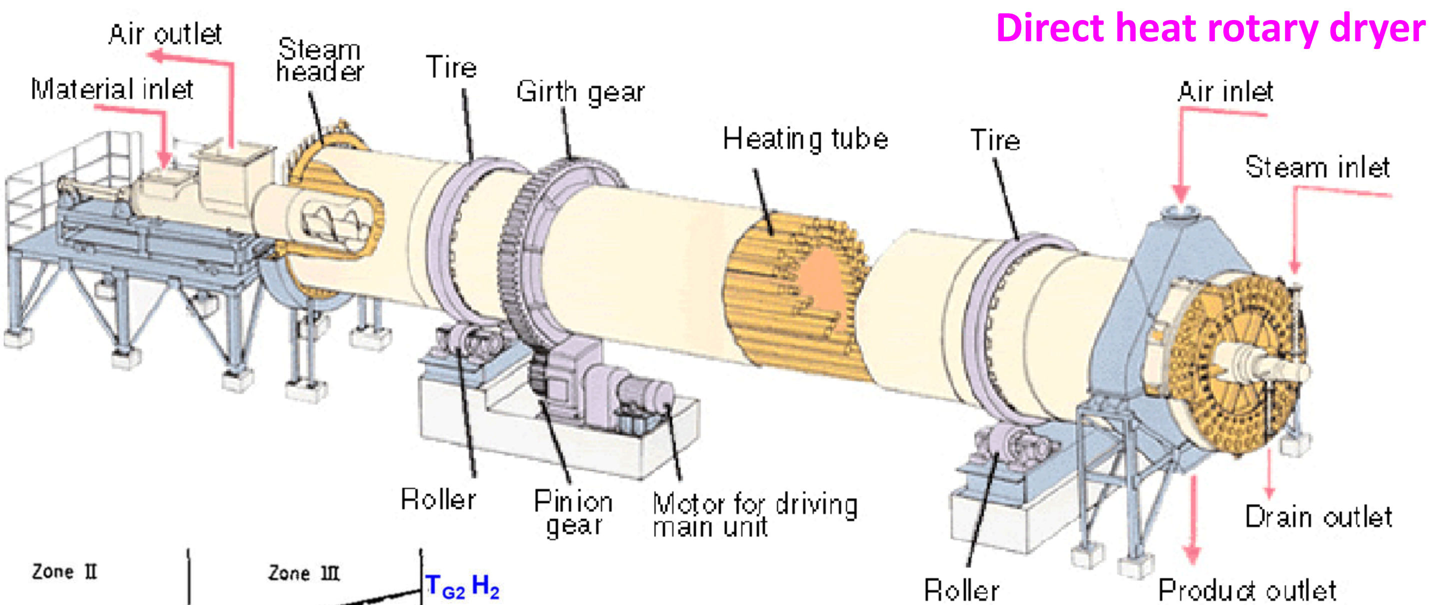
Topic 4.3. Continuous Drying

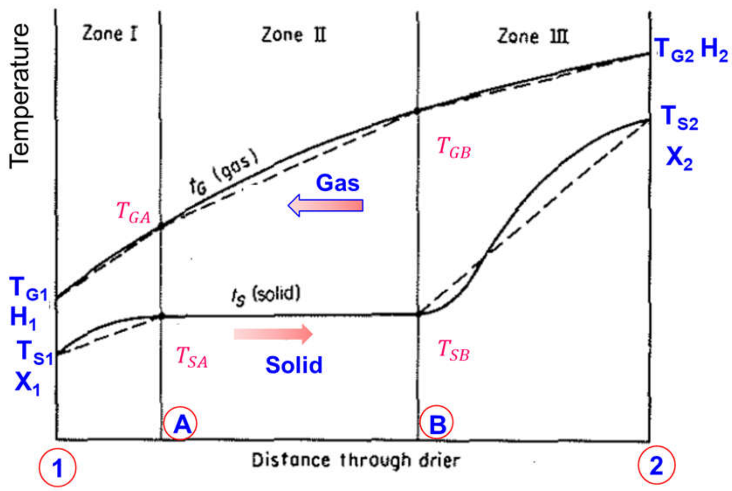
This lecture

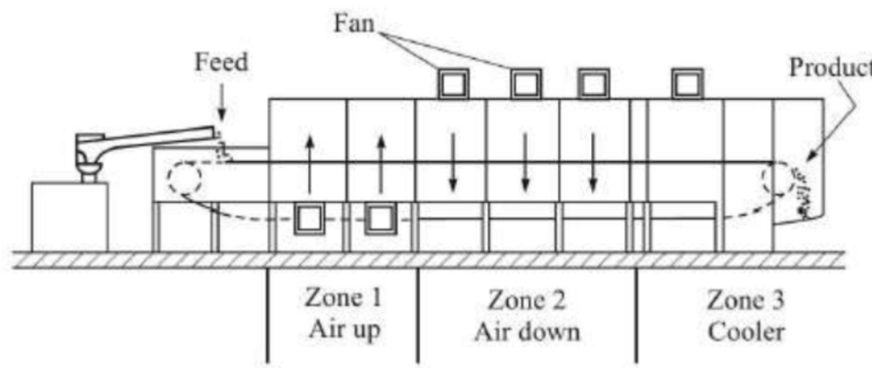
- ✓ Overview and definitions
- ✓ Continuous Drying

This topic was obtained from the notes of professor Zayed Hamamreh, ChE-University of Jordan

TEMPERATURE PATTERNS IN DRYERS

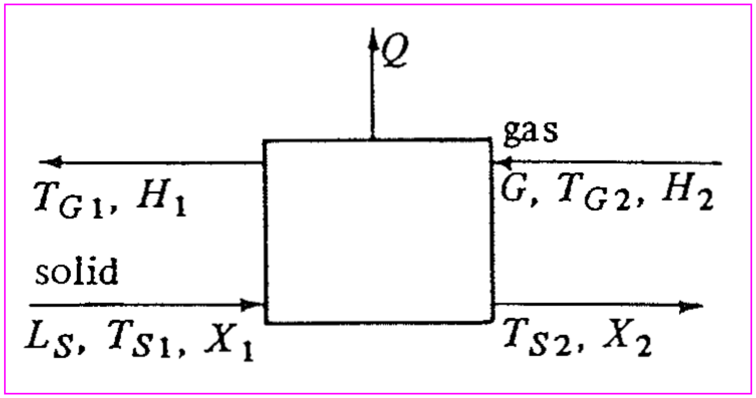






Balances for countercurrent continuous dryers

G: Mass flow rate of **dry air**
L_S: Mass flow rate of dry solid
X: Free moisture content.
H: Humidity
T_S: Wet solid temperature
T_G: dry gas temperature



- **Steady-state material balance on the moisture:**

$$L_S(X_1 - X_2) = G(H_1 - H_2)$$

- **Steady-state heat balance on dryer:**

$$L_S(H'_{S1} - H'_{S2}) = G(H_{y1} - H_{y2}) + Q$$

H'_{S1} and H'_{S2} : Enthalpies of wet solid in kJ/kg dry solid at T_{S1} and T_{S2}, respectively

H_{y1} and H_{y2} : Enthalpies of humid air in kJ/kg dry air at T_{G1} and T_{G2}, respectively.

