

*Transport Processes
and Unit Operations*

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*Transport Processes
and Unit Operations*

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*Dedicated to the memory of my beloved mother, Helen,
for her love and encouragement*

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Preface

In this third edition, the main objectives and the format of the first and second editions remain the same. The sections on momentum transfer have been greatly expanded, especially in the sections covering differential equations of momentum transfer. This now allows full coverage of the transport processes of momentum, heat, and mass transfer. Also, a section on adsorption and an expanded chapter on membrane processes have been added to the unit operations sections.

The field of chemical engineering involved with physical and physical-chemical changes of inorganic and organic materials, and to some extent biological materials, is overlapping more and more with the other process engineering fields of ceramic engineering, process metallurgy, agricultural food engineering, wastewater treatment (civil) engineering, and bioengineering. The principles of momentum, heat, and mass transport and the unit operations are used in these processing fields.

The principles of momentum transfer and heat transfer have been taught to all engineers. The study of mass transfer has been limited primarily to chemical engineers. However, engineers in other fields have become more interested in mass transfer in gases, liquids, and solids.

Since chemical and other engineering students must study so many topics today, a more unified introduction to the transport processes of momentum, heat, and mass transfer and to the applications of unit operations is provided. In this text the principles of the transport processes are covered first, and then the unit operations. To accomplish this, the text is divided into two main parts.

PART I: Transport Processes: Momentum, Heat, and Mass

This part, dealing with fundamental principles, includes the following chapters: 1. Introduction to Engineering Principles and Units; 2. Principles of Momentum Transfer and Overall Balances; 3. Principles of Momentum Transfer and Applications; 4. Principles of Steady-State Heat Transfer; 5. Principles of Unsteady-State Heat Transfer; 6.

PART 2: Unit Operations

This part, on applications, covers the following unit operations: 8. Evaporation; 9. Drying of Process Materials; 10. Stage and Continuous Gas-Liquid Separation Processes (humidification, absorption); 11. Vapor-Liquid Separation Processes (distillation); 12. Liquid-Liquid and Fluid-Solid Separation Processes (adsorption, ion exchange, extraction, leaching, crystallization); 13. Membrane Separation Processes (dialysis, gas separation, reverse osmosis, ultrafiltration); 14. Mechanical-Physical Separation Processes (filtration, settling, centrifugal separation, mechanical size reduction).

In Chapter 1 elementary principles of mathematical and graphical methods, laws of chemistry and physics, material balances, and heat balances are reviewed. Many, especially chemical engineers, may be familiar with most of these principles and may omit all or parts of this chapter.

A few topics, involved primarily with the processing of biological materials, may be omitted at the discretion of the reader or instructor: Sections 5.5, 6.4, 8.7, 9.11, and 9.12. Over 230 example or sample problems and over 500 homework problems on all topics are included in the text. Some of the homework problems are concerned with biological systems, for those readers who are especially interested in that area.

This text may be used for a course of study using any of the following five suggested plans. In all plans, Chapter 1 may or may not be included.

1. *Study of transport processes of momentum, heat, and mass and unit operations.* In this plan, most of the complete text covering the principles of the transport processes in Part 1 and the unit operations in Part 2 are covered. This plan could be applicable primarily to chemical engineering and also to other process engineering fields in a one and one-half year course of study at the junior and/or senior level.

2. *Study of transport processes of momentum, heat, and mass and selected unit operations.* Only the elementary sections of Part 1 (the principles chapters—2, 3, 4, 5, 6, and 7) are covered, plus selected unit operations topics in Part 2 applicable to a particular field in a two-semester or three-quarter course. Those in wastewater treatment engineering, food process engineering, and process metallurgy could follow this plan.

3. *Study of transport processes of momentum, heat, and mass.* The purpose of this plan in a two-quarter or two-semester course is to obtain a basic understanding of the transport processes of momentum, heat, and mass transfer. This involves studying sections of the principles chapters—2, 3, 4, 5, 6, and 7 in Part 1—and omitting Part 2, the applied chapters on unit operations.

4. *Study of unit operations.* If the reader has had courses in the transport processes of momentum, heat, and mass, Chapters 2 through 7 can be omitted and only the unit operations chapters in Part 2 studied in a one-semester or two-quarter course. This plan could be used by chemical and certain other engineers.

5. *Study of mass transfer.* For those such as chemical or mechanical engineers who have had momentum and heat transfer, or those who desire only a background in mass transfer in a one-quarter or one-semester course, Chapters 6, 7, and 10 would be covered. Chapters 9, 11, 12, and 13 might be covered optionally, depending on the needs of the reader.

The SI (Système International d'Unités) system of units has been adopted by the scientific community. Because of this, the SI system of units has been adopted in this text for use in the equations, example problems, and homework problems. However, the most important equations derived in the text are also given in a dual set of units, SI and English, when different. Many example and homework problems are also given using English units.

Christie J. Geankoplis

PART 2

Unit Operations

CHAPTER 8

Evaporation

8.1 INTRODUCTION

8.1A Purpose

In Section 4.8 we discussed the case of heat transfer to a boiling liquid. An important instance of this type of heat transfer occurs quite often in the process industries and is given the general name *evaporation*. In evaporation the vapor from a boiling liquid solution is removed and a more concentrated solution remains. In the majority of cases the unit operation evaporation refers to the removal of water from an aqueous solution.

Typical examples of evaporation are concentration of aqueous solutions of sugar, sodium chloride, sodium hydroxide, glycerol, glue, milk, and orange juice. In these cases the concentrated solution is the desired product and the evaporated water is normally discarded. In a few cases, water, which has a small amount of minerals, is evaporated to give a solids-free water which is used as boiler feed, for special chemical processes, or for other purposes. Evaporation processes to evaporate seawater to provide drinking water have been developed and used. In some cases, the primary purpose of evaporation is to concentrate the solution so that upon cooling, salt crystals will form and be separated. This special evaporation process, termed *crystallization*, is discussed in Chapter 12.

8.1B Processing Factors

The physical and chemical properties of the solution being concentrated and of the vapor being removed have a great effect on the type of evaporator used and on the pressure and temperature of the process. Some of these properties which affect the processing methods are discussed next.

1. Concentration in the liquid. Usually, the liquid feed to an evaporator is relatively dilute, so its viscosity is low, similar to water, and relatively high heat-transfer coefficients are obtained. As evaporation proceeds, the solution may become very concentrated and quite viscous, causing the heat-transfer coefficient to drop markedly. Adequate circulation and/or turbulence must be present to keep the coefficient from becoming too low.

2. *Solubility.* As solutions are heated and concentration of the solute or salt increases, the solubility limit of the material in solution may be exceeded and crystals may form. This may limit the maximum concentration in solution which can be obtained by evaporation. In Fig. 8.1-1 some solubilities of typical salts in water are shown as a function of temperature. In most cases the solubility of the salt increases with temperature. This means that when a hot concentrated solution from an evaporator is cooled to room temperature, crystallization may occur.

3. *Temperature sensitivity of materials.* Many products, especially food and other biological materials, may be temperature-sensitive and degrade at higher temperatures or after prolonged heating. Such products are pharmaceutical products; food products such as milk, orange juice, and vegetable extracts; and fine organic chemicals. The amount of degradation is a function of the temperature and the length of time.

4. *Foaming or frothing.* In some cases materials composed of caustic solutions, food solutions such as skim milk, and some fatty acid solutions form a foam or froth during boiling. This foam accompanies the vapor coming out of the evaporator and entrainment losses occur.

5. *Pressure and temperature.* The boiling point of the solution is related to the pressure of the system. The higher the operating pressure of the evaporator, the higher the temperature at boiling. Also, as the concentration of the dissolved material in solution increases by evaporation, the temperature of boiling may rise. This phenomenon is called *boiling-point rise* or *elevation* and is discussed in Section 8.4. To keep the temperatures low in heat-sensitive materials, it is often necessary to operate under 1 atm pressure, i.e., under vacuum.

6. *Scale deposition and materials of construction.* Some solutions deposit solid materials called *scale* on the heating surfaces. These could be formed by decomposition products or solubility decreases. The result is that the overall heat-transfer coefficient decreases and the evaporator must eventually be cleaned. The materials of construction of the evaporator are important to minimize corrosion.

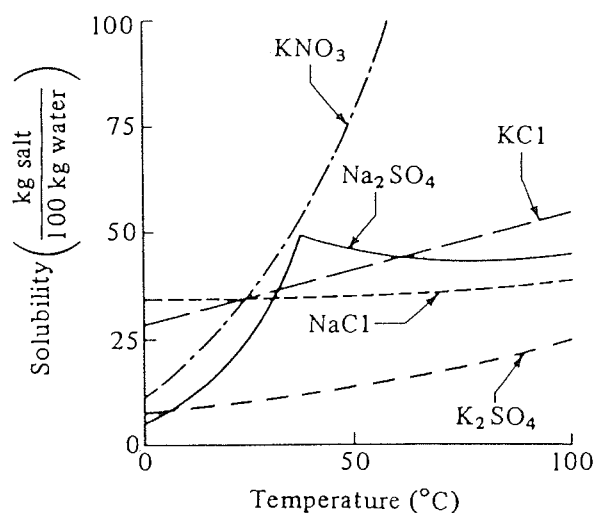


FIGURE 8.1-1. Solubility curves for some typical salts in water.

8.2 TYPES OF EVAPORATION EQUIPMENT AND OPERATION METHODS

8.2A General Types of Evaporators

In evaporation, heat is added to a solution to vaporize the solvent, which is usually water. The heat is generally provided by the condensation of a vapor such as steam on one side of a metal surface with the evaporating liquid on the other side. The type of equipment used depends primarily on the configuration of the heat-transfer surface and on the means employed to provide agitation or circulation of the liquid. These general types are discussed below.

1. *Open kettle or pan.* The simplest form of evaporator consists of an open pan or kettle in which the liquid is boiled. The heat is supplied by condensation of steam in a jacket or in coils immersed in the liquid. In some cases the kettle is direct-fired. These evaporators are inexpensive and simple to operate, but the heat economy is poor. In some cases, paddles or scrapers for agitation are used.

2. *Horizontal-tube natural circulation evaporator.* The horizontal-tube natural circulation evaporator is shown in Fig. 8.2-1a. The horizontal bundle of heating tubes is similar to the bundle of tubes in a heat exchanger. The steam enters into the tubes, where it condenses. The steam condensate leaves at the other end of the tubes. The boiling-liquid solution covers the tubes. The vapor leaves the liquid surface, often goes through some deentraining device such as a baffle to prevent carryover of liquid droplets, and leaves out the top. This type is relatively cheap and is used for nonviscous liquids having high heat-transfer coefficients and liquids that do not deposit scale. Since liquid circulation is poor, they are unsuitable for viscous liquids. In almost all cases, this evaporator and the types discussed below are operated continuously, where the feed enters at a constant rate and the concentrate leaves at a constant rate.

3. *Vertical-type natural circulation evaporator.* In this type of evaporator, vertical rather than horizontal tubes are used, and the liquid is inside the tubes and the steam condenses outside the tubes. Because of boiling and decreases in density, the liquid rises in the tubes by natural circulation as shown in Fig. 8.2-1b and flows downward through a large central open space or downcomer. This natural circulation increases the heat-transfer coefficient. It is not used with viscous liquids. This type is often called the *short-tube evaporator*. A variation of this is the basket type, where vertical tubes are used, but the heating element is held suspended in the body so there is an annular open space as the downcomer. The basket type differs from the vertical natural circulation evaporator, which has a central instead of annular open space as the downcomer. This type is widely used in the sugar, salt, and caustic soda industries.

4. *Long-tube vertical-type evaporator.* Since the heat-transfer coefficient on the steam side is very high compared to that on the evaporating liquid side, high liquid velocities are desirable. In a long-tube vertical-type evaporator shown in Fig. 8.2-1c, the liquid is inside the tubes. The tubes are 3 to 10 m long and the formation of vapor bubbles inside the tubes causes a pumping action giving quite high liquid velocities. Generally, the liquid passes through the tubes only once and is not recirculated. Contact times can be quite low in this type. In some cases, as when the ratio of feed to evaporation rate is low, natural recirculation of the product through the evaporator is done by adding a large pipe connection between the outlet concentrate line and the feed line. This is widely used for producing condensed milk.

