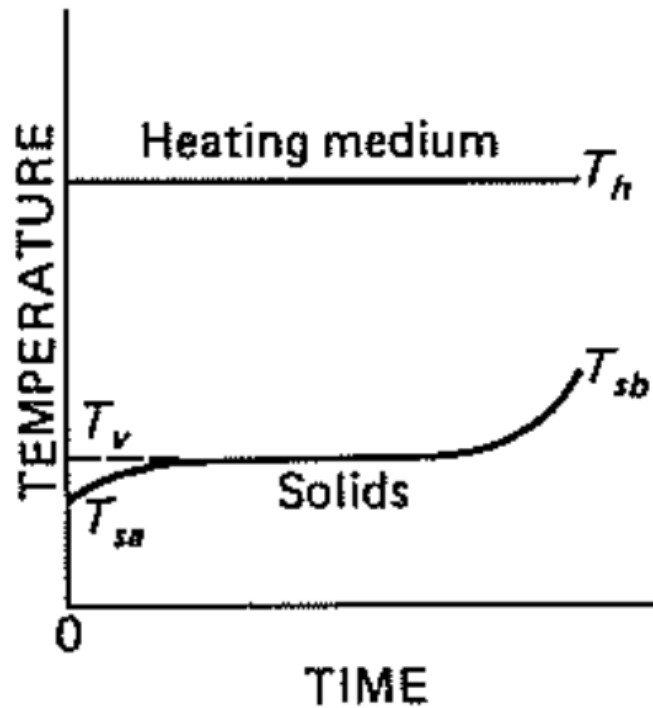


Topic 4.2. Fundamentals of Drying

This lecture

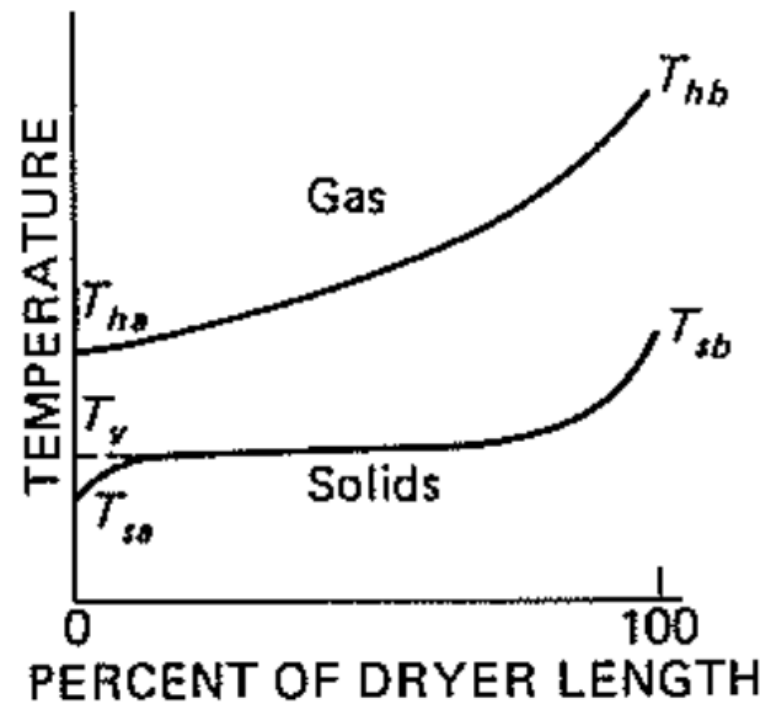
- ✓ Overview and definitions
- ✓ Drying applications

TEMPERATURE PATTERNS IN DRYERS



(a)

(a) batch dryer



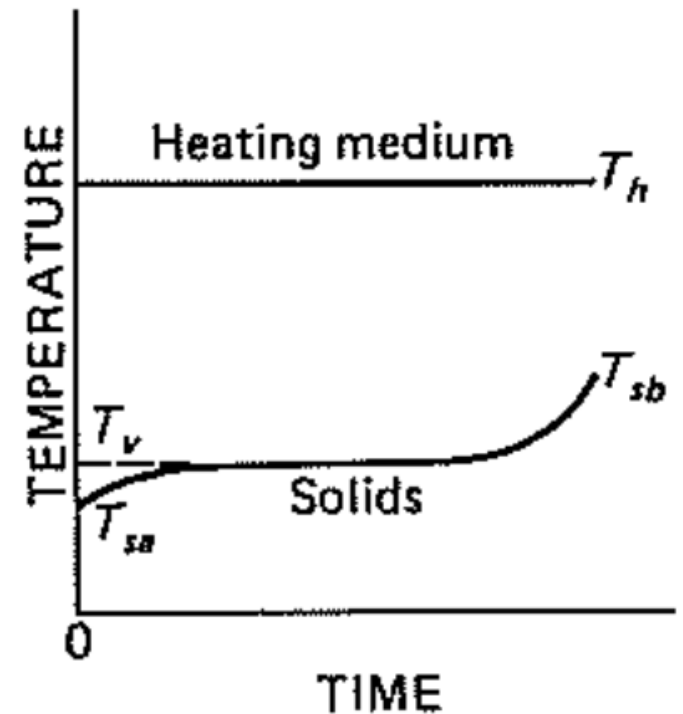
(b)

(b) continuous countercurrent adiabatic dryer

Batch Dryer

- The heating medium is at constant temperature
- The temperature of the **wet solids** rises quickly from its **initial value T_{sa}** to the **vaporization temperature T_v** .
- If the dryer operates **non-adiabatically** with no sweep gas, **T_v** is the **boiling point of the liquid** at the pressure inside the dryer.
- For **adiabatic operation** or If a sweep gas is used, **T_v** is at or near the wet-bulb temperature, **T_{wb}** of the gas (adiabatic-saturation temperature for air-water system).
- Drying occurs at **T_v** for a considerable period

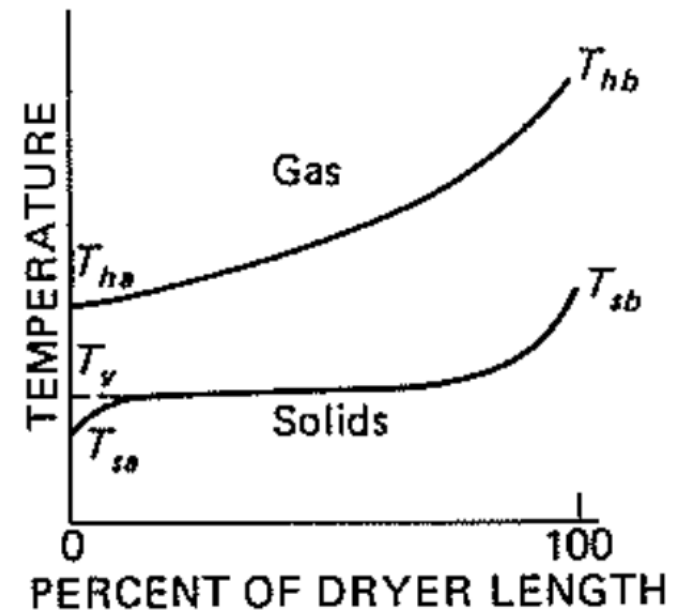
TEMPERATURE PATTERNS IN DRYERS



Continuous Dryer

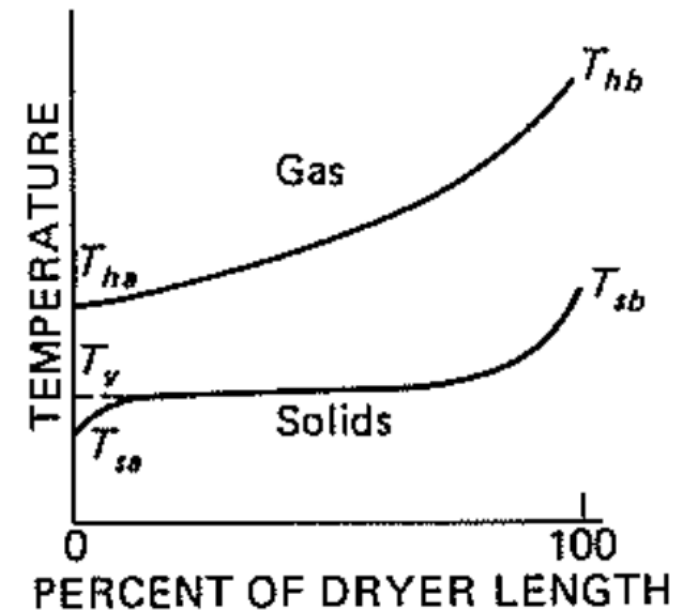
- The dryer operates at **steady-state**, **adiabatic**, **counter-current** conditions.
- The hot gas enters at temperature, T_{hb} and the hot solid leaves at temperature T_{sb} .
- The hot gas enters with low humidity; it cools rapidly at first, then more slowly as the temperature-difference driving force decreases.
- The solids are quickly heated from temperature T_{sa} to the **vaporization temperature** T_v .
- For **adiabatic operation** or If a sweep gas is used, T_v is at or near the wet-bulb temperature, T_{wb} of the gas (adiabatic-saturation temperature for air-water system).

