

Evaporation

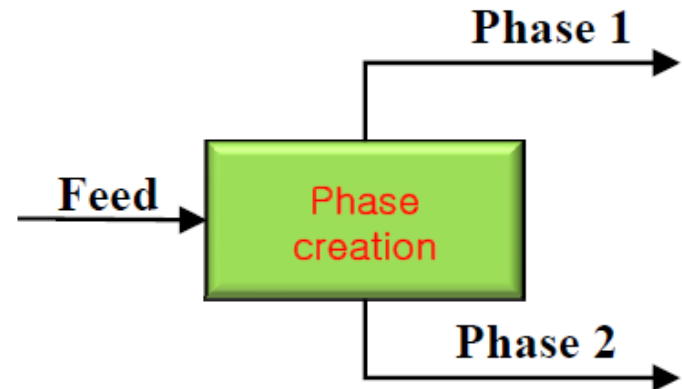
Contents

- Definition and purpose
- Factors affecting evaporation
- Types of evaporators
- Flow characteristics
- Heat Transfer in evaporators
- Methods of operation.
- Effects of processing variables on evaporator operation.
- Optimum boiling time.



Principal reference: Chapter 8 in C.J. Geankoplis, Transport Processes and Separation process Process principles., 4th Edition, Prentice-Hall.

Definition of Evaporation



- It is based on **phase-creation**.
- Heat transfer to aqueous solution to reach boiling (**ESA**).
- Vapor from boiling solution is removed and more concentrated solution remains inside.
- **Applications:** concentration of aqueous solutions of sugar, NaCl, NaOH, milk, orange juice, evaporation sea water to produce drinking water (desalination),....etc.
- The industrial equipment used for this purpose is called **evaporator**.
- **Desired product:**
 - usually, is the concentrated solution and water vapor is normally discarded. Sometimes, the water vapor as in desalination.

Processing factors affecting evaporation

1. Concentration in the liquid:

- Liquid feed to an evaporator is relatively dilute.
- So its viscosity is low, and heat-transfer coefficient high.
- As evaporation proceeds, the solution becomes concentrated.
- So viscosity increases and heat-transfer coefficient drops.
- Density and the boiling point of solution also increase.

2. Scale deposition:

- Some solutions deposit solid materials (called **scaling** or **fouling**) on the heating surfaces.
- The result is that the **overall heat-transfer coefficient** (U) may drastically decrease, leading to shut down of the evaporators for **cleaning purposes**.

Processing factors affecting evaporation

3. Solubility:

- As solution is heated, concentration of the solute in the solution increases.
- In case the solubility limit of the solute in solution is exceeded, then crystals may form.
- Solubility of the solute therefore determines the maximum concentration of the solute in the product stream.
- In most cases, the solubility of the solute increases with temperature. This means when a hot concentrated solution from an evaporator is cooled to room temperature, crystallization may occur.

Processing factors affecting evaporation

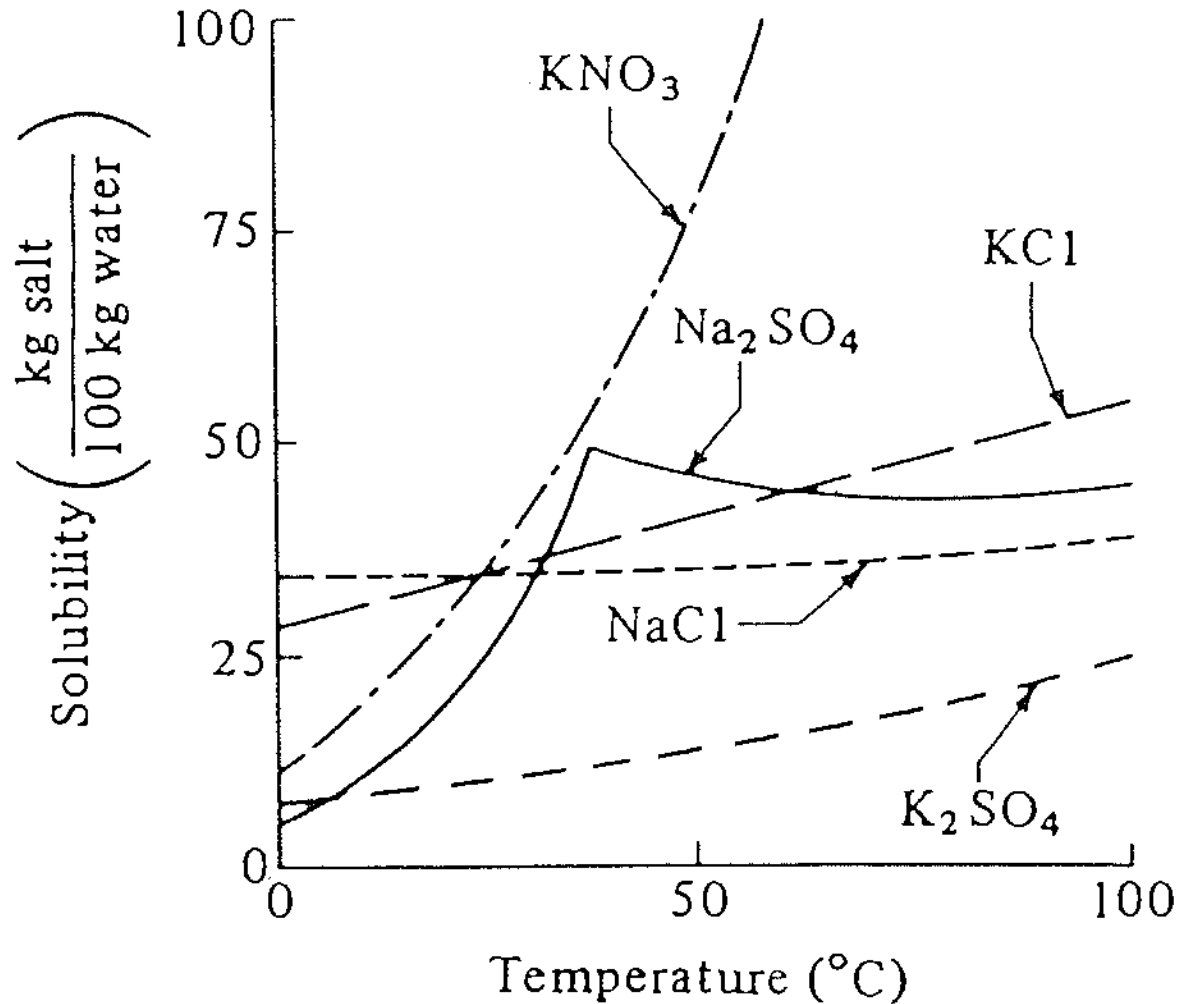


FIGURE 8.1-1. Solubility curves for some typical salts in water.

Processing factors affecting evaporation

4. Temperature sensitivity of materials:

- Pharmaceuticals products, fine chemicals and foods are damaged when heated to moderate temperatures for relatively short times.
- So special techniques are employed to reduce temperature of the liquid and time of heating during evaporation.

5. Foaming and frothing:

- Solutions like organic compounds tend to foam and froth during vaporization.
- The foam is carried away along with vapor leaving the evaporator.
- Entrainment losses occur.

Processing factors affecting evaporation

6. Pressure and temperature:

- The boiling point of the solution is related to the pressure of the system.
- The higher the operating pressure of the evaporator, the higher the temperature at boiling.
- Also, as the concentration of the dissolved material in solution increases by evaporation, the temperature of boiling may rise (a phenomenon known as **boiling point rise/elevation**).
- To keep the temperatures low in heat-sensitive materials, it is often necessary to operate below atmospheric pressure (that is, under vacuum).

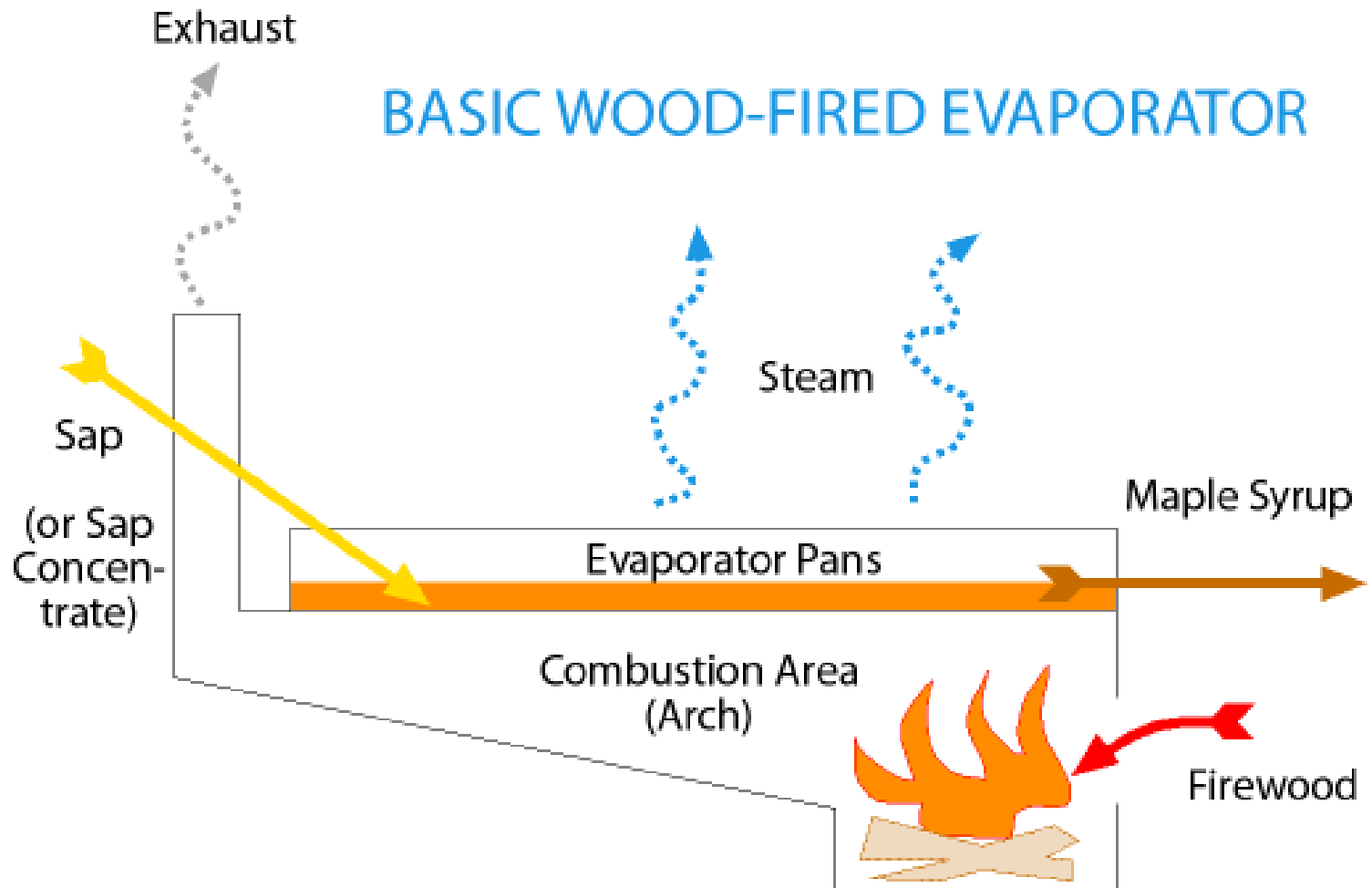
Processing factors affecting evaporation

7. Materials of construction:

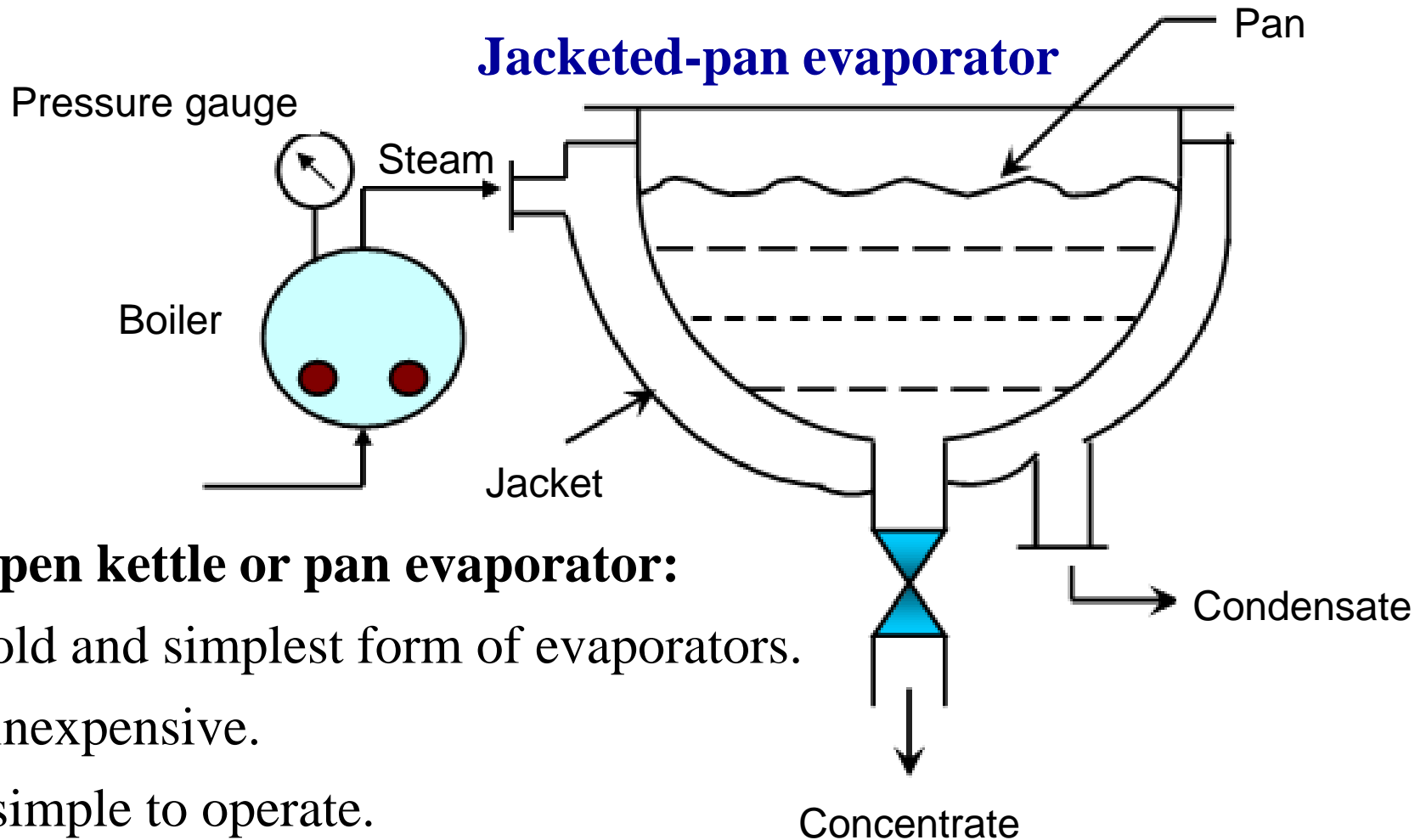
- Evaporators are made of some kind of stainless steel.
- However many solutions attack ferrous metals and are contaminated by them.
- Alloys of Copper, nickel, and stainless steels can also be used.

General types of evaporators

1. Open kettle or pan evaporator:



General types of evaporators



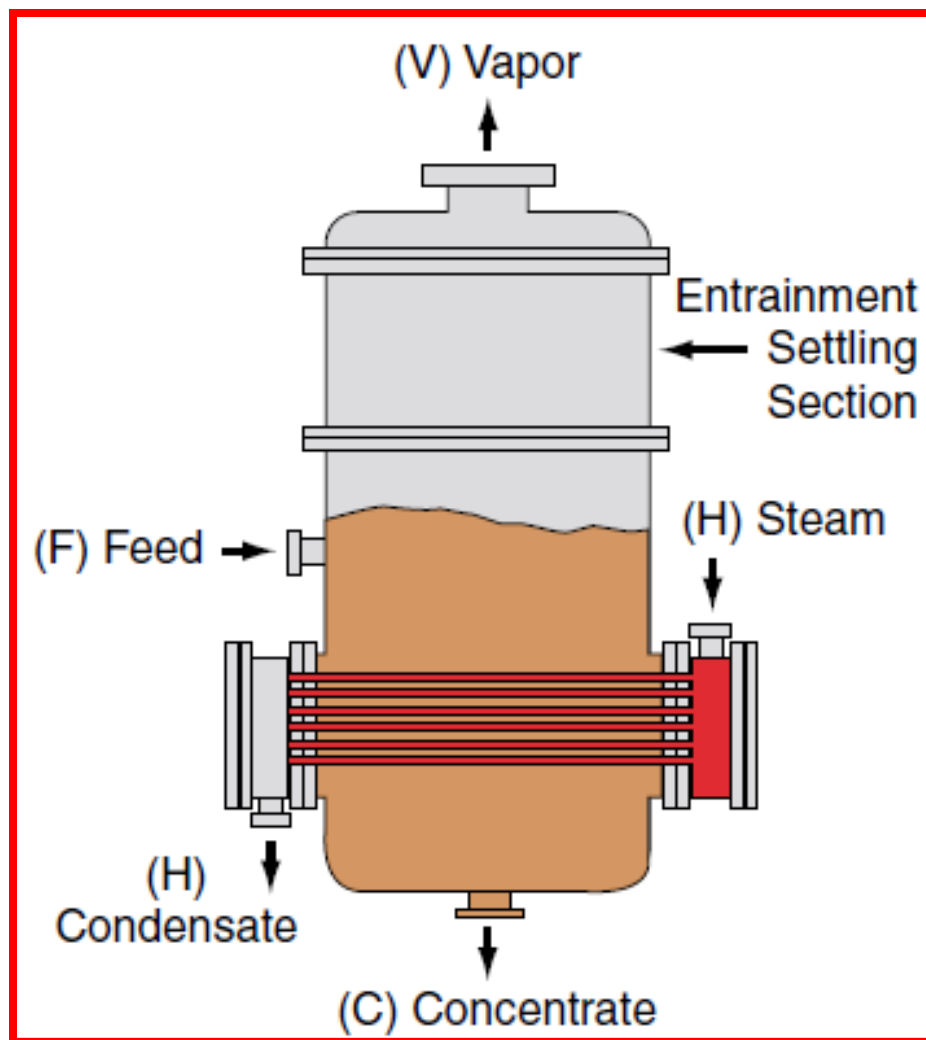
Open kettle or pan evaporator:

- old and simplest form of evaporators.
- inexpensive.
- simple to operate.
- very poor in heat economy.
- in some cases paddles and scrapers for agitation are used.

