

Topic 3.5. Design of cooling tower

Last lecture

- ✓ Psychrometric (humidity) terminology
- ✓ Plot processes on a psychrometric chart and analyze processes

This lecture

- ✓ humidity of mixed streams
- ✓ Gas-liquid contact operation

The content of this topic was obtained from notes of Professor Zayed Hammouri, ChE-UoJ

Fundamental relations for adiabatic operations

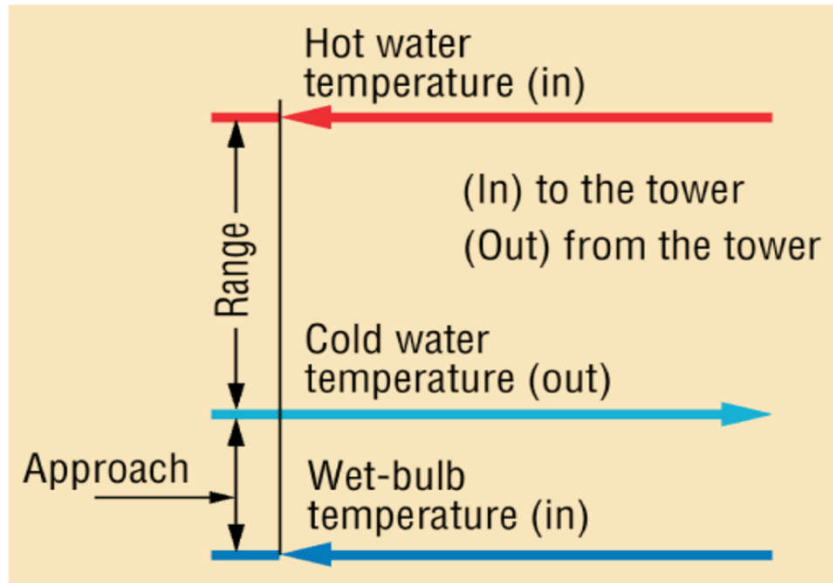
$$N_A M_A a_M dZ = -G'_S dY' = M_A F_G \left(\ln \frac{1 - \bar{p}_{A,i}/p_t}{1 - p_{A,G}/p_t} \right) a_M dZ$$

$$-G'_S C_S dt_G = h'_G a_H (t_G - t_i) dZ$$

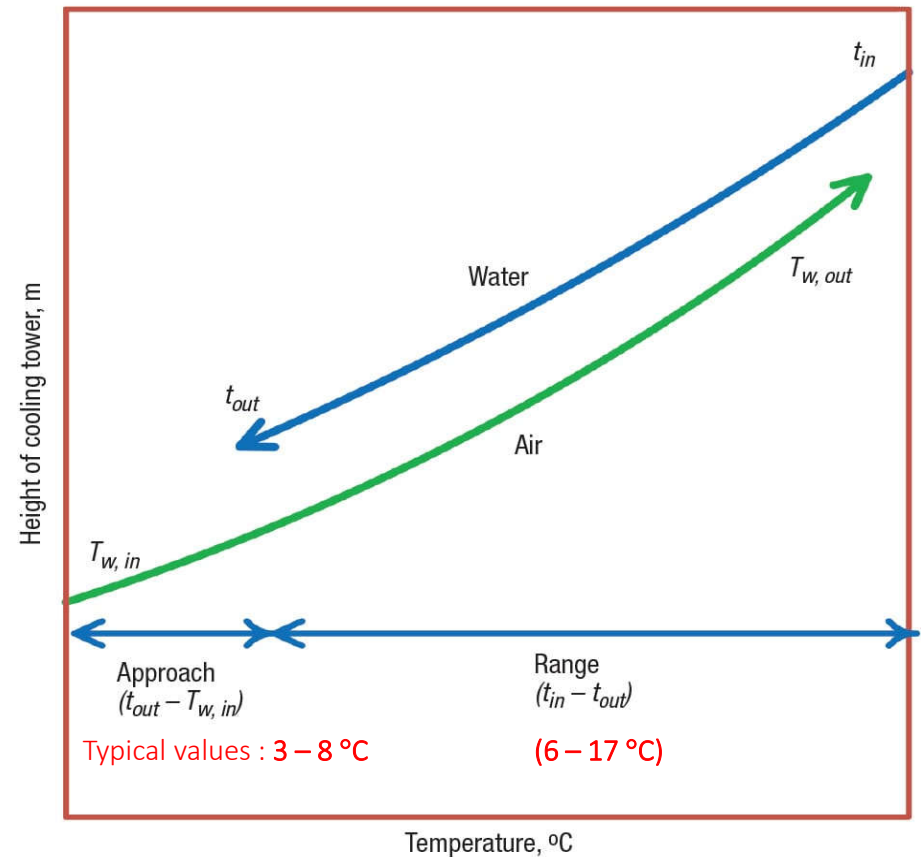
$$L' C_{A,L} dt_L = (G'_S C_{A,L} dY' - h_L a_H dZ)(t_i - t_L)$$

$$L' C_{A,L} dt_L = G'_S \left\{ C_S dt_G + [C_A(t_G - t_0) - C_{A,L}(t_L - t_0) + \lambda_0] dY' \right\}$$

Performance parameters in cooling towers



$$Effectivness = \frac{Range}{Range + Approach} \times 100$$



- There is no driving force for mass transfer in the liquid phase, since water is a pure liquid.
- The humidity driving force in the gas phase is

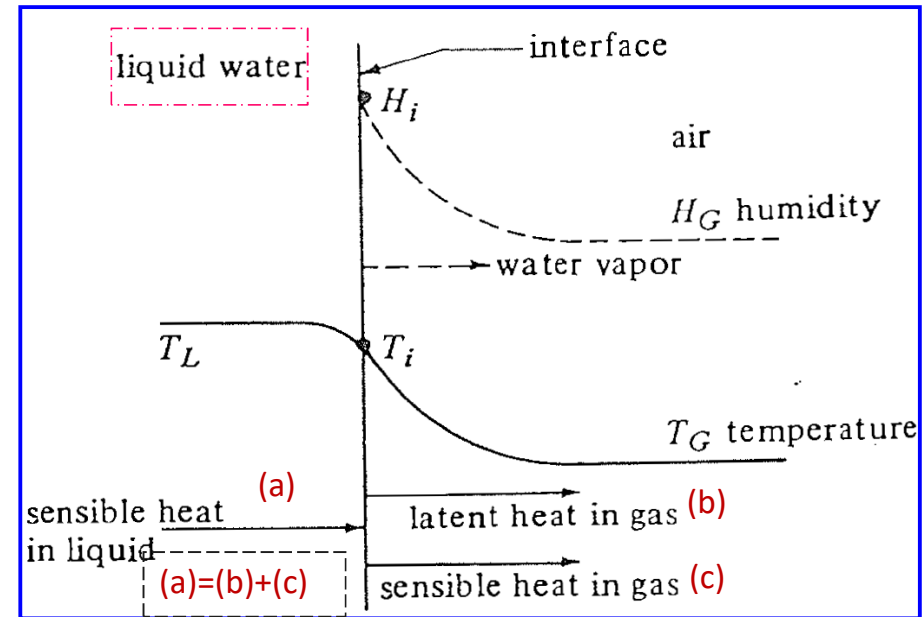
$$\Delta H = (H_i - H_G) \quad \text{kg } H_2O/\text{kg dry air}$$

- The temperature driving force is

$$\Delta T_L = T_L - T_i \quad \text{in the liquid phase}$$

and

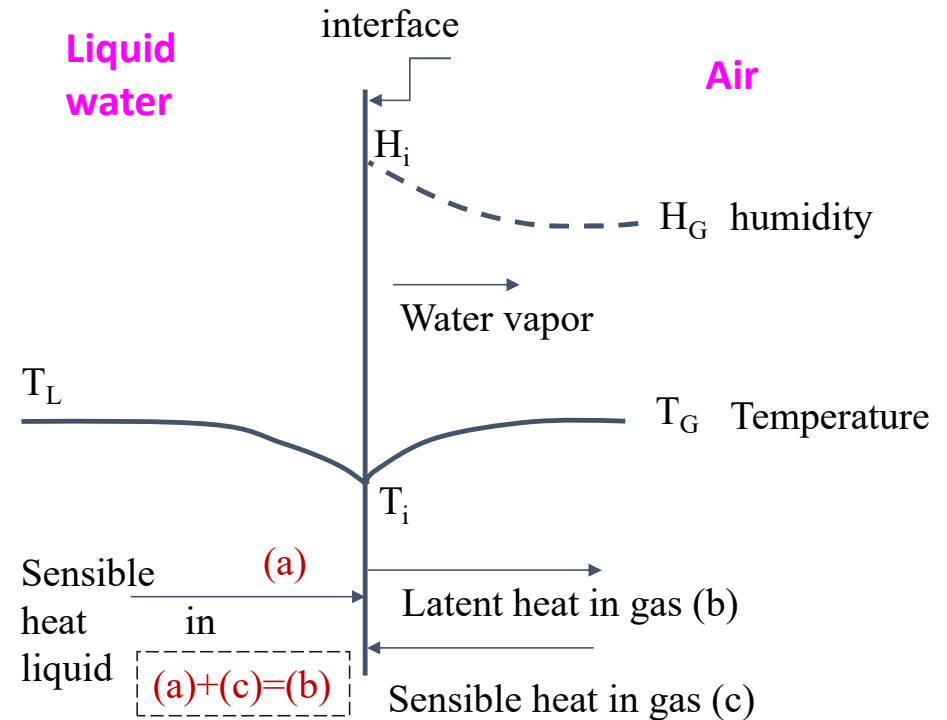
$$\Delta T_G = T_i - T_G \quad \text{in the gas phase.}$$



Temperature and humidity (concentration) profiles at the top of the cooling tower

- Latent heat leaves the interface in the water vapor, diffusing to the gas phase.
- The sensible heat flow from the liquid to the interface equals the sensible heat flow in the gas plus the latent heat flow in the gas

- In **the lower part of the tower**,
- The bulk water temperature (T_L) may be below the dry bulb temperature (T_G).
 - Then, the direction of sensible heat flow is reversed.