TEMPERATURE PATTERNS IN DRYERS



HEAT TRANSFER IN DRYER: Calculation of heat duty

1. Heat the feed (solids and liquid) to the vaporization temperature.
2. Vaporize the liquid.
3. Heat the solids to their final temperature.
4. Heat the vapor to its final temperature.



$$\frac{q_T}{\dot{m}_s} = c_{ps}(T_{sb} - T_{sa}) + X_a c_{pL}(T_v - T_{sa}) + (X_a - X_b)\lambda + X_b c_{pL}(T_{sb} - T_v) + (X_a - X_b) c_{pv}(T_{va} - T_v)$$

T_{sa} = feed temperature
 T_v = vaporization temperature
 T_{sb} = final solids temperature
 T_{va} = final vapor temperature
 λ = heat of vaporization

The heat transferred to the solids, liquid, and vapor:

$$q_T = UA \overline{\Delta T}$$

U = overall coefficient

A = heat-transfer area

$\overline{\Delta T}$ = average temperature difference

The area is not known due to the variable porosity of the solids

$$q_T = UaV \overline{\Delta T}$$

Ua = volumetric heat-transfer coefficient, Btu/ft³-h-°F or W/m³-°C

V = dryer volume, ft³ or m³

a is the heat-transfer area
per unit dryer volume

$$\overline{\Delta T} = \overline{\Delta T}_L = \frac{(T_{hb} - T_{wb}) - (T_{ha} - T_{wa})}{\ln[(T_{hb} - T_{wb})/(T_{ha} - T_{wa})]}$$

For the system water-air $T_{wb} = T_{wa}$

For a continuous adiabatic dryer the heat balance over gas phase

$$q_T = \dot{m}_g(1 + \mathcal{H}_b)c_{sb}(T_{hb} - T_{ha})$$

$$\begin{aligned}\dot{m}_g &= \text{mass rate of dry gas} \\ \mathcal{H}_b &= \text{humidity of gas at inlet} \\ c_{sb} &= \text{humid heat of gas at inlet humidity}\end{aligned}$$

In an adiabatic dryer T_v is the wet-bulb temperature of the gas and T_{hb} and T_{ha} are the inlet and exit gas temperatures

Calculation of heat transference coefficient

For a heat transfer from a gas to a single or isolated spherical particle

$$\frac{h_o D_p}{k_f} = 2.0 + 0.60 \left(\frac{D_p G}{\mu_f} \right)^{0.50} \left(\frac{c_p \mu_f}{k_f} \right)^{1/3}$$

HEAT-TRANSFER UNITS

Some adiabatic dryers, especially rotary dryers, are conveniently rated in terms of the number of heat-transfer units they contain. One heat-transfer unit is the section in which the temperature change in one phase equals the average driving force (temperature difference) in that section..

$$N_t = \ln \frac{T_{hb} - T_{wb}}{T_{ha} - T_{wb}}$$

$$N_t = \frac{T_{hb} - T_{ha}}{\Delta \bar{T}}$$

$$\Delta \bar{T} = \Delta \bar{T}_L = \frac{(T_{hb} - T_{wb}) - (T_{ha} - T_{wa})}{\ln[(T_{hb} - T_{wb})/(T_{ha} - T_{wa})]}$$

For the system water-air $T_{wb} = T_{wa}$  $N_t = \ln \frac{T_{hb} - T_{wb}}{T_{ha} - T_{wb}}$

MASS TRANSFER IN DRYER:

Vapor mass is transferred from the surface of the solid to the gas and sometimes through interior channels of the solid.

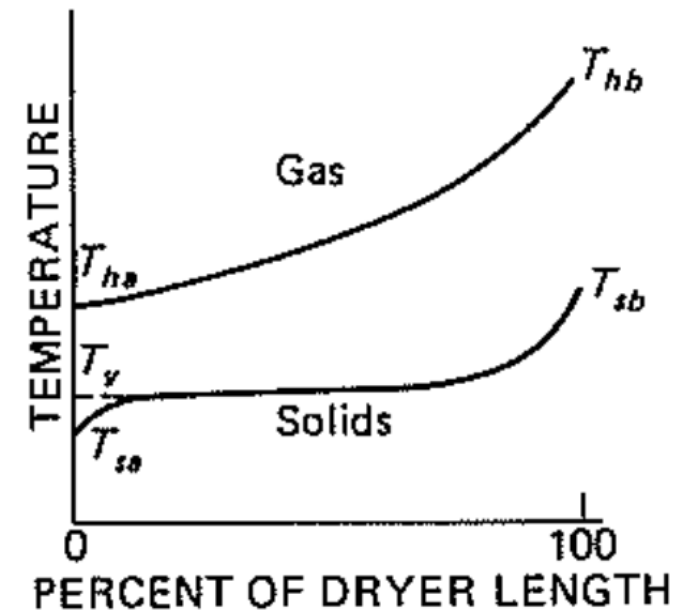
The resistance to mass transfer, not heat transfer, may control the drying rate.

The average rate of mass transfer \dot{m}_v

$$\dot{m}_v = \dot{m}_s(X_a - X_b)$$

If the gas enters at humidity \mathcal{H}_b , the exit humidity \mathcal{H}_a

$$\mathcal{H}_a = \mathcal{H}_b + \frac{\dot{m}_s(X_a - X_b)}{\dot{m}_g} = \mathcal{H}_b + \frac{\dot{m}_v}{\dot{m}_g}$$



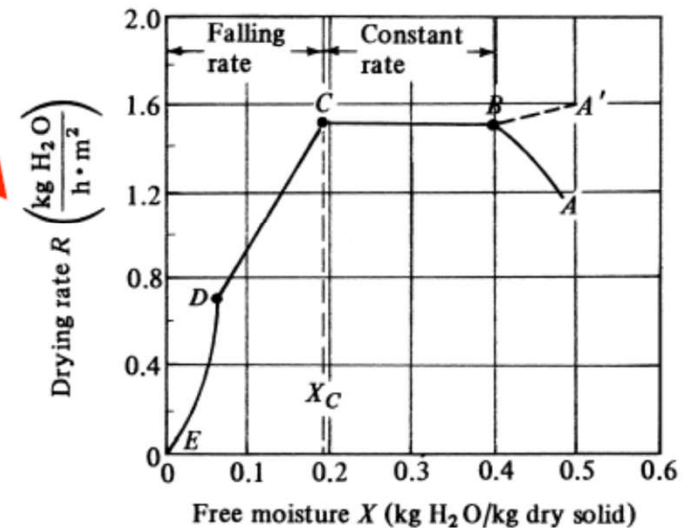
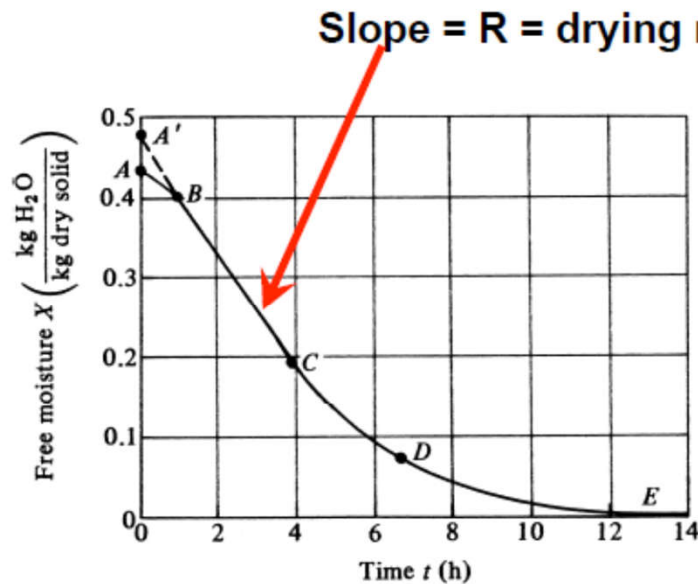
Rate of drying

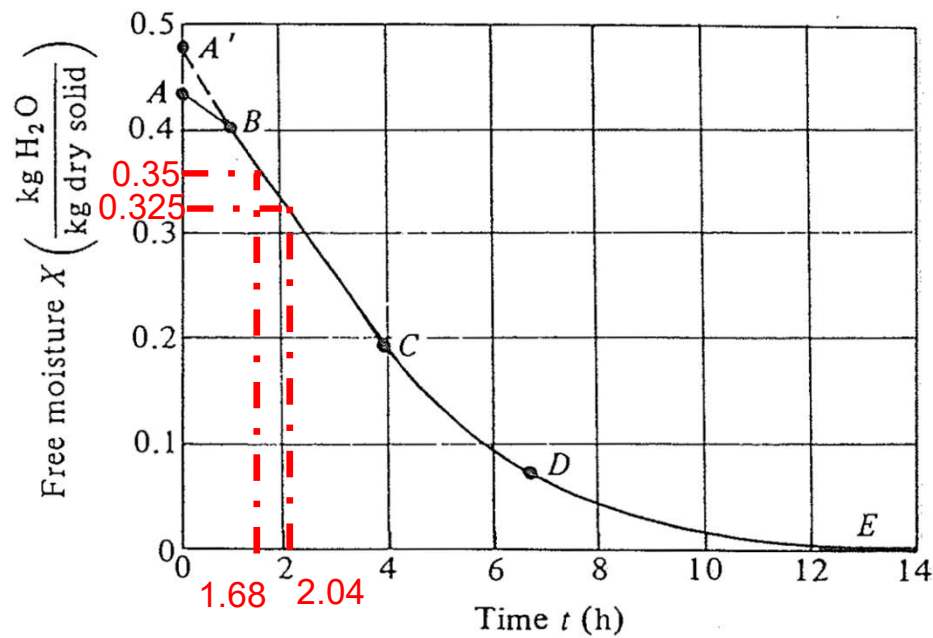
Drying rate: is the amount of solvent evaporated from solid per unit time and area

$$R = -\frac{L_s}{A} \frac{dX}{dt}$$

$\frac{\text{mass of water evaporated}}{(\text{surface area exposed for drying})(\text{time})}$

L_s : Mass of dry solids X : is the free moisture content A : is the surface area exposed for drying



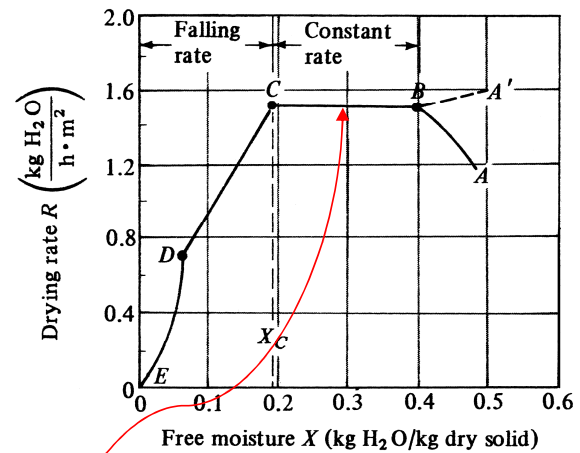


$$\frac{dX}{dt} = \frac{\Delta X}{\Delta t} = \frac{(0.35 - 0.325)}{(1.68 - 2.04)} = -0.07$$