General types of evaporators

2. Horizontal-tube evaporator:

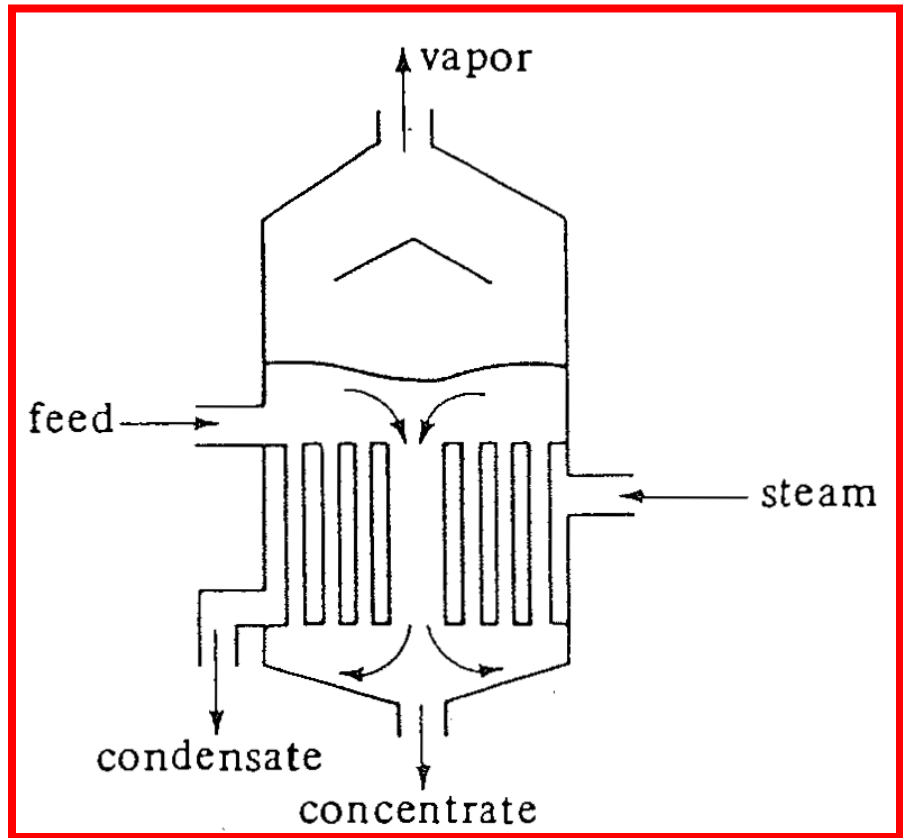
- old.
- relatively cheap.
- heating medium is inside the tubes.
- used for non-viscous liquids having high heat-transfer coefficients and liquids that do not deposit scales.
- poor liquid circulation; i.e. unsuitable for viscous liquids).



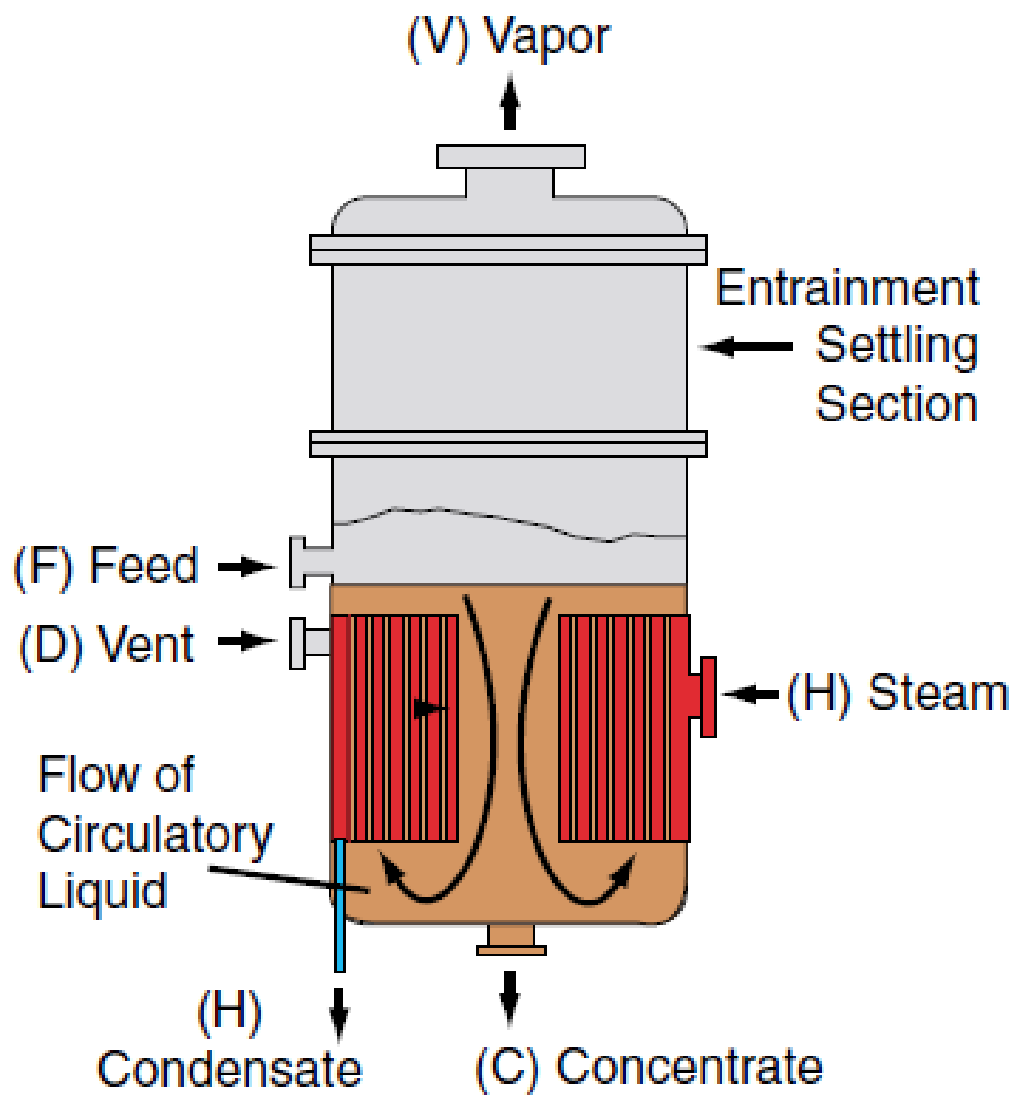
General types of evaporators

3. Vertical-type short-tube evaporator:

- liquid solution is inside the tubes.
- steam condenses outside the tubes.
- tubes are around 1-2 m long.
- used for non-viscous liquids having high heat-transfer coefficients and liquids that do not deposit scales.
- widely used in sugar, salt, and caustic soda industries.



General types of evaporators

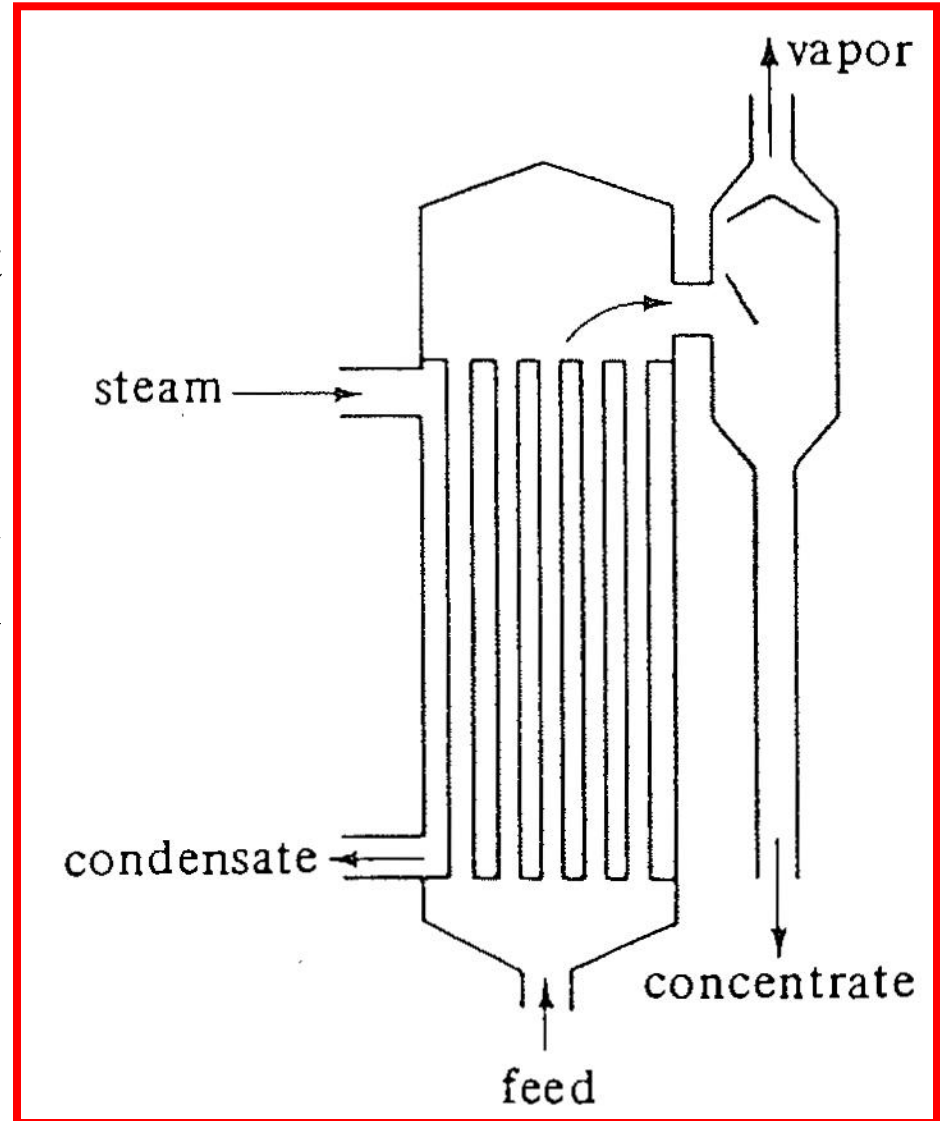


Vertical-type short-tube evaporator:

General types of evaporators

4. Vertical-type long-tube evaporator:

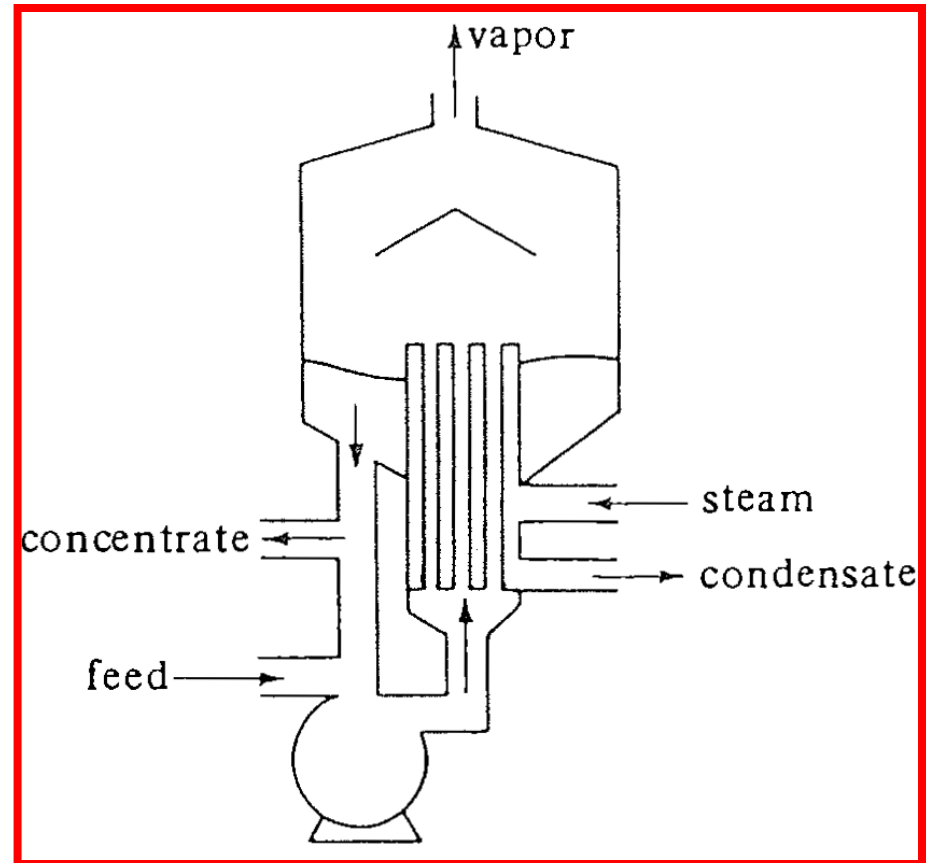
- **Rising film** evaporator.
- liquid is inside the tubes.
- high heat-transfer coefficient on the steam side.
- tubes are around 3-10 m long.
- formation of vapor bubble inside tubes gives quite high liquid velocities.
- contact times can be quite low.
- widely used for producing condensed milk.



General types of evaporators

5. Forced circulation evaporator:

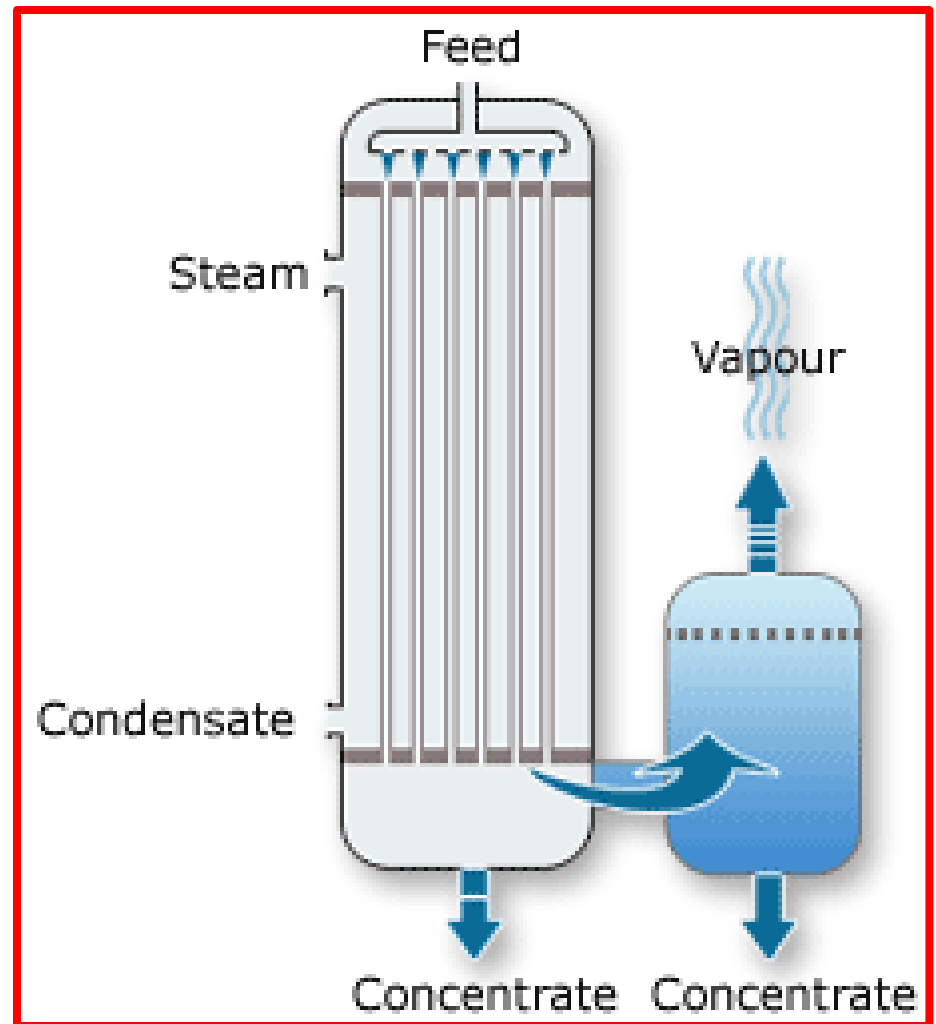
- similar to vertical log-tube type but with shorter tubes.
- high liquid film heat transfer coefficient (pumping action).
- in some cases, external horizontal type.
- useful for viscous liquids.



General types of evaporators

6. Falling-film evaporator:

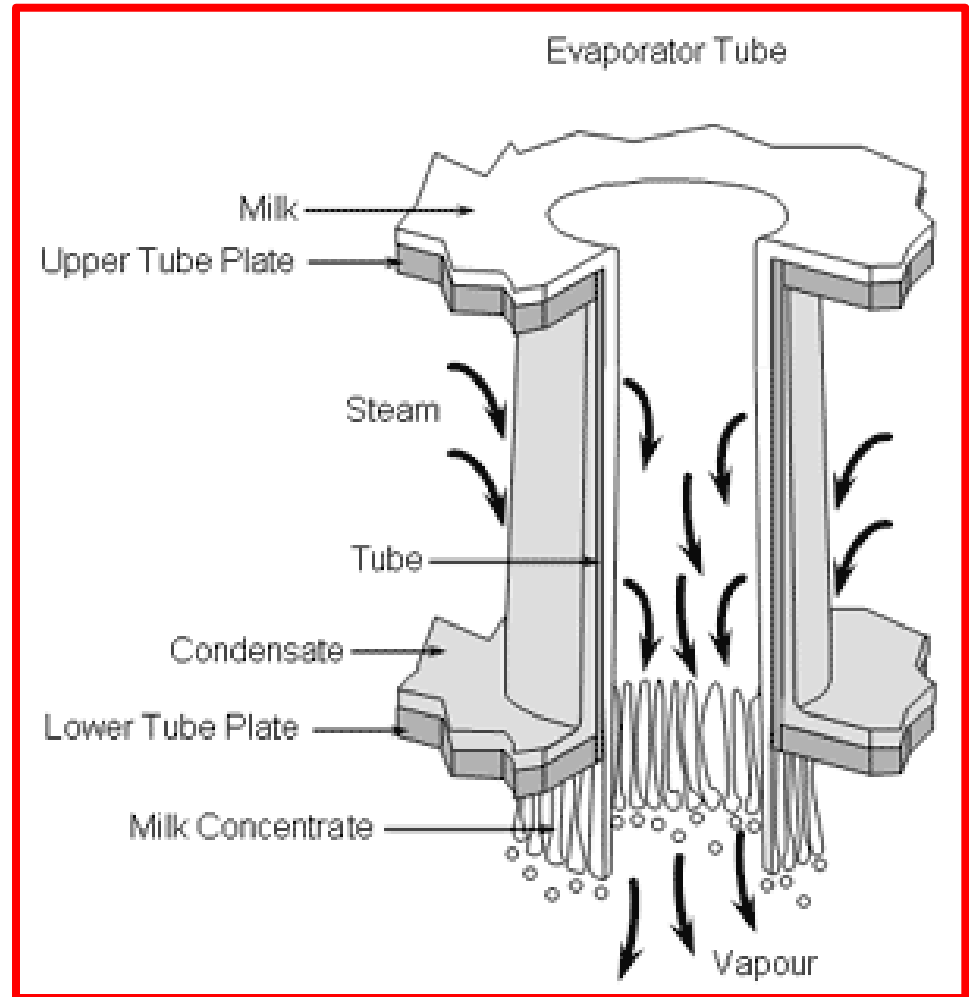
- upside down vertical long-tube evaporator.
- liquid solution flows down (by gravity) near the walls as a thin film inside tubes.
- vapor liquid separation occurs at the bottom.
- small holdup time (5-10 s).
- high heat-transfer coefficient.
- useful for heat-sensitive materials and viscous liquids.



General types of evaporators

Falling-film evaporator:

- falling film evaporators can be operated with very low temperature differences between the heating media and the boiling liquid.
- it is today the most frequently used.
- widely used for concentration fruit juices, dairy industry, ..etc.



General types of evaporators

Falling-film evaporator (cont.):

- however, falling film evaporators must be designed very carefully for each operating condition.
- sufficient wetting (film thickness) of the heating surface by liquid is extremely important for trouble-free operation of the plant.
- If the heating surfaces are not wetted sufficiently, dry patches will occur.
- the proper design of the feed distribution system in the head of the evaporator is critical to achieve full and even product wetting of the tubes.

