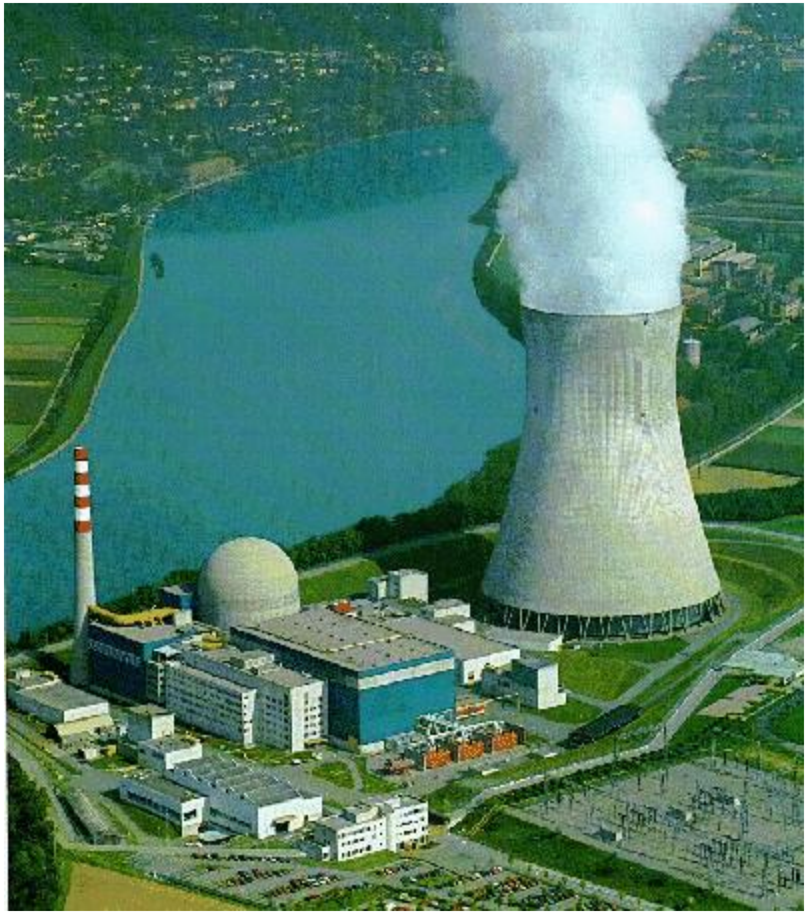
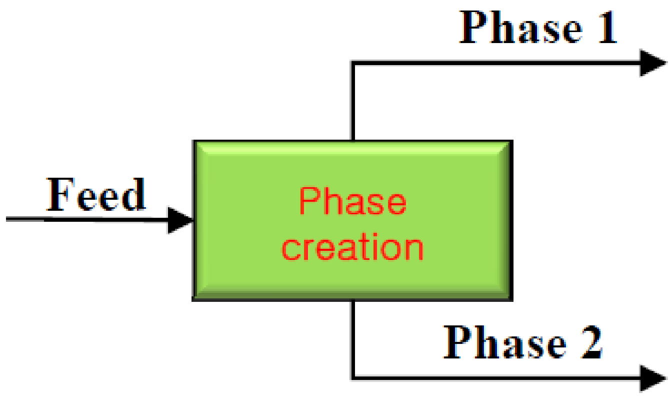
Humidification



Principal references: Chapter 9 (Drying) and Chapter 10 (Gas-liquid separation Processes) in C.J. Geankoplis book; Chapter 19 (Humidification operations) in McCabe et al. book; Chapter 7 (Humidification operations) in Treybal book ; Chapter 18 Seader & Roper book.

Overview and definitions

- **Based on phase-creation:**



- Humidification is based on **phase-creation** with **ESA**.
- It is a **gas-liquid** operations.
- **Gas** is pure air and **liquid** is pure water.
- **Basic principle:**
  - When a relatively warm liquid water is directly contacted with **unsaturated air**, some of liquid water will vaporized.
  - The liquid temperature will drop mainly because of the latent heat of vaporization.
- **Simply,** water vapor is added to air.
- The reverse of such operation is called **dehumidification** for the removal of water vapor from humid air (warm).

Overview and definitions

- **Industrial applications:**

Humidification/dehumidification are important for many industrial operations such as:

- Humidifying air for control of the moisture content of air in drying or air conditioning.
- Dehumidifying air where cold water condenses some water vapor from warm air.
- Water cooling where evaporation of water to the air cools water before reuse (**cooling towers**).

Psychrometric (humidity) terminology

- Remember that water **partial pressure**, ($P_A = y_A P$) is the pressure due to water vapor in the water-air mixture. Where y_A is the mole fraction of water vapor.

- In addition, the water **vapor pressure** (P_{AS}) is the absolute pressure exerted by (molecules of liquid water) on the gas phase in order to escape into the gas; it is a measure of volatility.

- The vapor pressure increases with temperature increase.

- The water vapor pressure can be taken from **steam tables** or estimated using , for example, the following **Antoine Eq**.:

$$\ln P_{AS}\,(\text{bar}) = 11.96481 - \frac{3984.923}{T(\text{K}) - 39.97}$$

Psychrometric (humidity) terminology

- **Molal humidity (H_m)**: moles of water vapor per unit mole of dry air. This is a direct measure of moisture content:

$$H_m = \frac{p_A}{P - p_A} = \frac{\text{moles of vapor}}{\text{moles of vapor-free (dry) gas}}$$

- **Absolute humidity or simply humidity (H)**: mass of water vapor per unit mass of dry air. This is another direct measure of moisture content:

$$H = \frac{\text{mass of vapor}}{\text{mass of dry gas}} = \frac{18.02}{28.97}\frac{p_A}{P - p_A}$$

- **Saturation humidity (H_s)**: absolute humidity at saturation ((P_A=P_{AS}):

$$H_S = \frac{18.02}{28.97}\frac{p_{AS}}{P - p_{AS}}$$

Psychrometric (humidity) terminology

- **Percentage humidity or saturation (H_p):** ratio of humidity to the saturation humidity multiplied by 100: $$H_P = 100\,\frac{H}{H_S}$$
- **Percentage relative humidity (H_R):** ratio of water vapor partial pressure to its saturation pressure multiplied by 100. It is another measure of the degree of saturation: $$H_R = 100\,\frac{p_A}{p_{AS}}$$
- Note that $H_R \neq H_P$ except at saturation

$$H_P = 100\,\frac{H}{H_S}$$

$$= (100)\,\frac{18.02}{28.97}\,\frac{p_A}{P - p_A}\Big/\frac{18.02}{28.97}\,\frac{p_{AS}}{P - p_{AS}} = \frac{p_A}{p_{AS}}\,\frac{P - p_{AS}}{P - p_A}\,(100)$$

- **At saturation:** $H_R = H_P = 100\%$

Psychrometric (humidity) terminology

EXAMPLE 9.3-1. Humidity from Vapor-Pressure Data

The air in a room is at 26.7°C (80°F) and a pressure of 101.325 kPa and contains water vapor with a partial pressure $p_A = 2.76$ kPa. Calculate the following.

(a) Humidity, H.

(b) Saturation humidity, H_S, and percentage humidity, H_P.

(c) Percentage relative humidity, H_R.

Solution:

$p_{AS} = 3.50$ kPa **(from steam tables)**

$$H = \frac{18.02}{28.97}\frac{p_A}{P - p_A} = \frac{18.02(2.76)}{28.97(101.3 - 2.76)} = 0.01742 \text{ kg } H_2O/\text{kg air}$$

$$H_S = \frac{18.02}{28.97}\frac{p_{AS}}{P - p_{AS}} = \frac{18.02(3.50)}{28.97(101.3 - 3.50)} = 0.02226 \text{ kg } H_2O/\text{kg air}$$

$$H_P = 100\frac{H}{H_S} = \frac{100(0.01742)}{0.02226} = 78.3\%$$

$$H_R = 100\frac{p_A}{p_{AS}} = \frac{100(2.76)}{3.50} = 78.9\%$$

Psychrometric (humidity) terminology

- **Dew point of air-water vapor mixture:** temperature at which the mixture is at saturated conditions ($P_A=P_{AS}$).
  - In the previous example if the partial pressure is 3.5 kPa. Then , the dew point will be 26.7 ^oC.
  - In other words, dew point is the temperature to which you must cool the air/vapor mixture to just obtain H_R=100% i.e. condensation just starts to occur.

- **Dry bulb temperature:** commonly measured temperature of water-air mixture by a thermometer. It indicates the amount of heat in the air and it is directly proportional to the mean kinetic energy of the air molecules.

Psychrometric (humidity) terminology

- **Humid heat of air-water vapor mixture (c_S):** amount of heat required to raise the temperature of 1 kg of mixture by 1 °C:

$$c_S = C_{pB} + H \times C_{pA}$$

- The heat capacity of water is assumed to be constant at , C_{pA} = 1.88 kJ/kg water vapor.
- The heat capacity of dry air is assumed to be constant at, C_{pB} = 1.005 kJ/kg dry air. Therefore,

$$c_S \text{ kJ/kg dry air} \cdot \text{K} = 1.005 + 1.88H$$

$$c_S \text{ btu/lb}_m \text{ dry air} \cdot {}^\circ\text{F} = 0.24 + 0.45H$$

Psychrometric (humidity) terminology

- **Humid volume of air-water vapor mixture (v_H):** total volume of 1 kg of the gas mixture at standard pressure and operating temperature (T). Using ideal gas law:

$$v_H \text{ m}^3/\text{kg dry air} = \frac{22.41}{273} T \text{ K} \left(\frac{1}{28.97} + \frac{1}{18.02} H \right)$$

$$= (2.83 \times 10^{-3} + 4.56 \times 10^{-3} H) T \text{ K}$$

$$v_H \text{ ft}^3/\text{lb}_m \text{ dry air} = \frac{359}{492} T {}^\circ\text{R} \left(\frac{1}{28.97} + \frac{1}{18.02} H \right)$$

$$= (0.0252 + 0.0405 H) T {}^\circ\text{R}$$

Psychrometric (humidity) terminology

- **Total enthalpy of air-water vapor mixture (H_y):** total enthalpy per 1 kg of the gas mixture:

$$H_y \text{ kJ/kg dry air} = c_S(T - T_0) + H\lambda_0$$

$$= (1.005 + 1.88H)(T - T_0 \,°\text{C}) + H\lambda_0$$

$$H_y \text{ btu/lb}_m \text{ dry air} = (0.24 + 0.45H)(T - T_0 \,°\text{F}) + H\lambda_0$$

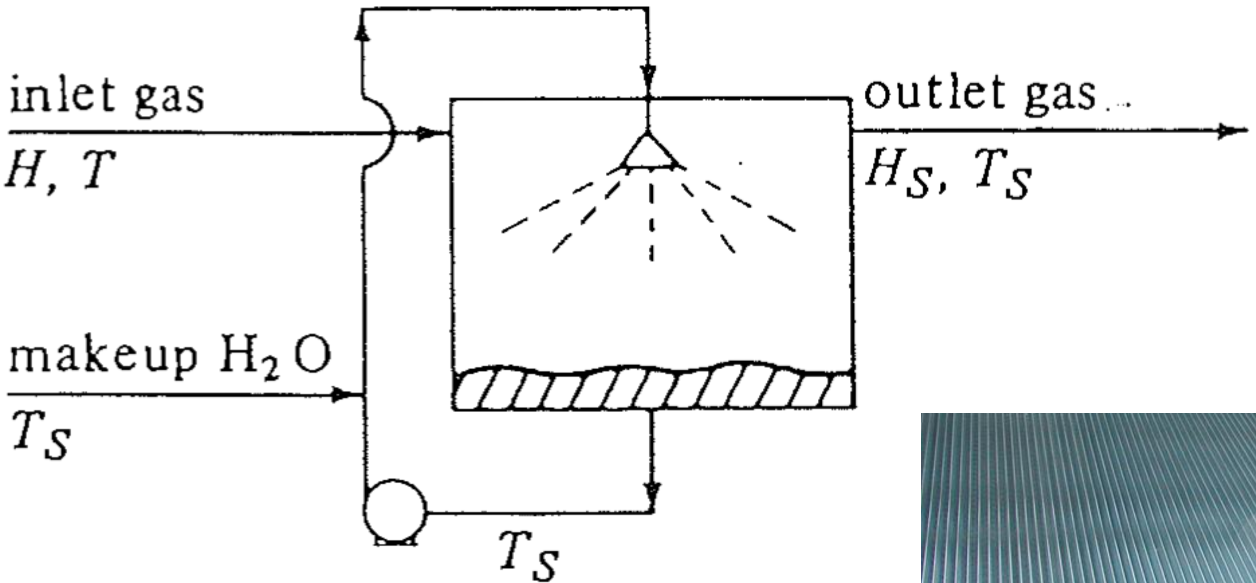
where λ_0 is the latent heat of vaporization of water at the base temperature, usually

T_0 =0 °C : λ_0=2502.3 kJ/kg water

T_0 =32 °F : λ_0=1075.8 Btu/lbm water

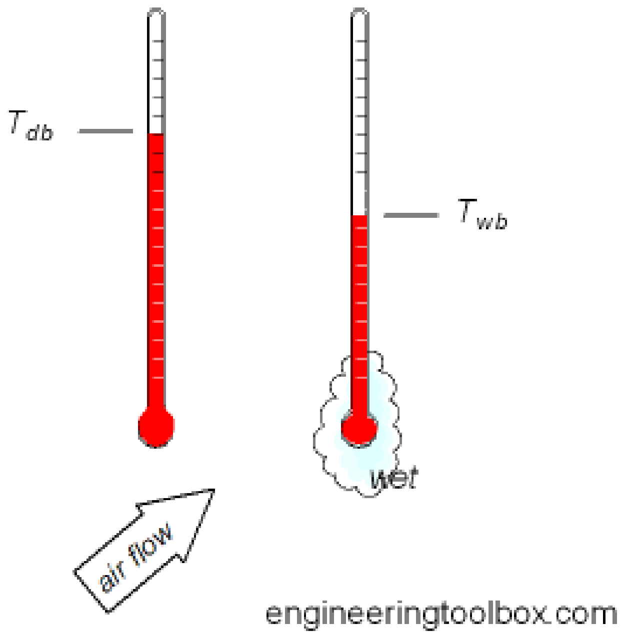
Psychrometric (humidity) terminology

- **Adiabatic saturation temperature T_s:** steady-state temperature attained when a continuous stream of air-water vapor mixture is contacted with large amount of water under adiabatic conditions.



Psychrometric (humidity) terminology

- **Wet bulb temperature T_{wb} or T_w:** steady-state non-equilibrium temperature reached when a small amount of water is contacted with a continuous stream of air under adiabatic conditions.



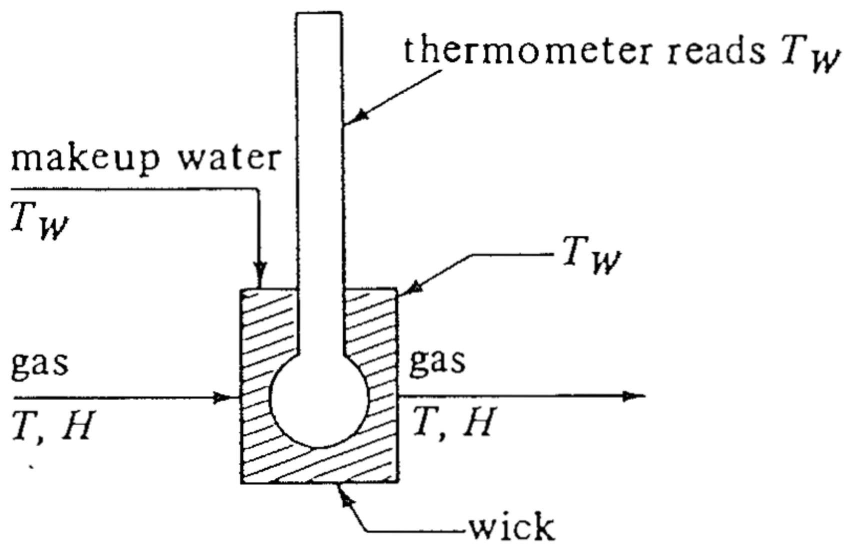


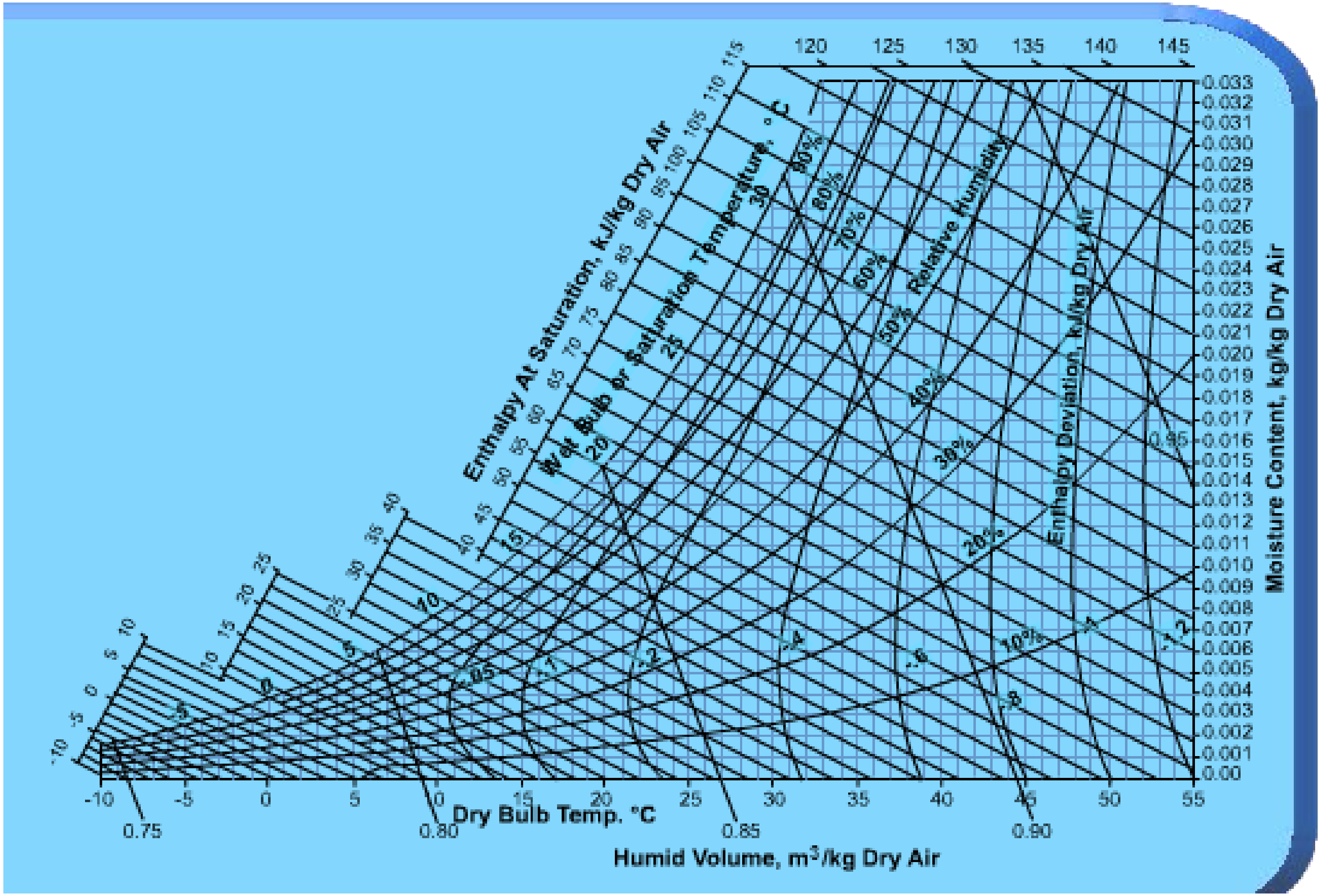
FIGURE 9.3-4. *Measurement of wet bulb temperature.*

Table 18.4 Definitions of quantities useful in psychrometry: A = moisture, B = moisture-free gas, ideal-gas conditions