$$L_s/A = 21.5 \text{ kg dry solid/m}^2$$

$$R = -21.5 (-0.07) = 1.493$$

$$R = -\frac{L_s}{A} \frac{dX}{dt}$$

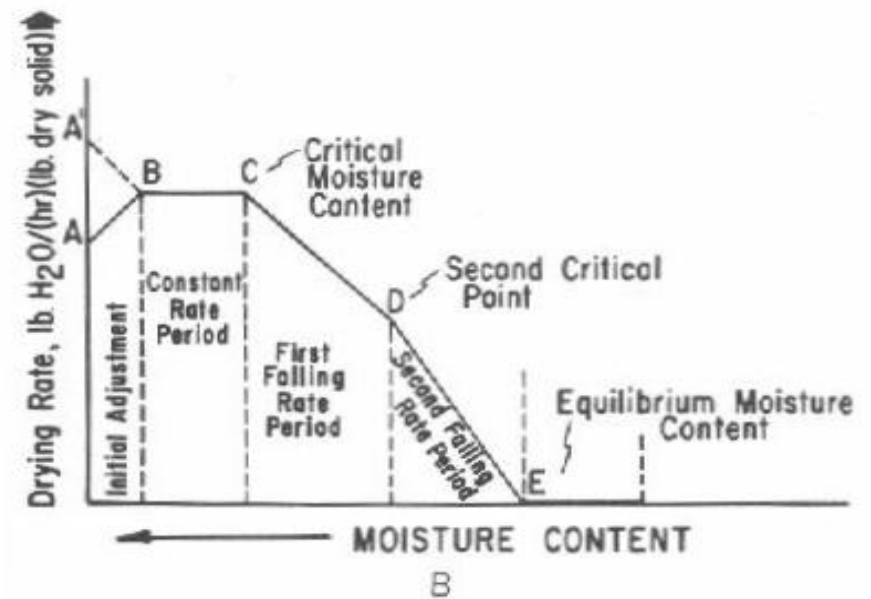
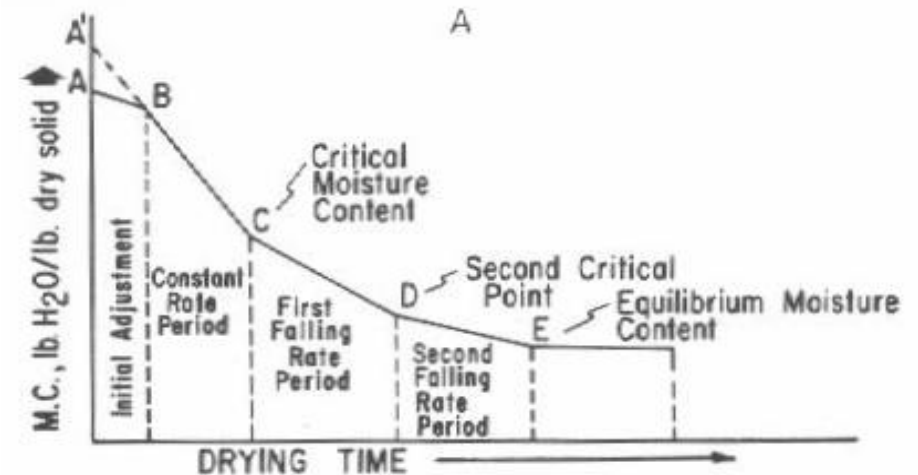


Behavior of solids during drying

1. Stage of increasing drying rate (A or A'→B) :

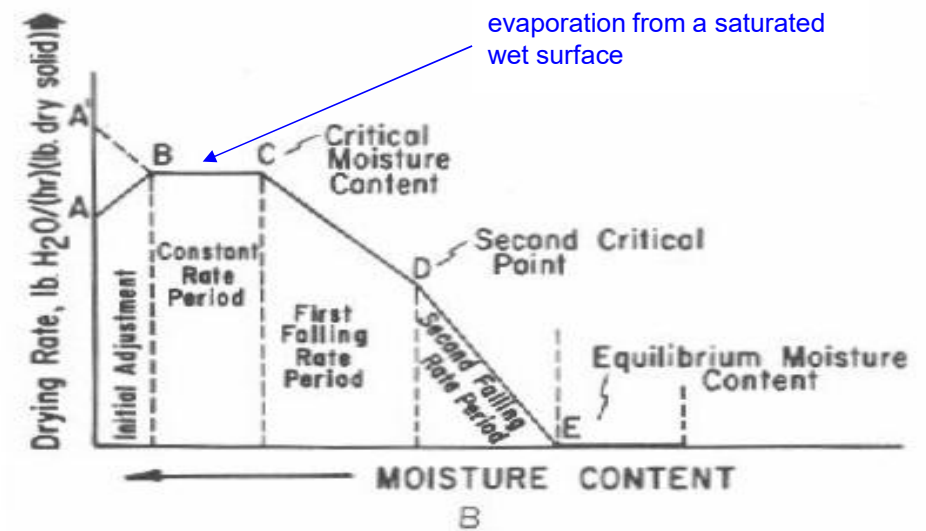
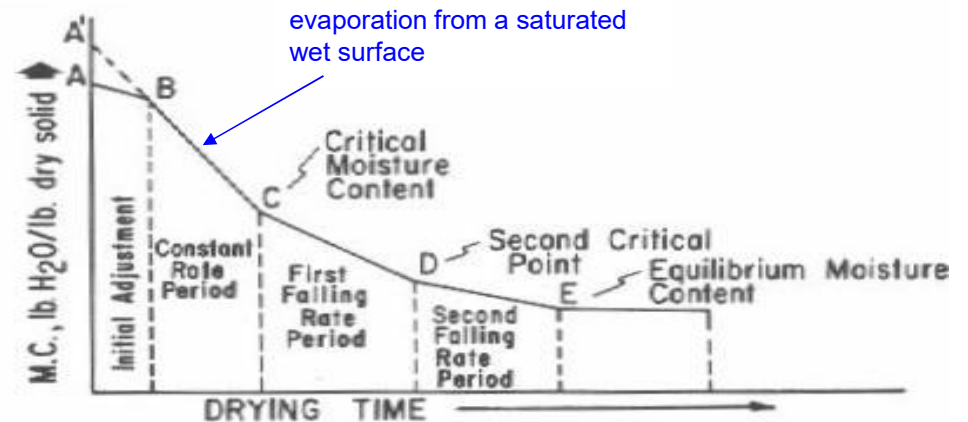
- At zero time, the free moisture content is shown at point A.
- **Point A (cold solid)**, it begins to absorb heat and increase in temperature
- At the same time, the moisture begins evaporating, and cools the drying solid, until the wet solid will reach equilibrium temperature.
- **Point A' (hot solid)**, the drying rate may decrease with evaporation.

2. Point B, After a period of initial adjustment, the rates of heating and cooling became equal and the temp. of drying material stabilizes.



3. Between point B and C: **Constant-rate period**

- The **evaporation rate** resulted from **free liquid surface**, Hence, the **evaporation** takes place from a saturated wet surface of the solid.
- The evaporated moisture from the surface is replaced by water from the interior of the solid **at a rate equal to the rate of evaporation**
- The rate of evaporation depends on the rate of **heat transfer to the drying surface** which remains constant at T_w and the rate of removal of the vapor.

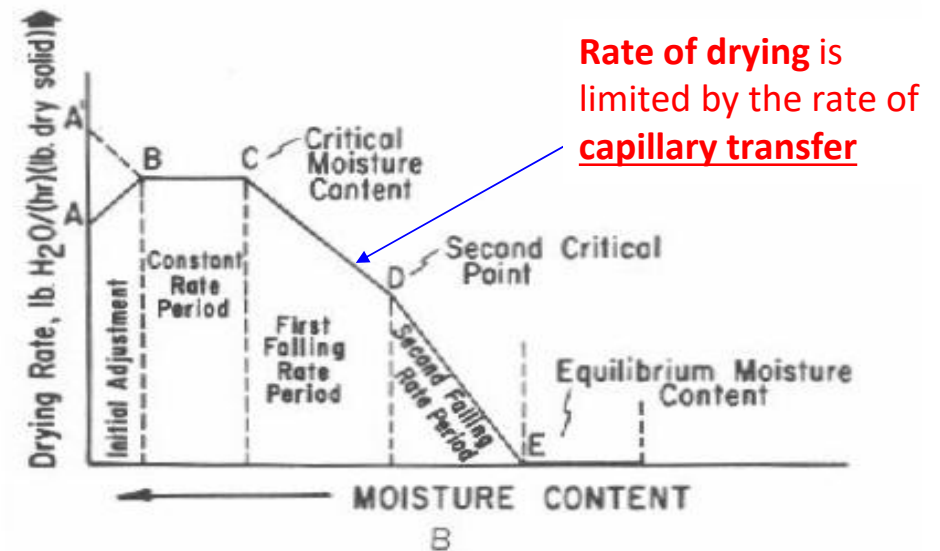
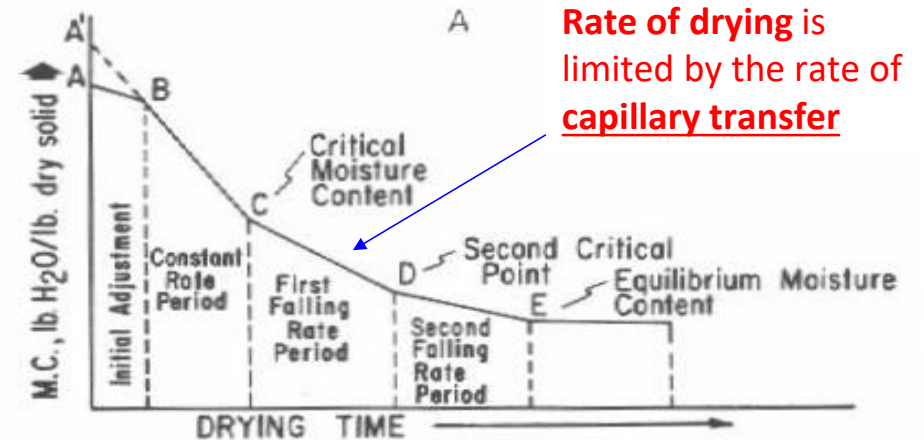


4. At Point C:

- Critical free moisture content, X_c , where the drying rate starts falling and surface temperature rises.
- Insufficient water on surface

5. Between point C and D:

- As moisture is removed from the surface, rate of water to surface is less than rate of evaporation from surface
- Under these conditions, **the rate of drying is limited by the rate of capillary transfer of the liquid to the **surface** of the **wet bed**,**
- Consequently, the **rate** of drying **decreases** continuously (decreases linearly with the free moisture content).
- The **time CD** is known as **first falling-rate period** (or unsaturated surface drying).