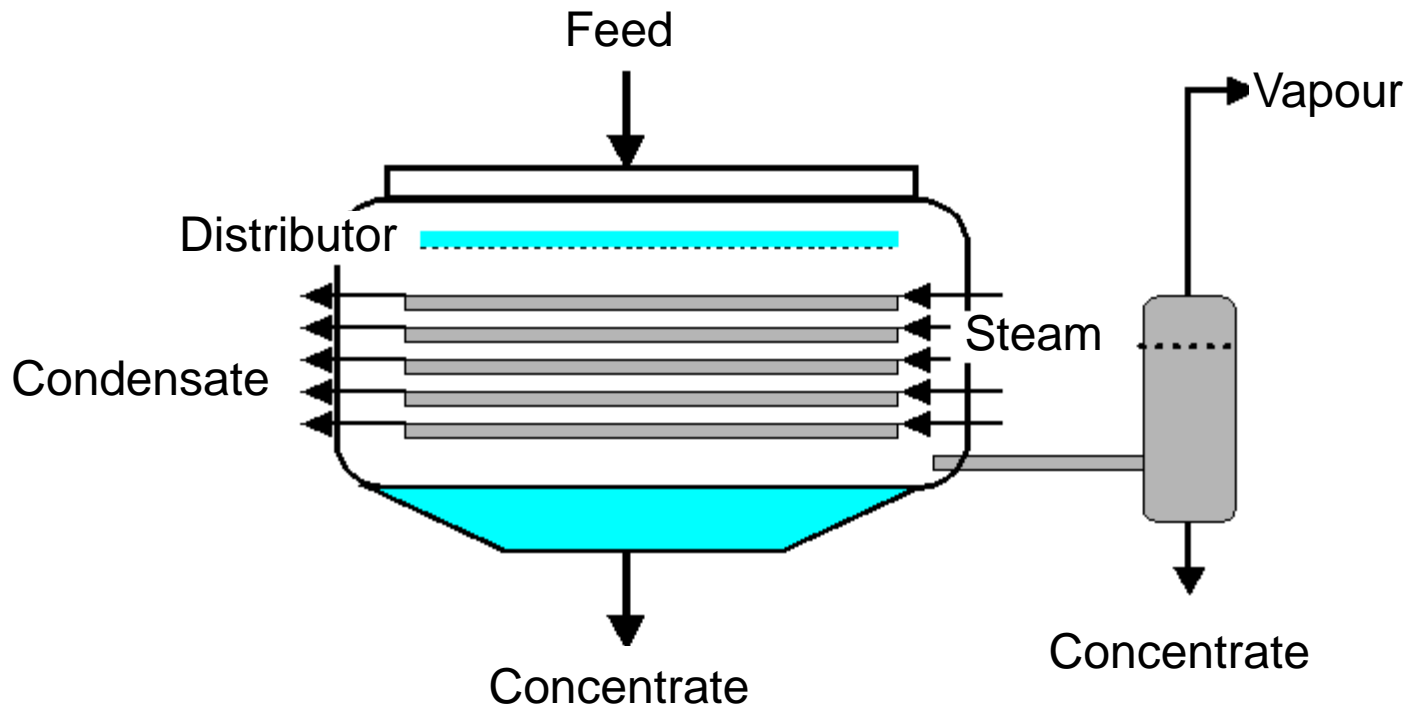
General types of evaporators

Falling-film evaporator (cont.):

- because of the low liquid holding volume in this type of unit, the falling film evaporator can be started up quickly and changed to cleaning mode or another product easily.
- falling film evaporators are highly responsive to alterations of parameters such as energy supply, vacuum, feed rate, concentrations, etc. When equipped with a well designed automatic control system they can produce a very consistent concentrated product.
- the tube length is typically 6 m to 11 m, but can be as short as 1.5 m to 3 m (for example, in deep vacuum applications).
- diameters are typically 20 mm to 64 mm.

General types of evaporators

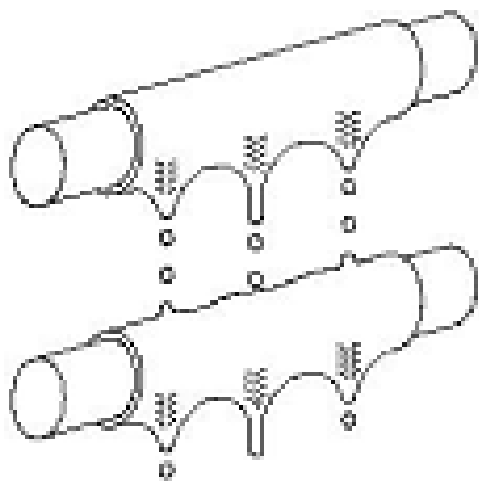
7. Horizontal falling-film evaporator:



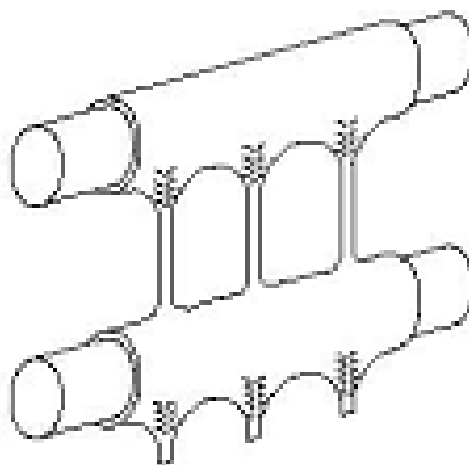
- liquid solution is evaporated at the outside of the tubes.
- it flows from one tube to the other in form of droplets, jets or as a continuous sheet.

General types of evaporators

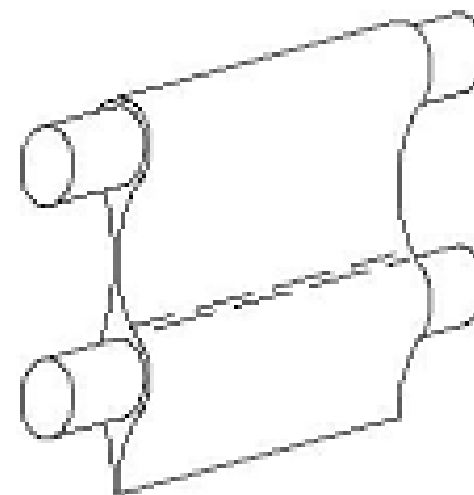
Horizontal falling-film evaporator:



Droplets form



Jets form



Sheet form

General types of evaporators

Horizontal falling-film evaporator (cont.:

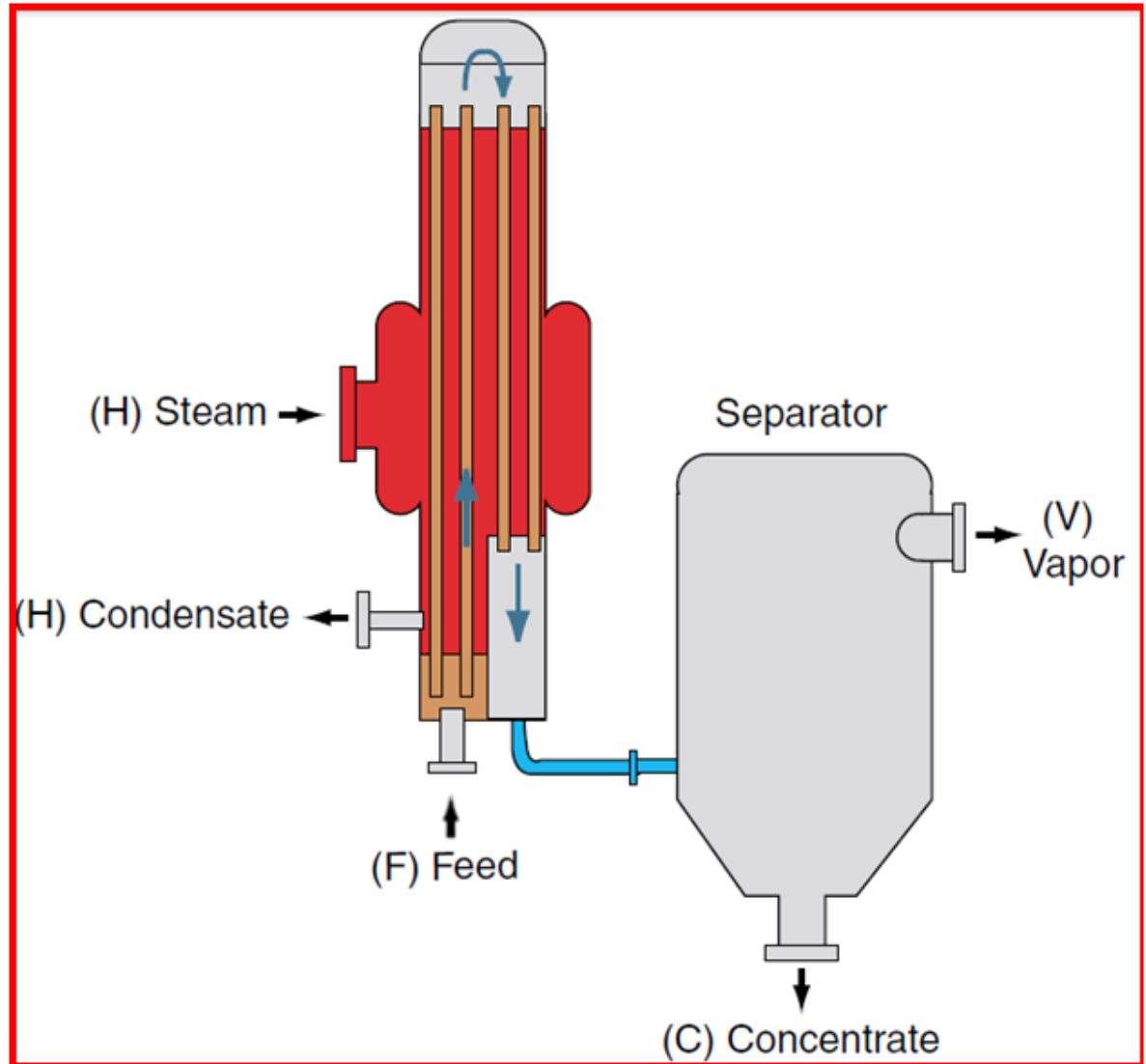
- due to the impinging effect when flowing from one tube to the other the heat transfer is higher compared to vertical falling film evaporators.
- in addition, this unit type can be operated with even lower pressure drops compared to the vertical design.
- it is also possible to design a higher heat transfer area for a given shell compared to the vertical units. Perforated plates or specially designed spray nozzles can be used in order to guarantee a even liquid distribution for each tube.
- cleaning of the outside tubes can be difficult, therefore this type of evaporators is not used for processes with tendency to foul.
- tube dimensions are typically 0.75 to 1".

General types of evaporators

8. Rising/falling film evaporator:

Best applied :

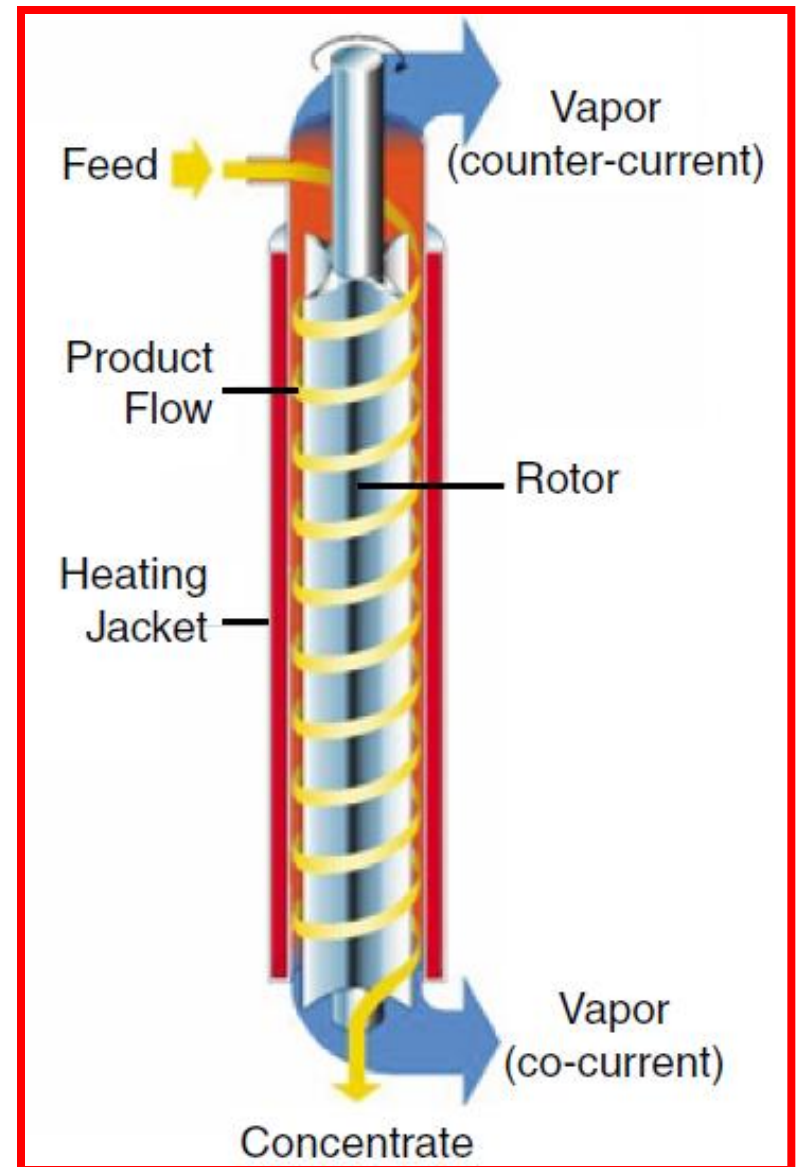
- for foamy liquids.
- When large evaporation load are required.



General types of evaporators

9. Agitated thin-film evaporator:

- falling-film evaporator with only a single jacketed tube containing an internal agitator.
- liquid solution spread downward into a turbulent film by vertical agitator blades.
- higher heat transfer coefficient.
- small residence time
- widely used for heat sensitive viscous materials such as antibiotics, gelatin, rubber latex
- high cost and small capacity.



General types of evaporators

Agitated thin-film evaporator:

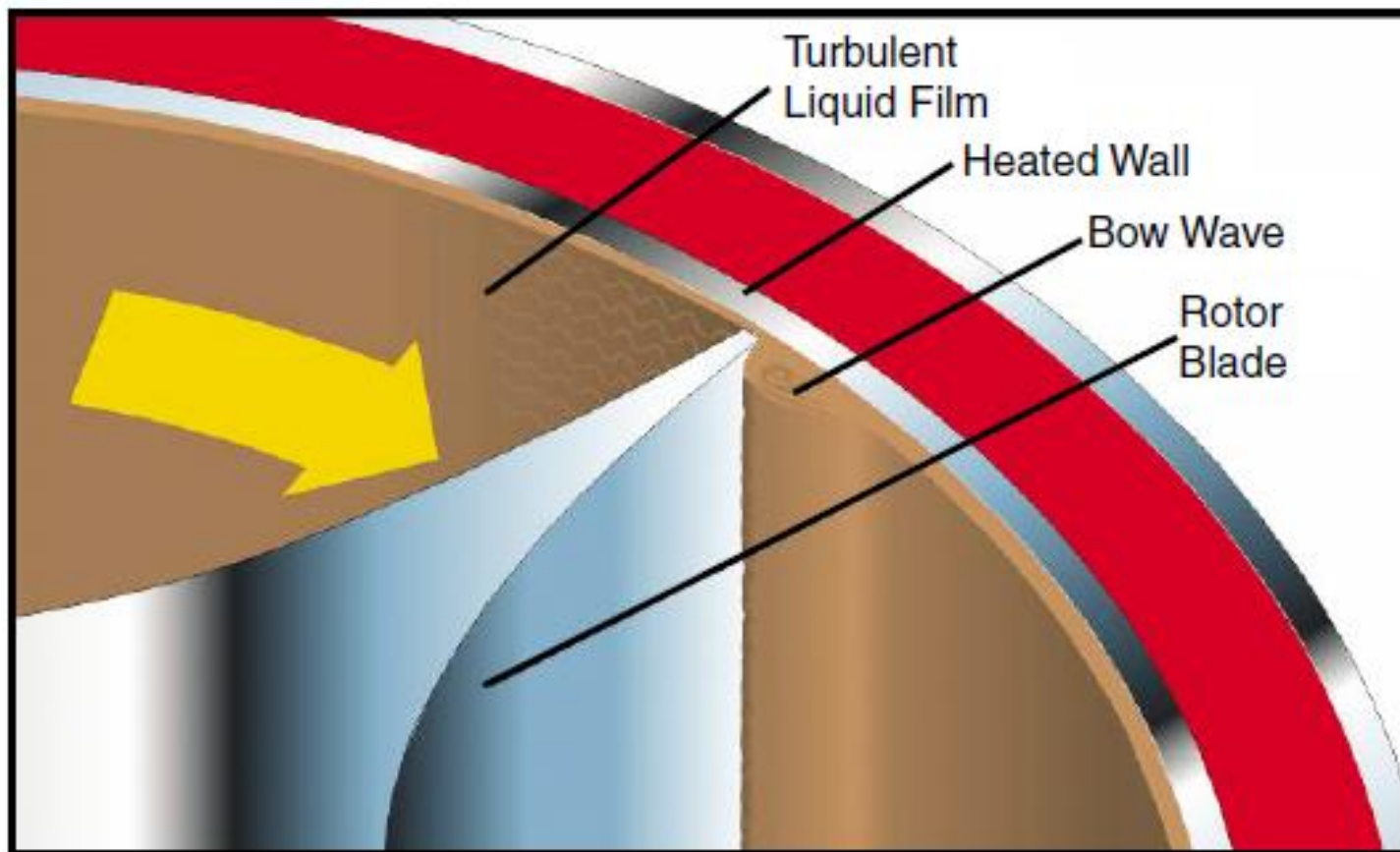
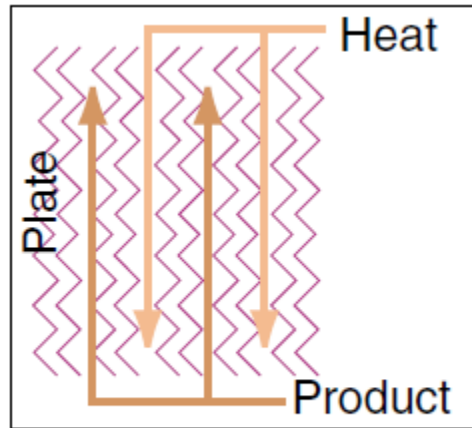


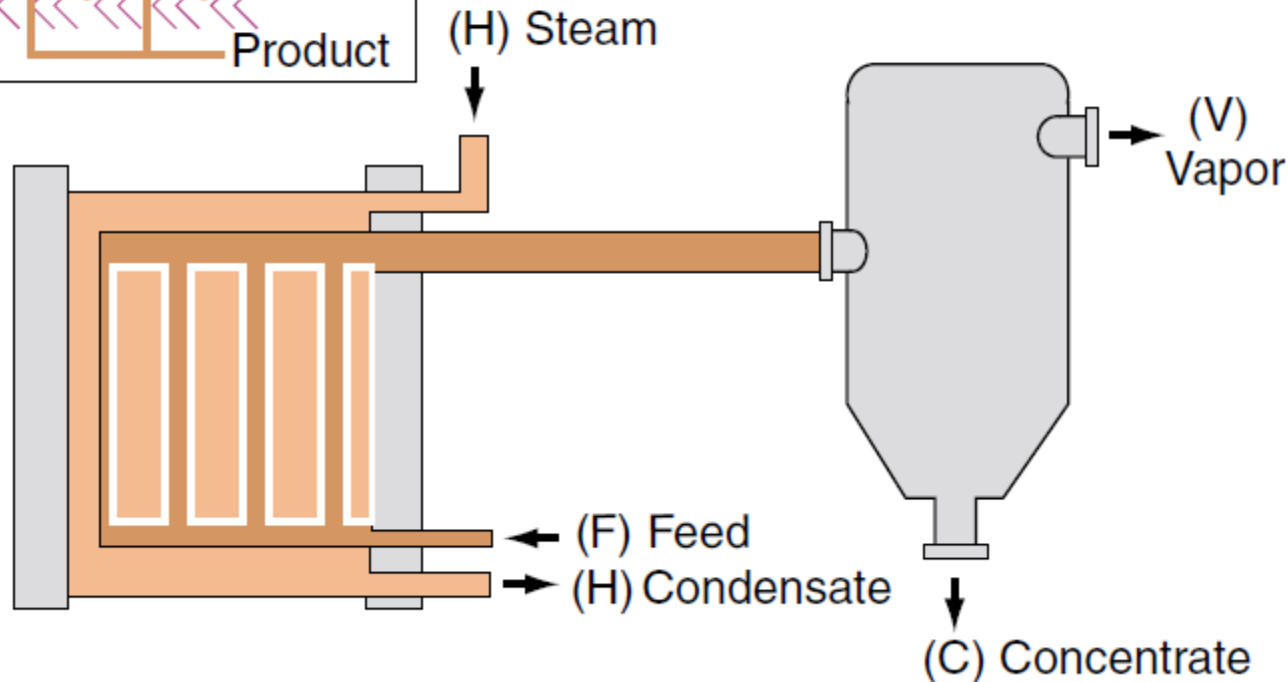
Figure 9. A rotor distributes the feed evenly over the inner circumference of the body of the wiped-film evaporator.

