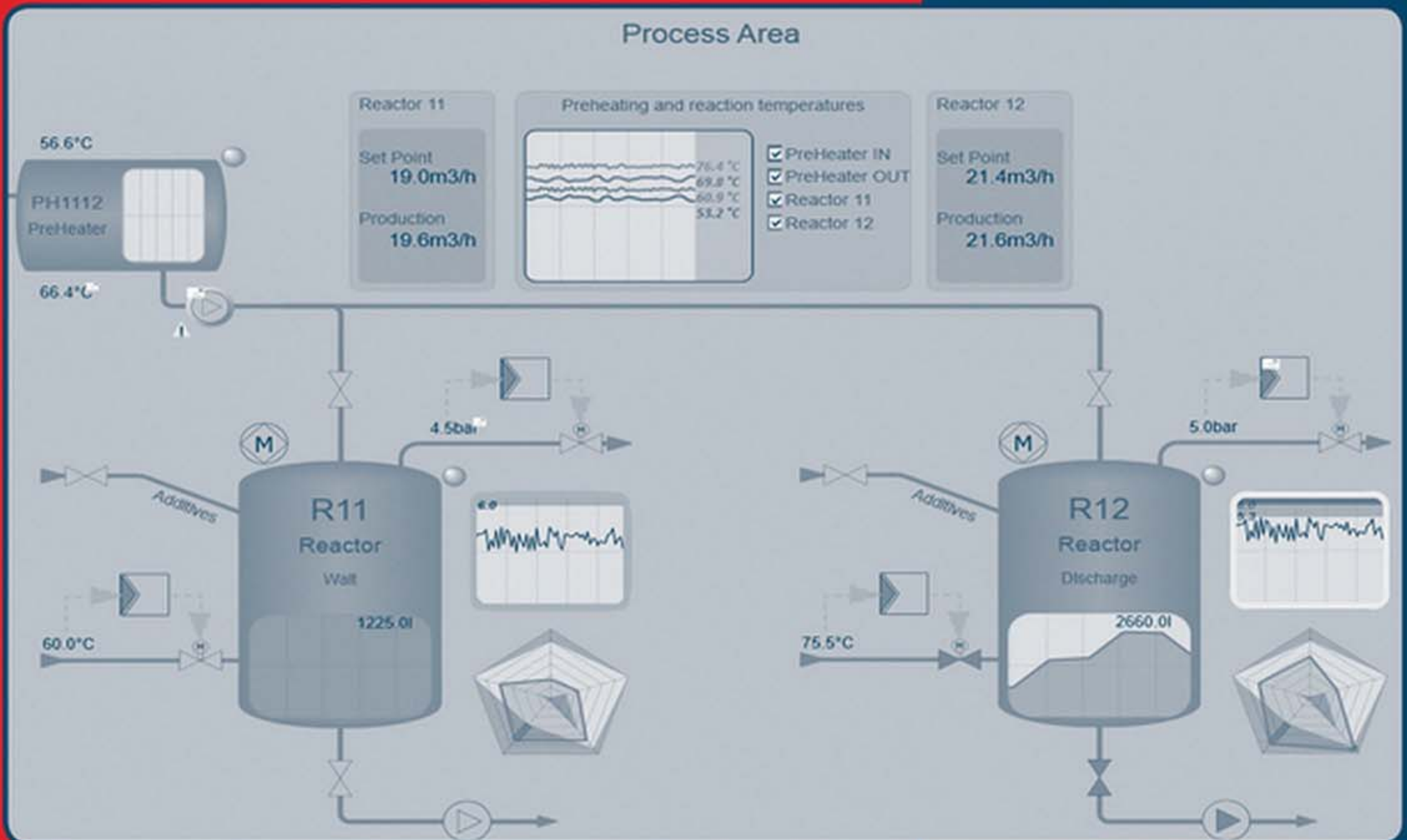
INSTRUCTOR'S SOLUTIONS MANUAL

Process Dynamics and Control

4th Edition



Seborg | Edgar | Mellichamp | Doyle

Chapter 2

2.1

a) Overall mass balance:

$$\frac{d(\rho V)}{dt} = w_1 + w_2 - w_3 \tag{1}$$

Energy balance:

$$C\frac{d\left[\rho V(T_3 - T_{ref})\right]}{dt} = w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) - w_3 C\left(T_3 - T_{ref}\right) \tag{2}$$

Because ρ = constant and $V = \overline{V}$ = constant, Eq. 1 becomes:

$$w_3 = w_1 + w_2 \tag{3}$$

b) From Eq. 2, substituting Eq. 3

$$\rho C\overline{V}\frac{d(T_3 - T_{ref})}{dt} = \rho C\overline{V}\frac{dT_3}{dt} = w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) - (w_1 + w_2)C\left(T_3 - T_{ref}\right) \tag{4}$$

Constants C and T_{ref} can be cancelled:

$$\rho\overline{V}\frac{dT_3}{dt} = w_1 T_1 + w_2 T_2 - (w_1 + w_2)T_3 \tag{5}$$

The simplified model now consists only of Eq. 5.

Degrees of freedom for the simplified model:

Parameters : ρ, $\overline{V}$

Variables : w_1, w_2, T_1, T_2, T_3
$N_E = 1$
$N_V = 5$

Thus, $N_F = 5 - 1 = 4$

Because w_1, w_2, T_1 and T_2 are determined by upstream units, we assume they are known functions of time:

$w_1 = w_1(t)$
$w_2 = w_2(t)$
$T_1 = T_1(t)$
$T_2 = T_2(t)$

Thus, N_F is reduced to 0.

2.2

Energy balance:

$$C_p \frac{d\left[\rho V (T - T_{ref})\right]}{dt} = wC_p(T_i - T_{ref}) - wC_p(T - T_{ref}) - UA_s(T - T_a) + Q$$

Simplifying

$$\rho V C_p \frac{dT}{dt} = wC_p T_i - wC_p T - UA_s(T - T_a) + Q$$
$$\rho V C_p \frac{dT}{dt} = wC_p(T_i - T) - UA_s(T - T_a) + Q$$

b) T increases if T_i increases and vice versa.

T decreases if w increases and vice versa if $(T_i - T) < 0$. In other words, if $Q > UA_s(T-T_a)$, the contents are heated, and $T > T_i$.

2.3

a) Mass Balances:

$$\rho A_1 \frac{dh_1}{dt} = w_1 - w_2 - w_3 \tag{1}$$

$$\rho A_2 \frac{dh_2}{dt} = w_2 \tag{2}$$

Flow relations:

Let P_1 be the pressure at the bottom of tank 1.

Let P_2 be the pressure at the bottom of tank 2.

Let P_a be the ambient pressure.

Then

$$w_2 = \frac{P_1 - P_2}{R_2} = \frac{\rho g}{g_c R_2}(h_1 - h_2) \tag{3}$$

$$w_3 = \frac{P_1 - P_a}{R_3} = \frac{\rho g}{g_c R_3} h_1 \tag{4}$$

b) Seven parameters: ρ, A_1, A_2, g, g_c, R_2, R_3

Five variables : h_1, h_2, w_1, w_2, w_3

Four equations

Thus $N_F = 5 - 4 = 1$

1 input = w_1 (specified function of time)
4 outputs = h_1, h_2, w_2, w_3

2.4

Assume constant liquid density, ρ . The mass balance for the tank is

$$\frac{d(\rho Ah + m_g)}{dt} = \rho(q_i - q)$$

Because ρ, A, and m_g are constant, this equation becomes

$$A\frac{dh}{dt} = q_i - q \tag{1}$$

The square-root relationship for flow through the control valve is

$$q = C_v\left(P_g + \frac{\rho gh}{g_c} - P_a\right)^{1/2} \tag{2}$$

From the ideal gas law,

$$P_g = \frac{(m_g / M)RT}{A(H-h)} \tag{3}$$

where T is the absolute temperature of the gas.

Equation 1 gives the unsteady-state model upon substitution of q from Eq. 2 and of P_g from Eq. 3:

$$A\frac{dh}{dt} = q_i - C_v\left[\frac{(m_g / M)RT}{A(H-h)} + \frac{\rho gh}{g_c} - P_a\right]^{1/2} \tag{4}$$

Because the model contains P_a, operation of the system is not independent of P_a. For an open system $P_g = P_a$ and Eq. 2 shows that the system is independent of P_a.

2.5

a) For linear valve flow characteristics,

$$w_a = \frac{P_d - P_1}{R_a}, \quad w_b = \frac{P_1 - P_2}{R_b}, \quad w_c = \frac{P_2 - P_f}{R_c} \tag{1}$$

Mass balances for the surge tanks

$$\frac{dm_1}{dt} = w_a - w_b, \qquad \frac{dm_2}{dt} = w_b - w_c \tag{2}$$

where m_1 and m_2 are the masses of gas in surge tanks 1 and 2, respectively.

If the ideal gas law holds, then

$$P_1 V_1 = \frac{m_1}{M} RT_1, \qquad P_2 V_2 = \frac{m_2}{M} RT_2 \tag{3}$$

where M is the molecular weight of the gas
T_1 and T_2 are the temperatures in the surge tanks.

Substituting for m_1 and m_2 from Eq. 3 into Eq. 2, and noticing that V_1, T_1, V_2, and T_2 are constant,

$$\frac{V_1 M}{RT_1} \frac{dP_1}{dt} = w_a - w_b \quad \text{and} \quad \frac{V_2 M}{RT_2} \frac{dP_2}{dt} = w_b - w_c \tag{4}$$

The dynamic model consists of Eqs. 1 and 4.

b) For adiabatic operation, Eq. 3 is replaced by

$$P_1 \left(\frac{V_1}{m_1} \right)^{\gamma} = P_2 \left(\frac{V_2}{m_2} \right)^{\gamma} = C, \text{ a constant} \tag{5}$$

or

$$m_1 = \left(\frac{P_1 V_1^{\gamma}}{C} \right)^{1/\gamma} \quad \text{and} \quad m_2 = \left(\frac{P_2 V_2^{\gamma}}{C} \right)^{1/\gamma} \tag{6}$$

Substituting Eq. 6 into Eq. 2 gives,

$$\frac{1}{\gamma} \left(\frac{V_1^{\gamma}}{C} \right)^{1/\gamma} P_1^{(1-\gamma)/\gamma} \frac{dP_1}{dt} = w_a - w_b$$

$$\frac{1}{\gamma}\left(\frac{V_2^{\ \gamma}}{C}\right)^{1/\gamma} P_2^{\ (1-\gamma)/\gamma}\frac{dP_2}{dt} = w_b - w_c$$

as the new dynamic model. If the ideal gas law were not valid, one would use an appropriate equation of state instead of Eq. 3.

2.6

a) Assumptions:

1. Each compartment is perfectly mixed.
2. ρ and C are constant.
3. No heat losses to ambient.

Compartment 1:
Overall balance (No accumulation of mass):

$$0 = \rho q - \rho q_1 \qquad \text{thus} \qquad q_1 = q \tag{1}$$

Energy balance (No change in volume):

$$V_1 \rho C \frac{dT_1}{dt} = \rho q C(T_i - T_1) - UA(T_1 - T_2) \tag{2}$$

Compartment 2:

Overall balance:

$$0 = \rho q_1 - \rho q_2 \qquad \text{thus} \qquad q_2 = q_1 = q \tag{3}$$

Energy balance:

$$V_2 \rho C \frac{dT_2}{dt} = \rho q C(T_1 - T_2) + UA(T_1 - T_2) - U_c A_c (T_2 - T_c) \tag{4}$$

b) Eight parameters: ρ, V_1, V_2, C, U, A, U_c, A_c
Five variables: T_i, T_1, T_2, q, T_c
Two equations: (2) and (4)

Thus $N_F = 5 - 2 = 3$

2 outputs = T_1, T_2
3 inputs = T_i, T_c, q (specify as functions of t)

c) Three new variables: c_i, c_1, c_2 (concentration of species A).
Two new equations: Component material balances on each compartment.
c_1 and c_2 are new outputs. c_i must be a known function of time.

2.7

As in Section 2.4.2, there are two equations for this system:

$$\frac{dV}{dt} = \frac{1}{\rho}(w_i - w)$$
$$\frac{dT}{dt} = \frac{w_i}{V\rho}\left(T_i - T\right) + \frac{Q}{\rho VC}$$

Results:

(a) Since w is determined by hydrostatic forces, we can substitute for this variable in terms of the tank volume as in Section 2.4.5 case 3.

$$\frac{dV}{dt} = \frac{1}{\rho}\left(w_i - C_v\sqrt{\frac{V}{A}}\right)$$
$$\frac{dT}{dt} = \frac{w_i}{\rho V}\left(T_i - T\right) + \frac{Q}{\rho VC}$$

This leaves us with the following:

5 variables: V, T, w_i, T_i, Q
4 parameters: C, ρ, C_v, A
2 equations

The degrees of freedom are $5 - 2 = 3$. To make sure the system is specified, we have:

2 output variables: T, V

2 manipulated variables: Q, w_i

1 disturbance variable: T_i

(b) In this part, two controllers have been added to the system. Each controller provides an additional equation. Also, the flow out of the tank is now a manipulated variable being adjusted by the controller. So, we have

4 parameters: C, ρ, T_{sp}, V_{sp}

6 variables: V, T, w_i, T_i, Q, w

4 equations

The degrees of freedom are $6-4=2$. To specify the two degrees of freedom, we set the variables as follows:

2 output variables: T, V

2 manipulated variables (determined by controller equations): Q, w

2 disturbance variables: T_i, w_i

2.8

Additional assumptions:

(i) Density of the liquid, ρ, and density of the coolant, ρ_J, are constant.
(ii) Specific heat of the liquid, C, and of the coolant, C_J, are constant.

Because V is constant, the mass balance for the tank is:

$$\rho \frac{dV}{dt} = q_F - q = 0; \ \text{thus } q = q_F$$

Energy balance for tank:

$$\rho VC \frac{dT}{dt} = q_F \rho C (T_F - T) - K q_J^{\,0.8} A (T - T_J) \tag{1}$$

Energy balance for the jacket:

$$\rho_J V_J C_J \frac{dT_J}{dt} = q_J \rho_J C_J (T_i - T_J) + K q_J^{\,0.8} A (T - T_J) \tag{2}$$

where A is the heat transfer area (in ft^2) between the process liquid and the coolant.

Eqs.1 and 2 comprise the dynamic model for the system.

2.9

Assume that the feed contains only A and B, and no C. Component balances for A, B, C over the reactor give.

$$V\frac{dc_A}{dt} = q_i c_{Ai} - qc_A - Vk_1 e^{-E_1/RT} c_A \tag{1}$$

$$V\frac{dc_B}{dt} = q_i c_{Bi} - qc_B + V(k_1 e^{-E_1/RT} c_A - k_2 e^{-E_2/RT} c_B) \tag{2}$$

$$V\frac{dc_C}{dt} = -qc_C + Vk_2 e^{-E_2/RT} c_B \tag{3}$$

An overall mass balance over the jacket indicates that $q_c = q_{ci}$ because the volume of coolant in jacket and the density of coolant are constant.

Energy balance for the reactor:

$$\frac{d\left[\left(Vc_A M_A S_A + Vc_B M_B S_B + Vc_C M_C S_C\right)T\right]}{dt} = \left(q_i c_{Ai} M_A S_A + q_i c_{Bi} M_B S_B\right)(T_i - T)$$
$$-UA(T - T_c) + (-\Delta H_1)Vk_1 e^{-E_1/RT} c_A + (-\Delta H_2)Vk_2 e^{-E_2/RT} c_B \tag{4}$$

where M_A, M_B, M_C are molecular weights of A, B, and C, respectively
S_A, S_B, S_C are specific heats of A, B, and C.
U is the overall heat transfer coefficient
A is the surface area of heat transfer

Energy balance for the jacket:

$$\rho_j S_j V_j \frac{dT_c}{dt} = \rho_j S_j q_{ci}(T_{ci} - T_c) + UA(T - T_c) \tag{5}$$

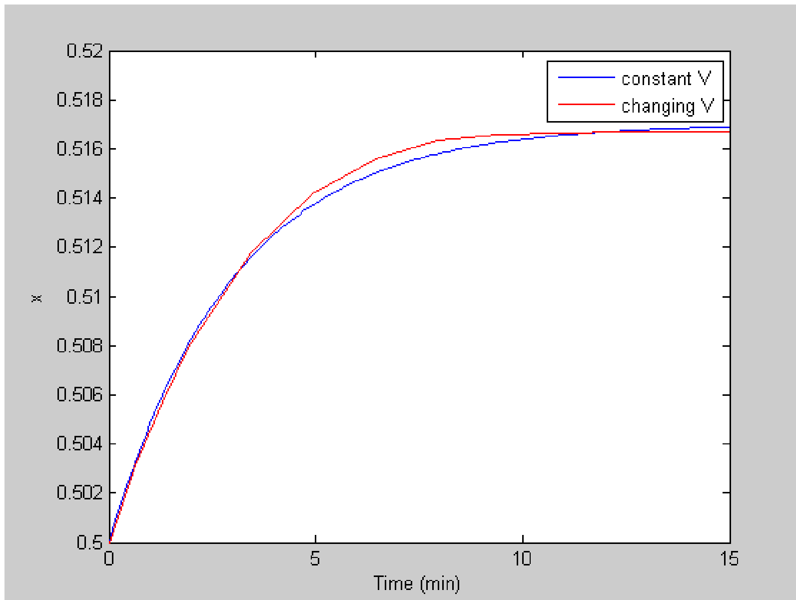
where:

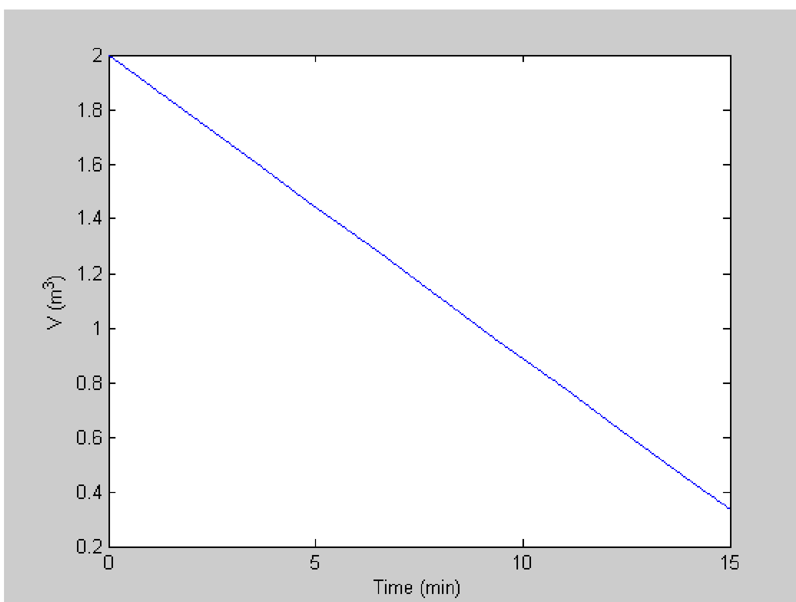
ρ_j, S_j are density and specific heat of the coolant.
V_j is the volume of coolant in the jacket.

Eqs. 1 - 5 represent the dynamic model for the system.

2.10

The plots should look as shown below:

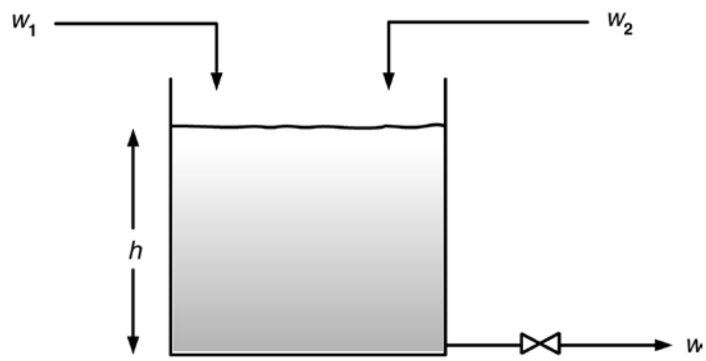




Notice that the functions are only good for t = 0 to t = 18, at which point the tank is completely drained. The concentration function blows up because the volume function is negative.

2.11

a)



Note that the only conservation equation required to find *h* is an overall mass balance:

$$\frac{dm}{dt} = \frac{d(\rho A h)}{dt} = \rho A \frac{dh}{dt} = w_1 + w_2 - w \quad (1)$$

Valve equation: $w = C_v' \sqrt{\frac{\rho g}{g_c} h} = C_v \sqrt{h}$ (2)

where $C_v = C_v' \sqrt{\frac{\rho g}{g_c}}$ (3)

Substituting the valve equation into the mass balance,

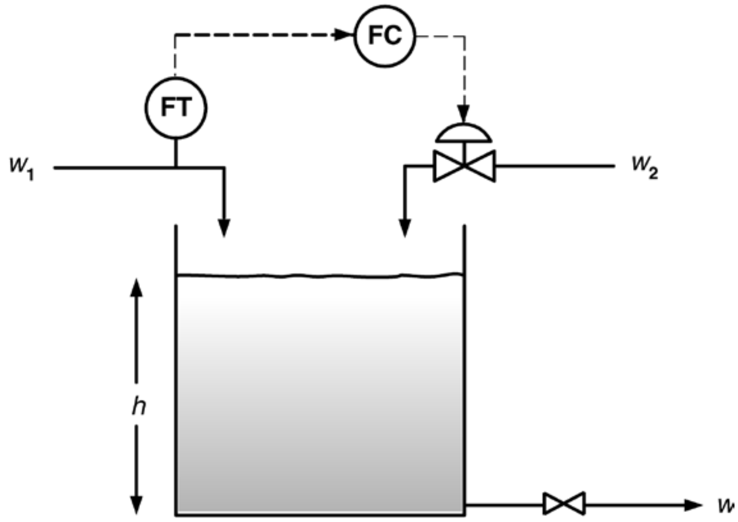
$$\frac{dh}{dt} = \frac{1}{\rho A}(w_1 + w_2 - C_v \sqrt{h}) \quad (4)$$

Steady-state model:

$$0 = \overline{w_1} + \overline{w_2} - C_v \sqrt{\overline{h}} \quad (5)$$

b) $C_v = \dfrac{\overline{w_1} + \overline{w_2}}{\sqrt{\overline{h}}} = \dfrac{2.0 + 1.2}{\sqrt{2.25}} = \dfrac{3.2}{1.5} = 2.13 \dfrac{\text{kg/s}}{\text{m}^{1/2}}$

c) Feedforward control



Rearrange Eq. 5 to get the feedforward (FF) controller relation,

$$w_2 = C_v\sqrt{\overline{h_R}} - w_1 \qquad \text{where } \overline{h_R} = 2.25 \text{ m}$$

$$w_2 = (2.13)(1.5) - w_1 = 3.2 - w_1 \qquad (6)$$

Note that Eq. 6, for a value of w_1 = 2.0, gives

w_2 = 3.2 –1.2 = 2.0 kg/s which is the desired value.

If the actual FF controller follows the relation, $w_2 = 3.2 - 1.1w_1$ (flow transmitter 10% higher), w_2 will change as soon as the FF controller is turned on,

w_2 = 3.2 –1.1 (2.0) = 3.2 – 2.2 = 1.0 kg/s

(instead of the correct value, 1.2 kg/s)

Then $C_v\sqrt{\bar{h}} = 2.13\sqrt{\bar{h}} = 2.0 + 1.0$

or $\sqrt{\bar{h}} = \dfrac{3}{2.13} = 1.408$ and $\bar{h}$ = 1.983 m (instead of 2.25 m)

Error in desired level = $\dfrac{2.25 - 1.983}{2.25} \times 100\% = 11.9\%$

The sensitivity does not look too bad in the sense that a 10% error in flow measurement gives ~12% error in desired level. Before making this conclusion, however, one should check how well the operating FF controller works for a change in w_1 (e.g., $\Delta w_1 = 0.4$ kg/s).

2.12

a) Model of tank (normal operation):

$$\rho A \frac{dh}{dt} = w_1 + w_2 - w_3 \qquad \text{(Below the leak point)}$$

$$A = \frac{\pi (2)^2}{4} = \pi = 3.14 \text{ m}^2$$

$$(800)(3.14)\frac{dh}{dt} = 120 + 100 - 200 = 20$$

$$\frac{dh}{dt} = \frac{20}{(800)(3.14)} = 0.007962 \ \text{ m/min}$$

Time to reach leak point (h = 1 m) = 125.6 min.

b) Model of tank with leak and w_1, w_2, w_3 constant:

$$\rho A \frac{dh}{dt} = 20 - \delta q_4 = 20 - \rho(0.025)\sqrt{h-1} = 20 - 20\sqrt{h-1} \quad , \quad h \geq 1$$

To check for overflow, one can simply find the level h_m at which dh/dt = 0. That is the maximum value of level when no overflow occurs.

$$0 = 20 - 20\sqrt{h_m - 1} \quad \text{or} \quad h_m = 2 \text{ m}$$

Thus, overflow does not occur for a leak occurring because $h_m < 2.25$ m.

2.13

<u>Model of process</u>

Overall material balance:

$$\rho A_T \frac{dh}{dt} = w_1 + w_2 - w_3 = w_1 + w_2 - C_v\sqrt{h} \tag{1}$$

Component:

$$\rho A_T \frac{d(hx_3)}{dt} = w_1x_1 + w_2x_2 - w_3x_3$$

$$\rho A_T h\frac{dx_3}{dt} + \rho A_T x_3 \frac{dh}{dt} = w_1x_1 + w_2x_2 - w_3x_3$$

Substituting for *dh/dt* (Eq. 1)

$$\rho A_T h\frac{dx_3}{dt} + x_3(w_1 + w_2 - w_3) = w_1x_1 + w_2x_2 - w_3x_3$$

$$\rho A_T h\frac{dx_3}{dt} = w_1(x_1 - x_3) + w_2(x_2 - x_3) \tag{2}$$

or $$\frac{dx_3}{dt} = \frac{1}{\rho A_T h}\left[w_1(x_1 - x_3) + w_2(x_2 - x_3)\right] \tag{3}$$

a) At initial steady state ,

$$\overline{w_3} = \overline{w_1} + \overline{w_2} = 120 + 100 = 220 \text{ Kg/min}$$

$$C_v = \frac{220}{\sqrt{1.75}} = 166.3$$

b) If x_1 is suddenly changed from 0.5 to 0.6 without changing flowrates, then level remains constant and Eq.3 can be solved analytically or numerically to find the time to achieve 99% of the x_3 response. From the material balance, the final value of x_3 = 0.555. Then,

$$\frac{dx_3}{dt} = \frac{1}{(800)(1.75)\pi}\left[120(0.6 - x_3) + 100(0.5 - x_3)\right]$$

$$= \frac{1}{(800)(1.75)\pi}\left[(72+50) - 220x_3)\right]$$

$$= 0.027738 - 0.050020x_3$$

Integrating,

$$\int_{x_{3o}}^{x_{3f}} \frac{dx_3}{0.027738 - 0.050020x_3} = \int_0^t dt$$

where x_{3o}=0.5 and x_{3f} =0.555 – (0.555)(0.01) = 0.549

Solving,

t = 47.42 min

c) If w_1 is changed to 100 kg/min without changing any other input variables, then x_3 will not change and Eq. 1 can be solved to find the time to achieve 99% of the h response. From the material balance, the final value of the tank level is h =1.446 m.

$$800\pi \frac{dh}{dt} = 100 + 100 - C_v\sqrt{h}$$

$$\frac{dh}{dt} = \frac{1}{800\pi}\left[200 - 166.3\sqrt{h}\quad\right]$$

$$= 0.079577 - 0.066169\sqrt{h}$$

where h_o=1.75 and h_f =1.446 + (1.446)(0.01) = 1.460

By using the MATLAB command ode45 ,

t = 122.79 min

Numerical solution of the ode is shown in Fig. S2.13

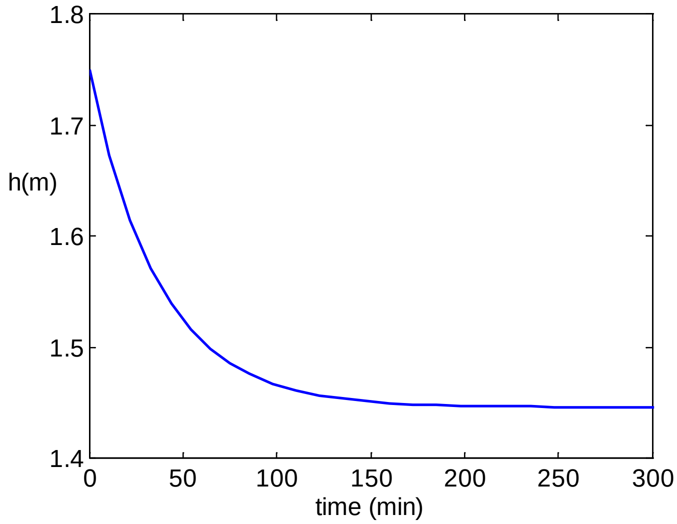


Figure S2.13. *Numerical solution of the ode for part c)*

d) In this case, both h and x_3 will be changing functions of time. Therefore, both Eqs. 1 and 3 will have to be solved simultaneously. Since concentration does not appear in Eq. 1, we would anticipate no effect on the h response.

2.14

a) The dynamic model for the chemostat is given by:

Cells: $$V\frac{dX}{dt} = Vr_g - FX \qquad \text{or} \qquad \frac{dX}{dt} = r_g - \left(\frac{F}{V}\right)X \qquad (1)$$

Product: $$V\frac{dP}{dt} = Vr_p - FP \qquad \text{or} \qquad \frac{dP}{dt} = r_p - \left(\frac{F}{V}\right)P \qquad (2)$$

Substrate: $$V\frac{dS}{dt} = F(S_f - S) - \frac{1}{Y_{X/S}}Vr_g$$

or

$$\frac{dS}{dt} = \left(\frac{F}{V}\right)(S_f - S) - \frac{1}{Y_{X/S}}r_g - \frac{1}{Y_{P/S}}r_P \qquad (3)$$

b) At steady state,

$$\frac{dX}{dt} = 0 \qquad \therefore \qquad r_g = DX$$

then,

$$\mu X = DX \qquad \therefore \qquad D = \mu \tag{4}$$

A simple feedback strategy can be implemented where the growth rate is controlled by manipulating the mass flow rate, *F*, so that *F/V* stays constant.

c) Washout occurs if dX/dt is negative for an extended period of time; that is,

$$r_g - DX < 0 \qquad \text{or} \qquad D > \mu$$

Thus, if $D > \mu$ the cells will be washed out.

d) At steady state, the dynamic model given by Eqs. 1, 2 and 3 becomes:

$$0 = r_g - DX \qquad\qquad DX = r_g \tag{5}$$

$$0 = r_p - DP \qquad\qquad DP = r_p \tag{6}$$

$$0 = D(S_f - S) - \frac{1}{Y_{X/S}} r_g \tag{7}$$

From Eq. 5,

$$DX = r_g \tag{8}$$

From Eq. 7

$$r_g = Y_{X/S}(S_f - S)D \tag{9}$$

Substituting Eq. 9 into Eq. 8,

$$DX = Y_{X/S}(S_f - S)D \tag{10}$$

From Eq. 4

$$S = \frac{DK_S}{\mu_{\max} - D}$$

Substituting these two equations into Eq. 10,

$$DX = Y_{X/S}\left(S_f - \frac{DK_S}{\mu_{\max} - D}\right)D \qquad (11)$$

For $Y_{x/s}$ = 0.5, S_f = 10, K_s = 1, X = 2.75, $\mu_{\max}$ = 0.2, the following plot can be generated based on Eq. 11.

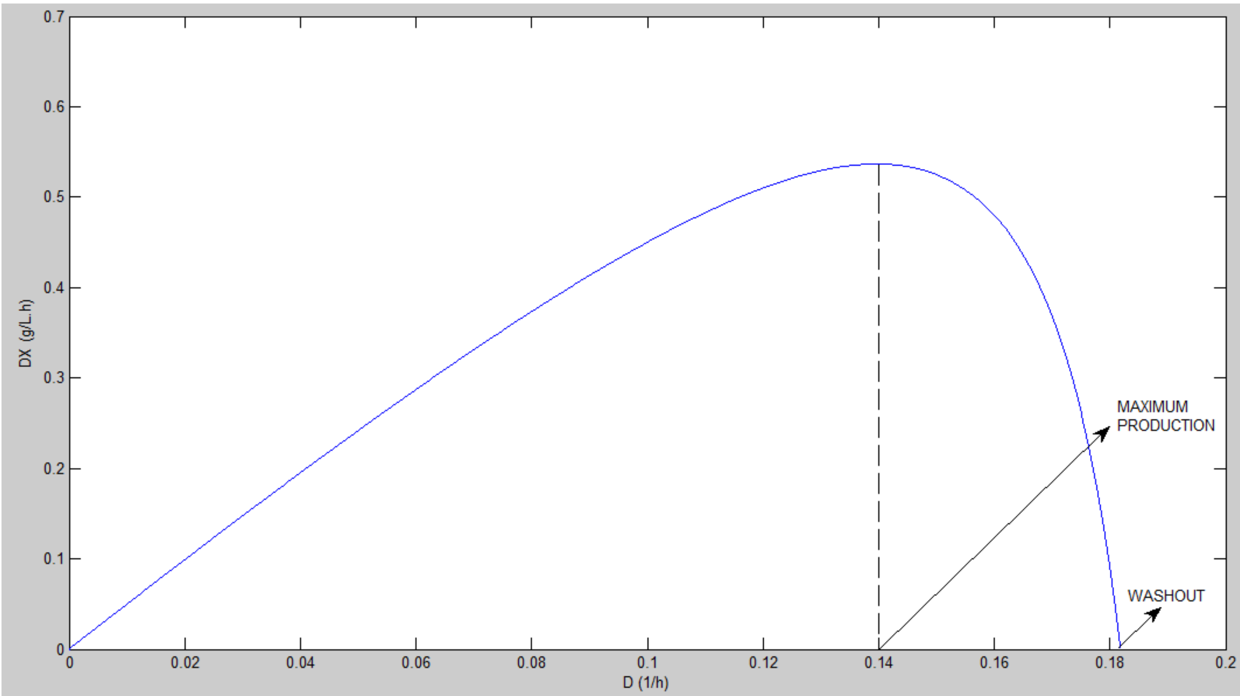


Figure S2.14. Steady-state cell production rate DX as a function of dilution rate D.

From Figure S2.14, washout occurs at D = 0.18 h^{-1} while the maximum production occurs at D = 0.14 h^{-1}. Notice that maximum and washout points are dangerously close to each other, so special care must be taken when increasing cell productivity by increasing the dilution rate.

2.15

a) We can assume that ρ and h are approximately constant. The dynamic model is given by:

$$r_d = -\frac{dM}{dt} = kAc_s \qquad (1)$$

Notice that:

$$M = \rho V \qquad \therefore \qquad \frac{dM}{dt} = \rho\frac{dV}{dt} \qquad (2)$$

$$V = \pi r^2 h \qquad \therefore \qquad \frac{dV}{dt} = (2\pi r h)\frac{dr}{dt} = A\frac{dr}{dt} \qquad (3)$$

Substituting (3) into (2) and then into (1),

$$-\rho A\frac{dr}{dt} = kAc_s \qquad \therefore \qquad -\rho\frac{dr}{dt} = kc_s$$

Integrating,

$$\int_{r_o}^{r} dr = -\frac{kc_s}{\rho}\int_0^t dt \qquad \therefore \qquad r(t) = r_o - \frac{kc_s}{\rho}t \qquad (4)$$

Finally,

$$M = \rho V = \rho\pi h r^2$$

then

$$M(t) = \rho\pi h\left(r_o - \frac{kc_s}{\rho}t\right)^2$$

b) The time required for the pill radius r to be reduced by 90% is given by Eq. 4:

$$0.1r_o = r_o - \frac{kc_s}{\rho}t \qquad \therefore \qquad t = \frac{0.9r_o\rho}{kc_s} = \frac{(0.9)(0.4)(1.2)}{(0.016)(0.5)} = 54\,\text{min}$$

Therefore, $t = 54\,\text{min}$.

2.16

For V = constant and F = 0, the simplified dynamic model is:

$$\frac{dX}{dt} = r_g = \mu_{max} \frac{S}{K_s + S} X$$

$$\frac{dP}{dt} = r_p = Y_{P/X} \mu_{max} \frac{S}{K_s + S} X$$

$$\frac{dS}{dt} = -\frac{1}{Y_{X/S}} r_g - \frac{1}{Y_{P/X}} r_P$$

Substituting numerical values:

$$\frac{dX}{dt} = 0.2 \frac{SX}{1+S}$$

$$\frac{dP}{dt} = (0.2)(0.2) \frac{SX}{1+S}$$

$$\frac{dS}{dt} = 0.2 \frac{SX}{1+S} \left[-\frac{1}{0.5} - \frac{0.2}{0.1} \right]$$

By using MATLAB, this system of differential equations can be solved. The time to achieve a 90% conversion of S is t = 22.15 h.

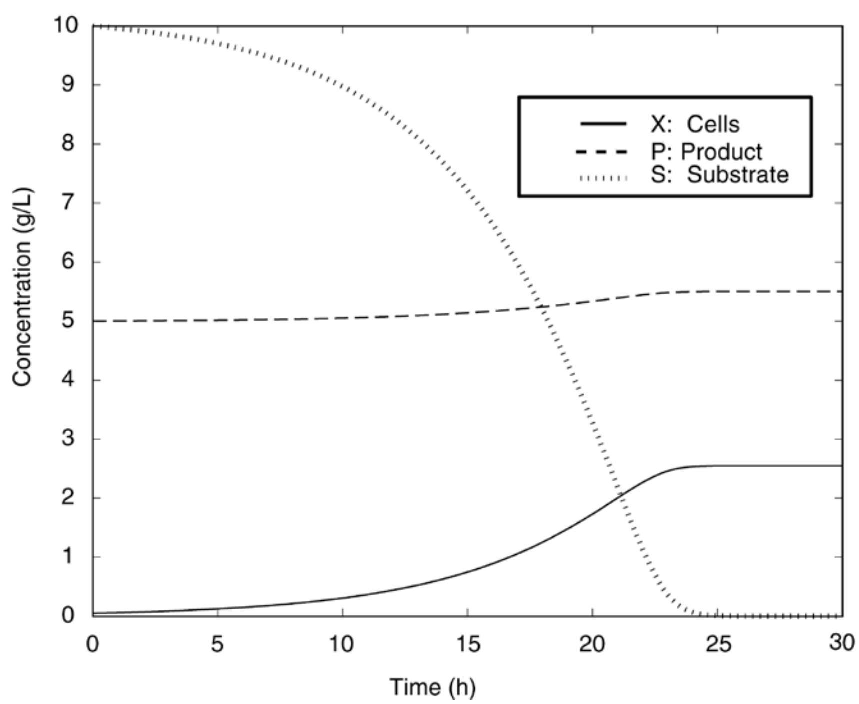


Figure S2.16. Fed-batch bioreactor dynamic behavior.

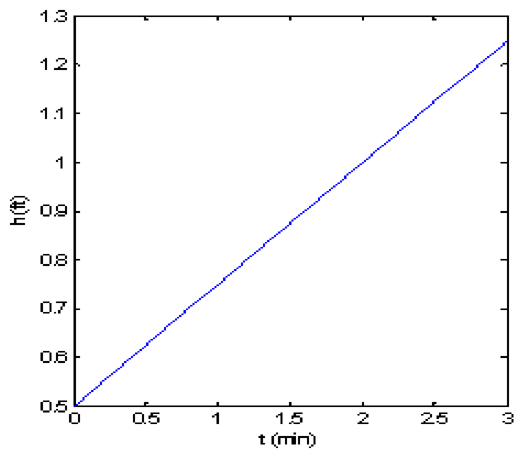
2.17

(a) Using a simple volume balance, for the system when the drain is closed (q = 0)

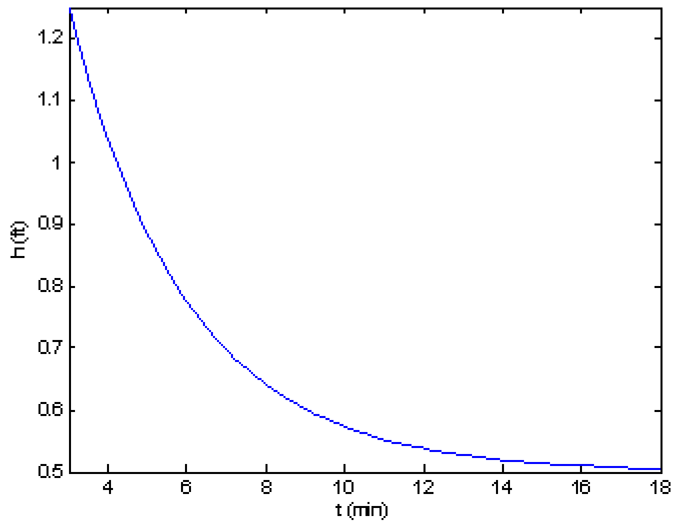
$$A\frac{dh}{dt} = q_1 \qquad (1)$$

Solving this ODE with the given initial condition gives a height that is increasing at a rate of 0.25 ft/min.

So the height in this time range will look like:

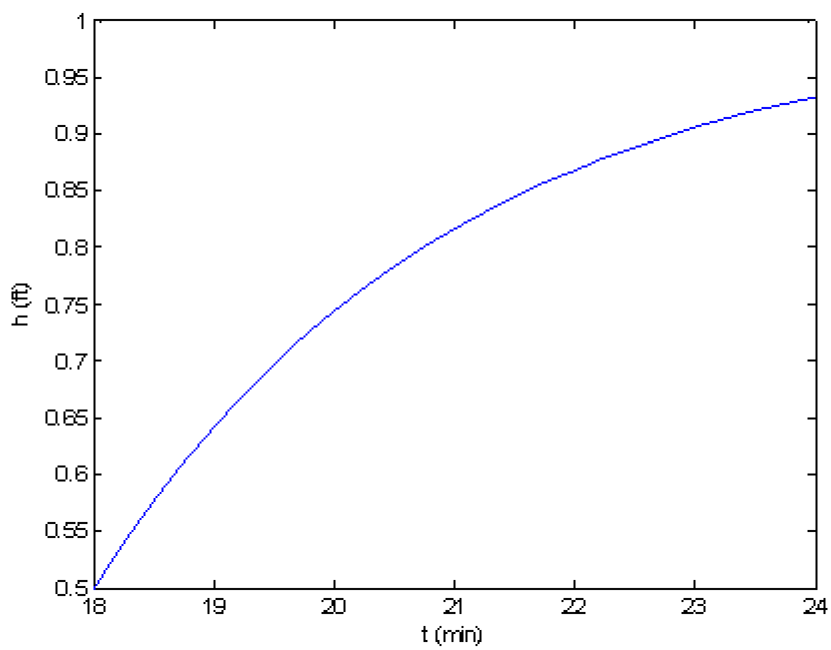


(b) the drain is opened for 15 mins; assume a time constant in a linear transfer function of 3 mins, so a steady state is essentially reached. ($3 \le t \le 18$). Assume that the process will return to its previous steady state in an exponential manner, reaching 63.2% of the response in three minutes.



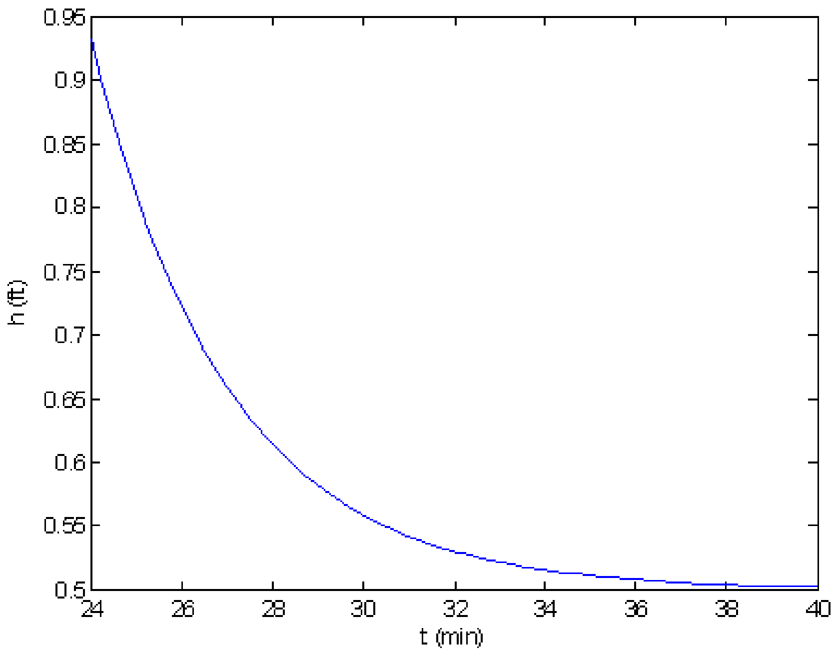
(c) the inflow rate is doubled for 6 minutes ($18 \le t \le 24$)

The height should rise exponentially towards a new steady state value double that of the steady state value in part b), but it should be apparent that the height does not reach this new steady state value at t = 24 min.. The new steady state would be 1 ft.

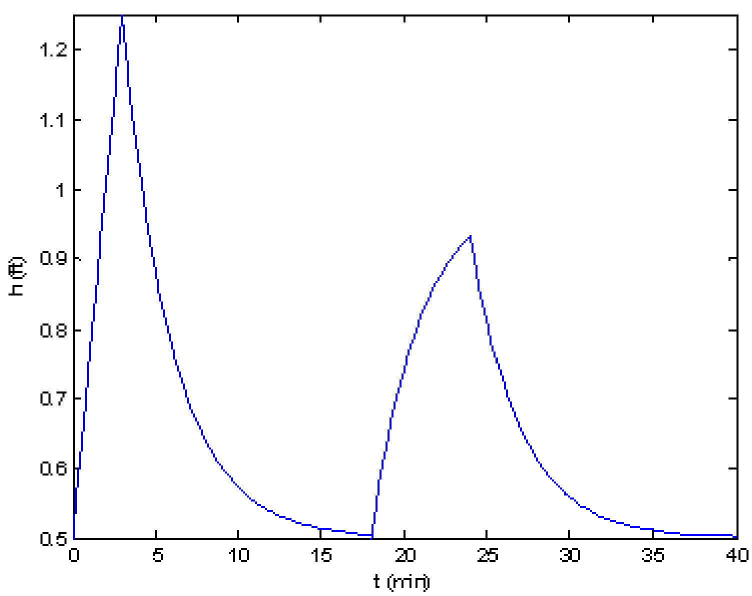


(d) the inflow rate is returned to its original value for 16 minutes ($24 \le t \le 40$)

The graph should show an exponential decrease to the previous steady state of 0.5 ft. The initial value should coincide with the final value from part (c).