- Heat capacity of dry solid (C_{pS}) and heat capacity of liquid water ($C_L \cong 4.187$ kJ/kg.K) can be used to calculate the enthalpy of wet solid at inlet and outlet:

$$H'_{S1} = C_{pS}(T_{S1} - T_0) + X_{T,1} C_L (T_{S1} - T_0)$$

$$H'_{S2} = C_{pS}(T_{S2} - T_0) + X_{T,2} C_L (T_{S2} - T_0)$$

where T_0 is base temperature has a convenient value of 0 ^oC

- The enthalpy of humid gas at inlet and outlet can be calculated from (**See humidification handout**):

$$H_{y1} = c_{S1}(T_{G1} - T_0) + \lambda_0 H_1$$

$$H_{y2} = c_{S2}(T_{G2} - T_0) + \lambda_0 H_2$$

$$c_{S1} = 1.005 + 1.88 H_1$$

$$c_{S2} = 1.005 + 1.88 H_2$$

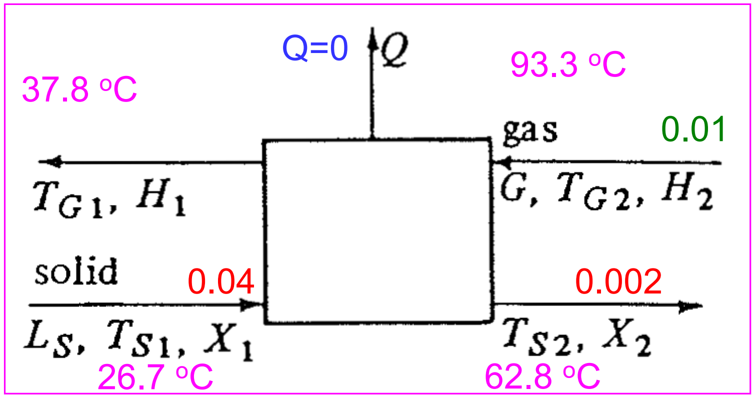
λ_0=2501 kJ/kg

- **Adiabatic drying**: $Q = 0$

Example 4.3.1 Drying rate during the constant-rate period

A continuous countercurrent dryer is being used to dry 453.6 kg dry solid/h containing 0.04 kg total moisture/kg dry solid to a value of 0.002 kg total moisture/kg dry solid. The granular solid enters at 26.7 °C and is to be discharged at 62.8 °C . The dry solid heat capacity is assumed to be constant at 1.465 kJ/(kg.K). Heating air enters the dryer at 93.3 °C with a humidity of 0.01 kg H2O/kg dry air and leaves at 37.8 °C. Calculate the air flow rate and the outlet humidity. Neglect heat losses in the dryer.

Solution



L_S=453.6 kg dry solid/h; Q=0

$X_{T,1}$= 0.04 kg H_2O/kg dry

$X_{T,2}$ =0.002 kg H_2O/kg dry

T_{S1}= 26.7 °C ; T_{S2}= 62.8 °C

T_{G1}= 37.8 °C ; T_{G2}= 93.3 °C ; H_2 = 0.01 kg H_2O/kg dry air

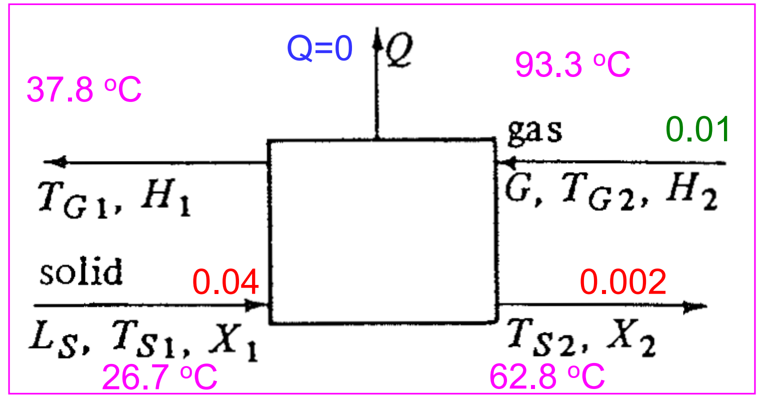
C_{pS}=1.465 kJ/(kg.K).

- Steady-state moisture material balance:

$$L_S(X_1 - X_2) = G(H_1 - H_2)$$

$$453.6(0.04 - 0.002) = G(H_1 - 0.01)$$

$$GH_1 - 0.01G = 17.237$$ **Eq. (1)**



- Steady-state heat balance:

$$L_S(H'_{S1} - H'_{S2}) = G(H_{y1} - H_{y2})$$

T_0 = 0 °C

$$H'_{S1} = C_{pS}(T_{S1} - T_0) + X_{T,1}C_L(T_{S1} - T_0) \rightarrow H'_{S1} = C_{pS}T_{S1} + X_{T,1}C_LT_{S1}$$

$$H'_{S2} = C_{pS}(T_{S2} - T_0) + X_{T,2}C_L(T_{S2} - T_0) \rightarrow H'_{S2} = C_{pS}T_{S2} + X_{T,2}C_LT_{S2}$$

$$H'_{S1} - H'_{S2} = C_{p_S}(T_{S1} - T_{S2}) + C_L\left[X_{T,1}T_{S1} - X_{T,2}T_{S2}\right]$$

T_0 = 0 °C

$$H'_{S1} - H'_{S2} = 1.465(26.7 - 62.8) + 4.187\left[(0.04)(26.7) - (0.002)(62.8)\right]$$
$$= -48.94 \text{ kJ/kg dry solid}$$

$$H_{y2} = c_{S2}(T_{G2} - T_0) + \lambda_0 H_2 = 1.0238(93.3 - 0) + 2501 \times 0.01 = 120.5\ \frac{\text{J}}{\text{kg}}$$

$$H_{y1} = c_{S1}(T_{G1} - T_0) + \lambda_0 H_1 = (1.005 + 1.88H_1)(37.8) + 2501H_1$$
$$= 37.99 + 2572H_1$$

$$c_{S2} = 1.005 + 1.88H_2 = 1.005 + 1.88(0.01) = 1.0238 \text{ kJ/kg dry air.K}$$

$$c_{S1} = 1.005 + 1.88H_1$$

- after substituting the above values in the steady state heat balance:

$$L_S(H'_{S1} - H'_{S2}) = G(H_{y1} - H_{y2})$$

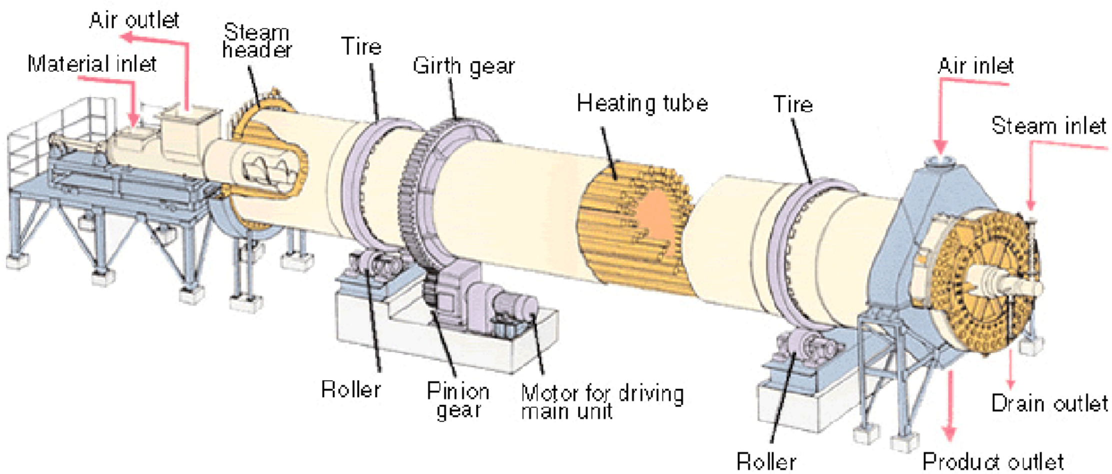
$$82.51G - 2572\,GH_1 = 22199$$ **Eq. (2)**

$$GH_1 - 0.01G = 17.237$$ **Eq. (1)**

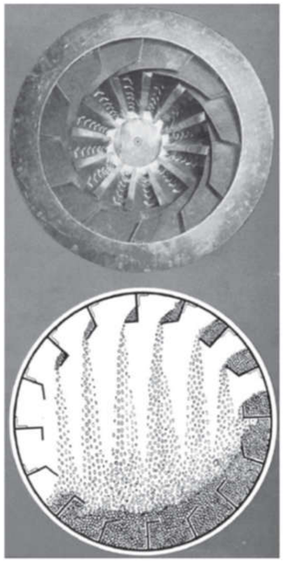
- **Solve Eqns. (1) and (2) simultaneously to get:**

G = 1171.8 kg dry air/h **,** H_1**=** 0.0247 kg water /kg air

Analysis and design of countercurrent continuous dryers

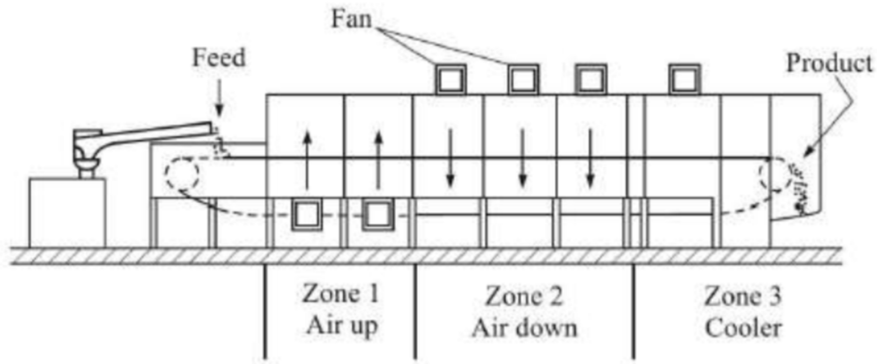


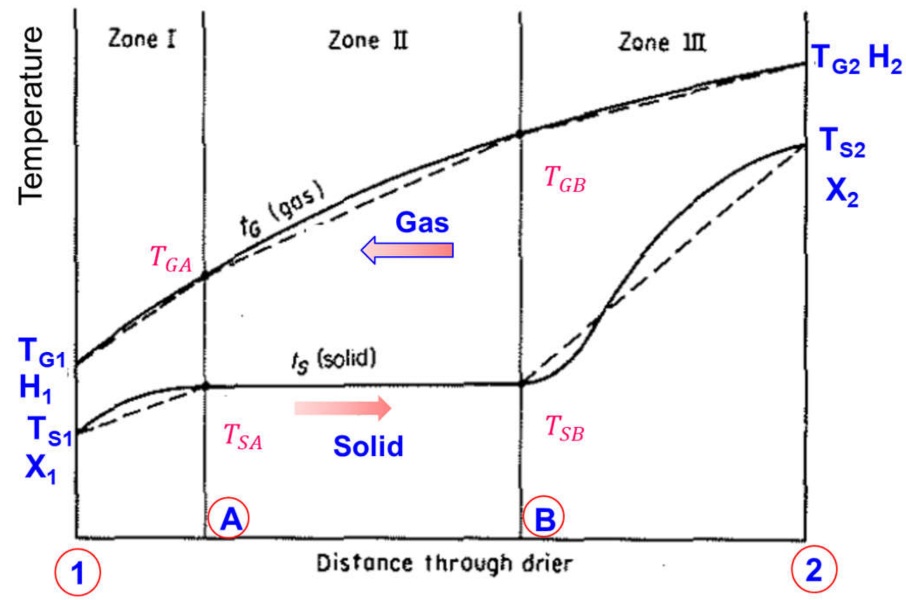
Rotary dryer



Zone I (1 → *A*: Preheat zone):

- The solid is heated up to the wet bulb or adiabatic saturation temperature (rate of heat transfer to the solid is balanced by the heat requirements for evaporation of moisture).
- Little evaporation occurs, thus this zone is usually ignored when drying performed at relatively low temperatures.
- The whole surface of the solid remains moist over zone I (as it happens during the constant rate drying period in a batch equipment).