5. *Falling-film-type evaporator.* A variation of the long-tube type is the falling-film evaporator, wherein the liquid is fed to the top of the tubes and flows down the walls as a thin film. Vapor-liquid separation usually takes place at the bottom. This type is widely used for concentrating heat-sensitive materials such as orange juice and other fruit juices, because the holdup time is very small (5 to 10 s or more) and the heat-transfer coefficients are high

6. *Forced-circulation-type evaporator.* The liquid-film heat-transfer coefficient can be increased by pumping to cause forced circulation of the liquid inside the tubes. This could be done in the long-tube vertical type in Fig. 8.2-1c by adding a pipe connection with a pump between the outlet concentrate line and the feed line. However, usually in a forced-circulation type, the vertical tubes are shorter than in the long-tube type, and this

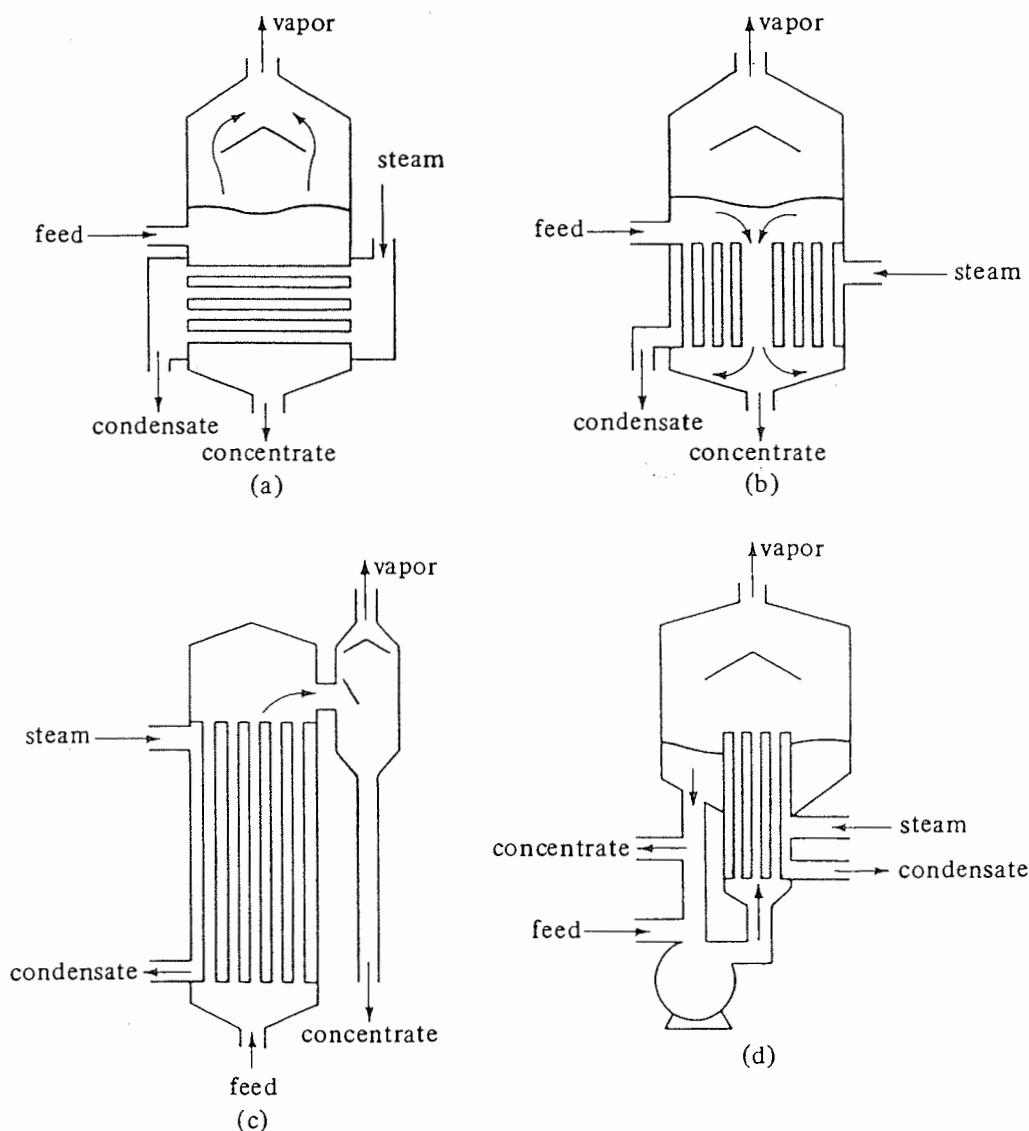


FIGURE 8.2-1. Different types of evaporators: (a) horizontal-tube type, (b) vertical-tube type, (c) long-tube vertical type, (d) forced-circulation type.

type is shown in Fig. 8.2-1d. Also, in other cases a separate and external horizontal heat exchanger is used. This type is very useful for viscous liquids.

7. *Agitated-film evaporator.* In an evaporator the main resistance to heat transfer is on the liquid side. One way to increase turbulence in this film, and hence the heat-transfer coefficient, is by actual mechanical agitation of this liquid film. This is done in a modified falling-film evaporator with only a single large jacketed tube containing an internal agitator. Liquid enters at the top of the tube and as it flows downward, it is spread out into a turbulent film by the vertical agitator blades. The concentrated solution leaves at the bottom and vapor leaves through a separator and out the top. This type of evaporator is very useful with highly viscous materials, since the heat-transfer coefficient is greater than in forced-circulation evaporators. It is used with heat-sensitive viscous materials such as rubber latex, gelatin, antibiotics, and fruit juices. However, it has a high cost and small capacity. For interested readers, Perry and Green (P2) give more detailed discussions and descriptions of evaporation equipment.

8. *Open-pan solar evaporator.* A very old but yet still used process is solar evaporation in open pans. Salt water is put in shallow open pans or troughs and allowed to evaporate slowly in the sun to crystallize the salt.

8.2B Methods of Operation of Evaporators

1. *Single-effect evaporators.* A simplified diagram of a single-stage or single-effect evaporator is given in Fig. 8.2-2. The feed enters at T_F K and saturated steam at T_S enters the heat-exchange section. Condensed steam leaves as condensate or drips. Since the solution in the evaporator is assumed to be completely mixed, the concentrated product and the solution in the evaporator have the same composition and temperature T_1 , which is the boiling point of the solution. The temperature of the vapor is also T_1 , since it is in equilibrium with the boiling solution. The pressure is P_1 , which is the vapor pressure of the solution at T_1 .

If the solution to be evaporated is assumed to be dilute and like water, then 1 kg of steam condensing will evaporate approximately 1 kg of vapor. This will hold if the feed entering has a temperature T_F near the boiling point.

The concept of an overall heat-transfer coefficient is used in the calculation of the rate of heat transfer in an evaporator. The general equation can be written

$$q = UA \Delta T = UA(T_S - T_1) \quad (8.2-1)$$

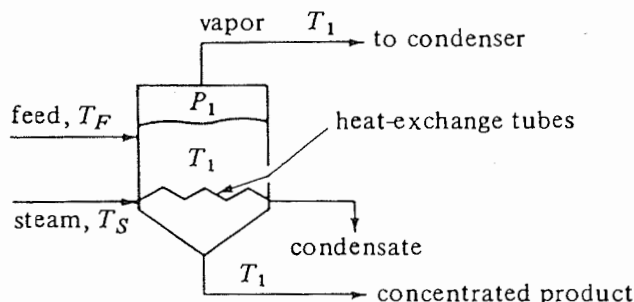


FIGURE 8.2-2. Simplified diagram of single-effect evaporator.

where q is the rate of heat transfer in W (btu/h), U is the overall heat-transfer coefficient in $W/m^2 \cdot K$ (btu/h $\cdot ft^2 \cdot ^\circ F$), A is the heat-transfer area in m^2 (ft^2), T_s is the temperature of the condensing steam in K ($^\circ F$), and T_1 is the boiling point of the liquid in K ($^\circ F$).

Single-effect evaporators are often used when the required capacity of operation is relatively small and/or the cost of steam is relatively cheap compared to the evaporator cost. However, for large-capacity operation, using more than one effect will markedly reduce steam costs.

2. Forward-feed multiple-effect evaporators. A single-effect evaporator as shown in Fig. 8.2-2 is wasteful of energy since the latent heat of the vapor leaving is not used but is discarded. However, much of this latent heat can be recovered and reused by employing multiple-effect evaporators. A simplified diagram of a forward-feed triple-effect evaporation system is shown in Fig. 8.2-3.

If the feed to the first effect is near the boiling point at the pressure in the first effect, 1 kg of steam will evaporate almost 1 kg of water. The first effect operates at a high-enough temperature so that the evaporated water serves as the heating medium to the second effect. Here, again, almost another kg of water is evaporated, which can be used as the heating medium to the third effect. As a very rough approximation, almost 3 kg of water will be evaporated for 1 kg of steam for a three-effect evaporator. Hence, the *steam economy*, which is kg vapor evaporated/kg steam used, is increased. This also approximately holds for a number of effects over three. However, this increased steam economy of a multiple-effect evaporator is gained at the expense of the original first cost of these evaporators.

In forward-feed operation as shown in Fig. 8.2-3, the fresh feed is added to the first effect and flows to the next in the same direction as the vapor flow. This method of operation is used when the feed is hot or when the final concentrated product might be damaged at high temperatures. The boiling temperatures decrease from effect to effect. This means that if the first effect is at $P_1 = 1$ atm abs pressure, the last effect will be under vacuum at a pressure P_3 .

3. Backward-feed multiple-effect evaporators. In the backward-feed operation shown in Fig. 8.2-4 for a triple-effect evaporator, the fresh feed enters the last and coldest effect and continues on until the concentrated product leaves the first effect. This method of reverse feed is advantageous when the fresh feed is cold, since a smaller amount of liquid must be heated to the higher temperatures in the second and first effects. However, liquid pumps

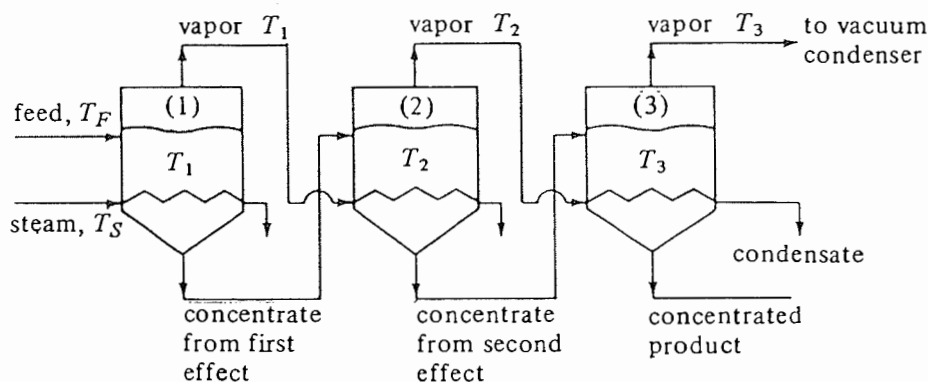


FIGURE 8.2-3. Simplified diagram of forward-feed triple-effect evaporator.

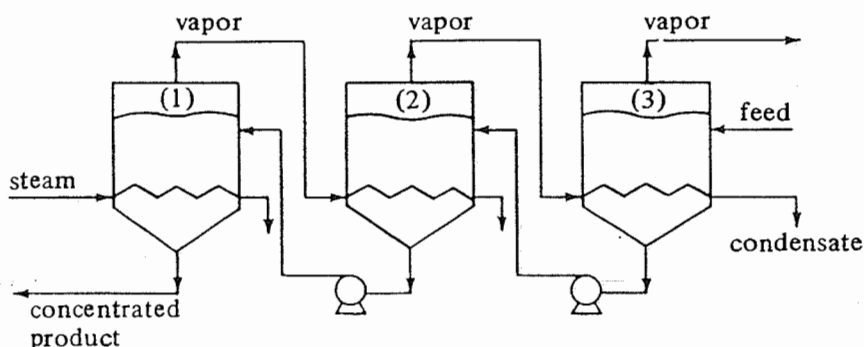


FIGURE 8.2-4. *Simplified diagram of backward-feed triple-effect evaporator.*

are used in each effect, since the flow is from low to high pressure. This method is also used when the concentrated product is highly viscous. The high temperatures in the early effects reduce the viscosity and give reasonable heat-transfer coefficients.

4. *Parallel-feed multiple-effect evaporators.* Parallel feed in multiple-effect evaporators involves the adding of fresh feed and the withdrawal of concentrated product from each effect. The vapor from each effect is still used to heat the next effect. This method of operation is mainly used when the feed is almost saturated and solid crystals are the product, as in the evaporation of brine to make salt.

8.3 OVERALL HEAT-TRANSFER COEFFICIENTS IN EVAPORATORS

The overall heat-transfer coefficient U in an evaporator is composed of the steam-side condensing coefficient, which has a value of about $5700 \text{ W/m}^2 \cdot \text{K}$ ($1000 \text{ btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$); the metal wall, which has a high thermal conductivity and usually has a negligible resistance; the resistance of the scale on the liquid side; and the liquid film coefficient, which is usually inside the tubes.

The steam-side condensing coefficient outside the tubes can be estimated using Eqs. (4.8-20)–(4.8-26). The resistance due to scale formation usually cannot be predicted. Increasing the velocity of the liquid in the tubes greatly decreases the rate of scale formation. This is one important advantage of forced-circulation evaporators. The scale can be salts, such as calcium sulfate and sodium sulfate, which decrease in solubility with an increase in temperature and hence tend to deposit on the hot tubes.

