Transport Processes and Unit Operations

NaR represent the ion-exchange sites on the resin filled with a proton H^+ and a sodium ion Na^+. It is assumed that all of the fixed number of sites are filled with H^+ or Na^+. At equilibrium,

$$K = \frac{[NaR][H^+]}{[Na^+][HR]} \tag{12.4-4}$$

Since the total concentration of the ionic groups $[\bar{R}]$ on the resin is fixed (B7),

$$[\bar{R}] = \text{constant} = [NaR] + [HR] \tag{12.4-5}$$

Combining Eqs. (12.4-4) and (12.4-5),

$$[NaR] = \frac{K[\bar{R}][Na^+]}{[H^+] + K[Na^+]} \tag{12.4-6}$$

If the solution is buffered so $[H^+]$ is constant, the equation above for sodium exchange or adsorption is similar to the Langmuir isotherm.

12.4C Design of Fixed-Bed Ion-Exchange Columns

The rate of ion exchange depends on mass transfer of ions from the bulk solution to the particle surface, diffusion of the ions in the pores of the solid to the surface, exchange of the ions at the surface, and diffusion of the exchange ions back to the bulk solution. This is similar to adsorption. The differential equations derived are also very similar to those for adsorption. The design methods used for ion exchange and adsorption are similar and are described in Section 12.3 for adsorption processes.

12.5 SINGLE-STAGE LIQUID–LIQUID EXTRACTION PROCESSES

12.5A Introduction to Extraction Processes

In order to separate one or more of the components in a mixture, the mixture is contacted with another phase. The two-phase pair can be gas–liquid, which was discussed in Chapter 10; vapor–liquid, which was covered in Chapter 11; liquid–liquid; or fluid–solid. In this section *liquid–liquid extraction separation processes* are considered first. Alternative terms are *liquid extraction* or *solvent extraction.*

In distillation the liquid is partially vaporized to create another phase, which is a vapor. The separation of the components depends on the relative vapor pressures of the substances. The vapor and liquid phases are similar chemically. In liquid–liquid extraction the two phases are chemically quite different, which leads to a separation of the components according to physical and chemical properties.

Solvent extraction can sometimes be used as an alternative to separation by distillation or evaporation. For example, acetic acid can be removed from water by distillation or by solvent extraction using an organic solvent. The resulting organic solvent–acetic acid solution is then distilled. Choice of distillation or solvent extraction would depend on relative costs (C7). In another example high-molecular-weight fatty acids can be separated from vegetable oils by extraction with liquid propane or by high-vacuum distillation, which is more expensive.

In the pharmaceutical industry products such as penicillin occur in fermentation mixtures that are quite complex, and liquid extraction can be used to separate the penicillin. Many metal separations are being done commercially by extraction of aqueous solutions, such as copper–iron, uranium–vanadium, and tantalum–columbium.

12.5B Equilibrium Relations in Extraction

1. *Phase rule.* Generally in a liquid–liquid system we have three components, A, B, and C, and two phases in equilibrium. Substituting into the phase rule, Eq. (10.2-1), the number of degrees of freedom is 3. The variables are temperature, pressure, and four concentrations. Four concentrations occur because only two of the three mass fraction concentrations in a phase can be specified. The third must make the total mass fractions total to 1.0, $x_A + x_B + x_C = 1.0$. If pressure and temperature are set, which is the usual case, then, at equilibrium, setting one concentration in either phase fixes the system.

2. *Triangular coordinates and equilibrium data.* Equilateral triangular coordinates are often used to represent the equilibrium data of a three-component system, since there are three axes. This is shown in Fig. 12.5-1. Each of the three corners represents a pure component, A, B, or C. The point M represents a mixture of A, B, and C. The perpendicular distance from the point M to the base AB represents the mass fraction x_C of C in the mixture at M, the distance to base CB the mass fraction x_A of A, and the distance to base AC the mass fraction x_B of B. Thus,

$$x_A + x_B + x_C = 0.40 + 0.20 + 0.40 = 1.0 \tag{12.5-1}$$

A common phase diagram where a pair of components A and B are partially miscible is shown in Fig. 12.5-2. Typical examples are methyl isobutyl ketone (A)–water

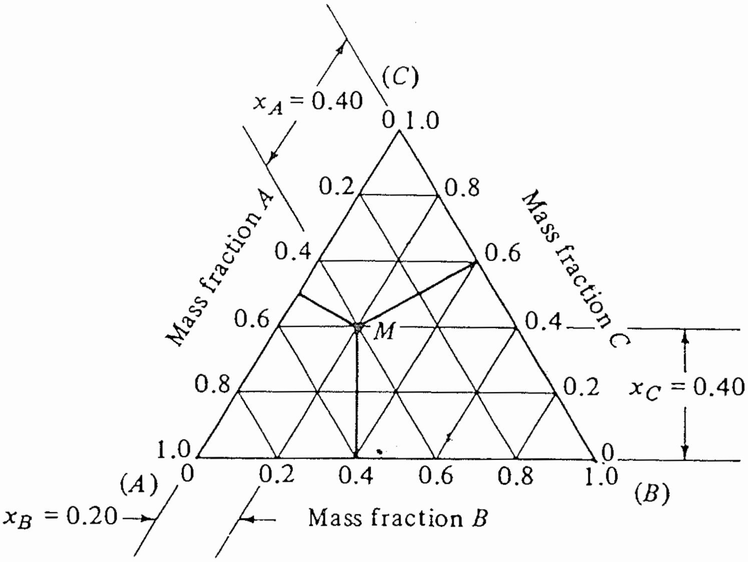


FIGURE 12.5-1. *Coordinates for a triangular diagram.*

(B)–acetone (C), water (A)–chloroform (B)–acetone (C), and benzene (A)–water (B)–acetic acid (C). Referring to Fig. 12.5-2, liquid C dissolves completely in A or in B. Liquid A is only slightly soluble in B and B slightly soluble in A. The two-phase region is included inside below the curved envelope. An original mixture of composition M will separate into two phases a and b which are on the equilibrium tie line through point M. Other tie lines are also shown. The two phases are identical at point P, the *Plait point.*

3. Equilibrium data on rectangular coordinates. Since triangular diagrams have some disadvantages because of the special coordinates, a more useful method of plotting the three component data is to use rectangular coordinates. This is shown in Fig. 12.5-3 for the system acetic acid (A)–water (B)–isopropyl ether solvent (C). Data are from the Appendix A.3 for this system. The solvent pair B and C are partially miscible. The concentration of the component C is plotted on the vertical axis and that of A on the horizontal axis. The concentration of component B is obtained by difference from Eqs. (12.5-2) or (12.5-3).

$$x_B = 1.0 - x_A - x_C \tag{12.5-2}$$

$$y_B = 1.0 - y_A - y_C \tag{12.5-3}$$

The two-phase region in Fig. 12.5-3 is inside the envelope and the one-phase region outside. A tie line gi is shown connecting the water-rich layer i, called the *raffinate layer*, and the ether-rich solvent layer g, called the *extract layer*. The raffinate composition is designated by x and the extract by y. Hence, the mass fraction of C is designated as y_C in the extract layer and as x_C in the raffinate layer. To construct the tie line gi using the equilibrium $y_A - x_A$ plot below the phase diagram, vertical lines to g and i are drawn.

EXAMPLE 12.5-1. Material Balance for Equilibrium Layers

An original mixture weighing 100 kg and containing 30 kg of isopropyl ether (C), 10 kg of acetic acid (A), and 60 kg water (B) is equilibrated and the equilibrium phases separated. What are the compositions of the two equilibrium phases?

Solution: The composition of the original mixture is $x_C = 0.30$, $x_A = 0.10$, and $x_B = 0.60$. This composition of $x_C = 0.30$ and $x_A = 0.10$ is plotted as point h on Fig. 12.5-3. The tie line gi is drawn through point h by trial and error. The composition of the extract (ether) layer at g is $y_A = 0.04$, $y_C = 0.94$, and $y_B = 1.00 - 0.04 - 0.94 = 0.02$ mass fraction. The raffinate (water) layer composition at i is $x_A = 0.12$, $x_C = 0.02$, and $x_B = 1.00 - 0.12 - 0.02 = 0.86$.