Temperature and humidity (concentration) profiles at the **bottom** of the cooling tower

Continuous countercurrent adiabatic water cooling

L = water flow, kg water/s·m²

T_L = water temperature, °C or K

G = dry air flow rate, kg/s·m²

T_G = air temperature, °C or K

H = Humidity of air, kg water/kg dry air*

H_y = enthalpy of air-water vapor mixture, J/kg dry air

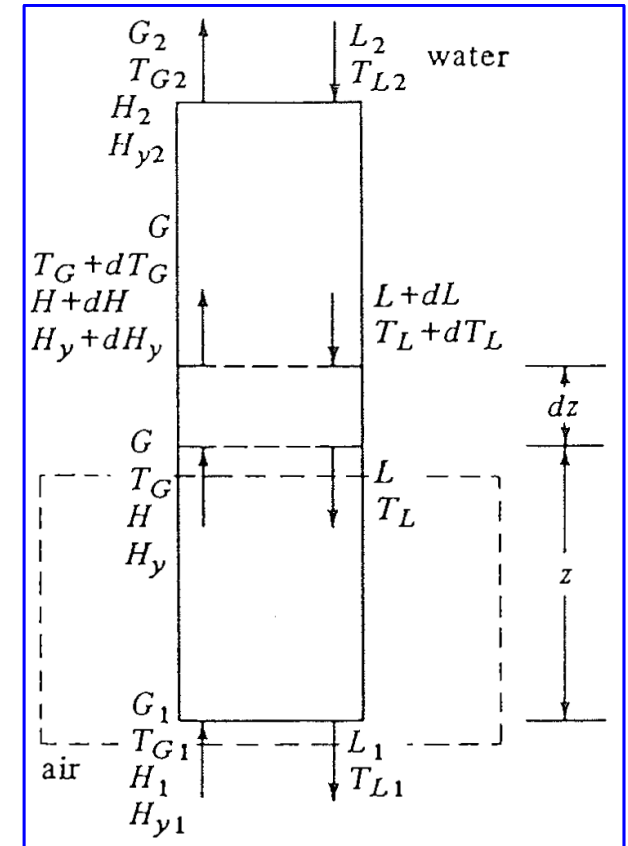
λ_o = latent heat of water, J/kg

c_s = humid heat = $c_L + c_G \cdot H$

The enthalpy, H_y given by:

$$\begin{aligned} H_y &= c_s(T - T_o) + H\lambda_o \\ &= (1.005 + 1.88H)10^3(T - 0) + 2.501 \times 10^6 H \end{aligned}$$

*Humidity, H can be retrieved from the humidity chart



➤ Assumptions

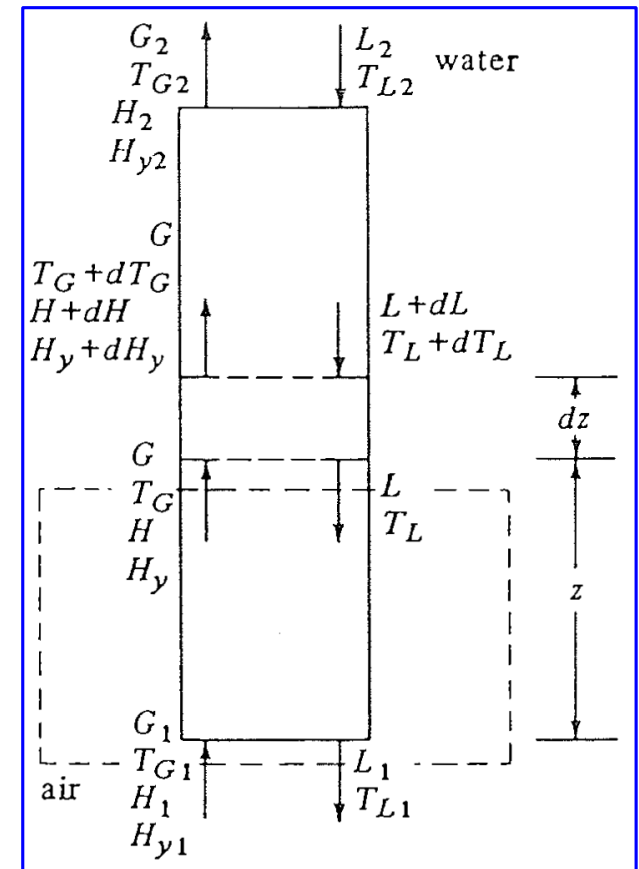
- i. Flow rate of gas and liquid water is assumed constant since only a small water evaporated (1-5%).
- ii. c_L is assumed constant at **$c_L = 4.187 \times 10^3 \text{ J/kg}\cdot\text{K}$**

➤ Perform the energy/heat balance:

Heat emitted = Heat absorbed

1) Heat balance for dashed line box making a heat balance for the **dz** column height :

→ Total sensible heat transferred from bulk fluid to interface;



$$L c_L dT_L = G dH_y = h_L \cdot a \cdot dz (T_L - T_i)$$

$$Lc_L dT_L = G dH_y = h_L \cdot a \cdot dz (T_L - T_i)$$

Liquid phase volumetric heat transfer coefficient

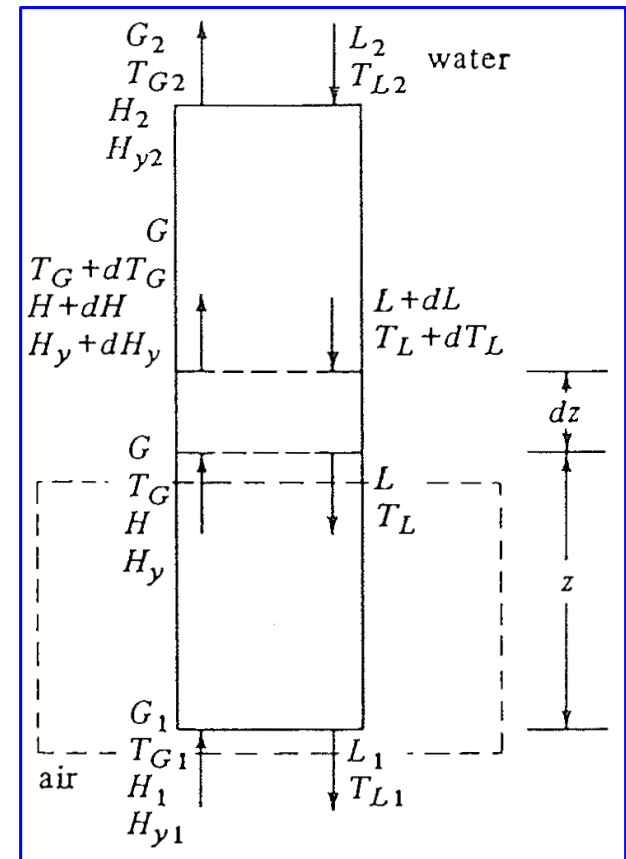
$$a = A/V \text{ (m}^2/\text{m}^3\text{)}$$

Considering the two terms to left, Integration

$$G (H_y - H_{y1}) = Lc_L (T_L - T_{L1})$$

- Rearrange the above Eq. to have the following **operating line Eq.:**

$$H_y = (H_{y1} - T_{L1} \frac{Lc_L}{G}) + \frac{Lc_L}{G} T_L$$

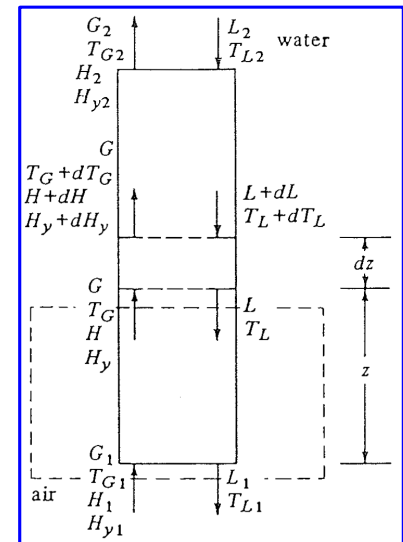


When plotted on a chart of H_y versus T_L , this Equation is a straight line

$$\text{Intercept} = H_{y1} - T_{L1} \frac{Lc_L}{G} \quad ; \quad \text{Slope} = \frac{Lc_L}{G}$$

- Also, making an overall heat balance over both ends of the tower,

$$G (H_{y2} - H_{y1}) = Lc_L (T_{L2} - T_{L1})$$



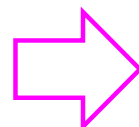
To draw the operating line we need either two points or one point and slope (Lc_L/G).

1. **Draw the equilibrium curve:** the enthalpy of saturated air versus the dew point temperature T_L using:

$$H_{yi} = c_S (T_L - T_0) + H_i \lambda_0$$

where the T_0 is the base temperature: $T_0 = 0^\circ\text{C}$: $\lambda_0 = 2502.3 \text{ kJ/kg water}$
 $T_0 = 32^\circ\text{F}$: $\lambda_0 = 1075.8 \text{ Btu/lbm water}$

$$\begin{aligned} c_S &= 1.005 + 1.88 H_i ; \text{kJ}/(\text{kg dry air} \cdot \text{K}) \\ &= 0.24 + 0.45 H_i ; \text{btu}/(\text{lbm dry air} \cdot ^\circ\text{F}) \end{aligned}$$

 *Enthalpy; $H_{yi} = (1.005 + 1.88 H_i) \times 10^3 (T - 0) + 2.501 \times 10^6 H_i$ J/kg air*

H_i is the saturated humidity picked up from the psychrometric chart at T_L .