For forced-circulation evaporators the coefficient h inside the tubes can be predicted if there is little or no vaporization inside the tube. The liquid hydrostatic head in the tubes prevents most boiling in the tubes. The standard equations for predicting the h value of liquids inside tubes can be used. Velocities used often range from 2 to 5 m/s (7 to 15 ft/s). The heat-transfer coefficient can be predicted from Eq. (4.5-8), but using a constant of 0.028 instead of 0.027 (B1). If there is some or appreciable boiling in part or all of the tubes, the use of the equation assuming no boiling will give conservative safe results (P1).

For long-tube vertical natural circulation evaporators the heat-transfer coefficient is more difficult to predict, since there is a nonboiling zone in the bottom of the tubes and a boiling zone in the top. The length of the nonboiling zone depends on the heat transfer in the two zones and the pressure drop in the boiling two-phase zone. The film heat-transfer

TABLE 8.3-1. *Typical Heat-Transfer Coefficients for Various Evaporators**
(B3, B4, L1, P2)

Type of Evaporator	Overall U	
	$W/m^2 \cdot K$	$Btu/h \cdot ft^2 \cdot ^\circ F$
Short-tube vertical, natural circulation	1100–2800	200–500
Horizontal-tube, natural circulation	1100–2800	200–500
Long-tube vertical, natural circulation	1100–4000	200–700
Long-tube vertical, forced circulation	2300–11 000	400–2000
Agitated film	680–2300	120–400

* Generally, nonviscous liquids have the higher coefficients and viscous liquids the lower coefficients in the ranges given.

coefficient in the nonboiling zone can be estimated using Eq. (4.5-8) with a constant of 0.028. For the boiling two-phase zone, a number of equations are given by Perry and Green (P2).

For short-tube vertical evaporators the heat-transfer coefficients can be estimated by using the same methods as for the long-tube vertical natural circulation evaporators. Horizontal-tube evaporators have heat-transfer coefficients of the same order of magnitude as the short-tube vertical evaporators.

For the agitated-film evaporator, the heat-transfer coefficient may be estimated using Eq. (4.13-4) for a scraped surface heat exchanger.

The methods given above are useful for actual evaporator design and/or for evaluating the effects of changes in operating conditions on the coefficients. In making preliminary designs or cost estimates, it is helpful to have available overall heat-transfer coefficients usually encountered in commercial practice. Some preliminary values and ranges of values for various types of evaporators are given in Table 8.3-1.

8.4 CALCULATION METHODS FOR SINGLE-EFFECT EVAPORATORS

8.4A Heat and Material Balances for Evaporators

The basic equation for solving for the capacity of a single-effect evaporator is Eq. (8.2-1), which can be written as

$$q = UA \Delta T \quad (8.4-1)$$

where ΔT K ($^\circ F$) is the difference in temperature between the condensing steam and the boiling liquid in the evaporator. In order to solve Eq. (8.4-1) the value of q in W (btu/h) must be determined by making a heat and material balance on the evaporator shown in Fig. 8.4-1. The feed to the evaporator is F kg/h (lb_m/h) having a solids content of x_F mass fraction, temperature T_F , and enthalpy h_F J/kg (btu/lb_m). Coming out as a liquid is the concentrated liquid L kg/h (lb_m/h) having a solids content of x_L , temperature T_1 , and enthalpy h_L . The vapor V kg/h (lb_m/h) is given off as pure solvent having a solids content of $y_V = 0$, temperature T_1 , and enthalpy H_V . Saturated steam entering is S kg/h (lb_m/h) and has a temperature of T_S and enthalpy of H_S . The condensed steam leaving of S kg/h is assumed usually to be at T_S , the saturation temperature, with an enthalpy of h_S . This

means that the steam gives off only its latent heat, which is λ , where

$$\lambda = H_S - h_S \quad (8.4-2)$$

Since the vapor V is in equilibrium with the liquid L , the temperatures of vapor and liquid are the same. Also, the pressure P_1 is the saturation vapor pressure of the liquid of composition x_L at its boiling point T_1 . (This assumes no boiling-point rise.)

For the material balance since we are at steady state, the rate of mass in = rate of mass out. Then, for a total balance,

$$F = L + V \quad (8.4-3)$$

For a balance on the solute (solids) alone,

$$F x_F = L x_L \quad (8.4-4)$$

For the heat balance, since the total heat entering = total heat leaving,
heat in feed + heat in steam

$$= \text{heat in concentrated liquid} + \text{heat in vapor} + \text{heat in condensed steam} \quad (8.4-5)$$

This assumes no heat lost by radiation or convection. Substituting into Eq. (8.4-5),

$$F h_F + S H_S = L h_L + V H_V + S h_S \quad (8.4-6)$$

Substituting Eq. (8.4-2) into (8.4-6),

$$F h_F + S \lambda = L h_L + V H_V \quad (8.4-7)$$

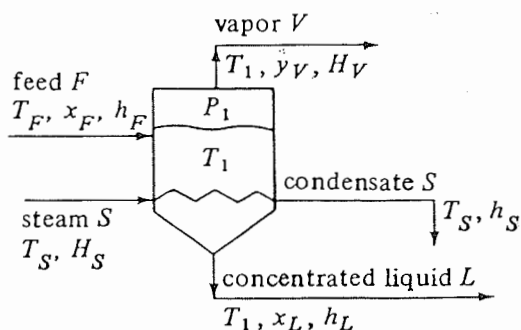
The heat q transferred in the evaporator is then

$$q = S(H_S - h_S) = S \lambda \quad (8.4-8)$$

In Eq. (8.4-7) the latent heat λ of steam at the saturation temperature T_S can be obtained from the steam tables in Appendix A.2. However, the enthalpies of the feed and products are often not available. These enthalpy-concentration data are available for only a few substances in solution. Hence, some approximations are made in order to make a heat balance. These are as follows.

1. It can be demonstrated as an approximation that the latent heat of evaporation of 1 kg mass of the water from an aqueous solution can be obtained from the steam tables using the temperature of the boiling solution T_1 (exposed surface temperature) rather than the equilibrium temperature for pure water at P_1 .
2. If the heat capacities c_{pF} of the liquid feed and c_{pL} of the product are known, they can be used to calculate the enthalpies. (This neglects heats of dilution, which in most cases are not known.)

FIGURE 8.4-1. Heat and mass balance for single-effect evaporator.



EXAMPLE 8.4-1. Heat-Transfer Area in Single-Effect Evaporator

A continuous single-effect evaporator concentrates 9072 kg/h of a 1.0 wt % salt solution entering at 311.0 K (37.8°C) to a final concentration of 1.5 wt %. The vapor space of the evaporator is at 101.325 kPa (1.0 atm abs) and the steam supplied is saturated at 143.3 kPa. The overall coefficient $U = 1704 \text{ W/m}^2 \cdot \text{K}$. Calculate the amounts of vapor and liquid product and the heat-transfer area required. Assume that, since it is dilute, the solution has the same boiling point as water.

Solution: The flow diagram is the same as that in Fig. 8.4-1. For the material balance, substituting into Eq. (8.4-3),

$$F = L + V \quad (8.4-3)$$

$$9072 = L + V$$

Substituting into Eq. (8.4-4) and solving,

$$Fx_F = Lx_L \quad (8.4-4)$$

$$9072(0.01) = L(0.015)$$

$$L = 6048 \text{ kg/h of liquid}$$

Substituting into Eq. (8.4-3) and solving,

$$V = 3024 \text{ kg/h of vapor}$$

The heat capacity of the feed is assumed to be $c_{pF} = 4.14 \text{ kJ/kg} \cdot \text{K}$. (Often, for feeds of inorganic salts in water, the c_p can be assumed approximately as that of water alone.) To make a heat balance using Eq. (8.4-7), it is convenient to select the boiling point of the dilute solution in the evaporator, which is assumed to be that of water at 101.32 kPa, $T_1 = 373.2 \text{ K}$ (100°C), as the datum temperature. Then H_V is simply the latent heat of water at 373.2 K, which from the steam tables in Appendix A.2 is 2257 kJ/kg (970.3 btu/lb_m). The latent heat λ of the steam at 143.3 kPa [saturation temperature $T_S = 383.2 \text{ K}$ (230°F)] is 2230 kJ/kg (958.8 btu/lb_m).

The enthalpy of the feed can be calculated from

$$h_F = c_{pF}(T_F - T_1) \quad (8.4-9)$$

Substituting into Eq. (8.4-7) with $h_L = 0$, since it is at the datum of 373.2 K,

$$9072(4.14)(311.0 - 373.2) + S(2230) = 6048(0) + 3024(2257)$$

$$S = 4108 \text{ kg steam/h}$$

The heat q transferred through the heating surface area A is, from Eq. (8.4-8),

$$q = S(\lambda) \quad (8.4-8)$$

$$q = 4108(2230)(1000/3600) = 2\,544\,000 \text{ W}$$

Substituting into Eq. (8.4-1), where $\Delta T = T_S - T_1$,

$$q = 2\,544\,000 = UA \Delta T = 1704(A)(383.2 - 373.2)$$

Solving, $A = 149.3 \text{ m}^2$.

8.4B Effects of Processing Variables on Evaporator Operation

1. *Effect of feed temperature.* The inlet temperature of the feed has a large effect on the operation of the evaporator. In Example 8.4-1 the feed entering was at a cold temper-

ature of 311.0 K compared to the boiling temperature of 373.2 K. About $\frac{1}{4}$ of the steam used for heating was used to heat the cold feed to the boiling point. Hence, only about $\frac{3}{4}$ of the steam was left for vaporization of the feed. If the feed is under pressure and enters the evaporator at a temperature above the boiling point in the evaporator, additional vaporization is obtained by the flashing of part of the entering hot feed. Preheating the feed can reduce the size of evaporator heat-transfer area needed.

2. *Effect of pressure.* In Example 8.4-1 the pressure of 101.32 kPa abs was used in the vapor space of the evaporator. This set the boiling point of the solution at 373.2 K and gave a ΔT for use in Eq. (8.4-1) of $383.2 - 373.2$, or 10 K. In many cases a larger ΔT is desirable, since, as the ΔT increases, the heating-surface area A and cost of evaporator decrease. To reduce the pressure below 101.32 kPa, i.e., to be under vacuum, a condenser and vacuum pump can be used. For example, if the pressure were reduced to 41.4 kPa, the boiling point of water would be 349.9 K and the new ΔT would be $383.2 - 349.9$, or 33.3 K. A large decrease in heating-surface area would be obtained.

3. *Effect of steam pressure.* Using higher pressure, saturated steam increases the ΔT , which decreases the size and cost of the evaporator. However, high-pressure steam is more costly and also is often more valuable as a source of power elsewhere. Hence, overall economic balances are really needed to determine the optimum steam pressures.

8.4C Boiling-Point Rise of Solutions

In the majority of cases in evaporation the solutions are not such dilute solutions as those considered in Example 8.4-1. In most cases, the thermal properties of the solution being evaporated may differ considerably from those of water. The concentrations of the solutions are high enough so that the heat capacity and boiling point are quite different from that of water.

For strong solutions of dissolved solutes the boiling-point rise due to the solutes in the solution usually cannot be predicted. However, a useful empirical law known as *Dühring's rule* can be used. In this rule a straight line is obtained if the boiling point of a solution in $^{\circ}\text{C}$ or $^{\circ}\text{F}$ is plotted against the boiling point of pure water at the same pressure for a given concentration at different pressures. A different straight line is obtained for each given concentration. In Fig. 8.4-2 such a Dühring line chart is given for solutions of sodium hydroxide in water. It is necessary to know the boiling point of a given solution at only two pressures to determine a line.

EXAMPLE 8.4-2. Use of Dühring Chart for Boiling-Point Rise

As an example of use of the chart, the pressure in an evaporator is given as 25.6 kPa (3.72 psia) and a solution of 30% NaOH is being boiled. Determine the boiling temperature of the NaOH solution and the boiling-point rise BPR of the solution over that of water at the same pressure.

Solution: From the steam tables in Appendix A.2, the boiling point of water at 25.6 kPa is 65.6°C . From Fig. 8.4-2 for 65.6°C (150°F) and 30% NaOH, the boiling point of the NaOH solution is 79.5°C (175°F). The boiling-point rise is $79.5 - 65.6 = 13.9^{\circ}\text{C}$ (25°F).