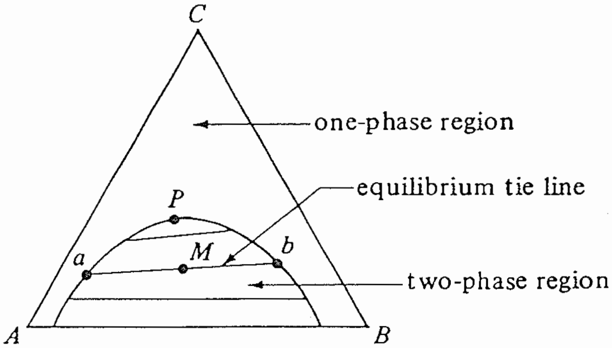


FIGURE 12.5-2. *Liquid–liquid phase diagram where components A and B are partially miscible.*

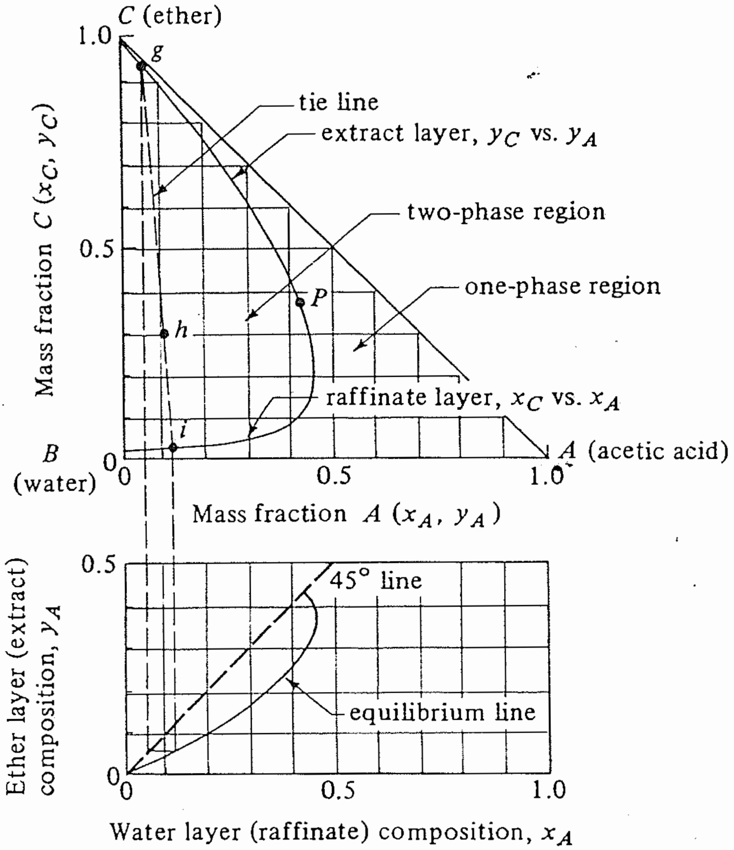


FIGURE 12.5-3. *Acetic acid (A)–water (B)–isopropyl ether (C) liquid–liquid phase diagram at 293 K (20°C).*

Another common type of phase diagram is shown in Fig. 12.5-4, where the solvent pairs B and C and also A and C are partially miscible. Examples are the system styrene (A)–ethylbenzene (B)–diethylene glycol (C) and the system chlorobenzene (A)–methyl ethyl ketone (B)–water (C).

12.5C Single-Stage Equilibrium Extraction

1. Derivation of lever-arm rule for graphical addition. This will be derived for use in the rectangular extraction-phase-diagram charts. In Fig. 12.5-5a two streams, L kg and V kg, containing components A, B, and C, are mixed (added) to give a resulting mixture stream M kg total mass. Writing an overall mass balance and a balance on A,

$$V + L = M \tag{12.5-4}$$

$$Vy_A + Lx_A = Mx_{AM} \tag{12.5-5}$$

where x_{AM} is the mass fraction of A in the M stream. Writing a balance for component C,

$$Vy_C + Lx_C = Mx_{CM} \tag{12.5-6}$$

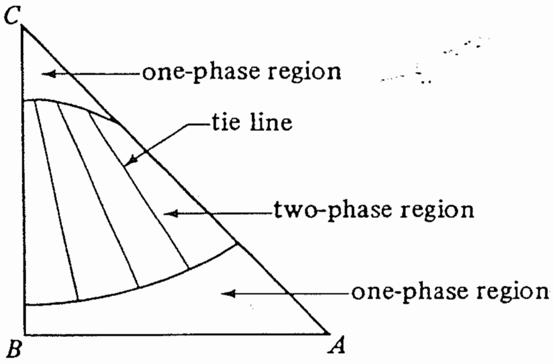


FIGURE 12.5-4. *Phase diagram where the solvent pairs B–C and A–C are partially miscible.*

Combining Eqs. (12.5-4) and (12.5-5),

$$\frac{L}{V} = \frac{y_A - x_{AM}}{x_{AM} - x_A} \tag{12.5-7}$$

Combining Eqs. (12.5-4) and (12.5-6),

$$\frac{L}{V} = \frac{y_C - x_{CM}}{x_{CM} - x_C} \tag{12.5-8}$$

Equating Eqs. (12.5-7) and (12.5-8) and rearranging,

$$\frac{x_C - x_{CM}}{x_A - x_{AM}} = \frac{x_{CM} - y_C}{x_{AM} - y_A} \tag{12.5-9}$$

This shows that points L, M, and V must lie on a straight line. By using the properties of similar right triangles,

$$\frac{L \text{ (kg)}}{V \text{ (kg)}} = \frac{\overline{VM}}{\overline{LM}} \tag{12.5-10}$$

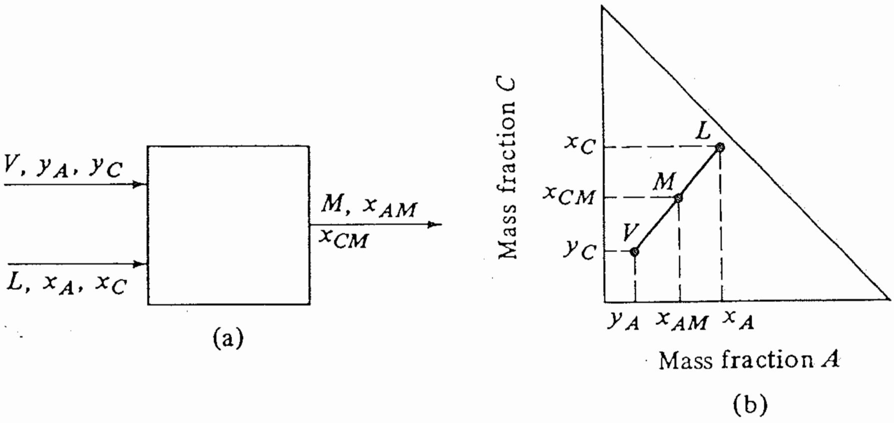


FIGURE 12.5-5. *Graphical addition and lever-arm rule: (a) process flow, (b) graphical addition.*

This is the lever-arm rule and states that kg L/kg V is equal to the length of line $\overline{VM}$/length of line $\overline{LM}$. Also,

$$\frac{L\ (\text{kg})}{M\ (\text{kg})} = \frac{\overline{VM}}{\overline{LV}} \tag{12.5-11}$$

These same equations also hold for kg mol and mol frac, lb_m, and so on.

EXAMPLE 12.5-2. Amounts of Phases in Solvent Extraction
The compositions of the two equilibrium layers in Example 12.5-1 are for the extract layer (V) $y_A = 0.04$, $y_B = 0.02$, and $y_C = 0.94$, and for the raffinate layer (L) $x_A = 0.12$, $x_B = 0.86$, and $x_C = 0.02$. The original mixture contained 100 kg and $x_{AM} = 0.10$. Determine the amounts of V and L.

Solution: Substituting into Eq. (12.5-4),

$$V + L = M = 100$$

Substituting into Eq. (12.5-5), where $M = 100$ kg and $x_{AM} = 0.10$,

$$V(0.04) + L(0.12) = 100(0.10)$$

Solving the two equations simultaneously, $L = 75.0$ and $V = 25.0$. Alternatively, using the lever-arm rule, the distance hg in Fig. 12.5-3 is measured as 4.2 units and gi as 5.8 units. Then by Eq. (12.5-11),

$$\frac{L}{M} = \frac{L}{100} = \frac{\overline{hg}}{\overline{gi}} = \frac{4.2}{5.8}$$

Solving, $L = 72.5$ kg and $V = 27.5$ kg, which is a reasonably close check on the material-balance method.

2. Single-stage equilibrium extraction. We now study the separation of A from a mixture of A and B by a solvent C in a single equilibrium stage. The process is shown in Fig. 12.5-6a, where the solvent, as stream V_2, enters and also the stream L_0. The streams are mixed and equilibrated and the exit streams L_1 and V_1 leave in equilibrium with each other.