General types of evaporators

10. Gasket plate-and-frame evaporator:





Typical application: stripping applications, removing monomers from polymers and deodorization



Evaporator selection

Operational category	Evaporator type	Feed condition ^a							Suitable for heat-sensitive products	Retention time ^b (s)	Holding volume ^c (m ³)
		Very viscous (above 2000 mN s/m ²)	Med. viscosity (100 – 1000 mN s/m ²)	Low viscosity to water (max. 100 mN s/m ²)	Foaming	Scaling or fouling	Crystal producing	Solids in suspension			
Recirculating	Calandria ^d (short vertical tube)								No	168	3.03
	Forced circulation								Yes	41.6	12.8
	Falling film								No ^e	Not available	Not available
	Natural circulation (thermo-siphon)								No ^e	16	10.1
Single pass	Agitated film (vertical or horizontal)								Yes	1.0	1.0
	Tubular (long tube)								Yes	Not available	Not available
	Falling film Rising film										
Single pass special type	Rising-Falling concentrator								Yes	0.45	0.79
	Plate (can be recirculating)								Yes	Not available	Not available

 = Applicable to conditions noted
  = Applicable over lower portion of range noted

a. Viscosities are at operating temperatures
 b. Based on agitated film evaporator = 1.0
 c. Based on agitated film evaporator = 1.0, proportioned to equal surface
 d. Special disengagement arrangement required for foamy liquids
 e. May be used in special cases

Reference: Coulson & Richardson Vol. 2, Ch. 14

Flow characteristics in vertical film flow

- The liquid film can be observed in different hydrodynamic conditions.
- This conditions are characterized by **film Reynolds number**, defined as follows:

$$Re_{\text{film}} = \frac{4 \times \text{Mass flow rate} / \text{circumference}}{\text{Liquid viscosity}} = \frac{4 (m/\pi D)}{\mu_L} = \frac{4m}{\pi D \mu_L}$$

m total mass flow rate of condensate (kg/s)

D tube diameter (m)

μ_L liquid viscosity (Pa.s)

Flow characteristics in vertical film flow

Pure laminar flow

- $Re_{\text{film}} < 30$
- This flow condition can hardly ever be encountered in technical processes.
- Only in very viscous flows this flow condition can be encountered. But even that in literature it is mentioned that wavy behavior was observed!.



$Re_{\text{film}} = 10$ (already wavy)

Flow characteristics in vertical film flow

Wavy laminar flow

- $Re_{\text{film}} < 1800$
- The thickness of a wavy laminar fluid film is reduced compared to a pure laminar film.
- Smaller average film thickness and increased partial turbulence yield a higher heat transfer compared to pure laminar flow conditions.



$$Re_{\text{film}} = 500$$

Flow characteristics in vertical film flow

Turbulent flow

$$Re_{\text{film}} > 1800$$

- Apart from the near to the wall laminar sub layer the flow is fully turbulent.

-In this region, heat transfer increases with increased turbulence which means with increased Reynolds number.



$$Re_{\text{film}} = 5000$$

Heat Transfer in evaporators

Design equation: $q = U A \Delta T = U A (T_S - T_1)$

T_S : Steam temperature

T_1 : Temperature in the evaporator

A: Surface area

- The overall heat transfer coefficient U consists of the following:
 - steam-side condensation coefficient.
 - a metal wall with small resistance (depending on steam pressure, wall thickness)
 - scale resistance on the process side.
 - a liquid film coefficient on the process side.

Heat Transfer in evaporators

- For laminar flow ($Re_{\text{film}} < 1800$), the steam-side condensation coefficient for **vertical** surfaces can be calculated by the following equation:

$$Nu = \frac{h L}{k_L} = 1.13 \left[\frac{\rho_L (\rho_L - \rho_V) g \lambda L^3}{\mu_L k_L \Delta T} \right]^{0.25}$$

- For laminar flow ($Re_{\text{film}} < 1800$), the steam-side condensation coefficient for **horizontal** surfaces can be calculated by the following equation:

$$Nu = \frac{h D}{k_L} = 0.725 \left[\frac{\rho_L (\rho_L - \rho_V) g \lambda D^3}{N \mu_L k_L \Delta T} \right]^{0.25}$$

Heat Transfer in evaporators

Nu	Nusselt number
h	heat transfer coefficient ($W/m^2.K$)
L	vertical height of tubes (m)
k_L	liquid thermal conductivity ($W/m.K$)
ρ_L	liquid density (kg/m^3)
ρ_V	vapour density (kg/m^3)
g	$= 9.8066 \text{ m/s}^2$
λ	latent heat (J/kg)
μ_L	liquid viscosity ($Pa.s$)
ΔT	$= T_{sat} - T_{wall}$ (K)
D	outside tube diameter (m)
N	Number of horizontal tubes

Heat Transfer in evaporators

- All physical properties of the liquid are evaluated at the film temperature

$$T_{\text{film}} = (T_{\text{sat}} + T_{\text{wall}})/2.$$

- λ (latent heat of condensation) is evaluated at T_{sat} .

- For turbulent flow ($Re_{\text{film}} > 1800$), the steam-side condensation coefficient for vertical surfaces can be calculated by the following equation:

$$Nu = \frac{h L}{k_L} = 0.0077 \left[\frac{\rho_L^2 g L^3}{\mu_L^2} \right]^{1/3} Re^{0.4}$$

Heat Transfer in evaporators

TABLE 8.3-1. *Typical Heat-Transfer Coefficients for Various Evaporators**
(B3, B4, L1, P2)

<i>Type of Evaporator</i>	<i>Overall U</i>	
	$W/m^2 \cdot K$	$Btu/h \cdot ft^2 \cdot ^\circ F$
Short-tube vertical, natural circulation	1100–2800	200–500
Horizontal-tube, natural circulation	1100–2800	200–500
Long-tube vertical, natural circulation	1100–4000	200–700
Long-tube vertical, forced circulation	2300–11 000	400–2000
Agitated film	680–2300	120–400

* Generally, nonviscous liquids have the higher coefficients and viscous liquids the lower coefficients in the ranges given.

Methods of operation of evaporators:

1. Single-effect evaporation:

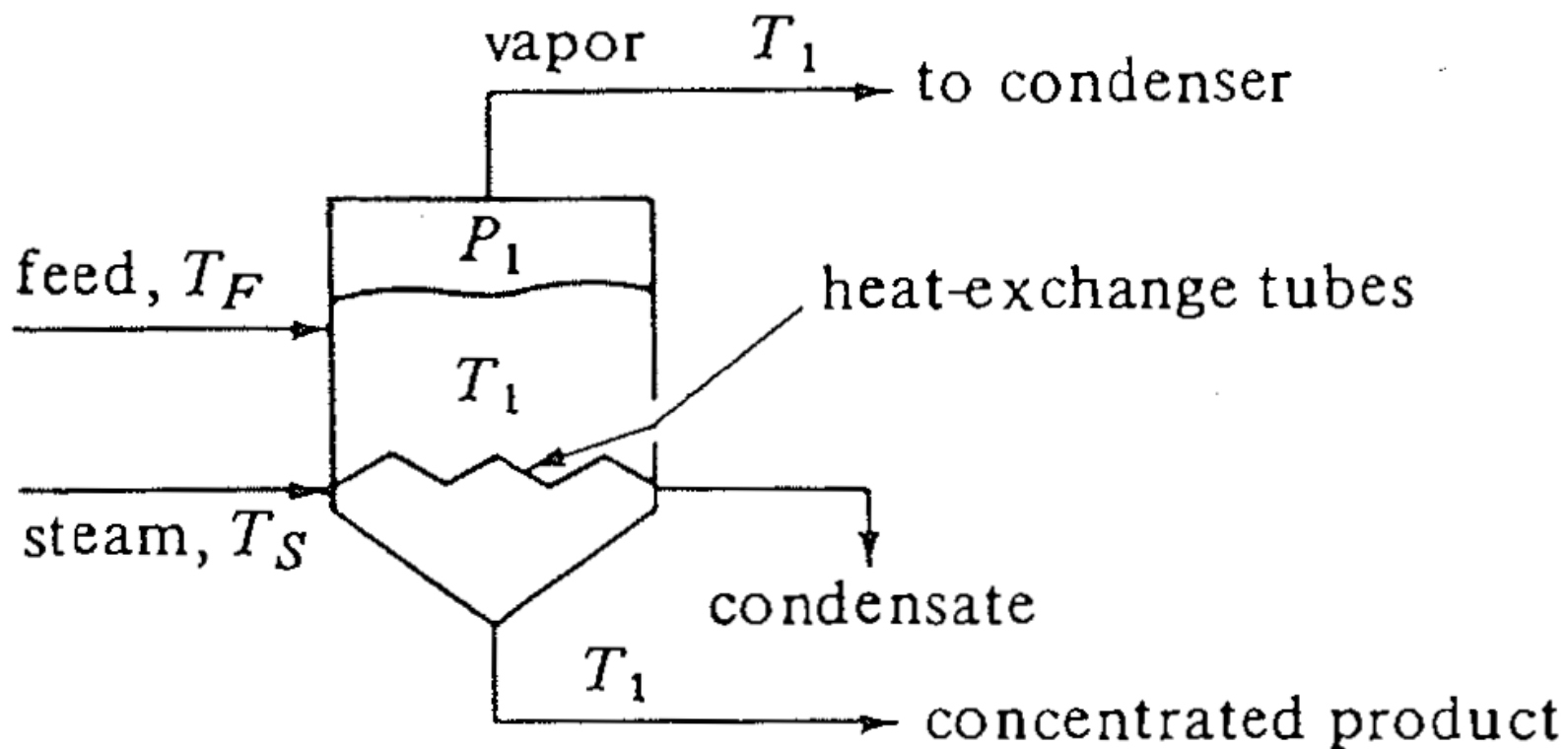


FIGURE 8.2-2. Simplified diagram of single-effect evaporator.

Methods of operation of evaporators:

Single-effect evaporator:

- When a **single evaporator** is used , the vapor from the boiling liquid is condensed and discarded. This is called single effect evaporation.
- It is simple but utilizes steam ineffectively.
- To evaporate 1 kg of water from the solution we require 1-1.3 kg of steam.
- Wasteful of energy since the latent heat of vapor leaving is discarded.
- Used when the required capacity of operation is relatively small and/or the cost of steam is relatively cheap compared to the evaporator cost.

Methods of operation of evaporators:

2. Forward-feed multiple-effect evaporator:

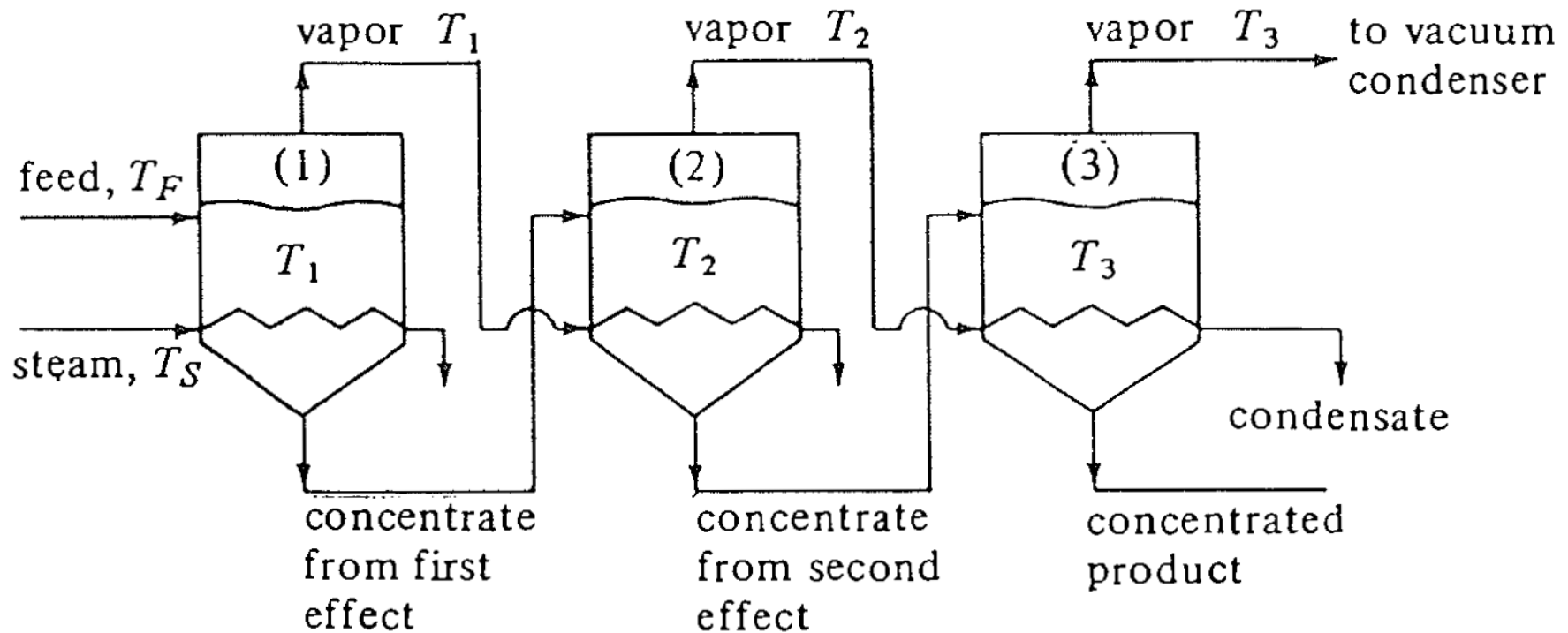
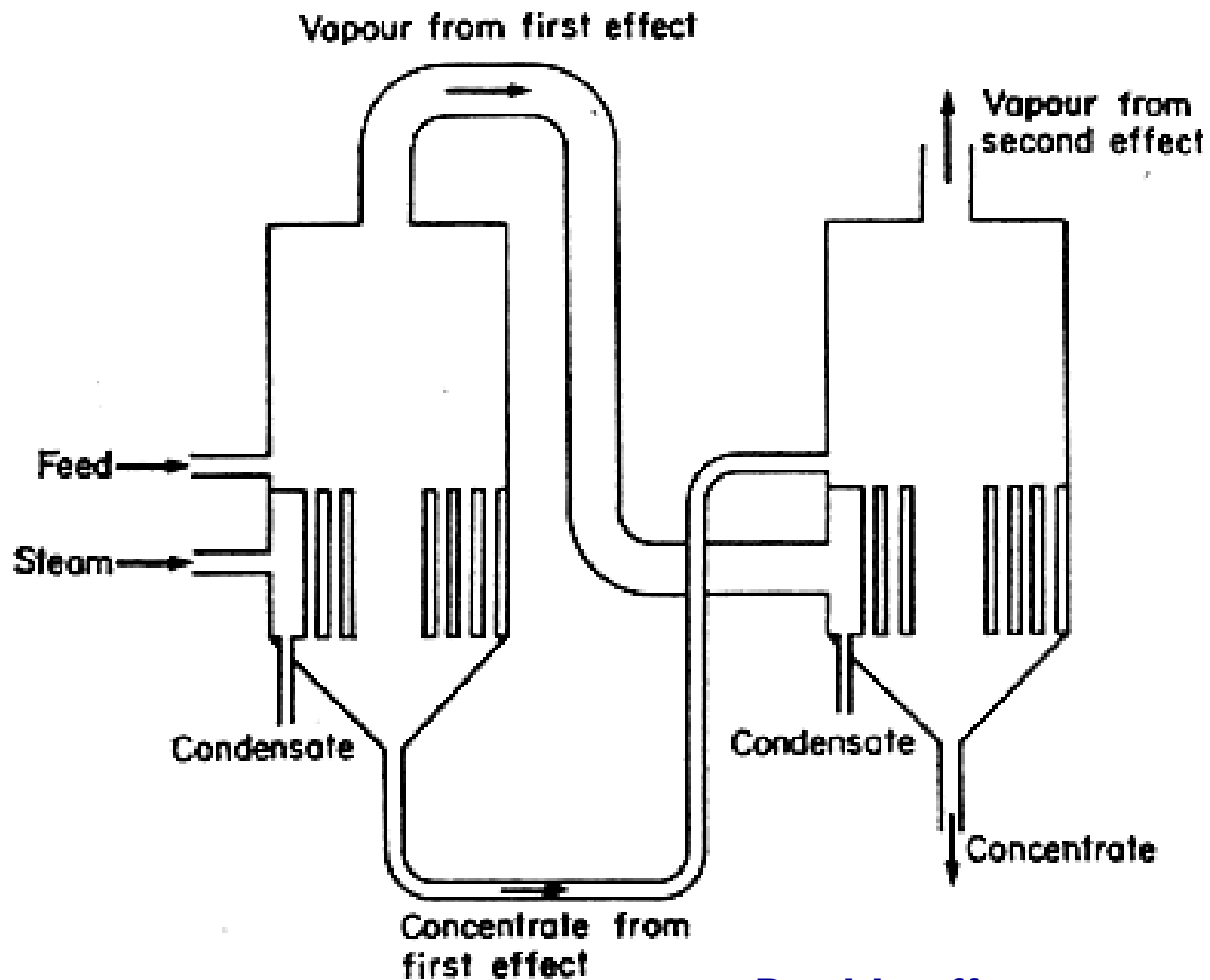


FIGURE 8.2-3. Simplified diagram of forward-feed triple-effect evaporator.

- The fresh feed is added to the first effect and flows to the next effect in the same direction as the vapor flows.

Methods of operation of evaporators:



Double-effect evaporator

Methods of operation of evaporators:

Forward-feed multiple-effect evaporator:

- Latent heat is recovered and reused.
- The vapor leaves the first evaporator is used to heat the second evaporator, and the vapor leaves the second evaporator is used to heat the third one, and so on.
- In rough approximation, around 3 kg of water will be evaporated for 1 kg of steam for three-effect evaporator.
- **Economy (kg vapor evaporated/kg steam used)** and the first cost of evaporation equipment increase with increasing effects.
- **Forward-feed** is used when the feed is hot or when the final concentrated product might be damaged at high temperature.
- The temperature and pressure decrease from first effect to second effect and so on.

