

Convective Mass Transfer

Definition of convective mass transfer:

The transport of material between a boundary surface and a moving fluid or between two immiscible moving fluids separated by a mobile interface

Convection is divided into two types:

1. Forced convection
2. Natural convection

Compare between forced and natural convection mass transfer?

Forced convection:

In this type the fluid moves under the influence of an external force (pressure difference) as in the case of transfer of liquids by pumps and gases by compressors.

Natural convection:

Natural convection currents develop if there is any variation in density within the fluid phase. The density variation may be due to temperature differences or to relatively large concentration differences.

The rate equation:

The rate equation for convective mass transfer (either forced or natural) is:

$$N_A = k_c \Delta c_A$$

N_A is the molar-mass flux of species A, measured relative to fixed spatial coordinates

k_c is the convective mass-transfer coefficient

Δc_A is the concentration difference between the boundary surface concentration and the average concentration of the diffusing species in the moving fluid stream

What is the mass transfer coefficient k_c ?

From the rate equation ($N_A = k_c \Delta c_A$) the mass transfer coefficient is the rate of mass transfer per unit area per unit driving force. It gives an indication to how fast is the mass transfer by convection.

What are the factors affecting k_c ?

The mass-transfer coefficient is related to:

- (1). The properties of the fluid,
- (2). The dynamic characteristics of the flowing fluid, and
- (3). The geometry of the specific system of interest

Evaluation of the mass transfer coefficient

There are four methods of evaluating convective mass-transfer coefficients. They are:

- (1). dimensional analysis coupled with experiment
- (2). analogy between momentum, energy, and mass transfer
- (3). exact laminar boundary-layer analysis
- (4). approximate boundary-layer analysis

(1). Dimensional analysis

Dimensionless analysis predicts the various dimensionless parameters that are helpful in correlating experimental data.

Significant parameters in convective mass transfer

A. Forced convection

Dimensionless parameters are often used to correlate convective transfer data. In momentum transfer Reynolds number and friction factor play a major role. In the correlation of convective heat transfer data, Prandtl (Pr) and Nusselt (Nu) numbers are important. Some of the same parameters, along with some newly defined dimensionless numbers, will be useful in the correlation of convective mass-transfer data.

The molecular diffusivities of the three transport process (momentum, heat and mass) have been defined as:

Momentum diffusivity	$\nu = \frac{\mu}{\rho}$
Thermal diffusivity	$\alpha = \frac{k}{\rho c_p}$
Mass diffusivity	D_{AB}

It can be shown that each of the diffusivities has the dimensions of $\left(\frac{L^2}{T}\right)$ hence; a ratio of any of the two of these must be dimensionless.

- The ratio of the momentum diffusivity to the thermal diffusivity is designated as the Prandtl Number

$$Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

- The analogous number in mass transfer is Schmidt number given as which represents the ratio of the momentum diffusivity to mass diffusivity

$$Sc = \frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}} = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$

- The ratio of the diffusivity of heat to the diffusivity of mass is designated the Lewis number, and is given by

$$Le = \frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}} = \frac{\alpha}{D_{AB}} = \frac{k}{\rho c_p D_{AB}}$$

Note: Lewis number is encountered in processes involving simultaneous convective transfer of mass and energy.

The ratio of molecular mass transport resistance to the convective mass transport resistance of the fluid. This ratio is generally known as the Sherwood number, Sh and analogous to the Nusselt number Nu, in heat transfer.

$$Sh = \frac{\text{molecular mass transport resistance}}{\text{convective mass transport resistance}} = \frac{k_c L}{D_{AB}}$$

Note: L is the characteristic length (it depends on the geometry)

Application of dimensionless analysis to mass transfer

Consider the transfer of mass from the walls of a circular conduit to a fluid flowing through the conduit. To get the dimensionless groups that will be used to predict the mass transfer coefficient, we will follow the steps of dimensional analysis:

1. Identify the important variables in the system

The important variables are: $d, V, \mu, \rho, D_{AB}, k_c$

2. List all the problem variables and parameters, along with their dimensions

Quantity	Dimensional expression
d	L
V	L/t
μ	M/Lt
ρ	M/L ³
D_{AB}	L ² /t
k_c	L/t

The number of dimensionless groups = number of variables – number of fundamental dimensions

The number of dimensionless groups = 6 – 3 = 3

3. Choose a set of reference variables

Choose a set of reference variables. The choice of variables is arbitrary, except that the following criteria must be satisfied:

- The number of reference variables must be equal to the minimum number of fundamental dimensions in the problem (in this case, three).
- No two reference variables should have exactly the same dimensions.
- All the dimensions that appear in the problem variables must also appear somewhere in the dimensions of the reference variables.

Note: In selection of the reference variable, try to select the variable of the simplest dimensions.