The equations for this process are the same as those given in Section 10.3 for a single

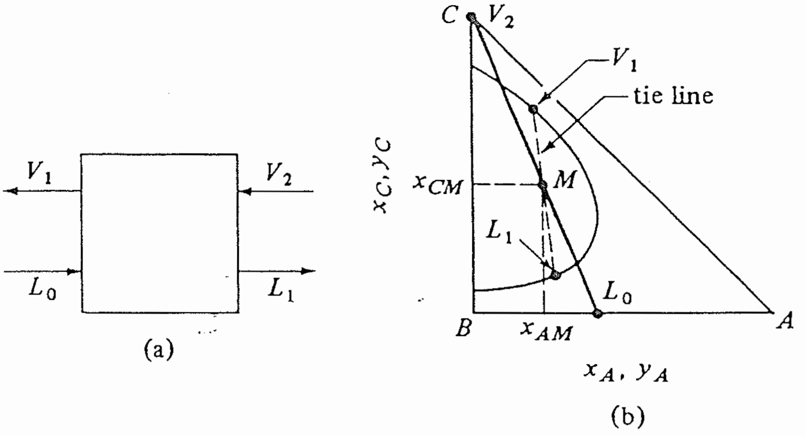


FIGURE 12.5-6. *Single-stage equilibrium liquid–liquid extraction: (a) process flow diagram, (b) plot on phase diagram.*

equilibrium stage where y represents the composition of the V streams and x the L streams.

$$L_0 + V_2 = L_1 + V_1 = M \tag{12.5-12}$$

$$L_0 x_{A0} + V_2 y_{A2} = L_1 x_{A1} + V_1 y_{A1} = M x_{AM} \tag{12.5-13}$$

$$L_0 x_{C0} + V_2 y_{C2} = L_1 x_{C1} + V_1 y_{C1} = M x_{CM} \tag{12.5-14}$$

Since $x_A + x_B + x_C = 1.0$, an equation for B is not needed. To solve the three equations, the equilibrium-phase diagram in Fig. 12.5-6b is used. Since the amounts and compositions of L_0 and V_2 are known, we can calculate values of M, x_{AM}, and x_{CM} from Eqs. (12.5-12)–(12.5-14). The points L_0, V_2, and M can be plotted as shown in Fig. 12.5-6b. Then using trial and error a tie line is drawn through the point M, which locates the compositions of L_1 and V_1. The amounts of L_1 and V_1 can be determined by substitution into Eqs. (12.5-12)–(12.5-14) or by using the lever-arm rule.

12.6 EQUIPMENT FOR LIQUID–LIQUID EXTRACTION

12.6A Introduction and Equipment Types

As in the separation processes of absorption and distillation, the two phases in liquid–liquid extraction must be brought into intimate contact with a high degree of turbulence in order to obtain high mass-transfer rates. After this contact of the two phases, they must be separated. In absorption and in distillation, this separation is rapid and easy because of the large difference in density between the gas or vapor phase and the liquid phase. In solvent extraction the density difference between the two phases is not large and separation is more difficult.

There are two main classes of solvent-extraction equipment, vessels in which mechanical agitation is provided for mixing, and vessels in which the mixing is done by the flow of the fluids themselves. The extraction equipment can be operated batchwise or operated continuously as in absorption and in distillation.

12.6B Mixer–Settlers for Extraction

To provide efficient mass transfer, a mechanical mixer is often used to provide intimate contact of the two liquid phases. One phase is usually dispersed into the other in the form of small droplets. Sufficient time of contact should be provided for the extraction to take place. Small droplets produce large interfacial areas and faster extraction. However, the droplets must not be so small that the subsequent settling time in the settler is too large.

The design and power requirements of baffled agitators or mixers have been discussed in detail in Section 3.4. In Fig. 12.6-1a a typical mixer–settler is shown, where the mixer or agitator is entirely separate from the settler. The feed of aqueous phase and organic phase are mixed in the mixer, and then the mixed phases are separated in the settler. In Fig. 12.6-1b a combined mixer–settler is shown, which is sometimes used in extraction of uranium salts or copper salts from aqueous solutions. Both types of mixer–settlers can be used in series for countercurrent or multiple-stage extraction.

12.6C Plate and Agitated Tower Contactors for Extraction

As discussed in Section 10.6 for plate absorption and distillation towers, somewhat similar types of devices are used for liquid–liquid contacting. In Fig. 12.6-2a a

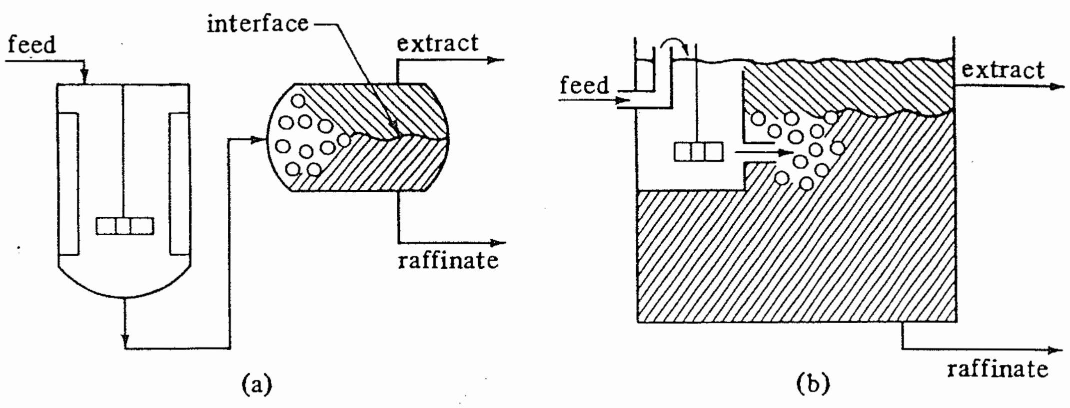


FIGURE 12.6-1. *Typical mixer–settlers for extraction: (a) separate mixer–settler, (b) combined mixer–settler.*

perforated-plate or sieve-tray extraction tower is shown wherein the rising droplets of the light solvent liquid are dispersed. The dispersed droplets coalesce below each tray and are then re-formed on each tray by passing through the perforations. The heavy aqueous liquid flows across each plate, where it is contacted by the rising droplets and then passes through the downcomer to the plate below.

In Fig. 12.6-2b an agitated extraction tower is shown. A series of paddle agitators mounted on a central rotating shaft provides the agitation for the two phases. Each agitator is separated from the next agitator by a calming section of wire mesh to encourage coalescence of the droplets and phase separation. This apparatus is essentially a series of mixer–settlers one above the other (C8, P1, T1). Another type is the Karr reciprocating-plate column, which contains a series of sieve trays with a large open area of 60% where the plates are moved up and down (C6, C8, L3). This is one of the few types of extraction towers that can be scaled up with reasonable accuracy (C6, K3).

12.6D Packed and Spray Extraction Towers

Packed and spray tower extractors give differential contacts, where mixing and settling proceed continuously and simultaneously (C8). In the plate-type towers or mixer–settler contactors, the extraction and settling proceeds in definite stages. In Fig. 12.6-3 the heavy liquid enters the top of the spray tower, fills the tower as the continuous phase, and flows out through the bottom. The light liquid enters through a nozzle distributor at the bottom which disperses or sprays the droplets upward. The light liquid coalesces at the top and flows out. In some cases the heavy liquid is sprayed downward into a rising light continuous phase.

A more effective type of tower is to pack the column with packing such as Raschig rings or Berl saddles, which cause the droplets to coalesce and redisperse at frequent intervals throughout the tower. Detailed discussions of flooding and construction of packed towers are given elsewhere (T1, P1).

12.7 CONTINUOUS MULTISTAGE COUNTERCURRENT EXTRACTION

12.7A Introduction

In Section 12.5 single-stage equilibrium contact was used to transfer the solute *A* from one liquid to the other liquid phase. To transfer more solute, the single-stage contact can