Quantity	Definition	Relationship	
Absolute, mass humidity	Moisture content of a gas by mass	$\mathcal{H} = \frac{M_A p_A}{M_B(P - p_A)}$	(18-5)
Molal humidity	Moisture content of a gas by mols	$\mathcal{H}_m = \frac{p_A}{P - p_A}$	(18-6)
Saturation humidity	Humidity at saturation	$\mathcal{H}_s = \frac{M_A P_A^s}{M_B(P - P_A^s)}$	(18-7)
Relative humidity (relative saturation as a percent)	Ratio of partial pressure of moisture to partial pressure of moisture at saturation	$\mathcal{H}_R = 100\% \times \frac{p_A}{P_A^s}$	(18-8)
Percentage humidity (percent saturation)	Ratio of humidity to humidity at saturation	$\mathcal{H}_P = 100\% \times \frac{\mathcal{H}}{\mathcal{H}_s}$	(18-9)
Humid volume	Volume of moisture–gas mixture per unit mass of moisture-free gas	$v_H = \frac{RT}{P}\left(\frac{1}{M_B} + \frac{\mathcal{H}}{M_A}\right)$	(18-10)
Humid heat	Specific heat of moisture–gas mixture per unit mass of moisture-free gas	$C_s = (C_P)_B + (C_P)_A \mathcal{H}$	(18-11)
Total enthalpy	Enthalpy of moisture–gas mixture per unit mass of moisture-free gas referred to temperature, T_o	$H = C_s(T - T_o) + \Delta H_o^{vap} \mathcal{H}$	(18-12)
Dew-point temperature	Temperature at which moisture begins to condense when mixture is cooled	T_{dew}	
Dry-bulb temperature	Temperature of mixture	T_d	
Wet-bulb temperature	Steady-state temperature attained by a wet-bulb thermometer	T_w	
Adiabatic-saturation temperature	Temperature attained when a gas is saturated with moisture in an adiabatic process	T_s	

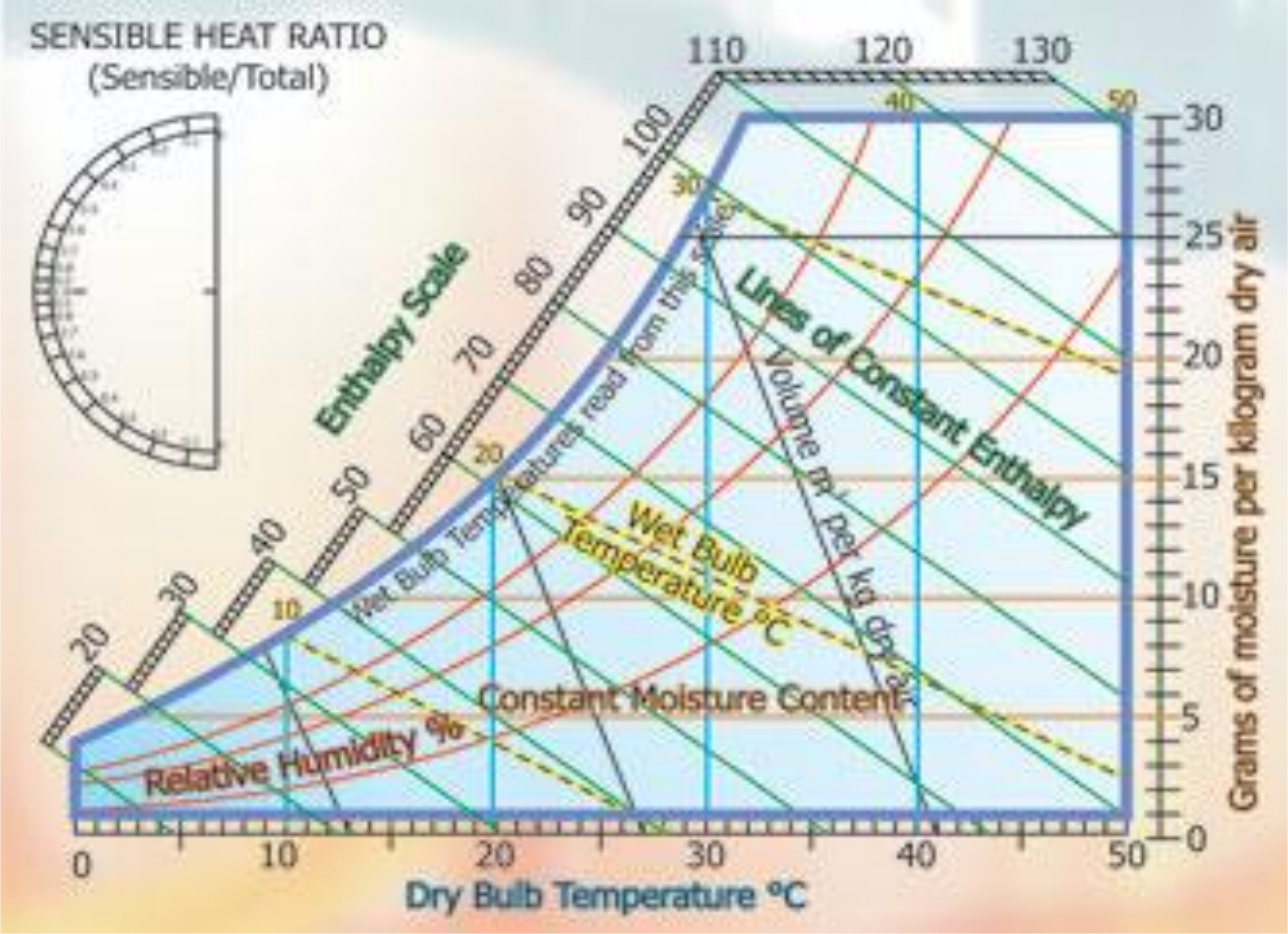
Psychrometric (humidity) charts

- **Visit:** http://www4.uwsp.edu/papersci/biasca/currentpages/

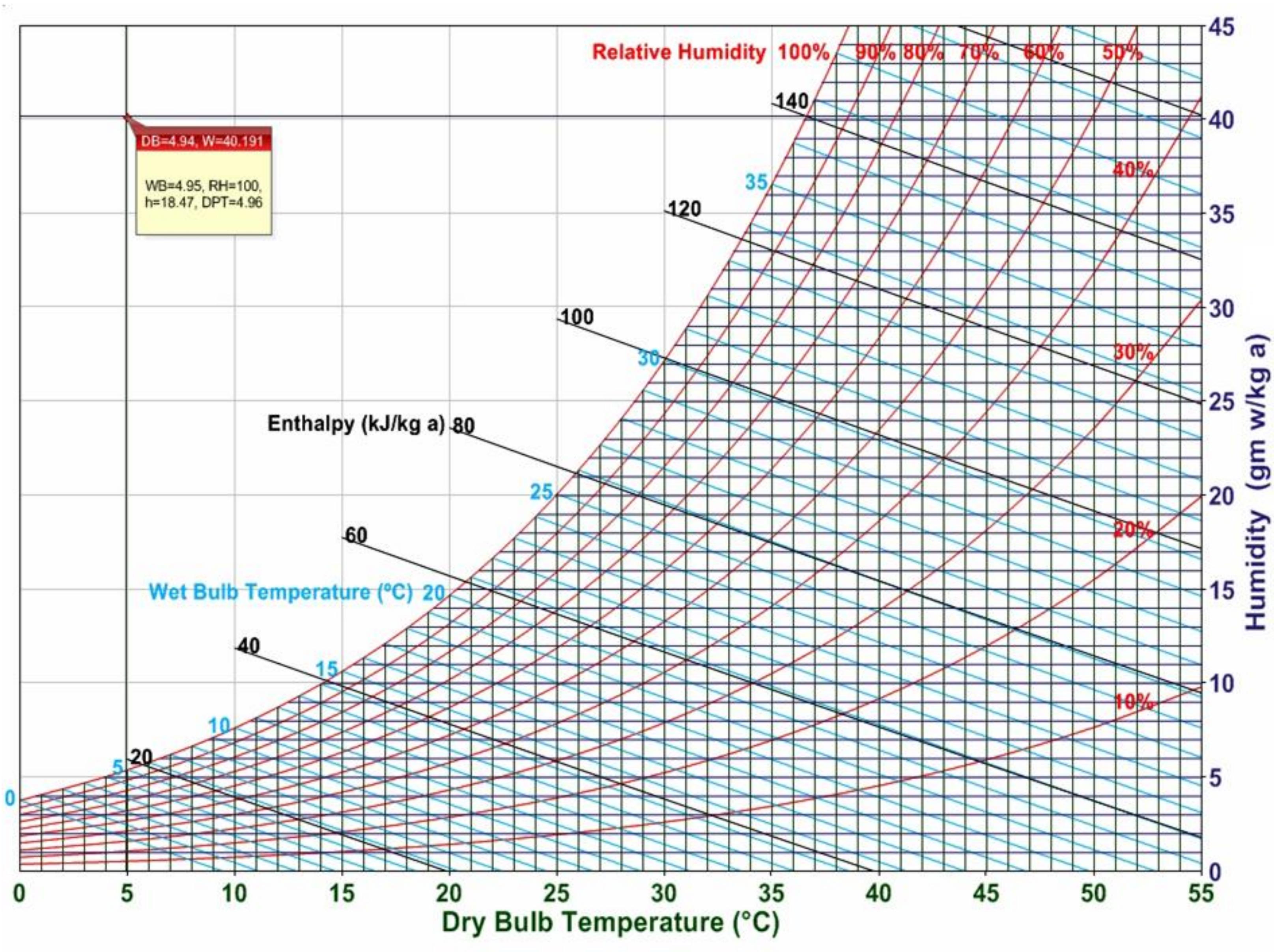


Psychrometric (humidity) charts

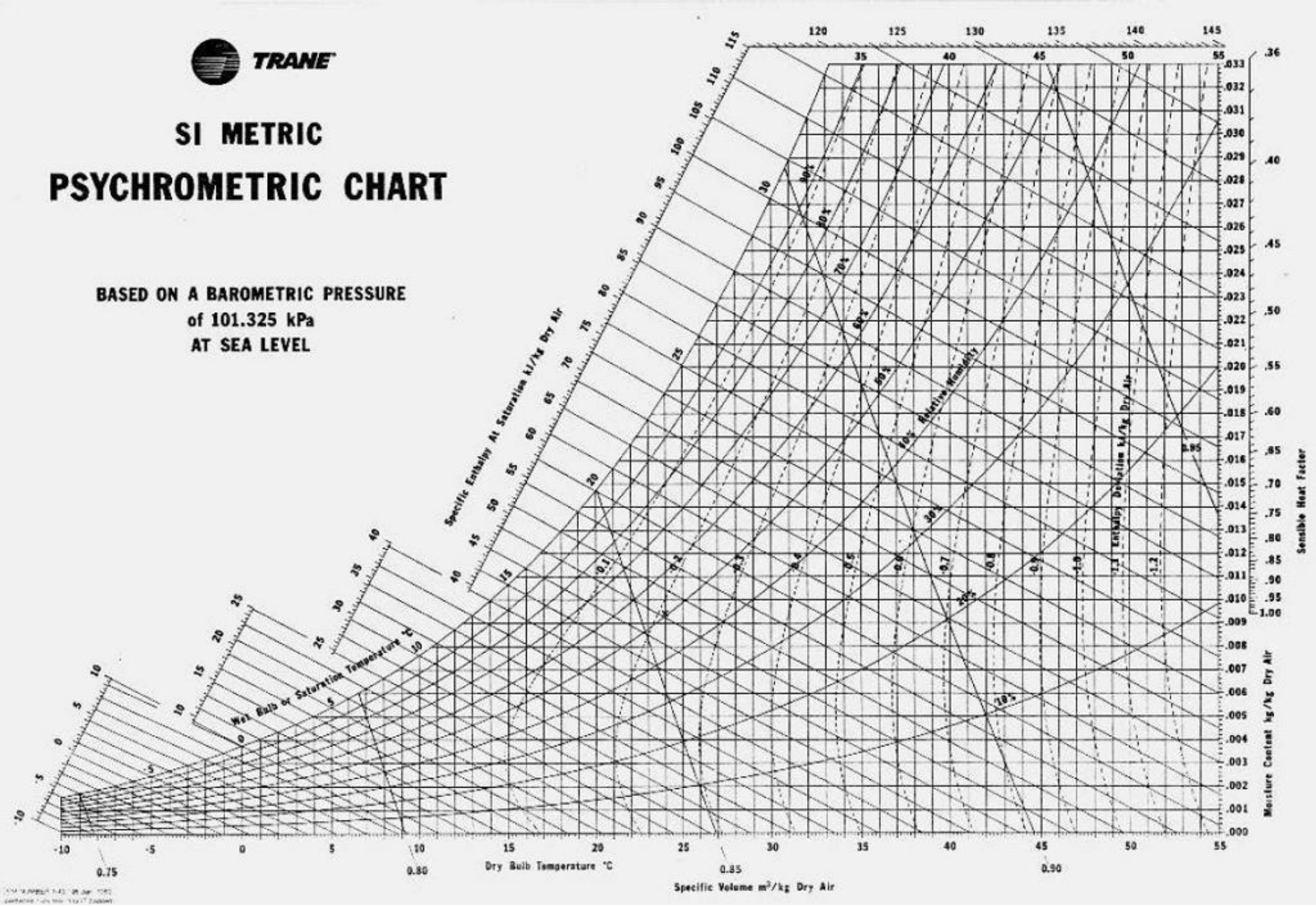
- **Visit:** http://www4.uwsp.edu/papersci/biasca/currentpages/