Putting all the graphs together would look like this:



2.18

Parameters (fixed by design process): m, C, m_e, C_e, h_e, A_e.

CVs: T and T_e.

Input variables (disturbance): w, T_i. Input variables (manipulated): Q.

Degrees of freedom = (11-6) (number of variables) – 2 (number of equations) = 3

The three input variables (w, T_i, Q) are assigned and the resulting system has zero degrees of freedom.

2.19

(a) First we simulate a step change in the vapor flow rate from 0.033 to 0.045 m^3/s.
The resulting plots of xD and xB are shown below.

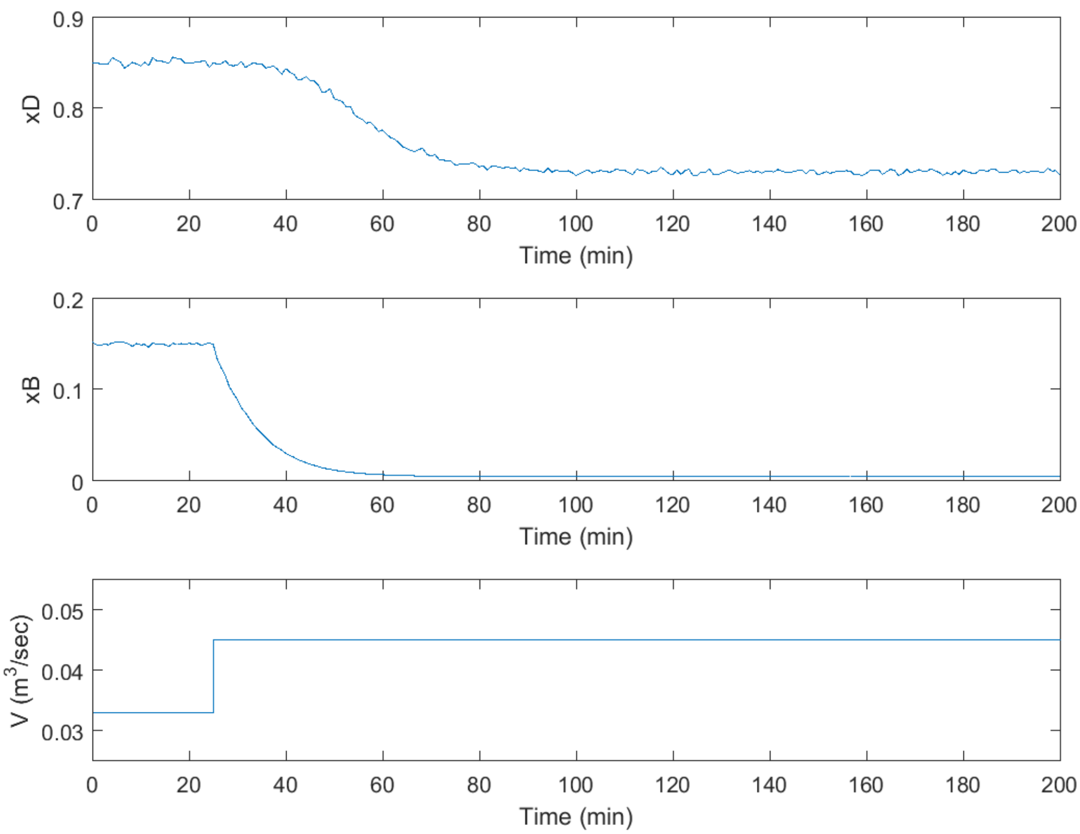


Figure: Plot of xD, xB, and V versus time for a step change in V from 0.033 to 0.045 m^3/s.

By examining the resulting data, we can find the steady-state values of xD and xB before and after the step change in V.

	Start	End	Change
xD	0.85	0.73	-0.12
xB	0.15	0.0050	-0.145

(b) Next we simulate a step change in the feed composition (zF) from 0.5 to 0.55. Note that the vapor flow rate, V, is still set at 0.045 m^3/s.

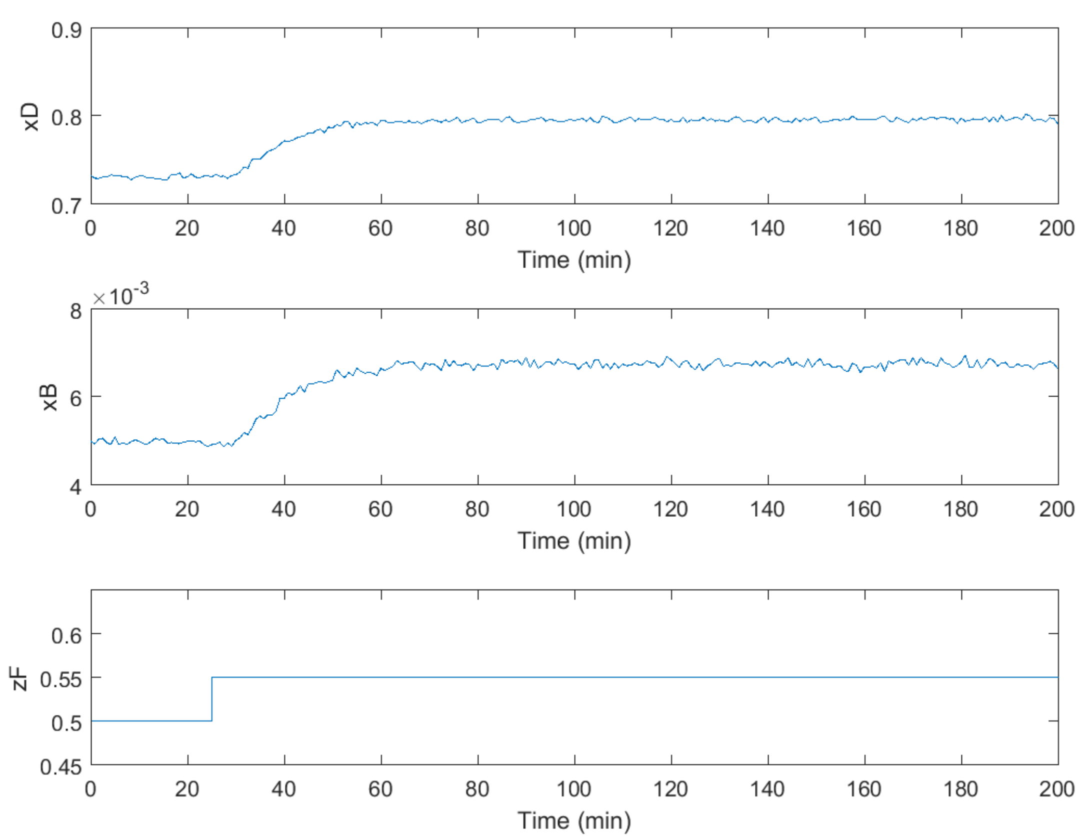


Figure: Plot of xD, xB, and zF versus time for a step change in zF from 0.5 to 0.55

By examining the resulting data, we can find the steady-state values of xD and xB before and after the step change in zF.

	Start	End	Change
xD	0.73	0.80	+0.066
xB	0.0050	0.0068	+0.0018

(c) Increasing V causes xD and xB to decrease, while increasing zF causes both xD and xB to increase. The magnitude of the effect is greater for changing V than for changing zF. When changing V, xB changes more quickly than xD.

2.20

(a) First we simulate a step change in the Fuel Gas Purity (FG_pur) from 1 to 0.95.
The resulting plots of Oxygen Exit Concentration (C_O2) and Hydrocarbon Outlet Temperature (T_HC) are shown below.

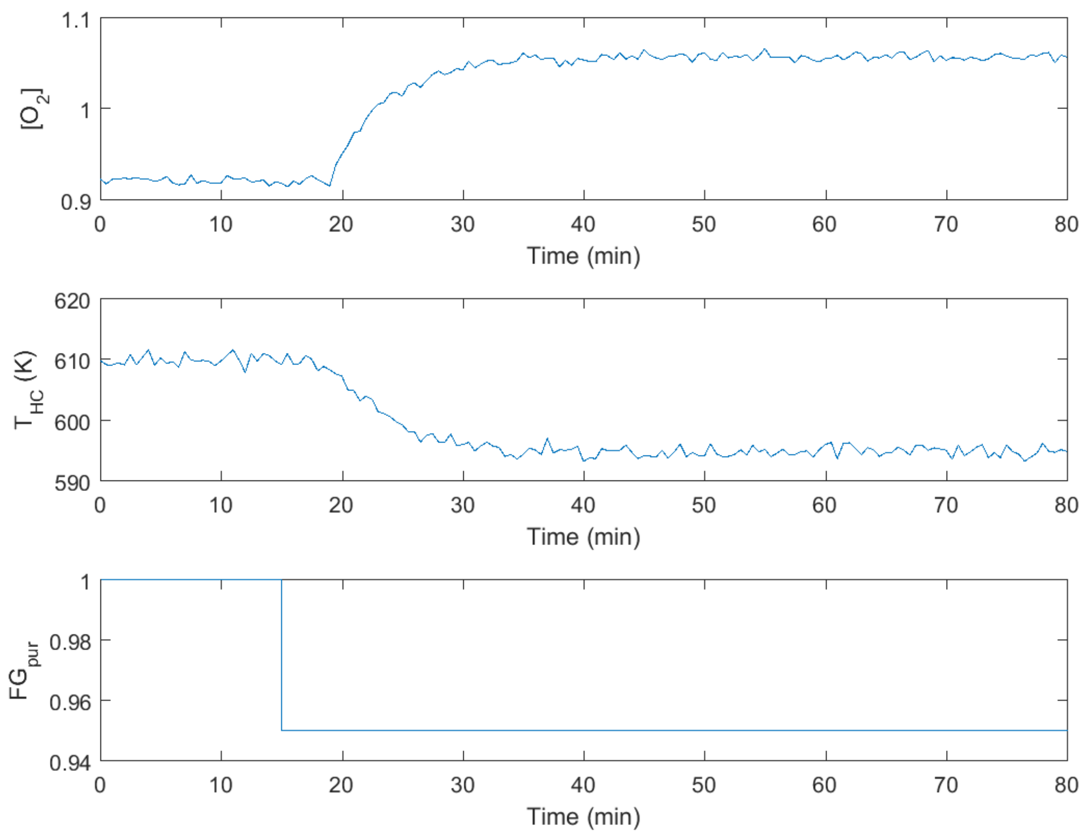


Figure: Plot of C_O2, T_HC, and FG_pur versus time for a step change in FG_pur from 1 to 0.95.

By examining the resulting data, we can find the steady-state values of C_O2 and T_HC before and after the step change in FG_pur.

	Start	End	Change
C_O2	0.92	1.06	0.14
T_HC	609	595	-14

(b) Next we simulate a step change in the Hydrocarbon Flow Rate (F_HC_sp) from 0.035 to 0.0385. Note that the Fuel Gas Purity, FG_pur, is still set at 0.95.

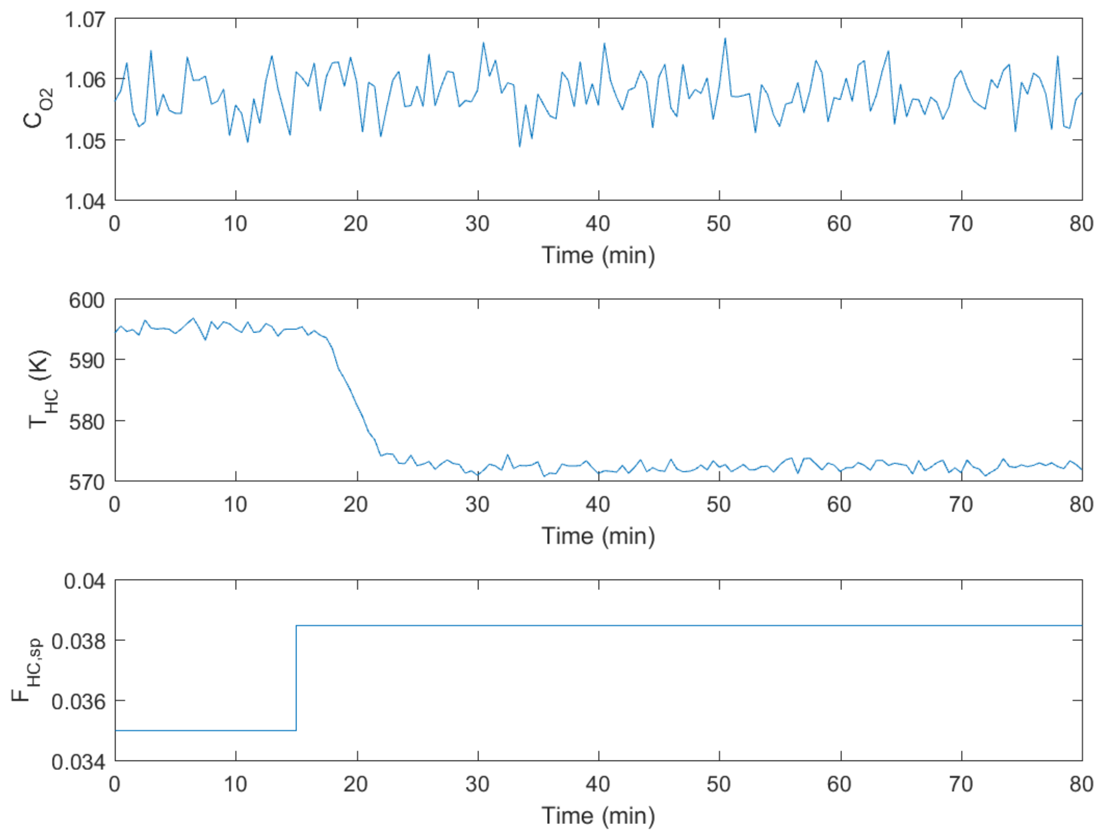


Figure: Plot of C_O2, T_HC, and F_HC_sp versus time for a step change in F_HC_sp from 0.035 to 0.0385.

By examining the resulting data, we can find the steady-state values of C_O2 and T_HC before and after the step change in F_HC_sp.

	Start	End	Change
C_O2	1.06	1.06	0
T_HC	595	572	-23

(c) Decreasing FG_pur causes C_O2 to increase, while T_HC decreases. Increasing F_HC_sp causes T_HC to decrease while C_O2 stays the same. The change in T_HC occurs more quickly when changing F_HC_sp versus changing FG_pur.

2.21

The key to this problem is solving the mass balance of the tank in each part.

Mass balance:

$$\frac{d}{dt}(\rho A h) = \rho q_i - \rho q_o$$

- ρ (density) and A (tank cross-sectional area) are constants, therefore:

$$A\frac{dh}{dt} = q_i - q_o$$

- The problem specifies q_o is linearly related to the tank height

$$q_o = \frac{1}{R}h$$

$$A\frac{dh}{dt} = q_i - \frac{1}{R}h$$

- Next, we can obtain R (valve constant) from the steady state information in the problem

$$\frac{dh}{dt} = 0 \quad \text{at steady state}$$

$$0 = \bar{q}_i - \frac{1}{R}\bar{h}$$

$$0 = 2 - \frac{1}{R}(1)$$

$$\therefore \quad \frac{1}{R} = 2 \quad R = 0.5 \quad \frac{\text{ft}^2}{\text{min}}$$

- In addition, we can find that

$$\tau = AR = (4)\left(\frac{1}{2}\right) = 2 \text{ min}$$

<u>Part a</u>

$$A \frac{dh}{dt} = q_i - q_o \qquad \text{(Mass Balance)}$$

$$4 \frac{dh}{dt} = 2 \qquad \text{(Separable ODE)}$$

$$\int dh = \int \frac{1}{2} \, dt$$

$$h(t) = \frac{1}{2} t + C \qquad h(0) = 1$$

$$\boxed{h(t) = \frac{1}{2} t + 1 \qquad 0 \le t < 3}$$

Part b

$$A\frac{dh}{dt} = q_i - \frac{1}{R}h \qquad \text{(Mass Balance)}$$

$$4\frac{dh}{dt} = 2 - 2h$$

$$\frac{dh}{dt} + \frac{1}{2}h = \frac{1}{2} \qquad \text{(Solution by integrating factor = } e^{t/2}\text{)}$$

$$\int d(e^{t/2}h) = \int \frac{1}{2}\, e^{t/2}\ dt$$

$$he^{t/2} = 1e^{t/2} + c \qquad h(3) = 2.5$$

$$h = 1 + ce^{-t/2}$$

$$2.5 = 1 + ce^{-3/2}$$

$$c = 1.5e^{3/2}$$

$$\boxed{h(t) = 1 + (1.5)e^{-(t-3)/2} \qquad 3 \le t < 18}$$

Part c

$$4\frac{dh}{dt} = 4 - 2h \qquad \text{(Mass balance)}$$

$$\frac{dh}{dt} + \frac{1}{2}h = 1 \qquad \text{(Solution by integrating factor)}$$

$$\int d(e^{t/2}h) = \int 1e^{t/2}dt \qquad h(18) = 1$$

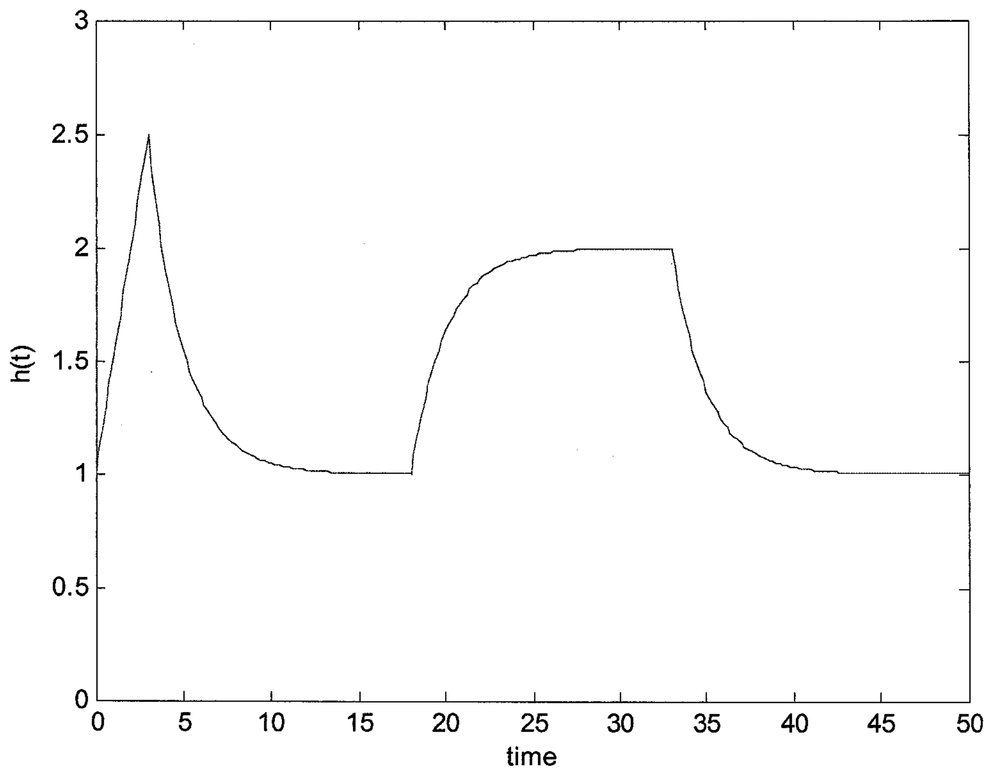
- Method is same as part b.

$$\boxed{h(t) = 2 - e^{-(t-18)/2} \qquad 18 \le t < 33}$$

Part d

Same as part b with h (33) = 2

$$\boxed{h(t) = 1 + e^{-(t-33)/2} \qquad 33 \le t \le 50}$$



2.22

To solve the problem, we start by writing the mass balance for each tank 1-4.
To write the mass balance for each tank, we start with the most general form, where the change in mass in the tank over time is equal to the mass flowing into the tank minus the mass flowing out of the tank. The general form of the equations are shown below, where i represents the tank number (1, 2, 3, 4). The mass can be written as the density multiplied by the tank volume, and the mass flow rates can be written as the density multiplied by the volumetric flow rate.

$$\frac{d(\rho V_i)}{dt} = \rho q_{in,i} - \rho q_{out,i}$$

With density assumed constant over time, it can be pulled out of the derivative. Also, we write the volume of the tank as the height of liquid in the tank, h_i, multiplied by the cross-sectional tank area, A_i.

$$\frac{\rho A_i d(h_i)}{dt} = \rho q_{in,i} - \rho q_{out,i}$$
$$\frac{A_i d(h_i)}{dt} = q_{in,i} - q_{out,i}$$

The flow exiting each tank through the bottom can be written as:

$$q_{exit,i} = C_i \sqrt{h_i}$$

Where C_i is the proportionality constant for each tank.

Results:

a) The final equations for the height of liquid in each tank are shown below.

$$\frac{dh_1}{dt} = -\frac{C_1}{A_1}\sqrt{h_1} + \frac{C_3}{A_1}\sqrt{h_3} + \frac{\gamma_1}{A_1}F_1 \quad (1)$$

$$\frac{dh_2}{dt} = -\frac{C_2}{A_2}\sqrt{h_2} + \frac{C_4}{A_2}\sqrt{h_4} + \frac{\gamma_2}{A_2}F_2 \quad (2)$$

$$\frac{dh_3}{dt} = -\frac{C_3}{A_3}\sqrt{h_3} + \frac{(1-\gamma_2)}{A_3}F_2 \quad (3)$$

$$\frac{dh_4}{dt} = -\frac{C_4}{A_4}\sqrt{h_4} + \frac{(1-\gamma_1)}{A_4}F_1 \quad (4)$$

b) Now we can substitute $\gamma_1 = \gamma_2 = 0.5$

$$\frac{dh_1}{dt} = -\frac{C_1}{A_1}\sqrt{h_1} + \frac{C_3}{A_1}\sqrt{h_3} + \frac{0.5}{A_1}F_1$$

$$\frac{dh_2}{dt} = -\frac{C_2}{A_2}\sqrt{h_2} + \frac{C_4}{A_2}\sqrt{h_4} + \frac{0.5}{A_2}F_2$$

$$\frac{dh_3}{dt} = -\frac{C_3}{A_3}\sqrt{h_3} + \frac{0.5}{A_3}F_2$$

$$\frac{dh_4}{dt} = -\frac{C_4}{A_4}\sqrt{h_4} + \frac{0.5}{A_4}F_1$$

The differential equations for the tank heights are coupled, so the heights cannot be solved for or controlled independently. F_1 and F_2 can be used to control h_3 and h_4 independently, but h_1 and h_2 will be affected in an uncontrolled manner.

c) In the extreme case where $\gamma_1 = \gamma_2 = 0$, we get:

$$\frac{dh_1}{dt} = -\frac{C_1}{A_1}\sqrt{h_1} + \frac{C_3}{A_1}\sqrt{h_3}$$

$$\frac{dh_2}{dt} = -\frac{C_2}{A_2}\sqrt{h_2} + \frac{C_4}{A_2}\sqrt{h_4}$$

$$\frac{dh_3}{dt} = -\frac{C_3}{A_3}\sqrt{h_3} + \frac{F_2}{A_3}$$

$$\frac{dh_4}{dt} = -\frac{C_4}{A_4}\sqrt{h_4} + \frac{F_1}{A_4}$$

These equations make sense with the process diagram because now F_1 and F_2 only affect tanks h_3 and h_4 directly (they no longer flow into tanks 1 and 2 at all). However, F_1 and F_2 indirectly affect tanks 1 and 2 through h_3 and h_4.

Chapter 3 ©

3.1

(a)

$$f(t) = 5 + e^{-3t} + te^{-4t}$$

Transform each term using rules 2, 5, and 7 from Table 3.1, respectively.

$$F(s) = \frac{5}{s} + \frac{1}{s+3} + \frac{1}{(s+4)^2}$$

(b)

$$f(t) = \sin(4t) + (t-3)S(t-3) + e^{-(t-3)}S(t-3) + \frac{5}{t}$$

- To transform sin(4t), use rule 14 from Table 3.1
- To transform (t-3)S(t-3) use rules 3 and 26 together. To use rule 26, set $f(t) = t$ and $t_0 = 3$.
- To transform $e^{-(t-3)}S(t-3)$ use rules 5 and 26 together. To use rule 26, set $f(t) = e^{-t}$ and $t_0 = 3$.
- Note that there is no Laplace transform for 1/t.

$$F(s) = \frac{4}{s^2+16} + \frac{e^{-3s}}{s^2} + \frac{e^{-3s}}{s+1} + 5\mathcal{L}\left(\frac{1}{t}\right)$$

(c)

$$f(t) = e^{-t}\cos(4t) + \frac{t}{5}$$

- To transform the first term, use rule 18 from Table 3.1
- To transform the second term, use rule 3 from Table 3.1

$$F(s) = \frac{s+1}{(s+1)^2+16} + \frac{1}{5s^2}$$

(d)

$$f(t) = S(t-1)\cos(4(t-1)) + t^2$$

- To transform the first term, use rules 15 and 26 together. To use rule 26, set $f(t) = \cos(4t)$ and $t_0 = 1$.
- To transform the second term, use rule 4 of Table 3.1.

$$F(s) = e^{-s}\frac{s}{s^2+16} + \frac{2}{s^3}$$

3.2

Break the pulse into three step functions. First, a step up to 10 at t=0. Then, a step down by 8 at t=1. Finally, a step down by 2 at t=3:

$f(t) = 10\ S(t) - 8\ S(t-1) - 2\ S(t-3)$

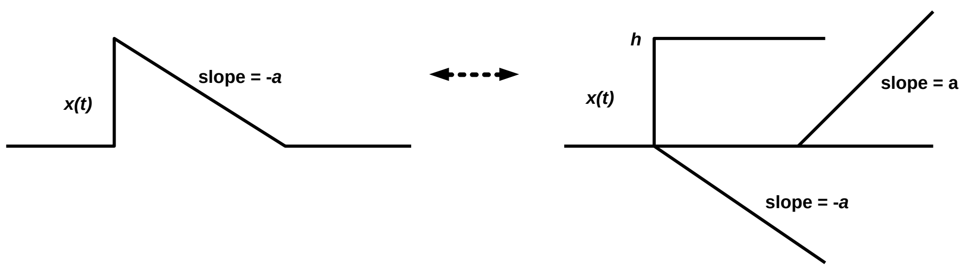
$$F(s) = = \frac{1}{s}\left(10 - 8e^{-s} - 2\ e^{-3s}\right)$$

3.3

a) Pulse width is obtained when x(t) = 0. Since x(t) = h – at

t_ω : $h - at_\omega = 0$ or $t_\omega = h/a$

b)



$x(t) = hS(t) - atS(t) + a(t - t_\omega)\ S(t-t_\omega)$

c) $$X(s) = \frac{h}{s} - \frac{a}{s^2} + \frac{ae^{-st_\omega}}{s^2} = \frac{h}{s} + \frac{e^{-st_\omega} - 1}{s^2}$$

d) Area under pulse = $h\ t_\omega/2$

3.4

(a) Laplace transform on the ODE gives:

$$\mathcal{L}\left(\frac{d^2y}{dt^2}\right) + 6\mathcal{L}\left(\frac{dy}{dt}\right) + 8\mathcal{L}(y) = 3b\mathcal{L}(e^{-2t})$$

$$s^2Y(s) - sy(0) - y'(0) + 6sY(s) - 6y(0) + 8Y(s) = 3b\frac{1}{s+2}$$

$$s^2Y(s)+6sY(s)+8Y(s)=\frac{3b}{s+2}$$

Thus:

$$Y(s)=\frac{3b}{(s+2)(s^2+6s+8)}=\frac{3b}{(s+2)^2(s+4)}=\frac{a_1}{(s+2)^2}+\frac{a_2}{(s+2)}+\frac{a_3}{(s+4)}$$

Regardless of the numerical values of a_1, a_2 and a_3, the inverse Laplace transform indicates that *y(t)* includes e^{-2t}, te^{-2t}, and e^{-4t}.

(b) When $u = ct$, Laplace transform gives:

$$s^2Y(s)+6sY(s)+8Y(s)=\frac{3c}{s^2}$$

$$Y(s)=\frac{3c}{s^2(s+2)(s+4)}=\frac{a_1}{s}+\frac{a_2}{s^2}+\frac{a_3}{(s+2)}+\frac{a_4}{(s+4)}$$

Regardless of the numerical values of a_1, a_2, a_3, and a_4, the inverse Laplace transform indicates *y(t)* includes a_1, t, e^{-2t}, and e^{-4t}.

3.5

$$T(t) = 20\, S(t) + \frac{55}{30} t\, S(t) - \frac{55}{30}(t\text{-}30)\, S(t\text{-}30)$$

$$T(s)=\frac{20}{s}+\frac{55}{30}\frac{1}{s^2} - \frac{55}{30}\frac{1}{s^2}e^{-30s} = \frac{20}{s}+\frac{55}{30}\frac{1}{s^2}\left(1-e^{-30s}\right)$$

3.6

a) $$X(s)=\frac{s(s+1)}{(s+2)(s+3)(s+4)}=\frac{\alpha_1}{s+2}+\frac{\alpha_2}{s+3}+\frac{\alpha_3}{s+4}$$

$$\alpha_1=\left.\frac{s(s+1)}{(s+3)(s+4)}\right|_{s=-2}=1$$

$$\alpha_2=\left.\frac{s(s+1)}{(s+2)(s+4)}\right|_{s=-3}=-6$$

$$\alpha_3 = \left.\frac{s(s+1)}{(s+2)(s+3)}\right|_{s=-4} = 6$$

$$X(s) = \frac{1}{s+2} - \frac{6}{s+3} + \frac{6}{s+4} \qquad \text{and} \qquad x(t) = e^{-2t} - 6e^{-3t} + 6e^{-4t}$$

b) $$X(s) = \frac{s+2}{(s+1)^2} = \frac{\alpha_1}{s+1} + \frac{\alpha_2}{(s+1)^2} \tag{1}$$

$$\alpha_2 = (s+2)\big|_{s=-1} = 1$$

In Eq. 1, substitute any $s \neq -1$ to determine α_1. Arbitrarily using $s=0$, Eq. 1 gives

$$\frac{2}{1^2} = \frac{\alpha_1}{1} + \frac{1}{1^2} \qquad \text{or} \qquad \alpha_1 = 1$$

$$X(s) = \frac{1}{s+1} + \frac{1}{(s+1)^2} \quad \text{and} \qquad x(t) = e^{-t} + te^{-t}$$

c) $$X(s) = \frac{1}{s^2+s+1} = \frac{1}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{1}{(s+b)^2 + \omega^2}$$

where $b = \frac{1}{2}$ and $\omega = \frac{\sqrt{3}}{2}$

$$x(t) = \frac{1}{\omega} e^{-bt} \sin \omega t = \frac{2}{\sqrt{3}} e^{-\frac{t}{2}} \sin \frac{\sqrt{3}}{2} t$$

d) $$X(s) = \frac{s+1}{s(s+4)(s+3)} e^{-0.5s}$$

To invert, first ignore the time delay term. Using the Heaviside expansion with the partial fraction expansion,

$$\hat{X}(s) = \frac{s+1}{s(s+4)(s+3)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+3}$$

Multiply by s and let $s \to 0$

$$A = \frac{1}{(4)(3)} = \frac{1}{12}$$

Multiply by (s+4) and let $s \to -4$

$$B = \frac{-4+1}{(-4)(-4+3)} = \frac{-3}{(-4)(-1)} = \frac{-3}{4}$$

Multiply by (s+3) and let $s \to$-3

$$C = \frac{-3+1}{(-3)(-3+4)} = \frac{-2}{(-3)(1)} = \frac{2}{3}$$

Then

$$\hat{X}(s) = \frac{1/12}{s} + \frac{-3/4}{s+4} + \frac{2/3}{s+3}$$

$$\hat{x}(t) = \frac{1}{12} - \frac{3}{4}e^{-4t} + \frac{2}{3}e^{-3t}$$

Using the Real Translation Theorem,

$$x(t) = \hat{x}(t-0.5) = \frac{1}{12} - \frac{3}{4}e^{-4(t-0.5)} + \frac{2}{3}e^{-3(t-0.5)}$$

for $t \geq 0.5$

3.7

a)

$$Y(s) = \frac{6(s+1)}{s^2(s+1)} = \frac{6}{s^2} = \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}$$

$$\alpha_2 = s^2 \frac{6}{s^2}\bigg|_{s=0} = 6 \qquad \alpha_1 = 0$$

$$Y(s) = \frac{6}{s^2}$$

b) $$Y(s) = \frac{12(s+2)}{s\ (s^2+9)} = \frac{\alpha_1}{s} + \frac{\alpha_2 s + \alpha_3}{s^2+9}$$

Multiplying both sides by $s(s^2+9)$

$$12(s+2) = \alpha_1(s^2+9) + (\alpha_2 s + \alpha_3)(s) \qquad \text{or}$$
$$12s + 24 = (\alpha_1 + \alpha_2)s^2 + \alpha_3 s + 9\alpha_1$$

Equating coefficients of like powers of s,

s^2: $\alpha_1 + \alpha_2 = 0$
s^1: $\alpha_3 = 12$
s^0: $9\alpha_1 = 24$

Solving simultaneously,

$$\alpha_1 = \frac{8}{3} \quad , \quad \alpha_2 = \frac{-8}{3} \quad , \quad \alpha_3 = 12$$

$$Y(s) = \frac{8}{3}\frac{1}{s} + \frac{\left(-\frac{8}{3}s + 12\right)}{s^2+9}$$

c) $$Y(s) = \frac{(s+2)(s+3)}{(s+4)(s+5)(s+6)} = \frac{\alpha_1}{s+4} + \frac{\alpha_2}{s+5} + \frac{\alpha_3}{s+6}$$

$$\alpha_1 = \left.\frac{(s+2)(s+3)}{(s+5)(s+6)}\right|_{s=-4} = 1$$

$$\alpha_2 = \left.\frac{(s+2)(s+3)}{(s+4)(s+6)}\right|_{s=-5} = -6$$

$$\alpha_3 = \left.\frac{(s+2)(s+3)}{(s+4)(s+5)}\right|_{s=-6} = 6$$

$$Y(s) = \frac{1}{s+4} - \frac{6}{s+5} + \frac{6}{s+6}$$

d) $$Y(s) = \frac{1}{\left[(s+1)^2+1\right]^2 (s+2)} = \frac{1}{(s^2+2s+2)^2(s+2)}$$

$$= \frac{\alpha_1 s + \alpha_2}{s^2 + 2s + 2} + \frac{\alpha_3 s + \alpha_4}{(s^2 + 2s + 2)^2} + \frac{\alpha_5}{s+2}$$

Multiplying both sides by $(s^2 + 2s + 2)^2 (s+2)$ gives

$1 = \alpha_1 s^4 + 4\alpha_1 s^3 + 6\alpha_1 s^2 + 4\alpha_1 s + \alpha_2 s^3 + 4\alpha_2 s^2 + 6\alpha_2 s + 4\alpha_2 + \alpha_3 s^2 + 2\alpha_3 s + \alpha_4 s + 2\alpha_4 + \alpha_5 s^4 + 4\alpha_5 s^3 + 8\alpha_5 s^2 + 8\alpha_5 s + 4\alpha_5$

Equating coefficients of like power of *s*,

s^4 : $\alpha_1 + \alpha_5 = 0$

s^3 : $4\alpha_1 + \alpha_2 + 4\alpha_5 = 0$

s^2 : $6\alpha_1 + 4\alpha_2 + \alpha_3 + 8\alpha_5 = 0$

s^1 : $4\alpha_1 + 6\alpha_2 + 2\alpha_3 + \alpha_4 + 8\alpha_5 = 0$

s^0 : $4\alpha_2 + 2\alpha_4 + 4\alpha_5 = 1$

Solving simultaneously:

$\alpha_1 = -1/4$ $\alpha_2 = 0$ $\alpha_3 = -1/2$ $\alpha_4 = 0$ $\alpha_5 = 1/4$

$$Y(s) = \frac{-1/4s}{s^2 + 2s + 2} + \frac{-1/2s}{(s^2 + 2s + 2)^2} + \frac{1/4}{s+2}$$

3.8

a) From Eq. 3-66

$$\mathfrak{L}\left[\int_0^t f(t^*)dt^*\right] = \frac{1}{s}F(s)$$

we know that $\mathfrak{L}\left[\int_0^t e^{-\tau}d\tau\right] = \frac{1}{s}\ \mathfrak{L}\left[e^{-t}\right] = \frac{1}{s(s+1)}$

$\therefore$ Laplace transforming yields

$$s^2X(s) + 4X(s) + 3X(s) = \frac{2}{s(s+1)}$$

or $(s^2 + 4s + 3)\, X(s) = \dfrac{2}{s(s+1)}$

$$X(s) = \frac{2}{s(s+1)^2(s+3)}$$

Performing partial fraction expansion and taking the inverse Laplace transform (either manually or using a symbolic software program), we get:

$$x(t) = \frac{2}{3} - te^{-t} - \frac{e^{-3t}}{6} - \frac{e^{-t}}{2}$$

b) Applying the Final Value Theorem (note that the theorem is applicable here)

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \frac{2}{(s+1)^2(s+3)} = \frac{2}{3}$$

3.9

i) $Y(s) = \dfrac{2}{s(s^2+4s)} = \dfrac{2}{s^2(s+4)} = \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{s+4}$

$\therefore$ $y(t)$ will contain terms of form: constant, t, e^{-4t}

ii) $Y(s) = \dfrac{2}{s(s^2+4s+3)} = \dfrac{2}{s(s+1)(s+3)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{s+3}$

$\therefore$ $y(t)$ will contain terms of form: constant, e^{-t}, e^{-3t}

iii) $Y(s) = \dfrac{2}{s(s^2+4s+4)} = \dfrac{2}{s(s+2)^2} = \dfrac{A}{s} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+2}$

$\therefore$ $y(t)$ will contain terms of form: constant, e^{-2t} , te^{-2t}

iv) $Y(s) = \dfrac{2}{s(s^2+4s+8)}$

$$s^2 + 4s + 8 = (s^2 + 4s + 4) + (8 - 4) = (s+2)^2 + 2^2$$

$$Y(s) = \frac{2}{s[(s+2)^2 + 2^2]}$$

$\therefore$ $y(t)$ will contain terms of form: constant, $e^{-2t}\sin 2t$, $e^{-2t}\cos 2t$

v) $$Y(s)=\frac{2(s+1)}{s(s^2+4)}=\frac{2(s+1)}{s(s^2+2^2)}=\frac{A}{s}+\frac{Bs}{s^2+2^2}+\frac{C}{s^2+2^2}$$

$$A=\lim_{s\to 0}\frac{2(s+1)}{(s^2+4)}=\frac{1}{2}$$

$2(s+1) = A(s^2+4) + Bs(s) + Cs$

$2s+2 = As^2 + 4A + Bs^2 + Cs$

Equating coefficients on like powers of s

s^2: $0 = A + B \rightarrow B = -A = -\frac{1}{2}$

s^1: $2 = C \rightarrow C = 2$

s^0: $2 = 4A \rightarrow A = \frac{1}{2}$

$$\therefore \quad Y(s)=\frac{1/2}{s}+\frac{-(1/2)s}{s^2+2^2}+\frac{2}{s^2+2^2}$$

$$y(t) = \frac{1}{2}-\frac{1}{2}\cos 2t+\frac{2}{2}\sin 2t$$

$$y(t) = \frac{1}{2}(1-\cos 2t)+\sin 2t$$

3.10

a) Laplace transform of the equation gives

$$s^3X(s)+2s^2X(s)+2sX(s)+X(s)=\frac{3}{s}$$

$$X(s)=\frac{3}{s(s^3+2s^2+2s+1)}=\frac{3}{s(s+1)(s+\frac{1}{2}+\frac{\sqrt{3}}{2}j)(s+\frac{1}{2}-\frac{\sqrt{3}}{2}j)}$$

The denominator of $[sX(s)]$ contains complex factors so that $x(t)$ is oscillatory, and the denominator vanishes at real values of $s= -1$ and -½ which are all <0; thus $x(t)$ is converges. See Fig. S3.10a.

b) $$s^2X(s)-X(s)=\frac{2}{s-1}$$

$$X(s)=\frac{2}{(s-1)(s^2-1)}=\frac{2}{(s-1)^2(s+1)}$$

The denominator contains no complex factors; thus $x(t)$ is not oscillatory. The denominator vanishes at s=1 ≥0; $x(t)$ is divergent. See Fig. S3.10b.

c) $$s^3 X(s) + X(s) = \frac{1}{s^2+1}$$

$$X(s) = \frac{1}{(s^2+1)(s^3+1)} = \frac{1}{(s+j)(s-j)(s+1)(s-\frac{1}{2}+\frac{\sqrt{3}}{2}j)(s-\frac{1}{2}-\frac{\sqrt{3}}{2}j)}$$

The denominator contains complex factors; $x(t)$ is oscillatory. The denominator vanishes at real s = 0 and s= ½; thus $x(t)$ is not convergent. See Fig. S3.10c.

d) $$s^2 X(s) + sX(s) = \frac{4}{s}$$

$$X(s) = \frac{4}{s(s^2+s)} = \frac{4}{s^2(s+1)}$$

The denominator of $[sX(s)]$ contains no complex factors; $x(t)$ is not oscillatory. The denominator of $[sX(s)]$ vanishes at s = 0; $x(t)$ is not convergent. See Fig. S3.10d.