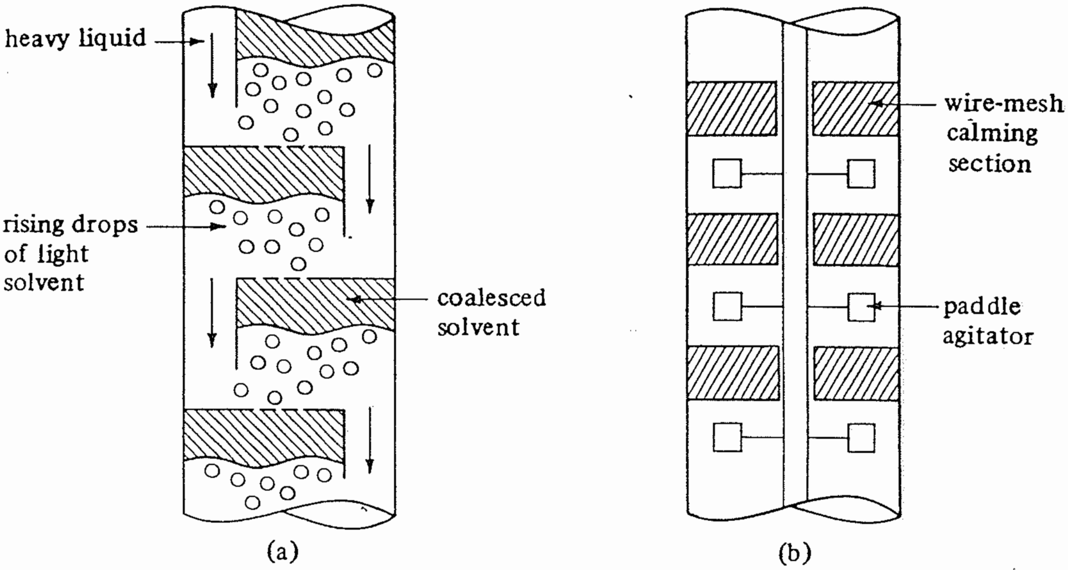


FIGURE 12.6-2. *Extraction towers: (a) perforated-plate or sieve-tray tower, (b) agitated extraction tower.*

be repeated by contacting the exit L_1 stream with fresh solvent V_2 in Fig. 12.5-6. In this way a greater percentage removal of the solute A is obtained. However, this is wasteful of the solvent stream and also gives a dilute product of A in the outlet solvent extract streams. In order to use less solvent and to obtain a more concentrated exit extract stream, countercurrent multistage contacting is often used.

Many of the fundamental equations of countercurrent gas absorption and of rectification are the same or similar to those used in countercurrent extraction. Because of the frequently high solubility of the two liquid phases in each other, the equilibrium relationships in extraction are more complicated than in absorption and distillation.

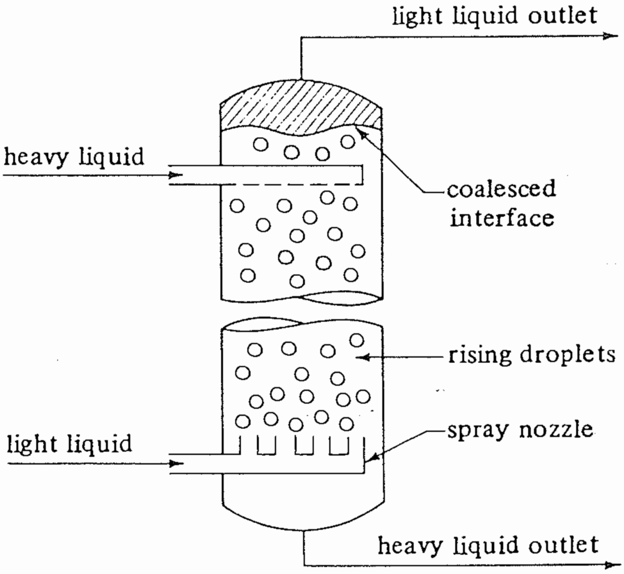


FIGURE 12.6-3. *Spray-type extraction tower.*

12.7B Continuous Multistage Countercurrent Extraction

1. Countercurrent process and overall balance. The process flow for this extraction process is the same as given previously in Fig. 10.3-2 and is shown in Fig. 12.7-1. The feed stream containing the solute A to be extracted enters at one end of the process and the solvent stream enters at the other end. The extract and raffinate streams flow countercurrently from stage to stage, and the final products are the extract stream V_1 leaving stage 1 and the raffinate stream L_N leaving stage N.

Making an overall balance on all the N stages,

$$L_0 + V_{N+1} = L_N + V_1 = M \tag{12.7-1}$$

where M represents total kg/h (lb_m/h) and is a constant, L_0 the inlet feed flow rate in kg/h, V_{N+1} the inlet solvent flow rate in kg/h, V_1 the exit extract stream, and L_N the exit raffinate stream. Making an overall component balance on component C,

$$L_0 x_{C0} + V_{N+1} y_{CN+1} = L_N x_{CN} + V_1 y_{C1} = M x_{CM} \tag{12.7-2}$$

Combining Eqs. (12.7-1) and (12.7-2) and rearranging,

$$x_{CM} = \frac{L_0 x_{C0} + V_{N+1} y_{CN+1}}{L_0 + V_{N+1}} = \frac{L_N x_{CN} + V_1 y_{C1}}{L_N + V_1} \tag{12.7-3}$$

A similar balance on component A gives

$$x_{AM} = \frac{L_0 x_{A0} + V_{N+1} y_{AN+1}}{L_0 + V_{N+1}} = \frac{L_N x_{AN} + V_1 y_{A1}}{L_N + V_1} \tag{12.7-4}$$

Equations (12.7-3) and (12.7-4) can be used to calculate the coordinates of point M on the phase diagram that ties together the two entering streams L_0 and V_{N+1} and the two exit streams V_1 and L_N. Usually, the flows and compositions of L_0 and V_{N+1} are known and the desired exit composition x_{AN} is set. If we plot points L_0, V_{N+1}, and M as in Fig. 12.7-2, a straight line must connect these three points. Then L_N, M, and V_1 must lie on one line. Also, L_N and V_1 must also lie on the phase envelope, as shown. These balances also hold for lb_m and mass fraction, kg mol and mol fractions, and so on.

EXAMPLE 12.7-1. Material Balance for Countercurrent Stage Process
Pure solvent isopropyl ether at the rate of $V_{N+1} = 600$ kg/h is being used to extract an aqueous solution of $L_0 = 200$ kg/h containing 30 wt % acetic acid (A) by countercurrent multistage extraction. The desired exit acetic acid concentration in the aqueous phase is 4%. Calculate the compositions and amounts of the ether extract V_1 and the aqueous raffinate L_N. Use equilibrium data from Appendix A.3.

Solution: The given values are $V_{N+1} = 600$, $y_{AN+1} = 0$, $y_{CN+1} = 1.0$, $L_0 = 200$, $x_{A0} = 0.30$, $x_{B0} = 0.70$, $x_{C0} = 0$, and $x_{AN} = 0.04$. In Fig. 12.7-3, V_{N+1}

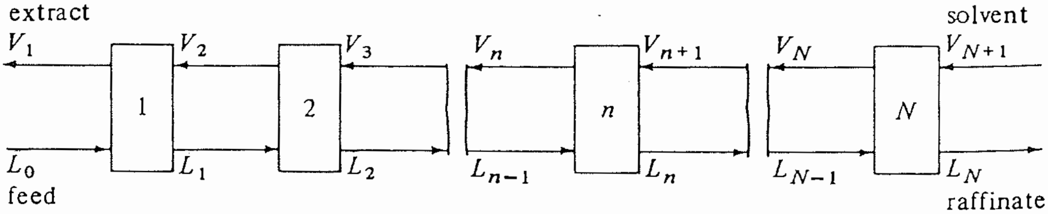
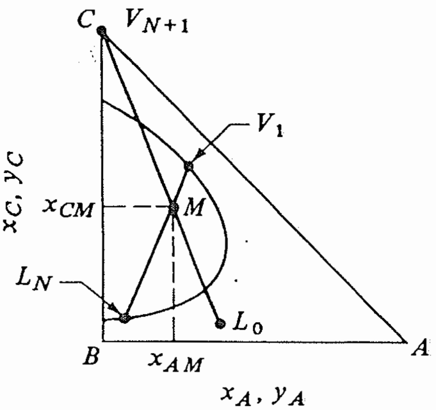


FIGURE 12.7-1. *Countercurrent-multistage-extraction-process flow diagram.*

FIGURE 12.7-2. *Use of the mixture point M for overall material balance in countercurrent solvent extraction.*



and L_0 are plotted. Also, since L_N is on the phase boundary, it can be plotted at $x_{AN} = 0.04$. For the mixture point M, substituting into Eqs. (12.7-3) and (12.7-4),

$$x_{CM} = \frac{L_0 x_{C0} + V_{N+1} y_{CN+1}}{L_0 + V_{N+1}} = \frac{200(0) + 600(1.0)}{200 + 600} = 0.75 \qquad (12.7\text{-}3)$$

$$x_{AM} = \frac{L_0 x_{A0} + V_{N+1} y_{AN+1}}{L_0 + V_{N+1}} = \frac{200(0.30) + 600(0)}{200 + 600} = 0.075 \qquad (12.7\text{-}4)$$

Using these coordinates, the point M is plotted in Fig. 12.7-3. We locate V_1 by drawing a line from L_N through M and extending it until it intersects the phase boundary. This gives $y_{A1} = 0.08$ and $y_{C1} = 0.90$. For L_N a value of $x_{CN} = 0.017$ is obtained. By substituting into Eqs. (12.7-1) and (12.7-2) and solving, $L_N = 136$ kg/h and $V_1 = 664$ kg/h.

