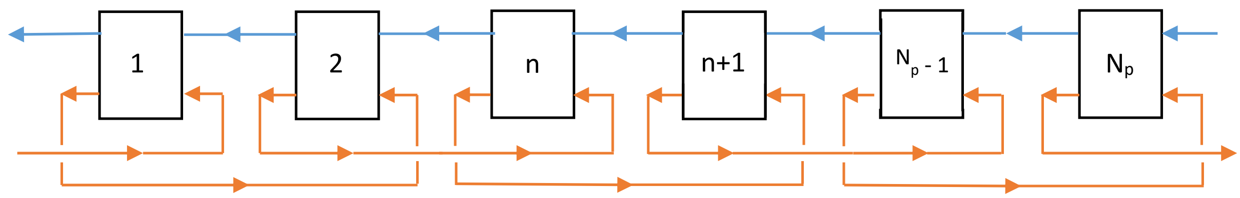
Counter Current Operations

- Most efficient arrangement [least number of stages for a given separation].
- Each stage is identical in its action to a cocurrent process; however, the cascade (battery) has the characteristics of a counter current process.
- Passing streams are not in equilibrium
- Streams leaving a stage are in equilibrium



Operating Lines:

Operating line for any equilibrium stage (n)

INPUT (X_{n-1}, Y_{n+1}) OUTPUT (X_n, Y_n)

Mole fractions: $R_{n-1}.x_{n-1} + E_{n+1}.y_{n+1} = R_n.x_n + E_n.y_n$

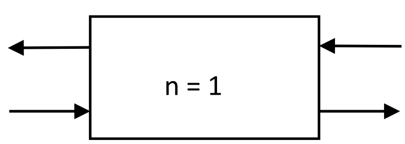
Mole ratios: $R_s.X_{n-1} + E_s.Y_{n+1} = R_s.X_n + E_s.Y_n$

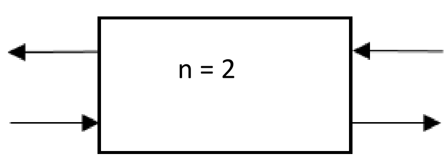
$$R_s(X_{n-1} - X_n) = E_s(Y_n - Y_{n+1})$$

$$E_s\,Y_n - R_s\,X_{n-1} + R_s\,X_n = E_s\,Y_{n+1}$$

$$Y_{n+1} = +\frac{R_s}{E_s}X_n + \frac{E_s.Y_n - R_s.X_{n-1}}{E_s}$$

For example, take the first two stages:





$$Y_2 = +\frac{R_s}{E_s}X_1 + \frac{E_s\,Y_1 - R_s\,X_0}{E_s} \qquad Y_3 = +\frac{R_s}{E_s}X_2 + \frac{E_s\,Y_2 - R_s\,X_1}{E_s}$$