SI METRIC PSYCHROMETRIC CHART

BASED ON A BAROMETRIC PRESSURE
of 101.325 kPa
AT SEA LEVEL

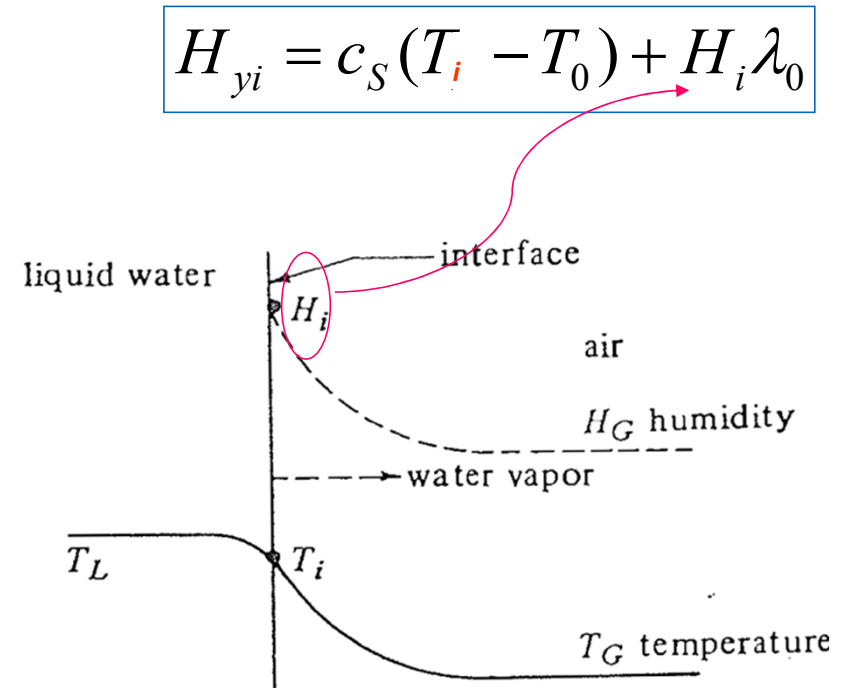
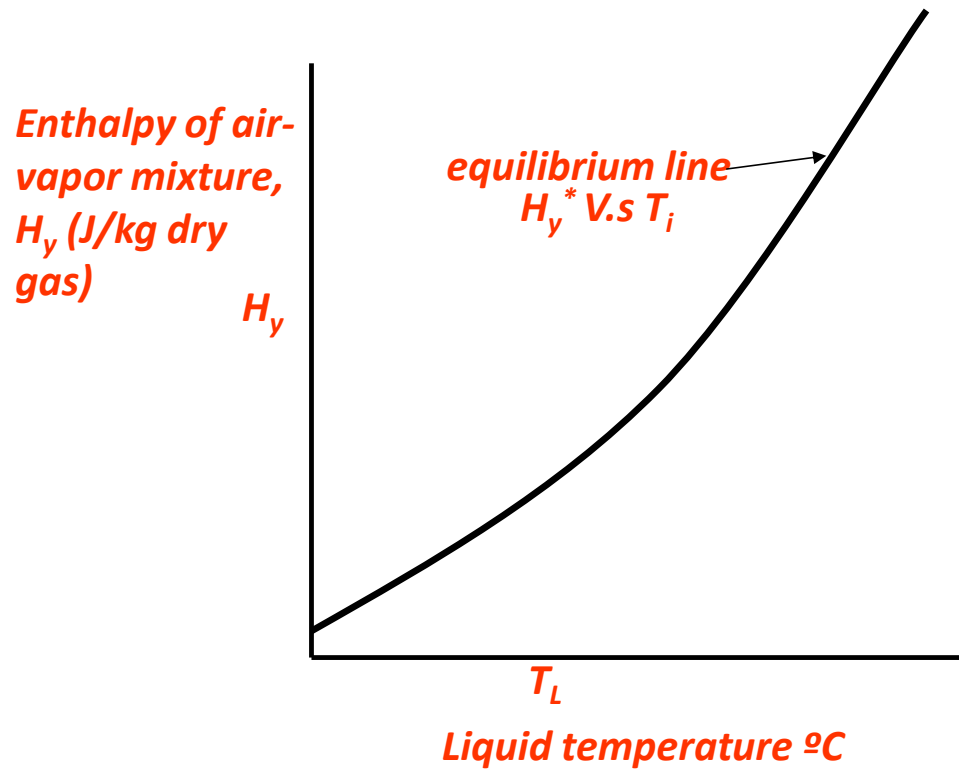
$H_{yi} = 84 \text{ kJ/kg DA}$

$0.0225 \text{ kg W/kg DA}$

$T_{L \text{ dew}} = 26.6^\circ \text{C}$

TABLE 10.5-1. *Enthalpies of Saturated Air–Water Vapor Mixtures*
(0°C Base Temperature)

H_y				H_y			
T_L		<i>btu</i>	<i>J</i>	T_L		<i>btu</i>	<i>J</i>
°F	°C	<i>lb_m dry air</i>	<i>kg dry air</i>	°F	°C	<i>lb_m dry air</i>	<i>kg dry air</i>
60	15.6	18.78	43.68×10^3	100	37.8	63.7	148.2×10^3
80	26.7	36.1	84.0×10^3	105	40.6	74.0	172.1×10^3
85	29.4	41.8	97.2×10^3	110	43.3	84.8	197.2×10^3
90	32.2	48.2	112.1×10^3	115	46.1	96.5	224.5×10^3
95	35.0	55.4	128.9×10^3	140	60.0	198.4	461.5×10^3



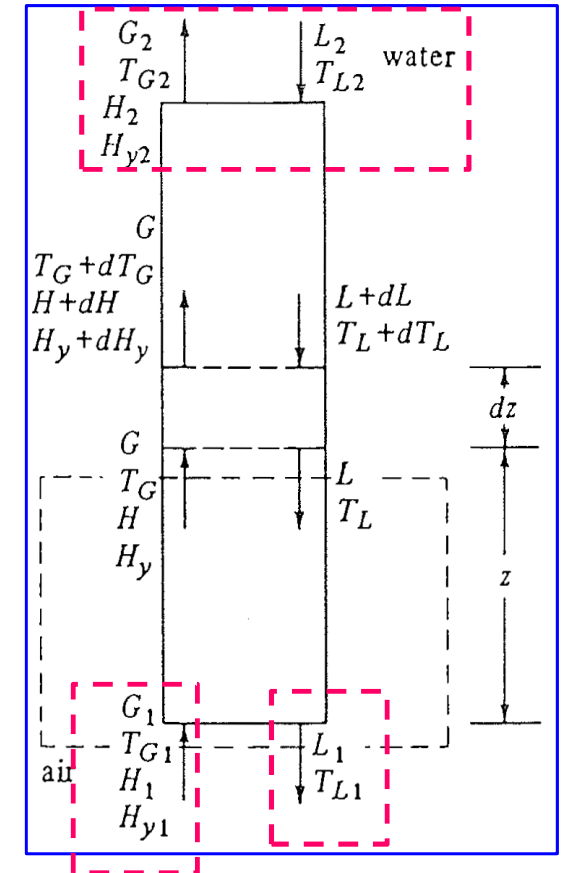
➤ Draw the **operating line Equation**

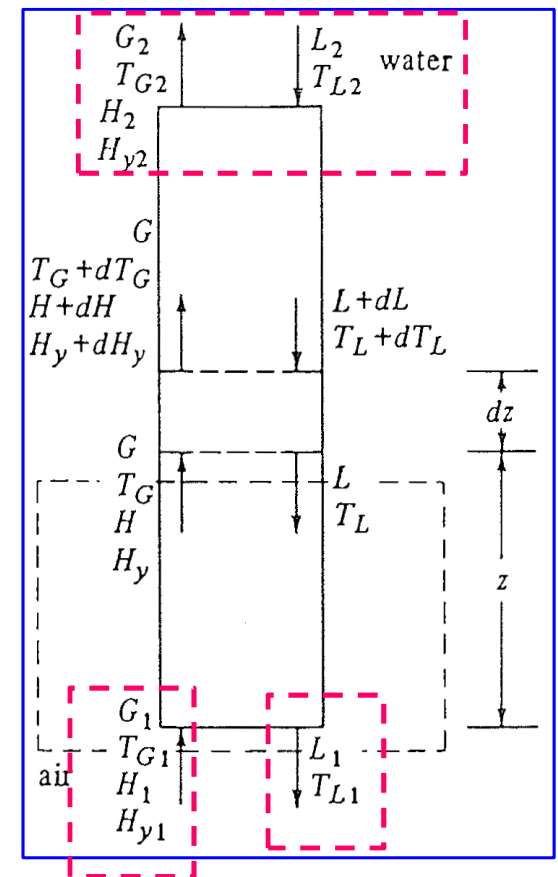
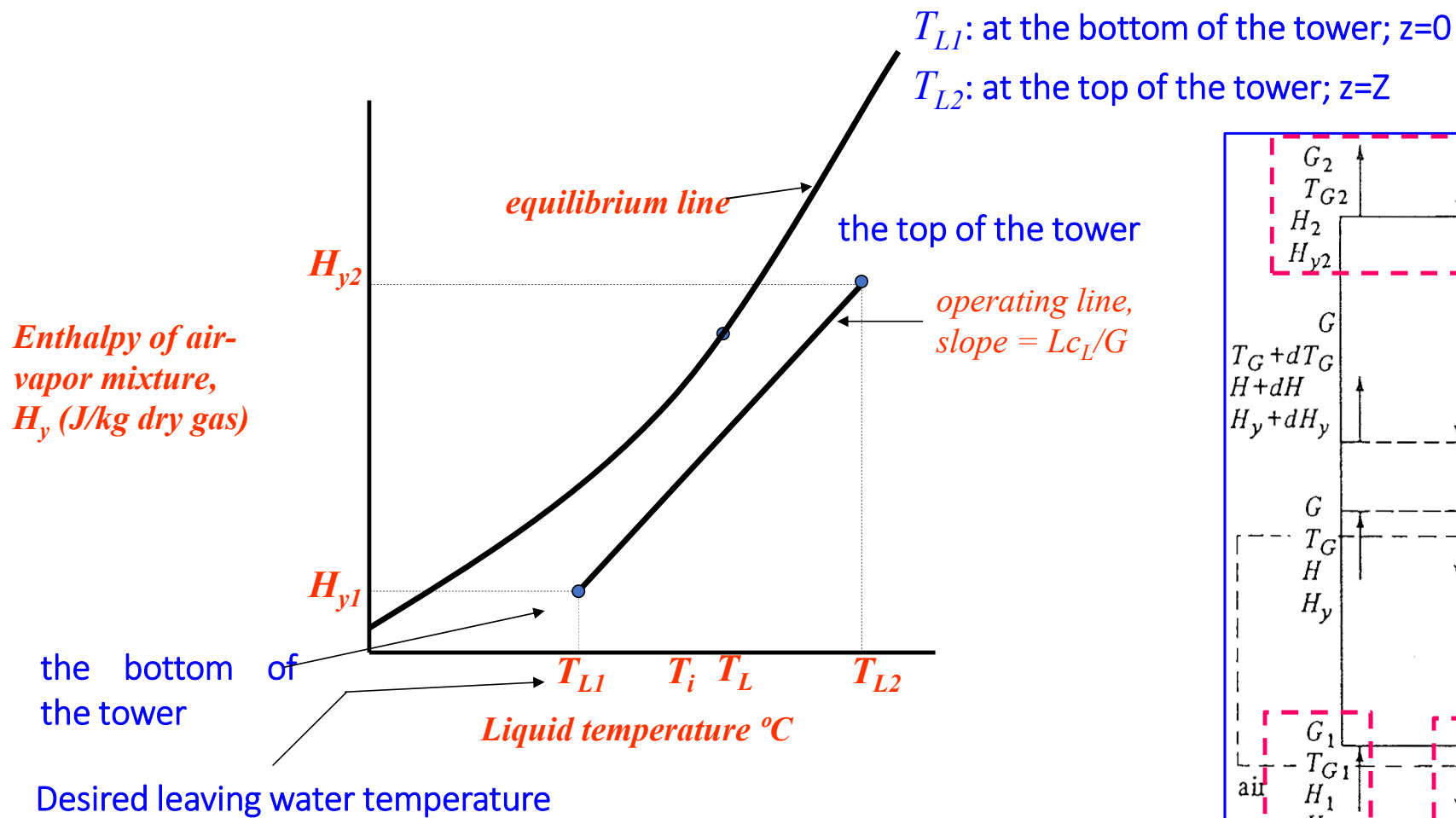
$$H_y = (H_{y1} - T_{L1} \frac{Lc_L}{G}) + \frac{Lc_L}{G} T_L$$

