flash drum should be used.

$\frac{L}{D} < 3$  (بكره قصير) ⇒ Means that surge capacity should be increased

$\frac{L}{D} > 5$  (طويل) ⇒ horizontal drum should be used

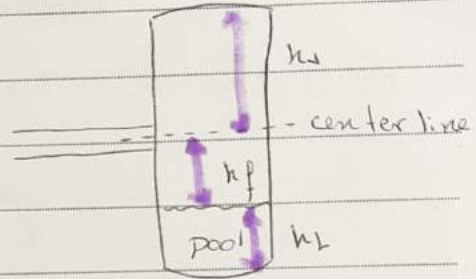
\* Surge volume: liquid holding time of approximately 10 minutes should be allowed

⇒ high of center line of feed line above max level of the liquid pool,  $h_f$

$$h_f = 12 \text{ in} + \frac{1}{2} \text{ diameter of feed line}$$

if  $h_f > 18 \text{ in} \Rightarrow$  ~~vertical~~ ~~is~~

if  $h_f < 18 \text{ in} \Rightarrow h_f = 18 \text{ in}$



⇒ Length of liquid pool,  $h_L$

$$h_L = \frac{\text{surge}}{\pi D^2_{\text{drum}} / 4}$$

$$L_{\text{tot}} = h_u + h_f + h_L$$

check on ratio  $\left( \frac{L}{D_{\text{drum}}} \right)$   $3 < \frac{L}{D} < 5$   
horizontal or vertical split is ~~is~~





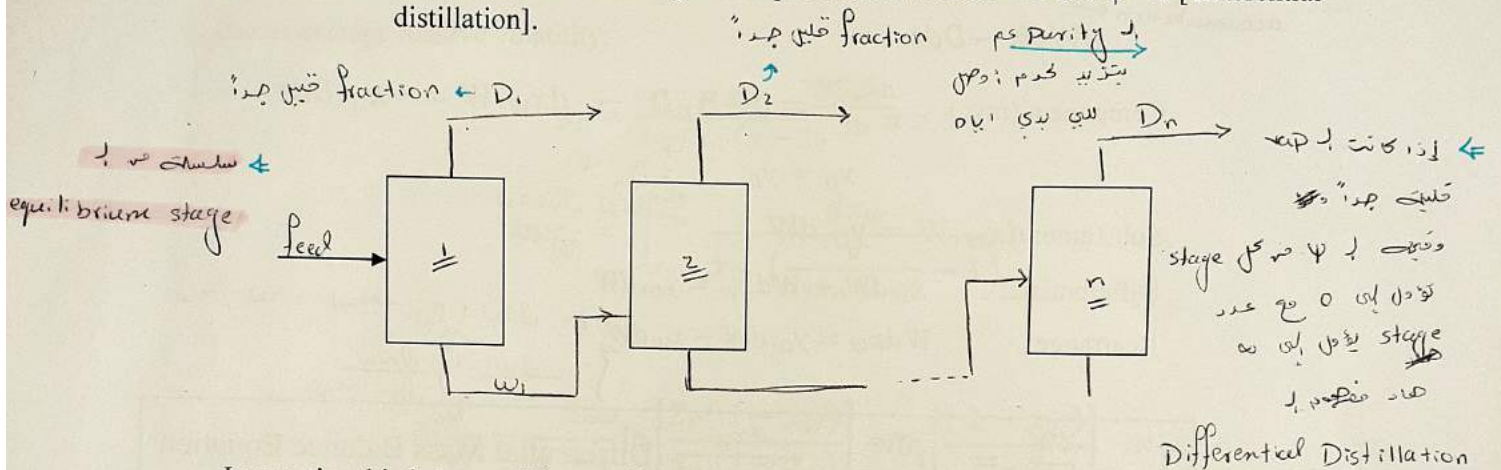
## Batch Distillation:

### Used when:

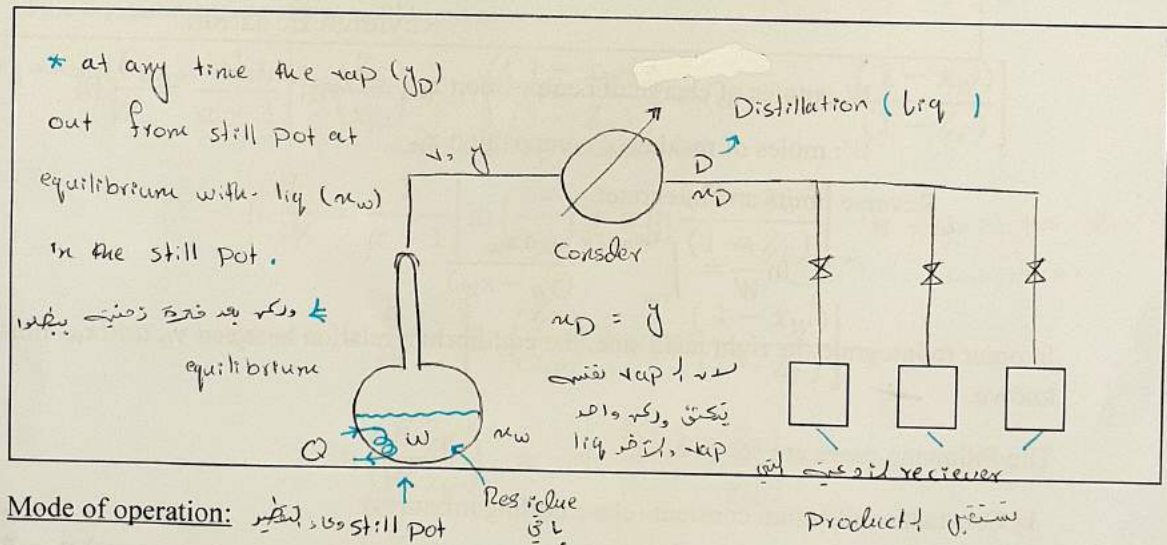
- The required operating capacity is very small
- Batch equipment offer more operating flexibility [feed fluctuations]

### Differential distillation [Simple distillation]

**Definition:** It is the limit of a multistage flash process in which  $n \rightarrow \infty$  and  $\psi \rightarrow 0$  [differential distillation].



In practice this is impossible to achieve and can only be approximated:



### Mode of operation:

- Still pot is initially charged with feed [Charge]
- Heat is supplied at a constant rate, and the charge is boiled
- Vapors are withdrawn immediately and collected in cuts as required

### Features:

- The first cut will be richest in mvc ➤ more volatile
- Operation is unsteady  $\rightarrow$  product composition changes with time
- At any instant, the vapor leaving the still pot is assumed to be in equilibrium with the liquid in the pot (residue)  $\rightarrow y_D$  in equilibrium with  $x_w$



## Material Balances:

### Binary Mixtures:

General: Rate of material = Rate of Material - Rate of Material  
 Accumulation IN OUT

input  $\Rightarrow$  close system (o)  
 Total:  $\frac{dW}{dt} = 0 - D$  D: distillate flow rate mole/hr  
 accumulation  $\leftarrow$   
 $dW = -Ddt$

Component (mvc):  $\frac{dx_W \cdot W}{dt} = 0 - x_D D \rightarrow dx_W \cdot W = -x_D \cdot Ddt$   
 $x_D = y_D$

Substitute:  $dx_W \cdot W = y_D \cdot dW$  لا تفتي بتركيز بيبي

Differentiate:  $x_W dW + W dx_W = y_D \cdot dW$

Rearrange:  $W dx_W = y_D \cdot dW - x_W dW \Rightarrow \int_{W_0}^W \frac{dW}{W} = \int_{x_{W_0}}^{x_W} \frac{y_D}{y_D - x_W} dx_W$

$$\int_{W_0=F}^W \frac{dW}{W} = \int_{x_{W_0}=Z_F}^{x_W} \frac{dx_W}{(y_D - x_W)}$$

Differential Mass Balance Equation

Rayleigh Equation

$W_0$ : moles of charge of composition  $x_{W_0} = Z_F$  initial concentration of the charge

$W$ : moles of residue of composition  $x_W$

Reverse limits and integrate:

$$\ln \frac{F}{W} = \int_{x_W}^{x_{W_0}=Z_F} \frac{dx_W}{(y_D - x_W)}$$

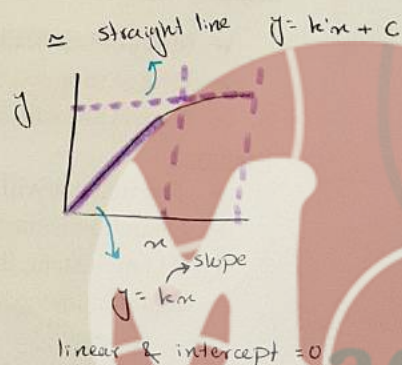
In order to integrate the right hand side, the equilibrium relation between  $y_D$  and  $x_W$  must be known.