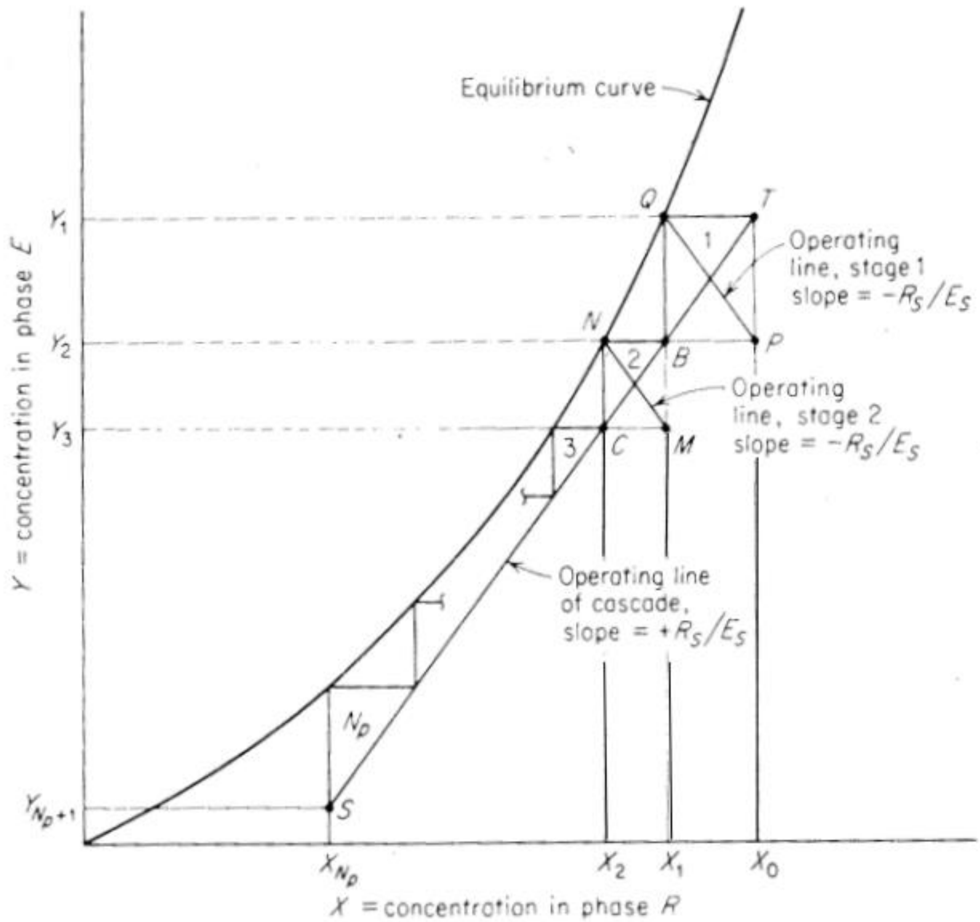


Figure 5.15 Countercurrent multistage cascade, solute transfer from phase R to phase E.

<u>Notes:</u>

- All individual stages have parallel operating lines of slope $=-\frac{R_s}{E_s}$
- Points fall on equilibrium curve. They are exit streams from stage n [, ……]
- Points these are inlet streams to stage n [, …….]
- Points are compositions of passing streams [ ]. Lines like , have the same slope $= +\frac{R_s}{E_s}$ and they all fall on the same line .

<u>Operating line for the cascade:</u>

Entire cascade: $$R_s\left(X_o - X_{N_p}\right) = E_s\left(Y_1 - Y_{N_p+1}\right)$$

For any stage n: $$R_s(X_o - X_n) = E_s\,(Y_1 - Y_{n+1})$$

$$Y_{n+1} = +\frac{R_s}{E_s}\,X_n + \frac{E_s\,Y_1 - R_s\,X_o}{E_s}$$, Slope ; This line also coincides with .

Special Case:

For most cases, because of either a curved operating line or equilibrium curve, the relation between number of stages, compositions, and flow ratio must be determined graphically, as shown. For the *special* case where both are straight, however, with the equilibrium curve continuing straight to the origin of the distribution graph, an analytical solution can be developed which will be most useful.

If the equilibrium-curve slope is $m = \frac{Y_{n+1}}{X_{n+1}}$ (straight line), and if an absorption factor (Extraction factor) A is defined as:

$$A = \frac{R_S}{mE_S},$$

then based on material balance and equilibrium data, we can obtain the following equation:

$$X_n = \left(X_0 - \frac{Y_{N_p+1}/m - AX_{N_p}}{1 - A} \right) A^n + \frac{Y_{N_p+1}/m - AX_{N_p}}{1 - A}$$

Therefore knowing terminal concentrations, X_n can be calculated.

In the opposite case where the transfer is from E to R, we define a stripping factor S as:

$$S = \frac{1}{A}$$

Kremser Equations: (Kremser-Brown-Souders)

In the case where $n = N_p$, then:

For transfer from R to E (stripping of R)

$A \neq 1$:

$$\frac{X_0 - X_{N_p}}{X_0 - Y_{N_p+1}/m} = \frac{(1/A)^{N_p+1} - 1/A}{(1/A)^{N_p+1} - 1}$$

$$N_p = \frac{\log\left[\dfrac{X_0 - Y_{N_p+1}/m}{X_{N_p} - Y_{N_p+1}/m}(1 - A) + A\right]}{\log 1/A}$$

$A = 1$:

$$\frac{X_0 - X_{N_p}}{X_0 - Y_{N_p+1}/m} = \frac{N_p}{N_p + 1}$$

$$N_p = \frac{X_0 - X_{N_p}}{X_{N_p} - Y_{N_p+1}/m}$$

For transfer from E to R (absorption into R). A similar treatment yields:

$A \neq 1$:

$$\frac{Y_{N_p+1} - Y_1}{Y_{N_p+1} - mX_0} = \frac{A^{N_p+1} - A}{A^{N_p+1} - 1}$$

$$N_p = \frac{\log\left[\dfrac{Y_{N_p+1} - mX_0}{Y_1 - mX_0}\left(1 - \dfrac{1}{A}\right) + \dfrac{1}{A}\right]}{\log A}$$

$A = 1$:

$$\frac{Y_{N_p+1} - Y_1}{Y_{N_p+1} - mX_0} = \frac{N_p}{N_p + 1}$$

$$N_p = \frac{Y_{N_p+1} - Y_1}{Y_1 - mX_0}$$

These are called the Kremser-Brown-Souders (or simply Kremser) equations, after those who derived them for gas absorption [7, 14] although apparently Turner [16] had used them earlier for leaching and solids washing. They are plotted in Fig. 5.16, which then becomes very convenient for quick solutions.

