



# Mass Transfer Operations

## Convective Mass Transfer Correlations

### Content

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## Mass Transfer Parameters: Definitions



where

$Sh$  = Sherwood number =  $kL/D_{AB}$

$Re_L$  = Reynolds number =  $Lv\rho/\mu$

$Sc$  = Schmidt number =  $\mu / (\rho D_{AB})$

$k$  = overall mass transfer coefficient

$L$  = length of sheet

$D_{AB}$  = diffusivity of A in B

$v$  = velocity of air

$\mu$  = viscosity of air

$\rho$  = density of air, and

$\mu/\rho$  = kinematic viscosity of air.

$\Gamma$  is the mass rate of liquid flow per unit of film width in the  $x$  direction.

$$j_D = \frac{k_c}{v_\infty} (Sc)^{2/3}$$



# Flow Inside Circular Pipes



Fluid motion	Range of conditions	Equation
1. Inside circular pipes	$Re = 4000 - 60\,000$	$j_D = 0.023 Re^{-0.17}$
	$Sc = 0.6 - 3000$	$Sh = 0.023 Re^{0.83} Sc^{1/3}$
	$Re = 10\,000 - 400\,000$ $Sc > 100$	$j_D = 0.0149 Re^{-0.12}$ $Sh = 0.0149 Re^{0.88} Sc^{1/3}$

For *laminar* flow of a fluid through a tube, with a Reynolds number range of  $10 < Re < 2000$ , the appropriate mass-transfer correlation is

$$Sh = 1.86 \left( \frac{v_{\infty} D^2}{L D_{AB}} \right)^{1/3} = 1.86 \left( \frac{D}{L} \frac{v_{\infty} D}{v} \cdot \frac{v}{D_{AB}} \right)^{1/3} = 1.86 \left( \frac{D}{L} Re Sc \right)^{1/3} \quad (30-19)$$

where  $L$  the length of the pipe and  $v_{\infty}$  is the bulk average velocity.

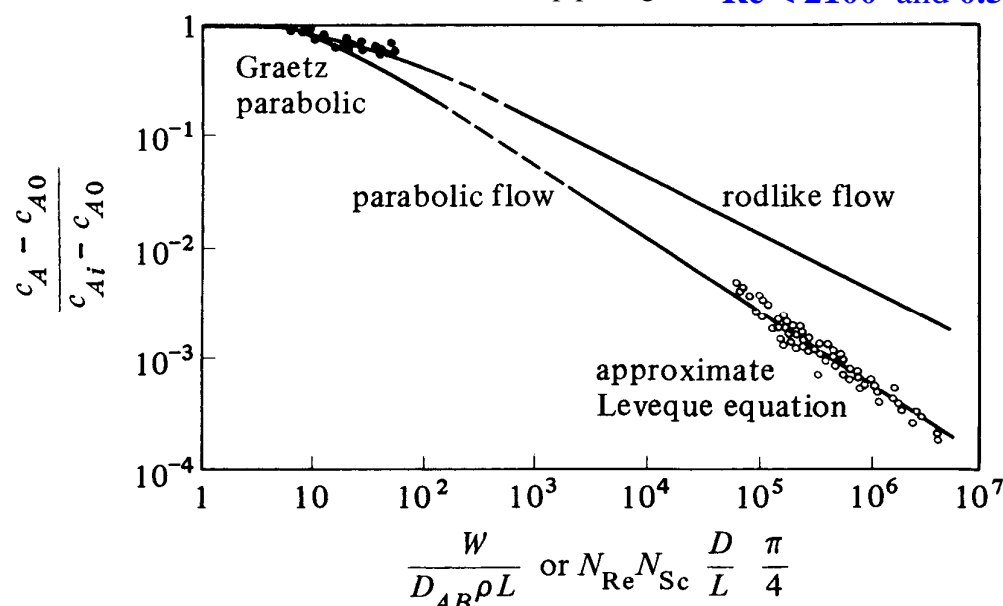
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# Flow Inside Circular Pipes



**Figure 7.3-2:** Mass transfer from the inner walls of a pipe to gases  $Re < 2100$  and  $0.5 < Sc < 3.0$



$c_A$  is the exit concentration,  $c_{A0}$  inlet concentration, and  $c_{Ai}$  concentration at the interface between the wall and the gas. The dimensionless abscissa is  $W/D_{AB} \rho L$  or  $N_{Re} N_{Sc} (D/L)(\pi/4)$ , where  $W$  is flow in kg/s and  $L$  is length of mass-transfer section in m.

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# Flow Parallel to Flat Plates



2. Unconfined flow parallel to flat plates‡	Transfer begins at leading edge $Re_x < 50\,000$	$Sh_L = \frac{k_c L}{D_{AB}} = 0.664 Re_L^{1/2} Sc^{1/3}$
	$Re_x = 5 \times 10^5 - 3 \times 10^7$ $Sc = 0.7 - 380$	$Sh_L = \frac{k_c L}{D_{AB}} = 0.0365 Re_L^{0.8} Sc^{1/3}$
	$Re_x = 2 \times 10^4 - 5 \times 10^5$ $Sc = 0.7 - 380$	Between above and $Nu = 0.0027 Re_x Pr_0^{0.43} \left( \frac{Pr_0}{Pr_i} \right)^{0.25}$
3. Confined gas flow parallel to a flat plate in a duct		
	$Re_c = 2600 - 22\,000$	$j_D = 0.11 Re_c^{-0.29}$

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# Liquid film in Wetted Wall



4. Liquid film in wetted-wall tower, transfer between liquid and gas
- $Re = 0 - 1200$ ,  $k_{L,av} = \left( \frac{6 D_{AB} \Gamma}{\pi \rho \delta L} \right)^{1/2}$
- $Re = 4\Gamma/\mu$  less than 100,  $Sh_{av} \approx 3.41$
- $3,000 < Re' < 40,000$ ;  $0.5 < Sc < 3$ ,  $j_D = 0.0328 (Re')^{-0.23}$
- $Re = \frac{4\Gamma}{\mu} = 1300 - 8300$ ,  $Sh = (1.76 \times 10^{-5}) \left( \frac{4\Gamma}{\mu} \right)^{1.506} Sc^{0.5}$

The film thickness is then

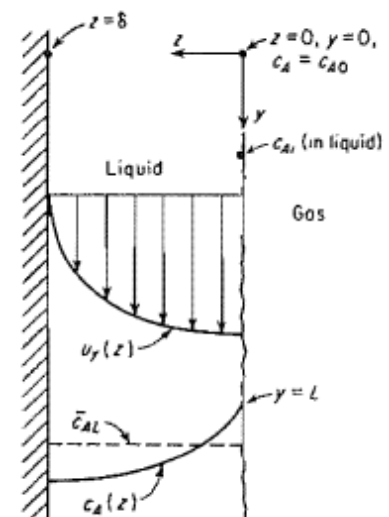
$$\delta = \left( \frac{3 \bar{u}_y \mu}{\rho g} \right)^{1/2} = \left( \frac{3 \mu \Gamma}{\rho^2 g} \right)^{1/3}$$

The Reynolds number of the liquid flowing down the tube is defined as

$$Re_L = \frac{4\Gamma}{\mu_L} = \frac{4w}{\pi D \mu_L}$$

where  $w$  is the mass flow rate of liquid,  $D$  is the inner diameter of the cylindrical column, and  $\Gamma$  is the mass flow rate of liquid per unit wetted perimeter of the column.

OR the mass rate of liquid flow per unit of film width in the  $x$  direction.



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## Past Single Sphere

For mass transfer into liquid streams, the equation of Brian and Hales<sup>4</sup>

$$Sh = \frac{k_L D}{D_{AB}} = (4 + 1.21 Pe_{AB}^{2/3})^{1/2} \quad Pe_{AB} = Re Sc$$

correlates data where the mass-transfer Peclet number,  $Pe_{AB}$ , is less than 10,000.

For Peclet numbers greater than 10,000, Levich<sup>5</sup> recommends the simpler relationship

$$Sh = \frac{k_L D}{D_{AB}} = 1.01 Pe_{AB}^{1/3}$$

For mass transfer into gas streams, the Frössling equation<sup>6</sup>

$$Sh = \frac{k_c D}{D_{AB}} = 2 + 0.552 Re^{1/2} Sc^{1/3}$$

Reynolds number range of 1500 to 12 000 under a Schmidt number range of 0.6 to 1.85.



## Past Single Sphere

➤ For very low Reynold's number, the Sherwood number should approach a value of 2.

$$\frac{k'_c D_p}{D_{AB}} = N_{Sh} = 2.0$$

The following correlation of Steinberger and Treybal<sup>8</sup> is recommended when the transfer occurs in the presence of natural convection

$$Sh = Sh_o + 0.347(Re Sc^{1/2})^{0.62}$$

where  $Sh_o$  is dependent on  $Gr Sc$

$$Sh_o = 2 + 0.569(GrSc)^{0.25} \quad Gr Sc \leq 10^8$$

$$Sh_o = 2 + 0.0254(GrSc)^{1/3} (Sc)^{0.244} \quad Gr Sc \geq 10^8$$

the Grashof number is defined as

$$Gr = \frac{D^3 \rho g \Delta \rho}{\mu^2}$$

where density,  $\rho$ , and viscosity,  $\mu$ , are taken at the bulk conditions of the flowing fluid, and  $\Delta \rho$  is the positive density difference between the two phases in contact. The prediction for  $Sh$  is valid when  $2 \leq Re \leq 3 \times 10^4$  and  $0.6 \leq Sc \leq 3200$ .



For gas bubble diameters ( $d_b$ ) less than 2.5 mm, use

$$Sh = \frac{k_L d_b}{D_{AB}} = 0.31 Gr^{1/3} Sc^{1/3}$$

For bubble diameters greater or equal to 2.5 mm, use

$$Sh = \frac{k_L d_b}{D_{AB}} = 0.42 Gr^{1/3} Sc^{1/2}$$

In the above correlations, the Grashof number is defined as

$$Gr = \frac{d_b^3 \rho_L g \Delta \rho}{\mu_L^2}$$

where  $\Delta \rho$  is the difference of the density of the liquid and the density of the gas inside the bubble, with density ( $\rho_L$ ) and viscosity ( $\mu_L$ ) determined at the bulk average properties of the liquid mixture. For dilute solutions, the fluid properties of the solvent approximate the fluid properties of the liquid mixture. The diffusion coefficient  $D_{AB}$  is with respect to dissolved gaseous solute  $A$  in solvent  $B$ .

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## Perpendicular to Single Cylinders

5. Perpendicular to single cylinders	$Re = 400-25\ 000$	$\frac{k_G P (Sc)^{0.56}}{G_M} = \frac{k_c (Sc)^{0.56}}{v_\infty} = 0.281 (Re_D)^{-0.4}$	:
	$Sc = 0.6-2.6$		
	$Re' = 0.1-10^5$		
	$Pr = 0.7-1500$	$Nu = (0.35 + 0.34 Re^{0.5} + 0.15 Re^{0.58}) Pr^{0.3}$	:

$P$  is the system total pressure and  $G_M$  is the superficial molar velocity of the gas flowing normal to the cylinder in units of  $\text{kg mol/m}^2 \cdot \text{s}$ . The Reynolds number for flow normal to a solid

cylinder,  $Re_D$ , is defined as

$$Re_D = \frac{\rho v_\infty D}{\mu}$$

where  $D$  is cylinder diameter,  $v_\infty$  is the fluid velocity normal to the solid cylinder, and  $\rho$  and  $\mu$  for the gas stream evaluated at the film average temperature.

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## Perpendicular to Single Cylinders



- Experimental data have been obtained for mass transfer from single cylinders when the flow is perpendicular to the cylinder. The cylinders are long and mass transfer to the ends of the cylinder is not considered.

$$J_D = 0.600(N_{Re})^{-0.487}$$

For gases  $0.6 < Sc < 2.6$

For liquid  $1000 < Sc < 3000$

$50 < Re < 50000$

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## Mass Transfer to and From Packed Beds



- For a Reynolds number range of 10-10000 for gases in a packed bed of spheres

$$J_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069}$$

where

$$N_{Re} = D_p v' \rho / \mu$$

The void fraction in a bed is  $\varepsilon$ ,  
range from 0.3 to 0.5

$D_p$  is diameter of the spheres

$v'$  is the superficial mass average velocity in the empty tube without packing.

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7. Through fixed beds of pellets §  $Re'' = 90-4000$   
 $Sc = 0.6$

$$j_D = j_H = \frac{2.06}{\epsilon} Re''^{-0.575}$$

$Re'' = 5000-10\ 300$   
 $Sc = 0.6$

$$j_D = 0.95 j_H = \frac{20.4}{\epsilon} Re''^{-0.815}$$

$Re'' = 0.0016-55$   
 $Sc = 168-70\ 600$

$$j_D = \frac{1.09}{\epsilon} Re''^{-2/3}$$

$Re'' = 5-1500$   
 $Sc = 168-70\ 600$

$$j_D = \frac{0.250}{\epsilon} Re''^{-0.31}$$

† Average mass-transfer coefficients throughout, for constant solute concentrations at the phase surface. Generally, fluid properties are evaluated at the average conditions between the phase surface and the bulk fluid. The heat-mass-transfer analogy is valid throughout.

‡ Mass-transfer data for this case scatter badly but are reasonably well represented by setting  $j_D = j_H$ .

§ For fixed beds, the relation between  $\epsilon$  and  $d_p$  is  $a = 6(1 - \epsilon)/d_p$ , where  $a$  is the specific solid surface, surface per volume of bed.



## Mass Transfer to Small Particles

### Mass transfer to small particles <0.6 mm

- The following equation has been shown to hold to predict mass-transfer coefficients from small gas bubbles such as oxygen or air to the liquid phase or from the liquid phase to the surface of small catalyst particles, microorganisms, other solids, or liquid drops

$$k'_L = \underbrace{\frac{2D_{AB}}{D_p}}_{\text{the molecular diffusion term}} + \underbrace{0.31 N_{Sc}^{-2/3} \left( \frac{\Delta \rho \mu_c g}{\rho_c^2} \right)^{1/3}}_{\text{free fall or rise of the sphere by gravitational forces}}$$

where  $D_{AB}$  is the diffusivity of the solute  $A$  in solution in  $m^2/s$ ,  $D_p$  is the diameter of the gas bubble or the solid particle in  $m$ ,  $\mu_c$  is the viscosity of the solution in  $kg/m \cdot s$ ,  $g = 9.80665 m/s^2$ ,  $\Delta \rho = (\rho_c - \rho_p)$  or  $(\rho_p - \rho_c)$ ,  $\rho_c$  is the density of the continuous phase in  $kg/m^3$ , and  $\rho_p$  is the density of the gas or solid particle. The value of  $\Delta \rho$  is always positive.



## Mass transfer to large gas bubbles > 2.5 mm.

- For large gas bubbles or liquid drops > 2.5 mm, the mass-transfer coefficient can be predicted by

$$k'_L = 0.42 N_{Sc}^{-0.5} \left( \frac{\Delta \rho \mu_c g}{\rho_c^2} \right)^{1/3}$$

- Large gas bubbles are produced when pure liquids are aerated in mixing vessels and sieve-plate columns



## Mass transfer in Fluidized Beds of Spheres

- For fluidized beds of spheres and for gases and liquids

$$J_D = \frac{0.4548}{\varepsilon} N_{Re}^{-0.4069} \quad 10 < Re < 4000$$

$$\varepsilon J_D = 1.1068 N_{Re}^{-0.72} \quad 1 < Re < 10$$