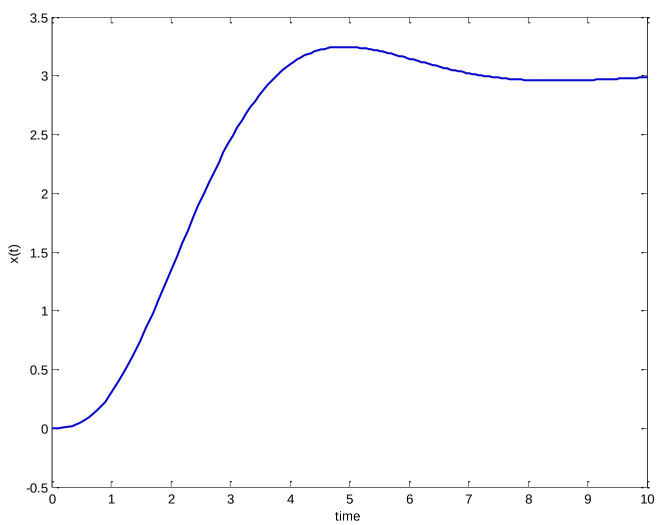


Figure S3.10a. *Simulation of X(s) for case a)*

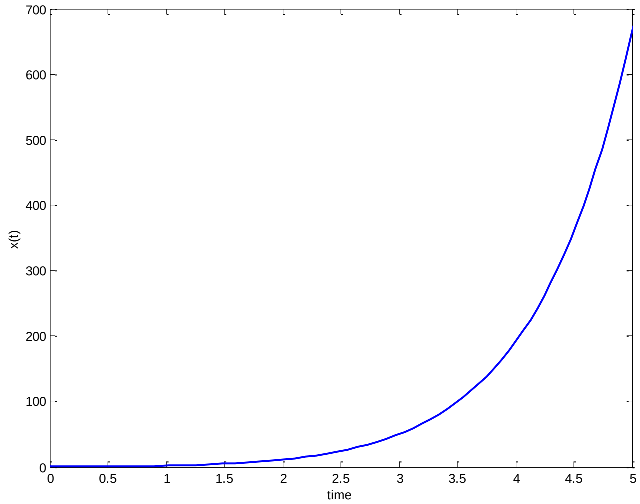


Figure S3.10b. *Simulation of X(s) for case b)*

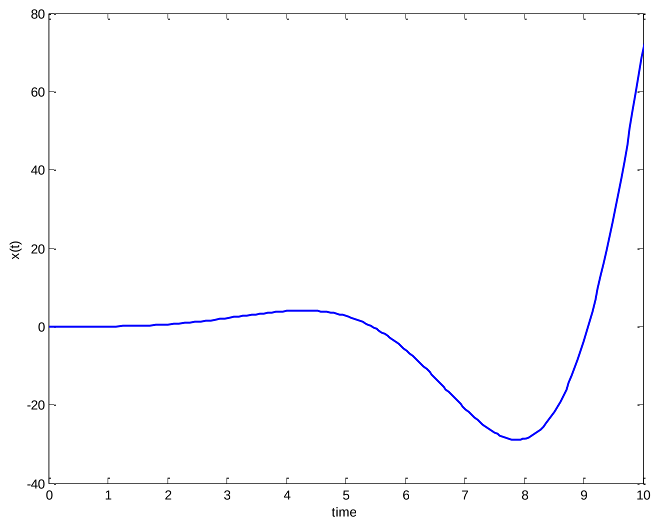


Figure S3.10c. *Simulation of X(s) for case c)*

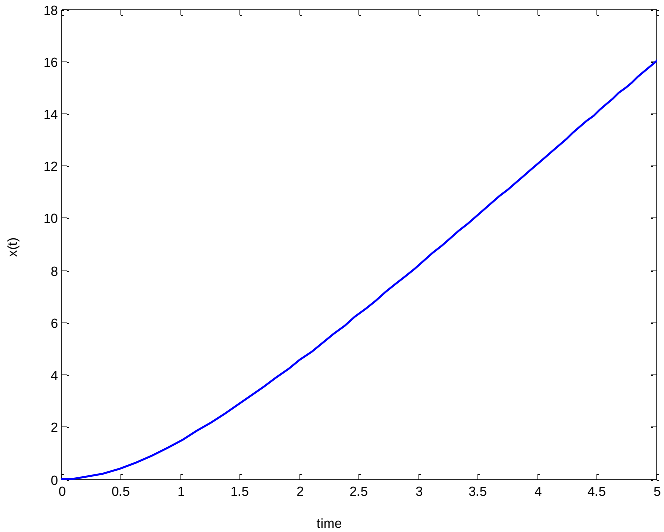


Figure S3.10d. *Simulation of X(s) for case d)*

3.11

Since the time function in the solution is not a function of initial conditions, take the Laplace transform with:

$$x(0) = \frac{dx(0)}{dt} = 0$$

$$\tau_1\tau_2 s^2 X(s) + (\tau_1+\tau_2)sX(s) + X(s) = KU(s)$$

$$X(s) = \frac{K}{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + 1}U(s)$$

Factoring the denominator

$$X(s) = \frac{K}{(\tau_1 s+1)(\tau_2 s+1)}U(s)$$

a) If $u(t) = a\,S(t)$ then $U(s)=\dfrac{a}{s}$

$$X_a(s) = \frac{Ka}{s(\tau_1 s+1)(\tau_2 s+1)} \qquad\qquad \tau_1 \neq \tau_2$$

$$x_a(t) = f_a(\,S(t),\quad e^{-t/\tau_1},\quad e^{-t/\tau_2})$$

b) If $u(t) = be^{-t/\tau}$ then $U(s) = \dfrac{b\tau}{\tau_s + 1}$

$$X_b(s) = \frac{Kb\tau}{(\tau s+1)(\tau_1 s+1)(\tau_2 s+1)} \qquad \tau \neq \tau_1 \neq \tau_2$$

$x_b(t) = f_b(e^{-t/\tau}\ ,\ e^{-t/\tau_1},\ \ e^{-t/\tau_2})$

c) If $u(t) = ce^{-t/\tau}$ where $\tau = \tau_1$, then $U(s) = \dfrac{\tau c}{\tau_1 s + 1}$

$$X_c(s) = \frac{Kc\tau}{(\tau_1 s+1)^2(\tau_2 s+1)}$$

$x_c(t) = f_c(e^{-t/\tau_1},\ t\,e^{-t/\tau_1},\ \ e^{-t/\tau_2})$

d) If $u(t) = d \sin \omega t$ then $U(s) = \dfrac{d\omega}{s^2 + \omega^2}$

$$X_d(s) = \frac{Kd}{(s^2+\omega^2)(\tau_1 s+1)(\tau_2 s+1)}$$

$x_d(t) = f_d(e^{-t/\tau_1},\ \ e^{-t/\tau_2}, \sin \omega t, \cos \omega t)$

3.12

a) $\dfrac{dx^3}{dt^3} + 4x = e^t$ with $\dfrac{d^2x(0)}{dt^2} = \dfrac{dx(0)}{dt} = x(0) = 0$

Take the Laplace transform of the equation:

$$s^3 X(s) + 4X(s) = \frac{1}{s-1}$$

$$X(s) = \frac{1}{(s-1)(s^3+4)} = \frac{1}{(s-1)(s+1.59)(s-0.79+1.37j)(s-0.79-1.37j)}$$

$$= \frac{\alpha_1}{s-1} + \frac{\alpha_2}{s+1.59} + \frac{\alpha_3 + j\beta_3}{s-0.79+1.37j} + \frac{\alpha_3 - j\beta_3}{s-0.79-1.37j}$$

$$\alpha_1 = \left.\frac{1}{(s^3+4)}\right|_{s=1} = \frac{1}{5}$$

$$\alpha_2 = \left.\frac{1}{(s-1)(s-0.79+1.37j)(s-0.79-1.37j)}\right|_{s=-1.59} = -\frac{1}{19.6}$$

$$\alpha_3 + j\beta_3 = \left.\frac{1}{(s-1)(s+1.59)(s-0.79-1.37j)}\right|_{s=0.79-1.37j} = -0.74-0.59j$$

$$X(\mathrm{s}) = \frac{\frac{1}{5}}{s-1} + \frac{-\frac{1}{19.6}}{s+1.59} + \frac{-0.074-0.059j}{s-0.79+1.37j} + \frac{-0.074+0.059j}{s-0.79-1.37j}$$

$$x(t) = \frac{1}{5}e^t - \frac{1}{19.6}e^{-1.59t} - 2e^{0.79t}(0.074\cos 1.37t + 0.059\sin 1.37t)$$

b) $\dfrac{dx}{dt} - 12x = \sin 3t$ with $x(0) = 0$

$$sX(\mathrm{s}) - 12X(s) = \frac{3}{s^2+9}$$

$$X(s) = \frac{3}{(s^2+9)(s\ \ -12)} = \frac{3}{(s+3j)(s-3j)(s-12)}$$

$$= \frac{\alpha_1 + j\beta_1}{s+3j} + \frac{\alpha_1 - j\beta_1}{s-3j} + \frac{\alpha_3}{s-12}$$

$$\alpha_1 + j\beta_1 = \left.\frac{3}{(s-3j)(s-12)}\right|_{s=-3j} = \frac{3}{-18+72j} = -\frac{1}{102} - \frac{4}{102}j$$

$$\alpha_3 = \left.\frac{3}{(s^2+9)}\right|_{s=12} = \frac{1}{51}$$

$$X(s) = \frac{-\frac{1}{102}-\frac{4}{102}j}{s+3j} + \frac{-\frac{1}{102}+\frac{4}{102}j}{s-3j} + \frac{\frac{1}{51}}{s-12}$$

$$x(t) = -\frac{1}{51}(\cos 3t + 4\sin 3t) + \frac{1}{51}e^{12t}$$

c) $$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+25x=e^{-t} \qquad \text{with} \qquad \frac{dx(0)}{dt}=x(0)=0$$

$$s^2X(s)+6sX(s)+25X(s)+X(s)=\frac{1}{s+1} \quad \text{or} \quad X(s)=\frac{1}{(s+1)(s^2+6s+25)}$$

$$X(s)=\frac{1}{(s+1)(s+3+4j)(s+3-4j)}=\frac{\alpha_1}{s+1}+\frac{\alpha_2+\beta_2 j}{s+3+4j}+\frac{\alpha_2-\beta_2 j}{s+3-4j}$$

$$\alpha_1=\left.\frac{1}{(s^2+6s+25)}\right|_{s=-1}=\frac{1}{20}$$

$$\alpha_2+j\beta_2=\left.\frac{1}{(s+1)(s+3-4j)}\right|_{s=-3-4j}=-\frac{1}{40}-\frac{1}{80}j$$

$$X(s)=\frac{\frac{1}{20}}{s+1}+\frac{-\frac{1}{40}-\frac{1}{80}j}{s+3+4j}+\frac{-\frac{1}{40}-\frac{1}{80}j}{s+3-4j}$$

$$x(t)=\frac{1}{20}e^{-t}-e^{-3t}(\frac{1}{20}\cos 4t+\frac{1}{40}\sin 4t)$$

3.13

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+y(t)=4\frac{d(x-2)}{dt}-x(t-2)$$

Take the Laplace transform assuming zero initial conditions:

$s^2Y(s) + 3sY(s) + Y(s) = 4\, e^{-2s}sX(s) - e^{-2s}X(s)$

Rearranging,

$$\frac{Y(s)}{X(s)}=G(s)=\frac{-(1-4s)e^{-2s}}{s^2+3s+1} \qquad (1)$$

a) The standard form of the denominator is : $\tau^2s^2 + 2\zeta\tau s + 1$

From (1) , $\tau = 1$, $\zeta = 1.5$

Thus the system will exhibit overdamped and non-oscillatory responses.

b) Steady-state gain

$$K = \lim_{s \to 0} G(s) = -1 \qquad \text{(from (1))}$$

c) For a step change in x

$$X(s) = \frac{1.5}{s} \quad \text{and} \quad Y(s) = \frac{-(1-4s)e^{-2s}}{(s^2+3s+1)}\ \frac{1.5}{s}$$

Therefore, $\hat{y}(t) = -1.5 \ + \ 1.5e^{-1.5t}\cosh(1.11t) \ + \ 7.38e^{-1.5t}\sinh(1.11t)$
Using MATLAB-Simulink, $y(t)$= $\hat{y}(t-2)$ is shown in Fig. S3.13

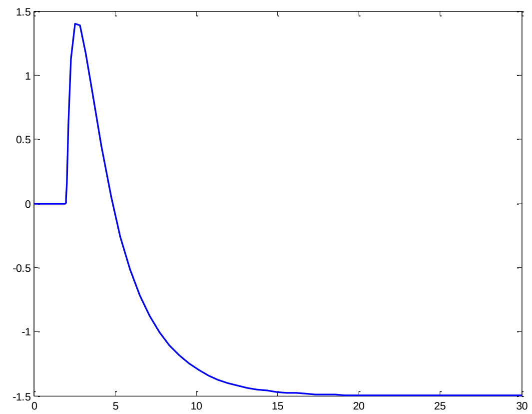


Figure S3.13. *Output variable for a step change in x of magnitude 1.5*

3.14

First, take the Laplace transform of each term in the equation

$$\mathcal{L}\left(\frac{d^2y}{dt^2}\right) = s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - s$$

$$\mathcal{L}\left(5\frac{dy}{dt}\right) = 5(sY(s) - y(0)) = 5sY(s) - 5$$

$$\mathcal{L}(6y) = 6Y(s)$$

$$\mathcal{L}(7) = \frac{7}{s}$$

The final transformed equation is:

$$Y(s^2+5s+6)-s-5=\frac{7}{s}$$
$$Y(s^2+5s+6)=\frac{s^2+5s+7}{s}$$
$$Y=\frac{s^2+5s+7}{s(s+2)(s+3)}$$

Now perform partial fraction expansion.

$$\frac{s^2+5s+7}{s(s+2)(s+3)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+3}$$
$$A=\frac{7}{6},B=-\frac{1}{2},C=\frac{1}{3}$$
$$X(s)=\frac{7}{6s}-\frac{1}{2(s+2)}+\frac{1}{3(s+3)}$$
$$x(t)=\frac{7}{6}-\frac{1}{2}e^{-2t}+\frac{1}{3}e^{-3t}$$

3.15

$$f(t)=hS(t)-hS(t-1/h)$$
$$\frac{dx}{dt}+4x=h[S(t)-S(t-1/h)] \qquad , \quad x(0)=0$$

Take the Laplace transform,

$$sX(s)+4X(s)=h\left(\frac{1}{s}-\frac{e^{-s/h}}{s}\right)$$
$$X(s)=h(1-e^{-s/h})\frac{1}{s(s+4)}=h(1-e^{-s/h})\left[\frac{\alpha_1}{s}+\frac{\alpha_2}{s+4}\right]$$
$$\alpha_1=\left.\frac{1}{s+4}\right|_{s=0}=\frac{1}{4} \quad , \quad \alpha_2=\left.\frac{1}{s}\right|_{s=-4}=-\frac{1}{4}$$
$$X(s)=\frac{h}{4}(1-e^{-s/h})\left[\frac{1}{s}-\frac{1}{s+4}\right]$$
$$=\frac{h}{4}\left[\frac{1}{s}-\frac{e^{-s/h}}{s}-\frac{1}{s+4}+\frac{e^{-s/h}}{s+4}\right]$$

$$x(t) = \begin{cases} 0 & t < 0 \\ \frac{h}{4}(1-e^{-4t}) & 0 < t < 1/h \\ \frac{h}{4}\left[e^{-4(t-1/h)} - e^{-4t}\right] & t > 1/h \end{cases}$$

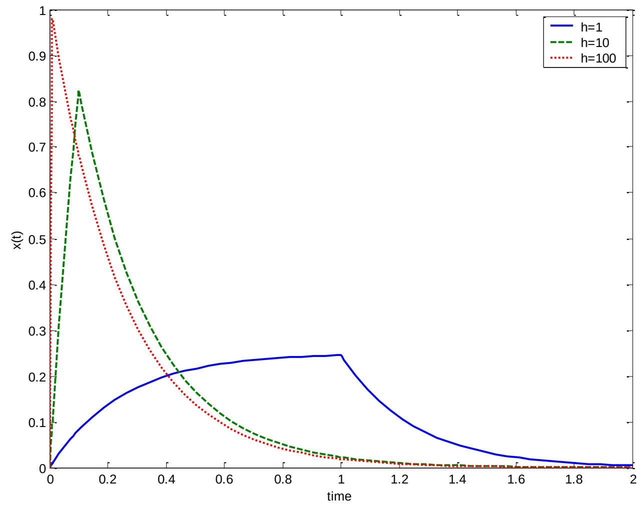


Figure S3.15. *Solution for values h= 1, 10 and 100*

3.16

a) Take the Laplace transform:

$$\left[s^2Y(s) - sy(0) - y'(0)\right] + 6\left[sY(s) - y(0)\right] + 9Y(s) = \frac{s}{s^2+1}$$

$$(s^2 + 6s + 9)Y(s) - s(1) - 2 - (6)(1) = \frac{s}{s^2+1}$$

$$(s^2 + 6s + 9)Y(s) = \frac{s}{s^2+1} + s + 8$$

$$(s^2 + 6s + 9)Y(s) = \frac{s + s^3 + s + 8s^2 + 8}{s^2+1}$$

$$Y(s) = \frac{s^3+8s^2+2s+8}{(s+3)^2(s^2+1)}$$

To find $y(t)$ we have to expand $Y(s)$ into its partial fractions

$$Y(s) = \frac{A}{(s+3)^2} + \frac{B}{s+3} + \frac{Cs}{s^2+1} + \frac{D}{s^2+1}$$

$y(t) = Ate^{-3t} + Be^{-3t} + C\cos t + D\sin t$

b) $$Y(s) = \frac{s+1}{s(s^2+4s+8)}$$

Since $\frac{4^2}{4} < 8$, there are complex factors.

$\therefore$ complete the square in denominator

$s^2 + 4s + 8 = s^2 + 4s + 4 + 8-4$

$= s^2 + 4s + 4 + 4 = (s+2)^2 + (2)^2$ $\{ b = 2 \;,\; \omega=2\}$

$\therefore$ Partial fraction expansion gives

$$Y(s) = \frac{A}{s} + \frac{B(s+2)}{s^2+4s+8} + \frac{C}{s^2+4s+8} = \frac{s+1}{s(s^2+4s+8)}$$

Multiply by s and let $s \rightarrow 0$

$A=1/8$

Multiply by $s(s^2+4s+8)$

$A(s^2+4s+8) + B(s+2)s + Cs = s + 1$

$As^2 + 4As + 8A + Bs^2 + 2Bs + Cs = s + 1$

s^2: $A + B = 0 \rightarrow B = -A = -\frac{1}{8}$

s^1: $4A + 2B + C = 1 \rightarrow C = 1 + 2\left(\frac{1}{8}\right) - 4\left(\frac{1}{8}\right) = \frac{3}{4}$

s^0: $8A = 1 \rightarrow A = \frac{1}{8}$ (This checks with above result)

$$Y(s) = \frac{1/8}{s} + \frac{(-1/8)(s+2)}{(s+2)^2 + 2^2} + \frac{3/4}{(s+2)^2 + 2^2}$$

$$y(t) = \left(\frac{1}{8}\right) - \left(\frac{1}{8}\right) e^{-2t} \cos 2t + \left(\frac{3}{8}\right) e^{-2t} \sin 2t$$

3.17

Laplace transform of the system of ODEs gives:

$$\mathcal{L}\left(\frac{dy_1}{dt}\right) + \mathcal{L}(y_2) = \mathcal{L}(e^{-t})$$

$$\mathcal{L}\left(\frac{dy_2}{dt}\right) + 3\mathcal{L}(y_2) = 2\mathcal{L}(y_1)$$

$$sY_1 + Y_2 = \frac{1}{s+1} \qquad (1)$$

$$sY_2 + 3Y_2 = 2Y_1 \qquad (2)$$

Next solve Equation 2 for Y_2 in terms of Y_1

$$Y_2(s+3) = 2Y_1$$

$$Y_2 = \frac{2Y_1}{s+3} \qquad (3)$$

Substitute equation 3 into equation 1 and solve for Y_1

$$sY_1 + \frac{2Y_1}{s+3} = \frac{1}{s+1}$$

$$Y_1\left(s + \frac{2}{s+3}\right) = \frac{1}{s+1}$$

$$Y_1 = \frac{1}{(s+1)(s+\frac{2}{s+3})}$$

Expand using partial fractions:

$$Y_1 = \frac{s+3}{(s+1)^2(s+2)} = \frac{1}{s+2} - \frac{1}{s+1} + \frac{2}{(s+1)^2}$$

Now go back and substitute into equation 3 to get Y_2 and expand using partial fractions:

$$Y_2 = \frac{2Y_1}{s+3} = \frac{2}{(s+1)^2(s+2)} = \frac{2}{s+2} - \frac{2}{s+1} + \frac{2}{(s+1)^2}$$

Finally, get both time-domain solutions using the inverse Laplace transform:

$$y_1(t) = e^{-2t} - e^{-t} + 2te^{-t}$$
$$y_2(t) = 2(e^{-2t} - e^{-t} + te^{-t})$$

3.18

$$V\frac{dc}{dt} + qc = qc_i$$

Since *V* and *q* are constant, taking Laplace transforms give

$$sVC(s) + qC(s) = q\,C_i(s)$$

Note that $c(t = 0) = 0$

Also, $c_i(t) = 0 \quad , \quad t \le 0$
$c_i(t) = \bar{c}_i \quad , \quad t > 0$

Taking Laplace transform of the input function, a constant, gives

$$C_i(s) = \frac{\bar{c}_i}{s}$$

so that

$$sVC(s) + qC(s) = q\frac{\bar{c}_i}{s} \qquad \text{or} \qquad C(s) = \frac{q\bar{c}_i}{(sV+q)s}$$

Dividing numerator and denominator by *q*

$$C(s) = \frac{\bar{c}_i}{\left(\frac{V}{q}s+1\right)s}$$

Use Transform pair #3 in Table 3.1 to invert ($\tau = V/q$)

$$c(t) = \bar{c}_i \left(1 - e^{-\frac{q}{V}t}\right)$$

Using MATLAB, the concentration response is shown in Fig. S3.18. (Consider $V = 2\ \text{m}^3$, c_i=50 Kg/m^3 and $q = 0.4\ \text{m}^3\text{/min}$)

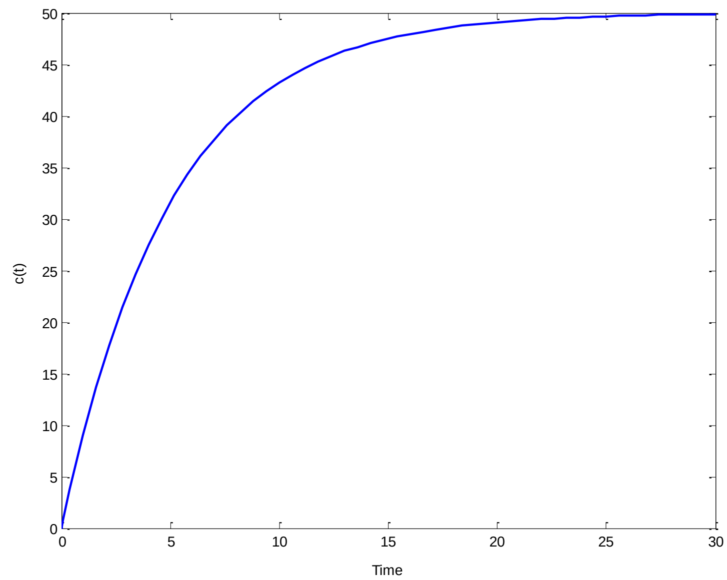


Figure S3.18. *Concentration response of the reactor effluent stream.*

3.19

(a) Take the Laplace transform of each term, taking into account that all initial conditions are zero:

$$s^2Y - sy(0) - y'(0) + 5sY - 5y(0) + Y = 8sU - 8u(0) + U$$
$$s^2Y + 5sY + Y = U(8s+1)$$
$$U(s) = \frac{1}{s}$$
$$Y(s^2 + 5s + 1) = \frac{8s+1}{s}$$
$$Y = \frac{8s+1}{s(s^2+5s+1)}$$

Now use symbolic mathematical software (ex. Mathematica) to solve for $y(t)$.

InverseLaplaceTransform[(8*s+1)/(s*(s^2+5*s+1)),s,t]

$$\text{g[t\_]:=}\frac{1}{41}\left(-21e^{-\frac{5+\sqrt{21}}{2}t}-11\sqrt{21}e^{-\frac{5+\sqrt{21}}{2}t}-21e^{\frac{\sqrt{21}-5}{2}t}+11\sqrt{21}e^{\frac{\sqrt{21}-5}{2}t}+42\right)$$

Plot[g[t],{t,0,100},AxesLabel→{time,Y},PlotRange→{{0,100},{0 ,2}}]

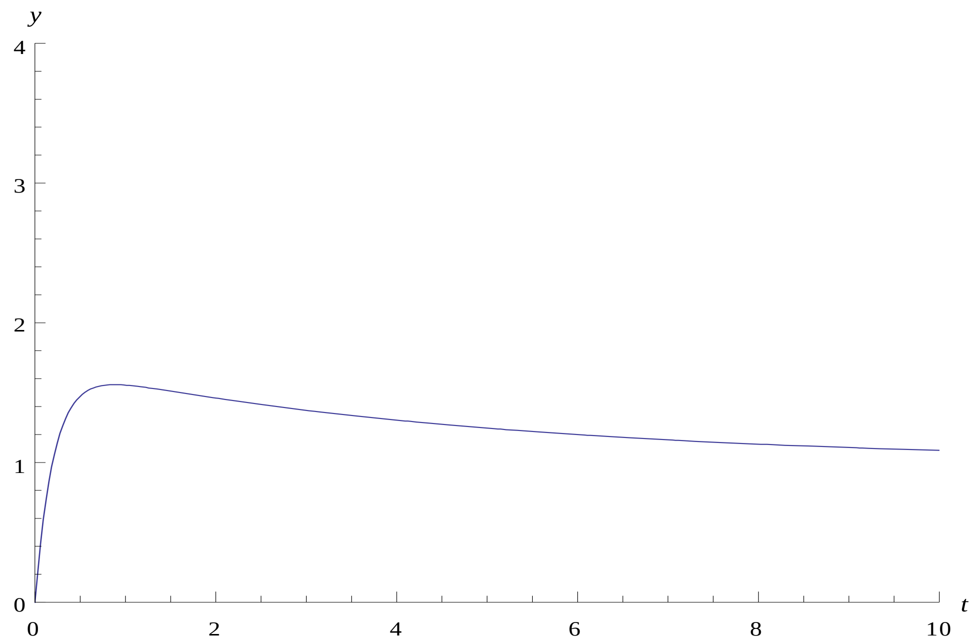


Figure S3.19a: Tank level response to a unit step change in flow rate.

(b) Define the time when $y(t)$ reaches its maximum as t_{max}. This time occurs when $y'(t)$=0. Solve for this time using Mathematica and find that t_{max}=0.877 and $y(t_{max})$=1.558. Therefore, the tank will not overflow.

(c) Now find the general solution for any input step size, M (the solution is denoted in this case as $Y_M(s)$ and $y_M(t)$ for clarity). The input $U(s)$ is M/s.

$$U(s) = \frac{M}{s}$$

$$Y_M(s^2+5s+1) = \frac{M(8s+1)}{s}$$

$$Y_M = \frac{M(8s+1)}{s(s^2+5s+1)} = MY$$

Y_M is the previous Y, multiplied by the size of the step, M. Since M is a constant, taking the inverse Laplace transform gives:

$$y_M(t) = My(t)$$

Now solve the equation:

$$y_M(t_{\max}) = 2.5 = My(t_{\max}) = M(1.558)$$

$$M = \frac{2.5}{1.558} = 1.605$$

The maximum step change in the flow rate into the tank that will not overflow the tank is 1.605.

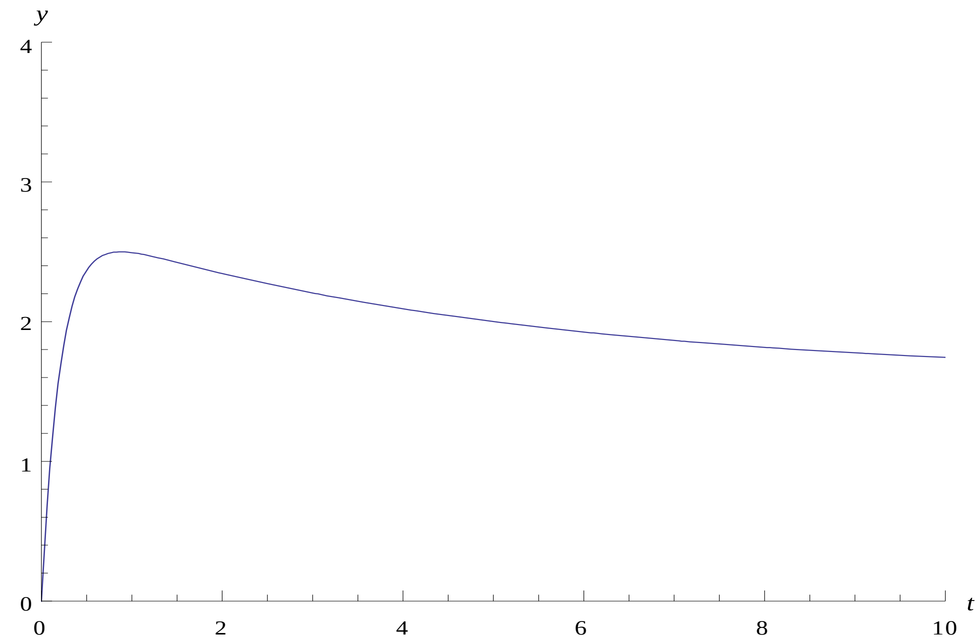


Figure S3.19b: Tank level response to a 1.605 step change in flow rate.

3.20

a) Given constant volumes, overall balances on the three tanks indicate that the flow rate out of each tank is equal to q

Component balance for tracer over each tank,

$$V_1 \frac{dc_1}{dt} = q\,(c_i - c_1)$$

$$V_2 \frac{dc_2}{dt} = q\,(c_1 - c_2)$$

$$V_3 \frac{dc_3}{dt} = q\,(c_2 - c_3)$$

b) Taking Laplace transform of above equations and eliminating $C_1(s)$ and $C_2(s)$ gives

$$C_3(s) = \frac{\left(\frac{q}{V_1}\right)\left(\frac{q}{V_2}\right)\left(\frac{q}{V_3}\right)}{\left(s+q/V_1\right)\left(s+q/V_2\right)\left(s+q/V_3\right)} C_i(s)$$

Since $c_i(t) = \delta(t), \qquad C_i(s) = 1$

1. $V_1 = V_2 = V_3 = V$

$$C_3(s) = \frac{(q/V)^3}{(s+q/V)^3} = \frac{\alpha_1}{(s+q/V)} + \frac{\alpha_2}{(s+q/V)^2} + \frac{\alpha_3}{(s+q/V)^3}$$

$$c_3(t) = \alpha_1 e^{-(q/V)t} + \alpha_2 t e^{-(q/V)t} + \alpha_3 t^2 e^{-(q/V)t}$$

2. $V_1 \neq V_2 \neq V_3 \neq V_1$

$$c_3(t) = \alpha_4 e^{-(q/V_1)t} + \alpha_5 e^{-(q/V_2)t} + \alpha_6 e^{-(q/V_3)t}$$

(c) Yes, amount of tracer can be calculated by measuring $c_3(t)$,

amount of tracer = $\int_0^\infty q c_3(t) dt$, which can be evaluated numerically

3.21

Start with the Laplace version of the equations from Exercise 3.20:

$$C_3(s) = \frac{\left(\frac{q}{V_1}\right)\left(\frac{q}{V_2}\right)\left(\frac{q}{V_3}\right)}{\left(s+q/V_1\right)\left(s+q/V_2\right)\left(s+q/V_3\right)} C_i(s)$$

Since V_1=V_2=V_3, this equation reduces to:

$$C_3(s) = \frac{\left(\frac{q}{V}\right)^3}{\left(s+q/V\right)^3} C_i(s) \qquad (1)$$

where $c_i(t)$ is a pulse of magnitude A and width t_w. A pulse can be described by the sum of two step functions. The first will be a step function of magnitude A at time 0. The second will be a step function of $-A$ at $t=t_w$.

$$c_i(t) = AS(t) - AS(t - t_w)$$
$$C_i(s) = \frac{A}{s} - \frac{A}{s} e^{-t_w s} = A\left(\frac{1 - e^{-t_w s}}{s}\right) \qquad (2)$$

Now substitute Equation (2) into Equation (1). For simplicity, define a new variable $f=q/V$.

$$C_3(s) = \frac{A(f)^3(1 - e^{-t_w s})}{s(s+f)^3}$$

Now use a symbolic mathematics software to find the inverse Laplace transform, giving $c_3(t)$. The solution is formulated as a function of t, f, A, and t_w. Then as an example, we plot $c_3(t)$ for f=1/20, A=10, and t_w=1.

In Mathematica, take the inverse Laplace transform:

```
InverseLaplaceTransform[A * f^3
        * (1 − Exp[−tw * s])/(s * (s + f)^3) , s, t]
```

The solution:

$c_3(t)$ =

$$\frac{1}{2} A\left(e^{-ft}(-2 + 2e^{ft} - 2ft - f^2t^2) - e^{f(-t+\text{tw})}\left(-2 + 2e^{f(t-\text{tw})} - 2f(t - \text{tw}) - f^2(t-\text{tw})^2\right)\text{HeavisideTheta}[t - \text{tw}]\right)$$

Define the function in terms of the parameters:

$$g[\text{t\_}, \text{f\_}, \text{A\_}, \text{tw\_}] := \frac{1}{2} A(e^{-ft}(-2 + 2e^{ft} - 2ft - f^2t^2) - e^{f(-t+\text{tw})}(-2 + 2e^{f(t-\text{tw})} - 2f(t - \text{tw}) - f^2(t - \text{tw})^2)\text{HeavisideTheta}[t - \text{tw}])$$

Then plot the concentration over time, assuming f=1/20, A=10, and t_w=1.

```
Plot[g[t, 1/20 , 10,1], {t, 0,200}, AxesLabel → {time, C3}]
```

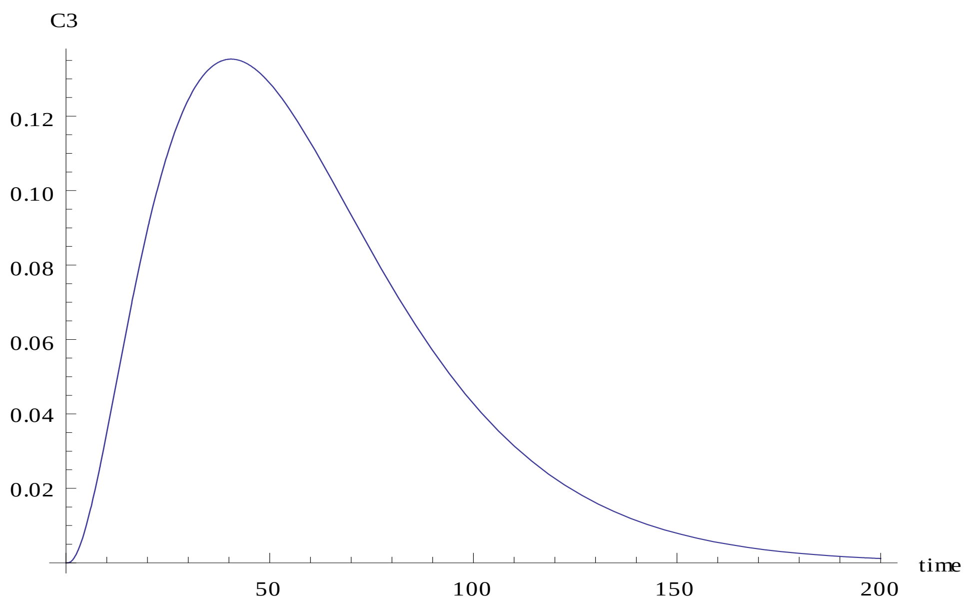


Figure S3.21: Plot of c_3 over time in response to a pulse in c_i of amplitude 10 and width 1, with f=1/20.

3.22

Solve this problem using a symbolic software program such as Mathematica. The following script will solve the problem (note that only 4 of the 5 possible initial conditions on y and its derivatives are included, otherwise the problem is over-specified).

$$\text{DSolve}[\{y''''[x] + 16 * y'''[x] + 86 * y''[x] + 176 * y'[x] + 105 * y[x] == 1, y[0] == 0, y'[0] == 0, y''[0] == 0, y'''[0] == 0\}, y[x], x]$$

Running this script will give the result:

$$\{\{y[x] \to \frac{e^{-7x}(-1+e^x)^4(5+20e^x+29e^{2x}+16e^{3x})}{1680}\}\}$$

Use the Expand[ ] command to expand this solution into its individual terms.

$$\{\{y[x] \to \frac{1}{105} + \frac{e^{-7x}}{336} - \frac{e^{-5x}}{80} + \frac{e^{-3x}}{48} - \frac{e^{-x}}{48}\}\}$$

If desired, the fractions can be approximated as decimals:

$$y(t) = 0.003e^{-7t} - 0.0125e^{-5t} + 0.021e^{-3t} - 0.021e^{-t} + 0.01$$

Chapter 4 ©

4.1

$$\frac{Y(s)}{U(s)} = \frac{d}{bs+c}$$

a) Gain K can be obtained by setting $s = 0$

$$K = \frac{d}{c}$$

Alternatively, the transfer function can be placed in the standard gain/time constant form by dividing the numerator and denominator by c:

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s+1}, \text{ where } K = \frac{d}{c} \text{ and } \tau = \frac{b}{c}.$$

b) In order to determine the boundedness of the output response, consider a step input of magnitude M. Then use $U(s) = M/s$ and

$$Y(s) = \frac{K}{\tau s+1}\frac{M}{s}$$

From Table 3.1, the step response is:

$$y(t) = KM(1-e^{-t/\tau})$$

By inspection, this response will be bounded only if $\tau > 0$, or equivalently, only if $b/c > 0$.

4.2

a) K=3

b) τ=10

c) We use the Final Value Theorem to find the value of y(t) when $t\to\infty$.

$$Y(s) = \frac{12e^{-s}}{s(10s+1)}$$

$$sY(s) = \frac{12e^{-s}}{(10s+1)}$$

$$\lim_{s\to 0}\frac{12e^{-s}}{(10s+1)} = 12$$

From the Final Value Theorem, $y(t) = 12$ when $t\to\infty$

d) $y(t) = 12(1-e^{-(t-1)/10})$, then $y(10) = 12(1-e^{-9/10}) = 7.12$

7.12/12=0.593.

e) Again use the final value theorem.

$$Y(s) = \frac{3e^{-s}}{(10s+1)} \frac{(1-\mathrm{e}^{-s})}{s}$$

$$sY(s) = \frac{3e^{-s}(1-e^{-s})}{(10s+1)}$$

$$\lim_{s\to 0} \frac{3e^{-s}(1-e^{-s})}{(10s+1)} = \frac{3(1-1)}{1} = 0$$

From the Final Value Theorem, $y(t) = 0$ when $t\to\infty$

f)

$$Y(s) = \frac{3e^{-s}}{(10s+1)}1$$

$$sY(s) = \frac{3se^{-s}}{(10s+1)}$$

$$\lim_{s\to 0} \frac{3se^{-s}}{(10s+1)} = 0$$

From the Final Value Theorem, $y(t)= 0$ when $t\to\infty$

g) $$Y(s) = \frac{3e^{-s}}{(10s+1)} \frac{10}{(s^2+4)} = \frac{30e^{-s}}{(10s+1)(s^2+4)}$$ then

$$y(t) = 30S(t-1)\left(\frac{10}{401} e^{\frac{-(t-1)}{10}} + \frac{1}{802}(\sin(2(t-1)) - 20\cos(2(t-1))) \right)$$

The sinusoidal input produces a sinusoidal output and $y(t)$ does not have a limit when $t\to\infty$.

These solutions can be verified by using mathematical software such as Mathematica or Simulink.

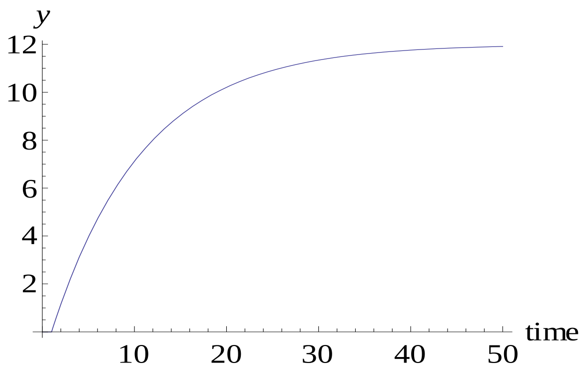


Fig. S4.2a. *Output for parts c) and d).*

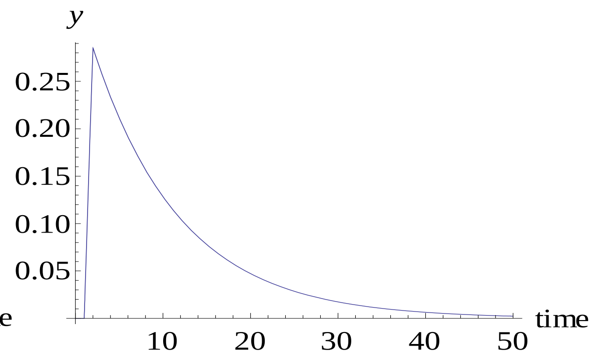


Fig. S4.2b. *Output for part e).*

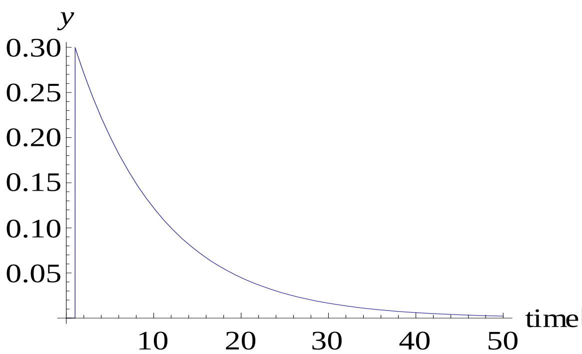


Fig. S4.2c. *Output for part f).*

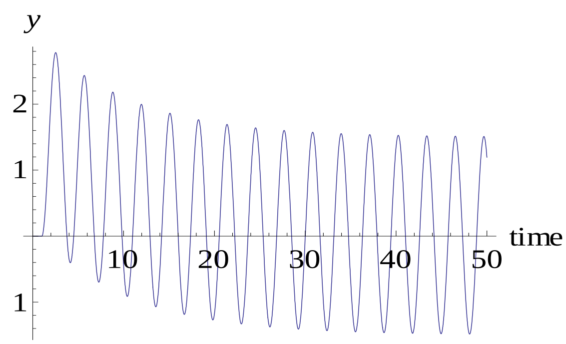


Fig. S4.2d. *Output for part g).*

4.3

The transfer function for the pressure transmitter is given by,

$$\frac{P'_m(s)}{P'(s)} = \frac{1}{10s+1} \tag{1}$$

and $P'(s) = 15/s$ for the step change from 35 to 50 psi. Substituting (1) and rearranging gives:

$$P'_m(s) = \frac{1}{10s+1}\,\frac{15}{s}$$

From item #13 in Table 3.1, the step response is given by:

$$P'_m(t) = 15\,(1 - e^{-t/10}) \tag{2}$$

Let t_a be the time that the alarm sounds. Then,

$$P'_m(t_a) = 45 - 35 = 10 \text{ psi} \tag{3}$$

Substituting (3) and $t=t_a$ into (2) and solving gives t_a = 11s. Thus, the alarm will sound 11 seconds after 1:30PM.

4.4

From Exercise 4.2,

$$\frac{Y(s)}{U(s)} = \frac{3e^{-s}}{10s+1}$$

Rearrange,

$$Y(s)[10s+1] = 3e^{-s}U(s) \qquad (2)$$

Take $\mathcal{L}^{-1}$ of (2),

$$10\frac{dy}{dt} + y = 3u(t-1) \qquad (3)$$

Take $\mathcal{L}$ of (3) for $y(0)$=4,

$$10[sY(s)-4] + Y(s) = 3e^{-s}U(s)$$

Substitute $U(s) = 2/s$ and rearrange to give,

$$10sY\text{-}40\text{+}Y\text{=}\frac{6e^{-s}}{s}$$

$$Y(10s+1) = \frac{6e^{-s}}{s} + 40$$

Partial fraction expansion:

$$Y(s) = e^{-s}\frac{6}{s(10s+1)} + \frac{40}{(10s+1)}$$

$$\frac{6}{s(10s+1)} = \frac{a_1}{s} + \frac{a_2}{10s+1}$$

Find α_1: Multiply by s and set $s = 0 \quad \Rightarrow \quad \alpha_1 = 6$

Find α_2: Multiply by $10s+1$ and set $s = -0.1 \quad \Rightarrow \quad \alpha_2 = 60$

$$Y(s) = e^{-s}\left(\frac{6}{s} + \frac{6}{s+0.1}\right) + \frac{4}{(s+0.1)}$$

Take $\mathcal{L}^{-1}$,

$$y(t) = 6S(t-1)(1+e^{-(t-1)/10}) + 4e^{-t/10}$$

Check: At t=0, $y(0)$=4.

4.5

a) $2\dfrac{dy_1}{dt}$ = -2y_1 – 3y_2 + 2u_1 (1)

$\dfrac{dy_2}{dt}$ = 4y_1 – 6y_2 + 2u_1 + 4u_2 (2)

Taking Laplace transform of the above equations and rearranging,

$(2s+2)Y_1(s) + 3Y_2(s) = 2U_1(s)$ (3)

$-4\, Y_1(s) + (s+6)Y_2(s)=2U_1(s) + 4U_2(s)$ (4)

Solving Eqs. (3) and (4) simultaneously for Y_1(s) and Y_2(s),

$$Y_1(s) = \frac{(2s+6)U_1(s) - 12U_2(s)}{2s^2+14s+24} = \frac{2(s+3)U_1(s) - 12U_2(s)}{2(s+3)(s+4)}$$

$$Y_2(s) = \frac{(4s+12)U_1(s) - (8s+8)U_2(s)}{2s^2+14s+24} = \frac{4(s+3)U_1(s) + 8(s+1)U_2(s)}{2(s+3)(s+4)}$$

Therefore,

$$\frac{Y_1(s)}{U_1(s)} = \frac{1}{s+4} \quad , \quad \frac{Y_1(s)}{U_2(s)} = \frac{-6}{(s+3)(s+4)}$$

$$\frac{Y_2(s)}{U_1(s)} = \frac{2}{s+4} \quad , \quad \frac{Y_2(s)}{U_2(s)} = \frac{4(s+1)}{(s+3)(s+4)}$$

4.6

a)

Taking the $\mathcal{L}^{-1}$ gives,

$x'(t) = 0.09e^{-t/10}$ and $x(t) = \bar{x} + x'(t) = 0.3 + 0.09e^{-t/10}$

The intial values are $x'(0) = 0.09$ and

$x(0) = x'(0) + \bar{x} = 0.09 + 0.3 = 0.39.$

The plot of the concentration response is shown in Fig. S4.6.

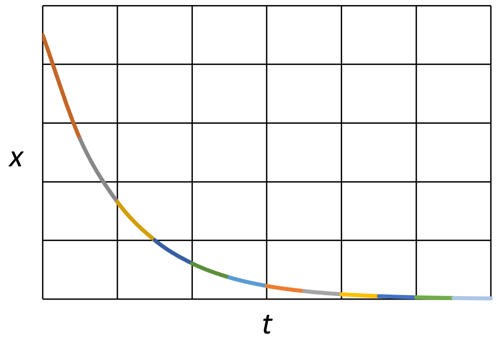


Fig. S4.6. Transient response.

The transfer function is given by:

$$\frac{X'(s)}{X_i'(s)} = \frac{0.6}{10s+1}$$

For the impulse input, $x_i'(t) = 1.5\delta(t)$, and from Table 3.1, $X_i'(s)$=1.5. Thus,

$$X'(s) = \frac{0.9}{10s+1}$$

b) Initial Value Theorem:

$$x'(0) = \lim_{s\to\infty} sX'(s) = \frac{0.9}{10} = 0.09$$

Thus, $x(0) = x'(0) + \bar{x} = 0.09 + 0.3 = 0.39$

c) For the steady-state condition,

$$x(0) = \bar{x} = 0.3$$

d) As indicated in the plot, the impulse response is discontinuous at t=0. The results for parts (a) and (b) give the values of $x(0)$ for $t=0^+$ while the result for (c) gives the value for $t=0^-$.