2. Stage-to-stage calculations for countercurrent extraction. The next step after an overall balance has been made is to go stage by stage to determine the concentrations at each stage and the total number of stages N needed to reach L_N in Fig. 12.7-1.

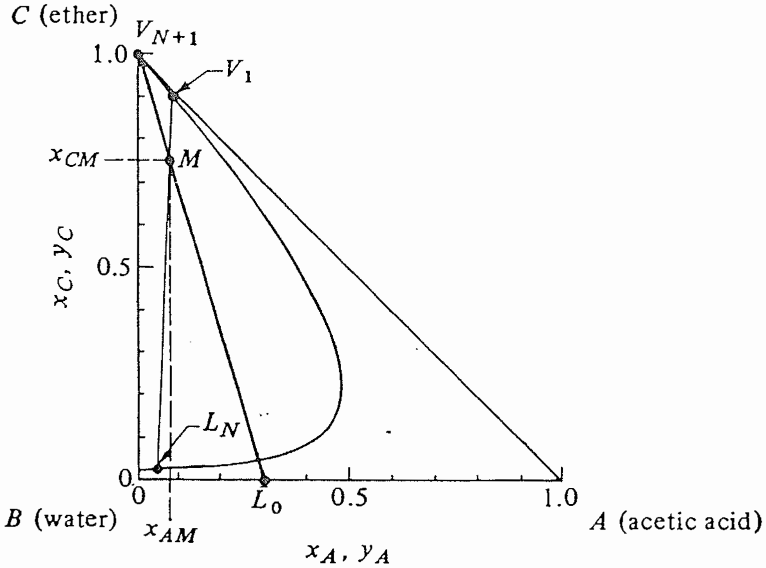


FIGURE 12.7-3. *Method to perform overall material balance for Example 12.7-1.*

Making a total balance on stage 1,

$$L_0 + V_2 = L_1 + V_1 \tag{12.7-5}$$

Making a similar balance on stage n,

$$L_{n-1} + V_{n+1} = L_n + V_n \tag{12.7-6}$$

Rearranging Eq. (12.7-5) to obtain the difference Δ in flows,

$$L_0 - V_1 = L_1 - V_2 = \Delta \tag{12.7-7}$$

This value of Δ in kg/h is constant and for all stages,

$$\Delta = L_0 - V_1 = L_n - V_{n+1} = L_N - V_{N+1} = \cdots \tag{12.7-8}$$

This also holds for a balance on component A, B, or C.

$$\Delta x_\Delta = L_0 x_0 - V_1 y_1 = L_n x_n - V_{n+1} y_{n+1} = L_N x_N - V_{N+1} y_{N+1} = \cdots \tag{12.7-9}$$

Combining Eqs. (12.7-8) and (12.7-9) and solving for x_Δ,

$$x_\Delta = \frac{L_0 x_0 - V_1 y_1}{L_0 - V_1} = \frac{L_n x_n - V_{n+1} y_{n+1}}{L_n - V_{n+1}} = \frac{L_N x_N - V_{N+1} y_{N+1}}{L_N - V_{N+1}} \tag{12.7-10}$$

where x_Δ is the x coordinate of point Δ.

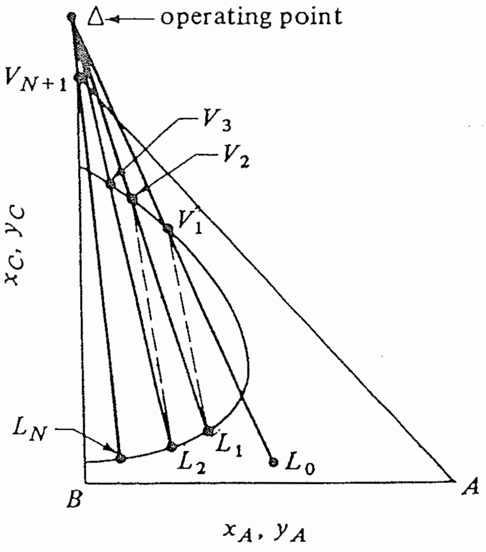
Equations (12.7-7) and (12.7-8) can be written as

$$L_0 = \Delta + V_1 \qquad L_n = \Delta + V_{n+1} \qquad L_N = \Delta + V_{N+1} \tag{12.7-11}$$

From Eq. (12.7-11), we see that L_0 is on a line through Δ and V_1, L_n is on a line through Δ and V_{n+1}, and so on. This means Δ is a point common to all streams passing each other, such as L_0 and V_1, L_n and V_{n+1}, L_N and V_{N+1}, and so on. The coordinates to locate this Δ operating point are given for $x_{C\Delta}$ and $x_{A\Delta}$ in Eq. (12.7-10). Since the end points V_{N+1}, L_N or V_1, and L_0 are known, x_Δ can be calculated and point Δ located. Alternatively, the Δ point is located graphically in Fig. 12.7-4 as the intersection of lines $L_0 V_1$ and $L_N V_{N+1}$.

In order to step off the number of stages using Eq. (12.7-11) we start at L_0 and draw the line $L_0 \Delta$, which locates V_1 on the phase boundary. Next a tie line through V_1 locates

FIGURE 12.7-4. *Operating point Δ and number of theoretical stages needed for countercurrent extraction.*



L_1, which is in equilibrium with V_1. Then line $L_1\Delta$ is drawn giving V_2. The tie line V_2L_2 is drawn. This stepwise procedure is repeated until the desired raffinate composition L_N is reached. The number of stages N is obtained to perform the extraction.

EXAMPLE 12.7-2. Number of Stages in Countercurrent Extraction
Pure isopropyl ether of 450 kg/h is being used to extract an aqueous solution of 150 kg/h with 30 wt % acetic acid (A) by countercurrent multistage extraction. The exit acid concentration in the aqueous phase is 10 wt %. Calculate the number of stages required.

Solution: The known values are $V_{N+1} = 450$, $y_{AN+1} = 0$, $y_{CN+1} = 1.0$, $L_0 = 150$, $x_{A0} = 0.30$, $x_{B0} = 0.70$, $x_{C0} = 0$, and $x_{AN} = 0.10$. The points V_{N+1}, L_0, and L_N are plotted in Fig. 12.7-5. For the mixture point M, substituting into Eqs. (12.7-3) and (12.7-4), $x_{CM} = 0.75$ and $x_{AM} = 0.075$. The point M is plotted and V_1 is located at the intersection of line L_NM with the phase boundary to give $y_{A1} = 0.072$ and $y_{C1} = 0.895$. This construction is not shown. (See Example 12.7-1 for construction of lines.)

The lines L_0V_1 and L_NV_{N+1} are drawn and the intersection is the operating point Δ as shown. Alternatively, the coordinates of Δ can be calculated from Eq. (12.7-10) to locate point Δ. Starting at L_0 we draw line $L_0\Delta$, which locates V_1. Then a tie line through V_1 locates L_1 in equilibrium with V_1. (The tie-line data are obtained from an enlarged plot such as the bottom of Fig. 12.5-3.) Line $L_1\Delta$ is next drawn locating V_2. A tie line through V_2 gives L_2. A line $L_2\Delta$ gives V_3. A final tie line gives L_3, which has gone beyond the desired L_N. Hence, about 2.5 theoretical stages are needed.

3. *Minimum solvent rate.* If a solvent rate V_{N+1} is selected at too low a value a limiting

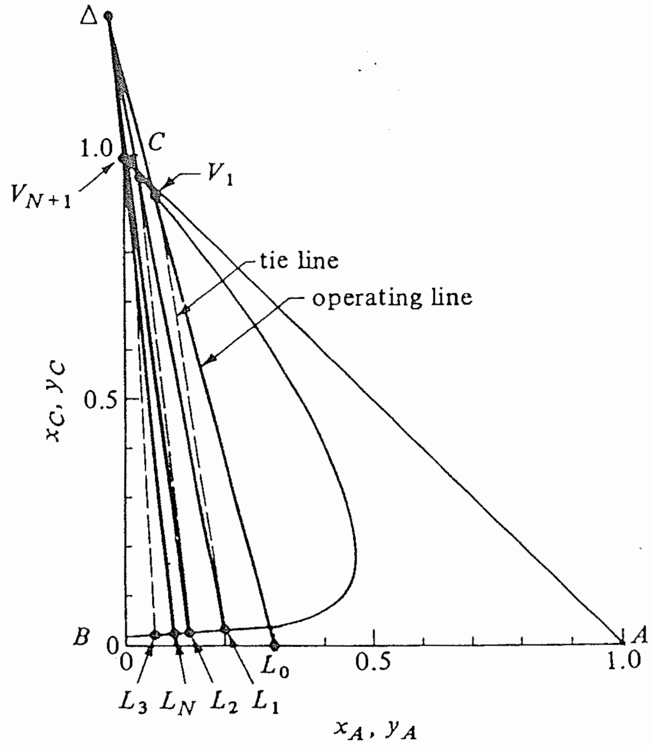


FIGURE 12.7-5. *Graphical solution for countercurrent extraction in Example 12.7-2.*

case will be reached with a line through Δ and a tie line being the same. Then an infinite number of stages will be needed to reach the desired separation. The minimum amount of solvent is reached. For actual operation a greater amount of solvent must be used.