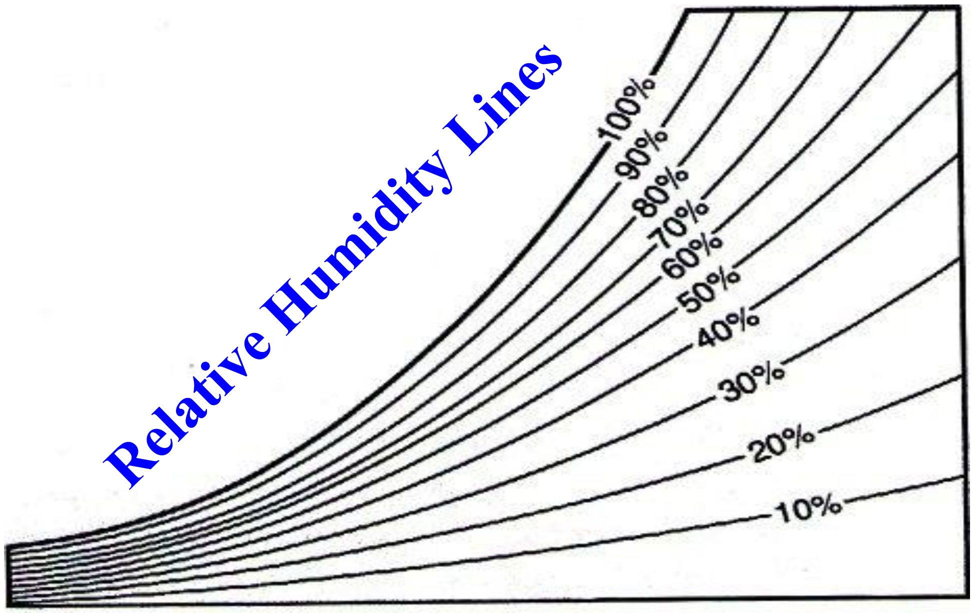
Psychrometric (humidity) charts



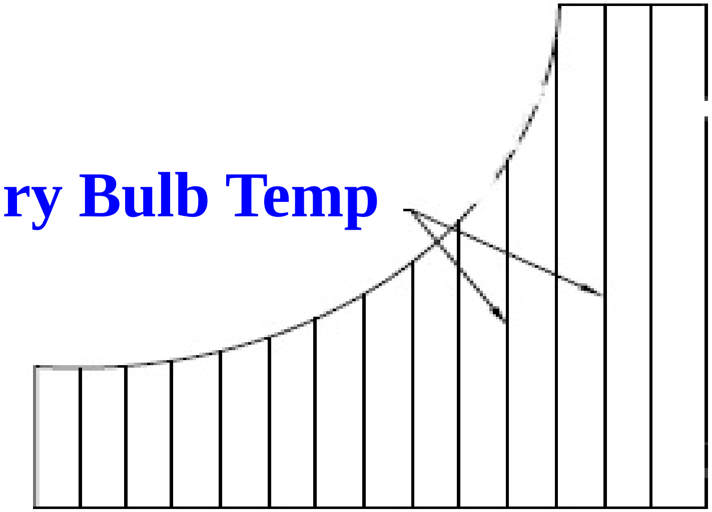
Psychrometric (humidity) charts

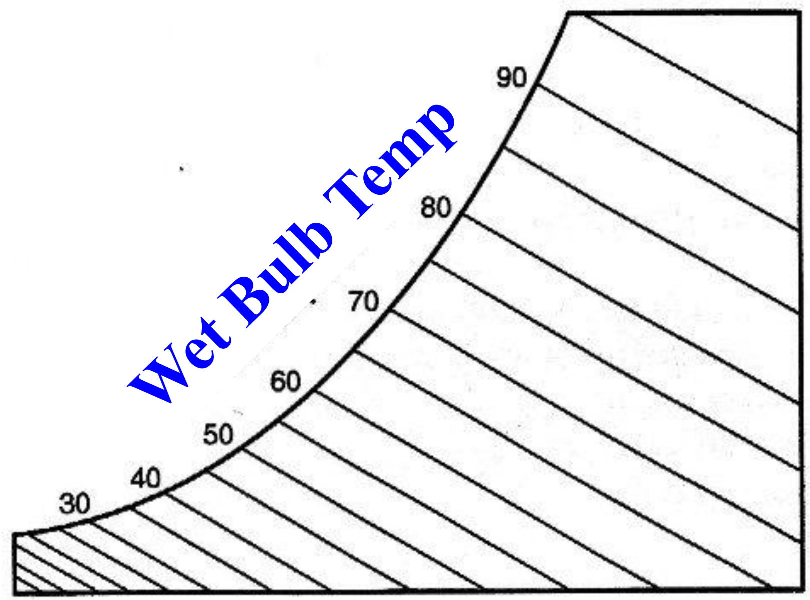


Psychrometric (humidity) chart



Dry Bulb Temp





Psychrometric (humidity) chart

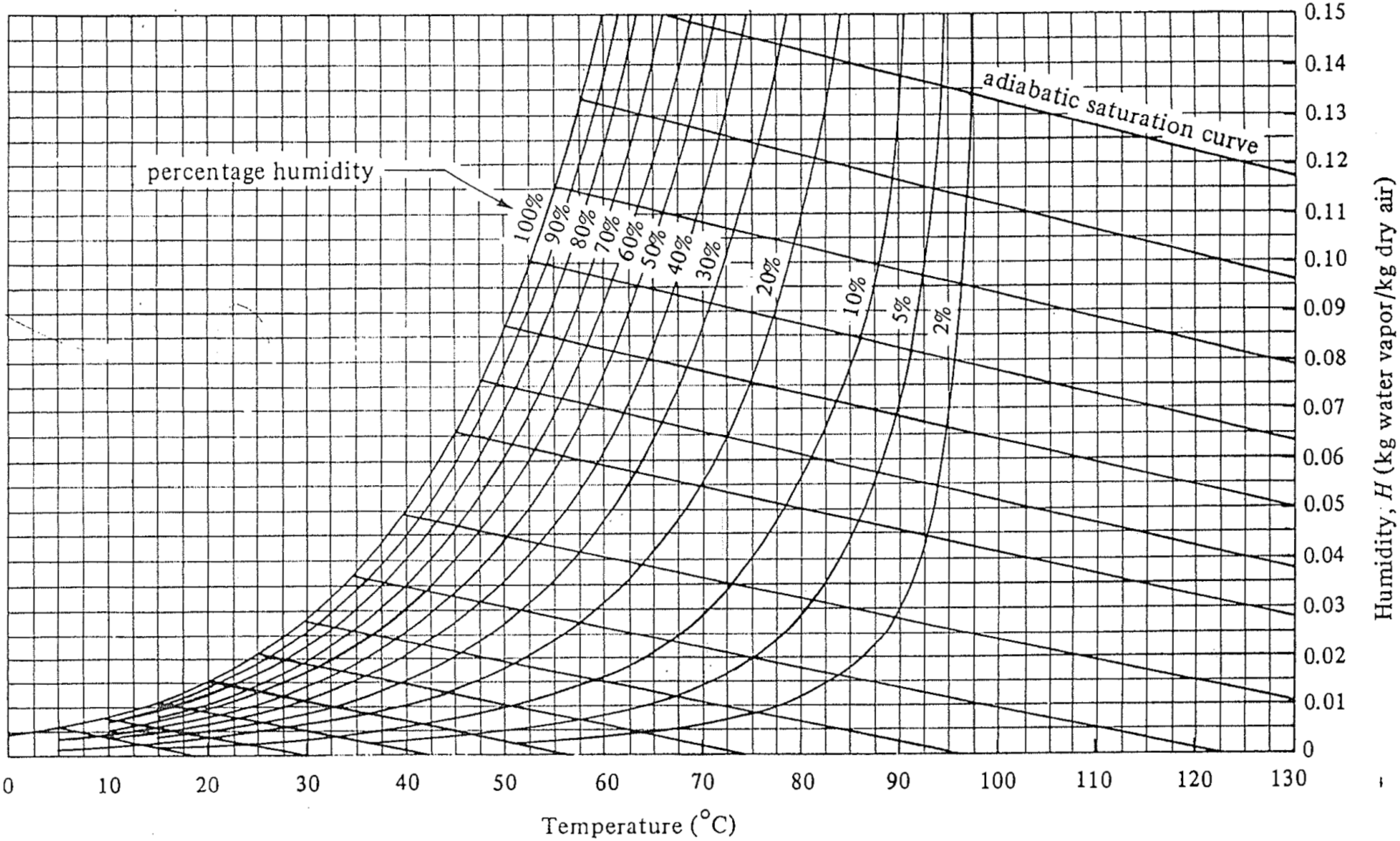
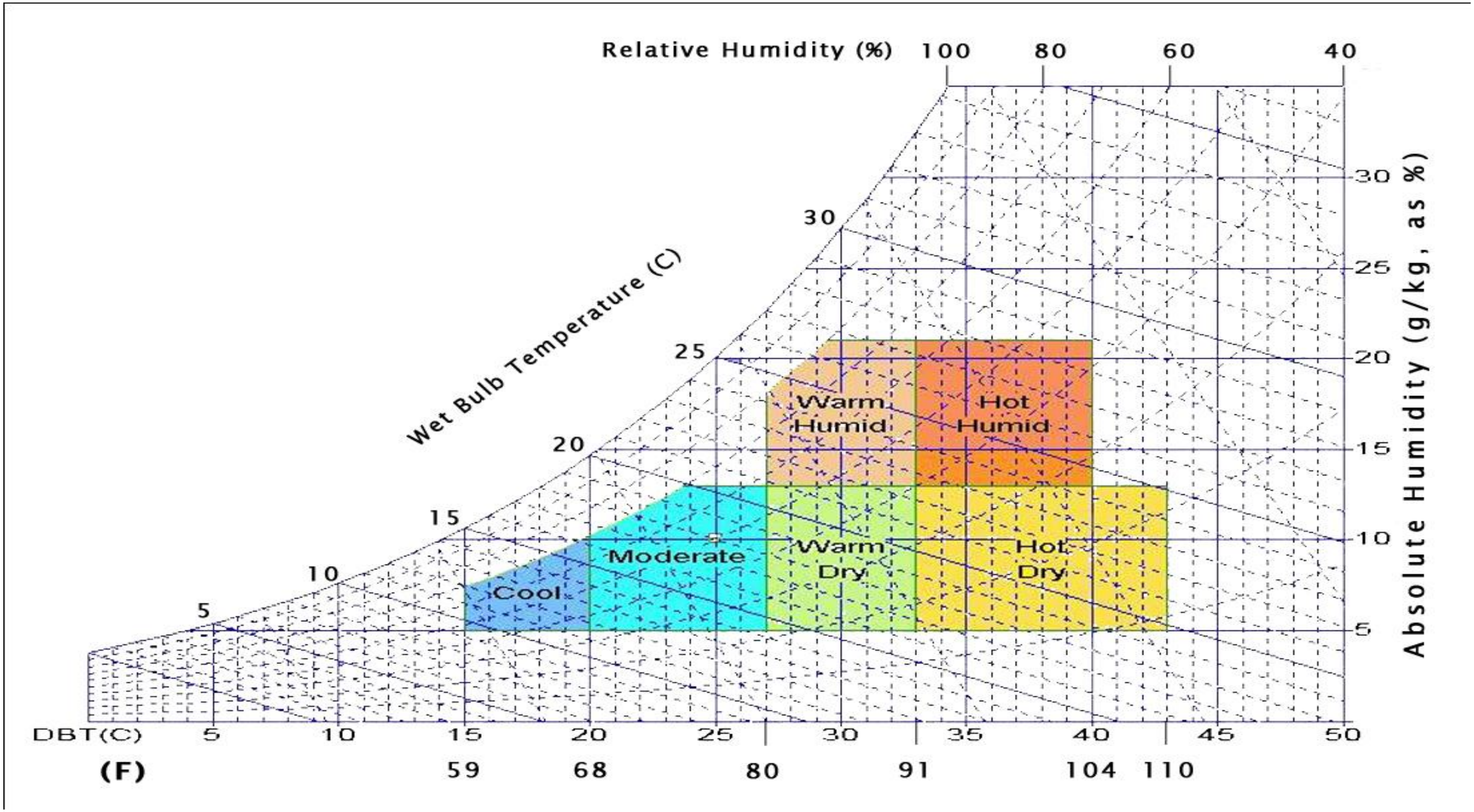


FIGURE 9.3-2. *Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)*

Psychrometric (humidity) chart

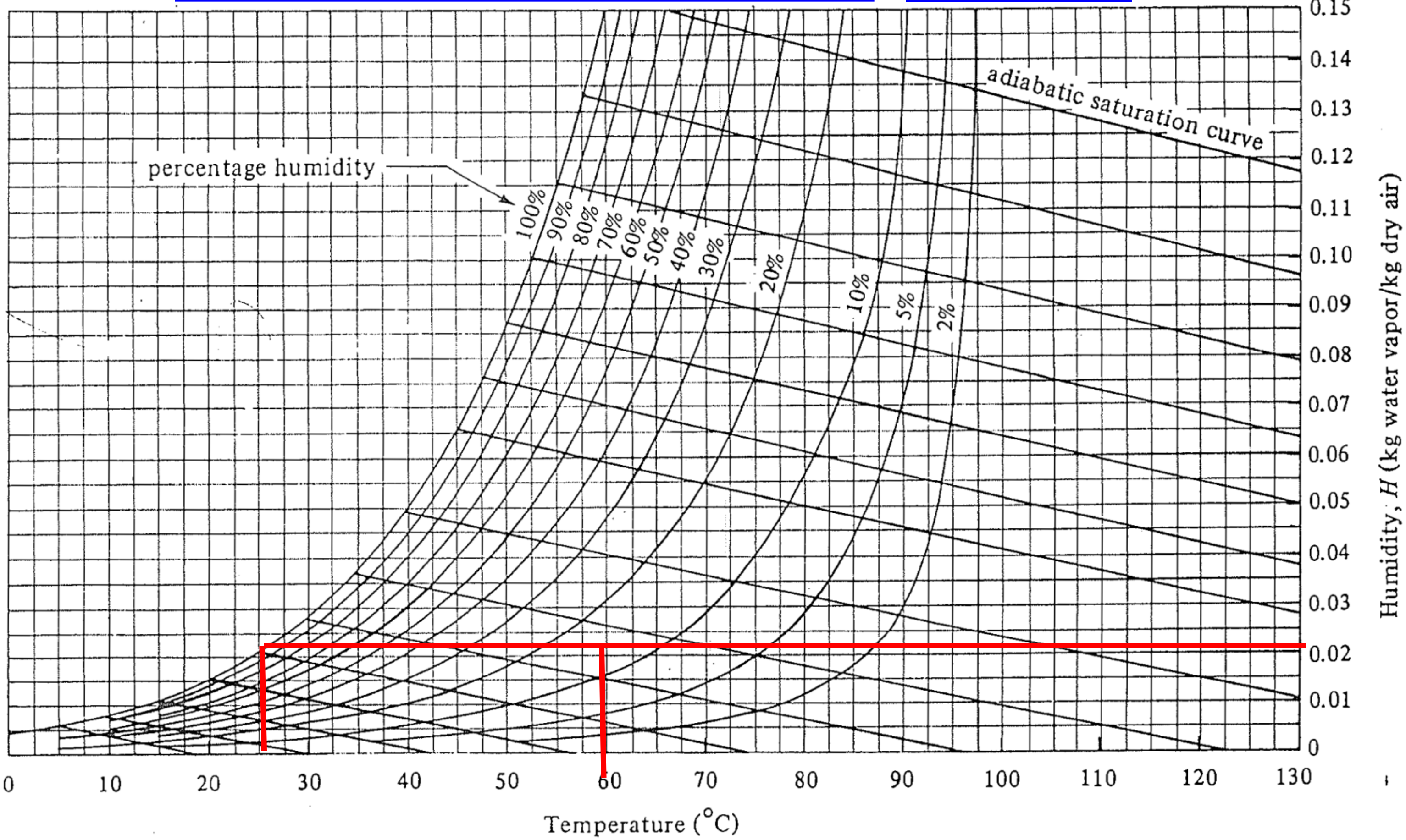
Where our ambient location?



EXAMPLE 9.3-2. Use of Humidity Chart

Air entering a dryer has a temperature (dry bulb temperature) of 60°C (140°F) and a dew point of 26.7°C (80°F). Using the humidity chart, determine the actual humidity H, percentage humidity H_P, humid heat c_S, and the humid volume v_H in SI and English units.

Solution $H = 0.0225$ kg H_2O/kg dry air $H_P = 14\%$



Psychrometric (humidity) chart

EXAMPLE 9.3-2.

$$c_S \text{ kJ/kg dry air} \cdot \text{K} = 1.005 + 1.88H$$

$$c_s = 1.005 + 1.88(0.0225)$$

$$= 1.047 \text{ kJ/kg dry air} \cdot \text{K} \quad \text{or} \quad 1.047 \times 10^3 \text{ J/kg} \cdot \text{K}$$

$$c_s = 0.24 + 0.45(0.0225)$$

$$= 0.250 \text{ btu/lb}_m \text{ dry air} \cdot {}^\circ\text{F} \qquad \text{(English)}$$

$$v_H = (2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.0225)(60 + 273)$$

$$= 0.977 \text{ m}^3\text{/kg dry air}$$

In English units,

$$v_H = (0.0252 + 0.0405 \times 0.0225)(460 + 140) = 15.67 \text{ ft}^3\text{/lb}_m \text{ dry air}$$

Psychrometric (humidity) chart

EXAMPLE 9.3-3. Adiabatic Saturation of Air

An air stream at 87.8°C having a humidity $H = 0.030$ kg H_2O/kg dry air is contacted in an adiabatic saturator with water. It is cooled and humidified to 90% saturation.

(a) What are the final values of H and T?

(b) For 100% saturation, what would be the values of H and T?

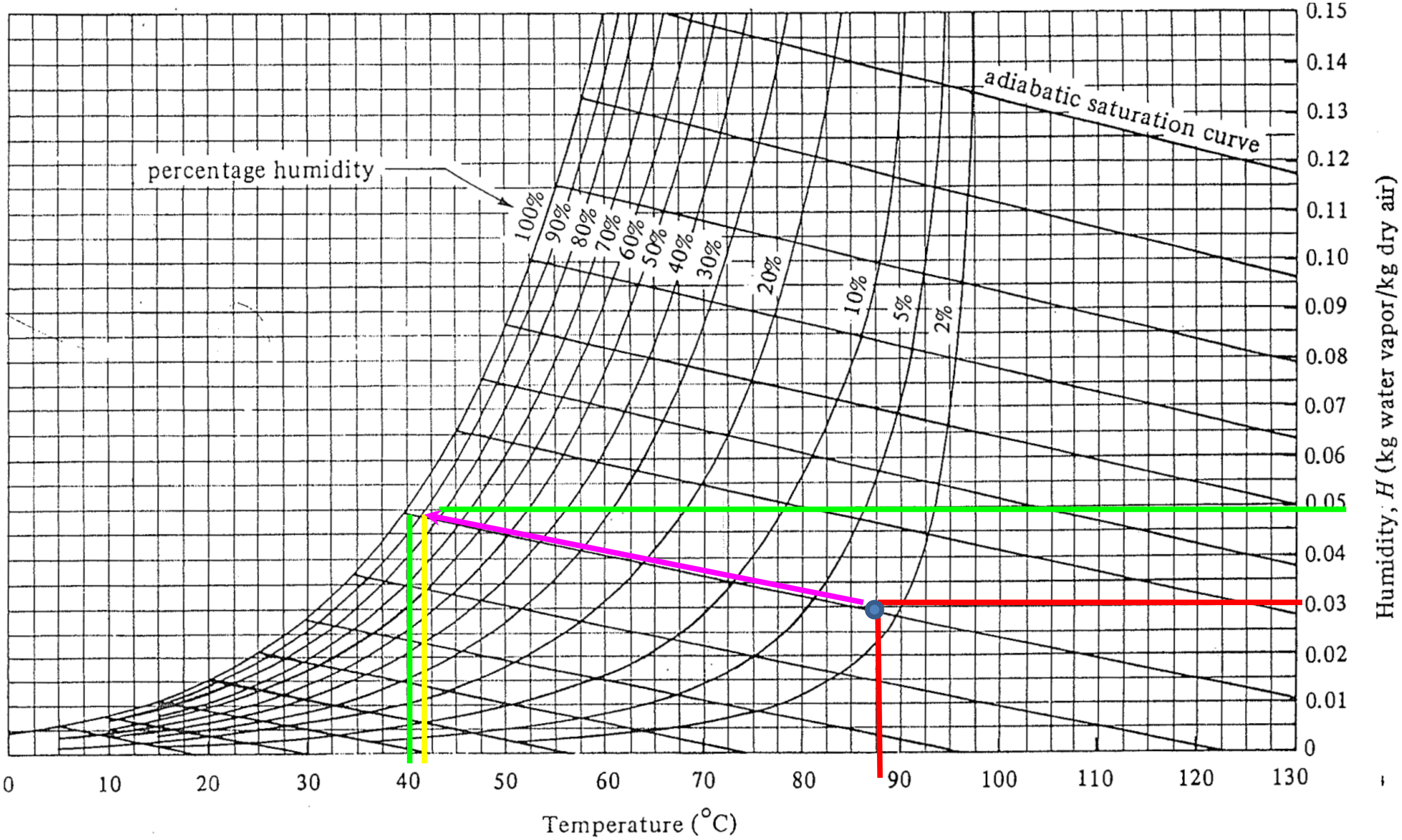
Psychrometric (humidity) chart

EXAMPLE 9.3-3.

Solution:

(a) $H = 0.0500$ kg H_2O/kg dry air $T = 42.5°C$

(b) $T = 40.5°C$, $H = 0.0505$ kg H_2O/kg dry air



Psychrometric (humidity) chart

EXAMPLE 9.3-4. Wet Bulb Temperature and Humidity

A water vapor–air mixture having a dry bulb temperature of $T = 60°C$ is passed over a wet bulb as shown in Fig. 9.3-4, and the wet bulb temperature obtained is $T_W = 29.5°C$. What is the humidity of the mixture?

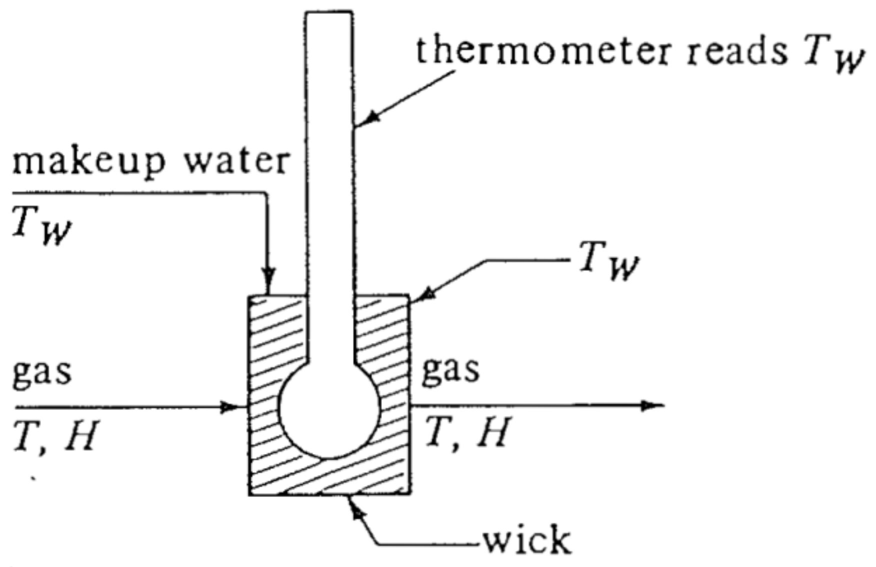


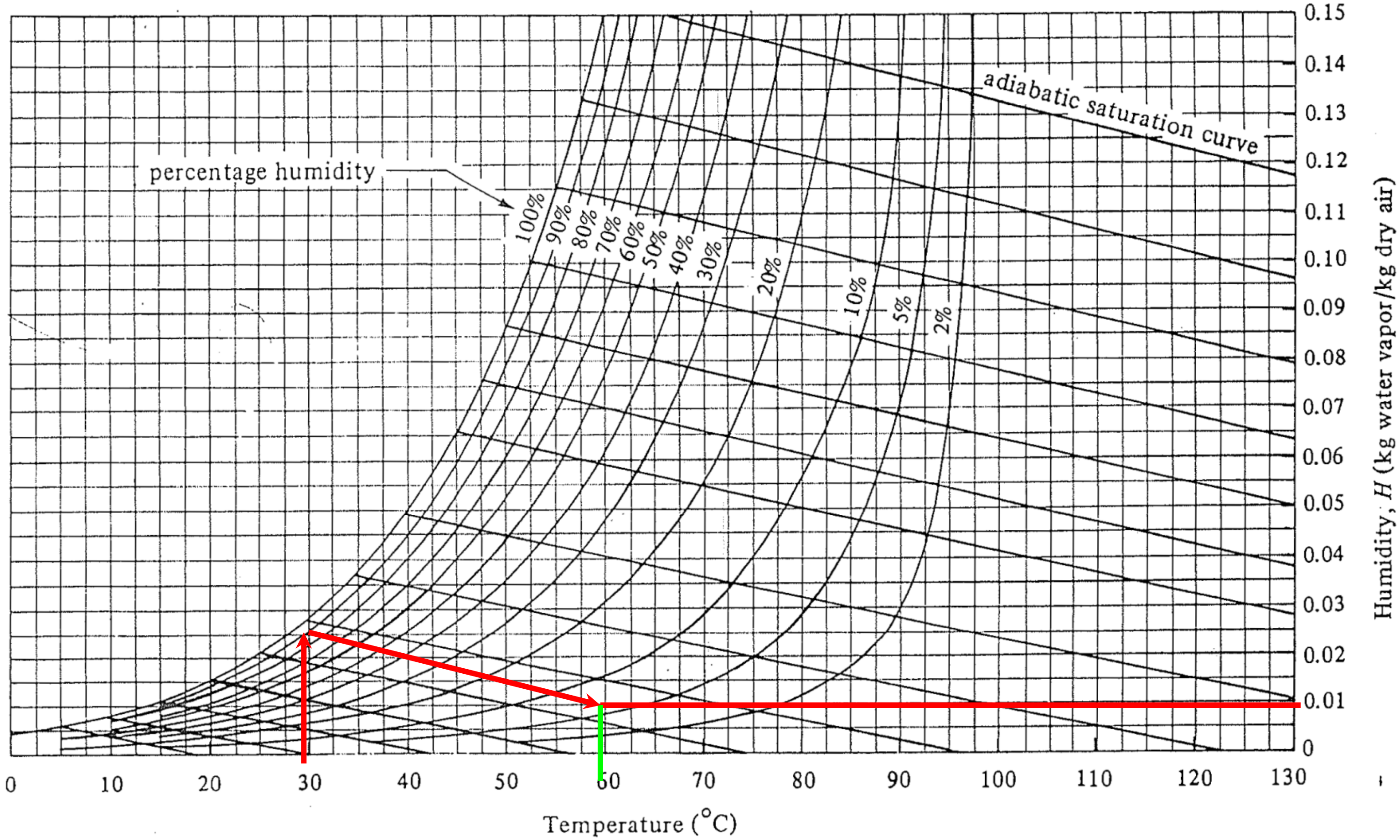
FIGURE 9.3-4. *Measurement of wet bulb temperature.*

Some psychrometric processes

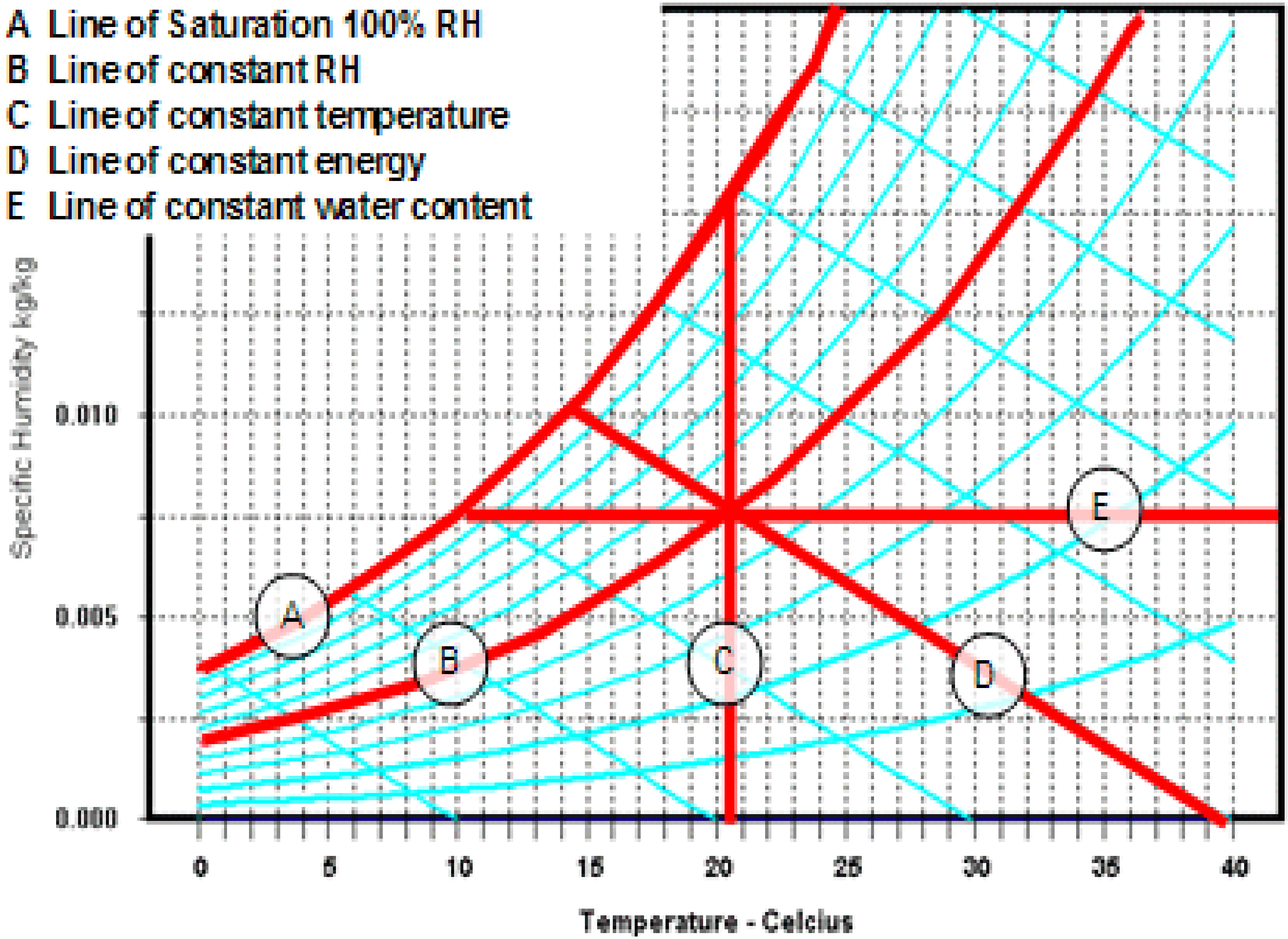
EXAMPLE 9.3-4.