- Knowing the entering air conditions T_{G1} and H_1 , the enthalpy of this air H_{y1} is calculated
- The point H_{y1} and T_{L1} (desired leaving water temperature) is plotted as one point on the operating line (T_{L1} and H_{y1})

$$\text{Intercept} = H_{y1} - T_{L1} \frac{Lc_L}{G} \quad ; \quad \text{Slope} = \frac{Lc_L}{G}$$

- Knowing T_{G2} and H_2 , the enthalpy of this air H_{y2} can be also calculated and the point (T_{L2} and H_{y2}) is plotted as a second point on the operating line





- We know from **mass transfer course**, that the flux, N_A , kmol water evaporating/s.m²:

$$N_A = k_y (y_{A,i} - y_{A,G}) = k_G (P_{A,i} - P_{A,G})$$

$$k_y = k_G P$$

where k_G is gas-phase film mass transfer coefficient in kgmol/(s.m².Pa), $P_{A,i}$ and $P_{A,G}$ is the water vapor partial pressure at the interface and in the bulk gas-phase, respectively. While y is water vapor mole fraction.

- The **mass-transfer interfacial area** between air and water droplets **is not known**.
- This film mass-transfer interfacial area is different from the surface area of packing. Here, **a quantity (a_M)**, defined as interfacial area per volume of packing section, is combined with the gas-phase mass transfer coefficient, k_G , to give a **volumetric film mass transfer coefficients** defined as ($k_G a_M$) in kgmol/(s.m³.Pa).

- Now the volumetric diffusion rate of water vapor, $N_{A,vol}$ is:

$$N_{A,vol} = k_y a_M (y_{A,i} - y_{A,G}) = k_G a_M (P_{A,i} - P_{A,G})$$

- The relationship between humidity and mole fraction is:**

$$y = \frac{H / M_A}{1 / M_B + H / M_A}$$

- where M_A and M_B is the molecular weight of water vapor and air, respectively.
- Since H is small, an approximation of the relationship is:

$$y \cong \frac{M_B H}{M_A} \quad \xrightarrow{N_{A,vol} = k_y a_M (y_{A,i} - y_{A,G})} \quad N_{A,vol} = \frac{M_B}{M_A} k_y a_M (H_i - H_G)$$

H_i is the humidity of the gas at the interface in kg water/kg dry air, and
 H_G is the humidity of the gas in the bulk gas phase in kg water/kg dry air

Note that $M_B k_y a_M = k_H a_M$

$$k_G a_M [=] \text{kgmol}/(\text{s.m}^3.\text{Pa})$$

$$k_H a_M [=] \text{kg}/(\text{s.m}^3)$$

$$k_y a_M [=] \text{kgmol}/(\text{s.m}^3)$$

$$Lc_L dT_L = GdH_y = h_L \cdot a \cdot dz (T_L - T_i)$$

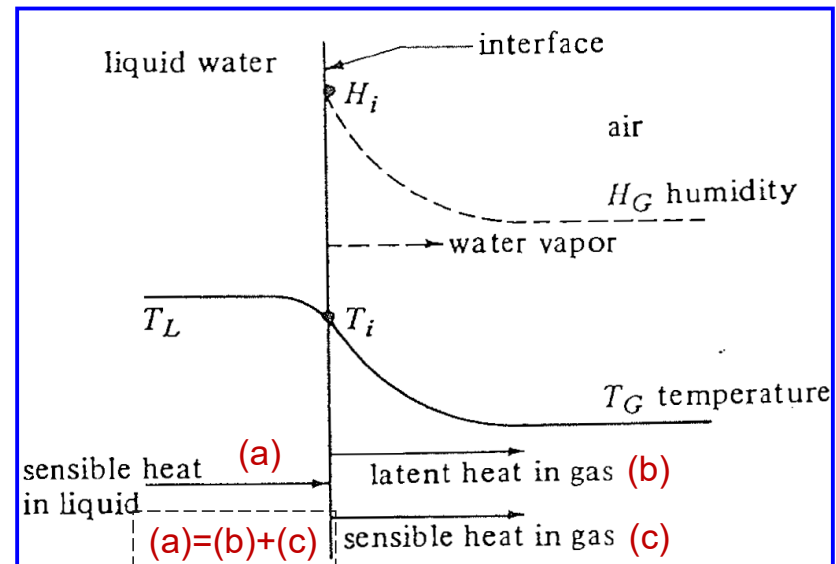
Liquid phase volumetric
heat transfer coefficient

$$a = A/V \text{ (m}^2/\text{m}^3\text{)}$$

- The sensible heat flow from the liquid to the interface = the sensible heat flow in the gas + the latent heat flow in the gas

$$(a) = (b) + (c)$$

Temperature and concentration profiles



- The latent heat in the water vapor being transferred over volume $dv=Adz$ height column is:

$$dQ_\lambda = N_{A,vol} \lambda_0 M_A dV$$

$$dV = dzA$$

$$N_{A,vol} = M_B k_y a_M (H_i - H_G) / M_A$$

$$dQ_\lambda = M_B k_y a_M (H_i - H_G) \lambda_0 Adz$$

or $dQ_\lambda = \mathbf{k_H a_M (H_i - H_G) \lambda_0 Adz}$

- Further, the rate of sensible heat transfer (**convective heat transfer rate in gas phase**) over volume $dv = Adz$ is:

$$\begin{aligned} dq_s &= \mathbf{G c_s dT_G} \\ &= \mathbf{h_G \cdot a_{H,G} \cdot (T_i - T_G) \cdot dz} \end{aligned}$$

$$dQ = dQ_{\lambda} + dq_s$$

$$dq = \frac{dQ}{A} = \left[M_B k_y a_M (H_i - H_G) \lambda_0 + h_G a_{H,G} (T_i - T_G) \right] dz$$

✓ *Keep in mind that*

$$dq = h_L a_{H,L} (T_L - T_i) dz$$

- It is found that for water vapor-air mixture the **experimental** value of which is called the **psychrometric ratio** is **closed** to **humid heat** c_s :

$$(h_G a_{H,G} / M_B k_y a_M)$$

$$c_s \cong \frac{h_G a_{H,G}}{M_B k_y a_M} \xrightarrow{k_y = k_G P} c_s \cong \frac{h_G a_{H,G}}{M_B P k_G a_M} \quad \text{(Lewis relation)}$$

- Using the above Lewis relation:

$$dq = \left[M_B k_y a_M (H_i - H_G) \lambda_0 + h_G a_{H,G} (T_i - T_G) \right] dz$$

$$c_S \cong \frac{h_G a_{H,G}}{M_B P k_G a_M}$$

$$dq = M_B P k_G a_M \left[H_i \lambda_0 + c_S T_i - (c_S T_G + \lambda_0 H_G) \right] dz$$

- **Adding and subtracting $c_S T_0$ inside the bracket of the above Eq.:**

$$dq = M_B P k_G a_M \left[c_S (T_i - T_0) + H_i \lambda_0 - (c_S (T_G - T_0) + \lambda_0 H_G) \right] dz$$

$$H_y = c_s(T_G - T_0) + \lambda_0 H_G$$

Enthalpy of water vapor-air mixture at T_G

$$H_{yi} = c_s(T_i - T_0) + H_i \lambda_0$$

Enthalpy of water vapor-air mixture at T_i

$$\longrightarrow dq = M_B P k_G a_M [H_{yi} - H_y] dz$$

But

$$dq = G dH_y$$

$$\longrightarrow Z = \frac{G}{M_B P k_G a_M} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y}$$

Design Eq. of the cooling tower

$$Z = \underbrace{\frac{G}{M_B P k_G a_M}}_{\text{HTU}} \int_{H_{y1}}^{H_{y2}} \underbrace{\frac{dH_y}{H_{yi} - H_y}}_{\text{NTU}} \equiv (\text{HTU})(\text{NTU})$$