The procedure to obtain this minimum is as follows. A tie line is drawn through point L_0 (Fig. 12.7-4) to intersect the extension of line $L_N V_{N+1}$. Other tie lines to the left of this tie line are drawn including one through L_N to intersect the line $L_N V_{N+1}$. The intersection of a tie line on line $L_N V_{N+1}$ which is nearest to V_{N+1} represents the $\Delta_{\min}$ point for minimum solvent. The actual position of Δ used must be closer to V_{N+1} than $\Delta_{\min}$ for a finite number of stages. This means that more solvent must be used. Usually, the tie line through L_0 represents the $\Delta_{\min}$.

12.7C Countercurrent-Stage Extraction with Immiscible Liquids

If the solvent stream V_{N+1} contains components A and C and the feed stream L_0 contains A and B and components B and C are relatively immiscible in each other, the stage calculations are made more easily. The solute A is relatively dilute and is being transferred from L_0 to V_{N+1}.

Referring to Fig. 12.7-1 and making an overall balance for A over the whole system and then over the first n stages,

$$L'\left(\frac{x_0}{1-x_0}\right) + V'\left(\frac{y_{N+1}}{1-y_{N+1}}\right) = L'\left(\frac{x_N}{1-x_N}\right) + V'\left(\frac{y_1}{1-y_1}\right) \qquad (12.7\text{-}12)$$

$$L'\left(\frac{x_0}{1-x_0}\right) + V'\left(\frac{y_{n+1}}{1-y_{n+1}}\right) = L'\left(\frac{x_n}{1-x_n}\right) + V'\left(\frac{y_1}{1-y_1}\right) \qquad (12.7\text{-}13)$$

where $L' =$ kg inert B/h, $V' =$ kg inert C/h, $y =$ mass fraction A in V stream, and $x =$ mass fraction A in L stream. This Eq. (12.7-13) is an operating-line equation whose slope $\cong L'/V'$. If y and x are quite dilute, the line will be straight when plotted on an xy diagram.

The number of stages are stepped off as shown previously in cases in distillation and absorption.

If the equilibrium line is relatively dilute, then since the operating line is essentially straight, the analytical Eqs. (10.3-21)–(10.3-26) given in Section 10.3D can be used to calculate the number of stages.

EXAMPLE 12.7-3. Extraction of Nicotine with Immiscible Liquids.
An inlet water solution of 100 kg/h containing 0.010 wt fraction nicotine (A) in water is stripped with a kerosene stream of 200 kg/h containing 0.0005 wt fraction nicotine in a countercurrent stage tower. The water and kerosene are essentially immiscible in each other. It is desired to reduce the concentration of the exit water to 0.0010 wt fraction nicotine. Determine the theoretical number of stages needed. The equilibrium data are as follows (C5), with x the weight fraction of nicotine in the water solution and y in the kerosene.

x	y	x	y
0.001010	0.000806	0.00746	0.00682
0.00246	0.001959	0.00988	0.00904
0.00500	0.00454	0.0202	0.0185

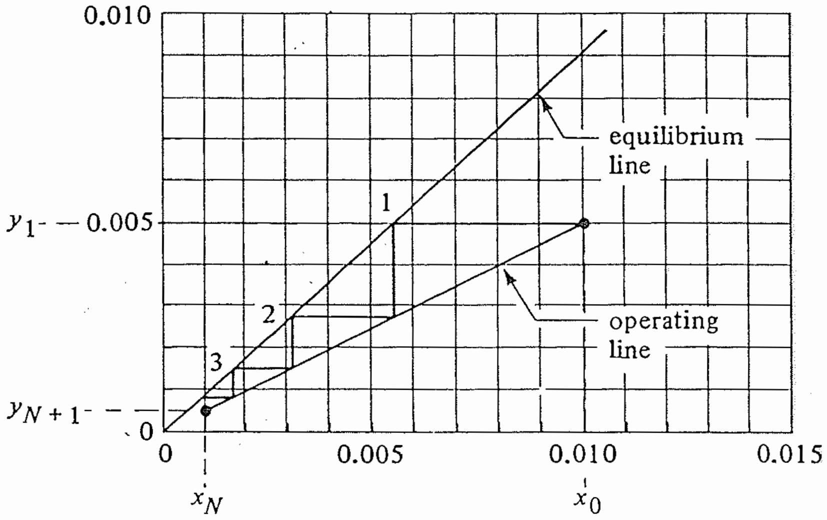


FIGURE 12.7-6. *Solution for extraction with immiscible liquids in Example 12.7-3.*

Solution: The given values are $L_0 = 100$ kg/h, $x_0 = 0.010$, $V_{N+1} = 200$ kg/h, $y_{N+1} = 0.0005$, $x_N = 0.0010$. The inert streams are

$$L' = L(1 - x) = L_0(1 - x_0) = 100(1 - 0.010) = 99.0 \text{ kg water/hr}$$

$$V' = V(1 - y) = V_{N+1}(1 - y_{N+1}) = 200(1 - 0.0005) = 199.9 \text{ kg kerosene/hr}$$

Making an overall balance on A using Eq. (12.7-12) and solving, $y_1 = 0.00497$. These end points on the operating line are plotted in Fig. 12.7-6. Since the solutions are quite dilute, the line is straight. The equilibrium line is also shown. The number of stages are stepped off, giving $N = 3.8$ theoretical stages.

12.8 INTRODUCTION AND EQUIPMENT FOR LIQUID–SOLID LEACHING

12.8A Leaching Processes

1. Introduction. Many biological and inorganic and organic substances occur in a mixture of different components in a solid. In order to separate the desired solute constituent or remove an undesirable solute component from the solid phase, the solid is contacted with a liquid phase. The two phases are in intimate contact and the solute or solutes can diffuse from the solid to the liquid phase, which causes a separation of the components originally in the solid. This process is called *liquid–solid leaching* or simply *leaching.* The term *extraction* is also used to describe this unit operation, although it also refers to liquid–liquid extraction. In leaching when an undesirable component is removed from a solid with water, the process is called *washing.*

2. Leaching processes for biological substances. In the biological and food processing industries, many products are separated from their original natural structure by liquid–solid leaching. An important process is the leaching of sugar from sugar beets with hot water. In the production of vegetable oils, organic solvents such as hexane, acetone, and mixed-suspension system, the crystallizer is at steady state and contains a well-mixed-suspension magma with no product classification and no solids entering with the feed.

12.12D Models for Crystallizers

In order to analyze data from a mixed-suspension crystallizer, an overall theory combining the effects of nucleation rate, growth rate, heat balance, and material balance is needed. Some progress has been made and an idealized model has been investigated by Randolph and Larson and their co-workers (R1, R2, R3, M2, S2, P1, P3, C2). Their equations are rather complicated but allow determination of some fundamental factors of growth rate and nucleation rate from experimental data.

A crystal product sample from the actual crystallizer is first obtained and a screen analysis run. The slurry density and retention time in the crystallizer are also needed. By converting the size analysis to a population density of crystals of various sizes and plotting the data, the nucleation rate and the growth rate in mm/h can be obtained for the actual conditions tested in the crystallizer. Then experiments can be conducted to determine the effects of operation effects on growth rate and nucleation rate. A sample calculation for this method is given elsewhere (P1). A contact nucleation model for the design of magma crystallizers has been developed (B6) based on single-particle-contact nucleation experiments (C4, M3). Larson (L2, P1) gives examples of design of crystallizing systems.

PROBLEMS

12.1-1. *Equilibrium Isotherm for Glucose Adsorption.* Equilibrium isotherm data for adsorption of glucose from an aqueous solution to activated alumina are as follows (H3):

c (g/cm^3)	0.0040	0.0087	0.019	0.027	0.094	0.195
q (g solute/g alumina)	0.026	0.053	0.075	0.082	0.123	0.129

Determine the isotherm that fits the data and give the constants of the equation using the given units.

Ans. Langmuir isotherm, $q = 0.145c/(0.0174 + c)$

12.2-1. *Batch Adsorption for Phenol Solution.* A wastewater solution having a volume of 2.5 m^3 contains 0.25 kg phenol/m^3 of solution. This solution is mixed thoroughly in a batch process with 3.0 kg of granular activated carbon until equilibrium is reached. Use the isotherm from Example 12.2-1 and calculate the final equilibrium values and the percent phenol extracted.

