McCabe-Thiele Method:

This method is based on:

① Assumption of constant molal overflow (CMO), which results in straight line operating lines on molal basis.

② Constant column pressure [sets equilibrium data] This makes it possible to use a $y-x$ diagram for a graphical solution.

Operating lines:

Enriching Section.

for stage n

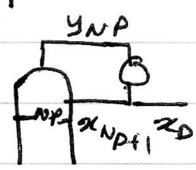
$$y_{n-1} = \frac{L}{V} x_n + \frac{D}{V} \cdot x_D.$$

$$V = D(R+1)$$

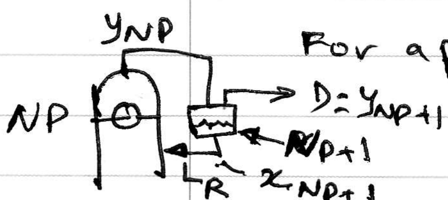
$$y_{n-1} = \frac{L}{D(R+1)} \cdot x_n + \frac{D}{D(R+1)} \cdot x_D.$$

$$\boxed{y_n = \frac{R}{R+1} \cdot x_{n+1} + \frac{1}{R+1} \cdot x_D}$$

This line intersects the 45° line at $y_n = x_{n+1} = x_D$.



For a total condenser: $y_{NP} = x_{NP+1} = x_D$



For a partial condenser: $y_{NP} \neq x_{NP+1} \neq x_D$

$$\left.\begin{aligned} x_D &= y_{NP+1} \\ x_{NP+1} &= f(x_D) \\ &= f(y_{NP+1}) \end{aligned}\right.$$

$$y_{NP} = \frac{L}{V} \cdot x_{NP+1} + \frac{D}{V} \cdot x_D$$

Exhausting Section:

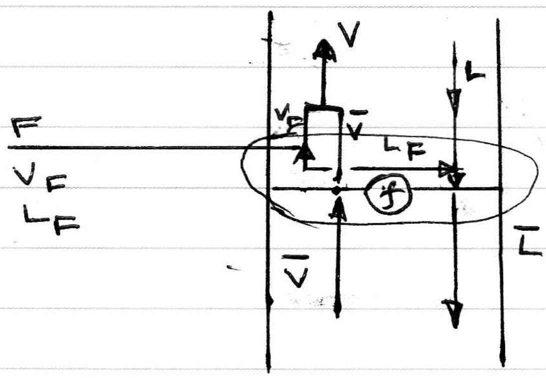
$$y_m = \frac{\bar{L}}{\bar{V}} \cdot x_{m+1} - \frac{B}{\bar{V}} \cdot x_B$$

This line intersects the 45° line at

$$y_m = x_{m+1} = x_B.$$

Note: For a saturated liquid feed the two operating lines intersect at x_f.

Feed line (q-line)



$$F + L + \bar{V} = V + \bar{L}$$

$$(\bar{V} - V) = (\bar{L} - L) - F$$

Ⓐ $\bar{L} = L + qF$ $\qquad$ q : fraction of feed which is liq.

Ⓑ $\bar{V} = V - (1-q)F$

Stripping $\qquad y = \frac{\bar{L}}{\bar{V}} \cdot x - \frac{B}{\bar{V}} \cdot x_B$

Enriching $\qquad - \left(y = \frac{L}{V} x + \frac{D}{V} \cdot x_D\right)$

$$(\bar{V} - V)\, y = (\bar{L} - L) \cdot x - (B x_B + D x_D)$$

Ⓑ → $(\bar{V} - V) = (q-1) \cdot F$

Ⓐ → $(\bar{L} - L) = qF$

$B x_B + D x_D = z_F \cdot F$

substitute:

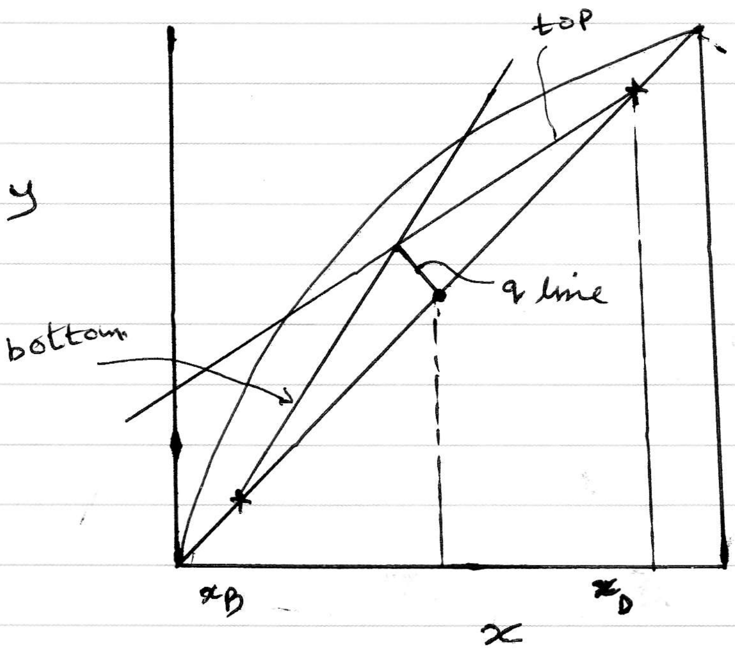
$(q-1)\cancel{F}y = q\cancel{F}x - \cancel{F}z_F$

$$\boxed{y = \frac{q}{q-1} \cdot x - \frac{z_F}{(q-1)}}$$ q-line equation.