In Perry and Green (P2) a chart is given to estimate the boiling-point rise of a large number of common aqueous solutions used in chemical and biological processes. In addition to the common salts and solutes, such as NaNO_3 , NaOH, NaCl, and H_2SO_4 , the biological solutes sucrose, citric acid, kraft solution, and glycerol are given. These

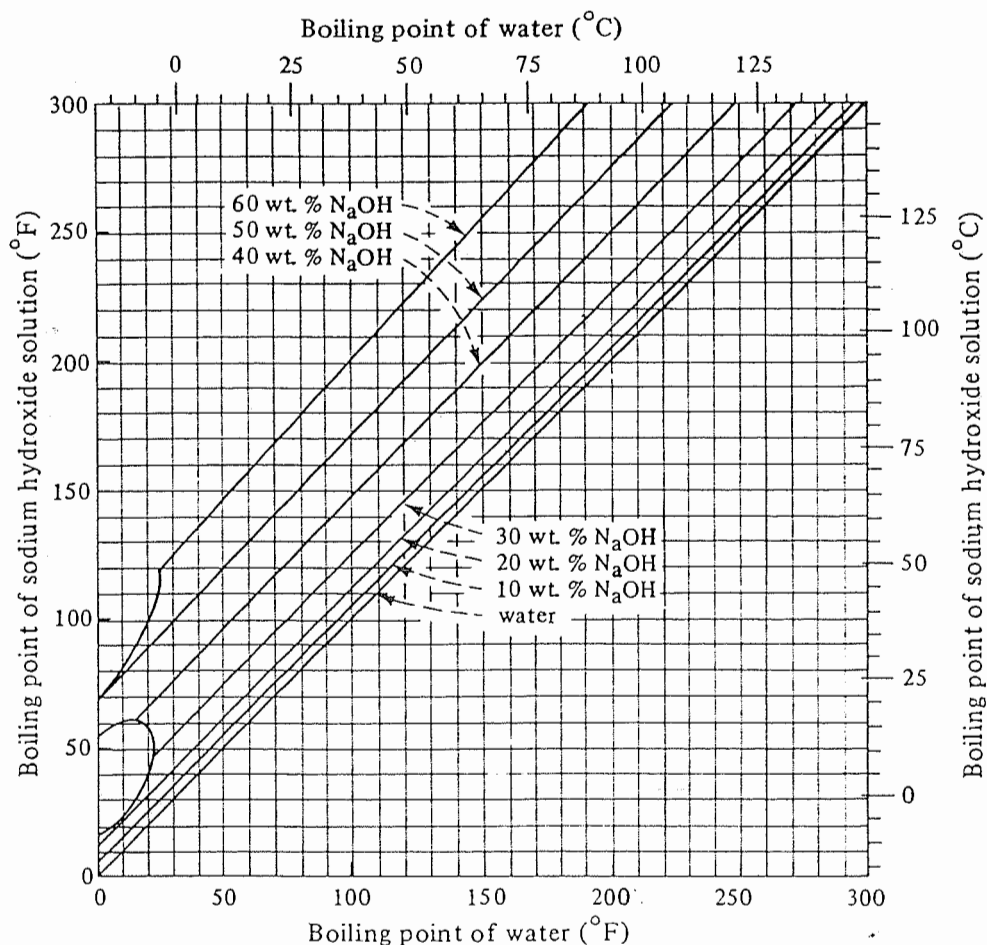


FIGURE 8.4-2. Dühring lines for aqueous solutions of sodium hydroxide.

biological solutes have quite small boiling-point-rise values compared to those of common salts.

8.4D Enthalpy-Concentration Charts of Solutions

If the heat of solution of the aqueous solution being concentrated in the evaporator is large, neglecting it could cause errors in the heat balances. This heat-of-solution phenomenon can be explained as follows. If pellets of NaOH are dissolved in a given amount of water, it is found that a considerable temperature rise occurs; i.e., heat is evolved, called *heat of solution*. The amount of heat evolved depends on the type of substance and on the amount of water used. Also, if a strong solution of NaOH is diluted to a lower concentration, heat is liberated. Conversely, if a solution is concentrated from a low to a high concentration, heat must be added.

In Fig. 8.4-3 an enthalpy-concentration chart for NaOH is given (M1) where the enthalpy is in kJ/kg (btu/lb_m) solution, temperature in °C (°F), and concentration in weight fraction NaOH in solution. Such enthalpy-concentration charts are usually not made for solutions having negligible heats of solution, since the heat capacities can be

easily used to calculate enthalpies. Also, such charts are available for only a few solutions.

The enthalpy of the liquid water in Fig. 8.4-3 is referred to the same datum or reference state as in the steam tables, i.e., liquid water at 0°C (273 K). This means that enthalpies from the figure can be used with those in the steam tables. In Eq. (8.4-7) values for h_F and h_L can be taken from Fig. 8.4-3 and values for λ and H_V from the steam tables. The uses of Fig. 8.4-3 will be best understood in the following example.

EXAMPLE 8.4-3. Evaporation of an NaOH Solution

An evaporator is used to concentrate 4536 kg/h (10000 lb_m/h) of a 20% solution of NaOH in water entering at 60°C (140°F) to a product of 50% solids. The pressure of the saturated steam used is 172.4 kPa (25 psia) and the pressure in the vapor space of the evaporator is 11.7 kPa (1.7 psia). The overall heat-transfer coefficient is 1560 W/m² · K (275 btu/h · ft² · °F). Calculate the steam used, the steam economy in kg vaporized/kg steam used, and the heating surface area in m².

Solution: The process flow diagram and nomenclature are the same as given in Fig. 8.4-1. The given variables are $F = 4536$ kg/h, $x_F = 0.20$ wt fraction, $T_F = 60^\circ\text{C}$, $P_1 = 11.7$ kPa, steam pressure = 172.4 kPa, and $x_L = 0.50$ wt fraction. For the overall material balance, substituting into Eq. (8.4-3),

$$F = 4536 = L + V \quad (8.4-3)$$

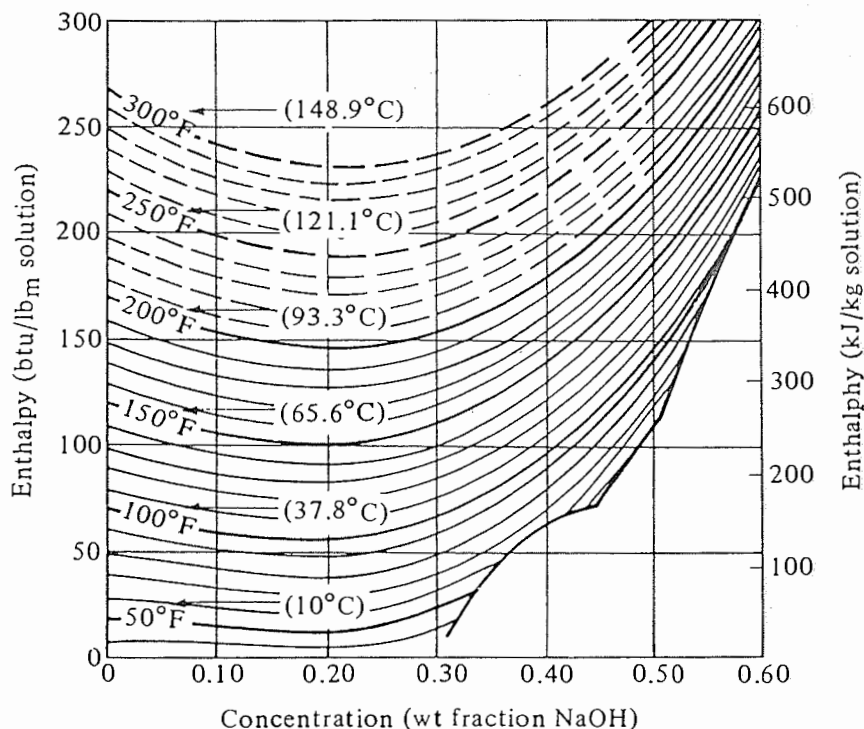


FIGURE 8.4-3. Enthalpy-concentration chart for the system NaOH-water. [Reference state liquid water at 0°C (273 K) or 32°F.] [From W. L. McCabe, *Trans. A.I.Ch.E.*, 31, 129 (1935). With permission.]

Substituting into Eq. (8.4-4) and solving (8.4-3) and (8.4-4) simultaneously,

$$Fx_F = Lx_L \quad (8.4-4)$$

$$4536(0.20) = L(0.50)$$

$$L = 1814 \text{ kg/h} \quad V = 2722 \text{ kg/h}$$

To determine the boiling point T_1 of the 50% concentrated solution, we first obtain the boiling point of pure water at 11.7 kPa from the steam tables, Appendix A.2, as 48.9°C (120°F). From the Dühring chart, Fig. 8.4-2, for a boiling point of water of 48.9°C and 50% NaOH, the boiling point of the solution is $T_1 = 89.5^\circ\text{C}$ (193°F). Hence,

$$\text{boiling-point rise} = T_1 - 48.9 = 89.5 - 48.9 = 40.6^\circ\text{C} \text{ (73°F)}$$

From the enthalpy-concentration chart (Fig. 8.4-3), for 20% NaOH at 60°C (140°F), $h_f = 214 \text{ kJ/kg}$ (92 btu/lb_m). For 50% NaOH at 89.5°C (193°F), $h_L = 505 \text{ kJ/kg}$ (217 btu/lb_m).

For the superheated vapor V at 89.5°C (193°F) and 11.7 kPa [superheated 40.6°C (73°F) since the boiling point of water is 48.9°C (120°F) at 11.7 kPa], from the steam tables, $H_v = 2667 \text{ kJ/kg}$ (1147 btu/lb_m). An alternative method to use to calculate the H_v is to first obtain the enthalpy of saturated vapor at 48.9°C (120°F) and 11.7 kPa of 2590 kJ/kg (1113.5 btu/lb_m). Then using a heat capacity of 1.884 kJ/kg·K for superheated steam with the superheat of $(89.5 - 48.9)^\circ\text{C} = (89.5 - 48.9)^\circ\text{K}$,

$$H_v = 2590 + 1.884(89.5 - 48.9) = 2667 \text{ kJ/kg}$$

For the saturated steam at 172.4 kPa, the saturation temperature from the steam tables is 115.6°C (240°F) and the latent heat is $\lambda = 2214 \text{ kJ/kg}$ (952 btu/lb_m).

Substituting into Eq. (8.4-7) and solving for S ,

$$Fh_f + S\lambda = Lh_L + VH_v \quad (8.4-7)$$

$$4535(214) + S(2214) = 1814(505) + 2722(2667)$$

$$S = 3255 \text{ kg steam/h}$$

Substituting into Eq. (8.4-8),

$$q = S\lambda = 3255(2214) \left(\frac{1}{3600} \right) = 2002 \text{ kW}$$

Substituting into Eq. (8.4-1) and solving,

$$2002(1000) = 1560(A)(115.6 - 89.5)$$

Hence, $A = 49.2 \text{ m}^2$. Also, steam economy = $2722/3255 = 0.836$.

8.5 CALCULATION METHODS FOR MULTIPLE-EFFECT EVAPORATORS

8.5A Introduction

In evaporation of solutions in a single-effect evaporator, a major cost is the cost of the steam used to evaporate the water. A single-effect evaporator is wasteful of steam costs,

since the latent heat of the vapor leaving the evaporator is usually not used. However, to reduce this cost, multiple-effect evaporators are used which recover the latent heat of the vapor leaving and reuse it.

A three-effect evaporator, discussed briefly in Section 8.2B, is shown in Fig. 8.2-3. In this system each effect in itself acts as a single-effect evaporator. In the first effect raw steam is used as the heating medium to this first effect, which is boiling at temperature T_1 and pressure P_1 . The vapor removed from the first effect is used as the heating medium, condensing in the second effect and vaporizing water at temperature T_2 and pressure P_2 in this effect. To transfer heat from the condensing vapor to the boiling liquid in this second effect, the boiling temperature T_2 must be less than the condensing temperature. This means that the pressure P_2 in the second effect is lower than P_1 in the first effect. In a similar manner, vapor from the second effect is condensed in heating the third effect. Hence, pressure P_3 is less than P_2 . If the first effect is operating at 1 atm abs pressure, the second and third effects will be under vacuum.

In the first effect, raw dilute feed is added and it is partly concentrated. Then this partly concentrated liquid (Fig. 8.2-3) flows to the second evaporator in series, where it is further concentrated. This liquid from the second effect flows to the third effect for final concentration.

When a multiple-effect evaporator is at steady-state operation, the flow rates and rate of evaporation in each effect are constant. The pressures, temperatures, and internal flow rates are automatically kept constant by the steady-state operation of the process itself. To change the concentration in the final effect, the feed rate to the first effect must be changed. The overall material balance made over the whole system and over each evaporator itself must be satisfied. If the final solution is too concentrated, the feed rate is increased, and vice versa. Then the final solution will reach a new steady state at the desired concentration.