Methods of operation of evaporators:

3. Backward-feed multiple-effect evaporator:

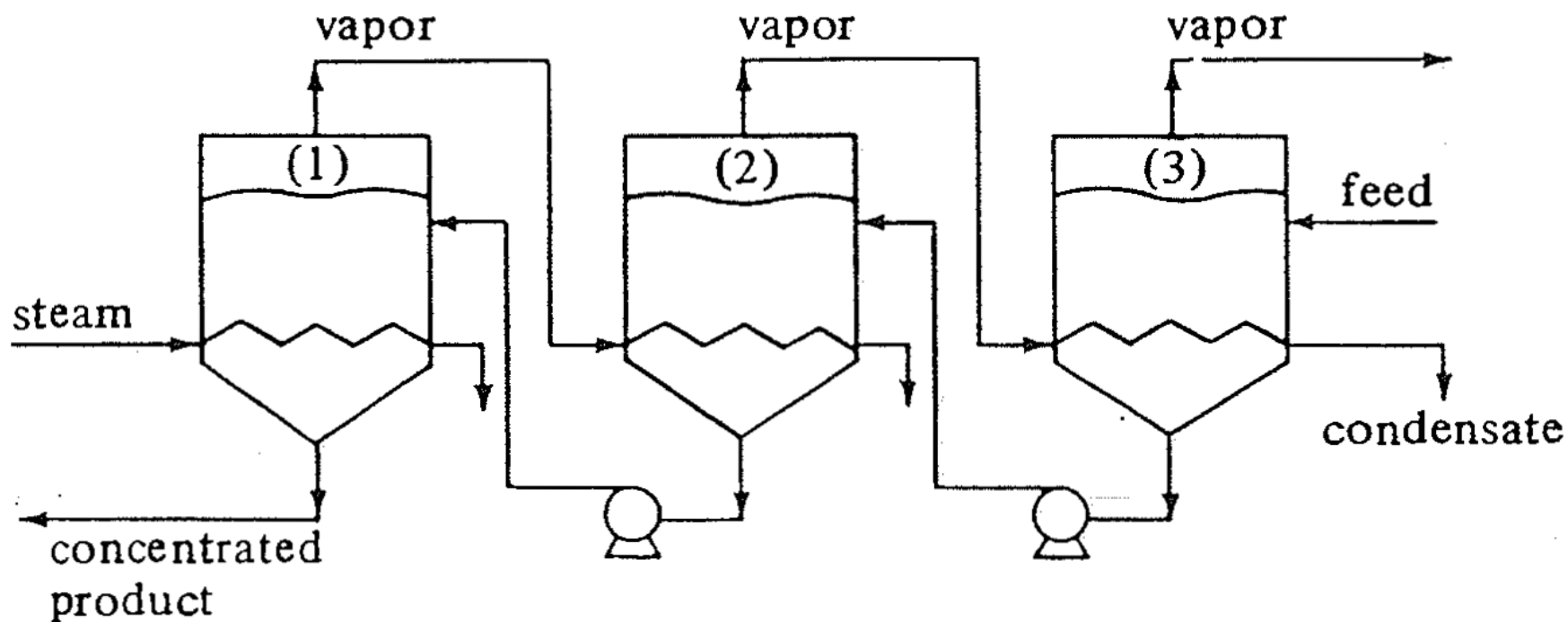


FIGURE 8.2-4. Simplified diagram of backward-feed triple-effect evaporator.

- The fresh feed enters the last and the coldest effect and the concentrate product leaves the first effect.

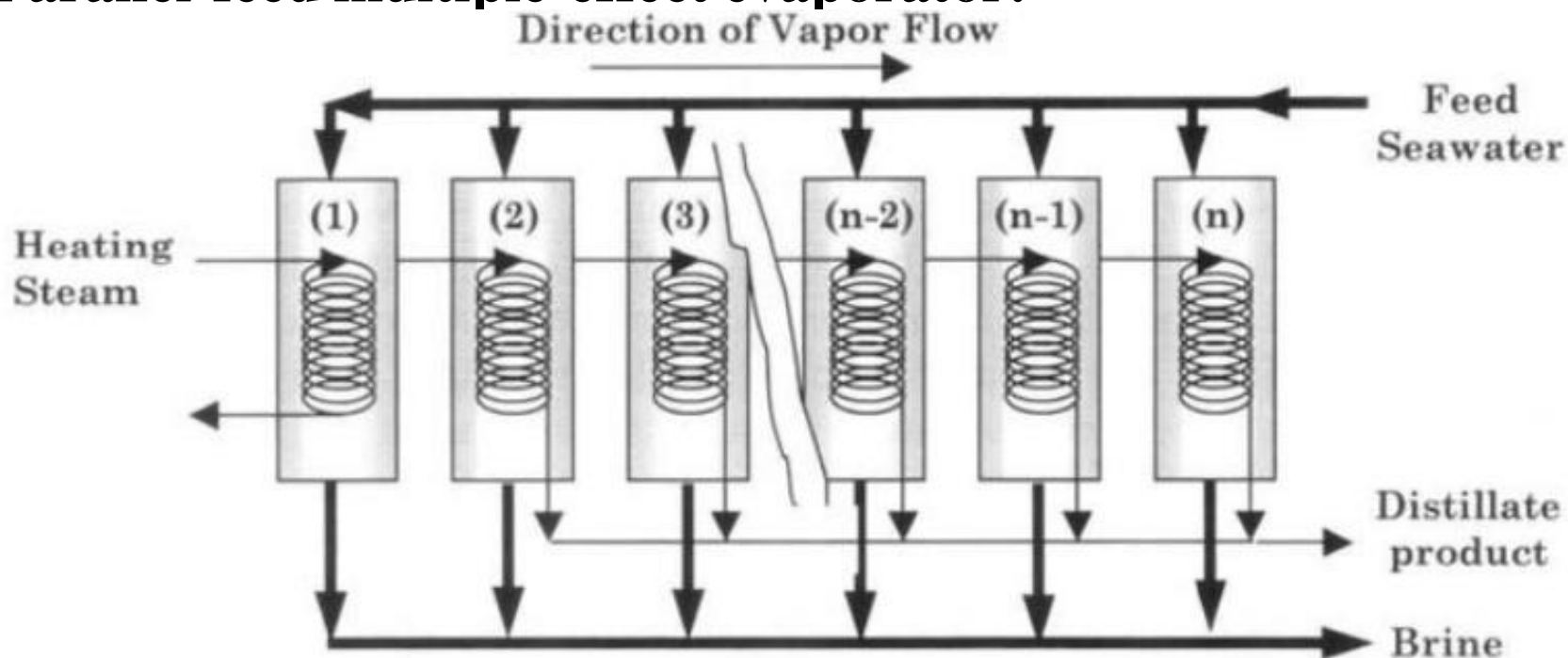
Methods of operation of evaporators:

Backward-feed multiple-effect evaporator:

- Reverse feed is advantageous when fresh feed is cold (smaller amount of liquid must be heated to higher temperatures in the second and first effects).
- Liquid solution pumps are used in each effect since the flow is from low to high pressure.
- Effective when the concentrated product is highly viscous.

Methods of operation of evaporators:

4. Parallel-feed multiple-effect evaporator:



- Addition of fresh feed and withdrawal of concentrated product from each effect.
- The vapor from each effect is still used to heat the next effect.
- Mainly used when the feed is almost saturated and solid crystals are the product, e.g., evaporation of brine to make salt.

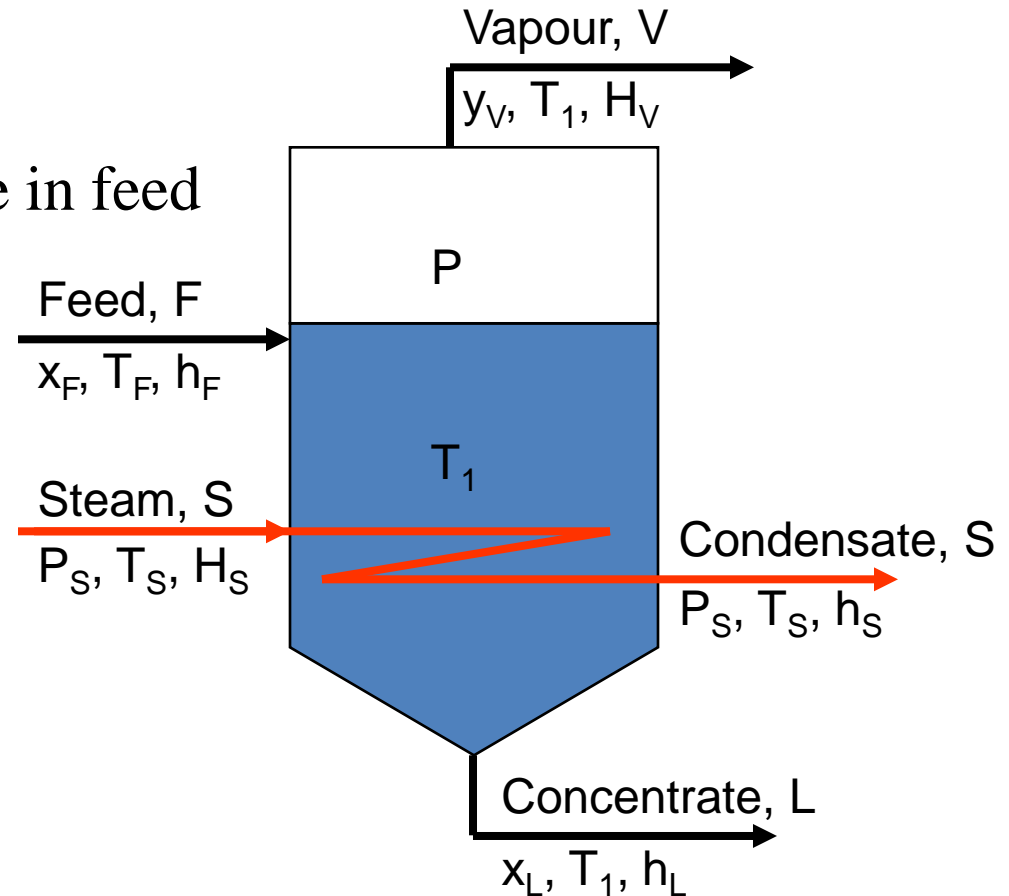
Calculation methods for single-effect evaporators

Feed:

F mass flow rate
 x_F mass fraction of solute in feed
 T_F temperature of feed
 h_F enthalpy of feed

Steam:

S mass flow rate
 P_S steam pressure
 T_S steam temperature
 H_S enthalpy of steam
 h_S enthalpy of condensate



P : pressure in the evaporator
 T_1 : temperature in the evaporator

Calculation methods for single-effect evaporators

Vapor leaving the evaporator:

V	mass flow rate
y_V	mass fraction of solute in vapor
T_1	temperature of vapor
H_V	enthalpy of vapor

Concentrate leaving the evaporator:

L	mass flow rate
x_L	mass fraction of solute in concentrate
T_1	temperature of concentrate
h_L	enthalpy of concentrate

Calculation methods for single-effect evaporators

Overall material balance:

$$F = L + V$$

Solute mass balance:

$$F x_F = L x_L + V y_V$$

If the vapor is free of solute:

$$F x_F = L x_L$$

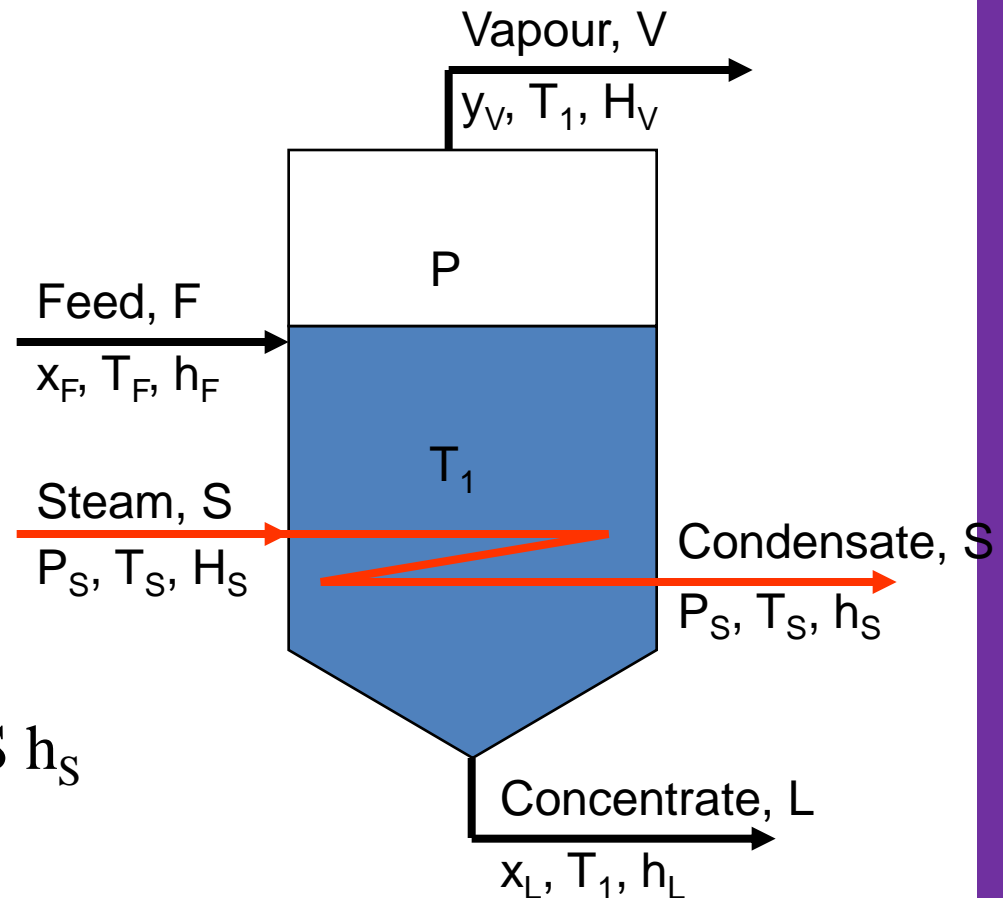
Heat balance:

$$F h_F + S H_S = L h_L + V H_V + S h_s$$

Rewriting:

$$F h_F + S (H_S - h_s) = L h_L + V H_V$$

$$F h_F + S \lambda = L h_L + V H_V \quad \text{where } \lambda = H_S - h_s$$



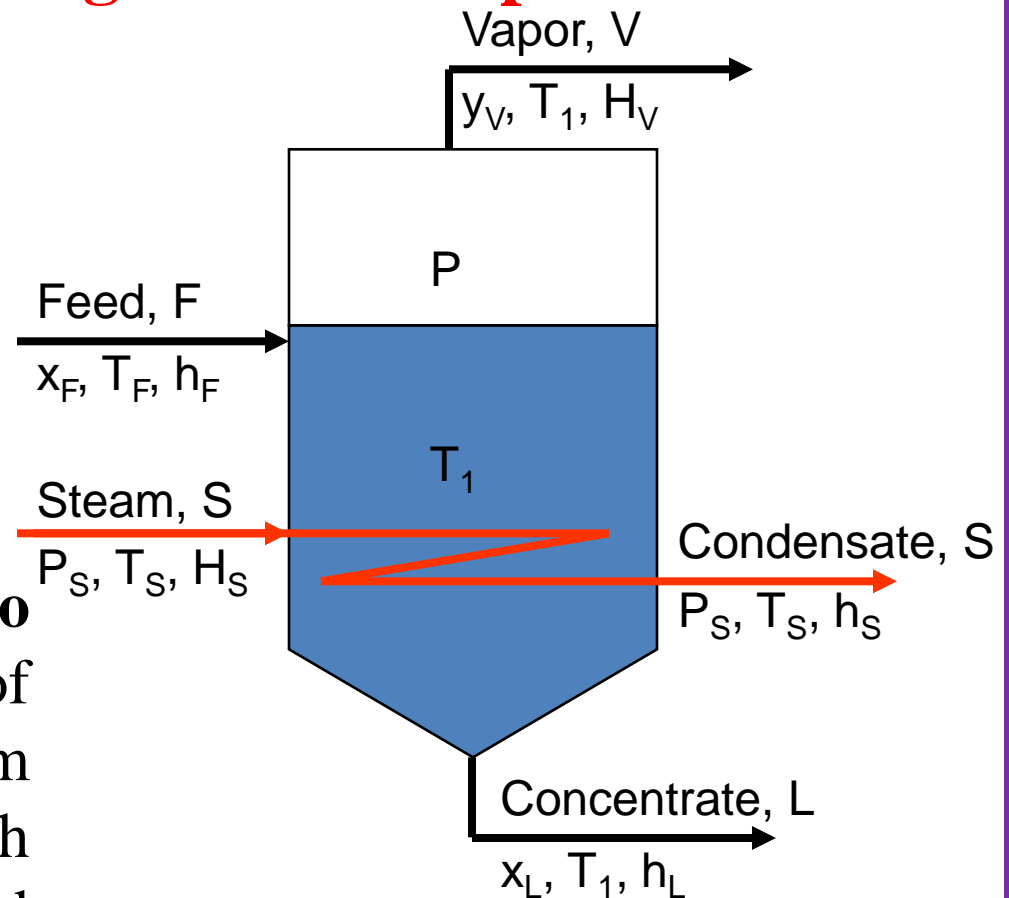
Calculation methods for single-effect evaporators

- Energy lost by the steam

$$q = S \lambda = S (H_S - h_S)$$

- In case of **no energy loss to the environment**, q amount of energy gets transferred from steam to the solution through the tube wall of area A and overall heat transfer coefficient U . Therefore,

$$q = U A \Delta T = U A (T_S - T_1)$$



$$\text{Steam economy} = V/S$$

(kg vaporized/kg steam used)

Calculation methods for single-effect evaporators

Example 8.4.1. A continuous single-effect evaporator concentrates 9072 kg/h of a 1.0 wt % salt solution entering at 37.8 °C to a final concentration of 1.5 wt %.

The vapor space of the evaporator is at 101.325 kPa (1.0 atm abs) and the steam supplied is saturated at 143.3 kPa. The overall heat-transfer coefficient $U = 1704 \text{ W/m}^2\cdot\text{K}$. The heat capacity of the feed is assumed to be $C_p = 4.14 \text{ kJ/(kg}\cdot\text{K)}$.

Calculate the amounts of vapor and liquid products, steam economy, the heat-transfer area required. Assumed that, since it is dilute, the solution has the same boiling point as water.

Calculation methods for single-effect evaporators

Example 8.4.1.

Overall material balance:

$$F = L + V \quad (1)$$

Solute mass balance:

$$F x_F = L x_L \text{ (no solute in the vapor)} \quad (2)$$

$$F = 9072 \text{ kg/h}$$

$$x_F = 1 \text{ wt \%} = 0.01 \text{ kg solute / kg feed}$$

$$x_L = 1.5 \text{ wt \%} = 0.015 \text{ kg solute / kg liquid product}$$

Solve Eqns. (1) and (2) to have :

$$\mathbf{L = 6048 \text{ kg/h}}$$

$$\mathbf{V = 3024 \text{ kg/h}}$$

Calculation methods for single-effect evaporators

Example 8.4.1.

Heat balance:

$$F h_F + S \lambda = L h_L + V H_V$$

where $\lambda = H_S - h_S$

$$q = S \lambda = U A \Delta T = U A (T_S - T_1) \rightarrow A = S \lambda / [U (T_S - T_1)]$$

$$\begin{aligned} S \lambda &= L h_L + V H_V - F h_F \\ &= (F - V) h_L + V H_V - F h_F \\ &= F (h_L - h_F) + V (H_V - h_L) \\ &= F C_p (T_1 - T_F) + V (\text{Latent heat at } 101.325 \text{ kPa}) \end{aligned}$$

Calculation methods for single-effect evaporators

Example 8.4.1.

$$F = 9072 \text{ kg/h}; L = 6048 \text{ kg/h}, V = 3024 \text{ kg/h} ; T_F = 37.8^\circ\text{C}$$

$$U = 1704 \text{ W/m}^2.\text{K} ; C_p = 4.14 \text{ kJ}/(\text{kg}.\text{K})$$

From steam tables (Appendix A.2):

$$T_1 = \text{saturated temperature at } P (= 101.325 \text{ kPa}) = 100^\circ\text{C}$$

$$T_s = \text{saturated steam temperature at } P_s (= 143.3 \text{ kPa}) = 110^\circ\text{C}$$

Latent heat of vaporization at 101.325 kPa

$$= H_V - h_L = 2676.1 - 419.04 = 2257.1 \text{ kJ/kg}$$

Latent heat of vaporization at 143.3 kPa:

$$\lambda = H_s - h_s = 2691.5 - 461.3 = 2230.2 \text{ kJ/kg}$$

Calculation methods for single-effect evaporators

Example 8.4.1.

$$F = 9072 \text{ kg/h}; L = 6048 \text{ kg/h}, V = 3024 \text{ kg/h}; T_F = 37.8^\circ\text{C}$$

$$U = 1704 \text{ W/m}^2\cdot\text{K}; T_1 = 100^\circ\text{C}; T_S = 110^\circ\text{C}$$

$$C_p = 4.14 \text{ kJ/kg}\cdot\text{K}$$

$$\lambda = 2230.2 \text{ kJ/kg (at 143.3 kPa)}$$

$$\text{Latent heat of vaporization} = 2257.1 \text{ kJ/kg (at 101.325 kPa)}$$

$$\begin{aligned} q &= S \lambda = F C_p (T_1 - T_F) + V (\text{Latent heat at 101.325 kPa}) \\ &= 9161583 \text{ kJ/h} = 2544.9 \text{ kW} \end{aligned}$$

$$S = q / \lambda = 9161583 / 2230.2 = 4108.0 \text{ kg/h}$$

$$\text{Steam economy} = V / S = 0.736$$

$$A = q / [U (T_S - T_1)] = 149.3 \text{ m}^2$$

Effects of processing variables on evaporator operation:

Effect of feed temperature:

- The inlet temperature of the feed has a large effect on the evaporator operation.
- When feed is not at its boiling point, steam is needed first to heat the feed to its boiling point and then to evaporate it.
- Preheating the feed can reduce the size of evaporator heat-transfer area.