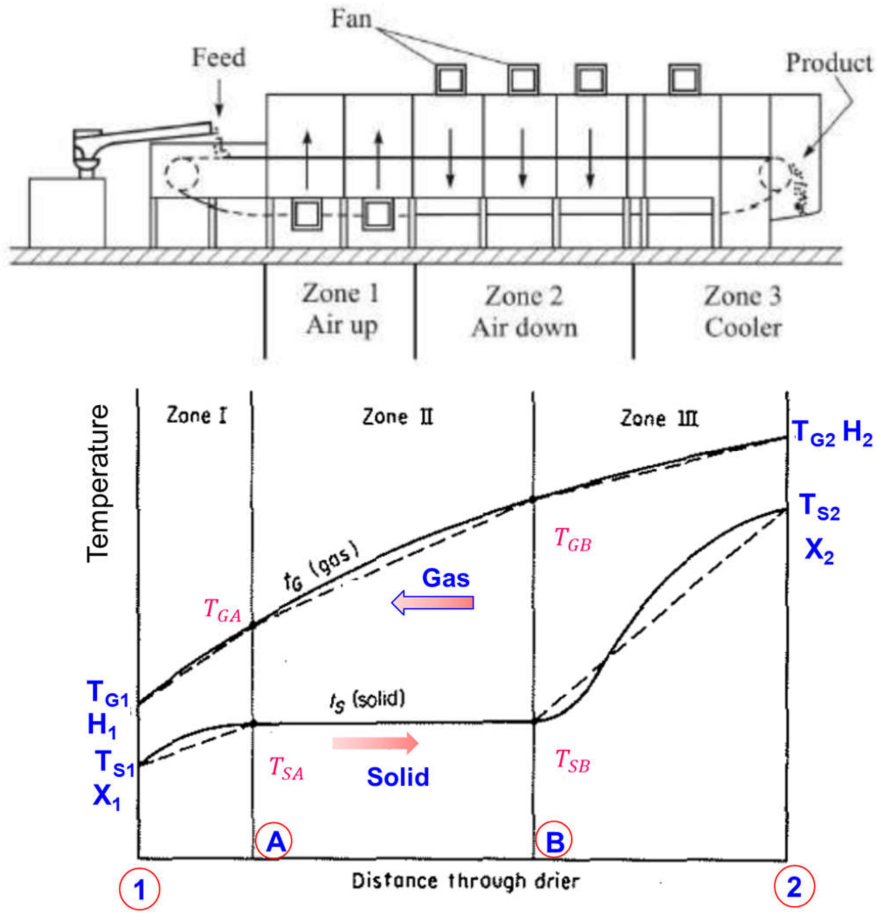


Typical gas and solid temperature profiles in a countercurrent rotary dryer

Zone II: $A \rightarrow B$:

- The equilibrium temperature of the solid remains substantially constant (at the wet bulb temperature of the air) while surface and unbound moisture are evaporated.

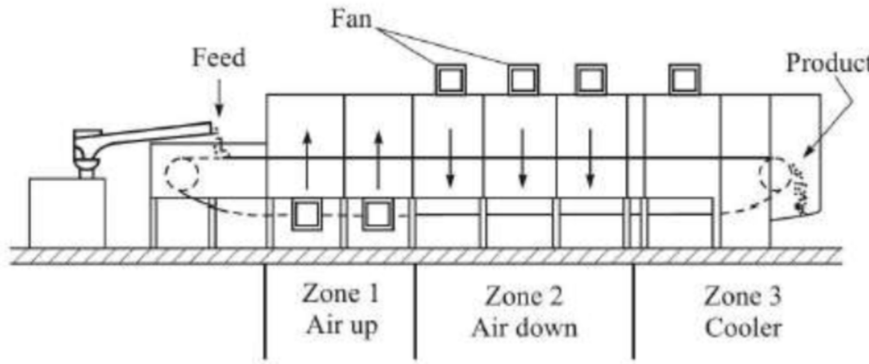
- At point B, the critical moisture of the solid is reached

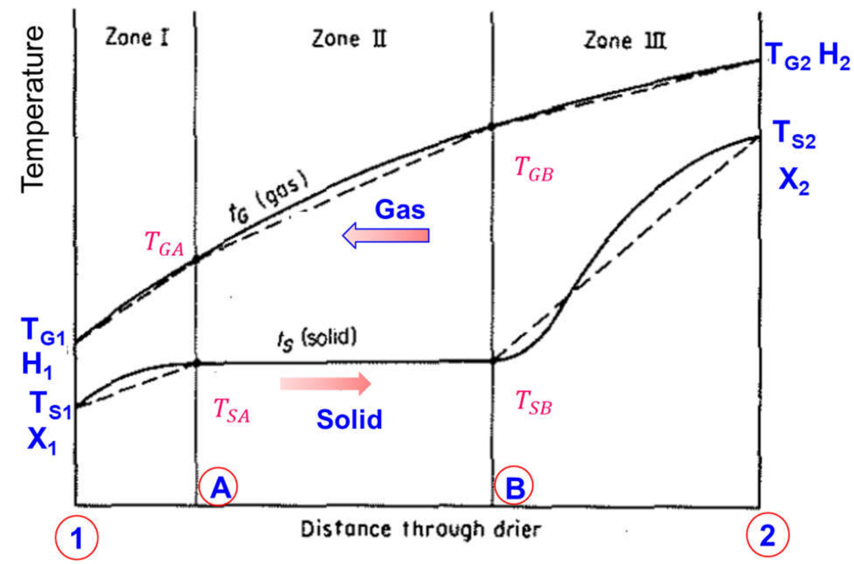


Typical gas and solid temperature profiles in a countercurrent rotary dryer

Zone III, $B \rightarrow 2$:

- Unsaturated surface drying and evaporation of bound moisture occur.
- Assuming that the heat-transfer coefficients remain essentially constant, the decreased rate of evaporation in this region results in increased solid temperature,
- The discharge temperature of the solid approaches the inlet temperature of the gas.





Typical gas and solid temperature profiles in a countercurrent rotary dryer

Differential balances for countercurrent continuous dryers

Assumptions:

- Adiabatic operation (*the losses Q = 0.0)*
- Heat transfer only from the gas (by convection), and neglecting any indirect heat transfer between the solid and the drier itself

Then, the loss in heat from the gas is equal to dq_G to that which is transferred to the solid d*q and the losses Q.*

$$dq_G = dq + dQ$$

$$dq = U\,dS\,(T_G - T_S) = Ua(T_G - T_S)\,\mathrm{dZ}$$

Where:

U = overall heat-transfer coefficient between gas and solid

T_G - T_s = temperature difference for heat transfer

S = interfacial surface/drier cross section

a = interfacial surface/drier volume

Also, $dq_G = -\,G\,c_s dT_G$

For adiabatic process, *the losses Q = 0.0* ⇨ $dq_G = dq + dQ$ ⇨ $dq_G = dq$

$$\rightarrow \quad -G\,c_s dT_G = Ua(T_G - T_S)\,\mathrm{dZ}$$

where

dT_G: is the temperature drop experienced by the gas as a result of transfer of heat to the solid,

c_s: is the humid heat.

$$dN_{tOG} = \frac{dT_G}{T_G - T_S} = \frac{Ua\,\mathrm{dZ}}{G\,c_s}$$

- if the heat-transfer coefficient is constant, NTU is:

$$N_{tOG} = \frac{\Delta T_G}{\Delta T_m} = \frac{L}{H_{tOG}}$$

Where HTU: $$H_{tOG} = \frac{G\,c_s}{Ua}$$

where

N_{tOG} = Number of heat-transfer units

H_{tOG} = Length of heat-transfer unit

ΔT_G = change in gas temperature owing to heat transfer to solid only

ΔT_m = appropriate average temperature difference between gas and solid (Log mean average)