8.5B Temperature Drops and Capacity of Multiple-Effect Evaporators

1. *Temperature drops in multiple-effect evaporators.* The amount of heat transferred per hour in the first effect of a triple-effect evaporator with forward feed as in Fig. 8.2-3 will be

$$q_1 = U_1 A_1 \Delta T_1 \quad (8.5-1)$$

where ΔT_1 is the difference between the condensing steam and the boiling point of the liquid, $T_s - T_1$. Assuming that the solutions have no boiling-point rise and no heat of solution and neglecting the sensible heat necessary to heat the feed to the boiling point, approximately all the latent heat of the condensing steam appears as latent heat in the vapor. This vapor then condenses in the second effect, giving up approximately the same amount of heat.

$$q_2 = U_2 A_2 \Delta T_2 \quad (8.5-2)$$

This same reasoning holds for q_3 . Then since $q_1 = q_2 = q_3$, then, approximately,

$$U_1 A_1 \Delta T_1 = U_2 A_2 \Delta T_2 = U_3 A_3 \Delta T_3 \quad (8.5-3)$$

Usually, in commercial practice the areas in all effects are equal and

$$\frac{q}{A} = U_1 \Delta T_1 = U_2 \Delta T_2 = U_3 \Delta T_3 \quad (8.5-4)$$

Hence, the temperature drops ΔT in a multiple-effect evaporator are approximately inversely proportional to the values of U . Calling $\sum \Delta T$ as follows for no boiling-point rise,

$$\sum \Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3 = T_s - T_3 \quad (8.5-5)$$

Note that $\Delta T_1 \text{ } ^\circ\text{C} = \Delta T_1 \text{ K}$, $\Delta T_2 \text{ } ^\circ\text{C} = \Delta T_2 \text{ K}$, and so on. Since ΔT_1 is proportional to $1/U_1$, then

$$\Delta T_1 = \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3} \quad (8.5-6)$$

Similar equations can be written for ΔT_2 and ΔT_3 .

2. *Capacity of multiple-effect evaporators.* A rough estimate of the capacity of a three-effect evaporator compared to a single effect can be obtained by adding the value of q for each evaporator.

$$q = q_1 + q_2 + q_3 = U_1 A_1 \Delta T_1 + U_2 A_2 \Delta T_2 + U_3 A_3 \Delta T_3 \quad (8.5-7)$$

If we make the assumption that the value of U is the same in each effect and the values of A are equal, Eq. (8.5-7) becomes

$$q = UA(\Delta T_1 + \Delta T_2 + \Delta T_3) = UA \Delta T \quad (8.5-8)$$

where $\Delta T = \sum \Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3 = T_s - T_3$.

If a single-effect evaporator is used with the same area A , the same value of U , and the same total temperature drop ΔT , then

$$q = UA \Delta T \quad (8.5-9)$$

This, of course, gives the same capacity as for the multiple-effect evaporators. Hence, the increase in steam economy obtained by using multiple-effect evaporators is obtained at the expense of reduced capacity.

8.5C Calculations for Multiple-Effect Evaporators

In doing calculations for a multiple-effect evaporator system, the values to be obtained are usually the area of the heating surface in each effect, the kg of steam per hour to be supplied, and the amount of vapor leaving each effect, especially the last effect. The given or known values are usually as follows: (1) steam pressure to first effect, (2) final pressure in vapor space of the last effect, (3) feed conditions and flow to first effect, (4) the final concentration in the liquid leaving the last effect, (5) physical properties such as enthalpies and/or heat capacities of the liquid and vapors, and (6) the overall heat-transfer coefficients in each effect. Usually, the areas of each effect are assumed equal.

The calculations are done using material balances, heat balances, and the capacity equations $q = UA \Delta T$ for each effect. A convenient way to solve these equations is by trial and error. The basic steps to follow are given as follows for a triple-effect evaporator.

8.5D Step-by-Step Calculation Methods for Triple-Effect Evaporators

1. From the known outlet concentration and pressure in the last effect, determine the boiling point in the last effect. (If a boiling-point rise is present, this can be determined from a Dühring line plot.)

- Determine the total amount of vapor evaporated by an overall material balance. For this first trial apportion this total amount of vapor among the three effects. (Usually, equal vapor produced in each effect, so that $V_1 = V_2 = V_3$ is assumed for the first trial.) Make a total material balance on effects 1, 2, and 3 to obtain L_1 , L_2 , and L_3 . Then calculate the solids concentration in each effect by a solids balance on each effect.
- Using Eq. (8.5-6), estimate the temperature drops ΔT_1 , ΔT_2 , and ΔT_3 in the three effects. Any effect that has an extra heating load, such as a cold feed, requires a proportionately larger ΔT . Then calculate the boiling point in each effect.

[If a boiling-point rise (BPR) in $^{\circ}\text{C}$ is present, estimate the pressure in effects 1 and 2 and determine the BPR in each of the three effects. Only a crude pressure estimate is needed since BPR is almost independent of pressure. Then the $\sum \Delta T$ available for heat transfer without the superheat is obtained by subtracting the sum of all three BPRs from the overall ΔT of $T_s - T_3$ (saturation). Using Eq. (8.5-6), estimate ΔT_1 , ΔT_2 , and ΔT_3 . Then calculate the boiling point in each effect.]

- Using heat and material balances in each effect, calculate the amount vaporized and the flows of liquid in each effect. If the amounts vaporized differ appreciably from those assumed in step 2, then steps 2, 3, and 4 can be repeated using the amounts of evaporation just calculated. (In step 2 only the solids balance is repeated.)
- Calculate the value of q transferred in each effect. Using the rate equation $q = UA \Delta T$ for each effect, calculate the areas A_1 , A_2 , and A_3 . Then calculate the average value A_m by

$$A_m = \frac{A_1 + A_2 + A_3}{3} \quad (8.5-10)$$

If these areas are reasonably close to each other, the calculations are complete and a second trial is not needed. If these areas are not nearly equal, a second trial should be performed as follows.

- To start trial 2, use the new values of L_1 , L_2 , L_3 , V_1 , V_2 , and V_3 calculated by the heat balances in step 4 and calculate the new solids concentrations in each effect by a solids balance on each effect.
- Obtain new values of $\Delta T'_1$, $\Delta T'_2$, and $\Delta T'_3$ from

$$\Delta T'_1 = \frac{\Delta T_1 A_1}{A_m} \quad \Delta T'_2 = \frac{\Delta T_2 A_2}{A_m} \quad \Delta T'_3 = \frac{\Delta T_3 A_3}{A_m} \quad (8.5-11)$$

The sum $\Delta T'_1 + \Delta T'_2 + \Delta T'_3$ must equal the original $\sum \Delta T$. If not, proportionately readjust all $\Delta T'$ values so that this is so. Then calculate the boiling point in each effect.

[If a boiling point rise is present, then using the new concentrations from step 6, determine the new BPRs in the three effects. This gives a new value of $\sum \Delta T$ available for heat transfer by subtracting the sum of all three BPRs from the overall ΔT . Calculate the new values of $\Delta T'$ by Eq. (8.5-11). The sum of the $\Delta T'$ values just calculated must be readjusted to this new $\sum \Delta T$ value. Then calculate the boiling point in each effect.] Step 7 is essentially a repeat of step 3 but using Eq. (8.5-11) to obtain a better estimate of the $\Delta T'$ values.

- Using the new $\Delta T'$ values from step 7, repeat the calculations starting with step 4. Two trials are usually sufficient so that the areas are reasonably close to being equal.

EXAMPLE 8.5-1. Evaporation of Sugar Solution in a Triple-Effect Evaporator

A triple-effect forward-feed evaporator is being used to evaporate a sugar solution containing 10 wt % solids to a concentrated solution of 50%. The boiling-point rise of the solutions (independent of pressure) can be estimated

from $\text{BPR}^{\circ}\text{C} = 1.78x + 6.22x^2$ ($\text{BPR}^{\circ}\text{F} = 3.2x + 11.2x^2$), where x is wt fraction of sugar in solution (K1). Saturated steam at 205.5 kPa (29.8 psia) [121.1°C (250°F) saturation temperature] is being used. The pressure in the vapor space of the third effect is 13.4 kPa (1.94 psia). The feed rate is 22 680 kg/h (50 000 lb_m/h) at 26.7°C (80°F). The heat capacity of the liquid solutions is (K1) $c_p = 4.19 - 2.35x$ kJ/kg·K (1.0 - 0.56x btu/lb_m·°F). The heat of solution is considered to be negligible. The coefficients of heat transfer have been estimated as $U_1 = 3123$, $U_2 = 1987$, and $U_3 = 1136$ W/m²·K or 550, 350, and 200 btu/h·ft²·°F. If each effect has the same surface area, calculate the area, the steam rate used, and the steam economy.

Solution: The process flow diagram is given in Fig. 8.5-1. Following the eight steps outlined, the calculations are as follows.

Step 1. For 13.4 kPa (1.94 psia), the saturation temperature is 51.67°C (125°F) from the steam tables. Using the equation for BPR for evaporator number 3 with $x = 0.5$,

$$\text{BPR}_3 = 1.78x + 6.22x^2 = 1.78(0.5) + 6.22(0.5)^2 = 2.45^{\circ}\text{C} (4.4^{\circ}\text{F})$$

$$T_3 = 51.67 + 2.45 = 54.12^{\circ}\text{C} (129.4^{\circ}\text{F})$$

Step 2. Making an overall and a solids balance to calculate the total amount vaporized ($V_1 + V_2 + V_3$) and L_3 ,

$$F = 22\,680 = L_3 + (V_1 + V_2 + V_3)$$

$$Fx_F = 22\,680(0.1) = L_3(0.5) + (V_1 + V_2 + V_3)(0)$$

$$L_3 = 4536 \text{ kg/h (10 000 lb}_m\text{/h)}$$

$$\text{total vaporized} = (V_1 + V_2 + V_3) = 18\,144 \text{ kg/h (40 000 lb}_m\text{/h)}$$

Assuming equal amount vaporized in each effect, $V_1 = V_2 = V_3 = 6048$ kg/h (13 333 lb_m/h). Making a total material balance on effects 1, 2, and 3 and solving,

$$(1) \quad F = 22\,680 = V_1 + L_1 = 6048 + L_1, \quad L_1 = 16\,632 \text{ kg/h (33 667 lb}_m\text{/h)}$$

$$(2) \quad L_1 = 16\,632 = V_2 + L_2 = 6048 + L_2, \quad L_2 = 10\,584 (23\,334)$$

$$(3) \quad L_2 = 10\,584 = V_3 + L_3 = 6048 + L_3, \quad L_3 = 4536 (10\,000)$$

Making a solids balance on effects 1, 2, and 3 and solving for x ,

$$(1) \quad 22\,680(0.1) = L_1x_1 = 16\,632(x_1), \quad x_1 = 0.136$$

$$(2) \quad 16\,632(0.136) = L_2x_2 = 10\,584(x_2), \quad x_2 = 0.214$$

$$(3) \quad 10\,584(0.214) = L_3x_3 = 4536(x_3), \quad x_3 = 0.500 \quad (\text{check balance})$$

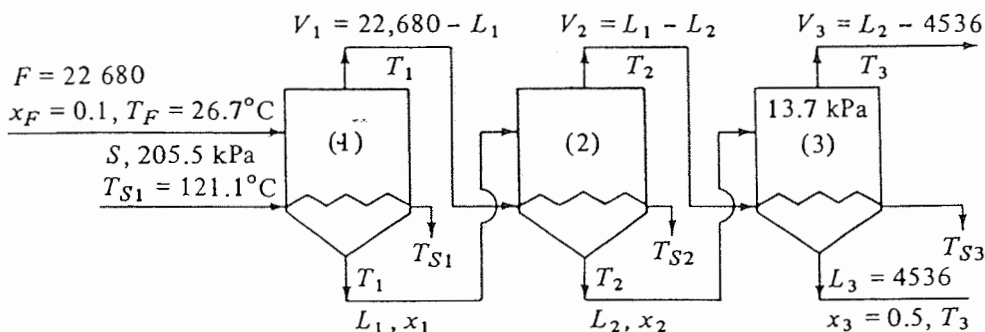


FIGURE 8.5-1. Flow diagram for triple-effect evaporation for Example 8.5-1.