Solution The wet bulb temperature of 29.5°C can be assumed to be the same as the adiabatic saturation temperature T_S

$T = 60°C$, $H = 0.0135$ kg H_2O/kg dry air.



Some psychrometric processes



Some psychrometric processes

(A) Humidification only

(B) Heating and humidification

(C) Sensible heating only

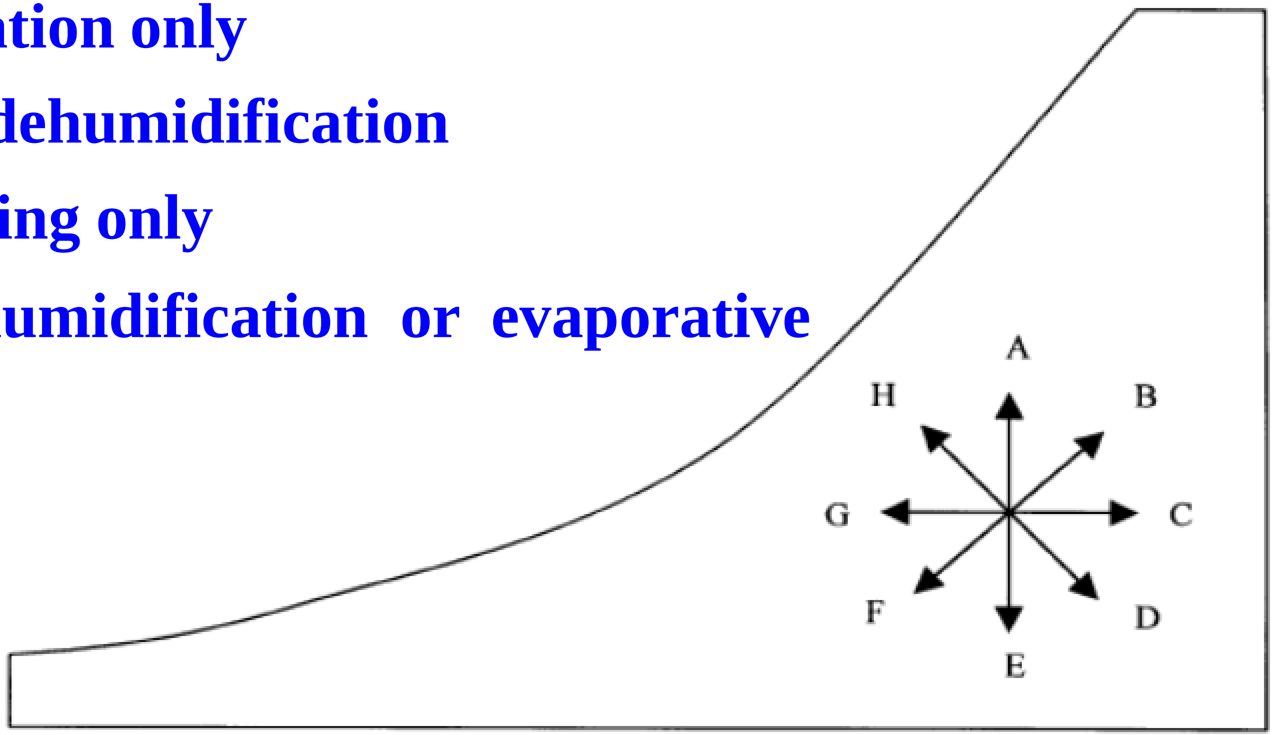
(D) Adiabatic heating and Dehumidification

(E) Dehumidification only

(F) Cooling and dehumidification

(G) Sensible cooling only

(H) Adiabatic humidification or evaporative cooling



Some psychrometric processes

Example. Two and a half cubic meters of lumber is being dried at 60 °C dry bulb temperature and 52 °C wet bulb temperature. The drying rate of the lumber is 12.5 kg of water/h. If outside air is at 27°C dry bulb temperature and 80% percentage humidity.

a. Classify this psychometric process?

b. How much outside cubic meters of air are needed per minute to carry way the evaporated moisture?

Solution:

a. This is **Heating and humidification process** since it will involves simultaneous increase in both the dry bulb temperature and humidity of the air.

b. ΔH= (0.092–0.018) kg/kg dry air = 0.074 kg water/kg dry air

Mass flow rate of dry air = drying rate/ΔH = 168.9 kg dry air/h

Humid volume, $v_H = [2.83\times10^{-3}+4.56\times10^{-3}(0.018)](27+273)$

$=0.87\ m^3/kg$ dry air

Volumetric flow rate of dry air = (168.9)(0.87)/60 = 2.45 m^3/min

Some psychrometric processes

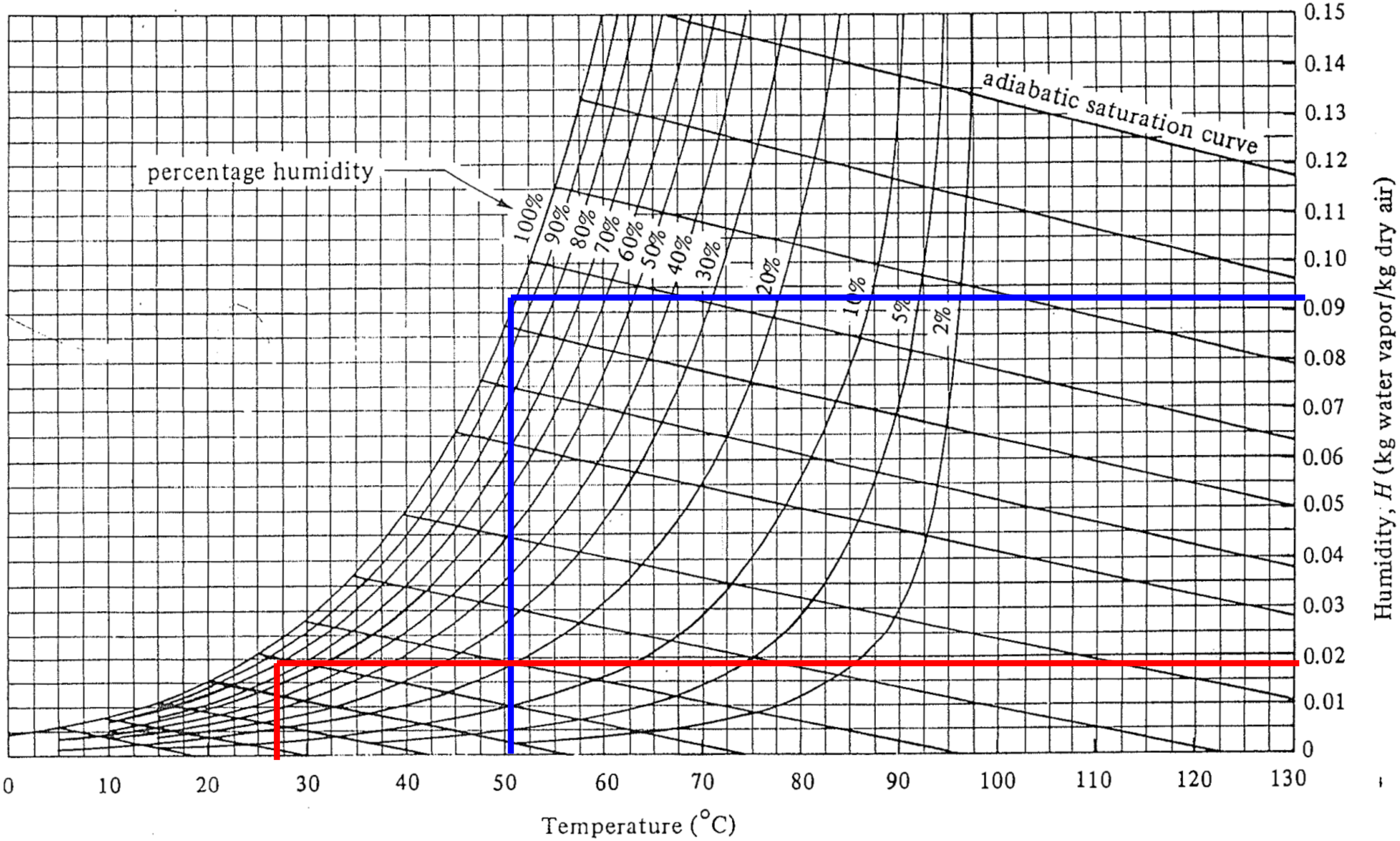


FIGURE 9.3-2. *Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)*

Some psychrometric processes

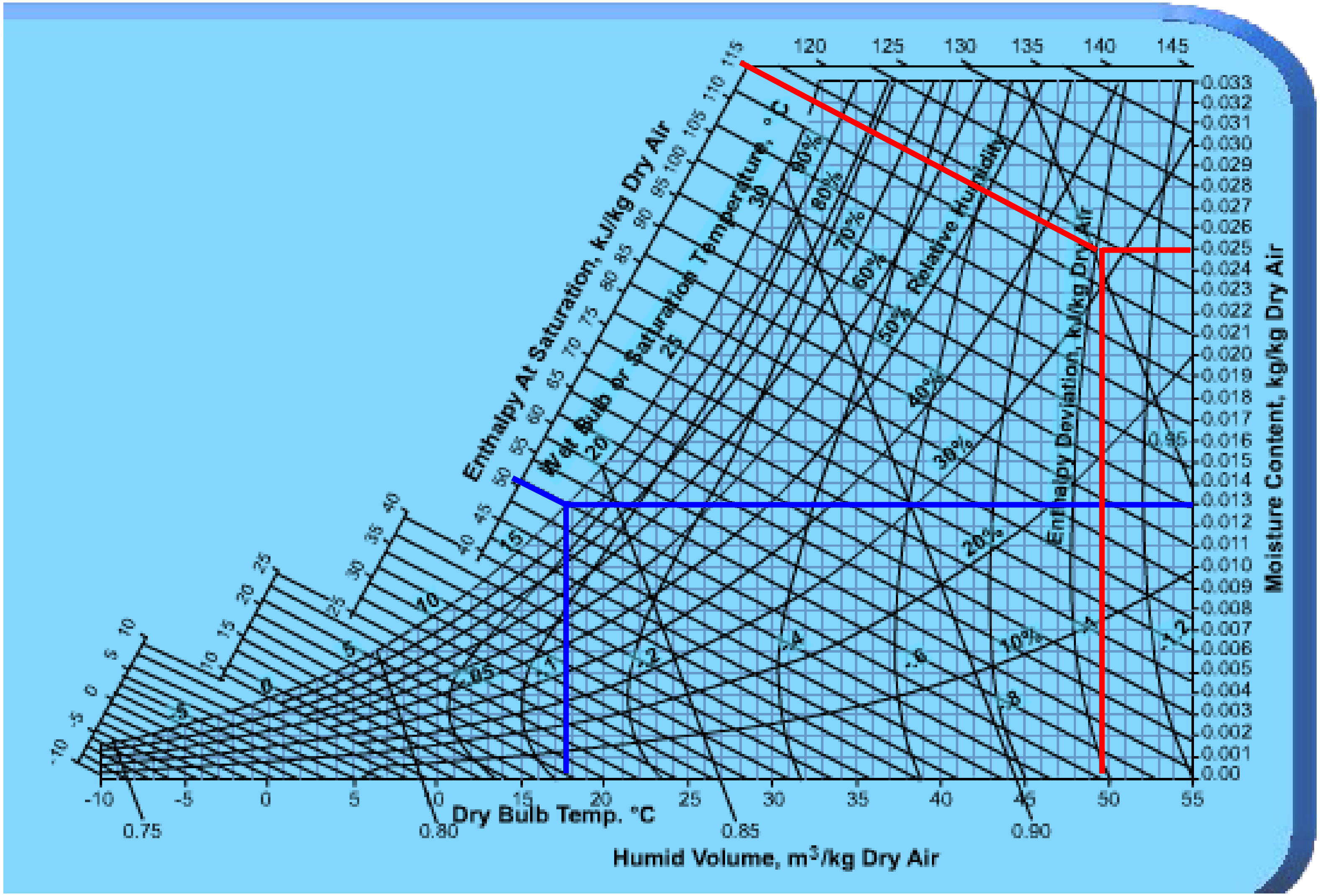
Example. Moist air at 50°C dry bulb temperature and 32% relative humidity enters the cooling coil of a dehumidification kiln heat pump system and is cooled to a temperature of 18°C. If the condensate rate is 4 kg water/h. Determine the kW of refrigeration required.

Solution:

This is **Cooling and dehumidification** since it will involves simultaneous decrease in both the dry bulb temperature and humidity of the air.

Assume the final state at 18 ^{o}C is at saturation.

Some psychrometric processes



Some psychrometric processes

$\Delta H = (0.025 - 0.013) = 0.012$ kg water/kg dry air

Mass flow rate of air= drying rate/ $\Delta H = 4/0.012 = 333.3$ kg dry air/h

$H_{y1} = 50$ kJ/kg dry air

$H_{y2} = 115$ kJ/kg dry air

$\Delta H_y = (115 - 50) = 65$ kJ/kg dry air.

$Q = \Delta H_y \times 333.3 = 65 \times 333.3 = 21664$ kJ /h = 6.0 kW

Cooling towers

- **Cooling tower** is one of the most important humidification applications which is used to lower the temperature of re-circulated water used for condensers and heat exchangers in chemical plants, power plants, and air conditioning units.
- It depends on the principle that when a warm liquid (e.g. water) is contacted with unsaturated gas (air), part of the liquid evaporates and the water temperature will drop (**water can NOT be cooled below the wet bulb temperature**).
- Cooling towers are **packed columns** (corrugated sheets) through which air passes by **natural convection** or **fans** and brought in contact with warm water introduced by **spray nozzles**.
- **In a typical cooling water**, warm water flows **countercurrent** to an air stream. Typically, the warm water enters the top of packed tower and cascades down through the packing, leaving at the bottom.

Types of cooling towers

A. Natural draft cooling tower

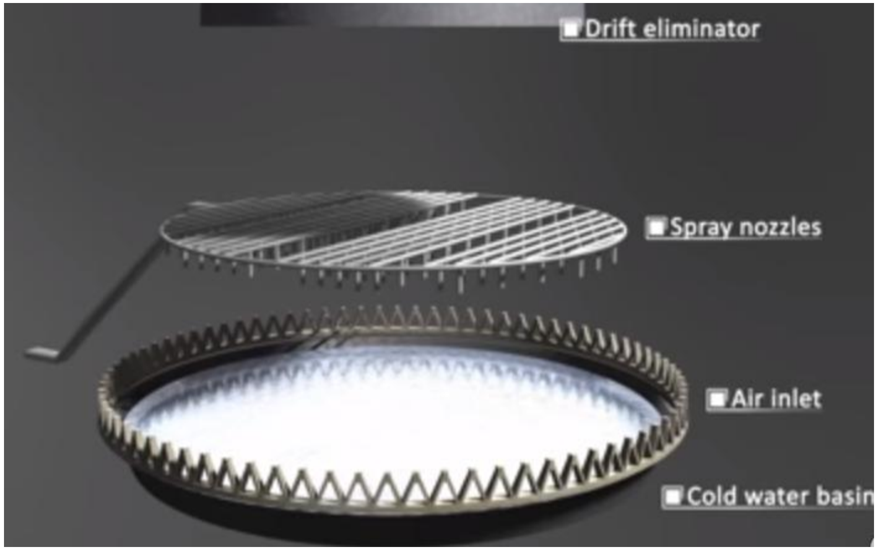
Types of cooling towers

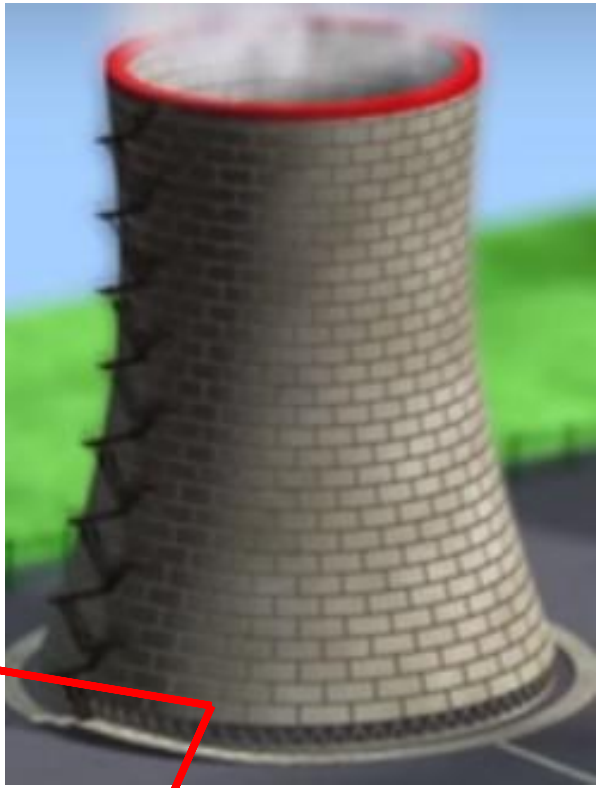
A. Natural draft cooling tower

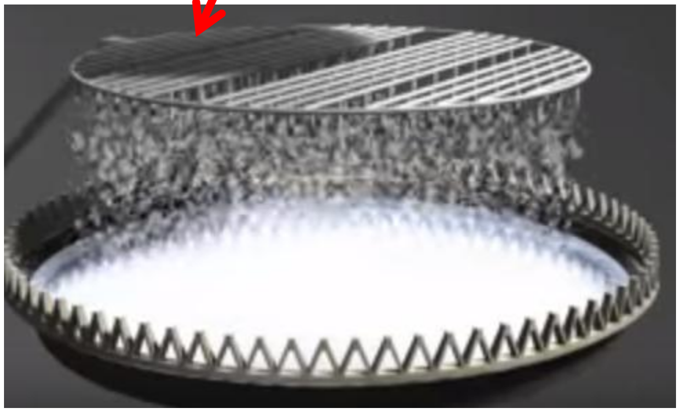
- Fresh cool atmospheric air is drawn into the tower from bottom. Thus, Air passes by natural convection (cold air displaces hot air)
- Hot air moves upward through tower.
- Concrete towers are typically about 400 ft (120 m) high.
- Used for large heat duty such as nuclear and power plants.
- Fill (packing) in the bottom part of the tower to enhance the intimate contact surface area between gas and liquid.
- No fans are used.

