Topic 2.3. Design of Evaporators

Last lecture

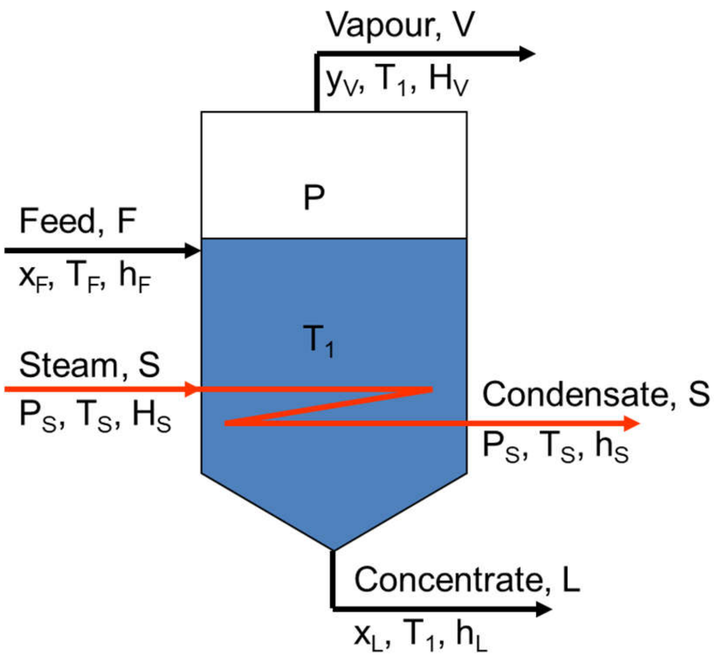
- ✓ Explain how the overall heat transfer coefficient is calculated for evaporation
- ✓ Perform heat and material balances on evaporation units and processes

This lecture

- ✓ Design of Multi-effect evaporators
- ✓ Examples

Given information for evaporators design

1. Steam pressure to the first effect
2. Final pressure in the vapor space of the last effect
3. Feed conditions and flow to the first effect
4. Final concentration in the liquid leaving the last effect
5. Physical properties such as enthalpies and/or heat capacities of the liquid and vapors
6. Overall heat-transfer coefficients in each effect
7. The areas of each effect are assumed equal



Step-by-Step Calculation Methods for Triple-Effect Evaporators

The calculations are done using material balances, heat balances, and the capacity equations: **q = U A ΔT** for each effect using trial and error calculations

1. Given the **concentration** and **pressure** in the last effect ⇨ Calculate the **boiling point** in the last effect using **Dühring-line plot**

2. Assume for the 1st guess the amount of vapor from each effect is equal

$$V_1 = V_2 = V_3$$

so $$V_{Total} = V_1 + V_2 + V_3$$

⇨ Perform overall and component balance to calculate the amount of liquid from the 3rd evaporator, L_3