4.7

The simplified stage concentration model becomes

$$H\frac{dx_1}{dt} = L(x_0 - x_1) + V(y_2 - y_1) \quad (1)$$

$$y_1 = a_0 + a_1x_1 + a_2x_1^2 + a_3x_1^3 \quad (2)$$

a) Let the right-hand side of Eq. 1 be denoted as $f(L, x_0, x_1, V, y_1, y_2)$

$$H\frac{dx_1}{dt} = f(L, x_0, x_1, V, y_1, y_2) = \left(\frac{\partial f}{\partial L}\right)_s L' + \left(\frac{\partial f}{\partial x_0}\right)_s x_0' + \left(\frac{\partial f}{\partial x_1}\right)_s x_1'$$
$$+\left(\frac{\partial f}{\partial V}\right)_s V' + \left(\frac{\partial f}{\partial y_1}\right)_s y_1' + \left(\frac{\partial f}{\partial y_2}\right)_s y_2'$$

Substituting for the partial derivatives and noting that $\frac{dx_1}{dt} = \frac{dx_1'}{dt}$:

$$H\frac{dx_1'}{dt} = (\bar{x}_0 - \bar{x}_1)L' + \bar{L}x_0' - \bar{L}x_1' + (\bar{y}_2 - \bar{y}_1)V' + \bar{V}y_2' - \bar{V}y_1' \tag{3}$$

Similarly,

$$y_1' = g(x_1) = \left(\frac{\partial g}{\partial x_1}\right)_s x_1' = (a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})x_1' \tag{4}$$

b) For constant liquid and vapor flow rates, $L' = V' = 0$

Taking Laplace transforms of Eqs. 3 and 4,

$$HsX_1'(s) = \bar{L}X_0'(s) - \bar{L}X_1'(s) + \bar{V}Y_2'(s) - \bar{V}Y_1'(s) \tag{5}$$

$$Y_1'(s) = (a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})X_1'(s) \tag{6}$$

From Eqs. 5 and 6, the desired transfer functions are:

$$\frac{X_1'(s)}{X_0'(s)} = \frac{\frac{\bar{L}}{H}\tau}{\tau s + 1} \quad , \quad \frac{X_1'(s)}{Y_2'(s)} = \frac{\frac{\bar{V}}{H}\tau}{\tau s + 1}$$

$$\frac{Y_1'(s)}{X_0'(s)} = \frac{(a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})\frac{\bar{L}}{H}\tau}{\tau s + 1}$$

$$\frac{Y_1'(s)}{Y_2'(s)} = \frac{(a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})\frac{\bar{V}}{H}\tau}{\tau s + 1}$$

where:

$$\tau = \frac{H}{\bar{L} + \bar{V}(a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})}$$

4.8

The material balance is,

$$\frac{d(\rho Ah)}{dt} = w_i - Rh^{1.5}$$

or

$$\frac{dh}{dt} = \frac{1}{\rho A} w_i - \frac{R}{\rho A} h^{1.5}$$

Use a Taylor series expansion to linearize

$$\frac{dh}{dt} = \left[\frac{1}{\rho A}\overline{w}_i - \frac{R}{\rho A}\overline{h}^{1.5}\right] + \frac{1}{\rho A}(w_i - \overline{w}_i) - \frac{1.5R\overline{h}^{0.5}}{\rho A}(h - \overline{h})$$

Since the bracketed term is identically zero at steady state,

$$\frac{dh'}{dt} = \frac{1}{\rho A} w_i' - \frac{1.5R\overline{h}^{0.5}}{\rho A} h'$$

Rearranging

$$\frac{\rho A}{1.5R\overline{h}^{0.5}} \frac{dh'}{dt} + h' = \frac{1}{1.5R\overline{h}^{0.5}} w_i'$$

Thus,

$$\frac{H'(s)}{W_i'(s)} = \frac{K}{\tau s + 1}$$

where,

$$K = \frac{1}{1.5R\overline{h}^{0.5}} = \frac{\overline{h}}{1.5R\overline{h}^{1.5}} = \frac{\overline{h}}{1.5\overline{w}} = \frac{[height]}{[flowrate]}$$

$$\tau = \frac{\rho A}{1.5R\overline{h}^{0.5}} = \frac{\rho A\overline{h}}{1.5R\overline{h}^{1.5}} = \frac{\rho \overline{V}}{1.5\overline{w}} = \frac{[mass]}{[mass/time]} = [time]$$

4.9

a) The model for the system is given by

$$mC\frac{dT}{dt} = wC(T_i - T) + h_p A_p (T_w - T) \tag{2-51}$$

$$m_w C_w \frac{dT_w}{dt} = h_s A_s (T_s - T_w) - h_p A_p (T_w - T) \tag{2-52}$$

Assume that m, m_w, C, C_w, h_p, h_s, A_p, A_s, and w are constant. Rewriting the above equations in terms of deviation variables, and noting that

$$\frac{dT}{dt} = \frac{dT'}{dt} \quad , \quad \frac{dT_w}{dt} = \frac{dT'_w}{dt}$$

$$mC\frac{dT'}{dt} = wC(T'_i - T') + h_p A_p (T'_w - T')$$

$$m_w C_w \frac{dT'_w}{dt} = h_s A_s (0 - T'_w) - h_p A_p (T'_w - T')$$

Taking Laplace transforms and rearranging,

$$(mCs + wC + h_p A_p)T'(s) = wCT'_i(s) + h_p A_p T'_w(s) \tag{1}$$

$$(m_w C_w s + h_s A_s + h_p A_p)T'_w(s) = h_p A_p T'(s) \tag{2}$$

Substituting in Eq. 1 for $T'_w(s)$ from Eq. 2,

$$(mCs + wC + h_p A_p)T'(s) = wCT'_i(s) + h_p A_p \frac{h_p A_p}{(m_w C_w s + h_s A_s + h_p A_p)} T'(s)$$

Therefore,

$$\frac{T'(s)}{T'_i(s)} = \frac{wC(m_w C_w s + h_s A_s + h_p A_p)}{(mCs + wC + h_p A_p)(m_w C_w s + h_s A_s + h_p A_p) - (h_p A_p)^2}$$

b) The gain is $\left[\dfrac{T'(s)}{T'_i(s)}\right]_{s=0} = \dfrac{wC(h_s A_s + h_p A_p)}{wC(h_s A_s + h_p A_p) + h_s A_s h_p A_p}$

c) No, the gain would be expected to be one only if the tank were insulated so that $h_p A_p = 0$. For the heated tank, the gain is not one because heat input changes as T changes.

4.10

Additional assumptions

1. Perfect mixing in the tank
2. Constant density ρ and specific heat C.
3. T_i is constant.

Energy balance for the tank,

$$\rho VC\frac{dT}{dt} = wC(T_i - T) + Q - (\bar{U} + bv^2)A(T - T_a)$$

Let the right-hand side be denoted by $f(T,v)$,

$$\rho VC\frac{dT}{dt} = f(T,v) = \left(\frac{\partial f}{\partial T}\right)_s T' + \left(\frac{\partial f}{\partial v}\right)_s v' \qquad (1)$$

$$\left(\frac{\partial f}{\partial T}\right)_s = -wC - (\bar{U} + b\bar{v}^2)A$$

$$\left(\frac{\partial f}{\partial v}\right)_s = -2\bar{v}bA(\bar{T} - T_a)$$

Substituting for the partial derivatives in Eq. 1 and noting that $\dfrac{dT}{dt} = \dfrac{dT'}{dt}$

$$\rho VC\frac{dT'}{dt} = -\left[wC + (\bar{U} + b\bar{v}^2)A\right]T' - 2\bar{v}bA(\bar{T} - T_a)v'$$

$$\rho VC\frac{dT'}{dt} + \left[wC + (\bar{U} + b\bar{v}^2)A\right]T' = -2\bar{v}bA(\bar{T} - T_a)v'$$

Taking the Laplace transform and rearranging

$$\rho VCsT' + \left[wC + (\bar{U} + b\bar{v}^2)A\right]T' = -2\bar{v}bA(\bar{T} - T_a)V'$$

$$\left[\rho VCs + \left[wC + (\bar{U} + b\bar{v}^2)A\right]\right]T' = -2\bar{v}bA(\bar{T} - T_a)V'$$

$$T' = \frac{-2\bar{v}bA(\bar{T} - T_a)}{\left[\rho VCs + \left[wC + (\bar{U} + b\bar{v}^2)A\right]\right]}V'$$

$$\frac{T'(s)}{V'(s)} = \frac{\dfrac{2\bar{v}bA(\bar{T} - T_a)}{wC + (\bar{U} + b\bar{v}^2)A}}{\left[\dfrac{\rho VC}{wC + (\bar{U} + b\bar{v}^2)A}\right]s + 1}$$

4.11

a) Mass balances on the surge tanks:

$$\frac{dm_1}{dt} = w_1 - w_2 \quad (1)$$

$$\frac{dm_2}{dt} = w_2 - w_3 \quad (2)$$

Ideal gas law:

$$P_1V_1 = \frac{m_1}{M}RT \quad (3)$$

$$P_2V_2 = \frac{m_2}{M}RT \quad (4)$$

Flows (Ohm's law: $I = \frac{E}{R} = \frac{\text{Driving Force}}{\text{Resistance}}$)

$$w_1 = \frac{1}{R_1}(P_c - P_1) \quad (5)$$

$$w_2 = \frac{1}{R_2}(P_1 - P_2) \quad (6)$$

$$w_3 = \frac{1}{R_3}(P_2 - P_h) \quad (7)$$

Degrees of freedom:

- number of parameters : 8 (V_1, V_2, M, R, T, R_1, R_2, R_3)
- number of variables : 9 (m_1, m_2, w_1, w_2, w_3, P_1, P_2, P_c, P_h)
- number of equations : 7

∴ number of degrees of freedom that must be eliminated = 9 – 7 = 2

Because P_c and P_h are known functions of time (i.e., inputs), $N_F = 0$.

b) Model Development

Substitute (3) into (1) : $\frac{MV_1}{RT}\frac{dP_1}{dt} = w_1 - w_2$ (8)

Substitute (4) into (2) : $\frac{MV_2}{RT}\frac{dP_2}{dt} = w_2 - w_3$ (9)

Substitute (5) and (6) into (8):

$$\frac{MV_1}{RT}\frac{dP_1}{dt} = \frac{1}{R_1}(P_c - P_1) - \frac{1}{R_2}(P_1 - P_2)$$

$$\frac{MV_1}{RT}\frac{dP_1}{dt} = \frac{1}{R_1}P_c(t) - (\frac{1}{R_1} + \frac{1}{R_2})P_1 + \frac{1}{R_2}P_2 \qquad (10)$$

Substitute (6) and (7) into (9):

$$\frac{MV_2}{RT}\frac{dP_2}{dt} = \frac{1}{R_2}(P_1 - P_2) - \frac{1}{R_3}(P_2 - P_h)$$

$$\frac{MV_2}{RT}\frac{dP_2}{dt} = \frac{1}{R_2}P_1 - (\frac{1}{R_2} + \frac{1}{R_3})P_2 + \frac{1}{R_3}P_h(t) \qquad (11)$$

Note that $\frac{dP_1}{dt} = f_1(P_1, P_2)$ from Eq. 10

$\frac{dP_2}{dt} = f_2(P_1, P_2)$ from Eq. 11

This system has the following characteristics:

(i) 2nd-order denominator (2 differential equations)

(ii) Zero-order numerator (See Example 4.7 in text)

(iii) The gain of $\frac{W_3'(s)}{P_c'(s)}$ is not equal to unity. (It cannot be because the units for the two variables are different).

4.12

(a) First write the steady-state equations:

$$0 = wC(\bar{T}_i - \bar{T}) + h_e A_e(\bar{T}_e - \bar{T})$$

$$0 = \bar{Q} - h_e A_e(\bar{T}_e - \bar{T})$$

Now subtract the steady-state equations from the dynamic equations

$$mC\frac{dT}{dt} = wC\left[(T_i - \bar{T}_i) - (T - \bar{T})\right] + h_e A_e\left[(T_e - \bar{T}_e) - (T - \bar{T})\right] \qquad (1)$$

$$m_e C_e\frac{dT_e}{dt} = (Q - \bar{Q}) - h_e A_e\left[(T_e - \bar{T}_e) - (T - \bar{T})\right] \qquad (2)$$

Note that $dT/dt = dT'/dt$ and $dT_e/dt = dT_e'/dt$. Substitute deviation variables; then multiply (1) by 1/wC and (2) by $1/(h_e A_e)$.

$$\frac{m}{w}\frac{dT'}{dt} = -(T' - T'_i) + \frac{h_e A_e}{wC}(T'_e - T') \qquad (3)$$

$$\frac{m_e C_e}{h_e A_e}\frac{dT'_e}{dt} = \frac{Q'}{h_e A_e} - (T'_e - T') \qquad (4)$$

Eliminate one of the output variables, T'(s) or $T'_e(s)$, by solving (4) for it, and substituting into (3). Because $T'_e(s)$ is the intermediate variable, remove it. Then rearranging gives:

$$\left[\frac{m m_e C_e}{w h_e A_e} s^2 + \left(\frac{m_e C_e}{h_e A_e} + \frac{m_e C_e}{wC} + \frac{m}{w}\right) s + 1\right] T'(s)$$

$$= \left(\frac{m_e C_e}{h_e A_e} s + 1\right) T'_i(s) + \frac{1}{wC} Q'(s)$$

Because both inputs influence the dynamic behavior of T', it is necessary to develop two transfer functions for the model. The effect of Q' on T' can be derived by assuming that T_i is constant at its nominal steady-state value, $\bar{T}_i$. Thus, T'_i = 0 and the previous equation can be rearranged as:

$$\frac{T'(s)}{Q'(s)} = \frac{1/wC}{b_2 s^2 + b_1 s + 1} = G_1(s) \qquad (T'_i(s) = 0)$$

Similarly, the effect of T'_i on T' is obtained by assuming that $Q=\bar{Q}$ (that is, Q'=0):

$$\frac{T'(s)}{T'_i(s)} = \frac{\frac{m_e C_e}{h_e A_e} s + 1}{b_2 s^2 + b_1 s + 1} = G_2(s) \qquad (Q'(s) = 0)$$

where

b_1 is defined to be $\frac{m_e C_e}{h_e A_e} + \frac{m_e C_e}{wC} + \frac{m}{w}$

b_2 is defined to be $\frac{m m_e C_e}{w h_e A_e}$

By the superposition principle, the effect of simultaneous changes in both inputs is given by

$$T'(s) = G_1(s) Q'(s) + G_2(s) T'_i(s)$$

(b)

The limiting behavior of $m_e C_e$ going to zero has $b_2 = 0$ and $b_1 = m/w$ and simplifies the last equation to

$$T'(s) = \frac{1/wC}{\frac{m}{w}s + 1} Q'(s) + \frac{1}{\frac{m}{w}s + 1} T'_i(s)$$

4.13

A mass balance yields:

$$\frac{dm}{dt} = \rho q_i - \rho q \qquad (1)$$

The mass accumulation term can be written, noting that dV=Adh=w_tLdh, as

$$\frac{dm}{dt} = \rho \frac{dV}{dt} = \rho w_t L \frac{dh}{dt} \qquad (2)$$

where $w_t L$ represents the changing surface are of the liquid. Substituting (2) into (1) and simplifying gives:

$$w_t L \frac{dh}{dt} = q_i - q \qquad (3)$$

The geometric construction indicates that $w_t/2$ is the length of one side of a right triangle whose hypotenuse is R. Thus, $w_t/2$ is related to the level h by

A mass balance yields:

$$\frac{\mathrm{w}_t}{2} = \sqrt{R^2 - (R-h)^2}$$

After rearrangement,

$$w_t = 2\sqrt{(D-h)h} \qquad (4)$$

with D = 2R (diameter of the tank). Substituting (4) into (3) yields a nonlinear dynamic model for the tank with q_i and q as inputs:

$$\frac{dh}{dt} = \frac{1}{2L\sqrt{(D-h)h}}(q_i - q)$$

To linearize this equation about the operating point $(h = \bar{h})$, let

$$f = \frac{q_i - q}{2L\sqrt{(D-h)h}}$$

Then

$$\left(\frac{\partial f}{\partial q_i}\right)_s = \frac{1}{2L\sqrt{(D-\bar{h})\bar{h}}}$$

$$\left(\frac{\partial f}{\partial q}\right)_s = \frac{-1}{2L\sqrt{(D-\bar{h})\bar{h}}}$$

$$\left(\frac{\partial f}{\partial h}\right)_s = (\bar{q}_i - \bar{q})\left[\frac{\partial}{\partial h}\left(\frac{1}{2L\sqrt{(D-\bar{h})\bar{h}}}\right)\right]_s = 0$$

The last partial derivative is zero, because $\bar{q}_i - \bar{q}$ from the steady-state relation, and the derivative term in brackets is finite for all 0<*h*<*D*. Consequently, the linearized model of the process, after substitution of deviation variables, is

$$\frac{dh'}{dt} = \frac{1}{2L\sqrt{(D-\bar{h})\bar{h}}}(q'_i - q')$$

Recall that the term $2L\sqrt{(D-h)h}$ in the previous equation represents the variable surface area of the tank. The linearized model treats this quantity as a constant that depends on the nominal (steady-state) operating level. Consequently, operation of the horizontal cylindrical tank for small variations in level around the stead-state value would be much like that of any tank with equivalent but constant liquid surface. For example, a vertical cylindrical tank with diameter *D*' has a surface are of liquid in the tank equal to $\pi(D')^2/4 = 2L\sqrt{(D-\bar{h})\bar{h}}$. Note that the coefficient $\frac{1}{2}L\sqrt{(D-\bar{h})\bar{h}}$ is infinite for $\bar{h}=0$ or for $\bar{h}=D$ and is a minimum at $\bar{h}=D/2$. Thus, for large variations in level, this equation would not be a good approximation, because *dh*/*dt* is independent of *h* in the linearized model. In these cases, the horizontal and vertical tanks would operate very differently.

4.14

Assumptions

1. Perfectly mixed reactor
2. Constant fluid properties and heat of reaction.

a) Component balance for A,

$$V\frac{dc_A}{dt} = q(c_{A_i} - c_A) - Vk(T)c_A \tag{1}$$

Energy balance for the tank,

$$\rho VC\frac{dT}{dt} = \rho qC(T_i - T) + (-\Delta H)Vk(T)c_A \tag{2}$$

Since a transfer function with respect to c_{Ai} is desired, assume the other inputs, namely *q* and T_i, to be constant.

Linearize (1) and (2) and note that $\frac{dc_A}{dt} = \frac{dc'_A}{dt}$, $\frac{dT}{dt} = \frac{dT'}{dt}$,

$$V\frac{dc'_A}{dt} = qc'_{Ai} - (q + Vk(\bar{T}))c'_A - V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}T' \tag{3}$$

$$\rho VC\frac{dT'}{dt} = -\left(\rho qC + \Delta HV\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right)T' + (-\Delta H)Vk(\bar{T})c'_A \tag{4}$$

Taking the Laplace transforms and rearranging,

$$\left[Vs+q+Vk(\bar{T})\right]C'_A(s) = qC'_{Ai}(s) - V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}T'(s) \qquad (5)$$

$$\left[\rho VCs+\rho qC-(-\Delta H)V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right]T'(s) = (-\Delta H)Vk(\bar{T})C'_A(s) \qquad (6)$$

Substituting for $C'_A(s)$ from Eq. 5 into Eq. 6 and rearranging,

$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{-\Delta HVk(\bar{T})q}{\left[Vs+q+Vk(\bar{T})\right]\left[\rho VCs+\rho qC-(-\Delta H)V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right]+(-\Delta H)\bar{c}_A V^2k^2(\bar{T})\frac{20000}{\bar{T}^2}} \qquad (7)$$

$\bar{c}_A$ is obtained from the steady-state version of Eq.1,

$$\bar{c}_A = \frac{q\bar{c}_{Ai}}{q+Vk(\bar{T})} = 0.001155 \ \text{ mol/cu.ft.}$$

Substituting the numerical values of $\bar{T}$, ρ, *C*, ($-\Delta H$), *q*, *V*, $\bar{c}_A$ into Eq. 7 and simplifying gives,

$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{11.38}{(0.0722s+1)(50s+1)}$$

b) The gain *K* of the above transfer function is equal to $\left[\frac{T'(s)}{C'_{Ai}(s)}\right]_{s=0}$,

$$K = \frac{0.15766\ \bar{q}}{\left(\frac{\bar{q}}{1000}-3.153\times10^6\frac{\bar{c}_A}{\bar{T}^2}\right)\left(\frac{\bar{q}}{1000}+13.84\right)+4.364.10^7\frac{\bar{c}_A}{\bar{T}^2}} \qquad (8)$$

It is obtained by setting *s*=0 in Eq. 7 and substituting numerical values for ρ, *C*, ($-\Delta H$), *V*. Evaluating sensitivities gives,

$$\frac{dK}{d\bar{q}} = \frac{K}{\bar{q}} - \frac{K^2}{0.15766\bar{q}}\left[2\frac{\bar{q}}{10^6}+0.01384-3153\frac{\bar{c}_A}{\bar{T}^2}\right] = -6.50\times10^{-4}$$

$$\frac{dK}{d\bar{T}} = -\frac{K^2}{3.153}\left[\left(\frac{\bar{q}}{1000}+13.84\right)\left(\frac{3.153\times10^6\bar{c}_A\times2}{\bar{T}^3}\right)-\frac{2\times4.364\times10^7\bar{c}_A}{\bar{T}^3}\right]$$
$$= -2.57\times10^{-5}$$

$$\frac{dK}{d\bar{c}_{Ai}} = \frac{dK}{d\bar{c}_A}\times\frac{d\bar{c}_A}{d\bar{c}_{Ai}}$$

$$= \frac{-K^2}{0.15766\bar{q}}\left[-\left(\frac{\bar{q}}{1000}+13.84\right)\left(\frac{3.153\times10^6}{\bar{T}^2}\right)+\frac{4.364\times10^7}{\bar{T}^2}\right]\left(\frac{\bar{q}}{\bar{q}+13840}\right)$$

$$= 8.87\times10^{-3}$$

4.15

Assumptions**:**

1. Constant physical properties
2. Perfect mixing

Dynamic model: Balances on cell mass and substrate concentration

$$\frac{dX}{dt} = \mu(S)X - DX = f_1(S, X, D) \tag{1}$$

$$\frac{dS}{dt} = -\mu(S)X / Y_{X/S} + D(S_f - S) = f_2(S, X, D, S_f) \tag{2}$$

where:

$\mu(S)X$ is defined as $\dfrac{\mu_m S}{K_s + S}X$,

D is defined as $\dfrac{F}{V}$

Linearization of (1) about the nominal steady state gives a linearized model of the form:

$$\frac{dX'}{dt} \approx \left.\frac{\partial f_1}{\partial S}\right|_{ss} S' + \left.\frac{\partial f_1}{\partial X}\right|_{ss} X' + \left.\frac{\partial f_1}{\partial D}\right|_{ss} D'$$

$$\frac{dX'}{dt} \approx \left(\frac{\mu_m(K_s+\bar{S}) - \mu_m\bar{S}}{(K_s+\bar{S})^2}\bar{X}\right)S' + \left(\frac{\mu_m\bar{S}}{K_s+\bar{S}} - \bar{D}\right)X' - \bar{X}\,D' \tag{3}$$

Linearization of (2) about the nominal steady state:

$$\frac{dS'}{dt} \approx \left.\frac{\partial f_2}{\partial S}\right|_{ss} S' + \left.\frac{\partial f_2}{\partial X}\right|_{ss} X' + \left.\frac{\partial f_2}{\partial D}\right|_{ss} D' + \left.\frac{\partial f_2}{\partial S_f}\right|_{ss} S_f'$$

$$\frac{dS'}{dt} \approx \left(-\frac{1}{Y_{X/S}}\frac{\mu_m(K_s+\bar{S}) - \mu_m\bar{S}}{(K_s+\bar{S})^2}\bar{X} - \bar{D}\right)S' + \left(-\frac{1}{Y_{X/S}}\frac{\mu_m\bar{S}}{K_s+\bar{S}}\right)X' + (\bar{S}_f - \bar{S})D' + \bar{D}S_f'$$

(4)

Substituting the numerical values gives:

$$\frac{dX'}{dt} = 0.113S' \;-\; 2.25D'$$

$$\frac{dS'}{dt} = -0.326\,S' - 0.2\,X' - 9D' + 0.1S'_f$$

Taking Laplace transforms, assuming steady state initially:

$$sX'(s) = 0.113S'(s) \;-\; 2.25D'(s)$$

$$sS'(s) = -0.326\,S'(s) - 0.2\,X'(s) - 9D'(s) + 0.1S'_f(s)$$

In order to derive the transfer function between X and D, assume that S_f is constant at its nominal steady-state value, $S_f(t) = \bar{S}_f$; thus $S'_f = 0$. Rearranging gives,

$$X'(s) = \frac{0.113}{s}S'(s) \;-\; \frac{2.25}{s}D'(s) \qquad (5)$$

$$S'(s) = \left(\frac{-0.2}{s+0.326}\right)X'(s) - \left(\frac{9}{s+0.126}\right)D'(s) \qquad (6)$$

Substitute (6) into (5) and rearrange gives,

$$\frac{X'(s)}{D'(s)} = \frac{-(2.25s+1.7)}{s^2+0.326s+0.0226} \qquad (7)$$

Rearrange (7) to a standard form:

$$\frac{X'(s)}{D'(s)} = \frac{K(\tau_a s+1)}{\tau^2 s^2 + 2\tau\zeta s + 1}$$

where:

$$K = -77.4 \text{ g h/L}$$
$$\tau_a = 0.778 \text{ h}$$
$$\tau = 6.65 \text{ h}$$
$$\zeta = 1.08$$

Note that the step response will be overdamped because $\zeta > 1$.

Chapter 5

5.1

No, the time required for the output Y(s) to reach steady state does not depend on the magnitude of the step input in U, it only depends on the time constant τ_i and delay θ . Since the Laplace transform of a step change is *M/s*, we have:

$$Y(s)=G(s)U(s)=\frac{KM}{s(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)(\tau_4 s+1)}e^{-\theta s}$$

The inverse Laplace transform takes the following form:

$$Y(t)=KMu(t-\theta)\left[\begin{array}{l}\frac{\tau_1^2}{(\tau_1-\tau_2)(\tau_1-\tau_3)(\tau_1-\tau_4)}e^{-t/\tau_1}-\frac{\tau_2^2}{(\tau_1-\tau_2)(\tau_2-\tau_3)(\tau_2-\tau_4)}e^{-t/\tau_2}-\\ \frac{\tau_3^2}{(\tau_1-\tau_3)(\tau_3-\tau_2)(\tau_3-\tau_4)}e^{-t/\tau_3}-\frac{\tau_4^2}{(\tau_1-\tau_4)(\tau_4-\tau_2)(\tau_4-\tau_3)}e^{-t/\tau_4}\end{array}\right]$$

As shown in above equation, the settling time is not related to the magnitude of input signal M.

5.2

(a) For a step change in input of magnitude *M*:

$$y(t)=KM\left(1-e^{-t/\tau}\right)+y(0)$$

We note that $KM=y(\infty)-y(0)=500-100=400^\circ C$

Then $K=\dfrac{400^\circ C}{(2-1)Kw}=400^\circ C/Kw$

At time $t=4$, $y(4)=400^\circ C$; thus, $\dfrac{400-100}{500-100}=1-e^{-4/\tau}$, or $\tau=2.89\,\text{min}$

$$\therefore\frac{T'(s)}{P'(s)}=\frac{400}{2.89s+1}\left[^\circ C/Kw\right]$$

(b) For an input ramp change with slope $a=0.5Kw/\text{min}$:

$$Ka=400\times0.5=200^\circ C/min$$

This maximum rate of change will occur as soon as the transient has died out, i.e., after 5×2.89 min $\approx$ 15 min have elapsed.

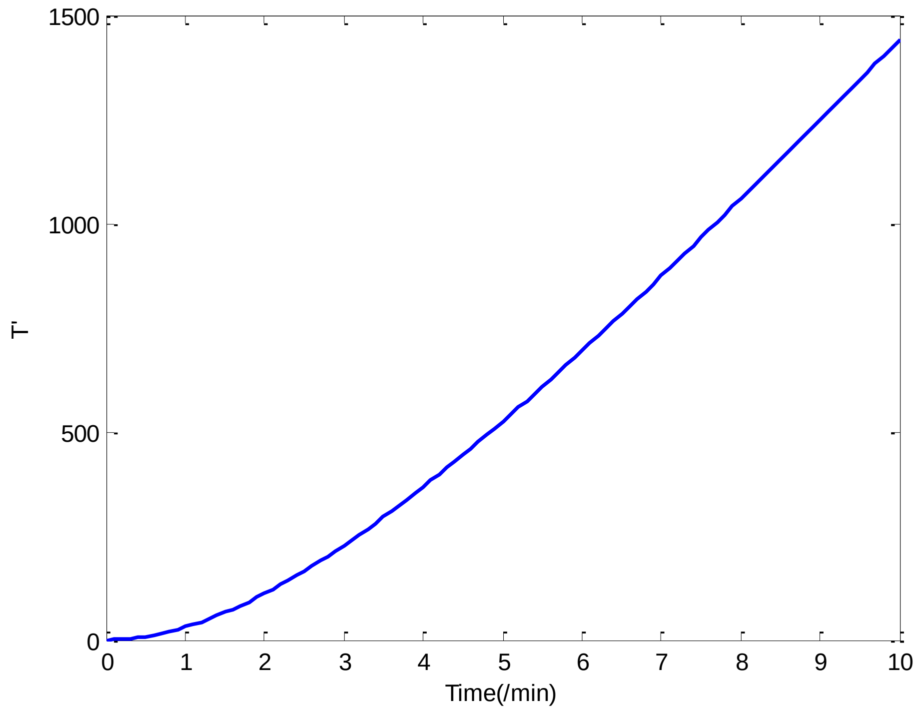


Figure S5.2. *Temperature response for a ramp input of magnitude 0.5 Kw/min.*

5.3

The contaminant concentration c increases according to this expression:

$$c(t) = 5 + 0.2t$$

Using deviation variables and Laplace transforming,

$$c'(t) = 0.2t \qquad \text{or} \qquad C'(s) = \frac{0.2}{s^2}$$

Hence

$$C'_m(s) = \frac{1}{10s+1} \cdot \frac{0.2}{s^2}$$

and applying Eq. 5-21

$$c'_m(t) = 2(e^{-t/10} - 1) + 0.2t$$

As soon as $c'_m(t) \geq 2$ ppm the alarm sounds. Therefore,

Δt = 18.4 s (starting from the beginning of the ramp input)

The time at which the actual concentration exceeds the limit (t = 10 s) is subtracted from the previous result to obtain the requested Δt .

$$\Delta t = 18.4 - 10.0 = 8.4 \text{ s}$$

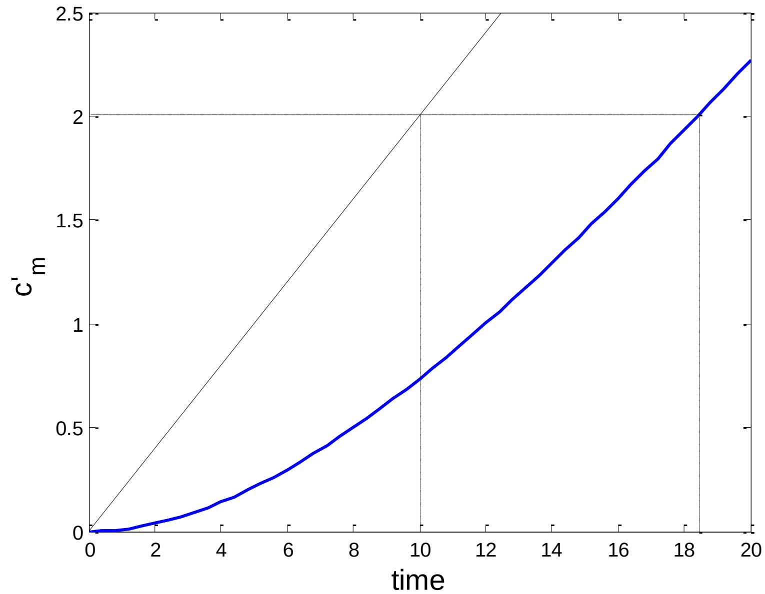


Figure S5.3. *Concentration response for a ramp input of magnitude 0.2 Kw/min.*

5.4

a) Using deviation variables, the rectangular pulse is

$$c'_F = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t < 2 \\ 0 & 2 \le t \le \infty \end{cases}$$

Laplace transforming this input yields

$$C'_F(s) = \frac{2}{s}\left(1 - e^{-2s}\right)$$

The input is then given by

$$C'(s) = \frac{8}{s(2s+1)} - \frac{8e^{-2s}}{s(2s+1)}$$

and from Table 3.1 the time domain function is

$$c'(t) = 8(1 - e^{-t/2}) - 8(1 - e^{-(t-2)/2})\, S\,(t-2)$$

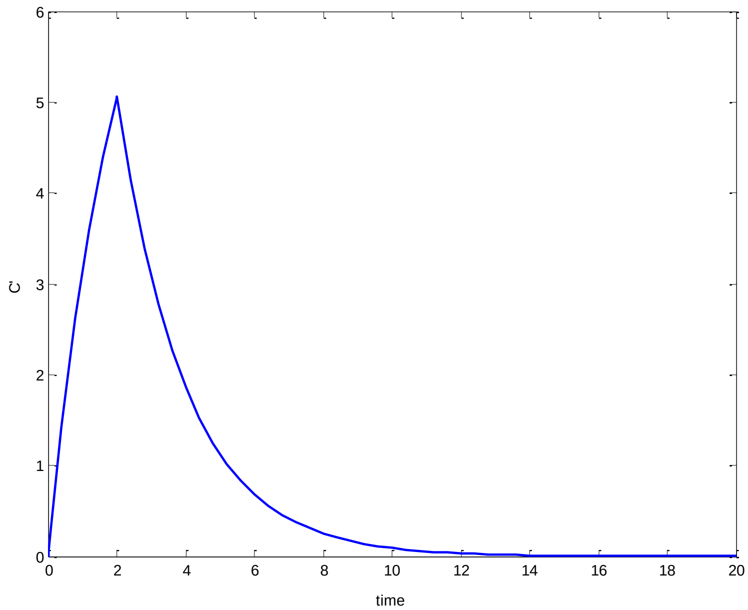


Figure S5.4. *Exit concentration response for a rectangular input.*

b) By inspection of Eq. 1, the time at which this function will reach its maximum value is 2, so maximum value of the output is given by

$$c'(2) = 8(1 - e^{-1}) - 8(1 - e^{-0/2})\,S(0)$$

and since the second term is zero, $c'(2) = 5.057$

c) By inspection, the steady state value of $c'(t)$ will be zero, since this is a first-order system with no integrating poles and the input returns to zero. To obtain $c'(\infty)$, simplify the function derived in a) for all time greater than 2, yielding

$$c'(t) = 8(e^{-(t-2)/2} - e^{-t/2})$$

which will obviously converge to zero.

Substituting $c'(t) = 0.05$ in the previous equation and solving for *t* gives

$$t = 9.233$$

5.5

a) Energy balance for the thermocouple,

$$mC\frac{dT}{dt} = hA(T_s - T)$$

where m is mass of thermocouple
C is heat capacity of thermocouple
h is heat transfer coefficient
A is surface area of thermocouple
t is time in sec

Substituting numerical values in (1) and noting that

$$\overline{T}_s = \overline{T} \quad \text{and} \quad \frac{dT}{dt} = \frac{dT'}{dt},$$

$$15\frac{dT'}{dt} = T_s' - T'$$

Taking Laplace transform, $\dfrac{T'(s)}{T_s'(s)} = \dfrac{1}{15s+1}$

b) $T_s(t) = 23 + (80 - 23)\, S(t)$

$$\overline{T}_s = \overline{T} = 23$$

From $t = 0$ to $t = 20$,

$$T_s'(t) = 57\, S(t) \qquad , \qquad T_s'(s) = \frac{57}{s}$$

$$T'(s) = \frac{1}{15s+1}T_s'(s) = \frac{57}{s(15s+1)}$$

Applying inverse Laplace Transform,

$$T'(t) = 57(1 - e^{-t/15})$$

Then

$$T(t) = T'(t) + \overline{T} = 23 + 57(1 - e^{-t/15})$$

Since $T(t)$ increases monotonically with time, maximum $T = T(20)$.

Maximum $T(t) = T(20) = 23 + 57\ (1\text{-}e^{-20/15}) = 64.97$ °C

c)

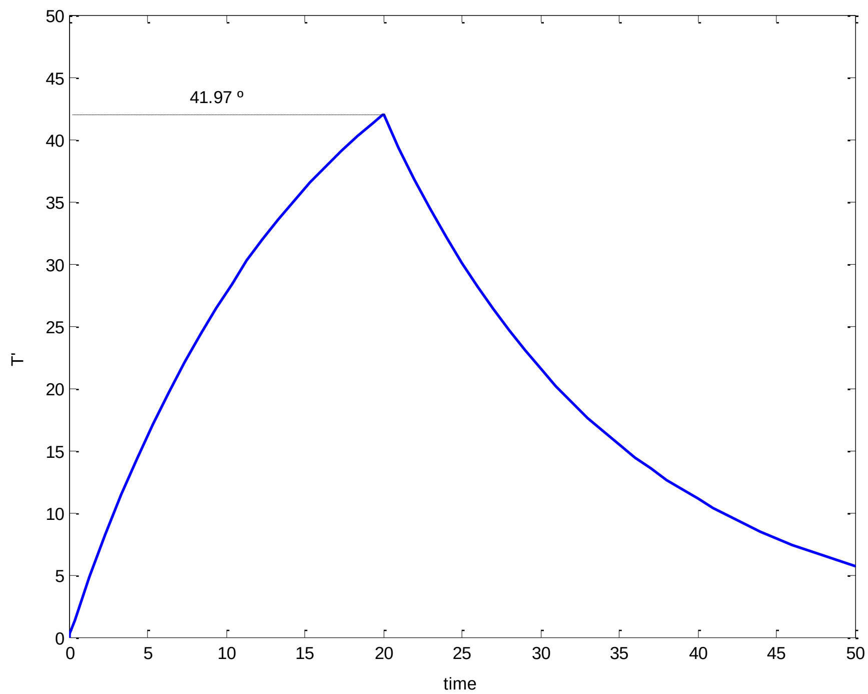


Figure S5.5. *Thermocouple output for parts b) and c)*

5.6

(a)

$$Y(s) = G(s)U(s) = \frac{10}{(5s+1)(3s+1)}\frac{M}{s}$$

$$Y(s) = (\frac{a_1}{5s+1} + \frac{a_2}{3s+1} + \frac{a_3}{s})M$$

Partial fraction expansion:

$a_1 = 125$, $a_2 = -45$, $a_3 = 10$.

$$Y(s) = (\frac{125}{5s+1} - \frac{45}{3s+1} + \frac{10}{s})M$$

Inverse Laplace:

$$y(t) = (25e^{-t/5} - 15e^{-t/3} + 10)M$$

Then,

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty}(25e^{-t/5} - 15e^{-t/3} + 10)M = 10M$$

Or, final value theorem from Chapter 3 applies:

$$\lim_{t\to\infty} y(t) = \lim_{s\to o} sY(s) = \lim_{s\to o} sG(s)U(s) = \lim_{s\to o} s\frac{10}{(5s+1)(3s+1)}\frac{M}{s} = 10M$$

(b)

$$Y(s) = G(s)U(s) = \frac{10}{(5s+1)(3s+1)}$$

$$Y(s) = \frac{a_1}{5s+1} + \frac{a_2}{3s+1}$$

Partial fraction expansion:
a_1 = 25, a_2 = -15.

$$Y(s) = \frac{25}{5s+1} - \frac{15}{3s+1}$$

Inverse Laplace:

$$y(t) = 5e^{-t/5} - 5e^{-t/3}$$

Then,

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} 5e^{-t/5} - 5e^{-t/3} = 0$$

Or, final value theorem from Chapter 3 applies:

$$\lim_{t\to\infty} y(t) = \lim_{s\to o} sY(s) = \lim_{s\to o} sG(s)U(s) = \lim_{s\to o} s\frac{10}{(5s+1)(3s+1)} = 0$$

(c)

$$Y(s) = G(s)U(s) = \frac{10}{(5s+1)(3s+1)}\frac{1}{s^2+1}$$

$$Y(s) = \frac{a_1}{5s+1} + \frac{a_2}{3s+1} + \frac{a_3 + b_3 j}{s+j} + \frac{a_3 - b_3 j}{s-j}$$

Partial fraction expansion:
a_1 = 625/26, a_2 = -27/2, a_3 = -2/13, b_3 = 7/26.

$$Y(s) = \frac{625/26}{5s+1} - \frac{27/2}{3s+1} + \frac{-2/13 + 7/26j}{s+j} + \frac{-2/13 - 7/26j}{s-j}$$

Inverse Laplace:

$$y(t) = 125/26e^{-t/5} - 9/2e^{-t/3} - 7/13\sin t - 4/13\cos t$$

Then, $\lim_{t\to\infty} y(t)$ does not converge.

(d)

$$Y(s) = G(s)U(s) = \frac{10}{(5s+1)(3s+1)}\frac{1}{s}(1-e^{-t_w s})$$

According to part (a), we have:

$$y(t) = (25e^{-t/5} - 15e^{-t/3} + 10) - S(t-t_w)(25e^{-(t-10)/5} - 15e^{-(t-10)/3} + 10)$$

Then,

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty}(25e^{-t/5} - 15e^{-t/3} + 10) - (25e^{-(t-10)/5} - 15e^{-(t-10)/3} + 10) = 0$$

5.7

Assume that at steady state the temperature indicated by the sensor T_m is equal to the actual temperature at the measurement point T. Then,

$$\frac{T'_m(s)}{T'(s)} = \frac{K}{\tau s+1} = \frac{1}{1.5s+1}$$

$$\overline{T}_m = \overline{T} = 350^\circ C$$

$$T'_m(t) = 15\sin\omega t$$

where $\omega = 2\pi \times 0.1$ rad/min = 0.628 rad/min

At large times when $t/\tau >> 1$, Eq. 5-26 shows that the amplitude of the sensor signal is

$$A_m = \frac{A}{\sqrt{\omega^2\tau^2+1}}$$

where A is the amplitude of the actual temperature at the measurement point.

Therefore $\quad A = 15\sqrt{(0.628)^2(1.5)^2+1}$ = 20.6°C

Maximum $T = \overline{T} + A$ =350 + 20.6 = 370.6

Maximum T_{center} = 3 (max T) – 2 T_{wall}

$$= (3\times 370.6)–(2\times 200) = 711.8°C$$

Therefore, the catalyst will not sinter instantaneously, but will sinter if operated for several hours.

5.8

a) Assume that q is constant. Material balance over the tank,

$$A\frac{dh}{dt} = q_1 + q_2 - q$$

Writing in deviation variables and taking Laplace transform

$$AsH'(s) = Q_1'(s) + Q_2'(s)$$

$$\frac{H'(s)}{Q_1'(s)} = \frac{1}{As}$$

b) $q_1'(t) = 5\ S(t) – 5S(t\text{-}12)$

$$Q_1'(s) = \frac{5}{s} - \frac{5}{s}e^{-12s}$$

$$H'(s) = \frac{1}{As}Q_1'(s) = \frac{5/A}{s^2} - \frac{5/A}{s^2}e^{-12s}$$

$$h'(t) = \frac{5}{A}t\,S(t) - \frac{5}{A}(t-12)\,S(t\text{-}12)$$

$$h(t) = \begin{cases} 4 + \dfrac{5}{A}t = 4 + 0.177t & 0 \le t \le 12 \\ 4 + \left(\dfrac{5}{A}\times 12\right) = 6.122 & 12 < t \end{cases}$$

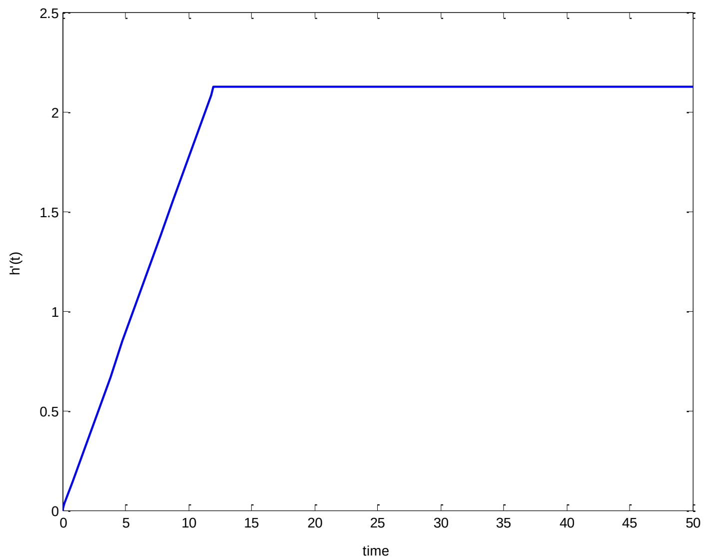


Figure S5.8a. *Liquid level response for part b)*

c) $\bar{h} = 6.122\ \text{ft}$ at the new steady state $t \geq 12$

d) $q_1'(t) = 10S(t) - 5S(t-12)$

$$Q_1'(s) = \frac{10}{s} - \frac{5}{s}e^{-12s}$$

$$H'(s) = \frac{10/A}{s^2} - \frac{5/A}{s^2}e^{-12s}$$

$$h(t) = \begin{cases} 4 + \dfrac{10}{A}t = 4 + 0.354t & 0 \leq t \leq 12 \\ 6.122 + 0.177t & t \geq 12 \end{cases}$$

The liquid level will keep increasing and there will be no steady-state value of liquid level $\bar{h}$.

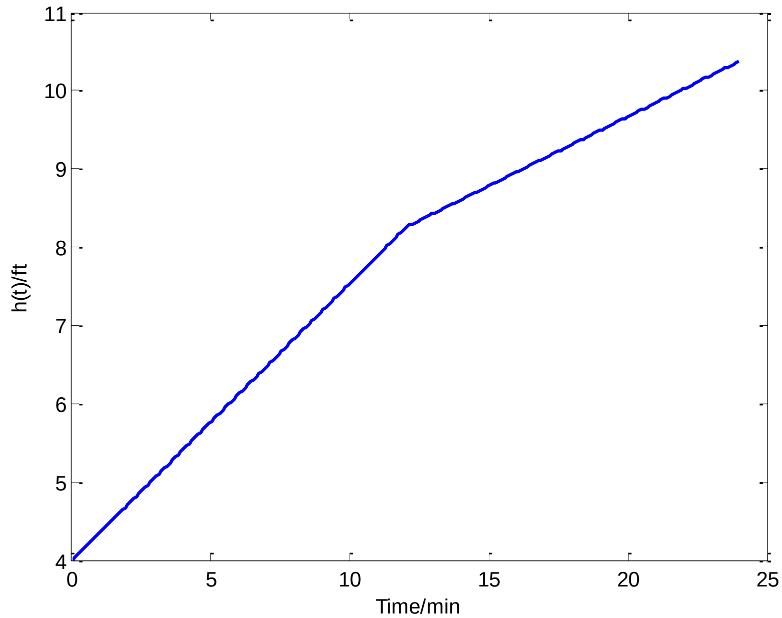


Figure S5.8b. *Liquid level response for part d)*

5.9

a) Material balance over tank 1.

$$A\frac{dh}{dt} = C(q_i - 8.33h)$$

where $A = \pi \times (4)^2/4 = 12.6 \text{ ft}^2$

$$C = 0.1337 \quad \frac{\text{ft}^3/\text{min}}{\text{USGPM}}$$

$$AsH'(s) = CQ_i'(s) - (C \times 8.33)H'(s)$$

$$\frac{H'(s)}{Q_i'(s)} = \frac{0.12}{11.28s + 1}$$

For tank 2,

$$A\frac{dh}{dt} = C(q_i - q)$$

$$AsH'(s) = CQ_i'(s) \quad , \qquad \frac{H'(s)}{Q_i'(s)} = \frac{0.011}{s}$$

b) $Q_i'(s) = 20/s$

For tank 1, $H'(s) = \dfrac{2.4}{s(11.28s+1)} = \dfrac{2.4}{s} - \dfrac{27.1}{11.28s+1}$

$$h(t) = 6 + 2.4(1 - e^{-t/11.28})$$

For tank 2, $H'(s) = 0.22/s^2$

$$h(t) = 6 + 0.22t$$

c) For tank 1, $h(\infty) = 6 + 2.4 - 0 = 8.4$ ft

For tank 2, $h(\infty) = 6 + (0.22 \times \infty) = \infty$ ft

d) For tank 1, $8 = 6 + 2.4(1 - e^{-t/11.28})$

$h = 8$ ft at $t = 20.1$ min

For tank 2, $8 = 6 + 0.22t$

$h = 8$ ft at $t = 9.4$ min

Tank 2 overflows first, at 9.4 min.

e) The red line (h'(t)=2 ft, or h(t)= 8 ft) shows that tank 2 overflows first at 9.4 min.

