Faculty of Engineering and Technology
Department of Chemical Engineering
University Of Jordan

Combined Heat and Mass Transfer

Lec 8: Drying-Part 1

Content

Overview and definitions, Drying applications, Equilibrium Moisture Content (EMC) of materials, Drying Fundamentals, Batch Drying, Experimental drying curve, Behavior of solids during drying, Drying in the constant-rate period, Calculation Methods for Falling-Rate Drying Period, Continuous Drying

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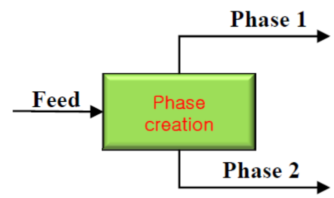
Content

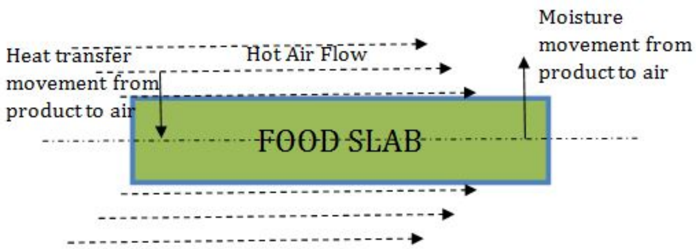
- Overview and definitions
- Drying applications
- Equilibrium Moisture Content (EMC) of materials
- Drying Fundamentals
- Batch Drying
  - Experimental drying curve
  - Behavior of solids during drying
  - Drying in the constant-rate period
  - Calculation Methods for Falling-Rate Drying Period
- Continuous Drying

Principal references:
Chapter 9 (Drying) in C.J. Geankoplis book;
Chapter 18 Seader et al. book.

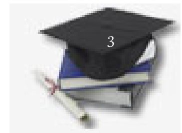
Overview and definitions

- **Drying (Dehydration)**: removal of moisture (either water or other volatile compounds) from wet solids, slurries, and pastes to give solid products.
- **Unlike evaporation**, **relatively small amounts** of water are evaporated.
- Drying is based on **phase-creation with ESA = thermal energy**.
- It is a **gas-liquid-solid** operation.





Heat and moisture flow during hot air drying of food slab



Drying applications

- It is the final separating step in many processes, especially after a filtration step
  - Pharmaceuticals
  - Biological materials, including foods and milk
  - Crops, grains and cereal products
  - Lumber, pulp and paper products
  - Catalysts
  - Detergents

Drying applications

- **Examples on the moisture content of the final dried product:**
  - Dried salt contains about 0.5 wt% water.
  - Coal contains about 4 wt% water.
  - Many food products contain about 5 wt% water.
- **Drying can be expensive**, especially **when large amounts of water**, must be evaporated.
- Therefore, it is important, before drying, to remove as much moisture as possible by **mechanical means** such as **expression**; **filtration**; **settling**; and **centrifugation**.

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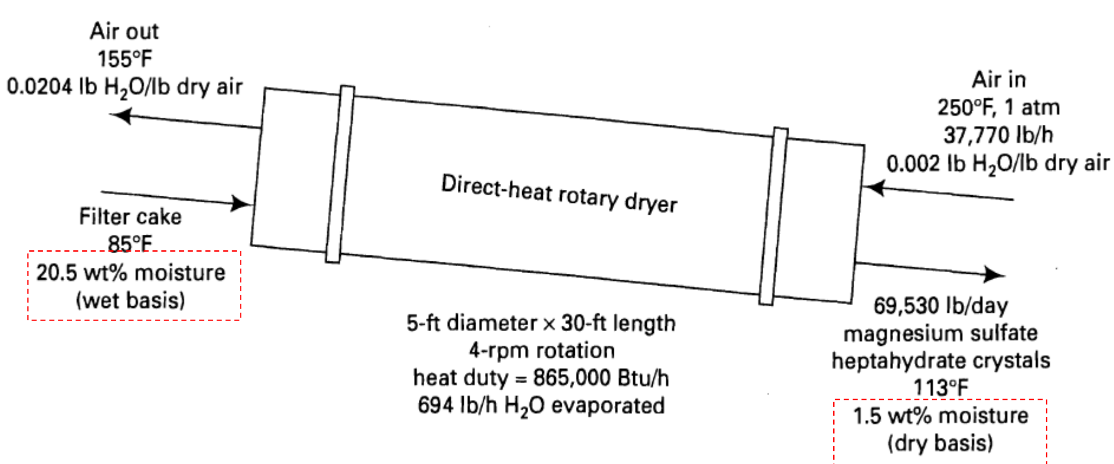
Drying applications

Industrial example: Drying of magnesium sulfate heptahydrate filter cake



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Drying applications

➤ Why drying?

- Make the materials easily handled (packaging).
- Reduce carrying and shipping cost.
- Control the quality of a product:
  - ✓ Stabilizes flavor and prolongs shelf-life in foods.
  - ✓ Preserves product from bacterial growth and avoid the damage of the materials.
- Reduces corrosion.

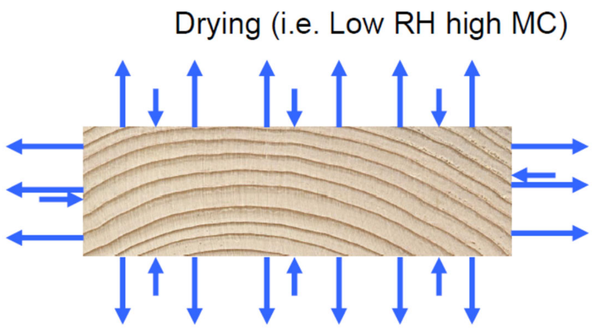
Difference between drying and evaporation

- ➤ In **drying** processes, the main operation usually carried out on **solid materials,** e.g. powders, or products.

Drying	Evaporation
○ The removal of relatively **small amounts** of water from **solids**	○ Include the **removal of large amounts of water** from solutions.
○ In most cases, involves the **removal** of water at **temperatures below** its boiling **point**	➤ The **removal** of water **by boiling** a solution.
○ **Water** is usually removed by **circulating air** over **the material** in order to **carry away** the **water vapour**	➤ Water is removed from the material as **pure water** vapor mixed with **other gases**.

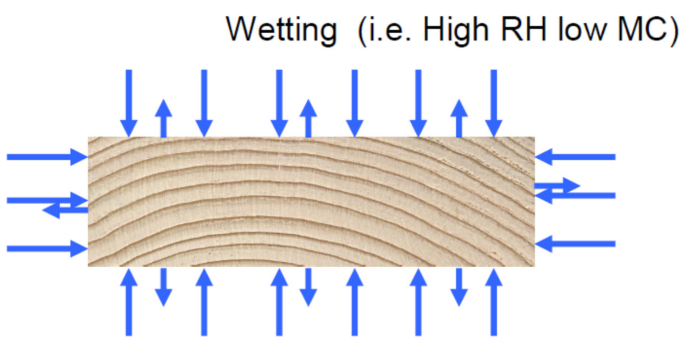
Equilibrium Moisture Content (EMC) of materials

- Equilibrium data for moist solids are commonly given as relationships between the relative humidity of the gas (or ***absolute humidity***) and the liquid content of the solid, in mass of liquid per unit mass of dry solid.
- When a wet solid is brought into contact with air of lower humidity than that corresponding to the moisture content of the solid, as shown by the humidity-equilibrium curve, the solid tends to lose moisture and dry to equilibrium with the air.



Equilibrium Moisture Content (EMC) of materials

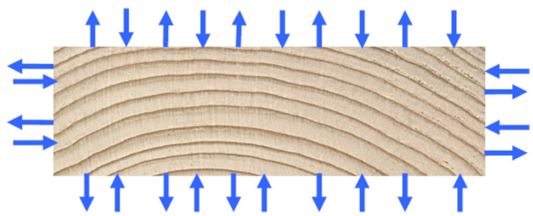
- When the air is more humid than the solid in equilibrium with it, the solid absorbs moisture from the air until equilibrium is attained.



Equilibrium Moisture Content (EMC) of materials

- When a wet solid containing moisture is brought into contact with a stream of air having a constant humidity **H** and temperature (a large excess of air is used, so its conditions remain constant), and for sufficiently long exposure, the solid will attain a definite moisture content.

In Equilibrium (i.e. @ EMC)

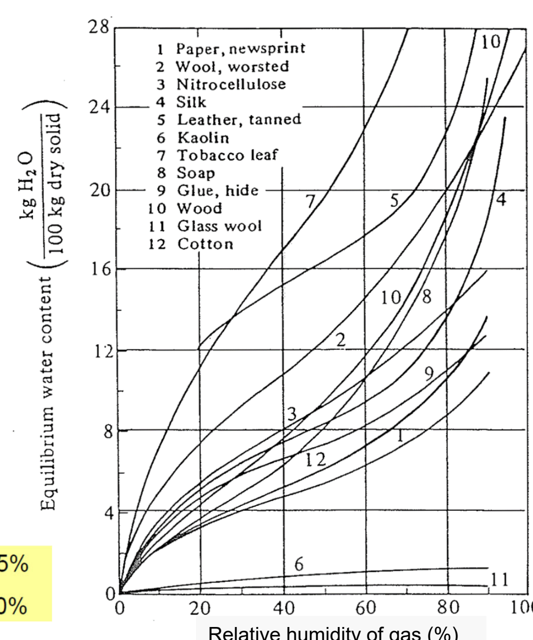


- This is known as the equilibrium moisture content of the material under the specified humidity and temperature of the air.

✓ The equilibrium moisture content of a solid material is that portion of the water in the wet solid that cannot be removed by the inlet air, because of the humidity of the latter.

Equilibrium Moisture Content (EMC) of materials

- It is the lowest moisture content obtainable at equilibrium under the drying conditions used.
- Expressed on a dry basis **(kg of water per kg of moisture-free solid) i.e. kg W/kg dry solid.**
- It depends on the 1) structure of the solid, 2) the temperature of the gas and 3) the moisture content of the gas.
- Varies greatly with the type of material for any given % relative humidity.
- Decreases with an increase in temperature
- It may change with temperature. For example, for wood @ 50° F. & 75% RH - EMC = 14.5%
  @ 150° F. & 75% RH – EMC = 12.0%
- However, it can be assumed constant for moderate temperature ranges.



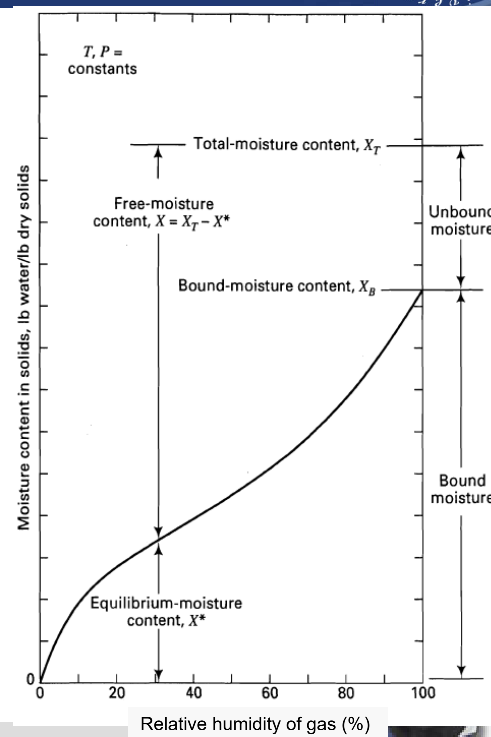
Typical equilibrium moisture content of some solids at 25 °C:

Equilibrium Moisture Content (EMC) of materials

➤ The nature of water in solid material:

1. Bound moisture (X_B):

- If the EMC curve for a given material is continued to intersect with 100% relative humidity line, this equilibrium moisture is called bound water with moisture content X_B.
- It represents the part of the moisture present in a wet solid which may be adsorbed on surfaces of the solid or be adsorbed within its structure to such an extent to prevent it from developing its full vapor pressure and from being easily removed by evaporation, i.e. the vapor pressure of moisture over a moist solid becomes less than the vapor pressure of pure water at that temperature
- It is more difficult to remove than unbound water.