$$F = L_3 + V_{Total}$$

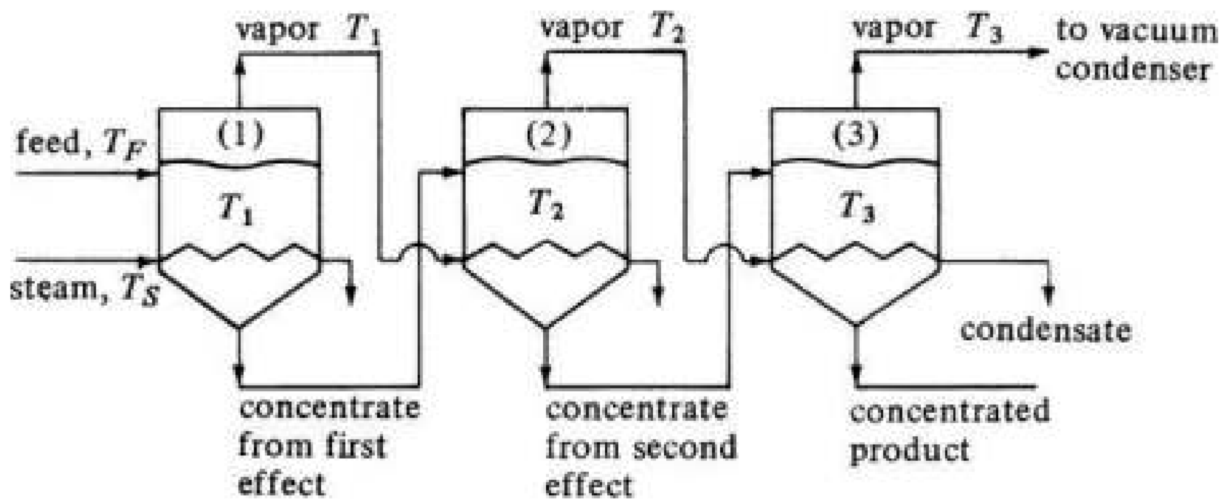
$$F\, x_F = L_3\, x_{L3}$$

Step-by-Step Calculation Methods for Triple-Effect Evaporators

Based on the obtained L_3 and V_1 , V_2 , V_3

Do material balance around each effect to get L_1 and L_2

$$F = L_1 + V_1$$
$$L_1 = L_2 + V_2$$



Make a solids balance on effects 1, 2, and 3 and solving for x

$$F\ x_F = L_1\ x_1$$
$$L_1\ x_1 = L_2\ x_2$$
$$L_2\ x_2 = L_3\ x_3$$

Based on the obtained

x_1 , x_2 , x_3

Calculate the BPR for each effect

- $BPR1\,°C = 1.78\,x_1 + 6.22\,x_1^2$
- $BPR2\,°C = 1.78\,x_2 + 6.22\,x_2^2$
- $BPR3\,°C = 1.78\,x_3 + 6.22\,x_3^2$

3. $\sum\Delta T$ available for heat transfer without the superheat is obtained by subtracting the sum of all three BPRs from the overall ΔT of TS – T3 (saturation)

$$\sum \Delta T = T_S - T_3 - BPR1 - BPR2 - BPR3$$

Calculate ΔT for each effect

$$\Delta T_1 = \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Delta T_2 = \sum \Delta T \frac{1/U_2}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Delta T_3 = \sum \Delta T \frac{1/U_3}{1/U_1 + 1/U_2 + 1/U_3}$$

- If a cold feed enters effect number 1, this effect requires more heat.

A first estimate is done by increasing ΔT1 and lowering ΔT2 and ΔT3 Proportionately,

- Calculate the boiling point in each effect.
- estimate the pressure in effects 1 and 2 and determine the BPR in each of the three effect

For saturation condition $\sum \Delta T = T_S - T_3 - BPR1 - BPR2 - BPR3$

$$\Delta T_1 = \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Delta T_2 = \sum \Delta T \frac{1/U_2}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Delta T_3 = \sum \Delta T \frac{1/U_3}{1/U_1 + 1/U_2 + 1/U_3}$$

4. Using heat and material balances in each effect, calculate the amount vaporized and the flows of liquid in each effect

   - If the amounts vaporized differ appreciably from those assumed in step 2, then steps 2, 3, and 4 can be repeated using the amounts of evaporation just calculated
     (In step 2, only the solids balance is repeated)

5. Calculate the value of q transferred in each effect.

   - Using the rate equation *q = U A ΔT* for each effect, calculate the areas A1, A2, and A3

   - The average area $$A_{avg} = \frac{A_1 + A_2 + A_3}{3}$$

- If these areas are reasonably close to each other, the calculations are complete and a second trial is not needed ⇨ Stop (You are done)

- If these areas are not almost equal, then do another trial as follows:

6. use the new values of L1, L2, L3, V1, V2, and V3 calculated by the heat balances in step 4 and calculate the new solids concentration in each effect by a solids balance on each effect

$$\Delta T_1' = \frac{\Delta T_1 \, A_1}{A_{avg}} \qquad \Delta T_2' = \frac{\Delta T_2 \, A_2}{A_{avg}} \qquad \Delta T_3' = \frac{\Delta T_3 \, A_3}{A_{avg}}$$

If $\sum \Delta T = \Delta T_1' + \Delta T_2' + \Delta T_3'$ Calculate the boiling point in each effect

If $\sum \Delta T \neq \Delta T_1' + \Delta T_2' + \Delta T_3'$ Proportionately readjust all ΔT′ values to have it equal

- Determine the new BPRs in the three effects using the new concentrations from step 6

7. To get a new value of ∑ΔT available for heat transfer, subtract the sum of all three BPRs from the overall ΔT

$$\Delta T_1' = \frac{\Delta T_1\, A_1}{A_{avg}} \qquad \Delta T_2' = \frac{\Delta T_2\, A_2}{A_{avg}} \qquad \Delta T_3' = \frac{\Delta T_3\, A_3}{A_{avg}}$$

- Now the sum of the $\Delta T'$ values just calculated must be readjusted to the new $\sum \Delta T$ value, then calculate the boiling point in each effect

8. Using the new $\Delta T'$ values from step 7, repeat the calculations starting with step 4. Two trials are usually sufficient so that the areas are reasonably close to being equal.

Example 3.1 Design of a triple-effect evaporator for sugar solution

A triple-effect forward-feed evaporator is being used to evaporate a sugar solution containing **10 wt %** solids to a concentrated solution of **50%**. The boiling-point rise of the solutions can be estimated from

$$BPR\ °C = 1.78\,x + 6.22\,x^2$$

where x is wt fraction of sugar in solution. Saturated steam at **205.5 kPa** (**121.1°C** saturation temperature) is being used. The pressure in the vapor space of the third effect is **13.4 kPa**. The feed rate is **22,680 kg/h** at **26.7°C**. The heat capacity of the liquid solutions is $c_p = 4.19 - 2.35\,x$ kJ/kg·K. The heat of solution is considered to be negligible. The coefficients of heat transfer have been estimated as

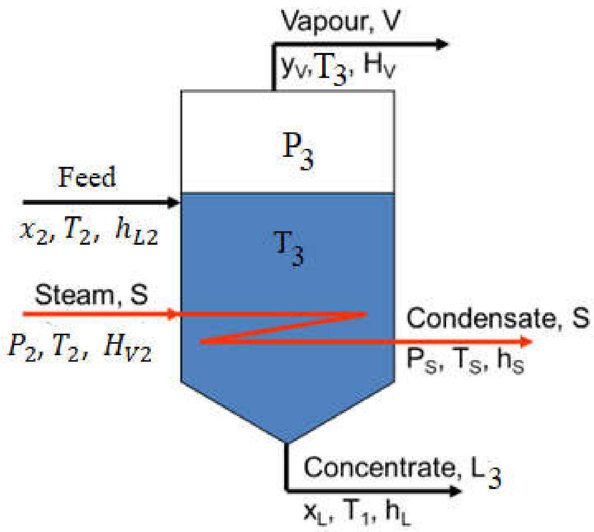
$$U_1 = 3123,\ \ U_2 = 1987,\ \ U_3 = 1136\ \ \mathrm{W/m^2 \cdot K}$$

If each effect has the same surface area, calculate the area, the steam rate used, and the steam economy