Effect of steam pressure:

- High pressure provides high T_s values, and hence $T_s - T_1$ will increase.
- High pressure steam is however more costly.
- Therefore, overall economic balances must be considered to determine the optimum steam pressure.

Effects of processing variables on evaporator operation:

Effect of pressure:

- Pressure in the evaporator sets the boiling point of the solution (T_1).
- Steam pressure determines the steam temperature (T_s)
- Since $q = U A (T_s - T_1)$, larger values of $(T_s - T_1)$ will help reduce the heat-transfer area needed and hence the cost of evaporator.
- Vacuum can be maintained in the solution side using a vacuum pump.
- For example, if the pressure in the previous example is reduced to 47.4 kPa, boiling point of water reduces to 80 °C and that would increase the $(T_s - T_1)$ from 10 °C to 30 °C. A large decrease in heat-transfer area would be obtained.

Boiling point rise (BPR) of solutions:

- or **Boiling point elevation (BPE)** of solutions

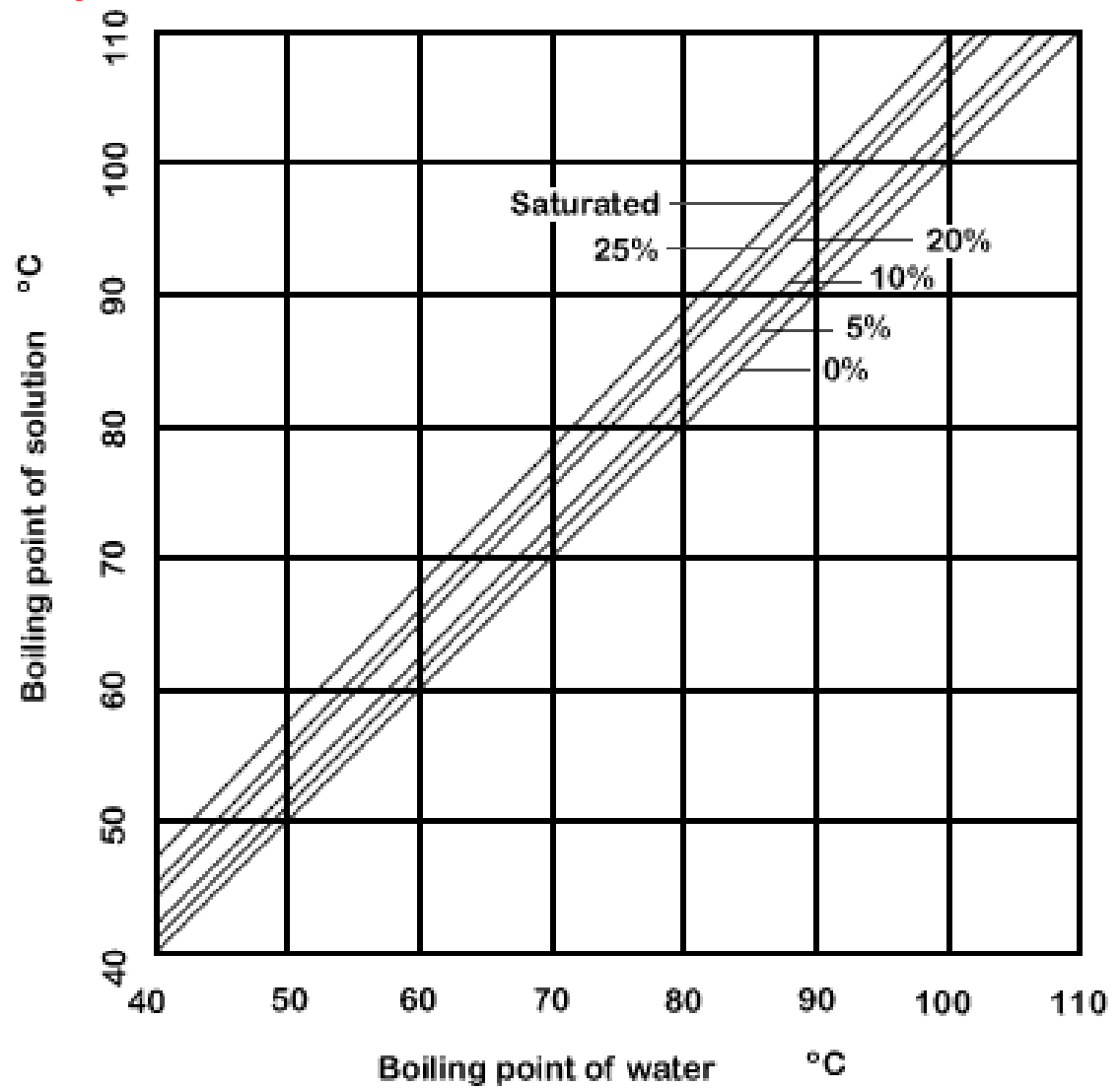
$$\text{BPE or BPR} = \text{BP}_{\text{solution}} - \text{BP}_{\text{solvent}}$$

-In Example 8.4.1, the solution is assumed to be dilute enough to be considered to have the same thermal properties as water. **It is not true always.**

- For concentrated solutions, heat capacity and boiling point are quite different from that of water.

- **Duhring's rule** is an empirical law that relates the boiling point of a solution to the boiling point of the solvent at different pressures for a solution of given concentration.

Boiling point rise (BPR) of solutions:



Dühring lines for boiling point of sodium chloride solutions

Boiling point rise (BPR) of solutions:

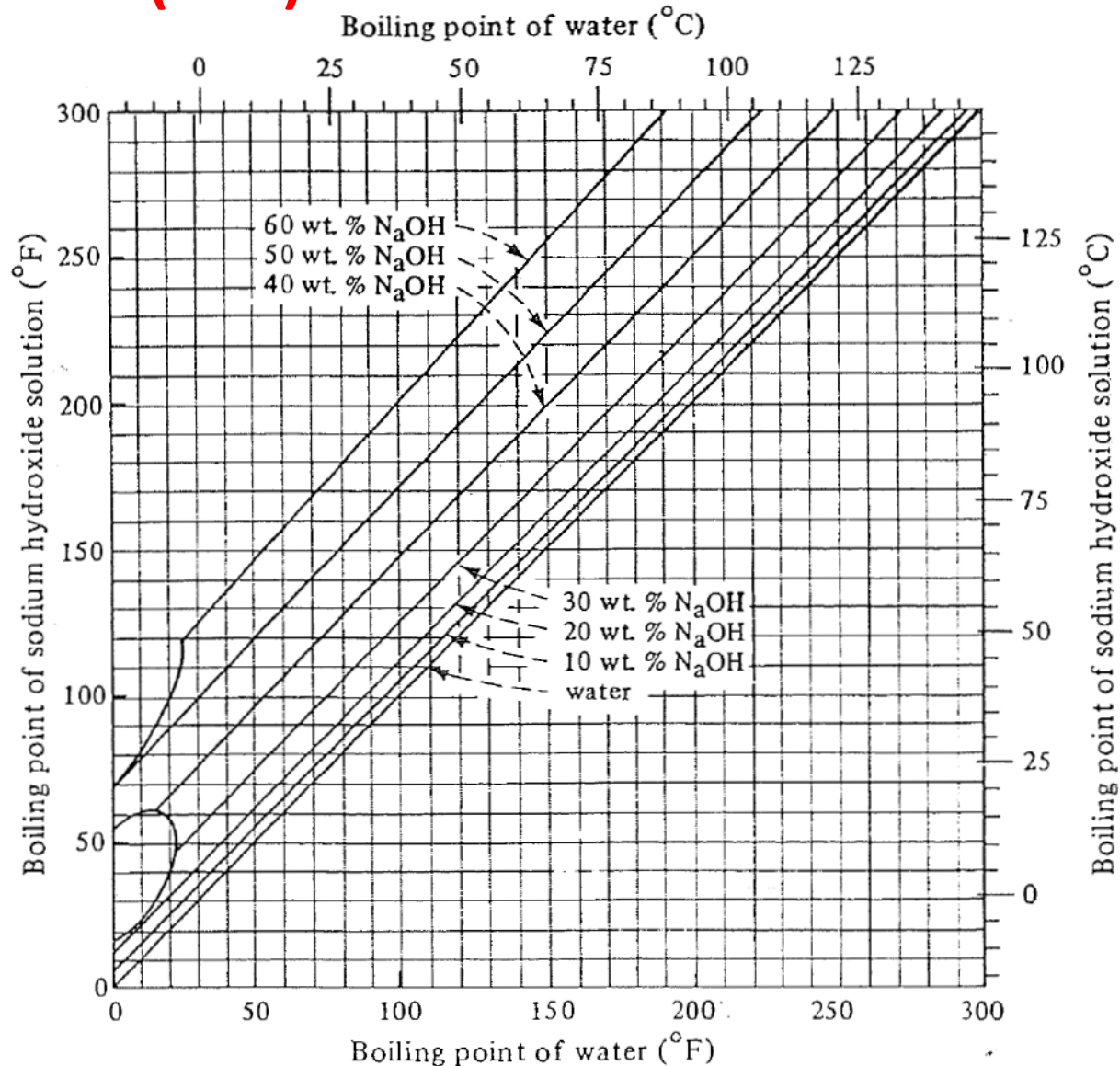


FIGURE 8.4-2. Dühring lines for aqueous solutions of sodium hydroxide.

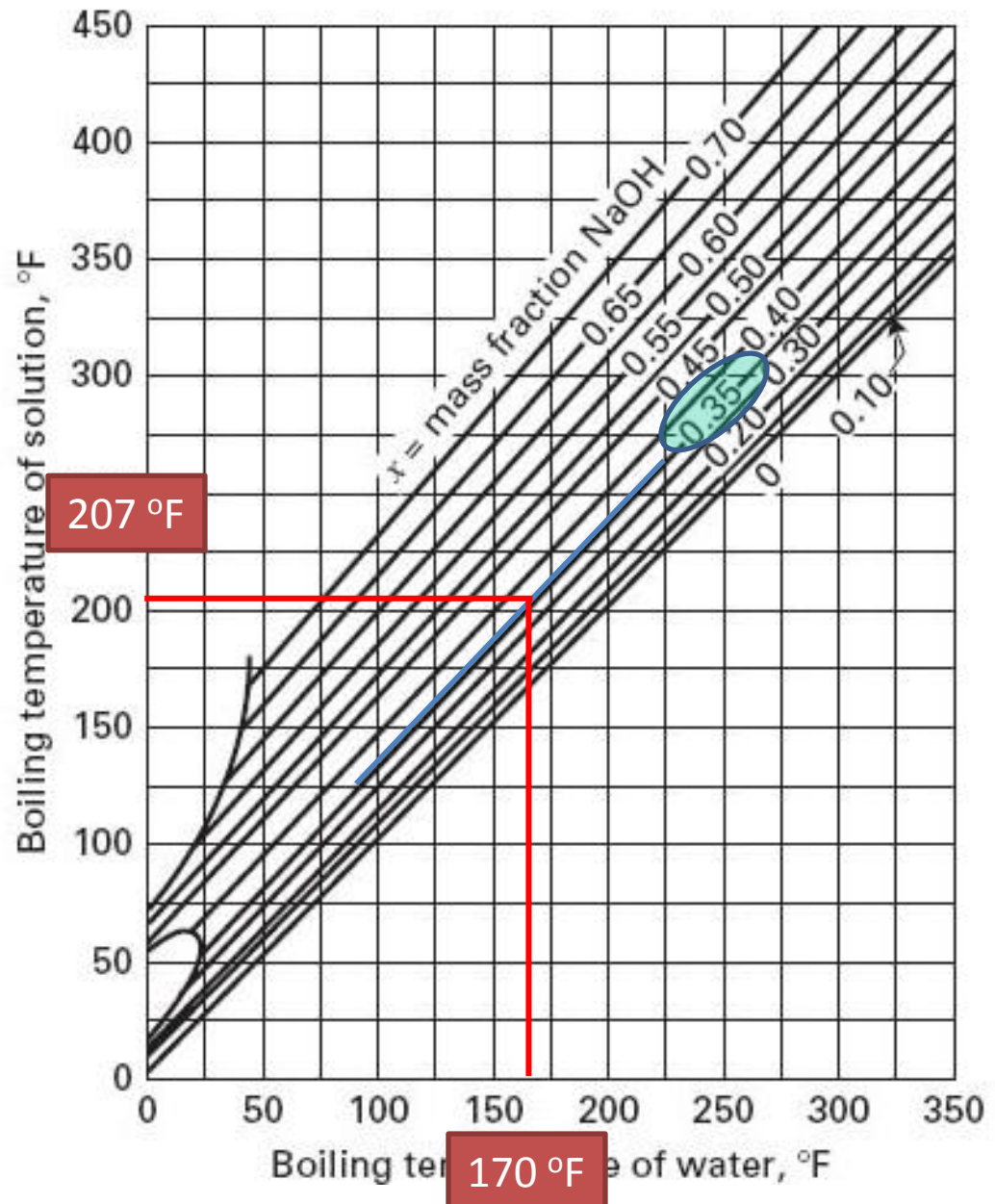
itions

Example: Approximate BPE for 35 wt% NaOH solution at 6 psia.

Using steam tables (Appendix A.2) at 6 psia:
Boiling point of pure water ; **$BP_w = 170\text{ }^{\circ}\text{F}$**

Duhring chart at
 $BP_w = 170\text{ }^{\circ}\text{F}$ and 35% \Rightarrow
Boiling point of solution
 $BP_{sol} = 210\text{ }^{\circ}\text{F}$

$BPE = 207 - 170 = 37\text{ }^{\circ}\text{F}$



Boiling point rise (BPR) of solutions:

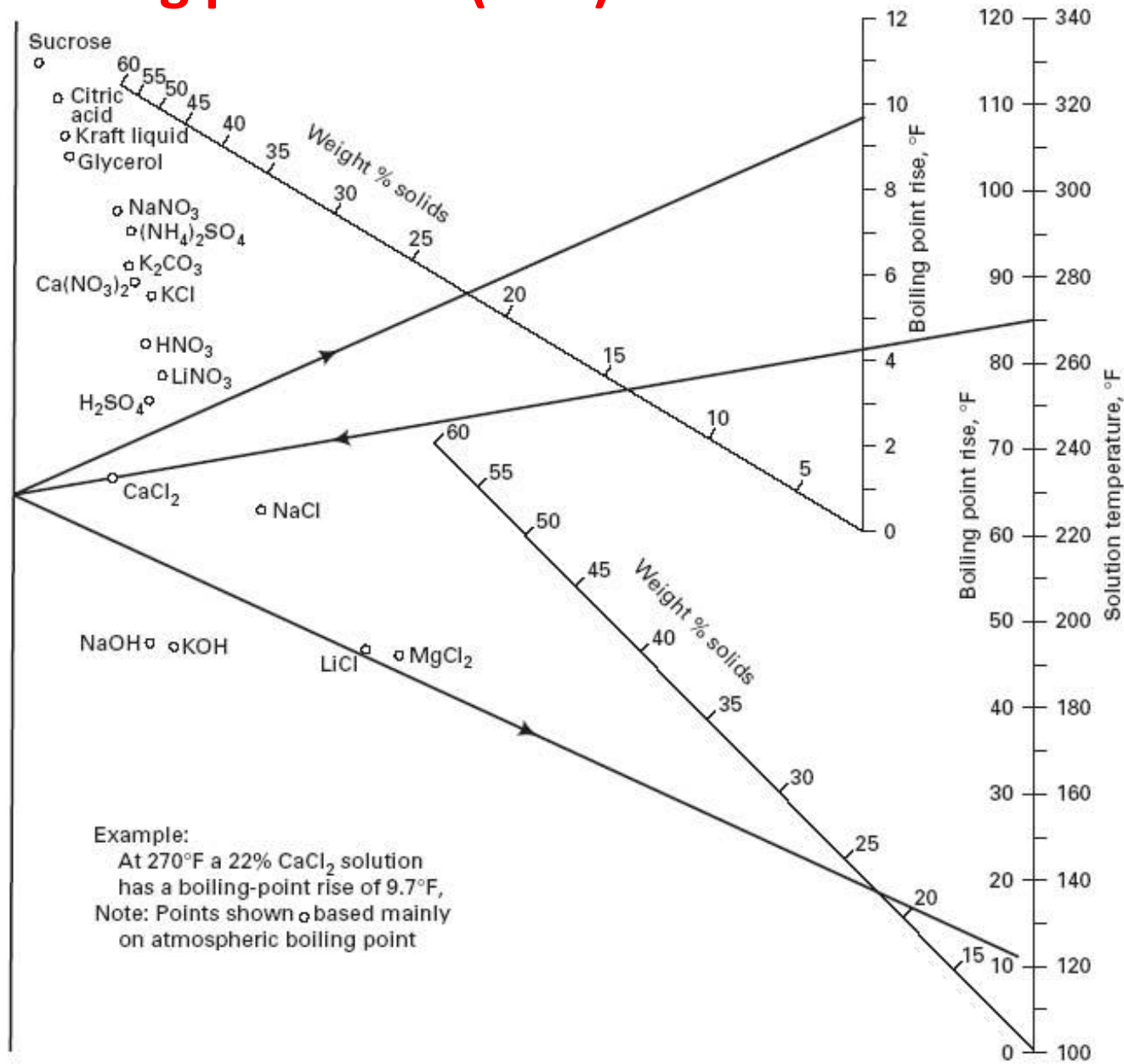


Figure 17.33 Nomograph for boiling-point elevation of aqueous solutions.

[From *Perry's Chemical Engineers' Handbook*, 6th ed., R.H. Perry, D.W. Green, and J.O. Maloney, eds., McGraw-Hill, New York (1984) with permission.]