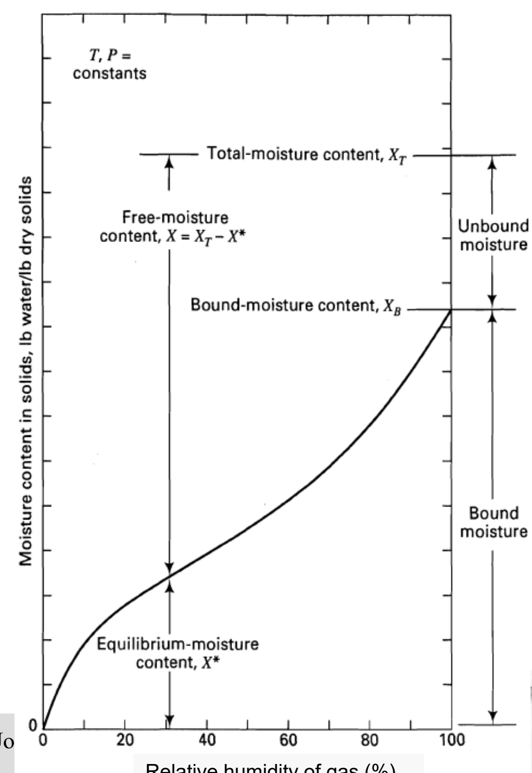
Equilibrium Moisture Content (EMC) of materials

2. Unbound moisture :

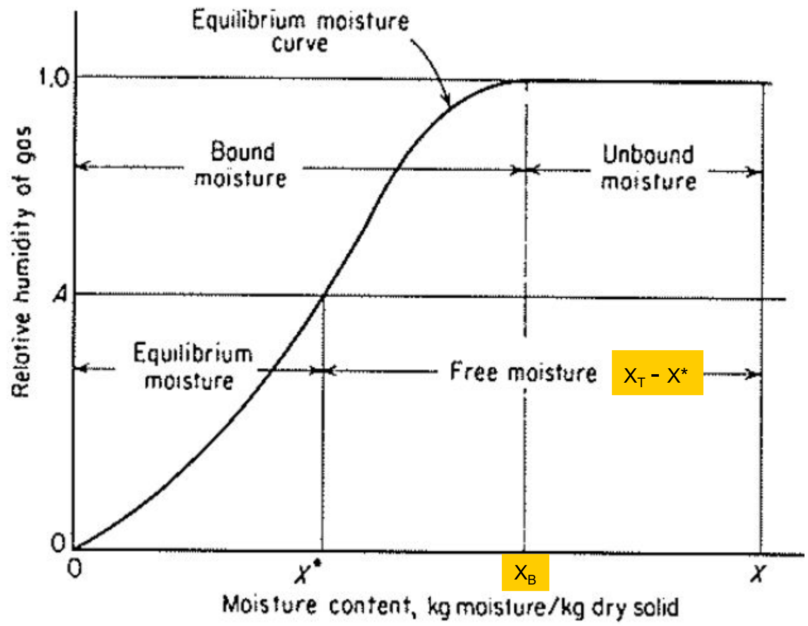
- When the solid material contains more water than indicated by the intersection with the line H_R=100%.
- Exerts a vapor pressure equivalent to that of the free liquid.

3. Free moisture (X):

- The moisture above the equilibrium moisture content. This is the moisture that can be removed by drying under given percent relative humidity.



Equilibrium Moisture Content (EMC) of materials



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Characteristics of Air

- **Three** characteristics of air that are necessary for successful drying when the material is moist are:
  1. A moderately high dry-bulb temperature
  2. A low RH
  3. A high air velocity.

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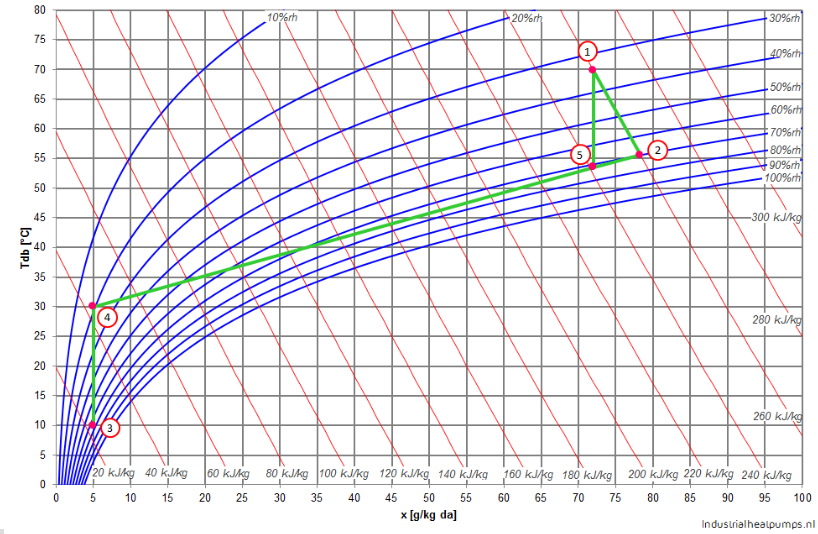
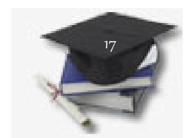
Relative humidity (RH) of air

- Air at a given temperature is capable of taking up water vapour until it is saturated (at 100% RH).
- If the **temperature is raised** then the air will be able to take up **more** moisture and the **relative humidity falls**.





Relative humidity (RH) of air

- The RH of air is dependent not only on the amount of moisture in the air, but also on its temperature, as the amount of water required to saturate air is itself dependent on temperature.
- It should be noted that in convective drying, where warm air is passed over the surface of a wet solid, the relative humidity may rise during the drying process as a result of **two separate factors:-**
  1. Uptake of evaporated water vapor from the wet solid.
  2. The cooling of the supply air as it transfers heat to the wet solid (evaporative cooling).
- If the cooling is **excessive** the temperature of the air may fall to a value known as the **dew point,** when liquid water will condense and be deposited.

Loss of water from wet solids

- Unbound water is easily lost by evaporation until the equilibrium moisture content of the solid is reached.
- Once the solid reaches its EMC, extending the time of drying will not change the moisture content as an equilibrium situation has been reached.
- The only way **to reduce** the moisture content is to **reduce the RH** of the ambient air.

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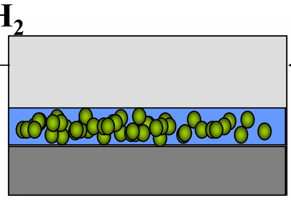
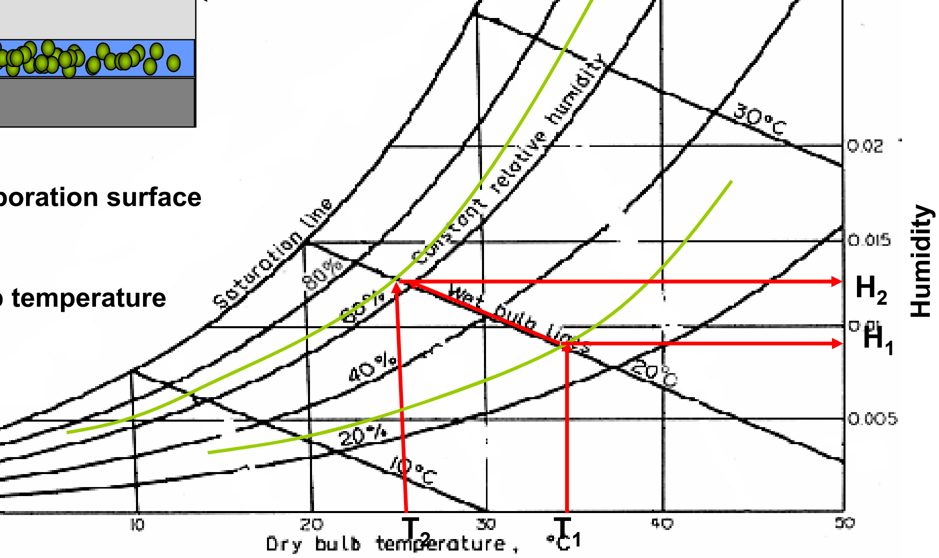
Importance of pyschrometric analysis for drying



Assumption: evaporation surface is a liquid film

Constant wet-bulb temperature process



Wet bulb temp. =20°C, dry bulb temp. = 30°C,humidity = ?

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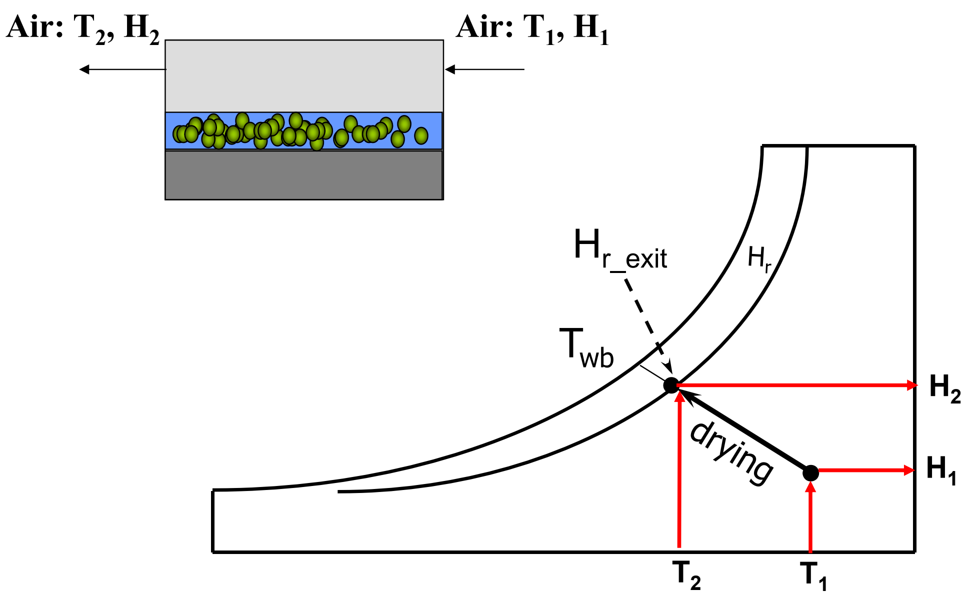
Importance of pyschrometric analysis for drying



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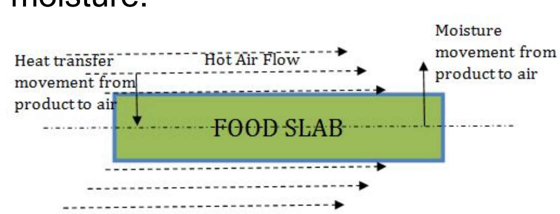
Drying Fundamentals

- The rate of drying is controlled by air temperature, humidity, & air velocity.
- Drying involves both heat and mass transfer.
- When hot air is blown over a wet food:
  - Heat must be transferred to the surface of the material in order to provide the latent heat of evaporation for the moisture.
  - Mass transfer :
    - ✓ A water vapour pressure gradient is established from the moist interior of the food to +the dry air.



    - ✓ This gradient provides the 'driving force' for water removal from the material
    - ✓ Diffusion of water through through a boundary film of air surrounding the food and is carried away by the moving air.

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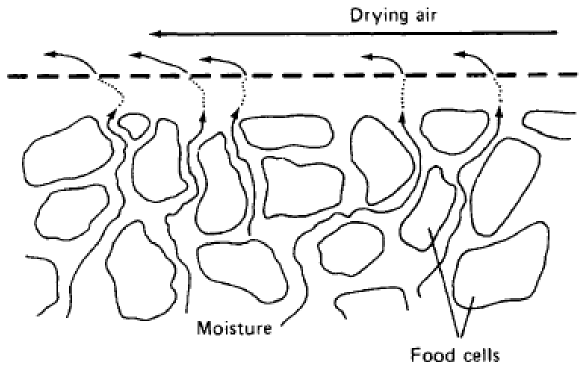
Drying Fundamentals



Movement of moisture during drying.

- The boundary film acts as a barrier to both **heat transfer** and **water vapour removal** (mass transfer) during drying.
- The **thickness** of the film is determined primarily by the **air velocity**; if the velocity is **low**, the boundary film is **thicker.**
- This reduces the **heat transfer coefficient** & the **rate of removal** of water vapour.

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Drying Fundamentals

- Water vapour leaves the surface of the food and increases the humidity of the surrounding air, to cause a reduction in the water vapour pressure gradient and hence the rate of drying.
- The **faster** the air, the **thinner** the boundary film & hence the **faster** the **rate of drying**.

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Drying Fundamentals

- In general terms, drying can be described by four physical mechanisms of transfer operating simultaneously.
  i. External heat transfer: from outside to the product surface, energy is generally brought by conduction or convection.
  ii. Internal heat transfer: within the product to conduct the necessary energy to transform water into vapor, energy is transmitted by conduction
  iii. The phase transformation of the water or organic solvent from a liquid or liquid-like state to a vapour state.
  iv. The transfer of the vapour generated away from the material and out of the drying equipment (pressure/vacuum)

- Conducting a drying study can help you determine where the major resistances to drying occur with the product, and the best types of drying systems to deliver the characteristics desired in pharmaceutical products.

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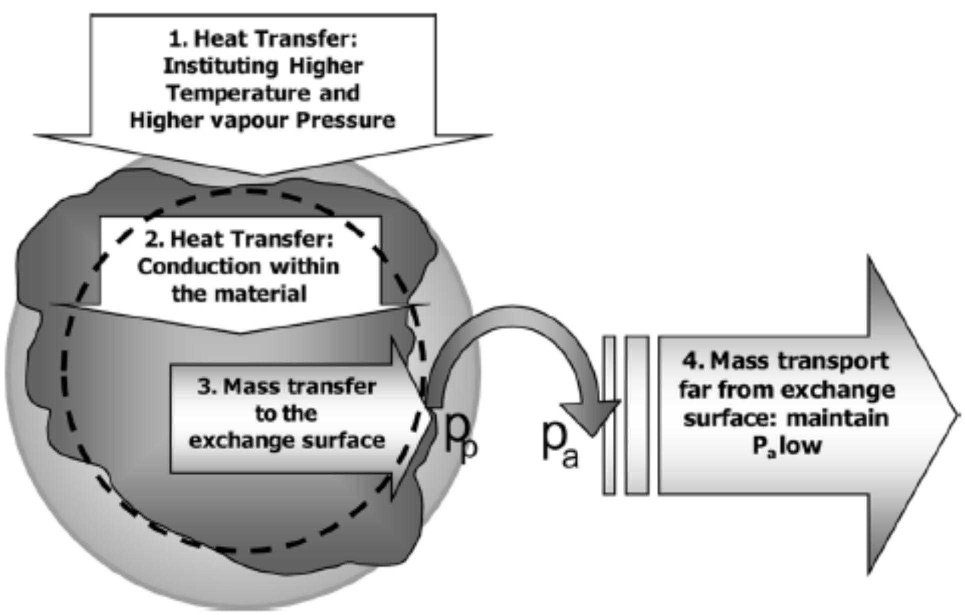
Drying Fundamentals



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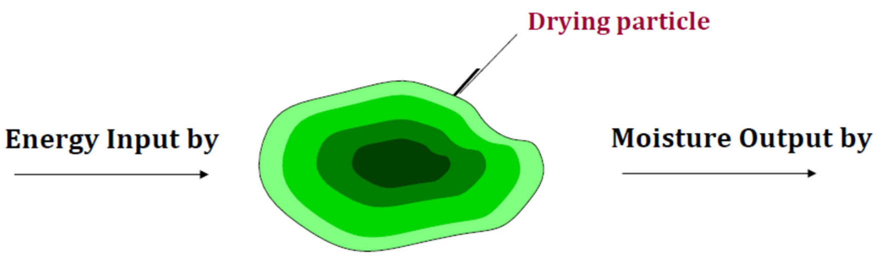
Drying Fundamentals



- Conduction
- Convection
- Radiation
- Microwave and Radio Frequency Fields
- Combined mode

- Liquid movement due to capillary forces
- Diffusion due to concentration gradients
- Liquid vapor flow due to pressure differences
- Vapor diffusion due to vapor pressure differences, concentration differences
- Knudsen diffusion – (depending on pore diameter)
- Vaporization-condensation mechanism

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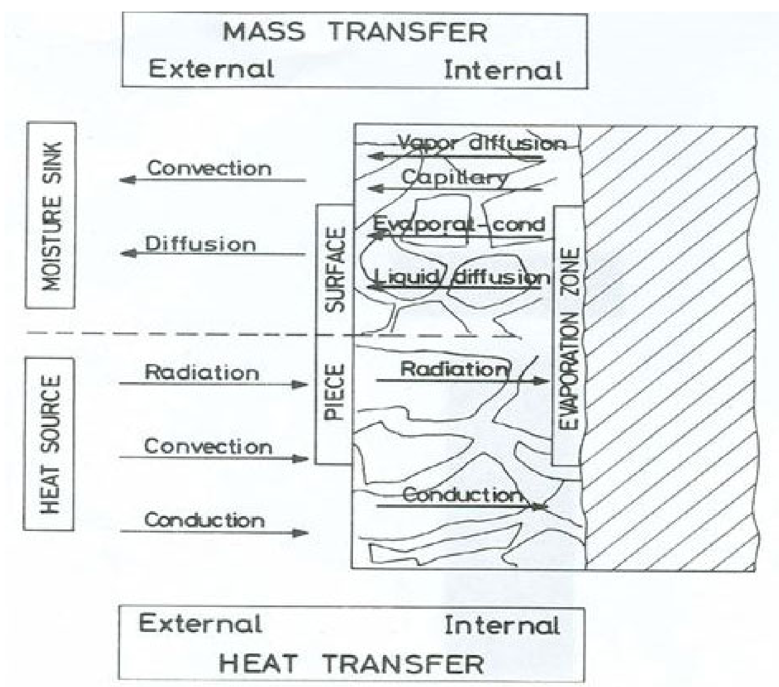
Drying Fundamentals



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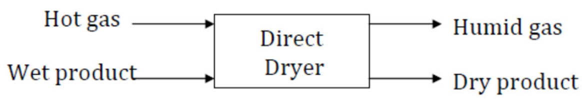
Drying Fundamentals

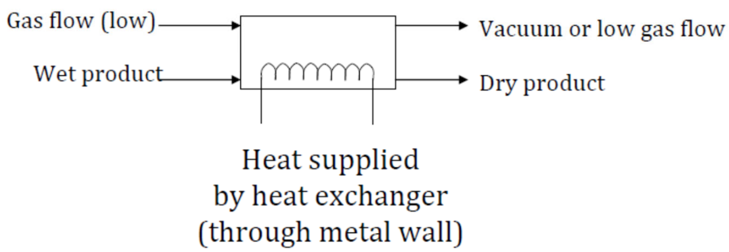
Heat Transfer

I. Direct (Convective)



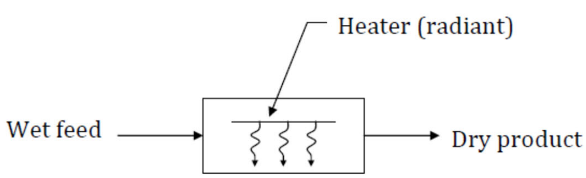
Drying medium directly contacts material to be dried and carries evaporated moisture.

II. Indirect (Contact, Conduction)



Drying Fundamentals

III. Radiant



Vacuum or low gas flow to carry evaporated moisture away.

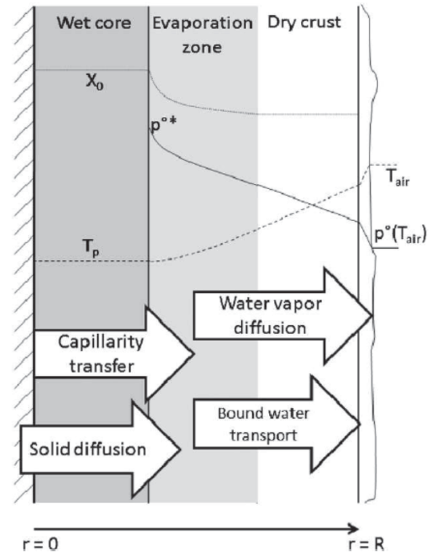
IV. Microwave or RF

Electromagnetic energy absorbed selectively by water (volumetric heating)

Drying Fundamentals

Mechanism of moisture movement within the solid

1. Liquid diffusion:
   - Diffusion of liquid moisture may result because of concentration gradients between the depths of the solid, where the concentration is high, and the surface where it is low.
2. Capillary movement:
   - Unbound moisture in granular and porous solids such as clays, sands, paint pigments, and the like, move through the capillaries and interstices of the solids by a mechanism involving surface tension.



Internal transfer in a porous particle

Drying Fundamentals

- The capillaries extend from small reservoirs of moisture in the solid to the drying surface.
- As drying proceeds, at first the moisture moves by the capillary to the surface sufficiently rapidly to maintain uniformly wetted surface and the rate drying is constant.
- The water is replaced by air entering the solid through relatively few opening and cracks.
- The surface moisture is eventually drawn to spaces between the granules of the surface, the wetted area at the surface decreases, and the unsaturated surface drying period follows.

Drying Fundamentals

- The subsurface reservoirs eventually dry up, the liquid surface recedes into the capillaries, evaporation occurs below the surface in a zone or plane which gradually recedes deeper into the solid, and a second falling rate period results.
- During this period, diffusion of vapor within the solid will occur from the place vaporization to the surface.

3. Especially if heat is supplied to one surface of a solid while drying proceeds from another, the moisture may evaporate beneath the surface and diffuse outward as a vapor. Moisture particles in granular solids, which have been isolated from the main portion of the moisture flowing through capillaries, may also be evaporated below the surface.
4. Pressure: Owing to shrinkage of outside layers of a solid on drying, moisture may be squeezed to the surface.

Different techniques of drying of solids (Drying methods)

- The following points should be considered before the **selection of the suitable** drying method:

  1- Heat sensitivity the material being dried.

  2- Physical characteristics of the material.

  3- Nature of the liquid to be removed.

  4- The scale of the operation.

  5- Available **sources** of heat (steam, electrical).

Different techniques of drying of solids (Drying methods)

- **The general principles for efficient drying can be summarized as follows**

1- Large surface area for heat transfer.

2- Efficient heat transfer per unit area (to supply sufficient latent heat of vaporization or heat of sublimation in case of freeze-drying)

3- Efficient mass transfer of evaporated water through any surrounding boundary layers, i.e. sufficient turbulence to minimize boundary layer thickness.

4- Efficient vapor removal, i.e. low relative humidity air at adequate velocity.

In summary

Mass transfer

- Bring liquid from interior of product to surface
- Vapourization of liquid at/near the surface
- Transport of vapour into the bulk gas phase

Heat transfer from bulk gas phase to solid phase:

- portion of it used to vapourize the liquid (latent heat)
- portion remains in the solid as (sensible heat)

  - Key point: heat to vapourize the liquid is adiabatically provided by the air stream. Air is cooled as a result of this evaporation
  - The ΔH_{vap} is a function of the temperature at which it occurs:
    - 2501 kJ/kg at 0°C
    - 2260 kJ/kg at 100°C
    - Linearly interpolate over this range (small error though)

Classification of solids on drying behavior

1- **Granular or crystalline solids** (water is held in shallow and open surface pores as well as in interstitial spaces between particles that are easily accessible to the surface).

Ex: calcium sulfate, zinc oxide, magnesium oxide

2- **Amorphous, fibrous or gelatinous solids** (moisture is an integral part of the molecular structure as well as being physically entrapped in fine capillaries and small interior pores).

Ex: starch, insulin and aluminum hydroxide.

Note: Amorphous solids are difficult to dry than granular or crystalline solids.

(Drying) Moisture removal in these 2 categories

Moisture in crystalline solids is lost with little hindrance by either gravitational or capillary force.

(Inorganic substances – not effected by heat unless temp. is high enough to hydrate, represent by high constant rate period while the equilibrium moisture contents are close to zero).

Moisture movement in amorphous is slow, such that liquid diffuses through structural obstacles caused by molecular configuration.

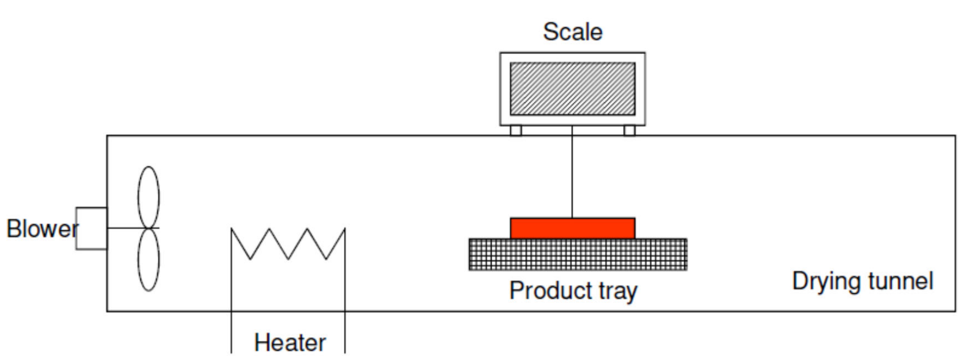
(The second drying rate period is high since it depends on diffusion rate of water through solids. Equilibrium moisture is high because most of water remains associated within the molecular interstitial spaces of substance.

The drying of these substances requires: low temp., reduced pressure and increase airflow.

Batch Drying

Experimental drying curve

- The material to be dried is placed on the tray.
- The tray is suspended from a balance and exposed to air flow in drying tunnel.
- Water removed/time can be easily determined



Experimental drying curve

- During drying of wet solids of mass (L) which contains dry solid of mass (L_s), the free moisture content (X) will decrease with time.
- To obtain X versus t curve, do the following:

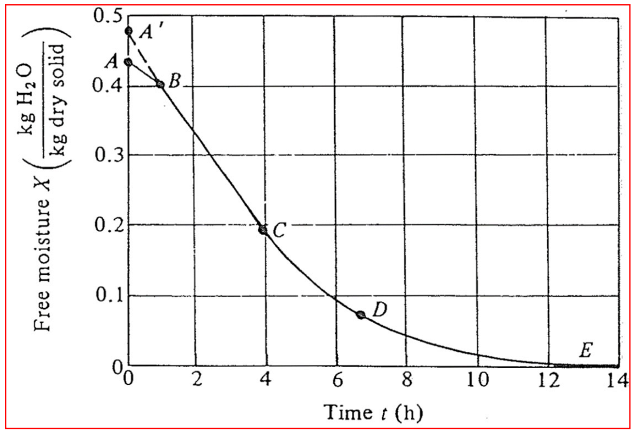
i. At each time, calculate the total moisture as $X_T = \frac{L - L_s}{L_s} [=] \frac{\text{kg total water}}{\text{kg dry solid}}$

ii. Calculate the free moisture content as

$$X = X_T - X^* = \frac{\text{kg free water}}{\text{kg dry solid}}$$

Get it from the the EMC curve

Draw X versus t:



"Experimental drying curve for certain material"

Drying rate

- **Definition of drying rate (R):**

$$R = -\frac{L_s}{A}\frac{dX}{dt} [=] \frac{\text{mass of water evaporated}}{(\text{surface area exposed for drying})\ \text{time}\ )}$$

where A: is the drying surface area

L: is the mass of wet solids which contains **dry solid of mass (L_s)**

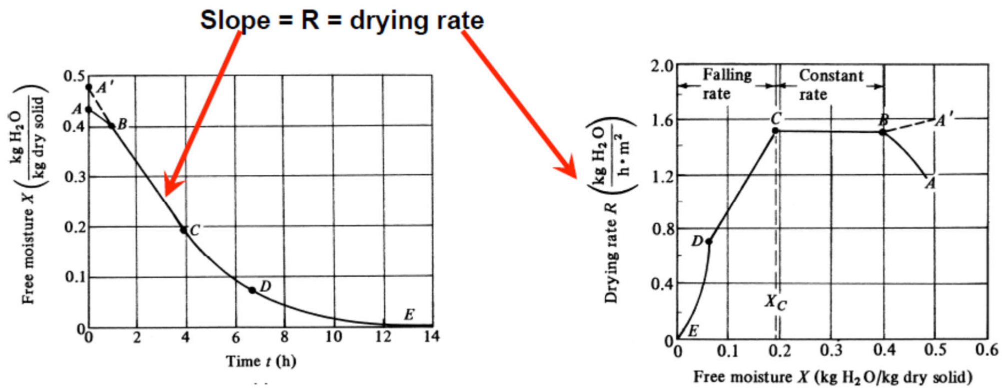
X **:** is the free moisture content

- **Experimental drying rate curve:**

  - Use the experimental drying curve, X versus t, to get slope values, *dX/dt,* at various points (**differential method**).
  - At each point, calculate R.
  - Draw R versus X to obtain the typical curve shown in in the next slide.

Drying rate

To obtain as rate of drying,R :

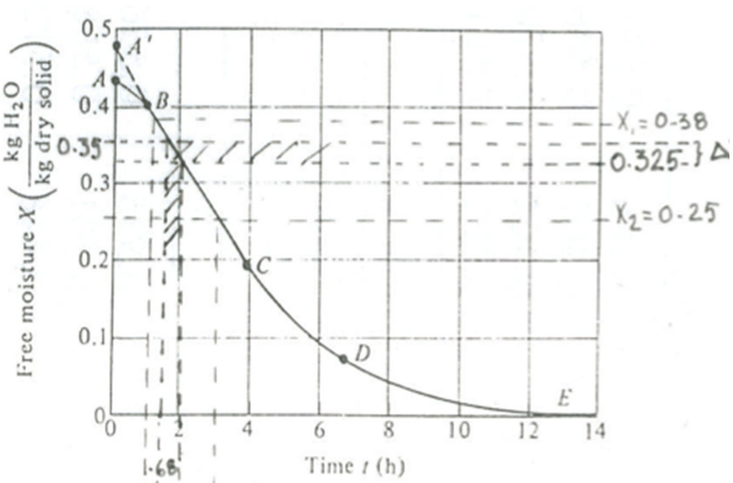


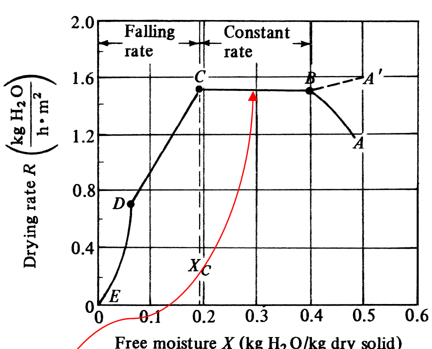
Get slopes of tangents at different values of t :

$$R = -\frac{L_s}{A}\frac{dX}{dt} [=] \frac{\text{mass of water evaporated}}{\text{(surface area exposed for drying)(time)}}$$

where A is the drying surface area

Drying rate





$$\frac{dX}{dt} = \frac{\Delta X}{\Delta t} = \frac{(0.35 - 0.325)}{(1.68 - 2.04)} = -0.07$$

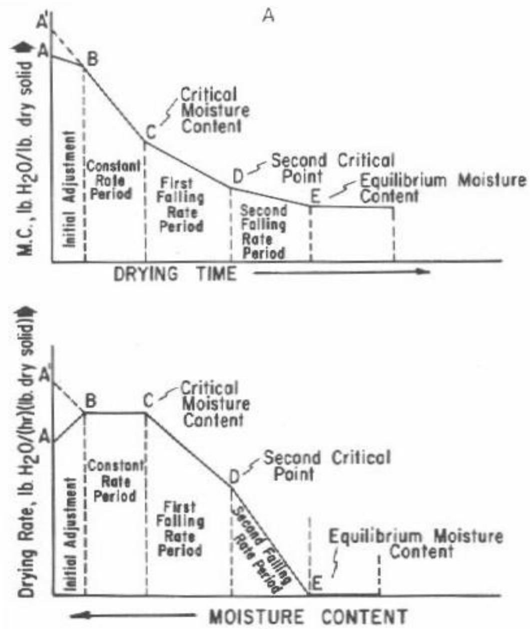
$L_s/A = 21.5$ kg dry solid/m^2 .

R = -21.5 (-0.07) = 1.493

Behavior of solids during drying

1. Stage of increasing drying rate (A or A′→B) :
   - At zero time, the free moisture content is shown at point A.
   - At the beginning, the solid is usually at a colder temperature, it begins to absorb heat and increase in temperature
   - At the same time, the moisture begins evaporating, and cools the drying solid, until the wet solid will reach equilibrium temperature.
   - Alternatively, if the wet solid is quite hot, the drying rate may decrease to start at point A′.

At point B, After a period of initial adjustment, the rates of heating and cooling became equal and the temp. of drying material stabilizes.



"Experimental drying rate curve for certain material"

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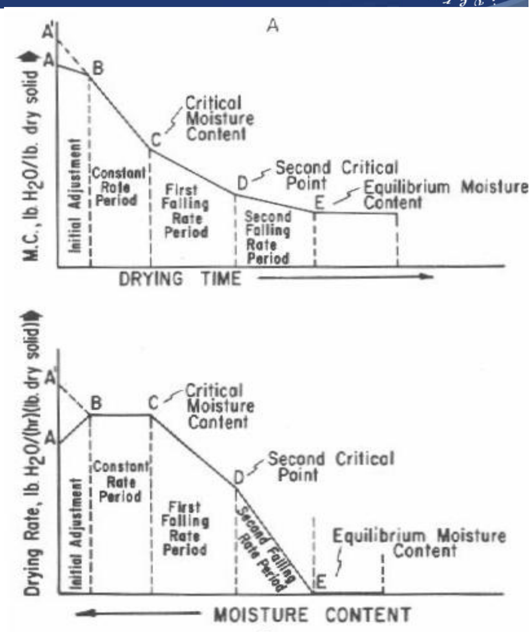
Behavior of solids during drying

- Between point B and C:
- It is found that the evaporation rate from the drying bed is similar to that of the solvent alone from a free liquid surface under the same conditions, Hence, the evaporation takes place from a saturated wet surface of the solid,
- The moisture evaporating from the surface is replaced by water from the interior of the solid at a rate equal to the rate of evaporation
- The rate of drying is constant (**known as the constant-rate period**)
- Controlling factors and hence the rate of evaporation depends on the rate of heat transfer to the drying surface which remains constant at T_W and the rate of removal of the vapour.
- Linear trend between X(t) and t and thereby, constant drying rate is obtained.



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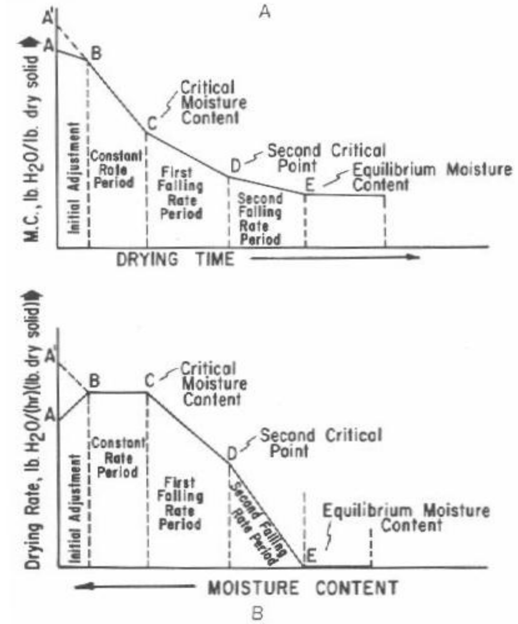
Behavior of solids during drying

- **At Point C:**
  - Critical free moisture content, X_C , where the drying rate starts falling and surface temperature rises.
  - Insufficient water on surface
- **Between point C and D:**
  - As moisture is removed from the surface, Rate of water to surface is less that rate of evaporation from surface
  - Under these conditions, **the rate of drying** will be limited by the rate of **capillary transfer** of the liquid to the **surface** of the **wet bed**,
  - Consequently, the **rate** of drying **decreases** continuously (decreases linearly with the free moisture content).
  - The time CD is known as first falling-rate period (or unsaturated surface drying).



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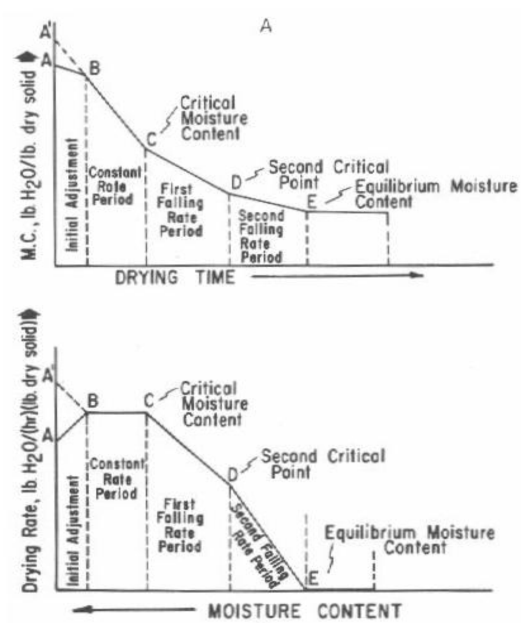
Behavior of solids during drying

At Point D,

- The film of continuous water is completely evaporated, hence,
- Surface completely dry and
- The rate of drying depends on rate of diffusion of moisture to the surface of solid. This point is known as **second critical point.**

- **Between point D and E,**
  - Any moisture that remains within the drying bed at the end of the first falling – rate period is unable to move,
  - Thus, drying cannot take place on the surface. But depends on the movement of the vapor through the pores of the bed to the surface, in general by molecular diffusion



"Experimental drying rate curve for certain material"

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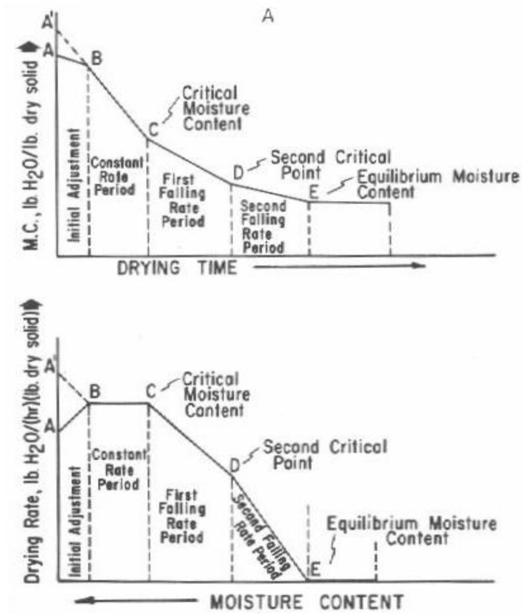
Behavior of solids during drying

- **The time DE is called second falling rate period.**
- The rate of drying falls rapidly than the first falling rate,
- Not linear trend

At point E,

- Free moisture content becomes zero (the total moisture content reaches the equilibrium value X*).
- Temp. And moisture content remain constant.
- No further drying occur, i.E. Continued drying is waste of time and energy.

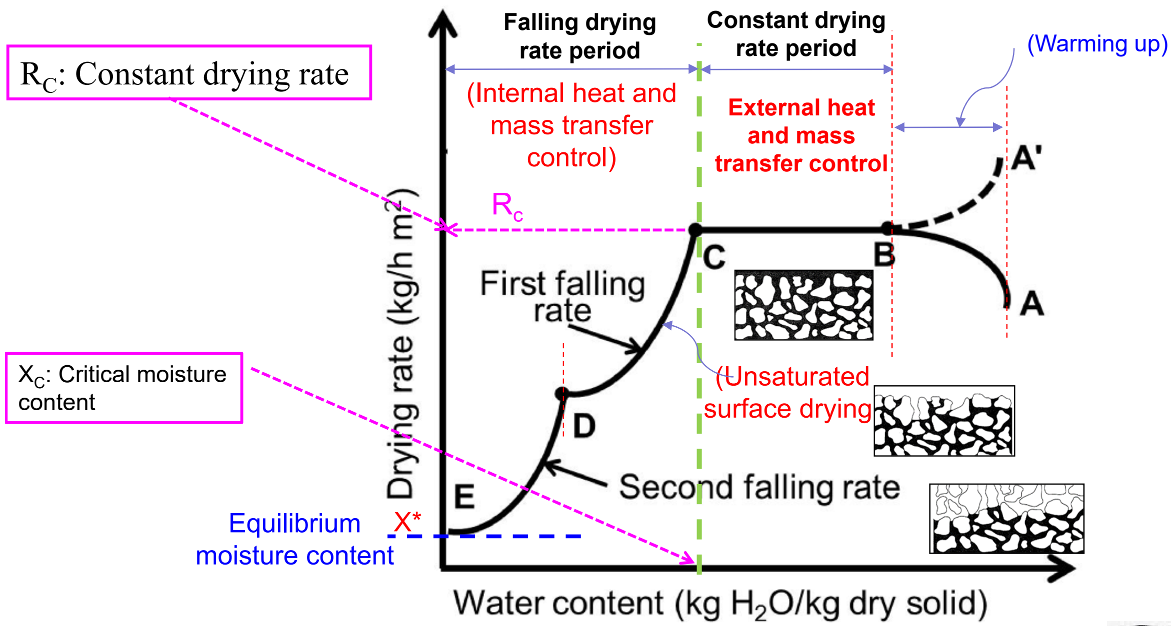


"Experimental drying rate curve for certain material"

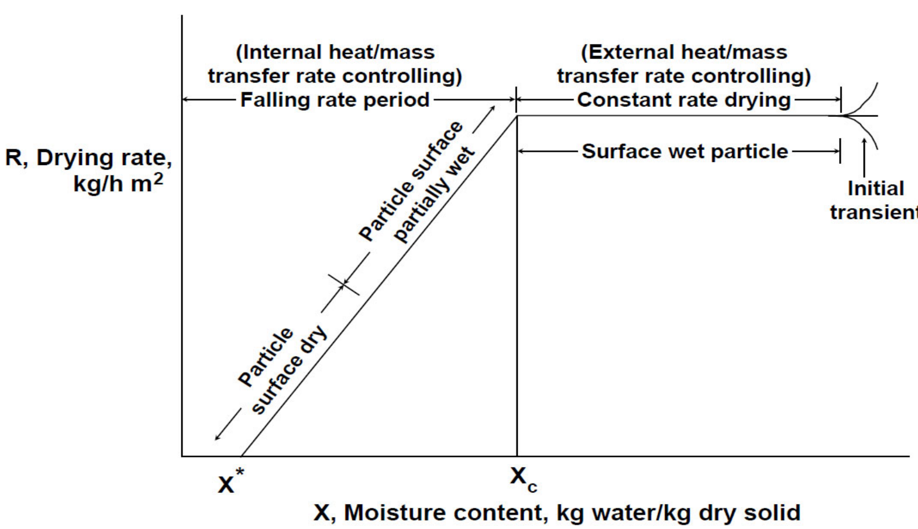
Behavior of solids during drying

- For some materials being dried, (C→D) may be missing completely or it may represent the whole falling rate period (C →E).
- In the absence of experimental information, Second stage of falling drying ate (D→E) , is modeled as linear

Behavior of solids during drying

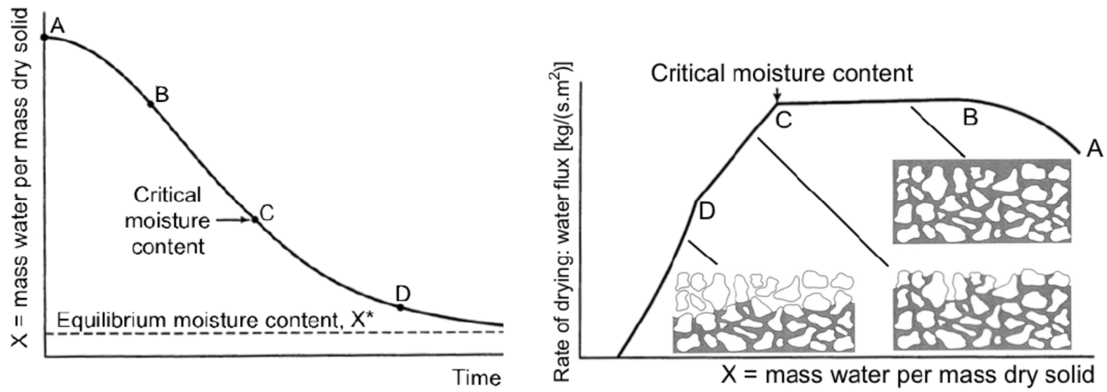


Behavior of solids during drying



Typical textbook batch drying rate curve under constant drying conditions

Summary



- A → B: initial phase as solid heats up
- B → C: constant-rate drying
- C → D: first falling-rate drying
- D → end: second falling-rate drying

Heat/mass transfer viewpoints: (direct-heating drying)

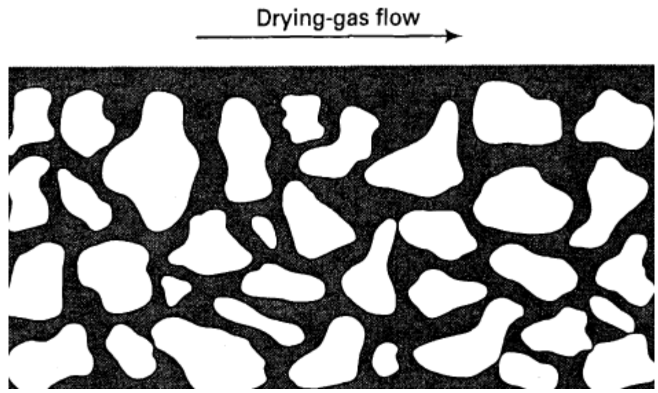
- **Heat transfer:**
  - From bulk gas phase to solid phase.
  - Large portion of it used to vaporize the liquid (latent heat).
  - Small portion remains in the solid as (sensible heat).
- Mass transfer:
  - Bring liquid from interior of solid to surface.
  - Vaporization of liquid at/near the surface.
  - Transport of vapor into the bulk gas phase.

Key points:

- Heat to vaporize the liquid is adiabatically provided by the air stream.
- Air is cooled as a result of this evaporation.
- The latent heat of vaporization, λ_w , is a function of the temperature at which it occurs: λ_w = 2501 kJ/kg at 0 ^{o}C and 2260 kJ/kg at 100 ^{o}C. Interpolate over this range (small error though)

Drying in the constant-rate period

- The surface of the solid is initially very wet and a continuous film of water exists on the drying surface.



- This water is entirely **unbounded water** and the water acts as if the solid were not present.
- The rate of drying (evaporation) is **independent of the solid** and is essentially the same as the rate from free liquid surface.

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Drying in the constant-rate period

- In the constant drying-rate period:
  - rate-limiting step: heat and mass transfer through boundary layer at the solid surface
  - The solid is able to provide water to the surface a fast rate
- Heat transfer during constant drying
  - In constant-rate drying region the wet surface continually supplies moisture.
  - All the heat provided is taken up to evaporate liquid

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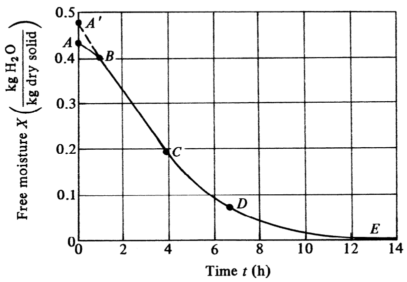
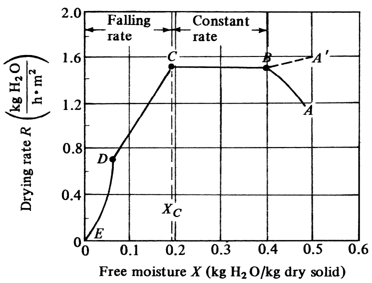
Calculation methods for constant-rate drying period

To determine the time required for drying from X_1 to X_2:

1. Experimental drying curve
2. Predicted mass-and-heat coefficients



(a)



(b)

Experimental drying curve:

Under similar conditions to actual process

1. Drying curve X vs. t
2. Rate-of-drying curve R vs. X

Calculation methods for constant-rate drying period

A. Method using experimental X(t) curve

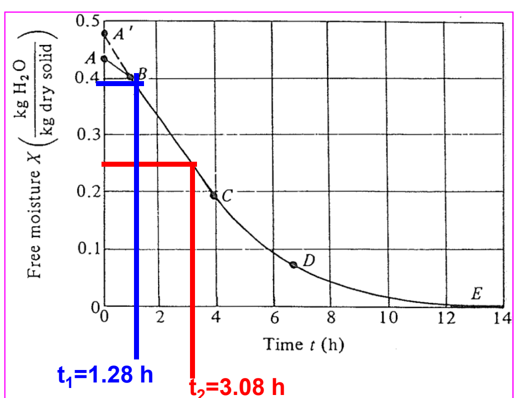
Example. A wet solid whose drying curve is represented in the previous slide is to be dried from free moisture content of 0.38 to 0.25 kg water/kg dry solid. Estimate the drying time period required.

Solution. From X versus t plot:

at X_1=0.38 read t_1=1.28 h

at X_2=0.25 read t_2=3.08 h

$t = t_2 - t_1 = 3.08 - 1.28 = 1.80$ h



Calculation methods for constant-rate drying period.

B. Method using experimental R versus X curve

- Instead of using the drying curve, the rate of drying can be used

The rate of drying R is defined as

$$R = -\frac{L_S}{A}\frac{dX}{dt}$$

- This can be rearranged and integrated over time interval to dry from X_1 at t_1 = 0 to X_2 at t_2 = t,

$$t = \int_{t_1=0}^{t_2=t} dt = \frac{L_S}{A}\int_{X_2}^{X_1}\frac{dX}{R}$$

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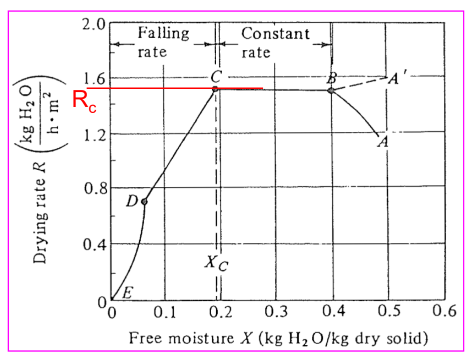
Calculation methods for constant-rate drying period.

And R is a constant = R_c

$$R = \text{constant} = R_c = -\frac{L_s}{A}\frac{dX}{dt}$$

Integrate

$$t = t_2 - t_1 = -\frac{L_s}{A}\frac{\Delta X}{R_c} = \frac{L_s}{A}\frac{X_1 - X_2}{R_c}$$



Or

$$t = \frac{L_S}{AR_c}(X_1 - X_2)$$

where A: is the drying surface area

L_s : is the mass of **dry solid of mass**

X **:** is the free moisture content

R_C: Constant drying rate

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Example

Use drying rate curve to resolve the previous example. Note that $L_s/A=21.5$ kg dry solid/m^2 .

Solution. From the above R(X) curve, at $X_1=0.38$ and $X_2=0.25$, the drying rate is constant: $R_c=1.51$ kg/h.m^2

$$t=\frac{L_s}{A}\frac{X_1-X_2}{R_c}=21.5\frac{0.38-0.25}{1.51}=1.85\,\mathrm{h}$$

This is closed to the results obtained using X(t) curve.

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Calculation methods for constant-rate drying period.

C. Method using convection heat transfer only

- In constant-rate drying region the wet surface continually supplies moisture.
- l All the heat provided is taken up to evaporate liquid

$$(\text{Water flux})\quad(\lambda_w)\quad=\quad\text{Heat flux}$$

- $\text{Water flux}=\dfrac{\text{mass of water removed}}{(\text{time})(\text{area})}=-\dfrac{L_s}{A}\dfrac{dX}{dt}=\dfrac{1}{A}\dfrac{d(m_w)}{dt}$
- $X=$ mass of water remaining per mass dry solid
- $A=$ surface area of solid exposed
- $L_s=$ mass of dry solid
- $m_w=$ mass of water evaporated out of solid

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Calculation methods for constant-rate drying period

$$(\text{Water flux})(\Delta H_{vap}) = \text{Heat flux}$$

$$\frac{1}{A}\frac{d(m_w)}{dt} \times (\lambda_w) = \frac{\text{driving force}}{\text{resistance}} = \frac{(T_{air} - T_{solid\ surface})}{1/h}$$

$$\frac{d(m_w)}{dt} = \frac{(h)(A)(T_{db} - T_{wb})}{\lambda_w}$$

$$\int_{m_{w,0}}^{m_{w,f}} d(m_w) = \Delta M_{water} = \int_{t_0}^{t_f} \frac{(h)(A)(T_{db} - T_{wb})}{\lambda_w}\, dt$$

$$\frac{(\Delta M_{water})\ (\lambda_w)}{(h)(A)(T_{db} - T_{wb})} = \text{time to remove } \Delta M_{water}$$

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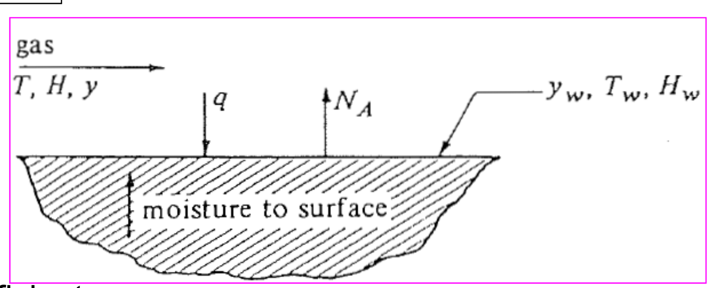
Calculation methods for constant-rate drying period

Or, assuming that the heat is transferred by convection only:

- The rate of convective heat transfer is: $Q = h_c A(T\text{-}T_w)$
- The flux of water vapor from the surface is (**see humidification**):

$$N_A = k_y(y_w\text{-}y) \cong k_y \frac{M_B}{M_A}(H_w\text{-}H) \longrightarrow R_c = N_A M_A = k_y M_B (H_w\text{-}H)$$

T: dry bulb temperature
T_w: wet bulb temperature
H_w: Saturated humidity **at T_w**
H: Humidity at T
h_c: convection heat transfer coefficient.



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Calculation methods for constant-rate drying period

- The heat required to vaporize water is:

$$Q_\lambda = (R_c A)\lambda_w = k_y M_B (H_w\text{-}H) A\lambda_w$$

- Neglecting the small sensible heat changes, this rate of heat is equal to the rate of convection heat transfer:

$$Q = Q_\lambda \Rightarrow h_c A(T\text{-}T_w) = (R_c A)\lambda_w = k_y M_B (H_w\text{-}H) A\lambda_w$$

$$R_c = h(T\text{-}T_w)/\lambda_w \qquad R_c = k_y M_B (H_w\text{-}H)$$

→ We need either convective heat or mass-transfer coefficient to get R_c .

→ Then, the required drying time in the constant drying rate region is:

$$t = \frac{L_s}{A}\frac{X_1 - X_2}{R_c} = \frac{L_s}{A}\lambda_w \frac{X_1 - X_2}{h_c(T\text{-}T_w)} = \frac{L_s}{A}\frac{X_1 - X_2}{k_y M_B (H_w\text{-}H)}$$

Calculation methods for constant-rate drying period

- **The following convective heat-transfer correlations can be used:**

1. Parallel flow to surface:

$$h_c(\mathrm{W/m^2.^oC}) = 0.0204 G^{0.8}$$

where **G=ρ*u*** is the humid air mass velocity in **kg/(m².h)**

• The above correlation is applicable for:

- Air temperature in the range: 45-150 °C
- Humid air velocity, u , in the range: 0.61-7.6 m/s or mass velocity (G=ρ*u*) of 2450-29300 kg/(h.m²):

2. Perpendicular flow to surface (impingement):

$$h_c(\mathrm{W/m^2.^oC}) = 1.17 G^{0.37}$$

where G=ρ*u* is the air mass velocity in kg/(m².h)

• The above correlation is applicable for:

- Humid air temperature in the range: 45-150 °C
- Humid air velocity, u , in the range: 0.9-4.6 m/s or mass velocity (G=ρ*v*) of 3900-19500 kg/(h.m²):

Example

An insoluble wet granular material is dried in a pan of 0.457×0.457 m and 25.4 mm deep. The material is 25.4 mm deep in the pan, and the sides and bottom can be considered to be insulated. Heat transfer is by convection from an air stream flowing parallel to the surface at a velocity of 6.1 m/s. The air is at 65.6 °C and has a humidity of 0.010 kg water/kg dry air. Estimate the rate of drying for constant-rate period.

$$R_c = h_c(T\text{-}T_w)/\lambda_w = ?$$

Solution. T= 65.6 °C. H=0.010 kg water/kg dry air.
From psychrometric chart:
- The wet bulb temperature is: T_w =28.9 °C
- The saturated humidity **at T_w** is: H_w =0.026 kg water/kg dry air.
- The humid volume of the gas mixture is

$$\begin{aligned} v_H &= (2.83\times10^{-3} + 4.56\times10^{-3} H)T \\ &= (2.83\times10^{-3} + 4.56\times10^{-3}\times0.01)(65.6+273.15) = 0.974\,\text{m}^3/\text{kg dry air} \end{aligned}$$

Solution

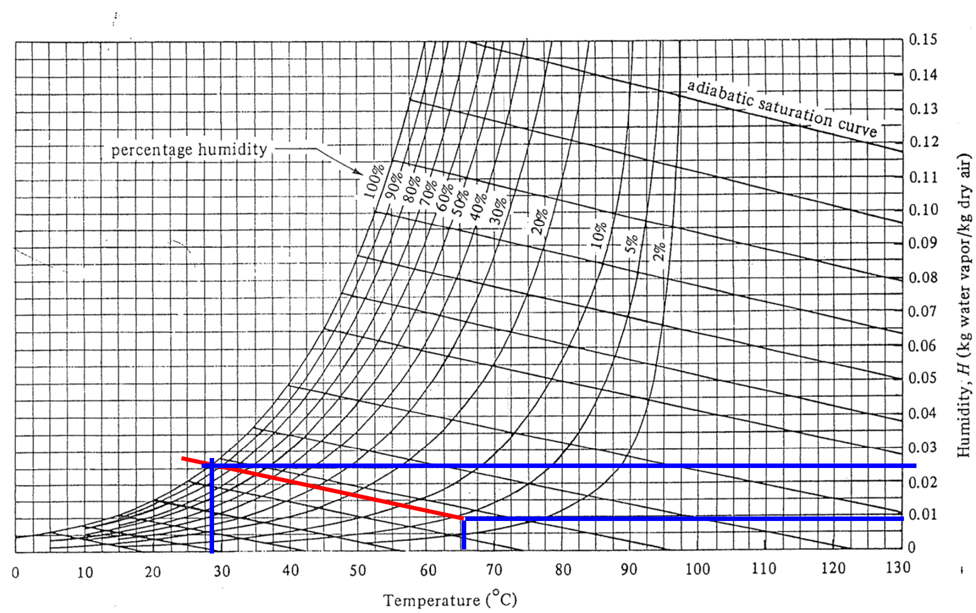


FIGURE 9.3-2. *Humidity chart for mixtures of air and water vapor at a total pressure of 101.325 kPa (760 mm Hg). (From R. E. Treybal, Mass-Transfer Operations, 3rd ed. New York: McGraw-Hill Book Company, 1980. With permission.)*

Solution Cont.d

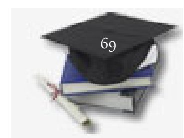
Take basis 1 kg dry gas→ 0.01 kg water→ 1.01 kg mixture. Thus,

$$v = 0.974\,\frac{\text{m}^3\,\text{mixture}}{\text{kg dry air}}\,\frac{1\,\text{kg dry air}}{1.01\,\text{kg mixture}} = 0.964\,\frac{\text{m}^3\,\text{mixture}}{\text{kg mixture}}$$

$$\rho = 1/v = 1.037\,\frac{\text{kg}}{\text{m}^3}$$

$$G = \rho u = (1.037)(6.1) = 6.3257\,\text{kg/(m}^2\text{.s)} = 22772\,\text{kg/(m}^2\text{.h)}$$

$$h_c = 0.0204 G^{0.8} = 0.0204(22772)^{0.8} = 62.45\,\text{W/m}^2.^\circ\text{C}$$



Solution Cont.d

The latent heat of vaporization, λ_w , at T_w=28.9 °C can be taken from steam tables or by interpolation using λ_w =2501 kJ/kg at 0 °C and λ_w = 2260 kJ/kg at 100 °C. By interpolation **λ_w =2431 kJ/kg**.

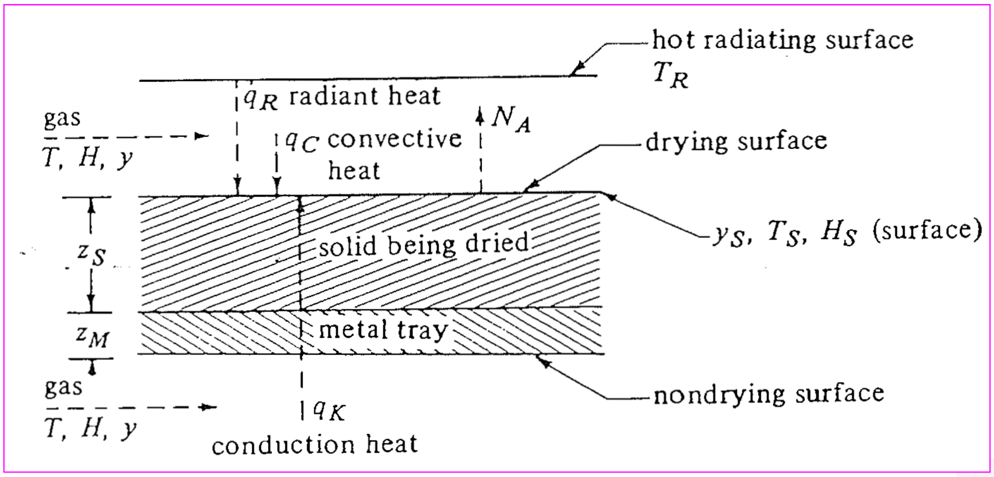
$$R_c = h_c(T\text{-}T_w)/\lambda_w = (62.45)(65.6-28.9)/(2431\times10^3)$$
$$= 9.428\times10^{-4}\,\text{kg water vapor/(s.m}^2) = 3.39\,\text{kg water vapor/(h.m}^2)$$

The total drying rate $= R_c A = (3.39)(0.457^2) = 0.708\,\text{kg water vapor/h}$

Calculation methods for constant-rate drying period.

D. Conduction, radiation, and convection heat transfer:

This general case takes into consideration the heat transfer by simultaneous convection, conduction and radiation:



Calculation methods for constant-rate drying period.

- The rate of convective heat transfer is:

$$Q_C = h_c A(T\text{-}T_s)$$

- The rate of radiant heat transfer is:

$$Q_R = h_R A(T_R\text{-}T_S)$$

where h_R is the radiant heat transfer coefficient defined as:

$$h_R = \varepsilon(5.676\times10^{-8})\frac{T_R^{\,4} - T_S^{\,4}}{T_R - T_S}$$

And ε is the solid emissivity

Calculation methods for constant-rate drying period.

- The rate of conduction heat transfer is:

$$Q_K = U_K A(T\text{-}T_S)$$

where U_K is the heat transfer coefficient defined as:

$$U_K \cong \frac{1}{1/h_c + Z_M/k_M + Z_S/k_S}$$

where Z_M and Z_S are the metal and solid thickness, respectively; k_M and k_S are the thermal conductivity of metal and solid , respectively.

Calculation methods for constant-rate drying period.

- Now the total rate of heat transfer to the drying surface is:

$$Q = Q_C + Q_R + Q_K \longrightarrow Q = A\left[(h_C + U_K)(T - T_s) + h_R(T_R - T_S)\right]$$

- This is total heat required to vaporized water, hence:

$$Q = Q_\lambda \quad \text{where} \quad Q_\lambda = (R_c A)\lambda_S$$

- The flux of water vapor from the solid surface is :

$$N_A = k_y(y_S\text{-}y) \cong k_y \frac{M_B}{M_A}(H_S\text{-}H) \longrightarrow R_c = N_A M_A = k_y M_B (H_S\text{-}H)$$

Calculation methods for constant-rate drying period.

- Then: $Q_\lambda = k_y M_B (H_S - H) A \lambda_S$

- Finally: $$R_c = \frac{(h_C + U_K)(T - T_s) + h_R (T_R - T_S)}{\lambda_S} = k_y M_B (H_S - H)$$

- To calculate R_c, the surface temperature T_S must be determined first as follows:
  - Rearranging the previous equation as:

$$(1 + \frac{U_K}{h_C})(T - T_s) + \frac{h_R}{h_C}(T_R - T_S) = \frac{\lambda_S}{h_C / (k_y M_B)}(H_S - H)$$

Calculation methods for constant-rate drying period.

- Since : $\frac{h_c}{M_B k_y} \cong c_S$ **Psychrometric ratio (Lewis rule)**

The Eq. becomes: $$(1 + \frac{U_K}{h_C})(T - T_s) + \frac{h_R}{h_C}(T_R - T_S) = \frac{\lambda_S}{c_S}(H_S - H)$$

Drying surface temperature Eq.

(Trial-and-error solution)

Usually $T_S > T_w$

Example

In the previous example, suppose that the metal bottom of the tray has a thickness of 0.610 mm and its thermal conductivity is 43.3 W/(m.K). The thermal conductivity of the wet solid can be assumed as 0.865 W/(m.K). The top drying surface, also receives direct radiation from upper steam heated pipe whose surface temperature is 93.3 °C. The emissivity of solid is 0.92. **Estimate the rate of drying for constant-rate period**.

Solution. Z_M=0.00061 m; k_M=43.3 W/m.K; k_S=0.865 W/m.K
T= 65.6 °C =338.75 K, H=0.010 kg water/kg dry air;
ε=0.92; T_R=93.3 °C=366.45 K

From previous example:

- The wet bulb temperature is: T_w =28.9 °C
- h_c = 62.45 W/(m^2.K)
- The wet solid is 25.4 mm deep→ Z_S = 0.0254 m

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Solution

- Calculate :

$$U_K \cong \frac{1}{1/h_c + z_m/k_M + z_S/k_S} = \frac{1}{1/62.45 + 0.00061/43.3 + 0.0254/0.865}$$
$$= 22.04 \text{ W/m}^2\text{.K}$$

$$c_S = (1.005 + 1.88H)\times 10^3 = (1.005 + 1.88\times 0.01)\times 10^3$$
$$= 1024 \text{ J/(kg dry air.K)}$$

- Express the radiant heat transfer coefficient as function of T_S:

$$h_R = \varepsilon(5.676\times 10^{-8})\frac{T_R^{\ 4} - T_S^{\ 4}}{T_R - T_S} = (0.92)(5.676\times 10^{-8})\frac{366.45^4 - T_S^{\ 4}}{366.45 - T_S}$$
$$= 5.222\times 10^{-8}\frac{1.803\times 10^{10} - T_S^{\ 4}}{366.45 - T_S}$$

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Solution Cont.d

• The drying surface temperature Eq. is:

$$(1+\frac{U_K}{h_C})(T-T_s)+\frac{h_R}{h_C}(T_R-T_S)=\frac{\lambda_S}{c_S}(H_S\text{-}H)$$

• Substitute H, T, h_c, U_K and c_S values and h_R expression in the above drying surface temperature Eq. to have:

$$(1+\frac{22.04}{62.45})(338.75-T_s)+\frac{5.222\times10^{-8}\,\dfrac{1.803\times10^{10}-T_S{}^4}{366.45-T_S}}{62.45}(366.45-T_S)$$
$$=\frac{\lambda_S}{1024}(H_S-0.01)$$

(Trial-and-error solution to get Ts)

Solution Cont.d

Trial-and- error solution as follows:

- Assume trial value of T_S that is a little bit greater than T_w .
  - Get λ_s (from steam tables for example).
  - Get saturated humidity, H_s, from the humidity chart.
- Calculate new T_S value from drying surface temperature equation.
- Compare the new T_S with the old value.
- Continue until there is is no significant difference in T_s.

• Start with, for example, **initial guess value of T_s =28.9 °C = 308.15 K** to get the following final results:

T_s =32.8 °C = 305.95 K: H_s=0.032 kg water/kg dry air
λ_S =2423×10^3 J/kg, h_R=8 W/m^2.K

$$R_c=\frac{(h_C+U_K)(T-T_s)+h_R(T_R-T_S)}{\lambda_S}=1.343\times10^{-3}\ \text{kg/m}^2.\text{s}$$
$$=4.83\ \text{kg/m}^2.\text{h}$$

Effect of process variables on the constant-rate drying

- Drying rate R_C increases with:

$$R_c = h(T\text{-}T_w)/\lambda_w$$

- increasing air velocity (u). This is due the increase in the heat transfer coefficient (h). However, its effect is less important when radiation and conduction are present.
- decreasing gas humidity (H). This is due to the decrease in wet bulb temperature (T_w).

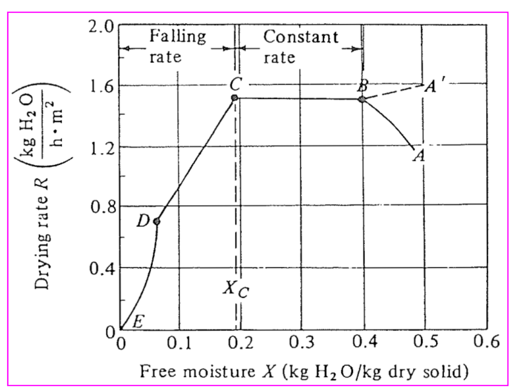
$$R_c = k_y M_B (H_w\text{-}H)$$

- increasing the gas temperature (T). Note that T_w also increases with increasing T but not as the increase in the gas temperature itself.

→ The thickness of solid being dried will NOT affect R_c when the heat is transferred by convection only. In contrast, the corresponding drying time will be longer as such thickness increases since the amount of dry solid (L_s) is directly proportional with solid thickness.

Calculation Methods for Falling-Rate Drying Period

- In the falling-rate drying period, $R = R(X)$
  - The rate of drying, *R* is not constant but decreases when drying proceeds past the critical free moisture content X_c.
  - When the free moisture content X is zero, the rate drops to zero.
  - R varies, hence it can not be integrated as in the constant rate period



The rate of drying R is defined as

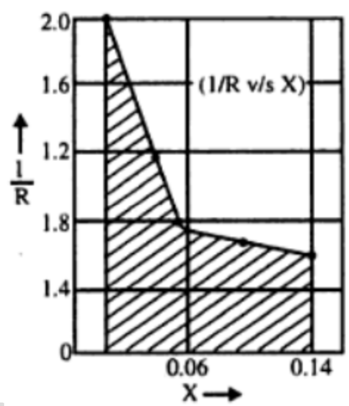
$$R = -\frac{L_S}{A}\frac{dX}{dt}$$

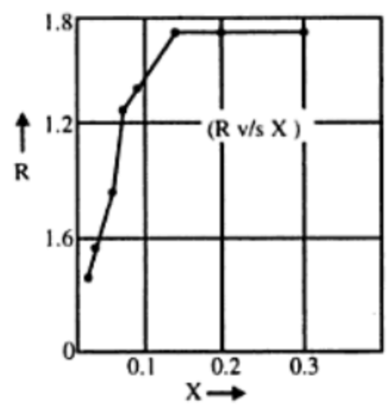
Calculation Methods for Falling-Rate Drying Period

- This can be rearranged and integrated over time interval to dry from X_1 at t_1 = 0 to X_2 at t_2 = t,

$$t = \int_{t_1=0}^{t_2=t} dt = \frac{L_S}{A}\int_{X_2}^{X_1}\frac{dX}{R}$$

Graphical integration plotting 1/R versus X and determining the <u>area under the curve.</u>





Calculation Methods for Falling-Rate Drying Period

Numerical integration

Trapezoidal rule (2-point):

$$\int_0^{X_1} f(x)dx = \frac{h}{2}[f(X_0)+f(X_1)]$$

$$h = X_1 - X_0$$

Simpson's one-third rule (3-point):

$$\int_0^{X_2} f(x)dx = \frac{h}{3}[f(X_0)+4f(X_1)+f(X_2)]$$

$$h = \frac{X_2 - X_0}{2} \qquad X_1 = X_0 + h$$

Simpson's three-eights rule (4-point):

$$X_1 = X_0 + h \quad X_2 = X_0 + 2h$$

$$\int_0^{X_3} f(x)dx = \frac{3}{8}h[f(X_0)+3f(X_1)+3f(X_2)+f(X_3)] \qquad h = \frac{X_3 - X_0}{3}$$

Simpson's five-point quadrature :

$$\int_0^{X_4} f(x)dx = \frac{h}{3}[f(X_0)+4f(X_1)+2f(X_2)+4f(X_3)+f(X_4)] \qquad h = \frac{X_4 - X_0}{4}$$

Example

A batch of wet solid whose drying rate curve is represented below is to be dried from a free moisture content of 0.38 to 0.04 kg H_2O/kg dry solid. The weight of dry solid is 399 kg and the top drying surface area is 18.58 m^2. **Calculate the time required for this drying**.

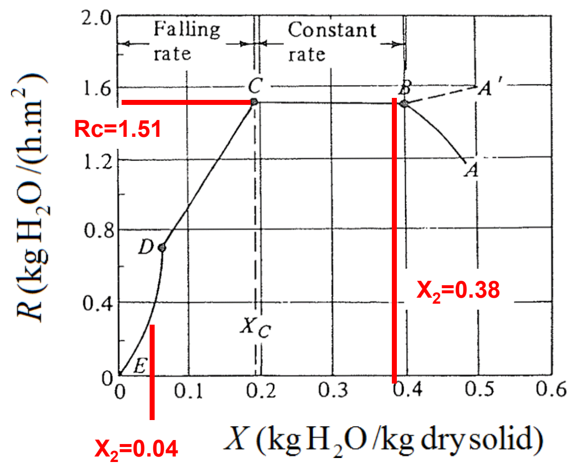
Solution:

$$X_1 = 0.38 \, \text{kg H}_2\text{O/kg dry solid}$$

$$X_2 = 0.04 \, \text{kg H}_2\text{O/kg dry solid}$$

$$L_s / A = 399/18.58$$

$$= 21.5 \, \text{kg dry solid/m}^2$$



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Solution Cont.d

$$t = -\frac{L_s}{A}\int_{X_1}^{X_2}\frac{dX}{R} = -\frac{L_s}{A}\left[\int_{X_1}^{X_C}\frac{dX}{R_C} + \int_{X_C}^{X_2}\frac{dX}{R}\right] = -\frac{L_s}{A}\left[\frac{X_c - X_1}{R_C} + \int_{X_C}^{X_2}\frac{dX}{R}\right]$$

- From draying rate curve:

$$X_c = 0.195 \, \text{kg H}_2\text{O/kg dry solid} \qquad R_c = 1.51 \, \text{kg H}_2\text{O/(m}^2\text{.h)}$$

- $\int_{X_C=0.195}^{X_2=0.04}\frac{dX}{R}$ can be integrated numerically or graphically. To perform this, read values of R and calculate 1/R:

- Using Trapezoidal rule:

$$\int_{X_C=0.195}^{X_2=0.04}\frac{dX}{R} \cong -0.189$$

X	R	1/R	X	R	1/R
0.195	1.51	0.663	0.065	0.71	1.41
0.150	1.21	0.826	0.050	0.37	2.70
0.100	0.90	1.11	0.040	0.27	3.70

- Finally the drying time is:

$$t = -21.5\left[\frac{0.195 - 0.38}{1.51} - 0.189\right] = 6.7 \, \text{h}$$

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Calculation Methods for Falling-Rate Drying Period

Drying rate varies linearly through the whole falling-rate:

- Sometimes, the drying rate through the whole falling rate period is approximated by **linear relation** as:

$$R \cong aX + b$$

- To find the constants a and b, use the following conditions :

$$X = 0 \quad ; \quad R = 0$$

$$X = X_c \; ; \quad R = R_c$$

to get

$$R = \frac{R_c}{X_c} X$$

- With this approximation , the drying period required to reduce moisture content from X_C to some moisture content X is :

$$t = -\frac{L_s}{A}\int_{X_C}^{X}\frac{dX}{R} \xrightarrow[\text{Integrate}]{R = R_c X / X_c} t = -\frac{L_s}{A}\frac{X_c}{R_c}\ln\left(\frac{X}{Xc}\right)$$

Example

Assume that the drying rate varies linearly through the whole falling-rate period in the previous example. Estimate the drying period.

Solution:

$$X_1 = 0.38\,\text{kg}\,H_2O/\text{kg dry solid}$$

$$X_2 = 0.04\,\text{kg}\,H_2O/\text{kg dry solid}$$

$$X_c = 0.195\,\text{kg}\,H_2O/\text{kg dry solid} \qquad R_c = 1.51\,\text{kg}\,H_2O/(\text{m}^2.\text{h})$$

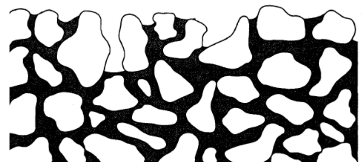
$$L_s / A = 21.5\ \text{kg dry solid}/\text{m}^2$$

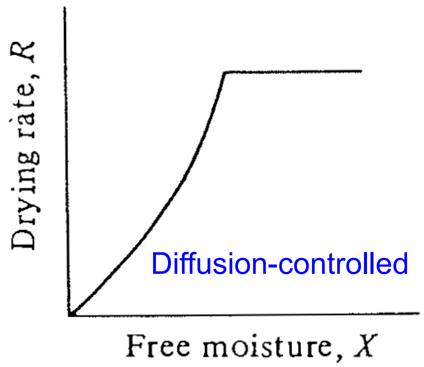
$$t \cong -\frac{L_s}{A}\left[\frac{X_c - X_1}{R_C} + \int_{X_C}^{X_2}\frac{dX}{R}\right] = -\frac{L_s}{A}\left[\frac{X_c - X_1}{R_C} + \frac{X_c}{R_C}\ln\left(\frac{X_2}{X_C}\right)\right] = 7.03\,\text{h}$$

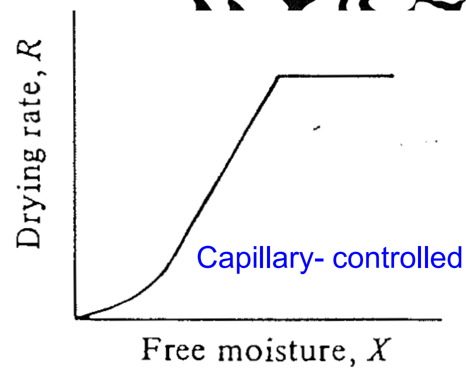
Calculation methods for falling-rate drying period.

The drying rate is controlled by liquid diffusion/capillary flow:

- Rate of drying is affected strongly by the mechanism of liquid movement from some depths of the solid to the drying surface.
- There are two important mechanisms:
  - Liquid diffusion.
  - Capillary movement in porous solid.







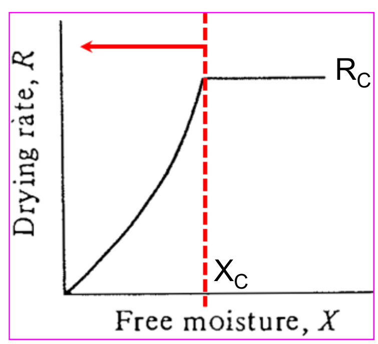
Calculation methods for falling-rate drying period.

Drying is controlled by liquid diffusion:

- Unsteady-state one dimensional diffusion:

$$\frac{\partial X}{\partial t} = D_L \frac{\partial^2 X}{\partial z^2}$$

where D_L is the liquid diffusion coefficient and z is the distance in the solid.



- Solving the above differential equation, with uniform initial moisture distribution of X_1 and for long drying time, gives :

$$\frac{\overline{X}(t)}{X_1} \cong \frac{8}{\pi^2} e^{-D_L t\left(\frac{\pi}{2z_1}\right)^2} \longrightarrow t \cong \frac{4z_1^2}{\pi^2 D_L} \ln\left(\frac{8X_1}{\pi^2 \overline{X}}\right) \longrightarrow R \cong \frac{L_s}{A}\frac{\pi^2 D_L}{4z_1^2}\overline{X}$$

Calculation methods for falling-rate drying period.

$\bar{X}$: Average free moisture content at time t.

z_1= 0.5 Z (Drying from top and bottom parallel faces of the slab).

z_1=Z (Drying from the top face only).

Example

The experimental diffusion coefficient of moisture in a given wood at 27 °C is 2.97×10^{-6} m²/h. Large planks of wood 25.4 mm thick are dried from both sides by air having relative humidity of 17.5%. At 27 °C, The EMC curve where the moisture content is in in kg H_2O/kg dry wood is given below. **Calculate the time required to dry the wood from average total moisture content of 0.29 to 0.09 H_2O/kg dry wood.**

Solution.

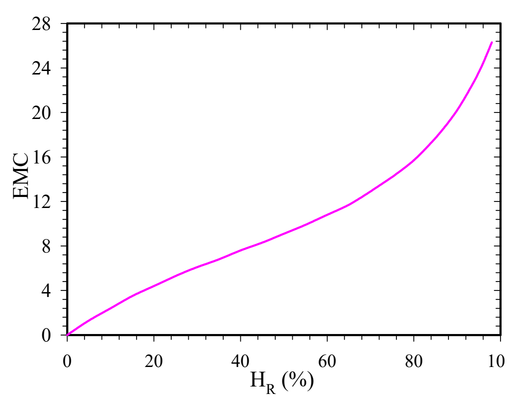
$D_L=2.97\times10^{-6}$ m²/h

Z=0.0254 m

H_R=17.5%

Initial: $X_{T,1}$= 0.29 kg H_2O/kg dry (total)

Final: X_T=0.09 kg H_2O/kg dry (total)



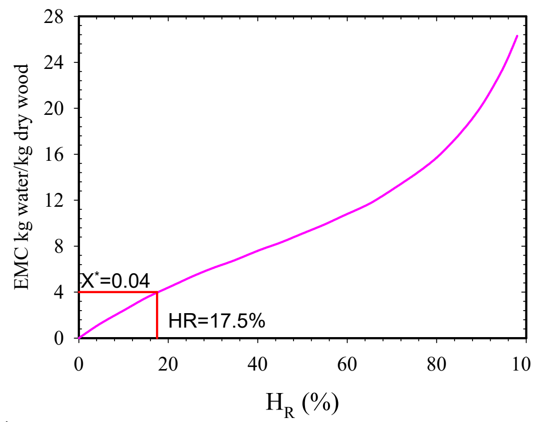
Solution cont.d

- From the equilibrium moisture content curve at H_R=17.5% is X^*=0.04
- Initial free moisture content:

$$X_1 = X_{T,1} - X^* = 0.29 - 0.04 = 0.25$$

- Final average free moisture content:

$$\overline{X} = X_T - X^* = 0.09 - 0.04 = 0.05$$



- Since, the wood slab is dried from both sides:

$z_1=0.5 \times Z=0.5\times0.0254=0.0127$ m

$$t \cong \frac{4z_1^2}{\pi^2 D_L}\ln\left(\frac{8X_1}{\pi^2 \overline{X}}\right) = \frac{4(0.1027)^2}{\pi^2(2.97\times10^{-6})}\ln\left(\frac{8(0.25)}{\pi^2(0.05)}\right) = 30.8\,\text{h}$$

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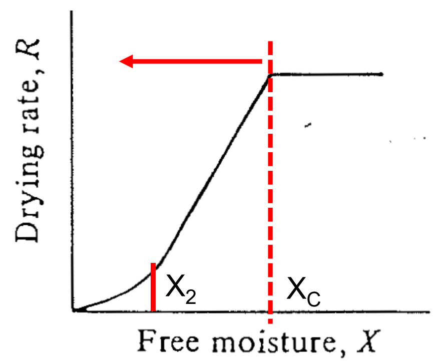
Calculation methods for falling-rate drying period.

Drying is controlled by capillary movement

- **Driving force: interfacial tension between water and solid.**
- Linear trend from Xc→X_2 and another linear trend from where X_c→0



- **Effect of shrinkage on drying rate:**

- Development of a hard layer on the surface which is impervious to the flow of liquid or vapor moisture.
- Rigid solids do NOT shrink appreciably, but colloidal and fibrous materials such as vegetables and other food stuff do undergo shrinkage.
- Shrinkage slow down the drying rate.
- Dry with moist air to decrease the shrinkage effect.

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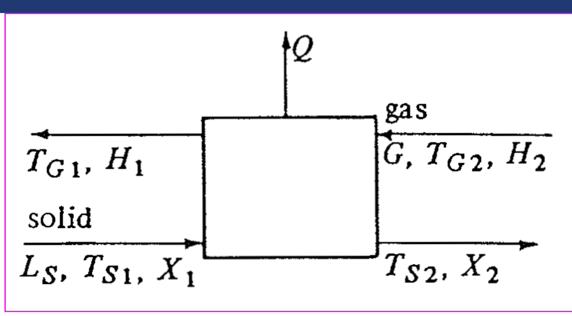
Continuous Drying

Balances for countercurrent continuous dryers:



G: Mass flow rate of **dry air**
L_S: Mass flow rate of dry solid
X: Free moisture content.
H: Humidity
T_S: Wet solid temperature
T_G: dry gas temperature

- **Steady-state material balance on the moisture:**

$$L_S(X_1 - X_2) = G(H_1 - H_2)$$

- **Steady-state heat balance on dryer:**

$$L_S(H'_{S1} - H'_{S2}) = G(H_{y1} - H_{y2}) + Q$$

H'_{S1} and H'_{S2} : entlapyies of wet solid in kJ/kg dry solid at T_{S1} and T_{S2}, respectively

H_{y1} and H_{y2} : entlapyies of humid air in kJ/kg dry air at T_{G1} and T_{G2}, respectively.

Balances for countercurrent continuous dryers

- Heat capacity of dry solid (C_{pS}) and heat capacity of liquid water ($C_L \cong 4.187$ kJ/kg.K) can be used to calculate the enthalpy of wet solid at inlet and outlet :

$$H'_{S1} = C_{pS}(T_{S1} - T_0) + X_{T,1} C_L (T_{S1} - T_0)$$

$$H'_{S2} = C_{pS}(T_{S2} - T_0) + X_{T,2} C_L (T_{S2} - T_0)$$

where T_0 is base temperature has a convenient value of 0 ^oC

- The enthalpy of humid gas at inlet and outlet can be calculated from (**See humidification handout**):

$$H_{y1} = c_{S1}(T_{G1} - T_0) + \lambda_0 H_1 \qquad c_{S1} = 1.005 + 1.88 H_1$$

$$H_{y2} = c_{S2}(T_{G2} - T_0) + \lambda_0 H_2 \qquad c_{S2} = 1.005 + 1.88 H_2$$

λ_0=2501 kJ/kg

- **Adiabatic drying**: Q = 0
- **Remark.** the difference in the free moisture content equals the difference in the free moisture content. **Why?**

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Example

Continuous countercurrent dryer is being used to dry 453.6 kg dry solid/h containing 0.04 kg total free moisture/kg dry solid to a value of 0.002 kg total ~~free~~ moisture/kg dry solid. The granular solid enters at 26.7 ^oC and is to be discharged at 62.8 ^oC . The dry solid heat capacity is assumed to be constant at 1.465 kJ/(kg.K). Heating air enters the dryer at 93.3 ^oC with a humidity of 0.01 kg H_2O/kg dry air and leaves at 37.8 ^oC. Calculate the air flow rate and the outlet humidity. Neglect heat losses in the dryer.

Solution:

L_S=453.6 kg dry solid/h; Q=0

$X_{T,1}$= 0.04 kg H_2O/kg dry

$X_{T,2}$ =0.002 kg H_2O/kg dry

T_{S1}= 26.7 ^oC ; T_{S2}= 62.8 ^oC

T_{G1}= 37.8 ^oC ; T_{G2}= 93.3 ^oC ; H_2 = 0.01 kg H_2O/kg dry air

C_{pS}=1.465 kJ/(kg.K).

G=?
H_1=?

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Solution Cont.d

- Steady-state moisture material balance:

$$L_S(X_1 - X_2) = G(H_1 - H_2) \rightarrow 453.6(0.04 - 0.002) = G(H_1 - 0.01)$$

$$GH_1 - 0.01G = 17.237$$ **Eq. (1)**

- Steady-state heat balance: $L_S(H'_{S1} - H'_{S2}) = G(H_{y1} - H_{y2})$

$$H'_{S1} = C_{pS}(T_{S1} - T_0) + X_{T,1}C_L(T_{S1} - T_0) \rightarrow H'_{S1} = C_{pS}T_{S1} + X_{T,1}C_L T_{S1}$$

T_0 = 0 °C

$$H'_{S2} = C_{pS}(T_{S2} - T_0) + X_{T,2}C_L(T_{S2} - T_0) \rightarrow H'_{S2} = C_{pS}T_{S2} + X_{T,2}C_L T_{S2}$$

$$H'_{S1} - H'_{S2} = C_{pS}(T_{S1} - T_{S2}) + C_L\left[X_{T,1}T_{S1} - X_{T,2}T_{S2}\right]$$

$$H'_{S1} - H'_{S2} = 1.465(26.7 - 62.8) + 4.187\left[(0.04)(26.7) - (0.002)(62.8)\right] = -48.94 \text{ kJ/kg dry air}$$

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Solution Cont.d

$$c_{S2} = 1.005 + 1.88H_2 = 1.005 + 1.88(0.01) = 1.0238 \text{ kJ/kg dry air.K}$$

$$H_{y2} = c_{S2}(T_{G2} - T_0) + \lambda_0 H_2 = 1.0238(93.3 - 0) + 2501 \times 0.01 = 120.5 \frac{\text{J}}{\text{kg}}$$

$$c_{S1} = 1.005 + 1.88H_1$$

$$H_{y1} = c_{S1}(T_{G1} - T_0) + \lambda_0 H_1 = (1.005 + 1.88H_1)(37.8) + 2501H_1 = 37.99 + 2572H_1$$

- after substituting the above values in the steady state heat balance:

$$82.51G - 2572GH_1 = 22199$$ **Eq. (2)**

- **Solve Eqns. (1) and (2) simultaneously to get:**

G=1171.8 kg dry air/h , H_1=0.0247 kg water /kg air

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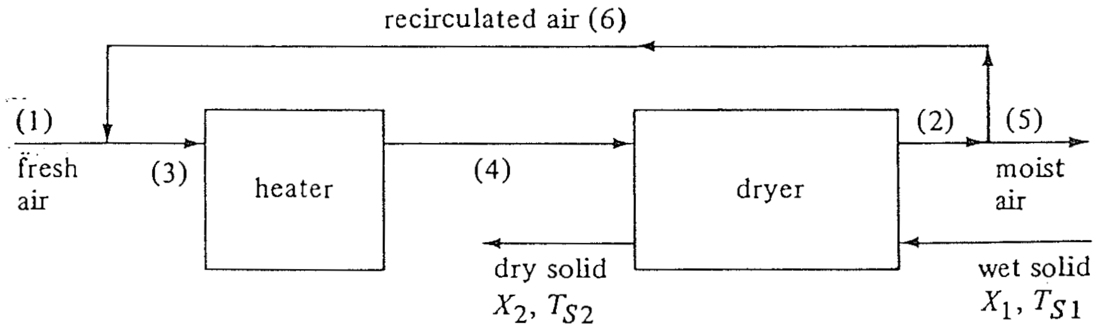
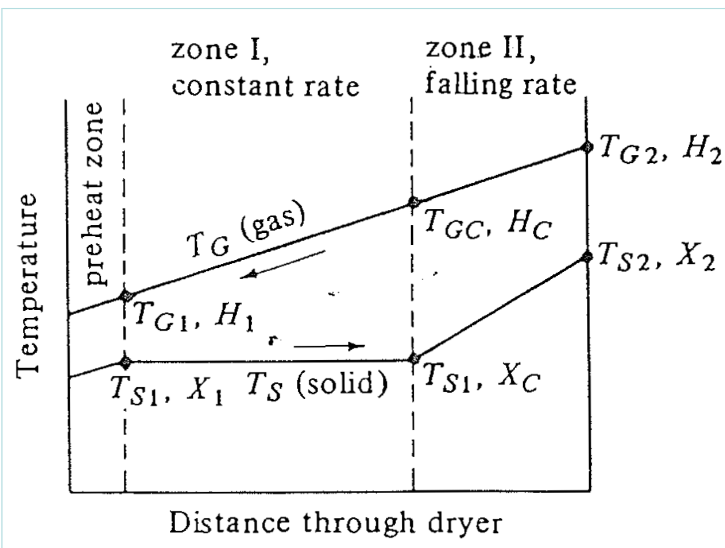
Air circulation in countercurrent continuous dryers:



- **Purpose of air circulation:**
  to control the humidity of the entering air and the wet bulb temperature in order to minimize heating cost of air.

Temperature profiles countercurrent continuous dryers:



- **Preheat zone:** the solid is heated up to the wet bulb or adiabatic saturation temperature. Little evaporation occurs, thus this zone is usually ignored.

Drying time in countercurrent continuous dryers:

- **Zone I (constant rate zone):**
- If heat is transferred by convection only, the drying rate is:

$$R_c = h(T\text{-}T_w)/\lambda_w = k_y M_B (H_w\text{-}H)$$

- Differential material balance on moisture:

$$L_s dX = G dH$$

- Drying time is:

$$t = -\frac{L_s}{A}\int_{X_1}^{X_C}\frac{dX}{R_C} = -\frac{G}{Ak_y M_B}\int_{H_1}^{H_C}\frac{dH}{H_w - H}$$

Drying time in countercurrent continuous dryers:

- H_W is constant since Tw is constant. Thus, the above Eq. can be integrated to have:

$$t = \frac{G}{Ak_y M_B}\ln\frac{H_W - H_C}{H_W - H_1} = \frac{G}{Ak_y M_B}\frac{H_1 - H_C}{\Delta H_{LM}}$$

$$\Delta H_{LM} = \frac{(H_W - H_C) - (H_W - H_1)}{\ln\dfrac{H_W - H_C}{H_W - H_1}}$$

Log mean humidity difference

- Apply moisture material balance over the whole Zone II to get H_c:

$$L_S(X_C - X_2) = G(H_C - H_2) \longrightarrow H_C = H_2 + \frac{L_S}{G}(X_C - X_2)$$

Drying time in countercurrent continuous dryers:

- **Zone II (falling rate zone):**
  - Linear drying rate: $R = R_c X / X_C$
  - Convection heat transfer : $R_c = h(T\text{-}T_w)/\lambda_w = k_y M_B (H_w\text{-}H)$
  - Differential material balance of moisture: $L_s dX = GdH$
  - Drying time is:

$$t = -\frac{L_s}{A}\int_{X_c}^{X_2}\frac{dX}{R} = -\frac{X_C L_s}{A}\int_{X_c}^{X_2}\frac{dX}{R_c} = -\frac{GX_C}{Ak_y M_B}\int_{H_c}^{H_2}\frac{dH}{X(H_w - H)}$$

- Moisture material balance over the zone from X_2 to X:

$$L_S(X - X_2) = G(H - H_2) \rightarrow X = X_2 + \frac{G}{L_S}(H - H_2)$$

Drying time in countercurrent continuous dryers:

Zone II (falling rate zone):

$$t = -\frac{GX_C}{Ak_y M_B}\int_{H_c}^{H_2} dH \Big/ \left[\left(X_2 + \frac{G}{L_S}(H - H_2)\right)(H_w - H)\right]$$

- Integrate to get :

$$t = \frac{GX_C}{Ak_y M_B}\frac{1}{(H_W - H_2)G/L_s + X_2}\ln\frac{X_C(H_W - H_2)}{X_2(H_W - H_C)}$$

- Again, H_c can be calculated from:

$$H_C = H_2 + \frac{L_S}{G}(X_C - X_2)$$