Solution

Given the **concentration** and **pressure** in the last effect

$$x_L = 0.5$$
$$P_3 = 13.4\ kPa$$



Calculate the **boiling point** in the last effect using **Dühring-line plot**

- At $P_3 = 13.4\ kPa$ use steam tables the saturation temperature is

$$T_{V3} = 51.67\ °C$$

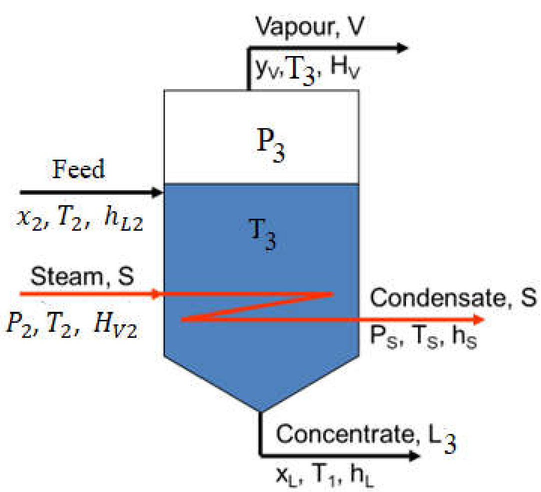
- $BPR\ °C = 1.78\,x + 6.22\,x^2$

$$BPR\ °C = 1.78\,(0.5) + 6.22\,(0.5)^2$$
$$= 54.12\ °C$$

Assume for the 1st guess the amount of vapor from each effect is equal

$$V_1 = V_2 = V_3$$

so $$V_{Total} = V_1 + V_2 + V_3$$



$$F = L_3 + V_{Total}$$

$$22{,}680 = L_3 + V_{Total}$$

$$F\, x_F = L_3\, x_{L3}$$

$$22{,}680\,(0.1) = L_3(0.5)$$

$$L_3 = 4536\ kg/h$$

$$V_{Total} = 18{,}144\ kg/h$$

$$V_i = \frac{V_{Total}}{3} = \frac{18{,}144}{3}$$

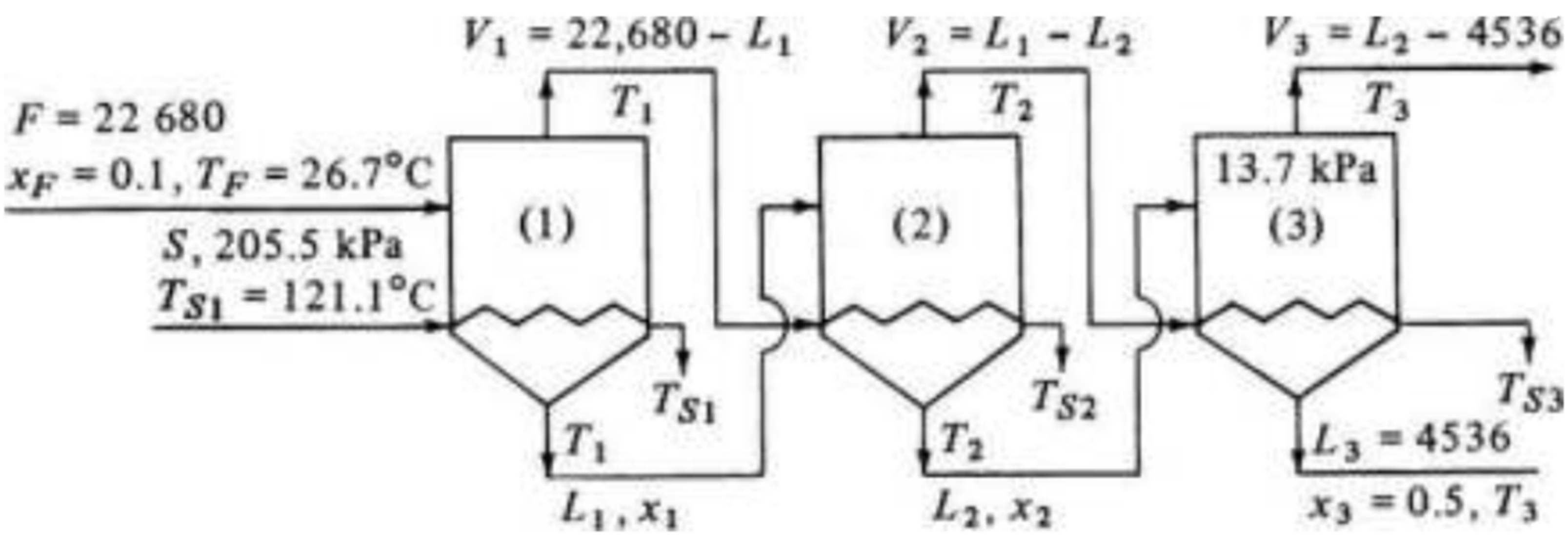
$$V_1 = V_2 = V_3 = 6048\ kg/h$$

Based on the obtained L_3 and V_1, V_2, V_3

$$F = L_1 + V_1$$
$$L_1 = L_2 + V_2$$

$$F = L_1 + V_1$$
$$22{,}680 = L_1 + 6048$$
$$L_1 = 16{,}632\ kg/h$$

$$L_1 = L_2 + V_2$$
$$16{,}632 = L_2 + 6048$$
$$L_2 = 10{,}584\ kg/h$$



Make a solids balance on effects 1, 2, and 3 and solving for x

$$F\ x_F = L_1\ x_1$$

$$L_1\ x_1 = L_2\ x_2$$

$$L_2\ x_2 = L_3\ x_3$$

$$22{,}680\ (0.1) = 16{,}632\ (\ x_1)$$

$$x_1 = 0.136$$

$$16{,}632\ (0.136) = 10{,}584\ (\ x_2)$$

$$x_2 = 0.214$$

$$10{,}584\ 0.215 = 4536\ (\ x_3)$$

$$x_3 = 0.5$$

Based on the obtained

$$x_1\ , x_2\ , x_3$$

- $BPR1\ °C\ =\ 1.78\ x_1\ +\ 6.22\ x_1^2$
- $BPR2\ °C\ =\ 1.78\ x_2\ +\ 6.22\ x_2^2$
- $BPR3\ °C\ =\ 1.78\ x_3\ +\ 6.22\ x_3^2$

$$BPR1\ = 0.36\ °C$$

$$BPR2\ = 0.65\ °C$$

$$BPR3\ = 2.45\ °C$$

∑ΔT available for heat transfer without the superheat

$$\sum \Delta T = T_S - T_3 - BPR1 - BPR2 - BPR3$$

$$\sum \Delta T = 121.1 - 51.67 - 0.36 - 0.65 - 2.45$$

$$\Rightarrow \sum \Delta T = 65.97°C$$

$$\Delta T_1 = \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Delta T_2 = \sum \Delta T \frac{1/U_2}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Delta T_3 = \sum \Delta T \frac{1/U_3}{1/U_1 + 1/U_2 + 1/U_3}$$

$$\Rightarrow \Delta T_1 = 12.4°C$$

$$\Delta T_2 = 19.5°C$$

$$\Delta T_3 = 34.07°C$$

since a cold feed enters effect number 1, this effect requires more heat;

Increasing ΔT1 and lowering ΔT2 and ΔT3 proportionately as a first estimate

$$\Delta T_1 = 15.56\ °C \qquad \Delta T_2 = 18.34\ °C \qquad \Delta T_3 = 32.07\ °C$$

$$\begin{aligned} T_1 &= T_{s1} - \Delta T_1 \\ &= 121.1 - 15.56 \\ &= 105.54\ °C \end{aligned}$$

T_{s1} is the condensing temperature of the saturated steam to effect 1

$$\begin{aligned} T_2 &= T_1 - \Delta T_2 - BPR1 \\ &= 105.54 - 18.34 - 0.36 \\ &= 86.84\ °C \end{aligned}$$

$$\begin{aligned} T_{s2} &= T_1 - BPR1 \\ &= 105.54 - 0.36 \\ &= 105.18\ °C \end{aligned}$$