In this case the reference variables will be: d, V and ρ

$$d = L$$

$$V = L/t$$

$$\rho = M/L^3$$

4. Solve the dimensional equations for the dimensions (L, m and t) in terms of the reference variables

$$L = d$$

$$t = d/V$$

$$M = \rho d^3$$

5. Write the dimensional equations for each of the remaining variables in terms of the reference variables

$\mu = \frac{M}{Lt} = \frac{\rho d^3}{d(d/V)} = \rho V d$	(1)
$D_{AB} = \frac{L^2}{t} = \frac{d^2}{d/V} = V d$	(2)
$k_c = \frac{L}{t} = \frac{d}{d/V} = V$	(3)

Divide equation (1) by equation (2) and equation (3) by equation (2)

$\frac{\mu}{D_{AB}} = \rho$	(4)
$\frac{k_c}{D_{AB}} = \frac{1}{d}$	(5)

6. The resulting equations are each a dimensional identity, so dividing one side by the other results in one dimensionless group from each equation

From equations (1), (4) and (5)

$$N_1 = \frac{\rho V d}{\mu} = Re$$

$$N_2 = \frac{\mu}{\rho D_{AB}} = Sc$$

$$N_3 = \frac{k_c d}{D_{AB}} = Sh$$

These three dimensionless groups can now be used as the primary variables to define the system behavior in place of the original six variables (i.e. number of variables is reduced).

The above groups can be written in the form of dimensionless equation represent the system:

$$Sh = f(Re, Sc)$$

$$Sh = a Re^\alpha Sc^\beta$$

The above equation is a mass transfer correlation and the constants in the correlation are determined experimentally.

There are different forms of correlations depending on the hydrodynamic conditions (laminar or turbulent flow) and the geometry of the solid surface.

Convective mass transfer correlations

Extensive data have been obtained for the transfer of mass between a moving fluid and certain shapes, such as flat plates, spheres and cylinders. The techniques employed include sublimation of a solid, vaporization of a liquid into a moving stream of air and the dissolution of a solid into water. By correlating the data in terms of dimensionless parameters, these empirical equations can be extended to other moving fluids and geometrically similar surfaces.

Table (1) shows the dimensionless numbers used in correlating mass-transfer data (L = characteristic length)

Mass transfer to plates, spheres, and cylinders

Mass transfer correlations for flat plate:

Correlation	Conditions
$Sh_L = 0.664 Re_L^{0.5} Sc^{0.33}$	(laminar) $Re_L < 2 \times 10^5$
$Sh_L = 0.0365 Re_L^{0.8} Sc^{0.33}$	(turbulent) $Re_L > 2 \times 10^5$

Local and average mass transfer coefficient

For a fluid flowing over a flat plate of total length L and width W the average mass transfer coefficient is denoted by \bar{k}_c and at a distance x from the leading edge the local mass transfer efficient is denoted by k_c .

Table (1) dimensionless numbers used in correlating mass-transfer data (L = characteristic length)

Name	Symbol	Dimensionless Group
Reynolds number	Re	$\frac{v_{\infty} \rho L}{\mu} = \frac{v_{\infty} L}{\nu}$
Sherwood number	Sh	$\frac{k_c L}{D_{AB}}$
Schmidt number	Sc	$\frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$
Lewis number	Le	$\frac{\alpha}{D_{AB}} = \frac{k}{\rho c_p D_{AB}}$
Prandtl number	Pr	$\frac{\nu}{\alpha} = \frac{\mu c_p}{k}$
Peclet number	Pe _{AB}	$\frac{v_{\infty} L}{D_{AB}} = \text{Re Sc}$
Stanton number	St _{AB}	$\frac{k_c}{v_{\infty}}$
Grashof number	Gr	$\frac{L^3 \rho g \Delta \rho}{\mu^2}$
Mass transfer <i>j</i> -factor	<i>j_D</i>	$\frac{k_c}{v_{\infty}} (\text{Sc})^{2/3}$
Heat transfer <i>j</i> -factor	<i>j_H</i>	$\frac{h}{\rho c_p v_{\infty}} (\text{Pr})^{2/3}$

Derive the relation between \bar{k}_c and k_c ?

Assume a fluid passing over a flat surface of length L and width W

$$\text{rate of mass transfer} = \bar{k}_c W L (c_{A_s} - c_{A_{\infty}}) = \int k_c (c_{A_s} - c_{A_{\infty}}) dA$$

$$A = Wx$$

$$dA = Wdx$$

$$\bar{k}_c W L (c_{A_s} - c_{A_{\infty}}) = \int_0^L k_c W (c_{A_s} - c_{A_{\infty}}) dx$$

$$\therefore \bar{k}_c = \frac{1}{L} \int_0^L k_c dx$$

The local mass transfer coefficient can be evaluated from the flowing correlations:

Correlation	Conditions
$Sh_x = 0.332 Re_x^{0.5} Sc^{0.33}$	(laminar) $Re_x < 2 \times 10^5$
$Sh_x = 0.0292 Re_x^{0.8} Sc^{0.33}$	(turbulent) $Re_x > 2 \times 10^5$

Example (1):

If the local mass transfer coefficient is evaluated from the following correlation for laminar flow: $Sh_x = 0.332 Re_x^{0.5} Sc^{0.33}$. Derive the corresponding equation (correlation) to calculate the average mass transfer coefficient. Repeat it for turbulent flow if $Sh_x = 0.0292 Re_x^{0.8} Sc^{0.33}$.