The following cases are considered:

1] Constant equilibrium constant (close boiling mixtures)

$y = kx$  ;  $k > 1$

$$\ln \frac{F}{W} = \frac{1}{k-1} \ln \frac{Z_F}{x_W}$$



ask :: believe & recieve



If  $k$  varies slightly with composition, an average value can be used in the concentration range.

- Local equilibrium constant:  $y = k'x + c$

$$\ln \frac{F}{W} = \frac{1}{k' - 1} \ln \left( \frac{Z_F(k' - 1) + c}{x_W(k' - 1) + c} \right)$$

2] Use of average relative volatility:

$$y_D = \frac{\alpha x_W}{1 + x_W(\alpha - 1)} \quad \alpha > 1$$

$$\ln \frac{F}{W} = \int_{x_W}^{Z_F} \frac{dx_W}{x_W \left( \frac{\alpha}{1 + x_W(\alpha - 1)} - 1 \right)}$$

$$\ln \frac{F}{W} = \frac{1}{\alpha - 1} \ln \left[ \frac{Z_F(1 - x_W)}{x_W(1 - Z_F)} \right] + \ln \left[ \frac{(1 - x_W)}{(1 - Z_F)} \right]$$

This equation can be rearranged to give:

$$\ln \frac{F}{W} = \frac{1}{\alpha - 1} \left[ \ln \left( \frac{Z_F}{x_W} \right) + \ln \left( \frac{(1 - x_W)}{(1 - Z_F)} \right) + (\alpha - 1) \ln \left( \frac{(1 - x_W)}{(1 - Z_F)} \right) \right]$$

$$\ln \frac{F}{W} = \frac{1}{\alpha - 1} \left[ \ln \left( \frac{Z_F}{x_W} \right) + \alpha \ln \left( \frac{(1 - x_W)}{(1 - Z_F)} \right) \right]$$

more volatile component (A) B = (1 - A) less volatile component

$$(\alpha - 1) \ln \frac{F}{W} = \left[ \ln \left( \frac{Z_F}{x_W} \right) + \alpha \ln \left( \frac{(1 - x_W)}{(1 - Z_F)} \right) \right]$$

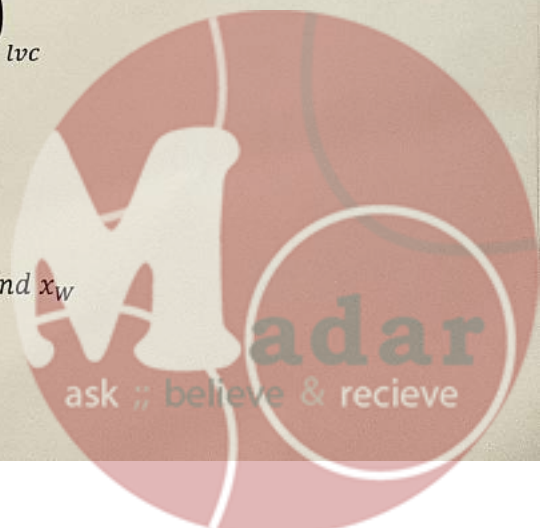
$$\ln \left( \frac{F Z_F}{W x_W} \right) = \alpha \ln \frac{F(1 - Z_F)}{W(1 - x_W)}$$

$$\ln \left( \frac{F Z_F}{W x_W} \right)_{mvc} = \alpha \ln \left( \frac{F Z_F}{W x_W} \right)_{lvc}$$

3] Graphical Integration:

Plot  $\frac{1}{(y_D - x_W)}$  vs  $x_W$

$\Rightarrow \ln \left( \frac{F}{W} \right) = \text{area under the curve between } Z_F \text{ and } x_W$



➔ **Multicomponent Mixtures (ideal solutions):** Reference component

The equations used for binary mixtures can be used for each component in the mixture:

For any component  $i$  with reference component  $j$ :

$$\ln \left( \frac{F Z_{i,F}}{W x_{i,W}} \right) = \alpha_{i,j} \ln \left( \frac{F Z_{j,F}}{W x_{j,W}} \right)$$

$$\sum_{i=1}^c x_{i,W} = 1$$

**Average composition:**

- The total amount of vapor (distillate) is not in equilibrium with the residue
- Overall vapor composition (composited distillate composition, average composition) is obtained from material balance as follows:

$$F Z_{i,F} = D (y_{i,D})_{Average} + W x_{i,W}$$

$$D = F - W$$

$$F Z_{i,F} = (F - W)(y_{i,D})_{Average} + W x_{i,W}$$

$$(y_{i,D})_{Average} = \frac{F Z_{i,F} - W x_{i,W}}{(F - W)}$$





ABET: Ex. Comparison of Flash distillation and Differential distillation:

A liquid containing 50 mole% Benzene (A), 25 mole % Toluene (B) and 25 mole % O-Xylene.