T_{s2} is the condensing temperature of the saturated steam to effect 2

$$T_3 = T_2 - \Delta T_3 - BPR2$$
$$= 105.54 - 18.34 - 0.36$$
$$= 86.84\ ℃$$

$$T_{s3} = T_2 - BPR2$$
$$= 86.84 - 0.65$$
$$= 86.19\ ℃$$

T_{s3} is the condensing temperature of the saturated steam to effect 3

The temperatures in the three effects are as follows

Effect 1	Effect 2	Effect 3	Condenser
$T_{s1} = 121.1℃$	$T_{s2} = 105.18℃$	$T_{s3} = 96.19℃$	$T_{s4} = 51.67℃$
$T_1 = 105.54\ ℃$	$T_2 = 86.84\ ℃$	$T_3 = 54.12\ ℃$	

The heat capacity of the liquid in each effect is calculated from the equation

$$\boldsymbol{c_p = 4.19 - 2.35(x)}$$

$Feed: \quad c_p = 4.19 - 2.35(0.1) = 3.955 \dfrac{kJ}{kg.K}$

$L1: \quad c_p = 4.19 - 2.35(0.136) = 3.869 \dfrac{kJ}{kg.K}$

$L3: \quad c_p = 4.19 - 2.35(0.5) = 3.015 \dfrac{kJ}{kg.K}$

$L2: \quad c_p = 4.19 - 2.35(0.214) = 3.684 \dfrac{kJ}{kg.K}$

The values of the enthalpy *H* of the various vapor streams relative to water at 0°C as a datum are obtained from the steam table as follows:

Effect 1

$T_1 = 105.54\ °C \qquad T_{s2} = 105.18°C \qquad BPR1 = 0.36\ °C \qquad T_{s1} = 121.1°C$

$H_1 = H_{s2}\ (sat\ enthalpy\ at T_{s2}) + 1.884\ (BPR1\ \ superheated)$

$H_1 = 2684 + 1.884\ (0.36) = 2685\ kJ/kg$

$$\lambda_1 = H_{s1}\ (sat\ vap) - h_{s1}\ (sat\ liq\ at T_{s1})$$

$$\lambda_1 = 2708 - 508 = 2200\ kJ/kg$$

Effect 2

$T_1 = 86.84$ $T_{s3} = 86.19$ $BPR_2 = 0.65$

$$H_2 = H_{s3} + c_{p\ steam}\ (BPR) = 2654 + 1.884(0.65) = 2655\ kJ/kg$$

$$\lambda_{s2} = H_1 - H_{s2} = 2685 - 441 = 2244\ kJ/kg$$

Effect 3

$T_3 = 54.12$ $T_{s4} = 51.67$ $BPR_3 = 2.45$

$$H_3 = H_{s4} + c_{p\ steam}\ (BPR) = 2595 + 1.884(2.45) = 2600\ kJ/kg$$

$$\lambda_{s3} = H_2 - H_{s3} = 2655 - 361 = 2294\ kJ/kg$$

$V_1 = F - L_1 = 22680 - L_1$ $V_2 = L_1 - L_2$ $V_3 = L_2 - L_3 = L_2 - 4536$

Write a heat balance on each effect

Effect 1 $F c_{pF}(T_F - 0) + S\lambda_{s1} = L_1 c_{p1}(T_1 - 0) + V_1 H_1$

$22680\ (3.955)\ (26.7 - 0) + \boldsymbol{S}(2200) = \boldsymbol{L_1}(3.969)(105.54 - 0) + (22680 - \boldsymbol{L_1})(2685)$

Effect 2 $L_1 c_{p1}(T_1 - 0) + V_1\lambda_{s2} = L_2 c_{p2}(T_2 - 0) + V_2 H_2$

$\boldsymbol{L_1}\ (3.869)\ (105.54 - 0) + (22680 - \boldsymbol{L_1})(2244) = \boldsymbol{L_2}(3.684)(86.84 - 0) + (\boldsymbol{L_1} - \boldsymbol{L_2})(2655)$