12.3-1. *Scale-Up of Laboratory Adsorption Column Data.* Using the break-point time and other results from Example 12.3-1, do as follows:

(a) The break-point time for a new column is to be 8.5 h. Calculate the new total length of the column required, column diameter, and the fraction of total capacity used up to the break point. The flow rate is to remain constant at 754 cm^3/s.

(b) Use the same conditions as part (a), but the flow rate is to be increased to 2000 cm^3/s.

Ans. (a) $H_T = 27.2$ cm, 0.849 fraction; (b) $D = 6.52$ cm

12.3-2. *Drying of Nitrogen and Scale-Up of Column.* Using molecular sieves, water vapor was removed from nitrogen gas in a packed bed (C3) at 28.3°C. The column height was 0.268 m, with the bulk density of the solid bed being equal to 712.8 kg/m^3. The initial water concentration in the solid was 0.01 kg water/kg solid and the mass velocity of the nitrogen gas was 4052 kg/m$^2\cdot$ h.

The initial water concentration in the gas was $c_o = 926 \times 10^{-6}$ kg water/kg nitrogen. The breakthrough data are as follows.

t (h)	0	9	9.2	9.6	10	10.4
c (kg H_2O/kg $N_2 \times 10^6$)	<0.6	0.6	2.6	21	91	235
t (h)	10.8	11.25	11.5	12.0	12.5	12.8
c (kg H_2O/kg $N_2 \times 10^6$)	418	630	717	855	906	926

A value of $c/c_o = 0.02$ is desired at the break point. Do as follows.

(a) Determine the break-point time, the fraction of total capacity used up to the break point, the length of the unused bed, and the saturation loading capacity of the solid.

(b) For a proposed column length $H_T = 0.40$ m, calculate the break-point time and fraction of total capacity used.

Ans. (a) $t_b = 9.58$ h, fraction used = 0.878

12.4-1. ***Scale-Up of Ion-Exchange Column.*** An ion-exchange process using a resin to remove copper ions from aqueous solution is conducted in a 1.0-in.-diameter column 1.2 ft high. The flow rate is 1.5 gph and the break point occurred at 7.0 min. Integrating the breakthrough curve gives a ratio of usable capacity to total capacity of 0.60. Design a new tower that will be 3.0 ft high and operating at 4.5 gph. Calculate the new tower size and break-point time.

Ans. $t_b = 24.5$ min, $D = 1.732$ in.

12.4-2. ***Height of Tower in Ion-Exchange.*** In a given run using a flow rate of 0.2 m^3/h in an ion-exchange tower with a column height of 0.40 m, the break point occurred at 8.0 min. The ratio of usable capacity to total equilibrium capacity is 0.65. What is the height of a similar column operating for 13.0 min to the break point at the same flow rate?

12.4-3. ***Ion Exchange of Copper in Column.*** An ion-exchange column containing 99.3 g of amberlite ion-exchange resin was used to remove Cu^{2+} from a solution where $c_o = 0.18$ M $CuSO_4$. The tower height = 30.5 cm and the diameter = 2.59 cm. The flow rate was 1.37 cm^3 solution/s to the tower. The breakthrough data are shown below.

t (s)	420	480	510	540	600	660
c (g mol Cu/L)	0	0.0033	0.0075	0.0157	0.0527	0.1063
t (s)	720	780	810	870	900	
c (g mol Cu/L)	0.1433	0.1634	0.1722	0.1763	0.180	

The concentration desired at the break point is $c/c_o = 0.010$. Determine the break-point time, fraction of total capacity used up to the break point, length of unused bed, and the saturation loading capacity of the solid.

12.5-1. ***Composition of Two Liquid Phases in Equilibrium.*** An original mixture weighing 200 kg and containing 50 kg of isopropyl ether, 20 kg of acetic acid, and 130 kg of water is equilibrated in a mixer–settler and the phases separated. Determine the amounts and compositions of the raffinate and extract layers. Use equilibrium data from Appendix A.3.

12.5-2. ***Single-Stage Extraction.*** A single-stage extraction is performed in which 400 kg of a solution containing 35 wt % acetic acid in water is contacted with 400 kg of pure isopropyl ether. Calculate the amounts and compositions of the extract and raffinate layers. Solve for the amounts both algebraically and by the lever-arm rule. What percent of the acetic acid is removed? Use equilibrium data from Appendix A.3.

Ans. $L_1 = 358$ kg, $x_{B1} = 0.715$, $x_{C1} = 0.03$, $V_1 = 442$ kg, $y_{A1} = 0.11$, $y_{C1} = 0.86$, 34.7% removed.

12.5-3. ***Single-Stage Extraction with Unknown Composition.*** A feed mixture weighing 200 kg of unknown composition containing water, acetic acid, and isopropyl ether is contacted in a single stage with 280 kg of a mixture containing 40 wt % acetic acid, 10 wt % water, and 50 wt % isopropyl ether. The resulting raffinate layer weighs 320 kg and contains 29.5 wt % acetic acid, 66.5 wt % water, and 4.0 wt % isopropyl ether. Determine the original composition of the feed mixture and the composition of the resulting extract layer. Use equilibrium data from Appendix A.3.

Ans. $x_{A0} = 0.030, x_{B0} = 0.955, y_{A1} = 0.15$

12.5-4. ***Extraction of Acetone in a Single Stage.*** A mixture weighing 1000 kg contains 23.5 wt % acetone and 76.5 wt % water and is to be extracted by 500 kg methyl isobutyl ketone in a single-stage extraction. Determine the amounts and compositions of the extract and raffinate phases. Use equilibrium data from Appendix A.3.

12.7-1. ***Multiple-Stage Extraction with Fresh Solvent in Each Stage.*** Pure water is to be used to extract acetic acid from 400 kg of a feed solution containing 25 wt % acetic acid in isopropyl ether. Use equilibrium data from Appendix A.3.

(a) If 400 kg of water is used, calculate the percent recovery in the water solution in a one-stage process.

(b) If a multiple four-stage system is used and 100 kg fresh water is used in each stage, calculate the overall percent recovery of the acid in the total outlet water. (*Hint*: First, calculate the outlet extract and raffinate streams for the first stage using 400 kg of feed solution and 100 kg of water. For the second stage, 100 kg of water contacts the outlet organic phase from the first stage. For the third stage, 100 kg of water contacts the outlet organic phase from the second stage, and so on.)

12.7-2. ***Overall Balance in Countercurrent Stage Extraction.*** An aqueous feed of 200 kg/h containing 25 wt % acetic acid is being extracted by pure isopropyl ether at the rate of 600 kg/h in a countercurrent multistage system. The exit acid concentration in the aqueous phase is to contain 3.0 wt % acetic acid. Calculate the compositions and amounts of the exit extract and raffinate streams. Use equilibrium data from Appendix A.3.

12.7-3. ***Minimum Solvent and Countercurrent Extraction of Acetone.*** An aqueous feed solution of 1000 kg/h containing 23.5 wt % acetone and 76.5 wt % water is being extracted in a countercurrent multistage extraction system using pure methyl isobutyl ketone solvent at 298–299 K. The outlet water raffinate will contain 2.5 wt % acetone. Use equilibrium data from Appendix A.3.

(a) Calculate the minimum solvent that can be used. [*Hint*: In this case the tie line through the feed L_0 represents the condition for minimum solvent flow rate. This gives $V_{1\,\text{min}}$. Then draw lines $L_N V_{1\,\text{min}}$ and $L_0 V_{N+1}$ to give the mixture point M_{min} and the coordinate $x_{AM\,\text{min}}$. Using Eq. (12.7-4), solve for $V_{N+1\,\text{min}}$, the minimum value of the solvent flow rate V_{N+1}.]

(b) Using a solvent flow rate of 1.5 times the minimum, calculate the number of theoretical stages.

12.7-4. ***Countercurrent Extraction of Acetic Acid and Minimum Solvent.*** An aqueous feed solution of 1000 kg/h of acetic acid–water solution contains 30.0 wt % acetic acid and is to be extracted in a countercurrent multistage process with pure isopropyl ether to reduce the acid concentration to 2.0 wt % acid in the final raffinate. Use equilibrium data in Appendix A.3.

(a) Calculate the minimum solvent flow rate that can be used. (*Hint*: See Problem 12.7-3 for the method to use.)

(b) If 2500 kg/h of ether solvent is used, determine the number of theoretical stages required. (*Note*: It may be necessary to replot on an expanded scale the concentrations at the dilute end.)