- The liquid is flash vaporized at 1 atm and 100°C. What is the fraction vaporized ( $\psi$ ) ? And what is the vapor composition?
- Under the same pressure and with the same ( $\psi$ ), the liquid is to be differentially distilled. Calculate the distillate and residue compositions.

reference comp ←

| Component | $Z_{i,F}$ | $P_{\text{mmHg at } 100^\circ\text{C}}$ | Flash $\psi=0.325$ |           | Differential distillation |                          |           |
|-----------|-----------|-----------------------------------------|--------------------|-----------|---------------------------|--------------------------|-----------|
|           |           |                                         | $y_{i,D}$          | $x_{i,W}$ | $\alpha$<br>(100°C)       | $(y_{i,D})_{\text{Ave}}$ | $x_{i,W}$ |
| A         | 0.50      | 1370                                    | 0.715              | 0.397     | 2.49                      |                          |           |
| B         | 0.25      | 550                                     | 0.198              | 0.274     | 1.00                      |                          |           |
| C         | 0.25      | 220                                     | 0.087              | 0.329     | 0.364                     |                          |           |
|           |           | $\Sigma$                                | 1.000              | 1.000     | $\Sigma$                  | 1.000                    | 1.000     |

Basis:  $F=100$  moles  $\rightarrow D=32.5$  moles and  $W=67.5$  moles

For A: 
$$\ln\left(\frac{100 \cdot 0.5}{67.5 \cdot x_{A,W}}\right) = 2.49 \ln\left(\frac{100 \cdot 0.25}{67.5 \cdot x_{B,W}}\right)$$

For C: 
$$\ln\left(\frac{100 \cdot 0.25}{67.5 \cdot x_{C,W}}\right) = 0.364 \ln\left(\frac{100 \cdot 0.25}{67.5 \cdot x_{B,W}}\right)$$

$$x_{A,W} + x_{B,W} + x_{C,W} = 1.000$$

Solve Simultaneously.





## Continuous Fractionation (Rectification):

### Fractionation Process:

There are three methods used in distillation, each offering a varying degree of separation:

- Flash or equilibrium distillation
- Differential distillation
- Continuous Fractionation

batch & flask

در کل ۳ روش برای جداسازی داریم  
۱- یکبارگانه (Batch) که در یک ظرف انجام می‌دهند  
۲- تفاقی (Differential) که در یک ظرف انجام می‌دهند  
۳- پیوسته (Continuous) که در یک ظرف انجام می‌دهند

معمولاً در یک ظرف

multistage  
vertical cylindrical pressure vessel  
تجهیز چند مرحله‌ای  
ظرف استوانه‌ای عمودی  
در فشار و دما

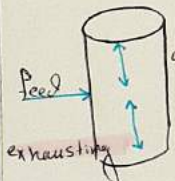
Fractionation is the most important as it offers a greater degree of separation. It is the most widely used unit operation process in the chemical industry. It is a multistage counter-current operation, where the liquid flows counter current to vapor with the feed introduced at some intermediate point. Some of the vapor is condensed (mainly lvc) and some of the liquid is vaporized (mainly mvc). This partial condensation and vaporization continuously enriches the vapor phase with the mvc, with the net result of a better separation.

وجود دارد به نحوی که  
در یک ظرف  
تفاوت دما  
تفاوت دما

The operation takes place in a vertical cylindrical pressure vessel (called distillation column or tower, or a fractionator), divided into compartments by a series of perforated plates which permit the upward flow of vapor. The fractionator consists of two main sections:

- Top section above the feed point called absorption, enriching, or rectifying section
- Bottom section below the feed point called exhausting or stripping section.