This line is the locus of the points of intersection of stripping and enriching operating lines

Slope = $\frac{q}{q-1}$ intercept = $-\frac{z_F}{q-1}$

and passes through $y = x = z_F$

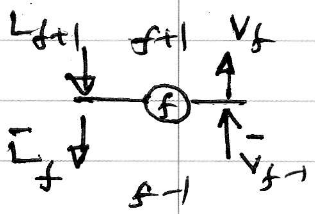


<u>Thermal condition of feed:</u>

From MB $(\bar{V} - V) = (\bar{L} - L) - F$

Enthalpy: $F h_F + L_{f+1} h_{L_{f+1}} + \bar{V}_{f-1} h_{\bar{V},f-1} = V_f h_{V,f} + \bar{L}_f h_{\bar{L}_f}$

Based on assumption of CMO



$$L_{f+1} = L \;;\; \bar{L}_f = \bar{L} \qquad V_f = V \;;\; \bar{V}_{f-1} = \bar{V}$$

$$h_{V_f} = h_{\bar{V},f-1} \qquad h_{L,f+1} = h_{\bar{L}f}$$

$$\therefore \; F h_F + (\bar{V} - V) h_{V_f} = (\bar{L} - L) . h_{\bar{L},f}$$

Substitute for $(\bar{V} - V) = (\bar{L} - L) - F$.

$$\Rightarrow \; F h_F + (\bar{L} - L) . h_{V_f} - F h_{V_f} = (\bar{L} - L) . h_{\bar{L}f}$$

$$(\bar{L} - L)(h_{V_f} - h_{\bar{L}f}) = F (h_{V_f} - h_F).$$

$$\frac{\bar{L} - L}{F} = \frac{\overbrace{h_{V_f} - h_F}^{\text{Enthalpy to vaporize 1 mole of feed.}}}{\underbrace{h_{V_f} - h_{\bar{L}f}}_{\text{Latent heat of vaporisation.}}} = q$$

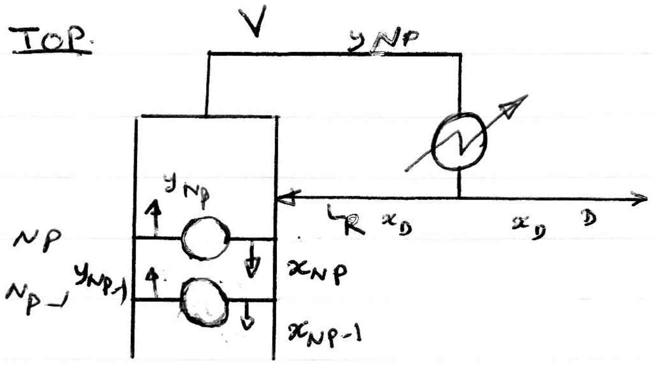
Sat Vap (h_{V_f}), sat liq ($h_{\bar{L}f}$)

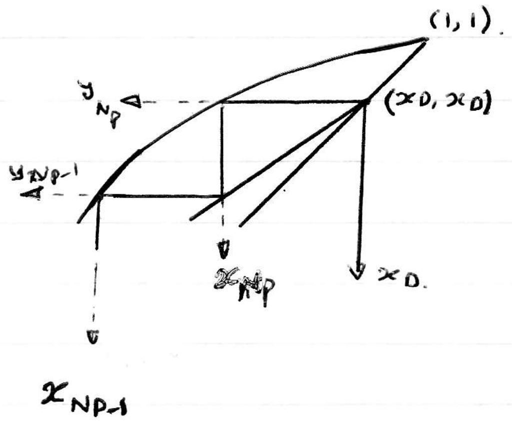
Values of q:

q	Feed Condition	Slope of q-line
$q > 1$	cooled liquid	> 1
$q = 1$	Saturated liquid	∞
$0 < q < 1.0$	vapor + liquid.	< 1
$q = 0$	Saturated vapor	0
$q < 0$	superheated vapor	$1 > \text{slope} > 0$

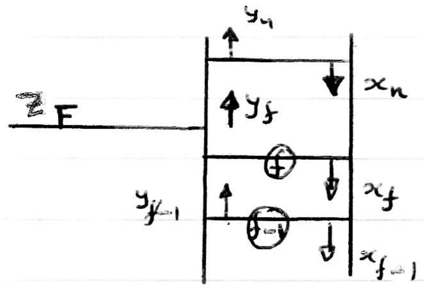
Important portions in fractionation column:

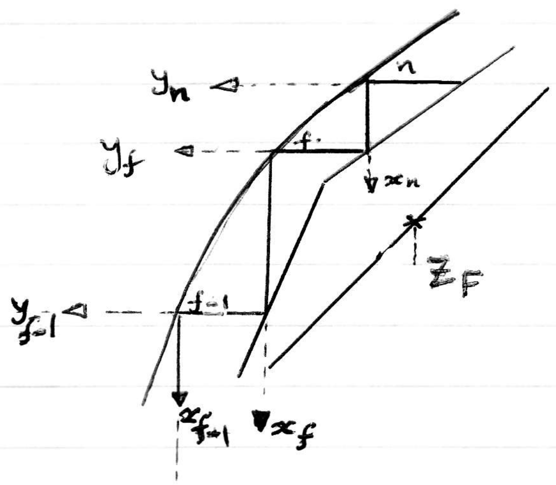
Top.





Feed.





Reboiler and bottom stage: