Transport Processes and Unit Operations

CHAPTER 9

Drying of Process Materials

9.1 INTRODUCTION AND METHODS OF DRYING

9.1A Purposes of Drying

The discussions of drying in this chapter are concerned with the removal of water from process materials and other substances. The term *drying* is also used to refer to removal of other organic liquids, such as benzene or organic solvents, from solids. Many of the types of equipment and calculation methods discussed for removal of water can also be used for removal of organic liquids.

Drying, in general, usually means removal of relatively small amounts of water from material. Evaporation refers to removal of relatively large amounts of water from material. In evaporation the water is removed as vapor at its boiling point. In drying the water is usually removed as a vapor by air.

In some cases water may be removed mechanically from solid materials by presses, centrifuging, and other methods. This is cheaper than drying by thermal means for removal of water, which will be discussed here. The moisture content of the final dried product varies depending upon the type of product. Dried salt contains about 0.5% water, coal about 4%, and many food products about 5%. Drying is usually the final processing step before packaging and makes many materials, such as soap powders and dyestuffs, more suitable for handling.

Drying or dehydration of biological materials, especially foods, is used as a preservation technique. Microorganisms that cause food spoilage and decay cannot grow and multiply in the absence of water. Also, many enzymes that cause chemical changes in food and other biological materials cannot function without water. When the water content is reduced below about 10 wt %, the microorganisms are not active. However, it is usually necessary to lower the moisture content below 5 wt % in foods to preserve flavor and nutrition. Dried foods can be stored for extended periods of time.

Some biological materials and pharmaceuticals, which may not be heated for ordinary drying, may be freeze-dried as discussed in Section 9.11. Also, in Section 9.12, sterilization of foods and other biological materials is discussed, which is another method often employed to preserve such materials.

9.1B General Methods of Drying

Drying methods and processes can be classified in several different ways. Drying processes can be classified as *batch*, where the material is inserted into the drying equipment and drying proceeds for a given period of time, or as *continuous*, where the material is continuously added to the dryer and dried material continuously removed.

Drying processes can also be categorized according to the physical conditions used to add heat and remove water vapor: (1) in the first category, heat is added by direct contact with heated air at atmospheric pressure, and the water vapor formed is removed by the air; (2) in vacuum drying, the evaporation of water proceeds more rapidly at low pressures, and the heat is added indirectly by contact with a metal wall or by radiation (low temperatures can also be used under vacuum for certain materials that may discolor or decompose at higher temperatures); and (3) in freeze drying, water is sublimed from the frozen material.

9.2 EQUIPMENT FOR DRYING

9.2A Tray Dryer

In *tray dryers*, which are also called shelf, cabinet, or compartment dryers, the material, which may be a lumpy solid or a pasty solid, is spread uniformly on a metal tray to a depth of 10 to 100 mm. Such a typical tray dryer, shown in Fig. 9.2-1, contains removable trays loaded in a cabinet.

Steam-heated air is recirculated by a fan over and parallel to the surface of the trays. Electrical heat is also used, especially for low heating loads. About 10 to 20% of the air passing over the trays is fresh air, the remainder being recirculated air.

After drying, the cabinet is opened and the trays are replaced with a new batch of trays. A modification of this type is the tray–truck type, where trays are loaded on trucks which are pushed into the dryer. This saves considerable time, since the trucks can be loaded and unloaded outside the dryer.

In the case of granular materials, the material can be loaded on screens which are the bottom of each tray. Then in this through-circulation dryer, heated air passes through the permeable bed, giving shorter drying times because of the greater surface area exposed to the air.

9.2B Vacuum-Shelf Indirect Dryers

Vacuum-shelf dryers are indirectly heated batch dryers similar to tray dryers. Such a dryer consists of a cabinet made of cast-iron or steel plates fitted with tightly fitted doors

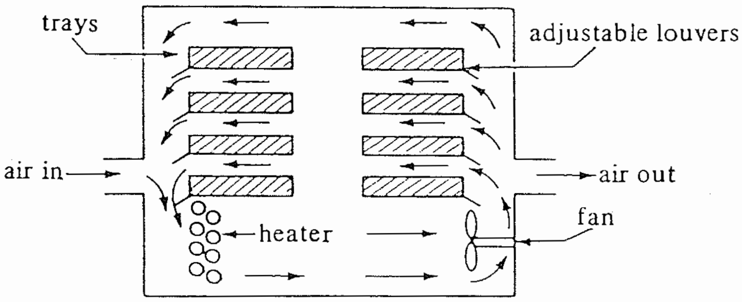


FIGURE 9.2-1. *Tray or shelf dryer.*

so that it can be operated under vacuum. Hollow shelves of steel are fastened permanently inside the chamber and are connected in parallel to inlet and outlet steam headers. The trays containing the solids to be dried rest upon the hollow shelves. The heat is conducted through the metal walls and added by radiation from the shelf above. For low-temperature operation, circulating warm water is used instead of steam for furnishing the heat to vaporize the moisture. The vapors usually pass to a condenser.

These dryers are used to dry expensive, or temperature-sensitive, or easily oxidizable materials. They are useful for handling materials with toxic or valuable solvents.

9.2C Continuous Tunnel Dryers

Continuous tunnel dryers are often batch truck or tray compartments operated in series, as shown in Fig. 9.2-2a. The solids are placed on trays or on trucks which move continuously through a tunnel with hot gases passing over the surface of each tray. The hot air flow can be countercurrent, cocurrent, or a combination. Many foods are dried in this way.

When granular particles of solids are to be dried, perforated or screen-belt continuous conveyors are often used, as in Fig. 9.2-2b. The wet granular solids are conveyed as a layer 25 to about 150 mm deep on a screen or perforated apron while heated air is blown upward through the bed, or downward. The dryer consists of several sections in series, each with a fan and heating coils. A portion of the air is exhausted to the atmosphere by a fan. In some cases pasty materials can be preformed into cylinders and placed on the bed to be dried.

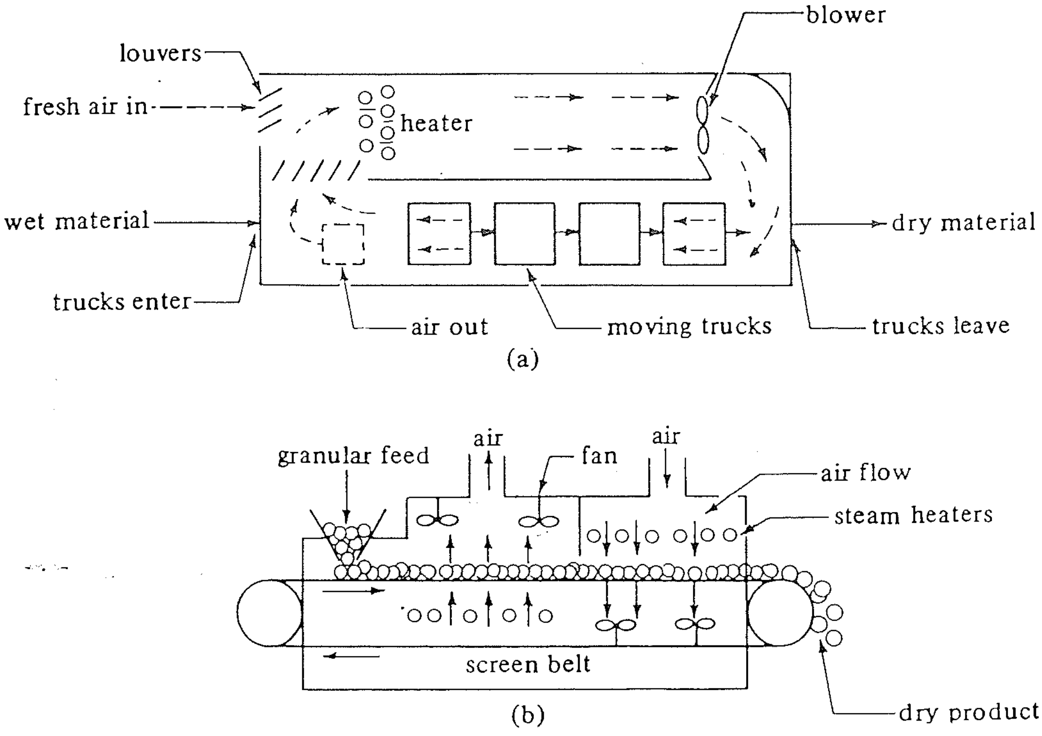


FIGURE 9.2-2. *Continuous tunnel dryers: (a) tunnel dryer trucks with countercurrent air flow, (b) through-circulation screen conveyor dryer.*

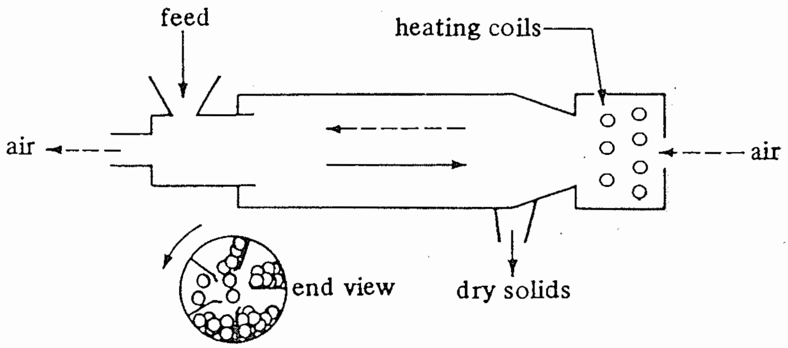


FIGURE 9.2-3. *Schematic drawing of a direct-heat rotary dryer.*

9.2D Rotary Dryers

A *rotary dryer* consists of a hollow cylinder which is rotated and usually slightly inclined toward the outlet. The wet granular solids are fed at the high end as shown in Fig. 9.2-3 and move through the shell as it rotates. The heating shown is by direct contact with hot gases in countercurrent flow. In some cases the heating is by indirect contact through the heated wall of the cylinder.

The granular particles move forward slowly a short distance before they are showered downward through the hot gases as shown. Many other variations of this rotary dryer are available, and these are discussed elsewhere (P1).

9.2E Drum Dryers

A *drum dryer* consists of a heated metal roll shown in Fig. 9.2-4, on the outside of which a thin layer of liquid or slurry is evaporated to dryness. The final dry solid is scraped off the roll, which is revolving slowly.

Drum dryers are suitable for handling slurries or pastes of solids in fine suspension and for solutions. The drum functions partly as an evaporator and also as a dryer. Other variations of the single-drum type are twin rotating drums with dip feeding or with top feeding to the two drums. Potato slurry is dried using drum dryers, to give potato flakes.

9.2F Spray Dryers

In a *spray dryer* a liquid or slurry solution is sprayed into a hot gas stream in the form of a mist of fine droplets. The water is rapidly vaporized from the droplets, leaving particles

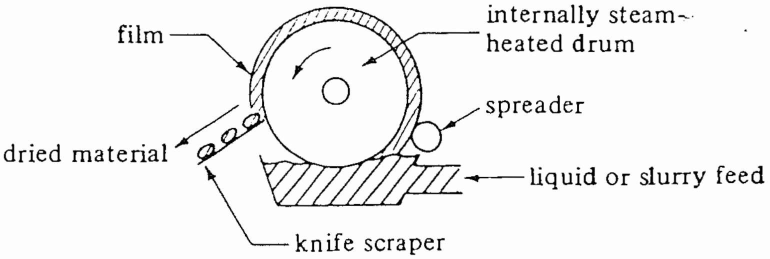


FIGURE 9.2-4. *Rotary-drum dryer.*

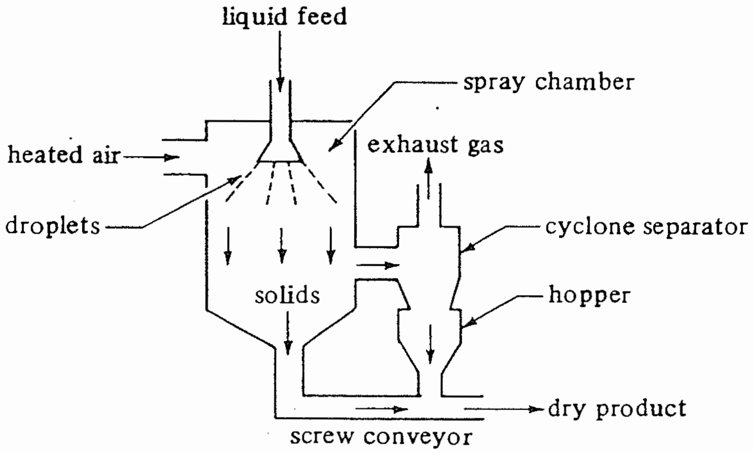


FIGURE 9.2-5. *Process flow diagram of spray-drying apparatus.*

of dry solid which are separated from the gas stream. The flow of gas and liquid in the spray chamber may be countercurrent, cocurrent, or a combination.

The fine droplets are formed from the liquid feed by spray nozzles or high-speed rotating spray disks inside a cylindrical chamber, as in Fig. 9.2-5. It is necessary to ensure that the droplets or wet particles of solid do not strike and stick to solid surfaces before drying has taken place. Hence, large chambers are used. The dried solids leave at the bottom of the chamber through a screw conveyor. The exhaust gases flow through a cyclone separator to remove any fines. The particles produced are usually light and quite porous. Dried milk powder is made from spray-drying milk.

9.2G Drying of Crops and Grains

In the drying of grain from a harvest, the grain contains about 30 to 35% moisture and for safe storage for about 1 year should be dried to about 13 wt % moisture (H1). A typical continuous-flow dryer is shown in Fig. 9.2-6. In the drying bin the thickness of the layer of grain is 0.5 m or less, through which the hot air passes. Unheated air in the bottom section cools the dry grain before it leaves. Other types of crop dryers and storage bins are described by Hall (H1).

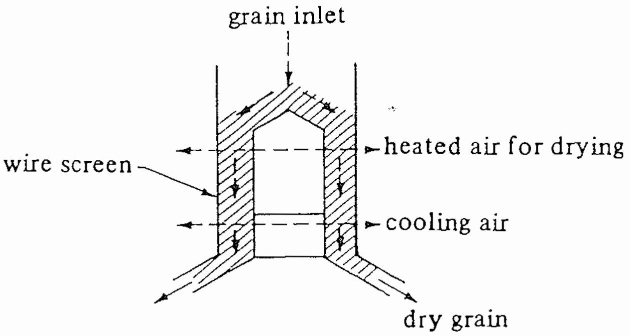


FIGURE 9.2-6. *Vertical continuous-flow grain dryer.*

9.3 VAPOR PRESSURE OF WATER AND HUMIDITY

9.3A Vapor Pressure of Water

1. Introduction. In a number of the unit operations and transport processes it is necessary to make calculations involving the properties of mixtures of water vapor and air. These calculations involve knowledge of the concentration of water vapor in air under various conditions of temperature and pressure, the thermal properties of these mixtures, and the changes occurring when this mixture is brought into contact with water or with wet solids in drying.

Humidification involves the transfer of water from the liquid phase into a gaseous mixture of air and water vapor. *Dehumidification* involves the reverse transfer, whereby water vapor is transferred from the vapor state to the liquid state. Humidification and dehumidification can also refer to vapor mixtures of materials such as benzene, but most practical applications occur with water. To better understand humidity, it is first necessary to discuss the vapor pressure of water.

2. Vapor pressure of water and physical states. Pure water can exist in three different physical states: solid ice, liquid, and vapor. The physical state in which it exists depends on the pressure and temperature.

Figure 9.3-1 illustrates the various physical states of water and the pressure–temperature relationships at equilibrium. In Fig. 9.3-1 the regions of the solid, liquid, and vapor states are shown. Along the line *AB*, the phases liquid and vapor coexist. Along line *AC*, the phases ice and liquid coexist. Along line *AD*, ice and vapor coexist. If ice at point (1) is heated at constant pressure, the temperature rises and the physical condition is shown moving horizontally. As the line crosses *AC*, the solid melts, and on crossing *AB* the liquid vaporizes. Moving from point (3) to (4), ice sublimes (vaporizes) to a vapor without becoming a liquid.

Liquid and vapor coexist in equilibrium along the line *AB*, which is the vapor-pressure line of water. Boiling occurs when the vapor pressure of the water is equal to the total pressure above the water surface. For example, at 100°C (212°F) the vapor pressure of water is 101.3 kPa (1.0 atm), and hence it will boil at 1 atm pressure. At 65.6°C (150°F), from the steam tables in Appendix A.2, the vapor pressure of water is 25.7 kPa (3.72 psia). Hence, at 25.7 kPa and 65.6°C, water will boil.

If a pan of water is held at 65.6°C in a room at 101.3 kPa abs pressure, the vapor pressure of water will again be 25.7 kPa. This illustrates an important property of the

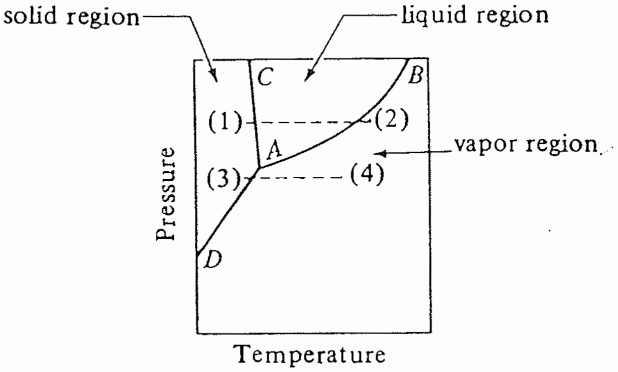


FIGURE 9.3-1. *Phase diagram for water.*

vapor pressure of water, which is not influenced by the presence of an inert gas such as air; i.e., the vapor pressure of water is essentially independent of the total pressure of the system.

9.3B Humidity and Humidity Chart

1. Definition of humidity. The humidity H of an air–water vapor mixture is defined as the kg of water vapor contained in 1 kg of dry air. The humidity so defined depends only on the partial pressure p_A of water vapor in the air and on the total pressure P (assumed throughout this chapter to be 101.325 kPa, 1.0 atm abs, or 760 mm Hg). Using the molecular weight of water (A) as 18.02 and of air as 28.97, the humdiity H in kg H_2O/kg dry air or in English units as lb H_2O/lb dry air is as follows:

$$H\,\frac{\text{kg } H_2O}{\text{kg dry air}} = \frac{p_A}{P - p_A}\,\frac{\text{kg mol } H_2O}{\text{kg mol air}} \times \frac{18.02 \text{ kg } H_2O}{\text{kg mol } H_2O} \times \frac{1}{28.97 \text{ kg air/kg mol air}}$$

$$H = \frac{18.02}{28.97}\,\frac{p_A}{P - p_A} \tag{9.3-1}$$

Saturated air is air in which the water vapor is in equilibrium with liquid water at the given conditions of pressure and temperature. In this mixture the partial pressure of the water vapor in the air–water mixture is equal to the vapor pressure p_{AS} of pure water at the given temperature. Hence, the saturation humidity H_S is

$$H_S = \frac{18.02}{28.97}\,\frac{p_{AS}}{P - p_{AS}} \tag{9.3-2}$$

2. Percentage humidity. The percentage humidity H_P is defined as 100 times the actual humidity H of the air divided by the humidity H_S if the air were saturated at the same temperature and pressure.

$$H_P = 100\,\frac{H}{H_S} \tag{9.3-3}$$

3. Percentage relative humidity. The amount of saturation of an air–water vapor mixture is also given as percentage relative humidity H_R using partial pressures.

$$H_R = 100\,\frac{p_A}{p_{AS}} \tag{9.3-4}$$

Note that $H_R \neq H_P$, since H_P expressed in partial pressures by combining Eqs. (9.3-1), (9.3-2), and (9.3-3) is

$$H_P = 100\,\frac{H}{H_S} = (100)\,\frac{18.02}{28.97}\,\frac{p_A}{P - p_A}\Big/\frac{18.02}{28.97}\,\frac{p_{AS}}{P - p_{AS}} = \frac{p_A}{p_{AS}}\,\frac{P - p_{AS}}{P - p_A}\,(100) \tag{9.3-5}$$

This, of course, is not the same as Eq. (9.3-4).

> *EXAMPLE 9.3-1. Humidity from Vapor-Pressure Data*
> The air in a room is at 26.7°C (80°F) and a pressure of 101.325 kPa and contains water vapor with a partial pressure $p_A = 2.76$ kPa. Calculate the following.
> (a) Humidity, H.
> (b) Saturation humidity, H_S, and percentage humidity, H_P.
> (c) Percentage relative humidity, H_R.

Solution: From the steam tables at 26.7°C, the vapor pressure of water is $p_{AS} = 3.50$ kPa (0.507 psia). Also, $p_A = 2.76$ kPa and $P = 101.3$ kPa (14.7 psia). For part (a), using Eq. (9.3-1),

$$H = \frac{18.02}{28.97}\frac{p_A}{P - p_A} = \frac{18.02(2.76)}{28.97(101.3 - 2.76)} = 0.01742 \text{ kg } H_2O/\text{kg air}$$

For part (b), using Eq. (9.3-2), the saturation humidity is

$$H_S = \frac{18.02}{28.97}\frac{p_{AS}}{P - p_{AS}} = \frac{18.02(3.50)}{28.97(101.3 - 3.50)} = 0.02226 \text{ kg } H_2O/\text{kg air}$$

The percentage humidity, from Eq. (9.3-3), is

$$H_P = 100\frac{H}{H_S} = \frac{100(0.01742)}{0.02226} = 78.3\%$$

For part (c), from Eq. (9.3-4), the percentage relative humidity is

$$H_R = 100\frac{p_A}{p_{AS}} = \frac{100(2.76)}{3.50} = 78.9\%$$

4. *Dew point of an air–water vapor mixture.* The temperature at which a given mixture of air and water vapor would be saturated is called the *dew-point temperature* or simply the *dew point.* For example, at 26.7°C (80°F), the saturation vapor pressure of water is $p_{AS} = 3.50$ kPa (0.507 psia). Hence, the dew point of a mixture containing water vapor having a partial pressure of 3.50 kPa is 26.7°C. If an air–water vapor mixture is at 37.8°C (often called the dry bulb temperature, since this is the actual temperature a dry thermometer bulb would indicate in this mixture) and contains water vapor of $p_A = 3.50$ kPa, the mixture would not be saturated. On cooling to 26.7°C, the air would be saturated, i.e., at the dew point. On further cooling, some water vapor would condense, since the partial pressure cannot be greater than the saturation vapor pressure.

5. *Humid heat of an air–water vapor mixture.* The humid heat c_S is the amount of heat in J (or kJ) required to raise the temperature of 1 kg of dry air plus the water vapor present by 1 K or 1°C. The heat capacity of air and water vapor can be assumed constant over the temperature ranges usually encountered at 1.005 kJ/kg dry air · K and 1.88 kJ/kg water vapor · K, respectively. Hence, for SI and English units,

$$\begin{aligned} c_S \text{ kJ/kg dry air} \cdot \text{K} &= 1.005 + 1.88H \qquad \text{(SI)} \\ c_S \text{ btu/lb}_m \text{ dry air} \cdot {}^\circ\text{F} &= 0.24 + 0.45H \qquad \text{(English)} \end{aligned} \tag{9.3-6}$$

[In some cases c_S will be given as $(1.005 + 1.88H)10^3$ J/kg · K.]

6. *Humid volume of an air–water vapor mixture.* The humid volume v_H is the total volume in m^3 of 1 kg of dry air plus the vapor it contains at 101.325 kPa (1.0 atm) abs pressure and the given gas temperature. Using the ideal gas law,

$$\begin{aligned} v_H \text{ m}^3/\text{kg dry air} &= \frac{22.41}{273}\, T \text{ K}\left(\frac{1}{28.97} + \frac{1}{18.02}H\right) \\ &= (2.83 \times 10^{-3} + 4.56 \times 10^{-3}H)T \text{ K} \\ v_H \text{ ft}^3/\text{lb}_m \text{ dry air} &= \frac{359}{492}\, T^\circ\text{R}\left(\frac{1}{28.97} + \frac{1}{18.02}H\right) \\ &= (0.0252 + 0.0405H)T^\circ\text{R} \end{aligned} \tag{9.3-7}$$

For a saturated air–water vapor mixture, $H = H_S$, and v_H is the saturated volume.

7. Total enthalpy of an air–water vapor mixture. The total enthalpy of 1 kg of air plus its water vapor is H_y J/kg or kJ/kg dry air. If T_0 is the datum temperature chosen for both components, the total enthalpy is the sensible heat of the air–water vapor mixture plus the latent heat λ_0 in J/kg or kJ/kg water vapor of the water vapor at T_0. Note that $(T - T_0)$°C = $(T - T_0)$ K and that this enthalpy is referred to liquid water.

$$\begin{aligned} H_y \text{ kJ/kg dry air} &= c_S(T - T_0) + H\lambda_0 = (1.005 + 1.88H)(T - T_0\,{}^\circ\text{C}) + H\lambda_0 \\ H_y \text{ btu/lb}_m \text{ dry air} &= (0.24 + 0.45H)(T - T_0\,{}^\circ\text{F}) + H\lambda_0 \end{aligned} \tag{9.3-8}$$

If the total enthalpy is referred to a base temperature T_0 of 0°C (32°F), the equation for H_y becomes

$$\begin{aligned} H_y \text{ kJ/kg dry air} &= (1.005 + 1.88H)\,(T\,{}^\circ\text{C} - 0) + 2501.4H \qquad \text{(SI)} \\ H_y \text{ btu/lb}_m \text{ dry air} &= (0.24 + 0.45H)\,(T\,{}^\circ\text{F} - 32) + 1075.4H \qquad \text{(English)} \end{aligned} \tag{9.3-9}$$

8. Humidity chart of air–water vapor mixtures. A convenient chart of the properties of air–water vapor mixtures at 1.0 atm abs pressure is the humidity chart in Fig. 9.3-2. In this figure the humidity H is plotted versus the actual temperature of the air–water vapor mixture (dry bulb temperature).

The curve marked 100% running upward to the right gives the saturation humidity H_S as a function of temperature. In Example 9.3-1, for 26.7°C H_S was calculated as 0.02226 kg H_2O/kg air. Plotting this point of 26.7°C (80°F) and $H_S = 0.02226$ on Fig. 9.3-2, it falls on the 100% saturated line.

Any point below the saturation line represents unsaturated air–water vapor mixtures. The curved lines below the 100% saturation line and running upward to the right represent unsaturated mixtures of definite percentage humidity H_P. Going downward vertically from the saturation line at a given temperature, the line between 100% saturation and zero humidity H (the bottom horizontal line) is divided evenly into 10 increments of 10% each.

All the percentage humidity lines H_P mentioned and the saturation humidity line H_S can be calculated from the data of vapor pressure of water.

EXAMPLE 9.3-2. Use of Humidity Chart

Air entering a dryer has a temperature (dry bulb temperature) of 60°C (140°F) and a dew point of 26.7°C (80°F). Using the humidity chart, determine the actual humidity H, percentage humidity H_P, humid heat c_S, and the humid volume v_H in SI and English units.

Solution: The dew point of 26.7°C is the temperature when the given mixture is at 100% saturation. Starting at 26.7°C, Fig. 9.3-2, and drawing a vertical line until it intersects the line for 100% humidity, a humidity of $H = 0.0225$ kg H_2O/kg dry air is read off the plot. This is the actual humidity of the air at 60°C. Stated in another way, if air at 60°C and having a humidity $H = 0.0225$ is cooled, its dew point will be 26.7°C. In English units, $H = 0.0225$ lb H_2O/lb dry air.

Locating this point of $H = 0.0225$ and $t = 60$°C on the chart, the percentage humidity H_P is found to be 14%, by linear interpolation vertically between the 10 and 20% lines. The humid heat for $H = 0.0225$ is, from

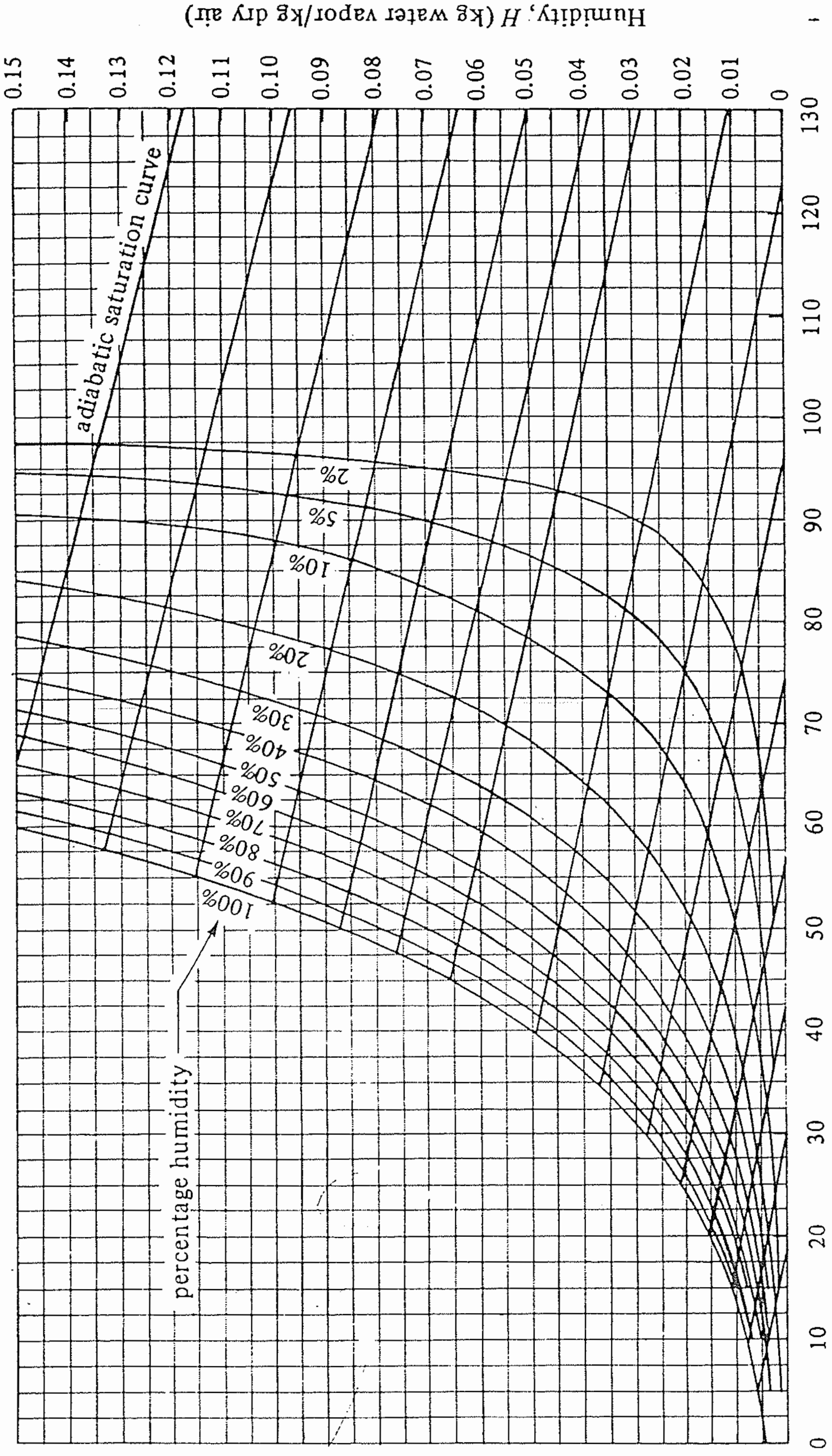


FIGURE 9.3-2. *Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)*

Eq. (9.3-6),

$$c_s = 1.005 + 1.88(0.0225)$$
$$= 1.047 \text{ kJ/kg dry air}\cdot\text{K} \quad \text{or} \quad 1.047 \times 10^3 \text{ J/kg}\cdot\text{K}$$
$$c_s = 0.24 + 0.45(0.0225)$$
$$= 0.250 \text{ btu/lb}_m \text{ dry air}\cdot{}^\circ\text{F} \qquad \text{(English)}$$

The humid volume at 60°C (140°F), from Eq. (9.3-7), is

$$v_H = (2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.0225)(60 + 273)$$
$$= 0.977 \text{ m}^3\text{/kg dry air}$$

In English units,

$$v_H = (0.0252 + 0.0405 \times 0.0225)(460 + 140) = 15.67 \text{ ft}^3\text{/lb}_m \text{ dry air}$$

9.3C Adiabatic Saturation Temperatures

Consider the process shown in Fig. 9.3-3, where the entering gas of air–water vapor mixture is contacted with a spray of liquid water. The gas leaves having a different humidity and temperature and the process is adiabatic. The water is recirculated, with some makeup water added.

The temperature of the water being recirculated reaches a steady-state temperature called the *adiabatic saturation temperature*, T_S. If the entering gas at temperature T having a humidity of H is not saturated, T_S will be lower than T. If the contact between the entering gas and the spray of droplets is enough to bring the gas and liquid to equilibrium, the leaving air is saturated at T_S, having a humidity H_S.

Writing an enthalpy balance (heat balance) over the process, a datum of T_S is used. The enthalpy of the makeup H_2O is then zero. This means that the total enthalpy of the entering gas mixture = enthalpy of the leaving gas mixture, or, using Eq. (9.3-8),

$$c_S(T - T_S) + H\lambda_S = c_S(T_S - T_S) + H_S\lambda_S \tag{9.3-10}$$

Or, rearranging, and using Eq. (9.3-6) for c_S,

$$\frac{H - H_S}{T - T_S} = -\frac{c_S}{\lambda_S} = \frac{1.005 + 1.88H}{\lambda_S} \qquad \text{(SI)}$$

$$\frac{H - H_S}{T - T_S} = \frac{0.24 + 0.45H}{\lambda_S} \qquad \text{(English)} \tag{9.3-11}$$

Equation (9.3-11) is the equation of an adiabatic humidification curve when plotted

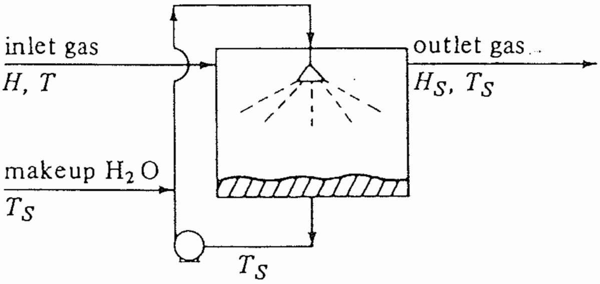


FIGURE 9.3-3. *Adiabatic air–water vapor saturator.*

on Fig. 9.3-2, which passes through the point H_S and T_S on the 100% saturation curve and other points of H and T. These series of lines, running upward to the left, are called *adiabatic humidification lines* or *adiabatic saturation lines.* Since c_S contains the term H, the adiabatic lines are not quite straight when plotted on the humidity chart.

If a given gas mixture at T_1 and H_1 is contacted for a sufficiently long time in an adiabatic saturator, it will leave saturated at H_{S1} and T_{S1}. The values of H_{S1} and T_{S1} are determined by following the adiabatic saturation line going through point T_1, H_1 until it intersects the 100% saturation line. If contact is not sufficient, the leaving mixture will be at a percentage saturation less than 100 but on the same line.

EXAMPLE 9.3-3. Adiabatic Saturation of Air
An air stream at 87.8°C having a humidity $H = 0.030$ kg H_2O/kg dry air is contacted in an adiabatic saturator with water. It is cooled and humidified to 90% saturation.

(a) What are the final values of H and T?
(b) For 100% saturation, what would be the values of H and T?

Solution: For part (a), the point $H = 0.030$ and $T = 87.8$°C is located on the humidity chart. The adiabatic saturation curve through this point is followed upward to the left until it intersects the 90% line at 42.5°C and $H = 0.0500$ kg H_2O/kg dry air.

For part (b), the same line is followed to 100% saturation, where $T = 40.5$°C and $H = 0.0505$ kg H_2O/kg dry air.

9.3D Wet Bulb Temperature

The adiabatic saturation temperature is the steady-state temperature attained when a large amount of water is contacted with the entering gas. The *wet bulb temperature* is the steady-state nonequilibrium temperature reached when a small amount of water is contacted under adiabatic conditions by a continuous stream of gas. Since the amount of liquid is small, the temperature and humidity of the gas are not changed, contrary to the adiabatic saturation case, where the temperature and humidity of the gas are changed.

The method used to measure the wet bulb temperature is illustrated in Fig. 9.3-4, where a thermometer is covered by a wick or cloth. The wick is kept wet by water and is immersed in a flowing stream of air–water vapor having a temperature of T (dry bulb temperature) and humidity H. At steady state, water is evaporating to the gas stream. The wick and water are cooled to T_W and stay at this constant temperature. The latent heat of evaporation is exactly balanced by the convective heat flowing from the gas stream at T to the wick at a lower temperature T_W.

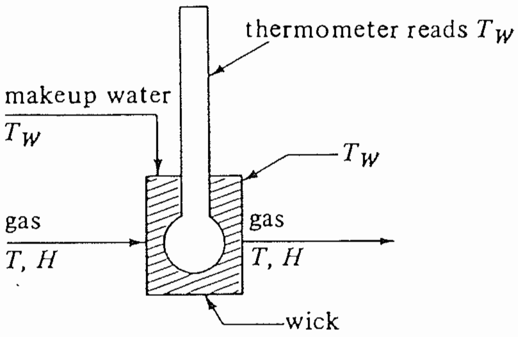


FIGURE 9.3-4. *Measurement of wet bulb temperature.*

A heat balance on the wick can be made. The datum temperature is taken at T_W. The amount of heat lost by vaporization, neglecting the small sensible heat change of the vaporized liquid and radiation, is

$$q = M_A N_A \lambda_W A \tag{9.3-12}$$

where q is kW(kJ/s), M_A is molecular weight of water, N_A is kg mol H_2O evaporating/s · m², A is surface area m², and λ_W is the latent heat of vaporization at T_W in kJ/kg H_2O. In English units, q is btu/h, N_A is lb mol/h · ft², and λ_W is btu/lb$_m$ H_2O. The flux N_A is

$$N_A = \frac{k'_y}{x_{BM}}(y_W - y) = k_y(y_W - y) \tag{9.3-13}$$

where k'_y is the mass-transfer coefficient in kg mol/s · m² · mol frac, x_{BM} is the log mean inert mole fraction of the air, y_W is the mole fraction of water vapor in the gas at the surface, and y is the mole fraction in the gas. For a dilute mixture $x_{BM} \cong 1.0$ and $k'_y \cong k_y$. The relation between H and y is

$$y = \frac{H/M_A}{1/M_B + H/M_A} \tag{9.3-14}$$

where M_B is the molecular weight of air and M_A the molecular weight of H_2O. Since H is small, as an approximation,

$$y \cong \frac{HM_B}{M_A} \tag{9.3-15}$$

Substituting Eq. (9.3-15) into (9.3-13) and then substituting the resultant into Eq. (9.3-12),

$$q = M_B k_y \lambda_W (H_W - H)A \tag{9.3-16}$$

The rate of convective heat transfer from the gas stream at T to the wick at T_W is

$$q = h(T - T_W)A \tag{9.3-17}$$

where h is the heat-transfer coefficient in kW/m² · K (btu/h · ft² · °F).

Equating Eq. (9.3-16) to (9.3-17) and rearranging,

$$\frac{H - H_W}{T - T_W} = -\frac{h/M_B k_y}{\lambda_W} \tag{9.3-18}$$

Experimental data on the value of $h/M_B k_y$, called the *psychrometric ratio*, show that for water vapor–air mixtures, the value is approximately 0.96–1.005. Since this value is close to the value of c_S in Eq. (9.3-11), approximately 1.005, Eqs. (9.3-18) and (9.3-11) are almost the same. This means that the adiabatic saturation lines can also be used for wet bulb lines with reasonable accuracy. (Note that this is only true for water vapor and not for others, such as benzene.) Hence, the wet bulb determination is often used to determine the humidity of an air–water vapor mixture.

EXAMPLE 9.3-4. Wet Bulb Temperature and Humidity

A water vapor–air mixture having a dry bulb temperature of $T = 60°C$ is passed over a wet bulb as shown in Fig. 9.3-4, and the wet bulb temperature obtained is $T_W = 29.5°C$. What is the humidity of the mixture?

Solution: The wet bulb temperature of 29.5°C can be assumed to be the same as the adiabatic saturation temperature T_S, as discussed. Following the adiabatic saturation curve of 29.5°C until it reaches the dry bulb temperature of 60°C, the humidity is $H = 0.0135$ kg H_2O/kg dry air.

9.4 EQUILIBRIUM MOISTURE CONTENT OF MATERIALS

9.4A Introduction

As in other transfer processes, such as mass transfer, the process of drying of materials must be approached from the viewpoint of the equilibrium relationships and also the rate relationships. In most of the drying apparatus discussed in Section 9.2, material is dried in contact with an air–water vapor mixture. The equilibrium relationships between the air–water vapor and the solid material will be discussed in this section.

An important variable in the drying of materials is the humidity of the air in contact with a solid of given moisture content. Suppose that a wet solid containing moisture is brought into contact with a stream of air having a constant humidity H and temperature. A large excess of air is used, so its conditions remain constant. Eventually, after exposure of the solid sufficiently long for equilibrium to be reached, the solid will attain a definite moisture content. This is known as the equilibrium moisture content of the material under the specified humidity and temperature of the air. The moisture content is usually expressed on a dry basis as kg of water per kg of moisture-free (bone-dry) solid or kg H_2O/100 kg dry solid; in English units as lb H_2O/100 lb dry solid.

For some solids the value of the equilibrium moisture content depends on the direction from which equilibrium is approached. A different value of the equilibrium moisture content is obtained according to whether a wet sample is allowed to dry by desorption or whether a dry sample adsorbs moisture by adsorption. For drying calculations it is the desorption equilibrium that is the larger value and is of particular interest.

9.4B Experimental Data of Equilibrium Moisture Content for Inorganic and Biological Materials

1. Typical data for various materials. If the material contains more moisture than its equilibrium value in contact with a gas of a given humidity and temperature, it will dry until it reaches its equilibrium value. If the material contains less moisture than its equilibrium value, it will adsorb water until it reaches its equilibrium value. For air having 0% humidity, the equilibrium moisture value of all materials is zero.

The equilibrium moisture content varies greatly with the type of material for any given percent relative humidity, as shown in Fig. 9.4-1 for some typical materials at room temperature. Nonporous insoluble solids tend to have equilibrium moisture contents which are quite low, as shown for glass wool and kaolin. Certain spongy, cellular materials of organic and biological origin generally show large equilibrium moisture contents. Examples of this in Fig. 9.4-1 are wool, leather, and wood.

2. Typical food materials. In Fig. 9.4-2 the equilibrium moisture contents of some typical food materials are plotted versus percent relative humidity. These biological materials also show large values of equilibrium moisture contents. Data in this figure and in Fig. 9.4-1 for biological materials show that at high percent relative humidities of about 60 to 80%, the equilibrium moisture content increases very rapidly with increases of relative humidity.

In general, at low relative humidities the equilibrium moisture content is greatest for food materials high in protein, starch, or other high-molecular-weight polymers and lower for food materials high in soluble solids. Crystalline salts and sugars and also fats generally adsorb small amounts of water.

3. Effect of temperature. The equilibrium moisture content of a solid decreases some-

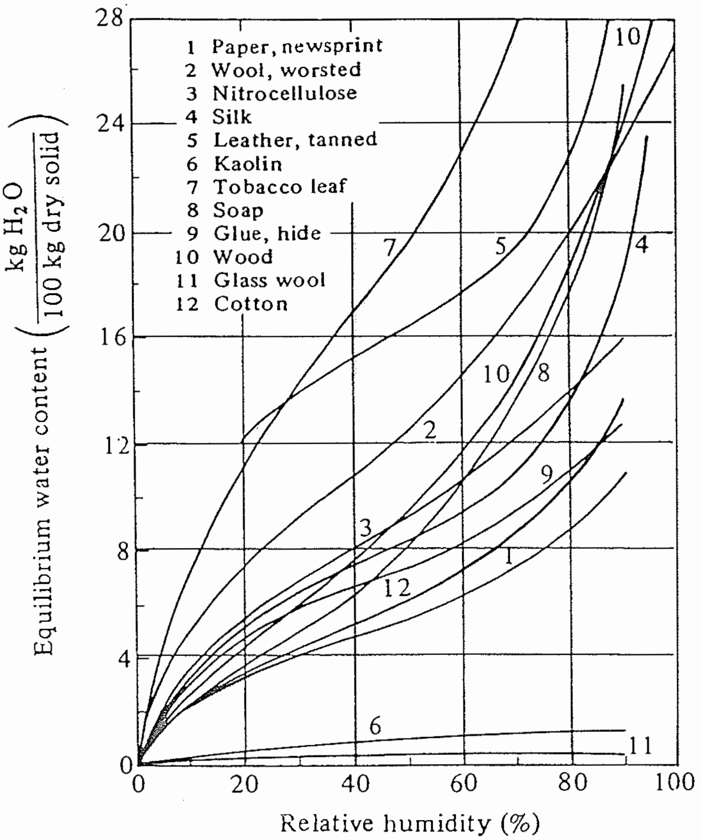


FIGURE 9.4-1. *Typical equilibrium moisture contents of some solids at approximately 298 K (25°C). [From National Research Council, International Critical Tables, Vol. II. New York: McGraw-Hill Book Company, 1929. Reproduced with permission of the National Academy of Sciences.]*

what with an increase in temperature. For example, for raw cotton at a relative humidity of 50%, the equilibrium moisture content decreased from 7.3 kg H_2O/100 kg dry solid at 37.8°C (311 K) to about 5.3 at 93.3°C (366.5 K), a decrease of about 25%. Often for moderate temperature ranges, the equilibrium moisture content will be assumed constant when experimental data are not available at different temperatures.

At present, theoretical understanding of the structure of solids and surface phenomena does not enable us to predict the variation of equilibrium moisture content of various materials from first principles. However, by using models such as those used for adsorption isotherms of multilayers of molecules and others, attempts have been made to correlate experimental data. Henderson (H2) gives an empirical relationship between equilibrium moisture content and percent relative humidity for some agricultural materials. In general, empirical relationships are not available for most materials, and equilibrium moisture contents must be determined experimentally. Also, equilibrium moisture relationships often vary from sample to sample of the same kind of material.

9.4C Bound and Unbound Water in Solids

In Fig. 9.4-1, if the equilibrium moisture content of a given material is continued to its intersection with the 100% humidity line, the moisture is called *bound water*. This water

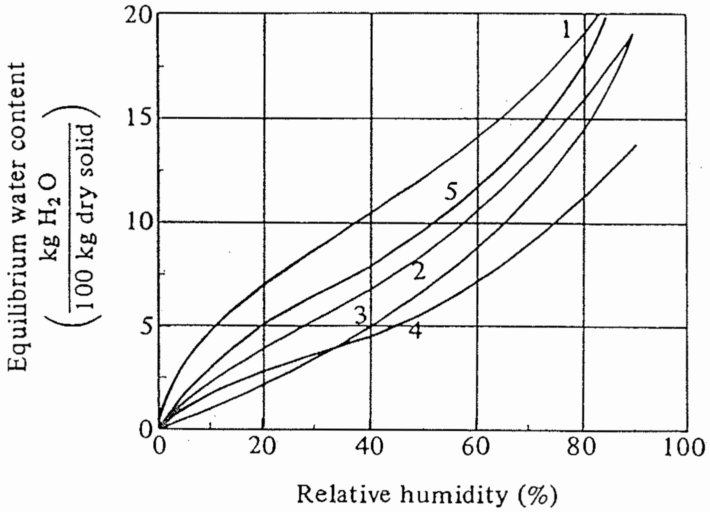


FIGURE 9.4-2. *Typical equilibrium moisture contents of some food materials at approximately 298 K (25°C): (1) macaroni, (2) flour, (3) bread, (4) crackers, (5) egg albumin. [Curve (5) from ref. (E1). Curves (1) to (4) from National Research Council, International Critical Tables, Vol. II. New York: McGraw-Hill Book Company, 1929. Reproduced with permission of the National Academy of Sciences.]*

in the solid exerts a vapor pressure less than that of liquid water at the same temperature. If such a material contains more water than indicated by intersection with the 100% humidity line, it can still exert only a vapor pressure as high as that of ordinary water at the same temperature. This excess moisture content is called *unbound water*, and it is held primarily in the voids of the solid. Substances containing bound water are often called *hygroscopic materials*.

As an example, consider curve 10 for wood in Fig. 9.4-1. This intersects the curve for 100% humidity at about 30 kg H_2O/100 kg dry solid. Any sample of wood containing less than 30 kg H_2O/100 kg dry solid contains only bound water. If the wood sample contained 34 kg H_2O/100 kg dry solid, 4 kg H_2O would be unbound and 30 kg H_2O bound per 100 kg dry solid.

The bound water in a substance may exist under several different conditions. Moisture in cell or fiber walls may have solids dissolved in it and have a lower vapor pressure. Liquid water in capillaries of very small diameter will exert a lowered vapor pressure because of the concave curvature of the surface. Water in natural organic materials is in chemical and physical–chemical combination.

9.4D Free and Equilibrium Moisture of a Substance

Free moisture content in a sample is the moisture above the equilibrium moisture content. This free moisture is the moisture that can be removed by drying under the given percent relative humidity. For example, in Fig. 9.4-1 silk has an equilibrium moisture content of 8.5 kg H_2O/100 kg dry material in contact with air of 50% relative humidity and 25°C. If a sample contains 10 kg H_2O/100 kg dry material, only 10.0 − 8.5, or 1.5, kg H_2O/100 kg dry material is removable by drying, and this is the free moisture of the sample under these drying conditions.

In many texts and references, the moisture content is given as percent moisture on a dry basis. This is exactly the same as the kg H_2O/100 kg dry material multiplied by 100.

9.5 RATE OF DRYING CURVES

9.5A Introduction and Experimental Methods

1. Introduction. In the drying of various types of process materials from one moisture content to another, it is usually desired to estimate the size of dryer needed, the various operating conditions of humidity and temperature for the air used, and the time needed to perform the amount of drying required. As discussed in Section 9.4, equilibrium moisture contents of various materials cannot be predicted and must be determined experimentally. Similarly, since our knowledge of the basic mechanisms of rates of drying is quite incomplete, it is necessary in most cases to obtain some experimental measurements of drying rates.

2. Experimental determination of rate of drying. To experimentally determine the rate of drying for a given material, a sample is usually placed on a tray. If it is a solid material it should fill the tray so that only the top surface is exposed to the drying air stream. By suspending the tray from a balance in a cabinet or duct through which the air is flowing, the loss in weight of moisture during drying can be determined at different intervals without interrupting the operation.

In doing batch-drying experiments, certain precautions should be observed to obtain usable data under conditions that closely resemble those to be used in the large-scale operations. The sample should not be too small in weight and should be supported in a tray or frame similar to the large-scale one. The ratio of drying to nondrying surface (insulated surface) and the bed depth should be similar. The velocity, humidity, temperature, and direction of the air should be the same and constant to simulate drying under constant drying conditions.

9.5B Rate of Drying Curves for Constant-Drying Conditions

1. Conversion of data to rate-of-drying curve. Data obtained from a batch-drying experiment are usually obtained as W total weight of the wet solid (dry solid plus moisture) at different times t hours in the drying period. These data can be converted to rate-of-drying data in the following ways. First, the data are recalculated. If W is the weight of the wet solid in kg total water plus dry solid and W_S is the weight of the dry solid in kg,

$$X_t = \frac{W - W_S}{W_S} \frac{\text{kg total water}}{\text{kg dry solid}} \left(\frac{\text{lb total water}}{\text{lb dry solid}}\right) \tag{9.5-1}$$

For the given constant drying conditions, the equilibrium moisture content X^* kg equilibrium moisture/kg dry solid is determined. Then the free moisture content X in kg free water/kg dry solid is calculated for each value of X_t.

$$X = X_t - X^* \tag{9.5-2}$$

Using the data calculated from Eq. (9.5-2), a plot of free moisture content X versus time t in h is made as in Fig. 9.5-1a. To obtain the rate-of-drying curve from this plot, the slopes of the tangents drawn to the curve in Fig. 9.5-1a can be measured, which give values of dX/dt at given values of t. The rate R is calculated for each point by

$$R = -\frac{L_S}{A}\frac{dX}{dt} \tag{9.5-3}$$

where R is drying rate in kg $H_2O/h \cdot m^2$, L_S kg of dry solid used, and A exposed surface area for drying in m^2. In English units, R is lb_m $H_2O/h \cdot ft^2$, L_S is lb_m dry solid, and A is ft^2. For obtaining R from Fig. 9.5-1a, a value of L_S/A of 21.5 kg/m^2 was used. The drying-rate curve is then obtained by plotting R versus the moisture content, as in Fig. 9.5-1b.

Another method to obtain the rate-of-drying curve is to first calculate the weight loss ΔX for a Δt time. For example, if $X_1 = 0.350$ at a time $t_1 = 1.68$ h and $x_2 = 0.325$ at a time $t_2 = 2.04$ h, $\Delta X/\Delta t = (0.350 - 0.325)/(2.04 - 1.68)$. Then, using Eq. (9.5-4) and

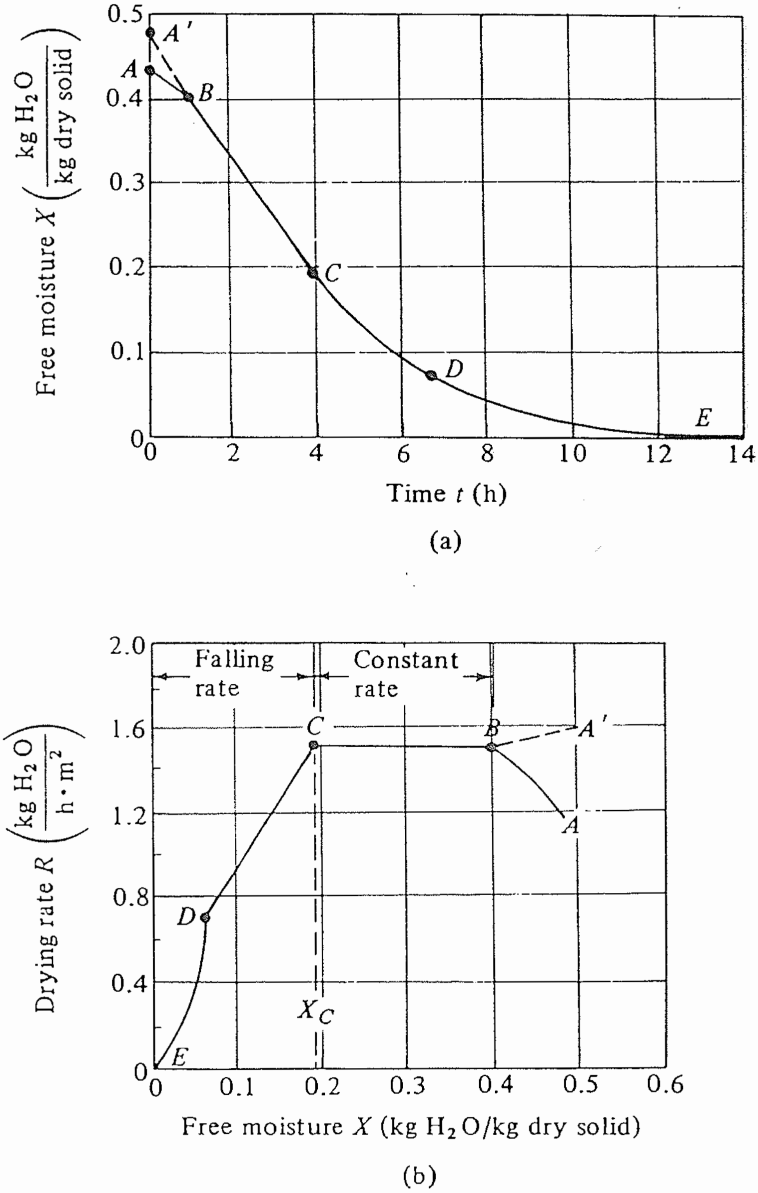


FIGURE 9.5-1. *Typical drying-rate curve for constant drying conditions: (a) plot of data as free moisture versus time, (b) rate of drying curve as rate versus free moisture content.*

$L_S/A = 21.5$,

$$R = -\frac{L_S}{A}\frac{\Delta X}{\Delta t} = 21.5\left(\frac{0.350 - 0.325}{2.04 - 1.68}\right) = 1.493$$

This rate R is the average over the period 1.68 to 2.04 h and should be plotted at the average concentration $X = (0.350 + 0.325)/2 = 0.338$.

2. *Plot of rate-of-drying curve.* In Fig. 9.5-lb the rate-of-drying curve for constant-drying conditions is shown. At zero time the initial free moisture content is shown at point A. In the beginning the solid is usually at a colder temperature than its ultimate temperature, and the evaporation rate will increase. Eventually at point B the surface temperature rises to its equilibrium value. Alternatively, if the solid is quite hot to start with, the rate may start at point A'. This initial unsteady-state adjustment period is usually quite short and it is often ignored in the analysis of times of drying.

From point B to C in Fig. 9.5-la the line is straight, and hence the slope and rate are constant during this period. This *constant-rate-of-drying period* is shown as line BC in Fig. 9.5-lb.

At point C on both plots, the drying rate starts to decrease in the *falling-rate period* until it reaches point D. In this first falling-rate period, the rate shown as line CD in Fig. 9.5-lb is often linear.

At point D the rate of drying falls even more rapidly, until it reaches point E, where the equilibrium moisture content is X^* and $X = X^* - X^* = 0$. In some materials being dried, the region CD may be missing completely or it may constitute all of the falling-rate period.

9.5C Drying in the Constant-Rate Period

Drying of different solids under different constant conditions of drying will often give curves of different shapes in the falling-rate period, but in general the two major portions of the drying-rate curve—constant-rate period and falling-rate period—are present.

In the constant-rate drying period, the surface of the solid is initially very wet and a continuous film of water exists on the drying surface. This water is entirely unbound water and the water acts as if the solid were not present. The rate of evaporation under the given air conditions is independent of the solid and is essentially the same as the rate from a free liquid surface. Increased roughness of the solid surface, however, may lead to higher rates than from a flat surface.

If the solid is porous, most of the water evaporated in the constant-rate period is supplied from the interior of the solid. This period continues only as long as the water is supplied to the surface as fast as it is evaporated. Evaporation during this period is similar to that in determining the wet bulb temperature, and in the absence of heat transfer by radiation or conduction, the surface temperature is approximately that of the wet bulb temperature.

9.5D Drying in the Falling-Rate Period

Point C in Fig. 9.5-1b is at the *critical free moisture content* X_C. At this point there is insufficient water on the surface to maintain a continuous film of water. The entire surface is no longer wetted, and the wetted area continually decreases in this first falling-rate period until the surface is completely dry at point D in Fig. 9.5-1b.

The second falling-rate period begins at point D when the surface is completely dry. The plane of evaporation slowly recedes from the surface. Heat for the evaporation is transferred through the solid to the zone of vaporization. Vaporized water moves through the solid into the air stream.

In some cases no sharp discontinuity occurs at point D, and the change from partially wetted to completely dry conditions at the surface is so gradual that no sharp change is detectable.

The amount of moisture removed in the falling-rate period may be relatively small but the time required may be long. This can be seen in Fig. 9.5-1. The period BC for constant-rate drying lasts for about 3.0 h and reduces X from 0.40 to about 0.19, a reduction of 0.21 kg H_2O/kg dry solid. The falling-rate period CE lasts about 9.0 h and reduces X only from 0.19 to 0.

9.5E Moisture Movements in Solids During Drying in the Falling-Rate Period

When drying occurs by evaporation of moisture from the exposed surface of a solid, moisture must move from the depths of the solid to the surface. The mechanisms of the movement affect the drying during the constant-rate and falling-rate periods. Some of the theories advanced to explain the various types of falling-rate curves will be briefly reviewed.

1. Liquid diffusion theory. In this theory diffusion of liquid moisture occurs when there is a concentration difference between the depths of the solid and the surface. This method of transport of moisture is usually found in nonporous solids where single-phase solutions are formed with the moisture, such as in paste, soap, gelatin, and glue. This is also found in drying the last portions of moisture from clay, flour, wood, leather, paper, starches, and textiles. In drying many food materials, the movement of water in the falling-rate period occurs by diffusion.

The shapes of the moisture distribution curves in the solid at given times are qualitatively consistent with use of the unsteady-state diffusion equations given in Chapter 7. The moisture diffusivity D_{AB} usually decreases with decreased moisture content, so that the diffusivities are usually average values over the range of concentrations used. Materials drying in this way are usually said to be drying by diffusion, although the actual mechanisms may be quite complicated. Since the rate of evaporation from the surface is quite fast, i.e., the resistance is quite low, compared to the diffusion rate through the solid in the falling-rate period, the moisture content at the surface is at the equilibrium value.

The shape of a diffusion-controlled curve in the falling-rate period is similar to Fig. 9.5-2a. If the initial constant-rate drying is quite high, the first falling-rate period of

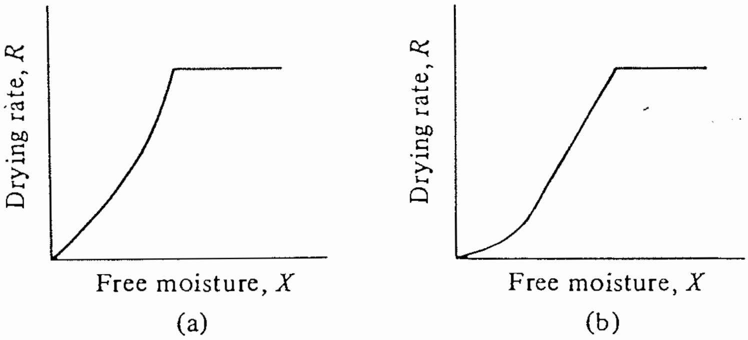


FIGURE 9.5-2. *Typical drying-rate curves: (a) diffusion-controlled falling-rate period, (b) capillary-controlled falling-rate period in a fine porous solid.*

unsaturated surface evaporation may not appear. If the constant-rate drying is quite low, the period of unsaturated surface evaporation is usually present in region *CD* in Fig. 9.5-1b and the diffusion-controlled curve is in region *DE*. Equations for calculating drying in this period where diffusion controls are given in Section 9.9. Also, Problem 7.1-4 for the drying of clay and Problem 7.1-6 for the drying of wood using diffusion theory are given in the Chapter 7 Problems.

2. *Capillary movement in porous solids.* When granular and porous solids such as clays, sand, soil, paint pigments, and minerals are being dried, unbound or free moisture moves through the capillaries and voids of the solids by *capillary action*, not by diffusion. This mechanism, involving surface tension, is similar to the movement of oil in a lamp wick.

A porous solid contains interconnecting pores and channels of varying pore sizes. As water is evaporated, a meniscus of liquid water is formed across each pore in the depths of the solid. This sets up capillary forces by the interfacial tension between the water and solid. These capillary forces provide the driving force for moving water through the pores to the surface. Small pores develop greater forces than those developed by large pores.

At the beginning of the falling-rate period at point *C* in Fig. 9.5-1b, the water is being brought to the surface by capillary action, but the surface layer of water starts to recede below the surface. Air rushes in to fill the voids. As the water is continuously removed, a point is reached where there is insufficient water left to maintain continuous films across the pores, and the rate of drying suddenly decreases at the start of the second falling-rate period at point *D*. Then the rate of diffusion of water vapor in the pores and rate of conduction of heat in the solid may be the main factors in drying.

In fine pores in solids, the rate-of-drying curve in the second falling-rate period may conform to the diffusion law and the curve is concave upward, as shown in Fig. 9.5-2b. For very porous solids, such as a bed of sand, where the pores are large, the rate-of-drying curve in the second falling-rate period is often straight, and hence the diffusion equations do not apply.

3. *Effect of shrinkage.* A factor often greatly affecting the drying rate is the shrinkage of the solid as moisture is removed. Rigid solids do not shrink appreciably, but colloidal and fibrous materials such as vegetables and other foodstuffs do undergo shrinkage. The most serious effect is that there may be developed a hard layer on the surface which is impervious to the flow of liquid or vapor moisture and slows the drying rate; examples are clay and soap. In many foodstuffs, if drying occurs at too high a temperature, a layer of closely packed shrunken cells, which are sealed together, forms at the surface. This presents a barrier to moisture migration and is known as *case hardening*. Another effect of shrinkage is to cause the material to warp and change its structure. This can happen in drying wood.

Sometimes to decrease these effects of shrinkage, it is desirable to dry with moist air. This decreases the rate of drying so that the effects of shrinkage on warping or hardening at the surface are greatly reduced.

9.6 CALCULATION METHODS FOR CONSTANT-RATE DRYING PERIOD

9.6A Method Using Experimental Drying Curve

1. *Introduction.* Probably the most important factor in drying calculations is the length of time required to dry a material from a given initial free moisture content X_1 to a final moisture content X_2. For drying in the constant-rate period, we can estimate the time

needed by using experimental batch drying curves or by using predicted mass- and heat-transfer coefficients.

2. Method using drying curve. To estimate the time of drying for a given batch of material, the best method is based on actual experimental data obtained under conditions where the feed material, relative exposed surface area, gas velocity, temperature, and humidity are essentially the same as in the final drier. Then the time required for the constant-rate period can be determined directly from the drying curve of free moisture content versus time.

EXAMPLE 9.6-1. Time of Drying from Drying Curve

A solid whose drying curve is represented by Fig. 9.5-1a is to be dried from a free moisture content $X_1 = 0.38$ kg H_2O/kg dry solid to $X_2 = 0.25$ kg H_2O/kg dry solid. Estimate the time required.

Solution: From Fig. 9.5-1a for $X_1 = 0.38$, t_1 is read off as 1.28 h. For $X_2 = 0.25$, $t_2 = 3.08$ h. Hence, the time required is

$$t = t_2 - t_1 = 3.08 - 1.28 = 1.80 \text{ h.}$$

3. Method using rate-of-drying curve for constant-rate period. Instead of using the drying curve, the rate-of-drying curve can be used. The drying rate R is defined by Eq. (9.5-3) as

$$R = -\frac{L_S}{A}\frac{dX}{dt} \tag{9.5-3}$$

This can be rearranged and integrated over the time interval to dry from X_1 at $t_1 = 0$ to X_2 at $t_2 = t$.

$$t = \int_{t_1=0}^{t_2=t} dt = \frac{L_S}{A}\int_{X_2}^{X_1}\frac{dX}{R} \tag{9.6-1}$$

If the drying takes place within the constant-rate period so that both X_1 and X_2 are greater than the critical moisture content X_C, then $R = \text{constant} = R_C$. Integrating Eq. (9.6-1) for the constant-rate period,

$$t = \frac{L_S}{AR_C}(X_1 - X_2) \tag{9.6-2}$$

EXAMPLE 9.6-2. Drying Time from Rate-of-Drying Curve

Repeat Example 9.6-1 but use Eq. (9.6-2) and Fig. 9.5-1b.

Solution: As given previously, a value of 21.5 for L_S/A was used to prepare Fig. 9.5-1b from 9.5-1a. From Fig. 9.5-1b, $R_C = 1.51$ kg $H_2O/h \cdot m^2$. Substituting into Eq. (9.6-2),

$$t = \frac{L_S}{AR_C}(X_1 - X_2) = \frac{21.5}{1.51}(0.38 - 0.25) = 1.85 \text{ h}$$

This is close to the value of 1.80 h of Example 9.6-1.

9.6B Method Using Predicted Transfer Coefficients for Constant-Rate Period

1. Introduction. In the constant-rate period of drying, the surfaces of the grains of solid in contact with the drying air flow remain completely wetted. As stated previously, the rate of evaporation of moisture under a given set of air conditions is independent of the type of solid and is essentially the same as the rate of evaporation from a free liquid surface under the same conditions. However, surface roughness may increase the rate of evaporation.

During this constant-rate period, the solid is so wet that the water acts as if the solid were not there. The water evaporated from the surface is supplied from the interior of the solid. The rate of evaporation from a porous material occurs by the same mechanism as that occurring at a wet bulb thermometer, which is essentially constant-rate drying.

2. *Equations for predicting constant-rate drying.* Drying of a material occurs by mass transfer of water vapor from the saturated surface of the material through an air film to the bulk gas phase or environment. The rate of moisture movement within the solid is sufficient to keep the surface saturated. The rate of removal of the water vapor (drying) is controlled by the rate of heat transfer to the evaporating surface, which furnishes the latent heat of evaporation for the liquid. At steady state, the rate of mass transfer balances the rate of heat transfer.

To derive the equation for drying, we neglect heat transfer by radiation to the solid surface and also assume no heat transfer by conduction from metal pans or surfaces. In Section 9.8, convection and radiation will also be considered. Assuming only heat transfer to the solid surface by convection from the hot gas to the surface of the solid and mass transfer from the surface to the hot gas (Fig. 9.6-1), we can write equations which are the same as those for deriving the wet bulb temperature T_W in Eq. (9.3-18).

The rate of convective heat transfer q in W (J/s, btu/h) from the gas at T°C (°F) to the surface of the solid at T_W °C, where $(T - T_W)$°C $= (T - T_W)$ K is

$$q = h(T - T_W)A \tag{9.6-3}$$

where h is the heat-transfer coefficient in W/m$^2 \cdot$ K (btu/h $\cdot$ ft$^2 \cdot$ °F) and A is the exposed drying area in m^2 (ft^2). The equation of the flux of water vapor from the surface is the same as Eq. (9.3-13) and is

$$N_A = k_y(y_W - y) \tag{9.6-4}$$

Using the approximation from Eq. (9.3-15) and substituting into Eq. (9.6-4),

$$N_A = k_y \frac{M_B}{M_A}(H_W - H) \tag{9.6-5}$$

The amount of heat needed to vaporize N_A kg mol/s $\cdot$ m^2 (lb mol/h $\cdot$ ft^2) water, neglecting the small sensible heat changes, is the same as Eq. (9.3-12).

$$q = M_A N_A \lambda_W A \tag{9.6-6}$$

where λ_W is the latent heat at T_W in J/kg (btu/lb$_m$).

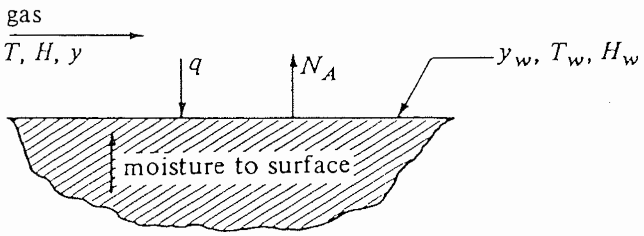


FIGURE 9.6-1. *Heat and mass transfer in constant-rate drying.*

Equating Eqs. (9.6-3) and (9.6-6) and substituting Eq. (9.6-5) for N_A,

$$R_C = \frac{q}{A\lambda_W} = \frac{h(T - T_W)}{\lambda_W} = k_y M_B (H_W - H) \qquad \textbf{(9.6-7)}$$

Equation (9.6-7) is identical to Eq. (9.3-18) for the wet bulb temperature. Hence, in the absence of heat transfer by conduction and radiation, the temperature of the solid is at the wet bulb temperature of the air during the constant-rate drying period. Hence, the rate of drying R_C can be calculated using the heat-transfer equation $h(T - T_W)/\lambda_W$ or the mass-transfer equation $k_y M_B (H_W - H)$. However, it has been found more reliable to use the heat-transfer equation (9.6-8), since an error in determining the interface temperature T_W at the surface affects the driving force $(T - T_W)$ much less than it affects $(H_W - H)$.

$$R_C \text{ kg } H_2O/h \cdot m^2 = \frac{h}{\lambda_W} (T - T_W \text{ °C})(3600) \qquad \text{(SI)}$$

$$R_C \text{ lb}_m \text{ } H_2O/h \cdot ft^2 = \frac{h}{\lambda_W} (T - T_W \text{ °F}) \qquad \text{(English)} \qquad \textbf{(9.6-8)}$$

To predict R_C in Eq. (9.6-8), the heat-transfer coefficient must be known. For the case where the air is flowing parallel to the drying surface, Eq. (4.6-3) can be used for air. However, because the shape of the leading edge of the drying surface causes more turbulence, the following can be used for an air temperature of 45–150°C and a mass velocity G of 2450–29 300 kg/h · m^2 (500–6000 lb$_m$/h · ft^2) or a velocity of 0.61–7.6 m/s (2–25 ft/s).

$$h = 0.0204 G^{0.8} \qquad \text{(SI)}$$

$$h = 0.0128 G^{0.8} \qquad \text{(English)} \qquad \textbf{(9.6-9)}$$

where in SI units G is $v\rho$ kg/h · m^2 and h is W/m^2 · K. In English units, G is in lb$_m$/h · ft^2 and h in btu/h · ft^2 · °F. When air flows perpendicular to the surface for a G of 3900–19 500 kg/h · m^2 or a velocity of 0.9–4.6 m/s (3–15 ft/s),

$$h = 1.17 G^{0.37} \qquad \text{(SI)}$$

$$h = 0.37 G^{0.37} \qquad \text{(English)} \qquad \textbf{(9.6-10)}$$

Equations (9.6-8) to (9.6-10) can be used to estimate the rate of drying during the constant-rate period. However, when possible, experimental measurements of the drying rate are preferred.

To estimate the time of drying during the constant-rate period, substituting Eq. (9.6-7) into (9.6-2),

$$t = \frac{L_S \lambda_W (X_1 - X_2)}{Ah(T - T_W)} = \frac{L_S(X_1 - X_2)}{Ak_y M_B(H_W - H)} \qquad \textbf{(9.6-11)}$$

EXAMPLE 9.6-3. Prediction of Constant-Rate Drying

An insoluble wet granular material is dried in a pan 0.457 × 0.457 m (1.5 × 1.5 ft) and 25.4 mm deep. The material is 25.4 mm deep in the pan, and the sides and bottom can be considered to be insulated. Heat transfer is by convection from an air stream flowing parallel to the surface at a velocity of 6.1 m/s (20 ft/s). The air is at 65.6°C (150°F) and has a humidity of 0.010 kg H_2O/kg dry air. Estimate the rate of drying for the constant-rate period using SI and English units.

Solution: For a humidity $H = 0.010$ and dry bulb temperature of 65.6°C and using the humidity chart, Fig. 9.3-2, the wet bulb temperature T_W is found as 28.9°C (84°F) and $H_W = 0.026$ by following the adiabatic saturation line (the same as the wet bulb line) to the saturated humidity. Using Eq. (9.3-7) to calculate the humid volume,

$$\begin{aligned} v_H &= (2.83 \times 10^{-3} + 4.56 \times 10^{-3} H)T \\ &= (2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.01)(273 + 65.6) \\ &= 0.974 \text{ m}^3/\text{kg dry air} \end{aligned}$$

The density for 1.0 kg dry air + 0.010 kg H_2O is

$$\rho = \frac{1.0 + 0.010}{0.974} = 1.037 \text{ kg/m}^3 \ (0.0647 \text{ lb}_m/\text{ft}^3)$$

The mass velocity G is

$$G = v\rho = 6.1(3600)(1.037) = 22\,770 \text{ kg/h} \cdot \text{m}^2$$

$$G = v\rho = 20(3600)(0.0647) = 4660 \text{ lb}_m/\text{h} \cdot \text{ft}^2$$

Using Eq. (9.6-9),

$$h = 0.0204G^{0.8} = 0.0204(22\,770)^{0.8} = 62.45 \text{ W/m}^2 \cdot \text{K}$$

$$h = 0.0128G^{0.8} = 0.0128(4660)^{0.8} = 11.01 \text{ btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

At $T_W = 28.9$°C (84°F), $\lambda_W = 2433$ kJ/kg (1046 btu/lb$_m$) from steam tables.

Substituting into Eq. (9.6-8), noting that (65.6 − 28.9)°C = (65.6 − 28.9) K,

$$\begin{aligned} R_C &= \frac{h}{\lambda_W}(T - T_W)(3600) = \frac{62.45}{2433 \times 1000}(65.6 - 28.9)(3600) \\ &= 3.39 \text{ kg/h} \cdot \text{m}^2 \end{aligned}$$

$$R_C = \frac{11.01}{1046}(150 - 84) = 0.695 \text{ lb}_m/\text{h} \cdot \text{ft}^2$$

The total evaporation rate for a surface area of 0.457 × 0.457 m^2 is

$$\begin{aligned} \text{total rate} = R_C A &= 3.39(0.457 \times 0.457) = 0.708 \text{ kg H}_2\text{O/h} \\ &= 0.695(1.5 \times 1.5) = 1.564 \text{ lb}_m \text{ H}_2\text{O/h} \end{aligned}$$

9.6C Effect of Process Variables on Constant-Rate Period

As stated previously, experimental measurements of the drying rate are usually preferred over using the equations for prediction. However, these equations are quite helpful to predict the effect of changing the drying process variables when limited experimental data are available.

1. Effect of air velocity. When conduction and radiation heat transfer are not present, the rate R_C of drying in the constant-rate region is proportional to h and hence to $G^{0.8}$ as given by Eq. (9.6-9) for air flow parallel to the surface. The effect of gas velocity is less important when radiation and conduction are present.

2. Effect of gas humidity. If the gas humidity H is decreased for a given T of the gas, then from the humidity chart the wet bulb temperature T_W will decrease. Then using Eq. (9.6-7), R_C will increase. For example, if the original conditions are R_{C1}, T_1, T_{W1}, H_1, and

H_{W1}, then if H_1 is changed to H_2 and H_{W1} is changed to H_{W2}, R_{C2} becomes

$$R_{C2} = R_{C1} \frac{T - T_{W2}}{T - T_{W1}} \frac{\lambda_{W1}}{\lambda_{W2}} = R_{C1} \frac{H_{W2} - H_2}{H_{W1} - H_1} \tag{9.6-12}$$

However, since $\lambda_{W1} \cong \lambda_{W2}$,

$$R_{C2} = R_{C1} \frac{T - T_{W2}}{T - T_{W1}} = R_{C1} \frac{H_{W2} - H_2}{H_{W1} - H_1} \tag{9.6-13}$$

3. Effect of gas temperature. If the gas temperature T is increased, T_W is also increased some, but not as much as the increase in T. Hence, R_C increases as follows:

$$R_{C2} = R_{C1} \frac{T_2 - T_{W2}}{T_1 - T_{W1}} = R_{C1} \frac{H_{W2} - H_2}{H_{W1} - H_1} \tag{9.6-14}$$

4. Effect of thickness of solid being dried. For heat transfer by convection only, the rate R_C is independent of the thickness x_1 of the solid. However, the time t for drying between fixed moisture contents X_1 and X_2 will be directly proportional to the thickness x_1. This is shown by Eq. (9.6-2), where increasing the thickness with a constant A will directly increase the amount of L_S kg dry solid.

5. Experimental effect of process variables. Experimental data tend to bear out the conclusions reached on the effects of material thickness, humidity, air velocity, and $T - T_W$.

9.7 CALCULATION METHODS FOR FALLING-RATE DRYING PERIOD

9.7A Method Using Graphical Integration

In the falling-rate drying period as shown in Fig. 9.5-1b, the rate of drying R is not constant but decreases when drying proceeds past the critical free moisture content X_C. When the free moisture content X is zero, the rate drops to zero.

The time for drying for any region between X_1 and X_2 has been given by Eq. (9.6-1).

$$t = \frac{L_S}{A} \int_{X_2}^{X_1} \frac{dX}{R} \tag{9.6-1}$$

If the rate is constant, Eq. (9.6-1) can be integrated to give Eq. (9.6-2). However, in the falling-rate period, R varies. For any shape of falling-rate drying curve, Eq. (9.6-1) can be graphically integrated by plotting $1/R$ versus X and determining the area under the curve.

EXAMPLE 9.7-1. Graphical Integration in Falling-Rate Drying Period

A batch of wet solid whose drying-rate curve is represented by Fig. 9.5-1b is to be dried from a free moisture content of $X_1 = 0.38$ kg H_2O/kg dry solid to $X_2 = 0.04$ kg H_2O/kg dry solid. The weight of the dry solid is $L_S = 399$ kg dry solid and $A = 18.58$ m^2 of top drying surface. Calculate the time for drying. Note that $L_S/A = 399/18.58 = 21.5$ kg/m^2.

Solution: From Fig. 9.5-1b, the critical free moisture content is $X_C = 0.195$ kg H_2O/kg dry solid. Hence, the drying is in the constant-rate and falling-rate periods.

For the constant-rate period, $X_1 = 0.38$ and $X_2 = X_C = 0.195$. From Fig. 9.5-1b, $R_C = 1.51\ \text{kg}\ H_2O/\text{h} \cdot \text{m}^2$. Substituting into Eq. (9.6-2),

$$t = \frac{L_S}{AR_C}(X_1 - X_2) = \frac{399(0.38 - 0.195)}{(18.58)(1.51)} = 2.63\ \text{h}$$

For the falling-rate period, reading values of R for various values of X from Fig. 9.5-1b, the following table is prepared:

X	R	$1/R$	X	R	$1/R$
0.195	1.51	0.663	0.065	0.71	1.41
0.150	1.21	0.826	0.050	0.37	2.70
0.100	0.90	1.11	0.040	0.27	3.70

In Fig. 9.7-1 a plot of $1/R$ versus X is made and the area under the curve from $X_1 = 0.195$ (point C) to $X_2 = 0.040$ is determined:

$$\text{area} = A_1 + A_2 + A_3 = (2.5 \times 0.024) + (1.18 \times 0.056) + (0.84 + 0.075)$$
$$= 0.189$$

Substituting into Eq. (9.6-1),

$$t = \frac{L_S}{A}\int_{X_2}^{X_1} \frac{dX}{R} = \frac{399}{18.58}(0.189) = 4.06\ \text{h}$$

The total time is 2.63 + 4.06 = 6.69 h.

9.7B Calculation Methods for Special Cases in Falling-Rate Region

In certain special cases in the falling-rate region, the equation for the time for drying, Eq. (9.6-1), can be integrated analytically.

1. Rate is a linear function of X. If both X_1 and X_2 are less than X_C and the rate R is linear in X over this region,

$$R = aX + b \tag{9.7-1}$$

where a is the slope of the line and b is a constant. Differentiating Eq. (9.7-1) gives $dR = a\,dX$. Substituting this into Eq. (9.6-1),

$$t = \frac{L_S}{aA}\int_{R_2}^{R_1} \frac{dR}{R} = \frac{L_S}{aA}\ln\frac{R_1}{R_2} \tag{9.7-2}$$

Since $R_1 = aX_1 + b$ and $R_2 = aX_2 + b$,

$$a = \frac{R_1 - R_2}{X_1 - X_2} \tag{9.7-3}$$

Substituting Eq. (9.7-3) into (9.7-2),

$$t = \frac{L_S(X_1 - X_2)}{A(R_1 - R_2)}\ln\frac{R_1}{R_2} \tag{9.7-4}$$

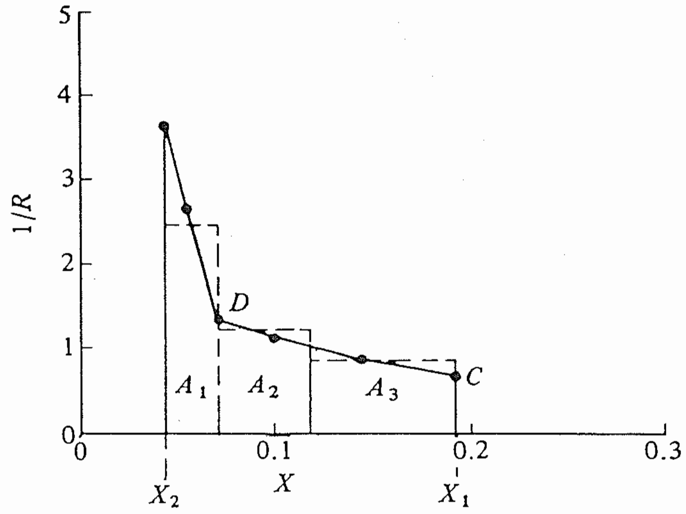


FIGURE 9.7-1. *Graphical integration for falling-rate period in Example 9.7-1.*

2. Rate is a linear function through origin. In some cases a straight line from the critical moisture content passing through the origin adequately represents the whole falling-rate period. In Fig. 9.5-1b this would be a straight line from C to E at the origin. Often for lack of more detailed data, this assumption is made. Then, for a straight line through the origin, where the rate of drying is directly proportional to the free moisture content,

$$R = aX \tag{9.7-5}$$

Differentiating, $dX = dR/a$. Substituting into Eq. (9.6-1),

$$t = \frac{L_S}{aA}\int_{R_2}^{R_1}\frac{dR}{R} = \frac{L_S}{aA}\ln\frac{R_1}{R_2} \tag{9.7-6}$$

The slope a of the line is R_C/X_C, and for $X_1 = X_C$ at $R_1 = R_C$,

$$t = \frac{L_S X_C}{AR_C}\ln\frac{R_C}{R_2} \tag{9.7-7}$$

Noting also that $R_C/R_2 = X_C/X_2$,

$$t = \frac{L_S X_C}{AR_C}\ln\frac{X_C}{X_2} \tag{9.7-8}$$

or

$$R = R_C\frac{X}{X_C} \tag{9.7-9}$$

EXAMPLE 9.7-2. Approximation of Straight Line for Falling-Rate Period
Repeat Example 9.7-1, but as an approximation assume a straight line of the rate R versus X through the origin from point X_C to $X = 0$ for the falling-rate period.

Solution: $R_C = 1.51$ kg $H_2O/h \cdot m^2$ and $X_C = 0.195$. Drying in the falling-rate region is from X_C to $X_2 = 0.040$. Substituting into Eq. (9.7-8),

$$t = \frac{L_S X_C}{AR_C} \ln \frac{X_C}{X_2} = \frac{399(0.195)}{18.58(1.51)} \ln \frac{0.195}{0.040}$$

$$= 4.39 \text{ h}$$

This value of 4.39 h compares with the value of 4.06 h obtained in Example 9.7-1 by graphical integration.

9.8 COMBINED CONVECTION, RADIATION, AND CONDUCTION HEAT TRANSFER IN CONSTANT-RATE PERIOD

9.8A Introduction

In Section 9.6B an equation was derived for predicting the rate of drying in the constant-rate period. Equation (9.6-7) was derived assuming heat transfer to the solid by convection only from the surrounding air to the drying surface. Often the drying is done in an enclosure, where the enclosure surface radiates heat to the drying solid. Also, in some cases the solid may be resting on a metal tray, and heat transfer by conduction through the metal to the bottom of the solid may occur.

9.8B Derivation of Equation for Convection, Conduction, and Radiation

In Fig. 9.8-1 a solid material being dried by a stream of air is shown. The total rate of heat transfer to the drying surface is

$$q = q_C + q_R + q_K \tag{9.8-1}$$

where q_C is the convective heat transfer from the gas at T°C to the solid surface at T_S°C in W (J/s), q_R is the radiant heat transfer from the surface at T_R to T_S in W (J/s), and q_K is the rate of heat transfer by conduction from the bottom in W. The rate of convective

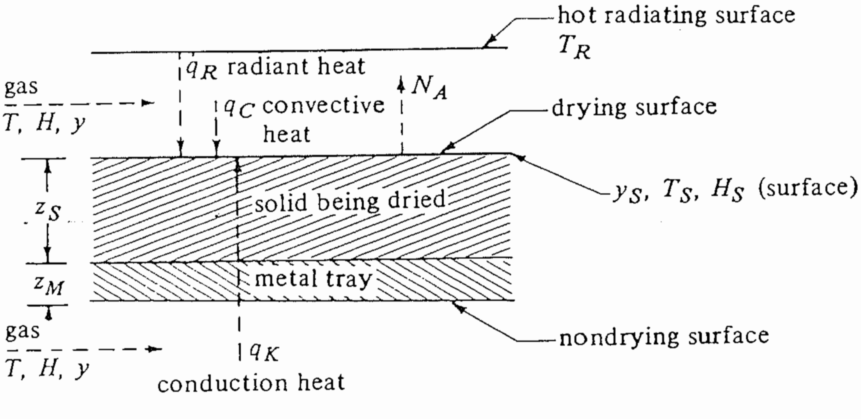


FIGURE 9.8-1. *Heat and mass transfer in drying a solid from the top surface.*

heat transfer is similar to Eq. (9.6-3) and is as follows, where $(T - T_S)°\text{C} = (T - T_S)\ \text{K}$,

$$q_C = h_C(T - T_S)A \tag{9.8-2}$$

where A is the exposed surface area in m^2. The radiant heat transfer is

$$q_R = h_R(T_R - T_S)A \tag{9.8-3}$$

where h_R is the radiant-heat-transfer coefficient defined by Eq. (4.10-10).

$$h_R = \varepsilon(5.676)\frac{\left(\frac{T_R}{100}\right)^4 - \left(\frac{T_S}{100}\right)^4}{T_R - T_S} \tag{4.10-10}$$

Note that in Eq. (4.10-10) T_R and T_S are in K. For the heat transfer by conduction from the bottom, the heat transfer is first by convection from the gas to the metal plate, then by conduction through the metal, and finally by conduction through the solid. Radiation to the bottom of the tray is often small if the tray is placed above another tray, and it will be neglected here. Also, if the gas temperatures are not too high, radiation from the top surface to the tray will be small. Hence, the heat by radiation should not be overemphasized. The heat by conduction is

$$q_K = U_K(T - T_S)A \tag{9.8-4}$$

$$U_K = \frac{1}{1/h_C + z_M/k_M + z_S/k_S} \tag{9.8-5}$$

where z_M is the metal thickness in m, k_M the metal thermal conductivity in W/m · K, z_S the solid thickness in m, and k_S the solid thermal conductivity. The value of h_C in Eq. (9.8-4) is assumed to be the same as in Eq. (9.8-2).

The equation for the rate of mass transfer is similar to Eq. (9.6-5) and is

$$N_A = k_y \frac{M_B}{M_A}(H_S - H) \tag{9.8-6}$$

Also, rewriting Eq. (9.6-6),

$$q = M_A N_A \lambda_S A \tag{9.8-7}$$

Combining Eqs. (9.8-1), (9.8-2), (9.8-3), (9.8-4), (9.8-6), and (9.8-7),

$$R_C = \frac{q}{A\lambda_S} = \frac{(h_C + U_K)(T - T_S) + h_R(T_R - T_S)}{\lambda_S} = k_y M_B(H_S - H) \tag{9.8-8}$$

This equation can be compared to Eq. (9.6-7), which gives the wet bulb temperature T_W when radiation and conduction are absent. Equation (9.8-8) gives surface temperature T_S greater than the wet bulb temperature T_W. Equation (9.8-8) must also intersect the saturated humidity line at T_S and H_S, and $T_S > T_W$ and $H_S > H_W$. The equation must be solved by trial and error.

To facilitate solution of Eq. (9.8-8), it can be rearranged (T1) to the following:

$$\frac{(H_S - H)\lambda_S}{h_C/k_y M_B} = \left(1 + \frac{U_K}{h_C}\right)(T - T_S) + \frac{h_R}{h_C}(T_R - T_S) \tag{9.8-9}$$

The ratio $h_C/k_y M_B$ was shown in the wet bulb derivation of Eq. (9.3-18) to be approximately c_S in Eq. (9.3-6).

$$c_S = (1.005 + 1.88H)10^3\ \text{J/kg} \cdot \text{K} \tag{9.3-6}$$

EXAMPLE 9.8-1. Constant-Rate Drying When Radiation and Convection Are Present

An insoluble granular material wet with water is being dried in a pan 0.457 × 0.457 m and 25.4 mm deep. The material is 25.4 mm deep in the metal pan, which has a metal bottom with thickness $z_M = 0.610$ mm having a thermal conductivity $k_M = 43.3$ W/m · K. The thermal conductivity of the solid can be assumed as $k_S = 0.865$ W/m · K. Heat transfer is by convection from an air stream flowing parallel to the top drying surface and the bottom metal surface at a velocity of 6.1 m/s and having a temperature of 65.6°C and humidity $H = 0.010$ kg H_2O/kg dry air. The top surface also receives direct radiation from steam-heated pipes whose surface temperature $T_R = 93.3$°C. The emissivity of the solid is $\varepsilon = 0.92$. Estimate the rate of drying for the constant-rate period.

Solution: Some of the given values are as follows:

$$T = 65.6°\text{C}, \qquad z_S = 0.0254 \text{ m}, \qquad k_M = 43.3, \qquad k_S = 0.865$$

$$z_M = 0.00061 \text{ m} \qquad \varepsilon = 0.92, \qquad H = 0.010$$

The velocity, temperature, and humidity of air are the same as Example 9.6-3 and the convective coefficient was predicted as $h_C = 62.45$ W/m^2 · K.

The solution of Eq. (9.8-9) is by trial and error. The temperature T_S will be above the wet bulb temperature of $T_W = 28.9$°C and will be estimated as $T_S = 32.2$°C. Then $\lambda_S = 2424$ kJ/kg from the steam tables. To predict h_R from Eq. (4.10-10) for $\varepsilon = 0.92$, $T_I = 93.3 + 273.2 = 366.5$ K, and $T_2 = 32.2 + 273.2 = 305.4$ K,

$$h_R = (0.92)(5.676)\frac{(366.5/100)^4 - (305.4/100)^4}{366.5 - 305.4} = 7.96 \text{ W/m}^2 \cdot \text{K}$$

Using Eq. (9.8-5),

$$U_K = \frac{1}{1/h_C + z_M/k_M + z_S/k_S} = \frac{1}{1/62.45 + 0.00061/43.3 + 0.0254/0.865}$$

$$= 22.04 \text{ W/m}^2 \cdot \text{K}$$

From Eq. (9.3-6),

$$c_S = (1.005 + 1.88\text{H})10^3 = (1.005 + 1.88 \times 0.010)10^3$$

$$= 1.024 \times 10^3 \text{ J/kg} \cdot \text{K}$$

This can be substituted for $(h_C/k_y M_B)$ into Eq. (9.8-9). Also, substituting other knowns,

$$\frac{(H_S - 0.01)\lambda_S}{1.024 \times 10^3} = (1 + 22.04/62.45)(65.6 - T_S)$$

$$+ (7.96/62.45)(93.3 - T_S)$$

$$= 1.353(65.6 - T_S) + 0.1275(93.3 - T_S) \qquad \textbf{(9.8-10)}$$

For T_S assumed as 32.2°C, $\lambda_S = 2424 \times 10^3$ J/kg. From the humidity chart for $T_S = 32.2$°C, the saturation humidity $H_S = 0.031$. Substituting into Eq. (9.8-10) and solving for T_S,

$$\frac{(0.031 - 0.010)(2424 \times 10^3)}{1.024 \times 10^3} = 1.353(65.6 - T_S) + 0.1275(93.3 - T_S)$$

$$T_S = 34.4°\text{C}$$

For the second trial, assuming that $T_S = 32.5°C$, $\lambda_S = 2423 \times 10^3$ and H_S from the humidity chart at saturation is 0.032. Substituting into Eq. (9.8-10) while assuming that h_R does not change appreciably, a value of $T_S = 32.8°C$ is obtained. Hence, the final value is 32.8°C. This is 3.9°C greater than the wet bulb temperature of 28.9°C in Example 9.6-3, where radiation and conduction were absent.

Using Eq. (9.8-8),

$$R_C = \frac{(h_C + U_K)(T - T_S) + h_R(T_R - T_S)}{\lambda_S}(3600)$$

$$= \frac{(62.45 + 22.04)(65.6 - 32.8) + 7.96(93.3 - 32.8)}{2423 \times 10^3}(3600)$$

$$= 4.83 \text{ kg/h} \cdot \text{m}^2$$

This compares with 3.39 kg/h · m^2 for Example 9.6-3 for no radiation or conduction.

9.9 DRYING IN FALLING-RATE PERIOD BY DIFFUSION AND CAPILLARY FLOW

9.9A Introduction

In the falling-rate period, the surface of the solid being dried is no longer completely wetted, and the rate of drying steadily falls with time. In Section 9.7 empirical methods were used to predict the time of drying. In one method the actual rate of drying curve was graphically integrated to determine the time of drying.

In another method an approximate straight line from the critical free moisture content to the origin at zero free moisture was assumed. Here the rate of drying was assumed to be a linear function of the free moisture content. The rate of drying R is defined by Eq. (9.5-3).

$$R = -\frac{L_S}{A}\frac{dX}{dt} \tag{9.5-3}$$

When R is a linear function of X in the falling-rate period,

$$R = aX \tag{9.7-5}$$

where a is a constant. Equating Eq. (9.7-5) to Eq. (9.5-3),

$$R = -\frac{L_S}{A}\frac{dX}{dt} = aX \tag{9.9-1}$$

Rearranging,

$$\frac{dX}{dt} = -\frac{aA}{L_S}X \tag{9.9-2}$$

In many instances, however, as mentioned briefly in Section 9.5E, the rate of moisture movement in the falling-rate period is governed by the rate of internal movement of the liquid by liquid diffusion or by capillary movement. These two methods of moisture movement will be considered in more detail and the theories related to experimental data in the falling-rate region.

9.9B Liquid Diffusion of Moisture in Drying

When liquid diffusion of moisture controls the rate of drying in the falling-rate period, the equations for diffusion described in Chapter 7 can be used. Using the concentrations as X kg free moisture/kg dry solid instead of concentrations kg mol moisture/m^3, Fick's second law for unsteady-state diffusion, Eq. (7.10-10), can be written as

$$\frac{\partial X}{\partial t} = D_L \frac{\partial^2 X}{\partial x^2} \tag{9.9-3}$$

where D_L is the liquid diffusion coefficient in m^2/h and x is distance in the solid in m.

This type of diffusion is often characteristic of relatively slow drying in nongranular materials such as soap, gelatin, and glue, and in the later stages of drying of bound water in clay, wood, textiles, leather, paper, foods, starches, and other hydrophilic solids.

A major difficulty in analyzing diffusion drying data is that the initial moisture distribution is not uniform throughout the solid at the start if a drying period at constant rate precedes this falling-rate period. During diffusion-type drying, the resistance to mass transfer of water vapor from the surface is usually very small, and the diffusion in the solid controls the rate of drying. Then the moisture content at the surface is at the equilibrium value X^*. This means that the free moisture content X at the surface is essentially zero.

Assuming that the initial moisture distribution is uniform at $t = 0$, Eq. (9.9-3) may be integrated by the methods in Chapter 7 to give the following:

$$\frac{X_t - X^*}{X_{t1} - X^*} = \frac{X}{X_1} = \frac{8}{\pi^2}\left[e^{-D_L t(\pi/2x_1)^2} + \tfrac{1}{9}e^{-9D_L t(\pi/2x_1)^2} + \tfrac{1}{25}e^{-25D_L t(\pi/2x_1)^2} + \cdots\right] \tag{9.9-4}$$

where X = average free moisture content at time t h, X_1 = initial free moisture content at $t = 0$, X^* = equilibrium free moisture content, $x_1 = \frac{1}{2}$ the thickness of the slab when drying occurs from the top and the bottom parallel faces, and x_1 = total thickness of slab if drying only from the top face.

Equation (9.9-4) assumes that D_L is constant, but D_L is rarely constant; it varies with moisture content, temperature, and humidity. For long drying times, only the first term in Eq. (9.9-4) is significant, and the equation becomes

$$\frac{X}{X_1} = \frac{8}{\pi^2} e^{-D_L t(\pi/2x_1)^2} \tag{9.9-5}$$

Solving for the time of drying,

$$t = \frac{4x_1^2}{\pi^2 D_L} \ln \frac{8X_1}{\pi^2 X} \tag{9.9-6}$$

In this equation if the diffusion mechanism starts at $X = X_C$, then $X_1 = X_C$. Differentiating Eq. (9.9-6) with respect to time and rearranging,

$$\frac{dX}{dt} = -\frac{\pi^2 D_L X}{4x_1^2} \tag{9.9-7}$$

Multiplying both sides by $-L_S/A$,

$$R = -\frac{L_S}{A}\frac{dX}{dt} = \frac{\pi^2 L_S D_L}{4x_1^2 A} X \tag{9.9-8}$$

Hence, Eqs. (9.9-7) and (9.9-8) state that when internal diffusion controls for long times, the rate of drying is directly proportional to the free moisture X and the liquid diffusivity and that the rate of drying is inversely proportional to the thickness squared.

Or, stated as the time of drying between fixed moisture limits, the time varies directly as the square of the thickness. The drying rate should be independent of gas velocity and humidity.

EXAMPLE 9.9-1. Drying Slabs of Wood When Diffusion of Moisture Controls

The experimental average diffusion coefficient of moisture in a given wood is $2.97 \times 10^{-6}\ m^2/h$ ($3.20 \times 10^{-5}\ ft^2/h$). Large planks of wood 25.4 mm thick are dried from both sides by air having a humidity such that the equilibrium moisture content in the wood is $X^* = 0.04$ kg H_2O/kg dry wood. The wood is to be dried from a total average moisture content of $X_{t1} = 0.29$ to $X_t = 0.09$. Calculate the time needed.

Solution: The free moisture content $X_1 = X_{t1} - X^* = 0.29 - 0.04 = 0.25$, $X = X_t - X^* = 0.09 - 0.04 = 0.05$. The half-slab thickness $x_1 = 25.4/(2 \times 1000) = 0.0127$ m. Substituting into Eq. (9.9-6),

$$t = \frac{4x_1^2}{\pi^2 D_L} \ln \frac{8X_1}{\pi^2 X} = \frac{4(0.0127)^2}{\pi^2(2.97 \times 10^{-6})} \ln \frac{8 \times 0.25}{\pi^2 \times 0.05}$$

$$= 30.8 \text{ h}$$

Alternatively, Fig. 5.3-13 for the average concentration in a slab can be used. The ordinate $E_a = X/X_1 = 0.05/0.25 = 0.20$. Reading off the plot a value of $0.56 = D_L t/x_1^2$, substituting, and solving for t,

$$t = \frac{x_1^2(0.56)}{D_L} = \frac{(0.0127)^2(0.56)}{2.97 \times 10^{-6}} = 30.4 \text{ h}$$

9.9C Capillary Movement of Moisture in Drying

Water can flow from regions of high concentrations to those of low concentrations as a result of capillary action rather than by diffusion if the pore sizes of granular materials are suitable.

The capillary theory (P1) assumes that a packed bed of nonporous spheres contains a void space between the spheres called *pores*. As water is evaporated, capillary forces are set up by the interfacial tension between the water and solid. These forces provide the driving force for moving the water through the pores to the drying surface.

A modified form of Poiseuille's equation for laminar flow can be used in conjunction with the capillary-force equation to derive an equation for the rate of drying when flow is by capillary movement. If the moisture movement follows the capillary-flow equations, the rate of drying R will vary linearly with X. Since the mechanism of evaporation during this period is the same as in the constant-rate period, the effects of the variables of the drying gas of gas velocity, temperature of the gas, humidity of the gas, and so on, will be the same as for the constant-rate drying period.

The defining equation for the rate of drying is

$$R = -\frac{L_S}{A}\frac{dX}{dt} \tag{9.5-3}$$

For the rate R varying linearly with X given previously,

$$R = R_C \frac{X}{X_C} \tag{9.7-9}$$

$$t = \frac{L_S X_C}{A R_C} \ln \frac{X_C}{X_2} \tag{9.7-8}$$

We define t as the time when $X = X_2$ and

$$L_S = x_1 A \rho_S \tag{9.9-9}$$

where ρ_S = solid density kg dry solid/m^3. Substituting Eq. (9.9-9) and $X = X_2$ into Eq. (9.7-8),

$$t = \frac{x_1 \rho_S X_C}{R_C} \ln \frac{X_C}{X} \tag{9.9-10}$$

Substituting Eq. (9.6-7) for R_C,

$$t = \frac{x_1 \rho_S \lambda_W X_C}{h(T - T_W)} \ln \frac{X_C}{X} \tag{9.9-11}$$

Hence, Eqs. (9.9-10) and (9.9-11) state that when capillary flow controls in the falling-rate period, the rate of drying is inversely proportional to the thickness. The time of drying between fixed moisture limits varies directly as the thickness and depends upon the gas velocity, temperature, and humidity.

9.9D Comparison of Liquid Diffusion and Capillary Flow

To determine the mechanism of drying in the falling-rate period, the experimental data obtained of moisture content at various times using constant drying conditions are often analyzed as follows. The *unaccomplished moisture change*, defined as the ratio of free moisture present in the solid after drying for t hours to the total free moisture content present at the start of the falling-rate period, X/X_C, is plotted versus time on semilog paper. If a straight line is obtained, such as curve B in Fig. 9.9-1 using the upper scale for the abscissa, then either Eqs. (9.9-4)–(9.9-6) for diffusion are applicable or Eqs. (9.9-10) and (9.9-11) for capillary flow are applicable.

If the relation for capillary flow applies, the slope of the falling-rate drying line B in Fig. 9.9-1 is related to Eq. (9.9-10), which contains the constant drying rate R_C. The value of R_C is calculated from the measured slope of the line, which is $-R_C/x_1\rho_S X_C$, and if it agrees with the experimental value of R_C in the constant drying period or the predicted value of R_C, the moisture movement is by capillary flow.

If the values of R_C do not agree, the moisture movement is by diffusion and the slope of line B in Fig. 9.9-1 from Eq. (9.9-6) should equal $-\pi^2 D_L/4x_1^2$. In actual practice, however, the diffusivity D_L is usually less at small moisture contents than at large moisture contents, and an average value of D_L is usually determined experimentally over the moisture range of interest. A plot of Eq. (9.9-4) is shown as line A, where $\ln(X/X_1)$ or $\ln(X/X_C)$ is plotted versus $D_L t/x_1^2$. This is the same plot as Fig. 5.3-13 for a slab and shows a curvature in the line for values of X/X_C between 1.0 and 0.6 and a straight line for $X/X_C < 0.6$.

When the experimental data show that the movement of moisture follows the diffusion law, the average experimental diffusivities can be calculated as follows for different concentration ranges. A value of X/X_C is chosen at 0.4, for example. From an experimental plot similar to curve B, Fig. 9.9-1, the experimental value of t is obtained. From curve A at $X/X_C = 0.4$, the theoretical value of $(D_L t/x_1^2)_{\text{theor}}$ is obtained. Then, by substituting the known values of t and x_1 into Eq. (9.9-12), the experimental average value of D_L over the range X/X_C = 1.0–0.4 is obtained.

$$D_L = \left(\frac{D_L t}{x_1^2}\right)_{\text{theor}} \frac{x_1^2}{t} \tag{9.9-12}$$

This is repeated for various values of X/X_C. Values of D_L obtained for $X/X_C > 0.6$ are in error because of the curvature of line A.

EXAMPLE 9.9-2. Diffusion Coefficient in the Tapioca Root

Tapioca flour is obtained from drying and then milling the tapioca root. Experimental data on drying thin slices of the tapioca root 3 mm thick on both sides in the falling-rate period under constant drying conditions are tabulated below. The time $t = 0$ is the start of the falling-rate period.

X/X_C	t (h)	X/X_C	t (h)	X/X_C	t (h)
1.0	0	0.55	0.40	0.23	0.94
0.80	0.15	0.40	0.60	0.18	1.07
0.63	0.27	0.30	0.80		

It has been determined that the data do not follow the capillary-flow equation but appear to follow the diffusion equation. Plot the data as X/X_C versus t on semilog coordinates and determine the average diffusivity of the moisture up to a value of $X/X_C = 0.20$.

Solution: In Fig. 9.9-2 the data are plotted as X/X_C on the log scale versus t on a linear scale and a smooth curve drawn through the data. At $X/X_C =$

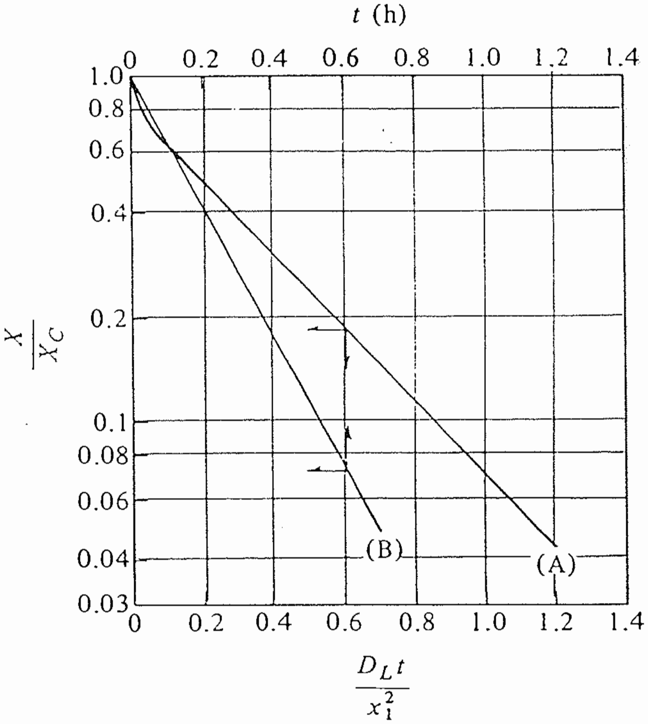


FIGURE 9.9-1. *Plot of equations for falling-rate period: (A) Eq. (9.9-4) for moisture movement by diffusion, (B) Eq. (9.9-10) for moisture movement by capillary flow. (From R. H. Perry and C. H. Chilton, Chemical Engineers Handbook, 5th ed. New York: McGraw-Hill Book Company, 1973. With permission.)*

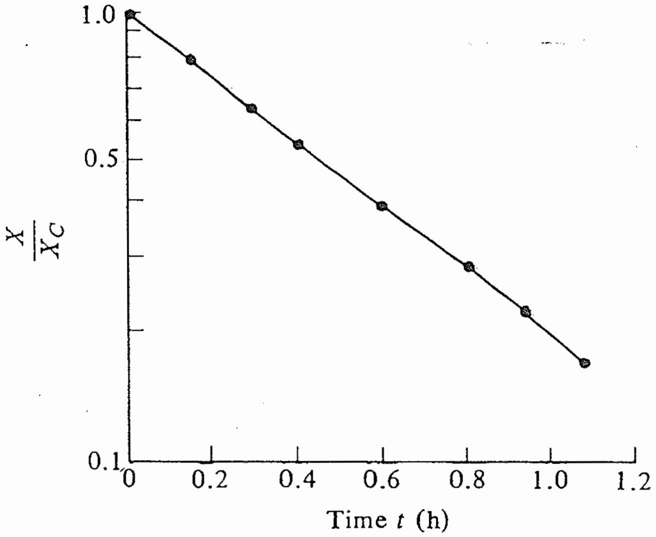


FIGURE 9.9-2. *Plot of drying data for Example 9.9-2.*

0.20, a value of $t = 1.02$ h is read off the plot. The value of $x_1 = 3$ mm/2 = 1.5 mm for drying from both sides. From Fig. 9.9-1, line A, for $X/X_C = 0.20$, $(D_L t/x_1^2)_{\text{theor}} = 0.56$. Then substituting into Eq. (9.9-12),

$$D_L = (D_L t/x_1^2)_{\text{theor}} \frac{x_1^2}{t} = \frac{0.56(1.5/1000)^2}{1.02 \times 3600} = 3.44 \times 10^{-10} \text{ m}^2/\text{s}$$

9.10 EQUATIONS FOR VARIOUS TYPES OF DRYERS

9.10A Through Circulation Drying in Packed Beds

For through circulation drying where the drying gas passes upward or downward through a bed of wet granular solids, both a constant-rate period and a falling-rate period of drying may result. Often the granular solids are arranged on a screen so that the gas passes through the screen and through the open spaces or voids between the solid particles.

1. Derivation of equations. To derive the equations for this case, no heat losses will be assumed, so the system is adiabatic. The drying will be for unbound moisture in the wet granular solids. We shall consider a bed of uniform cross-sectional area A m^2, where a gas flow of G kg dry gas/h · m^2 cross section enters with a humidity of H_1. By a material balance on the gas at a given time, the gas leaves the bed with a humidity H_2. The amount of water removed from the bed by the gas is equal to the rate of drying at this time.

$$R = G(H_2 - H_1) \tag{9.10-1}$$

where R = kg H$_2$O/h · m^2 cross section and G = kg dry air/h · m^2 cross section.

In Fig. 9.10-1 the gas enters at T_1 and H_1 and leaves at T_2 and H_2. Hence, the temperature T and humidity H both vary through the bed. Making a heat balance over the short section dz m of the bed,

$$dq = -Gc_S A\ dT \tag{9.10-2}$$

where A = m^2 cross-sectional area, q is the heat-transfer rate in W (J/s), and c_S is the humid heat of the air–water vapor mixture in Eq. (9.3-6). Note that G in this equation is in kg/s · m^2. The heat-transfer equation gives

$$dq = haA\ dz(T - T_W) \tag{9.10-3}$$

where T_W = wet bulb temperature of solid, h is the heat-transfer coefficient in W/m^2 · K, and a is m^2 surface area of solids/m^3 bed volume. Equating Eq. (9.10-2) to (9.10-3), rearranging, and integrating,

$$\frac{ha}{Gc_S}\int_0^z dz = -\int_{T_1}^{T_2} \frac{dT}{T - T_W} \tag{9.10-4}$$

$$\frac{haz}{Gc_S} = \ln \frac{T_1 - T_W}{T_2 - T_W} \tag{9.10-5}$$

where z = bed thickness = x_1 m.

For the constant-rate period of drying by air flowing parallel to a surface, Eq. (9.6-11) was derived.

$$t = \frac{L_S \lambda_W (X_1 - X_2)}{Ah(T - T_W)} = \frac{L_S(X_1 - X_2)}{Ak_y M_B(H_W - H)} \tag{9.6-11}$$

Using Eq. (9.9-9) and the definition of a, we obtain

$$\frac{L_S}{A} = \frac{\rho_S}{a} \tag{9.10-6}$$

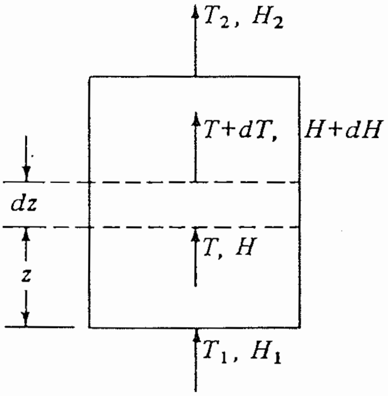
Substituting Eq. (9.10-6) into (9.6-11) and setting $X_2 = X_C$ for drying to X_C, we obtain the equation for through circulation drying in the constant-rate period.

$$t = \frac{\rho_S \lambda_W (X_1 - X_C)}{ah(T - T_W)} = \frac{\rho_S(X_1 - X_C)}{ak_y M_B(H_W - H)} \tag{9.10-7}$$

In a similar manner, Eq. (9.7-8) for the falling-rate period, which assumes that R is proportional to X, becomes, for through circulation drying,

$$t = \frac{\rho_S \lambda_W X_C \ln (X_C/X)}{ah(T - T_W)} = \frac{\rho_S X_C \ln (X_C/X)}{ak_y M_B(H_W - H)} \tag{9.10-8}$$

FIGURE 9.10-1. *Heat and material balances in a through circulation dryer in a packed bed.*



Both Eqs. (9.10-7) and (9.10-8), however, hold only for one point in the bed in Fig. 9.10-1; since the temperature T of the gas varies throughout the bed. Hence, in a manner similar to the derivation in heat transfer, a log mean temperature difference can be used as an approximation for the whole bed in place of $T - T_W$ in Eqs. (9.10-7) and (9.10-8).

$$(T - T_W)_{LM} = \frac{(T_1 - T_W) - (T_2 - T_W)}{\ln [(T_1 - T_W)/(T_2 - T_W)]} = \frac{T_1 - T_2}{\ln [(T_1 - T_W)/(T_2 - T_W)]} \tag{9.10-9}$$

Substituting Eq. (9.10-5) for the denominator of Eq. (9.10-9) and also substituting the value of T_2 from Eq. (9.10-5) into (9.10-9),

$$(T - T_W)_{LM} = \frac{(T_1 - T_W)(1 - e^{-haz/Gc_S})}{haz/Gc_S} \tag{9.10-10}$$

Substituting Eq. (9.10-10) into (9.10-7) for the constant-rate period and setting $x_1 = z$,

$$t = \frac{\rho_S \lambda_W x_1(X_1 - X_C)}{Gc_S(T_1 - T_W)(1 - e^{-hax_1/Gc_S})} \tag{9.10-11}$$

Similarly, for the falling-rate period an approximate equation is obtained.

$$t = \frac{\rho_S \lambda_W x_1 X_C \ln (X_C/X)}{Gc_S(T_1 - T_W)(1 - e^{-hax_1/Gc_S})} \tag{9.10-12}$$

A major difficulty with the use of Eq. (9.10-12) is that the critical moisture content is not easily estimated. Different forms of Eqs. (9.10-11) and (9.10-12) can also be derived, using humidity instead of temperature (T1).

2. Heat-transfer coefficients. For through circulation drying, where the gases pass through a bed of wet granular solids, the following equations for estimating h for adiabatic evaporation of water can be used (G1, W1).

$$\left.\begin{aligned} h &= 0.151 \frac{G_t^{0.59}}{D_p^{0.41}} \quad \text{(SI)} \\ h &= 0.11 \frac{G_t^{0.59}}{D_p^{0.41}} \quad \text{(English)} \end{aligned}\right\} \frac{D_p G_t}{\mu} > 350 \tag{9.10-13}$$

$$\left.\begin{aligned} h &= 0.214 \frac{G_t^{0.49}}{D_p^{0.51}} \quad \text{(SI)} \\ h &= 0.15 \frac{G_t^{0.49}}{D_p^{0.51}} \quad \text{(English)} \end{aligned}\right\} \frac{D_p G_t}{\mu} < 350 \tag{9.10-14}$$

where h is in W/m$^2 \cdot$ K, D_p is diameter in m of a sphere having the same surface area as the particle in the bed, G_t is the total mass velocity entering the bed in kg/h $\cdot$ m^2, and μ is viscosity in kg/m $\cdot$ h. In English units, h is btu/h $\cdot$ ft$^2 \cdot$ °F, D_p is ft, G_t is lb$_m$/h $\cdot$ ft^2, and μ is lb$_m$/ft $\cdot$ h.

3. Geometry factors in a bed. To determine the value of a, m^2 surface area/m^3 of bed, in a packed bed for spherical particles having a diameter D_p m,

$$a = \frac{6(1 - \varepsilon)}{D_p} \tag{9.10-15}$$

where ε is the void fraction in the bed. For cylindrical particles,

$$a = \frac{4(1-\varepsilon)(h + 0.5D_c)}{D_c h} \tag{9.10-16}$$

where D_c is diameter of cylinder in m and h is length of cylinder in m. The value of D_p to use in Eqs. (9.10-13) and (9.10-14) for a cylinder is the diameter of a sphere having the same surface area as the cylinder, as follows:

$$D_p = (D_c h + 0.5D_c^2)^{1/2} \tag{9.10-17}$$

4. Equations for very fine particles. The equations derived for the constant- and falling-rate periods in packed beds hold for particles of about 3–19 mm in diameter in shallow beds about 10–65 mm thick (T1, M1). For very fine particles of 10–200 mesh (1.66–0.079 mm) and bed depth greater than 11 mm, the interfacial area a varies with the moisture content. Empirical expressions are available to estimate a and the mass-transfer coefficient (T1, A1).

EXAMPLE 9.10-1. Through Circulation Drying in a Bed

A granular paste material is extruded into cylinders with a diameter of 6.35 mm and length of 25.4 mm. The initial total moisture content $X_{t1} = 1.0$ kg H_2O/kg dry solid and the equilibrium moisture is $X^* = 0.01$. The density of the dry solid is 1602 kg/m^3 (100 lb$_m$/ft^3). The cylinders are packed on a screen to a depth of $x_1 = 50.8$ mm. The bulk density of the dry solid in the bed is $\rho_S = 641$ kg/m^3. The inlet air has a humidity $H_1 = 0.04$ kg H_2O/kg dry air and a temperature $T_1 = 121.1°C$. The gas superficial velocity is 0.811 m/s and the gas passes through the bed. The total critical moisture content is $X_{tC} = 0.50$. Calculate the total time to dry the solids to $X_t = 0.10$ kg H_2O/kg dry solid.

Solution: For the solid,

$$X_1 = X_{t1} - X^* = 1.00 - 0.01 = 0.99 \text{ kg } H_2O\text{/kg dry solid}$$

$$X_C = X_{tC} - X^* = 0.50 - 0.01 = 0.49$$

$$X = X_t - X^* = 0.10 - 0.01 = 0.09$$

For the gas, $T_1 = 121.1°C$ and $H_1 = 0.04$ kg H_2O/kg dry air. The wet bulb temperature $T_W = 47.2°C$ and $H_W = 0.074$. The solid temperature is at T_W if radiation and conduction are neglected. The density of the entering air at 121.1°C and 1 atm is as follows.

$$v_H = (2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.04)(273 + 121.1) \quad \textbf{(9.3-7)}$$

$$= 1.187 \text{ m}^3\text{/kg dry air}$$

$$\rho = \frac{1.00 + 0.04}{1.187} = 0.876 \text{ kg dry air} + H_2O\text{/m}^3$$

The mass velocity of the dry air is

$$G = v\rho\left(\frac{1.0}{1.0 + 0.04}\right) = 0.811(3600)(0.876)\left(\frac{1}{1.04}\right) = 2459 \text{ kg dry air/h} \cdot \text{m}^2$$

Since the inlet $H_1 = 0.040$ and the outlet will be less than 0.074, an approximate average H of 0.05 will be used to calculate the total average mass velocity. The approximate average G_t is

$$G_t = 2459 + 2459(0.05) = 2582 \text{ kg air} + H_2O\text{/h} \cdot \text{m}^2$$

For the packed bed, the void fraction ε is calculated as follows for 1 m^3 of bed containing solids plus voids. A total of 641 kg dry solid is present. The density of the dry solid is 1602 kg dry solid/m^3 solid. The volume of the solids in 1 m^3 of bed is then 641/1602, or 0.40 m^3 solid. Hence, $\varepsilon = 1 - 0.40 = 0.60$. The solid cylinder length $h = 0.0254$ m. The diameter $D_c = 0.00635$ m. Substituting into Eq. (9.10-16),

$$a = \frac{4(1-\varepsilon)(h + 0.5D_c)}{D_c h} = \frac{4(1-0.6)[0.0254 + 0.5(0.00635)]}{0.00635(0.0254)}$$

$$= 283.5 \text{ m}^2 \text{ surface area/m}^3 \text{ bed volume}$$

To calculate the diameter D_p of a sphere with the same area as the cylinder using Eq. (9.10-17),

$$D_p = (D_c h + 0.5D_c^2)^{1/2} = [0.00635 \times 0.0254 + 0.5(0.00635)^2]^{1/2}$$

$$= 0.0135 \text{ m}$$

The bed thickness $x_1 = 50.8$ mm $= 0.0508$ m.

To calculate the heat-transfer coefficient, the Reynolds number is first calculated. Assuming an approximate average air temperature of 93.3°C, the viscosity of air is $\mu = 2.15 \times 10^{-5}$ kg/m · s $= 2.15 \times 10^{-5}$ (3600) $= 7.74 \times 10^{-2}$ kg/m · h. The Reynolds number is

$$N_{\text{Re}} = \frac{D_p G_t}{\mu} = \frac{0.0135(2582)}{7.74 \times 10^{-2}} = 450$$

Using Eq. (9.10-13),

$$h = 0.151 \frac{G_t^{0.59}}{D_p^{0.41}} = \frac{0.151(2582)^{0.51}}{(0.0135)^{0.41}} = 90.9 \text{ W/m}^2 \cdot \text{K}$$

For $T_W = 47.2$°C, $\lambda_W = 2389$ kJ/kg, or 2.389×10^6 J/kg (1027 btu/lb_m), from steam tables. The average humid heat, from Eq. (9.3-6), is

$$c_S = 1.005 + 1.88H = 1.005 + 1.88(0.05) = 1.099 \text{ kJ/kg dry air} \cdot \text{K}$$

$$= 1.099 \times 10^3 \text{ J/kg} \cdot \text{K}$$

To calculate the time of drying for the constant-rate period using Eq. (9.10-11) and $G = 2459/3600 = 0.6831$ kg/s · m^2,

$$t = \frac{\rho_S \lambda_W x_1 (X_1 - X_C)}{G c_S (T_1 - T_W)(1 - e^{-hax_1/Gc_S})}$$

$$= \frac{641(2.389 \times 10^6)(0.0508)(0.99 - 0.49)}{(0.683)(1.099 \times 10^3)(121.1 - 47.2)[1 - e^{-(90.9 \times 283.5 \times 0.0508)/(0.683 \times 1.099 \times 10^3)}]}$$

$$= 850 \text{ s} = 0.236 \text{ h}$$

For the time of drying for the falling-rate period, using Eq. (9.10-12),

$$t = \frac{\rho_S \lambda_W x_1 X_C \ln (X_C/X)}{G c_S (T_1 - T_W)(1 - e^{-hax_1/Gc_S})}$$

$$= \frac{641(2.389 \times 10^6)(0.0508)(0.49) \ln (0.49/0.09)}{(0.6831)(1.099 \times 10^3)(121.1 - 47.2)[1 - e^{-(90.9 \times 283.5 \times 0.0508)/(0.683 \times 1.099 \times 10^3)}]}$$

$$= 1412 \text{ s} = 0.392 \text{ h}$$

$$\text{total time } t = 0.236 + 0.392 = 0.628 \text{ h}$$

9.10B Tray Drying with Varying Air Conditions

For drying in a compartment or tray dryer where the air passes in parallel flow over the surface of the tray, the air conditions do not remain constant. Heat and material balances similar to those for through circulation must be made to determine the exit-gas temperature and humidity.

In Fig. 9.10-2 air is shown passing over a tray. It enters having a temperature of T_1 and humidity H_1 and leaves at T_2 and H_2. The spacing between the trays is b m and dry air flow is G kg dry air/s · m² cross-sectional area. Writing a heat balance over a length dL_t of tray for a section 1 m wide,

$$dq = Gc_S(1 \times b)\,dT \tag{9.10-18}$$

The heat-transfer equation is

$$dq = h(1 \times dL_t)(T - T_W) \tag{9.10-19}$$

Rearranging and integrating,

$$\frac{hL_t}{Gc_S b} = \ln \frac{T_1 - T_W}{T_2 - T_W} \tag{9.10-20}$$

Defining a log mean temperature difference similar to Eq. (9.10-10) and substituting into Eqs. (9.6-11) and (9.7-8), we obtain the following. For the constant-rate period,

$$t = \frac{x_1 \rho_S L_t \lambda_W (X_1 - X_C)}{Gc_S b(T_1 - T_W)(1 - e^{-hL_t/Gc_S b})} \tag{9.10-21}$$

For the falling-rate period, an approximate equation is obtained.

$$t = \frac{x_1 \rho_S L_t \lambda_W X_C \ln (X_C/X)}{Gc_S b(T_1 - T_W)(1 - e^{-hL_t/Gc_S b})} \tag{9.10-22}$$

9.10C Material and Heat Balances for Continuous Dryers

1. Simple heat and material balances. In Fig. 9.10-3 a flow diagram is given for a continuous-type dryer where the drying gas flows countercurrently to the solids flow. The solid enters at a rate of L_S kg dry solid/h, having a free moisture content X_1 and a

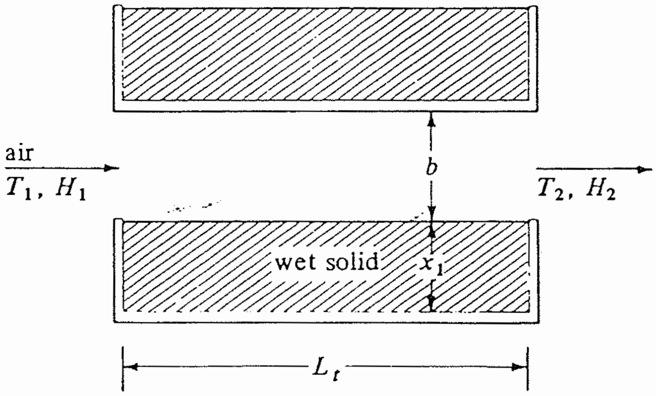
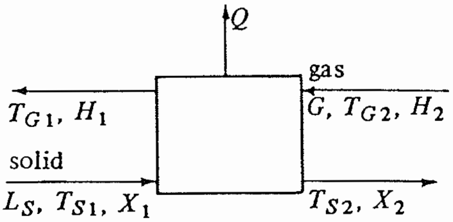


FIGURE 9.10-2. *Heat and material balances in a tray dryer.*

FIGURE 9.10-3. *Process flow for a counter-current continuous dryer.*



temperature T_{S1}. It leaves at X_2 and T_{S2}. The gas enters at a rate G kg dry air/h, having a humidity H_2 kg H_2O/kg dry air and a temperature of T_{G2}. The gas leaves at T_{G1} and H_1.

For a material balance on the moisture,

$$GH_2 + L_S X_1 = GH_1 + L_S X_2 \tag{9.10-23}$$

For a heat balance a datum of T_0 °C is selected. A convenient temperature is 0°C (32°F). The enthalpy of the wet solid is composed of the enthalpy of the dry solid plus that of the liquid as free moisture. The heat of wetting is usually neglected. The enthalpy of the gas H'_G in kJ/kg dry air is

$$H'_G = c_S(T_G - T_0) + H\lambda_0 \tag{9.10-24}$$

where λ_0 is the latent heat of water at T_0 °C, 2501 kJ/kg (1075.4 btu/lb$_m$) at 0°C, and c_S is the humid heat, given as kJ/kg dry air · K.

$$c_S = 1.005 + 1.88H \tag{9.3-6}$$

The enthalpy of the wet solid H'_S in kJ/kg dry solid, where $(T_S - T_0)$°C $= (T_S - T_0)$ K, is

$$H'_S = c_{pS}(T_S - T_0) + Xc_{pA}(T_S - T_0) \tag{9.10-25}$$

where c_{pS} is the heat capacity of the dry solid in kJ/kg dry solid · K and c_{pA} is the heat capacity of liquid moisture in kJ/kg H_2O · K. The heat of wetting or adsorption is neglected.

A heat balance on the dryer is

$$GH'_{G2} + L_S H'_{S1} = GH'_{G1} + L_S H'_{S2} + Q \tag{9.10-26}$$

where Q is the heat loss in the dryer in kJ/h. For an adiabatic process $Q = 0$, and if heat is added, Q is negative.

EXAMPLE 9.10-2. Heat Balance on a Dryer

A continuous countercurrent dryer is being used to dry 453.6 kg dry solid/h containing 0.04 kg total moisture/kg dry solid to a value of 0.002 kg total moisture/kg dry solid. The granular solid enters at 26.7°C and is to be discharged at 62.8°C. The dry solid has a heat capacity of 1.465 kJ/kg · K, which is assumed constant. Heating air enters at 93.3°C, having a humidity of 0.010 kg H_2O/kg dry air, and is to leave at 37.8°C. Calculate the air flow rate and the outlet humidity, assuming no heat losses in the dryer.

Solution: The flow diagram is given in Fig. 9.10-3. For the solid, $L_S =$ 453.6 kg/h dry solid, $c_{pS} = 1.465$ kJ/kg dry solid · K, $X_1 = 0.040$ kg H_2O/kg dry solid, $c_{pA} = 4.187$ kJ/kg H_2O · K, $T_{S1} = 26.7$°C, $T_{S2} = 62.8$°C, $X_2 =$ 0.002. (Note that X values used are X_t values.) For the gas, $T_{G2} = 93.3$°C, $H_2 = 0.010$ kg H_2O/kg dry air, and $T_{G1} = 37.8$°C.

Making a material balance on the moisture using Eq. (9.10-23),

$$GH_2 + L_S X_1 = GH_1 + L_S X_2$$

$$G(0.010) + 453.6(0.040) = GH_1 + 453.6(0.002) \tag{9.10-27}$$

For the heat balance, the enthalpy of the entering gas at 93.3°C using 0°C as a datum is, by Eq. (9.10-24), $\Delta T\,°\mathrm{C} = \Delta T$ K, and $\lambda_0 = 2501$ kJ/kg, from the steam tables,

$$\begin{aligned} H'_{G2} &= c_S(T_{G2} - T_0) + H_2\lambda_0 \\ &= [1.005 + 1.88(0.010)](93.3 - 0) + 0.010(2501) \\ &= 120.5 \text{ kJ/kg dry air} \end{aligned}$$

For the exit gas,

$$\begin{aligned} H'_{G1} &= c_S(T_{G1} - T_0) + H_1\lambda_0 \\ &= (1.005 + 1.88H_1)(37.8 - 0) + H_1(2501) = 37.99 + 2572H_1 \end{aligned}$$

For the entering solid using Eq. (9.10-25),

$$\begin{aligned} H'_{S1} &= c_{pS}(T_{S1} - T_0) + X_1 c_{pA}(T_{S1} - T_0) \\ &= 1.465(26.7 - 0) + 0.040(4.187)(26.7 - 0) = 43.59 \text{ kJ/kg dry solid} \\ H'_{S2} &= c_{pS}(T_{S2} - T_0) + X_2 c_{pA}(T_{S2} - T_0) \\ &= 1.465(62.8 - 0) + 0.002(4.187)(62.8 - 0) = 92.53 \text{ kJ/kg} \end{aligned}$$

Substituting into Eq. (9.10-26) for the heat balance with $Q = 0$ for no heat loss,

$$G(120.5) + 453.6(43.59) = G(37.99 + 2572H_1) + 453.6(92.53) + 0 \tag{9.10-28}$$

Solving Eqs. (9.10-27) and (9.10-28) simultaneously,

$$G = 1166 \text{ kg dry air/h} \qquad H_1 = 0.0248 \text{ kg H}_2\text{O/kg dry air}$$

2. Air recirculation in dryers. In many dryers it is desired to control the wet bulb temperature at which the drying of the solid occurs. Also, since steam costs are often important in heating the drying air, recirculation of the drying air is sometimes used to reduce costs and control humidity. Part of the moist hot air leaving the dryer is recirculated (recycled) and combined with the fresh air. This is shown in Fig. 9.10-4. Fresh air having a temperature T_{G1} and humidity H_1 is mixed with recirculated air at T_{G2} and H_2 to give air at T_{G3} and H_3. This mixture is heated to T_{G4} with $H_4 = H_3$. After drying, the air leaves at a lower temperature T_{G2} and a higher humidity H_2.

The following material balances on the water can be made. For a water balance on

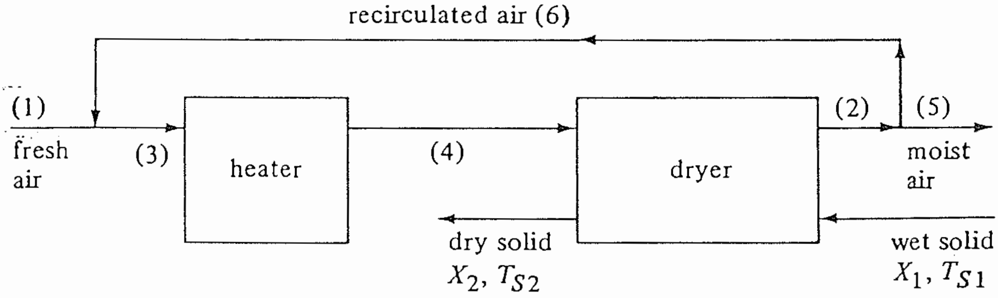


FIGURE 9.10-4. *Process flow for air recirculation in drying.*

the heater, noting that $H_6 = H_5 = H_2$,

$$G_1 H_1 + G_6 H_2 = (G_1 + G_6) H_4 \tag{9.10-29}$$

Making a water balance on the dryer,

$$(G_1 + G_6) H_4 + L_S X_1 = (G_1 + G_6) H_2 + L_S C_2 \tag{9.10-30}$$

In a similar manner heat balances can be made on the heater and dryer and on the overall system.

9.10D Continuous Countercurrent Drying

1. Introduction and temperature profiles. Drying continuously offers a number of advantages over batch drying. Smaller sizes of equipment can often be used and the product has a more uniform moisture content. In a continuous dryer the solid is moved through the dryer while in contact with a moving gas stream that may flow parallel or countercurrent to the solid. In countercurrent adiabatic operation, the entering hot gas contacts the leaving solid, which has been dried. In parallel adiabatic operation, the entering hot gas contacts the entering wet solid.

In Fig. 9.10-5 typical temperature profiles of the gas T_G and the solid T_S are shown for a continuous countercurrent dryer. In the preheat zone, the solid is heated up to the wet bulb or adiabatic saturation temperature. Little evaporation occurs here, and for low-temperature drying this zone is usually ignored. In the constant-rate zone, I, unbound and surface moisture are evaporated and the temperature of the solid remains essentially constant at the adiabatic saturation temperature if heat is transferred by convection. The rate of drying would be constant here but the gas temperature is changing and also the humidity. The moisture content falls to the critical value X_C at the end of this period.

In zone II, unsaturated surface and bound moisture are evaporated and the solid is dried to its final value X_2. The humidity of the entering gas entering zone II is H_2 and it rises to H_C. The material-balance equation (9.10-23) may be used to calculate H_C as follows.

$$L_S(X_C - X_2) = G(H_C - H_2) \tag{9.10-31}$$

where L_S is kg dry solid/h and G is kg dry gas/h.

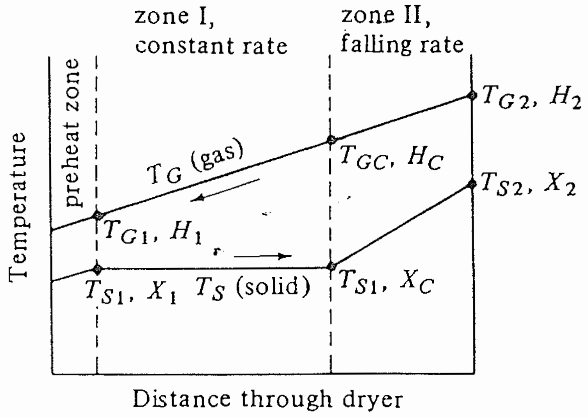


FIGURE 9.10-5. *Temperature profiles for a continuous countercurrent dryer.*

2. *Equation for constant-rate period.* The rate of drying in the constant-rate region in zone I would be constant if it were not for the varying gas conditions. The rate of drying in this section is given by an equation similar to Eq. (9.6-7).

$$R = k_y M_B (H_W - H) = \frac{h}{\lambda_W}(T_G - T_W) \tag{9.10-32}$$

The time for drying is given by Eq. (9.6-1) using limits between X_1 and X_C.

$$t = \left(\frac{L_S}{A}\right) \int_{X_C}^{X_1} \frac{dX}{R} \tag{9.10-33}$$

where A/L_S is the exposed drying surface m^2/kg dry solid. Substituting Eq. (9.10-32) into (9.10-33) and $(G/L_S)\,dH$ for dX,

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right) \frac{1}{k_y M_B} \int_{H_C}^{H_1} \frac{dH}{H_W - H} \tag{9.10-34}$$

where $G =$ kg dry air/h, $L_S =$ kg dry solid/h, and $A/L_S =$ m^2/kg dry solid. This can be integrated graphically.

For the case where T_W or H_W is constant for adiabatic drying, Eq. (9.10-34) can be integrated.

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right) \frac{1}{k_y M_B} \ln \frac{H_W - H_C}{H_W - H_1} \tag{9.10-35}$$

The above can be modified by use of a log mean humidity difference.

$$\Delta H_{LM} = \frac{(H_W - H_C) - (H_W - H_1)}{\ln\,[(H_W - H_C)/(H_W - H_1)]} = \frac{H_1 - H_C}{\ln\,[(H_W - H_C)/(H_W - H_1)]} \tag{9.10-36}$$

Substituting Eq. (9.10-36) into (9.10-35), an alternative equation is obtained.

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right) \frac{1}{k_y M_B} \frac{H_1 - H_C}{\Delta H_{LM}} \tag{9.10-37}$$

From Eq. (9.10-31), H_C can be calculated as follows.

$$H_C = H_2 + \frac{L_S}{G}(X_C - X_2) \tag{9.10-38}$$

3. *Equation for falling-rate period.* For the situation where unsaturated surface drying occurs, H_W is constant for adiabatic drying, the rate of drying is directly dependent upon X as in Eq. (9.7-9), and Eq. (9.10-32) applies.

$$R = R_C \frac{X}{X_C} = k_y M_B (H_W - H) \frac{X}{X_C} \tag{9.10-39}$$

Substituting Eq. (9.10-39) into (9.6-1),

$$t = \left(\frac{L_S}{A}\right) \frac{X_C}{k_y M_B} \int_{X_2}^{X_C} \frac{dX}{(H_W - H)X} \tag{9.10-40}$$

Substituting $G\,dH/L_S$ for dX and $(H - H_2)G/L_S + X_2$ for X,

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right) \frac{X_C}{k_y M_B} \int_{H_2}^{H_C} \frac{dH}{(H_W - H)[(H - H_2)G/L_S + X_2]} \tag{9.10-41}$$

$$t = \frac{G}{L_S}\left(\frac{L_S}{A}\right) \frac{X_C}{k_y M_B} \frac{1}{(H_W - H_2)G/L_S + X_2} \ln \frac{X_C(H_W - H_2)}{X_2(H_W - H_C)} \tag{9.10-42}$$

Again, to calculate H_C, Eq. (9.10-38) can be used.

These equations for the two periods can also be derived using the last part of Eq. (9.10-32) and temperatures instead of humidities.

9.11 FREEZE DRYING OF BIOLOGICAL MATERIALS

9.11A Introduction

Certain foodstuffs, pharmaceuticals, and biological materials, which may not be heated even to moderate temperatures in ordinary drying, may be freeze-dried. The substance to be dried is usually frozen by exposure to very cold air. In freeze drying the water is removed as a vapor by sublimation from the frozen material in a vacuum chamber. After the moisture sublimes to a vapor, it is removed by mechanical vacuum pumps or steam jet ejectors.

As a rule, freeze drying produces the highest-quality food product obtainable by any drying method. A prominent factor is the structural rigidity afforded by the frozen substance when sublimation occurs. This prevents collapse of the remaining porous structure after drying. When water is added later, the rehydrated product retains much of its original structural form. Freeze drying of biological and food materials also has the advantage of little loss of flavor and aroma. The low temperatures involved minimize the degradative reactions which normally occur in ordinary drying processes. However, freeze drying is an expensive form of dehydration for foods because of the slow drying rate and the use of vacuum.

Since the vapor pressure of ice is very small, freeze drying requires very low pressures or high vacuum. If the water were in a pure state, freeze drying at or near 0°C (273 K) at a pressure of 4580 μm (4.58 mm Hg abs) could be performed. (See Appendix A.2 for the properties of ice.) However, since the water usually exists in a solution or a combined state, the material must be cooled below 0°C to keep the water in the solid phase. Most freeze drying is done at -10°C (263 K) or lower at pressures of about 2000 μm or less.

9.11B Derivation of Equations for Freeze Drying

In the freeze-drying process the original material is composed of a frozen core of material. As the ice sublimes, the plane of sublimation, which started at the outside surface, recedes and a porous shell of material already dried remains. The heat for the latent heat of sublimation of 2838 kJ/kg (1220 btu/lb$_m$) ice is usually conducted inward through the layer of dried material. In some cases it is also conducted through the frozen layer from the rear. The vaporized water vapor is transferred through the layer of dried material. Hence, heat and mass transfer are occurring simultaneously.

In Fig. 9.11-1 a material being freeze-dried is pictured. Heat by conduction, convection, and/or radiation from the gas phase reaches the dried surface and is then transferred by conduction to the ice layer. In some cases heat may also be conducted through the frozen material to reach the sublimation front or plane. The total drying time must be long enough so that the final moisture content is below about 5 wt % to prevent degradation of the final material on storage. The maximum temperatures reached in the dried food and reached in the frozen food must be low enough to keep degradation to a minimum.

The most widely used freeze-drying process is based upon the heat of sublimation

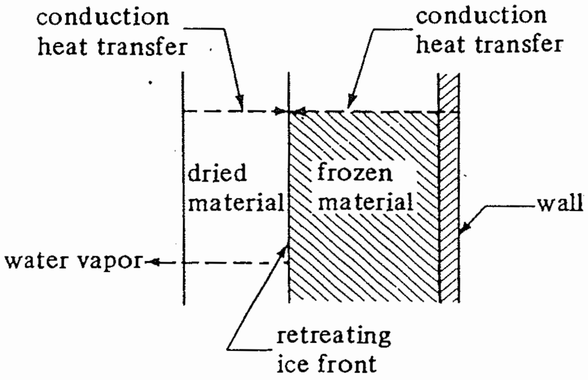


FIGURE 9.11-1. *Heat and mass transfer in freeze drying.*

being supplied from the surrounding gases to the sample surface. Then the heat is transferred by conduction through the dried material to the ice surface. A simplified model by Sandall et al. (S1) is shown in Fig. 9.11-2.

The heat flux to the surface of the material in Fig. 9.11-2 occurs by convection and in the dry solid by conduction to the sublimation surface. The heat flux to the surface is equal to that conducted through the dry solid, assuming pseudo steady state.

$$q = h(T_e - T_s) = \frac{k}{\Delta L}(T_s - T_f) \tag{9.11-1}$$

where q is heat flux in W (J/s), h is external heat-transfer coefficient in W/m$^2 \cdot$K, T_e is external temperature of the gas in °C, T_s is surface temperature of the dry solid in °C, T_f is temperature of the sublimation front or ice layer in °C, k is thermal conductivity of the dry solid in W/m·K, and ΔL is the thickness of the dry layer in m. Note that $(T_s - T_f)$°C $= (T_s - T_f)$ K.

In a similar manner, the mass flux of the water vapor from the sublimation front is

$$N_A = \frac{D'}{RT\,\Delta L}(p_{fw} - p_{sw}) = k_g(p_{sw} - p_{ew}) \tag{9.11-2}$$

where N_A is flux of water vapor in kg mol/s · m^2, k_g is external mass-transfer coefficient in

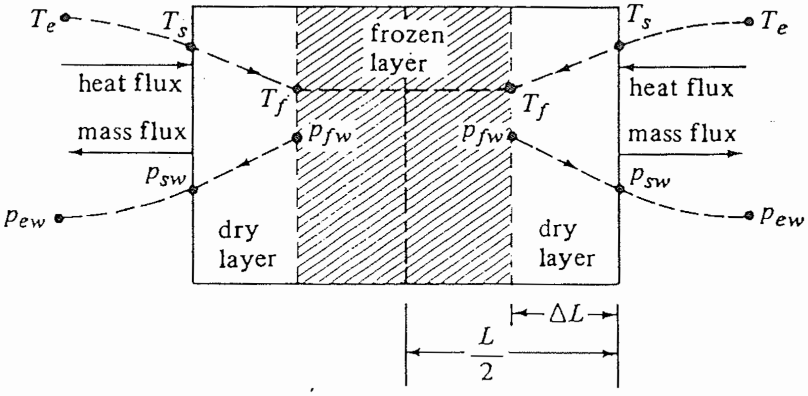


FIGURE 9.11-2. *Model for uniformly retreating ice front in freeze drying.*

kg mol/s · m^2 · atm, p_{sw} is partial pressure of water vapor at the surface in atm, p_{ew} is partial pressure of water vapor in the external bulk gas phase in atm, T is the average temperature in the dry layer, D' is an average effective diffusivity in the dry layer in m^2/s, and p_{fw} is the partial pressure of water vapor in equilibrium with the sublimation ice front in atm.

Equation (9.11-1) can be rearranged to give

$$q = \frac{1}{1/h + \Delta L/k}(T_e - T_f) \tag{9.11-3}$$

Also, Eq. (9.11-2) can be rearranged to give

$$N_A = \frac{1}{1/k_g + RT\ \Delta L/D'}(p_{fw} - p_{ew}) \tag{9.11-4}$$

The coefficients h and k_g are determined by the gas velocities and characteristics of the dryer and hence are constant. The values of T_e and p_{ew} are set by the external operating conditions. The values of k and D' are determined by the nature of the dried material.

The heat flux and mass flux at pseudo steady state are related by

$$q = \Delta H_s N_A \tag{9.11-5}$$

where ΔH_s is the latent heat of sublimation of ice in J/kg mol. Also, p_{fw} is uniquely determined by T_f, since it is the equilibrium vapor pressure of ice at that temperature; or

$$p_{fw} = f(T_f) \tag{9.11-6}$$

Substituting Eqs. (9.11-3) and (9.11-4) into (9.11-5),

$$\frac{1}{1/h + \Delta L/k}(T_e - T_f) = \Delta H_s \frac{1}{1/k_g + RT\ \Delta L/D'}(p_{fw} - p_{ew}) \tag{9.11-7}$$

Also, substituting Eqs. (9.11-1) and (9.11-4) into (9.11-5),

$$\frac{1}{\Delta L/k}(T_s - T_f) = \Delta H_s \frac{1}{1/k_g + RT\ \Delta L/D'}(p_{fw} - p_{ew}) \tag{9.11-8}$$

As T_e and, hence, T_s are raised to increase the rate of drying, two limits may possibly be reached. First, the outer surface temperature T_s cannot go too high because of thermal damage. Second, the temperature T_f must be kept well below the melting point. For the situation where $k/\Delta L$ is small compared to k_g and $D'/RT\ \Delta L$, the outer-surface temperature limit will be encountered first as T_s is raised. To further increase the drying rate, k must be raised. Hence, the process is considered to be heat-transfer-controlled. Most commercial freeze-drying processes are heat-transfer-controlled (K1).

In order to solve the given equations, ΔL is related to x, the fraction of the original free moisture remaining.

$$\Delta L = (1 - x)\frac{L}{2} \tag{9.11-9}$$

The rate of freeze drying can be related to N_A by

$$N_A = \frac{L}{2}\frac{1}{M_A V_S}\left(-\frac{dx}{dt}\right) \tag{9.11-10}$$

where M_A is molecular weight of water, V_S is the volume of solid material occupied by a unit kg of water initially ($V_S = 1/X_0 \rho_S$), X_0 is initial free moisture content in kg H_2O/kg dry solid, and ρ_S is bulk density of dry solid in kg/m^3.

Combining Eqs. (9.11-3), (9.11-5), (9.11-9), and (9.11-10), we obtain, for heat transfer,

$$\frac{L}{2} \frac{\Delta H_s}{M_A V_S} \left(-\frac{dx}{dt} \right) = \frac{1}{1/h + (1-x)L/2k} (T_e - T_f) \tag{9.11-11}$$

Similarly for mass transfer,

$$\frac{L}{2} \frac{1}{M_A V_S} \left(-\frac{dx}{dt} \right) = \frac{1}{1/k_g + RT(1-x)L/2D'} (p_{fw} - p_{ew}) \tag{9.11-12}$$

Integrating Eq. (9.11-11) between the limits of $t = 0$ at $x_1 = 1.0$ and $t = t$ at $x_2 = x_2$, the equation for the time of drying to x_2 is as follows for h being very large (negligible external resistance):

$$t = \frac{L^2}{4kV_S} \frac{\Delta H_s}{M_A} \frac{1}{T_e - T_f} \left(x_1 - x_2 - \frac{x_1^2}{2} + \frac{x_2^2}{2} \right) \tag{9.11-13}$$

where $\Delta H_s/M_A$ is heat of sublimation in J/kg H_2O. For $x_2 = 0$, the slab is completely dry.

Assuming that the physical properties and mass- and heat-transfer coefficients are known, Eq. (9.11-8) can be used to calculate the ice sublimation temperature T_f when the environment temperature T_e and the environment partial pressure p_{ew} are set. Since h is very large, $T_e \cong T_s$. Then Eq. (9.11-8) can be solved for T_f since T_f and p_{fw} are related by the equilibrium-vapor-pressure relation, Eq. (9.11-6). In Eq. (9.11-8) the value to use for T can be approximated by $(T_f + T_s)/2$.

The uniformly retreating ice-front model was tested by Sandall et al. (S1) against actual freeze-drying data. The model satisfactorily predicted the drying times for removal of 65–90% of the total initial water (S1, K1). The temperature T_f of the sublimation interface did remain essentially constant as assumed in the derivation. However, during removal of the last 10–35% of the water, the drying rate slowed markedly and the actual time was considerably greater than the predicted for this period.

The effective thermal conductivity k in the dried material has been found to vary significantly with the total pressure and with the type of gas present. Also, the type of material affects the value of k (S1, K1). The effective diffusivity D' of the dried material is a function of the structure of the material, Knudsen diffusivity, and molecular diffusivity (K1).

9.12 UNSTEADY-STATE THERMAL PROCESSING AND STERILIZATION OF BIOLOGICAL MATERIALS

9.12A Introduction

Materials of biological origin are usually not as stable as most inorganic and some organic materials. Hence, it is necessary to use certain processing methods to preserve biological materials, especially foods. Physical and chemical processing methods for preservation can be used such as drying, smoking, salting, chilling, freezing, and heating. Freezing and chilling of foods were discussed in Section 5.5 as methods of slowing the spoilage of biological materials. Also, in Section 9.11, freeze drying of biological materials was discussed.

An important method is heat or thermal processing, whereby contaminating microorganisms that occur primarily on the outer surface of foods and cause spoilage and health problems are destroyed. This leads to longer storage times of the food and other biological materials. A common method for preservation is to heat seal cans of food. Also, thermal processing is used to sterilize aqueous fermentation media to be used in fermentation processes so that organisms which do not survive are unable to compete with the organism that is to be cultured.

The sterilization of food materials by heating destroys bacteria, yeast, molds, and so on, which cause the spoilage and also destroys pathogenic (disease-producing) organisms, which may produce deadly toxins if not destroyed. The rate of destruction of microorganisms varies with the amount of heating and the type of organism. Some bacteria can exist in a vegetative growing form and in a dormant or spore form. The spore forms are much more resistant to heat. This mechanism of heat resistance is not clear.

For foods it is desired to kill essentially all the spores of *Clostridium botulinum*, which produces a toxin that is a deadly poison. Complete sterility with respect to this spore is the purpose of thermal processing. Since *Cl. botulinum* is so dangerous and often difficult to use, other spores, such as *Bacillus stearothermophilus*, which is a nonpathogenic organism of similar heat resistance, are often used for testing the heat-treating processes (A2, C1).

Temperature has a great effect on the growth rate of microorganisms, which have no temperature-regulating mechanism. Each organism has a certain optimal temperature range in which it grows best. If any microorganism is heated to a sufficiently high temperature for a sufficient time, it will be rendered sterile or killed.

The exact mechanism of thermal death of vegetative bacteria and spores is still somewhat uncertain. It is thought, however, to be due to the breakdown of the enzymes, which are essential to the functioning of the living cell (B1).

9.12B Thermal Death-Rate Kinetics of Microorganisms

The destruction of microorganisms by heating means loss of viability and not destruction in the physical sense. If it is assumed that inactivation of a single enzyme in a cell will inactivate the cell, then in a suspension of organisms of a single species at a constant temperature, the death rate can be expressed as a first-order kinetic equation (A2). The rate of destruction (number dying per unit time) is proportional to the number of organisms.

$$\frac{dN}{dt} = -kN \tag{9.12-1}$$

where N is the number of viable organisms at a given time, t is time in min, and k is a reaction velocity constant in min^{-1}. The reaction velocity constant is a function of temperature and the type of microorganism.

After rearranging, Eq. (9.12-1) can be integrated as follows:

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_{t=0}^{t} k\,dt \tag{9.12-2}$$

$$\ln \frac{N_0}{N} = kt \tag{9.12-3}$$

where N_0 is the original number at $t = 0$ and N is the number at time t. Often N_0 is called the *contamination level* (original number of contaminating microbes before sterilization) and N the sterility level. Also, Eq. (9.12-3) can be written as

$$N = N_0 e^{-kt} \tag{9.12-4}$$

Sometimes microbiologists use the term *decimal reduction time D*, which is the time in min during which the original number of viable microbes is reduced by 1/10. Substituting into Eq. (9.12-4),

$$\frac{N}{N_0} = \frac{1}{10} = e^{-kD} \tag{9.12-5}$$

Taking the $\log_{10}$ of both sides and solving for D,

$$D = \frac{2.303}{k} \tag{9.12-6}$$

Combining Eqs. (9.12-3) and (9.12-6),

$$t = D \log_{10} \frac{N_0}{N} \tag{9.12-7}$$

If the $\log_{10} (N/N_0)$ is plotted versus t, a straight line should result from Eq. (9.12-3). Experimental data bear this out for vegetative cells and approximately for spores. Data for the vegetative cell *E. coli* (A1) at constant temperature follow this logarithmic death-rate curve. Bacterial spore plots sometimes deviate somewhat from the logarithmic rate of death, particularly during a short period immediately following exposure to heat. However, for thermal-processing purposes for use with spores such as *Cl. botulinum*, a logarithmic-type curve is used.

To experimentally measure the microbial death rate, the spore or cell suspension in a solution is usually sealed in a capillary or test tube. A number of these tubes are then suddenly dipped into a hot bath for a given time. Then they are removed and immediately chilled. The number of viable organisms before and after exposure to the high temperature is then usually determined biologically with a plate count.

The effect of temperature on the reaction-rate constant k may be expressed by an Arrhenius-type equation.

$$k = ae^{-E/RT} \tag{9.12-8}$$

where a = an empirical constant, R is the gas constant in kJ/g mol · K (cal/g · mol · K), T is absolute temperature in K, and E is the activation energy in kJ/g mol (cal/g mol). The value of E is in the range 210 to about 418 kJ/g mol (50–100 kcal/g mol) for vegetative cells and spores (A2) and much less for enzymes and vitamins.

Substituting Eq. (9.12-8) into (9.12-2) and integrating,

$$\ln \frac{N_0}{N} = a \int_{t=0}^{t} e^{-E/RT}\, dt \tag{9.12-9}$$

At constant temperature T, Eq. (9.12-9) becomes (9.12-3). Since k is a function of temperature, the decimal reduction time D, which is related to k by Eq. (9.12-6), is also a function of temperature. Hence, D is often written as D_T to show that it is temperature-dependent.

9.12C Determination of Thermal Process Time for Sterilization

For canned foods, *Cl. botulinum* is the primary organism to be reduced in number (S2). It has been established that the minimum heating process should reduce the number of the spores by a factor of 10^{-12}. This means that since D is the time to reduce the original number by 10^{-1}, substituting $N/N_0 = 10^{-12}$ into Eq. (9.12-4) and solving for t,

$$t = 12\,\frac{2.303}{k} = 12D \tag{9.12-10}$$

This means that the time t is equal to $12D$ (often called the *12D concept*). This time in Eq. (9.12-10) to reduce the number by 10^{-12} is called the ***thermal death time***. Usually, the sterility level N is a number much less than one organism. These times do not represent complete sterilization but a mathematical concept which has been found empirically to give effective sterilization.

Experimental data of thermal death rates of ***Cl. botulinum***, when plotted as the decimal reduction time D_T at a given T versus the temperature T in °F on a semilog plot, give essentially straight lines over the range of temperatures used in food sterilization (S2). A typical thermal destruction curve is shown in Fig. 9.12-1. Actually, by combining Eqs. (9.12-6) and (9.12-8), it can be shown that the plot of $\log_{10} D_T$ versus $1/T$ (T in degrees absolute) is a straight line, but over small ranges of temperature a straight line is obtained when $\log_{10} D_T$ is plotted versus T°F or °C.

In Fig. 9.12-1 the term z represents the temperature range in °F for a 10 : 1 change in D_T. Since the plot is a straight line, the equation can be represented as

$$\log_{10} D_{T_2} - \log_{10} D_{T_1} = \frac{1}{z}(T_1 - T_2) \tag{9.12-11}$$

Letting $T_1 = 250$°F (121.1°C), which is the standard temperature against which thermal processes are compared, and calling $T_2 = T$, Eq. (9.12-11) becomes

$$D_T = D_{250} \cdot 10^{(250-T)/z} \tag{9.12-12}$$

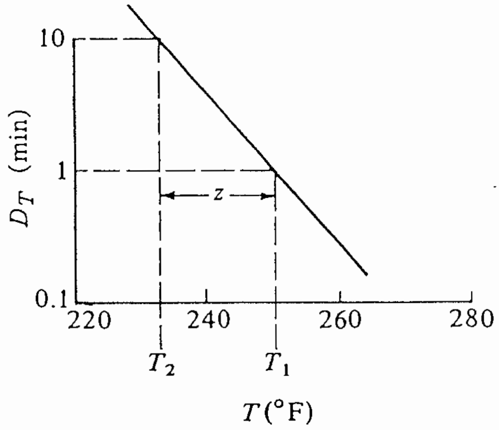
For the organism *Cl. botulinum* the experimental value of $z = 18$°F. This means that each increase in temperature of 18°F (10°C) will increase the death rate by a factor of 10. This compares with the factor of 2 for many chemical reactions for an 18°F increase in temperature.

Using Eq. (9.12-7),

$$t = D_T \log_{10} \frac{N_0}{N} \tag{9.12-7}$$

Substituting $T = 250$°F (121.1°C) as the standard temperature into this equation and

FIGURE 9.12-1. *Thermal destruction curve: plot of decimal reduction time versus temperature.*



substituting F_0 for t,

$$F_0 = D_{250} \log_{10} \frac{N_0}{N} \tag{9.12-13}$$

where the F_0 value of a process is the time t in min at 250°F that will produce the same degree of sterilization as the given process at its temperature T. Combining Eqs. (9.12-7), (9.12-12), and (9.12-13), the F_0 of the given process at temperature T is

$$\begin{aligned} F_0 &= t \cdot 10^{(T^\circ\text{C} - 121.1)/(z^\circ\text{C})} && \text{(SI)} \\ F_0 &= t \cdot 10^{(T^\circ\text{F} - 250)/(z^\circ\text{F})} && \text{(English)} \end{aligned} \tag{9.12-14}$$

This is the F_0 value in min for the given thermal process at a given constant temperature T°F and a given time t in min. Values for F_0 and z for adequate sterilization with *Cl. botulinum* vary somewhat with the type of food. Data are tabulated by Stumbo (S2) and Charm (C2) for various foods and microorganisms.

The effects of different but successive sterilization processes in a given material are additive. Hence, for several different temperature stages T_1, T_2, and so on, each having different times $t_1, t_2, \ldots$, the F_0 values for each stage are added to give the total F_0.

$$F_0 = t_1 \cdot 10^{(T_1 - 250)/z} + t_2 \cdot 10^{(T_2 - 250)/z} + \cdots \quad \text{(English)} \tag{9.12-15}$$

EXAMPLE 9.12-1. Sterilization of Cans of Food

Cans of a given food were heated in a retort for sterilization. The F_0 for *Cl. botulinum* in this type of food is 2.50 min and $z = 18$°F. The temperatures in the center of a can (the slowest-heating region) were measured and were approximately as follows, where the average temperature during each time period is listed: t_1 (0–20 min), $T_1 = 160$°F; t_2 (20–40 min), $T_2 = 210$°F; t_3 (40–73 min), $T_3 = 230$°F. Determine if this sterilization process is adequate. Use English and SI units.

Solution: For the three time periods the data are as follows:

$$t_1 = 20 - 0 = 20 \text{ min}, \quad T_1 = 160^\circ\text{F } (71.1^\circ\text{C}), \quad z = 18^\circ\text{F } (10^\circ\text{C})$$

$$t_2 = 40 - 20 = 20 \text{ min}, \quad T_2 = 210^\circ\text{F } (98.9^\circ\text{C})$$

$$t_3 = 73 - 40 = 33 \text{ min}, \quad T_3 = 230^\circ\text{F } (110^\circ\text{C})$$

Substituting into Eq. (9.12-15) and solving using English and SI units,

$$\begin{aligned} F_0 &= t_1 \cdot 10^{(T_1 - 250)/z} + t_2 \cdot 10^{(T_2 - 250)/z} + t_3 \cdot 10^{(T_3 - 250)/z} \qquad (9.12\text{-}15) \\ &= (20)10^{(160-250)/18} + (20)10^{(210-250)/18} + (33)10^{(230-250)/18} \\ &= 0.0020 + 0.1199 + 2.555 = 2.68 \text{ min} \qquad \text{(English)} \\ &= (20)10^{(71.1-121.1)/10} + (20)10^{(98.9-121.1)/10} + (33)10^{(110-121.1)/10} \\ &= 2.68 \text{ min} \qquad \text{(SI)} \end{aligned}$$

Hence, this thermal processing is adequate since only 2.50 min is needed for complete sterilization. Note that the time period at 160°F (71.1°C) contributes an insignificant amount to the final F_0. The major contribution is at 230°F (110°C), which is the highest temperature.

In the general case when cans of food are being sterilized in a retort, the temperature is not constant for a given time period but varies continuously with time. Hence, Eq. (9.12-15) can be modified and written for a continuously varying temperature T by taking small time increments of dt min for each value of T and summing. The final equation is

$$F_0 = \int_{t=0}^{t=t} 10^{(T°\text{F}-250)/(z°\text{F})}\, dt \qquad \text{(English)}$$

$$F_0 = \int_{t=0}^{t=t} 10^{(T°\text{C}-121.1)/(z°\text{C})}\, dt \qquad \text{(SI)} \tag{9.12-16}$$

This equation can be used as follows. Suppose that the temperature of a process is varying continuously and a graph or a table of values of T versus t is known or is calculated by the unsteady-state methods given in Chapter 5. The Eq. (9.12-16) can be graphically integrated by plotting values of $10^{(T-250)/z}$ versus t and taking the area under the curve.

In many cases the temperature of a process that is varying continuously with time is determined experimentally by measuring the temperature in the slowest-heating region. In cans this is the center of the can. Methods given in Chapter 5 for unsteady-state heating of short, fat cylinders by conduction can be used to predict the center temperature of the can as a function of time. However, these predictions can be somewhat in error, since physical and thermal properties of foods are difficult to measure accurately and often can vary. Also, trapped air in the container and unknown convection effects can affect the accuracy of predictions.

EXAMPLE 9.12-2. Thermal Process Evaluation by Graphical Integration
In the sterilization of a canned purée, the temperature in the slowest-heating region (center) of the can was measured giving the following time–temperature data for the heating and holding time. The cooling time data will be neglected as a safety factor.

t (min)	T (°F)	t (min)	T (°F)
0	80 (26.7°C)	40	225 (107.2°C)
15	165 (73.9)	50	230.5 (110.3)
25	201 (93.9)	64	235 (112.8)
30	212.5 (100.3)		

The F_0 value of *Cl. botulinum* is 2.45 min and z is 18°F. Calculate the F_0 value of the process above and determine if the sterilization is adequate.

Solution: In order to use Eq. (9.12-16), the values of $10^{(T-250)/z}$ must be calculated for each time. For $t = 0$ min, $T = 80°\text{F}$, and $z = 18°\text{F}$,

$$10^{(T-250)/z} = 10^{(80-250)/18} = 3.6 \times 10^{-10}$$

For $t = 15$ min, $T = 165°\text{F}$

$$10^{(165-250)/18} = 0.0000189$$

For $t = 25$ min, $T = 201°\text{F}$

$$10^{(201-250)/18} = 0.00189$$

For $t = 30$ min,

$$10^{(212.5-250)/18} = 0.00825$$

For $t = 40$ min,

$$10^{(225-250)/18} = 0.0408$$

For $t = 50$ min,

$$10^{(230.5-250)/18} = 0.0825$$

For $t = 64$ min,

$$10^{(235-250)/18} = 0.1465$$

These values are plotted versus t in Fig. 9.12-2. The areas of the various rectangles shown are

$$\begin{aligned} F_0 &= A_1 + A_2 + A_3 + A_4 \\ &= 10(0.0026) + 10(0.0233) + 10(0.0620) + 14(0.1160) \\ &= 0.026 + 0.233 + 0.620 + 1.621 = 2.50 \text{ min} \end{aligned}$$

The process value of 2.50 min is greater than the required 2.45 min and the sterilization is adequate.

9.12D Sterilization Methods Using Other Design Criteria

In types of thermal processing which are not necessarily involved with sterilization of foods, other types of design criteria are used. In foods the minimum heat process should reduce the number of spores by a factor of 10^{-12}, i.e., $N/N_0 = 10^{-12}$. However, in other batch sterilization processes, such as in the sterilization of fermentation media, other criteria are often used. Often the equation for k, the reaction velocity constant for the specific organism to be used, is available.

$$k = ae^{-E/RT} \tag{9.12-8}$$

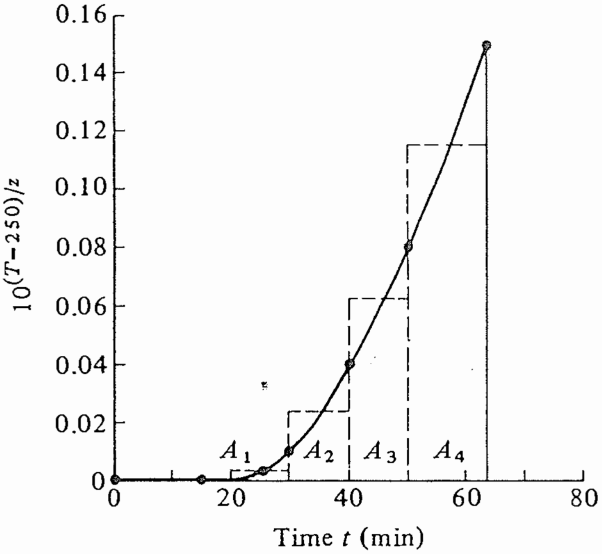


FIGURE 9.12-2. *Graphical integration for Example 9.12-2.*

Then Eq. (9.12-9) is written as

$$\nabla = \ln \frac{N_0}{N} = a \int_{t=0}^{t} e^{-E/RT}\, dt = \int_{t=0}^{t} k\, dt \qquad \textbf{(9.12-17)}$$

where ∇ is the design criterion. Usually, the contamination level N_0 is available and either the sterility level N is the unknown or the time of sterilization at a given temperature is the unknown. In either case a graphical integration is usually done, where $ae^{-E/RT}$ is plotted versus t and the area under the curve is obtained.

In sterilization of food in a container, the time required to render the material safe is calculated at the slowest-heating region of the container (usually the center). Other regions of the container are usually heated to higher temperatures and are overtreated. Hence, another method used is based on the probability of survival in the whole container. These details are given by others (C2, S2). In still another processing method, a short-time, continuous-flow process is used instead of a batch process in a container (B2).

9.12E Pasteurization

The term *pasteurization* is used today to apply to mild heat treatment of foods that is less drastic than sterilization. It is used to kill organisms that are relatively low in thermal resistance compared to those for which the more drastic sterilization processes are designed to eliminate. Pasteurization usually involves killing vegetative microorganisms and not heat-resistant spores.

The most common process is the pasteurization of milk to kill *Mycobacterium tuberculosis*, which is a non-spore-forming bacterium. This pasteurization does not sterilize the milk but kills the *M. tuberculosis* and reduces the other bacterial count sufficiently so that the milk can be stored if refrigerated.

For the pasteurization of such foods as milk, fruit juices, and beer, the same mathematical and graphical procedures covered for sterilization processes in this section are used to accomplish the degree of sterilization desired in pasteurization (B1, S2). The times involved are much shorter and the temperatures used in pasteurization are much lower. Generally, the F_0 value is given at 150°F (65.6°C) or a similar temperature rather than at 250°F as in sterilization. Also, the concept of the z value is employed, in which a rise in temperature of z°F will increase the death rate by a factor of 10. An F_0 value written as F_{150}^{9} means the F value at 150°F with a z value of 9°F (S2).

In pasteurizing milk, batch and continuous processes are used. The U.S. health regulations specify two equivalent sets of conditions, where in one the milk is held at 145°F (62.8°C) for 30 min and in the other at 161°F (71.7°C) for 15 s.

The general equations used for pasteurization are similar to sterilization and can be written as follows. Rewriting Eq. (9.12-13),

$$F_{T_1}^{z} = D_{T_1} \log_{10} \frac{N_0}{N} \qquad \textbf{(9.12-18)}$$

Rewriting Eq. (9.12-14),

$$F_{T_1}^{z} = t \cdot 10^{(T - T_1)/z} \qquad \textbf{(9.12-19)}$$

where T_1 is the standard temperature being used such as 150°F, z is the value of z in °F for a tenfold increase in death rate, and T is the temperature of the actual process.

EXAMPLE 9.12-3. ***Pasteurization of Milk***

A typical F value given for the thermal processing of milk in a tubular heat exchanger is $F_{150}^{9} = 9.0$ min and $D_{150} = 0.6$ min. Calculate the reduction in the number of viable cells for these conditions.

Solution: The z value is 9°F (5°C) and the temperature of the process is 150°F (65.6°C). Substituting into Eq. (9.12-18) and solving,

$$F_{150}^{9} = 9.0 = 0.6 \log_{10} \frac{N_0}{N}$$

$$\frac{N_0}{N} = \frac{10^{15}}{1}$$

This gives a reduction in the number of viable cells of 10^{15}.

9.12F Effects of Thermal Processing on Food Constituents

Thermal processing is used to cause the death of various undesirable microorganisms, but it also causes undesirable effects, such as the reduction of certain nutritional values. Ascorbic acid (vitamin C) and thiamin and riboflavin (vitamins B_1 and B_2) are partially destroyed by thermal processing. The reduction of these desirable constituents can also be given kinetic parameters such as F_0 and z values in the same way as for sterilization and pasteurization. Examples and data are given by Charm (C2).

These same kinetic methods of thermal death rates can also be applied to predict the time for detecting a flavor change in a food product. Dietrich et al. (D1) determined a curve for the number of days to detect a flavor change of frozen spinach versus temperature of storage. The data followed Eq. (9.12-8) and a first-order kinetic relation.

PROBLEMS

9.3-1. *Humidity from Vapor Pressure.* The air in a room is at 37.8°C and a total pressure of 101.3 kPa abs containing water vapor with a partial pressure p_A = 3.59 kPa. Calculate:
(a) Humidity.
(b) Saturation humidity and percentage humidity.
(c) Percentage relative humidity.

9.3-2. *Percentage and Relative Humidity.* The air in a room has a humidity H of 0.021 kg H_2O/kg dry air at 32.2°C and 101.3 kPa abs pressure. Calculate:
(a) Percentage humidity H_P.
(b) Percentage relative humidity H_R.

Ans. (a) $H_P = 67.5\%$; (b) $H_R = 68.6\%$

9.3-3. *Use of the Humidity Chart.* The air entering a dryer has a temperature of 65.6°C (150°F) and dew point of 15.6°C (60°F). Using the humidity chart, determine the actual humidity and percentage humidity. Calculate the humid volume of this mixture and also calculate c_S using SI and English units.

Ans. $H = 0.0113$ kg H_2O/kg dry air, $H_P = 5.3\%$,
$c_S = 1.026$ kJ/kg · K (0.245 btu/lb$_m$ · °F),
$v_H = 0.976$ m^3 air + water vapor/kg dry air

9.3-4. *Properties of Air to a Dryer.* An air–water vapor mixture going to a drying process has a dry bulb temperature of 57.2°C and a humidity of 0.030 kg H_2O/kg dry air. Using the humidity chart and appropriate equations, determine the percentage humidity, saturation humidity at 57.2°C, dew point, humid heat, and humid volume.

9.3-5. *Adiabatic Saturation Temperature.* Air at 82.2°C and having a humidity $H = 0.0655$ kg H_2O/kg dry air is contacted in an adiabatic saturator with water. It leaves at 80% saturation.
(a) What are the final values of H and T°C?
(b) For 100% saturation, what would be the values of H and T?

Ans. (a) $H = 0.079$ kg H_2O/kg dry air, $T = 52.8$°C

9.3-6. ***Adiabatic Saturation of Air.*** Air enters an adiabatic saturator having a temperature of 76.7°C and a dew-point temperature of 40.6°C. It leaves the saturator 90% saturated. What are the final values of H and T°C?

9.3-7. ***Humidity from Wet and Dry Bulb Temperatures.*** An air–water vapor mixture has a dry bulb temperature of 65.6°C and a wet bulb temperature of 32.2°C. What is the humidity of the mixture?

Ans. $H = 0.0175$ kg H_2O/kg dry air

9.3-8. ***Humidity and Wet Bulb Temperature.*** The humidity of an air–water vapor mixture is $H = 0.030$ kg H_2O/kg dry air. The dry bulb temperature of the mixture is 60°C. What is the wet bulb temperature?

9.3-9. ***Dehumidification of Air.*** Air having a dry bulb temperature of 37.8°C and a wet bulb of 26.7°C is to be dried by first cooling to 15.6°C to condense water vapor and then heating to 23.9°C.

(a) Calculate the initial humidity and percentage humidity.

(b) Calculate the final humidity and percentage humidity. [*Hint*: Locate the initial point on the humidity chart. Then go horizontally (cooling) to the 100% saturation line. Follow this line to 15.6°C. Then go horizontally to the right to 23.9°C.]

Ans. (b) $H = 0.0115$ kg H_2O/kg dry air, $H_P = 60\%$

9.3-10. ***Cooling and Dehumidifying Air.*** Air entering an adiabatic cooling chamber has a temperature of 32.2°C and a percentage humidity of 65%. It is cooled by a cold water spray and saturated with water vapor in the chamber. After leaving, it is heated to 23.9°C. The final air has a percentage humidity of 40%.

(a) What is the initial humidity of the air?

(b) What is the final humidity after heating?

9.6-1. ***Time for Drying in Constant-Rate Period.*** A batch of wet solid was dried on a tray dryer using constant drying conditions and a thickness of material on the tray of 25.4 mm. Only the top surface was exposed. The drying rate during the constant-rate period was $R = 2.05$ kg $H_2O/h \cdot m^2$ (0.42 lb_m $H_2O/h \cdot ft^2$). The ratio L_S/A used was 24.4 kg dry solid/m^2 exposed surface (5.0 lb_m dry solid/ft^2). The initial free moisture was $X_1 = 0.55$ and the critical moisture content $X_C = 0.22$ kg free moisture/kg dry solid.

Calculate the time to dry a batch of this material from $X_1 = 0.45$ to $X_2 = 0.30$ using the same drying conditions but a thickness of 50.8 mm, with drying from the top and bottom surfaces. (*Hint*: First calculate L_S/A for this new case.)

Ans. $t = 1.785$ h

9.6-2. ***Prediction of Effect of Process Variables on Drying Rate.*** Using the conditions in Example 9.6-3 for the constant-rate drying period, do as follows.

(a) Predict the effect on R_C if the air velocity is only 3.05 m/s.

(b) Predict the effect if the gas temperature is raised to 76.7°C and H remains the same.

(c) Predict the effect on the time t for drying between moisture contents X_1 to X_2 if the thickness of material dried is 38.1 mm instead of 25.4 mm and the drying is still in the constant-rate period.

Ans. (a) $R_C = 1.947$ kg $H_2O/h \cdot m^2$ (0.399 lb_m $H_2O/h \cdot ft^2$)
(b) $R_C = 4.21$ kg $H_2O/h \cdot m^2$

9.6-3. ***Prediction in Constant-Rate Drying Region.*** A granular insoluble solid material wet with water is being dried in the constant-rate period in a pan 0.61 m × 0.61 m and the depth of material is 25.4 mm. The sides and bottom are insulated. Air flows parallel to the top drying surface at a velocity of 3.05 m/s and has a dry bulb temperature of 60°C and wet bulb temperature of 29.4°C. The pan contains 11.34 kg of dry solid having a free moisture content of 0.35 kg H_2O/kg dry solid and the material is to be dried in the constant-rate period to 0.22 kgH_2O/kg dry solid.

(a) Predict the drying rate and the time in hours needed.
(b) Predict the time needed if the depth of material is increased to 44.5 mm.

9.6-4. ***Drying a Filter Cake in the Constant-Rate Region.*** A wet filter cake in a pan 1 ft × 1 ft square and 1 in. thick is dried on the top surface with air at a wet bulb temperature of 80°F and a dry bulb of 120°F flowing parallel to the surface at a velocity of 2.5 ft/s. The dry density of the cake is 120 lb_m/ft^3 and the critical free moisture content is 0.09 lb H_2O/lb dry solid. How long will it take to dry the material from a free moisture content of 0.20 lb H_2O/lb dry material to the critical moisture content?

Ans. $t = 13.3$ h

9.7-1. ***Graphical Integration for Drying in Falling-Rate Region.*** A wet solid is to be dried in a tray dryer under steady-state conditions from a free moisture content of $X_1 = 0.40$ kg H_2O/kg dry solid to $X_2 = 0.02$ kg H_2O/kg dry solid. The dry solid weight is 99.8 kg dry solid and the top surface area for drying is 4.645 m^2. The drying-rate curve can be represented by Fig. 9.5-1b.

(a) Calculate the time for drying using graphical integration in the falling-rate period.
(b) Repeat but use a straight line through the origin for the drying rate in the falling-rate period.

Ans. (a) t(constant rate) = 2.91 h, t(falling rate) = 6.36 h, t(total) = 9.27 h

9.7-2. ***Drying Tests with a Foodstuff.*** In order to test the feasibility of drying a certain foodstuff, drying data were obtained in a tray dryer with air flow over the top exposed surface having an area of 0.186 m^2. The bone-dry sample weight was 3.765 kg dry solid. At equilibrium after a long period, the wet sample weight was 3.955 kg H_2O + solid. Hence, 3.955 − 3.765, or 0.190, kg of equilibrium moisture was present. The following sample weights versus time were obtained in the drying test.

Time (h)	*Weight (kg)*	*Time (h)*	*Weight (kg)*	*Time (h)*	*Weight (kg)*
0	4.944	2.2	4.554	7.0	4.019
0.4	4.885	3.0	4.404	9.0	3.978
0.8	4.808	4.2	4.241	12.0	3.955
1.4	4.699	5.0	4.150		

(a) Calculate the free moisture content X kg H_2O/kg dry solid for each data point and plot X versus time. (*Hint*: For 0 h, 4.944 − 0.190 − 3.765 = 0.989 kg free moisture in 3.765 kg dry solid. Hence, $X = 0.989/3.765$.)
(b) Measure the slopes, calculate the drying rates R in kg $H_2O/h \cdot m^2$, and plot R versus X.
(c) Using this drying-rate curve, predict the total time to dry the sample from $X = 0.20$ to $X = 0.04$. Use graphical integration for the falling-rate period. What is the drying rate R_C in the constant-rate period and X_C?

Ans. (c) $R_C = 0.996$ kg $H_2O/h \cdot m^2$
$X_C = 0.12$, $t = 4.1$ h (total)

9.7-3. ***Prediction of Drying Time.*** A material was dried in a tray-type batch dryer using constant drying conditions. When the initial free moisture content was 0.28 kg free moisture/kg dry solid, 6.0 h was required to dry the material to a free moisture content of 0.08 kg free moisture/kg dry solid. The critical free moisture content is 0.14. Assuming a drying rate in the falling-rate region where the rate is a straight line from the critical point to the origin, predict the time to dry a sample from a free moisture content of 0.33 to 0.04 kg free moisture/kg dry solid. (*Hint*: First use the analytical equations for the constant-rate and the linear falling-rate periods with the known total time of 6.0 h. Then use the same equations for the new conditions.)

9.8-1. ***Drying of Biological Material in Tray Dryer.*** A granular biological material wet with water is being dried in a pan 0.305 × 0.305 m and 38.1 mm deep. The material is 38.1 mm deep in the pan, which is insulated on the sides and the bottom. Heat transfer is by convection from an air stream flowing parallel to the top surface at a velocity of 3.05 m/s, having a temperature of 65.6°C and humidity $H = 0.010$ kg H_2O/kg dry air. The top surface receives radiation from steam-heated pipes whose surface temperature $T_R = 93.3°C$. The emissivity of the solid is $\varepsilon = 0.95$. It is desired to keep the surface temperature of the solid below 32.2°C so that decomposition will be kept low. Calculate the surface temperature and the rate of drying for the constant-rate period.

Ans. $T_S = 31.3°C, R_C = 2.583$ kg $H_2O/h \cdot m^2$

9.8-2. ***Drying When Radiation, Conduction, and Convection Are Present.*** A material is granular and wet with water and is being dried in a layer 25.4 mm deep in a batch-tray dryer pan. The pan has a metal bottom having a thermal conductivity of $k_M = 43.3$ W/m · K and a thickness of 1.59 mm. The thermal conductivity of the solid is $k_S = 1.125$ W/m · K. The air flows parallel to the top exposed surface and the bottom metal at a velocity of 3.05 m/s and a temperature of 60°C and humidity $H = 0.010$ kg H_2O/kg dry solid. Direct radiation heat from steam pipes having a surface temperature of 104.4°C falls on the exposed top surface, whose emissivity is 0.94. Estimate the surface temperature and the drying rate for the constant-rate period.

9.9-1. ***Diffusion Drying in Wood.*** Repeat Example 9.9-1 using the physical properties given but with the following changes.

(a) Calculate the time needed to dry the wood from a total moisture of 0.22 to 0.13. Use Fig. 5.3-13.

(b) Calculate the time needed to dry planks of wood 12.7 mm thick from $X_{t1} = 0.29$ to $X_t = 0.09$. Compare with the time needed for 25.4 mm thickness.

Ans. (b) $t = 7.60$ h (12.7 mm thick)

9.9-2. ***Diffusivity in Drying Tapioca Root.*** Using the data given in Example 9.9-2, determine the average diffusivity of the moisture up to a value of $X/X_C = 0.50$.

9.9-3. ***Diffusion Coefficient.*** Experimental drying data of a typical nonporous biological material obtained under constant drying conditions in the falling-rate region are tabulated below.

X/X_C	$t(h)$	X/X_C	$t(h)$
1.00	0	0.17	11.4
0.65	2.50	0.10	14.0
0.32	7.00	0.06	16.0

Drying from one side occurs with the material having a thickness of 10.1 mm. The data appear to follow the diffusion equation. Determine the average diffusivity over the range $X/X_C = 1.0–0.10$.

9.10-1. ***Drying a Bed of Solids by Through Circulation.*** Repeat Example 9.10-1 for drying of a packed bed of wet cylinders by through circulation of the drying air. Use the same conditions except that the air velocity is 0.381 m/s.

9.10-2. ***Derivation of Equation for Through Circulation Drying.*** Different forms of Eqs. (9.10-11) and (9.10-12) can be derived using humidity and mass-transfer equations rather than temperature and heat-transfer equations. This can be done by writing a mass-balance equation similar to Eq. (9.10-2) for a heat balance and a mass-transfer equation similar to Eq. (9.10-3).

(a) Derive the final equation for the time of drying in the constant-rate period using humidity and mass-transfer equations.

(b) Repeat for the falling-rate period.

Ans. (a) $t = \dfrac{\rho_S x_1 (X_1 - X_C)}{G(H_W - H_1)(1 - e^{-k_y M_B a x_1/G})}$

9.10-3. ***Through Circulation Drying in the Constant-Rate Period.*** Spherical wet catalyst pellets having a diameter of 12.7 mm are being dried in a through circulation dryer. The pellets are in a bed 63.5 mm thick on a screen. The solids are being dried by air entering with a superficial velocity of 0.914 m/s at 82.2°C and having a humidity $H = 0.01$ kg H_2O/kg dry air. The dry solid density is determined as 1522 kg/m^3, and the void fraction in the bed is 0.35. The initial free moisture content is 0.90 kg H_2O/kg solid and the solids are to be dried to a free moisture content of 0.45, which is above the critical free moisture content. Calculate the time for drying in this constant-rate period.

9.10-4. ***Material and Heat Balances on a Continuous Dryer.*** Repeat Example 9.10-2, making heat and material balances, but with the following changes. The solid enters at 15.6°C and leaves at 60°C. The gas enters at 87.8°C and leaves at 32.2°C. Heat losses from the dryer are estimated as 2931 W.

9.10-5. ***Drying in a Continuous Tunnel Dryer.*** A rate of feed of 700 lb$_m$ dry solid/h containing a free moisture content of $X_1 = 0.4133$ lb H_2O/lb dry solid is to be dried to $X_2 = 0.0374$ lb H_2O/lb dry solid in a continuous counterflow tunnel dryer. A flow of 13 280 lb$_m$ dry air/h enters at 203°F with an $H_2 = 0.0562$ lb H_2O/lb dry air. The stock enters at the wet bulb temperature of 119°F and remains essentially constant in temperature in the dryer. The saturation humidity at 119°F from the humidity chart is $H_W = 0.0786$ lb H_2O/lb dry air. The surface area available for drying is $(A/L_S) = 0.30$ ft^2/lb$_m$ dry solid.

A small batch experiment was performed using constant drying conditions, air velocity, and temperature of the solid approximately the same as in the continuous dryer. The equilibrium critical moisture content was found to be $X_C = 0.0959$ lb H_2O/lb dry solid, and the experimental value of $k_y M_B$ was found as 30.15 lb$_m$ air/h · ft^2. In the falling-rate period, the drying rate was directly proportional to X.

For the continuous dryer, calculate the time in the dryer in the constant-rate zone and in the falling-rate zone.

Ans. $H_C = 0.0593$ lb H_2O/lb dry air, $H_1 = 0.0760$ lb H_2O/lb dry air, $t = 4.24$ h in the constant-rate zone, $t = 0.47$ h in the falling-rate zone

9.10-6. ***Air Recirculation in a Continuous Dryer.*** The wet feed material to a continuous dryer contains 50 wt % water on a wet basis and is dried to 27 wt % by countercurrent air flow. The dried product leaves at the rate of 907.2 kg/h. Fresh air to the system is at 25.6°C and has a humidity of $H = 0.007$ kg H_2O/kg dry air. The moist air leaves the dryer at 37.8°C and $H = 0.020$ and part of it is recirculated and mixed with the fresh air before entering a heater. The heated mixed air enters the dryer at 65.6°C and $H = 0.010$. The solid enters at 26.7°C and leaves at 26.7°C. Calculate the fresh-air flow, the percent air leaving the dryer that is recycled, the heat added in the heater, and the heat loss from the dryer.

Ans. 32 094 kg fresh dry air/h, 23.08% recycled, 440.6 kW in heater

9.12-1. *Sterilizing Canned Foods.* In a sterilizing retort, cans of a given food were heated and the average temperature in the center of a can is approximately 98.9°C for the first 30 min. The average temperature for the next period is 110°C. If the F_0 for the spore organism is 2.50 min and $z = 10$°C, calculate the time of heating at 110°C to make the process safe.

Ans. 29.9 min

9.12-2. *Temperature Effect on Decimal Reduction Time.* Prove by combining Eqs. (9.12-6) and (9.12-8) that a plot of $\log_{10} D_T$ versus $1/T$ (T in degrees absolute) is a straight line.

9.12-3. *Thermal Process Time for Pea Purée.* For cans of pea purée, the $F_0 = 2.45$ min and $z = 9.94$°C (C2). Neglecting heat-up time, determine the process time for adequate sterilization at 112.8°C at the center of the can.

Ans. $t = 16.76$ min

9.12-4. *Process Time for Adequate Sterilization.* The F_0 value for a given canned food is 2.80 min and z is 18°F (10°C). The center temperatures of a can of this food when heated in a retort were as follows for the time periods given: t_1 (0–10 min), $T_1 = 140$°F; t_2 (10–30 min), $T_2 = 185$°F; t_3 (30–50 min), $T_3 = 220$°F; t_4 (50–80 min), $T_4 = 230$°F; t_5 (80–100 min), $T_5 = 190$°F. Determine if adequate sterilization is obtained.

9.12-5. *Process Time and Graphical Integration.* The following time–temperature data were obtained for the heating, holding, and cooling of a canned food product in a retort, the temperature being measured in the center of the can.

t(min)	*T(°F)*	*t(min)*	*T(°F)*
0	110 (43.3°C)	80	232 (111.1)
20	165 (73.9)	90	225 (107.2)
40	205 (96.1)	100	160 (71.1)
60	228 (108.9)		

The F_0 value used is 2.60 min and z is 18°F (10°C). Calculate the F_0 value for this process and determine if the thermal processing is adequate. Use SI and English units.

9.12-6. *Sterility Level of Fermentation Medium.* The aqueous medium in a fermentor is being sterilized and the time–temperature data obtained are as follows.

Time (min)	0	10	20	25	30	35
Temperature (°C)	100	110	120	120	110	100

The reaction velocity constant k in min^{-1} for the contaminating bacterial spores can be represented as (A1)

$$k = 7.94 \times 10^{38} e^{-(68.7 \times 10^3)/1.987T}$$

where $T = \text{K}$. The contamination level $N_0 = 1 \times 10^{12}$ spores. Calculate the sterility level N at the end and V.

9.12-7. *Time for Pasteurization of Milk.* Calculate the time in min at 62.8°C for pasteurization of milk. The F_0 value to be used at 65.6°C is 9.0 min. The z value is 5°C.

Ans. $t = 32.7$ min

9.12-8. ***Reduction in Number of Viable Cells in Pasteurization.*** In a given pasteurization process the reduction in the number of viable cells used is 10^{15} and the F_0 value used is 9.0 min. If the reduction is to be increased to 10^{16} because of increased contamination, what would be the new F_0 value?

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CHAPTER 10

Stage and Continuous Gas–Liquid Separation Processes

10.1 TYPES OF SEPARATION PROCESSES AND METHODS

10.1A Introduction

Many chemical process materials and biological substances occur as mixtures of different components in the gas, liquid, or solid phase. In order to separate or remove one or more of the components from its original mixture, it must be contacted with another phase. The two phases are brought into more or less intimate contact with each other so that a solute or solutes can diffuse from one to the other. The two bulk phases are usually only somewhat miscible in each other. The two-phase pair can be gas–liquid, gas–solid, liquid–liquid, or liquid–solid. During the contact of the two phases the components of the original mixture redistribute themselves between the two phases. The phases are then separated by simple physical methods. By choosing the proper conditions and phases, one phase is enriched while the other is depleted in one or more components.

10.1B Types of Separation Processes

1. Absorption. When the two contacting phases are a gas and a liquid, the unit operation is called *absorption.* A solute A or several solutes are absorbed from the gas phase into a liquid phase in absorption. This process involves molecular and turbulent diffusion or mass transfer of solute A through a stagnant nondiffusing gas B into a stagnant liquid C. An example is absorption of ammonia A from air B by the liquid water C. Usually, the exit ammonia–water solution is distilled to recover relatively pure ammonia.

Another example is absorbing SO_2 from the flue gases by absorption in alkaline solutions. In the hydrogenation of edible oils in the food industry, hydrogen gas is bubbled into oil and absorbed. The hydrogen in solution then reacts with the oil in the presence of a catalyst. The reverse of absorption is called *stripping* or *desorption*, and the same theories and basic principles hold. An example is the steam stripping of nonvolatile oils, in which the steam contacts the oil and small amounts of volatile components of the oil pass out with the steam.

When the gas is pure air and the liquid is pure water, the process is called *humidification*. *Dehumidification* involves removal of water vapor from air.

2. Distillation. In the *distillation* process, a volatile vapor phase and a liquid phase that vaporizes are involved. An example is distillation of an ethanol–water solution, where the vapor contains a concentration of ethanol greater than in the liquid. Another example is distillation of an ammonia–water solution to produce a vapor richer in ammonia. In the distillation of crude petroleum, various fractions, such as gasoline, kerosene, and heating oils, are distilled off.

3. Liquid–liquid extraction. When the two phases are liquids, where a solute or solutes are removed from one liquid phase to another liquid phase, the process is called *liquid–liquid extraction*. One example is extraction of acetic acid from a water solution by isopropyl ether. In the pharmaceutical industry, antibiotics in an aqueous fermentation solution are sometimes removed by extraction with an organic solvent.

4. Leaching. If a fluid is being used to extract a solute from a solid, the process is called *leaching*. Sometimes this process is also called *extraction*. Examples are leaching copper from solid ores by sulfuric acid and leaching vegetable oils from solid soybeans by organic solvents such as hexane. Vegetable oils are also leached from other biological products, such as peanuts, rape seeds, and sunflower seeds. Soluble sucrose is leached by water extraction from sugar cane and beets.

5. Membrane processing. Separation of molecules by the use of membranes is a relatively new unit operation and is becoming more important. The relatively thin, solid membrane controls the rate of movement of molecules between two phases. It is used to remove salt from water, purify gases, in food processing, and so on.

6. Crystallization. Solute components soluble in a solution can be removed from the solution by adjusting the conditions, such as temperature or concentration, so that the solubility of one or more solute components is exceeded and they *crystallize* out as a solid phase. Examples of this separation process are crystallization of sugar from solution and crystallization of metal salts in the processing of metal ore solutions.

7. Adsorption. In an adsorption process one or more components of a liquid or gas stream are adsorbed on the surface or in the pores of a solid adsorbent and a separation is obtained. Examples include removal of organic compounds from polluted water, separation of paraffins from aromatics, and removal of solvents from air.

10.1C Processing Methods

Several methods of processing are used in the separations discussed above. The two phases, such as gas and liquid, or liquid and liquid, can be mixed together in a vessel and then separated. This is a *single-stage process*. Often the phases are mixed in one stage, separated, and then contacted again in a *multiple-stage process*. These two methods can be carried out batchwise or continuously. In still another general method, the two phases can be contacted continuously in a packed tower.

In this chapter, humidification and absorption will be considered; in Chapter 11, distillation; in Chapter 12, adsorption, liquid–liquid extraction, leaching, and crystallization, and in Chapter 13, membrane processes. In all of these processes the equilibrium relations between the two phases being considered must be known. This is discussed for gas–liquid systems in Section 10.2 and for the other systems in Chapters 11, 12, and 13.

10.2 EQUILIBRIUM RELATIONS BETWEEN PHASES

10.2A Phase Rule and Equilibrium

In order to predict the concentration of a solute in each of two phases in equilibrium, experimental equilibrium data must be available. Also, if the two phases are not at equilibrium, the rate of mass transfer is proportional to the driving force, which is the departure from equilibrium. In all cases involving equilibria, two phases are involved, such as gas–liquid or liquid–liquid. The important variables affecting the equilibrium of a solute are temperature, pressure, and concentration.

The equilibrium between two phases in a given situation is restricted by the phase rule:

$$F = C - P + 2 \tag{10.2-1}$$

where P is the number of phases at equilibrium, C the number of total components in the two phases when no chemical reactions are occurring, and F the number of variants or degrees of freedom of the system. For example, for the gas–liquid system of CO_2–air–water, there are two phases and three components (considering air as one inert component). Then, by Eq. (10.2-1),

$$F = C - P + 2 = 3 - 2 + 2 = 3$$

This means that there are 3 degrees of freedom. If the total pressure and the temperature are set, only one variable is left that can be arbitrarily set. If the mole fraction composition x_A of CO_2 (A) in the liquid phase is set, the mole fraction composition y_A or pressure p_A in the gas phase is automatically determined.

The phase rule does not tell us the partial pressure p_A in equilibrium with the selected x_A. The value of p_A must be determined experimentally. The two phases can, of course, be gas–liquid, liquid–solid, and so on. For example, the equilibrium distribution of acetic acid between a water phase and an isopropyl ether phase has been determined experimentally for various conditions.

10.2B Gas–Liquid Equilibrium

1. Gas–liquid equilibrium data. To illustrate the obtaining of experimental gas–liquid equilibrium data, the system SO_2–air–water will be considered. An amount of gaseous SO_2, air, and water are put in a closed container and shaken repeatedly at a given temperature until equilibrium is reached. Samples of the gas and liquid are analyzed to give the partial pressure p_A in atm of SO_2 (A) in the gas and mole fraction x_A in the liquid. Figure 10.2-1 shows a plot of data from Appendix A.3 of the partial pressure p_A of SO_2 in the vapor in equilibrium with the mole fraction x_A of SO_2 in the liquid at 293 K (20°C).

2. Henry's law. Often the equilibrium relation between p_A in the gas phase and x_A can be expressed by a straight-line Henry's law equation at low concentrations.

$$p_A = Hx_A \tag{10.2-2}$$

where H is the Henry's law constant in atm/mole fraction for the given system. If both sides of Eq. (10.2-2) are divided by total pressure P in atm,

$$y_A = H'x_A \tag{10.2-3}$$

where H' is the Henry's law constant in mole frac gas/mole frac liquid and is equal to H/P. Note that H' depends on total pressure, whereas H does not.

In Fig. 10.2-1 the data follow Henry's law up to a concentration x_A of about 0.005, where $H = 29.6$ atm/mol frac. In general, up to a total pressure of about 5×10^5 Pa (5 atm) the value of H is independent of P. Data for some common gases with water are given in Appendix A.3.

EXAMPLE 10.2-1. Dissolved Oxygen Concentration in Water
What will be the concentration of oxygen dissolved in water at 298 K when the solution is in equilibrium with air at 1 atm total pressure? The Henry's law constant is 4.38×10^4 atm/mol fraction.

Solution: The partial pressure p_A of oxygen (A) in air is 0.21 atm. Using Eq. (10.2-2),

$$0.21 = Hx_A = 4.38 \times 10^4 x_A$$

Solving, $x_A = 4.80 \times 10^{-6}$ mol fraction. This means that 4.80×10^{-6} mol O_2 is dissolved in 1.0 mol water plus oxygen or 0.000853 part O_2/100 parts water.

10.3 SINGLE AND MULTIPLE EQUILIBRIUM CONTACT STAGES

10.3A Single-Stage Equilibrium Contact

In many operations of the chemical and other process industries, the transfer of mass from one phase to another occurs, usually accompanied by a separation of the components of the mixture, since one component will be transferred to a larger extent than will another component.

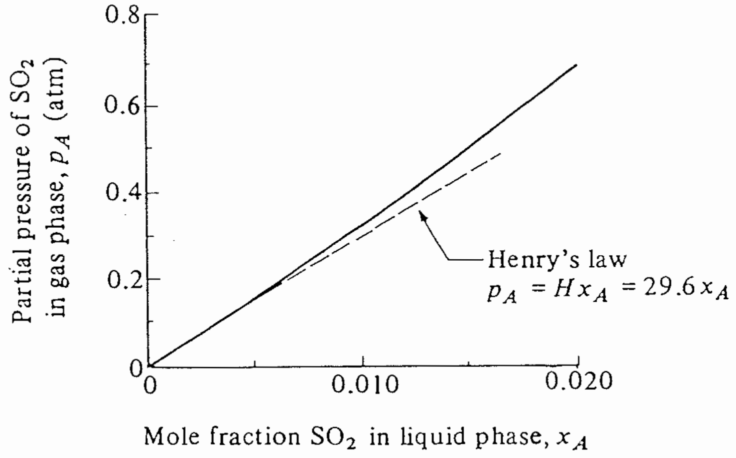
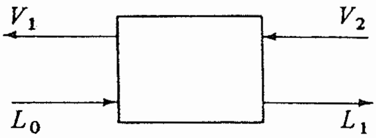


FIGURE 10.2-1. *Equilibrium plot for SO_2–water system at 293 K (20°C).*

FIGURE 10.3-1. *Single-stage equilibrium process.*



A single-stage process can be defined as one in which two different phases are brought into intimate contact with each other and then are separated. During the time of contact, intimate mixing occurs and the various components diffuse and redistribute themselves between the two phases. If mixing time is long enough, the components are essentially at equilibrium in the two phases after separation and the process is considered a single equilibrium stage.

A single equilibrium stage can be represented as in Fig. 10.3-1. The two entering phases, L_0 and V_2, of known amounts and compositions, enter the stage, mixing and equilibration occur, and the two exit streams L_1 and V_1 leave in equilibrium with each other. Making a total mass balance,

$$L_0 + V_2 = L_1 + V_1 = M \tag{10.3-1}$$

where L is kg (lb$_m$), V is kg, and M is total kg.

Assuming that three components, A, B, and C, are present in the streams and making a balance on A and C,

$$L_0 x_{A0} + V_2 y_{A2} = L_1 x_{A1} + V_1 y_{A1} = M x_{AM} \tag{10.3-2}$$

$$L_0 x_{C0} + V_2 y_{C2} = L_1 x_{C1} + V_1 y_{C1} = M x_{CM} \tag{10.3-3}$$

An equation for B is not needed since $x_A + x_B + x_C = 1.0$. The mass fraction of A in the L stream is x_A and y_A in the V stream. The mass fraction of A in the M stream is x_{AM}.

To solve the three equations, the equilibrium relations between the components must be known. In Section 10.3B, this will be done for a gas–liquid system and in Chapter 11 for a vapor–liquid system. Note that Eqs. (10.3-1)–(10.3-3) can also be written using mole units, with L and V having units of moles and x_A and y_A units of mole fraction.

10.3B Single-Stage Equilibrium Contact for Gas–Liquid System

In the usual gas–liquid system the solute A is in the gas phase V, along with inert air B and in the liquid phase L along with inert water C. Assuming that air is essentially insoluble in the water phase and that water does not vaporize to the gas phase, the gas phase is a binary A–B and the liquid phase is a binary A–C. Using moles and mole fraction units, Eq. (10.3-1) holds for a single-stage process for the total material balance. Since component A is the only component that redistributes between the two phases, a balance on A can be written as follows.

$$L'\left(\frac{x_{A0}}{1 - x_{A0}}\right) + V'\left(\frac{y_{A2}}{1 - y_{A2}}\right) = L'\left(\frac{x_{A1}}{1 - x_{A1}}\right) + V'\left(\frac{y_{A1}}{1 - y_{A1}}\right) \tag{10.3-4}$$

where L' is moles inert water C and V' is moles inert air B. Both L' and V' are constant and usually known.

To solve Eq. (10.3-4), the relation between y_{A1} and x_{A1} in equilibrium is given by Henry's law.

$$y_{A1} = H'x_{A1} \qquad \textbf{(10.3-5)}$$

If the solution is not dilute, equilibrium data in the form of a plot of p_A or y_A versus x_A must be available, such as in Fig. 10.2-1.

EXAMPLE 10.3-1. Equilibrium Stage Contact for CO_2–Air–Water
A gas mixture at 1.0 atm pressure abs containing air and CO_2 is contacted in a single-stage mixer continuously with pure water at 293 K. The two exit gas and liquid streams reach equilibrium. The inlet gas flow rate is 100 kg mol/h, with a mole fraction of CO_2 of $y_{A2} = 0.20$. The liquid flow rate entering is 300 kg mol water/h. Calculate the amounts and compositions of the two outlet phases. Assume that water does not vaporize to the gas phase.

Solution: The flow diagram is the same as given in Fig. 10.3-1. The inert water flow is $L' = L_0 = 300$ kg mol/h. The inert air flow V' is obtained from Eq. (10.3-6).

$$V' = V(1 - y_A) \qquad \textbf{(10.3-6)}$$

Hence, the inert air flow is $V' = V_2(1 - y_{A2}) = 100(1 - 0.20) = 80$ kg mol/h. Substituting into Eq. (10.3-4) to make a balance on CO_2 (A),

$$300\left(\frac{0}{1-0}\right) + 80\left(\frac{0.20}{1-0.20}\right) = 300\left(\frac{x_{A1}}{1-x_{A1}}\right) + 80\left(\frac{y_{A1}}{1-y_{A1}}\right) \qquad \textbf{(10.3-7)}$$

At 293 K, the Henry's law constant from Appendix A.3 is $H = 0.142 \times 10^4$ atm/mol frac. Then $H' = H/P = 0.142 \times 10^4/1.0 = 0.142 \times 10^4$ mol frac gas/mol frac liquid. Substituting into Eq. (10.3-5),

$$y_{A1} = 0.142 \times 10^4 x_{A1} \qquad \textbf{(10.3-8)}$$

Substituting Eq. (10.3-8) into (10.3-7) and solving, $x_{A1} = 1.41 \times 10^{-4}$ and $y_{A1} = 0.20$. To calculate the total flow rates leaving,

$$L_1 = \frac{L'}{1 - x_{A1}} = \frac{300}{1 - 1.41 \times 10^{-4}} = 300 \text{ kg mol/h}$$

$$V_1 = \frac{V'}{1 - y_{A1}} = \frac{80}{1 - 0.20} = 100 \text{ kg mol/h}$$

In this case, since the liquid solution is so dilute, $L_0 \cong L_1$.

10.3C Countercurrent Multiple-Contact Stages

1. Derivation of general equation. In Section 10.3A we used single-stage contact to transfer the solute A between the V and L phases. In order to transfer more solute from, say, the V_1 stream, the single-stage contact can be repeated by again contacting the V_1 stream leaving the first stage with fresh L_0. This can be repeated using multiple stages. However, this is wasteful of the L_0 stream and gives a dilute product in the outlet L_1 streams. To conserve use of the L_0 stream and to get a more concentrated product, countercurrent multiple-stage contacting is generally used. This is somewhat similar to countercurrent heat transfer in a heat exchanger, where the outlet heated stream approaches more closely the temperature of the inlet hot stream.

The process flow diagram for a countercurrent stage process is shown in Fig. 10.3-2. The inlet L stream is L_0 and the inlet V stream is V_{N+1} instead of V_2 as for a single-stage in Fig. 10.3-1. The outlet product streams are V_1 and L_N and the total number of stages is N. The component A is being exchanged between the V and L streams. The V stream is

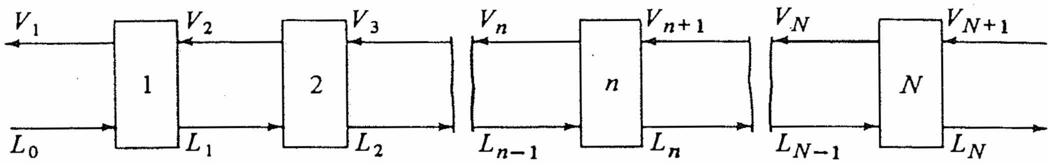


FIGURE 10.3-2. *Countercurrent multiple-stage process.*

composed mainly of component B and the L stream of component C. Components B and C may or may not be somewhat miscible in each other. The two-phase system can be gas–liquid, vapor–liquid, liquid–liquid, or other.

Making a total overall balance on all stages,

$$L_0 + V_{N+1} = L_N + V_1 = M \tag{10.3-9}$$

where V_{N+1} is mol/h entering, L_N is mol/h leaving the process, and M is the total flow. Note in Fig. 10.3-2 that any two streams leaving a stage are in equilibrium with each other. For example, in stage n, V_n and L_n are in equilibrium. For an overall component balance on A, B, or C,

$$L_0 x_0 + V_{N+1} y_{N+1} = L_N x_N + V_1 y_1 = M x_M \tag{10.3-10}$$

where x and y are mole fractions. Flows in kg/h (lb_m/h) and mass fraction can also be used in these equations.

Making a total balance over the first n stages,

$$L_0 + V_{n+1} = L_n + V_1 \tag{10.3-11}$$

Making a component balance over the first n stages,

$$L_0 x_0 + V_{n+1} y_{n+1} = L_n x_n + V_1 y_1 \tag{10.3-12}$$

Solving for y_{n+1} in Eq. (10.3-12).

$$y_{n+1} = \frac{L_n x_n}{V_{n+1}} + \frac{V_1 y_1 - L_0 x_0}{V_{n+1}} \tag{10.3-13}$$

This is an important material-balance equation, often called an *operating line*. It relates the concentration y_{n+1} in the V stream with x_n in the L stream passing it. The terms V_1, y_1, L_0, and x_0 are constant and usually known or can be determined from Eqs. (10.3-9)–(10.3-12).

2. *Countercurrent contact with immiscible streams.* An important case where the solute A is being transferred occurs when the solvent stream V contains components A and B with no C and the solvent stream L contains A and C with no B. The two streams L and V are immiscible in each other with only A being transferred. When Eq. (10.3-13) is plotted on an xy plot (x_A and y_A of component A) as in Fig. 10.3-3, it is often curved, since the slope L_n/V_{n+1} of the operating line varies if the L and V streams vary from stage to stage.

In Fig. 10.3-3 is plotted the equilibrium line that relates the compositions of two streams leaving a stage in equilibrium with each other. To determine the number of ideal stages required to bring about a given separation or reduction of the concentration of A from y_{N+1} to y_1, the calculation is often done graphically. Starting at stage 1, y_1 and x_0 are on the operating line, Eq. (10.3-13), plotted in the figure. The vapor y_1 leaving is in equilibrium with the leaving x_1 and both compositions are on the equilibrium line. Then y_2 and x_1 are on the operating line and y_2 is in equilibrium with x_2, and so on. Each stage is represented by a step drawn on Fig. 10.3-3. The steps are continued on the graph until y_{N+1} is reached. Alternatively, we can start at y_{N+1} and draw the steps going to y_1.

If the streams L and V are dilute in component A, the streams are approximately constant and the slope L_n/V_{n+1} of Eq. (10.3-13) is nearly constant. Hence, the operating line is essentially a straight line on an xy plot. In distillation, where only components A and B are present, Eq. (10.3-13) also holds for the operating line, and this will be covered in Chapter 11. Cases where A, B, and C are appreciably soluble in each other often occur in liquid–liquid extraction and will be discussed in Chapter 12.

EXAMPLE 10.3-2. Absorption of Acetone in a Countercurrent Stage Tower

It is desired to absorb 90% of the acetone in a gas containing 1.0 mol % acetone in air in a countercurrent stage tower. The total inlet gas flow to the tower is 30.0 kg mol/h, and the total inlet pure water flow to be used to absorb the acetone is 90 kg mol H_2O/h. The process is to operate isothermally at 300 K and a total pressure of 101.3 kPa. The equilibrium relation for the acetone (A) in the gas–liquid is $y_A = 2.53x_A$. Determine the number of theoretical stages required for this separation.

Solution: The process flow diagram is similar to Fig. 10.3-3. Given values are $y_{AN+1} = 0.01$, $x_{A0} = 0$, $V_{N+1} = 30.0$ kg mol/h, and $L_0 = 90.0$ kg mol/h. Making an acetone material balance,

$$\text{amount of entering acetone} = y_{AN+1}V_{N+1} = 0.01(30.0) = 0.30 \text{ kg mpl/h}$$

$$\text{entering air} = (1 - y_{AN+1})V_{N+1} = (1 - 0.01)(30.0)$$

$$= 29.7 \text{ kg mol air/h}$$

$$\text{acetone leaving in } V_1 = 0.10(0.30) = 0.030 \text{ kg mol/h}$$

$$\text{acetone leaving in } L_N = 0.90(0.30) = 0.27 \text{ kg mol/h}$$

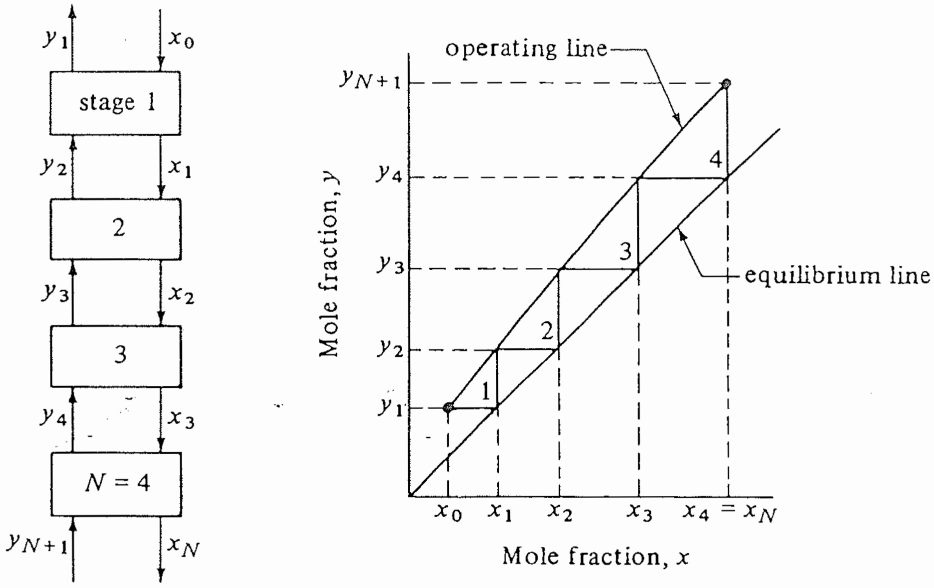


FIGURE 10.3-3. *Number of stages in a countercurrent multiple-stage contact process.*

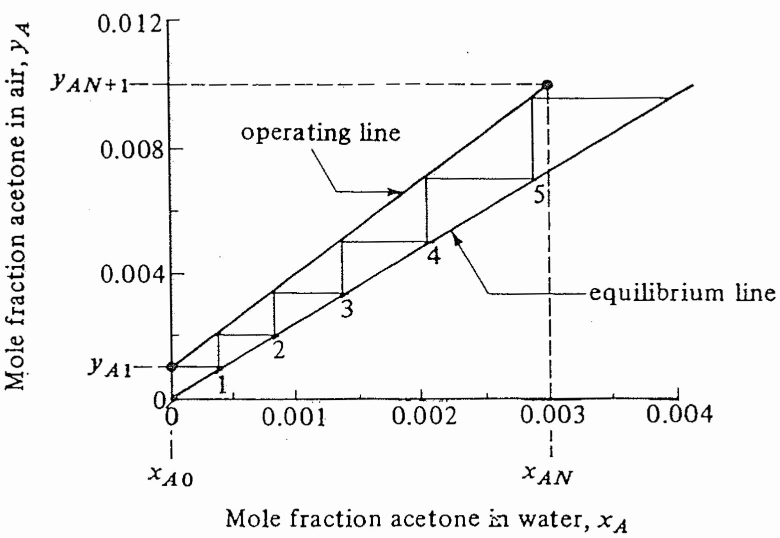


FIGURE 10.3-4. *Theoretical stages for countercurrent absorption in Example 10.3-2.*

$$V_1 = 29.7 + 0.03 = 29.73 \text{ kg mol air + acetone/h}$$

$$y_{A1} = \frac{0.030}{29.73} = 0.00101$$

$$L_N = 90.0 + 0.27 = 90.27 \text{ kg mol water + acetone/h}$$

$$x_{AN} = \frac{0.27}{90.27} = 0.00300$$

Since the flow of liquid varies only slightly from $L_0 = 90.0$ at the inlet to $L_N = 90.27$ at the outlet and V from 30.0 to 29.73, the slope L_n/V_{n+1} of the operating line in Eq. (10.3-13) is essentially constant. This line is plotted in Fig. 10.3-4 and the equilibrium relation $y_A = 2.53x_A$ is also plotted. Starting at point y_{A1}, x_{A0}, the stages are drawn as shown. About 5.2 theoretical stages are required.

10.3D Analytical Equations for Countercurrent Stage Contact

When the flow rates V and L in a countercurrent process are essentially constant, the operating-line equation (10.3-13) becomes straight. If the equilibrium line is also a straight line over the concentration range, simplified analytical expressions can be derived for the number of equilibrium stages in a countercurrent stage process.

Referring again to Fig. 10.3-2, Eq. (10.3-14) is an overall component balance on component A.

$$L_0 x_0 + V_{N+1} y_{N+1} = L_N x_N + V_1 y_1 \tag{10.3-14}$$

Rearranging,

$$L_N x_N - V_{N+1} y_{N+1} = L_0 x_0 - V_1 y_1 \tag{10.3-15}$$

Making a component balance for A on the first n stages,

$$L_0 x_0 + V_{n+1} y_{n+1} = L_n x_n + V_1 y_1 \tag{10.3-16}$$

Rearranging,

$$L_0 x_0 - V_1 y_1 = L_n x_n - V_{n+1} y_{n+1} \tag{10.3-17}$$

Equating Eq. (10.3-15) to (10.3-17),

$$L_n x_n - V_{n+1} y_{n+1} = L_N x_N - V_{N+1} y_{N+1} \tag{10.3-18}$$

Since the molar flows are constant, $L_n = L_N = \text{constant} = L$ and $V_{n+1} = V_{N+1} = \text{constant} = V$. Then Eq. (10.3-18) becomes

$$L(x_n - x_N) = V(y_{n+1} - y_{N+1}) \tag{10.3-19}$$

Since y_{n+1} and x_{n+1} are in equilibrium, and the equilibrium line is straight, $y_{n+1} = mx_{n+1}$. Also $y_{N+1} = mx_{N+1}$. Substituting mx_{n+1} for y_{n+1} and calling $A = L/mV$, Eq. (10.3-19) becomes

$$x_{n+1} - Ax_n = \frac{y_{N+1}}{m} - Ax_N \tag{10.3-20}$$

where A is an absorption factor and is constant.

All factors on the right-hand side of Eq. (10.3-20) are constant. This equation is a linear first-order difference equation and can be solved by the calculus of finite-difference methods (G1, M1). The final derived equations are as follows.

For transfer of solute A from phase L to V (stripping),

$$\frac{x_0 - x_N}{x_0 - (y_{N+1}/m)} = \frac{(1/A)^{N+1} - (1/A)}{(1/A)^{N+1} - 1} \tag{10.3-21}$$

$$N = \frac{\log\left[\dfrac{x_0 - (y_{N+1}/m)}{x_N - (y_{N+1}/m)}(1 - A) + A\right]}{\log(1/A)} \tag{10.3-22}$$

When $A = 1$,

$$N = \frac{x_0 - x_N}{x_N - (y_{N+1}/m)} \tag{10.3-23}$$

For transfer of solute A from phase V to L (absorption),

$$\frac{y_{N+1} - y_1}{y_{N+1} - mx_0} = \frac{A^{N+1} - A}{A^{N+1} - 1} \tag{10.3-24}$$

$$N = \frac{\log\left[\dfrac{y_{N+1} - mx_0}{y_1 - mx_0}\left(1 - \dfrac{1}{A}\right) + \dfrac{1}{A}\right]}{\log A} \tag{10.3-25}$$

When $A = 1$,

$$N = \frac{y_{N+1} - y_1}{y_1 - mx_0} \tag{10.3-26}$$

Often the term A is called the *absorption factor* and S the *stripping factor*, where $S = 1/A$. These equations can be used with any consistent set of units such as mass flow and mass fraction or molar flow and mole fraction. Such series of equations are often called *Kremser equations* and are convenient to use. If A varies slightly from the inlet to the outlet, the geometric average of the two values can be used, with the value of m at the dilute end being used for both values of A.

EXAMPLE 10.3-3. Number of Stages by Analytical Equation.
Repeat Example 10.3-2 but use the Kremser analytical equations for countercurrent stage processes.

Solution: At one end of the process at stage 1, $V_1 = 29.73$ kg mol/h, $y_{A1} = 0.00101$, $L_0 = 90.0$, and $x_{A0} = 0$. Also, the equilibrium relation is $y_A = 2.53x_A$ where $m = 2.53$. Then,

$$A_1 = \frac{L}{mV} = \frac{L_0}{mV_1} = \frac{90.0}{2.53 \times 29.73} = 1.20$$

At stage N, $V_{N+1} = 30.0$, $y_{AN+1} = 0.01$, $L_N = 90.27$, and $x_{AN} = 0.00300$.

$$A_N = \frac{L_N}{mV_{N+1}} = \frac{90.27}{2.53 \times 30.0} = 1.19$$

The geometric average $A = \sqrt{A_1 A_N} = \sqrt{1.20 \times 1.19} = 1.195$.

The acetone solute is transferred from the V to the L phase (absorption). Substituting into Eq. (10.3-25),

$$N = \frac{\log\left[\dfrac{0.01 - 2.53(0)}{0.00101 - 2.53(0)}\left(1 - \dfrac{1}{1.195}\right) + \dfrac{1}{1.195}\right]}{\log (1.195)} = 5.04 \text{ stages}$$

This compares closely with 5.2 stages obtained using the graphical method.

10.4 MASS TRANSFER BETWEEN PHASES

10.4A Introduction and Equilibrium Relations

1. Introduction to interphase mass transfer. In Chapter 7 we considered mass transfer from a fluid phase to another phase, which was primarily a solid phase. The solute A was usually transferred from the fluid phase by convective mass transfer and through the solid by diffusion. In the present section we shall be concerned with the mass transfer of solute A from one fluid phase by convection and then through a second fluid phase by convection. For example, the solute may diffuse through a gas phase and then diffuse through and be absorbed in an adjacent and immiscible liquid phase. This occurs in the case of absorption of ammonia from air by water.

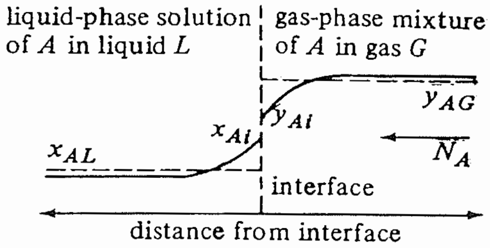
The two phases are in direct contact with each other, such as in a packed, tray, or spray-type tower, and the interfacial area between the phases is usually not well defined. In two-phase mass transfer, a concentration gradient will exist in each phase, causing mass transfer to occur. At the interface between the two fluid phases, equilibrium exists in most cases.

2. Equilibrium relations. Even when mass transfer is occurring equilibrium relations are important to determine concentration profiles for predicting rates of mass transfer. In Section 10.2 the equilibrium relation in a gas–liquid system and Henry's law were discussed. In Section 7.1C a discussion covered equilibrium distribution coefficients between two phases. These equilibrium relations will be used in discussion of mass transfer between phases in this section.

10.4B Concentration Profiles in Interphase Mass Transfer

In the majority of mass-transfer systems, two phases, which are essentially immiscible in each other, are present and also an interface between these two phases. Assuming the solute A is diffusing from the bulk gas phase G to the liquid phase L, it must pass through phase G, through the interface, and then into phase L in series. A concentration gradient

FIGURE 10.4-1. *Concentration profile of solute A diffusing through two phases.*



must exist to cause this mass transfer through the resistances in each phase, as shown in Fig. 10.4-1. The average or bulk concentration of A in the gas phase in mole fraction units is y_{AG}, where $y_{AG} = p_A/P$, and x_{AL} in the bulk liquid phase in mole fraction units.

The concentration in the bulk gas phase y_{AG} decreases to y_{Ai} at the interface. The liquid concentration starts at x_{Ai} at the interface and falls to x_{AL}. At the interface, since there would be no resistance to transfer across this interface, y_{Ai} and x_{Ai} are in equilibrium and are related by the equilibrium distribution relation

$$y_{Ai} = f(x_{Ai}) \tag{10.4-1}$$

where y_{Ai} is a function of x_{Ai}. They are related by an equilibrium plot such as Fig. 10.1-1. If the system follows Henry's law, $y_A P$ or p_A and x_A are related by Eq. (10.2-2) at the interface.

Experimentally, the resistance at the interface has been shown to be negligible for most cases of mass transfer where chemical reactions do not occur, such as absorption of common gases from air to water and extraction of organic solutes from one phase to another. However, there are some exceptions. Certain surface-active compounds may concentrate at the interface and cause an "interfacial resistance" that slows down the diffusion of solute molecules. Theories to predict when interfacial resistance may occur are still obscure and unreliable.

10.4C Mass Transfer Using Film Mass-Transfer Coefficients and Interface Concentrations

1. Equimolar counterdiffusion. For equimolar counterdiffusion the concentrations of Fig. 10.4-1 can be plotted on an xy diagram in Fig. 10.4-2. Point P represents the bulk phase compositions x_{AG} and x_{AL} of the two phases and point M the interface concentrations y_{Ai} and x_{Ai}. For A diffusing from the gas to liquid and B in equimolar counterdiffusion from liquid to gas,

$$N_A = k'_y(y_{AG} - y_{Ai}) = k'_x(x_{Ai} - x_{AL}) \tag{10.4-2}$$

where k'_y is the gas-phase mass-transfer coefficient in kg mol/s · m^2 · mol frac (g mol/s · cm^2 · mol frac, lb mol/h · ft^2 · mol frac) and k'_x the liquid-phase mass-transfer coefficient in kg mol/s · m^2 · mol frac (g mol/s · cm^2 · mol frac, lb mol/h · ft^2 · mol frac). Rearranging Eq. (10.4-2),

$$-\frac{k'_x}{k'_y} = \frac{y_{AG} - y_{Ai}}{x_{AL} - x_{Ai}} \tag{10.4-3}$$

The driving force in the gas phase is $(y_{AG} - y_{Ai})$ and in the liquid phase it is $(x_{Ai} - x_{AL})$. The slope of the line PM is $-k'_x/k'_y$. This means if the two film coefficients k'_x and k'_y are known, the interface compositions can be determined by drawing line PM with a slope $-k'_x/k'_y$ intersecting the equilibrium line.

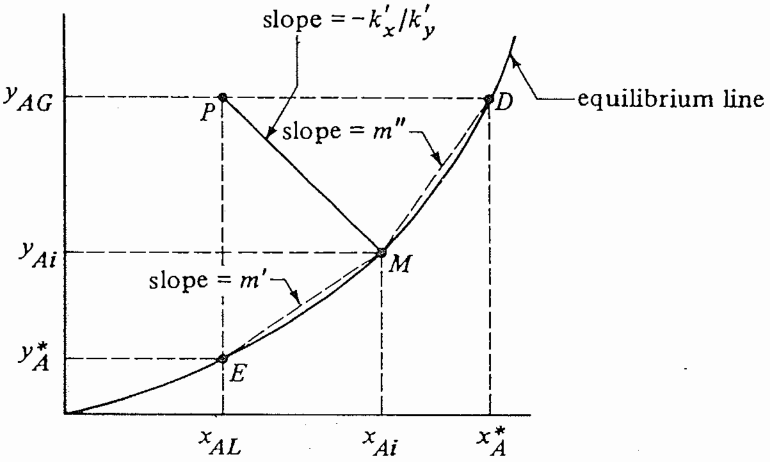


FIGURE 10.4-2. *Concentration driving forces and interface concentrations in interphase mass transfer (equimolar counterdiffusion).*

The bulk-phase concentrations y_{AG} and x_{AL} can be determined by simply sampling the mixed bulk gas phase and sampling the mixed bulk liquid phase. The interface concentrations are determined by Eq. (10.4-3).

2. Diffusion of A through stagnant or nondiffusing B. For the common case of A diffusing through a stagnant gas phase and then through a stagnant liquid phase, the concentrations are shown in Fig. 10.4-3, where P again represents bulk-phase compositions and M interface compositions. The equations for A diffusing through a stagnant gas and then through a stagnant liquid are

$$N_A = k_y(y_{AG} - y_{Ai}) = k_x(x_{Ai} - x_{AL}) \tag{10.4-4}$$

Now,

$$k_y = \frac{k'_y}{(1 - y_A)_{iM}} \qquad k_x = \frac{k'_x}{(1 - x_A)_{iM}} \tag{10.4-5}$$

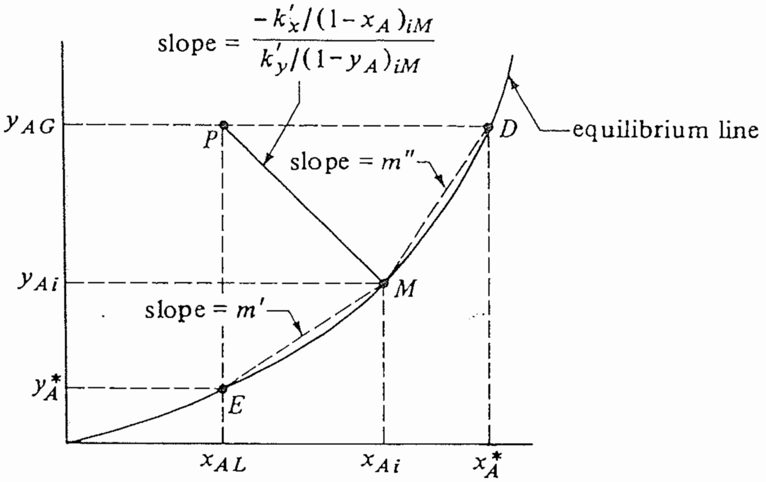


FIGURE 10.4-3. *Concentration driving forces and interface concentrations in interphase mass transfer (A diffusing through stagnant B).*

where

$$(1-y_A)_{iM} = \frac{(1-y_{Ai})-(1-y_{AG})}{\ln[(1-y_{Ai})/(1-y_{AG})]} \tag{10.4-6}$$

$$(1-x_A)_{iM} = \frac{(1-x_{AL})-(1-x_{Ai})}{\ln[(1-x_{AL})/(1-x_{Ai})]} \tag{10.4-7}$$

Then,

$$N_A = \frac{k'_y}{(1-y_A)_{iM}}(y_{AG}-y_{Ai}) = \frac{k'_x}{(1-x_A)_{iM}}(x_{Ai}-x_{AL}) \tag{10.4-8}$$

Note that $(1-y_A)_{iM}$ is the same as y_{BM} of Eq. (7.2-11) but is written for the interface, and $(1-x_A)_{iM}$ is the same as x_{BM} of Eq. (7.2-11). Using Eq. (10.4-8) and rearranging,

$$\frac{-k'_x/(1-x_A)_{iM}}{k'_y/(1-y_A)_{iM}} = \frac{y_{AG}-y_{Ai}}{x_{AL}-x_{Ai}} \tag{10.4-9}$$

The slope of the line PM in Fig. 10.4-3 to obtain the interface compositions is given by the left-hand side of Eq. (10.4-9). This differs from the slope of Eq. (10.4-3) for equimolar counterdiffusion by the terms $(1-y_A)_{iM}$ and $(1-x_A)_{iM}$. When A is diffusing through stagnant B and the solutions are dilute, $(1-y_A)_{iM}$ and $(1-x_A)_{iM}$ are close to 1.

A trial-and-error method is needed to use Eq. (10.4-9) to get the slope, since the left-hand side contains y_{Ai} and x_{Ai} that are being sought. For the first trial $(1-y_A)_{iM}$ and $(1-x_A)_{iM}$ are assumed to be 1.0 and Eq. (10.4-9) is used to get the slope and y_{Ai} and x_{Ai} values. Then for the second trial, these values of y_{Ai} and x_{Ai} are used to calculate a new slope to get new values of y_{Ai} and x_{Ai}. This is repeated until the interface compositions do not change. Three trials are usually sufficient.

EXAMPLE 10.4-1. Interface Compositions in Interphase Mass Transfer

The solute A is being absorbed from a gas mixture of A and B in a wetted-wall tower with the liquid flowing as a film downward along the wall. At a certain point in the tower the bulk gas concentration $y_{AG} = 0.380$ mol fraction and the bulk liquid concentration is $x_{AL} = 0.100$. The tower is operating at 298 K and 1.013×10^5 Pa and the equilibrium data are as follows:

x_A	y_A	x_A	y_A
0	0	0.20	0.131
0.05	0.022	0.25	0.187
0.10	0.052	0.30	0.265
0.15	0.087	0.35	0.385

The solute A diffuses through stagnant B in the gas phase and then through a nondiffusing liquid.

Using correlations for dilute solutions in wetted-wall towers, the film mass-transfer coefficient for A in the gas phase is predicted as $k_y = 1.465 \times 10^{-3}$ kg mol A/s · m² · mol frac (1.08 lb mol/h · ft² · mol frac) and for the liquid phase as $k_x = 1.967 \times 10^{-3}$ kg mol A/s · m² · mol frac (1.45 lb mol/h · ft² · mol frac). Calculate the interface concentrations y_{Ai} and x_{Ai} and the flux N_A.

Solution: Since the correlations are for dilute solutions, $(1-y_A)_{iM}$ and

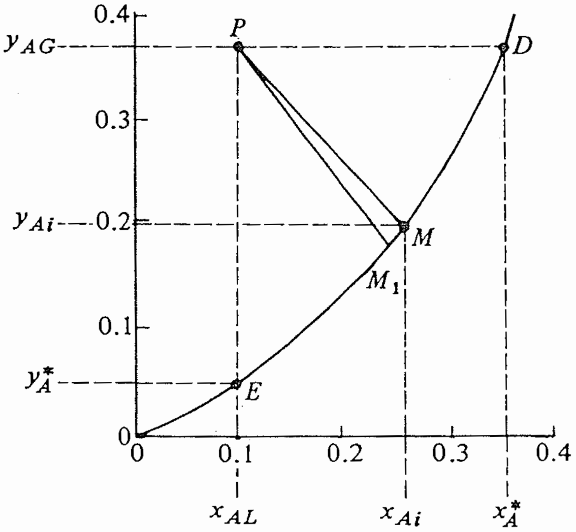


FIGURE 10.4-4. *Location of interface concentrations for Example 10.4-1.*

$(1 - x_A)_{iM}$ are approximately 1.0 and the coefficients are the same as k'_y and k'_x. The equilibrium data are plotted in Fig. 10.4-4. Point P is plotted at $y_{AG} = 0.380$ and $x_{AL} = 0.100$. For the first trial $(1 - y_A)_{iM}$ and $(1 - x_A)_{iM}$ are assumed as 1.0 and the slope of line PM is, from Eq. (10.4-9),

$$-\frac{k'_x/(1 - x_A)_{iM}}{k'_y/(1 - y_A)_{iM}} = -\frac{1.967 \times 10^{-3}/1.0}{1.465 \times 10^{-3}/1.0} = -1.342$$

A line through point P with a slope of -1.342 is plotted in Fig. 10.4-4 intersecting the equilibrium line at M_1, where $y_{Ai} = 0.183$ and $x_{Ai} = 0.247$.

For the second trial we use y_{Ai} and x_{Ai} from the first trial to calculate the new slope. Substituting into Eqs. (10.4-6) and (10.4-7),

$$(1 - y_A)_{iM} = \frac{(1 - y_{Ai}) - (1 - y_{AG})}{\ln [(1 - y_{Ai})/(1 - y_{AG})]}$$

$$= \frac{(1 - 0.183) - (1 - 0.380)}{\ln [(1 - 0.183)/(1 - 0.380)]} = 0.715$$

$$(1 - x_A)_{iM} = \frac{(1 - x_{AL}) - (1 - x_{Ai})}{\ln [(1 - x_{AL})/(1 - x_{Ai})]}$$

$$= \frac{(1 - 0.100) - (1 - 0.247)}{\ln [(1 - 0.100)/(1 - 0.247)]} = 0.825$$

Substituting into Eq. (10.4-9) to obtain the new slope,

$$-\frac{k'_x/(1 - x_A)_{iM}}{k'_y/(1 - y_A)_{iM}} = -\frac{1.967 \times 10^{-3}/0.825}{1.465 \times 10^{-3}/0.715} = -1.163$$

A line through point P with a slope of -1.163 is plotted and intersects the equilibrium line at M, where $y_{Ai} = 0.197$ and $x_{Ai} = 0.257$.

Using these new values for the third trial, the following values are calculated:

$$(1 - y_A)_{iM} = \frac{(1 - 0.197) - (1 - 0.380)}{\ln [(1 - 0.197)/(1 - 0.380)]} = 0.709$$

$$(1 - x_A)_{iM} = \frac{(1 - 0.100) - (1 - 0.257)}{\ln [(1 - 0.100)/(1 - 0.257)]} = 0.820$$

$$-\frac{k'_x/(1 - x_A)_{iM}}{k'_y/(1 - y_A)_{iM}} = -\frac{1.967 \times 10^{-3}/0.820}{1.465 \times 10^{-3}/0.709} = -1.160$$

This slope of -1.160 is essentially the same as the slope of -1.163 for the second trial. Hence, the final values are $y_{Ai} = 0.197$ and $x_{Ai} = 0.257$ and are shown as point M.

To calculate the flux, Eq. (10.4-8) is used.

$$N_A = \frac{k'_y}{(1 - y_A)_{iM}} (y_{AG} - y_{Ai}) = \frac{1.465 \times 10^{-3}}{0.709} (0.380 - 0.197)$$

$$= 3.78 \times 10^{-4} \text{ kg mol/s} \cdot \text{m}^2$$

$$N_A = \frac{1.08}{0.709} (0.380 - 0.197) = 0.2785 \text{ lb mol/h} \cdot \text{ft}^2$$

$$N_A = \frac{k'_x}{(1 - x_A)_{iM}} (x_{Ai} - x_{AL}) = \frac{1.967 \times 10^{-3}}{0.820} (0.257 - 0.100)$$

$$= 3.78 \times 10^{-4} \text{ kg mol/s} \cdot \text{m}^2$$

Note that the flux N_A through each phase is the same as in the other phase, which should be the case at steady state.

10.4D Overall Mass-Transfer Coefficients and Driving Forces

1. Introduction. Film or single-phase mass-transfer coefficients k'_y and k'_x or k_y and k_x are often difficult to measure experimentally, except in certain experiments designed so that the concentration difference across one phase is small and can be neglected. As a result, overall mass-transfer coefficients K'_y and K'_x are measured based on the gas phase or liquid phase. This method is used in heat transfer, where overall heat-transfer coefficients are measured based on inside or outside areas instead of film coefficients.

The overall mass transfer K'_y is defined as

$$N_A = K'_y(y_{AG} - y_A^*) \qquad (10.4\text{-}10)$$

where K'_y is based on the overall gas-phase driving force in kg mol/s · m^2 · mol frac, and y_A^* is the value that would be in equilibrium with x_{AL}, as shown in Fig. 10.4-2. Also, K'_x is defined as

$$N_A = K'_x(x_A^* - x_{AL}) \qquad (10.4\text{-}11)$$

where K'_x is based on the overall liquid-phase driving force in kg mol/s · m^2 · mol frac and x_A^* is the value that would be in equilibrium with y_{AG}.

2. Equimolar counterdiffusion and/or diffusion in dilute solutions. Equation (10.4-2) holds for equimolar counterdiffusion, or when the solutions are dilute, Eqs. (10.4-8) and (10.4-2) are identical.

$$N_A = k'_y(y_{AG} - y_{Ai}) = k'_x(x_{Ai} - x_{AL}) \qquad (10.4\text{-}2)$$

From Fig. 10.4-2,

$$y_{AG} - y_A^* = (y_{AG} - y_{Ai}) + (y_{Ai} - y_A^*) \qquad (10.4\text{-}12)$$

Between the points E and M the slope m' can be given as

$$m' = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_{AL}} \tag{10.4-13}$$

Solving Eq. (10.4-13) for $(y_{Ai} - y_A^*)$ and substituting into Eq. (10.4-12),

$$y_{AG} - y_A^* = (y_{AG} - y_{Ai}) + m'(x_{Ai} - x_{AL}) \tag{10.4-14}$$

Then on substituting Eqs. (10.4-10) and (10.4-2) into (10.4-14) and canceling out N_A,

$$\frac{1}{K'_y} = \frac{1}{k'_y} + \frac{m'}{k'_x} \tag{10.4-15}$$

The left-hand side of Eq. (10.4-15) is the total resistance based on the overall gas driving force and equals the gas film resistance $1/k'_y$ plus the liquid film resistance m'/k'_x.

In a similar manner from Fig. 10.4-2,

$$x_A^* - x_{AL} = (x_A^* - x_{Ai}) + (x_{Ai} - x_{AL}) \tag{10.4-16}$$

$$m'' = \frac{y_{AG} - y_{Ai}}{x_A^* - x_{Ai}} \tag{10.4-17}$$

Proceeding as before,

$$\frac{1}{K'_x} = \frac{1}{m''k'_y} + \frac{1}{k'_x} \tag{10.4-18}$$

Several special cases of Eqs. (10.4-15) and (10.4-18) will now be discussed. The numerical values of k'_x and k'_y are very roughly similar. The values of the slopes m' or m'' are very important. If m' is quite small, so that the equilibrium curve in Fig. 10.4-2 is almost horizontal, a small value of y_A in the gas will give a large value of x_A in equilibrium in the liquid. The gas solute A is then very soluble in the liquid phase, and hence the term m'/k'_x in Eq. (10.4-15) is very small. Then,

$$\frac{1}{K'_y} \cong \frac{1}{k'_y} \tag{10.4-19}$$

and the major resistance is in the gas phase, or the "gas phase is controlling." The point M has moved down very close to E, so that

$$y_{AG} - y_A^* \cong y_{AG} - y_{Ai} \tag{10.4-20}$$

Similarly, when m'' is very large, the solute A is very insoluble in the liquid, $1/(m''k'_y)$ becomes small, and

$$\frac{1}{K'_x} \cong \frac{1}{k'_x} \tag{10.4-21}$$

The "liquid phase is controlling" and $x_{Ai} \cong x_A^*$. Systems for absorption of oxygen or CO_2 from air by water are similar to Eq. (10.4-21).

3. Diffusion of A through stagnant or nondiffusing B. For the case of A diffusing through nondiffusing B, Eqs. (10.4-8) and (10.4-14) hold and Fig. 10.4-3 is used.

$$N_A = \frac{k'_y}{(1 - y_A)_{iM}}(y_{AG} - y_{Ai}) = \frac{k'_x}{(1 - x_A)_{iM}}(x_{Ai} - x_{AL}) \tag{10.4-8}$$

$$y_{AG} - y_A^* = (y_{AG} - y_{Ai}) + m'(x_{Ai} - x_{AL}) \tag{10.4-14}$$

We must, however, define the equations for the flux using overall coefficients as follows:

$$N_A = \left[\frac{K'_y}{(1-y_A)_{\bullet M}}\right](y_{AG} - y_A^*) = \left[\frac{K'_x}{(1-x_A)_{\bullet M}}\right](x_A^* - x_{AL}) \qquad \textbf{(10.4-22)}$$

The bracketed terms are often written as follows:

$$K_y = \frac{K'_y}{(1-y_A)_{\bullet M}} \qquad K_x = \frac{K'_x}{(1-x_A)_{\bullet M}} \qquad \textbf{(10.4-23)}$$

where K_y is the overall gas mass-transfer coefficient for A diffusing through stagnant B and K_x the overall liquid mass-transfer coefficient. These two coefficients are concentration-dependent. Substituting Eqs. (10.4-8) and (10.4-22) into (10.4-14), we obtain

$$\frac{1}{K'_y/(1-y_A)_{\bullet M}} = \frac{1}{k'_y/(1-y_A)_{iM}} + \frac{m'}{k'_x/(1-x_A)_{iM}} \qquad \textbf{(10.4-24)}$$

where

$$(1-y_A)_{\bullet M} = \frac{(1-y_A^*) - (1-y_{AG})}{\ln\,[(1-y_A^*)/(1-y_{AG})]} \qquad \textbf{(10.4-25)}$$

Similarly, for K'_x,

$$\frac{1}{K'_x/(1-x_A)_{\bullet M}} = \frac{1}{m''k'_y/(1-y_A)_{iM}} + \frac{1}{k'_x/(1-x_A)_{iM}} \qquad \textbf{(10.4-26)}$$

where

$$(1-x_A)_{\bullet M} = \frac{(1-x_{AL}) - (1-x_A^*)}{\ln\,[(1-x_{AL})/(1-x_A^*)]} \qquad \textbf{(10.4-27)}$$

It should be noted that the relations derived here also hold for any two-phase system where y stands for the one phase and x for the other phase. For example, for the extraction of the solute acetic acid (A) from water (y phase) by isopropyl ether (x phase), the same relations will hold.

EXAMPLE 10.4-2. Overall Mass-Transfer Coefficients from Film Coefficients

Using the same data as in Example 10.4-1, calculate the overall mass-transfer coefficient K'_y, the flux, and the percent resistance in the gas and liquid films. Do this for the case of A diffusing through stagnant B.

Solution: From Fig. 10.4-4, $y_A^* = 0.052$, which is in equilibrium with the bulk liquid $x_{AL} = 0.10$. Also, $y_{AG} = 0.380$. The slope of chord m' between E and M from Eq. (10.4-13) is, for $y_{Ai} = 0.197$ and $x_{Ai} = 0.257$,

$$m' = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_{AL}} = \frac{0.197 - 0.052}{0.257 - 0.100} = 0.923$$

From Example 10.4-1,

$$\frac{k'_y}{(1-y_A)_{iM}} = \frac{1.465 \times 10^{-3}}{0.709} \qquad \frac{k'_x}{(1-x_A)_{iM}} = \frac{1.967 \times 10^{-3}}{0.820}$$

Using Eq. (10.4-25),

$$(1-y_A)_{\bullet M} = \frac{(1-y_A^*) - (1-y_{AG})}{\ln\,[(1-y_A^*)/(1-y_{AG})]}$$

$$= \frac{(1-0.052) - (1-0.380)}{\ln\,[(1-0.052)/(1-0.380)]} = 0.773$$

Then, using Eq. (10.4-24),

$$\frac{1}{K'_y/0.773} = \frac{1}{1.465 \times 10^{-3}/0.709} + \frac{0.923}{1.967 \times 10^{-3}/0.820}$$

$$= 484.0 + 384.8 = 868.8$$

Solving, $K'_y = 8.90 \times 19^{-4}$. The percent resistance in the gas film is $(484.0/868.8)100 = 55.7\%$ and 44.3% in the liquid film. The flux is as follows, using Eq. (10.4-22):

$$N_A = \frac{K'_y}{(1 - y_A)_{*M}}(y_{AG} - y_A^*) = \frac{8.90 \times 10^{-4}}{0.773}(0.380 - 0.052)$$

$$= 3.78 \times 10^{-4} \text{ kg mol/s} \cdot \text{m}^2$$

This, of course, is the same flux value as was calculated in Example 10.4-1 using the film equations.

4. Discussion of overall coefficients. If the two-phase system is such that the major resistance is in the gas phase as in Eq. (10.4-19), then to increase the overall rate of mass transfer, efforts should be centered on increasing the gas-phase turbulence, not the liquid-phase turbulence. For a two-phase system where the liquid film resistance is controlling, turbulence should be increased in this phase to increase rates of mass transfer.

To design mass-transfer equipment, the overall mass-transfer coefficient is synthesized from the individual film coefficients, as discussed in this section.

10.5 CONTINUOUS HUMIDIFICATION PROCESSES

10.5A Introduction and Types of Equipment for Humidification

1. Introduction to gas–liquid contactors. When a relatively warm liquid is directly contacted with gas that is unsaturated, some of the liquid is vaporized. The liquid temperature will drop mainly because of the latent heat of evaporation. This direct contact of a gas with a pure liquid occurs most often in contacting air with water. This is done for the following purposes: humidifying air for control of the moisture content of air in drying or air conditioning; dehumidifying air, where cold water condenses some water vapor from warm air; and water cooling, where evaporation of water to the air cools warm water.

In Chapter 9 the fundamentals of humidity and adiabatic humidification were discussed. In this section the performance and design of continuous air–water contactors is considered. The emphasis is on cooling of water, since this is the most important type of process in the process industries. There are many cases in industry in which warm water is discharged from heat exchangers and condensers when it would be more economical to cool and reuse it than to discard it.

2. Towers for water cooling. In a typical water-cooling tower, warm water flows countercurrently to an air stream. Typically, the warm water enters the top of a packed tower and cascades down through the packing, leaving at the bottom. Air enters at the bottom of the tower and flows upward through the descending water. The tower packing often consists of slats of wood or plastic or of a packed bed. The water is distributed by troughs and overflows to cascade over slat gratings or packing that provide large interfacial areas of contact between the water and air in the form of droplets and films of water. The flow of air upward through the tower can be induced by the buoyancy of the warm air in the tower (natural draft) or by the action of a fan. Detailed descriptions of towers are given in other texts (B1, T1).

The water cannot be cooled below the wet bulb temperature. The driving force for the evaporation of the water is approximately the vapor pressure of the water less the vapor pressure it would have at the wet bulb temperature. The water can be cooled only to the wet bulb temperature, and in practice it is cooled to about 3 K or more above this. Only a small amount of water is lost by evaporation in cooling water. Since the latent heat of vaporization of water is about 2300 kJ/kg, a typical change of about 8 K in water temperature corresponds to an evaporation loss of about 1.5%. Hence, the total flow of water is usually assumed to be constant in calculations of tower size.

In humidification and dehumidification, intimate contact between the gas phase and liquid phase is needed for large rates of mass transfer and heat transfer. The gas-phase resistance controls the rate of transfer. Spray or packed towers are used to give large interfacial areas and to promote turbulence in the gas phase.

10.5B Theory and Calculation of Water-Cooling Towers

1. Temperature and concentration profiles at interface. In Fig. 10.5-1 the temperature profile and the concentration profile in terms of humidity are shown at the water–gas interface. Water vapor diffuses from the interface to the bulk gas phase with a driving force in the gas phase of $(H_i - H_G)$ kg H_2O/kg dry air. There is no driving force for mass transfer in the liquid phase, since water is a pure liquid. The temperature driving force is $T_L - T_i$ in the liquid phase and $T_i - T_G$ K or °C in the gas phase. Sensible heat flows from the bulk liquid to the interface in the liquid. Sensible heat also flows from the interface to the gas phase. Latent heat also leaves the interface in the water vapor, diffusing to the gas phase. The sensible heat flow from the liquid to the interface equals the sensible heat flow in the gas plus the latent heat flow in the gas.

The conditions in Fig. 10.5-1 occur at the upper part of the cooling tower. In the lower part of the cooling tower the temperature of the bulk water is higher than the wet bulb temperature of the air but may be below the dry bulb temperature. Then the direction of the sensible heat flow in Fig. 10.5-1 is reversed.

2. Rate equations for heat and mass transfer. We shall consider a packed water-cooling

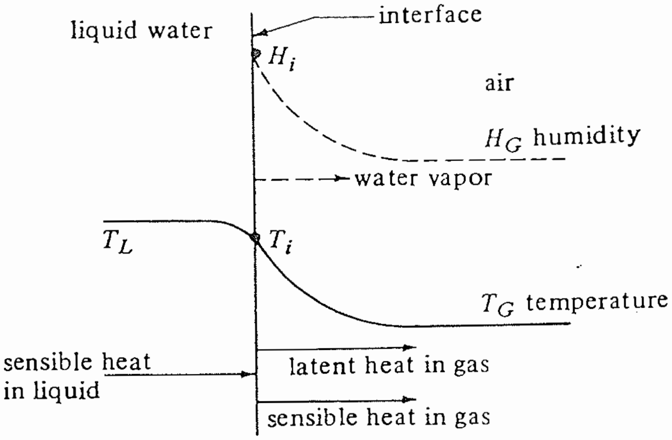


FIGURE 10.5-1. *Temperature and concentration profiles in upper part of cooling tower.*

tower with air flowing upward and water countercurrently downward in the tower. The total interfacial area between the air and water phases is unknown, since the surface area of the packing is not equal to the interfacial area between the water droplets and the air. Hence, we define a quantity a, defined as m^2 of interfacial area per m^3 volume of packed section, or m^2/m^3. This is combined with the gas-phase mass-transfer coefficient k_G in kg mol/s · m^2 · Pa or kg mol/s · m^2 · atm to give a volumetric coefficient $k_G a$ in kg mol/s · m^3 volume · Pa or kg mol/s · m^3 · atm (lb mol/h · ft^3 · atm).

The process is carried out adiabatically and the various streams and conditions are shown in Fig. 10.5-2, where

L = water flow, kg water/s · m^2 (lb_m/h · ft^2)

T_L = temperature of water, °C or K (°F)

G = dry air flow, kg/s · m^2 (lb_m/h · ft^2)

T_G = temperature of air, °C or K (°F)

H = humidity of air, kg water/kg dry air (lb water/lb dry air)

H_y = enthalpy of air–water vapor mixture, J/kg dry air (btu/lb_m dry air)

The enthalpy H_y as given in Eq. (9.3-8) is

$$H_y = c_S(T - T_0) + H\lambda_0 = (1.005 + 1.88H)10^3(T - 0) + 2.501 \times 10^6 H \quad \text{(SI)}$$

$$H_y = c_S(T - T_0) + H\lambda_0 = (0.24 + 0.45H)(T - 32) + 1075.4H \quad \text{(English)} \tag{9.3-8}$$

The base temperature selected is 0°C or 273 K (32°F). Note that $(T - T_0)°C = (T - T_0)$ K.

Making a total heat balance for the dashed-line box shown in Fig. 10.5-2, an operating line is obtained.

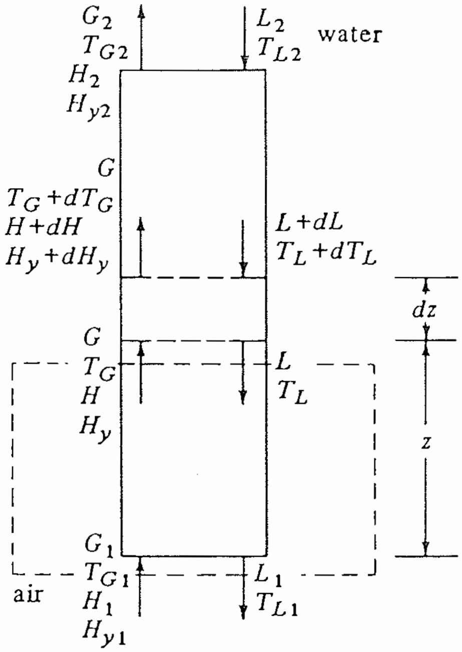


FIGURE 10.5-2. *Continuous countercurrent adiabatic water cooling.*

$$G(H_y - H_{y1}) = Lc_L(T_L - T_{L1}) \tag{10.5-1}$$

This assumes that L is essentially constant, since only a small amount is evaporated. The heat capacity c_L of the liquid is assumed constant at 4.187×10^3 J/kg · K (1.00 btu/lb$_m$ · °F). When plotted on a chart of H_y versus T_L, this Eq. (10.5-1) is a straight line with a slope of Lc_L/G. Making an overall heat balance over both ends of the tower,

$$G(H_{y2} - H_{y1}) = Lc_L(T_{L2} - T_{L1}) \tag{10.5-2}$$

Again making a heat balance for the dz column height and neglecting sensible heat terms compared to the latent heat,

$$Lc_L \, dT_L = G \, dH_y \tag{10.5-3}$$

The total sensible heat transfer from the bulk liquid to the interface is (refer to Fig. 10.5-1)

$$Lc_L \, dT_L = G \, dH_y = h_L a \, dz(T_L - T_i) \tag{10.5-4}$$

where $h_L a$ is the liquid-phase volumetric heat-transfer coefficient in W/m^3 · K (btu/h · ft^3 · °F) and T_i is the interface temperature.

For adiabatic mass transfer the rate of heat transfer due to the latent heat in the water vapor being transferred can be obtained from Eq. (9.3-16) by rearranging and using a volumetric basis.

$$\frac{q_\lambda}{A} = M_B k_G a P \lambda_0 (H_i - H_G) \, dz \tag{10.5-5}$$

where q_λ/A is in W/m^2 (btu/h · ft^2), M_B = molecular weight of air, $k_G a$ is a volumetric mass-transfer coefficient in the gas in kg mol/s · m^3 · Pa, P = atm pressure in Pa, λ_0 is the latent heat of water in J/kg water, H_i is the humidity of the gas at the interface in kg water/kg dry air, and H_G is the humidity of the gas in the bulk gas phase in kg water/kg dry air. The rate of sensible heat transfer in the gas is

$$\frac{q_S}{A} = h_G a(T_i - T_G) \, dz \tag{10.5-6}$$

where q_S/A is in W/m^2 and $h_G a$ is a volumetric heat-transfer coefficient in the gas in W/m^3 · K.

Now from Fig. 10.5-1, Eq. (10.5-4) must equal the sum of Eqs. (10.5-5) and (10.5-6).

$$G dH_y = M_B k_G a P \lambda_0 (H_i - H_G) \, dz + h_G a(T_i - T_G) \, dz \tag{10.5-7}$$

Equation (9.3-18) states that

$$\frac{h_G a}{M_B k_y a} \cong c_S \tag{10.5-8}$$

Substituting $Pk_G a$ for $k_y a$,

$$\frac{h_G a}{M_B P k_G a} \cong c_S \tag{10.5-9}$$

Substituting Eq. (10.5-9) into Eq. (10.5-7) and rearranging,

$$G \, dH_y = M_B k_G a P \, dz \, [(c_S T_i + \lambda_0 H_i) - (c_S T_G + \lambda_0 H_G)] \tag{10.5-10}$$

Adding and subtracting $c_S T_0$ inside the brackets,

$$G \, dH_y = M_B k_G a P \, dz\{c_S(T_i - T_0) + H_i \lambda_0 - [c_S(T_G - T_0) + H_G \lambda_0]\} \tag{10.5-11}$$

The terms inside the braces are $(H_{yi} - H_y)$, and Eq. (10.5-11) becomes

$$G\,dH_y = M_B k_G aP\,dz(H_{yi} - H_y) \tag{10.5-12}$$

Integrating, the final equation to use to calculate the tower height is

$$\int_0^z dz = z = \frac{G}{M_B k_G aP} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y} \tag{10.5-13}$$

If Eq. (10.5-4) is equated to Eq. (10.5-12) and the result rearranged,

$$-\frac{h_L a}{k_G a M_B P} = \frac{H_{yi} - H_y}{T_i - T_L} \tag{10.5-14}$$

10.5C Design of Water-Cooling Tower Using Film Mass-Transfer Coefficients

The tower design is done using the following steps.

1. The enthalpy of saturated air H_{yi} is plotted versus T_i on an H versus T plot as shown in Fig. 10.5-3. This enthalpy is calculated with Eq. (9.3-8) using the saturation humidity from the humidity chart for a given temperature, with 0°C (273 K) as a base temperature. Calculated values are tabulated in Table 10.5-1.
2. Knowing the entering air conditions T_{G1} and H_1, the enthalpy of this air H_{y1} is calculated from Eq. (9.3-8). The point H_{y1} and T_{L1} (desired leaving water temperature) is plotted in Fig. 10.5-3 as one point on the operating line. The operating line is plotted with a slope Lc_L/G and ends at point T_{L2}, which is the entering water temperature. This gives H_{y2}. Alternatively, H_{y2} can be calculated from Eq. (10.5-2).
3. Knowing $h_L a$ and $k_G a$, lines with a slope of $-h_L a/k_G aM_B P$ are plotted as shown in Fig. 10.5-3. From Eq. (10.5-14) point P represents H_y and T_L on the operating line, and point M represents H_{yi} and T_i, the interface conditions. Hence, line MS or $H_{yi} - H_y$ represents the driving force in Eq. (10.5-13).

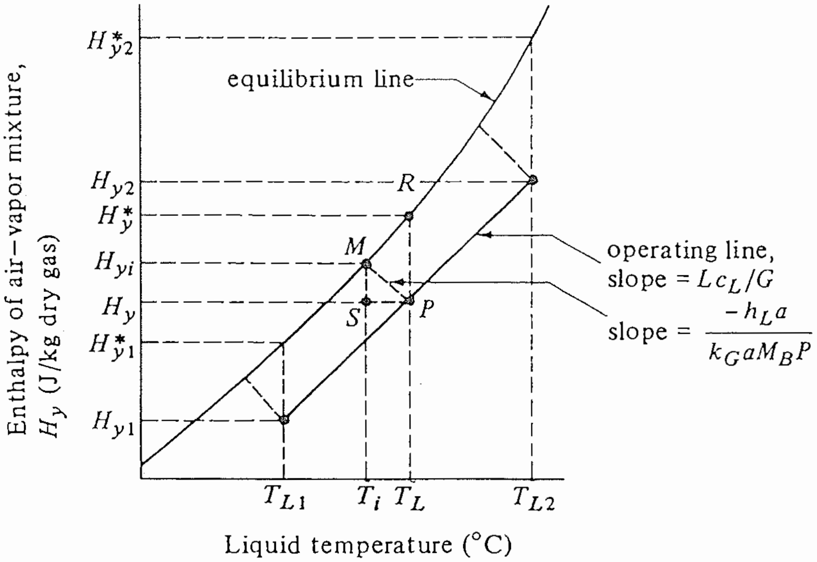


FIGURE 10.5-3. *Temperature enthalpy diagram and operating line for water-cooling tower.*

TABLE 10.5-1. *Enthalpies of Saturated Air–Water Vapor Mixtures (0°C Base Temperature)*

T_L		H_y		T_L		H_y	
		btu	J			btu	J
°F	°C	lb_m dry air	kg dry air	°F	°C	lb_m dry air	kg dry air
60	15.6	18.78	43.68×10^3	100	37.8	63.7	148.2×10^3
80	26.7	36.1	84.0×10^3	105	40.6	74.0	172.1×10^3
85	29.4	41.8	97.2×10^3	110	43.3	84.8	197.2×10^3
90	32.2	48.2	112.1×10^3	115	46.1	96.5	224.5×10^3
95	35.0	55.4	128.9×10^3	140	60.0	198.4	461.5×10^3

4. The driving force $H_{yi} - H_y$ is computed for various values of T_L between T_{L1} and T_{L2}. Then by plotting $1/(H_{yi} - H_y)$ versus H_y from H_{y1} to H_{y2}, a graphical integration is performed to obtain the value of the integral in Eq. (10.5-13). Finally, the height z is calculated from Eq. (10.5-13).

10.5D Design of Water-Cooling Tower Using Overall Mass-Transfer Coefficients

Often, only an overall mass-transfer coefficient $K_G a$ in kg mol/s · m^3 · Pa or kg mol/s · m^3 · atm is available, and Eq. (10.5-13) becomes

$$z = \frac{G}{M_B K_G a P} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_y^* - H_y} \tag{10.5-15}$$

The value of H_y^* is determined by going vertically from the value of H_y at point P up to the equilibrium line to give H_y^* at point R, as shown in Fig. 10.5-3. In many cases the experimental film coefficients $k_G a$ and $h_L a$ are not available. The few experimental data available indicate that $h_L a$ is quite large and the slope of the lines $-h_L a/(k_G a M_B P)$ in Eq. (10.5-14) would be very large and the value of H_{yi} would approach that of H_y^* in Fig. 10.5-3.

The tower design using the overall mass-transfer coefficient is done using the following steps.

1. The enthalpy–temperature data from Table 10.5-1 are plotted as shown in Fig. 10.5-3.
2. The operating line is calculated as in steps 1 and 2 for the film coefficients and plotted in Fig. 10.5-3.
3. In Fig. 10.5-3 point P represents H_y and T_L on the operating line and point R represents H_y^* on the equilibrium line. Hence, the vertical line RP or $H_y^* - H_y$ represents the driving force in Eq. (10.5-15).
4. The driving force $H_y^* - H_y$ is computed for various values of T_L between T_{L1} and T_{L2}. Then by plotting $1/(H_y^* - H_y)$ versus H_y from H_{y1} to H_{y2}, a graphical integration is performed to obtain the value of the integral in Eq. (10.5-15). Finally, the height z is obtained from Eq. (10.5-15).

If experimental cooling data in an actual run in a cooling tower with known height z are available, then using Eq. (10.5-15), the experimental value of $K_G a$ can be obtained.

EXAMPLE 10.5-1. Design of Water-Cooling Tower Using Film Coefficients

A packed countercurrent water-cooling tower using a gas flow rate of $G = 1.356$ kg dry air/s · m^2 and a water flow rate of $L = 1.356$ kg water/s · m^2 is to cool the water from $T_{L2} = 43.3°\text{C}$ (110°F) to $T_{L1} = 29.4°\text{C}$ (85°F). The entering air at 29.4°C has a wet bulb temperature of 23.9°C. The mass-transfer coefficient $k_G a$ is estimated as 1.207×10^{-7} kg mol/s · m^3 · Pa and $h_L a/k_G a M_B P$ as 4.187×10^4 J/kg · K (10.0 btu/lb$_m$ · °F). Calculate the height of packed tower z. The tower operates at a pressure of 1.013×10^5 Pa.

Solution: Following the steps outlined, the enthalpies from the saturated air–water vapor mixtures from Table 10.5-1 are plotted in Fig. 10.5-4. The inlet air at $T_{G1} = 29.4°\text{C}$ has a wet bulb temperature of 23.9°C. The humidity from the humidity chart is $H_1 = 0.0165$ kg H$_2$O/kg dry air. Substituting into Eq. (9.3-8), noting that $(29.4 - 0)°\text{C} = (29.4 - 0)$ K,

$$H_{y1} = (1.005 + 1.88 \times 0.0165)10^3(29.4 - 0) + 2.501 \times 10^6(0.0165)$$
$$= 71.7 \times 10^3 \text{ J/kg}$$

The point $H_{y1} = 71.7 \times 10^3$ and $T_{L1} = 29.4°\text{C}$ is plotted. Then substituting into Eq. (10.5-2) and solving,

$$1.356(H_{y2} - 71.7 \times 10^3) = 1.356(4.187 \times 10^3)(43.3 - 29.4)$$

$H_{y2} = 129.9 \times 10^3$ J/kg dry air (55.8 btu/lb$_m$). The point $H_{y2} = 129.9 \times 10^3$ and $T_{L2} = 43.3°\text{C}$ is also plotted, giving the operating line. Lines with slope $-h_L a/k_G a M_B P = -41.87 \times 10^3$ J/kg · K are plotted giving H_{yi} and H_y values, which are tabulated in Table 10.5-2 along with derived values as shown. Values of $1/(H_{yi} - H_y)$ are plotted versus H_y and the area under the curve from $H_{y1} = 71.7 \times 10^3$ to $H_{y2} = 129.9 \times 10^3$ is

$$\int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y} = 1.82$$

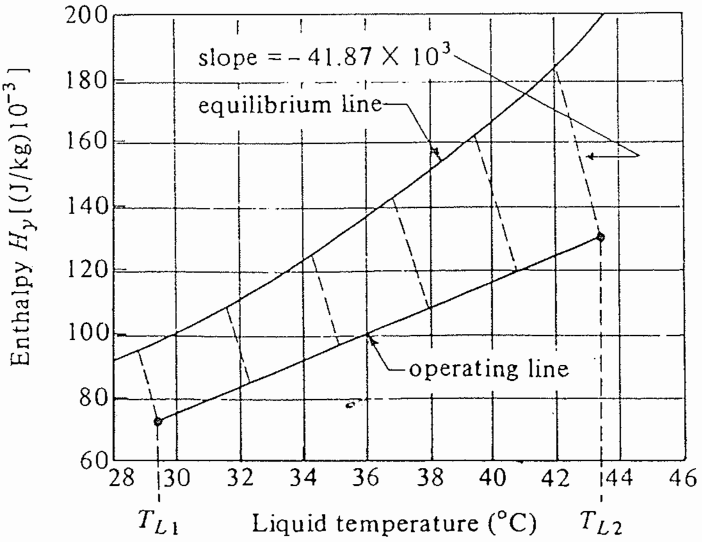


FIGURE 10.5-4. *Graphical solution of Example 10.5-1.*

TABLE 10.5-2. *Enthalpy Values for Solution to Example 10.5-1 (enthalpy in J/kg dry air)*

H_{yi}	H_y	$H_{yi} - H_y$	$1/(H_{yi} - H_y)$
94.4×10^3	71.7×10^3	22.7×10^3	4.41×10^{-5}
108.4×10^3	83.5×10^3	24.9×10^3	4.02×10^{-5}
124.4×10^3	94.9×10^3	29.5×10^3	3.39×10^{-5}
141.8×10^3	106.5×10^3	35.3×10^3	2.83×10^{-5}
162.1×10^3	118.4×10^3	43.7×10^3	2.29×10^{-5}
184.7×10^3	129.9×10^3	54.8×10^3	1.82×10^{-5}

Substituting into Eq. (10.5-13),

$$z = \frac{G}{M_B k_G aP} \int \frac{dH_y}{H_{yi} - H_y} = \frac{1.356}{29(1.207 \times 10^{-7})(1.013 \times 10^5)} (1.82)$$

$$= 6.98 \text{ m } (22.9 \text{ ft})$$

10.5E Minimum Value of Air Flow

Often the air flow G is not fixed but must be set for the design of the cooling tower. As shown in Fig. 10.5-5 for a minimum value of G, the operating line MN is drawn through the point H_{y1} and T_{L1} with a slope that touches the equilibrium line at T_{L2}, point N. If the equilibrium line is quite curved, line MN could become tangent to the equilibrium line at a point farther down the equilibrium line than point N. For the actual tower, a value of G greater than G_{min} must be used. Often, a value of G equal to 1.3 to 1.5 times G_{min} is used.

10.5F Design of Water-Cooling Tower Using Height of a Transfer Unit

Often another form of the film mass-transfer coefficient is used in Eq. (10.5-13):

$$z = H_G \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_{yi} - H_y} \tag{10.5-16}$$

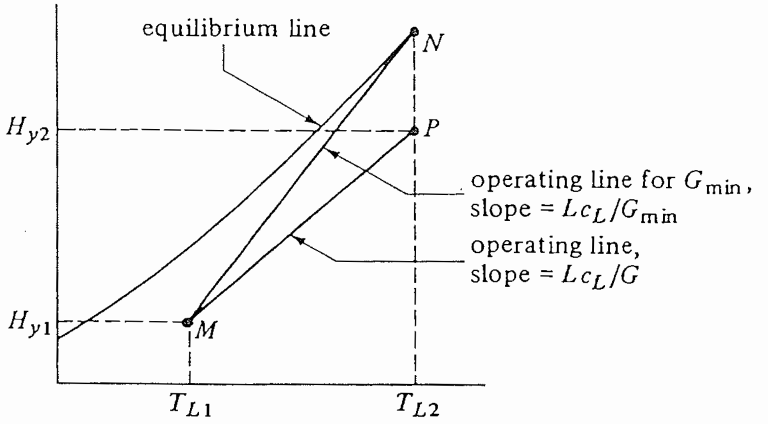


FIGURE 10.5-5. *Operating-line construction for minimum gas flow.*

$$H_G = \frac{G}{M_B k_G a P} \qquad (10.5\text{-}17)$$

where H_G is the height of a gas enthalpy transfer unit in m and the integral term is called the number of transfer units. The term H_G is often used since it is less dependent upon flow rates than $k_G a$.

Often another form of the overall mass-transfer coefficient $K_G a$ in kg mol/s · m^3 · Pa or kg mol/s · m^3 · atm is used and Eq. (10.5-15) becomes

$$z = \frac{G}{M_B K_G a P} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_y^* - H_y} = H_{OG} \int_{H_{y1}}^{H_{y2}} \frac{dH_y}{H_y^* - H_y} \qquad (10.5\text{-}18)$$

where H_{OG} is the height of an overall gas enthalpy transfer unit in m. The value of H_y^* is determined by going vertically from the value of H_y up to the equilibrium line as shown in Fig. 10.5-3. This method should be used only when the equilibrium line is almost straight over the range used. However, the H_{OG} is often used even if the equilibrium line is somewhat curved because of the lack of film mass-transfer coefficient data.

10.5G Temperature and Humidity of Air Stream in Tower

The procedures outlined above do not yield any information on the changes in temperature and humidity of the air–water vapor stream through the tower. If this information is of interest, a graphical method by Mickley (M2) is available. The equation used for the graphical method is derived by first setting Eq. (10.5-6) equal to $Gc_S\, dT_G$ and combining it with Eqs. (10.5-12) and (10.5-9) to yield Eq. (10.5-19).

$$\frac{dH_y}{dT_G} = \frac{H_{yi} - H_y}{T_i - T_G} \qquad (10.5\text{-}19)$$

10.5H Dehumidification Tower

For the cooling or humidification tower discussed, the operating line lies below the equilibrium line and water is cooled and air humidified. In a dehumidification tower cool water is used to reduce the humidity and temperature of the air that enters. In this case the operating line is above the equilibrium line. Similar calculation methods are used (T1).