Types of cooling towers

A. Natural draft cooling tower







- **Drift eliminators**: capture droplets in air stream.
- **Cold water basin**: receives water at bottom of tower
- **Air inlet**: entry point of air
- **Spray nozzles**: spray water to wet the fill

Types of cooling towers

B. Mechanical draft cooling tower

- Air is passed mechanically by a fan.
- Water falls over packing (fill) surfaces to maximum heat/mass transfer.
- Cooling rates depend on many parameters.
- The gas-phase resistance controls the rate of transfer.
- Uniform distribution of air is important and is controlled by the fan location and cooling tower arrangement.
- Three common types:

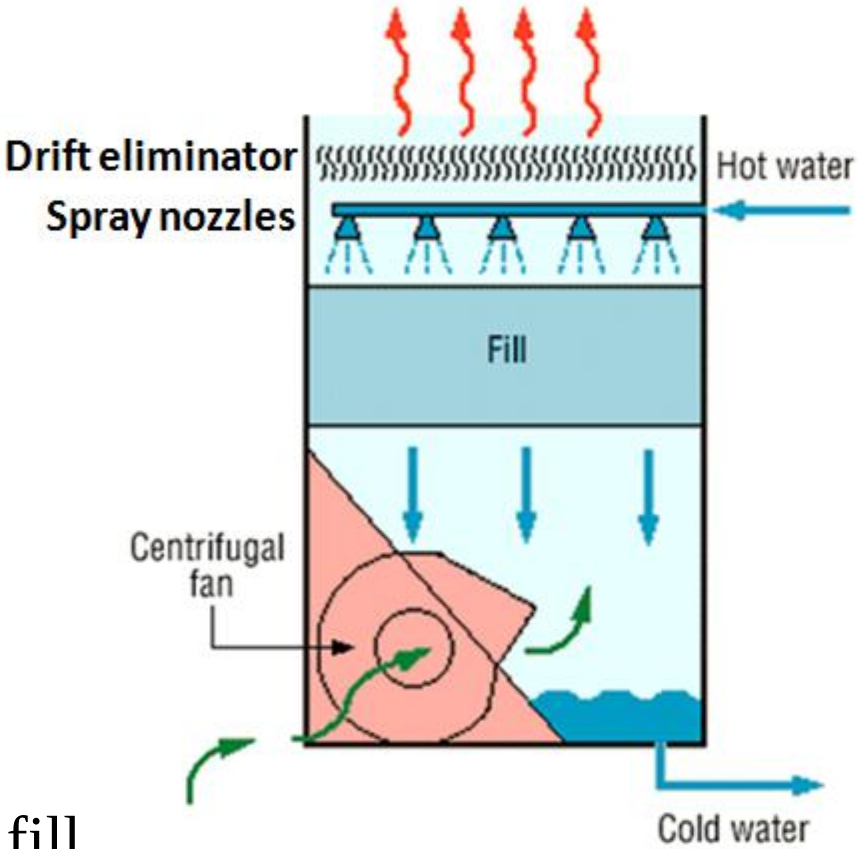
1. Forced draft cooling tower

2. Induced draft counter flow cooling tower

3. Induced draft cross flow cooling tower

Types of cooling towers

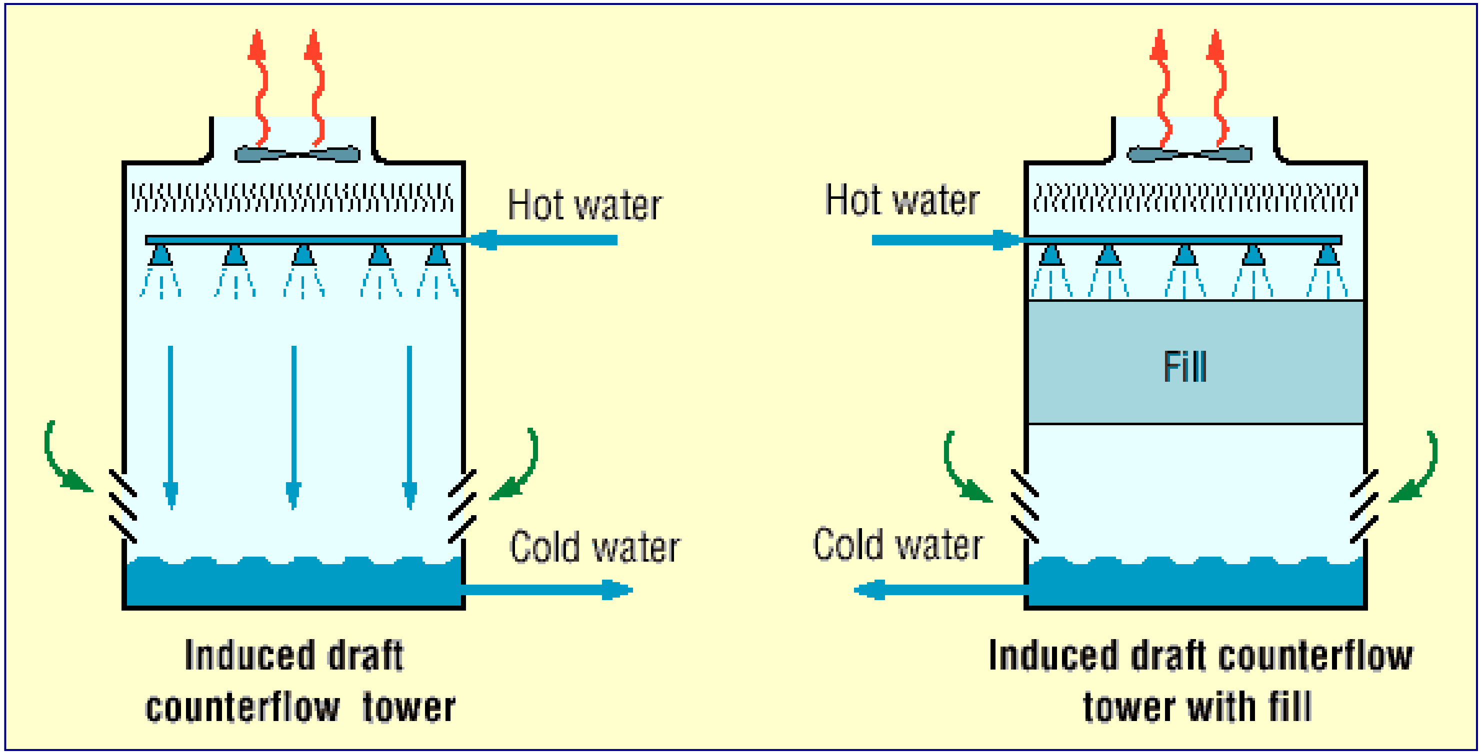
1. Forced draft cooling tower



- Water enters top and passes over fill.
- Air blown through tower by centrifugal fan at air inlet.
- Suitable for high air resistance.
- Air recirculation due to high air-entry and low air-exit velocities

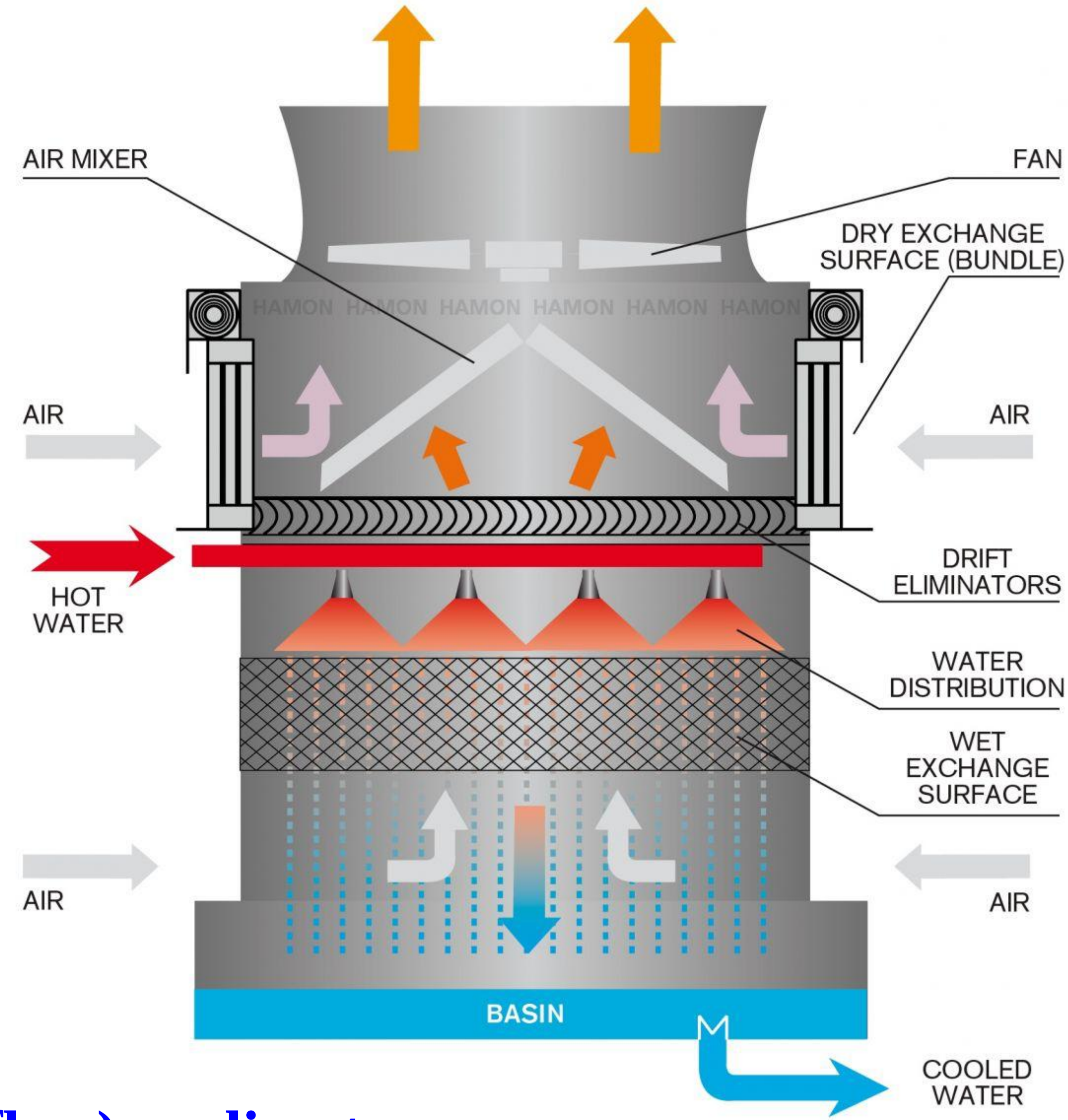
Types of cooling towers

2. Induced draft (counter-flow) cooling tower



- Hot water enters at the top
- Air enters at bottom and exits at top
- Uses forced and induced draft fans
- Less recirculation than forced draft towers

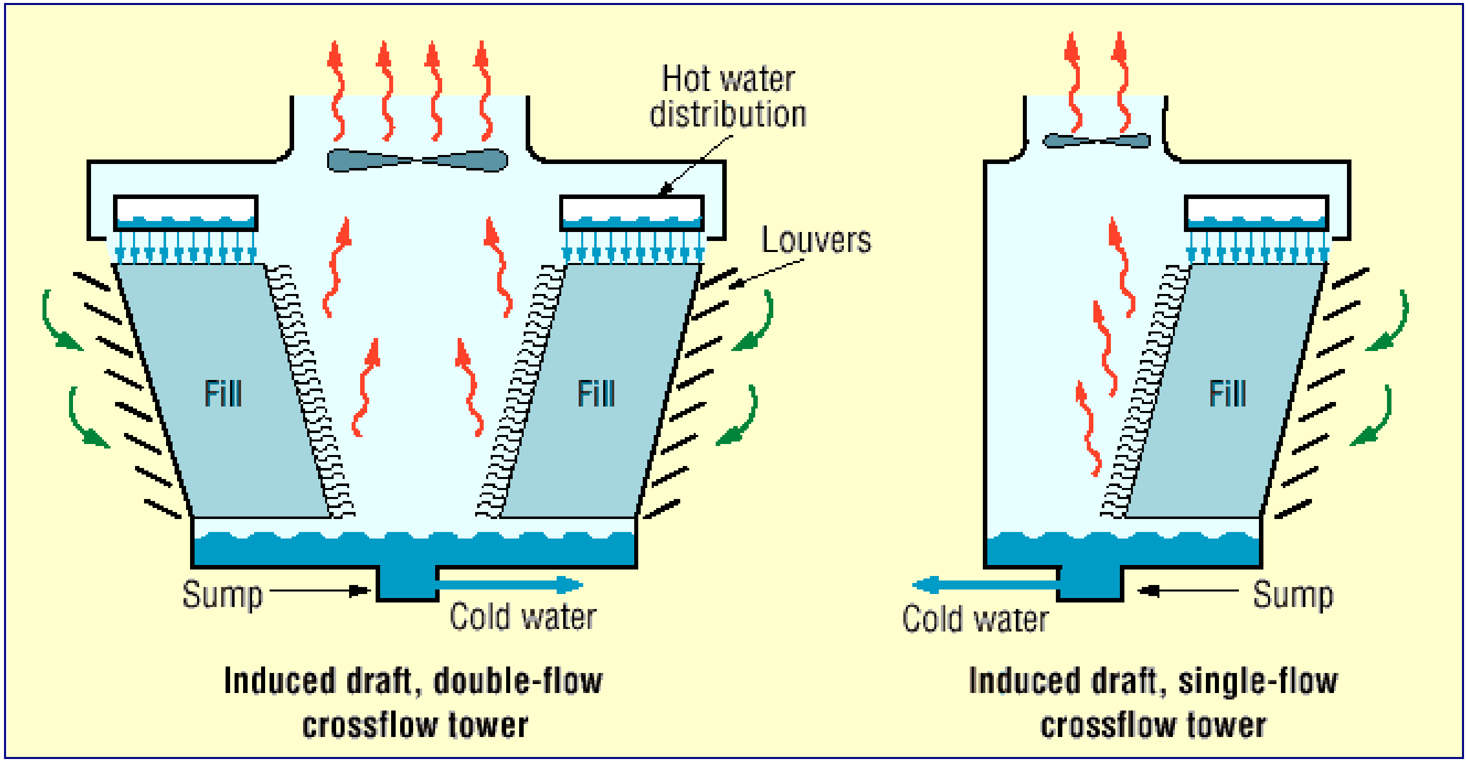
Types of cooling towers



Induced draft (counter-flow) cooling tower

Types of cooling towers

2. Induced draft (cross-flow) cooling tower



- Water enters top and passes over fill
- Air enters on one side or opposite sides
- Induced draft fan draws air across fill
- **Louvers:** equalize air flow into the fill and retain water within tower

Measured parameters in cooling towers

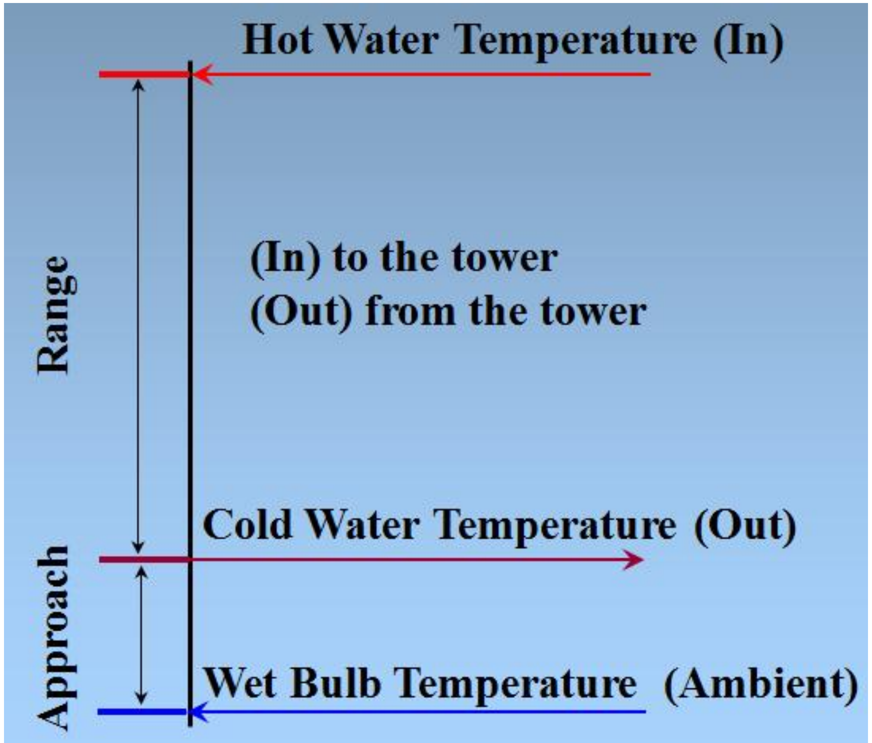
- **Wet bulb temperature of air**
- **Dry bulb temperature of air**
- **Cooling tower inlet water temperature**
- **Cooling tower outlet water temperature**
- **Water flow rate**
- **Air flow rate**
- **Electrical readings of pump and fan motors**

Performance parameters in cooling towers

1. **Range**
2. **Approach**
3. **Effectiveness**
4. **Liquid/Gas ratio**
5. **Cooling capacity**
6. **Evaporation loss**
7. **Air pressure drop**

- Typical **air pressure drop** for counter-flow cooling tower **< 250 Pa**

Performance parameters in cooling towers



- **Range**

  Typical values of the Range:

  6 – 17 °C = 10– 30 °F

- **Approach**

  Typical values of the Approach:

  3–8 °C= 5– 15 °F

- **Effectiveness (%) = 100×Range / (Range + Approach)**

- **Cooling capacity:** $\text{Cooling capacity} = \dot{m}_{water} \times C_L (-\text{Range})$

→ High Range, low Approach, high effectiveness, and high cooling capacity means **good performance.**

Performance parameters in cooling towers

- **Liquid /Gas ratio (L/G):**

L:Water mass rate per cross-sectional area of the tower.

G: Air mass flow rate per cross-sectional area of the tower.

Typical values:

L = 0.7-3.5 kg/m².s G= 1.6-2.8 kg/m².s

- **Evaporation loss:** ratio of mass evaporated to the inlet water flow rate:

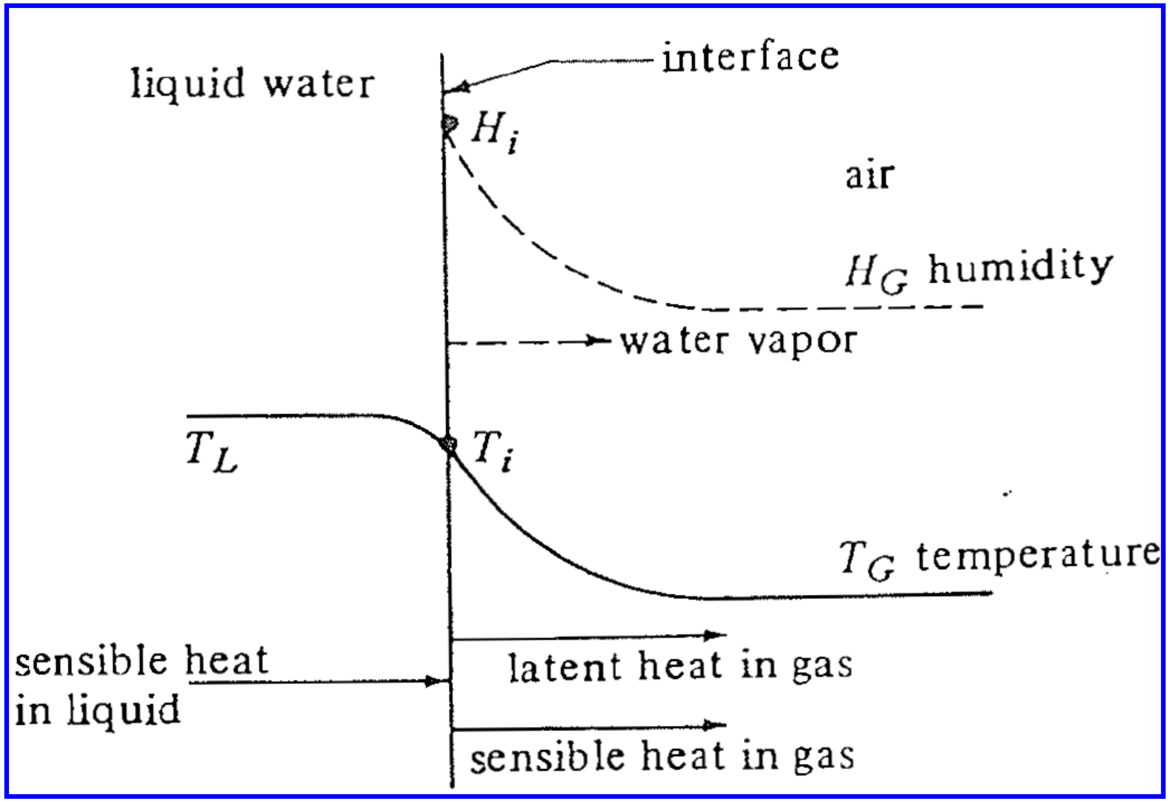
$$\lambda_{water}\dot{\mathrm{m}}_{\mathrm{evaprated}} = \dot{m}_{water,in} \times c_L(-\mathrm{Range})$$

$$\mathrm{Evapration\,loss}\,(\%) = 100\frac{\dot{\mathrm{m}}_{\mathrm{evaprated}}}{\dot{m}_{water,in}} = 100\frac{c_L(-\mathrm{Range})}{\lambda_{water}}$$

where c_L~4.187 kJ/(kg.K), λ_{water} ~2300 kJ/kg. For a Range of 20 °C, **the evaporation loss is around 2%.** This is small evaporation loss. Hence, the flow rate of water, and thereby, the flow rate of water, are assumed to be constant in calculation tower size.