HTU \equiv Height of a transfer unit

NTU \equiv Number of transfer units

- The enthalpy, H_{yi} , at the interface temperature T_i is determined from:

$$dq = M_B P k_G a_M [H_{yi} - H_y] dz$$




$$dq = h_L a_{H,L} (T_L - T_i) dz$$

$$\frac{H_{yi} - H_y}{T_i - T_L} = - \frac{h_L a_{H,L}}{M_B P k_G a_M}$$

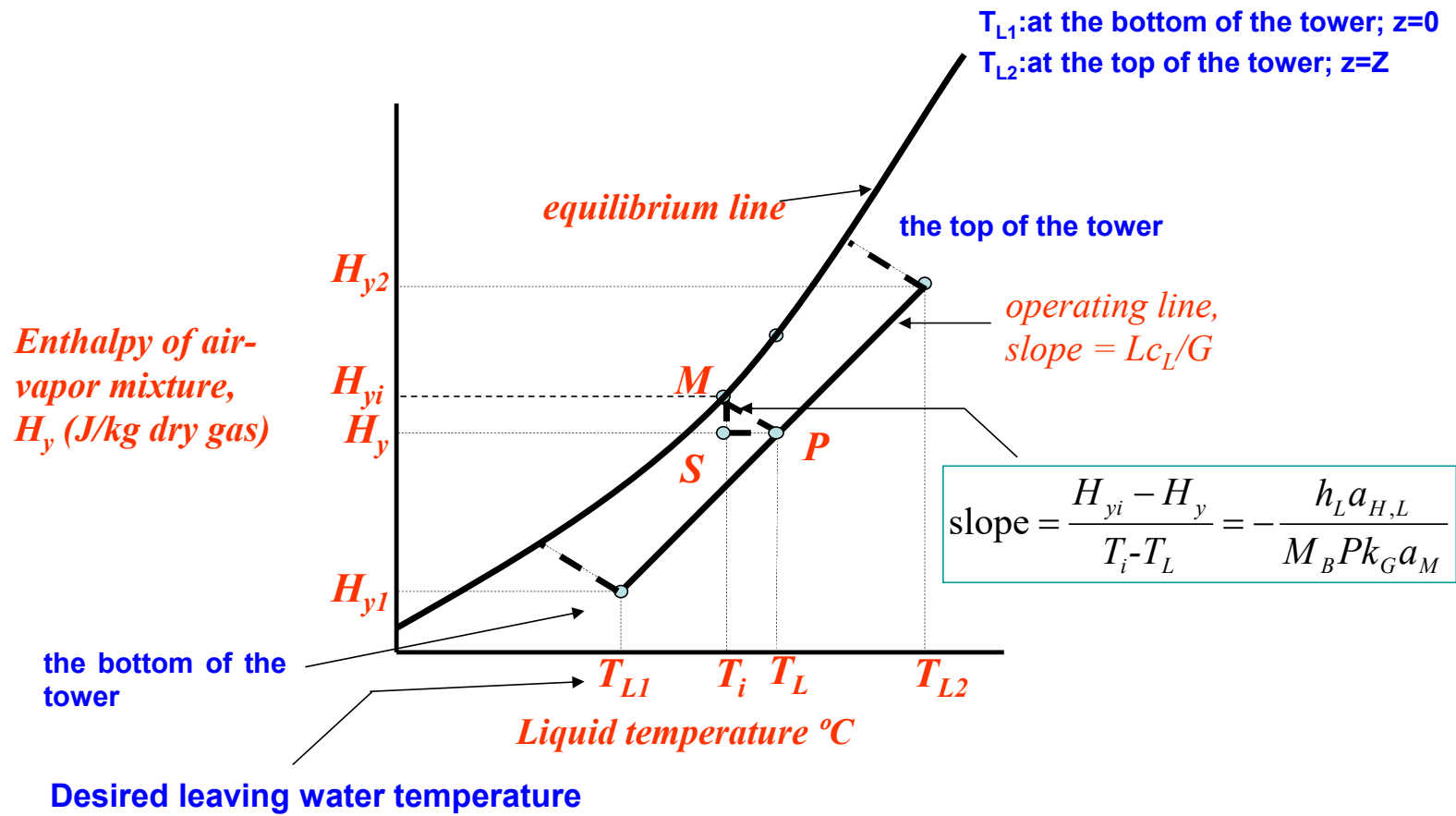
Note that $Lc_L dT_L = GdH_y$, $dq = GdH_y$

and $dq = M_B Pk_G a_M [H_{yi} - H_y] dz$

Hence, $Lc_L dT_L = M_B Pk_G a_M [H_{yi} - H_y] dz$



$$Z = \underbrace{\frac{Lc_L}{M_B Pk_G a_m}}_{\text{HTU}} \underbrace{\int_{T_{L1}}^{T_{L2}} \frac{dT_L}{H_{yi} - H_y}}_{\text{NTU}}$$



Design procedure of water cooling tower using film mass transfer coefficients

1. Draw the equilibrium curve:

- The enthalpy of saturated air H_{yi} is plotted versus T_i on an H versus T plot. This enthalpy is calculated using the equation

$$H_{yi} = (1.005 + 1.88 H_i) \times 10^3 (T - 0) + 2.501 \times 10^6 H_i \quad \text{J/kg air}$$

H_i is the saturated humidity picked up from the psychrometric chart for a given temperature.

2. Draw the operating line:

- Use the operating line equation (enthalpy vs Temp)

$$H_y = (H_{y1} - T_{L1} \frac{Lc_L}{G}) + \frac{Lc_L}{G} T_L$$

and/or the overall steady-state heat balance over the entire cooling tower:

$$G(H_{y2} - H_{y1}) = Lc_L(T_{L2} - T_{L1})$$

H_{y1} and H_{y2} is the gas mixture enthalpy at T_{G1} and T_{G2} , respectively.

→ To draw the operating line we need either two points or one point and slope (Lc_L/G).

3. Draw lines with slope: (Lewis relation)

$$\frac{H_{yi} - H_y}{T_i - T_L} = - \frac{h_L a_{H,L}}{M_B P k_G a_M} = \text{Slope} = \frac{H_{yi1} - H_{y1}}{T_{i1} - T_{L1}} = \frac{H_{yi2} - H_{y2}}{T_{i2} - T_{L2}}$$

- Select some value of T_i and read H_{yi} from the equilibrium curve.
- Select some value of T_L and calculate H_y from the above equation.
- Draw a line pass through the points (T_i, H_{yi}) and (T_L, H_y) this line must have slope of $h_L a_{H,L} / (M_B P k_G a_M)$.
- At 6 to 8 locations, draw parallel lines (slope= $- h_L a_{H,L} / (M_B P k_G a_M)$) from T_{L1} to T_{L2} to read enthalpies H_{yi} from equilibrium curve.

4. Calculate the number of transfer units (NTU):

- Use Enthalpy vs. T_L graph to find the driving force $H_{yi}-H_y$ for various T_L value from T_{L1} to T_{L2} .
- Calculate $1/(H_{yi}-H_y)$ for various T_L value from T_{L1} to T_{L2} .
- Perform graphical or numerical integration to calculate NTU:

$$\text{NTU} = \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y}$$

5. Calculate the height of a transfer unit number of transfer units (HTU):

$$\text{HTU} = \frac{G}{M_B P k_G a_M}$$

6. Calculate the height of the cooling tower: $Z = (\text{HTU})(\text{NTU})$

Example 3.5.1 Effectiveness of a cooling tower

A packed countercurrent water-cooling tower using gas flow rate of $1.356 \text{ kg dry air}/(\text{s.m}^2)$ and water flow rate of $1.356 \text{ kg}/(\text{s.m}^2)$. The water is cooled from 43.3 to 29.4 °C. The entering air at 29.4 °C has a wet bulb temperature of 23.9 °C. The gas film mass-transfer coefficient is estimated as $1.207 \times 10^{-7} \text{ kmol}/(\text{s.m}^3.\text{Pa})$. The term $h_L a_{HL}/M_B P k_G a_M$ has a value of $41.87 \text{ kJ}/(\text{kg.K})$. The tower operates at 1 atm . Calculate the Range, The approach, the tower effectiveness, and the height of the packed tower.

Solution

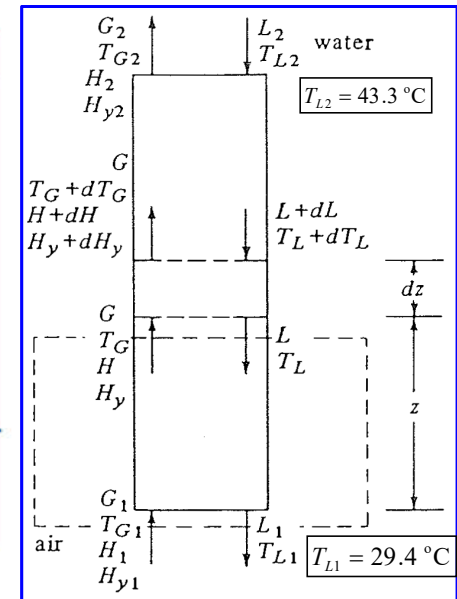
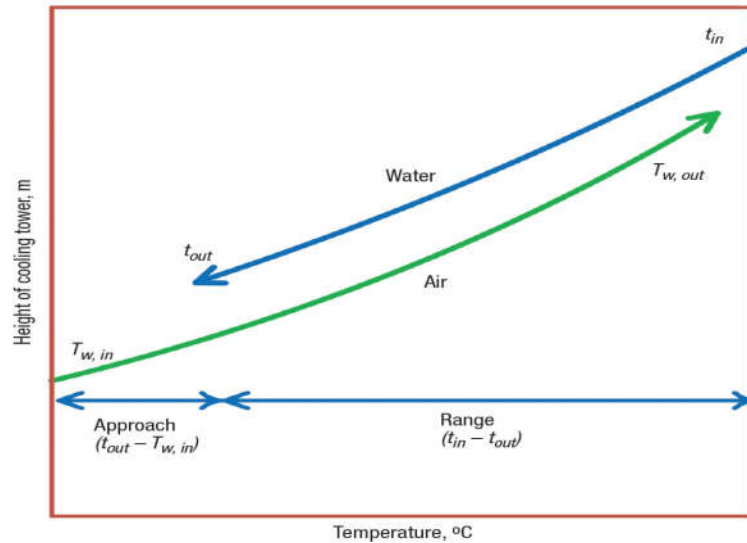
$$G = L = 1.356 \text{ kg}/(\text{s.m}^2)$$