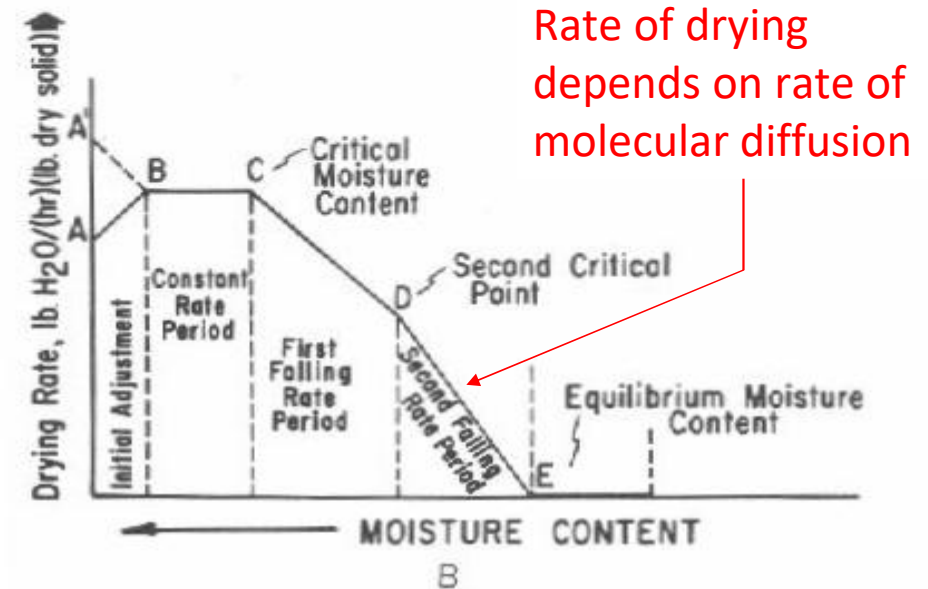
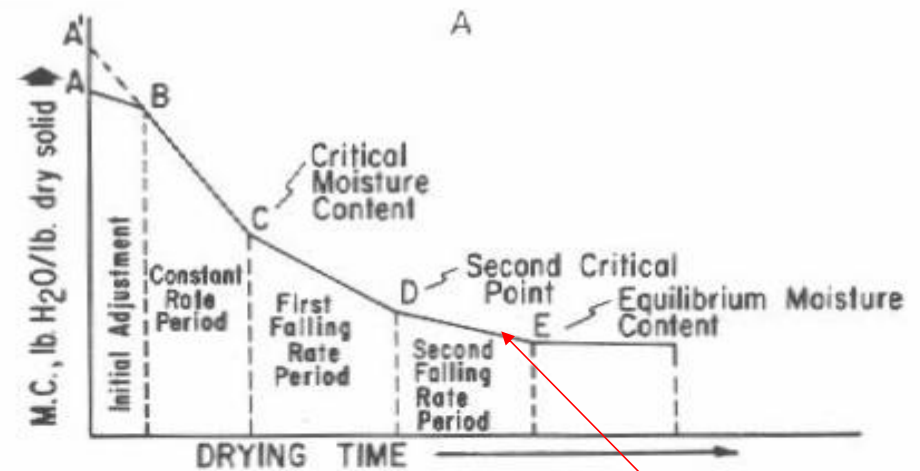


6. At Point D,

- The film of continuous water is completely evaporated,
- Surface completely dry
- The rate of drying depends on rate of diffusion of moisture to the surface of solid. This point is known as **second critical point**.

7. Between point D and E,

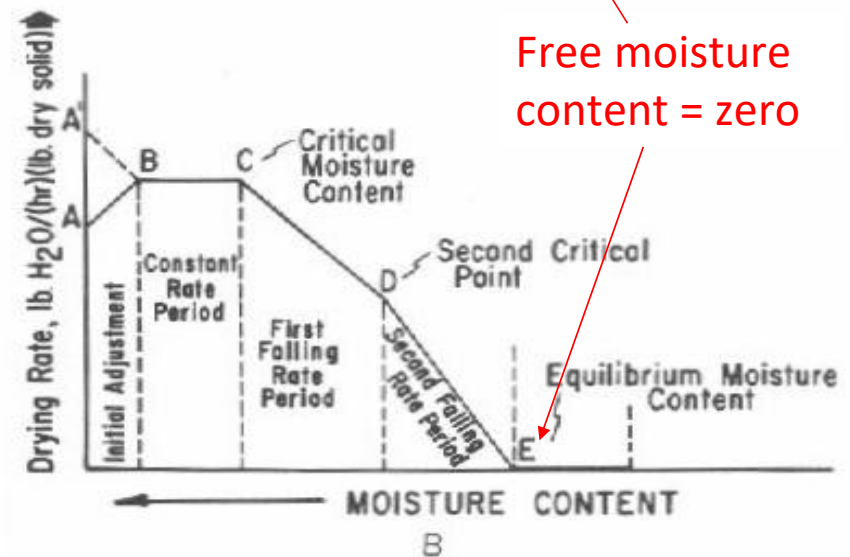
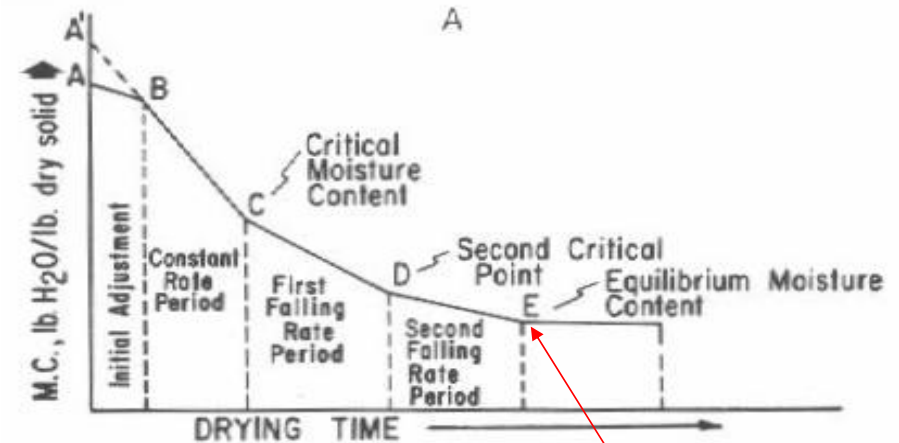
- Any moisture that remains within the drying bed at the end of the first falling-rate period is unable to move,
- Thus, drying cannot take place on the surface. But depends on the movement of the vapor through the pores of the bed to the surface, in general by molecular diffusion



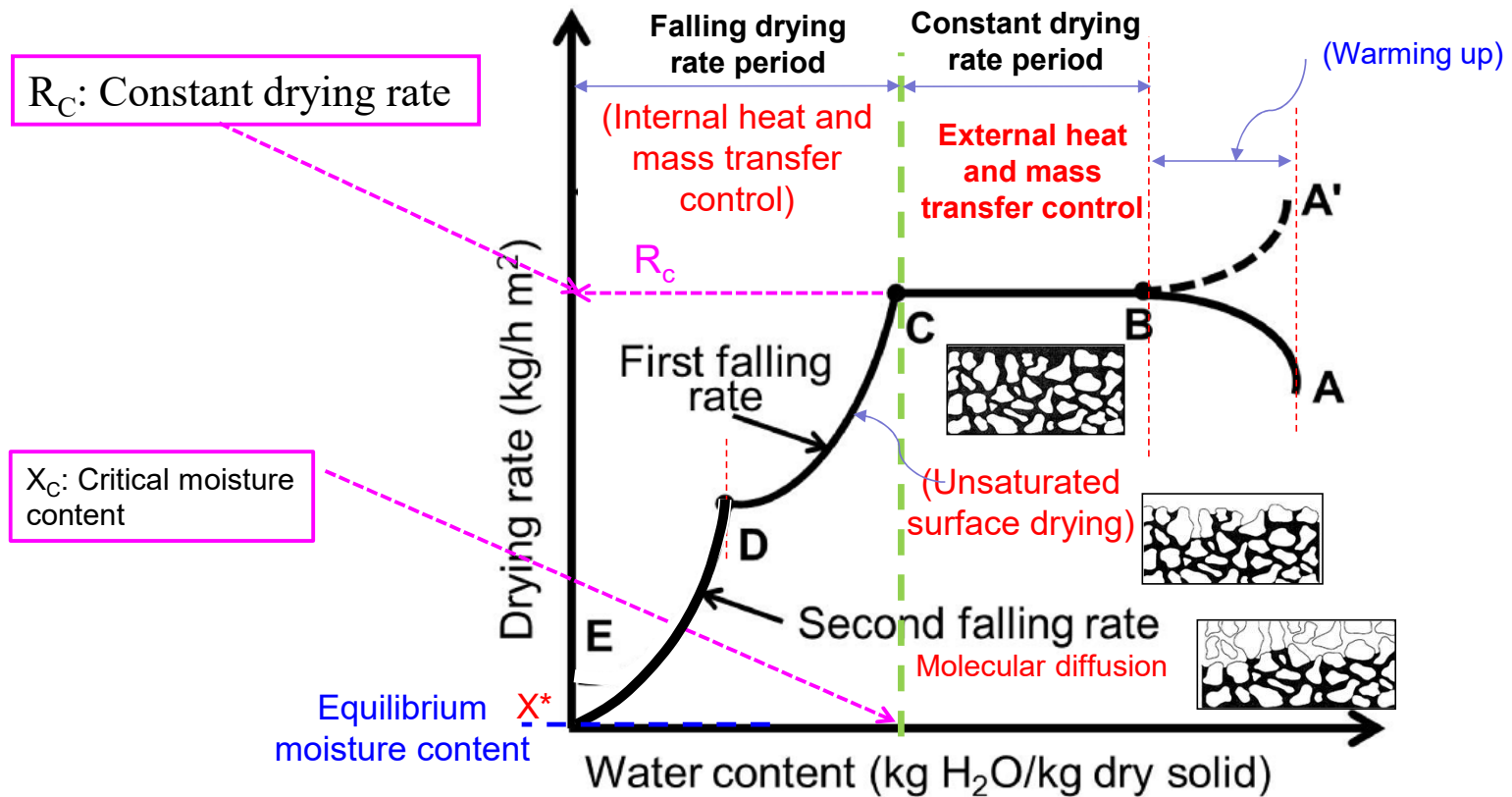
- The time DE is called **second falling rate period**.
- The rate of drying falls rapidly than the first falling rate,
- No linear trend