Example (2):

If the local Sherwood number for the laminar layer that is formed over a flat plate is

$$Sh_x = 0.332 Re_x^{0.5} Sc^{0.33}$$

and for the turbulent layer is

$$Sh_x = 0.0292 Re_x^{0.8} Sc^{0.33}$$

evaluate

- The value of the mass-transfer coefficient at a point where the Reynolds number is 70,000
- The average film-transfer coefficient, \bar{k}_c , for the flat plate from the leading edge to the point where the Reynolds number is 70,000.
- The value of the mass-transfer coefficient at a point where the Reynolds number is 700,000

Given:

- $D_{AB} = 2.5 \times 10^{-5} \frac{m^2}{s}$
- $\nu = 1.5 \times 10^{-5} \frac{m^2}{s}$

Mass transfer correlations for single sphere:

$$Sh = 2 + C Re^m Sc^{0.33}$$

C and m are constants and their values depend on the type of fluid, the range of Re and the range of Sc. The characteristic length in Sh and Re is the sphere diameter.

Ex: For mass transfer from a single sphere into gas streams:

$$Sh = 2 + 0.552 Re^{0.5} Sc^{0.33}$$

The above equation is used for $2 < Re < 800$ and $0.6 < Sc < 2.7$

For mass transfer into liquid streams: for ($Pe < 10000$)

$$Sh = (4 + 1.21 Pe^{2/3})^{1/2}$$

for ($Pe > 10000$)

$$Sh = 1.01 Pe^{1/3}$$

The above three equations are used to describe forced convection mass-transfer coefficients only when the effects of free or natural convection are negligible

Example (3):

The mass flux from a 5 cm diameter naphthalene ball placed in stagnant air at 40°C and atmospheric pressure, is $1.47 \times 10^{-3} \frac{\text{mol}}{\text{m}^2 \cdot \text{s}}$. Assume the vapor pressure of naphthalene to be 0.15 atm at 40 °C and negligible bulk concentration of naphthalene in air. If air starts blowing across the surface of naphthalene ball at 3 m/s by what factor will the mass transfer rate increase, all other conditions remaining the same?

For mass transfer from a single sphere into gas streams:

$$Sh = 2 + 0.552 Re^{0.5} Sc^{0.33}$$

The viscosity and density of air are $1.8 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}}$ and $1.123 \frac{\text{kg}}{\text{m}^3}$, respectively and the gas constant is $82.06 \frac{\text{cm}^3 \cdot \text{atm}}{\text{mol} \cdot \text{K}}$.

Solution:

$$\text{The factor of increase in mass transfer} = \frac{\text{Mass flux in moving air}}{\text{Mass flux in stagnant air}} = \frac{k_{c1} \Delta c}{k_{c2} \Delta c} = \frac{k_{c1}}{k_{c2}}$$

For k_{c2} :

$$1.47 \times 10^{-3} = k_{c2} \left(\frac{p_v}{RT} - 0.0 \right) = k_{c2} \times \frac{0.15 \times 1000}{0.08206 \times 313}$$

$$k_{c2} = 2.52 \times 10^{-4} \text{ m/s}$$

For k_{c1} :

$$Sh = 2 + 0.552 Re^{0.5} Sc^{0.33}$$

$$\frac{k_{c1} \times 5 \times 10^{-2}}{D_{AB}} = 2 + 0.552 \left(\frac{1.123 \times 3 \times 0.05}{1.8 \times 10^{-5}} \right)^{0.5} \left(\frac{1.8 \times 10^{-5}}{1.123 \times D_{AB}} \right)^{0.33}$$

$D_{AB}??$

For stagnant air ($Re = 0.0$)

$$Sh = 2$$

$$\frac{k_{c_2} \times 5 \times 10^{-2}}{D_{AB}} = 2$$

$$2.52 \times 10^{-4}$$

$$\frac{2.52 \times 10^{-4} \times 5 \times 10^{-2}}{D_{AB}} = 2$$

$$D_{AB} = 6.2925 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\therefore k_{c_1} = 0.0102 \text{ m/s}$$

The factor of increase = $\frac{k_{c_1}}{k_{c_2}} = \frac{0.0102}{2.52 \times 10^{-4}} = 40.5$

Mass transfer correlations for single cylinder

Several investigators have studied the sublimation from a solid cylinder into air flowing normal to its axis. Additional results on the dissolution of solid cylinders into a turbulent water stream have been reported.

$$\frac{k_c Sc^{0.56}}{V_\infty} = 0.281 Re_D^{-0.4}$$

For the conditions: $400 < Re_D < 25000$ and $0.6 < Sc < 2.6$

Flow through pipes:

For mass transfer from the inner wall of a tube to a moving fluid:

$$Sh = 0.032 Re^{0.8} Sc^{0.33}$$

For the conditions: $2000 < Re_D < 35000$ and $1000 < Sc < 2260$

Example (4):

A solid disc of benzoic acid 3 cm in diameter is spin at 20 rpm and 25 °C. Calculate the rate of dissolution in a large volume of water. Diffusivity of benzoic acid in water is $1 \times 10^{-5} \frac{\text{cm}^2}{\text{s}}$, and solubility is 0.003 g/cm³. The following mass transfer correlation is applicable: $Sh = 0.62 Re^{0.5} Sc^{0.33}$

Where $Re = \frac{d^2 \omega \rho}{\mu}$ and ω is the angular speed in radians/time.

(2) Mass, heat and momentum transfer analogy

Analogies among mass, heat and momentum transfer have their origin either in the mathematical description of the effects or in the physical parameters used for quantitative description.