Theory and calculation of water cooling towers

- Temperature and concentration profiles:



- **These profiles occur in the upper part of the cooling tower.**
- **In the lower part**, the bulk water temperature (T_L) my be below the dry bulb temperature (T_G). Then, the direction of sensible heat flow is reversed.

Theory and calculation of water cooling towers

- **Continuous countercurrent adiabatic water cooling:**

- Define the following:

L = water flow, kg water/s · m^2 (lb$_m$/h · ft^2)

T_L = temperature of water, °C or K (°F)

G = dry air flow, kg/s · m^2 (lb$_m$/h · ft^2)

T_G = temperature of air, °C or K (°F)

H = humidity of air, kg water/kg dry air

H_y = enthalpy of air–water vapor mixture, J/kg dry air

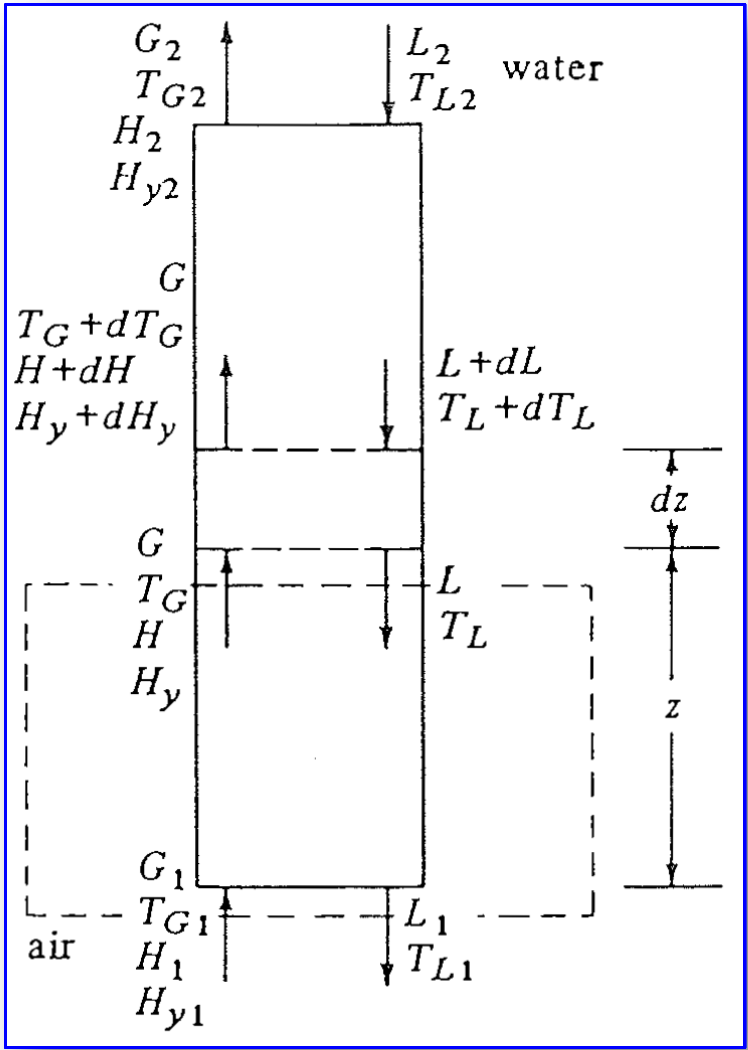
Theory and calculation of water cooling towers

▪ Continuous countercurrent adiabatic water cooling:

• Making steady-state heat balance over volume **dv =Adz** height column a gives:

$$dQ = AGdH_y = ALc_L dT_L$$

$$dq = \frac{dQ}{A} = GdH_y = Lc_L dT_L$$



where *A*: Cross-section area of the column, *dQ* is the rate of heat transferred from hot water to cold water-vapor phase over *dv* volume, and *dq* is the corresponding heat flux.

Theory and calculation of water cooling towers

▪ Continuous countercurrent adiabatic water cooling:

• We know from **mass transfer course,** that the flux, N_A, for species A (water vapor) diffusing through stagnant gas (air) is:

$$N_A = k_y(y_{A,i}\text{-}y_{A,G}) = k_G(P_{A,i}\text{-}P_{A,G})$$

$$k_y = k_G P$$

where k_G is gas-phase film mass transfer coefficient in kgmol/(s.m^2.Pa), $P_{A,i}$ and $P_{A,G}$ is the water vapor partial pressure at the interface and in the bulk gas-phase, respectively. While y is water vapor mole fraction.

• The **mass-transfer interfacial area** between air and water droplets **is not known**.

• This film mass-transfer interfacial area is different from the surface area of packing. Here, **a quantity (a_M)**, defined as interfacial area per volume of packing section, is combined with the gas-phase mass transfer coefficient, k_G , to give a **volumetric film mass transfer coefficients** defined as (**k_G a_M**) in kgmol/(s.m^3.Pa).

Theory and calculation of water cooling towers

▪ Continuous countercurrent adiabatic water cooling:

- Now the volumetric diffusion rate of water vapor, $N_{A,vol}$ is:

$$N_{A,vol} = k_y a_M (y_{A,i} - y_{A,G}) = k_G a_M (P_{A,i} - P_{A,G})$$

- **The relationship between humidity and mole fraction is:**

$$y = \frac{H / M_A}{1/M_B + H / M_A}$$

where M_A and M_B is the molecular weight of water vapor and air, respectively.

- Since H is small, an approximation of the relationship is:

$$y \cong \frac{M_B H}{M_A}$$

$$N_{A,vol} = k_y a_M (y_{A,i} - y_{A,G})$$

$$\longrightarrow \quad N_{A,vol} = \frac{M_B}{M_A} k_y a_M (H_i - H_G)$$

Theory and calculation of water cooling towers

- **Continuous countercurrent adiabatic water cooling:**

  - The latent heat in the water vapor being transferred over volume dv=Adz height column is:

$$dQ_\lambda = N_{A,vol}\lambda_0 M_A dV$$

$$dV = dzA$$

$$N_{A,vol} = M_B k_y a_M (H_i\text{-}H_G)/M_A$$

$$dQ_\lambda = M_B k_y a_M (H_i\text{-}H_G)\lambda_0 A dz$$

Theory and calculation of water cooling towers

▪ Continuous countercurrent adiabatic water cooling:

• **In analogy to mass transfer**, the **volumetric convective heat-transfer rates in the gas and liquid phases** are:

$$Q_{vol,L} = h_L a_{H,L} (T_L - T_i) \qquad Q_{vol,G} = h_G a_{H,G} (T_i - T_G)$$

where:

$Q_{vol,L}$ and $Q_{vol,G}$: volumetric convective heat transfer rates in the gas- and liquid phase, respectively, in W/m^3.

$a_{H,L}$ and $a_{H,G}$: liquid-gas phase heat-transfer interfacial area per volume of packing section in m^2/m^3, respectively

$a_{H,L} h_L$ and $a_{H,G} h_G$: liquid- and gas phase volumetric heat transfer coefficient in $W/(m^3.^oC)$.

Theory and calculation of water cooling towers

▪ Continuous countercurrent adiabatic water cooling:

- The **convective heat transfer rate in gas phase** over volume dv =Adz is:

$$dQ_{conv} = Q_{vol,G} dv$$

$$Q_{vol,G} = h_G a_{H,G} (T_i - T_G)$$

$$dV = dzA$$

$$dQ_{conv} = h_G a_{H,G} (T_i - T_G) dzA$$

- and the total heat transfer in the gas phase over dv volume is:

$$dQ = dQ_{\lambda} + dQ_{conv}$$

$$dQ_{\lambda} = M_B k_y a_M (H_i - H_G) \lambda_0 A dz$$

$$dQ_{conv} = h_G a_{H,G} (T_i - T_G) dzA$$

$$dq = \frac{dQ}{A} = \left[M_B k_y a_M (H_i - H_G) \lambda_0 + h_G a_{H,G} (T_i - T_G) \right] dz$$

Theory and calculation of water cooling towers

▪ Continuous countercurrent adiabatic water cooling:

• It is found that for water vapor-air mixture the experimental value of $(h_G a_{H,G} / M_B k_y a_M)$ which is called the **psychrometric ratio** is closed to **humid heat** c_S:

$$c_S \cong \frac{h_G a_{H,G}}{M_B k_y a_M} \xrightarrow{k_y = k_G P} c_S \cong \frac{h_G a_{H,G}}{M_B P k_G a_M}$$ **(Lewis relation)**

• Using the above Lewis relation:

$$dq = \left[M_B k_y a_M (H_i - H_G)\lambda_0 + h_G a_{H,G}(T_i - T_G) \right] dz$$

$$c_S \cong \frac{h_G a_{H,G}}{M_B P k_G a_M}$$

$$dq = M_B P k_G a_M \left[H_i \lambda_0 + c_S T_i - (c_S T_G + \lambda_0 \mathrm{H}_G) \right] dz$$

Theory and calculation of water cooling towers

- **Continuous countercurrent adiabatic water cooling:**

$$dq = M_B P k_G a_M \left[H_i \lambda_0 + c_S T_i - (c_S T_G + \lambda_0 \mathrm{H_G})\right] dz$$

- **adding and subtracting $c_S T_0$ inside the bracket of the above Eq.:**

$$dq = M_B P k_G a_M \left[c_S (T_i - T_0) + H_i \lambda_0 - (c_S (T_G - T_0) + \lambda_0 H_G)\right] dz$$

$$H_y = c_S (T_G - T_0) + \lambda_0 H_G$$ Enthalpy of water vapor-air mixture at T_G

$$H_{yi} = c_S (T_i - T_0) + H_i \lambda_0$$ Enthalpy of water vapor-air mixture at T_i

$$dq = M_B P k_G a_M \left[H_{yi} - H_y)\right] dz$$

$$dq = G dH_y$$

$$Z = \frac{G}{M_B P k_G a_M} \int_{H_{y1}}^{H_{y2}} \frac{dHy}{H_{yi} - H_y}$$

Design Eq. of the cooling tower

Theory and calculation of water cooling towers

- **Design equation of the continuous countercurrent adiabatic water cooling:**

$$Z = \underbrace{\frac{G}{M_B P k_G a_M}}_{\text{HTU}} \underbrace{\int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y}}_{\text{NTU}} \equiv (\text{HTU})(\text{NTU})$$

$$\text{HTU} \equiv \text{Height of a transfer unit}$$

$$\text{NTU} \equiv \text{Number of transfer units}$$

- **The enthalpy, H_{yi}, at the interface temperature T_i is determined from:**

$$dq = M_B P k_G a_M \left[H_{yi} - H_y \right) \right] dz$$

$$dq = h_L a_{H,L} (T_L \text{-} T_i) dz$$

$$\frac{H_{yi} - H_y}{T_i \text{-} T_L} = -\frac{h_L a_{H,L}}{M_B P k_G a_M}$$

Theory and calculation of water cooling towers

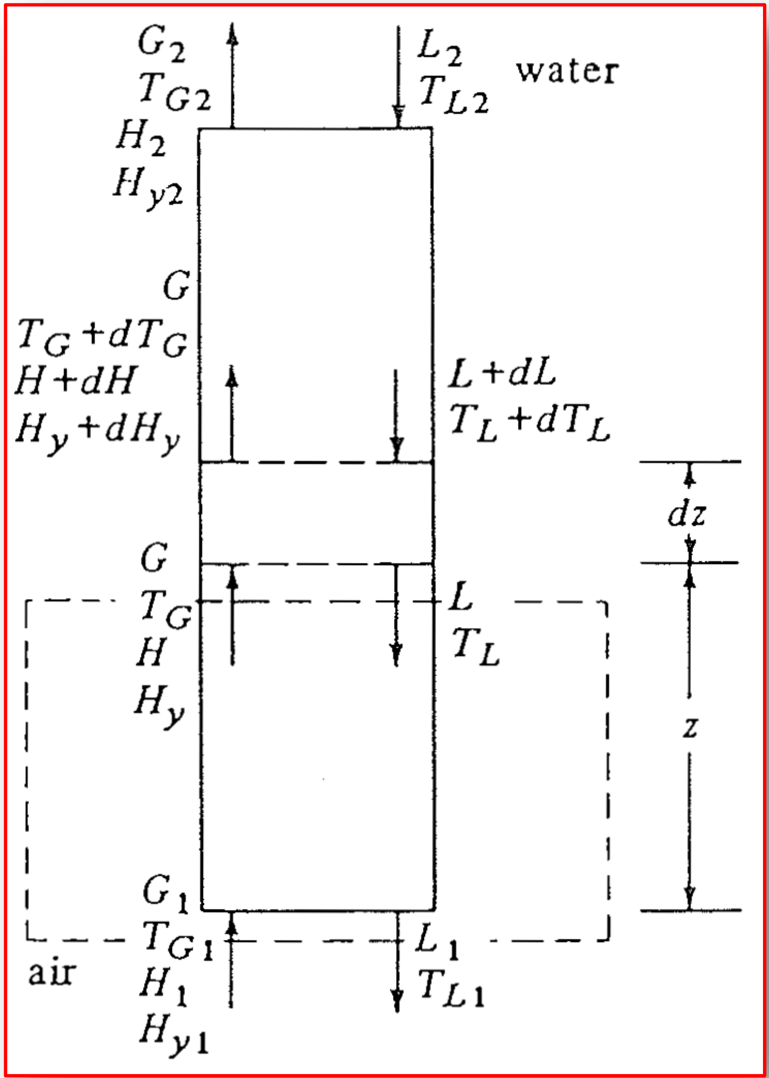
▪ Continuous countercurrent adiabatic water cooling:

- Making steady-state total heat balance for dashed line box:

$$G(H_y - H_{y1}) = Lc_L(T_L - T_{L1})$$

- Note that the liquid water heat capacity is constant (c_L~4.187 kJ/[kg.K]) and the evaporation loss is small enough to have constant values of L and G.
- Rearrange the above Eq. to have the following **operating line Eq**.:

$$H_y = (H_{y1} - T_{L1}\frac{Lc_L}{G}) + \frac{Lc_L}{G}T_L$$



Theory and calculation of water cooling towers

- **Operating line equation:**

$$H_y = (H_{y1} - T_{L1}\frac{Lc_L}{G}) + \frac{Lc_L}{G}T_L$$

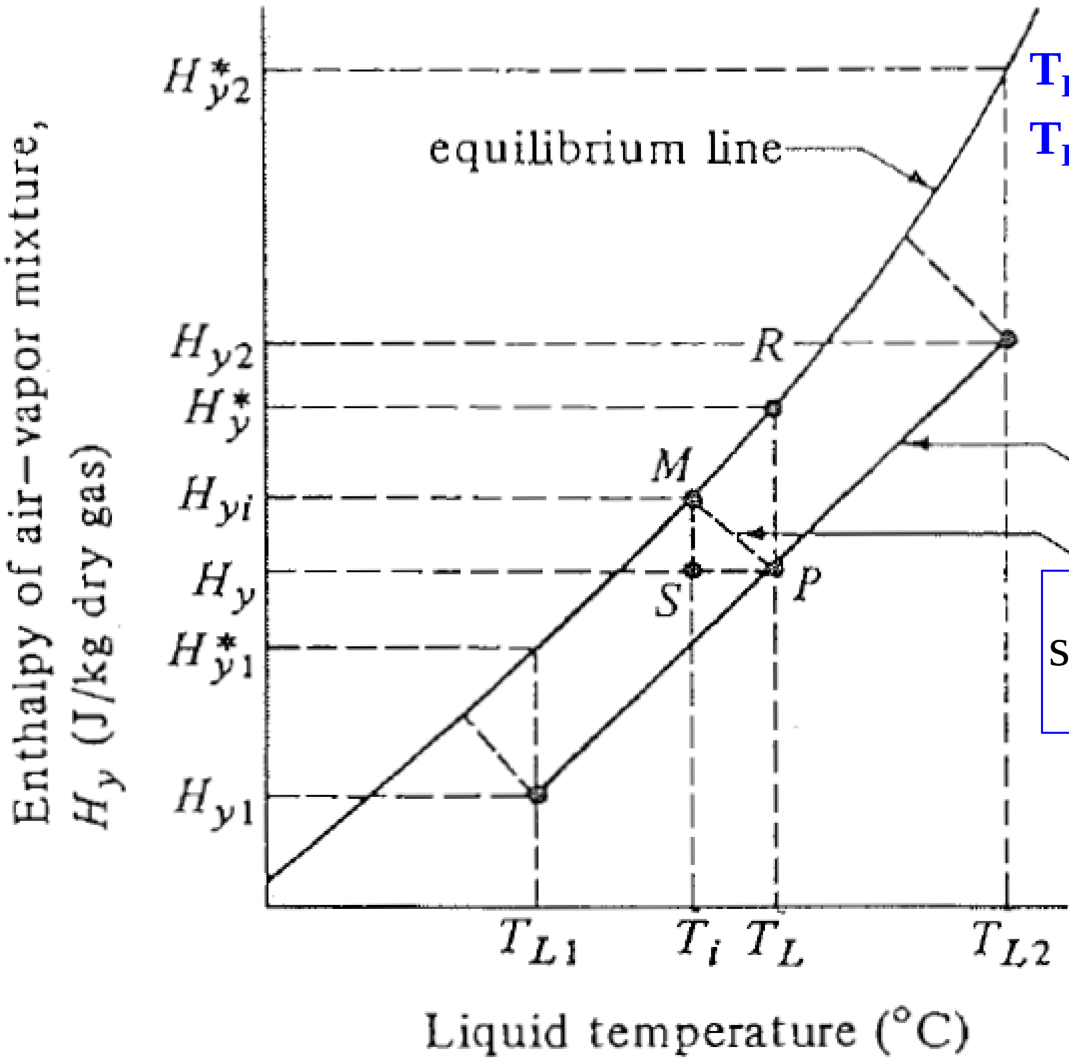
$$\text{Intercept} = H_{y1} - \text{T}_{L1}\frac{Lc_L}{G} \quad ; \quad \text{Slope} = \frac{Lc_L}{G}$$

• Also, remember that we can make steady-state heat balance over the entire cooling tower to have:

$$G(H_{y2} - H_{y1}) = Lc_L(T_{L2} - T_{L1})$$

Theory and calculation of water cooling towers

- **Continuous countercurrent adiabatic water cooling:**



T_{L1}: at the bottom of the tower; z=0

T_{L2}: at the top of the tower; z=Z

Operating line with:

$$\text{slope} = \frac{Lc_L}{G}$$

$$\text{slope} = \frac{H_{yi} - H_y}{T_i - T_L} = -\frac{h_L a_{H,L}}{M_B P k_G a_M}$$

Theory and calculation of water cooling towers

▪ Design of water cooling tower using film mass transfer coefficients:

1. Draw the equilibrium curve: the enthalpy of saturated air versus the dew point temperature T_L using:

$$H_{yi} = c_S(T_L - T_0) + H_i\lambda_0$$

where the T_0 is the base temperature:

T_0 =0 °C : λ_0=2502.3 kJ/kg water

T_0 =32 °F : λ_0=1075.8 Btu/lbm water

$$\begin{aligned} c_S &= 1.005 + 1.88H_i \text{ ; kJ/(kg dry air.K)} \\ &= 0.24 + 0.45H_i \text{ ; btu/(lbm dry air.}^\circ\text{F)} \end{aligned}$$

H_i is the saturated humidity picked up from the psychrometric chart at T_L.