$$T_{G1} = 29.4 \text{ °C}; T_{WB1} = 23.9 \text{ °C}$$

$$k_G a_M = 1.207 \times 10^{-7} \text{ kmol}/(\text{s.m}^3)$$

$$\text{Range} = T_{L2} - T_{L1} = 43.3 - 29.4 = 13.9 \text{ °C}$$

$$\text{Approach} = T_{L1} - T_{WB,1} = 29.4 - 23.9 = 5.5 \text{ °C}$$



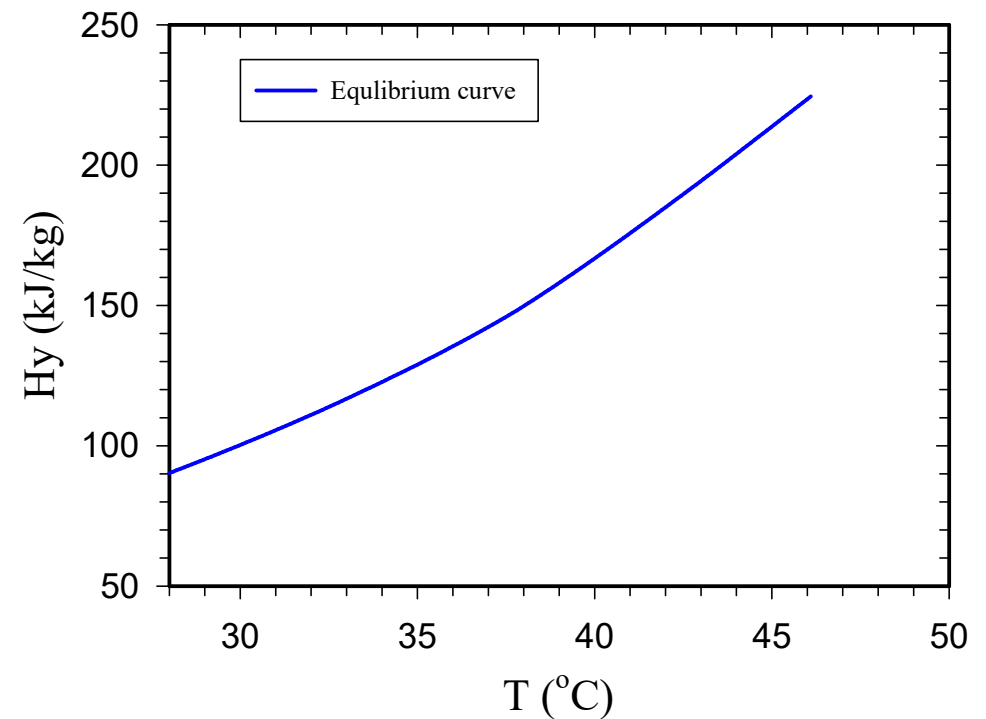
$$\text{Effectiveness} = 100 \times \text{Range} / (\text{Range} + \text{Approach}) = 71.6\%$$

Height of the packed tower.

1. **Draw the equilibrium curve:** use saturated humidity curve in the psychrometric chart and enthalpy Eq. to get:

$$H_{yi} = (1.005 + 1.88 H_i) \times 10^3 (T - 0) + 2.501 \times 10^6 H_i \quad \text{J/kg air}$$

T_L	H_{yi} , kJ/kg
15.6	43.7
26.7	84.0
29.4	97.2
32.2	112.1
35.0	128.9
37.8	148.2
40.6	172.1
43.3	197.2
46.1	224.5

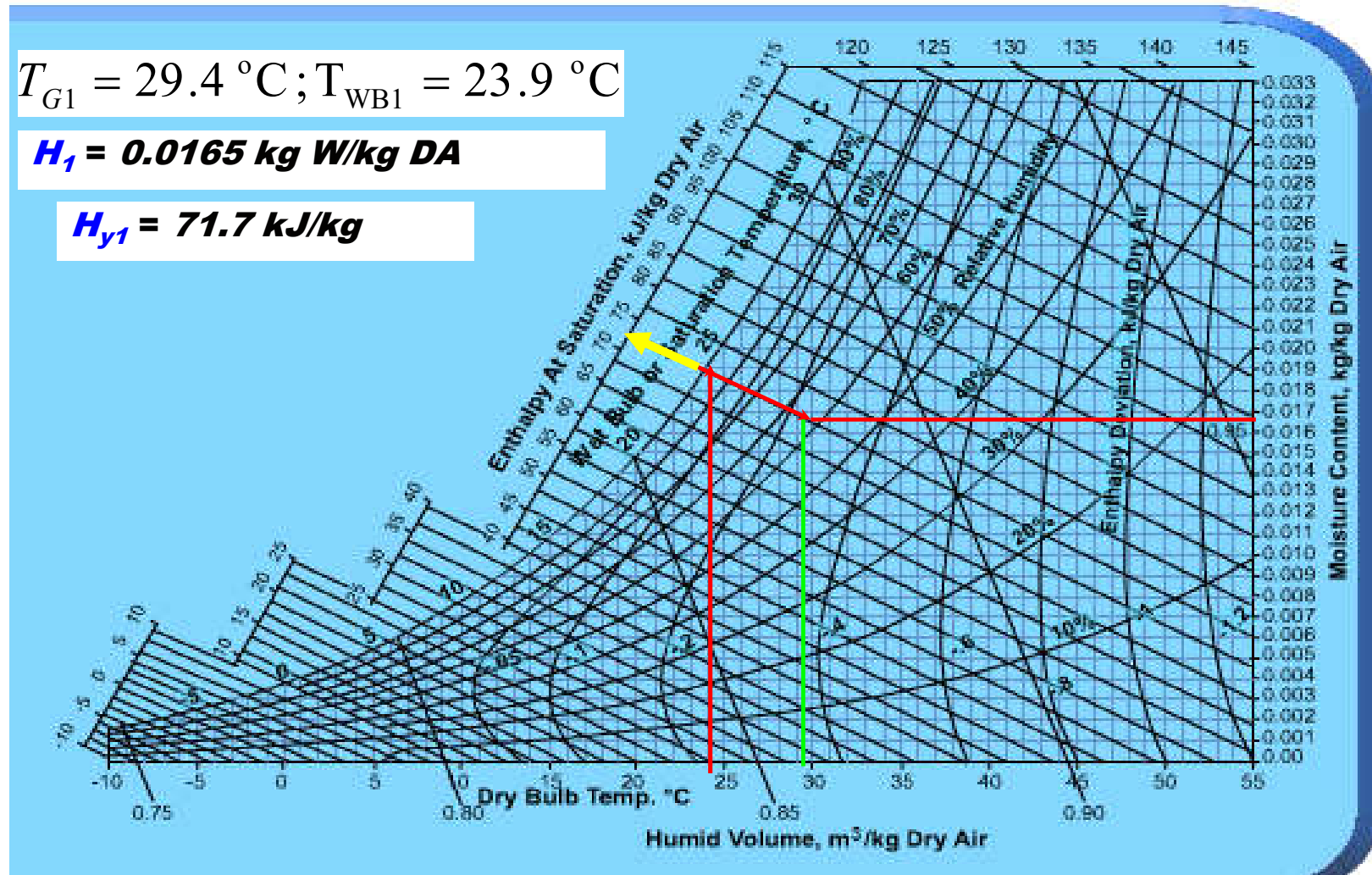


2. Draw the operating line:

$$T_{G1} = 29.4\text{ }^{\circ}\text{C}; T_{WB1} = 23.9\text{ }^{\circ}\text{C}$$

$$H_1 = 0.0165\text{ kg W/kg DA}$$

$$H_{y1} = 71.7\text{ kJ/kg}$$



Draw the operating line:

$$c_s = 1.005 + 1.88H_1 = 1.005 + 1.88(0.0165) = 1.036 \text{ kJ}/(\text{kg dry air.K})$$

$$H_{y1} = c_s(T_{G1} - T_0) + H_i\lambda_0 = 1.036(29.4 - 0) + (0.0165)(2502.3) = 71.7 \text{ kJ/kg}$$

Apply overall steady-state heat balance over the entire cooling to get H_{y2} :

$$G(H_{y2} - H_{y1}) = Lc_L(T_{L2} - T_{L1})$$

$$G = L = 1.356 \text{ kg}/(\text{s.m}^2)$$

$$c_L = 4.187 \text{ kJ}/(\text{kg.K})$$

$$T_{L1} = 29.4 \text{ }^\circ\text{C}$$

$$T_{L2} = 43.3 \text{ }^\circ\text{C}$$

$$H_{y1} = 71.7 \text{ kJ/kg}$$

$$H_{y2} = 129.9 \text{ kJ/kg}$$

We have two points enough to draw the operating line:

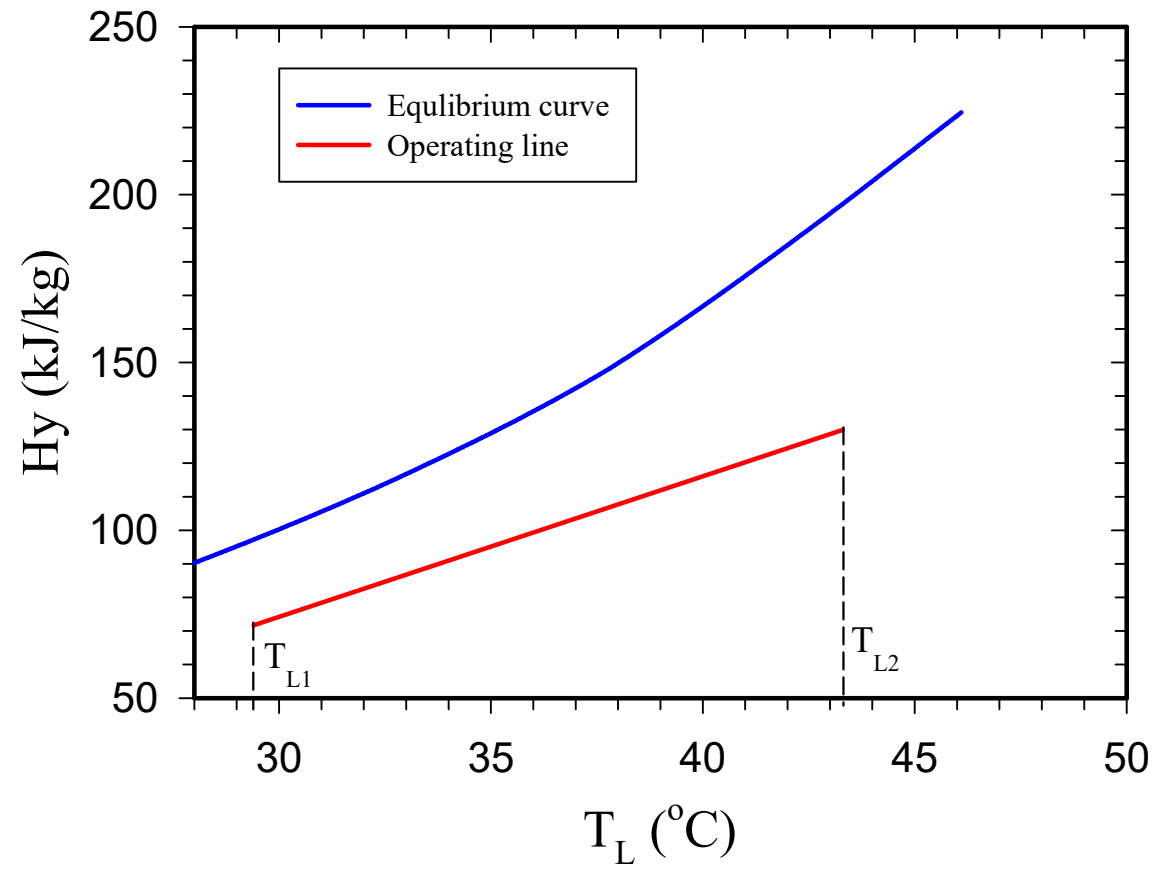
$$(T_{L1}, H_{y1}) = (29.4 \text{ }^\circ\text{C}, 71.7 \text{ kJ/kg})$$