Ans. (a) Minimum solvent flow rate $V_{N+1} = 1630$ kg/h; (b) 7.5 stages

12.7-5. ***Number of Stages in Countercurrent Extraction.*** Repeat Example 12.7-2 but use an exit acid concentration in the aqueous phase of 4.0 wt %.

12.7-6. ***Extraction with Immiscible Solvents.*** A water solution of 1000 kg/h containing 1.5 wt % nicotine in water is stripped with a kerosene stream of 2000 kg/h containing 0.05 wt % nicotine in a countercurrent stage tower. The exit water is to contain only 10% of the original nicotine, i.e., 90% is removed. Use equilibrium data from Example 12.7-3. Calculate the number of theoretical stages needed.

Ans. 3.7 stages

12.7-7. ***Analytical Equation for Number of Stages.*** Example 12.7-3 gives data for extraction of nicotine from water by kerosene where the two solvents are immiscible. A total of 3.8 theoretical stages were needed. Use the analytical equations (10.3-21)–(10.3-26) to calculate the number of theoretical stages and compare with the value obtained graphically.

12.7-8. ***Minimum Solvent Rate with Immiscible Solvents.*** Determine the minimum solvent kerosene rate to perform the desired extraction in Example 12.7-3. Using 1.25 times this minimum rate, determine the number of theoretical stages needed graphically and also by using Eqs. (10.3-21)–(10.3-26).

12.7-9. ***Stripping Nicotine from Kerosene.*** A kerosene flow of 100 kg/h contains 1.4 wt % nicotine and is to be stripped with pure water in a countercurrent multistage tower. It is desired to remove 90% of the nicotine. Using a water rate of 1.50 times the minimum, determine the number of theoretical stages required. (Use the equilibrium data from Example 12.7-3.)

12.8-1. ***Effective Diffusivity in Leaching Particles.*** In Example 12.8-1 a time of leaching of the solid particle of 3.11 h is needed to remove 80% of the solute. Do the following calculations.

(a) Using the experimental data, calculate the effective diffusivity, $D_{A\,\text{eff}}$.

(b) Predict the time to leach 90% of the solute from the 2.0 mm particle.

Ans. (a) $D_{A\,\text{eff}} = 1.0 \times 10^{-5}$ mm²/s; (b) $t = 5.00$ h

12.9-1. ***Leaching of Oil from Soybeans in a Single Stage.*** Repeat Example 12.9-1 for single-stage leaching of oil from soybeans. The 100 kg of soybeans contains 22 wt % oil and the solvent feed is 80 kg of solvent containing 3 wt % soybean oil.

Ans. $L_1 = 52.0$ kg, $y_{A1} = 0.239$, $V_1 = 50.0$ kg, $x_{A1} = 0.239$, $N_1 = 1.5$

12.9-2. ***Leaching a Soybean Slurry in a Single Stage.*** A slurry of flaked soybeans weighing a total of 100 kg contains 75 kg of inert solids and 25 kg of solution with 10 wt % oil and 90 wt % solvent hexane. This slurry is contacted with 100 kg of pure hexane in a single stage so that the value of N for the outlet underflow is 1.5 kg insoluble solid/kg solution retained. Calculate the amounts and compositions of the overflow V_1 and the underflow L_1 leaving the stage.

12.10-1. ***Constant Underflow in Leaching Oil from Meal.*** Use the same conditions as given in Example 12.10-1, but assume constant underflow of $N = 1.85$ kg solid/kg solution. Calculate the exit flows and compositions and the number of stages required. Compare with Example 12.10-1.

Ans. $y_{AN} = 0.111$, $x_{A1} = 0.623$, 4.3 stages

12.10-2. ***Effect of Less Solvent Flow in Leaching Oil from Meal.*** Use the same conditions as given in Example 12.10-1, but the inlet fresh solvent mixture flow rate per hour is decreased by 10%, to 1179 kg of benzene and 18 kg of oil. Calculate the number of stages needed.

12.10-3. ***Countercurrent Multistage Washing of Ore.*** A treated ore containing inert solid gangue and copper sulfate is to be leached in a countercurrent multistage extractor using pure water to leach the $CuSO_4$. The solid charge rate per hour consists of 10000 kg of inert gangue (B), 1200 kg of $CuSO_4$ (solute A), and 400 kg of water (C). The exit wash solution is to contain 92 wt % water and 8 wt % $CuSO_4$. A total of 95% of the $CuSO_4$ in the inlet ore is to be recovered. The underflow is constant at $N = 0.5$ kg inert gangue solid/kg aqueous solution. Calculate the number of stages required.

12.10-4. ***Countercurrent Multistage Leaching of Halibut Livers.*** Fresh halibut livers containing 25.7 wt % oil are to be extracted with pure ethyl ether to remove 95% of the oil in a countercurrent multistage leaching process. The feed rate is 1000 kg of fresh livers per hour. The final exit overflow solution is to contain 70 wt % oil. The retention of solution by the inert solids (oil-free liver) of the liver varies as follows (C1), where N is kg inert solid/kg solution retained and y_A is kg oil/kg solution.

N	y_A	N	y_A
4.88	0	1.67	0.6
3.50	0.2	1.39	0.81
2.47	0.4		

Calculate the amounts and compositions of the exit streams and the total number of theoretical stages.

12.10-5. ***Countercurrent Leaching of Flaked Soybeans.*** Soybean flakes containing 22 wt % oil are to be leached in a countercurrent multistage process to contain 0.8 kg oil/100 kg inert solid using fresh and pure hexane solvent. For every 1000 kg soybeans, 1000 kg hexane is used. Experiments (S1) give the following retention of solution with the solids in the underflow, where N is kg inert solid/kg solution retained and y_A is wt fraction of oil in solution.

N	y_A
1.73	0
1.52	0.20
1.43	0.30

Calculate the exit flows and compositions and the number of theoretical stages needed.

12.11-1. ***Crystallization of $Ba(NO_3)_2$.*** A hot solution of $Ba(NO_3)_2$ from an evaporator contains 30.6 kg $Ba(NO_3)_2$/100 kg H_2O and goes to a crystallizer where the solution is cooled and $Ba(NO_3)_2$ crystallizes. On cooling, 10% of the original water present evaporates. For a feed solution of 100 kg total, calculate the following.

(a) The yield of crystals if the solution is cooled to 290 K (17°C), where the solubility is 8.6 kg $Ba(NO_3)_2$/100 kg total water.

(b) The yield if cooled instead to 283 K, where the solubility is 7.0 kg $Ba(NO_3)_2$/100 kg total water.

Ans. (a) 17.47 kg $Ba(NO_3)_2$ crystals

12.11-2. ***Dissolving and Subsequent Crystallization.*** A batch of 1000 kg of KCl is dissolved in sufficient water to make a saturated solution at 363 K, where the solubility is 35 wt % KCl in water. The solution is cooled to 293 K, at which temperature its solubility is 25.4 wt %.

(a) What is the weight of water required for solution and the weight of crystals of KCl obtained?

(b) What is the weight of crystals obtained if 5% of the original water evaporates on cooling?

Ans. (a) 1857 kg water, 368 kg crystals; (b) 399 kg crystals

12.11-3. ***Crystallization of $MgSO_4 \cdot 7H_2O$.*** A hot solution containing 1000 kg of $MgSO_4$ and water having a concentration of 30 wt % $MgSO_4$ is cooled to 288.8 K, where crystals of $MgSO_4 \cdot 7H_2O$ are precipated. The solubility at 288.8 K is 24.5 wt % anhydrous $MgSO_4$ in the solution. Calculate the yield of crystals obtained if 5% of the original water in the system evaporates on cooling.

12.11-4. ***Heat Balance in Crystallization.*** A feed solution of 10 000 lb_m at 130°F containing 47.0 lb $FeSO_4$/100 lb total water is cooled to 80°F, where $FeSO_4 \cdot 7H_2O$ crystals are removed. The solubility of the salt is 30.5 lb $FeSO_4$/100 lb total water (P1). The average heat capacity of the feed solution is 0.70 btu/$lb_m \cdot$ °F. The heat of solution at 18°C is −4.4 kcal/g mol (−18.4 kJ/g mol) $FeSO_4 \cdot 7H_2O$ (P1). Calculate the yield of crystals and make a heat balance. Assume that no water is vaporized.

Ans. 2750 lb_m $FeSO_4 \cdot 7H_2O$ crystals, $q = -428\ 300$ btu ($-451\ 900$ kJ)

12.11-5. ***Effect of Temperature on Yield and Heat Balance in Crystallization.*** Use the conditions of Example 12.11-2, but the solution is cooled instead to 283.2 K, where the solubility is 30.9 kg $MgSO_4$/100 kg total water (P1). Calculate the effect on yield and the heat absorbed by using 283.2 K instead of 293.2 K for the crystallization.

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