- The volumetric heat transfer coefficient is calculated using the correlation

$$Ua\left(\frac{W}{m^3.K}\right) = 237\,\frac{\acute{G}^{0.68}}{d}$$

where $\acute{G} = G(1 + H)$

$\acute{G}$ = total gas mass flow rate (dry air + water vapor i.e. humidity) ($kg/m^2.s$),

d = dryer diameter (m)

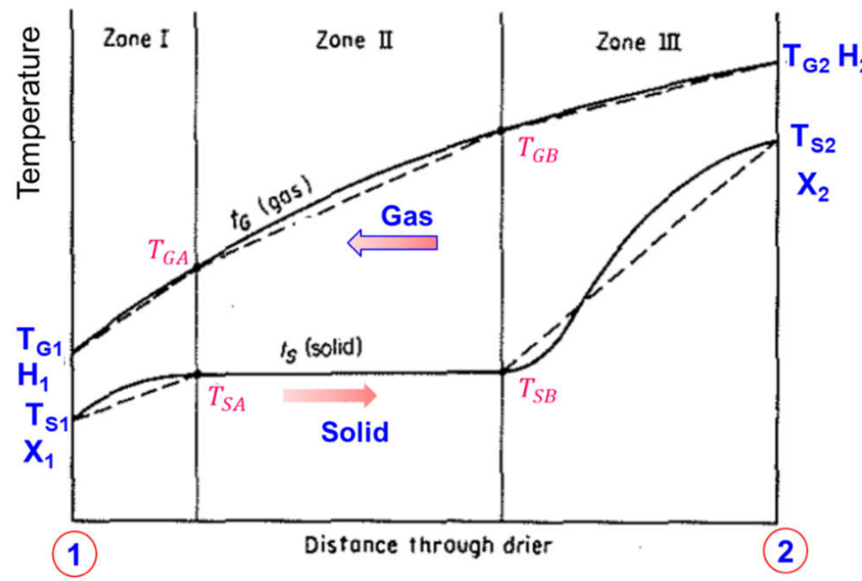
The total length of dryer is given by

$$\rightarrow \boxed{L = N_{tOG} \times H_{tOG}} = \frac{\Delta T_G}{\Delta T_m} \frac{G\, c_s}{Ua}$$

- For the **zone II**, for example, the number of heat transfer units is given by

$$(N_{tOG})_{II} = \frac{T_{GB} - T_{GA}}{(\Delta T_m)_{II}}$$

$$(\Delta T_m)_{II} = \frac{(T_{GB} - T_{SB}) - (T_{GA} - T_{SA})}{\ln \frac{(T_{GB} - T_{SB})}{(T_{GA} - T_{SA})}}$$



Example 4.3.2 Drying rate for falling rate period (nonporous solid)

A moist non-hygroscopic granular solid at 26 °C is to be dried from 20% initial moisture to 0.3% final moisture in a rotary dryer at a rate of 1500 kg/h. The hot air enters the dryer at 135 °C with a humidity of 0.015 and leaves at 60 °C . With condition that the temperature of the solid leaving the dryer must not exceed 100 °C and the air velocity must not exceed 1.5 m/s in order to avoid dust carry over. *Cps* = 0.85 kJ/kg.K. Find the diameter, length and other parameters of the dryer

Solution

Solid contains 20% initial moisture:

Mass flow of dry solid:

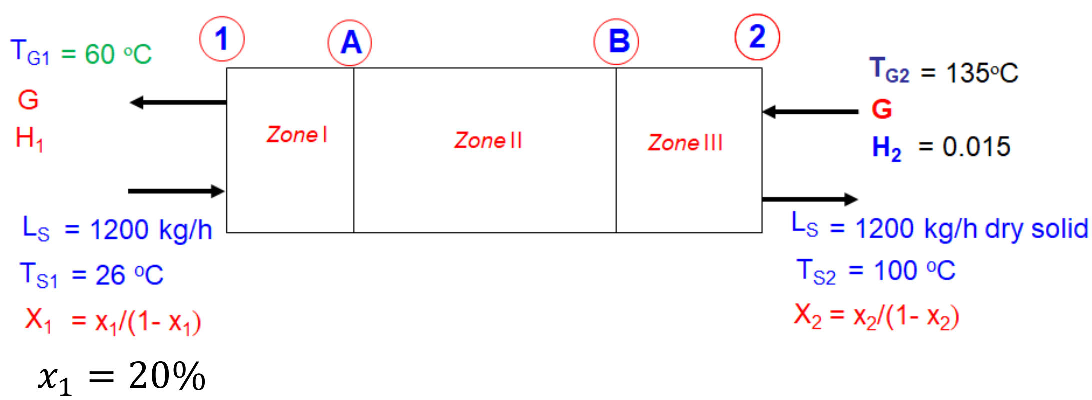
$L_S = 1500\ (1\text{-}0.2) = 1200$ kg/h,

Moisture in the wet solid:

$X_1 = 0.20/(1\text{-}0.20) = 0.25$

Moisture in the dry solid:

$X_2 = 0.003/(1\text{-}0.003) = 0.00301$



Water evaporated, $m_{w\ evaporated} = L_S\ (X_1 - X_2)$ = 1200 (0.25 – 0.00301) = 296.4 kg

Enthalpy of different streams (suppose ref temp = 0°C)

$$H'_{S1} = C_{pS}(T_{S1} - T_0) + X_{T,1}C_L(T_{S1} - T_0) \xrightarrow{T_0 = 0\ ^\circ C} H'_{S1} = C_{pS}T_{S1} + X_{T,1}C_L T_{S1}$$

$$H'_{S2} = C_{pS}(T_{S2} - T_0) + X_{T,2}C_L(T_{S2} - T_0) \xrightarrow{T_0 = 0\ ^\circ C} H'_{S2} = C_{pS}T_{S2} + X_{T,2}C_L T_{S2}$$

H'_{S1} = 0.85 (26) + 4.187 (0.25) (26) = 49.31 kJ/kg dry solid

H'_{S2} = 0.85 (100) + 4.187 (0.00301) (100) = 86.2 kJ/kg dry solid

$$c_{S2} = 1.005 + 1.88 H_2$$ = 1.005 + 1.88 (0.015) = 1.0332 kJ/kg dry air.K

$$H_{y2} = c_{S2}(T_{G2} - T_0) + \lambda_0 H_2$$ = 1.0332 (135-0) + 0.015 (2500) = 177 kJ/kg DA

$$c_{S1} = 1.005 + 1.88 H_1$$

$$H_{y1} = c_{S1}(T_{G1} - T_0) + \lambda_0 H_1$$ = (1.005 + 1.88 H_1) (60)+ 2500 H_1 = 60.3 + 2613 H_1