$$(T_{L2}, H_{y2}) = (43.3 \text{ }^\circ\text{C}, 129.9 \text{ kJ/kg})$$

Draw the operating line:

$$(T_{L1}, H_{y1}) = (29.4\text{ }^{\circ}\text{C}, 71.7\text{ kJ/kg})$$

$$(T_{L2}, H_{y2}) = (43.3\text{ }^{\circ}\text{C}, 129.9\text{ kJ/kg})$$



3. Draw lines with constant slope:

For example,

at $T_i = 35^\circ\text{C}$, from the equilibrium curve $H_{yi} = 128.9 \text{ kJ/kg}$.

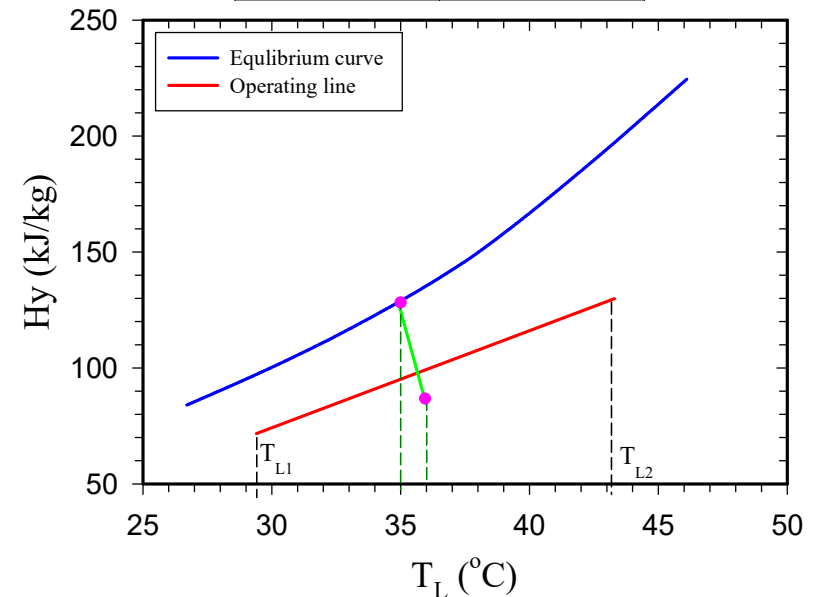
- at $T_L = 36^\circ\text{C}$, calculate H_y from:

$$\frac{H_{yi} - H_y}{T_i - T_L} = \frac{128.9 - H_y}{35 - 36} = -\frac{h_L a_{H,L}}{M_B P k_G a_M} = -41.87 \text{ kJ}/(\text{kg} \cdot \text{K})$$

$$H_y = 87.03 \text{ kJ/kg}$$

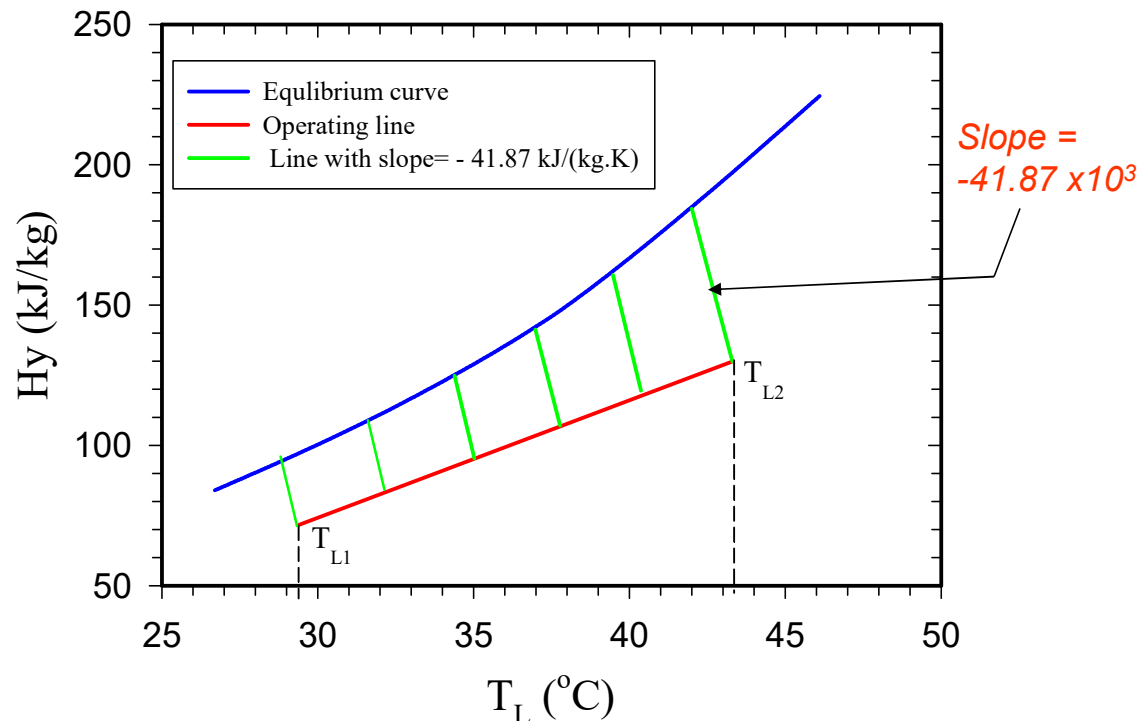
- Draw a line passes through the points $(35^\circ\text{C}, 128.9 \text{ kJ/kg})$ and $(36^\circ\text{C}, 87.03 \text{ kJ})$.

T_L	$H_{yi}, \text{ kJ/kg}$
15.6	43.7
26.7	84.0
29.4	97.2
32.2	112.1
35.0	128.9
37.8	148.2
40.6	172.1
43.3	197.2
46.1	224.5

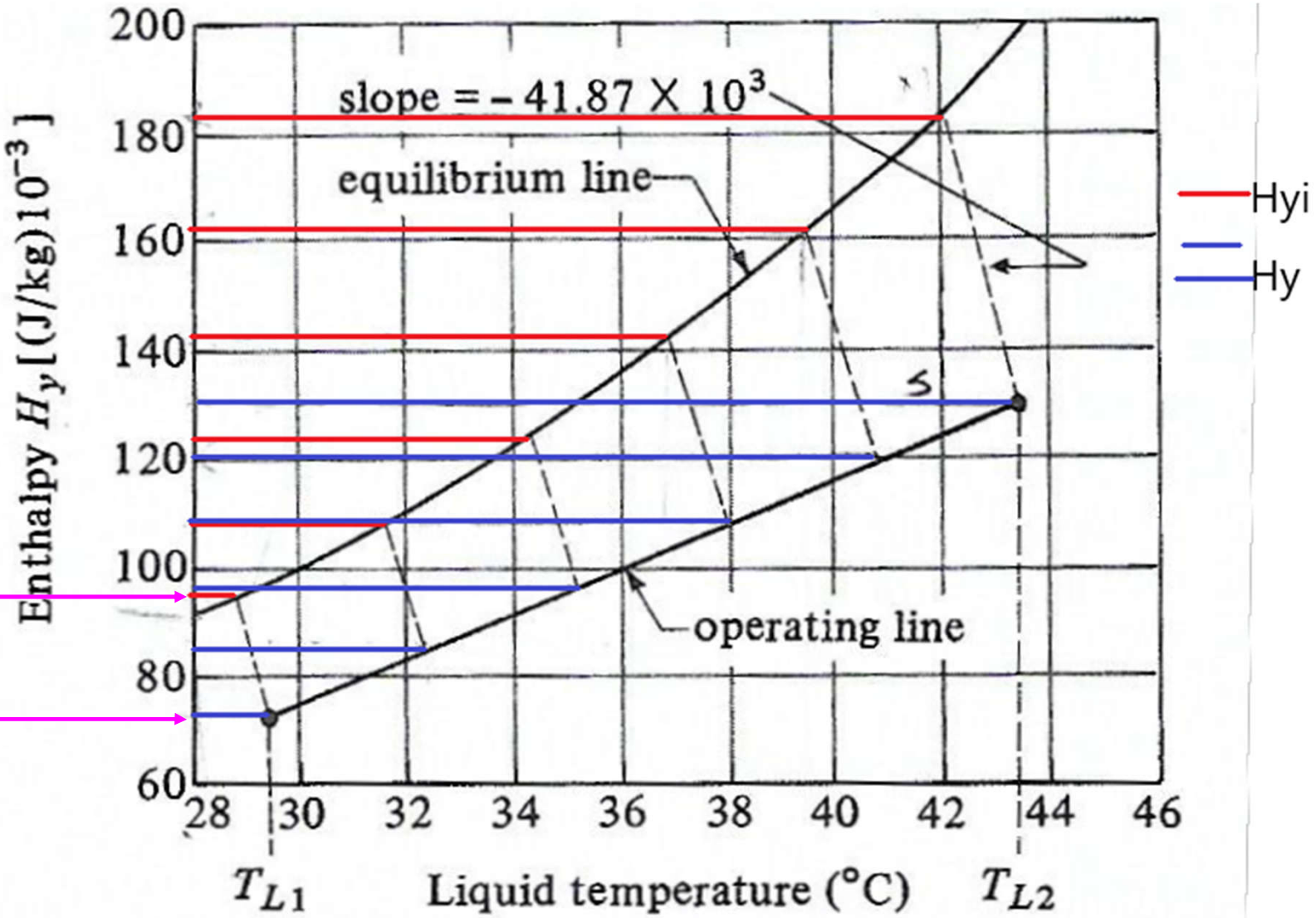


4. Calculate the number of transfer units (NTU):

- At 6 to 8 locations, draw parallel lines as shown below from T_{L1} to T_{L2} to read enthalpies H_{yi} from equilibrium curve