To explore those analogies, it could be understood that the diffusion of mass and conduction of heat obey very similar equations. In particular, diffusion in one dimension is described by the Fick's Law as

$$J_A = -D_{AB} \frac{dc_A}{dz}$$

Similarly, heat conduction is described by Fourier's law as

$$q = -k \frac{dT}{dz}$$

where k is the thermal conductivity.

The similar equation describing momentum transfer as given by Newton's law is

$$\tau = -\mu \frac{du}{dz}$$

where τ is the momentum flux (or shear stress) and μ is the viscosity of fluid.

At this point it has become conventional to draw an analogy among mass, heat and momentum transfer. Each process uses a simple law combined with a mass or energy or momentum balance.

In this section, we shall consider several analogies among transfer phenomenon which has been proposed because of the similarity in their mechanisms. The analogies are useful in understanding the transfer phenomena and as a satisfactory means for predicting behavior of systems for which limited quantitative data are available.

The similarity among the transfer phenomena and accordingly the existence of the analogies require that the following five conditions exist within the system

1. The physical properties are constant
2. There is no mass or energy produced within the system. This implies that there is no chemical reaction within the system
3. There is no emission or absorption of radiant energy.
4. There is no viscous dissipation of energy.
5. The velocity profile is not affected by the mass transfer. This implies there should be a low rate of mass transfer.

2.1. Reynolds Analogy

The first recognition of the analogous behavior of mass, heat and momentum transfer was reported by Osborne Reynolds in 1874. Although his analogy is limited in application, it served as the base for seeking better analogies.

Reynolds postulated that the mechanisms for transfer of momentum, energy and mass are identical. Accordingly,

$$\frac{k_c}{V_\infty} = \frac{h}{\rho V_\infty c_p} = \frac{f}{2}$$

Where: h is heat transfer coefficient, f is friction factor, V_∞ is velocity of free stream

The Reynolds analogy is interesting because it suggests a very simple relation between different transport phenomena. This relation is found to be accurate when Prandtl and Schmidt numbers are equal to one. This is applicable for mass transfer by means of turbulent eddies in gases. In this situation, we can estimate mass transfer coefficients from heat transfer coefficients or from friction factors.

2.2. Chilton – Colburn Analogy

Because the Reynolds analogy was practically useful, many authors tried to extend it to liquids. Chilton and Colburn, using experimental data, sought modifications to the Reynolds analogy that would not have the restrictions that Prandtl and Schmidt numbers must be equal to one. They defined for the j factor for mass transfer as

$$j_D = \frac{k_c}{V_\infty} Sc^{2/3}$$

and the analogous j factor for heat transfer as

$$j_H = St Pr^{2/3}$$

St is the Stanton number $St = \frac{Nu}{Re Pr} = \frac{h}{\rho V_{\infty} c_p}$

Based on data collected in both laminar and turbulent flow regimes, they found

$$j_D = j_H = \frac{f}{2}$$

This analogy is valid for gases and liquids within the range of $0.6 < Sc < 2500$ and $0.6 < Pr < 100$.

The Chilton-Colburn analogy has been observed to hold for much different geometry for example, flow over flat plates, flow in pipes, and flow around cylinders.

Example (5):

A stream of air at 100 kPa pressure and 300 K is flowing on the top surface of a thin flat sheet of solid naphthalene of length 0.2 m with a velocity of 20 m/sec. The other data are:

Mass diffusivity of naphthalene vapor in air = $6 \times 10^{-6} \text{ m}^2/\text{s}$

Kinematic viscosity of air = $1.5 \times 10^{-5} \text{ m}^2/\text{s}$

Concentration of naphthalene at the air-solid naphthalene interface = $1 \times 10^{-5} \text{ kmol}/\text{m}^3$

For heat transfer over a flat plate, convective heat transfer coefficient for laminar flow can be calculated by the equation:

$$Nu = 0.664 Re_L^{0.5} Pr^{0.33}$$

Required:

- (a) Develop an equation for Sh_L for laminar flow over a flat plate by using Chilton – Colburn analogy
- (b) the average mass transfer coefficient over the flat plate

(c) the rate of loss of naphthalene from the surface per unit width

Solution:

a. The analogous relation for mass transfer

From Chilton – Colburn analogy:

$$\frac{k_c}{V_\infty} Sc^{2/3} = \frac{h}{\rho V_\infty c_p} Pr^{2/3}$$

$$h = k_c \rho c_p \frac{Sc^{2/3}}{Pr^{2/3}}$$

$$\frac{k_c \rho c_p \frac{Sc^{2/3}}{Pr^{2/3}} L}{k} = 0.664 Re_L^{0.5} Pr^{0.33}$$

$$\left(\frac{k_c L}{D_{AB}}\right) \left(\frac{\mu c_p}{k}\right) \left(\frac{\rho D_{AB}}{\mu}\right) Sc^{2/3} = 0.664 Re_L^{0.5} Pr$$

$$Sh = 0.664 Re_L^{0.5} Sc^{0.33}$$

b. k_c is calculated from the mass transfer correlation

$$\frac{k_c \times 0.2}{6 \times 10^{-6}} = 0.664 \left(\frac{0.2 \times 20}{1.5 \times 10^{-5}}\right)^{0.5} \left(\frac{1.5 \times 10^{-5}}{6 \times 10^{-6}}\right)^{0.33}$$

$$k_c = 0.014 \text{ m/s}$$

c. Rate of loss of naphthalene = $k_c A (c_{A,s} - c_{A,\infty}) = k_c WL (c_{A,s} - c_{A,\infty})$

$$W = 1(\text{unit width})$$

$$\text{Rate of loss of naphthalene per unit width} = 1.04024 \times 10^{-7} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

Solved problems:

Problem 1:

In applying dimensional analysis to explain mass-transfer coefficient, one must consider the geometry involved, a variable to explain the flow characteristics of the moving stream, and the properties of the moving stream. Predict the variables that are necessary to explain the mass-transfer coefficient for a gas stream flowing over a flat plate and arrange these variables into dimensionless groups.