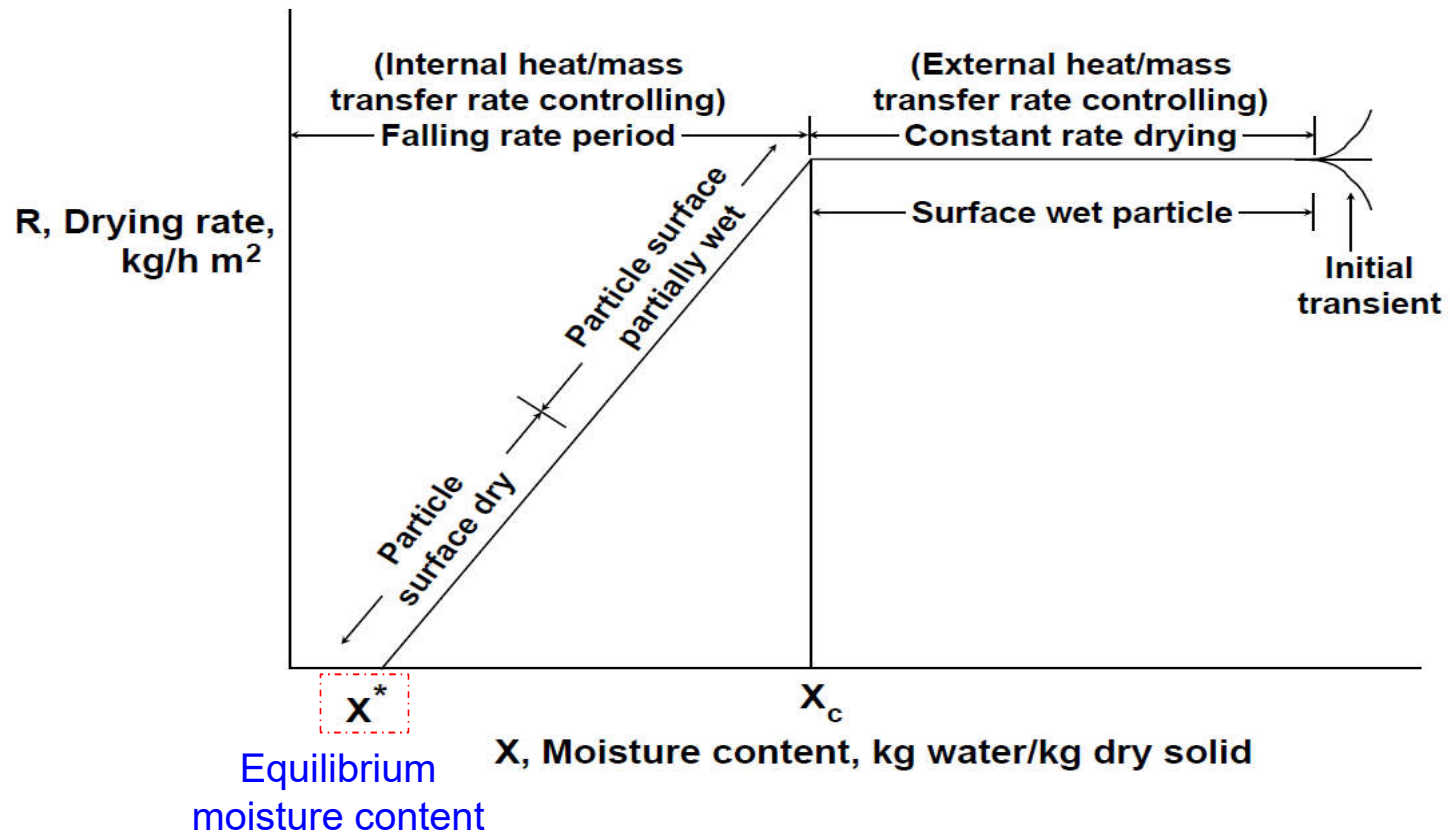
8. At point E,

- Free moisture content becomes zero (the total moisture content reaches the equilibrium value X^*).
- Temperature and moisture content remain constant.
- No further drying occur, i.e. continued drying is waste of time and energy.



Resistances affecting drying





Typical textbook batch drying rate curve under constant drying conditions

Calculation of constant-rate of drying

During the constant-rate period the drying rate per unit area R_c , can be estimated based on mass transfer or on heat transfer coefficients

$$\dot{m}_v = \frac{M_v k_y (y_i - y) A}{(1 - y)_L}$$

$$\dot{m}_v = \frac{h_y (T - T_i) A}{\lambda_i}$$

$$R = -\frac{L_s}{A} \frac{dX}{dt} \quad R_c = \frac{\dot{m}_v}{A}$$

$$t_c = \frac{m_s (X_1 - X_2)}{A R_c}$$

\dot{m}_v = rate of evaporation

A = drying area

h_y = heat-transfer coefficient

k_y = mass-transfer coefficient

M_v = molecular weight of vapor

T = temperature of gas

T_i = temperature at interface

y = mole fraction of vapor in gas

y_i = mole fraction of vapor at interface

λ_i = latent heat at temperature T_i

The heat transfer coefficients can be estimated:

- For air flowing **parallel with the surface** of a solid (based on the properties of air at 95°C; it applies for Reynolds numbers between 2600 and 22,000)

$$h_y = 8.8G^{0.8}/D_e^{0.2}$$

- For a **perpendicular flow** to the surface, at air velocities between 0.9 and 4.5 m/s,

$$h_y = 24.2G^{0.37}$$

h_y = heat-transfer coefficient, W/m²-°C

G = mass velocity, kg/s-m²

D_e = equivalent diameter of the airflow channel, m

The coefficients for the above equation in English units, with h in Btu/ft²-h-F, G in lb/ft²-h, and D_e in ft, the coefficient 8.8 becomes 0.01 and 24.2 becomes 0.37, respectively.

Example 4.2.1 Drying rate during the constant-rate period

A filter cake 24 in. (610 mm) square and 2 in. (51 mm) thick, supported on a screen, is dried from both sides with air at a wet-bulb temperature of 80°F (26.7°C) and a dry-bulb temperature of 120°F (48.9°C). The air flows parallel with the faces of the cake at a velocity of 3.5 ft/s (1.07 m/s). The dry density of the cake is 120 lb/ft³ (1922 kg/m³). The equilibrium-moisture content is negligible. Under the conditions of drying the critical moisture is 9% dry basis. Equivalent diameter D_e is equal to 2 ft. $\lambda_i = 1049 \text{ Btu/lb}$. Air density $\rho = 0.068 \text{ lb/ft}^3 = 1.096 \text{ kg/m}^3$ at 120 F.

- (a) What is the drying rate during the constant-rate period?
- (b) How long would it take to dry this material from an initial moisture content of 20% (dry basis) to a final moisture content of 10%?

Solution

The interface temperature T_i is the wet-bulb temperature of the air, 80°F.

The mass velocity of the air is $G = \rho v$

$$= 0.068 \times 3.6 \times 3600 \frac{s}{h} = 863 \frac{lb}{ft^2 \cdot h}$$

$$h_y = 0.01 G^{0.8} / De^{0.2} \quad \Rightarrow \quad h = 0.01 \times 863^{0.8} / 2^{0.2} = 1.94 \text{ Btu/ft}^2\text{-h-}^\circ\text{F}$$

(a) The drying rate during the constant-rate period, R_c

$$R_c = \frac{\dot{m}_v}{A} \quad \dot{m}_v = \frac{h_y(T - T_i)A}{\lambda_i} \quad \Rightarrow \quad R_c = \frac{1.94(120 - 80)}{1049} = 0.074 \text{ lb/ft}^2\text{-h}$$

(b) Since drying is from both faces, The dried area A is $2 \times \left(\frac{24}{12}\right)^2 = 8 \text{ ft}^2$.

The rate of drying \dot{m}_v

$$\dot{m}_v = 0.074 \times 8 = 0.59 \text{ lb/h}$$

The volume of the cake is $(24/12)^2 \times (2/12) = 0.667 \text{ ft}^3$, and the mass of bone-dry solids is $120 \times 0.667 = 80 \text{ lb}$. The quantity of moisture to be vaporized is $80(0.20 - 0.10) = 8 \text{ lb}$. Drying time t_T is therefore $8/0.59 = 13.5 \text{ h}$.

Example 4.2.2 Constant drying rate using heat transfer equations

An insoluble wet granular material is dried in a pan of 0.457 by 0.457 m and 25.4 mm deep. The material is 25.4 mm deep in the pan, and the sides and bottom can be considered to be insulated. Heat transfer is by convection from an air stream flowing parallel to the surface at a velocity of 6.1 m/s. The air is at 65.6 °C and has a humidity of 0.010 kg water/kg dry air. Estimate the rate of drying for constant-rate period.

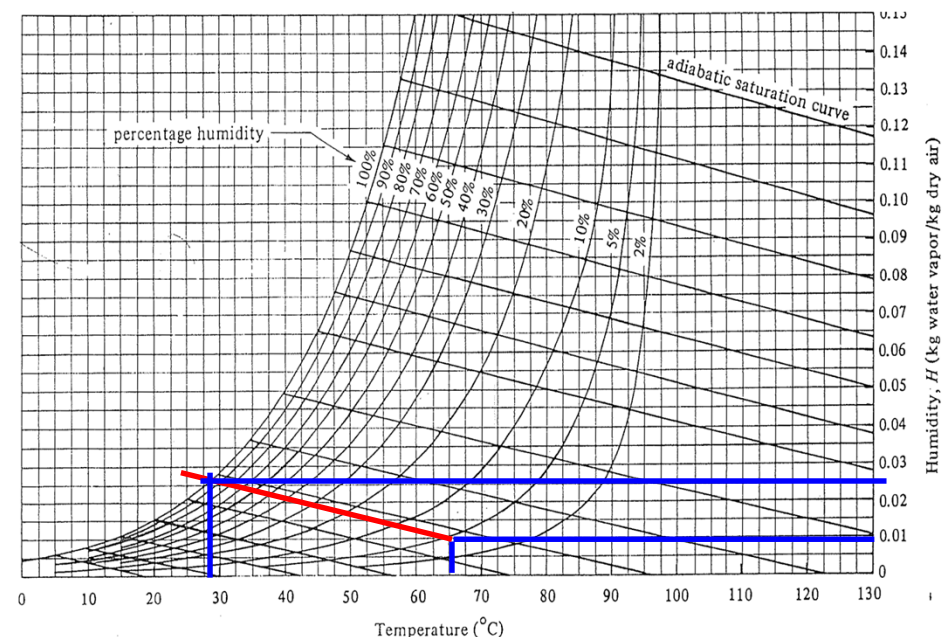
Solution

Entering air at $T = 65.6\text{ }^{\circ}\text{C}$. $H = 0.010\text{ kg water/kg dry air}$.