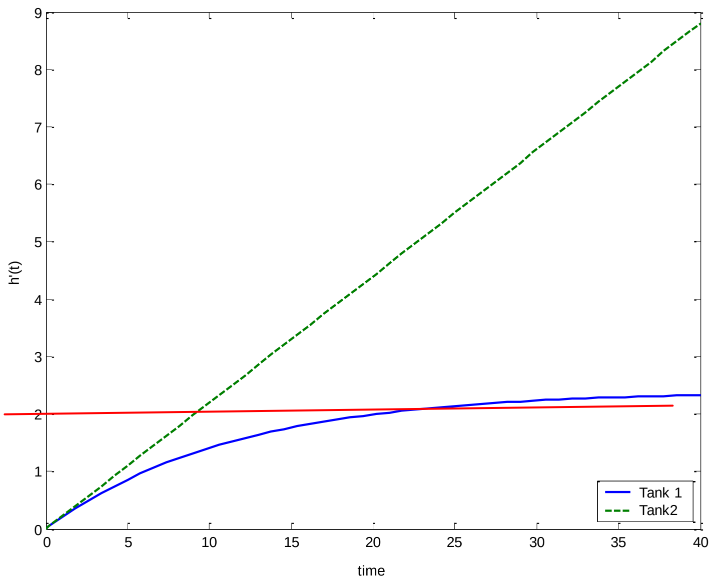


Figure S5.9. *Transient response in tanks 1 and 2 for a step input.*

5.10

a) The dynamic behavior of the liquid level is given by

$$\frac{d^2h'}{dt} + A\frac{dh'}{dt} + Bh' = C\,p'(t)$$

where

$$A = \frac{6\mu}{R^2\rho} \qquad B = \frac{3g}{2L} \qquad and \;\; C = \frac{3}{4\rho L}$$

Taking the Laplace Transform and assuming initial values = 0

$$s^2H'(s) + AsH'(s) + BH'(s) = C\,P'(s)$$

or $$H'(s) = \frac{C/B}{\frac{1}{B}s^2 + \frac{A}{B}s + 1}P'(s)$$

We want the previous equation to have the form

$$H'(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}P'(s)$$

Hence $K = C/B = \dfrac{1}{2\rho g}$

$$\tau^2 = \frac{1}{B} \qquad \text{then} \quad \tau = \sqrt{1/B} = \left(\frac{2L}{3g}\right)^{1/2}$$

$$2\zeta\tau = \frac{A}{B} \qquad \text{then} \quad \zeta = \frac{3\mu}{R^2\rho}\left(\frac{2L}{3g}\right)^{1/2}$$

b) The manometer response oscillates as long as $0 < \zeta < 1$ or

$$0 < \frac{3\mu}{R^2\rho}\left(\frac{2L}{3g}\right)^{1/2} < 1$$

If ρ is larger , then ζ is smaller and the response would be more oscillatory.

If μ is larger, then ζ is larger and the response would be less oscillatory.

5.11

$$Y(s) = \frac{KM}{s^2(\tau s+1)} = \frac{K_1}{s^2} + \frac{K_2}{s(\tau s+1)}$$

$$K_1\tau s + K_1 + K_2 s = KM$$

$$K_1 = KM$$
$$K_2 = -K_1\tau = -KM\tau$$

Hence

$$Y(s) = \frac{KM}{s^2} - \frac{KM\tau}{s(\tau s+1)}$$

or

$$y(t) = KMt - KM\tau\,(1-e^{-t/\tau})$$

After a long enough time, we can simplify to

$$y(t) \approx KMt - KM\tau \qquad \text{(linear)}$$

slope = KM
intercept = $-KM\tau$

That way we can get K and τ

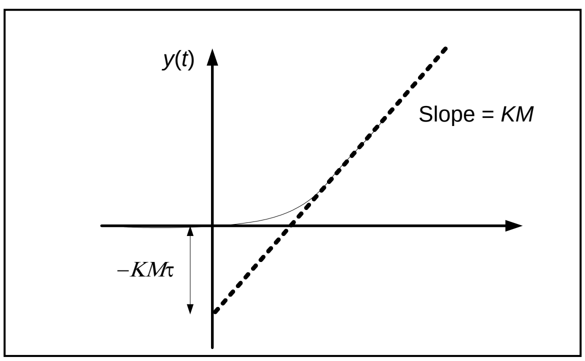


Figure S5.11. *Time domain response and parameter evaluation*

5.12

a) $\ddot{y} + K\dot{y} + 4y = x$

Assuming y(0) = $\dot{y}(0) = 0$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + Ks + 4} = \frac{0.25}{0.25s^2 + 0.25Ks + 1}$$

b) Characteristic equation is

$$s^2 + Ks + 4 = 0$$

The roots are $s = \dfrac{-K \pm \sqrt{K^2 - 16}}{2}$

$-10 \le K < -4$ Roots : positive real, distinct
Response : $A + Be^{t/\tau_1} + C\ e^{t/\tau_2}$

$K = -4$ Roots : positive real, repeated
Response : $A + Be^{t/\tau} + C\ e^{t/\tau}$

$-4 < K < 0$ Roots: complex with positive real part.
Response: $A + e^{\zeta t/\tau}(B \cos\sqrt{1-\zeta^2}\ \frac{t}{\tau} + C \sin\sqrt{1-\zeta^2}\ \frac{t}{\tau})$

$K = 0$ Roots: imaginary, zero real part.
Response: $A + B \cos t/\tau + C \sin t/\tau$

$0 < K < 4$ Roots: complex with negative real part.
Response: $A + e^{-\zeta t/\tau}(B \cos\sqrt{1-\zeta^2}\ \frac{t}{\tau} + C \sin\sqrt{1-\zeta^2}\ \frac{t}{\tau})$

$K = 4$ Roots: negative real, repeated.
Response: $A + Be^{-t/\tau} + C\ t\ e^{-t/\tau}$

$4 < K \le 10$ Roots: negative real, distinct
Response: $A + Be^{-t/\tau_1} + C\ e^{-t/\tau_2}$

Response will converge in region $0 < K \le 10$, and will not converge in region $-10 \le K \le 0$

5.13

a) The solution of a critically-damped second-order process to a step change of magnitude *M* is given by Eq. 5-50 in text.

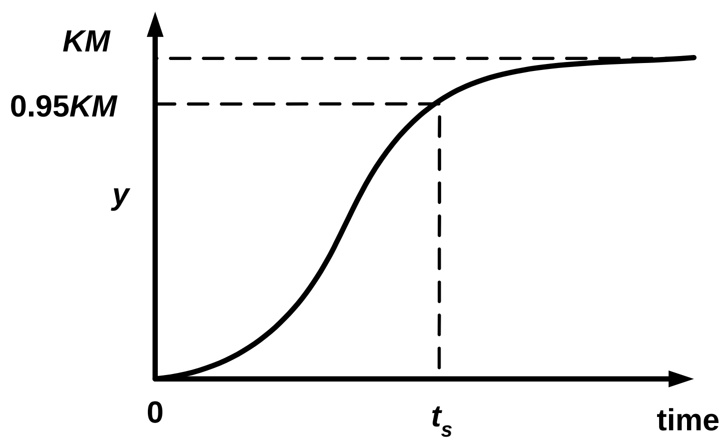
$$y(t) = KM\left[1-\left(1+\frac{t}{\tau}\right)e^{-t/\tau}\right]$$

Rearranging

$$\frac{y}{KM} = 1-\left(1+\frac{t}{\tau}\right)e^{-t/\tau}$$

$$\left(1+\frac{t}{\tau}\right)e^{-t/\tau} = 1-\frac{y}{KM}$$

When *y/KM* = 0.95, the response is 0.05 *KM* below the steady-state value.



$$\left(1+\frac{t_s}{\tau}\right)e^{-t/\tau} = 1-0.95 = 0.05$$

$$\ln\left(1+\frac{t_s}{\tau}\right)-\frac{t_s}{\tau} = \ln(0.05) = -3.00$$

Let $E = \ln\left(1+\frac{t_s}{\tau}\right) - \frac{t_s}{\tau} + 3$

and find value of $\frac{t_s}{\tau}$ that makes $E \approx 0$ by trial-and-error.

t_s/τ	E
4	0.6094
5	-0.2082
4.5	0.2047
4.75	-0.0008

$\therefore$ a value of $t = 4.75\tau$ is t_s, the settling time.

b) $Y(s) = \dfrac{Ka}{s^2(\tau s+1)^2} = \dfrac{a_1}{s} + \dfrac{a_2}{s^2} + \dfrac{a_3}{\tau s+1} + \dfrac{a_4}{(\tau s+1)^2}$

We know that the a_3 and a_4 terms are exponentials that go to zero for large values of time, leaving a linear response.

$$a_2 = \lim_{s\to 0} \frac{Ka}{(\tau s+1)^2} = Ka$$

Define $Q(s) = \dfrac{Ka}{(\tau s+1)^2}$

$$\frac{dQ}{ds} = \frac{-2Ka\tau}{(\tau s+1)^3}$$

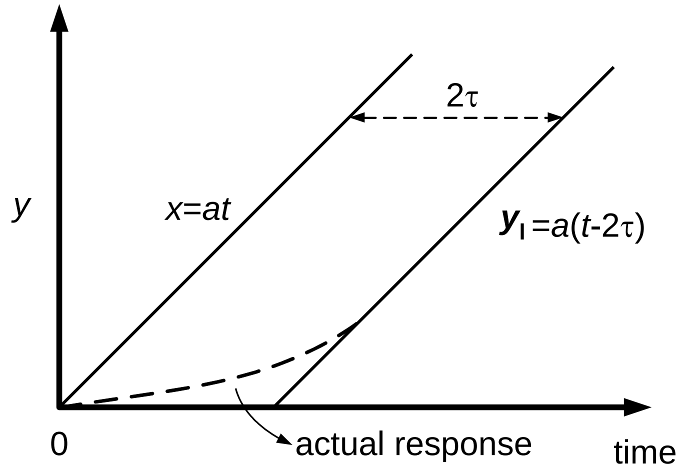
Then $a_1 = \dfrac{1}{1!} \lim_{s\to 0}\left[\dfrac{-2Ka\tau}{(\tau s+1)^3}\right]$

$$a_1 = -2\,Ka\tau$$

$\therefore$ the long-time response (after transients have died out) is

$$y_\ell(t) = Kat - 2Ka\tau = Ka(t-2\tau)$$
$$= a(t-2\tau) \qquad \text{for} \quad K = 1$$

and we see that the output lags the input by a time equal to 2τ.



(c) .

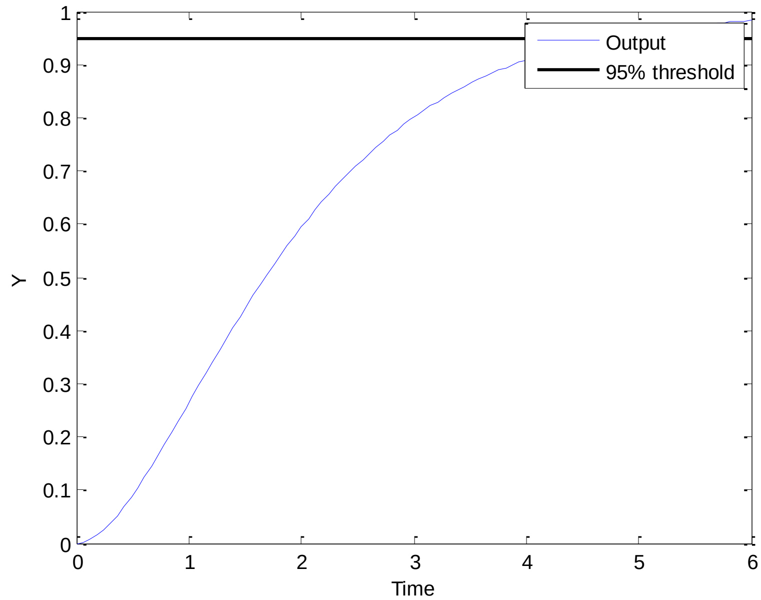


Figure S5.13a *Computer simulation results on part (a)*

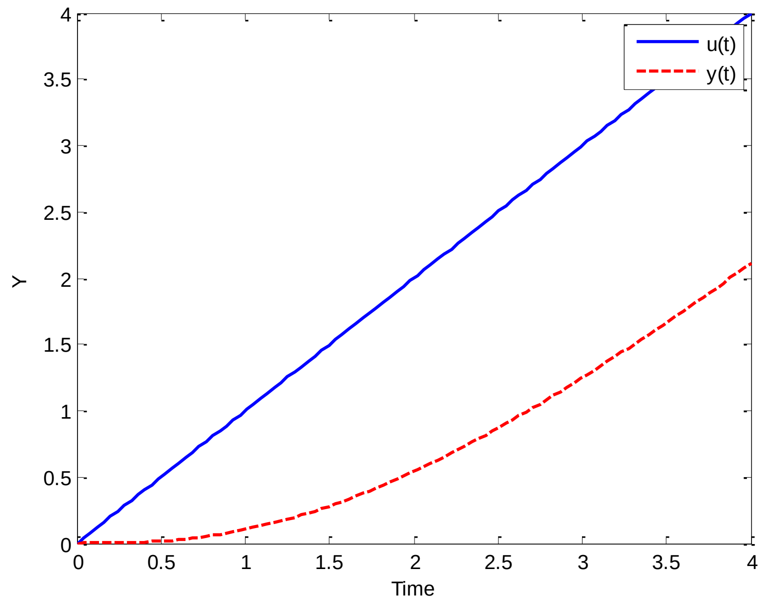


Figure S5.13b *Computer simulation results on part (b)*

5.14

a) $$\text{Gain} = \frac{11.2\text{mm} - 8\text{mm}}{31\text{psi} - 15\text{psi}} = 0.20\text{mm/psi}$$

$$\text{Overshoot} = \frac{12.7\text{mm} - 11.2\text{mm}}{11.2\text{mm} - 8\text{mm}} = 0.47$$

$$\text{Overshoot} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.47 \quad , \quad \zeta = 0.234$$

$$\text{Period} = \left(\frac{2\pi\tau}{\sqrt{1-\zeta^2}}\right) = 2.3\text{ sec}$$

$$\tau = 2.3\text{ sec} \times \frac{\sqrt{1-0.234^2}}{2\pi} = 0.356\text{ sec}$$

$$\frac{R'(s)}{P'(s)} = \frac{0.2}{0.127s^2 + 0.167s + 1} \tag{1}$$

b) From Eq. 1, taking the inverse Laplace transform,

$$0.127\ddot{R}' + 0.167\dot{R}' + R' = 0.2P'$$

$$\ddot{R}' = \ddot{R} \qquad \dot{R}' = \dot{R} \qquad R' = R\text{-}8 \qquad P' = P\text{-}15$$

$$0.127\,\ddot{R} + 0.167\,\dot{R} + R = 0.2\,P + 5$$

$$\ddot{R} + 1.31\,\dot{R} + 7.88\,R = 1.57\,P + 39.5$$

5.15

$$\frac{P'(s)}{T'(s)} = \frac{3}{(3)^2 s^2 + 2(0.7)(3)s + 1} \qquad \left[{}^\circ C/\text{kW}\right]$$

Note that the input change $p'(t) = 26 - 20 = 6$ kw

Since K is 3 °C/kW, the output change in going to the new steady state will be

$$\underset{t\to\infty}{T'} = (3^\circ C/\text{kW})(6\ \text{kW}) = 18^\circ \text{C}$$

a) Therefore the expression for $T(t)$ is Eq. 5-51

$$T(t) = 70^\circ + 18^\circ \left\{ 1 - e^{-\frac{0.7t}{3}} \left(\cos\left(\frac{\sqrt{1-(0.7)^2}}{3} t \right) + \frac{0.7}{\sqrt{1-(0.7)^2}} \sin\left(\frac{\sqrt{1-(0.7)^2}}{\tau} t \right) \right) \right\}$$

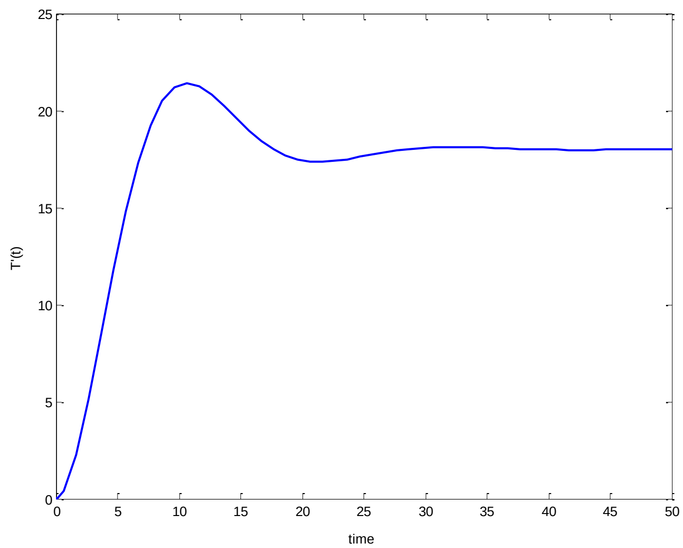


Figure S5.15. *Process temperature response for a step input*

b) The overshoot can obtained from Eq. 5-53 or Fig. 5.11. From Figure 5.11 we see that OS ≈ 0.05 for ζ=0.7. This means that maximum temperature is

$$T_{max} \approx 70° + (18)(1.05) = 70 + 18.9 = 88.9°$$

From Fig S5.15 we obtain a more accurate value.
The time at which this maximum occurs can be calculated by taking derivative of Eq. 5-51 or by inspection of Fig. 5.8. From the figure we see that t / τ = 3.8 at the point where an (interpolated) ζ=0.7 line would be.

$$\therefore \qquad t_{max} \approx 3.8 \text{ (3 min)} = 11.4 \text{ minutes}$$

5.16

For underdamped responses,

$$y(t) = KM\left\{1 - e^{-\zeta t/\tau}\left[\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right\} \qquad (5\text{-}51)$$

a) At the response peaks,

$$\frac{dy}{dt} = KM\left\{\frac{\zeta}{\tau}e^{-\zeta t/\tau}\left[\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right.$$

$$\left.-e^{-\zeta t/\tau}\left[-\frac{\sqrt{1-\zeta^2}}{\tau}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\tau}\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right\} = 0$$

Since $KM \neq 0$ and $e^{-\zeta t/\tau} \neq 0$

$$0 = \left(\frac{\zeta}{\tau} - \frac{\zeta}{\tau}\right)\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \left(\frac{\zeta^2}{\tau\sqrt{1-\zeta^2}} + \frac{\sqrt{1-\zeta^2}}{\tau}\right)\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)$$

$$0 = \sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) = \sin n\pi \quad , \quad t = n\frac{\pi\tau}{\sqrt{1-\zeta^2}}$$

where n is the number of the peak.

Time to the first peak, $t_p = \dfrac{\pi\tau}{\sqrt{1-\zeta^2}}$

b) Overshoot, OS = $\dfrac{y(t_p) - KM}{KM}$

$$\text{OS} = -\exp\left(\frac{-\zeta t}{\tau}\right)\left[\cos(\pi) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\pi)\right]$$

$$= \exp\left[\frac{-\zeta\tau\pi}{\tau\sqrt{1-\zeta^2}}\right] = \exp\left[\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right]$$

c) Decay ratio, DR = $\dfrac{y(t_{3p}) - KM}{y(t_p) - KM}$

where $y(t_{3p}) = \dfrac{3\pi\tau}{\sqrt{1-\zeta^2}}$ is the time to the third peak.

$$\text{DR} = \frac{KM\ e^{-\zeta t_{3p}/\tau}}{KM\ e^{-\zeta t_p/\tau}} = \exp\left[-\frac{\zeta}{\tau}(t_{3p} - t_p)\right] = \exp\left[-\frac{\zeta}{\tau}\left(\frac{2\pi\tau}{\sqrt{1-\zeta^2}}\right)\right]$$

$$= \exp\left[\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}\right] = (\text{OS})^2$$

d) Consider the trigonometric identity

sin (A+B) = sin A cos B + cos A sin B

Let B = $\left(\dfrac{\sqrt{1-\zeta^2}}{\tau}t\right)$, sin A = $\sqrt{1-\zeta^2}$, cos A = ζ

$$y(t) = KM\left\{1 - e^{-\zeta t/\tau}\frac{1}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2}\cos B + \zeta \sin B\right]\right\}$$

$$= KM\left\{1 - \frac{e^{-\zeta t/\tau}}{\sqrt{1-\zeta^2}}\sin(A+B)\right\}$$

Hence for $t \geq t_s$, the settling time,

$$\left|\frac{e^{-\zeta t/\tau}}{\sqrt{1-\zeta^2}}\right| \leq 0.05 \quad \text{, or} \quad t \geq -\ln\left(0.05\sqrt{1-\zeta^2}\right)\frac{\tau}{\zeta}$$

Therefore, $t_s \geq \frac{\tau}{\zeta} \ln\left(\frac{20}{\sqrt{1-\zeta^2}}\right)$

5.17

a) Assume underdamped second-order model (exhibits overshoot)

$$K = \frac{\Delta\text{output}}{\Delta\text{input}} = \frac{15-10 \text{ ft}}{210-180 \text{ gal/min}} = 1/6 \frac{\text{ft}}{\text{gal/min}}$$

$$\text{Fraction overshoot} = \frac{16.5-15}{15-10} = \frac{1.5}{5} = 0.3$$

From Fig 5.11, this corresponds (approx) to $\zeta = 0.35$

From Fig. 5.8 , $\zeta = 0.35$, we note that $t_p/\tau \approx 3.5$

Since t_p = 4 minutes (from problem statement, assuming first peak),

$$\tau = \frac{t_p\sqrt{1-\zeta^2}}{\pi} = 1.19 \text{ min}$$

$$\therefore \quad G_p(s) = \frac{1/6}{(1.19)^2 s^2 + 2(0.35)(1.19)s + 1} = \frac{0.17}{1.42s^2 + 0.83s + 1}$$

b) 4 minutes might not be the first peak (as shown in Figure 5.8); thus, the solution may be not unique.

5.18

(a)

$\tau=1, \zeta=0.5$.

Roots of denominator are: $s^2 + s + 1 = 0$, s = -0.50+0.87j and -0.50-0.87j.

Imaginary roots suggest oscillation.

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\frac{2}{s^2+s+1}\frac{2}{s} = \lim_{s\to 0}\frac{4}{s^2+s+1} = 4\,.$$

Time to first peak: $\pi\tau/\sqrt{1-\zeta^2}=3.6$

Overshoot: $2\times 2\times \exp(-\pi\zeta/\sqrt{1-\zeta^2})=0.652$

Period: $\dfrac{2\pi\tau}{\sqrt{1-\zeta^2}} = 7.25$

Figure can be sketched using Figure S5.8 in Chapter 5 for $\tau = 1, \zeta = 0.5$.

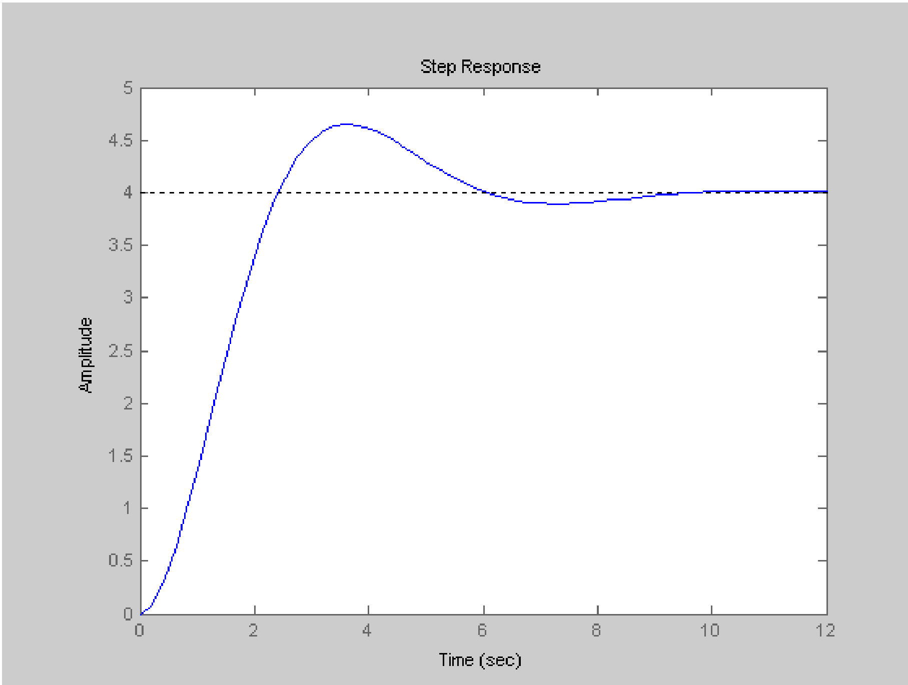


Figure S5.18 *Step response of G.*

(b)

Decay Ratio: $\exp(-2\pi\zeta/\sqrt{1-\zeta^2}) = 0.106$

5.19

a) For the original system,

$$A_1 \frac{dh_1}{dt} = Cq_i - \frac{h_1}{R_1}$$

$$A_2 \frac{dh_2}{dt} = \frac{h_1}{R_1} - \frac{h_2}{R_2}$$

where $A_1 = A_2 = \pi(3)^2/4 = 7.07\ \text{ft}^2$

$$C = 0.1337 \ \frac{\text{ft}^3/\text{min}}{\text{gpm}}$$

$$R_1 = R_2 = \frac{\bar{h}_1}{C\bar{q}_i} = \frac{2.5}{0.1337 \times 100} = 0.187 \frac{\text{ft}}{\text{ft}^3/\text{min}}$$

Using deviation variables and taking Laplace transforms,

$$\frac{H_1'(s)}{Q_i'(s)} = \frac{C}{A_1 s + \frac{1}{R_1}} = \frac{CR_1}{A_1 R_1 s + 1} = \frac{0.025}{1.32s + 1}$$

$$\frac{H_2'(s)}{H_1'(s)} = \frac{1/R_1}{A_2 s + \frac{1}{R_2}} = \frac{R_2/R_1}{A_2 R_2 s + 1} = \frac{1}{1.32s + 1}$$

$$\frac{H_2'(s)}{Q_i'(s)} = \frac{0.025}{(1.32s + 1)^2}$$

For step change in q_i of magnitude M,

$h'_{1\max} = 0.025M$

$h'_{2\max} = 0.025M$ since the second-order transfer function

$\frac{0.025}{(1.32s + 1)^2}$ is critically damped (ζ=1), not underdamped

Hence $M_{max} = \frac{2.5 \text{ ft}}{0.025 \text{ ft/gpm}} = 100 \text{ gpm}$

For the modified system,

$$A\frac{dh}{dt} = Cq_i - \frac{h}{R}$$

$A = \pi(4)^2/4 = 12.6 \text{ ft}^2$

$V = V_1 + V_2 = 2 \times 7.07\text{ft}^2 \times 5\text{ft} = 70.7\text{ft}^3$

$h_{max} = V/A = 5.62$ ft

$$R = \frac{\bar{h}}{C\bar{q}_i} = \frac{0.5 \times 5.62}{0.1337 \times 100} = 0.21 \frac{\text{ft}}{\text{ft}^3/\text{min}}$$

$$\frac{H'(s)}{Q_i'(s)} = \frac{C}{As + \frac{1}{R}} = \frac{CR}{ARs+1} = \frac{0.0281}{2.64s+1}$$

$$h'_{\max} = 0.0281M$$

$$M_{max} = \frac{2.81\,\text{ft}}{0.0281\,\text{ft/gpm}} = 100\,\text{gpm}$$

Hence, both systems can handle the same maximum step disturbance in q_i.

b) For step change of magnitude M, $Q_i'(s) = \frac{M}{s}$

For original system,

$$Q_2'(s) = \frac{1}{R_2} H_2'(s) = \frac{1}{0.187} \frac{0.025}{(1.32s+1)^2} \frac{M}{s}$$

$$= 0.134M\left[\frac{1}{s} - \frac{1.32}{(1.32s+1)} - \frac{1.32}{(1.32s+1)^2}\right]$$

$$q_2'(t) = 0.134M\left[1 - \left(1 + \frac{t}{1.32}\right)e^{-t/1.32}\right]$$

For modified system,

$$Q'(s) = \frac{1}{R} H'(s) = \frac{1}{0.21} \frac{0.0281}{(2.64s+1)} \frac{M}{s} = 0.134M\left[\frac{1}{s} - \frac{2.64}{2.64s+1}\right]$$

$$q'(t) = 0.134M\left[1 - e^{-t/2.64}\right]$$

Original system provides better damping since $q_2'(t) < q'(t)$ for $t < 3.4$.

c) Computer simulation result

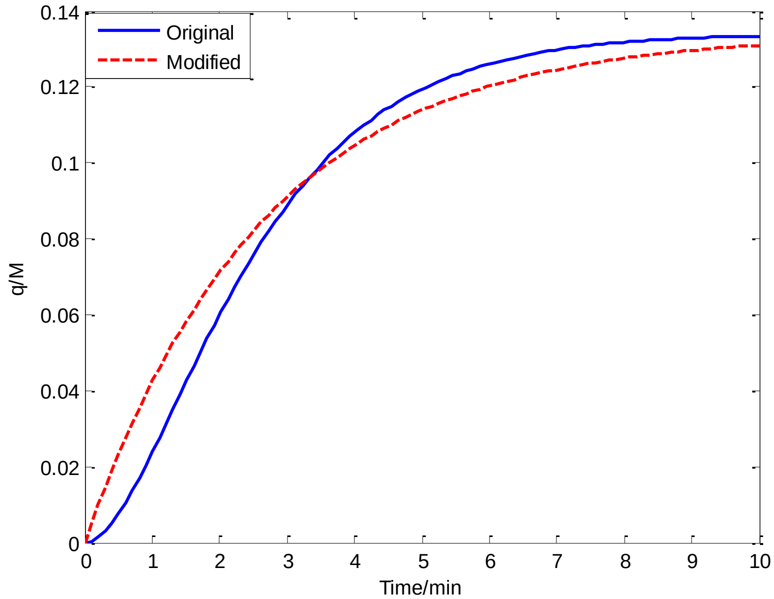


Figure S5.19 *Computer simulation results on part (b)*

5.20

a) Caustic balance for the tank,

$$\rho V \frac{dC}{dt} = w_1 c_1 + w_2 c_2 - wc$$

Since V is constant, $w = w_1 + w_2 = 10$ lb/min

For constant flows,

$$\rho V s C'(s) = w_1 C_1'(s) + w_2 C_2'(s) - wC'(s)$$

$$\frac{C'(s)}{C_1'(s)} = \frac{w_1}{\rho V s + w} = \frac{5}{(70)(7)s + 10} = \frac{0.5}{49s + 1}$$

$$\frac{C_m'(s)}{C'(s)} = \frac{K}{\tau s + 1} \quad , \quad K = (3\text{-}0)/3 = 1 \quad , \quad \tau \approx 6 \text{ sec} = 0.1 \text{ min}$$

(from the graph)

$$\frac{C_m'(s)}{C_1'(s)} = \frac{1}{(0.1s+1)} \frac{0.5}{(49s+1)} = \frac{0.5}{(0.1s+1)(49s+1)}$$

b) $C_1'(s) = \dfrac{3}{s}$

$$C_m'(s) = \frac{1.5}{s(0.1s+1)(49s+1)}$$

$$c_m'(t) = 1.5\left[1 + \frac{1}{(49-0.1)}(0.1e^{-t/0.1} - 49e^{-t/49})\right]$$

c) $$C_m'(s) = \frac{0.5}{(49s+1)}\frac{3}{s} = \frac{1.5}{s(49s+1)}$$

$$c_m'(t) = 1.5\left(1 - e^{-t/49}\right)$$

d) The responses in b) and c) are nearly the same. Hence the dynamics of the conductivity cell are negligible.

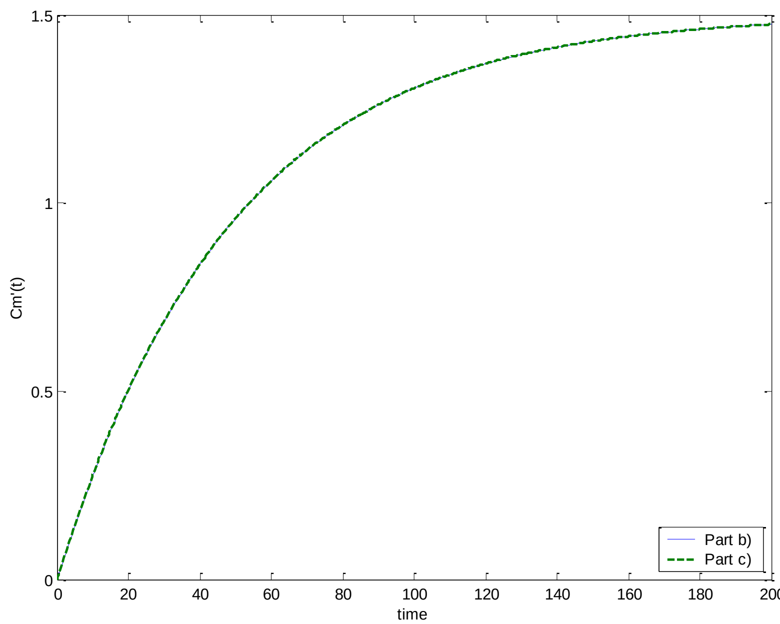


Figure S5.20 *Step responses for parts b) and c)*

5.21

Assumptions:
1) Perfectly mixed reactor
2) Constant fluid properties and heat of reaction

a) Component balance for A,

$$V\frac{dc_A}{dt} = q(c_{Ai} - c_A) - Vk(T)c_A \tag{1}$$

Energy balance for the tank,

$$\rho VC\frac{dT}{dt} = \rho qC(T_i - T) + (-\Delta H_R)Vk(T)c_A \tag{2}$$

Since a transfer function with respect to c_{Ai} is desired, assume the other inputs, namely q and T_i, are constant. Linearize (1) and (2) and note that

$$\frac{dc_A}{dt} = \frac{dc'_A}{dt} \quad , \quad \frac{dT}{dt} = \frac{dT'}{dt} ,$$

$$V\frac{dc'_A}{dt} = qc'_{Ai} - (q + Vk(\bar{T}))c'_A - V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}T' \tag{3}$$

$$\rho VC\frac{dT'}{dt} = -\left(\rho qC + \Delta H_R V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right)T' - \Delta H_R Vk(\bar{T})c'_A \tag{4}$$

Taking Laplace transforms and rearranging

$$\left[Vs + q + Vk(\bar{T})\right]C'_A(s) = qC'_{Ai}(s) - V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}T'(s) \tag{5}$$

$$\left[\rho VCs + \rho qC - (-\Delta H_R)V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right]T'(s) = (-\Delta H_R)Vk(\bar{T})C'_A(s) \tag{6}$$

Substituting $C'_A(s)$ from Eq. 5 into Eq. 6 and rearranging,

$$\frac{T'(s)}{C'_A(s)} = \frac{(-\Delta H_R)Vk(\bar{T})q}{\left[Vs + q + Vk(\bar{T})\right]\left[\rho VCs + \rho qC - (-\Delta H_R)V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right] + (-\Delta H_R)V^2\bar{c}_A k^2(\bar{T})\frac{20000}{\bar{T}^2}} \tag{7}$$

$\bar{c}_A$ is obtained from Eq. 1 at steady state,

$$\bar{c}_A = \frac{q\bar{c}_{Ai}}{q + Vk(\bar{T})} = 0.01159 \text{ lb mol/cu.ft.}$$

Substituting the numerical values of $\bar{T}$, ρ, C, $-\Delta H_R$, q, V, $\bar{c}_A$ into Eq. 7 and simplifying,

$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{12.69}{(0.082s+1)(5s+1)}$$

For step response, $C'_{Ai}(s) = 1/s$

$$T'(s) = \frac{12.69}{(0.082s+1)(5s+1)s}$$

$$T'(t) = 12.69\left[1 + \frac{1}{(50-0.082)}(0.082e^{-t/0.082} - 5e^{-t/5})\right]$$

A first-order approximation of the transfer function is

$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{12.69}{5s+1}$$

For step response, $T'(s) = \dfrac{12.69}{s(5s+1)}$ or $T'(t) = 12.69\left[1 - e^{-t/5}\right]$

The two step responses are very close to each other hence the approximation is valid. The ODE calculation indicates a slightly different gain due to linearization.

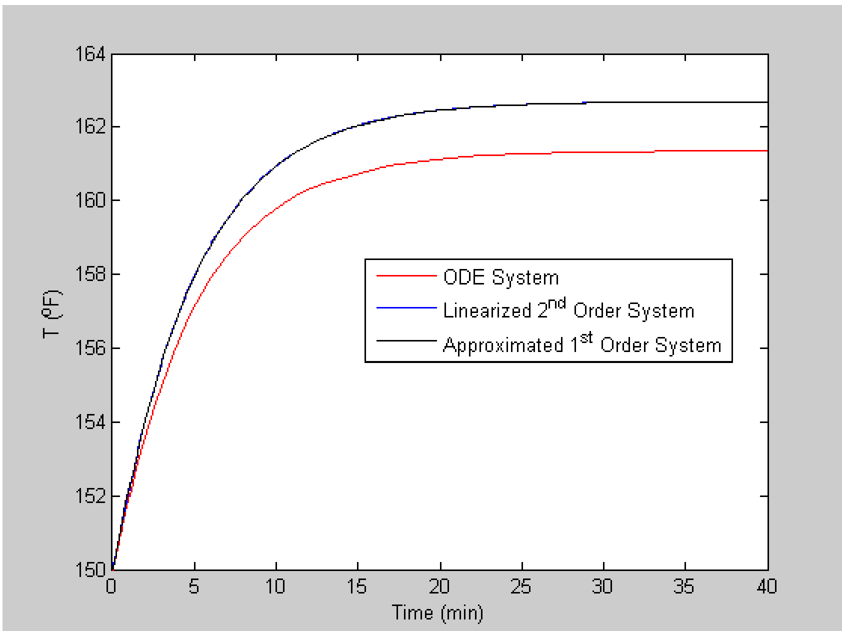


Figure S5.21 *Step responses for the ODE system, 2nd order t.f and 1st order approx.*

5.22

(a)
Step response of a first-order process is:

$$Y(s) = G(s)U(s) = \frac{K_1}{(\tau s+1)}\frac{M}{s}$$

Inverse Laplace gives:

$$y(t) = K_1 M(1-e^{-t/\tau}) \qquad (1)$$

Taylor series expansion at t = 0: $e^{-t/\tau}$ = 1- 1/τ × t. Substitute into Eq. (1):

$$y(t) = K_1 M(1-(1-t/\tau)) = \frac{K_1}{\tau}Mt \qquad (2)$$

Inverse Laplace on integrator $G_o(s) = \frac{K_o}{s}$:

$$y(t) = K_0 Mt \qquad (3)$$

Compared Eqs. 2 and 3, we conclude when t is close to zero, or t << τ, first order system can be approximated by integrator with:

$$K_0 = \frac{K_1}{\tau} \qquad (4)$$

(b)

From part (a), $K_0 = \frac{K_1}{\tau}$.

(c)
Eq. 3 shows the integrator step response in time domain. With the step test data, plot the data and approximate the slope of the line. Set the slope equal to K_0M and find K_0. The time delay would be estimated to be the time where the line intersects the x-axis.

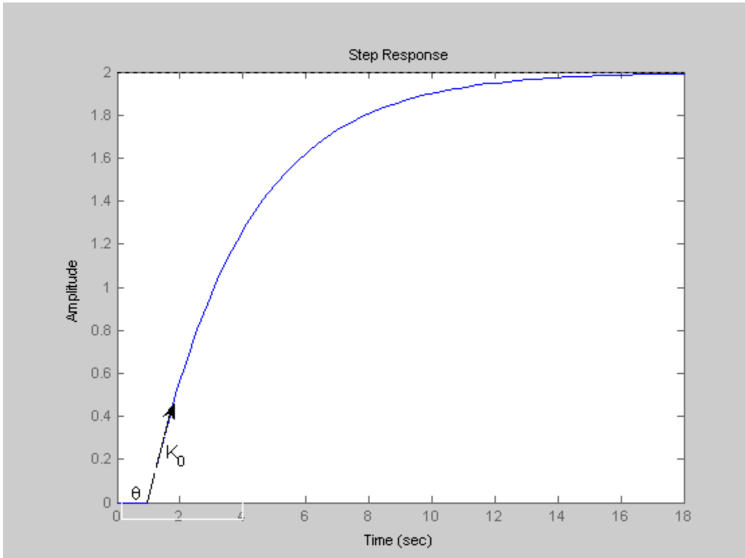


Figure S5.22 *Step response data to find delay and approximated integrator process gain.*

5.23

(a)

$$Y(s) = G(s)U(s) = \frac{5}{(3s+1)}\frac{(1-e^{-s})}{s}$$

Final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0}\left(s\frac{5}{(3s+1)}\frac{(1-e^{-s})}{s}\right) = \lim_{s\to 0}\frac{5(1-e^{-s})}{(3s+1)} = 0$$

(b)

$$Y(s) = G(s)U(s) = \frac{K_1}{(3s+1)}\frac{1}{s^2}$$

Final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0}\left(s\frac{5}{(3s+1)}\frac{1}{s^2}\right) = \lim_{s\to 0}\frac{5}{(3s+1)s} = \infty$$ Undefined.

(c)

For part (a), the heating rate returns to steady state after time 1, the tank temperature will gradually return to the steady state value once the hotter fluid is passed out of the stirred tank heater.

For part (b), the heating rate rises linearly with time and so does the outlet temperature. Physical limitations include element burnout, boiling of the liquid, and constraints on the amount of electrical power available. However, there should not be any short term physical limitations to the ramp, but it is an unsafe situation.

5.24

a)
From block algebra,

$$Y(s) = G_1(s)U(s) + G_2(s)U(s) + G_3(s)U(s)$$

or $$Y(s) = [G_1(s) + G_2(s) + G_3(s)]U(s)$$

After some simple operations, and by account that $U(s) = 1$, then

$$Y(s) = \left[\frac{1}{s} + \frac{4}{2s+1} + \frac{-3}{s+1}\right]U(s) = \frac{4s+1}{s(2s+1)(s+1)}$$

or $$Y(s) = \frac{4s+1}{(2s+1)(s+1)} \times \frac{1}{s} = \frac{4s+1}{2s^2+3s+1} \times \frac{1}{s}$$

Notice that this system is equivalent to a step input response of an overdamped

(ζ = 1.06) second-order transfer function with numerator dynamics (see Example 6.2 in your textbook).

For this example, $\tau_a > \tau_1$ (e.g., 4 > 2), so the response will exhibit some overshoot.

The system poles (-0.5, -1) lie in the LHP, so $y(t)$ will be bounded.

Finally,

$$y(0) = \lim_{s \to \infty} sY(s) = 0 \qquad\qquad y(\infty) = \lim_{s \to 0} sY(s) = 1$$

b)

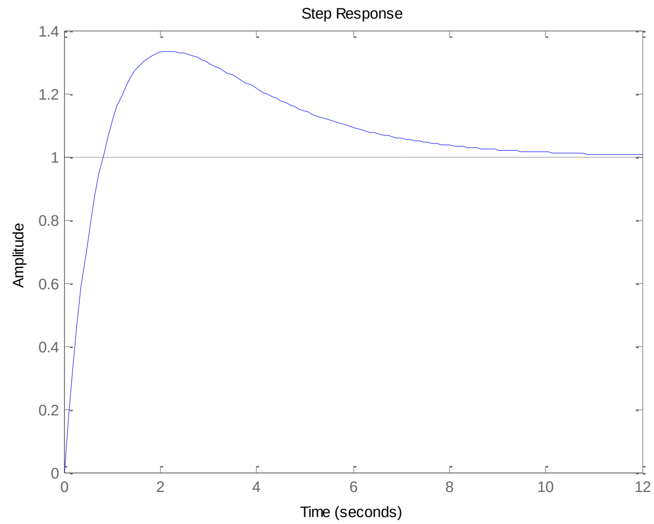


Figure S5.24 *Step response for part (a)*

5.25

For such an integrating process at steady state, any positive/negative step change in inlet flow will cause the tank level to increase/decrease with time. Thus, no new steady state will be attained, unless the tank overflows or empties.