Enthalpy-concentration charts of solutions:

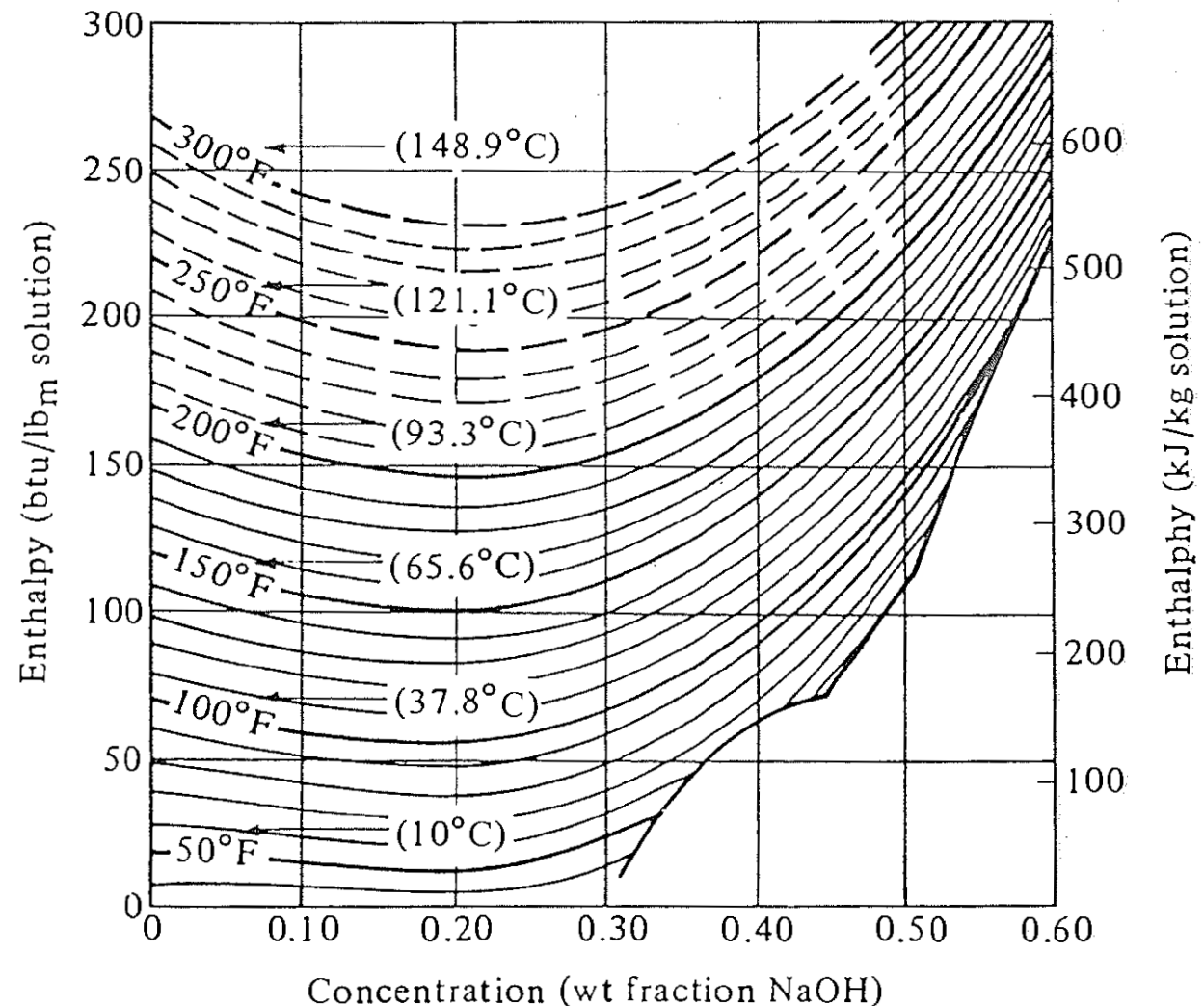
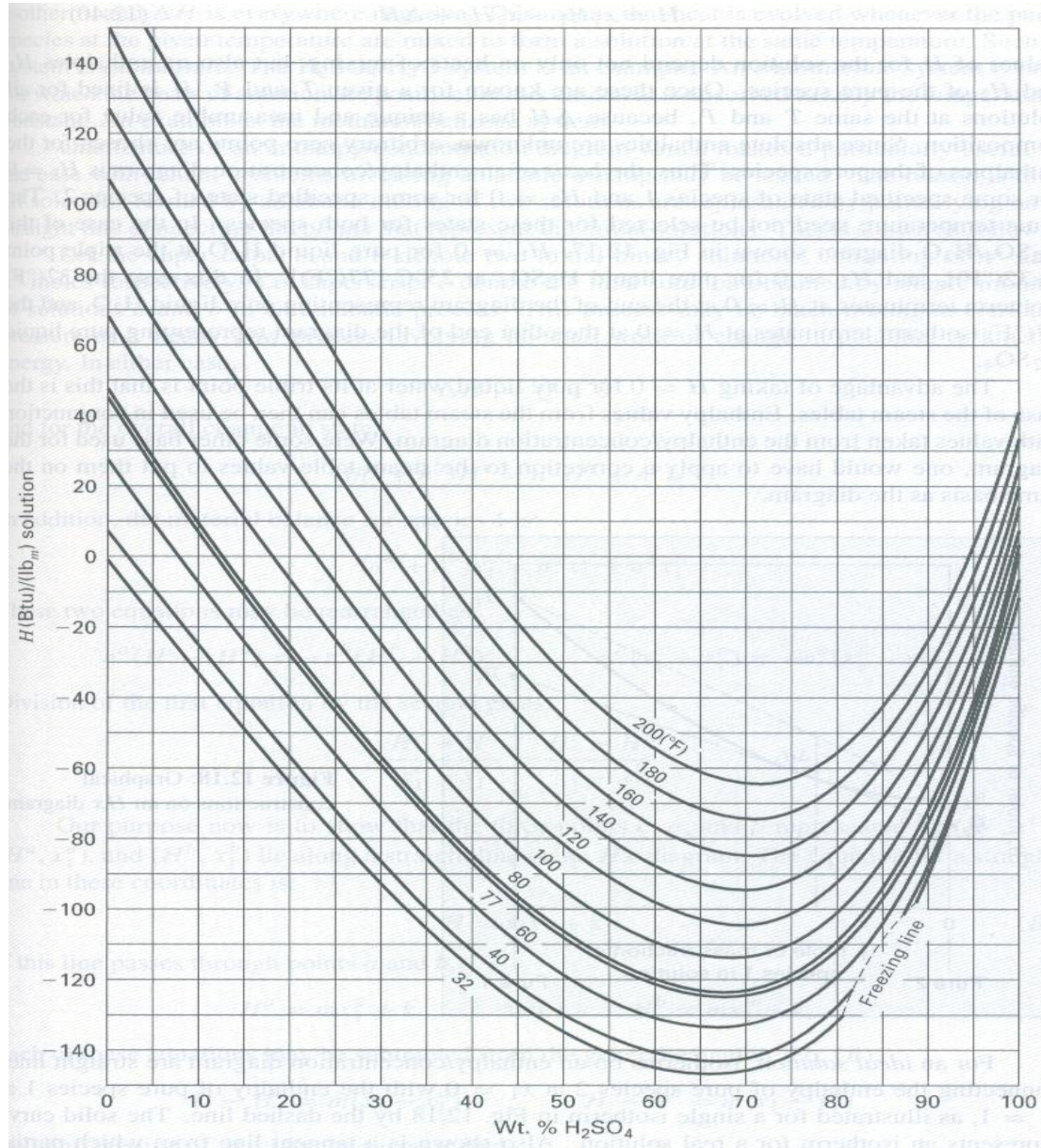


FIGURE 8.4-3. Enthalpy-concentration chart for the system NaOH-water. [Reference state liquid water at 0°C (273 K) or 32°F.] [From W. L. McCabe, *Trans. A.I.Ch.E.*, **31**, 129 (1935). With permission.]

Enthalpy-concentration charts of solutions:



Example 8.4.3. An evaporator is used to concentrate 4536 kg/h of a 20% NaOH solution entering at 60°C to a product of 50% solids. The pressure of the saturated steam used is 172.4 kPa and the vapor space pressure of the evaporator is at 11.7 kPa. The overall coefficient U is 1560 W/m².K. Calculate the steam used, the steam economy and the heating surface area.

Data provided:

$$F = 4536 \text{ kg/h}$$

$$x_F = 0.2$$

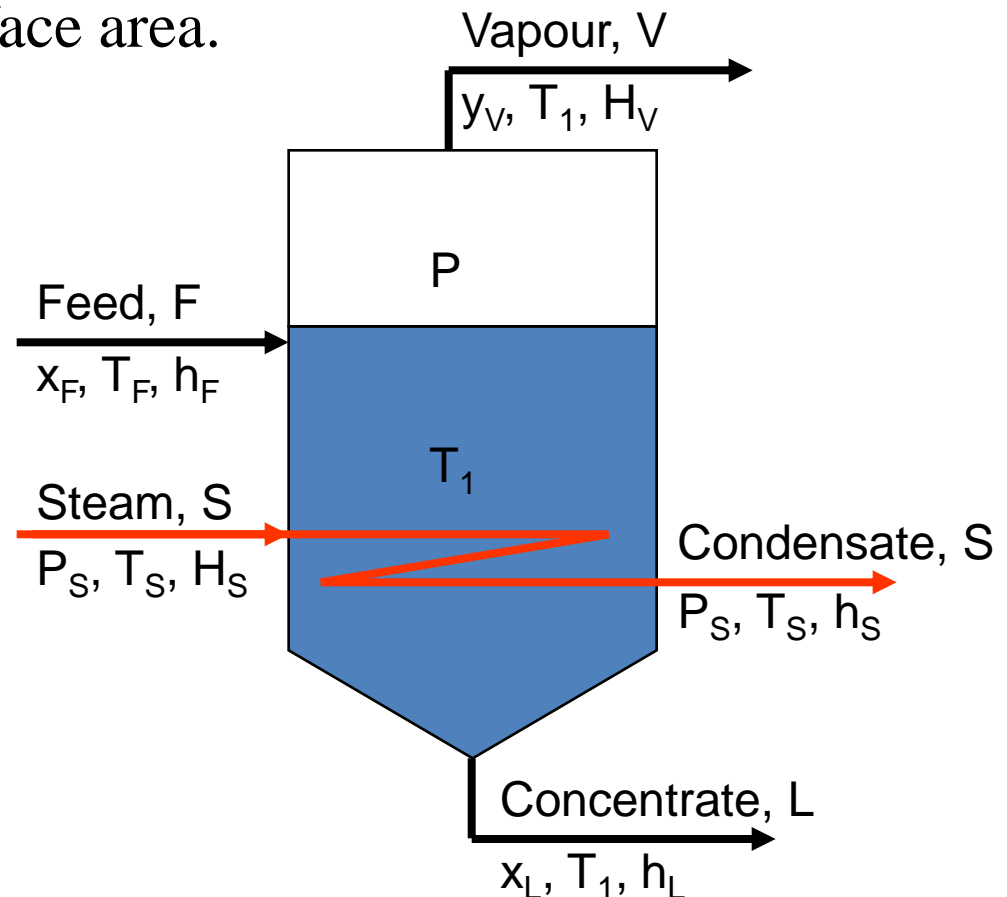
$$T_F = 60^\circ\text{C} = 140^\circ\text{F}$$

$$x_L = 0.5$$

$$P = 11.7 \text{ kPa}$$

$$P_S = 172.4 \text{ kPa}$$

$$U = 1560 \text{ W/m}^2\cdot\text{K}$$



Example 8.4.3.**Overall material balance:**

$$F = L + V \quad (1)$$

Solute mass balance:

$$F x_F = L x_L \quad (2)$$

$$F = 4536 \text{ kg/h} ; x_F = 0.2; x_L = 0.5$$

Solve Eqns. (1) and (2) to have **L = 1814 kg/h** and **V = 2722 kg/h****Heat balance:**

$$F h_F + S \lambda = L h_L + V H_V$$

From steam tables (Appendix A.2):

Boiling point of water at P (= 11.7 kPa) = 48.9 °C=120 °F

 T_S = saturated steam temperature at 172.4 kPa = 115.6 °C

Example 8.4.3.

h_F = enthalpy of 20% solution at 60°C = 214 kJ/kg
(using the enthalpy–concentration chart)

h_L = enthalpy of 50% solution at $T_1 = ?$
(using the enthalpy–concentration and boiling-point rise charts)

Using the boiling-point rise chart at boiling point of water = 120 °F and 50 wt% NaOH:

The boiling point of the solution, $T_1 = 89.5\text{ °C} = 193\text{ °F} >$ boiling point of water → The vapor is superheated.

h_L = enthalpy of 50% solution at 193 °F = 505 kJ/kg
(using the enthalpy–concentration chart)

H_V = enthalpy of superheated steam at (89.5°C and 11.7 kPa) = 2667 kJ/kg (using the superheated steam table)

Example 8.4.3.**From steam tables (Appendix A.2):**

For saturated steam at 172.4 kPa:

$$T_s = 115.6 \text{ }^{\circ}\text{C}$$

$$\lambda = H^v - H^l = 2214 \text{ kJ/kg}$$

Steam used = S

$$F h_F + S \lambda = L h_L + V H_V$$

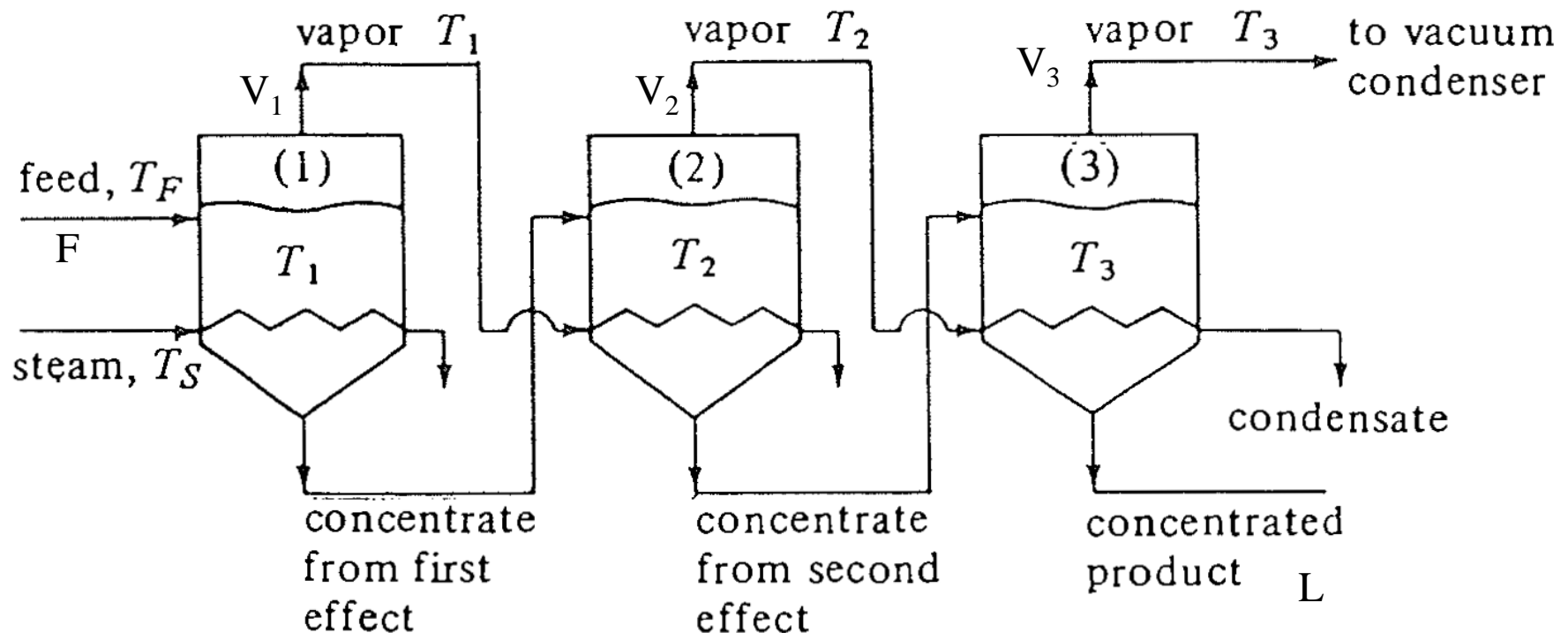
$$\rightarrow S = 3255 \text{ kg steam/h}$$

$$\text{Steam economy} = V / S = 2722 / 3255 = 0.836$$

$$A = S \lambda / U (T_s - T_1) = 49.2 \text{ m}^2$$

Exercise. Repeat Example 8.4.3 assuming that the thermal properties of the liquid in the evaporator can be approximated by those of water.

Calculation methods for multiple-effect evaporators



- Remember that mass/heat balances must be satisfied over each evaporator and over the whole process. For example, overall mass balance over the whole process is:

$$F = V_1 + V_2 + V_3 + L$$

Calculation methods for multiple-effect evaporators

- Heat balance for the first evaporator:

$$q_1 = U_1 A_1 \Delta T_1 = U_1 A_1 (T_s - T_1)$$

- Similarly, in the second evaporator, remembering that the "steam" in the second is the vapor from the first evaporator and that this will condense at approximately the same temperature as it boiled, since pressure changes are small,

$$q_2 = U_2 A_2 \Delta T_2 = U_2 A_2 (T_1 - T_2)$$

- Heat balance for the third evaporator:

$$q_3 = U_3 A_3 \Delta T_3 = U_3 A_3 (T_2 - T_3)$$

- and so on.

Calculation methods for multiple-effect evaporators

- Assuming that heat losses, boiling-point elevation of solution, and heat of solution are negligible, the feed is supplied at its boiling point to have approximately:

$$q_1 = q_2 = q_3 = \dots$$

That is, $U_1 A_1 \Delta T_1 = U_2 A_2 \Delta T_2 = U_3 A_3 \Delta T_3 = \dots$

- Further, if the evaporators are constructed so that $A_1 = A_2 = A_3$:.
 $q/A = U_1 \Delta T_1 = U_2 \Delta T_2 = U_3 \Delta T_3 = \dots$

That is, the temperature differences are inversely proportional to the overall heat transfer coefficients in the two effects.

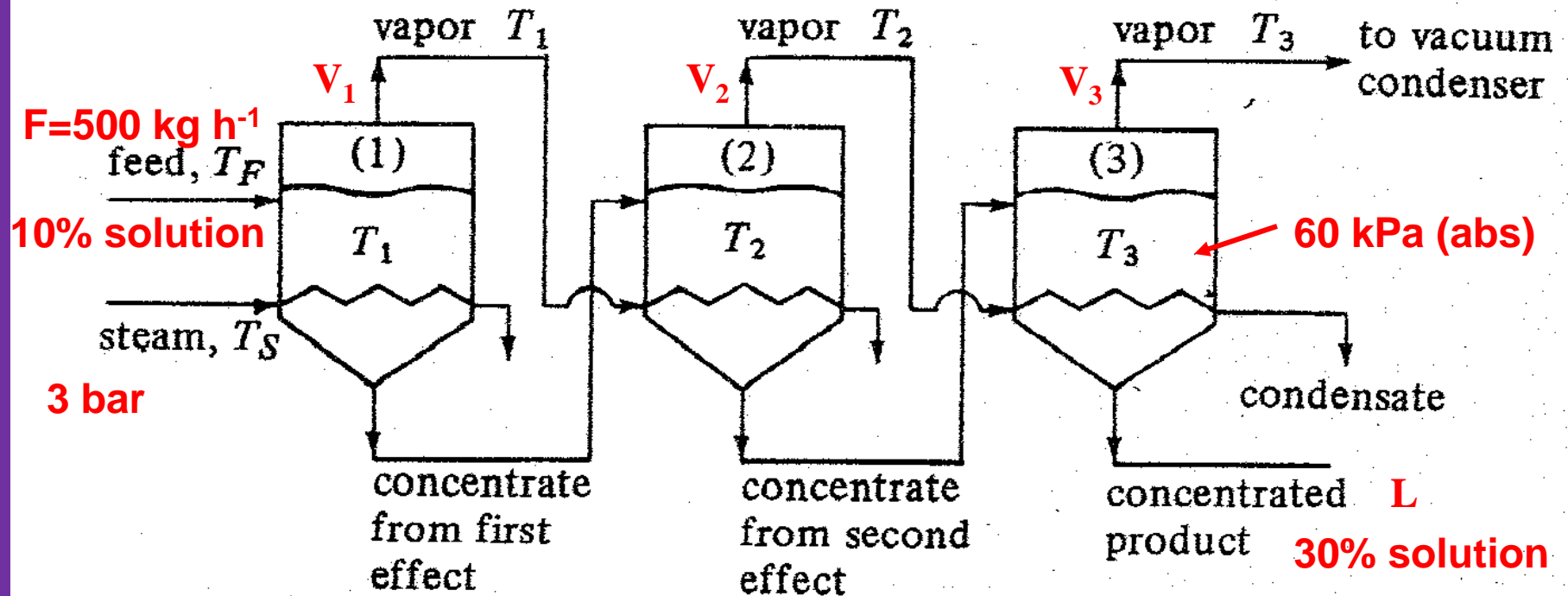
- Capacity of multiple-effect evaporator:** $q = q_1 + q_2 + q_3 + \dots$

Calculation methods for multiple-effect evaporators

Example. Estimate the requirements of steam and heat transfer surface, and the evaporating temperatures in each effect, for a triple effect evaporator evaporating 500 kg h^{-1} of a 10 wt% solution up to a 30 wt% solution.