در یک ظرف  
تفاوت دما  
تفاوت دما



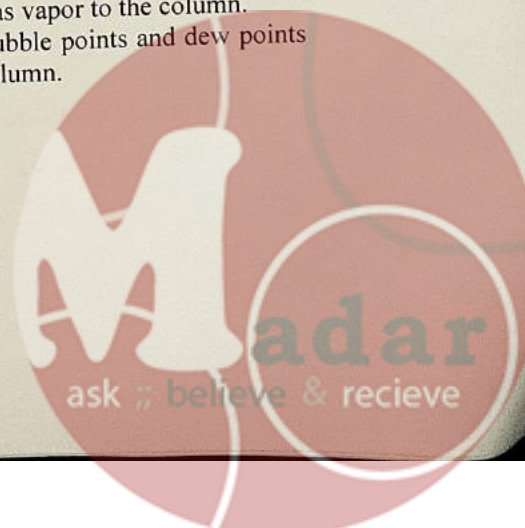
The feed is introduced continuously into the fractionator where it is eventually split into two products. One removed from the top of the column richer than the feed in the mvc (called distillate, top product, overheads product, or light product); the other product withdrawn from the bottom of the column weaker in the mvc (called bottom product, waste, residue or heavy product). The purities of the products depend on the liquid/gas ratio and the number of stages.

# of sections  
= # of feed + 1

In some cases, multiple feeds and side streams are available. Some distillation columns consist of the top section only and they are called rectification columns.

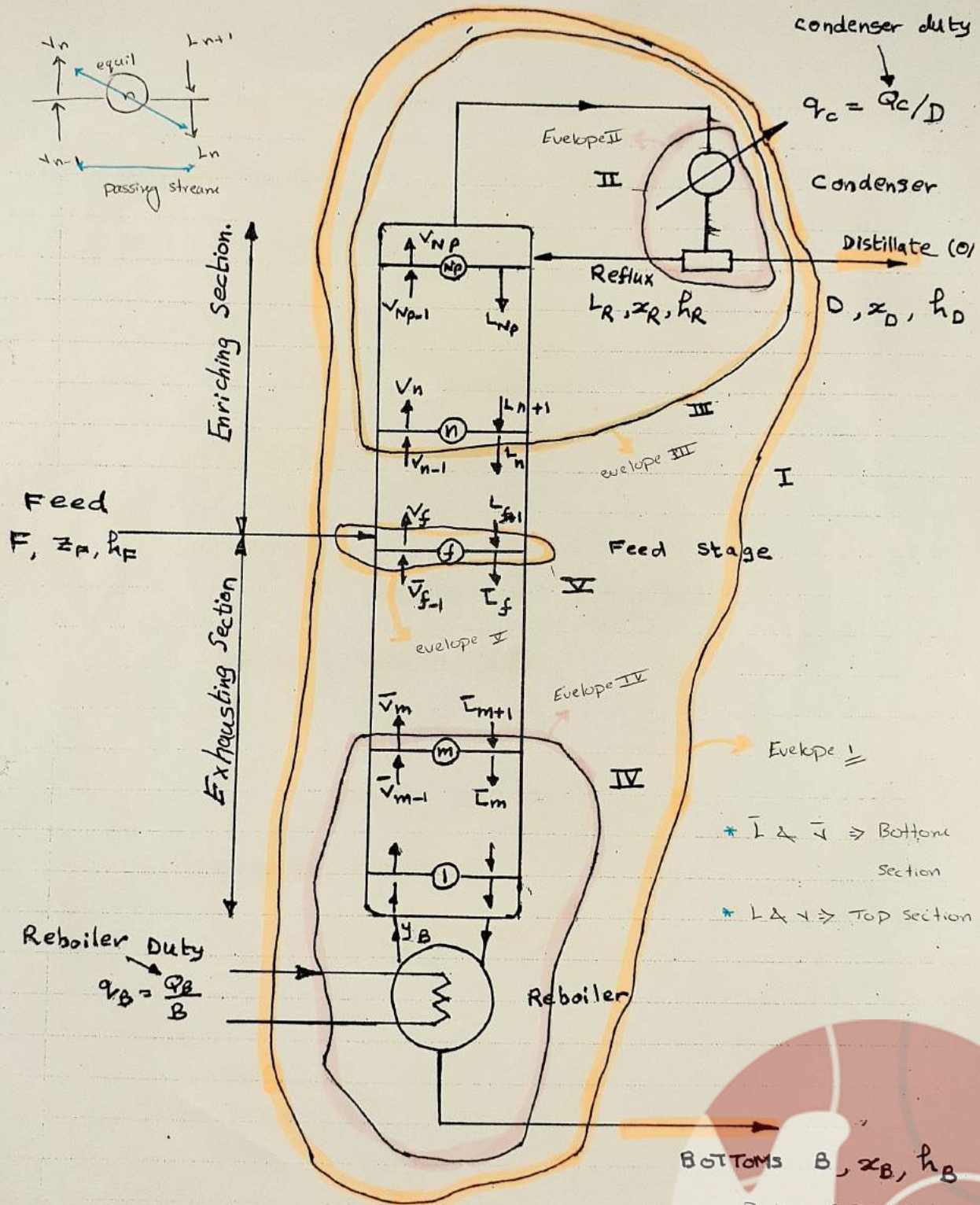
Vapors rising from the top are totally (or partially) condensed in a condenser and some of the liquid is returned to the top of the column. This liquid is called reflux. The ratio of reflux to distillate is called reflux ratio or sometimes external ratio.