Effect 3 $L_2 c_{p2}(T_2 - 0) + V_2\lambda_{s3} = L_3 c_{p3}(T_3 - 0) + V_3 H_3$

$\boldsymbol{L_2}\ (3.684)\ (86.84 - 0) + (\boldsymbol{L_1} - \boldsymbol{2})(2294) = 4536(3.015)(54.12 - 0) + (\boldsymbol{L_2} - 4536)(2600)$

Solving the last two equations simultaneously for L1 and L2, and substituting into the first equation

$L_1 = 17078\ kg/h \qquad L_2 = 11068\ kg/h \qquad L_3 = 4536\ kg/h$

$S = 8936\ kg/h \qquad V_1 = 5602\ kg/h \qquad V_2 = 6010\ kg/h \qquad V_3 = 6532\ kg/h$

$$q_1 = S\lambda_{S1} = \left(\frac{8936}{3600}\right)(2200 \times 1000) = 5.460 \times 10^6\ \text{W}$$

$$q_2 = V_1\lambda_{S2} = \left(\frac{5602}{3600}\right)(2244 \times 1000) = 3.492 \times 10^6\ \text{W}$$

$$q_3 = V_2\lambda_{S3} = \left(\frac{6010}{3600}\right)(2294 \times 1000) = 3.830 \times 10^6\ \text{W}$$

$$A_1 = \frac{q_1}{U_1\,\Delta T_1} = \frac{5.460 \times 10^6}{3123(15.56)} = 112.4\ \text{m}^2$$

$$A_2 = \frac{q_2}{U_2\,\Delta T_2} = \frac{3.492 \times 10^6}{1987(18.34)} = 95.8\ \text{m}^2$$

$$A_3 = \frac{q_3}{U_3\,\Delta T_3} = \frac{3.830 \times 10^6}{1136(32.07)} = 105.1\ \text{m}^2$$

$$A_m = 104.4\ \text{m}^2.$$

a second trial will be made starting with step 6

Step 6. Making a new solids balance on effects 1, 2, and 3, using the new L_1 = 17 078, L_2 = 11 068, and L_3 = 4536, and solving for x,

(1) $22\,680(0.1) = 17\,078(x_1),\ x_1 = 0.133$

(2) $17\,078(0.133) = 11\,068(x_2),\ x_2 = 0.205$

(3) $11\,068(0.205) = 4536(x_3),\quad x_3 = 0.500$ (check balance)

Step 7. The new **BPR** in each effect is then

(1) $\text{BPR}_1 = 1.78x_1 + 6.22x_1^2 = 1.78(0.133) + 6.22(0.133)^2 = 0.35°\text{C}$

(2) $\text{BPR}_2 = 1.78(0.205) + 6.22(0.205)^2 = 0.63°\text{C}$

(3) $\text{BPR}_3 = 1.78(0.5) + 6.22(0.5)^2 = 2.45°\text{C}$

$\sum \Delta T$ available $= 121.1 - 51.67 - (0.35 + 0.63 + 2.45) = 66.00°\text{C}$

$$\Delta T_1' = \frac{\Delta T_1 A_1}{A_m} = \frac{15.56(112.4)}{104.4} = 16.77\text{ K} = 16.77°\text{C}$$

$$\Delta T_2' = \frac{\Delta T_2 A_2}{A_m} = \frac{18.34(95.8)}{104.4} = 16.86°\text{C}$$

$$\Delta T_3' = \frac{\Delta T_3 A_3}{A_m} = \frac{32.07(105.1)}{104.4} = 32.34°\text{C}$$

$$\sum \Delta T = 16.77 + 16.86 + 32.34 = 65.97°\text{C}$$

These $\Delta T'$ values are readjusted so that $\Delta T_1' = 16.77$, $\Delta T_2' = 16.87$, $\Delta T_3' = 32.36$, and $\sum\Delta T = 16.77 + 16.87 + 32.36 = 66.00°\text{C}$. To calculate the actual boiling point of the solution in each effect,