10.6 ABSORPTION IN PLATE AND PACKED TOWERS

10.6A Equipment for Absorption and Distillation

1. Introduction to absorption. As discussed briefly in Section 10.1B, absorption is a mass-transfer process in which a vapor solute A in a gas mixture is absorbed by means of a liquid in which the solute is more or less soluble. The gas mixture consists mainly of an inert gas and the solute. The liquid also is primarily immiscible in the gas phase; i.e., its vaporization into the gas phase is relatively slight. A typical example is absorption of the solute ammonia from an air–ammonia mixture by water. Subsequently, the solute is recovered from the solution by distillation. In the reverse process of desorption or stripping, the same principles and equations hold.

Equilibrium relations for gas–liquid systems in absorption were discussed in Section 10.2, and such data are needed for design of absorption towers. Some data are tabulated in Appendix A.3. Other more extensive data are available in Perry and Green (P1).

2. Various types of tray (plate) towers for absorption and distillation. In order to efficiently contact the vapor and liquid in absorption and distillation, tray (plate) towers are often used. A very common type of tray contacting device is the sieve tray, which is shown schematically in Fig. 10.6-1a and in Section 11.4A for distillation.

1. *Sieve tray.* Essentially, the same type of sieve tray is used in gas absorption and in distillation. In the sieve tray, vapor bubbles up through simple holes in the tray through the flowing liquid. Hole sizes range from 3 to 12 mm in diameter, with 5 mm a common size. The vapor area of the holes varies between 5 to 15% of the tray area. The liquid is maintained on the tray surface and prevented from flowing down through the holes by the kinetic energy of the gas or vapor. The depth of liquid on the tray is maintained by an overflow, outlet weir. The overflow liquid flows into the downspout to the next tray below.
2. *Valve tray.* A modification of the sieve tray is the valve tray, which consists of openings in the tray and a lift-valve cover for each opening, providing a variable open area which is varied by the vapor flow inhibiting leakage of liquid down the opening at low vapor rates. Hence, this type of tray can operate at a greater range of flow rates than the sieve tray, with a cost of only about 20% higher than a sieve tray. The valve tray is being increasingly used today.
3. *Bubble-cap tray.* Bubble-cap trays, shown in Fig. 10.6-1b, have been used for over 100 years, but since 1950 they have been generally superseded by sieve-type or valve trays because of their cost, which is almost double that of sieve-type trays. In the bubble tray, the vapor or gas rises through the opening in the tray into the bubble caps. Then the gas flows through slots in the periphery of each cap and bubbles upward through the flowing liquid. Details and design procedures for many of these and other types of trays are given elsewhere (B2, P1, T1). The different types of tray efficiencies are discussed in Section 11.5.

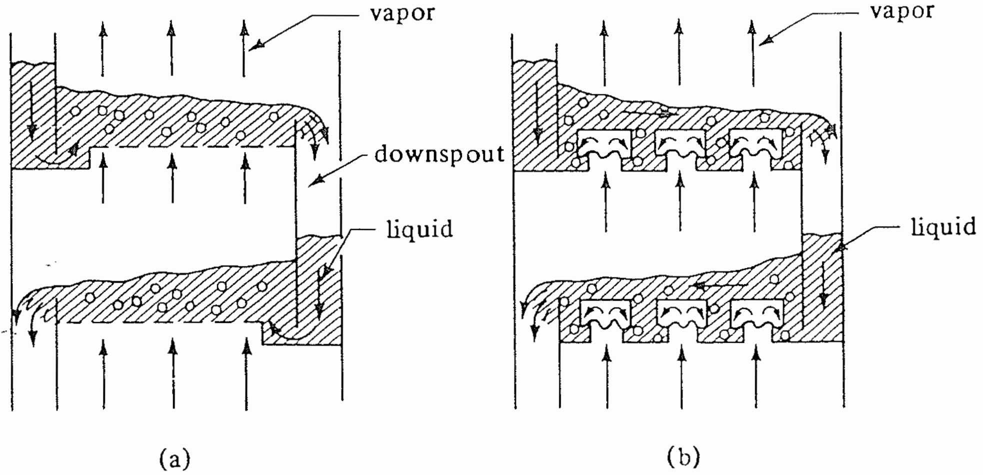


FIGURE 10.6-1. *Tray contacting devices: (a) detail of sieve-tray tower, (b) detail of bubble-cap tower tray.*

3. Packed towers for absorption and distillation. Packed towers are used for continuous countercurrent contacting of gas and liquid in absorption and also for vapor–liquid contacting in distillation. The tower in Fig. 10.6-2 consists of a cylindrical column containing a gas inlet and distributing space at the bottom, a liquid inlet and distributing device at the top, a gas outlet at the top, a liquid outlet at the bottom, and a packing or filling in the tower. The entering gas enters the distributing space below the packed section and rises upward through the openings or interstices in the packing and contacts the descending liquid flowing through the same openings. A large area of intimate contact between the liquid and gas is provided by the packing.

Many different types of tower packing have been developed and a number are used quite commonly. Common types of packing which are dumped at random in the tower are shown in Fig. 10.6-3. Such packings and other commercial packings are available in sizes of 3 mm to about 75 mm. Most of the tower packings are made of inert and cheap materials such as clay, porcelain, graphite, or plastic. High void spaces of 60 to 90% are characteristic of good packings. The packings permit relatively large volumes of liquid to pass countercurrently to the gas flow through the openings with relatively low pressure drops for the gas. These same types of packing are also used in vapor–liquid separation processes of distillation.

Stacked packing having sizes of 75 mm or so and larger is also used. The packing is stacked vertically, with open channels running uninterruptedly through the bed. The advantage of the lower pressure drop of the gas is offset in part by the poorer gas–liquid contact in stacked packings. Typical stacked packings are wood grids, drip-point grids, spiral partition rings, and others.

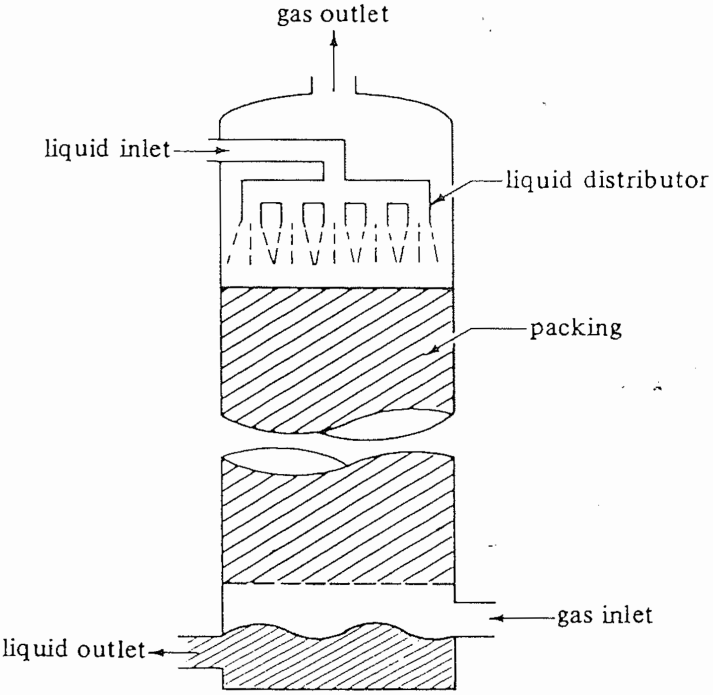


FIGURE 10.6-2. *Packed tower flows and characteristics for absorption.*

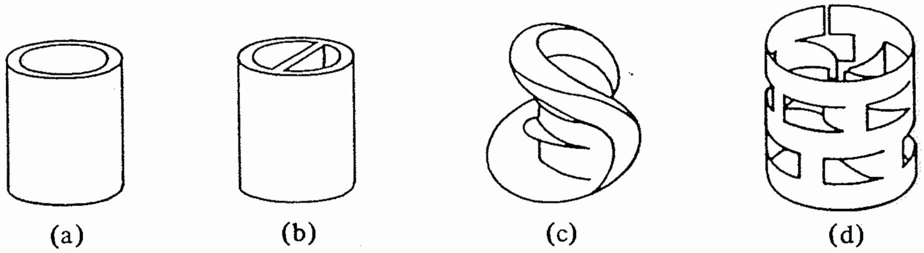


FIGURE 10.6-3. *Typical tower packings: (a) Raschig ring, (b) Lessing ring, (c) Berl saddle, (d) Pall ring.*

In a given packed tower with a given type and size of packing and with a definite flow of liquid, there is an upper limit to the rate of gas flow, called the *flooding velocity.* Above this gas velocity the tower cannot operate. At low gas velocities the liquid flows downward through the packing essentially uninfluenced by the upward gas flow. As the gas flow rate is increased at low gas velocities, the pressure drop is proportional to the flow rate to the 1.8 power. At a gas flow rate called the *loading point*, the gas starts to hinder the liquid downflow and local accumulations or pools of liquid start to appear in the packing. The pressure drop of the gas starts to rise at a faster rate. As the gas flow rate is increased, the liquid holdup or accumulation increases. At the flooding point, the liquid can no longer flow down through the packing and is blown out with the gas.

In an actual operating tower the gas velocity is well below flooding. The optimum economic gas velocity is about one half or so of the flooding velocity. It depends upon an economic balance between the cost of power and the fixed charges on the equipment cost (S1). Detailed design methods for predicting the pressure drop in various types of packing are given elsewhere (P1, L1, T1).

10.6B Design of Plate Absorption Towers

1. Operating-line derivation. A plate (tray) absorption tower has the same process flow diagram as the countercurrent multiple-stage process in Fig. 10.3-2 and is shown as a vertical tray tower in Fig. 10.6-4. In the case of solute A diffusing through a stagnant gas (B) and then into a stagnant fluid, as in the absorption of acetone (A) from air (B) by water, the moles of inert or stagnant air and inert water remain constant throughout the entire tower. If the rates are V' kg mol inert air/s and L' kg mol inert solvent water/s or in kg mol inert/s $\cdot$ m^2 units (lb mol inert/h $\cdot$ ft^2), an overall material balance on component A in Fig. 10.6-4 is

$$L'\left(\frac{x_0}{1-x_0}\right)+V'\left(\frac{y_{N+1}}{1-y_{N+1}}\right)=L'\left(\frac{x_N}{1-x_N}\right)+V'\left(\frac{y_1}{1-y_1}\right) \tag{10.6-1}$$

A balance around the dashed-line box gives

$$L'\left(\frac{x_0}{1-x_0}\right)+V'\left(\frac{y_{n+1}}{1-y_{n+1}}\right)=L'\left(\frac{x_n}{1-x_n}\right)+V'\left(\frac{y_1}{1-y_1}\right) \tag{10.6-2}$$

where x is the mole fraction A in the liquid, y the mole fraction of A in the gas, L_n the total moles liquid/s, and V_{n+1} the total moles gas/s. The total flows/s of liquid and of gas vary throughout the tower.

Equation (10.6-2) is the material balance or operating line for the absorption tower and is similar to Eq. (10.3-13) for a countercurrent-stage process, except that the inert streams L' and V' are used instead of the total flow rates L and V. Equation (10.6-2) relates the concentration y_{n+1} in the gas stream with x_n in the liquid stream passing it. The terms V', L', x_0, and y_1 are constant and usually known or can be determined.

2. Graphical determination of the number of trays. A plot of the operating-line equation (10.6-2) as y versus x will give a curved line. If x and y are very dilute, the denominators $1 - x$ and $1 - y$ will be close to 1.0, and the line will be approximately straight, with a slope $\cong L'/V'$. The number of theoretical trays are determined by simply stepping off the number of trays, as done in Fig. 10.3-3 for a countercurrent multiple-stage process.

EXAMPLE 10.6-1. Absorption of SO_2 in a Tray Tower

A tray tower is to be designed to absorb SO_2 from an air stream by using pure water at 293 K (68°F). The entering gas contains 20 mol % SO_2 and that leaving 2 mol % at a total pressure of 101.3 kPa. The inert air flow rate is 150 kg air/h · m^2, and the entering water flow rate is 6000 kg water/h · m^2. Assuming an overall tray efficiency of 25%, how many theoretical trays and actual trays are needed? Assume that the tower operates at 293 K (20°C).

Solution: First calculating the molar flow rates,

$$V' = \frac{150}{29} = 5.18 \text{ kg mol inert air/h} \cdot \text{m}^2$$

$$L' = \frac{6000}{18.0} = 333 \text{ kg mol inert water/h} \cdot \text{m}^2$$

Referring to Fig. 10.6-4, $y_{N+1} = 0.20$, $y_1 = 0.02$, and $x_0 = 0$. Substituting into Eq. (10.6-1) and solving for x_N,

$$333\left(\frac{0}{1-0}\right) + 5.18\left(\frac{0.20}{1-0.20}\right) = 333\left(\frac{x_N}{1-x_N}\right) + 5.18\left(\frac{0.02}{1-0.02}\right)$$

$$x_N = 0.00355$$

Substituting into Eq. (10.6-2), using V' and L' as kg mol/h · m^2 instead of

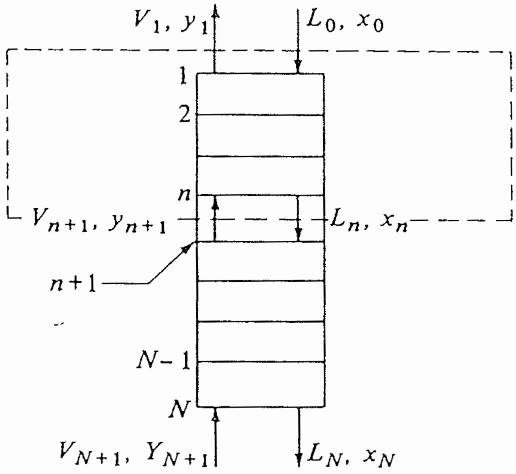


FIGURE 10.6-4. *Material balance in an absorption tray tower.*

kg mol/s · m^2,

$$333\left(\frac{0}{1-0}\right) + 5.18\left(\frac{y_{n+1}}{1-y_{n+1}}\right) = 333\left(\frac{x_n}{1-x_n}\right) + 5.18\left(\frac{0.02}{1-0.02}\right)$$

In order to plot the operating line, several intermediate points will be calculated. Setting $y_{n+1} = 0.07$ and substituting into the operating equation,

$$0 + 5.18\left(\frac{0.07}{1-0.07}\right) = 333\left(\frac{x_n}{1-x_n}\right) + 5.18\left(\frac{0.02}{1-0.02}\right)$$

Hence, $x_n = 0.000855$. To calculate another intermediate point, we set $y_{n+1} = 0.13$, and x_n is calculated as 0.00201. The two end points and the two intermediate points on the operating line are plotted in Fig. 10.6-5, as are the equilibrium data from Appendix A.3. The operating line is somewhat curved.

The number of theoretical trays is determined by stepping off the steps to give 2.4 theoretical trays. The actual number of trays is 2.4/0.25 = 9.6 trays.

10.6C Design of Packed Towers for Absorption

1. Operating-line derivation. For the case of solute A diffusing through a stagnant gas and then into a stagnant fluid, an overall material balance on component A in Fig. 10.6-6 for a packed absorption tower is

$$L'\left(\frac{x_2}{1-x_2}\right) + V'\left(\frac{y_1}{1-y_1}\right) = L'\left(\frac{x_1}{1-x_1}\right) + V'\left(\frac{y_2}{1-y_2}\right) \qquad (10.6\text{-}3)$$

where L' is kg mol inert liquid/s or kg mol inert liquid/s · m^2, V' is kg mol inert gas/s or kg mol inert gas/s · m^2, and y_1 and x_1 are mole fractions A in gas and liquid, respectively.

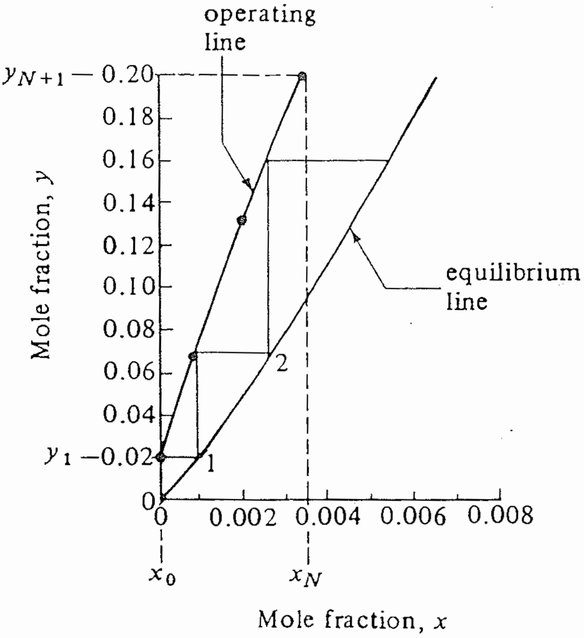


FIGURE 10.6-5. *Theoretical number of trays for absorption of SO_2 in Example 10.6-1.*

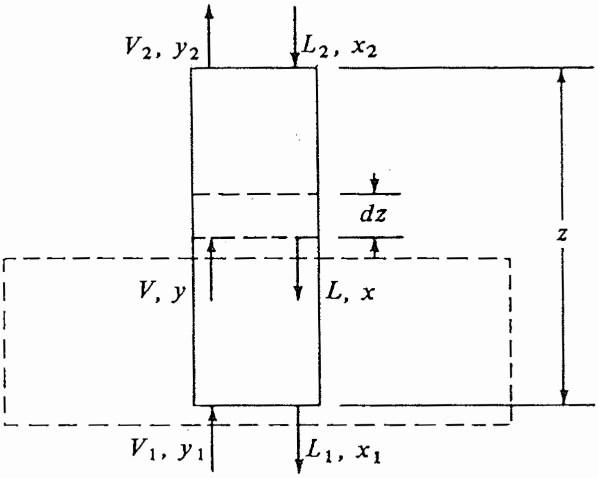


FIGURE 10.6-6. *Material balance for a countercurrent packed absorption tower.*

The flows L' and V' are constant throughout the tower, but the total flows L and V are not constant.

A balance around the dashed-line box in Fig. 10.6-6 gives the operating-line equation.

$$L'\left(\frac{x}{1-x}\right) + V'\left(\frac{y_1}{1-y_1}\right) = L'\left(\frac{x_1}{1-x_1}\right) + V'\left(\frac{y}{1-y}\right) \tag{10.6-4}$$

This equation, when plotted on yx coordinates, will give a curved line, as shown in Fig. 10.6-7a. Equation (10.6-4) can also be written in terms of partial pressure p_1 of A, where $y_1/(1-y_1) = p_1/(P-p_1)$, and so on. If x and y are very dilute, $(1-x)$ and $(1-y)$ can be taken as 1.0 and Eq. (10.6-4) becomes

$$L'x + V'y_1 \cong L'x_1 + V'y \tag{10.6-5}$$

This has a slope L'/V' and the operating line is essentially straight.

When the solute is being transferred from the L to the V stream, the process is called *stripping*. The operating line is below the equilibrium line, as shown in Fig. 10.6-7b.

2. Limiting and optimum L'/V' ratios. In the absorption process, the inlet gas flow V_1 (Fig. 10.6-6) and its composition y_1 are generally set. The exit concentration y_2 is also usually set by the designer and the concentration x_2 of the entering liquid is often fixed by process requirements. Hence, the amount of the entering liquid flow L_2 or L' is open to choice.

In Fig. 10.6-8 the flow V_1 and the concentrations y_2, x_2, and y_1 are set. When the operating line has a minimum slope and touches the equilibrium line at point P, the liquid flow L' is a minimum at L'_{min}. The value of x_1 is a maximum at $x_{1\,\text{max}}$ when L' is a minimum. At point P the driving forces $y - y^*$, $y - y_i$, $x^* - x$, or $x_i - x$ are all zero. To solve for L'_{min}, the values y_1 and $x_{1\,\text{max}}$ are substituted into the operating-line equation. In some cases if the equilibrium line is curved concavely downward, the minimum value of L is reached by the operating line becoming tangent to the equilibrium line instead of intersecting it.

The choice of the optimum L'/V' ratio to use in the design depends on an economic balance. In absorption, too high a value requires a large liquid flow, and hence a

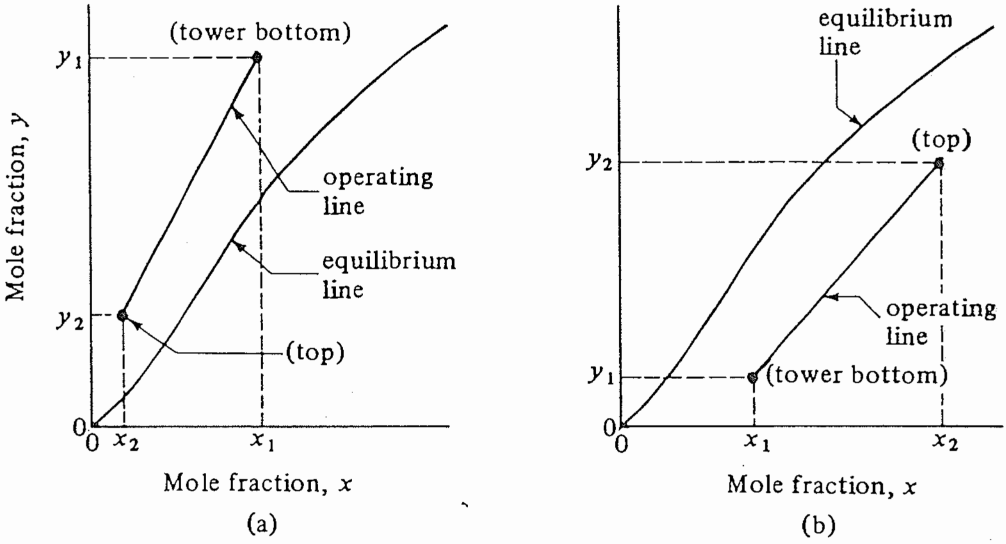


FIGURE 10.6-7. *Location of operating lines: (a) for absorption of A from V to L stream, (b) for stripping of A from L to V stream.*

large-diameter tower. The cost of recovering the solute from the liquid by distillation will be high. A small liquid flow results in a high tower, which is costly. As an approximation, the optimum liquid flow is obtained by using a value of about 1.5 for the ratio of the average slope of the operating line to that of the equilibrium line for absorption. This factor can vary depending on the value of the solute and tower type.

3. Film and overall mass-transfer coefficients in packed towers. As discussed in Section 10.5, it is very difficult to measure experimentally the interfacial area A m^2 between phases L and V. Also, it is difficult to measure the film coefficients k'_x and k'_y and the overall coefficients K'_x and K'_y. Usually, experimental measurements in a packed tower yield a volumetric mass-transfer coefficient that combines the interfacial area and mass-transfer coefficient.

Defining a as interfacial area in m^2 per m^3 volume of packed section, the volume of

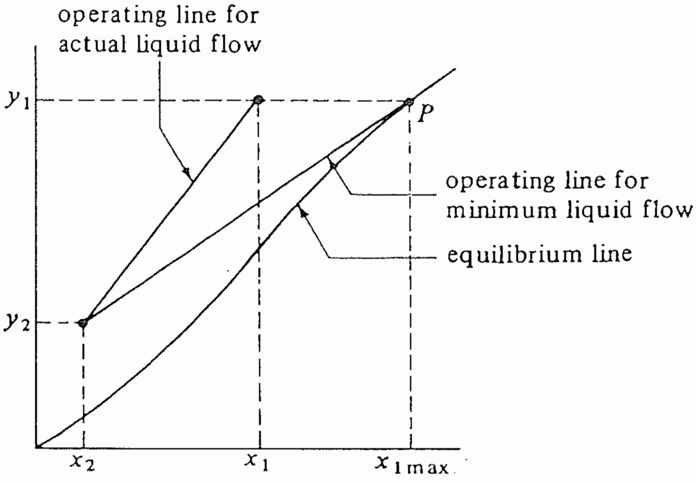


FIGURE 10.6-8. *Minimum liquid/gas ratio for absorption.*

packing in a height dz m (Fig. 10.6-6) is $S\,dz$ and

$$dA = aS\,dz \tag{10.6-6}$$

where S is m^2 cross-sectional area of tower. The volumetric film and overall mass-transfer coefficients are then defined as

$$k'_y a = \frac{\text{kg mol}}{\text{s}\cdot\text{m}^3\ \text{packing}\cdot\text{mol frac}} \qquad k'_x a = \frac{\text{kg mol}}{\text{s}\cdot\text{m}^3\ \text{packing}\cdot\text{mol frac}} \quad \text{(SI)}$$

$$K'_y a = \frac{\text{kg mol}}{\text{s}\cdot\text{m}^3\ \text{packing}\cdot\text{mol frac}} \qquad K'_x a = \frac{\text{kg mol}}{\text{s}\cdot\text{m}^3\ \text{packing}\cdot\text{mol frac}} \quad \text{(SI)}$$

$$k'_y a = \frac{\text{lb mol}}{\text{h}\cdot\text{ft}^3\ \text{packing}\cdot\text{mol frac}} \qquad k'_x a = \frac{\text{lb mol}}{\text{h}\cdot\text{ft}^3\ \text{packing}\cdot\text{mol frac}} \quad \text{(English)}$$

4. Design method for packed towers. For absorption of A from stagnant B, the operating-line equation (10.6-4) holds. For the differential height of tower dz in Fig. (10.6-6), the moles of A leaving V equal the moles entering L.

$$d(Vy) = d(Lx) \tag{10.6-7}$$

where V = kg mol total gas/s, L = kg mol total liquid/s, and $d(Vy) = d(Lx)$ = kg mol A transferred/s in height dz m. The kg mol A transferred/s from Eq. (10.6-7) must equal the kg mol A transferred/s from the mass-transfer equation for N_A. Equation (10.4-8) gives the flux N_A using the gas-film and liquid-film coefficients.

$$N_A = \frac{k'_y}{(1-y_A)_{iM}}(y_{AG} - y_{Ai}) = \frac{k'_x}{(1-x_A)_{iM}}(x_{Ai} - x_{AL}) \tag{10.4-8}$$

where $(1-y_A)_{iM}$ and $(1-x_A)_{iM}$ are defined by Eqs. (10.4-6) and (10.4-7). Multiplying the left-hand side of Eq. (10.4-8) by dA and the two right-side terms by $aS\,dz$ from Eq. (10.6-6),

$$N_A\,dA = \frac{k'_y a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\,dz = \frac{k'_x a}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})S\,dz \tag{10.6-8}$$

where $N_A\,dA$ = kg mol A transferred/s in height dz m (lb mol/h).

Equating Eq. (10.6-7) to (10.6-8) and using y_{AG} for the bulk gas phase and x_{AL} for the bulk liquid phase,

$$d(Vy_{AG}) = \frac{k'_y a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\,dz \tag{10.6-9}$$

$$d(Lx_{AL}) = \frac{k'_x a}{(1-x_A)_{iM}}(x_{Ai} - x_{AL})S\,dz \tag{10.6-10}$$

Since $V' = V(1-y_{AG})$ or $V = V'/(1-y_{AG})$,

$$d(Vy_{AG}) = d\left(\frac{V'}{(1-y_{AG})}y_{AG}\right) = V'd\left(\frac{y_{AG}}{1-y_{AG}}\right) = \frac{V'\,dy_{AG}}{(1-y_{AG})^2} \tag{10.6-11}$$

Substituting V for $V'/(1-y_{AG})$ in Eq. (10.6-11) and then equating Eq. (10.6-11) to (10.6-9),

$$\frac{V\,dy_{AG}}{1-y_{AG}} = \frac{k'_y a}{(1-y_A)_{iM}}(y_{AG} - y_{Ai})S\,dz \tag{10.6-12}$$

Repeating for Eq. (10.6-10) since $L = L'/(1 - x_{AL})$,

$$\frac{L\,dx_{AL}}{1 - x_{AL}} = \frac{k'_x a}{(1 - x_A)_{iM}}(x_{Ai} - x_{AL})S\,dz \tag{10.6-13}$$

Dropping the subscripts A, G, and L and integrating, the final equations are as follows using film coefficients:

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{k'_y aS}{(1 - y)_{iM}}(1 - y)(y - y_i)} \tag{10.6-14}$$

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{k'_x aS}{(1 - x)_{iM}}(1 - x)(x_i - x)} \tag{10.6-15}$$

In a similar manner, the final equations can be derived using overall coefficients.

$$z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{K'_y aS}{(1 - y)_{*M}}(1 - y)(y - y^*)} \tag{10.6-16}$$

$$z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{K'_x aS}{(1 - x)_{*M}}(1 - x)(x^* - x)} \tag{10.6-17}$$

In the general case, the equilibrium and the operating lines are usually curved and $k'_x a$, $k'_y a$, $K'_y a$, and $K'_x a$ vary somewhat with total gas and liquid flows. Then Eqs. (10.6-14)–(10.6-17) must be integrated graphically. The methods to do this for concentrated mixtures will be discussed in Section 10.7. Methods for dilute gases will be considered below.

10.6D Simplified Design Methods for Absorption of Dilute Gas Mixtures in Packed Towers

Since a considerable percentage of the absorption processes include absorption of a dilute gas A, these cases will be considered using a simplified design procedure.

The concentrations can be considered dilute for engineering design purposes when the mole fractions y and x in the gas and liquid streams are less than about 0.10, i.e., 10%. The flows will vary by less than 10% and the mass-transfer coefficients by considerably less than this. As a result, the average values of the flows V and L and the mass-transfer coefficients at the top and bottom of the tower can be taken outside the integral. Likewise, the terms $(1 - y)_{iM}/(1 - y)$, $(1 - y)_{*M}/(1 - y)$, $(1 - x)_{iM}/(1 - x)$, and $(1 - x)_{*M}/(1 - x)$ can be taken outside and average values of the values at the top and bottom of the tower used. (Often these terms are close to 1.0 and can be dropped out entirely.) Then Eqs. (10.6-14)–(10.6-17) become

$$z = \left[\frac{V}{k'_y aS}\,\frac{(1 - y)_{iM}}{1 - y}\right]_{\text{av}} \int_{y_2}^{y_1} \frac{dy}{y - y_i} \tag{10.6-18}$$

$$z = \left[\frac{L}{k'_x aS}\,\frac{(1 - x)_{iM}}{1 - x}\right]_{\text{av}} \int_{x_2}^{x_1} \frac{dx}{x_i - x} \tag{10.6-19}$$

$$z = \left[\frac{V}{K'_y aS}\frac{(1-y)_{*M}}{1-y}\right]_{av}\int_{y_2}^{y_1}\frac{dy}{y-y^*}$$

$$z = \left[\frac{L}{K'_x aS}\frac{(1-x)_{*M}}{1-x}\right]_{av}\int_{x_2}^{x_1}\frac{dx}{x^*-x}$$

Since the solutions are dilute, the operating line will be essentially straigh suming the equilibrium line is approximately straight over the range of concentra used, $(y - y_i)$ varies linearly with y and also with x.

$$y - y_i = ky + b$$

where k and b are constants. Therefore, the integral of Eq. (10.6-18) can be integra give the following.

$$\int_{y_2}^{y_1}\frac{dy}{y-y_i} = \frac{y_1 - y_2}{(y-y_i)_M}$$

where $(y - y_i)_M$ is the log mean driving force.

$$(y - y_i)_M = \frac{(y_1 - y_{i1}) - (y_2 - y_{i2})}{\ln\,[(y_1 - y_{i1})/(y_2 - y_{i2})]}$$

$$(y - y^*)_M = \frac{(y_1 - y_1^*) - (y_2 - y_2^*)}{\ln\,[(y_1 - y_1^*)/(y_2 - y_2^*)]}$$

If the term $(1 - y)_{iM}/(1 - y)$ is considered 1.0, then substituting Eq. (10.6-2 (10.6-18) and doing the same for Eqs. (10.6-19)–(10.6-21), the final results are as follo

$$\frac{V}{S}(y_1 - y_2) = k'_y az(y - y_i)_M$$

$$\frac{L}{S}(x_1 - x_2) = k'_x az(x_i - x)_M$$

$$\frac{V}{S}(y_1 - y_2) = K'_y az(y - y^*)_M$$

$$\frac{L}{S}(x_1 - x_2) = K'_x az(x^* - x)_M$$

where the left side is the kg mol absorbed/s · m^2 (lb mol/h · ft^2) by material balan the right-hand side is the rate equation for mass transfer. The value of V is the av $(V_1 + V_2)/2$ and of L is $(L_1 + L_2)/2$.

Equations (10.6-26) to (10.6-29) can be used in slightly different ways. The steps to follow are discussed below and shown in Fig. 10.6-9.

1. The operating-line equation (10.6-4) is plotted as in Fig. 10.6-9 as a straigh Calculate V_1, V_2, and $V_{av} = (V_1 + V_2)/2$; also calculate L_1, L_2, and $L_{av} = (L_1 +$
2. Average experimental values of the film coefficients $k'_y a$ and $k'_x a$ are available obtained from empirical correlations. The interface compositions y_{i1} and x_{i1} at y_1, x_1 in the tower are determined by plotting line P_1M_1 whose slope is calcul Eq. (10.6-30):

$$\text{slope} = -\frac{k'_x a/(1-x)_{iM}}{k'_y a/(1-y)_{iM}} = -\frac{k_x a}{k_y a}$$

$$\text{slope} \cong -\frac{k'_x a/(1-x_1)}{k'_y a/(1-y_1)} \qquad \textbf{(10.6-31)}$$

If terms $(1-x)_{iM}$ and $(1-y)_{iM}$ are used, the procedure is trial and error, as in Example 10.4-1. However, since the solutions are dilute, the terms $(1-x_1)$ and $(1-y_1)$ can be used in Eq. (10.6-31) without trial and error and with a small error in the slope. If the coefficients $k_y a$ and $k_x a$ are available for the approximate concentration range, they can be used, since they include the terms $(1-x)_{iM}$ and $(1-y)_{iM}$. For line P_2M_2 at the other end of the tower, values of y_{i2} and x_{i2} are determined using Eq. (10.6-30) or (10.6-31) and y_2 and x_2.

3. If the overall coefficient $K'_y a$ is being used, y_1^* and y_2^* are determined as shown in Fig. 10.6-9. If $K'_x a$ is used, x_1^* and x_2^* are obtained.
4. Calculate the log mean driving force $(y-y_i)_M$ by Eq. (10.6-24) if $k'_y a$ is used. For $K'_y a$, $(y-y^*)_M$ is calculated by Eq. (10.6-25). Using the liquid coefficients, the appropriate driving forces are calculated.
5. Calculate the column height z m by substituting into the appropriate form of Eqs. (10.6-26)–(10.6-29).

EXAMPLE 10.6-2. Absorption of Acetone in a Packed Tower

Acetone is being absorbed by water in a packed tower having a cross-sectional area of 0.186 m^2 at 293 K and 101.32 kPa (1 atm). The inlet air contains 2.6 mol % acetone and outlet 0.5%. The gas flow is 13.65 kg mol inert air/h (30.1 lb mol/h). The pure water inlet flow is 45.36 kg mol water/h (100 lb mol/h). Film coefficients for the given flows in the tower are $k'_y a = 3.78 \times 10^{-2}$ kg mol/s · m^3 · mol frac (8.50 lb mol/h · ft^3 · mol frac) and $k'_x a = 6.16 \times 10^{-2}$ kg mol/s · m^3 · mol frac (13.85 lb mol/h · ft^3 · mol frac). Equilibrium data are given in Appendix A.3.

(a) Calculate the tower height using $k'_y a$.
(b) Repeat using $k'_x a$.
(c) Calculate $K'_y a$ and the tower height.

Solution: From Appendix A.3 for acetone–water and $x_A = 0.0333$ mol frac, $p_A = 30/760 = 0.0395$ atm or $y_A = 0.0395$ mol frac. Hence, the equilibrium line is $y_A = mx_A$ or $0.0395 = m\,(0.0333)$. Then, $y = 1.186x$. This equi-

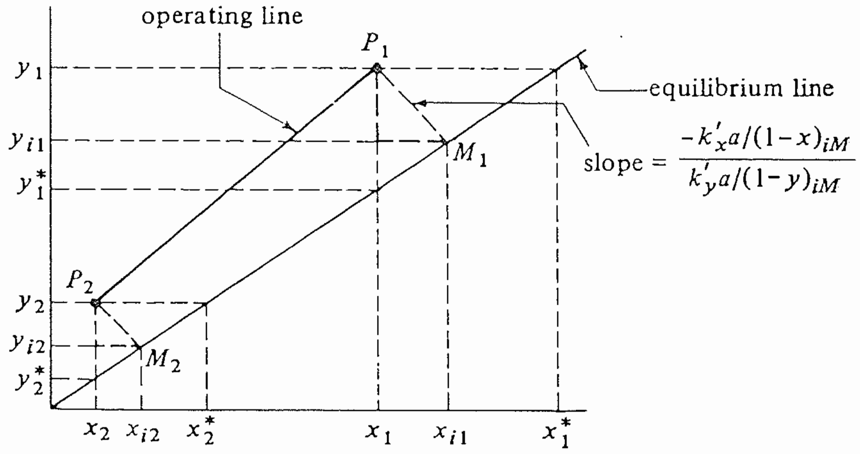


FIGURE 10.6-9. *Operating-line and interface compositions in a packed tower for absorption of dilute gases.*

librium line is plotted in Fig. 10.6-10. The given data are $L' = 45.36$ kg mol/h, $V' = 13.65$ kg mol/h, $y_1 = 0.026$, $y_2 = 0.005$, and $x_2 = 0$.

Substituting into Eq. (10.6-3) for an overall material balance using flow rates as kg mol/h instead of kg mol/s,

$$45.36\left(\frac{0}{1-0}\right) + 13.65\left(\frac{0.026}{1-0.026}\right) = 45.36\left(\frac{x_1}{1-x_1}\right) + 13.65\left(\frac{0.005}{1-0.005}\right)$$

$$x_1 = 0.00648$$

The points y_1, x_1 and y_2, x_2 are plotted in Fig. 10.6-10 and a straight line is drawn for the operating line.

Using Eq. (10.6-31) the approximate slope at y_1, x_1 is

$$\text{slope} \cong -\frac{k'_x a/(1-x_1)}{k'_y a/(1-y_1)} = -\frac{6.16 \times 10^{-2}/(1-0.00648)}{3.78 \times 10^{-2}/(1-0.026)} = -1.60$$

Plotting this line through y_1, x_1, the line intersects the equilibrium line at $y_{i1} = 0.0154$ and $x_{i1} = 0.0130$. Also, $y_1^* = 0.0077$. Using Eq. (10.6-30) to calculate a more accurate slope, the preliminary values of y_{i1} and x_{i1} will be used in the trial-and-error solution. Substituting into Eq. (10.4-6),

$$(1-y)_{iM} = \frac{(1-y_{i1})-(1-y_1)}{\ln[(1-y_{i1})/(1-y_1)]}$$

$$= \frac{(1-0.0154)-(1-0.026)}{\ln[(1-0.0154)/(1-0.020)]} = 0.979$$

Using Eq. (10.4-7),

$$(1-x)_{iM} = \frac{(1-x_1)-(1-x_{i1})}{\ln[(1-x_1)/(1-x_{i1})]}$$

$$= \frac{(1-0.00648)-(1-0.0130)}{\ln[(1-0.00648)/(1-0.0130)]} = 0.993$$

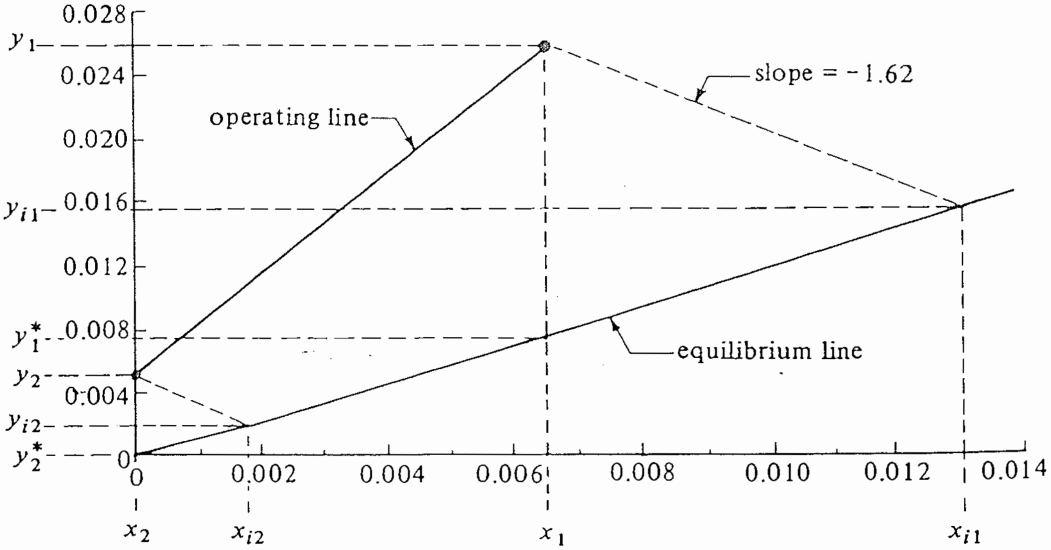


FIGURE 10.6-10. *Location of interface compositions for Example 10.6-2.*

Substituting into Eq. (10.6-30),

$$\text{slope} = -\frac{k'_x a/(1-x)_{iM}}{k'_y a/(1-y)_{iM}} = -\frac{6.16 \times 10^{-2}/0.993}{3.78 \times 10^{-2}/0.929} = -1.61$$

Hence, the approximate slope and interface values are accurate enough.

For the slope at point y_2, x_2,

$$\text{slope} \cong -\frac{k'_x a/(1-x_2)}{k'_y a/(1-y_2)} = -\frac{6.16 \times 10^{-2}/(1-0)}{3.78 \times 10^{-2}/(1-0.005)} = -1.62$$

The slope changes little in the tower. Plotting this line, $y_{i2} = 0.0020$, $x_{i2} = 0.0018$, and $y_2^* = 0$.

Substituting into Eq. (10.6-24),

$$(y - y_i)_M = \frac{(y_1 - y_{i1}) - (y_2 - y_{i2})}{\ln [(y_1 - y_{i1})/(y_2 - y_{i2})]}$$

$$= \frac{(0.026 - 0.0154) - (0.005 - 0.0020)}{\ln [(0.026 - 0.0154)/(0.005 - 0.0020)]} = 0.00602$$

To calculate the total molar flow rates in kg mol/s,

$$V_1 = \frac{V'}{1 - y_1} = \frac{13.65/3600}{1 - 0.026} = 3.893 \times 10^{-3} \text{ kg mol/s}$$

$$V_2 = \frac{V'}{1 - y_2} = \frac{13.65/3600}{1 - 0.005} = 3.811 \times 10^{-3} \text{ kg mol/s}$$

$$V_{av} = \frac{V_1 + V_2}{2} = \frac{3.893 \times 10^{-3} + 3.811 \times 10^{-3}}{2} = 3.852 \times 10^{-3} \text{ kg mol/s}$$

$$L' \cong L_1 \cong L_2 \cong L_{av} = \frac{45.36}{3600} = 1.260 \times 10^{-2} \text{ kg mol/s}$$

For part (a), substituting into Eq. (10.6-26) and solving,

$$\frac{V_{av}}{S}(y_1 - y_2) = k'_y a z (y - y_i)_M$$

$$\frac{3.852 \times 10^{-3}}{0.186}(0.0260 - 0.005) = (3.78 \times 10^{-2}) z (0.00602)$$

$$z = 1.911 \text{ m } (6.27 \text{ ft})$$

For part (b), using an equation similar to Eq. (10.6-24),

$$(x_i - x)_M = \frac{(x_{i1} - x_1) - (x_{i2} - x_2)}{\ln [(x_{i1} - x_1)/(x_{i2} - x_2)]}$$

$$= \frac{(0.0130 - 0.00648) - (0.0018 - 0)}{\ln [(0.0130 - 0.00648)/(0.0018 - 0)]} = 0.00368$$

Substituting into Eq. (10.6-27) and solving,

$$\frac{1.260 \times 10^{-2}}{0.186}(0.00648 - 0) = (6.16 \times 10^{-2}) z (0.00368)$$

$$z = 1.936 \text{ m}$$

This checks part (a) quite closely.

For part (c), substituting into Eq. (10.4-25) for point y_1, x_1,

$$(1-y)_{*M} = \frac{(1-y_1^* - (1-y_1)}{\ln [(1-y_1^*)/(1-y_1)]} = \frac{(1-0.0077)-(1-0.026)}{\ln [(1-0.0077)/(1-0.026)]} = 0.983$$

The overall mass-transfer coefficient $K'_y a$ at point y_1, x_1 is calculated by substituting into Eq. (10.4-24).

$$\frac{1}{K'_y a/(1-y)_{*M}} = \frac{1}{k'_y a/(1-y)_{iM}} + \frac{m'}{k'_x a/(1-x)_{iM}}$$

$$\frac{1}{K'_y a/0.983} = \frac{1}{3.78 \times 10^{-2}/0.979} + \frac{1.186}{6.16 \times 10^{-2}/0.993}$$

$$K'_y a = 2.183 \times 10^{-2} \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

Substituting into Eq. (10.6-25),

$$(y-y^*)_M = \frac{(y_1 - y_1^*) - (y_2 - y_2^*)}{\ln [(y_1 - y_1^*)/(y_2 - y_2^*)]}$$

$$= \frac{(0.0260-0.0077)-(0.0050-0)}{\ln [(0.0260-0.0077)/(0.0050-0)]} = 0.01025$$

Finally substituting into Eq. (10.6-28),

$$\frac{3.852 \times 10^{-3}}{0.186}(0.0260-0.0050) = (2.183 \times 10^{-2})z(0.01025)$$

$$z = 1.944 \text{ m}$$

This checks parts (a) and (b).