Steam is available at 3 bar (absolute) and the pressure in the evaporation space in the final effect is 60 kPa absolute. Assume that the overall heat transfer coefficients are 2270, 2000 and $1420 \text{ J m}^{-2} \text{ s}^{-1} \text{ }^{\circ}\text{C}^{-1}$ in the first, second and third effects, respectively. Assume feed enters at the boiling point. Neglect sensible heat effects and assume no boiling-point elevation, and assume equal heat transfer area in each effect.

Calculation methods for multiple-effect evaporators



- Vapor from all effects: $V = V_1 + V_2 + V_3$

Calculation methods for multiple-effect evaporators

- **Solute mass balance over the whole process:**

$$F x_F = L x_L \rightarrow (500)(0.1) = (0.3)(L) \rightarrow L = 166.7 \text{ kg/h}$$

- **Overall material balance over the whole process:**

$$F = L + V \rightarrow 500 = 166.7 + V \rightarrow V = 333.3 \text{ kg/h}$$

Stream	Solute	Solvent	Solution
Feed	50 kg/h	450 kg/h	500 kg/h
Concentrated product	50 kg/h	116.7 kg/h	166.7 kg/h
Vapor from all effects	0	333.3 kg/h	333.3 kg/h

Calculation methods for multiple-effect evaporators

Steam properties:

Data: Steam is available at 3 bar (absolute) and the pressure in the evaporation space in the final effect is 60 kPa absolute. (Neglect sensible heat effects and assume no boiling-point elevation). From steam tables:

Steam pressure	Saturation temperature	Latent heat of vaporization
3 bar (abs)	$T_s = 133.5^\circ\text{C}$	$\lambda_1 = 2164 \text{ kJ/kg}$
60 kPa (abs)	$T_3 = 86.0^\circ\text{C}$	2293 kJ/kg

Calculation methods for multiple-effect evaporators

Evaporator layout:

	First effect	Second effect	Third effect
Steam temperature	$T_s = 133.5^\circ\text{C}$	$T_1^\circ\text{C}$	$T_2^\circ\text{C}$
Solution temperature	$T_1^\circ\text{C}$	$T_2^\circ\text{C}$	86.0°C
Temperature driving force	$\Delta T_1 = 133.5 - T_1$	$\Delta T_2 = T_1 - T_2$	$\Delta T_1 = T_2 - 86.0$

Calculation methods for multiple-effect evaporators

Heat balances:

Data: $U_1=2270$, $U_2=2000$ and $U_3= 1420 \text{ J m}^{-2} \text{ s}^{-1} \text{ }^{\circ}\text{C}^{-1}$.

Assume equal heat transfer in each effect, $q_1 = q_2 = q_3$ which gives:

$$U_1 A_1 \Delta T_1 = U_2 A_2 \Delta T_2 = U_3 A_3 \Delta T_3$$

U_1 , U_2 and U_3 are given. A_1 , A_2 and A_3 can be found if ΔT_1 , ΔT_2 and ΔT_3 are known. Let us assume that the evaporators are so constructed that $A_1 = A_2 = A_3$, then we have

$$U_1 \Delta T_1 = U_2 \Delta T_2 = U_3 \Delta T_3$$

$$2270 (133.5 - T_1) = 2000 (T_1 - T_2) = 1420 (T_2 - 86.0)$$

There are two equations and two unknowns in the above expression. The equations can be solved to give the following:

$$T_1 = 120.8 \text{ }^{\circ}\text{C} \text{ and } T_2 = 106.3 \text{ }^{\circ}\text{C}$$

Calculation methods for multiple-effect evaporators

Properties in all effects:

	First effect	Second effect	Third effect
Steam temperature	133.5 °C	$T_1 = 120.8 \text{ °C}$	$T_2 = 106.3 \text{ °C}$
Solution temperature	$T_1 = 120.8 \text{ °C}$	$T_2 = 106.3 \text{ °C}$	86.0 °C
Temperature driving force	$\Delta T_1 = 12.7 \text{ °C}$	$\Delta T_2 = 14.4 \text{ °C}$	$\Delta T_1 = 20.3 \text{ °C}$
Heat transfer coefficient	$U_1 = 2270 \text{ J m}^{-2} \text{ s}^{-1} \text{ °C}^{-1}$	$U_2 = 2000 \text{ J m}^{-2} \text{ s}^{-1} \text{ °C}^{-1}$	$U_3 = 1420 \text{ J m}^{-2} \text{ s}^{-1} \text{ °C}^{-1}$
Latent heat of vapourization of steam	$\lambda_1 = 2164 \text{ kJ/kg}$ at 133.5 °C	$\lambda_2 = 2200 \text{ kJ/kg}$ at 120.8 °C	$\lambda_3 = 2240 \text{ kJ/kg}$ at 106.3 °C
Latent heat of vaporization of solution	2200 kJ/kg at 120.8 °C	2240 kJ/kg at 106.3 °C	2293 kJ/kg at 86.0 °C

Calculation methods for multiple-effect evaporators

Consider the first effect:

	First effect
Steam temperature	133.5°C
Solution temperature	$T_1 = 120.8^\circ\text{C}$
Temperature driving force	$\Delta T_1 = 12.7^\circ\text{C}$
Heat transfer coefficient	$U_1 = 2270 \text{ J m}^{-2} \text{ s}^{-1} \text{ }^\circ\text{C}^{-1}$
Latent heat of vaporization of steam	$\lambda_1 = 2164 \text{ kJ/kg}$
Latent heat of vaporization of solution	2200 kJ/kg

Steam used = ?

Assuming feed enters at the boiling point ($T_F = T_1$) then

$S_1 (\lambda_1) = F C_p (T_1 - T_F) + V_1$
(Latent heat of vaporization of solution)

$S_1 (\lambda_1) = V_1$ (Latent heat of vaporization of solution)

S_1 is the flow rate of steam used in the first effect and V_1 is the flow rate of vapor leaving the first effect. Therefore,

$$S_1 (2164) = V_1 (2200)$$

Calculation methods for multiple-effect evaporators

Consider the second effect:

	Second effect
Steam temperature	$T_1 = 120.8\text{ }^{\circ}\text{C}$
Solution temperature	$T_2 = 106.3\text{ }^{\circ}\text{C}$
Temperature driving force	$\Delta T_2 = 14.4\text{ }^{\circ}\text{C}$
Heat transfer coefficient	$U_2 = 2000\text{ J m}^{-2}\text{ s}^{-1}\text{ }^{\circ}\text{C}^{-1}$
Latent heat of vaporization of steam	$\lambda_2 = 2200\text{ kJ/kg}$
Latent heat of vaporization of solution	2240 kJ/kg

Steam used = ?

- Feed enters at the boiling point
- steam used in the second effect is the vapor leaving the first effect. Therefore,

$$V_1 (\lambda_2) = V_2 (\text{Latent heat of vaporization of solution})$$

where

V_2 is the flow rate of vapor leaving the second effect. Therefore,

$$V_1 (2200) = V_2 (2240)$$

Calculation methods for multiple-effect evaporators

Consider the third effect:

	Third effect
Steam temperature	$T_2 = 106.3\text{ }^{\circ}\text{C}$
Solution temperature	$86.0\text{ }^{\circ}\text{C}$
Temperature driving force	$\Delta T_1 = 20.3\text{ }^{\circ}\text{C}$
Heat transfer coefficient	$U_3 = 1420\text{ J m}^{-2}\text{ s}^{-1}\text{ }^{\circ}\text{C}^{-1}$
Latent heat of vapourization of steam	$\lambda_3 = 2240\text{ kJ/kg}$
Latent heat of vaporization of solution	2293 kJ/kg

Steam used = ?

- Feed enters at the boiling point
- steam used in the third effect is the vapor leaving the second effect.

Therefore,

$$V_2 (\lambda_3)$$

$$= V_3 (\text{Latent heat of vaporization of solution})$$

where

V_3 is the flow rate of vapour leaving the third effect.

Therefore,

$$V_2 (2240) = V_3 (2293)$$

Calculation methods for multiple-effect evaporators

Steam economy:

$$S_1 (2164) = V_1 (2200) = V_2 (2240) = V_3 (2293)$$

$$V_1 + V_2 + V_3 = 333 \text{ kg/h (from the mass balance).}$$

4 Equations by 4 unknowns

Solve

$$S_1 (2164/2200) + S_1 (2164/2240) + S_1 (2164/2293) = 333 \text{ kg/h}$$

$$2164 S_1 (1/2200 + 1/2240 + 1/2293) = 333 \text{ kg/h}$$

$$\rightarrow S_1 = 115.1 \text{ kg/h}$$

We could calculate the vapor flow rate as

$$V_1 = 113.2 \text{ kg/h}; \quad V_2 = 111.2 \text{ kg/h}; \quad V_3 = 108.6 \text{ kg/h}$$

$\text{Steam economy} = \text{kg vaporized} / \text{kg steam used} = 333 / 115.1 = 2.9$

Calculation methods for multiple-effect evaporators

Heat transfer area:

	First effect
Steam temperature	133.5 °C
Solution temperature	$T_1 = 120.8^\circ\text{C}$
Temperature driving force	$\Delta T_1 = 12.7^\circ\text{C}$
Heat transfer coefficient	$U_1 = 2270 \text{ J m}^{-2} \text{ s}^{-1} \text{ }^\circ\text{C}^{-1}$
Latent heat of vapourization of steam	$\lambda_1 = 2164 \text{ kJ/kg}$
Latent heat of vaporization of solution	2200 kJ/kg

$$A_1 = S_1 \lambda_1 / U_1 \Delta T_1$$

$$= (115 \text{ kg/h}) (2164 \text{ kJ/kg})$$

$$/ [2270 \text{ J m}^{-2} \text{ s}^{-1} \text{ }^\circ\text{C}^{-1} \times (12.7)^\circ\text{C}]$$

$$= (115 \times 2164 \times 1000 / 3600 \text{ J/s})$$

$$/ [2270 \times 12.7 \text{ J m}^{-2} \text{ s}^{-1}]$$

$$= \mathbf{2.4 \text{ m}^2}$$

Overall heat transfer area required

$$= A_1 + A_2 + A_3 = 3 \times A_1$$

$$= \mathbf{7.2 \text{ m}^2}$$

Calculation methods for multiple-effect evaporators

Exercise.

Repeat the previous example taking boiling point elevation into considerations and assuming equal vapor evaporation rate from the three effects. The solute is caustic soda.

Optimum boiling time:

- In evaporation, solids may come out of solution and form a deposit or scale on the heat transfer surfaces. This causes a gradual increase in the resistance to heat transfer.
- If the same temperature difference is maintained, the rate of evaporation decreases with time and it is necessary to **shut down the unit for cleaning at periodic intervals**.
- The longer the boiling time, the lower is the number of shutdowns which are required in a given period although the rate of evaporation would fall to very low levels and the cost per unit mass of material handled would become very high.
- A far better approach is to make a balance which gives a minimum number of shutdowns whilst maintaining an acceptable throughput.

Reference: Coulson, J.M.; and Richardson, J.F.; Chemical Engineering Volume 2, 5th Ed., Butterworth-Heinemann, 1999

Optimum boiling time:

- It has long been established that, with scale formation, the overall coefficient of heat transfer (U) may be expressed as a function of the boiling time (t) by an equation of the form:

$$1/U^2 = a t + b \quad (\text{where } a \text{ and } b \text{ are to be estimated})$$

The heat transfer rate is given by:

$$dQ/dt = U A \Delta T$$

Combining the above two expressions:

$$dQ/dt = A \Delta T (at + b)^{-0.5}$$

Integration of the above between 0 and Q_b and 0 and t_b gives

$$Q_b = (2 A \Delta T / a) [(at_b + b)^{0.5} - b^{0.5}]$$

where Q_b is the total heat transferred in the boiling time t_b .

Optimum boiling time to maximize heat transfer:

- Let us now optimize the boiling time so as to maximize the heat transferred and hence to maximize the solvent evaporated.
- If the time taken to empty, clean and refill the unit is t_c , then the total time for one cycle is $t = (t_b + t_c)$ and the number of cycles in a period t_p is $t_p / (t_b + t_c)$. The total heat transferred during this period is the product of the heat transferred per cycle and the number of cycles in the period or:

$$Q_P = Q_b t_p / (t_b + t_c) = (2 A \Delta T / a) [(at_b + b)^{0.5} - b^{0.5}] t_p / (t_b + t_c)$$

- The optimum value of the boiling time which gives the maximum heat transferred during this period is obtained by differentiating the above equation with respect to t_b and equating to zero which gives:

$$t_{b,\text{optimum}} = t_c + (2/a) (abt_c)^{0.5}$$

Optimum boiling time to minimize cost:

Take C_c as the cost of a shutdown and the variable cost during operation including a labour component as C_b , then the total cost during period t_p is:

$$C_T = (C_c + t_b C_b) t_p / (t_b + t_c)$$

Using $Q_P = (2 A \Delta T / a) [(at_b + b)^{0.5} - b^{0.5}] t_p / (t_b + t_c)$

we can write

$$C_T = (C_c + t_b C_b) a Q_P / \{ (2 A \Delta T [(at_b + b)^{0.5} - b^{0.5}]) \}$$

The optimum value of the boiling time which gives the minimum cost is obtained by differentiating the above equation and equating to zero which gives:

$$t_{b, \text{optimum}} = (C_c / C_b) + 2(abC_c C_b)^{0.5} / (aC_b)$$

Optimum boiling time and minimum cost:

Example. In an evaporator handling an aqueous salt solution, overall heat transfer coefficient U (kW/m².°C) is related to the boiling time t (s) by the following relation:

$$1/U^2 = 7 \times 10^{-5} t + 0.2$$

The heat transfer area is 40 m², the temperature driving force is 40°C and latent heat of vaporization of water is 2300 kJ/kg. Down-time for cleaning is 4.17 h, the cost of a shutdown is JD 600 and the operating cost during boiling is JD64.6 /hour.

Estimate the optimum boiling times to give:

- a) maximum throughput and
- b) minimum cost.

Optimum boiling time and minimum cost:

Data provided:

Since U is in $\text{kW}/\text{m}^2\cdot^\circ\text{C}$ and t is in s , a and b takes the following units:

$$a = 7 \times 10^{-5} \text{ m}^4 \cdot (^\circ\text{C})^2 / \text{kW}^2 \cdot \text{s} = 7 \times 10^{-5} \text{ m}^4 \cdot (^\circ\text{C})^2 \cdot \text{s} / \text{kJ}^2$$

$$b = 0.2 \text{ m}^4 \cdot (^\circ\text{C})^2 / (\text{kJ}/\text{s})^2;$$

Other data are given as

$$A = 40 \text{ m}^2;$$

$$\Delta T = 40^\circ\text{C};$$

Latent heat of vaporization of water = $2300 \text{ kJ}/\text{kg}$;

$$t_c = 4.17 = 4.17 \times 3600 \text{ s} = 15012 \text{ s};$$

$$C_c = \text{JD}600;$$

$$C_b = \text{JD}64.8/\text{h} = \text{JD } 0.018/\text{s}$$

Optimum boiling time and minimum cost:

A) For the case of maximum throughput:

$$\begin{aligned}
 t_{b,\text{optimum}} &= t_c + (2/a) (abt_c)^{0.5} \\
 &= (15012) + (2 / 0.00007) (0.00007 \times 0.2 \times 15012)^{0.5} \\
 &= 28110 \text{ s} = \mathbf{7.81 \text{ h}}
 \end{aligned}$$

Heat transferred during boiling:

$$\begin{aligned}
 Q_b &= (2 A \Delta T/a) [(at_b+b)^{0.5} - b^{0.5}] \\
 &= (2 \times 40 \times 40 / 0.00007) [(0.00007 \times 28110 + 0.2)^{0.5} - 0.2^{0.5}] \\
 &= \mathbf{46.9 \times 10^6 \text{ kJ per cycle}}
 \end{aligned}$$

Optimum boiling time and minimum cost:

Water evaporated during $t_{b, \text{optimum}}$

= **46.9×10^6 kJ per cycle** / Latent heat of vaporization of water

= $(46.9 \times 10^6 / 2300)$ kg = **20375 kg per cycle**

Cost of operation per cycle

$$C_T = (C_c + t_{b, \text{optimum}} C_b)$$

$$= (\text{JD } 600 + 28110 \text{ s} \times 0.018/\text{s})$$

$$= \text{JD } 1106 \text{ per cycle}$$

$$= \text{JD } 1106 \text{ per cycle} / \text{water evaporated per cycle}$$

$$= \text{JD } 1106 / 20374.8 \text{ kg per cycle}$$

$$= \text{JD } 0.054 \text{ per kg}$$

Optimum boiling time and minimum cost:

Rate of evaporation during boiling

$$= 20374.8 \text{ kg} / 28110 \text{ s} = 0.725 \text{ kg/s}$$

Mean rate of evaporation during the cycle

$$= 20374.8 \text{ kg} / (28110 \text{ s} + 15012 \text{ s}) = 0.473 \text{ kg/s}$$

Optimum boiling time and minimum cost:

B) For the case of minimum cost:

$$\begin{aligned}
 t_{b,\text{optimum}} &= (C_c / C_b) + 2(abC_c C_b)^{0.5} / (aC_b) \\
 &= (600 / 0.018) \\
 &\quad + 2(0.00007 \times 0.2 \times 600 \times 0.018)^{0.5} / (0.00007 \times 0.018) \\
 &= 52851 \text{ s} = \mathbf{14.7 \text{ h}}
 \end{aligned}$$

Heat transferred during boiling:

$$\begin{aligned}
 Q_b &= (2 A \Delta T / a) [(at_b + b)^{0.5} - b^{0.5}] \\
 &= (2 \times 40 \times 40 / 0.00007) [(0.00007 \times 52851 + 0.2)^{0.5} - 0.2^{0.5}] \\
 &= \mathbf{69.8 \times 10^6 \text{ kJ}}
 \end{aligned}$$

Optimum boiling time and minimum cost:

Water evaporated during $t_{b,optimum}$

$$= \mathbf{69.8 \times 10^6 \text{ kJ}} / \text{Latent heat of vaporization of water}$$

$$= (69.8 \times 10^6 / 2300) \text{ kg} = 30.35 \times 10^3 \text{ kg per cycle}$$

Cost of operation per cycle

$$C_T = (C_c + t_{b,optimum} C_b)$$

$$= (\text{JD } 600 + 52851 \text{ s} \times \text{JD}0.018/\text{s})$$

$$= \text{JD}1551 \text{ per cycle}$$

$$= \text{JD}1551 \text{ per cycle} / \text{water evaporated per cycle}$$

$$= \text{JD } 1551 \text{ per cycle} / 30.35 \times 10^3 \text{ kg per cycle}$$

$$= \mathbf{\text{JD } 0.051 \text{ per kg}}$$

Optimum boiling time and minimum cost:

Rate of evaporation during boiling

$$= 30.35 \text{ kg} \times 10^3 / 52851 \text{ s} = 0.574 \text{ kg/s}$$

Mean rate of evaporation during the cycle

$$= 30.35 \text{ kg} \times 10^3 / (52851 \text{ s} + 15012 \text{ s}) = 0.447 \text{ kg/s}$$

Optimum boiling time and minimum cost:

Summary:

	maximum throughput	minimum cost
Optimum boiling time	7.81 h	14.7 h
Heat transferred during boiling	46.9×10^6 kJ	69.8×10^6 kJ
Mean rate of evaporation per cycle	0.473 kg/s	0.447 kg/s
Cost of operation per cycle	JD 1106 per cycle	JD1551 per cycle
Cost of operation per kg of water evaporated	JD 0.054 per kg	JD 0.051 per kg