From psychrometric chart:

- The wet bulb temperature is: $T_w = 28.9\text{ }^{\circ}\text{C}$
- The saturated humidity **at** T_w is: $H_w = 0.026\text{ kg water/kg dry air}$.
- The humid volume of the gas mixture is

$$\begin{aligned} v_H &= (2.83 \times 10^{-3} + 4.56 \times 10^{-3} H) T \\ &= (2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.01)(65.6 + 273.15) = 0.974\text{ m}^3/\text{kg dry air} \end{aligned}$$



Take basis 1 kg dry gas

Entering air at $T = 65.6\text{ }^{\circ}\text{C}$. $H = 0.010\text{ kg}$

$\rightarrow 0.01\text{ kg water} \rightarrow 1.01\text{ kg mixture.}$

The humid volume of the gas mixture per kg dry gas

$$v_H = (2.83 \times 10^{-3} + 4.56 \times 10^{-3} H) T = 0.974 \text{ m}^3 / \text{kg dry air}$$

$$v = 0.974 \frac{\text{m}^3 \text{ mixture}}{\text{kg dry air}} \frac{1 \text{ kg dry air}}{1.01 \text{ kg mixture}} = 0.964 \frac{\text{m}^3 \text{ mixture}}{\text{kg mixture}}$$

$$\rho = 1/v = 1.037 \frac{\text{kg}}{\text{m}^3}$$

$$G = \rho u = (1.037)(6.1) = 6.3257 \text{ kg}/(\text{m}^2 \cdot \text{s}) = 22772 \text{ kg}/(\text{m}^2 \cdot \text{h})$$

$$h_c = 0.0204 G^{0.8} = 0.0204 (22772)^{0.8} = 62.45 \text{ W}/\text{m}^2 \cdot ^{\circ}\text{C}$$

The latent heat of vaporization, λ_w , at $T_w = 28.9\text{ }^{\circ}\text{C}$ can be taken from steam tables or by interpolation using $\lambda_w = 2501\text{ kJ/kg}$ at $0\text{ }^{\circ}\text{C}$ and $\lambda_w = 2260\text{ kJ/kg}$ at $100\text{ }^{\circ}\text{C}$.

By interpolation $\lambda_w = \mathbf{2431\text{ kJ/kg}}$.

$$R_c = \frac{\dot{m}_v}{A} \qquad \dot{m}_v = \frac{h_y(T - T_i)A}{\lambda_i}$$

$$R_c = h_c(T - T_w)/\lambda_w = (62.45)(65.6 - 28.9)/(2431 \times 10^3) \\ = 9.428 \times 10^{-4} \text{ kg water vapor/(s.m}^2\text{)} = 3.39 \text{ kg water vapor/(h.m}^2\text{)}$$

The total drying rate $= R_c A = (3.39)(0.457^2) = 0.708 \text{ kg water vapor/h}$

Rate of drying for nonporous solids based on diffusion

Assuming that the moisture is flowing by diffusion through the nonporous solid. The moisture distribution in a solid giving a falling-rate curve

$$\frac{X_T - X^*}{X_{T1} - X^*} = \frac{X}{X_1} = \frac{8}{\pi^2} \left(e^{-a_1 \beta} + \frac{1}{9} e^{-9a_1 \beta} + \frac{1}{25} e^{-25a_1 \beta} + \dots \right)$$

where $\beta = D'_v t_T / s^2$

$$a_1 = (\pi/2)^2$$

X_T = average total moisture content at time t_T h

X = average free-moisture content at time t_T h

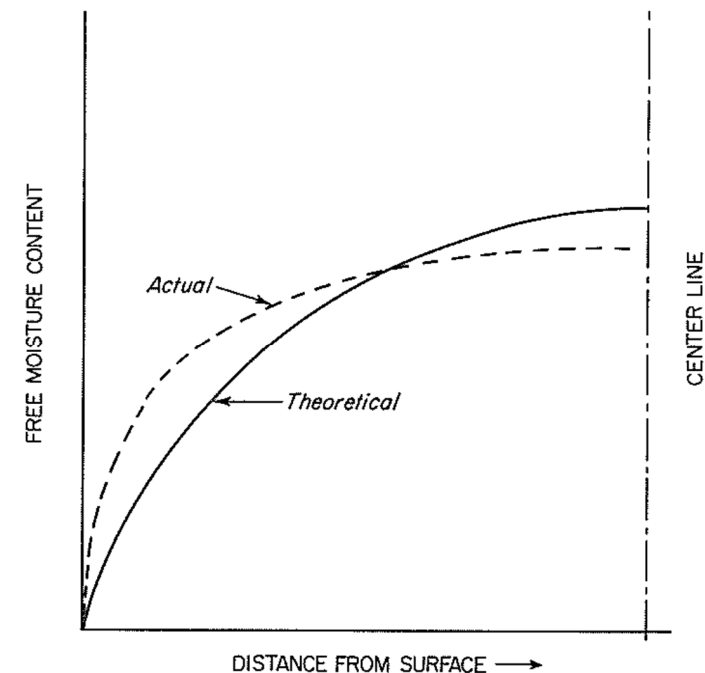
X^* = equilibrium-moisture content

X_{T1} = initial moisture content at start of drying when $t = 0$

X_1 = initial free-moisture content

D'_v = diffusivity of moisture through solid

s = one-half slab thickness



$$\frac{X_T - X^*}{X_{T1} - X^*} = \frac{X}{X_1} = \frac{8}{\pi^2} \left(e^{-a_1 \beta} + \frac{1}{9} e^{-9a_1 \beta} + \frac{1}{25} e^{-25a_1 \beta} + \dots \right)$$

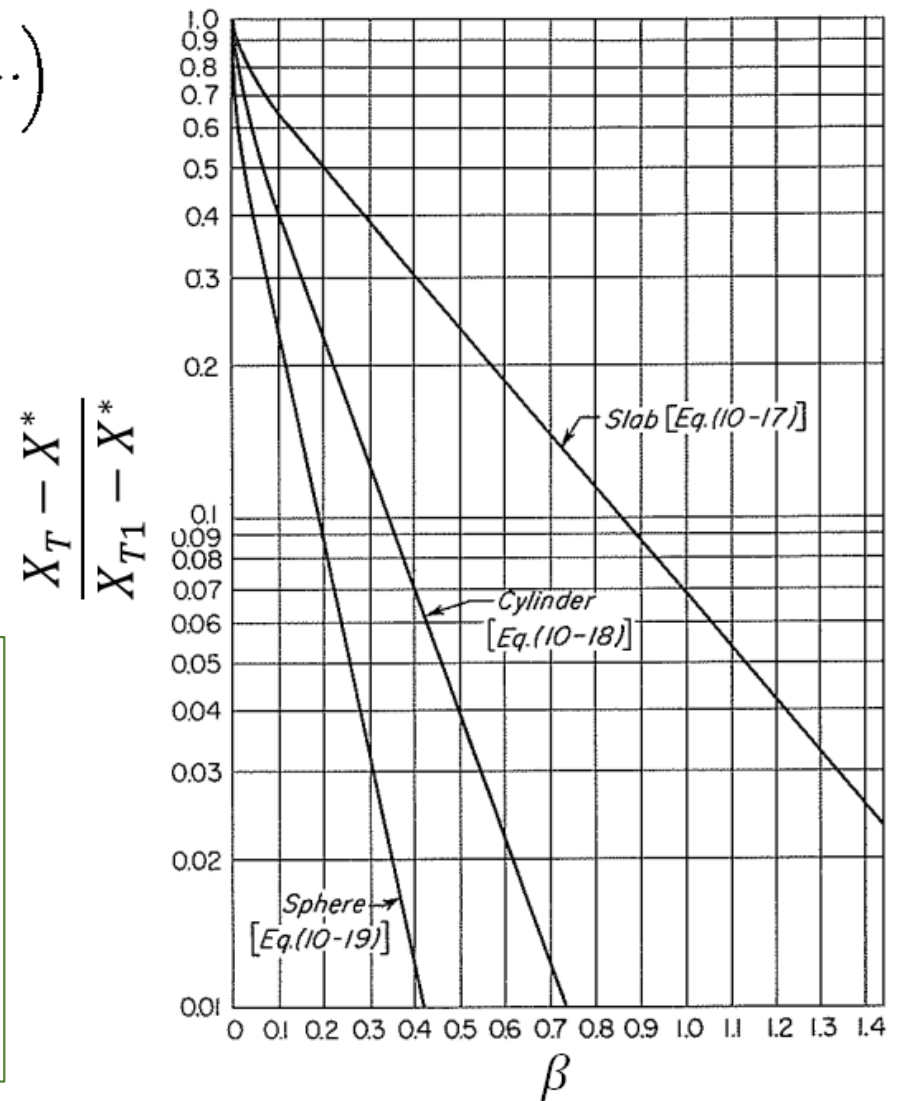
When β is greater than about 0.1

➡

$$t_T = \frac{4s^2}{\pi^2 D'_v} \ln \frac{8X_1}{\pi^2 X}$$

$$-\frac{dX}{dt} = \left(\frac{\pi}{2} \right)^2 \frac{D'_v}{s^2} X$$

$\beta = D'_v t_T / s^2$
 $a_1 = (\pi/2)^2$
 X_T = average total moisture content at time t_T h
 X = average free-moisture content at time t_T h
 X^* = equilibrium-moisture content
 X_{T1} = initial moisture content at start of drying when $t = 0$
 X_1 = initial free-moisture content
 D'_v = diffusivity of moisture through solid
 s = one-half slab thickness



Example 4.2.3 Drying rate for falling rate period (nonporous solid)

A piece of wood 25.4 mm thick is dried from an initial moisture content of 25% to a final moisture of 5% using air of negligible humidity.

If the diffusion coefficient of water within the wood, D'_v is $8.3 \times 10^{-6} \text{ cm}^2/\text{s}$, how long should it take to dry the wood?

Solution

The one-half thickness of the piece is

$$s = \frac{25.4/10}{2} = 1.27 \text{ cm}$$

$$t_T = \frac{4s^2}{\pi^2 D'_v} \ln \frac{8X_1}{\pi^2 X} \quad \Rightarrow \quad t_T = \frac{4(12.7/10)^2 \ln [(8 \times 0.25)/(\pi^2 \times 0.05)]}{\pi^2 (8.3 \times 10^{-6}) \times 3600} = 30.6 \text{ h}$$

Example 4.2.4. Drying rate for falling rate period (nonporous solid)

A piece of wood 25.4 mm thick is dried from an initial moisture content of 25% to a final moisture of 5% using air of negligible humidity.

If the diffusion coefficient of water within the wood, D'_v is $8.3 \times 10^{-6} \text{ cm}^2/\text{s}$, how long should it take to dry the wood?