10.6E Design of Packed Towers Using Transfer Units

Another and in some ways a more useful design method of packed towers is the use of the transfer unit concept. For the most common case of A diffusing through stagnant and nondiffusing B, Eqs. (10.6-14)–(10.6-17) can be rewritten as

$$z = H_G \int_{y_2}^{y_1} \frac{(1-y)_{iM}\, dy}{(1-y)(y-y_i)} \tag{10.6-32}$$

$$z = H_L \int_{x_2}^{x_1} \frac{(1-x)_{iM}\, dx}{(1-x)(x_i-x)} \tag{10.6-33}$$

$$z = H_{OG} \int_{y_2}^{y_1} \frac{(1-y)_{*M}\, dy}{(1-y)(y-y^*)} \tag{10.6-34}$$

$$z = H_{OL} \int_{x_2}^{x_1} \frac{(1-x)_{*M}\, dx}{(1-x)(x^*-x)} \tag{10.6-35}$$

where

$$H_G = \frac{V}{k'_y aS} = \frac{V}{k_y a(1-y)_{iM} S} \tag{10.6-36}$$

$$H_L = \frac{L}{k'_x aS} = \frac{L}{k_x a(1-x)_{iM} S} \tag{10.6-37}$$

$$H_{OG} = \frac{V}{K'_y aS} = \frac{V}{K_y a(1-y)_{*M} S} \tag{10.6-38}$$

$$H_{OL} = \frac{L}{K'_x aS} = \frac{L}{K_x a(1-x)_{*M} S} \tag{10.6-39}$$

The units of H are in m (ft). The H_G is the height of a transfer unit based on the gas film. The values of the heights of transfer units are more constant than the mass-transfer coefficients. For example, $k'_y a$ is often proportional to $V^{0.7}$, then $H_G \propto V^{1.0}/V^{0.7} \propto V^{0.3}$. The average values of the mass-transfer coefficients, $(1-y)_{iM}$, $(1-y)_{*M}$, $(1-x)_{iM}$, and $(1-x)_{*M}$, must be used in Eqs. (10.6-36)–(10.6-39).

The integrals on the right side of Eqs. (10.6-32)–(10.6-35) are the number of transfer units N_G, N_L, N_{OG}, and N_{OL}, respectively. The height of the packed tower is then

$$z = H_G N_G = H_L N_L = H_{OG} N_{OG} = H_{OL} N_{OL} \tag{10.6-40}$$

These equations are basically no different than those using mass-transfer coefficients. One still needs $k'_y a$ and $k'_x a$ to determine interface concentrations. Disregarding $(1-y)_{iM}/(1-y)$, which is near 1.0 in Eq. (10.6-32), the greater the amount of absorption $(y_1 - y_2)$ or the smaller the driving force $(y - y_i)$, the larger the number of transfer units N_G and the taller the tower.

When the solutions are dilute with concentrations below 10%, the terms $(1-y)_{iM}/(1-y)$, $(1-x)_{iM}/(1-x)$, $(1-y)_{*M}/(1-y)$, and $(1-x)_{*M}/(1-x)$ can be taken outside the integral and average values used. Often they are quite close to 1 and can be dropped out. The equations become

$$z = H_G N_G = H_G \left[\frac{(1-y)_{iM}}{1-y}\right]_{\text{av}} \int_{y_2}^{y_1} \frac{dy}{y - y_i} \tag{10.6-41}$$

$$z = H_L N_L = H_L \left[\frac{(1-x)_{iM}}{1-x}\right]_{\text{av}} \int_{x_2}^{x_1} \frac{dx}{x_i - x} \tag{10.6-42}$$

$$z = H_{OG} N_{OG} = H_{OG} \left[\frac{(1-y)_{*M}}{1-y}\right]_{\text{av}} \int_{y_2}^{y_1} \frac{dy}{y - y^*} \tag{10.6-43}$$

$$z = H_{OL} N_{OL} = H_{OL} \left[\frac{(1-x)_{*M}}{1-x}\right]_{\text{av}} \int_{x_2}^{x_1} \frac{dx}{x^* - x} \tag{10.6-44}$$

If the operating and equilibrium lines are both straight and the solutions dilute, the integral shown in Eq. (10.6-23) is valid.

$$\int_{y_2}^{y_1} \frac{dy}{y - y_i} = \frac{y_1 - y_2}{(y - y_i)_M} \tag{10.6-23}$$

This then can be substituted into Eq. (10.6-41) and similar expressions into Eqs. (10.6-42)–(10.6-44).

EXAMPLE 10.6-3. Use of Transfer Units for Packed Tower
Repeat Example 10.6-2 using transfer units and height of a transfer unit as follows.

(a) Use H_G and N_G to calculate tower height.
(b) Use H_{OG} and N_{OG} to calculate tower height.

Solution: For part (a), $k'_y a = 3.78 \times 10^{-2}$ kg mol/s·m³·mol frac from

Example 10.6-2. From Eq. (10.6-36),

$$H_G = \frac{V}{k'_y aS} \tag{10.6-36}$$

The average V is 3.852×10^{-3} kg mol/s and $S = 0.186\ \text{m}^2$. Substituting and solving,

$$H_G = \frac{3.852 \times 10^{-3}}{(3.78 \times 10^{-2})(0.186)} = 0.548\ \text{m}$$

Since the solution is dilute, the number of transfer units from Eq. (10.6-41) is

$$N_G = \left[\frac{(1-y)_{iM}}{1-y}\right]_{\text{av}} \int_{y_1}^{y_2} \frac{dy}{y - y_i} \tag{10.6-45}$$

The term in the brackets will be evaluated at point 1 and point 2. At point $y_1 = 0.026$, $y_{i1} = 0.0154$ from Example 10.6-2. Also, from Eq. (10.4-6) in Example 10.6-2, $(1-y)_{iM} = 0.979$. Also, $1 - y = 1 - 0.026 = 0.974$. Then at point 1,

$$\frac{(1-y)_{iM}}{1-y} = \frac{0.979}{0.974} = 1.005$$

At point $y_2 = 0.005$, $y_{i2} = 0.002$. Substituting into Eq. (10.4-6), $(1-y)_{iM} = 0.997$ and $1 - y = 1 - 0.005 = 0.995$. Then at point 2,

$$\frac{(1-y)_{iM}}{1-y} = \frac{0.997}{0.995} = 1.002$$

Hence, the average value of the term in the brackets in Eq. (10.6-45) is

$$\left[\frac{(1-y)_{iM}}{1-y}\right]_{\text{av}} = \frac{1.005 + 1.002}{2} = 1.003$$

Using Eq. (10.6-23), the integral is

$$\int_{y_2}^{y_1} \frac{dy}{y - y_i} = \frac{y_1 - y_2}{(y - y_i)_M} \tag{10.6-23}$$

From Example 10.6-2, $(y - y_i)_M = 0.00602$. Substituting Eq. (10.6-23) into Eq. (10.6-45),

$$N_G = \left[\frac{(1-y)_{iM}}{1-y}\right]_{\text{av}} \frac{y_1 - y_2}{(y - y_i)_M} \tag{10.6-46}$$

Substituting into Eq. (10.6-46),

$$N_G = (1.003)\left(\frac{0.026 - 0.005}{0.00602}\right) = 3.50 \text{ transfer units}$$

Finally, substituting into Eq. (10.6-40),

$$z = H_G N_G = (0.548)(3.50) = 1.918\ \text{m}$$

For part (b), using $K'_y a = 2.183 \times 10^{-2}$ kg mol/s · m^3 · mol frac from Example 10.6-2 and substituting into Eq. (10.6-38),

$$H_{OG} = \frac{V}{K'_y aS} = \frac{3.852 \times 10^{-3}}{(2.183 \times 10^{-2})(0.186)} = 0.949\ \text{m}$$

The number of transfer units in Eq. (10.6-43) becomes as follows when the integration similar to Eq. (10.6-23) is carried out.

$$N_{OG} = \left[\frac{(1-y)_{\cdot M}}{1-y}\right]_{av} \frac{y_1 - y_2}{(y - y^*)_M} \tag{10.6-47}$$

Substituting $(y - y^*)_M = 0.01025$ from Example 10.6-2 and the other knowns into Eq. (10.6-47) and calling the bracketed term 1.0,

$$N_{OG} = (1.0)\left(\frac{0.026 - 0.005}{0.01025}\right) = 2.05 \text{ transfer units}$$

Finally by Eq. (10.6-40),

$$z = H_{OG} N_{OG} = 0.949(2.05) = 1.945 \text{ m}$$

Note that the number of transfer units N_{OG} of 2.05 is not the same as N_G of 3.50.

10.7 ABSORPTION OF CONCENTRATED MIXTURES IN PACKED TOWERS

In Section 10.6D simplified design methods were given for absorption of dilute gases in packed towers when the mole fractions in the gas and liquid streams were less than about 10%. Straight operating lines and approximately straight equilibrium lines are obtained. In concentrated gas mixtures the operating line and usually the equilibrium line will be substantially curved and $k'_x a$ and $k'_y a$ may vary with total flows. Then the design equations (10.6-14)–(10.6-17) must be integrated graphically or numerically.

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{k'_y aS}{(1-y)_{iM}}(1-y)(y-y_i)} \tag{10.6-14}$$

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{k'_x aS}{(1-x)_{iM}}(1-x)(x_i-x)} \tag{10.6-15}$$

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V\,dy}{\dfrac{K'_y aS}{(1-y)_{\cdot M}}(1-y)(y-y^*)} \tag{10.6-16}$$

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L\,dx}{\dfrac{K'_x aS}{(1-x)_{\cdot M}}(1-x)(x^*-x)} \tag{10.6-17}$$

The detailed general steps to follow are as follows:

1. The operating-line equation (10.6-4) and the equilibrium line are plotted.
2. The values of the film coefficients $k'_y a$ and $k'_x a$ are obtained from empirical equations. These two film coefficients are functions of G_y^n, kg total gas/s · m^2, and G_x^m, kg total liquids/s · m^2, where n and m are in the range 0.2–0.8. Using the operating-line-equation values, total V and L are calculated for different values of y and x in the tower and converted to G_y and G_x. Then values of $k'_y a$ and $k'_x a$ are calculated. If the variation between $k'_y a$ or $k'_x a$ at the top and bottom of the tower is small, an average value can be used.

3. Starting with the tower bottom at point $P_1(y_1, x_1)$, the interface compositions y_{i1}, x_{i1} are determined by plotting a line P_1M_1 with a slope calculated by Eq. (10.6-30). This line intersects the equilibrium line at the interface concentrations at point M_1:

$$\text{slope} = -\frac{k'_x a/(1-x)_{iM}}{k'_y a/(1-y)_{iM}} = -\frac{k_x a}{k_y a} \tag{10.6-30}$$

where $(1-y)_{iM}$ and $(1-x)_{iM}$ are determined from Eqs. (10.4-6) and (10.4-7), respectively. This is trial and error. As a first trial, $(1-x_1)$ can be used for $(1-x)_{iM}$ and $(1-y_1)$ for $(1-y)_{iM}$. The values of y_{i1} and x_{i1} determined in the first trial are used in Eq. (10.6-30) for the second trial.
4. At point $P_2(y_2, x_2)$ determine a new slope using Eq. (10.6-30) repeating step 3. Do this for several intermediate points in the tower. This slope may vary throughout the tower.
5. Using the values of y_i and x_i determined, graphically integrate Eq. (10.6-14) to obtain the tower height by plotting $f(y)$, where $f(y)$ is as follows:

$$f(y) = \frac{V}{\dfrac{k'_y aS}{(1-y)_{iM}}(1-y)(y-y_i)} \tag{10.7-1}$$

versus y between y_2 and y_1. Then determine the area under the curve to give the tower height. If $k'_x a$ or other coefficients are used the appropriate functions indicated in Eqs. (10.6-15)–(10.6-17) are plotted. If a stream is quite dilute, $(1-y)_{iM}$ or $(1-x)_{iM}$ can be assumed to be 1.0.

EXAMPLE 10.7-1. Design of an Absorption Tower with a Concentrated Gas Mixture

A tower packed with 25.4-mm ceramic rings is to be designed to absorb SO_2 from air by using pure water at 293 K and 1.013×10^5 Pa abs pressure. The entering gas contains 20 mol % SO_2 and that leaving 2 mol %. The inert air flow is 6.53×10^{-4} kg mol air/s and the inert water flow is 4.20×10^{-2} kg mol water/s. The tower cross-sectional area is 0.0929 m². For dilute SO_2, the film mass-transfer coefficients at 293 K are for 25.4-mm (1-in.) rings (W1),

$$k'_y a = 0.0594 G_y^{0.7} G_x^{0.25} \qquad k'_x a = 0.152 G_x^{0.82}$$

where $k'_y a$ is kg mol/s · m³ · mol frac, $k'_x a$ is kg mol/s · m³ · mol frac, and G_x and G_y are kg total liquid or gas, respectively, per sec per m² tower cross section. Calculate the tower height.

Solution: The given data are $V' = 6.53 \times 10^{-4}$ kg mol air/s (5.18 lb mol/h), $L' = 4.20 \times 10^{-2}$ kg mol/s (333 lb mol/h), $y_1 = 0.20$, $y_2 = 0.02$, and $x_2 = 0$.

Substituting into the overall material-balance equation (10.6-3),

$$L'\left(\frac{x_2}{1-x_2}\right) + V'\left(\frac{y_1}{1-y_1}\right) = L'\left(\frac{x_1}{1-x_1}\right) + V'\left(\frac{y_2}{1-y_2}\right)$$

$$4.20 \times 10^{-2}\left(\frac{0}{1-0}\right) + 6.53 \times 10^{-4}\left(\frac{0.2}{1-0.2}\right) = 4.20 \times 10^{-2}\left(\frac{x_1}{1-x_1}\right) + 6.53 \times 10^{-4}\left(\frac{0.02}{1-0.02}\right)$$

Solving, $x_1 = 0.00355$. The operating line Eq. (10.6-4) is

$$4.20 \times 10^{-2}\left(\frac{x}{1-x}\right) + 6.53 \times 10^{-4}\left(\frac{0.2}{1-0.2}\right) = 4.20 \times 10^{-2}\left(\frac{0.00355}{1-0.00355}\right) + 6.53 \times 10^{-4}\left(\frac{y}{1-y}\right)$$

Setting $y = 0.04$ in the operating-line equation above and solving for x, $x = 0.000332$. Selecting other values of y and solving for x, points on the operating line were calculated as shown in Table 10.7-1 and plotted in Fig. 10.7-1 together with the equilibrium data from Appendix A.3.

The total molar flow V is calculated from $V = V'/(1 - y)$. At $y = 0.20$, $V = 6.53 \times 10^{-4}/(1 - 0.2) = 8.16 \times 10^{-4}$. Other values are calculated and tabulated in Table 10.7-1. The total mass flow G_y in kg/s · m^2 is equal to the mass flow of air plus SO$_2$ divided by the cross-sectional area.

$$G_y = \frac{6.53 \times 10^{-4}(29) \text{ kg air/s} + 6.53 \times 10^{-4}\left(\frac{y}{1-y}\right)(64.1) \text{ kg SO}_2\text{/s}}{0.0929 \text{ m}^2}$$

Setting $y = 0.20$,

$$G_y = \frac{6.53 \times 10^{-4}(29) + 6.53 \times 10^{-4}\left(\frac{0.2}{1-0.2}\right)64.1}{0.0929} = 0.3164 \text{ kg/s} \cdot \text{m}^2$$

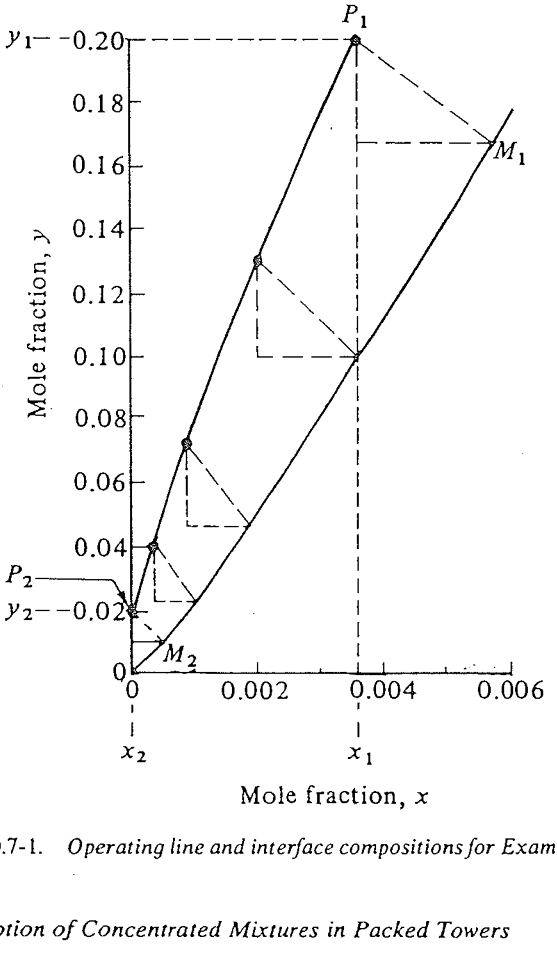


FIGURE 10.7-1. *Operating line and interface compositions for Example 10.7-1.*

Table 10.7-1. *Calculated Data for Example 10.7-1*

y	x	V	L	G_y	G_x	$k'_y a$	$k'_x a$	x_i	y_i	$1 - y$	$(1 - y)_{iM}$	$y - y_i$	$\dfrac{V}{\dfrac{k'_y aS}{(1 - y)_{iM}}(1 - y)(y - y_i)}$
0.02	0	6.66×10^{-4}	0.04200	0.2130	8.138	0.03398	0.848	0.00046	0.0090	0.980	0.985	0.0110	19.25
0.04	0.000332	6.80×10^{-4}	0.04201	0.2226	8.147	0.03504	0.849	0.00103	0.0235	0.960	0.968	0.0165	12.77
0.07	0.000855	7.02×10^{-4}	0.04203	0.2378	8.162	0.03673	0.850	0.00185	0.0476	0.930	0.941	0.0224	9.29
0.13	0.00201	7.51×10^{-4}	0.04208	0.2712	8.196	0.04032	0.853	0.00355	0.1015	0.870	0.885	0.0285	7.16
0.20	0.00355	8.16×10^{-4}	0.04215	0.3164	8.241	0.04496	0.857	0.00565	0.1685	0.800	0.816	0.0315	6.33

Similarly, G_y is calculated for all points and tabulated. For the liquid flow, $L = L'/(1 - x)$. Also, for the total liquid mass flow rate,

$$G_x = \frac{4.20 \times 10^{-2}(18) + 4.20 \times 10^{-2}\left(\frac{x}{1-x}\right)64.1}{0.0929}$$

Calculated values of L and G_x for various values of x are tabulated in Table 10.7-1.

To calculate values of $k'_x a$, for $x = 0$, $G_x = 8.138$ and

$$k'_x a = 0.152 G_x^{0.82} = 0.152(8.138)^{0.82} = 0.848 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

The rest of these values are calculated and given in Table 10.7-1. For the value of $k'_y a$, for $y = 0.02$, $G_y = 0.2130$, $G_x = 8.138$, and

$$k'_y a = 0.0594 G_y^{0.7} G_x^{0.25} = 0.0594(0.2130)^{0.7}(8.138)^{0.25}$$
$$= 0.03398 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

This and other calculated values of k'_y are tabulated. Note that these values vary considerably in the tower.

Next the interface compositions y_i and x_i must be determined for the given y and x values on the operating line. For the point $y_1 = 0.20$ and $x_1 = 0.00355$ we make a preliminary estimate of $(1 - y)_{iM} \cong 1 - y \cong 1 - 0.20 \cong 0.80$. Also, the estimate of $(1 - x)_{iM} \cong 1 - x \cong 1 - 0.00355 \cong 0.996$. The slope of the line P_1M_1 by Eq. (10.6-30) is approximately

$$\text{slope} = -\frac{k'_x a/(1 - x)_{iM}}{k'_y a/(1 - y)_{iM}} = -\frac{0.857/(0.996)}{0.04496/(0.80)} = -15.3$$

Plotting this on Fig. 10.7-1, $y_i = 0.1688$ and $x_i = 0.00566$. Using these values for the second trial in Eqs. (10.4-6) and (10.4-7),

$$(1 - y)_{iM} = \frac{(1 - 0.1688) - (1 - 0.20)}{\ln[(1 - 0.1688)/(1 - 0.20)]} = 0.816$$

$$(1 - x)_{iM} = \frac{(1 - 0.00355) - (1 - 0.00566)}{\ln[(1 - 0.00355)/(1 - 0.00566)]} = 0.995$$

The new slope by Eq. (10.6-30) is $(-0.857/0.995)/(0.04496/0.816) = -15.6$. Plotting, $y_i = 0.1685$ and $x_i = 0.00565$. This is shown as point M_1. This calculation is repeated until point y_2, x_2 is reached. The slope of Eq. (10.6-30) increases markedly in going up the tower, being -24.6 at the top of the tower. The values of y_i and x_i are given in Table 10.7-1.

In order to integrate Eq. (10.6-14), values of $(1 - y)$, $(1 - y)_{iM}$, and $(y - y_i)$ are needed and are tabulated in Table 10.7-1. Then for $y = 0.20$, $f(y)$ is calculated from Eq. (10.7-1).

$$f(y) = \frac{V}{\dfrac{k'_y aS}{(1 - y)_{iM}}(1 - y)(y - y_i)} = \frac{8.16 \times 10^{-4}}{\dfrac{0.04496(0.0929)}{0.816}(0.800)(0.0315)} = 6.33$$

This is repeated for other values of y. Then the function $f(y)$ is plotted versus y. The total area is then the sum of four rectangles:

$$\text{total area} = 0.312 + 0.418 + 0.318 + 0.540 = 1.588$$

Hence, the tower height is equal to the area by Eq. (10.6-14) and $z = 1.588$ m.

10.8 ESTIMATION OF MASS-TRANSFER COEFFICIENTS FOR PACKED TOWERS

10.8A Experimental Determination of Film Coefficients

The individual film mass-transfer coefficients $k'_y a$ and $k'_x a$ depend generally upon Schmidt number, Reynolds number, and the size and shape of the packing. The interactions among these factors are quite complex. Hence, the correlations for mass-transfer coefficients are highly empirical. The reliability of these correlations is not too satisfactory. Deviations of up to 25% are not uncommon. A main difficulty arises because an overall coefficient or resistance is measured experimentally that represents the two film resistances in series. To obtain the single-phase film coefficient, the experiment is so arranged that the other film resistance is negligible or can be approximately calculated.

To measure the liquid film mass-transfer coefficient $k'_x a$, a system for absorption or desorption of very insoluble gases such as O_2 or CO_2 in water is used. The experiment gives $K'_x a$, which equals $k'_x a$, since the gas-phase resistance is negligible.

To measure the gas-phase film coefficient $k'_y a$, we desire to use a system such that the solute is very soluble in the liquid and the liquid-phase resistance is negligible. Most such systems as NH_3–air–water have a liquid-phase resistance of about 10%. By subtracting this known liquid phase resistance (obtained by correcting $k'_x a$ data for absorption of CO_2 or O_2 to NH_3 data for $k'_x a$) from the overall resistance in Eq. (10.4-24), we obtain the coefficient $k'_y a$. Details of these are discussed elsewhere (G1, S1, S2).

10.8B Correlations for Film Coefficients

The experimental data for the gas film coefficient in dilute mixtures have been correlated in terms of H_G, where

$$H_G = \frac{V}{k'_y a S} \tag{10.6-36}$$

The empirical equation is as follows:

$$H_G = \alpha G_y^{\beta} G_x^{\gamma} N_{Sc}^{0.5} \tag{10.8-1}$$

where G_y = kg total gas/s · m^2; G_x = kg total liquid/s · m^2; and α, β, and γ are constants for a packing as given in Table 10.8-1 (T2). The temperature effect, which is small, is included in the Schmidt number $\mu/\rho D$, where μ is the viscosity of the gas mixture in kg/m · s, ρ the density in kg/m^3, and D the diffusivity of solute A in the gas in m^2/s. The coefficients $k'_y a$ and H_G can be shown to be independent of pressure.

Equation (10.8-1) can be used to correct existing data for absorption of solute A in a gas on a specific packing to absorption of solute E in the same system and the same mass-flow rates. This is done by Eq. (10.8-2).

$$H_{G(E)} = H_{G(A)}[N_{Sc(E)}/N_{Sc(A)}]^{0.5} \tag{10.8-2}$$

The correlations for liquid film coefficients in dilute mixtures show that H_L is independent of gas rate until loading occurs, as given by the following:

$$H_L = \theta \left(\frac{G_x}{\mu_L}\right)^{\eta} N_{Sc}^{0.5} \tag{10.8-3}$$

where H_L is in m, μ_L is liquid viscosity in kg/m · s, N_{Sc} is Schmidt number $\mu_L/\rho D$, ρ is

TABLE 10.8-1. *Gas Film Height of a Transfer Unit H_G in Meters**

Packing Type	α	β	γ	Range of Values G_y	 G_x
Raschig rings					
9.5 mm ($\frac{3}{8}$ in.)	0.620	0.45	−0.47	0.271–0.678	0.678–2.034
25.4 mm (1 in.)	0.557	0.32	−0.51	0.271–0.814	0.678–6.10
38.1 mm (1.5 in.)	0.830	0.38	−0.66	0.271–0.950	0.678–2.034
38.1 mm (1.5 in.)	0.689	0.38	−0.40	0.271–0.950	2.034–6.10
50.8 mm (2 in.)	0.894	0.41	−0.45	0.271–1.085	0.678–6.10
Berl saddles					
12.7 mm (0.5 in.)	0.541	0.30	−0.74	0.271–0.950	0.678–2.034
12.7 mm (0.5 in.)	0.367	0.30	−0.24	0.271–0.950	2.034–6.10
25.4 mm (1 in.)	0.461	0.36	−0.40	0.271–1.085	0.542–6.10
38.1 mm (1.5 in.)	0.652	0.32	−0.45	0.271–1.356	0.542–6.10

* $H_G = \alpha G_y^{\beta} G_x^{\gamma} N_{Sc}^{0.5}$, where G_y = kg total gas/s · m², G_x = kg total liquid/s · m², and $N_{Sc} = \mu/\rho D$.
Source: Data from Fellinger (P2) as given by R. E. Treybal, *Mass Transfer Operations.* New York: McGraw-Hill Book Company, 1955, p. 239. With permission.

liquid density in kg/m³, and D is diffusivity of solute A in the liquid in m²/s. Data are given in Table 10.8-2 for different packings. Equation (10.8-3) can be used to correct existing data on a given packing and solute to another solute.

EXAMPLE 10.8-1. Prediction of Film Coefficients for Ammonia Absorption

Predict H_G, H_L, and $K'_y a$ for absorption of NH_3 from water in a dilute solution in a packed tower with 25.4-mm Raschig rings at 303 K (86°F) and

TABLE 10.8-2. *Liquid Film Height of a Transfer Unit H_L in Meters**

Packing	θ	η	Range of G_x
Raschig rings			
9.5 mm ($\frac{3}{8}$ in.)	3.21×10^{-4}	0.46	0.542–20.34
12.7 mm (0.5 in.)	7.18×10^{-4}	0.35	0.542–20.34
25.4 mm (1 in.)	2.35×10^{-3}	0.22	0.542–20.34
38.1 mm (1.5 in.)	2.61×10^{-3}	0.22	0.542–20.34
50.8 mm (2 in.)	2.93×10^{-3}	0.22	0.542–20.34
Berl saddles			
12.7 mm (0.5 in.)	1.456×10^{-3}	0.28	0.542–20.34
25.4 mm (1 in.)	1.285×10^{-3}	0.28	0.542–20.34
38.1 mm (1.5 in.)	1.366×10^{-3}	0.28	0.542–20.34

* $H_L = \theta(G_x/\mu_L)^{\eta} N_{Sc}^{0.5}$, where G_x = kg total liquid/s · m², μ_L = viscosity of liquid in kg/m · s, and $N_{Sc} = \mu_L/\rho D$. G_y is less than loading.
Source: Based on data by Sherwood and Holloway (S3) as given by R. E. Treybal, *Mass Transfer Operations.* New York: McGraw-Hill Book Company, 1955, p. 237. With permission.

101.32 kPa pressure. The flow rates are $G_x = 2.543$ kg/s · m² and $G_y = 0.339$ kg/s · m².

Solution: From Appendix A.3 the equilibrium relation in a dilute solution is $0.0151 = m(0.0126)$ or $y = 1.20x$. Also, from Appendix A.3 for air, $\mu = 1.86 \times 10^{-5}$ kg/m · s. Density $\rho = 1.168$ kg/m³. The diffusivity of NH_3 in air at 273 K from Table 6.2-1 is 1.98×10^{-5} m²/s. Correcting to 303 K by Eq. (6.2-45), $D_{AB} = 2.379 \times 10^{-5}$ m²/s. Hence,

$$N_{Sc} = \frac{\mu}{\rho D} = \frac{1.86 \times 10^{-5}}{(1.168)(2.379 \times 10^{-5})} = 0.669$$

Substituting into Eq. (10.8-1) using data from Table 10.8-1,

$$H_G = \alpha G_y^{\beta} G_x^{\gamma} N_{Sc}^{0.5} = 0.557(0.339)^{0.32}(2.543)^{-0.51}(0.669)^{0.5} = 0.200 \text{ m}$$

The viscosity of water $= 1.1404 \times 10^{-3}$ kg/m · s at 15°C and 0.8007×10^{-3} at 30°C from Appendix A.2. The D_{AB} of NH_3 in water at 288 K (15°C) from Table 6.3-1 is 1.77×10^{-9} m²/s. Correcting this to 303 K (30°C), using Eq. (6.3-9),

$$D_{AB} = \left(\frac{1.1404 \times 10^{-3}}{0.8007 \times 10^{-3}}\right)\left(\frac{303}{288}\right)(1.77 \times 10^{-9}) = 2.652 \times 10^{-9} \text{ m}^2/\text{s}$$

Then, using $\rho = 996$ kg/m³ for water,

$$N_{Sc} = \frac{\mu}{\rho D} = \frac{0.8007 \times 10^{-3}}{(996)(2.652 \times 10^{-9})} = 303.1$$

Substituting into Eq. (10.8-3), using data from Table 10.8-2,

$$H_L = \theta\left(\frac{G_x}{\mu_L}\right)^{\eta} N_{Sc}^{0.5} = (2.35 \times 10^{-3})\left(\frac{2.543}{0.8007 \times 10^{-3}}\right)^{0.22} (303.1)^{0.5}$$
$$= 0.2412 \text{ m}$$

Converting to $k'_y a$ using Eq. (10.6-36),

$$k'_y a = \frac{V}{H_G S} = \frac{0.339/29}{0.200} = 0.0584 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

For $k'_x a$ using Eq. (10.6-37),

$$k'_x a = \frac{L}{H_L S} = \frac{2.543/18}{0.2412} = 0.586 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

Substituting into Eq. (10.4-24) for dilute solutions,

$$\frac{1}{K'_y a} = \frac{1}{k'_y a} + \frac{m'}{k'_x a} = \frac{1}{0.0584} + \frac{1.20}{0.586} = 17.12 + 2.048 = 19.168$$

$$K'_y a = 0.0522 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

Note that the percent resistance in the gas film is $(17.12/19.168)(100) = 89.3\%$.

PROBLEMS

10.2-1. ***Equilibrium and Henry's Law Constant.*** The partial pressure of CO_2 in air is 1.333×10^4 Pa and the total pressure is 1.133×10^5 Pa. The gas phase is in equilibrium with a water solution at 303 K. What is the value of x_A of CO_2 in equilibrium in the solution? See Appendix A.3 for the Henry's law constant.

Ans. $x_A = 7.07 \times 10^{-5}$ mol frac CO_2

10.2-2. ***Gas Solubility in Aqueous Solution.*** At 303 K the concentration of CO_2 in water is 0.90×10^{-4} kg CO_2/kg water. Using the Henry's law constant from Appendix A.3, what partial pressure of CO_2 must be kept in the gas to keep the CO_2 from vaporizing from the aqueous solution?

Ans. $p_A = 6.93 \times 10^3$ Pa (0.0684 atm)

10.2-3. ***Phase Rule for a Gas–Liquid System.*** For the system SO_2–air–water, the total pressure is set at 1 atm abs and the partial pressure of SO_2 in the vapor is set at 0.20 atm. Calculate the number of degrees of freedom, F. What variables are unspecified that can be arbitrarily set?

10.3-1. ***Equilibrium Stage Contact for Gas–Liquid System.*** A gas mixture at 2.026 $\times 10^5$ Pa total pressure containing air and SO_2 is contacted in a single-stage equilibrium mixer with pure water at 293 K. The partial pressure of SO_2 in the original gas is 1.52×10^4 Pa. The inlet gas contains 5.70 total kg mol and the inlet water 2.20 total kg mol. The exit gas and liquid leaving are in equilibrium. Calculate the amounts and compositions of the outlet phases. Use equilibrium data from Fig. 10.2-1.

Ans. $x_{A1} = 0.00495$, $y_{A1} = 0.0733$, $L_1 = 2.211$ kg mol, $V_1 = 5.69$ kg mol

10.3-2. ***Absorption in a Countercurrent Stage Tower.*** Repeat Example 10.3-2 using the same conditions but with the following change. Use a pure water flow to the tower of 108 kg mol H_2O/h, that is, 20% above the 90 used in Example 10.3-2. Determine the number of stages required graphically. Repeat using the analytical Kremser equation.

10.3-3. ***Stripping Taint from Cream by Steam.*** Countercurrent stage stripping is to be used to remove a taint from cream. The taint is present in the original cream to the stripper at a concentration of 20 parts per million (ppm). For every 100 kg of cream entering per unit time, 50 kg of steam will be used for stripping. It is desired to reduce the concentration of the taint in the cream to 1 ppm. The equilibrium relation between the taint in the steam vapor and the liquid cream is $y_A = 10x_A$, where y_A is ppm of taint in the steam and x_A ppm in the cream (E1). Determine the number of theoretical stages needed. [*Hint:* In this case, for stripping from the liquid (L) stream to the vapor (V) stream the operating line will be below the equilibrium line on the $y_A - x_A$ diagram. It is assumed that none of the steam condenses in the stripping. Use ppm in the material balances.]

Ans. Number stages = 1.85 (stepping down starting from the concentrated end)

10.4-1. ***Overall Mass-Transfer Coefficient from Film Coefficients.*** Using the same data as in Example 10.4-1, calculate the overall mass-transfer coefficients K'_x and K_x, the flux, and the percent resistance in the gas film.

Ans. $K'_x = 1.173 \times 10^{-3}$ kg mol/s · m² · mol frac, $K_x = 1.519 \times 10^{-3}$,
$N_A = 3.78 \times 10^{-4}$ kg mol/s · m², 36.7% resistance

10.4-2. ***Interface Concentrations and Overall Mass-Transfer Coefficients.*** Use the same equilibrium data and film coefficients k'_y and k'_x as in Example 10.4-1. However, use bulk concentrations of $y_{AG} = 0.25$ and $x_{AL} = 0.05$. Calculate the following.

(a) Interface concentrations y_{Ai} and x_{Ai} and flux N_A.
(b) Overall mass-transfer coefficients K'_y and K_y and flux N_A.
(c) Overall mass-transfer coefficient K'_x and flux N_A.

10.5-1. ***Countercurrent Water-Cooling Tower.*** A forced-draft countercurrent water-cooling tower is to cool water from 43.3 to 26.7°C. The air enters the bottom of the tower at 23.9°C with a wet bulb temperature of 21.1°C. The value of H_G for the flow conditions is $H_G = 0.533$ m. The heat-transfer resistance in the liquid phase will be neglected; i.e., h_L is very large. Hence, values of H_y^* should be used. Calculate the tower height needed if 1.5 times the minimum air rate is used.

10.5-2. ***Minimum Gas Rate and Height of Water-Cooling Tower.*** It is planned to cool water from 110°F to 85°F in a packed countercurrent water-cooling tower using entering air at 85°F with a wet bulb temperature of 75°F. The water flow is 2000 $\text{lb}_m/\text{h} \cdot \text{ft}^2$ and the air flow is 1400 lb_m $\text{air/h} \cdot \text{ft}^2$. The overall mass-transfer coefficient is $K_G a = 6.90 \text{ lb mol/h} \cdot \text{ft}^3 \cdot \text{atm}$.

(a) Calculate the minimum air rate that can be used.

(b) Calculate the tower height needed if the air flow of 1400 lb_m $\text{air/h} \cdot \text{ft}^2$ is used.

Ans. (a) $G_{\min} = 935 \text{ lb}_m \text{ air/h} \cdot \text{ft}^2$ ($4241 \text{ kg air/h} \cdot \text{m}^2$); (b) $z = 21.8$ ft (6.64 m)

10.5-3. ***Design of Water-Cooling Tower.*** Recalculate Example 10.5-1, but calculate the minimum air rate and use 1.75 times the minimum air rate.

10.5-4. ***Effect of Changing Air Conditions on Cooling Tower.*** For the cooling tower in Example 10.5-1, to what temperature would the water be cooled if the entering air enters at 29.4°C but the wet bulb temperature is 26.7°C? The same gas and liquid flow rates are used. The water enters at 43.3°C, as before. (*Hint:* In this case T_{L1} is the unknown. The tower height is the same as in Example 10.5-1. The slope of the operating line is as before. The solution is trial and error. Assume a value of T_{L1} that is greater than 29.4°C. Do the graphical integration to see if the same height z is obtained.)

10.6-1. ***Amount of Absorption in a Tray Tower.*** An existing tower contains the equivalent of 3.0 theoretical trays and is being used to absorb SO_2 from air by pure water at 293 K and 1.013×10^5 Pa. The entering gas contains 20 mol % SO_2 and the inlet air flow rate is 150 kg inert $\text{air/h} \cdot \text{m}^2$. The entering water rate is 6000 $\text{kg/h} \cdot \text{m}^2$. Calculate the outlet composition of the gas. (*Hint:* This is a trial-and-error solution. Assume an outlet gas composition of, say, $y_1 = 0.01$. Plot the operating line and determine the number of theoretical trays needed. If this number is not 3.0 trays, assume another value of y_1, and so on.)

Ans. $y_1 = 0.011$

10.6-2. ***Analytical Method for Number of Trays in Absorption.*** Use the analytical equations in Section 10.3 for countercurrent stage contact to calculate the number of theoretical trays needed for Example 10.6-1.

10.6-3. ***Absorption of Ammonia in a Tray Tower.*** A tray tower is to be used to remove 99% of the ammonia from an entering air stream containing 6 mol % ammonia at 293 K and 1.013×10^5 Pa. The entering pure water flow rate is 188 kg $H_2O/\text{h} \cdot \text{m}^2$ and the inert air flow is 128 kg $\text{air/h} \cdot \text{m}^2$. Calculate the number of theoretical trays needed. Use equilibrium data from Appendix A.3. For the dilute end of the tower, plot an expanded diagram to step off the number of trays more accurately.

Ans. $y_1 = 0.000639$ (exit), $x_N = 0.0260$ (exit), 3.8 theoretical trays

10.6-4. ***Minimum Liquid Flow in a Packed Tower.*** The gas stream from a chemical reactor contains 25 mol % ammonia and the rest inert gases. The total flow is 181.4 kg mol/h to an absorption tower at 303 K and 1.013×10^5 Pa pressure, where water containing 0.005 mol frac ammonia is the scrubbing liquid. The outlet gas concentration is to be 2.0 mol % ammonia. What is the minimum flow $L'_{\min}$? Using 1.5 times the minimum plot the equilibrium and operating lines.

Ans. $L'_{\min} = 262.6$ kg mol/h

10.6-5. ***Steam Stripping and Number of Trays.*** A relatively nonvolatile hydrocarbon oil contains 4.0 mol % propane and is being stripped by direct superheated steam in a stripping tray tower to reduce the propane content to 0.2%. The temperature is held constant at 422 K by internal heating in the tower at 2.026×10^5

Pa pressure. A total of 11.42 kg mol of direct steam is used for 300 kg mol of total entering liquid. The vapor–liquid equilibria can be represented by $y = 25x$, where y is mole fraction propane in the steam and x is mole fraction propane in the oil. Steam can be considered as an inert gas and will not condense. Plot the operating and equilibrium lines and determine the number of theoretical trays needed.

Ans. 5.6 theoretical trays (stepping down from the tower top)

10.6-6. ***Absorption of Ammonia in Packed Tower.*** A gas stream contains 4.0 mol % NH_3 and its ammonia content is reduced to 0.5 mol % in a packed absorption tower at 293 K and 1.013×10^5 Pa. The inlet pure water flow is 68.0 kg mol/h and the total inlet gas flow is 57.8 kg mol/h. The tower diameter is 0.747 m. The film mass-transfer coefficients are $k'_y a = 0.0739$ kg mol/s · m³ · mol frac and $k'_x a = 0.169$ kg mol/s · m³ · mol frac. Using the design methods for dilute gas mixtures, do as follows.

(a) Calculate the tower height using $k'_y a$.

(b) Calculate the tower height using $K'_y a$.

Ans. (a) $z = 2.362$ m (7.75 ft)

10.6-7. ***Tower Height Using Overall Mass-Transfer Coefficient.*** Repeat Example 10.6-2, using the overall liquid mass-transfer coefficient $K'_x a$ to calculate the tower height.

10.6-8. ***Experimental Overall Mass-Transfer Coefficient.*** In a tower 0.254 m in diameter absorbing acetone from air at 293 K and 101.32 kPa using pure water, the following experimental data were obtained. Height of 25.4-mm Raschig rings = 4.88 m, $V' = 3.30$ kg mol air/h, $y_1 = 0.01053$ mol frac acetone, $y_2 = 0.00072$, $L' = 9.03$ kg mol water/h, $x_1 = 0.00363$ mol frac acetone. Calculate the experimental value of $K_y a$.

10.6-9. ***Conversion to Transfer Unit Coefficients from Mass-Transfer Coefficients.*** Experimental data on absorption of dilute acetone in air by water at 80°F and 1 atm abs pressure in a packed tower with 25.4-mm Raschig rings were obtained. The inert gas flow was 95 lb_m air/h · ft² and the pure water flow was 987 lb_m/h · ft². The experimental coefficients are $k_G a = 4.03$ lb mol/h · ft³ · atm and $k_L a = 16.6$ lb mol/h · ft³ · lb mol/ft³. The equilibrium data can be expressed by $c_A = 1.37 p_A$, where c_A = lb mol/ft³ and p_A = atm partial pressure of acetone.

(a) Calculate the film height of transfer units H_G and H_L.

(b) Calculate H_{OG}.

Ans. (b) $H_{OG} = 0.957$ ft (0.292 m)

10.6-10. ***Height of Tower Using Transfer Units.*** Repeat Example 10.6-2 but use transfer units and calculate H_L, N_L, and tower height.

10.6-11. ***Experimental Value of H_{OG}.*** Using the experimental data given in Problem 10.6-8, calculate the number of transfer units N_{OG} and the experimental value of H_{OG}.

Ans. $H_{OG} = 1.265$ m

10.7-1. ***Liquid Film Coefficients and Design of SO_2 Tower.*** Using the data of Example 10.7-1, calculate the height of the tower using Eq. (10.6-15), which is based on the liquid film mass-transfer coefficient $k'_x a$. [*Note:* The interface values x_i have already been obtained. Use a graphical integration of Eq. (10.6-15).]

Ans. $z = 1.586$ m

10.7-2. ***Design of SO_2 Tower Using Overall Coefficients.*** Using the data of Example 10.7-1, calculate the tower height using the overall mass-transfer coefficient $K'_y a$. [*Hint:* Calculate $K'_y a$ at the top of the tower and at the bottom of the tower from the film coefficients. Then use a linear average of the two values for the design. Obtain the values of y^* from the operating and equilibrium line plot. Graphically integrate Eq. (10.6-16), keeping $K'_y a$ outside the integral.]

10.7-3. ***Height of Packed Tower Using Transfer Units.*** For Example 10.7-1 calculate the tower height using the H_G and the number of transfer units N_G. [*Hint:* Calculate H_G at the tower top using Eq. (10.6-36) and at the tower bottom. Use the linear average value for H_G. Calculate the number of transfer units N_G by graphical integration of the integral of Eq. (10.6-32). Then calculate the tower height.]

Ans. $H_G = 0.2036$ m (average value)

10.7-4. ***Design of Absorption Tower Using Transfer Units.*** The gas SO_2 is being scrubbed from a gas mixture by pure water at 303 K and 1.013×10^5 Pa. The inlet gas contains 6.00 mol % SO_2 and the outlet 0.3 mol % SO_2. The tower cross-sectional area of packing is 0.426 m^2. The inlet gas flow is 13.65 kg mol inert air/h and the inlet water flow is 984 kg mol inert water/h. The mass-transfer coefficients are $H_L = 0.436$ m and $k_G a = 6.06 \times 10^{-7}$ kg mol/s · m^3 · Pa and are to be assumed constant in the tower for the given concentration range. Use equilibrium data from Appendix A.3. By graphical integration, determine N_G. Calculate the tower height. (*Note:* The equilibrium line is markedly curved, so graphical integration is necessary even for this dilute mixture.)

Ans. $N_G = 8.47$ transfer units, $z = 1.311$ m

10.8-1. ***Prediction of Mass-Transfer Coefficients.*** Predict the mass-transfer coefficients H_G, H_L, $k'_x a$, $k'_y a$, and $K'_x a$ for absorption of CO_2 from air by water in the same packing and using 1.6 times the flow rates in Example 10.8-1 at 303 K. (*Hint:* Use equilibrium data from Appendix A.3 for Henry's law constants for CO_2 in water. Use diffusivity data for CO_2 in water from Table 6.3-1 and for CO_2 in air from Table 6.2-1. Correct data to 303 K.)

Ans. $H_G = 0.2186$ m, $H_L = 0.2890$ m

10.8-2. ***Correction of Film Mass-Transfer Coefficients.*** For absorption of dilute NH_3 from air by water at 20°C and 101.3 kPa abs pressure, experimental values are $H_G = 0.1372$ m and $H_L = 0.2103$ m for heights of transfer units. The flow rates are $G_x = 13\,770$ kg/h · m^2 and $G_y = 1343$ kg/h · m^2. For absorption of acetone from air by water under the same conditions in the same packing, predict $k'_y a$, $k'_x a$, and $K'_y a$. Use equilibrium data from Appendix A.3. Use the diffusivity for acetone in water from Table 6.3-1. The diffusivity of acetone in air at 0°C is 0.109×10^{-4} m^2/s at 101.3 kPa abs pressure.

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