Integrating processes do not have a steady-state gain in the usual sense. Note that G(0) is undefined because of dividing by zero.

$$K = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{K}{s} = \infty \qquad \text{Undefined.}$$

5.26

(a)

At time 0, T_m *(0)* and *T(0)* are the same which is t_0. Then *T* (bath temperature) follows a ramp:

$$T(t) = t + T_0 \qquad (1)$$

Define deviation variables: $T_m' = T_m - T_{ss} = T_m - T_0$; $T' = T - T_{ss} = T - T_0$, substitute these into Eq. 1 :

$$T'(t) = t \text{ and LT: } T'(s) = \frac{1}{s^2} \qquad (2)$$

As known, thermometer can be modeled by a first order system with time constant 0.1 and gain 1:

$$\frac{T_m'(s)}{T'(s)} = \frac{G}{\tau s + 1} = \frac{1}{0.1s + 1} \qquad (3) \quad \text{Eq. 5-19 in the book}$$

Combine Eqs. 2 and 3 :

$$T_m'(s) = \frac{1}{0.1s+1} T'(s) = \frac{1}{0.1s+1} \cdot \frac{1}{s^2} \qquad (4)$$

Apply PFE to Eq. 4:

$$T_m'(s) = \frac{1}{0.1s+1} T'(s) = \frac{1}{0.1s+1} \cdot \frac{1}{s^2} = \frac{a_1}{0.1s+1} + \frac{a_2}{s} + \frac{a_3}{s^2}$$

$$a_1 = \left. \frac{1}{s^2} \right|_{s=-10} = 0.01$$

$$a_3 = \left. \frac{1}{0.1s+1} \right|_{s=0} = 1$$

$$set \quad s = 10, \frac{1}{0.1s+1} \cdot \frac{1}{s^2} = 0.005; \frac{a_1}{0.1s+1} + \frac{a_2}{s} + \frac{a_3}{s^2} = 0.015 + 0.1a_2$$

$$a_2 = -0.1$$

As a result:

$$T_m'(s) = \frac{0.01}{0.1s+1} - \frac{0.1}{s} + \frac{1}{s^2} \qquad (5)$$

Use inverse LT to time domain:

$$T_m'(t) = 0.01 \frac{1}{0.1} \exp(-\frac{t}{0.1}) - 0.1 + t \qquad \text{Eq. 5-21 in the book}$$

$$T_m'(t) = 0.1(\exp(-10t) - 1) + t$$

At t = 0.1 min and t = 1.0 min after the change in *T(t)*, the difference would be:

$$T_m(t) = T_m(0) + 0.1(\exp(-10t) - 1) + t$$
$$T(t) = T(0) + t \qquad (6)$$
$$\Delta T_m = T_m(t) - T(t) = 0.1(\exp(-10t) - 1)$$

$$\Delta T_m(0.1) = -0.0632$$
$$\Delta T_m(0.1) = -0.1$$

(b)

By looking at Figure 5.5, the maximum difference occurs when t→∞ and the corresponding difference is: $\lim_{t\to\infty}\{0.1(\exp(-10t)-1)\} = -0.1$

(c)
For large time, exp(-10t) approaches zero and:
$T_m(t) = T_m(0) + 0.1(\exp(-10t) - 1) + t = T_m(0) + (t - 0.1)$
which indicates there is a 0.1 min time delay between measurement and true value after a long time.

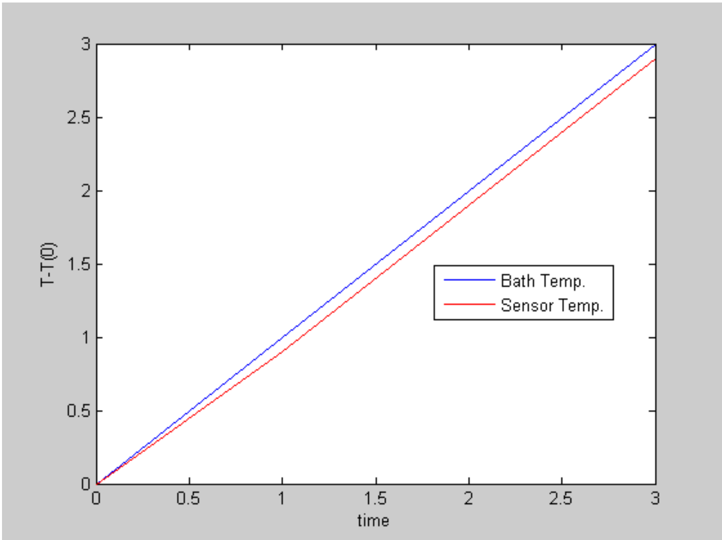


Figure S5.26 *T(t) and $T_m(t)$*

5.27

The temperature of the bath can be described as:

$$T(t) = 120 + 20S(t) - 40S(t-10) \qquad (1)$$

Define deviation variables as: $T'(t) = T(t) - T(0)$ and $T_m'(t) = T_m'(t) - T(0)$ where $T(0)$ = 120 °F. Transfer $T(t)$ into $T'(s)$:

$$T'(t) = T(t) - 120 = 20S(t) - 40S(t-10)$$

$$T'(s) = \frac{20}{s} - \frac{40}{s}e^{-10s} \quad (2)$$

According to the problem, the dynamics of the thermometer follow first order:

$$\frac{T_m'(s)}{T'(s)} = \frac{1}{s+1} \quad (3)$$

Combine Eqs. 2 and 3:

$$T_m'(s) = \frac{1}{0.1s+1}T'(s) = \frac{1}{s+1}\left(\frac{20}{s} - \frac{40}{s}e^{-10s}\right)$$

$$T_m'(s) = \left(\frac{-20}{s+1} + \frac{20}{s}\right) - \left(\frac{-40}{s+1} + \frac{40}{s}\right)e^{-10s}$$

Use inverse LT to time domain:

$$T_m'(t) = 20(1-\exp(-t)) - 40(1-\exp(-(t-10)))S(t-10) \quad (4)$$

Add $T(0)$ back:

$$T_m(t) = 20(1-\exp(-t)) - 40(1-\exp(-(t-10)))S(t-10) + 120 \quad (5)$$

when $0 < t < 10$s, $T_m(t) = 20(1-\exp(-t)) + 120$;

when $t > 10$s, $T_m(t) = 20(1-\exp(-t)) - 40(1-\exp(-(t-10))) + 120$

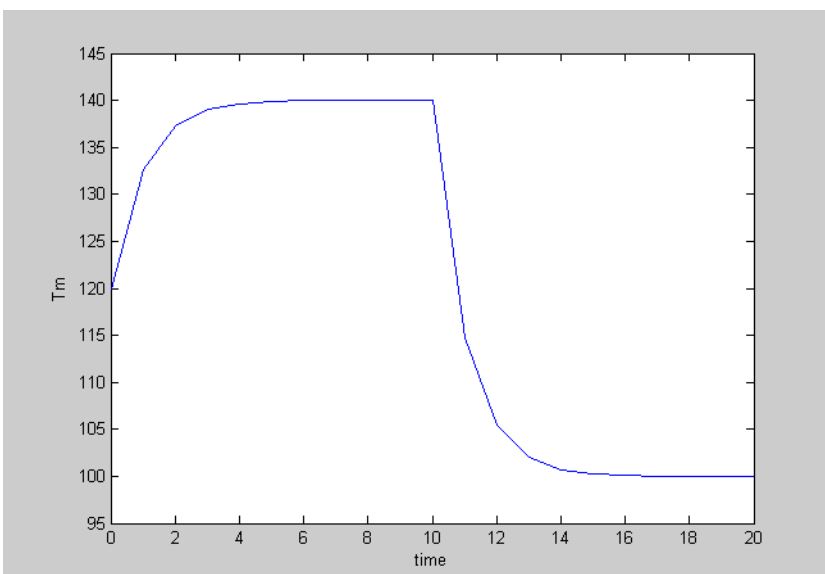


Figure S5.27 T_m *vs. time*

(b)

when t = 0.5 s, $T_m(0.5) = 20(1-\exp(-0.5)) + 120 = 127.87\text{F}$

when t = 15 s, $T_m(2) = 20(1-\exp(-2)) + 120 = 100.26\text{F}$

5.28

$$Y(s) = \frac{1}{s^2(2s+1)^3} = \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{\tau s+1} + \frac{a_4}{(\tau s+1)^2} + \frac{a_5}{(\tau s+1)^3}$$

We know that the a_3, a_4, a_5 terms are exponentials that go to zero for large values of time, leaving a linear response.

$$\therefore \frac{1}{s^2\left(2s+1\right)^3} = \frac{a_1}{s} + \frac{a_2}{s^2} \Leftrightarrow \frac{1}{\left(2s+1\right)^3} = a_1 s + a_2$$

$$\therefore a_2 = \lim_{s\to 0} \frac{1}{\left(2s+1\right)^3} = 1$$

Define $Q(s) = \dfrac{1}{(2s+1)^3}$

$\therefore \dfrac{dQ}{ds} = \dfrac{-6}{(2s+1)^4}$

Then $a_1 = \dfrac{1}{1!} \lim_{s\to 0} \left[\dfrac{-6}{(2s+1)^4} \right]$

(from Eq. 3-62)

$a_1 = -6$

$\therefore$ the long-time response (after transients have died out) is

$y_\ell(t) = t - 6$

We see that the output lags the input by a time equal to 6.

5.29

(a) Energy balance:

$$\rho V c_p \frac{dT}{dt} = UA(T_A - T)$$

where ρ is density of water

V is volume of water

c_p is heat capacity of water
U is heat transfer coefficient
A is surface area of tank
t is time in mins

Substituting numerical values in and noting that

$$\bar{T}_A = \bar{T}; \frac{dT}{dt} = \frac{dT'}{dt}$$

$$1000 \times \frac{\pi}{4} \times 0.5^2 \times 1 \times 4180 \times \frac{dT'}{dt} \leftarrow = 120 \times 60 \times \pi \times 0.5 \times 1 \times \left(T_A' - T'\right)$$

Taking Laplace transform:

$$\frac{T'(s)}{T_A'(s)} = \frac{1}{72.57s + 1}$$

$$T_A = 20 + \left(-15 - 20\right) S(t); \bar{T}_A = \bar{T} = 20$$

$$T_A' = -35 S(t) \Rightarrow T_A'(s) = -\frac{35}{s}$$

$$T'(s) = -\frac{35}{s(72.57s + 1)}$$

Applying inverse Laplace Transform to find when the water temperature reaches 0℃.

$$T'(t) = -35(1 - e^{-t/72.57}) \Rightarrow 0 - 20 = -35 \times (1 - e^{-t/72.57})$$

$$\therefore t = 61.45 \min$$

(b) Since the second stage involves a phase change with a constant temperature, thus, the time spent on phase change can be calculated based on the following equation:

$$\rho V \lambda = UA\left(T - T_A\right)t$$

$$1000 kg/m^3 \times \frac{\pi}{4} \times 0.5^2 \times 1 m^3 \times 334 \times 10^3 J/kg = 120 W/m^2 K \times \pi \times 0.5 \times 1 m^2 \times 15 K \times t$$

$$t = 386.6 \min$$

So the total time it takes to complete freeze the water in the tank is:

$$t_{total} = 61.45 + 386.6 \approx 448 \min$$

5.30

(a) From the results after 15 hr, we can see:
It is a first order system, the gain K is:

$$K = \frac{0 - (-1)K}{0 - (-1000)kW} = 10^{-3} K/kW$$

It takes 5τ to reach steady state, thus, the time constant

$$\tau = \frac{4}{5}hr = 0.8hr = 2880s$$

(b) The interval of step changes for the input should be larger, possibly greater than 4 hours.

(c) $\frac{T'(s)}{Q(s)} = \frac{K}{\tau s+1}; mc_p \frac{dT}{dt} = UA(T - T_a)$ Because the gain is small and the time constant is large, we can see that the mass, density, heat capacity and furnace height are all large.

Chapter 6

6.1

a)

$$G(s) = \frac{0.7\left(s^2 + 2s + 2\right)}{s^5 + 5s^4 + 9s^3 + 11s^2 + 8s + 6}$$

By using MATLAB, the poles and zeros are:

Zeros: (-1 +j) , (-1-j)

Poles: -3
-1
(0+j), (0-j)
(-1+j), (-1-j)

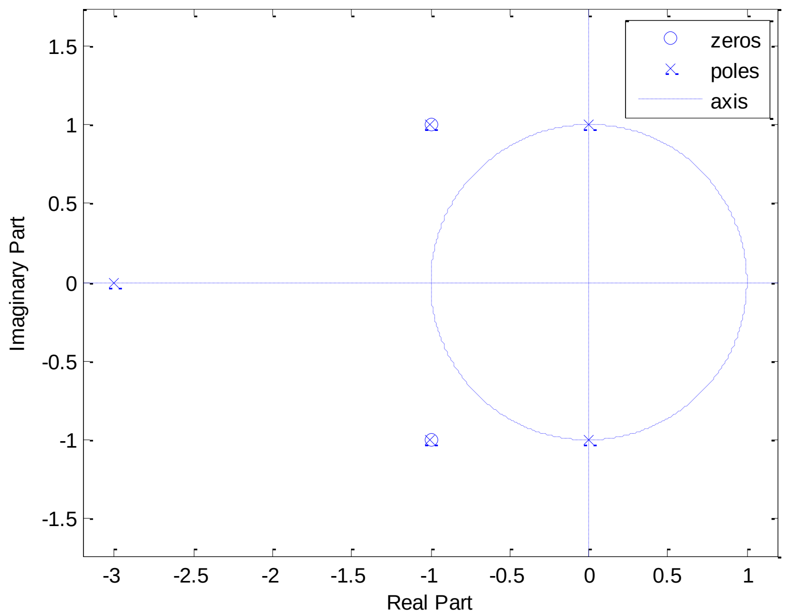


Figure S6.1. *Poles and zeros of G(s) plotted in the complex s plane.*

b) The process output will be bounded because there is no pole in the right half plane, but oscillations will be shown because of pure imagine roots.

c) Simulink results:

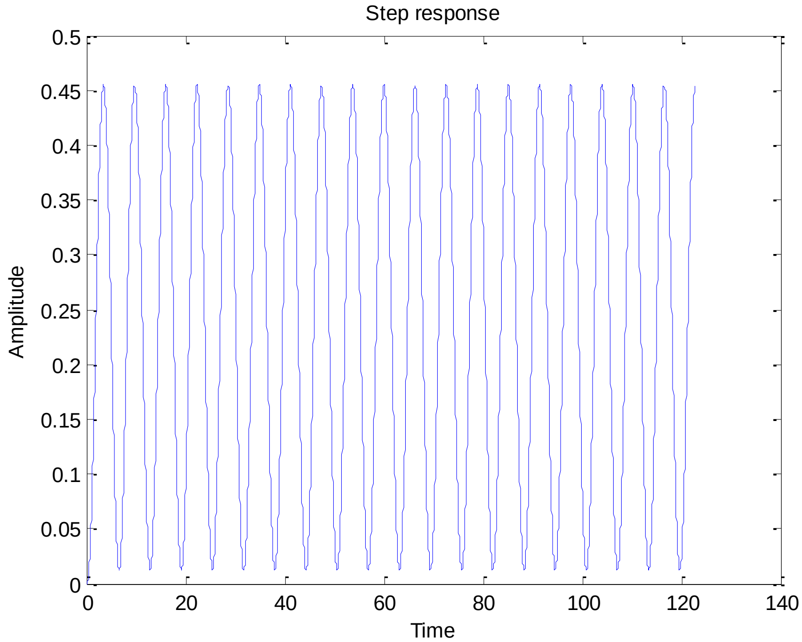


Figure S6.1c. *Response of the output to a unit step input.*

As shown in Fig. S6.1c, the system is stable but oscillations show up because of pure imaginary roots.

6.2

(a) Standard form: $G(s)=\dfrac{4(s+2)}{(0.5s+1)(2s+1)}e^{-5s}=\dfrac{8(0.5s+1)}{(0.5s+1)(2s+1)}e^{-5s}$

(b) Apply zero-pole cancellation:

$$G(s)=\frac{8}{(2s+1)}e^{-5s}$$

Gain =8; Pole=-0.5; Zeros=None

(c)

1/1 Padé approximation: $e^{-5s}=\dfrac{1-5/2s}{1+5/2s}$

The transfer function becomes

$$G(s)=\left(\frac{8}{2s+1}\right)\frac{(1-5/2s)}{(1+5/2s)}$$

Gain = 8; poles = -0.5, -0.4; zero = 0.4

6.3

$$\frac{Y(s)}{X(s)} = \frac{K(\tau_a s+1)}{(\tau_1 s+1)} \quad , \quad X(s) = \frac{M}{s}$$

From Eq. 6-13

$$y(t) = KM\left[1-\left(1-\frac{\tau_a}{\tau_1}\right)e^{-t/\tau_1}\right] = KM\left[1+\frac{\tau_a-\tau_1}{\tau_1}e^{-t/\tau_1}\right]$$

a) $$y(0^+) = KM\left[1+\frac{\tau_a-\tau_1}{\tau_1}\right] = \frac{\tau_a}{\tau_1}KM$$

b) Overshoot $\rightarrow y(t) > KM$

$$KM\left[1+\frac{\tau_a-\tau_1}{\tau_1}e^{-t/\tau_1}\right] > KM$$

or when, $\tau_a - \tau_1 > 0$, that is, $\tau_a > \tau_1$

$$\dot{y} = -KM\frac{(\tau_a-\tau_1)}{\tau_1^2}e^{-t/\tau_1} < 0 \quad for \quad KM > 0$$

c) Inverse response $\rightarrow y(t) < 0$

$$KM\left[1+\frac{\tau_a-\tau_1}{\tau_1}e^{-t/\tau_1}\right] < 0$$

$$\frac{\tau_a-\tau_1}{\tau_1} < -e^{+t/\tau_1} \quad \text{or} \quad \frac{\tau_a}{\tau_1} < 1-e^{+t/\tau_1} < 0 \quad \text{at } t = 0.$$

Therefore, $\tau_a < 0$.

6.4

$$\frac{Y(s)}{X(s)} = \frac{K(\tau_a s+1)}{(\tau_1 s+1)(\tau_2 s+1)} \quad , \quad \tau_1 > \tau_2, \quad X(s) = M/s$$

From Eq. 6-15

$$y(t) = KM\left[1+\frac{\tau_a-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1} - \frac{\tau_a-\tau_2}{\tau_1-\tau_2}e^{-t/\tau_2}\right]$$

a) Extremum $\Rightarrow \quad \dot{y}(t)=0$

$$KM\left[0-\frac{1}{\tau_1}\left(\frac{\tau_a-\tau_1}{\tau_1-\tau_2}\right)e^{-t/\tau_1}+\frac{1}{\tau_2}\left(\frac{\tau_a-\tau_2}{\tau_1-\tau_2}\right)e^{-t/\tau_2}\right]=0$$

$$\frac{1-\tau_a/\tau_2}{1-\tau_a/\tau_1}=e^{-t\left(\frac{1}{\tau_1}-\frac{1}{\tau_2}\right)}\geq 1 \quad \text{since} \quad \tau_1>\tau_2$$

b) Overshoot $\rightarrow \quad y(t) > KM$

$$KM\left[1+\frac{\tau_a-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1}-\frac{\tau_a-\tau_2}{\tau_1-\tau_2}e^{-t/\tau_2}\right]>KM$$

$$\frac{\tau_a-\tau_1}{\tau_a-\tau_2}>e^{-t\left(\frac{1}{\tau_2}-\frac{1}{\tau_1}\right)}>0, \quad \text{therefore} \quad \tau_a>\tau_1$$

c) Inverse response $\Rightarrow \quad \dot{y}(t)<0$ at $t = 0^+$

$$KM\left[0-\frac{1}{\tau_1}\left(\frac{\tau_a-\tau_1}{\tau_1-\tau_2}\right)e^{-t/\tau_1}+\frac{1}{\tau_2}\left(\frac{\tau_a-\tau_2}{\tau_1-\tau_2}\right)e^{-t/\tau_2}\right]<0 \quad \text{at} \quad t=0^+$$

$$-\frac{1}{\tau_1}\left(\frac{\tau_a-\tau_1}{\tau_1-\tau_2}\right)+\frac{1}{\tau_2}\left(\frac{\tau_a-\tau_2}{\tau_1-\tau_2}\right)<0$$

$$\frac{\tau_a\left(\frac{1}{\tau_2}-\frac{1}{\tau_1}\right)}{\tau_1-\tau_2}<0$$

Since $\tau_1 > \tau_2$, $\tau_a < 0$.

d) If an extremum in y exists, then from (a):

$$e^{-t\left(\frac{1}{\tau_1}-\frac{1}{\tau_2}\right)}=\left(\frac{1-\tau_a/\tau_2}{1-\tau_a/\tau_1}\right)$$

$$t=\frac{\tau_1\tau_2}{\tau_1-\tau_2}\ln\left(\frac{1-\tau_a/\tau_2}{1-\tau_a/\tau_1}\right)$$

6.5

Using 1/1 Padé approximation: $e^{-45s} \approx \dfrac{1-22.5s}{1+22.5s}$;

$$G_p(s) \approx \frac{1.4+13.5s}{(40s+1)(1+22.5s)}e^{-30s} = \frac{1.4(1+9.64s)}{(40s+1)(1+22.5s)}e^{-30s}$$

Gain = 1.4; $0 < \tau_a < \tau_1$, so it is an over damped process,

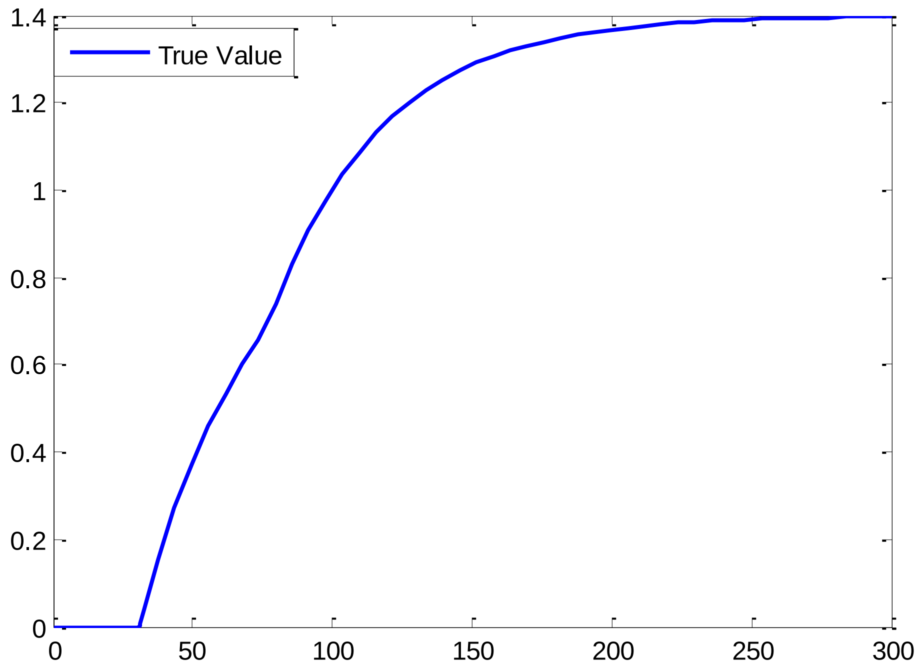


Figure S6.5. *Step response of the system.*

6.6

$$Y(s) = \frac{K_1}{s}U(s) + \frac{K_2}{\tau s+1}U(s) = \left[\frac{K_1}{s} + \frac{K_2}{\tau s+1}\right]U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{K_1\tau s + K_1 + K_2 s}{s(\tau s+1)} = \frac{(K_1\tau + K_2)s + K_1}{s(\tau s+1)}$$

Put in standard K/τ form for analysis:

$$G(s)=\frac{Y(s)}{U(s)}=\frac{K_1\left[\left(\tau+\frac{K_2}{K_1}\right)s+1\right]}{s(\tau s+1)}$$

a) Order of G(s) is 2 (maximum exponent on s in denominator is 2)

b) Gain of G(s) is K_1. Gain is negative if $K_1 < 0$.

c) Poles of G(s) are: $s_1 = 0$ and $s_2 = -1/\tau$

s_1 is on imaginary axis; s_2 is in the left hand plane.

d) The zero of G(s) is:

$$s_a=\frac{-1}{\left(\tau+\frac{K_2}{K_1}\right)}=\frac{-K_1}{K_1\tau+K_2}$$

If $\frac{K_1}{K_1\tau+K_2}<0$, the zero is in right half plane.

Two possibilities:

1. $K_1<0$ and $K_1\tau + K_2 >0$

e) Gain is negative if $K_1 < 0$

Then the zero is RHP if $K_1\tau + K_2 > 0$. This is the only possibility.

f) Constant term and $e^{-t/\tau}$ term.

g) If input is *M/s*, the output will contain a *t* term that is not bounded.

6.7

a) $p'(t)=(4-2)S(t)$, $P'(s)=\frac{2}{s}$

$$Q'(s)=\frac{-3}{20s+1}P'(s)=\frac{-3}{20s+1}\,\frac{2}{s}$$

$$Q'(t) = -6(1 - e^{-t/20})$$

b) $$R'(s) + Q'(s) = P_m'(s)$$

$$r'(t) + q'(t) = p_m'(t) = p_m(t) - p_m(0)$$

$$r'(t) = p_m(t) - 12 + 6(1 - e^{-t/20})$$

$$K = \frac{r'(t=\infty)}{p(t=\infty) - p(t=0)} = \frac{18 - 12 + 6(1-0)}{4-2} = 6$$

Overshoot,

$$\text{OS} = \frac{r'(t=15) - r'(t=\infty)}{r'(t=\infty)} = \frac{27 - 12 + 6(1 - e^{-15/20}) - 12}{12} = 0.514$$

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.514 \qquad , \qquad \zeta = 0.2$$

Period T for $r'(t)$ is equal to the period for $p_m(t)$ because $e^{-t/20}$ decreases monotonically.

Thus, $T = 50 - 15 = 35$

and $$\tau = \frac{T}{2\pi}\sqrt{1-\zeta^2} = 5.46$$

c) $$\frac{P_m'(s)}{P'(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} + \frac{K'}{\tau' s + 1}$$

$$= \frac{\left(K'\tau^2\right)s^2 + (K\tau' + 2K'\zeta\tau)s + (K + K')}{(\tau^2 s^2 + 2\zeta\tau s + 1)(\tau' s + 1)}$$

d) Overall process gain = $\left.\frac{P_m'(s)}{P'(s)}\right|_{s=0} = K + K' = 6 - 3 = 3\frac{\%}{\text{psi}}$

6.8

a) Transfer Function for the blending tank:

$$G_{bt}(s) = \frac{K_{bt}}{\tau_{bt} s + 1}$$

where $K_{bt} = \frac{q_{in}}{\sum q_i} \neq 1$ and $\tau_{bt} = \frac{2\text{m}^3}{1\text{m}^3/\text{min}} = 2\,\text{min}$

Transfer Function for the transfer line

$$G_{tl}(s) = \frac{K_{tl}}{(\tau_{tl} s + 1)^5}$$

where:

$$K_{tl} = 1$$

$$\tau_{tl} = \frac{0.1\text{m}^3}{5 \times 1\text{m}^3/\text{min}} = 0.02\,\text{min}$$

Thus,

$$\frac{C'_{out}(s)}{C'_{in}(s)} = \frac{K_{bt}}{(2s+1)(0.02s+1)^5}$$

which is a 6th-order transfer function.

b) Since $\tau_{bt} >> \tau_{tl}$ [2 >> 0.02], we can approximate $\frac{1}{(0.02s+1)^5}$ by $e^{-\theta s}$

where $\theta = \sum_{i=1}^{5}(0.02) = 0.1$

$$\therefore \quad \frac{C'_{out}(s)}{C'_{in}(s)} \approx \frac{K_{bt} e^{-0.1s}}{2s+1}$$

c) Because $\tau_{bt} \approx 100\ \tau_{tl}$, we anticipate that this approximate TF will yield results very close to those from the original TF (part (a)). This approximate TF is exactly the same as would have been obtained using a plug flow assumption for the transfer line. Thus we conclude that investing a lot of effort into obtaining an accurate dynamic model for the transfer line is not worthwhile in this case.

Note: if $\tau_{bt} \approx \tau_{tl}$, this conclusion would not be valid.

d) Simulink simulation

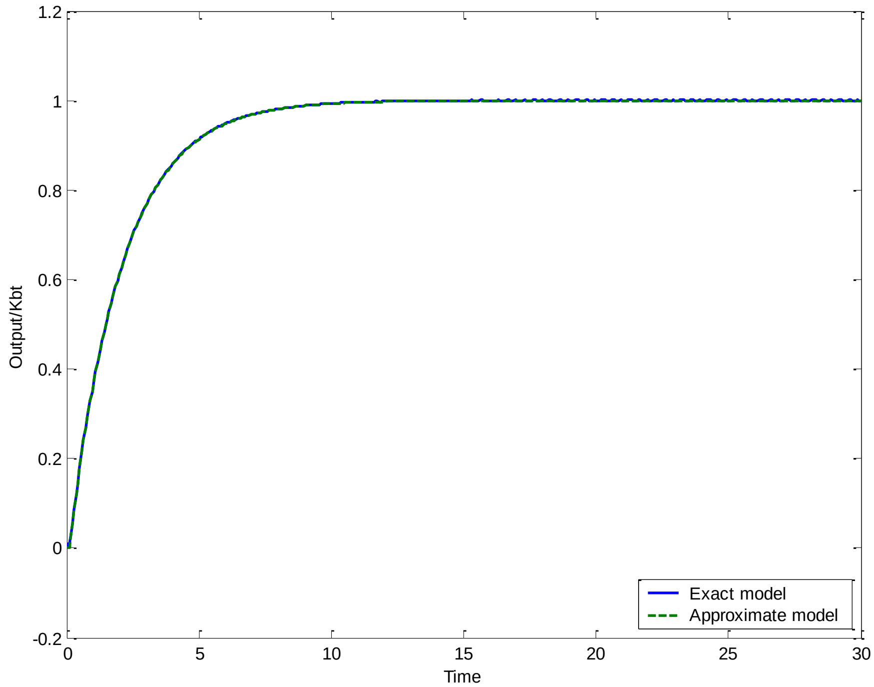


Figure S6.8. *Unit step responses for exact and approximate models.*

6.9

(a) $G(s) = \dfrac{320(1-4s)e^{-3s}}{24s^2+28s+4} = \dfrac{80(1-4s)e^{-3s}}{(6s+1)(s+1)}$

Gain= 80; time delay = 3; time constants $\tau_1 = 6, \tau_2 = 1$; poles = -1, -1/6; zeros = 0.25

(b) Since $\tau_a = -4 < 0;$ it will show an inverse response.

6.10

a) The transfer function for each tank is

$$\frac{C_i'(s)}{C_{i-1}'(s)} = \frac{1}{\left(\frac{V}{q}\right)s+1} \qquad ,i = 1, 2, \ldots, 5$$

where i represents the i^th tank.

c_o is the inlet concentration to tank 1.

V is the volume of each tank.

q is the volumetric flow rate.

$$\frac{C_5'(s)}{C_0'(s)} = \prod_{i=1}^{5}\left[\frac{C_i'(s)}{C_{i-1}'(s)}\right] = \left(\frac{1}{6s+1}\right)^5 ,$$

Then, by partial fraction expansion,

$$c_5(t) = 0.60 - 0.15\left[1 - e^{-t/6}\left\{1 + \frac{t}{6} + \frac{1}{2!}\left(\frac{t}{6}\right)^2 + \frac{1}{3!}\left(\frac{t}{6}\right)^3 + \frac{1}{4!}\left(\frac{t}{6}\right)^4\right\}\right]$$

b) Using Simulink,

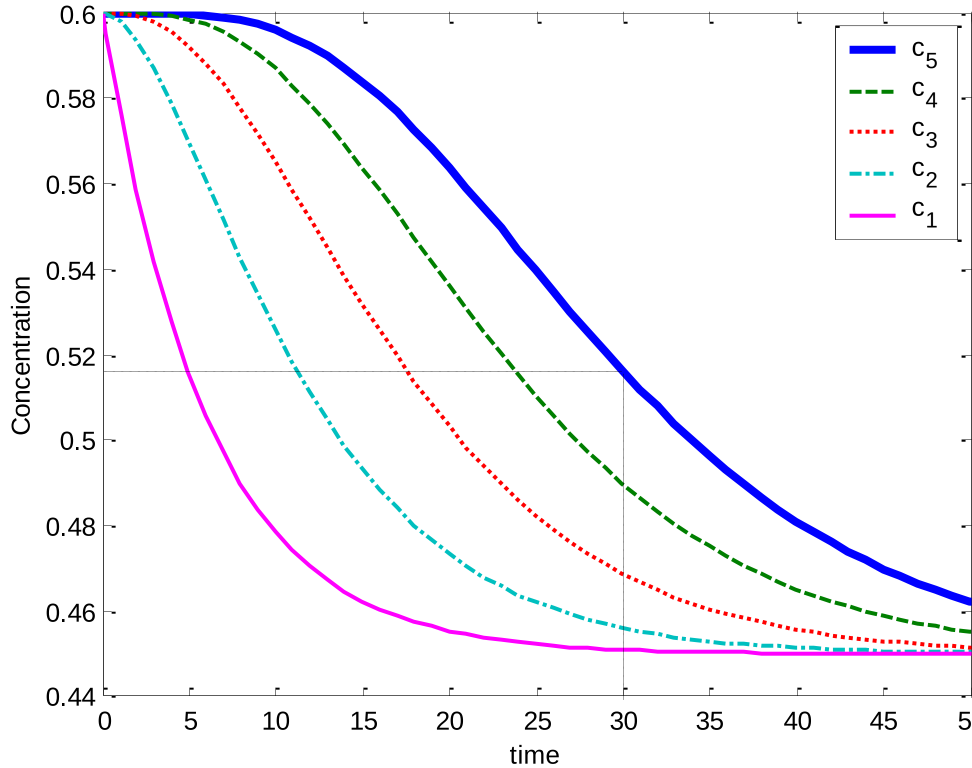


Figure S6.10. *Concentration step responses of the stirred tank.*

The value of the expression for $c_5(t)$ verifies the simulation results above:

$$c_5(30) = 0.60 - 0.15\left[1 - e^{-5}\left\{1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right\}\right] = 0.5161$$

6.11

First, consider then the undelayed response (with θ=0); then apply the Real Translation Theorem to find the desired delayed response.

Denote the undelayed response (for θ=0) by $\tilde{c}'_m(t)$. Then,

$$c'_m(t) = \tilde{c}'_m(t-\theta) \tag{1}$$

Taking the Laplace transforms give,

$$C'_m(s) = e^{-\theta s}\tilde{C}'_m(s) \tag{2}$$

The transfer functions for the delayed and undelayed systems are:

$$\frac{C'_m(s)}{C'(s)} = \frac{e^{-\theta s}}{\tau s+1} \tag{3}$$

$$\frac{\tilde{C}'_m(s)}{C'(s)} = \frac{1}{\tau s+1} \tag{4}$$

For the ramp input, $c'(t) = 2t$; from Table 3.1:

$$C'(s) = \frac{2}{s^2} \tag{5}$$

Substituting (5) into (4) and rearranging gives:

$$\tilde{C}'_m(s) = \left(\frac{1}{\tau s+1}\right)\left(\frac{2}{s^2}\right) \tag{6}$$

The corresponding response to the ramp input is given by Eq. 5-19 with K=1, a=2, and τ=10:

$$\tilde{c}'_m(t) = 20(e^{-t/10} - 1) + 2t \tag{7}$$

Let $\tilde{t}_a$ denote the time that the alarm goes on for the undelayed system; thus, the alarm lights up when $\tilde{c}_m(\tilde{t}_a) = 25$ min; i.e., when $\tilde{c}'_m(\tilde{t}_a) = 25 - 5 = 20$ min. .Substituting into (7) and solving for $\tilde{t}_a$ by trial and error gives

$$\tilde{t}_a = 9.24 \text{ min}$$

Let t_a denote the time that the alarm goes off for the system with time delay. It follows from the definition of a time delay that,

$$t_a = \tilde{t}_a + \theta = 9.24 + 2.00 = \boxed{11.24 \text{ min}}$$

6.12

a) Using Skogestad's method

$$G(s) = \frac{5e^{-(2+1+0.2)s}}{(12s+1)} = \frac{5e^{-3.2s}}{12s+1}$$

Using Simulink,

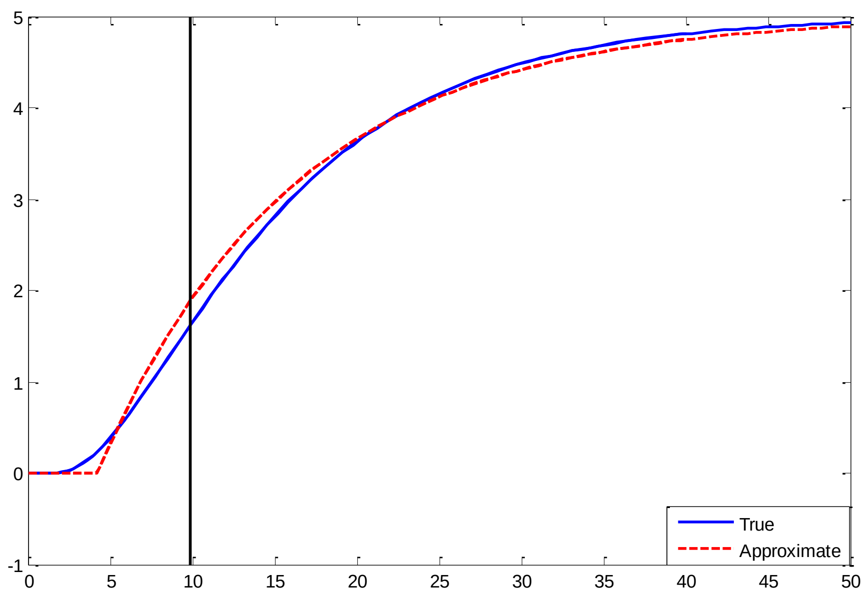


Figure S6.12 *Unit step responses for the exact and approximate models.*

(c) Maximum error =0.265, at *t*= 9.89s, and the location corresponding to the maximum error is graphically shown in above figure by black vertical line.

6.13

From the solution to Exercise 2.5(a) , the dynamic model for isothermal operation is

$$\frac{V_1 M}{RT_1}\frac{dP_1}{dt} = \frac{P_d - P_1}{R_a} - \frac{P_1 - P_2}{R_b} \qquad (1)$$

$$\frac{V_2 M}{RT_2}\frac{dP_2}{dt} = \frac{P_1 - P_2}{R_b} - \frac{P_2 - P_f}{R_c} \qquad (2)$$

Taking Laplace transforms, and noting that

$$P_f'(s) = 0$$

since P_f is constant,

$$P_1'(s) = \frac{K_b P_d'(s) + K_a P_2'(s)}{\tau_1 s + 1} \qquad (3)$$

$$P_2'(s) = \frac{K_c P_1'(s)}{\tau_2 s + 1} \qquad (4)$$

where

$$K_a = R_a /(R_a + R_b)$$
$$K_b = R_b /(R_a + R_b)$$

$$K_c = R_c /(R_b + R_c)$$

$$\tau_1 = \frac{V_1 M}{RT_1}\frac{R_a R_b}{(R_a + R_b)}$$

$$\tau_2 = \frac{V_2 M}{RT_2}\frac{R_b R_c}{(R_b + R_c)}$$

Substituting for $P_1'(s)$ from Eq. 3 into 4,

$$\frac{P_2'(s)}{P_d'(s)} = \frac{K_b K_c}{(\tau_1 s+1)(\tau_2 s+1) - K_a K_c} = \frac{\left(\dfrac{K_b K_c}{1-K_a K_c}\right)}{\left(\dfrac{\tau_1\tau_2}{1-K_a K_c}\right)s^2 + \left(\dfrac{\tau_1+\tau_2}{1-K_a K_c}\right)s + 1} \qquad (5)$$

Substituting for $P_2'(s)$ from Eq. 5 into 4,

$$\frac{P_1'(s)}{P_d'(s)} = \frac{\left(\dfrac{K_b}{1-K_a K_c}\right)(\tau_2 s+1)}{\left(\dfrac{\tau_1\tau_2}{1-K_a K_c}\right)s^2 + \left(\dfrac{\tau_1+\tau_2}{1-K_a K_c}\right)s + 1} \qquad (6)$$

To determine whether the system is overdamped or underdamped, consider the denominator of the transfer functions in Eqs. 5 and 6.

$$\tau^2 = \left(\frac{\tau_1\tau_2}{1-K_a K_c}\right) \quad , \quad 2\zeta\tau = \frac{\tau_1+\tau_2}{1-K_a K_c}$$

Therefore,

$$\zeta = \frac{1}{2}\frac{(\tau_1+\tau_2)}{(1-K_a K_c)}\frac{\sqrt{(1-K_a K_c)}}{\sqrt{\tau_1\tau_2}} = \frac{1}{2}\left(\sqrt{\frac{\tau_1}{\tau_2}} + \sqrt{\frac{\tau_2}{\tau_1}}\right)\frac{1}{\sqrt{(1-K_a K_c)}}$$

Since $x + 1/x \geq 2$ for all positive x,

$$\zeta \geq \frac{1}{\sqrt{(1-K_a K_c)}}$$

Since $K_a K_c \geq 0$,

$$\zeta \geq 1$$

Hence the system is overdamped.

6.14

Let $G(s) = \dfrac{4e^{-\theta s}}{(0.4s+1)^2(2s^2+3s+1)} = \dfrac{4e^{-\theta s}}{(0.4s+1)^2(2s+1)(s+1)}$

We want an approximate model of the form,

$$G_{approx}(s) = \frac{\tilde{K}e^{-\tilde{\theta}s}}{\tilde{\tau}s+1}$$

In order for the approximate model and the original models to have the same steady-state gain, we set $\boxed{\tilde{K}=4.}$

The largest time constant in $G(s)$ to neglect is 1. Thus,

$$\tilde{\tau} = 2 + \left(\frac{1}{2}\right)(1) = \boxed{2.5}$$

Approximate the smallest time constant by:

$$\frac{1}{0.4s+1} \approx e^{-0.4s}$$

Thus,

$$\tilde{\theta} \approx \theta + (\frac{1}{2})2 + 2(0.4) = \boxed{\theta + 1.8}$$

6.15

From Eqs. 6-71 and 6-72,

$$\zeta = \frac{R_2A_2 + R_1A_1 + R_2A_1}{2\sqrt{R_1R_2A_1A_2}} = \frac{1}{2}\left(\sqrt{\frac{R_1A_1}{R_2A_2}} + \sqrt{\frac{R_2A_2}{R_1A_1}}\right) + \frac{1}{2}\sqrt{\frac{R_2A_1}{R_1A_2}}$$

Since $x + \dfrac{1}{x} \geq 2$ for all positive x and since R_1, R_2, A_1, A_2 are positive

$$\zeta \geq \frac{1}{2}(2) + \frac{1}{2}\sqrt{\frac{R_2A_1}{R_1A_2}} \geq 1$$

6.16

a) Mass balance on Tank 2:

$$\rho A_2 \frac{dh_2}{dt} = \rho q_0 - \rho q_2$$

Dividing by ρ,

$$A_2 \frac{dh_2}{dt} = q_0 - q_2$$

For a linear resistance, (cf. Eq. 4-50),

$$q_2 = \frac{1}{R_2} h_2$$

Substitute,

$$A_2 \frac{dh_2}{dt} = q_0 - \frac{1}{R_2} h_2$$

or

$$A_2 R_2 \frac{dh_2}{dt} = R_2 q_0 - h_2$$

Introducing deviation variables and Laplace transforming yields

$$\frac{H_2'(s)}{Q_0'(s)} = \frac{R_2}{A_2 R_2 s + 1}$$

Because

$$Q_2'(s) = \frac{1}{R_2} H_2'(s)$$

we obtain,

$$\frac{Q_2'(s)}{Q_0'(s)} = \frac{1}{R_2} \frac{R_2}{A_2 R_2 s + 1} = \frac{1}{A_2 R_2 s + 1}$$

Letting $\tau_2 = A_2 R_2$

$$\frac{Q_2'(s)}{Q_0'(s)} = \frac{1}{\tau_2 s + 1}$$

b) Mass balances on the two tanks yield (after dividing by ρ, which is constant)

$$A_1 \frac{dh_1}{dt} = -q_1 \qquad A_2 \frac{dh_2}{dt} = q_0 + q_1 - q_2$$

Valve resistance relations:

$$q_1 = \frac{1}{R_1}(h_1 - h_2) \qquad q_2 = \frac{1}{R_2} h_2$$

c) These equations clearly describe an interacting second-order system; one or more transfer functions may contain a single zero (cf. Section 6.4). For the Q_2/Q_0 transfer function we know that the steady-state gain must be equal to one by physical arguments (the steady-state material balance around the two tank system is $\bar{q}_2 = \bar{q}_0$).

d) The response for Case (b) will be slower because this interacting system is second order, instead of first order.

6.17

The input is $T_i'(t) = 12 \sin \omega t$ where

$$\omega = \frac{2\pi \text{ radians}}{24 \text{ hours}} = 0.262 \text{ hr}^{-1}$$

The Laplace transform of the input is from Table 3.1,

$$T_i'(s) = \frac{12\omega}{s^2 + \omega^2}$$

Multiplying the transfer function by the input transform yields

$$T_i'(s) = \frac{(-72 + 36s)\omega}{(10s+1)(5s+1)(s^2+\omega^2)}$$

To invert, either (i) make a partial fraction expansion manually, or (ii) use the MATLAB residue function. The first method requires solution of a system of algebraic equations to obtain the coefficients of the four partial fractions. The second method requires that the numerator and denominator be defined as coefficients of descending powers of *s* prior to calling the MATLAB residue function:

<u>MATLAB Commands:</u>

```
>> b = [ 36*0.262  –72*0.262]
b =
    9.4320  –18.8640
>> a = conv([10 1], conv([5 1], [1 0 0.262^2]))
b =
    50.0000      15.0000          4.4322            1.0297            0.0686
>> [r,p,k] = residue(b,a)
r =
          6.0865 – 4.9668j
          6.0865 + 4.9668j
          38.1989
        –50.3718
p =
        –0.0000 – 0.2620j
        –0.0000 + 0.2620j
        –0.2000
        –0.1000
k =
          []
```

Note: the residue function re-computes all the poles (listed under p). They are, in reverse order: $p_1 = 0.1(\tau_1 = 10)$, $p = 0.2(\tau_2 = 5)$, and the two purely imaginary poles corresponding to the sine and cosine functions. The residues (listed under r) are exactly the coefficients of the corresponding poles; in other words, the coefficients that would have been obtained via a manual partial fraction expansion. In this case, we are not interested in the real poles since both of them yield exponential functions that go to 0 as $t \rightarrow \infty$.