Step 3. The BPR in each effect is calculated as follows:

$$(1) \text{ BPR}_1 = 1.78x_1 + 6.22x_1^2 = 1.78(0.136) + 6.22(0.136)^2 = 0.36^\circ\text{C} (0.7^\circ\text{F})$$

$$(2) \text{ BPR}_2 = 1.78(0.214) + 6.22(0.214)^2 = 0.65^\circ\text{C} (1.2^\circ\text{F})$$

$$(3) \text{ BPR}_3 = 1.78(0.5) + 6.22(0.5)^2 = 2.45^\circ\text{C} (4.4^\circ\text{F})$$

$$\begin{aligned} \sum \Delta T \text{ available} &= T_{S1} - T_3 \text{ (saturation)} - (\text{BPR}_1 + \text{BPR}_2 + \text{BPR}_3) \\ &= 121.1 - 51.67 - (0.36 + 0.65 + 2.45) = 65.97^\circ\text{C} (118.7^\circ\text{F}) \end{aligned}$$

Using Eq. (8.5-6) for ΔT_1 and similar equations for ΔT_2 and ΔT_3 ,

$$\begin{aligned} \Delta T_1 &= \sum \Delta T \frac{1/U_1}{1/U_1 + 1/U_2 + 1/U_3} = \frac{(65.97)(1/3123)}{(1/3123) + (1/1987) + (1/1136)} \\ \Delta T_1 &= 12.40^\circ\text{C} \quad \Delta T_2 = 19.50^\circ\text{C} \quad \Delta T_3 = 34.07^\circ\text{C} \end{aligned}$$

However, since a cold feed enters effect number 1, this effect requires more heat. Increasing ΔT_1 and lowering ΔT_2 and ΔT_3 proportionately as a first estimate,

$$\begin{aligned} \Delta T_1 &= 15.56^\circ\text{C} = 15.56 \text{ K} \quad \Delta T_2 = 18.34^\circ\text{C} = 18.34 \text{ K} \\ \Delta T_3 &= 32.07^\circ\text{C} = 32.07 \text{ K} \end{aligned}$$

To calculate the actual boiling point of the solution in each effect,

$$\begin{aligned} (1) \quad T_1 &= T_{S1} - \Delta T_1 \\ &= 121.1 - 15.56 = 105.54^\circ\text{C} \\ T_{S1} &= 121.1^\circ\text{C} \text{ (condensing temperature of saturated steam to effect 1)} \\ (2) \quad T_2 &= T_1 - \text{BPR}_1 - \Delta T_2 \\ &= 105.54 - 0.36 - 18.34 = 86.84^\circ\text{C} \\ T_{S2} &= T_1 - \text{BPR}_1 = 105.54 - 0.36 \\ &= 105.18^\circ\text{C} \text{ (condensing temperature of steam to effect 2)} \\ (3) \quad T_3 &= T_2 - \text{BPR}_2 - \Delta T_3 \\ &= 86.84 - 0.65 - 32.07 = 54.12^\circ\text{C} \\ T_{S3} &= T_2 - \text{BPR}_2 \\ &= 86.84 - 0.65 = 86.19^\circ\text{C} \text{ (condensing temperature of steam to effect 3)} \end{aligned}$$

The temperatures in the three effects are as follows:

Effect 1	Effect 2	Effect 3	Condenser
$T_{S1} = 121.1^\circ\text{C}$	$T_{S2} = 105.18$	$T_{S3} = 86.19$	$T_{S4} = 51.67$
$T_1 = 105.54$	$T_2 = 86.84$	$T_3 = 54.12$	

Step 4. The heat capacity of the liquid in each effect is calculated from the equation $c_p = 4.19 - 2.35x$.

$$\begin{aligned} F: \quad c_p &= 4.19 - 2.35(0.1) = 3.955 \text{ kJ/kg} \cdot \text{K} \\ L_1: \quad c_p &= 4.19 - 2.35(0.136) = 3.869 \\ L_2: \quad c_p &= 4.19 - 2.35(0.214) = 3.684 \\ L_3: \quad c_p &= 4.19 - 2.35(0.5) = 3.015 \end{aligned}$$

The values of the enthalpy H of the various vapor streams relative to water at 0°C as a datum are obtained from the steam table as follows:

Effect 1:

$$T_1 = 105.54^\circ\text{C}, T_{S2} = 105.18 \text{ (221.3}^\circ\text{F)}, \text{BPR}_1 = 0.36, T_{S1} = 121.1 \text{ (250}^\circ\text{F)}$$

$$\begin{aligned} H_1 &= H_{S2} \text{ (saturation enthalpy at } T_{S2}) + 1.884 \text{ (0.36}^\circ\text{C superheat)} \\ &= 2684 + 1.884(0.36) = 2685 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \lambda_{S1} &= H_{S1} \text{ (vapor saturation enthalpy)} - h_{S1} \text{ (liquid enthalpy at } T_{S1}) \\ &= (2708 - 508) = 2200 \text{ kJ/kg latent heat of condensation} \end{aligned}$$

Effect 2:

$$T_2 = 86.84^\circ\text{C}, T_{S3} = 86.19, \text{BPR}_2 = 0.65$$

$$H_2 = H_{S3} + 1.884(0.65) = 2654 + 1.884(0.65) = 2655 \text{ kJ/kg}$$

$$\lambda_{S2} = H_1 - h_{S2} = 2685 - 441 = 2244 \text{ kJ/kg}$$

Effect 3:

$$T_3 = 54.12^\circ\text{C}, T_{S4} = 51.67, \text{BPR}_3 = 2.45$$

$$H_3 = H_{S4} + 1.884(2.45) = 2595 + 1.884(2.45) = 2600 \text{ kJ/kg}$$

$$\lambda_{S3} = H_2 - h_{S3} = 2655 - 361 = 2294 \text{ kJ/kg}$$

(Note that the superheat corrections in this example are small and could possibly have been neglected. However, the corrections were used to demonstrate the method of calculation.) Flow relations to be used in heat balances are

$$V_1 = 22680 - L_1, \quad V_2 = L_1 - L_2, \quad V_3 = L_2 - 4536, \quad L_3 = 4536$$

Write a heat balance on each effect. Use 0°C as a datum since the values of H of the vapors are relative to 0°C (32°F) and note that $(T_F - 0)^\circ\text{C} = (T_F - 0) \text{ K}$ and $(T_1 - 0)^\circ\text{C} = (T_1 - 0) \text{ K}$,

$$(1) \quad Fc_p(T_F - 0) + S\lambda_{S1} = L_1c_p(T_1 - 0) + V_1H_1$$

Substituting the known values,

$$\begin{aligned} 22680(3.955)(26.7 - 0) + S(2200) &= L_1(3.869)(105.54 - 0) \\ &\quad + (22680 - L_1)(2685) \end{aligned}$$

$$\begin{aligned} (2) \quad L_1c_p(T_1 - 0) + V_1\lambda_{S2} &= L_2c_p(T_2 - 0) + V_2H_2 \\ L_1(3.869)(105.54 - 0) + (22680 - L_1)(2244) &= L_2(3.684)(86.84 - 0) \\ &\quad + (L_1 - L_2)(2655) \end{aligned}$$

$$\begin{aligned} (3) \quad L_2c_p(T_2 - 0) + V_2\lambda_{S3} &= L_3c_p(T_3 - 0) + V_3H_3 \\ L_2(3.684)(86.84 - 0) + (L_1 - L_2)(2294) &= 4536(3.015)(54.12 - 0) \\ &\quad + (L_2 - 4536)(2600) \end{aligned}$$

Solving the last two equations simultaneously for L_1 and L_2 and substituting into the first equation,

$$\begin{aligned} L_1 &= 17078 \text{ kg/h} & L_2 &= 11068 & L_3 &= 4536 \\ S &= 8936 & V_1 &= 5602 & V_2 &= 6010 & V_3 &= 6532 \end{aligned}$$

The calculated values of V_1 , V_2 , and V_3 are close enough to the assumed values so that steps 2, 3, and 4 do not need to be repeated. If the calculation were repeated, the calculated values of V_1 , V_2 , and V_3 would be used starting with step 2 and a solids balance in each effect would be made.

Step 5. Solving for the values of q in each effect and area,

$$q_1 = S \lambda_{s1} = \left(\frac{8936}{3600} \right) (2200 \times 1000) = 5.460 \times 10^6 \text{ W}$$

$$q_2 = V_1 \lambda_{s2} = \left(\frac{5602}{3600} \right) (2244 \times 1000) = 3.492 \times 10^6 \text{ W}$$

$$q_3 = V_2 \lambda_{s3} = \left(\frac{6010}{3600} \right) (2294 \times 1000) = 3.830 \times 10^6 \text{ W}$$

$$A_1 = \frac{q_1}{U_1 \Delta T_1} = \frac{5.460 \times 10^6}{3123(15.56)} = 112.4 \text{ m}^2$$

$$A_2 = \frac{q_2}{U_2 \Delta T_2} = \frac{3.492 \times 10^6}{1987(18.34)} = 95.8 \text{ m}^2$$

$$A_3 = \frac{q_3}{U_3 \Delta T_3} = \frac{3.830 \times 10^6}{1136(32.07)} = 105.1 \text{ m}^2$$

The average area $A_m = 104.4 \text{ m}^2$. The areas differ from the average value by less than 10% and a second trial is not really necessary. However, a second trial will be made starting with step 6 to indicate the calculation methods used.

Step 6. Making a new solids balance on effects 1, 2, and 3 using the new $L_1 = 17078$, $L_2 = 11068$, and $L_3 = 4536$ and solving for x ,

$$(1) \quad 22680(0.1) = 17078(x_1), \quad x_1 = 0.133$$

$$(2) \quad 17078(0.133) = 11068(x_2), \quad x_2 = 0.205$$

$$(3) \quad 11068(0.205) = 4536(x_3), \quad x_3 = 0.500 \text{ (check balance)}$$

Step 7. The new BPR in each effect is then

$$(1) \quad \text{BPR}_1 = 1.78x_1 + 6.22x_1^2 = 1.78(0.133) + 6.22(0.133)^2 = 0.35^\circ\text{C}$$

$$(2) \quad \text{BPR}_2 = 1.78(0.205) + 6.22(0.205)^2 = 0.63^\circ\text{C}$$

$$(3) \quad \text{BPR}_3 = 1.78(0.5) + 6.22(0.5)^2 = 2.45^\circ\text{C}$$

$$\sum \Delta T \text{ available} = 121.1 - 51.67 - (0.35 + 0.63 + 2.45) = 66.00^\circ\text{C}$$

The new values for ΔT are obtained using Eq. (8.5-11).

$$\Delta T'_1 = \frac{\Delta T_1 A_1}{A_m} = \frac{15.56(112.4)}{104.4} = 16.77 \text{ K} = 16.77^\circ\text{C}$$

$$\Delta T'_2 = \frac{\Delta T_2 A_2}{A_m} = \frac{18.34(95.8)}{104.4} = 16.86^\circ\text{C}$$

$$\Delta T'_3 = \frac{\Delta T_3 A_3}{A_m} = \frac{32.07(105.1)}{104.4} = 32.34^\circ\text{C}$$

$$\sum \Delta T = 16.77 + 16.86 + 32.34 = 65.97^\circ\text{C}$$

These $\Delta T'$ values are readjusted so that $\Delta T'_1 = 16.77$, $\Delta T'_2 = 16.87$, $\Delta T'_3 = 32.36$, and $\sum \Delta T = 16.77 + 16.87 + 32.36 = 66.00^\circ\text{C}$. To calculate the actual boiling point of the solution in each effect,

$$(1) \quad T_1 = T_{S1} - \Delta T'_1 = 121.1 - 16.77 = 104.33^\circ\text{C}, \quad T_{S1} = 121.1^\circ\text{C}$$

$$(2) \quad T_2 = T_1 - \text{BPR}_1 - \Delta T'_2 = 104.33 - 0.35 - 16.87 = 87.11^\circ\text{C}$$

$$T_{S2} = T_1 - \text{BPR}_1 = 104.33 - 0.35 = 103.98^\circ\text{C}$$

$$(3) \quad T_3 = T_2 - \text{BPR}_2 - \Delta T'_3 = 87.11 - 0.63 - 32.36 = 54.12^\circ\text{C}$$

$$T_{S3} = T_2 - \text{BPR}_2 = 87.11 - 0.63 = 86.48^\circ\text{C}$$

Step 8. Following step 4, the heat capacity of the liquid is $c_p = 4.19 - 2.35x$.

$$F: \quad c_p = 3.955 \text{ kJ/kg} \cdot \text{K}$$

$$L_1: \quad c_p = 4.19 - 2.35(0.133) = 3.877$$

$$L_2: \quad c_p = 4.19 - 2.35(0.205) = 3.708$$

$$L_3: \quad c_p = 3.015$$

The new values of the enthalpy H are as follows in each effect.

$$(1) \quad H_1 = H_{S2} + 1.884(^\circ\text{C superheat}) = 2682 + 1.884(0.35) = 2683 \text{ kJ/kg}$$

$$\lambda_{S1} = H_{S1} - h_{S1} = 2708 - 508 = 2200 \text{ kJ/kg}$$

$$(2) \quad H_2 = H_{S3} + 1.884(0.63) = 2654 + 1.884(0.63) = 2655 \text{ kJ/kg}$$

$$\lambda_{S2} = H_1 - h_{S2} = 2683 - 440 = 2243 \text{ kJ/kg}$$

$$(3) \quad H_3 = H_{S4} + 1.884(2.45) = 2595 + 1.884(2.45) = 2600 \text{ kJ/kg}$$

$$\lambda_{S3} = H_2 - h_{S3} = 2655 - 362 = 2293 \text{ kJ/kg}$$

Writing a heat balance on each effect,

$$(1) \quad 22680(3.955)(26.7 - 0) + S(2200) = L_1(3.877)(104.33 - 0) \\ + (22680 - L_1)(2683)$$

$$(2) \quad L_1(3.877)(104.33 - 0) + (22680 - L_1)(2243) = L_2(3.708)(87.11 - 0) \\ + (L_1 - L_2)(2655)$$

$$(3) \quad L_2(3.708)(87.11 - 0) + (L_1 - L_2)(2293) = 4536(3.015)(54.12 - 0) \\ + (L_2 - 4536)(2600)$$

Solving,

$$L_1 = 17005 \text{ kg/h} \quad L_2 = 10952 \quad L_3 = 4536 \quad S = 8960 \text{ (steam used)}$$

$$V_1 = 5675 \quad V_2 = 6053 \quad V_3 = 6416$$

Note that these values from trial 2 differ very little from the trial 1 results. Following step 5, and solving for q in each effect and A ,

$$q_1 = S\lambda_{S1} = \frac{8960}{3600} (2200 \times 1000) = 5.476 \times 10^6 \text{ W}$$

$$q_2 = V_1\lambda_{S2} = \frac{5675}{3600} (2243 \times 1000) = 3.539 \times 10^6 \text{ W}$$

$$q_3 = V_2 \lambda_{s3} = \frac{6053}{3600} (2293 \times 1000) = 3.855 \times 10^6 \text{ W}$$

$$A_1 = \frac{q_1}{U_1 \Delta T'_1} = \frac{5.476 \times 10^6}{3123(16.77)} = 104.6 \text{ m}^2$$

$$A_2 = \frac{q_2}{U_2 \Delta T'_2} = \frac{3.539 \times 10^6}{1987(16.87)} = 105.6 \text{ m}^2$$

$$A_3 = \frac{q_3}{U_3 \Delta T'_3} = \frac{3.855 \times 10^6}{1136(32.36)} = 104.9 \text{ m}^2$$

The average area $A_m = 105.0 \text{ m}^2$ to use in each effect. Note that this value of 105.0 m^2 is quite close to the average value of 104.4 m^2 from the first trial.

$$\text{steam economy} = \frac{V_1 + V_2 + V_3}{S} = \frac{5675 + 6053 + 6416}{8960} = 2.025$$

8.6 CONDENSERS FOR EVAPORATORS

8.6A Introduction

In multiple-effect evaporators the vapors from the last effect are usually leaving under vacuum, i.e., at less than atmospheric pressure. These vapors must be condensed and discharged as a liquid at atmospheric pressure. This is done by condensing the vapors using cooling water. The condenser can be a surface condenser, where the vapor to be condensed and the cooling liquid are separated by a metal wall, or a direct contact condenser, where the vapor and cooling liquid are mixed directly.