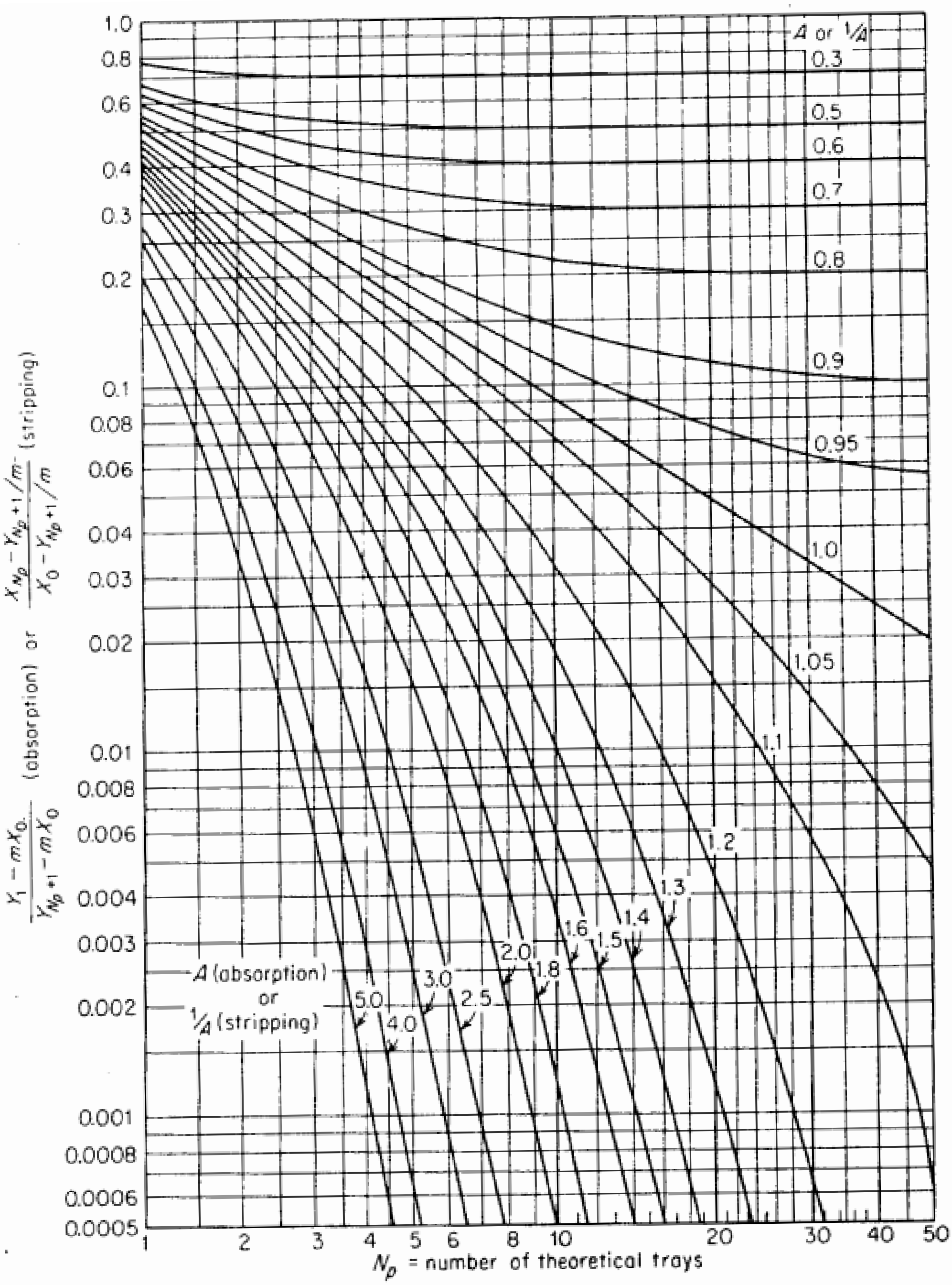


Figure 5.16 Number of theoretical stages for countercurrent cascades, with Henry's law equilibrium and constant absorption or stripping factors. [*After Hachmuth and Vance, Chem. Eng. Prog.*, **48**, *523, 570, 617 (1952).*]