- Steady-state moisture material balance:

$$\boxed{L_S(X_1 - X_2) = G(H_1 - H_2)}$$ → 1200 (0.25 - 0.00301) = G (H_1 – 0.015)

→ G H_1 – 0.015 G = 296.4 kg **Eq. (1)**

- Steady-state heat balance:

$$L_S(H'_{S1} - H'_{S2}) = G(H_{y1} - H_{y2})$$

$\rightarrow$ 1200 (49.31 - 86.2) = G (60.3 + 2613 H_1 – 177) **Eq. (2)**

- **Solve Eqns. (1) and (2) simultaneously to get:**

G = 10560 kg dry air/h , H_1= 0.04306 kg water /kg air

Calculation of the shell diameter

Humid volume of the inlet gas (135°C, H_2= 0.015), v_{H2} = 1.183 m^3/(kg dry air)

Humid volume of the exit gas (60°C, H_1= 0.04306), v_{H1} = 1.008 m^3/(kg dry air)

The maximum volumetric gas flow rate (this occurs at end 2)

$$= G_s v_{H2} = (10{,}560)(1.183) = 12{,}490\ \mathrm{m^3/h} \Rightarrow 3.47\ \mathrm{m^3/s}$$

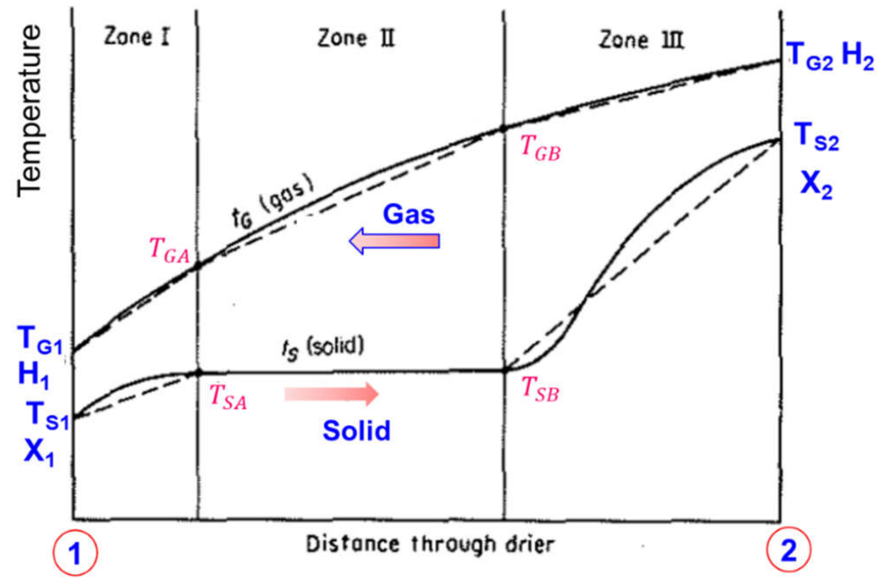
- Take the maximum superficial air velocity to be 1.2 m/s (this is 20% less than the maximum allowable velocity since part of the dryer is filled with the moving solid, and the entire cross-section is not available for gas flow).

$$(\pi d^2/4)(1.2) = 3.47 \Rightarrow d = 1.98\ \mathrm{m}$$

Select a 2-m diameter shell:

Calculation of the number of heat transfer units

- The dryer is considered to consist of three zones as shown in the Figure.
- The stage wise calculation of temperature and humidity or moisture content of the streams can be obtained by material and energy balance



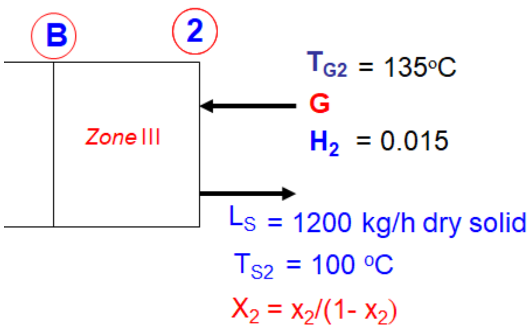
- ***Zone* III**: Only heating of the solid occurs in this zone; there is little water left for vaporization.

- At the boundary between zones III and II, the solid is at the wet-bulb temperature T_{SB}, of the air at that location.

Assume T_{SB} (= T_{SA} = T_{WB}) = 41 °C (at H = 0.015 (inlet humidity, only heating in zone III, i.e. H = constant as solid in this zone doesn't have free moisture) and air temperature entering this zone is:

T_{DA} = 115 °C

H = 0.015 kg/kg DA



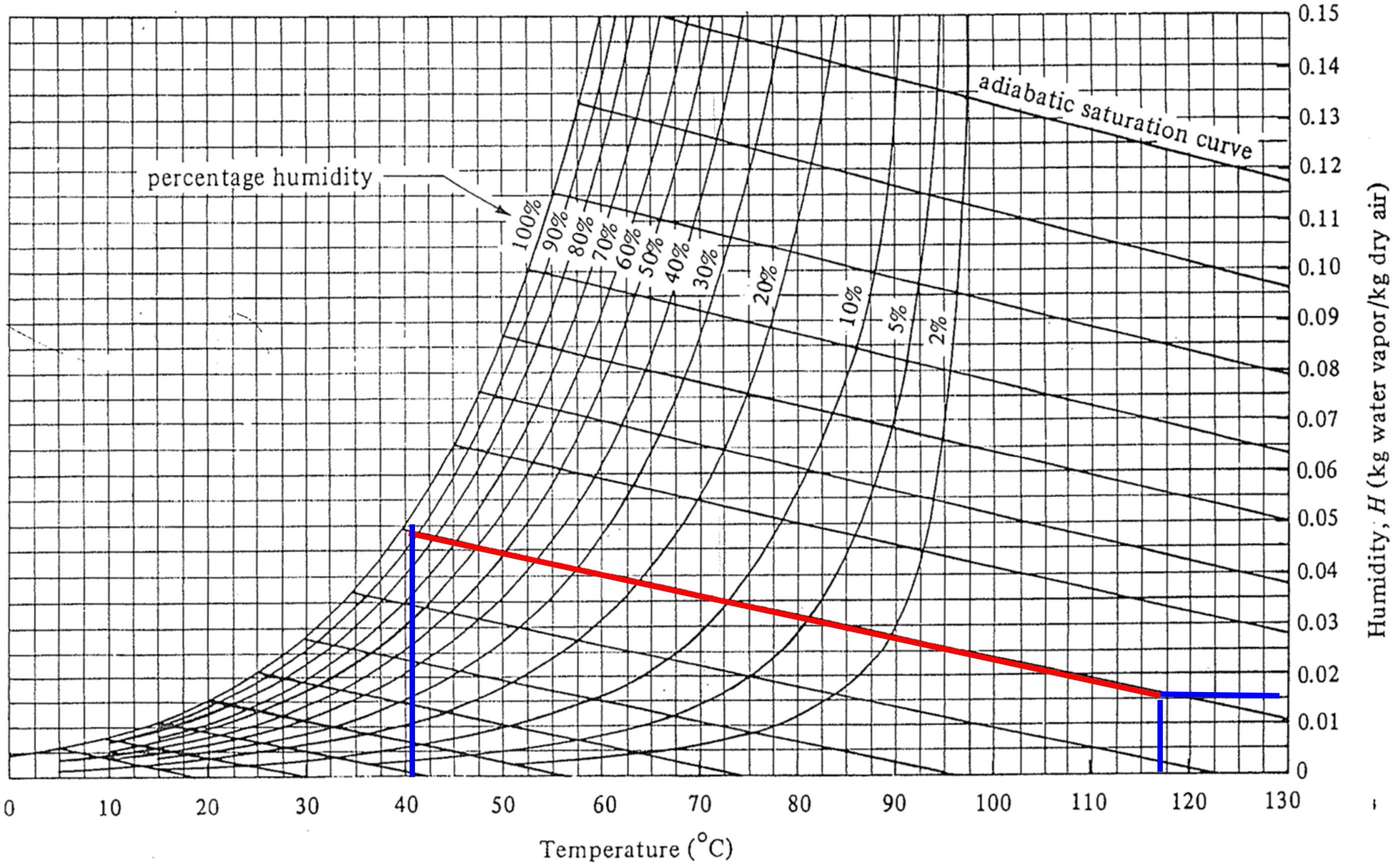


FIGURE 9.3-2. *Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)*

Enthalpy of the solid at the inlet to zone III

$$H'_{SB} = C_{p_S} T_{SB} + X_{T,1} C_L T_{SB}$$

$$H'_{sB} = [0.85 + (0.00301)(4.187)](41 - 0) = 35.37 \text{ kJ/(kg dry solid)}$$

Humid heat of the gas entering zone III

$$c_{SB} = 1.005 + 1.88\, H_B$$

$$c_{sB} = [1.005 + (1.88)(0.015)] = 1.033 \text{ kJ/kg·K}$$

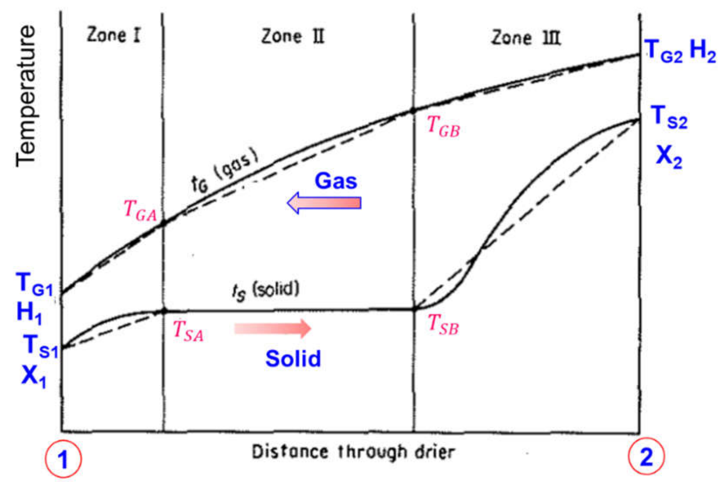
(this remains constant in zone III, since the humidity <u>does not</u> change in this section).

Heat balance over zone III

$$L_s(H'_{s2} - H'_{sB}) = G\, c_{sB} (T_{G2} - T_{GB})$$

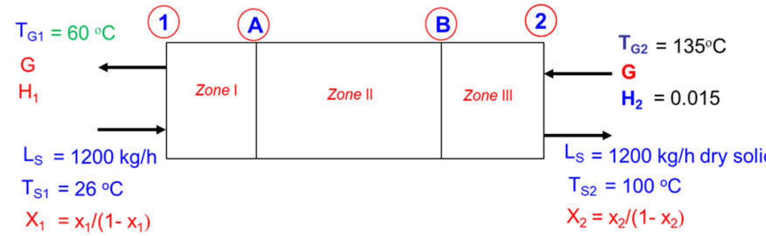
$$(1200)(86.2 - 35.37) = (10{,}560)(1.033)(135 - T_{GB})$$

$$\Rightarrow T_{GB} = 129°\text{C}$$



The wet bulb temperature of air entering zone II (129 °C and humidity of 0.015) is 41.3 °C.

This is fairly close to the guess value of 41 °C and T_{SB} (= T_{SA}) = 41 °C **is not changed**.



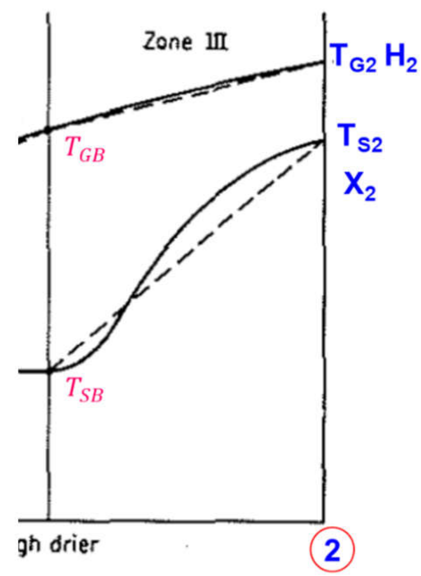
At the boundary B, $\Delta T_B = 129 - 41 = 88°C$;

at end 2, $\Delta T_2 = 135 - 100 = 35°C$

Log mean temperature in zone III, $(\Delta T)_m = \dfrac{88-35}{\ln(88/35)} = 57.5°C$

$$(N_{tOG})_{III} = \frac{T_{GB} - T_{GA}}{(\Delta T_m)_{III}}$$

Number of heat transfer units, $(N_{tG})_{III} = \dfrac{T_2 - T_{GB}}{(\Delta T)_m} = \dfrac{135-129}{57.5} = 0.104$



Zone II: In order to calculate $(N_{tG})_{II}$, we need the value of T_{GA}. This can be obtained by heat balance.

$$H_{yB} = [1.005 + 1.88Y_B](129 - 0) + (2500)(H_B) = 170.8 \text{ kJ/kg.} \quad (\text{since } H_B = 0.015)$$

$$H'_{SA} = C_{p_S} T_{SA} + X_{T,B} C_L T_{SA}$$

$$H'_{sA} = [0.85 + c_{ps}X_1](T_{sA} - 0) = [0.85 + (4.187)(0.25)](41) = 77.77 \text{ kJ/(kg dry solid)}$$