The liquid at the bottom is either heated by a coil placed at the base of the column or by an external reboiler; the more volatile material returns as vapor to the column. Inside the tower liquids and vapors are always at their bubble points and dew points respectively. Their flow rates are not constant along the column.





# Material and Energy Balances



Partial condenser : considered as a stage

Partial Reboiler : considered as a stage

Bottoms & Distillate  
 are not in equilibrium  
 with each other

ask :: believe & recieve

## Material Balances:

Envelope I :

$$F = D + B \quad \text{over all}$$

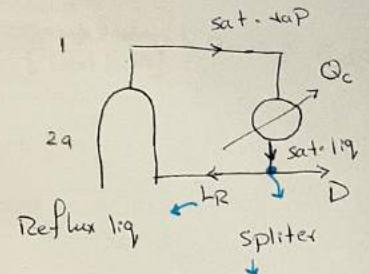
Envelope II :

$$V_{Np} = L_R + D \quad \text{condenser}$$

purity + composition values let  $R = \frac{L_R}{D}$  = reflux ratio.  
Product  $D$

$$V_{Np} = L_R + D \quad \leftarrow V_{Np} = RD + D$$

$$V_{Np} = D(R+1)$$



splitter has 2 outputs  
composition P & T of both are  
flow rate of both is 2b

Envelope III : (Down to stage n)

$$V_{n-1} = L_n + D$$

3 a

$$V_{n-1} - L_n = D$$

[Net Flow, const] 3b  
of vap up

Envelope IV : (up to stage m)

$$\bar{L}_{m+1} = \bar{V}_m + B$$

4 a

$$\bar{L}_{m+1} - \bar{V}_m = B$$

[net flow, const]  
of liq down

Envelope V : (Feed stage)

$$F + L_{f+1} + \bar{V}_{f-1} = V_f + \bar{L}_f$$

⇒ where :

- n ⇒ Top section
- m ⇒ Bottom section





Component Balances: (For any component)

D & B is comp + solvent

Envelope I

$$Z_F \cdot F = x_D \cdot D + x_B \cdot B$$

$$x_D = \frac{Z_F \cdot F - x_B \cdot B}{F - B}$$

$$x_B = \frac{Z_F \cdot F - x_D \cdot D}{F - D}$$

Enriching Section (Envelope III)

$$y_{n-1} \cdot V_{n-1} = x_n \cdot L_n + D \cdot x_D$$

[D & B: net flow]

Operating line for enriching section

$$y_{n-1} = \frac{L_n}{V_{n-1}} \cdot x_n + \frac{D x_D}{V_{n-1}}$$

op. line for enriching section

if cost  $\Rightarrow$  Operating line

if variables from stage to another stage  $\Rightarrow$  Operating curve

Stripping Section (Envelope IV)

$$x_{m+1} \cdot \bar{L}_{m+1} = y_m \cdot \bar{V}_m + x_B \cdot B$$

Operating line or curve

$$y_m = \frac{\bar{L}_{m+1}}{\bar{V}_m} \cdot x_{m+1} - \frac{B}{\bar{V}_m} \cdot x_B$$

Enthalpy Balances

(Calculation of  $Q_c, Q_B$ )

\* the separating agent is energy to create a new phase

Envelope II: (Condenser duty)

Specific enthalpy

$$V_{np} \cdot h_{np} = Q_c + L_R \cdot h_R + D \cdot h_D$$

Flowrate of solvent is  $x_D$

system is going up! in  $Q$  is

$$Q_c = V_{np} \cdot h_{np} - D(R \cdot h_R + h_D)$$

cooling water

$$Q_c = D[(R+1) \cdot h_{v,np} - (R \cdot h_R + h_D)]$$

$T_D$  or  $T_{LR}$

$$m c_p \Delta T$$

Total condenser

Rate of cooling water:

Partial condenser

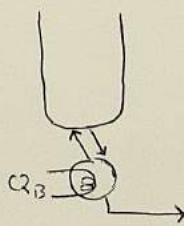
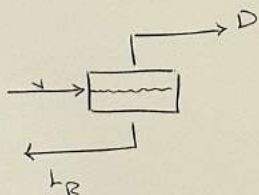
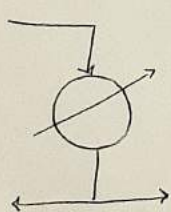
$$m = \frac{Q_c}{c_p \Delta T}$$