Theory and calculation of water cooling towers

- **Design of water cooling tower using film mass transfer coefficients:**

TABLE 10.5-1. *Enthalpies of Saturated Air–Water Vapor Mixtures (0°C Base Temperature)*

T_L		H_y		T_L		H_y	
		btu	J			btu	J
°F	°C	lb_m dry air	kg dry air	°F	°C	lb_m dry air	kg dry air
60	15.6	18.78	43.68×10^3	100	37.8	63.7	148.2×10^3
80	26.7	36.1	84.0×10^3	105	40.6	74.0	172.1×10^3
85	29.4	41.8	97.2×10^3	110	43.3	84.8	197.2×10^3
90	32.2	48.2	112.1×10^3	115	46.1	96.5	224.5×10^3
95	35.0	55.4	128.9×10^3	140	60.0	198.4	461.5×10^3

Theory and calculation of water cooling towers

- **Design of water cooling tower using film mass transfer coefficients:**

2. Draw the operating line:

- Use the operating line equation

$$H_y = (H_{y1} - T_{L1}\frac{Lc_L}{G}) + \frac{Lc_L}{G}T_L$$

and/or the overall steady-state heat balance over the entire cooling tower:

$$G(H_{y2} - H_{y1}) = Lc_L(T_{L2} - T_{L1})$$

H_{y1} and H_{y2} is the gas mixture enthalpy at T_{G1} and T_{G2} , respectively.

→ To draw the operating line we need either two points or one point and slope (Lc_L/G).

Theory and calculation of water cooling towers

- **Design of water cooling tower using film mass transfer coefficients:**

3. Draw lines with slope:

$$\frac{H_{yi}-H_{y}}{T_i\text{-}T_L}=-\frac{h_L a_{H,L}}{M_B P k_G a_M}=\text{Slope}=\frac{H_{yi1}-H_{y1}}{T_{i1}\text{-}T_{L1}}=\frac{H_{yi2}-H_{y2}}{T_{i2}\text{-}T_{L2}}$$

- Select some value of T_i and read H_{yi} from the equilibrium curve.
- Select some value of T_L and calculate H_y from the above equation.
- Draw a line pass through the points (T_i, H_{yi}) and (T_L, H_y) this line must have slope of $h_L a_{H,L}/(M_B P k_G a_M)$.
- At 6 to 8 locations, draw parallel lines (slope= $h_L a_{H,L}/(M_B P k_G a_M)$) from T_{L1} to T_{L2} to read enthalpies H_{yi} from equilibrium curve.

Theory and calculation of water cooling towers

4. Calculate the number of transfer units (NTU):

- Use Enthalpy vs. T_L graph to find the driving force H_{yi}-H_y for various T_L value from T_{L1} to T_{L2}.
- Calculate $1/(H_{yi}-H_y)$ for various T_L value from T_{L1} to T_{L2}.
- Perform graphical or numerical integration to calculate NTU:

$$\text{NTU} = \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y}$$

5. Calculate the height of a transfer unit umber of transfer units (HTU):

$$\text{HTU} = \frac{G}{M_B P k_G a_M}$$

6. Calculate the height of the cooling tower: $Z = (\text{HTU})(\text{NTU})$

Theory and calculation of water cooling towers

Example. A packed countercurrent water-cooling tower using gas flow rate of 1.356 kg dry air/(s.m^2) and water flow rate of 1.356 kg dry air/(s.m^2). The water is cooled from 43.3 to 29.4 ^{o}C. The entering air at 29.4 ^{o}C has a wet bulb temperature of 23.9 ^{o}C. The film mass-transfer coefficient is estimated as 1.207×10^{-7} kgmol/(s.m^3.Pa). $h_L a_{H,L}/(M_B P k_G a_M)$ has a value of 41.87 kJ/(kg.K)The tower operates at 1 atm. **Calculate the Range, The approach, the tower effectiveness, and the height of the packed tower.**

Solution: $G = L = 1.356 \text{ kg/(s.m}^2)$ $T_{L1} = 29.4\,^oC$ $T_{L2} = 43.3\,^oC$

$T_{G1} = 29.4\,^oC; T_{WB1} = 23.9\,^oC$ $k_G a_M = 1.207\times10^{-7} \text{ kmol/(s.m}^3)$

$$\text{Range} = T_{L2} - T_{L1} = 43.3 - 29.4 = 13.9\,^oC$$

$$\text{Approach} = T_{L1} - T_{WB,1} = 29.4 - 23.9 = 5.5\,^oC$$

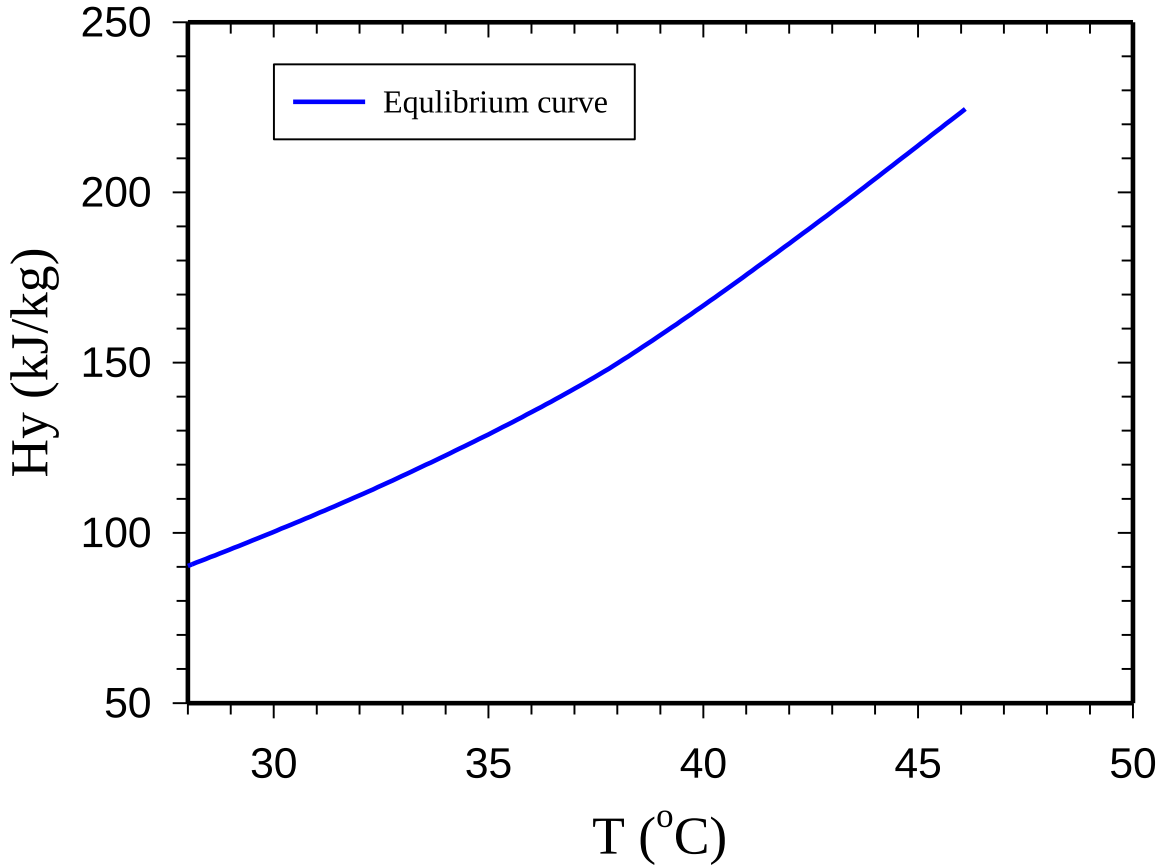
$$\text{Effectivness} = 100\times\text{Range}/0\,(\text{Range} + \text{Approach}) = 71.6\%$$

Theory and calculation of water cooling towers

Height of the packed tower.

1. **Draw the equilibrium curve:** use saturated humidity curve in the psychrometric chart and enthalpy Eq. to get:

T_L	H_{yi}, kJ/kg
15.6	43.7
26.7	84.0
29.4	97.2
32.2	112.1
35.0	128.9
37.8	148.2
40.6	172.1
43.3	197.2
46.1	224.5

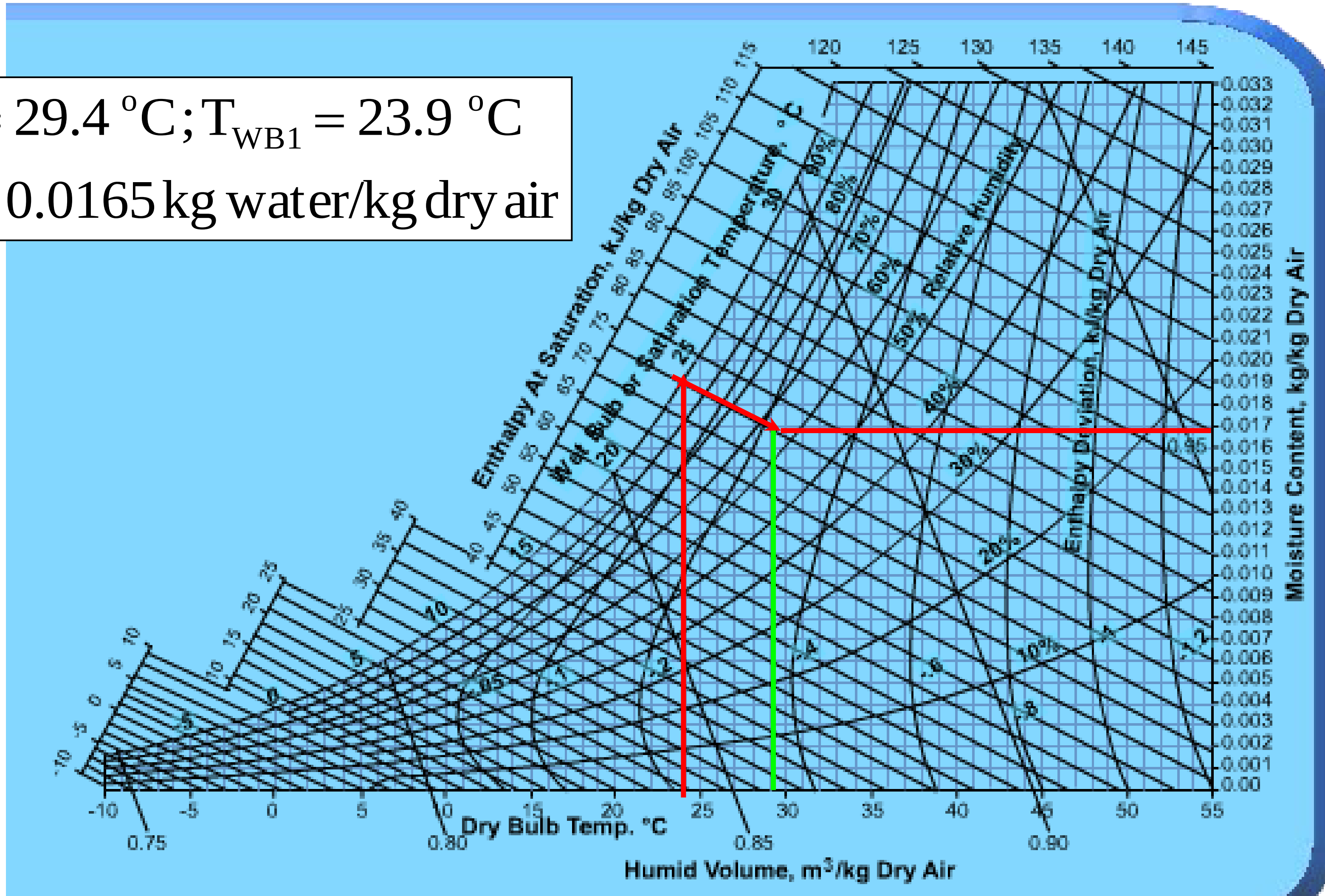


Theory and calculation of water cooling towers

2. Draw the operating line:

$$T_{G1} = 29.4\ ^{o}\mathrm{C};\ \mathrm{T_{WB1}} = 23.9\ ^{o}\mathrm{C}$$

$$\mathrm{H_1} = 0.0165\ \mathrm{kg\ water/kg\ dry\ air}$$



Theory and calculation of water cooling towers

2. Draw the operating line:

$$c_S = 1.005 + 1.88H_1 = 1.005 + 1.88(0.0165) = 1.036 \text{ kJ/(kg dry air.K)}$$

$$H_{y1} = c_S(T_{G1} - T_0) + H_i\lambda_0 = 1.036(29.4 - 0) + (0.0165)(2502.3) = 71.7 \text{ kJ/kg}$$

Apply overall steady-state heat balance over the entire cooling to get H_{y2}:

$$G(H_{y2} - H_{y1}) = Lc_L(T_{L2} - T_{L1})$$

$$G = L = 1.356 \text{ kg/(s.m}^2\text{)}$$

$$c_L = 4.187 \text{ kJ/(kg.K)}$$

$$T_{L1} = 29.4\,^{o}\text{C}$$

$$T_{L2} = 43.3\,^{o}\text{C}$$

$$H_{y1} = 71.7 \text{ kJ/kg}$$

$$H_{y2} = 129.9 \text{ kJ/kg}$$

We have two points enough to draw the operating line:

$$(T_{L1}, H_{y1}) = (29.4\,^{o}\text{C}, 71.7 \text{ kJ/kg})$$

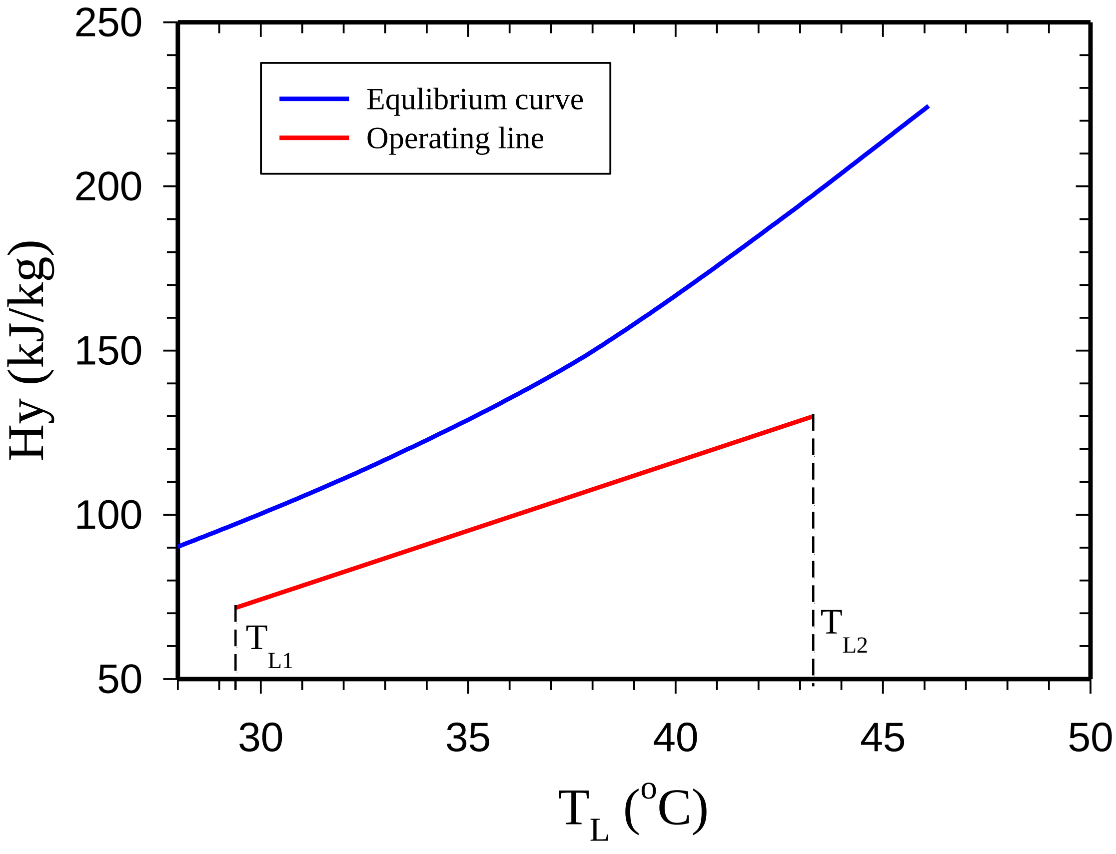
$$(T_{L2}, H_{y2}) = (43.3\,^{o}\text{C}, 129.9 \text{ kJ/kg})$$

Theory and calculation of water cooling towers

2. Draw the operating line:

$$(T_{L1}, H_{y1}) = (29.4\,^{o}C, 71.7\,kJ/kg)$$

$$(T_{L2}, H_{y2}) = (43.3\,^{o}C, 129.9\,kJ/kg)$$



Theory and calculation of water cooling towers

3. Draw lines with constant slope:

- For example, at T_i =35 °C, from the equilibrium curve H_{yi} =128.9 kJ/kg .
- at T_L= 36 °C, calculate H_y from:

$$\frac{H_{yi}-H_y}{T_i\text{-}T_L}=\frac{128.9-H_y}{35\text{-}36}=-\frac{h_L a_{H,L}}{M_B P k_G a_M}=-41.87\,\text{kJ/(kg.K)}$$

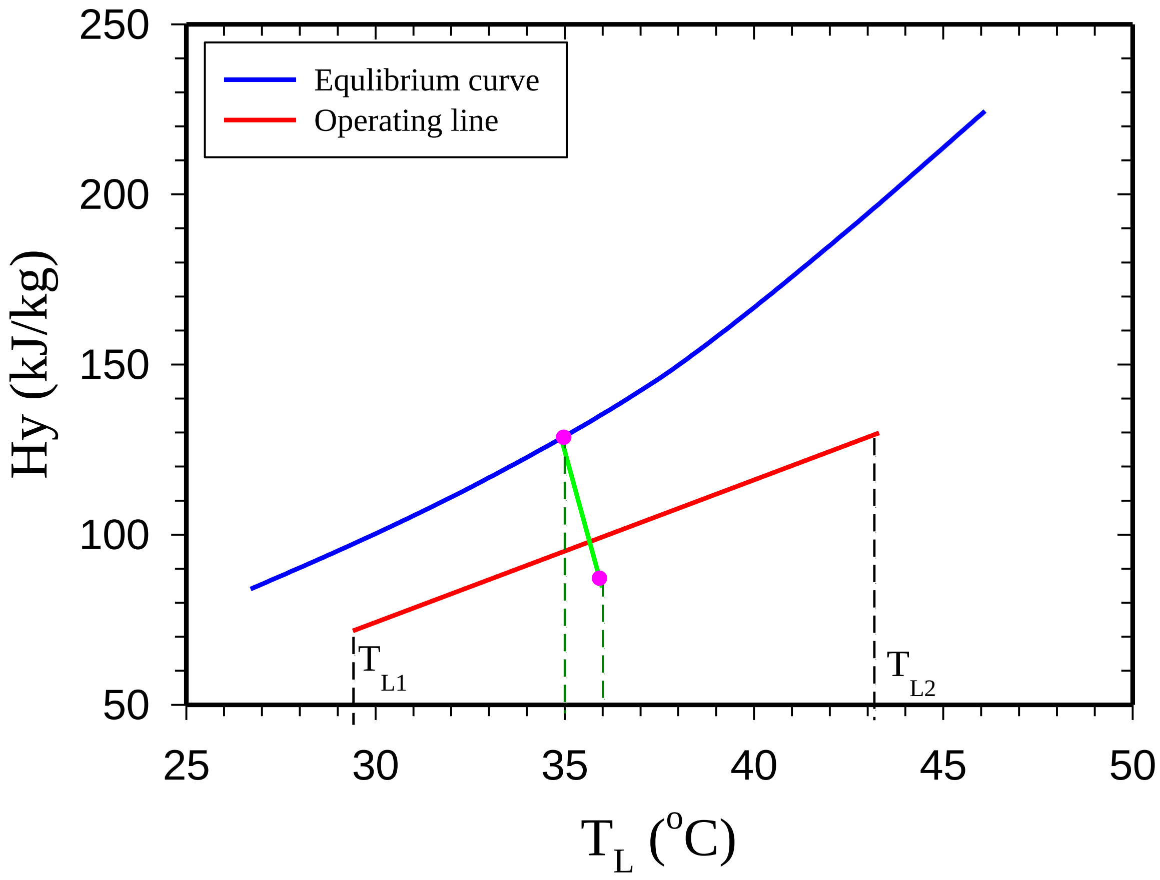
$$\downarrow$$

$$H_y = 87.03\,\text{kJ/kg}$$

- Draw a line passes through the points (35 °C, 128.9 kJ/kg) and (36 °C, 87.03 kJ).