Solution:

To get the dimensionless groups that will be used to predict the mass transfer coefficient, we will follow the steps of dimensional analysis:

1. Identify the important variables in the system

The important variables are: $L, V, \mu, \rho, D_{AB}, k_c$

2. List all the problem variables and parameters, along with their dimensions

Quantity	Dimensional expression
L	L
V	L/t
μ	M/Lt
ρ	M/L ³
D_{AB}	L ² /t
k_c	L/t

The number of dimensionless groups = number of variables – number of fundamental dimensions

The number of dimensionless groups = $6 - 3 = 3$

3. Choose a set of reference variables

Choose a set of reference variables. The choice of variables is arbitrary, except that the following criteria must be satisfied:

- The number of reference variables must be equal to the minimum number of fundamental dimensions in the problem (in this case, three).

- No two reference variables should have exactly the same dimensions.
- All the dimensions that appear in the problem variables must also appear somewhere in the dimensions of the reference variables.

Note: In selection of the reference variable, try to select the variable of the simplest dimensions.

In this case the reference variables will be: L, V and ρ

$$L = L$$

$$V = L/t$$

$$\rho = M/L^3$$

4. Solve the dimensional equations for the dimensions (L, m and t) in terms of the reference variables

$$L = L$$

$$t = d/V$$

$$M = \rho d^3$$

6. Write the dimensional equations for each of the remaining variables in terms of the reference variables

$\mu = \frac{M}{Lt} = \frac{\rho d^3}{L(L/V)} = \rho VL$	(1)
$D_{AB} = \frac{L^2}{t} = \frac{L^2}{L/V} = VL$	(2)
$k_c = \frac{L}{t} = \frac{L}{L/V} = V$	(3)

Divide equation (1) by equation (2) and equation (3) by equation (2)

$\frac{\mu}{D_{AB}} = \rho$	(4)
$\frac{k_c}{D_{AB}} = \frac{1}{L}$	(5)

7. The resulting equations are each a dimensional identity, so dividing one side by the other results in one dimensionless group from each equation

From equations (1), (4) and (5)

$$N_1 = \frac{\rho V d}{\mu} = Re$$

$$N_2 = \frac{\mu}{\rho D_{AB}} = Sc$$

$$N_3 = \frac{k_c d}{D_{AB}} = Sh$$

Problem 2:

In a mass transfer spray column, a liquid is sprayed into a gas stream, and mass is transferred between the liquid and gas phases. The mass of the drops that are formed from a spray nozzle is considered a function of the nozzle diameter, acceleration of gravity, and surface tension of the liquid against the gas, fluid density, fluid viscosity, fluid velocity, and the viscosity and density of the gas medium. Arrange these variables in dimensionless groups. Should any other variables have been included?

Note: surface tension has a unit of force per unit length or

Solution:

1. Identify the important variables in the system

The important variables are: $m, d, g, \sigma, \rho_L, \mu_L, V, \rho_g, \mu_g$

2. List all the problem variables and parameters, along with their dimensions

Quantity	Dimensional expression
m	M
d	L
g	L/t ²
σ	M/t ²
ρ_L	M/L ³
μ_L	M/Lt
V	L/t
ρ_g	M/L ³
μ_g	M/Lt

The number of dimensionless groups = number of variables – number of fundamental dimensions

The number of dimensionless groups = 9 – 3 = 6

3. Choose a set of reference variables

In this case the reference variables will be: d , V and ρ

$$d = L$$

$$V = L/t$$

$$\rho_L = M/L^3$$

4. Solve the dimensional equations for the dimensions (L, m and t) in terms of the reference variables

$$L = d$$

$$t = d/V$$

$$M = \rho d^3$$

5. Write the dimensional equations for each of the remaining variables in terms of the reference variables

Quantity	Dimensional expression	In terms of the reference variables
m	M	$\rho_L d^3$
g	L/t^2	$\frac{V^2}{d}$
σ	M/t^2	$\rho_L d V^2$
μ_L	M/Lt	$\rho_L d V$
ρ_g	M/L^3	ρ_L
μ_g	M/Lt	$\rho_L d V$

6. The resulting equations are each a dimensional identity, so dividing one side by the other results in one dimensionless group from each equation

N_1	$\left(\frac{\rho_L d^3}{m}\right)$
N_2	$\left(\frac{V^2}{gd}\right)$

N_3	$\left(\frac{\rho_L dV^2}{\sigma}\right)$
N_4	$\left(\frac{\rho_L dV}{\mu_L}\right)$
N_5	$\left(\frac{\rho_L}{\rho_g}\right)$
N_6	$\left(\frac{\rho_L dV}{\mu_g}\right)$

Problem 3:

Dry air, flowing at a velocity of 1.5 m/s, enters a 6-m- long, 0.15-m-diameter tube at 310 K and 1.013×10^5 Pa. The inner surface of the tube is lined with a felt material (diameter-to- roughness ratio, $\frac{D}{\epsilon}$, of 10,000) that is continuously saturated with water at 290 K. Assuming constant temperature of the air and the pipe wall, determine the exit concentration of water in the exit gas stream.

Given that:

Diffusivity of water in air at 300 K = $2.6 \times 10^{-5} \frac{m^2}{s}$

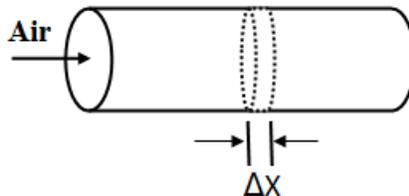
Kinematic viscosity of air at 300 K = $1.569 \times 10^{-5} \frac{m^2}{s}$

The vapor pressure of water at 290 K = 17.5 mm Hg

The gas constant = $0.08206 \frac{L \cdot atm}{mol \cdot K}$

The Fanning friction factor is given by: $f = 0.00791 Re^{0.12}$

Solution:



By making material balance on water over a control volume of air of a thickness Δx
 Water input with air + water transferred to air from the wall by convection = water out with air

$$c_A V \frac{\pi d^2}{4} \Big|_x + k_c (c_{A_s} - c_A) \pi d \Delta x = c_A V \frac{\pi d^2}{4} \Big|_{x+\Delta x}$$

$$\div \frac{\pi d^2}{4} \Delta x$$

$$\frac{c_A \Big|_{x+\Delta x} - c_A \Big|_x}{\Delta x} = \frac{4 k_c}{d V} (c_{A_s} - c_A)$$

Take the limits as Δx approaches zero

$$\frac{dc_A}{dx} = \frac{4 k_c}{d V} (c_{A_s} - c_A)$$

$$- \int_{c_{A_o}}^{c_{A_L}} \frac{-dc_A}{(c_{A_s} - c_A)} = \frac{4 k_c}{d V} \int_0^L dx$$

$\ln \left(\frac{c_{A_s} - c_{A_o}}{c_{A_s} - c_{A_L}} \right) = \frac{4 k_c}{d V} L$	(1)
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$$\ln \left(\frac{c_{A_s} - c_{A_o}}{c_{A_s} - c_{A_L}} \right) = \frac{4 k_c}{d V} L$$

$d = 0.15 \text{ m}$	$V = 1.5 \frac{\text{m}}{\text{s}}$
$L = 6 \text{ m}$	$c_{A_o} = 0 \text{ (dry air)}$
$c_{A_s} = \frac{p_v}{RT} = \frac{17.5}{0.08206 \times 290} = 0.735 \text{ mol/L}$	$k_c = ??? \text{ (from analogy)}$

From Chilton – Colburn analogy:

$$\frac{k_c}{V_\infty} Sc^{2/3} = \frac{f}{2}$$

$$f = 0.00791 Re^{0.12}$$

$$Re = \frac{Vd}{\nu} = \frac{1.5 \times 0.15}{1.569 \times 10^{-5}} = 1.434 \times 10^5$$

$$f = 0.00791 (1.434 \times 10^5)^{0.12} = 0.03288$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}} = \frac{1.569 \times 10^{-5}}{2.6 \times 10^{-5}} = 0.603$$

$$\therefore \frac{k_c}{V_\infty} (0.603)^{2/3} = \frac{0.03288}{2}$$

$$\therefore \frac{k_c}{V_\infty} = 0.023$$

Substitute in equation (1) we can get $c_{A_L} = 0.716 \text{ mol/L}$

Notes about this problem:

- a) If the equation $f = 0.00791 Re^{0.12}$ is not given, the friction factor can be obtained from Moody chart by using the value of the given surface roughness.
- b) The diffusion coefficient may be not given. In this case you have to use the Hirschfelder equation to calculate it.

Problem 4:

Air passes through a naphthalene tube, that has an inside diameter of 2.5 cm, flowing at a bulk velocity of 15 m/s. The air is at 283 K and an average pressure of $1.013 \times 10^5 \text{ Pa}$. Assuming that the change in pressure along the tube is negligible and that the naphthalene surface is at 283 K, determine the length of tube that is necessary to produce a naphthalene concentration in the exiting gas stream of $4.75 \times 10^{-4} \text{ mol/m}^3$. At 283 K, naphthalene has a vapor pressure of 3 Pa and a diffusivity in air of $5.4 \times 10^{-6} \text{ m}^2/\text{s}$.

Given that:

Diffusivity of naphthalene in air at 283 K = $5.4 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$

Kinematic viscosity of air at 283 K = $1.415 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$

The vapor pressure of water at 283 K = 3 Pa

The gas constant = $8.314 \frac{\text{m}^3 \cdot \text{Pa}}{\text{mol} \cdot \text{K}}$

The Fanning friction factor is given by: $f = 0.007 Re^{0.11}$

Solution:

By the same procedure as in the above problem get the equation:

$$\ln \left(\frac{c_{A_s} - c_{A_o}}{c_{A_s} - c_{A_L}} \right) = \frac{4 k_c}{d V} L$$