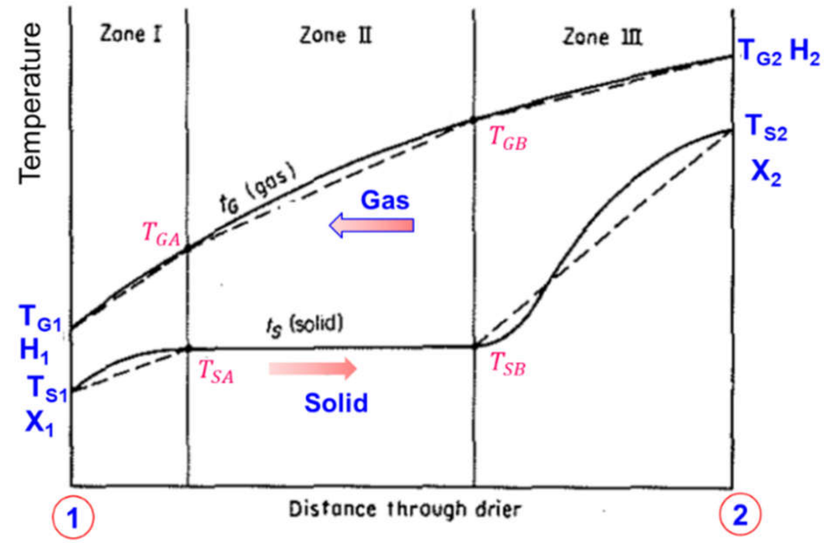
Enthalpy balance: $L_s(H'_{sB} - H'_{sA}) = G(H_{yB} - H_{yA})$

$$(1200)(35.37 - 77.77) = (10{,}560)(170.8 - H_{yA})$$

$$\Rightarrow H_{yA} = 175.6$$

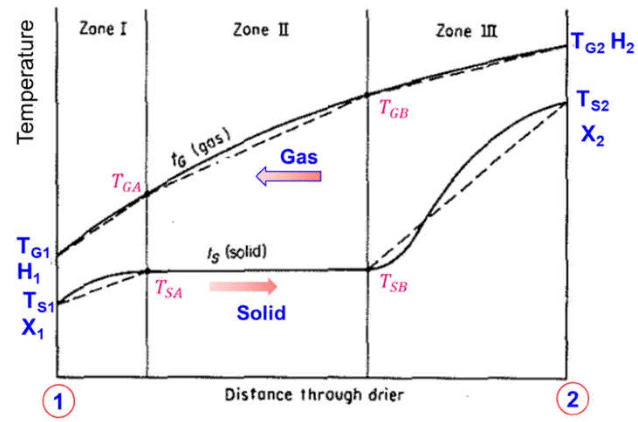
$$= [1.005 + (0.04306)(1.88)](T_{GA} - 0) + (0.04306)(2500)$$

$$\Rightarrow T_{GA} = 63°\text{C}$$



Temperature differences: At section A, $(\Delta T)_A = 63 - 41 = 22°C$; $(\Delta T)_B = 88°C$

$$(\Delta T)_m = \frac{88-22}{\ln(88/22)} = 47.6$$



Number of heat transfer units,

$$(N_{tG})_{II} = \frac{T_{GB} - T_{GA}}{(\Delta T)_m}$$

$$= \frac{129-63}{47.6} = 1.386$$

Zone I: $(\Delta T)_1 = 60 - 26 = 34°C$; $(\Delta T)_A = 22°C$; $(\Delta T)_m = \dfrac{34-22}{\ln(34/22)} = 27.5$

Number of heat transfer units, $(N_{tG})_I = \dfrac{T_{GA} - T_{G1}}{(\Delta T)_m} = \dfrac{63-60}{27.5} = 0.109$

Total number of heat transfer units $N_{tG} = 0.104 + 1.386 + 0.109 = 1.53$

Length of a transfer unit calculation

$$H_{tOG} = \frac{G\, c_s}{Ua}$$

Average gas mass flow rate

$$G' = [(10{,}560)(1.015) + (10{,}560)(1.04306)]/2 = 10{,}867 \text{ kg/h}$$

The gas mass flow rate, $G' = (10{,}867/3600)/(\pi/4)(2)^2 = 0.961 \text{ kg/m}^2\cdot\text{s}$

Volumetric heat transfer coefficient

$$U\bar{a} = \frac{237(G')^{0.67}}{d} = \frac{(237)(0.961)^{0.67}}{2} = 115 \text{ W/m}^3\cdot\text{K}$$

Humid heats at the ends: $c_{S2} = 1.005 + 1.88 H_2$

$= 1.005 + 1.88\ (0.015) = 1.0332$ kJ/kg DA.K

$$c_{S1} = 1.005 + 1.88 H_1$$

$= 1.005 + 1.88\ (0.04306) = 1.083$ kJ/kg DA.K

Average humid heat

$$c_S = (1.033 + 1.083)/2 = 1.058 \text{ kJ/kg}\cdot\text{K} = 1058 \text{ J/(kg dry air)(K)}$$

$$H_{tOG} = \frac{G' C_s}{U a} = \frac{(0.961)(1058)}{115} = 8.84 \text{ m}$$

Length of the dryer, $L = (N_{tG})(L_t) = (1.56)(8.84) = 13.8$ m

Select a 2 m diameter, 15 m long dryer

Thermodynamic and transport properties of the air-water system

Property	Expression
P_v	$P_v = 100\exp[27.0214 - (6887 / T_{abs}) - 5.32\ln(T_{abs} / 273.16)]$
Y	$Y = 0.622 RH\, P_v / (P - RH\, P_v)$
c_{pg}	$c_{pg} = 1.00926\times10^3 - 4.0403\times10^{-2} T + 6.1759\times10^{-4} T^2 - 4.097\times10^{-7} T^3$
k_g	$k_g = 2.425\times10^{-2} - 7.889\times10^{-5} T - 1.790\times10^{-8} T^2 - 8.570\times10^{-12} T^3$
ρ_g	$\rho_g = PM_g / (RT_{abs})$
μ_g	$\mu_g = 1.691\times10^{-5} + 4.984\times10^{-8} T - 3.187\times10^{-11} T^2 + 1.319\times10^{-14} T^3$
c_{pv}	$c_{pv} = 1.883 - 1.6737\times10^{-4} T + 8.4386\times10^{-7} T^2 - 2.6966\times10^{-10} T^3$
c_{pw}	$c_{pw} = 2.8223 + 1.1828\times10^{-2} T - 3.5043\times10^{-5} T^2 + 3.601\times10^{-8} T^3$

P_v vapor pressure of pure water, Pa

Y absolute air humidity, kg water vapor/kg dry air

c_p specific heat, J kg^{-1} K^{-1}

k_g thermal conductivity, W m^{-1} K^{-1}

T_{abs} absolute temperature, K

g gas

v vapor

w water

ρ_g density, kg m^{-3}

μ_g dynamic viscosity, kg m^{-1} s^{-1}