Theory and calculation of water cooling towers

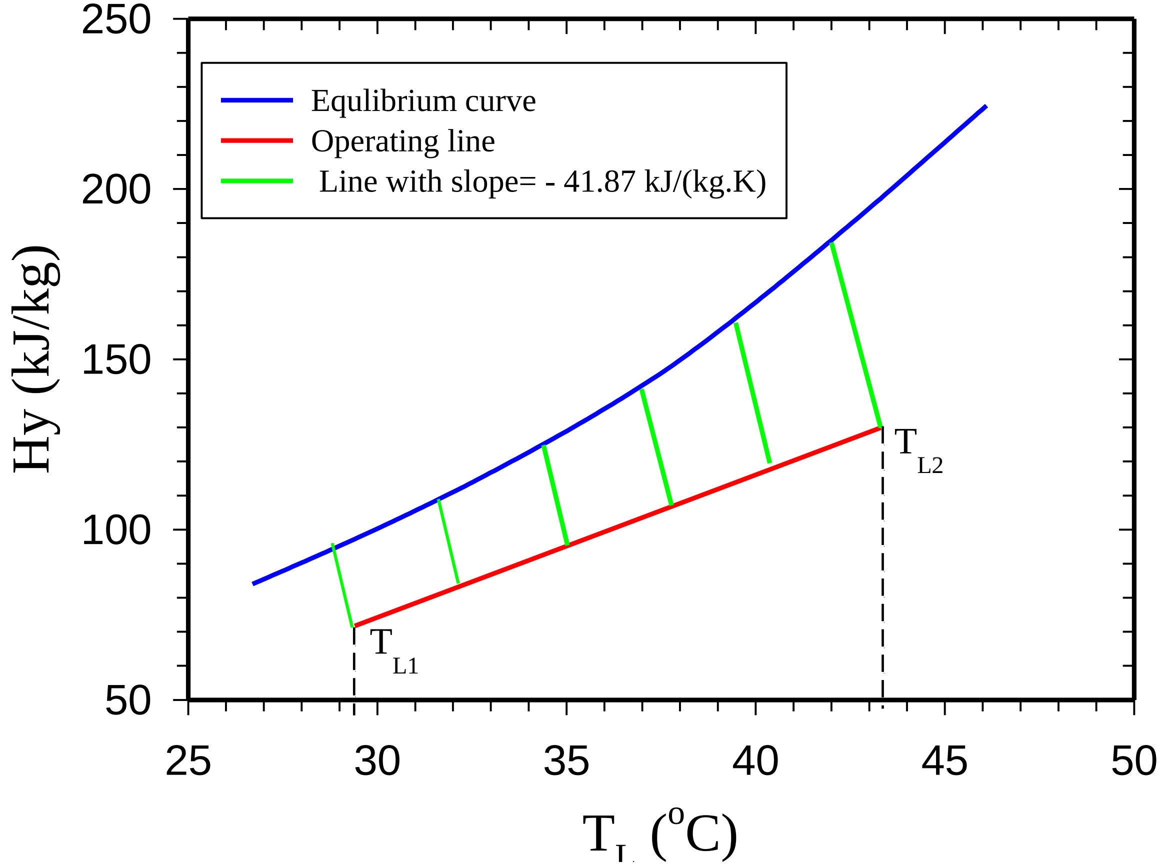
3. Draw lines with constant slope:



Theory and calculation of water cooling towers

4. Calculate the number of transfer units (NTU):

- At 6 to 8 locations, draw parallel lines as shown below from T_{L1} to T_{L2} to read enthalpies H_{yi} from equilibrium curve



Theory and calculation of water cooling towers

4. Calculate the number of transfer units (NTU):

H_{yi} (kJ/kg)	H_y (kJ/kg)	H_{yi}-H_y (kJ/kg)	1/(H_{yi} - H_y); (kg/kJ)
94.4	71.7	22.7	0.0441
108.4	83.5	24.9	0.0402
124.4	94.9	29.5	0.0339
141.8	106.5	35.3	0.0283
162.1	118.4	43.7	0.0229
184.7	129.9	54.8	0.0182

- Using Trapezoidal rule of numerical integration:

$$\mathrm{NTU} = \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y} \cong 1.82$$

Theory and calculation of water cooling towers

5. Calculate the height of a transfer unit (HTU):

$$\mathrm{HTU} = \frac{G}{M_B P k_G a_M} = \frac{1.356}{(29)(101325)(1.207\times10^{-7})} = 3.82\,\mathrm{m}$$

6. Calculate the height of the cooling tower:

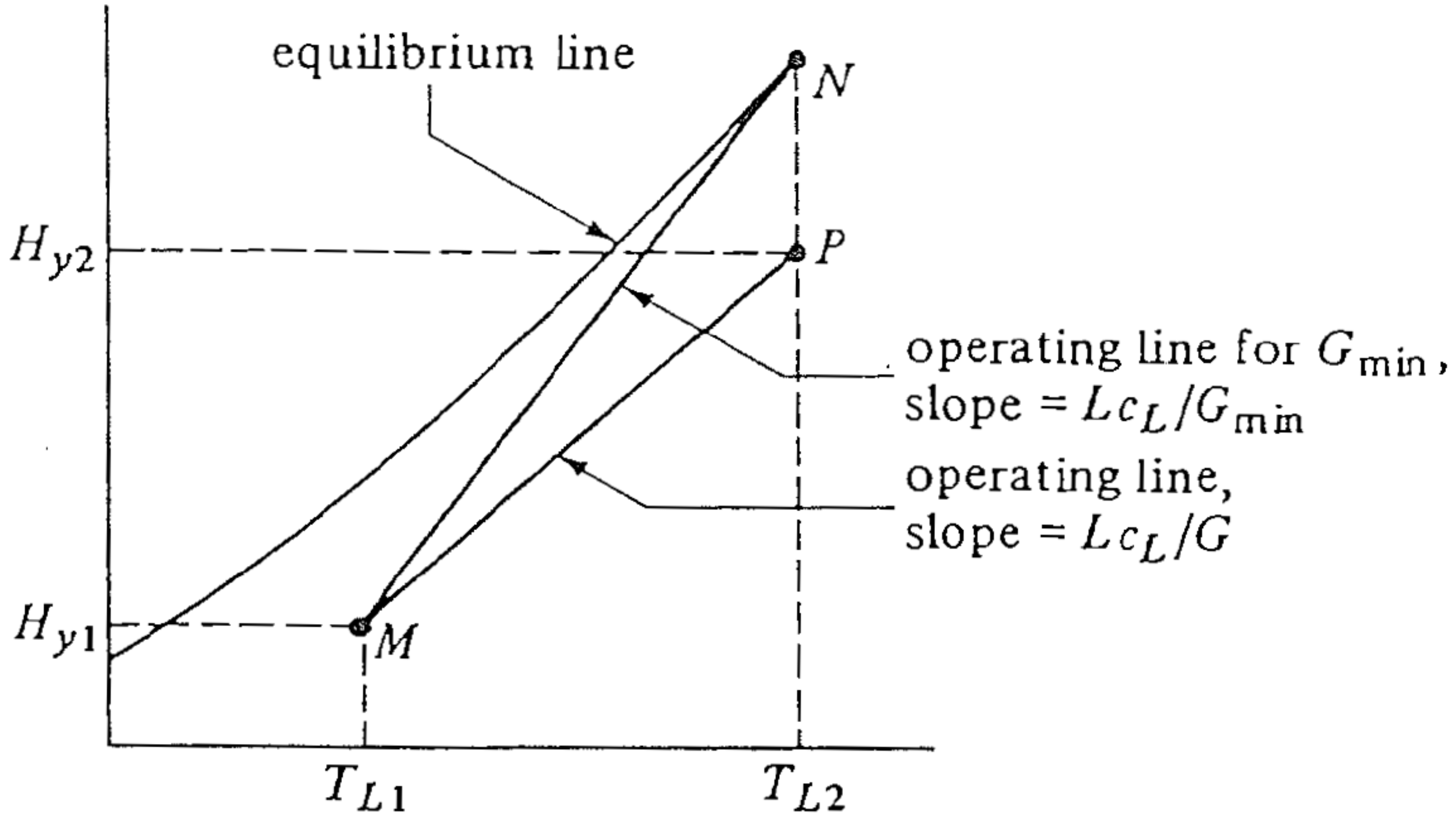
$$\mathrm{Z} = (\mathrm{HTU})(\mathrm{NTU}) = (3.82)(1.82) = 6.96\,\mathrm{m}$$

Theory and calculation of water cooling towers

▪ Minimum value of air flow G_{min}:

- Minimum air flow gives maximum slope of the operating line Eq.

$$H_y = (H_{y1} - T_{L1}\frac{Lc_L}{G}) + \frac{Lc_L}{G}T_L \longrightarrow Slope_{max} = \frac{Lc_L}{G\min}$$



Theory and calculation of water cooling towers

- **Minimum value of air flow G_{min}:**

- For actual cooling towers, a value of air flow rate greater than G_{min} must be used. **A reasonable value** of G is $(1.3\text{-}1.5)\times G_{min}$.

Example. Find the minimum air flow for previous example

$$\text{Slope}_{max} = \frac{H_{y2} - H_{y1}}{T_{L2} - T_{L1}}$$

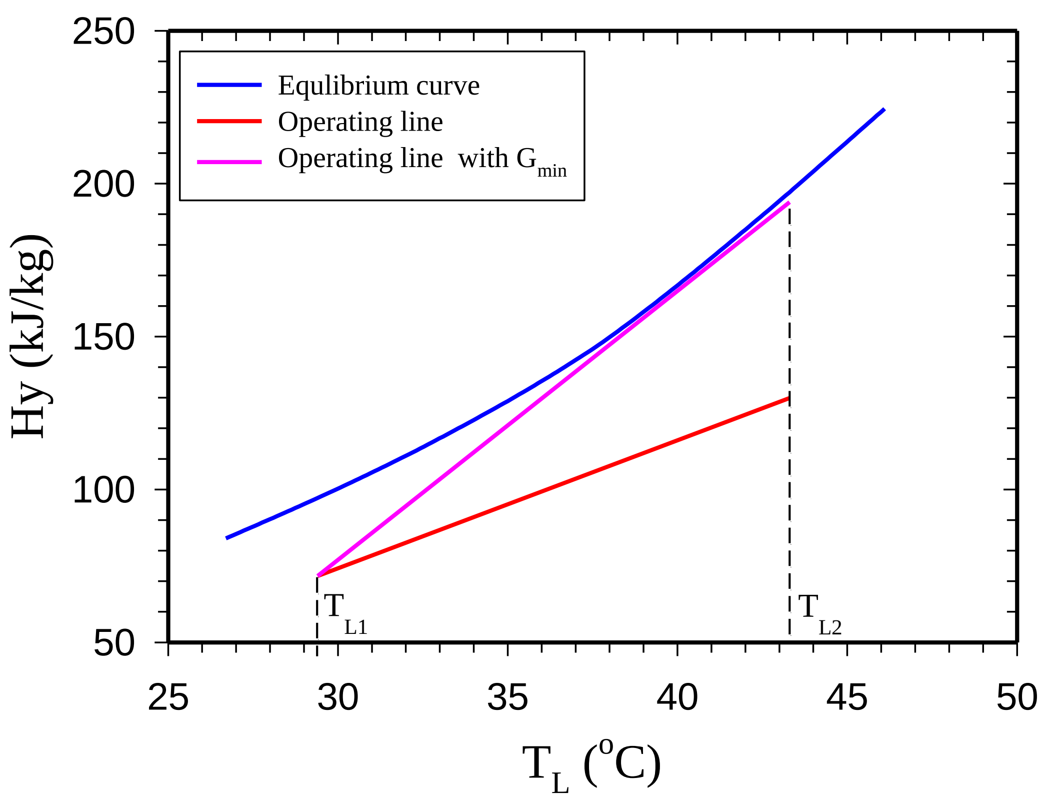
$$= \frac{194 - 71.7}{43.3 - 29.4}$$

$$= 8.8\,\text{kJ/(kg.K)}$$

$$\text{Slope}_{max} = \frac{Lc_L}{G_{min}}$$

$$G_{min} = \frac{Lc_L}{\text{Slope}_{max}}$$

$$= 0.64\,kg/(s.m^2)$$



Theory and calculation of water cooling towers

- **Design of water cooling tower using overall mass-transfer coefficients:**
  - Usually, the **overall volumetric mass-transfer coefficient** $K_G a_M$ is available and the design equation becomes:

$$Z = \underbrace{\frac{G}{M_B P K_G a_M}}_{\text{HTU}} \underbrace{\int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_y^* - H_y}}_{\text{NTU}} \equiv (\text{HTU})(\text{NTU})$$

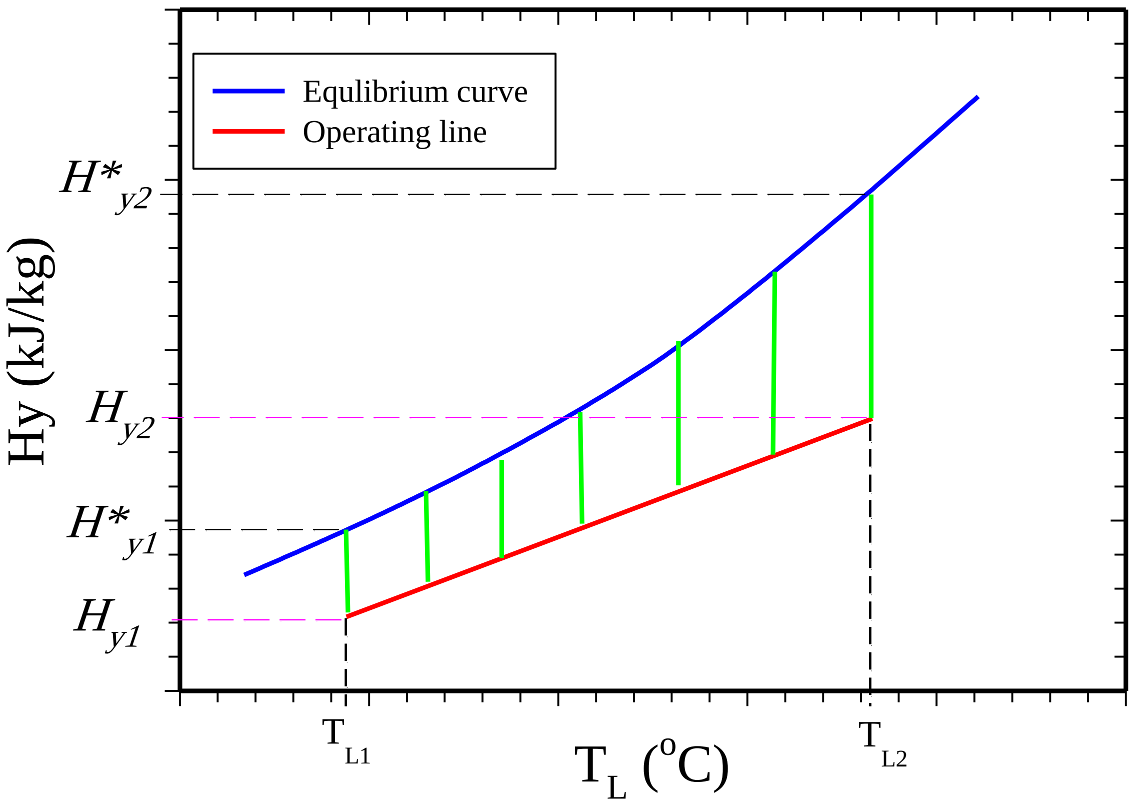
HTU ≡ Height of a transfer unit

NTU ≡ Number of transfer units

- Use the same procedure as in film mass-transfer coefficient except that the values of H^*_y is determined by going vertically from the point H_y on the operating line up to equilibrium curve.

Theory and calculation of water cooling towers

- **Design of water cooling tower using overall mass-transfer coefficients:**



- If the experimental cooling tower data with known height (Z) are available, then design equation can be used to determine the experimental value of the overall volumetric mass-transfer coefficient.

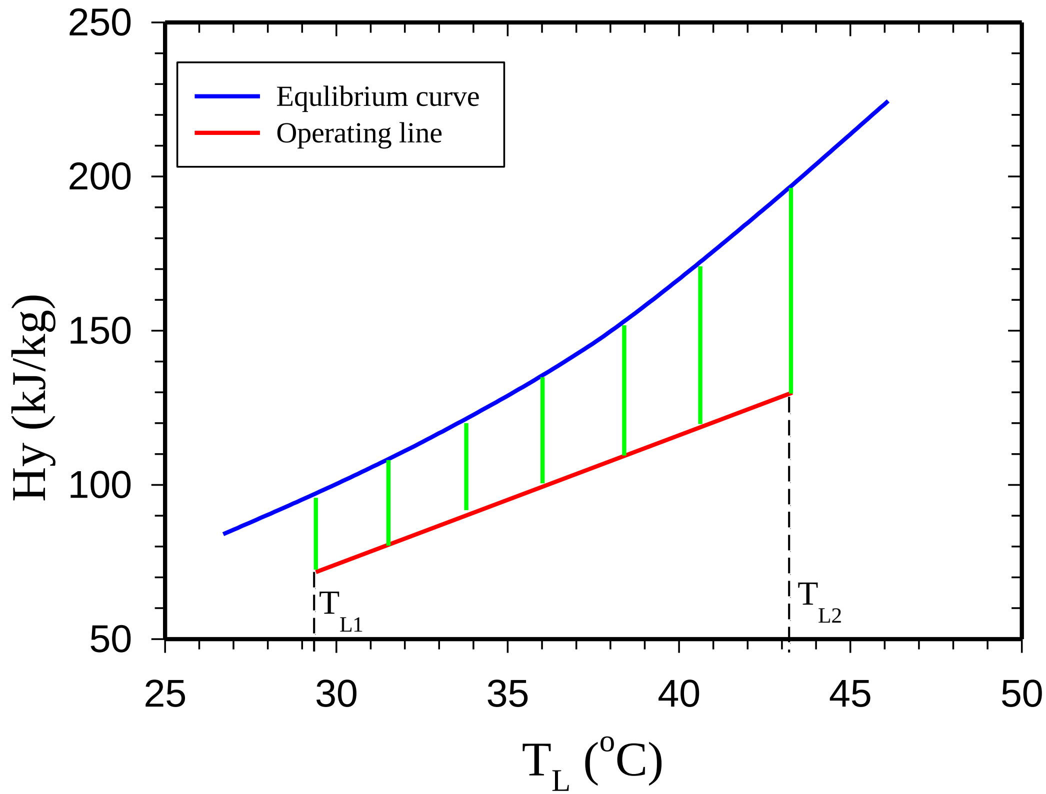
Theory and calculation of water cooling towers

- **Design of water cooling tower using overall mass-transfer coefficients:**

Example. Use the data and result of previous cooling tower example to estimate the overall mass-transfer coefficient $K_G a_M$.

Solution:

H_y (kJ/kg)	H^*_y (kJ/kg)
71.7	96.8
80.7	108.3
91.5	120.9
100.4	135.4
110.1	152.4
119.2	172.1
129.9	196.8



Theory and calculation of water cooling towers

- **Design of water cooling tower using overall mass-transfer coefficients:**

H_y (kJ/kg)	H^*_y (kJ/kg)	$H^*_y-H_y$ (kJ/kg)	$1/(H^*_y-H_y)$ (kg/kJ)
71.7	96.8	25.1	0.0398
80.7	108.3	27.6	0.0362
91.5	120.9	29.4	0.0340
100.4	135.4	35.0	0.0286
110.1	152.4	42.3	0.0236
119.2	172.1	52.9	0.0189
129.9	196.8	66.9	0.0149

Using Trapezoidal rule:

$$\mathrm{NTU} = \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_y^* - H_y} \cong 1.63$$

$$\mathrm{HTU} = \frac{Z}{\mathrm{NTU}} = \frac{6.96}{1.63} = 4.27\,\mathrm{m} = \frac{G}{M_B P (K_G a_M)}$$

$$K_G a_M = \frac{G}{(M_B)(P)(\mathrm{HTU})} = \frac{1.356}{(29)(101325)(4.27)} = 1.08\times10^{-7}\ \mathrm{kmol/(s.m^3.Pa)}$$

Dehumidification towers

- To reduce the humidity and temperature of air that enters.
- **For dehumidifier, the operating line must be above the equilibrium line.**
- Similar calculation methods are used.