$d = 2.5 \text{ cm}$	$V = 15 \frac{\text{m}}{\text{s}}$
$L = ??$	$c_{A_o} = 0(\text{pure air})$
$c_{A_s} = \frac{p_v}{RT} = \frac{3}{8.314 \times 283} = 1.75 \times 10^{-3} \text{ mol/m}^3$	$c_{A_L} = 4.75 \times 10^{-4} \text{ mol/m}^3$

From Chilton – Colburn analogy:

$$\frac{k_c}{V_\infty} Sc^{2/3} = \frac{f}{2}$$

$$f = 0.007 Re^{0.11}$$

$$Re = \frac{Vd}{\nu} = \frac{0.025 \times 15}{1.415 \times 10^{-5}} = 2.6501 \times 10^4$$

$$f = 0.007 (2.6501 \times 10^4)^{0.12} = 0.021$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}} = \frac{1.415 \times 10^{-5}}{5.4 \times 10^{-6}} = 2.62$$

$$\therefore \frac{k_c}{V_\infty} (2.62)^{2/3} = \frac{0.021}{2}$$

$$\therefore \frac{k_c}{V_\infty} = 0.00555$$

$$\ln \left(\frac{c_{As} - c_{Ao}}{c_{As} - c_{AL}} \right) = \frac{4 k_c}{d V} L$$

$$\ln \left(\frac{1.75 \times 10^{-3} - 0.0}{1.75 \times 10^{-3} - 4.75 \times 10^{-4}} \right) = \frac{4}{0.025} \times 0.00555 \times L$$

$$L = 1.9 \text{ m}$$

Problem 5:

Davis investigated heat transfer to fluids flowing in the annular section between concentric tubes. He proposed the following correlation for the heat-transfer coefficient when considering the film on the inner tube:

$$\frac{hd_1}{k} = 0.031 (Re_{d_1})^{0.8} (Pr)^{0.31} \left(\frac{\mu}{\mu_s} \right)^{0.14} \left(\frac{d_2}{d_1} \right)^{0.15}$$

where d_1 and d_2 are the outer diameter of the inner tube and the inner diameter of the outer tube, respectively, μ is the viscosity of the fluid at the bulk temperature of the fluid, and μ_s is the viscosity of the fluid at the heating surface temperature. Use this equation and the analogy between heat and mass transfer to get the corresponding mass transfer correlation that can be used to predict the mass-transfer coefficient from a naphthalene rod used to form the inner tube of an annular duct.

Solution:

From Chilton – Colburn analogy:

$$\frac{k_c}{V_\infty} Sc^{2/3} = \frac{h}{\rho V_\infty c_p} Pr^{2/3}$$

$$h = k_c \rho c_p \frac{Sc^{2/3}}{Pr^{2/3}}$$

$$\frac{k_c \rho c_p \frac{Sc^{2/3}}{Pr^{2/3}} d_1}{k} = 0.031 (Re_{d_1})^{0.8} (Pr)^{0.31} \left(\frac{\mu}{\mu_s}\right)^{0.14} \left(\frac{d_2}{d_1}\right)^{0.15}$$

$$\left(\frac{k_c d_1}{D_{AB}}\right) \left(\frac{\mu c_p}{k}\right) \left(\frac{\rho D_{AB}}{\mu}\right) Sc^{2/3} = 0.031 (Re_{d_1})^{0.8} Pr \left(\frac{\mu}{\mu_s}\right)^{0.14} \left(\frac{d_2}{d_1}\right)^{0.15}$$

$$\therefore \left(\frac{k_c d_1}{D_{AB}}\right) = 0.031 (Re_{d_1})^{0.8} Sc^{0.31} \left(\frac{\mu}{\mu_s}\right)^{0.14} \left(\frac{d_2}{d_1}\right)^{0.15}$$

Problem 6:

McAdams presented the heat-transfer equation for the turbulent flow of gases past a single sphere:

$$Nu = 0.37(Re_d)^{0.6} Pr^{0.33}$$

where d_p is the diameter of the sphere. Predict the equation you might use to correlate the mass-transfer coefficient from a single sphere into a turbulent gas stream.

Solution:

The same procedure as the above problem and we will get the equation:

$$Sh = 0.37(Re_d)^{0.6} Sc^{0.33}$$

Problem 7:

The boundary layer solution for a flat plate predicts the following equations:

For laminar flow is:

$$\frac{k_c x}{D_{AB}} = 0.332 Re_x^{0.5} Sc^{0.33}$$

and for the turbulent layer is

$$\frac{k_c x}{D_{AB}} = 0.0292 Re_x^{0.8} Sc^{0.33}$$

with the transition occurring at $Re_x = 2 \times 10^5$

Determine what percentage of the mass transfer occurs in the laminar zone of the flow over the flat plate if the Reynolds number at the end of the plate is $Re_L = 3 \times 10^6$