Solution

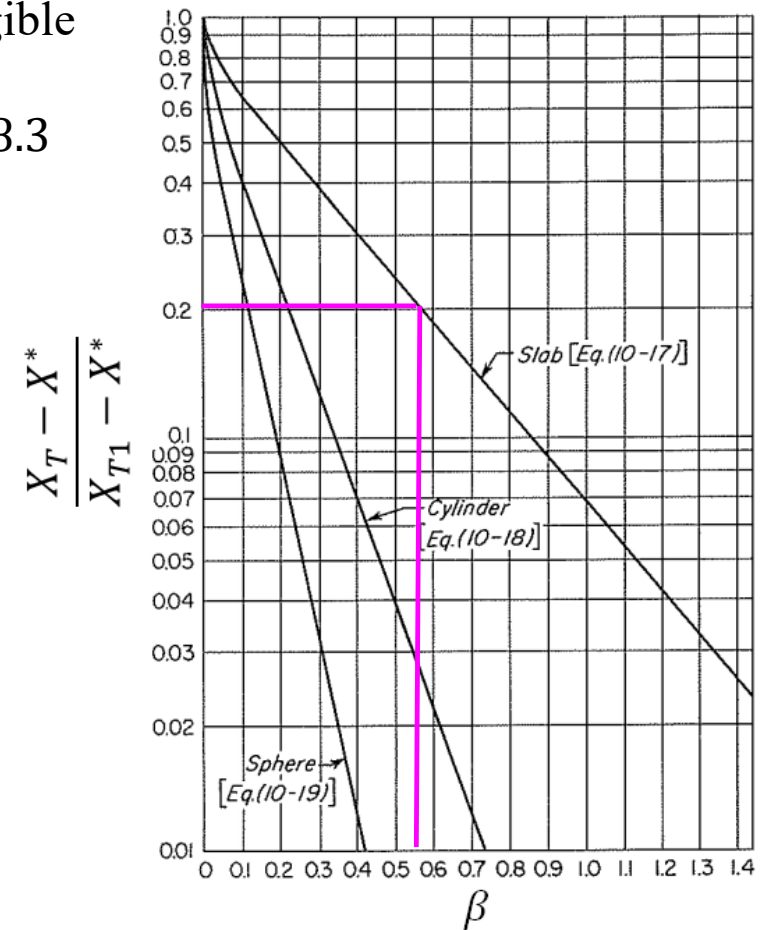
From graph at $\frac{X_T}{X_{T1}} = \frac{5}{25} = 0.2$, the value of β is 0.57

The one-half thickness of the piece is

$$s = \frac{25.5/10}{2} = 1.27 \text{ cm}$$

$$t_T = \frac{4s^2}{\pi^2 D'_v} \ln \frac{8X_1}{\pi^2 X}$$

$$t_T = \frac{4(12.7/10)^2 \ln [(8 \times 0.25)/(\pi^2 \times 0.05)]}{\pi^2(8.3 \times 10^{-6}) \times 3600} = 30.6 \text{ h}$$



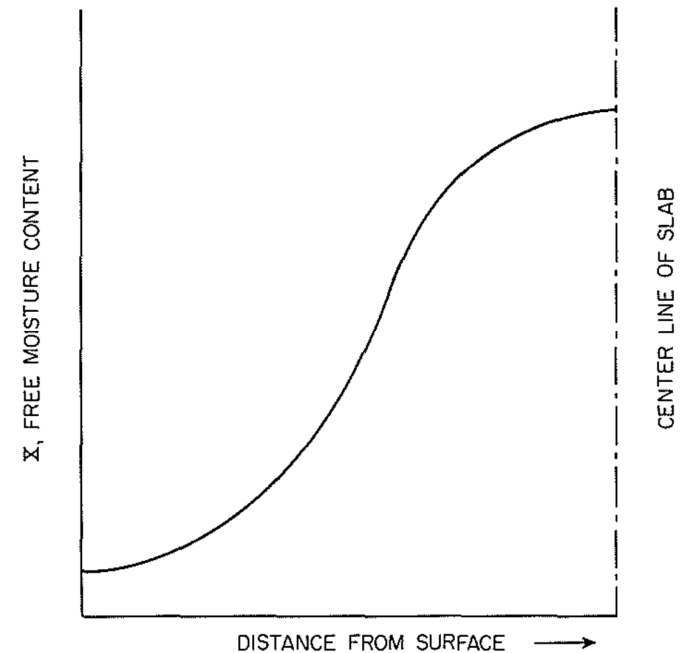
Rate of drying for porous solids based on diffusion

Moisture flows through porous solids by capillarity and to some extent by surface diffusion. The distribution of free moisture content is concave up followed by inflection point then down.

A porous material contains a complicated network of interconnecting pores and channels, the cross sections of which vary greatly.

$$R = -\frac{dm_v}{A dt} = -\frac{m_s dX}{A dt}$$

$$t_T = \frac{m_s}{A} \int_{x_2}^{x_1} \frac{dX}{R}$$

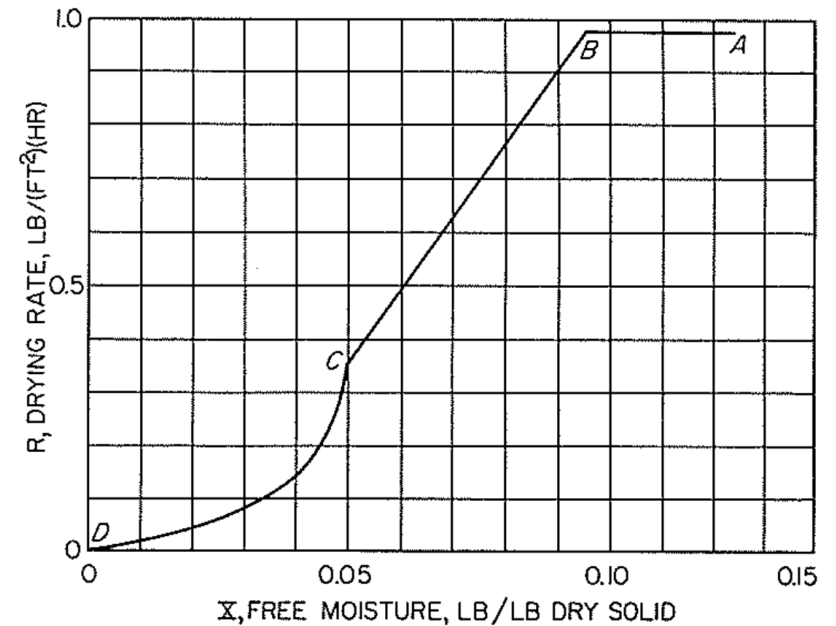


The falling rate period (**line BC**), the vaporization zone is at or near the surface, the rate-of-drying curve is usually linear.

Curve CD

The remaining water is located in small isolated areas in the corners and some of the pores. The rate of drying again suddenly decreases

Point C is called the second critical point



In the falling-rate period, when diffusion controls, R is linear in X ,
as with many porous solids, during the falling-rate period

$$R = aX + b$$

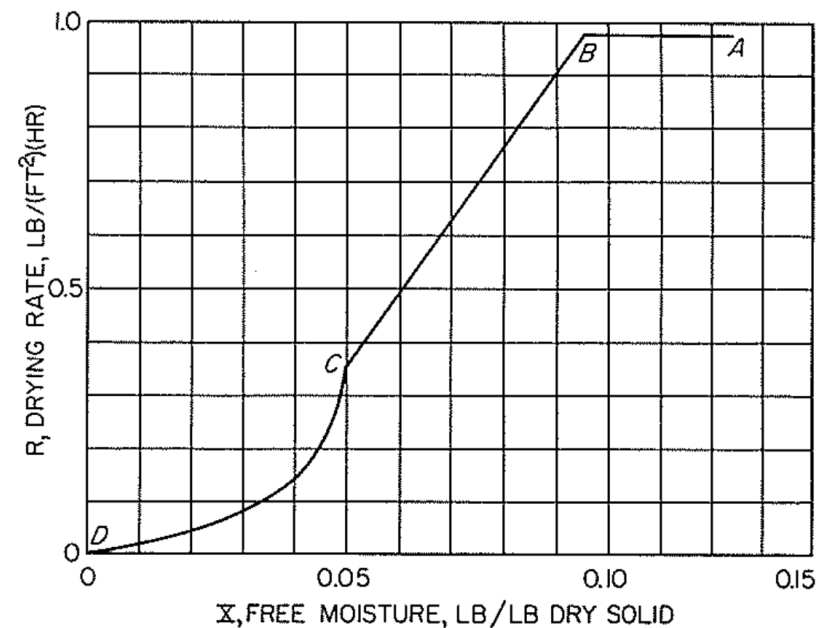
where a and b are constants

$$R = -\frac{L_s}{A} \frac{dX}{dt}$$

$$dR = a dX$$

the time required in the falling-rate period

$$t_f = \frac{m_s}{aA} \int_{R_2}^{R_1} \frac{dR}{R} = \frac{m_s}{aA} \ln \frac{R_1}{R_2}$$



$$t_f = \frac{m_s}{aA} \int_{R_2}^{R_1} \frac{dR}{R} = \frac{m_s}{aA} \ln \frac{R_1}{R_2}$$

where R_1 and R_2 are the ordinates for the initial and final moisture contents, respectively. The constant a is the slope of the rate-of-drying curve and may be written as