(1) $T_1 = T_{S1} - \Delta T'_1 = 121.1 - 16.77 = 104.33°C, T_{S1} = 121.1°C$

(2) $T_2 = T_1 - BPR_1 - \Delta T'_2 = 104.33 - 0.35 - 16.87 = 87.11°C$

$T_{S2} = T_1 - BPR_1 = 104.33 - 0.35 = 103.98°C$

(3) $T_3 = T_2 - BPR_2 - \Delta T'_3 = 87.11 - 0.63 - 32.36 = 54.12°C$

$T_{S3} = T_2 - BPR_2 = 87.11 - 0.63 = 86.48°C$

Step 8. Following step 4, the heat capacity of the liquid is $c_p = 4.19 - 2.35x$:

$$F: \quad c_p = 3.955 \text{ kJ/kg} \cdot \text{K}$$

$$L_1: \quad c_p = 4.19 - 2.35(0.133) = 3.877$$

$$L_2: \quad c_p = 4.19 - 2.35(0.205) = 3.708$$

$$L_3: \quad c_p = 3.015$$

The new values of the enthalpy H are as follows in each effect:

$$(1) \quad H_1 = H_{S2} + 1.884(^\circ\text{C superheat}) = 2682 + 1.884(0.35) = 2683 \text{ kJ/kg}$$

$$\lambda_{S1} = H_{S1} - h_{S1} = 2708 - 508 = 2200 \text{ kJ/kg}$$

(2) $H_2 = H_{S3} + 1.884(0.63) = 2654 + 1.884(0.63) = 2655$ kJ/kg

$\lambda_{S2} = H_1 - h_{S2} = 2683 - 440 = 2243$ kJ/kg

(3) $H_3 = H_{S4} + 1.884(2.45) = 2595 + 1.884(2.45) = 2600$ kJ/kg

$\lambda_{S3} = H_2 - h_{S3} = 2655 - 362 = 2293$ kJ/kg

Writing a heat balance on each effect,

(1) $22\,680(3.955)(26.7 - 0) + S(2200)$

$= L_1(3.877)(104.33 - 0) + (22\,680 - L_1)(2683)$

$$(2)\quad L_1(3.877)(104.33-0)+(22\,680-L_1)(2243)$$
$$=L_2(3.708)(87.11-0)+(L_1-L_2)(2655)$$
$$(3)\quad L_2(3.708)(87.11-0)+(L_1-L_2)(2693)$$
$$=4536(3.015)(54.12-0)+(L_2-4536)(2600)$$

Solving,

$L_1 = 17\,005$ kg/h $\quad L_2 = 10\,952 \quad L_3 = 4536 \quad S = 8960$ (steam used)

$V_1 = 5675 \quad V_2 = 6053 \quad V_3 = 6416$

$$q_1 = S\lambda_{S1} = \frac{8960}{3600}(2200 \times 1000) = 5.476 \times 10^6 \text{ W}$$

$$q_2 = V_1\lambda_{S2} = \frac{5675}{3600}(2243 \times 1000) = 3.539 \times 10^6 \text{ W}$$

$$q_3 = V_2\lambda_{S3} = \frac{6053}{3600}(2293 \times 1000) = 3.855 \times 10^6 \text{ W}$$

$$A_1 = \frac{q_1}{U_1\ \Delta T_1'} = \frac{5.476 \times 10^6}{3123(16.77)} = 104.6 \text{ m}^2$$

$$A_2 = \frac{q_2}{U_2\ \Delta T_2'} = \frac{3.539 \times 10^6}{1987(16.87)} = 105.6 \text{ m}^2$$

$$A_3 = \frac{q_3}{U_3\ \Delta T_3'} = \frac{3.855 \times 10^6}{1136(32.36)} = 104.9 \text{ m}^2$$

$A_m = 105.0 \text{ m}^2$.

$$\text{steam economy} = \frac{V_1 + V_2 + V_3}{S} = \frac{5675 + 6053 + 6416}{8960} = 2.025$$