Total boiler

$$\Delta T = T_{out} - T_{in}$$

$$T_{in} = 20-25^\circ C$$

partial boiler



Envelope I:

losses & dist

$$F \cdot h_F + Q_B = D h_D + B h_B + Q_C + Q_L$$

negligible if well insulated [Total losses]

$$Q_B = D(h_D + \frac{Q_C}{D}) + B \cdot h_B - F \cdot h_F + Q_L$$

Envelope III

$$\frac{Q_C}{D} = q$$

$$V_{n+1} \cdot h_{V,n+1} - L_n \cdot h_{L,n} = D \left( \frac{Q_C}{D} + h_D \right) = \Delta D \quad (P')$$

$$= D(q + h_D)$$

= net flow

P' & P''

⇒ source & sink

Envelope IV:

$$L_{m+1} \cdot h_{L,m+1} - V_m \cdot h_{V,m} = B(h_B - \frac{Q_C}{B}) = \Delta W \quad (P'')$$

$$= B(h_B - q_B)$$

= net flow

The above relations together with equilibrium relations give the necessary equations for the design of distillation columns.

At the end of design calculations, one hopes to have

- $N_p$
- Product flow rates, compositions, product flow rate, compositions
- Operating conditions (T, P)
- $Q_C, Q_B$  boiler & condensor





## Specifications:

Two types of problems may be encountered with any equipment, in this case distillation column:

- Design Problem
- Simulation Problem

**Design Problem:** Desired separation is set and column is designed that will achieve this separation

**Simulation Problem:** Column is already built and the procedure should predict how much separation can be achieved for a given feed.

For both problems we usually specify:

- ⇒ • Column pressure, (sets equilibrium data)
- ⇒ • Feed composition
- ⇒ • Feed flow rate.
- ⇒ • Feed temperature and pressure (or enthalpy or quality – state of the feed  $L_f$  and  $V_f$ ). ψ → ... , vap , liq
- ⇒ • Reflux temperature or enthalpy (usually saturated liquid) (Simulation Problem).

Other variables which must be set for adequate specification.

For a binary system the most usual specifications and the resulting calculated variables are:

| Specifications                                                                                                                                                                                                | Calculated variables                                                                                                                                             |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>Design:</b><br>- $x_D, x_B$ (mvc)<br>- $R$ ( $L_R/D$ )                                                                                                                                                     | $D, B, Q_c, Q_B, N_p$ , feed location.<br>Column Diameter.<br><span style="margin-left: 100px;">→ permissible velocity &amp; volumetric flow rate of vap</span>  |
| <b>Simulation:</b><br>- $N_p$ , feed location<br>- column diameter<br>- Reboiler size (gives max vapor)<br>and<br>- $x_D$ and $x_B$<br>or<br>- $R, x_D$ (or $x_B$ )<br>or<br>- $x_D$ (or $x_B$ ), $V=V_{max}$ | $R, B, D, Q_c, Q_R$ Check $v < v_{max}$<br>$x_B$ (or $x_D$ ), $B, D, Q_c, Q_R$ Check $v < v_{max}$<br>$R, x_B$ (or $x_D$ ), $B, D, Q_c, Q_R$ Check $v < v_{max}$ |



⇒ Two approaches may be used:

- **Approximate**: simple sequential, (adequate in many cases), graphs, simple algorithms
- **Rigorous**: requires detailed stage to stage calculations (simulation), handles all situations





## Binary Rectification:

### Approximate Methods:

⇒ Simplifying assumptions: (constant flowrate & pressure)

First assumption → 1. Constant molal overflows: valid if (over range of temperature and Pressure):

- heat of vaporization of both species of the binary system are equal  
مساوية إذا كانا من نفس البنية الكيميائية
- Heats of mixing, stage heat losses and sensible heat changes of both liquid and vapour are negligible  
تغيرات الحرارة في السائل والبخار مهملة

This assumption indicates that every mole of condensing vapour vaporizes exactly ONE mole of liquid.

This means that on molal basis in each section of a column;

$$\frac{\text{Moles of Liquid}}{\text{Moles of Vapor}} = \text{constant} ; L_n = L, V_n = V \text{ and } \bar{L}_m = \bar{L}, \bar{V}_m = \bar{V}$$

This results in a straight line operating line for each section.