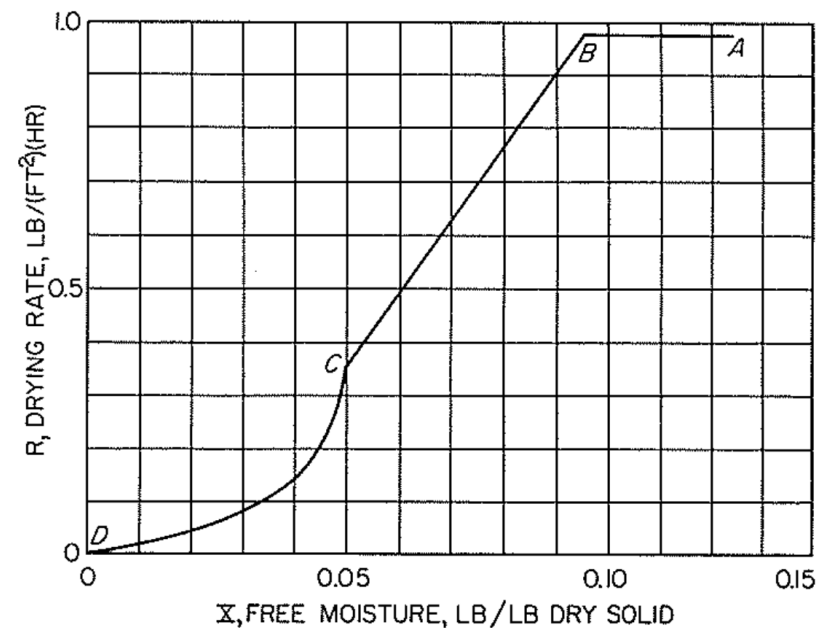
$$a = \frac{R_c - R'}{X_c - X'}$$

R_c = rate at first critical point

R' = rate at second critical point

X_c = free-moisture content at first critical point

X' = free-moisture content at second critical point



$$t_f = \frac{m_s}{aA} \int_{R_2}^{R_1} \frac{dR}{R} = \frac{m_s}{aA} \ln \frac{R_1}{R_2}$$

$$a = \frac{R_c - R'}{X_c - X'}$$

$$t_f = \frac{m_s(X_c - X')}{A(R_c - R')} \ln \frac{R_1}{R_2}$$

Also the drying time can be presented in term of moisture contents. The time required to reduce moisture content from X_c to some moisture content X is :

$$t = -\frac{L_s}{A} \int_{X_c}^X \frac{dX}{R} \xrightarrow[\text{Integrate}]{R = R_c X / X_c} t = -\frac{L_s}{A} \frac{X_c}{R_c} \ln \left(\frac{X}{X_c} \right)$$

Example 4.2.5. Estimation of drying rate

Assume that the drying rate varies linearly through the whole falling-rate period

$$X_1 = 0.38 \text{ kg H}_2\text{O /kg dry solid}$$

$$X_2 = 0.04 \text{ kg H}_2\text{O /kg dry solid}$$

$$X_c = 0.195 \text{ kg H}_2\text{O /kg dry solid}$$

$$R_c = 1.51 \text{ kg H}_2\text{O / (m}^2 \cdot \text{h)}$$

$$L_s / A = 21.5 \text{ kg dry solid /m}^2$$

Calculate the time required for this drying.

Solution

$$t \cong -\frac{L_s}{A} \left[\frac{X_c - X_1}{R_c} + \int_{X_c}^{X_2} \frac{dX}{R} \right] = -\frac{L_s}{A} \left[\frac{X_c - X_1}{R_c} + \frac{X_c}{R_c} \ln \left(\frac{X_2}{X_c} \right) \right] = 7.03 \text{ h}$$

Numerical calculation Methods for Falling-Rate Drying Period

Numerical integration

Trapezoidal rule (2-point):

$$\int_0^{X_1} f(x)dx = \frac{h}{2} [f(X_0) + f(X_1)]$$

$$h = X_1 - X_0$$

Simpson's one-third rule (3-point):

$$\int_0^{X_2} f(x)dx = \frac{h}{3} [f(X_0) + 4f(X_1) + f(X_2)]$$

$$h = \frac{X_2 - X_0}{2} \quad X_1 = X_0 + h$$

Simpson's three-eighths rule (4-point):

$$X_1 = X_0 + h \quad X_2 = X_0 + 2h$$

$$\int_0^{X_3} f(x)dx = \frac{3}{8} h [f(X_0) + 3f(X_1) + 3f(X_2) + f(X_3)]$$

$$h = \frac{X_3 - X_0}{3}$$

Simpson's five-point quadrature :

$$\int_0^{X_4} f(x)dx = \frac{h}{3} [f(X_0) + 4f(X_1) + 2f(X_2) + 4f(X_3) + f(X_4)]$$

$$h = \frac{X_4 - X_0}{4}$$

Example 4.2.6. Numerical estimation of drying rate

A batch of wet solid whose drying rate curve is represented aside, is to be dried from a free moisture content of 0.38 to 0.04 kg H₂O/kg dry solid. The weight of dry solid is 399 kg and the top drying surface area is 18.58 m².

Calculate the time required for this drying.

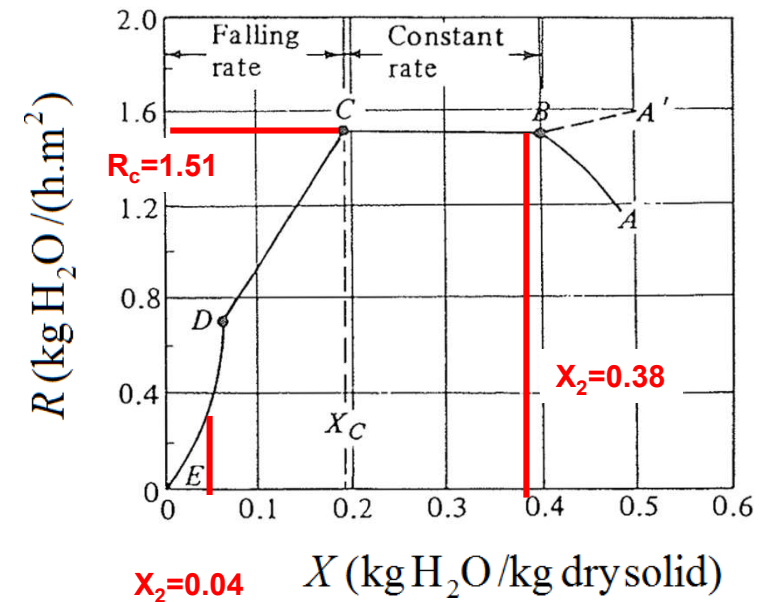
Solution

$$X_1 = 0.38 \text{ kg H}_2\text{O / kg dry solid}$$

$$X_2 = 0.04 \text{ kg H}_2\text{O / kg dry solid}$$

$$L_s / A = 399 / 18.58$$

$$= 21.5 \text{ kg dry solid / m}^2$$



$$t = -\frac{L_s}{A} \int_{X_1}^{X_2} \frac{dX}{R} = -\frac{L_s}{A} \left[\int_{X_1}^{X_c} \frac{dX}{R_c} + \int_{X_c}^{X_2} \frac{dX}{R} \right] = -\frac{L_s}{A} \left[\frac{X_c - X_1}{R_c} + \int_{X_c}^{X_2} \frac{dX}{R} \right]$$

▪ From draying rate curve:

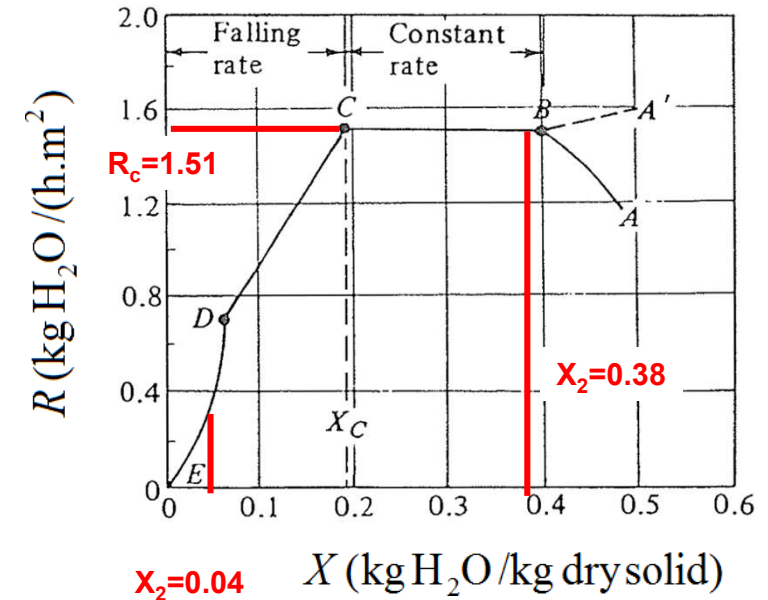
$$X_c = 0.195 \text{ kg H}_2\text{O / kg dry solid}$$

$$R_c = 1.51 \text{ kg H}_2\text{O / (m}^2 \cdot \text{h)}$$

- $\int_{X_C=0.195}^{X_2=0.04} \frac{dX}{R}$ can be integrated numerically or graphically. To perform this, read values of R and calculate $1/R$:

X	R	$1/R$	X	R	$1/R$
0.195	1.51	0.663	0.065	0.71	1.41
0.150	1.21	0.826	0.050	0.37	2.70
0.100	0.90	1.11	0.040	0.27	3.70

- Using Trapezoidal rule: $\int_{X_C=0.195}^{X_2=0.04} \frac{dX}{R} \cong -0.189$



- Finally the drying time is:

$$t = -21.5 \left[\frac{0.195 - 0.38}{1.51} - 0.189 \right] = 6.7 \text{ h}$$

Thermodynamic and transport properties of the air-water system

Property	Expression		
P_v	$P_v = 100 \exp[27.0214 - (6887 / T_{abs}) - 5.32 \ln(T_{abs} / 273.16)]$		
Y	$Y = 0.622 RH P_v / (P - RH P_v)$		
c_{pg}	$c_{pg} = 1.00926 \times 10^3 - 4.0403 \times 10^{-2} T + 6.1759 \times 10^{-4} T^2 - 4.097 \times 10^{-7} T^3$		
k_g	$k_g = 2.425 \times 10^{-2} - 7.889 \times 10^{-5} T - 1.790 \times 10^{-8} T^2 - 8.570 \times 10^{-12} T^3$		
ρ_g	$\rho_g = PM_g / (RT_{abs})$		
μ_g	$\mu_g = 1.691 \times 10^{-5} + 4.984 \times 10^{-8} T - 3.187 \times 10^{-11} T^2 + 1.319 \times 10^{-14} T^3$		
c_{pv}	$c_{pv} = 1.883 - 1.6737 \times 10^{-4} T + 8.4386 \times 10^{-7} T^2 - 2.6966 \times 10^{-10} T^3$		
c_{pw}	$c_{pw} = 2.8223 + 1.1828 \times 10^{-2} T - 3.5043 \times 10^{-5} T^2 + 3.601 \times 10^{-8} T^3$		
P_v	vapor pressure of pure water, Pa	T_{abs}	absolute temperature, K
Y	absolute air humidity, kg water vapor/kg dry air	g	gas
c_p	specific heat, J kg ⁻¹ K ⁻¹	v	vapor
k_g	thermal conductivity, W m ⁻¹ K ⁻¹	w	water
		ρ_g	density, kg m ⁻³
		μ_g	dynamic viscosity, kg m ⁻¹ s ⁻¹