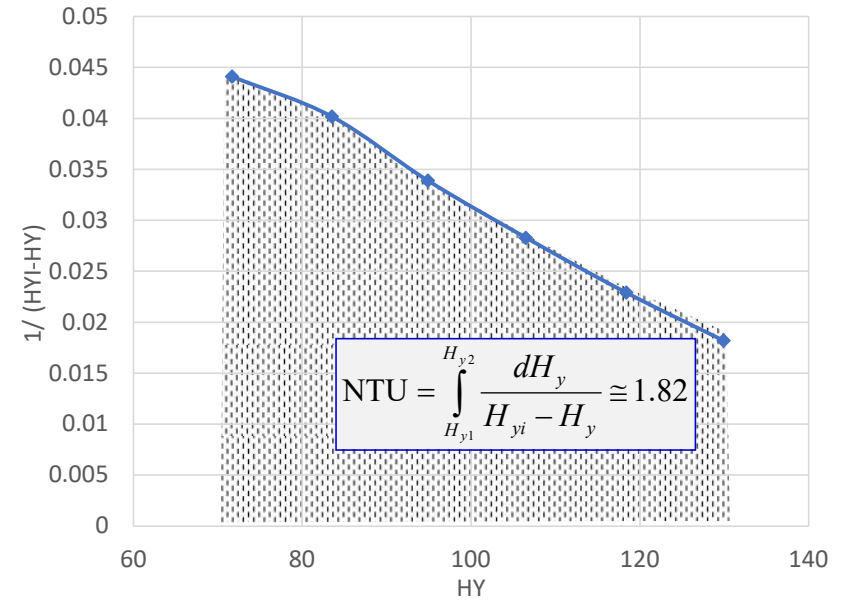


H_{yi} (kJ/kg)	H_y (kJ/kg)
94.4	71.7



4. Calculate the number of transfer units (NTU):

H_{yi} (kJ/kg)	H_y (kJ/kg)	$H_{yi} - H_y$ (kJ/kg)	$1/(H_{yi} - H_y)$; (kg/kJ)
94.4	71.7	22.7	0.0441
108.4	83.5	24.9	0.0402
124.4	94.9	29.5	0.0339
141.8	106.5	35.3	0.0283
162.1	118.4	43.7	0.0229
184.7	129.9	54.8	0.0182



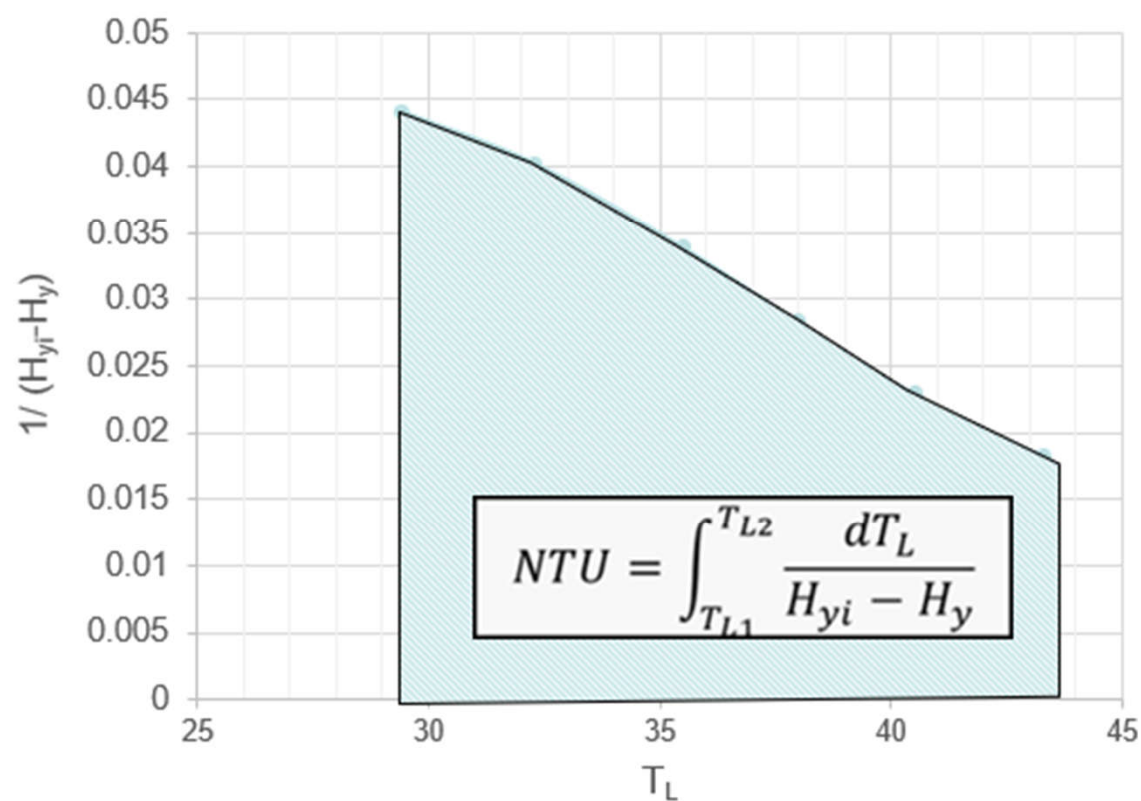
- Using Trapezoidal rule of numerical integration:

$$NTU = \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y} \cong 1.82$$

$$Z = \underbrace{\frac{G}{M_B P k_G a_M}}_{HTU} \underbrace{\int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y}}_{NTU} \equiv (HTU)(NTU)$$

$$z = \frac{Lc_L}{M_B P k_G a_m} \int_{T_{L1}}^{T_{L2}} \frac{dT_L}{H_{yi} - H_y}$$

$$HTU = \frac{Lc_L}{M_B P k_G a_m}$$



5. Calculate the height of a transfer unit (HTU):

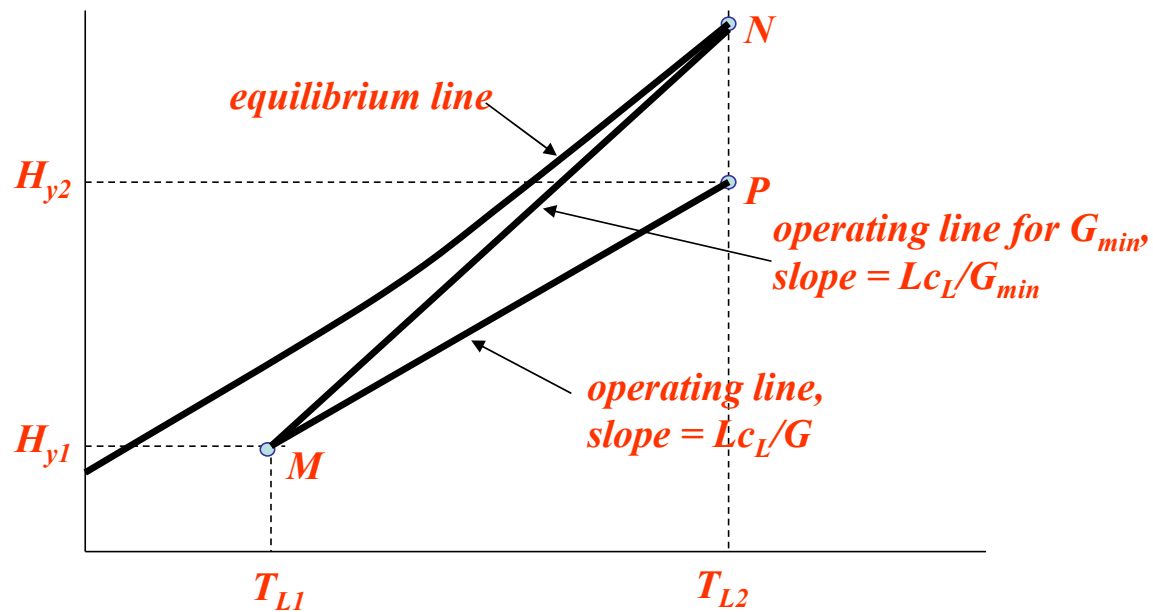
$$\text{HTU} = \frac{G}{M_B P k_G a_M} = \frac{1.356}{(29)(101325)(1.207 \times 10^{-7})} = 3.82 \text{ m}$$

6. Calculate the height of the cooling tower:

$$Z = (\text{HTU})(\text{NTU}) = (3.82)(1.82) = 6.96 \text{ m}$$

- Minimum air flow gives maximum slope of the operating line Eq.

$$H_y = (H_{y1} - T_{L1} \frac{Lc_L}{G}) + \frac{Lc_L}{G} T_L \longrightarrow \text{Slope}_{\max} = \frac{Lc_L}{G_{\min}}$$



▪ **Minimum value of air flow G_{\min} :**

- For actual cooling towers, a value of air flow rate greater than G_{\min} must be used.

A reasonable value of G is $(1.3-1.5) \times G_{\min}$.

Example. Find the minimum air flow for previous example

$$\begin{aligned} \text{Slope}_{\max} &= \frac{H_{y2} - H_{y1}}{T_{L2} - T_{L1}} \\ &= \frac{194 - 71.7}{43.3 - 29.4} \\ &= 8.8 \text{ kJ}/(\text{kg} \cdot \text{K}) \\ \text{Slope}_{\max} &= \frac{Lc_L}{G_{\min}} \\ G_{\min} &= \frac{Lc_L}{\text{Slope}_{\max}} \\ &= 0.64 \text{ kg}/(\text{s} \cdot \text{m}^2) \end{aligned}$$