8.6B Surface Condensers

Surface condensers are employed where actual mixing of the condensate with condenser cooling water is not desired. In general, they are shell and tube condensers with the vapor on the shell side and cooling water in multipass flow on the tube side. Noncondensable gases are usually present in the vapor stream. These can be air, CO_2 , N_2 , or another gas which may have entered as dissolved gases in the liquid feed or occur because of decomposition in the solutions. These noncondensable gases may be vented from any well-cooled point in the condenser. If the vapor being condensed is below atmospheric pressure, the condensed liquid leaving the surface condenser can be removed by pumping and the noncondensable gases by a vacuum pump. Surface condensers are much more expensive and use more cooling water, so they are usually not used in cases where a direct-contact condenser is suitable.

8.6C Direct-Contact Condensers

In *direct-contact condensers* cooling water directly contacts the vapors and condenses the vapors. One of the most common types of direct-contact condensers is the counter-current barometric condenser shown in Fig. 8.6-1. The vapor enters the condenser and is condensed by rising upward against a shower of cooling water droplets. The condenser is located on top of a long discharge tail pipe. The condenser is high enough above the discharge point in the tail pipe so that the water column established in the pipe more than compensates for the difference in pressure between the low absolute pressure in the

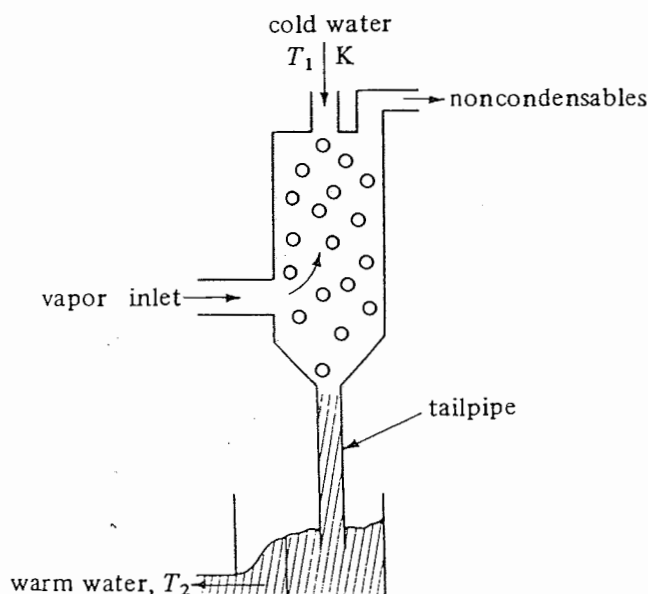


FIGURE 8.6-1. Schematic of barometric condenser.

condenser and the atmosphere. The water can then discharge by gravity through a seal pot at the bottom. A height of about 10.4 m (34 ft) is used.

The barometric condenser is inexpensive and economical of water consumption. It can maintain a vacuum corresponding to a saturated vapor temperature within about 2.8 K (5°F) of the water temperature leaving the condenser. For example, if the discharge water is at 316.5 K (110°F), the pressure corresponding to 316.5 + 2.8 or 319.3 K is 10.1 kPa (1.47 psia).

The water consumption can be estimated by a simple heat balance for a barometric condenser. If the vapor flow to the condenser is V kg/h at temperature T_s and the water flow is W kg/h at an entering temperature of T_1 and a leaving temperature of T_2 , the derivation is as follows.

$$VH_s + Wc_p(T_1 - 273.2) = (V + W)c_p(T_2 - 273.2) \quad (8.6-1)$$

where H_s is the enthalpy from the steam tables of the vapor at T_s K and the pressure in the vapor stream. Solving,

$$\frac{W}{V} = \frac{\text{kg water}}{\text{kg vapor}} = \frac{H_s - c_p(T_2 - 273.2)}{c_p(T_2 - T_1)} \quad (8.6-2)$$

The noncondensable gases can be removed from the condenser by a vacuum pump. The vacuum pump used can be a mechanical pump or a steam-jet ejector. In the ejector high-pressure steam enters a nozzle at high speed and entrains the noncondensable gases from the space under vacuum.

Another type of direct-contact condenser is the jet barometric condenser. High-velocity jets of water act both as a vapor condenser and as an entrainer of the noncondensables out of the tail pipe. Jet condensers usually require more water than the more common barometric condensers and are more difficult to throttle at low vapor rates.

8.7 EVAPORATION OF BIOLOGICAL MATERIALS

8.7A Introduction and Properties of Biological Materials

The evaporation of many biological materials often differs from evaporation of inorganic materials such as NaCl and NaOH and organic materials such as ethanol and acetic acid. Biological materials such as pharmaceuticals, milk, citrus juices, and vegetable extracts are usually quite heat-sensitive and often contain fine particles of suspended matter in the solution. Also, because of problems due to bacteria growth, the equipment must be designed for easy cleaning. Many biological materials in solution exhibit only a small boiling-point rise when concentrated. This is due to the fact that suspended solids in a fine dispersed form and dissolved solutes of large molecular weight contribute little to this rise.

The amount of degradation of biological materials on evaporation is a function of the temperature and the length of time. To keep the temperature low, the evaporation must be done under vacuum, which reduces the boiling point of the solution. To keep the time of contact low, the equipment must provide for a low holdup time (contact time) of the material being evaporated. Typical types of equipment used and some biological materials processed are given below. Detailed discussions of the equipment are given in Section 8.2.

1. Long-tube vertical evaporator. Condensed milk.
2. Falling-film evaporator. Fruit juices.
3. Agitated-film (wiped-film) evaporator. Rubber latex, gelatin, antibiotics, fruit juices.
4. Heat-pump cycle evaporator. Fruit juices, milk, pharmaceuticals.

8.7B Fruit Juices

In evaporation of fruit juices such as orange juice the problems are quite different from evaporation of a typical salt such as NaCl. The fruit juices are heat-sensitive and the viscosity increases greatly as concentration increases. Also, solid suspended matter in fruit juices has a tendency to cling to the heating surface and thus causes overheating, leading to burning and spoilage of the matter (B2).

To reduce this tendency to stick and to reduce residence time, high rates of circulation over the heat-transfer surface are necessary. Since the material is heat-sensitive, low-temperature operation is also necessary. Hence, a fruit juice concentration plant usually employs a single and not a multiple evaporation unit. Vacuum is used to reduce the temperature of evaporation.

A typical fruit juice evaporation system using the heat pump cycle is shown (P1, C1), which uses low-temperature ammonia as the heating fluid. A frozen concentrated citrus juice process is described by Charm (C1). The process uses a multistage falling-film evaporator. A major fault of concentrated orange juice is a flat flavor due to the loss of volatile constituents during evaporation. To overcome this, a portion of the fresh pulpy juice bypasses the evaporation cycle and is blended with the evaporated concentrate.

8.7C Sugar Solutions

Sugar (sucrose) is obtained primarily from the sugar cane and sugar beet. Sugar tends to caramelize if kept at high temperatures for long periods (B2). The general tendency is to

use short-tube evaporators of the natural circulation type. In the evaporation process of sugar solutions, the clear solution of sugar having a concentration of 10–13° Brix (10–13 wt %) is evaporated to 40–60° Brix (K1, S1).

The feed is first preheated by exhaust steam and then typically enters a six-effect forward-feed evaporator system. The first effect operates at a pressure in the vapor space of the evaporator of about 207 kPa (30 psia) [121.1°C (250°F) saturation temperature] and the last effect under vacuum at about 24 kPa (63.9°C saturation). Examples of the relatively small boiling-point rise of sugar solutions and the heat capacity are given in Example 8.5-1.

8.7D Paper-Pulp Waste Liquors

In the making of paper pulp in the sulfate process, wood chips are digested or cooked and spent black liquor is obtained after washing the pulp. This solution contains primarily sodium carbonate and organic sulfide compounds. This solution is concentrated by evaporation in a six-effect system (K1, S1).

8.8. EVAPORATION USING VAPOR RECOMPRESSION

8.8A Introduction

In the single-effect evaporator the vapor from the unit is generally condensed and discarded. In the multiple-effect evaporator, the pressure in each succeeding effect is lowered so that the boiling point of the liquid is lowered in each effect. Hence, there is a temperature difference created for the vapor from one effect to condense in the next effect and boil the liquid to form vapor.

In a single-effect vapor recompression (sometimes called vapor compression) evaporator the vapor is compressed so that its condensing or saturation temperature is increased. This compressed vapor is returned back to the heater of steam chest and condenses so that vapor is formed in the evaporator (B5, W1, Z1). In this manner the latent heat of the vapor is used and not discarded. The two types of vapor recompression units are the mechanical and the thermal type.

8.8B Mechanical Vapor Recompression Evaporator.

In a mechanical vapor recompression evaporator, a conventional single effect evaporator similar to that in Fig. 8.2-2 is used and is shown in Fig. 8.8-1. The cold feed is preheated by exchange with the hot outlet liquid product and then flows to the unit. The vapor coming overhead does not go to a condenser but is sent to a centrifugal or positive displacement compressor driven by an electric motor or steam. This compressed vapor or steam is sent back to the heat exchanger or steam chest. The compressed vapor condenses at a higher temperature than the boiling point of the hot liquid in the effect and a temperature difference is set up. Vapor is again generated and the cycle repeated.

Sometimes it is necessary to add a small amount of makeup steam to the vapor line before the compressor (B5, K2). Also, a small amount of condensate may be added to the compressed vapor to remove any superheat, if present.

Vapor recompression units generally operate at low optimum temperature differences of 5 to 10°C. Hence, large heat transfer areas are needed. These units usually have higher capital costs than multiple-effect units because of the larger area and the

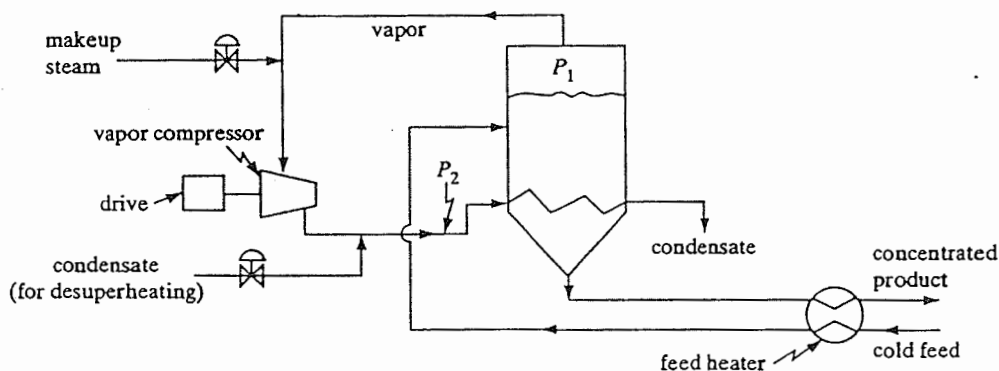


FIGURE 8.8-1. Simplified process flow for mechanical vapor recompression evaporator.

costs of the relatively expensive compressor and drive unit. The main advantage of vapor recompression units is in the lower energy costs. Using the steam equivalent of the power to drive the compressor, the steam economy is equivalent to a multiple-effect evaporator of up to 10 or more units (Z1).