Solution:

$$\% \text{ mass transfer occurs in the laminar zone} = \frac{N_{A_L}}{N_{A_L} + N_{A_t}} = \frac{N_{A_L}}{\bar{N}_A}$$

$$= \frac{\bar{k}_{c_{laminar}}}{\bar{k}_c}$$

$$\bar{k}_{c_{laminar}} = \frac{1}{L_t} \int_0^{L_{tr}} k_c dx$$

$$= \frac{1}{L_t} (0.332) Sc^{0.33} \left(\frac{V}{\nu}\right)^{0.5} \int_0^{L_{tr}} x^{0.5} dx$$

$$\bar{k}_{c_{laminar}} = \frac{0.664}{L_t} Sc^{0.33} Re_{tr}^{0.5} D_{AB}$$

$$\bar{k}_c = \frac{\int_0^{L_{tr}} k_{c_{laminar}} + \int_{L_{tr}}^L k_{c_{laminar}}}{L}$$

$$\int_{L_{tr}}^L k_{c_{turbulent}} = 0.0292 Sc^{0.33} \left(\frac{V}{\nu}\right)^{0.8} \int_{L_{tr}}^L x^{0.8} dx$$

$$= 0.0365 Sc^{0.33} (Re_L^{0.8} - Re_{L_{tr}}^{0.8}) D_{AB}$$

$$\bar{k}_c = \frac{0.664 Sc^{0.33} Re_{tr}^{0.5} D_{AB} + 0.0365 Sc^{0.33} (Re_L^{0.8} - Re_{L_{tr}}^{0.8}) D_{AB}}{L}$$

$$\text{laminar fraction} = \frac{\frac{0.664}{L_t} Sc^{0.33} Re_{tr}^{0.5} D_{AB}}{\frac{0.664 Sc^{0.33} Re_{tr}^{0.5} D_{AB} + 0.0365 Sc^{0.33} (Re_L^{0.8} - Re_{L_{tr}}^{0.8}) D_{AB}}{L}}$$

$$\text{laminar fraction} = \frac{0.664 Re_{tr}^{0.5}}{0.664 Re_{tr}^{0.5} + 0.0365 (Re_L^{0.8} - Re_{L_{tr}}^{0.8})} \frac{L}{L_t}$$

$$\begin{aligned} \text{laminar fraction} &= \frac{0.664 Re_{tr}^{0.5}}{0.664 Re_{tr}^{0.5} + 0.0365 (Re_L^{0.8} - Re_{Ltr}^{0.8})} \frac{Re_L}{Re_{Ltr}} \\ &= \frac{0.664 (2 \times 10^5)^{0.5}}{0.664 (2 \times 10^5)^{0.5} + 0.0365 ((3 \times 10^6)^{0.8} - (2 \times 10^5)^{0.8})} \frac{(3 \times 10^6)^{0.8}}{(2 \times 10^5)^{0.5}} \end{aligned}$$

Problem 8:

A very thin polymeric coating of thickness 0.1 mm uniformly coats a rectangular surface. The rectangular surface has a length of 20 cm and a width of 10 cm. The coating contains a solvent that must be evaporated away from the coating in order to cure the coating. Initially, there is 0.001 mole of solvent per cm³ of coating loaded in the coating. A heated plate just beneath the surface maintains the coating at a uniform temperature of 40 °C, and the vapor pressure exerted by the solvent is 0.05 atm at 40 °C. Air gently flows parallel to the surface at a velocity of 5.0 cm/s. The surrounding air at 1.0 atm total system pressure and 20 °C represents an “infinite sink” for mass transfer. You may neglect any molecular diffusion of the solvent through the very thin polymeric film and focus only on the convection aspects of the problem.

- What is the average mass transfer coefficient, k_c ?
- How long will it take for the solvent to completely evaporate from the coating?

Given:

The physical properties at 303 K are:

Kinematic viscosity $\nu = 0.158 \text{ cm}^2/\text{s}$

Density $\rho = 1.17 \times 10^{-3} \text{ g/cm}^3$

The diffusion coefficient of species in air at 30 °C, $D_{AB} = 1.025 \text{ cm}^2/\text{s}$

Correlation	Conditions
$Sh_L = 0.664 Re_L^{0.5} Sc^{0.33}$	(laminar) $Re_L < 2 \times 10^5$
$Sh_L = 0.0365 Re_L^{0.8} Sc^{0.33}$	(turbulent) $Re_L > 2 \times 10^5$

Solution:

- a) The mass transfer coefficient is determined from the correlation and depending on the nature of the flow

$$Re = \frac{\rho VL}{\mu} = \frac{VL}{\nu} = \frac{5 \times 20}{0.158} = 633 (\because \text{laminar flow})$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.158}{1.025} = 0.154$$

$$Sh_L = 0.664 (633)^{0.5} (0.154)^{0.33} = 9.16$$

$$\frac{k_c L}{D_{AB}} = 9.16$$

$$\therefore k_c = 0.469 \text{ cm/s}$$

- b) The time required for the solvent to completely evaporate from the coating

$$\text{time (s)} = \frac{\text{total moles of solvent}}{\text{rate of evaporation } \left(\frac{\text{mol}}{\text{s}}\right)}$$

$$\text{total moles of solvent} = 20 \times 10 \times 0.01 \times 0.001 = 0.002 \text{ mole}$$

$$\text{rate of evaporation} = k_c A (c_{A_s} - c_{A_\infty})$$

$$c_{A_s} = \frac{p_v}{RT} = \frac{0.05}{0.08206 \times 313} = 0.00194 \frac{\text{mol}}{\text{L}} = 1.94 \times 10^{-6} \frac{\text{mol}}{\text{cm}^3}$$

$$c_{A_\infty} = 0.0 \text{ (fresh air)}$$

$$\text{time} = \frac{0.002 \text{ mole}}{0.469 \times 10 \times 20 \times 1.94 \times 10^{-6} \left(\frac{\text{mol}}{\text{s}}\right)} = 10.99 \text{ sec}$$