Second assumption → 2. Distillation takes place at constant pressure (small variations of pressure inside the column)

بنج من حيث الضغط ثابت  
equilibrium relation

### Plate to plate calculations: Lewis-Sorel Method

Consider the following case for the distillation of a binary mixture:

→ Given Information:

$F, \bar{Z}, x_D, x_B, R, P_{\text{column}}$   
bottom  
reflux ratio

Feed, Distillate, Bottoms product and reflux are saturated liquids at their bubble points.

Required:  $N_p$  and Feed location.

# of theoretical stages

\* إذا بقي الضغط ثابت في كل stage  
bubble points



### Analysis:

- Operating lines give relations between vapour and liquid compositions of passing streams:

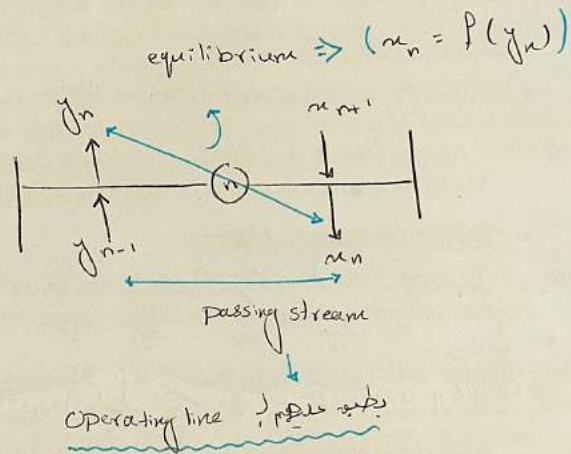
Top Rectifying Section:  $y_{n-1} = \frac{L}{V} \cdot x_n + \frac{D}{V} \cdot x_D$  net flow (D/V)

Bottom Stripping Section:  $y_m = \frac{L}{V} \cdot x_{m+1} - \frac{B}{V} \cdot x_B$  net flow down (B/V)

} between passing streams

- Equilibrium data give relations between liquid and vapour compositions leaving a stage

relative volatility or curve  $\Rightarrow x_n = f(y_n)$   
or curve fitting





\* The Feed is sat. liq

### Calculation Procedure:

- Use column balances to calculate  $D$ ,  $B$ ,  $L$  and  $V$ ,  $\bar{L}$ ,  $\bar{V}$

\* Over all component balance :

$$z_F F = n_D D + n_B B \rightarrow z_F F = n_D D + n_B (F - D)$$

$$\Rightarrow D = \frac{F(z_F - n_B)}{n_D - n_B}$$

$$\Rightarrow B = F - D \quad \text{or} \quad B = D \frac{(z_F - n_D)}{(n_B - z_F)}$$

- Starting with top specification, find  $y_{NP}$  from ... operating line ...

$$y_{n+1} = \frac{L}{V} x_n + \frac{D}{V} x_D \quad n = N_{p+1}$$

$$y_{NP} = \frac{L}{V} x_{NP+1} + \frac{D}{V} x_D$$

$y_{NP}$  &  $x_{NP+1}$

- Find  $x_{NP}$  from ... equilibrium data

$$x_{NP} = f(y_{NP})$$

- Repeat for stage ...  $N_{p-1}$

$$y_{NP-1} = \frac{L}{V} x_{NP} + \frac{D}{V} x_D$$

equilibrium relation  $x_{NP-1} = f(y_{NP-1})$

column composition of nuc & liq

- Repeat for stages below until  $x_n$  calculated is nearly the same as ...  $z_B$ .

bottom section / data top section &  $z_F$  no  $y_{NP}$  &  $x_{NP}$

- Change ... operating line ..., and starting with ...  $x_{n+1} = x_F$

$$y_{F,1} = \frac{L}{V} x_{F,1} + \frac{D}{V} x_D$$

below the feed stage

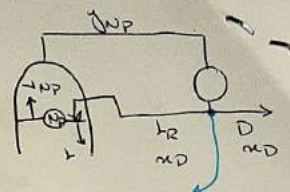
- Find  $x_{F,1}$  from ... equilibrium data

$$x_{F,1} = f(y_{F,1})$$

- Repeat for stages below until  $x_n$  calculated is nearly the same as ...  $z_B$ .

- Stop  $\Rightarrow$  operating line & equilibrium

stages on column



$$V = D + L_R$$

$$V = D + R \cdot D$$

$$\Rightarrow V = D(1 + R)$$

$$L = R \cdot D$$

$$\Rightarrow L = L + F = R \cdot D + F$$

$$\Rightarrow V = V$$