Some typical applications of mechanical vapor recompression units are evaporation of sea water to give distilled water, evaporation of kraft black liquor in the paper industry (L2), evaporation of heat-sensitive materials such as fruit juices, and crystallizing of salts having inverse solubility curves where the solubility decreases with increasing temperature (K2, M3).

Falling-film evaporators are well suited for vapor recompression systems (W1) because they operate at low-temperature-difference values and have very little entrained liquid which can cause problems in the compressor. Vapor recompression has been used in distillation towers where the overhead vapor is recompressed and used in the reboiler as the heating medium (M2).

8.8C Thermal Vapor Recompression Evaporator

A steam jet can also be used to compress the vapor in a thermal vapor recompression unit. The main disadvantages are the low efficiency of the steam jet, necessitating the removal of this excess heat, and the lack of flexibility to changes in process variables (M3). Steam jets are cheaper and more durable than mechanical compressors and can more easily handle large volumes of low-pressure vapors.

PROBLEMS

8.4-1. Heat-Transfer Coefficient in Single-Effect Evaporator. A feed of 4535 kg/h of a 2.0 wt % salt solution at 311 K enters continuously a single-effect evaporator and is being concentrated to 3.0%. The evaporation is at atmospheric pressure and the area of the evaporator is 69.7 m². Saturated steam at 383.2 K is supplied for heating. Since the solution is dilute, it can be assumed to have the same boiling point as water. The heat capacity of the feed can be taken as $c_p = 4.10 \text{ kJ/kg} \cdot \text{K}$. Calculate the amounts of vapor and liquid product and the overall heat-transfer coefficient U .

Ans. $U = 1823 \text{ W/m}^2 \cdot \text{K}$

8.4-2. Effects of Increased Feed Rate in Evaporator. Using the same area, value of U , steam pressure, evaporator pressure, and feed temperature as in Problem 8.4-1,

calculate the amounts of liquid and vapor leaving and the liquid outlet concentration if the feed rate is increased to 6804 kg/h.

Ans. $V = 1256 \text{ kg/h}$, $L = 5548 \text{ kg/h}$, $x_L = 2.45\%$

8.4-3. Effect of Evaporator Pressure on Capacity and Product Composition. Recalculate Example 8.4-1 but use an evaporator pressure of 41.4 kPa instead of 101.32 kPa abs. Use the same steam pressure, area A , and heat-transfer coefficient U in the calculations.

- Do this to obtain the new capacity or feed rate under these new conditions. The composition of the liquid product will be the same as before.
- Do this to obtain the new product composition if the feed rate is increased to 18 144 kg/h.

8.4-4. Production of Distilled Water. An evaporator having an area of 83.6 m^2 and a $U = 2270 \text{ W/m}^2 \cdot \text{K}$ is used to produce distilled water for a boiler feed. Tap water having 400 ppm of dissolved solids at 15.6°C is fed to the evaporator operating at 1 atm pressure abs. Saturated steam at 115.6°C is available for use. Calculate the amount of distilled water produced per hour if the outlet liquid contains 800 ppm solids.

8.4-5. Boiling-Point Rise of NaOH Solutions. Determine the boiling temperature of the solution and the boiling-point rise for the following cases.

- A 30% NaOH solution boiling in an evaporator at a pressure of 172.4 kPa (25 psia).
- A 60% NaOH solution boiling in an evaporator at a pressure of 3.45 kPa (0.50 psia).

Ans. (a) Boiling point = 130.6°C , boiling point rise = 15°C

8.4-6. Boiling-Point Rise of Biological Solutes in Solution. Determine the boiling-point rise for the following solutions of biological solutes in water. Use the figure in (P1), p. 11–31.

- A 30 wt % solution of citric acid in water boiling at 220°F (104.4°C).
- A 40 wt % solution of sucrose in water boiling at 220°F (104.4°C).

Ans. (a) Boiling-point rise = 2.2°F (1.22°C)

8.4-7. Effect of Feed Temperature on Evaporating an NaOH Solution. A single-effect evaporator is concentrating a feed of 9072 kg/h of a 10 wt % solution of NaOH in water to a product of 50% solids. The pressure of the saturated steam used is 42 kPa (gage) and the pressure in the vapor space of the evaporator is 20 kPa (abs). The overall heat-transfer coefficient is $1988 \text{ W/m}^2 \cdot \text{K}$. Calculate the steam used, the steam economy in kg vaporized/kg steam, and the area for the following feed conditions.

- Feed temperature of 288.8 K (15.6°C).
- Feed temperature of 322.1 K (48.9°C).

Ans. (a) $S = 8959 \text{ kg/h}$ of steam, $A = 295.4 \text{ m}^2$

8.4-8. Heat-Transfer Coefficient to Evaporate NaOH. In order to concentrate 4536 kg/h of an NaOH solution containing 10 wt % NaOH to a 20 wt % solution, a single-effect evaporator is being used with an area of 37.6 m^2 . The feed enters at 21.1°C (294.3 K). Saturated steam at 110°C (383.2 K) is used for heating and the pressure in the vapor space of the evaporator is 51.7 kPa. Calculate the kg/h of steam used and the overall heat-transfer coefficient.

8.4-9. Throughput of a Single-Effect Evaporator. An evaporator is concentrating $F \text{ kg/h}$ at 311 K of a 20 wt % solution of NaOH to 50%. The saturated steam used for heating is at 399.3 K . The pressure in the vapor space of the evaporator is 13.3 kPa abs. The overall coefficient is $1420 \text{ W/m}^2 \cdot \text{K}$ and the area is 86.4 m^2 . Calculate the feed rate F of the evaporator.

Ans. $F = 9072 \text{ kg/h}$

8.4-10. Surface Area and Steam Consumption of an Evaporator. A single-effect evaporator is concentrating a feed solution of organic colloids from 5 to 50 wt %. The

solution has a negligible boiling-point elevation. The heat capacity of the feed is $c_p = 4.06 \text{ kJ/kg} \cdot \text{K}$ ($0.97 \text{ btu/lb}_m \cdot ^\circ\text{F}$) and the feed enters at 15.6°C (60°F). Saturated steam at 101.32 kPa is available for heating, and the pressure in the vapor space of the evaporator is 15.3 kPa . A total of 4536 kg/h ($10000 \text{ lb}_m/\text{h}$) of water is to be evaporated. The overall heat-transfer coefficient is $1988 \text{ W/m}^2 \cdot \text{K}$ ($350 \text{ btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$). What is the required surface area in m^2 and the steam consumption?

- 8.4-11. Evaporation of Tomato Juice Under Vacuum.** Tomato juice having a concentration of 12 wt % solids is being concentrated to 25% solids in a film-type evaporator. The maximum allowable temperature for the tomato juice is 135°F , which will be the temperature of the product. The feed enters at 100°F . Saturated steam at 25 psia is used for heating. The overall heat-transfer coefficient U is $600 \text{ btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$ and the area A is 50 ft^2 . The heat capacity of the feed c_p is estimated as $0.95 \text{ btu/lb}_m \cdot ^\circ\text{F}$. Neglect any boiling-point rise if present. Calculate the feed rate of tomato juice to the evaporator.

- 8.4-12. Concentration of Cane Sugar Solution.** A single-effect evaporator is being used to concentrate a feed of $10000 \text{ lb}_m/\text{h}$ of a cane sugar solution at 80°F and containing a sugar content of 15° Brix (degrees Brix is wt % sugar) to 30° Brix for use in a food process. Saturated steam at 240°F is available for heating. The vapor space in the evaporator will be at 1 atm abs pressure. The overall $U = 350 \text{ btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$ and the heat capacity of the feed is $c_p = 0.91 \text{ btu/lb}_m \cdot ^\circ\text{F}$. The boiling-point rise can be estimated from Example 8.5-1. The heat of solution can be considered negligible and neglected. Calculate the area required for the evaporator and the amount of steam used per hour.

Ans. Boiling-point rise = 2.0°F (1.1°C), $A = 667 \text{ ft}^2$ (62.0 m^2)

- 8.5-1. Boiling Points in a Triple-Effect Evaporator.** A solution with a negligible boiling-point rise is being evaporated in a triple-effect evaporator using saturated steam at 121.1°C (394.3 K). The pressure in the vapor of the last effect is 25.6 kPa abs. The heat-transfer coefficients are $U_1 = 2840$, $U_2 = 1988$, and $U_3 = 1420 \text{ W/m}^2 \cdot \text{K}$ and the areas are equal. Estimate the boiling point in each of the evaporators.

Ans. $T_1 = 108.6^\circ\text{C}$ (381.8 K)

- 8.5-2. Evaporation of Sugar Solution in a Multiple-Effect Evaporator.** A triple-effect evaporator with forward feed is evaporating a sugar solution with negligible boiling-point rise (less than 1.0 K , which will be neglected) and containing 5 wt % solids to 25% solids. Saturated steam at 205 kPa abs is being used. The pressure in the vapor space of the third effect is 13.65 kPa . The feed rate is 22680 kg/h and the temperature 299.9 K . The liquid heat capacity is $c_p = 4.19 - 2.35x$, where c_p is in $\text{kJ/kg} \cdot \text{K}$ and x in wt fraction (K1). The heat-transfer coefficients are $U_1 = 3123$, $U_2 = 1987$, and $U_3 = 1136 \text{ W/m}^2 \cdot \text{K}$. Calculate the surface area of each effect if each effect has the same area, and the steam rate.

Ans. Area $A = 99.1 \text{ m}^2$, steam rate $S = 8972 \text{ kg/h}$

- 8.5-3. Evaporation in Double-Effect Reverse-Feed Evaporators.** A feed containing 2 wt % dissolved organic solids in water is fed to a double-effect evaporator with reverse feed. The feed enters at 100°F and is concentrated to 25% solids. The boiling-point rise can be considered negligible as well as the heat of solution. Each evaporator has a 1000-ft^2 surface area and the heat-transfer coefficients are $U_1 = 500$ and $U_2 = 700 \text{ btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$. The feed enters evaporator number 2 and steam at 100 psia is fed to number 1. The pressure in the vapor space of evaporator number 2 is 0.98 psia . Assume that the heat capacity of all liquid solutions is that of liquid water. Calculate the feed rate F and the product rate L_1 of a solution containing 25% solids. (Hint: Assume a feed rate of, say, $F = 1000 \text{ lb}_m/\text{h}$. Calculate the area. Then calculate the actual feed rate by multiplying 1000 by $1000/\text{calculated area}$.)

Ans. $F = 133800 \text{ lb}_m/\text{h}$ (60691 kg/h), $L_1 = 10700 \text{ lb}_m/\text{h}$ (4853 kg/h)

8.5-4. Concentration of NaOH Solution in Triple-Effect Evaporator. A forced-circulation triple-effect evaporator using forward feed is to be used to concentrate a 10 wt % NaOH solution entering at 37.8°C to 50%. The steam used enters at 58.6 kPa gage. The absolute pressure in the vapor space of the third effect is 6.76 kPa. The feed rate is 13 608 kg/h. The heat-transfer coefficients are $U_1 = 6246$, $U_2 = 3407$, and $U_3 = 2271 \text{ W/m}^2 \cdot \text{K}$. All effects have the same area. Calculate the surface area and steam consumption.

Ans. $A = 97.3 \text{ m}^2$, $S = 5284 \text{ kg steam/h}$

8.5-5. Triple-Effect Evaporator with Reverse Feed. A feed rate of 20 410 kg/h of 10 wt % NaOH solution at 48.9°C is being concentrated in a triple-effect reverse-feed evaporator to produce a 50% solution. Saturated steam at 178.3°C is fed to the first evaporator and the pressure in the third effect is 10.34 kPa abs. The heat-transfer coefficient for each effect is assumed to be $2840 \text{ W/m}^2 \cdot \text{K}$. Calculate the heat-transfer area and the steam consumption rate.

8.5-6. Evaporation of Sugar Solution in Double Effect Evaporator. A double-effect evaporator with reverse feed is used to concentrate 4536 kg/h of a 10 wt % sugar solution to 50%. The feed enters the second effect at 37.8°C. Saturated steam at 115.6°C enters the first effect and the vapor from this effect is used to heat the second effect. The absolute pressure in the second effect is 13.65 kPa abs. The overall coefficients are $U_1 = 2270$ and $U_2 = 1705 \text{ W/m}^2 \cdot \text{K}$. The heating areas for both effects are equal. Use boiling-point-rise and heat-capacity data from Example 8.5-1. Calculate the area and steam consumption.

8.6-1. Water Consumption and Pressure in Barometric Condenser. The concentration of NaOH solution leaving the third effect of a triple-effect evaporator is 50 wt %. The vapor flow rate leaving is 5670 kg/h and this vapor goes to a barometric condenser. The discharge water from the condenser leaves at 40.5°C. Assuming that the condenser can maintain a vacuum in the third effect corresponding to a saturated vapor pressure of 2.78°C above 40.5°C, calculate the pressure in the third effect and the cooling water flow to the condenser. The cooling water enters at 29.5°C. (Note: The vapor leaving the evaporator will be superheated because of the boiling-point rise.)

Ans. Pressure = 8.80 kPa abs, $W = 306\,200 \text{ kg water/h}$

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