The two complex poles are interpreted as the sine/cosine terms using Appendix L.

The coefficients of the periodic terms:

$$y(t) = a_1 e^{-bt} \cos \omega t + \frac{a_2}{\omega} e^{-bt} \sin \omega t + ...$$

b= 0, thus the exponential terms = 1. Using (L-13) and (L-15), $\omega = 0.264$.

$$y(t) = 12.136 \cos \omega t + 9.9336 \sin \omega t + ...$$

The amplitude of the composite output sinusoidal signal, for large values, of *t* is given by

$$A = \sqrt{(12.136)^2 + (9.9336)^2} = 15.7$$

Thus the amplitude of the output is 15.7° for the specified 12° amplitude input.

6.18

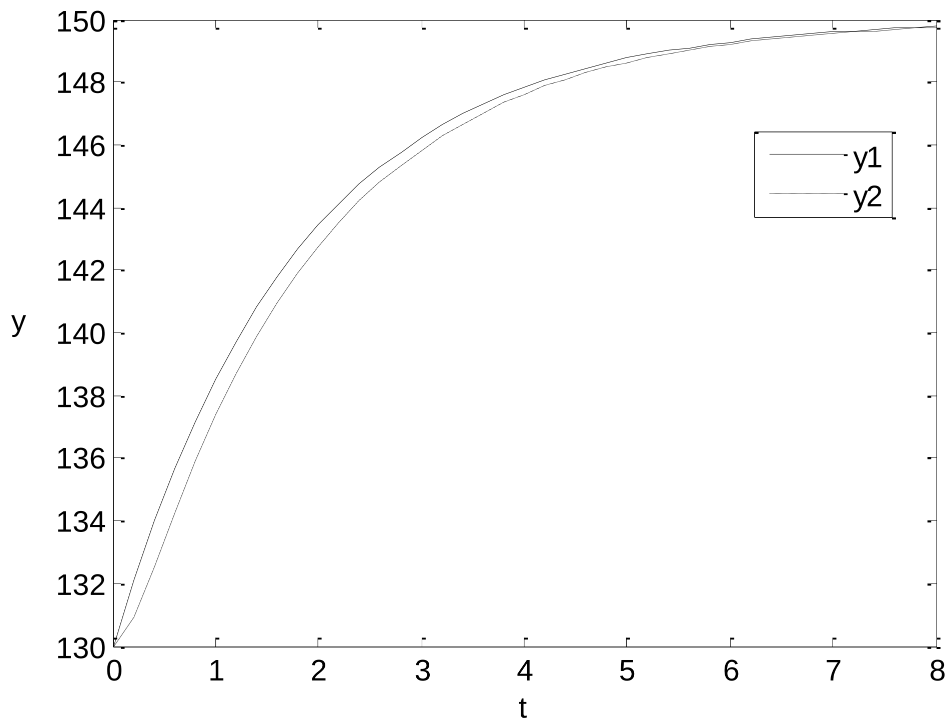


Figure S6.18 *Comparison between y1 and y2*

6.19

(a) The mathematical model is derived based on material balance:

$$V\frac{dc_1}{dt} = F_0c_0 + RF_2c_2 - F_1c_1 - Vkc_1$$

$$V\frac{dc_2}{dt} = F_1c_1 - (1+R)F_2c_2 - Vkc_2$$

Subtracting the steady-state equation and substituting deviation variables yields:

$$V\frac{dc_1^{'}}{dt} = F_0c_0^{'} + RF_2c_2^{'} - F_1c_1^{'} - Vkc_1^{'}$$

$$V\frac{dc_2^{'}}{dt} = F_1c_1^{'} - (1+R)F_2c_2^{'} - Vkc_2^{'}$$

(b) The transfer function model can be derived based on Laplace transform:

$$VsC_1^{'}(s) = F_0C_0^{'}(s) + RF_2C_2^{'}(s) - F_1C_1^{'}(s) - VkC_1^{'}(s)$$

$$VsC_2^{'}(s) = F_1C_1^{'}(s) - (1+R)F_2C_2^{'}(s) - VkC_2^{'}(s)$$

Solve above equations, we have:

$$C_2^{'}(s) = \frac{F_0F_1}{V^2s^2 + \left[2Vk + (F_1 + F_2 + RF_2)\right]Vs + F_1F_2 + Vk(1+R)F_2 + F_1Vk + V^2k^2}C_0^{'}(s)$$

(c) When $R \to 0$, we have:

$$C_2^{'}(s) = \frac{F_0F_1}{V^2s^2 + \left[2Vk + (F_1 + F_2)\right]Vs + F_1F_2 + VkF_2 + F_1Vk + V^2k^2}C_0^{'}(s)$$

$$= \frac{F_0F_1}{[Vs + Vk + F_1][Vs + Vk + F_2]}C_0^{'}(s)$$

which is equivalent to the transfer function of the two tanks connected in series.

(c) When k=0, Equation in (b) becomes:

$$C_2^{'}(s) = \frac{F_0F_1}{V^2s^2 + (F_1 + F_2 + RF_2)Vs + F_1F_2}C_0^{'}(s)$$

Since $F_1 = RF_2 + F_2, F_0 = F_2$

$$C_2^{'}(s) = \frac{F_0\left(RF_0 + F_0\right)}{V^2s^2 + \left(RF_0 + RF_0 + 2F_0\right)Vs + \left(RF_0 + F_0\right)F_0} C_0^{'}(s)$$

$$= \frac{F_0^2\left(1 + \dfrac{1}{R}\right)}{\dfrac{V^2s^2}{R} + \dfrac{2F_0}{R}Vs + 2F_0Vs + F_0^2 + \dfrac{F_0^2}{R}} C_0^{'}(s)$$

When $R \to \infty$, $C_2^{'}(s) = \dfrac{F_0}{2Vs + F_0} C_0^{'}(s)$, equivalent to a single tank with a volume = 2V.

The gain of above transfer function is 1.

6.20

The dynamic model for the process is given by Eqs. 2-45 and 2-46, which can be written as

$$\frac{dh}{dt} = \frac{1}{\rho A}(w_i - w) \tag{1}$$

$$\frac{dT}{dt} = \frac{w_i}{\rho Ah}(T_i - T) + \frac{Q}{\rho AhC} \tag{2}$$

where h is the liquid-level
A is the constant cross-sectional area

System outputs: h , T
System inputs : w, Q

Assume that w_i and T_i are constant. In Eq. 2, note that the nonlinear term $\left(h\dfrac{dT}{dt}\right)$ can be linearized as

$$\bar{h}\frac{dT'}{dt} + \frac{d\bar{T}}{dt}h'$$

or $\quad \bar{h}\dfrac{dT'}{dt}$ since $\quad \dfrac{d\bar{T}}{dt} = 0$

Then the linearized deviation variable form of (1) and (2) is

$$\frac{dh'}{dt} = -\frac{1}{\rho A}w'$$

$$\frac{dT'}{dt} = \frac{-w_i}{\rho A\bar{h}} T' + \frac{1}{\rho A\bar{h} C} Q'$$

Taking Laplace transforms and rearranging,

$$\frac{H'(s)}{W'(s)} = \frac{K_1}{s} \quad , \quad \frac{H'(s)}{Q'(s)} = 0 \quad , \quad \frac{T'(s)}{W'(s)} = 0 \quad , \quad \frac{T'(s)}{Q'(s)} = \frac{K_2}{\tau_2 s + 1}$$

where $K_1 = -\dfrac{1}{\rho A}$; and $K_2 = \dfrac{1}{w_i C}$, $\tau_2 = \dfrac{\rho A\bar{h}}{w_i}$

For an unit-step change in Q: $h(t) = \bar{h}$, $T(t) = \bar{T} + K_2(1 - e^{-t/\tau_2})$

For an unit step change in w: $h(t) = \bar{h} + K_1 t$, $T(t) = \bar{T}$

6.21

Additional assumptions:

(i) The density ρ and specific heat C of the liquid are constant.

(ii) The temperature of steam, T_s, is uniform over the entire heat transfer area.

(iii) The feed temperature T_F is constant (not needed in the solution).

Mass balance for the tank is

$$\frac{dV}{dt} = q_F - q \tag{1}$$

Energy balance for the tank is

$$\rho C \frac{d[V(T - T_{ref})]}{dt} = q_F \rho C(T_F - T_{ref}) - q\rho C(T - T_{ref}) + UA(T_s - T) \tag{2}$$

where T_{ref} is a constant reference temperature and A is the heat transfer area

Eq. 2 is simplified by substituting for $\frac{dV}{dt}$ from Eq. 1. Also, replace V by $A_T h$ (where A_T is the tank area) and replace A by $p_T h$ (where p_T is the perimeter of the tank). Then,

$$A_T \frac{dh}{dt} = q_F - q \tag{3}$$

$$\rho C A_T h \frac{dT}{dt} = q_F \rho C(T_F - T) + U p_T h (T_s - T) \tag{4}$$

Then, Eqs. 3 and 4 are the dynamic model for the system.

a) Making a Taylor series expansion of nonlinear terms in (4) and introducing deviation variables, Eqs. 3 and 4 become:

$$A_T \frac{dh'}{dt} = q'_F - q' \tag{5}$$

$$\rho C A_T \bar{h} \frac{dT'}{dt} = \rho C(T_F - \bar{T}) q'_F - (\rho C \bar{q}_F + U p_T \bar{h}) T' + U p_T \bar{h} T'_s + U p_T (\bar{T}_s - \bar{T}) h' \tag{6}$$

Taking Laplace transforms,

$$H'(s) = \frac{1}{A_T s} Q'_F(s) - \frac{1}{A_T s} Q'(s) \tag{7}$$

$$\left[\left(\frac{\rho C A_T \bar{h}}{\rho C \bar{q}_F + U p_T \bar{h}}\right) s + 1\right] T'(s) = \left[\frac{\rho C (T_F - \bar{T})}{\rho C \bar{q}_F + U p_T \bar{h}}\right] Q'_F(s)$$
$$+ \left[\frac{U p_T \bar{h}}{\rho C \bar{q}_F + U p_T \bar{h}}\right] T'_s(s) + \left[\frac{U p_T (\bar{T}_s - \bar{T})}{\rho C \bar{q}_F + U p_T \bar{h}}\right] H'(s) \tag{8}$$

Substituting for $H'(s)$ from (7) into (8) and rearranging gives

$$[A_T s]\left[\left(\frac{\rho C A_T \bar{h}}{\rho C \bar{q}_F + U p_T \bar{h}}\right) s + 1\right] T'(s) = \left[\frac{\rho C (T_F - \bar{T})}{\rho C \bar{q}_F + U p_T \bar{h}} A_T s\right] Q'_F(s)$$

$$+ \left[\frac{U p_T \bar{h} A_T s}{\rho C \bar{q}_F + U p_T \bar{h}}\right] T'_s(s) + \left[\frac{U p_T (\bar{T}_s - \bar{T})}{\rho C \bar{q}_F + U p_T \bar{h}}\right] [Q'_F(s) - Q'(s)] \tag{9}$$

Let $\quad \tau = \dfrac{\rho C A_T \bar{h}}{\rho C \bar{q}_F + U p_T \bar{h}}$

Then from Eq. 7

$$\frac{H'(s)}{Q_F'(s)} = \frac{1}{A_T s} \quad , \quad \frac{H'(s)}{Q'(s)} = -\frac{1}{A_T s} \quad , \quad \frac{H'(s)}{T_s'(s)} = 0$$

And from Eq. 9

$$\frac{T'(s)}{Q_F'(s)} = \frac{\left[\dfrac{U p_T (\bar{T}_s - \bar{T})}{\rho C \bar{q}_F + U p_T \bar{h}}\right]\left[\left(\dfrac{\rho C (T_F - \bar{T}) A_T}{U p_T (\bar{T}_s - \bar{T})}\right) s + 1\right]}{(A_T s)(\tau s + 1)}$$

$$\frac{T'(s)}{Q'(s)} = \frac{-\left[\dfrac{U p_T (\bar{T}_s - \bar{T})}{\rho C \bar{q}_F + U p_T \bar{h}}\right]}{(A_T s)(\tau s + 1)}$$

$$\frac{T'(s)}{T_s'(s)} = \frac{\left[\dfrac{U p_T \bar{h}}{\rho C \bar{q}_F + U p_T \bar{h}}\right]}{\tau s + 1}$$

Note:

$$\tau_2 = \frac{\rho C(T_F - \bar{T})A_T}{Up_T(\bar{T}_s - \bar{T})}$$ is the time constant in the numerator.

Because $T_F - \bar{T} < 0$ (heating) and $\bar{T}_s - T > 0$, τ_2 is negative, we can show this property by using Eq. 2 at steady state:

$$\rho C\bar{q}_F(T_F - \bar{T}) = -Up_T\bar{h}(\bar{T}_s - \bar{T})$$

or $$\rho C(T_F - \bar{T}) = \frac{-Up_T\bar{h}(\bar{T}_s - \bar{T})}{\bar{q}_F}$$

Substituting

$$\tau_2 = -\frac{\bar{h}A_T}{\bar{q}_F}$$

Let $\bar{V} = \bar{h}A_T$ so that $\tau_2 = -\dfrac{\bar{V}}{\bar{q}_F} = -$(initial residence time of tank)

For $\dfrac{T'(s)}{Q_F'(s)}$ and $\dfrac{T'(s)}{Q'(s)}$ the "gain" in each transfer function is

$$K = \left[\frac{Up_T(\bar{T}_s - \bar{T})}{A_T\left(\rho C\bar{q}_F + Up_T\bar{h}\right)}\right]$$

and must have the units of temperature/volume . (The integrator *s* has units of t^{-1}).

To simplify the transfer function gain, we can substitute

$$Up_T(\bar{T}_s - \bar{T}) = -\frac{\rho C\bar{q}_F(\bar{T}_F - \bar{T})}{\bar{h}}$$

from the steady-state relation. Then

$$K = \frac{-\rho C\bar{q}_F(\bar{T}_F - \bar{T})}{\bar{h}A_T\left(\rho C\bar{q}_F + Up_T\bar{h}\right)}$$

or $$K = \frac{\bar{T} - \bar{T}_F}{\bar{V}\left(1 + \dfrac{Up_T\bar{h}}{\rho C\bar{q}_F}\right)}$$

and the gain is positive since $\bar{T} - \bar{T}_F > 0$. Furthermore, it has dimensions of temperature/volume.

(The ratio $\dfrac{Up_T\bar{h}}{\rho C\bar{q}_F}$ is dimensionless).

(b) The $h - q_F$ transfer function is an integrator with a positive gain. Liquid level accumulates any changes in q_F, increasing for positive changes and vice-versa.

$h - q$ transfer function is an integrator with a negative gain. h accumulates changes in q, in the opposite direction, decreasing as q increases and vice versa.

$h - T_s$ transfer function is zero. Liquid level is independent of T_s and steam pressure P_s.

$T - q$ transfer function is second-order due to the interaction with liquid level; it is the product of an integrator and a first-order process.

$T - q_F$ transfer function is second-order due to the interaction with liquid level; it has numerator dynamics since q_F affects T directly as well if $T_F \neq \bar{T}$.

$T - T_s$ transfer function is first-order because there is no interaction with liquid level.

c) $h - q_F$: h increases continuously at a constant rate.

$h - q$: h decreases continuously at a constant rate.

$h - T_s$: h stays constant.

$T - q_F$: for $T_F < \bar{T}$, T decreases initially (inverse response) and then increases. After long times, T increases like a ramp function.

$T - q$: T decreases, eventually at a constant rate.

$T - T_s$: T increases with a first-order response and attains a new steady state.

6.22

a) The two-tank process is described by the following equations in deviation variables:

$$\frac{dh_1'}{dt} = \frac{1}{\rho A_1}\left[w_1' - \frac{1}{R}(h_1' - h_2' \right] \tag{1}$$

$$\frac{dh_2'}{dt} = \frac{1}{\rho A_2}\left[\frac{1}{R}(h_1' - h_2' \right] \tag{2}$$

Laplace transforming

$$\rho A_1 Rs H_1'(s) = RW_i'(s) - H_1'(s) + H_2'(s) \tag{3}$$

$$\rho A_2 Rs H_2'(s) = H_1'(s) - H_2'(s) \tag{4}$$

From (4)

$$(\rho A_2 Rs + 1) H_2'(s) = H_1'(s) \tag{5}$$

or

$$\frac{H_2'(s)}{H_1'(s)} = \frac{1}{\rho A_2 Rs + 1} = \frac{1}{\tau_2 s + 1} \tag{6}$$

where $\tau_2 = \rho A_2 R$

Returning to (3)

$$(\rho A_1 Rs + 1) H_1'(s) - H_2'(s) = RW_i'(s) \tag{7}$$

Substituting (6) with $\tau_1 = \rho A_1 R$

$$\left[(\tau_1 s + 1) - \frac{1}{\tau_2 s + 1} \right] H_1'(s) = RW_i'(s) \tag{8}$$

or

$$\left[(\tau_1\tau_2)s^2 + (\tau_1\tau_2)s\right]H_1^{'}(s) = R(\tau_2 s + 1)W_i^{'}(s) \tag{9}$$

$$\frac{H_1^{'}(s)}{W_1^{'}(s)} = \frac{R(\tau_2 s + 1)}{s\left[\tau_1\tau_2 s + (\tau_1 + \tau_2)\right]} \tag{10}$$

Dividing numerator and denominator by $(\tau_1 + \tau_2)$ to put into standard form

$$\frac{H_1^{'}(s)}{W_1^{'}(s)} = \frac{[R/(\tau_1 + \tau_2)](\tau_2 s + 1)}{s\left[\dfrac{\tau_1\tau_2}{\tau_1 + \tau_2} s + 1\right]} \tag{11}$$

Note that

$$K = \frac{R}{\tau_1 + \tau_2} = \frac{R}{\rho A_1 R + \rho A_2 R} = \frac{1}{\rho(A_1 + A_2)} = \frac{1}{\rho A} \tag{12}$$

since $A = A_1 + A_2$

Also, let

$$\tau_s = \frac{\tau_1\tau_2}{\tau_1 + \tau_2} = \frac{\rho^2 R^2 A_1 A_2}{\rho R(A_1 + A_2)} = \frac{\rho R A_1 A_2}{A} \tag{13}$$

so that

$$\frac{H_1^{'}(s)}{W_i^{'}(s)} = \frac{K(\tau_2 s + 1)}{s(\tau_3 s + 1)} \tag{14}$$

and

$$\frac{H_2^{'}(s)}{W_i^{'}(s)} = \frac{H_2^{'}(s)}{H_1^{'}(s)}\frac{H_1^{'}(s)}{W_i^{'}(s)} = \frac{1}{(\tau_2 s + 1)}\frac{K(\tau_2 s + 1)}{s(\tau_3 s + 1)}$$

$$= \frac{K}{s(\tau_3 s + 1)} \tag{15}$$

Transfer functions (6), (14) and (15) define the operation of the two-tank process.

The single-tank process is described by the following equation in deviation variables:

$$\frac{dh'}{dt} = \frac{1}{\rho A} w_i' \tag{16}$$

Note that $\bar{\omega}$, which is constant, subtracts out.

Laplace transforming and rearranging:

$$\frac{H'(s)}{W_i'(s)} = \frac{1/\rho A}{s} \tag{17}$$

Again

$$K = \frac{1}{\rho A}$$

$$\frac{H'(s)}{W_i'(s)} = \frac{K}{s} \tag{18}$$

which is the expected integral relationship with no zero.

b) For $A_1 = A_2 = A/2$

$$\left.\begin{aligned} \tau_2 &= \rho AR/2 \\ \tau_3 &= \rho AR/4 \end{aligned}\right\} \tag{19}$$

Thus $\tau_2 = 2\tau_3$

We have two sets of transfer functions:

<u>One-Tank Process</u>

$$\frac{H'(s)}{W_i'(s)} = \frac{K}{s}$$

<u>Two-Tank Process</u>

$$\frac{H_i'(s)}{W_i'(s)} = \frac{K(2\tau_3 s + 1)}{s(\tau_3 s + 1)}$$

$$\frac{H_2'(s)}{W_i'(s)} = \frac{K}{s(\tau_3 s + 1)}$$

Remarks:

- The gain $(K = 1/\rho A)$ is the same for all TFs.
- Each TF contains an integrating element.

- However, the two-tank TF's contain a pole $(\tau_3 s+1)$ that will "filter out" changes in level caused by changing $w_i(t)$.
- On the other hand, for this special case, we see that the zero in the first tank transfer function $(H_i^{'}(s)/W_i^{'}(s))$ is larger than the pole:

  $$2\,\tau_3 > \tau_3$$

  Thus we should make sure that amplification of changes in $h_1(t)$ caused by the zero do not more than cancel the beneficial filtering of the pole so as to cause the first compartment to overflow easily.

  Now look at more general situations of the two-tank case:

$$\frac{H_1^{'}(s)}{W_i^{'}(s)} = \frac{K(\rho A_2 Rs+1)}{s\left(\dfrac{\rho R A_1 A_2}{A}s+1\right)} = \frac{K(\tau_2 s+1)}{s(\tau_3 s+1)} \tag{20}$$

$$\frac{H_2^{'}(s)}{W_i^{'}(s)} = \frac{K}{s(\tau_3 s+1)} \tag{21}$$

For either $A_1 \to 0$ or $A_2 \to 0$,

$$\tau_3 = \frac{\rho R A_1 A_2}{A} \to 0$$

Thus the beneficial effect of the pole is lost as the process tends to look more like the first-order process.

c) The optimum filtering can be found by maximizing τ_3 with respect to A_1 (or A_2)

$$\tau_3 = \frac{\rho R A_1 A_2}{A} = \frac{\rho R A_1 (A-A_1)}{A}$$

Find max $\tau_3 : \dfrac{\partial \tau_3}{\partial A_1} = \dfrac{\rho R}{A}\left[(A-A_1)+A_1(-1)\right]$

Set to 0: $A - A_1 - A_1 = 0$

$$2A_1 = A$$

$$A_1 = A/2$$

Thus the maximum filtering action is obtained when $A_1 = A_2 = A/2$.

The ratio of τ_2/τ_3 determines the "amplification effect" of the zero on $h_1(t)$.

$$\frac{\tau_2}{\tau_3} = \frac{\rho A_2 R}{\frac{\rho R A_1 A_2}{A}} = \frac{A}{A_1}$$

As A_1 goes to 0, $\frac{\tau_2}{\tau_3} \to \infty$

Therefore, the influence of changes in $w_i(t)$ on $h_1(t)$ will be very large, leading to the possibility of overflow in the first tank.

Summing up:

The process designer would like to have $A_1 = A_2 = A/2$ in order to obtain the maximum filtering of $h_1(t)$ and $h_2(t)$. However, the process response should be checked for typical changes in $w_i(t)$ to make sure that h_1 does not overflow. If it does, area A_1 needs to be increased until it is not a problem.

Note that $\tau_2 = \tau_3$ when $A_1 = A$, thus a careful study (simulations) should be made before designing the partitioned tank. Otherwise, leave wellenough alone and use the non-partitioned tank.

6.23

The process transfer function is

$$\frac{Y(s)}{U(s)} = G(s) = \frac{K}{(0.1s+1)^2(4s^2+2s+1)}$$

where $K = K_1K_2$.

The quadratic term describes an underdamped 2nd-order system since

$$\tau^2 = 4 \qquad \rightarrow \qquad \tau = 2$$

$$2\zeta\tau = 2 \qquad \rightarrow \qquad \zeta = 0.5$$

a) For the second-order process element with τ_2 = 2 and this degree of underdamping $(\zeta = 0.5)$, the small time constant, critically damped 2nd-order process element (τ_1 = 0.1) will have little effect.

In fact, since 0.1 << τ_2 (= 2) we can approximate the critically damped element as $e^{-2\tau_1}$ so that

$$G(s) \approx \frac{Ke^{-0.2s}}{4s^2 + 2s + 1}$$

b) From Fig. 5.10 for $\zeta = 0.5$, $OS \approx 0.15$ or from Eq. 5-51

$$\text{Overshoot} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.163$$

Hence y_{max} = 0.163 KM + KM = 0.163 (1) (3) + 3 = 3.5

c) From Fig. 5.3, y_{max} occurs at t/τ = 3K or t_{max} = 6.8 for an underdamped 2nd-order process with $\zeta = 0.5$.

Adding in the effect of the time delay t' = 6.8 + 0.2 = 7.0

d) By using Simulink

τ_1 = 0.1:

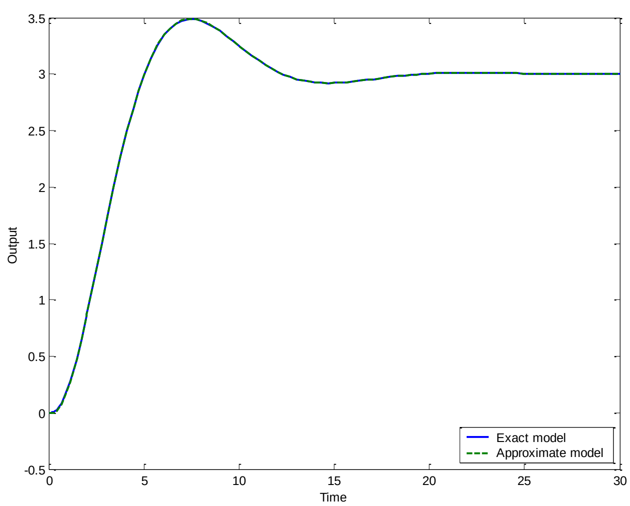


Figure S6.23a *Step response for exact and approximate models;* τ_1 = 0.1.

$\tau_1 = 1$:

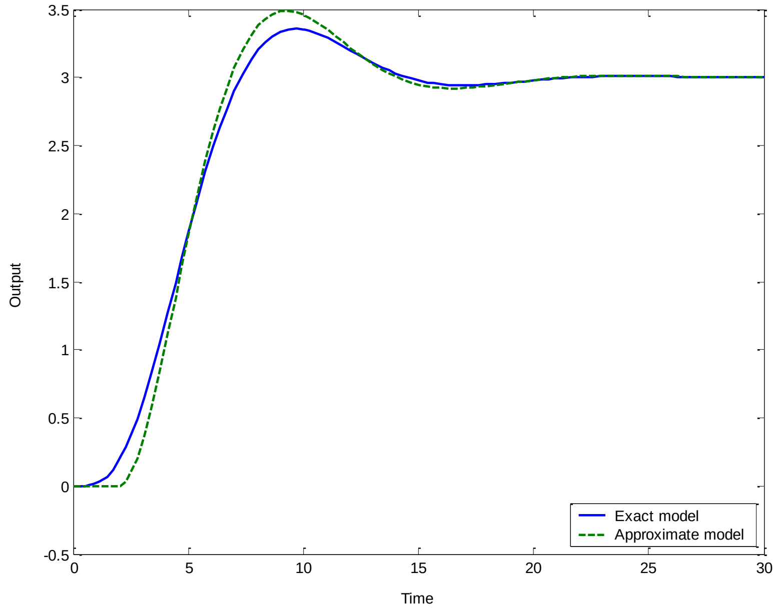


Figure S6.23b *Step responses for exact and approximate models;* $\tau_1 = 1$.

$\tau_1 = 5$:

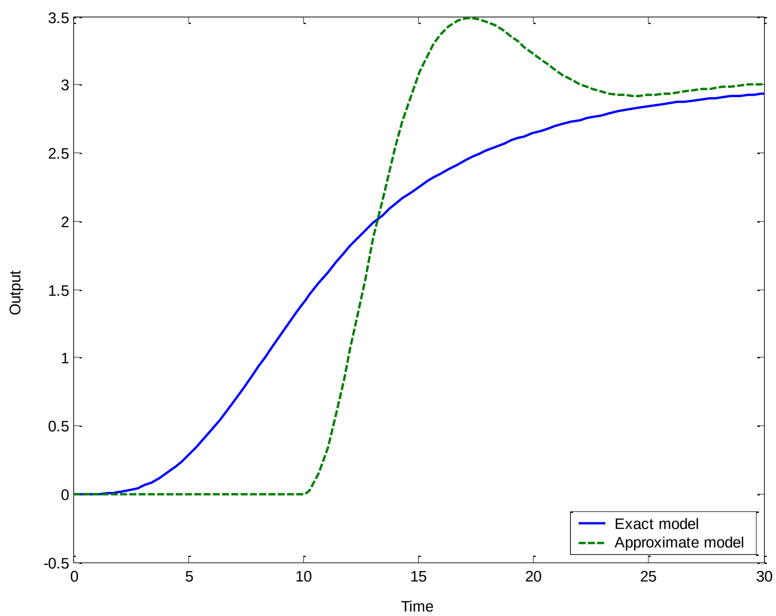


Figure S6.23c *Step response for exact and approximate models ;* $\tau_1 = 5$.

As is apparent from the plots, the smaller τ_1 is, the better the quality of the approximation. For large values of τ_1 (on the order of the underdamped element's time scale), the approximate model fails.

6.24

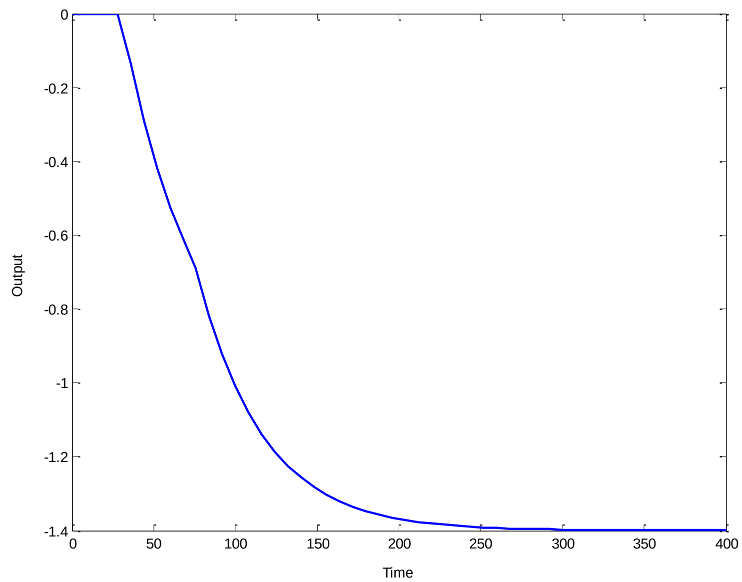
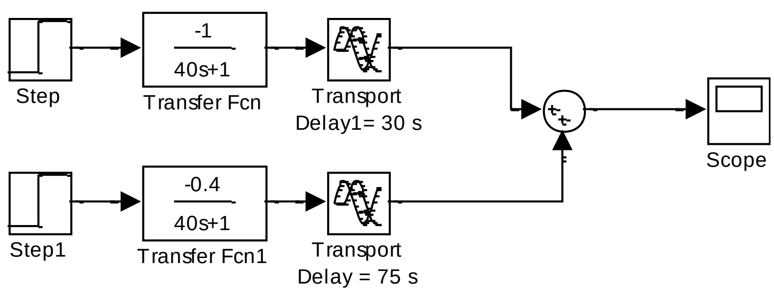


Fig. S6.24. *Unit step response in blood pressure.*

The Simulink- block diagram is shown below



The system appears to respond approximately as a first-order system or overdamped second-order process with time delay.

6.25

The system equations are:

$$A_1 \frac{dh_1'}{dt} = q_i' - \frac{1}{R_1} h_1' \qquad , \qquad q_1' = \frac{1}{R_1} h_1'$$

$$A_2 \frac{dh_2'}{dt} = \frac{1}{R_1} h_1' - \frac{1}{R_2} h_2' \qquad , \qquad q_2' = \frac{1}{R_2} h_2'$$

Using a state space representation,

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$$
$$\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du}$$

where $x = \begin{bmatrix} h_1' \\ h_2' \end{bmatrix}$, $u = q_i$ and $y = q_2'$

then,

$$\begin{bmatrix} \dfrac{dh_1'}{dt} \\ \dfrac{dh_2'}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{R_1 A_1} & 0 \\ \dfrac{1}{R_1 A_1} & -\dfrac{1}{R_2 A_2} \end{bmatrix} \begin{bmatrix} h_1' \\ h_2' \end{bmatrix} + \begin{bmatrix} \dfrac{1}{A_1} \\ 0 \end{bmatrix} q_i'$$

$$q_2' = \begin{bmatrix} 0 & \dfrac{1}{R_2} \end{bmatrix} \begin{bmatrix} h_1' \\ h_2' \end{bmatrix}$$

Therefore,

$$\boldsymbol{A} = \begin{bmatrix} -\dfrac{1}{R_1 A_1} & 0 \\ \dfrac{1}{R_1 A_1} & -\dfrac{1}{R_2 A_2} \end{bmatrix} \quad , \quad \boldsymbol{B} = \begin{bmatrix} \dfrac{1}{A_1} \\ 0 \end{bmatrix} \quad , \quad \boldsymbol{C} = \begin{bmatrix} 0 & \dfrac{1}{R_2} \end{bmatrix} \quad , \quad \boldsymbol{E} = 0$$

6.26

Applying numerical values, equations for the three-stage absorber are:

$$\frac{dx_1}{dt} = 0.881y_f - 1.173x_1 + 0.539x_2$$

$$\frac{dx_2}{dt} = 0.634x_1 - 1.173x_2 + 0.539x_3$$

$$\frac{dx_3}{dt} = 0.634x_2 - 1.173x_3 + 0.539x_f$$

$$y_i = 0.72x_i$$

Transforming into a state-space representation form:

$$\begin{bmatrix} \dfrac{dx_1}{dt} \\ \dfrac{dx_2}{dt} \\ \dfrac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} -1.173 & 0.539 & 0 \\ 0.634 & -1.173 & 0.539 \\ 0 & 0.634 & -1.173 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0.881 \\ 0 \\ 0 \end{bmatrix} y_f$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.72 & 0 & 0 \\ 0 & 0.72 & 0 \\ 0 & 0 & 0.72 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} y_f$$

Therefore, because x_f can be neglected in obtaining the desired transfer functions,

$$A = \begin{bmatrix} -1.173 & 0.539 & 0 \\ 0.634 & -1.173 & 0.539 \\ 0 & 0.634 & -1.173 \end{bmatrix} \quad B = \begin{bmatrix} 0.881 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.72 & 0 & 0 \\ 0 & 0.72 & 0 \\ 0 & 0 & 0.72 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying the MATLAB function *ss2tf* , the transfer functions are:

$$\frac{Y_1'(s)}{Y_f'(s)} = \frac{0.6343s^2 + 1.4881s + 0.6560}{s^3 + 3.5190s^2 + 3.443s + 0.8123}$$

$$\frac{Y_2'(s)}{Y_f'(s)} = \frac{0.4022s + 0.4717}{s^3 + 3.5190s^2 + 3.443s + 0.8123}$$

$$\frac{Y_2'(s)}{Y_f'(s)} = \frac{0.2550}{s^3 + 3.5190s^2 + 3.443s + 0.8123}$$

Chapter 7

7.1

In the absence of more accurate data, use a first-order transfer function:

$$\frac{T'(s)}{Q_i'(s)} = \frac{Ke^{-\theta s}}{\tau s+1}$$

$$K = \frac{T(\infty)-T(0)}{\Delta q_i} = \frac{(124.7-120)}{520-500} = 0.235\frac{^{\text{o}}\text{F}}{\text{gal/min}}$$

θ = 3:08 am – 3:05 am = 3 min

Assuming that the operator logs a 99% complete system response as "no change after 3:34 am", five time constants elapse between 3:08 and 3:34 am.

5τ = 3:34 min – 3:08 min = 26 min

τ = 26/5 min = 5.2 min

Therefore,

$$\frac{T'(s)}{Q_i'(s)} = \frac{0.235e^{-3s}}{5.2s+1}$$

To obtain a better estimate of the transfer function, the operator should log more data between the first change in T and the new steady state.

7.2

Process gain,

$$K = \frac{h(5.0)-h(0)}{\Delta q_i} = \frac{6.52-5.50}{30.1\times 0.1} = 0.339\frac{\text{min}}{\text{ft}^2}$$

a) Output at 63.2% of the total change

= 5.50 + 0.632(6.52-5.50) = 6.145 ft

Interpolating between h = 6.07 ft and h = 6.18 ft

$$\tau = 0.6 + \frac{(0.8-0.6)}{(6.18-6.07)}(6.145-6.07)\,\text{min} = 0.74\,\text{min}$$

b)

$$\left.\frac{dh}{dt}\right|_{t=0} \approx \frac{h(0.2)-h(0)}{0.2-0} = \frac{5.75-5.50}{0.2}\frac{\text{ft}}{\text{min}} = 1.25\frac{\text{ft}}{\text{min}}$$

Using Eq. 7-15,

$$\tau = \frac{KM}{\left(\left.\frac{dh}{dt}\right|_{t=0}\right)} = \frac{0.339\times(30.1\times 0.1)}{1.25} \approx 0.82\,\text{min}$$

c) The slope of the linear relation between t_i and $z_i \equiv \ln\left[1-\frac{h(t_i)-h(0)}{h(\infty)-h(0)}\right]$ gives an approximation of (-1/τ), according to Eq. 7-13.

Using $h(\infty) = h(5.0) = 6.52$, the values of z_i are

t_i	z_i	t_i	z_i
0.0	0.00	1.4	-1.92
0.2	-0.28	1.6	-2.14
0.4	-0.55	1.8	-2.43
0.6	-0.82	2.0	-2.68
0.8	-1.10	3.0	-3.93
1.0	-1.37	4.0	-4.62
1.2	-1.63	5.0	-∞

Then the slope of the least squares fit, using Eq. 7-6 is

$$slope = \left(-\frac{1}{\tau}\right) = \frac{13S_{tz}-S_tS_z}{13S_{tt}-(S_t)^2} \qquad (1)$$

where the datum at $t = 5.0$ has been ignored.

Using definitions,

$$S_t = 18.0 \qquad S_{tt} = 40.4$$
$$S_z = -23.5 \qquad S_{tz} = -51.1$$

Substituting in (1),

$$\left(-\frac{1}{\tau}\right) = -1.213 \qquad \tau = 0.82\,\text{min}$$

d)

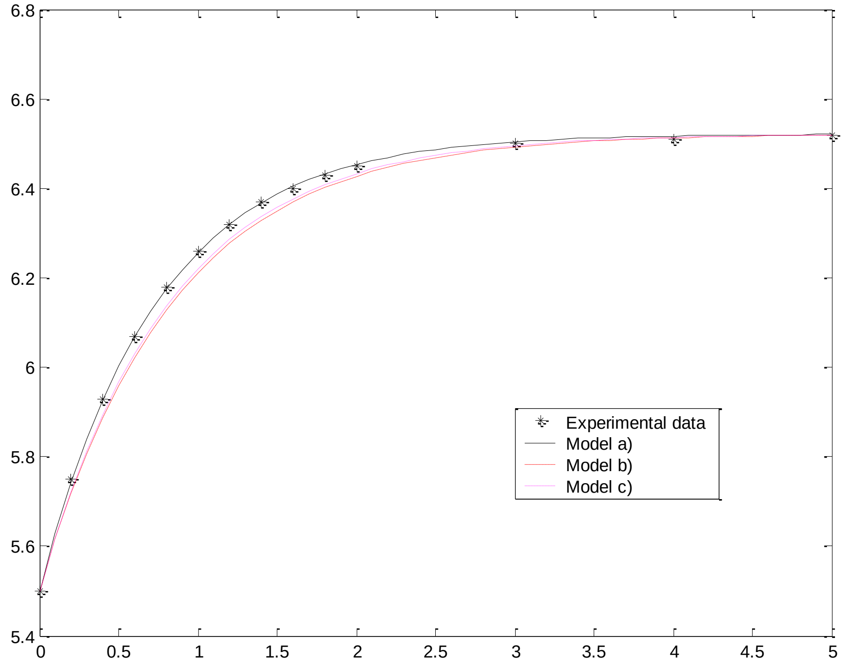


Figure S7.2 *Comparison between models a), b) and c) and the step response data.*

7.3

a)

$$\frac{T_1'(s)}{Q'(s)} = \frac{K_1}{\tau_1 s + 1} \qquad \frac{T_2'(s)}{T_1'(s)} = \frac{K_2}{\tau_2 s + 1}$$

$$\frac{T_2'(s)}{Q'(s)} = \frac{K_1 K_2}{(\tau_1 s + 1)(\tau_2 s + 1)} \approx \frac{K_1 K_2 e^{-\tau_2 s}}{\tau_1 s + 1} \qquad (1)$$

where the approximation follows from Eq. 6-58 and the fact that $\tau_1 > \tau_2$, as revealed by an inspection of the data.

$$K_1 = \frac{T_1(50) - T_1(0)}{\Delta q} = \frac{18.0 - 10.0}{85 - 82} = 2.667$$

$$K_2 = \frac{T_2(50) - T_2(0)}{T_1(50) - T_1(0)} = \frac{26.0 - 20.0}{18.0 - 10.0} = 0.75$$

Let z_1 and z_2 be the natural log of the fraction incomplete response for T_1 and T_2, respectively. Then,

$$z_1(t) = \ln\left[\frac{T_1(50) - T_1(t)}{T_1(50) - T_1(0)}\right] = \ln\left[\frac{18 - T_1(t)}{8}\right]$$

$$z_2(t) = \ln\left[\frac{T_2(50) - T_2(t)}{T_2(50) - T_2(0)}\right] = \ln\left[\frac{26 - T_2(t)}{6}\right]$$

A plot of z_1 and z_2 versus t is shown below. The slope of the z_1 plot is -0.333; hence $(1/-\tau_1) = -0.333$ and $\tau_1 = 3.0$

From the best-fit line for z_2 versus t, the projection intersects $z_2 = 0$ at $t \approx 1.15$. Hence $\tau_2 = 1.15$.

$$\frac{T_1'(s)}{Q'(s)} = \frac{2.667}{3s+1} \tag{2}$$

$$\frac{T_2'(s)}{T_1'(s)} = \frac{0.75}{1.15s+1} \tag{3}$$

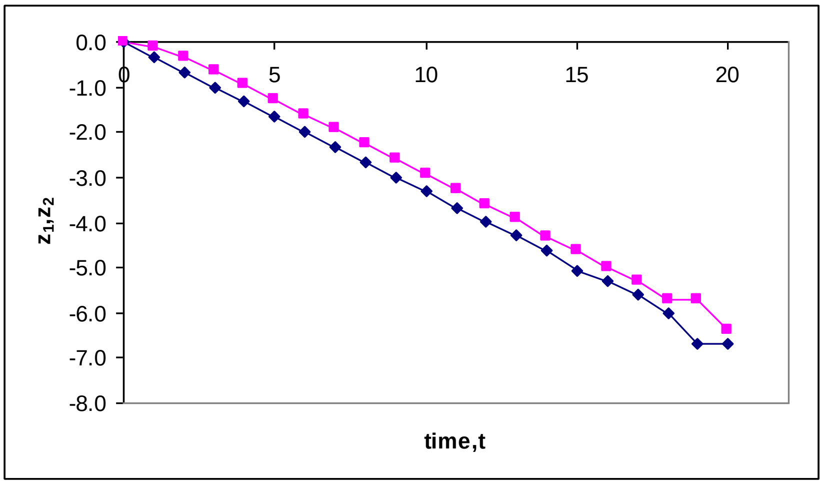


Figure S7.3a *z_1 and z_2 as a function of t*

b) Using Simulink-MATLAB, the following results are obtained:

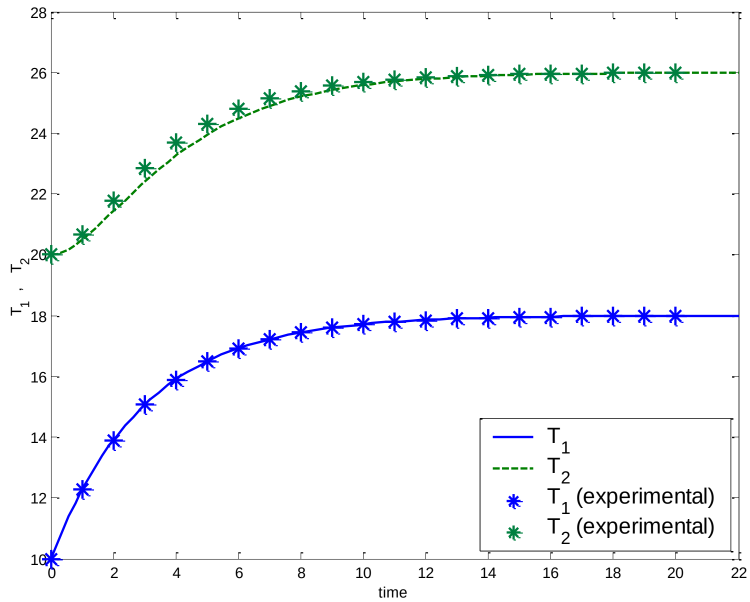


Figure S7.3b *Comparison of experimental data and models for a step change.*

7.4

$$Y(s) = G(s)\ X(s) = \frac{2}{(5s+1)(3s+1)(s+1)} \times \frac{1.5}{s}$$

Taking the inverse Laplace transform,

$$y(t) = (-75/8)\ \exp(-t/5) + (27/4)\ \exp\ (-t/3) - (3/8)\exp(-t) + 3 \qquad (1)$$

a) Fraction incomplete response

$$z(t) = \ln\left[1 - \frac{y(t)}{3}\right]$$

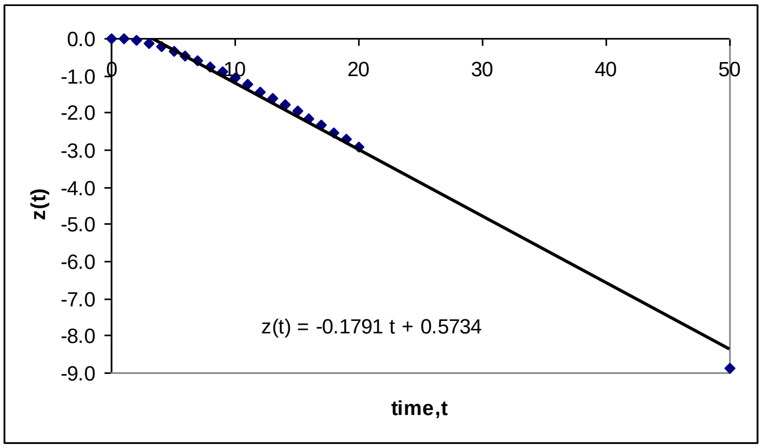


Figure S7.4a *Fraction incomplete response; linear regression*

From the plot: slope = - 0.179 and intercept ≈ 3.2

Hence,

$-1/\tau = -0.179$ and $\tau = 5.6$

$\theta = 3.2$

$$G(s) = \frac{2e^{-3.2s}}{5.6s+1}$$

b) In order to use Smith's method, find t_{20} and t_{60}:

$y(t_{20})= 0.2 \times 3 = 0.6$

$y(t_{60})= 0.6 \times 3 = 1.8$

Using either Eq. 1 or the plot of this equation, $t_{20} = 4.2$, $t_{60} = 9.0$

Using Fig. 7.7 for $t_{20}/t_{60} = 0.47$

$\zeta = 0.65$, $t_{60}/\tau = 1.75$, and $\tau = 5.14$

$$G(s) \approx \frac{2}{26.4s^2 + 6.68s + 1}$$

The models are compared in Fig. S7.4b:

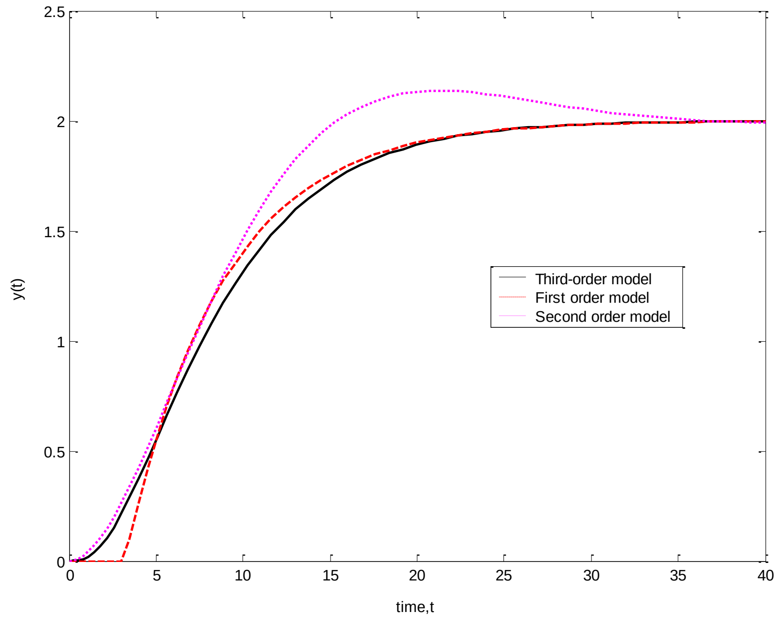


Figure S7.4b *Comparison of three models for a step input*

7.5

For a first-order plus time-delay model $G = \dfrac{1}{\tau s+1}e^{-\theta s}$, assume $\tau = 1; \theta = 0.1, 1, 10$, we have:

a) $\theta / \tau = 0.1$:

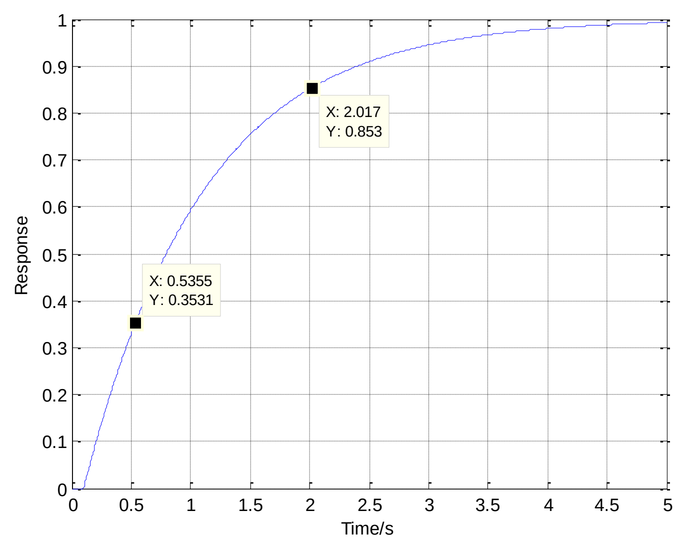


Figure S7.5a *Plot of the true data;* $\theta / \tau = 0.1$

So $t_1 = 0.5355; t_2 = 2.017;$

$\therefore \theta = 1.3t_1 - 0.29t_2 = 0.1112$

$\tau = 0.67\left(t_2 - t_1\right) = 0.9926$

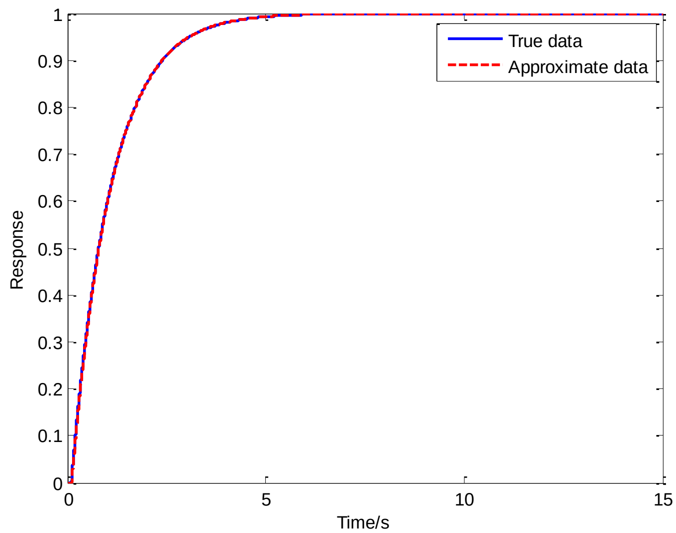


Figure S7.5b *Comparison of true data and approximate model*

Sum of squared error = 0.0232

b) $\theta / \tau = 1$:

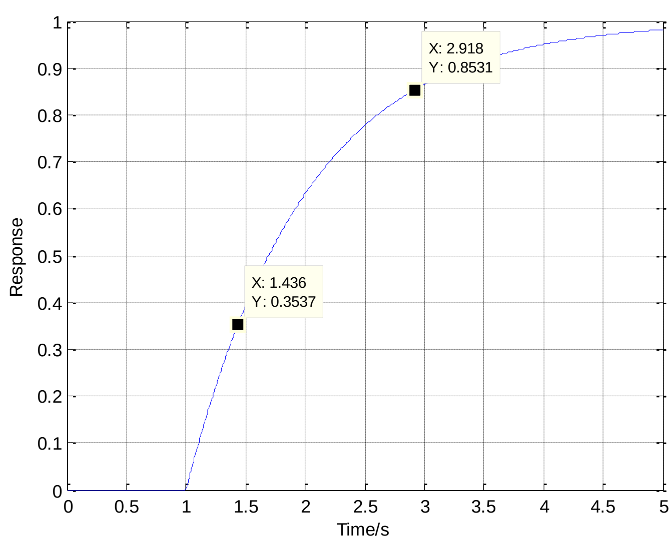


Figure S7.5c *Plot of the true data;* $\theta / \tau = 1$

So $t_1 = 1.436; t_2 = 2.918;$

$\therefore \theta = 1.3t_1 - 0.29t_2 = 1.021$

$\tau = 0.67\left(t_2 - t_1\right) = 0.9929$

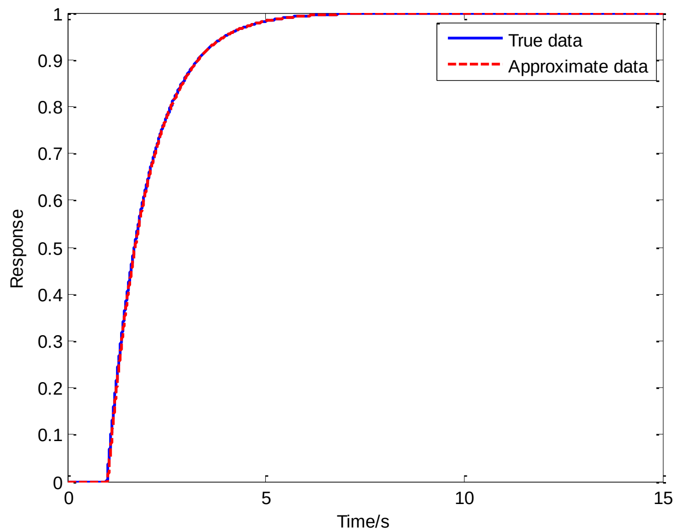


Figure S7.5d *Comparison of true data and approximate model*

Sum of squared error = 0.1050

c) $\theta / \tau = 10$:

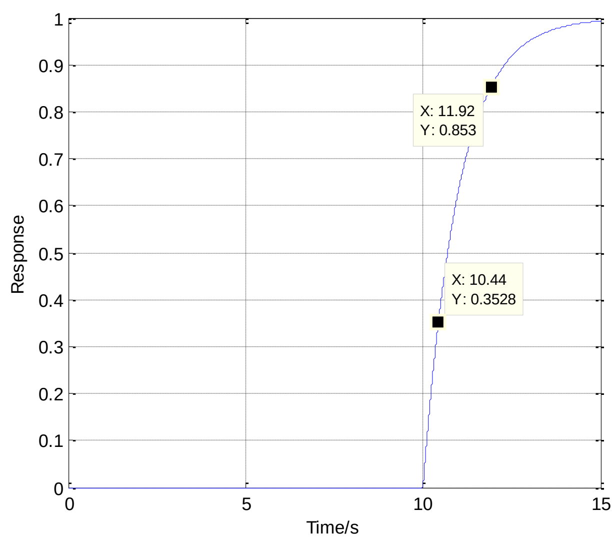


Figure S7.5e *Plot of the true data;* $\theta / \tau = 10$

So $t_1 = 10.44; t_2 = 11.92;$

$\therefore \theta = 1.3t_1 - 0.29t_2 = 10.12$

$\tau = 0.67(t_2 - t_1) = 0.9916$

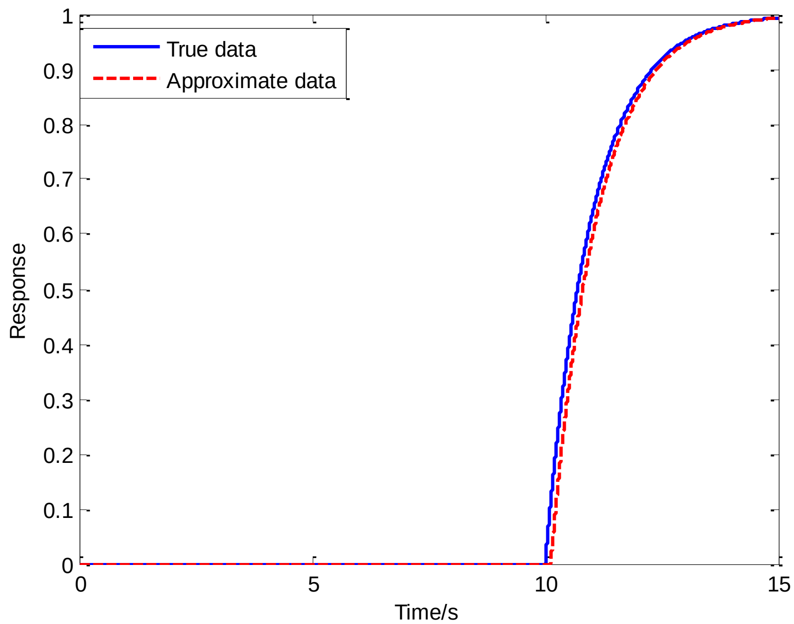


Figure S7.5f *Comparison of true data and approximate model*

Sum of squared error = 4.3070

7.6

a) Drawing a tangent at the inflection point which is roughly at $t \approx 5$, the intersection with $y(t) = 0$ line is at $t \approx 1$ and with the $y(t)=1$ line at $t \approx 14$. Hence $\theta = 1$ and $\tau = 14-1=13$

$$G_1(s) \approx \frac{e^{-s}}{13s+1}$$

b) Smith's method

From the plot, $t_{20} = 3.9$, $t_{60} = 9.6$; using Fig 7.7 for $t_{20}/t_{60} = 0.41$

$\zeta = 1.0$, $t_{60}/\tau = 2.0$, hence $\tau = 4.8$ and $\tau_1 = \tau_2 = \tau = 4.8$

$$G(s) \approx \frac{1}{(4.8s+1)^2}$$

Nonlinear regression

From Figure E7.5, we obtain these values (approximate):

Table *Output values from Figure E7.5*

Time	Output
0.0	0.0
2.0	0.1
4.0	0.2
5.0	0.3
7.0	0.4
8.0	0.5
9.0	0.6
11.0	0.7
14.0	0.8
17.5	0.9
30.0	1.0

For the step response of Eq. 5-48, the time constants were calculated so as to minimize the sum of the squares of the errors between data and model predictions. Use Excel Solver for this Optimization problem:

τ_1 =6.76 min and τ_2 = 6.95 min

$$G(s) \approx \frac{1}{(6.95s+1)(6.76s+1))}$$

The models are compared in Fig.S7.6:

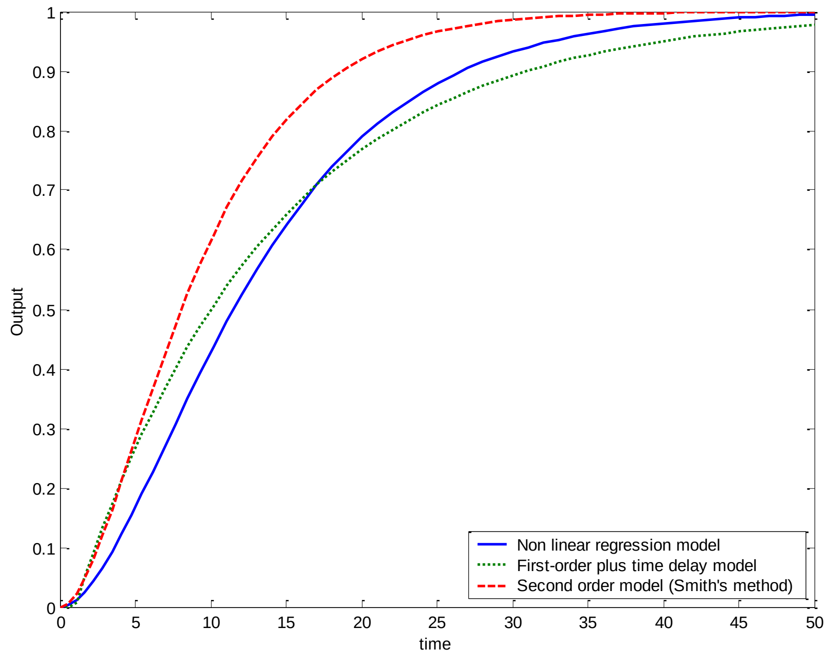


Figure S7.6 *Comparison of three models for unit step input*

7.7

a) From the plot, time delay θ = 4.0 min

Using Smith's method,

from the graph, $t_{20} + \theta \approx 5.6$, $t_{60} + \theta \approx 9.1$

$t_{20} = 1.6$, $t_{60} = 5.1$, $t_{20}/t_{60} = 1.6/5.1 = 0.314$

From Fig.7.7 , $\zeta = 1.63$, $t_{60}/\tau = 3.10$, $\tau = 1.645$

Using Eqs. 5-45 and 5-46, $\tau_1 = 4.81$ min , $\tau_2 = 0.56$ min

b) Overall transfer function

$$G(s) = \frac{10e^{-4s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad , \quad \tau_1 > \tau_2$$

Assuming plug-flow in the pipe with constant-velocity,

$$G_{pipe}(s) = e^{-\theta_p s} \quad , \quad \theta_p = \frac{3}{0.5} \times \frac{1}{60} = 0.1\min$$

Assuming that the thermocouple has unit gain and no time delay

$$G_{TC}(s) = \frac{1}{\tau_2 s + 1} \quad \text{since} \quad \tau_2 << \tau_1$$

Then

$$G_{HE}(s) = \frac{10e^{-3s}}{\tau_1 s + 1} \quad ,$$

so that,

$$G(s) = G_{HE}(s) G_{pipe}(s) G_{TC}(s) = \left(\frac{10e^{-3s}}{\tau_1 s + 1}\right)(e^{-0.1s})\left(\frac{1}{\tau_2 s + 1}\right)$$

7.8

(a) 63% response method

From inspection of the data, it is obvious that there is no time delay in the system (θ=0).

Time constant τ is estimated by the 63% response method:

$$h'(\tau) = 0.63\left(h'_{ss} - h'_0\right) = 0.63 * 20.3 = 12.78\,ft$$
$$h(\tau) = 12.78 + 10.4 = 23.18\,ft$$

From inspection at the data, $\tau \approx 270$ min.

The process gain is calculated as:

$$K = \frac{h'_{ss} - h'_0}{q - q_0} = \frac{20.3 - 0}{(4.8 - 1.5) \times 0.1337} = \frac{20.3}{0.4412} = 46 \text{ min}/\ ft^2$$

The estimated process model is:

$$\frac{H'(s)}{Q'(s)} = \frac{K}{\tau s + 1} = \frac{46}{2.70s + 1}$$

(b) Nonlinear regression

By using deviation variables, the first order tank can be expressed as

$$\frac{H^{'}(s)}{Q^{'}(s)} = \frac{K}{\tau s+1}$$

The inlet flow rate is quickly changed from 1.5 gallon/min to 4.8 gallon/min so it is a step change, Q'(s) 3.3/s:

$$H^{'}(s) = \frac{K}{\tau s+1}Q^{'}(s) = \frac{K}{\tau s+1}\frac{(4.8-1.5)\times 0.1337}{s} = \frac{K}{\tau s+1}\frac{0.4412}{s}$$

Apply the inverse Laplace transform:

$$h^{'}(t) = 0.4412K\left(1-e^{-t/\tau}\right)$$

By using EXCEL, the estimated model is:

$$\frac{H^{'}(s)}{Q^{'}(s)} = \frac{K}{\tau s+1} = \frac{46.31}{2.65s+1}$$

A comparison of the data and the two models is shown in Fig.S7.8.

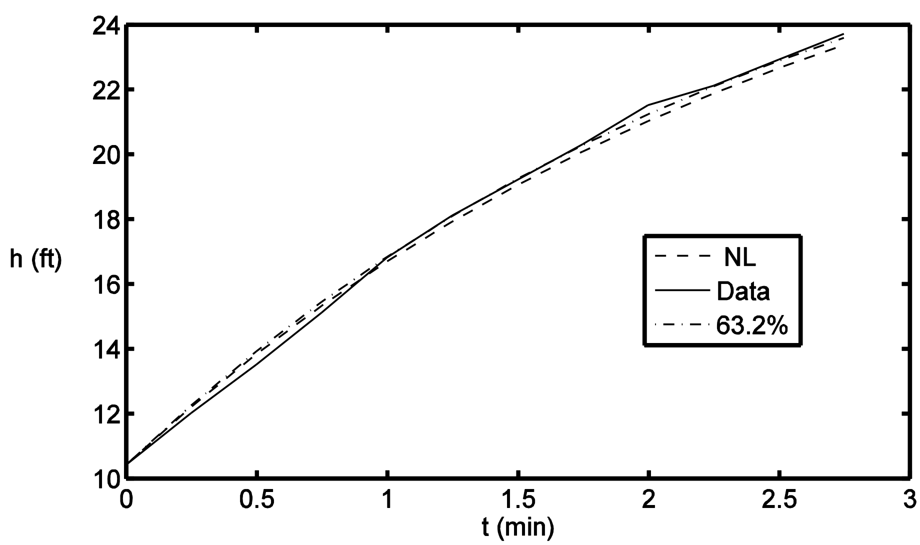


Figure S7.8 *A comparison of the step responses of the data and the two models.*

The Sum of Squared Errors for the two models are:

SSE (63.2%) = 0.75

SSE (NR) = 0.43

As indicated in Fig.S7.8, both methods fit the data well. The NR model is preferred due to its smaller SSE value.

7.9

$$K = \frac{\Delta y}{\Delta u} = \frac{3-0}{5-1} = 0.75$$

$\theta = 2$ (by inspection)

Use Smith's method to find τ_1 and τ_2.

$$y_{20} = y(0) + (0.2)(\Delta y) = 0 + (0.2)(3) = 0.6$$

From inspection of the data,

$$t_{20} = 4 - \theta = 2$$

Similarly,

$$y_{60} = y(0) + (0.6)(\Delta y) = 0 + (0.6)(3) = 1.8$$

$$t_{60} = 7 - \theta = 5$$

Therefore,

$$\frac{t_{20}}{t_{60}} = \frac{2}{5} = 0.4$$

From Fig. 7.7:

$$\zeta = 1.2$$

and

$$\frac{t_{60}}{\tau} = 2.1 \quad \Rightarrow \quad \tau = \frac{t_{60}}{2.1} = \frac{5}{2.1} = 2.38$$

Thus the transfer function can be written as:

$$G(s) = \frac{0.75e^{-2s}}{5.66s^2 + 5.71s + 1}$$

From (5-45) and (5-46) or by factoring (e.g., using MATLAB command *roots*) gives:

$$G(s) = \frac{0.75e^{-2s}}{(4.44s+1)(1.28s+1)}$$

7.10

Assume that $T(\infty) = T(13) = 890$ °C. The steady-state gain K is the change in output divided by the change in input:

$$K = \frac{890 - 850}{950 - 1000} = -0.8 \text{ °C/cfm}$$

Assume that the input change in air flow rate is made at $t = 2^+$ min so that the observed input first changes at $t = 3$ min ; the output first changes at $t = 5$ min. This means that the time delay is two sampling periods, i.e., $\theta = 2$ min. Why is θ=2 min, rather than θ=3 min? To understand this point, first consider a process with no time delay (θ=0). For a step change at $t = 2^+$ min, the first observed changes in the input and the output of this undelayed would occur at $t = 3$ min, because the output cannot change simultaneously due to the process dynamics. But for our process, the first changes are observed at $t = 5$ min which implies that $\theta = 2$ min.

Time constant τ can be obtained from the 63.2% response time:

$$T_{63.2\%} = 850 \text{ °C} + (890 - 850 \text{ °C})(0.632) = 875.3 \text{ °C}$$

Interpolating between $t = 7$ min and $t = 8$ min gives

$$t_{63.2\%} = \left(\frac{t(8) - t(7)}{T(8) - T(7)}\right)(T_{63.2\%} - T(7)) + t(7)$$

$$= \left(\frac{8 - 7}{878 - 873}\right)(875.3 - 873) + 7$$

$$= 7.46 \text{ min}$$

Then

$$t_{63.2\%} = \tau + \theta + t(0)$$

where $t(0)$=3, the time when the input first changes. Thus

$$\tau = t_{63.2\%} - \theta - t(0) = 7.46 - 1 - 3$$

$$\tau = 3.46$$

So the FOPTD model of the process is

$$G(s) = \frac{-0.8e^{-2s}}{3.46s+1}$$

7.11

(a) For a SOPTD model shown in below,

$$G(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}e^{-\theta s}$$

Based on visual inspection on the figure, it is an underdamped process, using Equation (5-51) we have gain

$$K = 1, \theta = 2, OS = \frac{1.5-1}{1-0} = 0.5 \Rightarrow \zeta = \sqrt{\frac{\left[\ln(OS)\right]^2}{\pi^2 + \left[\ln(OS)\right]^2}} = 0.1572 \approx 0.16 \quad ,$$

$$t_p = 5.5 - \theta = 5.5 - 2 = 3.5s$$

Based on Equation (5-50):

$$\tau = \frac{t_p\sqrt{1-\zeta^2}}{\pi} = \frac{5.5 * \sqrt{1-0.1572^2}}{\pi} = 1.7289 \approx 1.73$$

$$G(s) = \frac{1}{1.73^2 s + 2*0.16*1.73s + 1}e^{-2s} = \frac{1}{3s^2 + 0.55s + 1}e^{-2s}$$

An alternative method is to use the Smith's Method shown in Figure 7.7:

$$\theta = 2, K = 1, t_{20} = 2.6, t_{60} = 3.1$$

The adjusted times are employed for the actual graphical analysis:

$$t_{20}^{'} = t_{20} - \theta = 2.6 - 2 = 0.6$$

$$t_{60}^{'} = t_{60} - \theta = 3.1 - 2 = 1.1$$

$$\frac{t_{20}^{'}}{t_{60}^{'}} = 0.54$$

Based on Figure 7.7, we have $\zeta = 0.14, \frac{t_{60}}{\tau} = 1.3 \Rightarrow \tau = \frac{3.1}{1.3} = 2.38$

$$G(s) = \frac{1}{2.38^2 s + 2*0.14*2.38s + 1}e^{-2s} = \frac{1}{5.67s^2 + 0.67s + 1}e^{-2s}$$

(b) Because a damped oscillation occurs, this matches the features of SOPTD. FOPTD method does not allow for oscillation.

7.12

(a) For a FOPTD model shown in below:

$$G(s)=\frac{Ke^{-\theta s}}{\tau s+1}$$

Based on visual inspection, the gain $K=2$; $\theta=2.5$; when the response reaches 63.2% complete, i.e., 2*0.632=1.264, $\tau=t_{63.2\%}=5s$

(b)

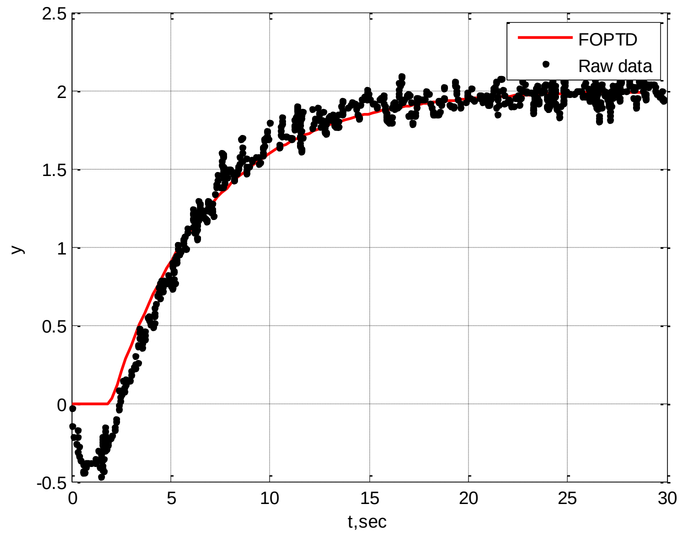


Figure S7.12 *The response of the derived FOPTD model*

(c) The inverse response at the initial state is caused by a right-half plan zero and is not captured by FOPTD model.

7.13

a) Replacing τ by 5, and *K* by 6 in Eq. 7-25

$$y(k)=e^{-\Delta t/5}y(k-1)+[1-e^{-\Delta t/5}]6u(k-1)$$

b) Replacing τ by 5, and *K* by 6 in Eq. 7-22

$$y(k) = (1 - \frac{\Delta t}{5}) y(k-1) + \frac{\Delta t}{5} 6u(k-1)$$

In the integrated results tabulated below for Δt = 0.1, the values are shown only at integer values of t, for comparison.

Table S7.13 *Integrated results for the first order differential equation*

t	***y*(*k*)** (exact)	***y*(*k*)** (Δt=1)	***y*(*k*)** (Δt=0.1)
0	3	3	3
1	2.456	2.400	2.451
2	5.274	5.520	5.296
3	6.493	6.816	6.522
4	6.404	6.653	6.427
5	5.243	5.322	5.251
6	4.293	4.258	4.290
7	3.514	3.408	3.505
8	2.877	2.725	2.864
9	2.356	2.180	2.340
10	1.929	1.744	1.912

Thus Δt = 0.1 does improve the finite difference model making it a more accurate approximation of the exact model.

7.14

To find a_1 and b_1, use the given first order model to minimize

$$J = \sum_{n=1}^{10} (y(k) - a_1 y(k-1) - b_1 x(k-1))^2$$

where y(k) denotes the data.

$$\frac{\partial J}{\partial a_1'} = \sum_{n=1}^{10} 2(y(k) - a_1 y(k-1) - b_1 x(k-1))(-y(k-1) = 0$$

$$\frac{\partial J}{\partial b_1} = \sum_{n=1}^{10} 2(y(k) - a_1 y(k-1) - b_1 x(k-1))(-x(k-1)) = 0$$

Solving simultaneously for a_1 and b_1 gives

$$a_1 = \frac{\sum_{n=1}^{10} y(k)y(k-1) - b_1 \sum_{n=1}^{10} y(k-1)x(k-1)}{\sum_{n=1}^{10} y(k-1)^2}$$

$$b_1 = \frac{\sum_{n=1}^{10} x(k-1)y(k) \sum_{n=1}^{10} y(k-1)^2 - \sum_{n=1}^{10} y(k-1)x(k-1) \sum_{n=1}^{10} y(k-1)y(k)}{\sum_{n=1}^{10} x(k-1)^2 \sum_{n=1}^{10} y(k-1)^2 - \left(\sum_{n=1}^{10} y(k-1)x(k-1)\right)^2}$$

Using the given data,

$$\sum_{n=1}^{10} x(k-1)y(k) = 35.212 \quad , \quad \sum_{n=1}^{10} y(k-1)y(k) = 188.749$$

$$\sum_{n=1}^{10} x(k-1)^2 = 14 \quad , \quad \sum_{n=1}^{10} y(k-1)^2 = 198.112$$

$$\sum_{n=1}^{10} y(k-1)x(k-1) = 24.409$$

Substituting into expressions for a_1' and b_1 gives

$a_1 = 0.8187$, $b_1 = 1.0876$

The fitted model is $y(k+1) = 0.8187y(k) + 1.0876x(k)$

or $$y(k) = 0.8187y(k-1) + 1.0876x(k-1) \qquad (1)$$

Let the first-order continuous transfer function be,

$$\frac{Y(s)}{X(s)} = \frac{K}{\tau s + 1}$$

For Eq. 7-34, the discrete model is

$$y(k) = e^{-\Delta t/\tau} y(k-1) + [1 - e^{-\Delta t/\tau}]Kx(k-1) \qquad (2)$$

Comparing Eqs. 1 and 2, for Δt=1, gives

$\tau = 5$ s and $K = 6$ volts

Hence, the continuous transfer function is

$$G(s) = \frac{6}{5s+1}$$

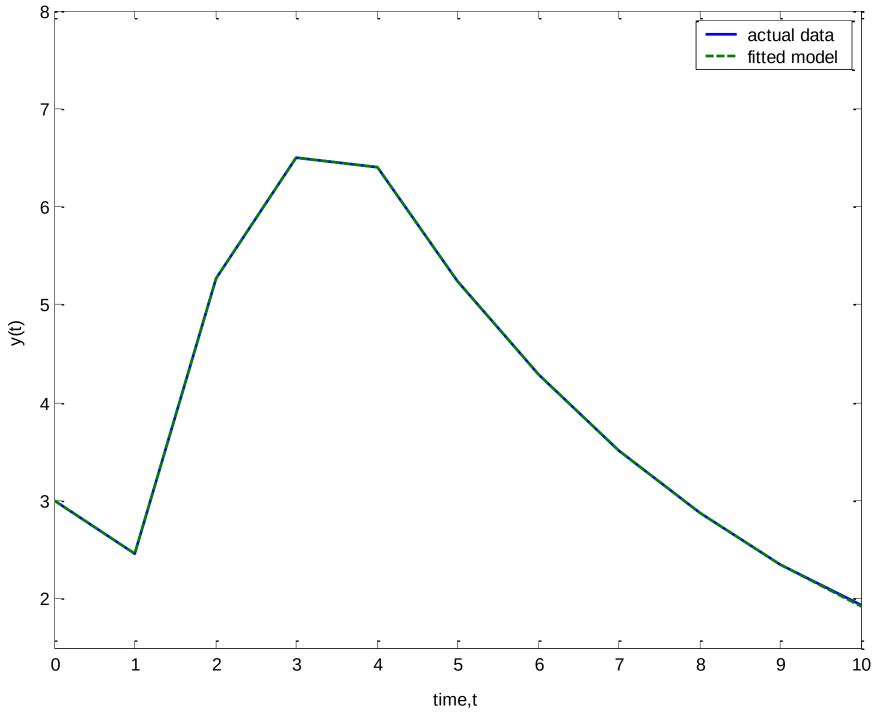


Figure S7.14 *Responses of the fitted model and the data*

7.15

a) FOPTD model:

Since K=1, using linear interpolation to find times corresponding to the 35.3% and 85.3% of response:

$$t_{35.3\%} = 2.89; t_{85.3\%} = 8.66$$

$$\therefore \theta = 1.3t_{35.3\%} - 0.29t_{85.3\%} = 1.24$$

$$\tau = 0.67\left(t_{85.3\%} - t_{35.3\%}\right) = 3.87$$

b) Discrete-time ARX model:

$y(k) = 0.911y(k-1) + 0.1329u(k-1)$ (since $u(k) = u(k-1) = 1$)

Thus:

$e^{-1/\tau} = 0.911; K\left(1 - e^{-1/\tau}\right) = 0.1329 \Rightarrow K = 1.49$

An alternative way to calculate K is to set $y(k) = y(k-1) = y_{ss}, u(k-1) = u_{ss} = 1$

$$y_{ss} = 0.911 y_{ss} + 0.1329 \Rightarrow K = \frac{0.1329}{0.089} = 1.49$$

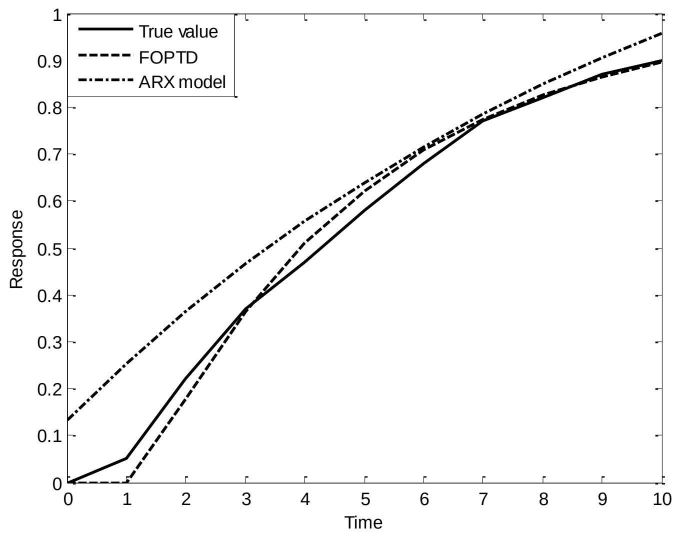


Figure S7.15. *Comparison of true data and model responses.*

The result obtained using the ARX model is different from that obtained using an FOPTD model, because the extra constraint "K=1" is not used. In other words, the discrete time data do not include the final steady-state value, so the calculation gives a different gain. If more data points are added on steady-state values, the result obtained using ARX model will converge to K=1.

7.16

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2) \qquad (1)$$

a) For the model in (1), the least squares parameter estimates are given by

$$\hat{\boldsymbol{\beta}} = \left(\boldsymbol{X}^T \boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \boldsymbol{Y} \qquad (2)$$

For the *basal1* dataset:

$$\hat{\boldsymbol{\beta}} = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix}, \quad \boldsymbol{X} = \begin{bmatrix} 4 & 0 & 2.5 & 0 \\ -4 & 4 & 2.5 & 2.5 \\ \vdots & \vdots & \vdots & \vdots \\ -213 & -211 & 2.5 & 2.5 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{Y} = \begin{bmatrix} -4 \\ -11 \\ \vdots \\ -214 \end{bmatrix}$$

Calculate parameter estimates using (2):

$$\boxed{\hat{\boldsymbol{\beta}} = \begin{bmatrix} 1.29 & -0.31 & -3.67 & 1.26 \end{bmatrix}^T}.$$

Next, we generate model predictions for the calibration data (dataset *basal1*) using past inputs and past model predictions, but **not past output data**. Figure S7.16a compares the calibration data and the model predictions, where $\Delta y = y - y(0)$. Metric S denotes the corresponding sum of squared errors,

$$S = \sum_{1}^{N} \left[y(k) - \hat{y}(k) \right]^2 \tag{2}$$

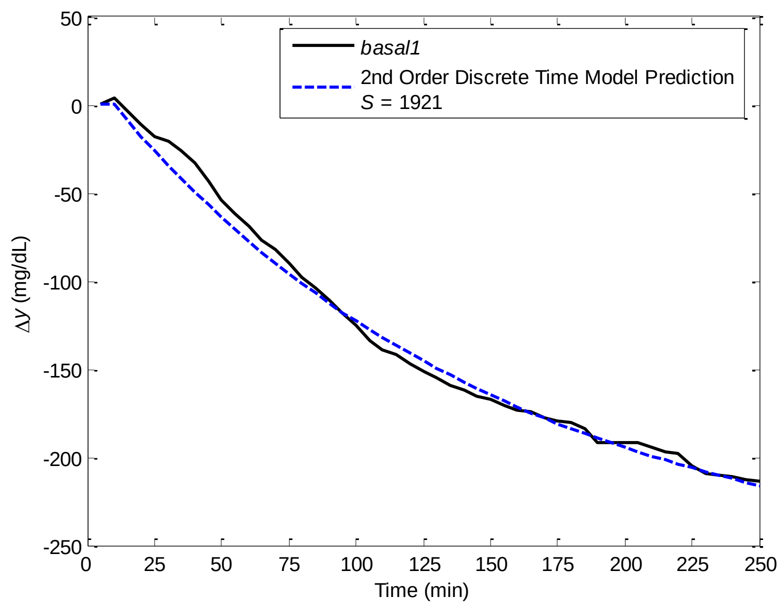


Figure S7.16a *Comparison of model predictions and calibration data for the 2nd order discrete-time model (S is the sum of squared errors in Eq. 2).*

b) The comparison of the validation data (dataset *basal2*) and the corresponding model predictions is shown in Figure S7.16b.

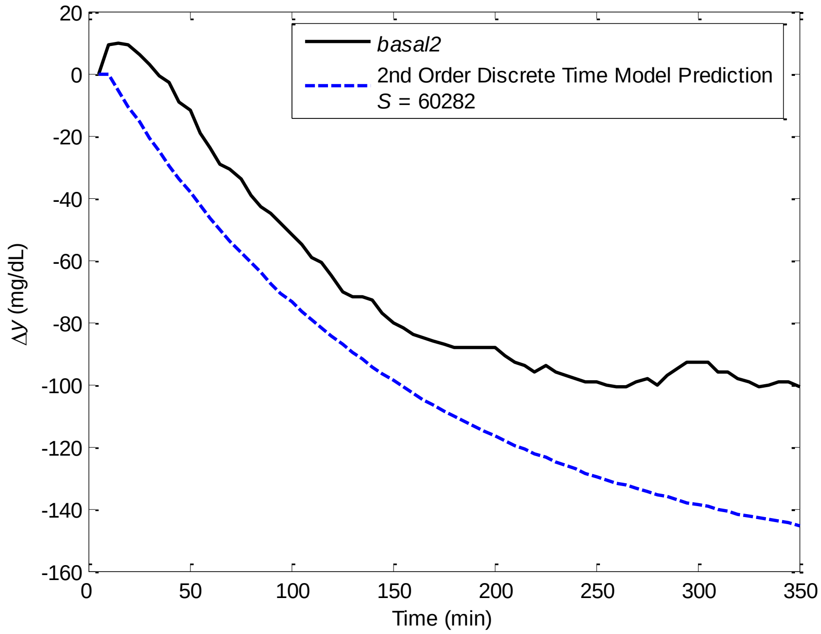


Figure S7.16b *Comparison of model predictions and validation data for the 2nd order discrete time model (S is the sum of squared errors in Eq. 2).*

b) Now, consider the first-order transfer function model

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \tag{3}$$

First, determine the steady-state gain, $K = \Delta y/\Delta u$. The output finally reaches a new steady state of about 250 mg/dL. For dataset *basal1*, the input change is Δu=2.5 units/day. Thus,

$$K = -\frac{250}{2.5} = -100 \quad \frac{\text{mg day}}{\text{dL units}}$$

To identify time constant τ, determine the time at which 63.2% of the total change has occurred. This corresponds to the time at which the output, Δy has a value of - 250 $\times$63.2% = - 158. For inspection of the data, τ = 134 min when Δy= 158 mg/dL.

The model predictions for the model in (3) can be calculated from the step response for a first-order transfer function in (5-18)

$$y(t) = KM(1 - e^{-t/\tau}) \tag{5-18)}$$

The input step sizes are M = 2.5 units/day for *basal*1 and 1.5 units/day for *basal2*.

Figures S7.16c and S7.16d show the model predictions for the calibration and validation data, respectively.

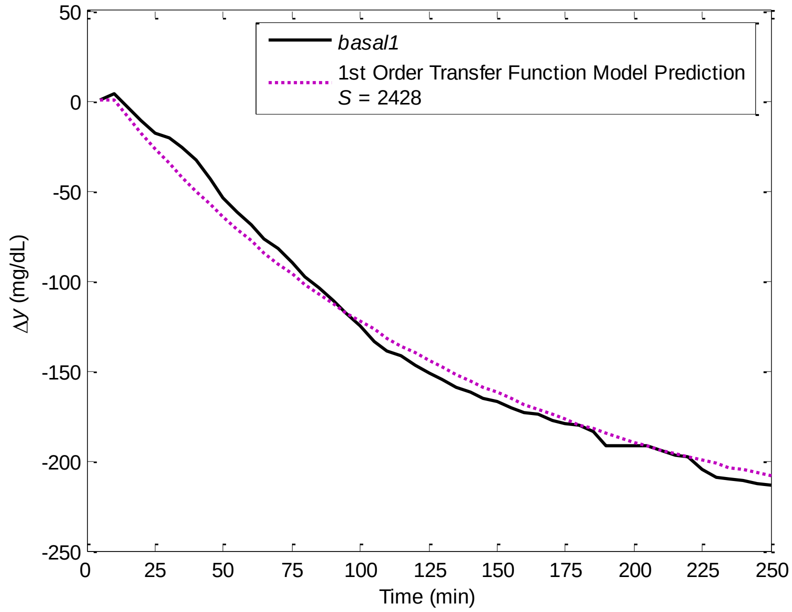


Figure S7.16c *Comparison of the model predictions and calibration data for the 1st order transfer function (S is the sum of squared errors in Eq. 2).*

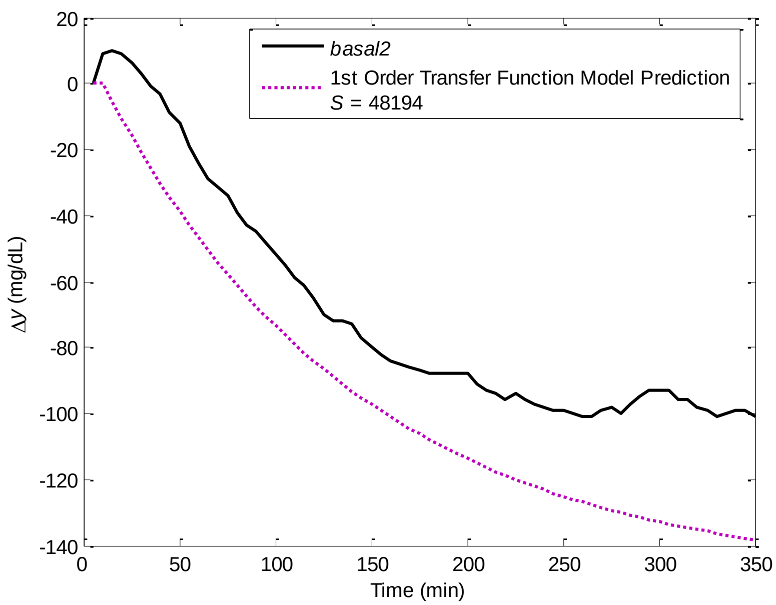


Figure S7.16d *Comparison of the model predictions and validation data for the 1st order transfer function (S is the sum of squared errors in Eq. 2).*

c) **Discussion of results**

Table S7.16 lists the calculated values of *S* for the two models.

Table S7.16. Average squared error for the model predictions.

	Modelh	
	Discrete-time	**Transfer function**
Calibration data (*basal1*)	1921	2428
Validation data (*basal2*)	60,282	48,194

The discrete-time model is more accurate than the transfer function model for the calibration data, which is not surprising because the former has more model parameters. Although, the transfer function model is more accurate for the validation dataset, neither model is very accurate for this dataset.

7.17

a) Fit a first-order model:

Let y = hydrocarbon exit temperature, T_{HC}
u = air flow rate, F_A

Note: There is a typo in the 1st printing. The step change in u should start at 17.9 m^3/min, not 17.0 m^3/min.

The step response data is shown in Fig. S7.17a. The step change in u from 17.9 to 21 m^3/min occurs at t = 14 min. By inspection of the noisy y data, the time delay is approximately $\theta = 4$ min.

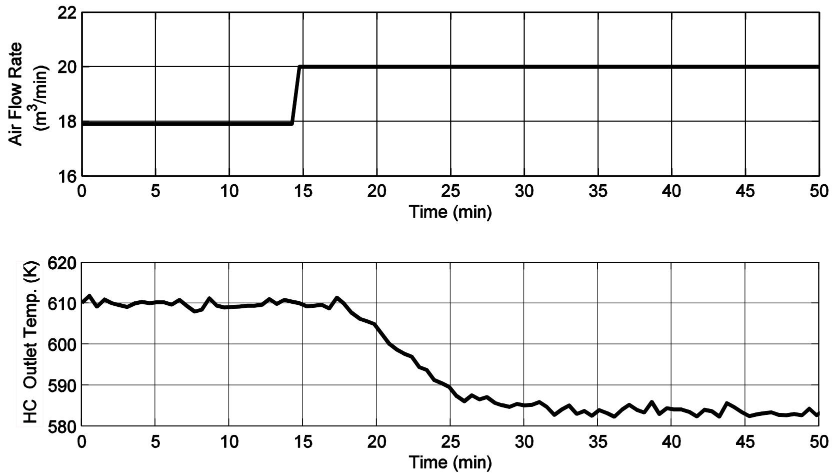


Figure S7.17a *Step response data for the furnace module.*

From the step response data, the following information can be obtained:

$$K = \frac{\Delta y}{\Delta u} = -\frac{25\,\text{K}}{2.1\ \text{m}^3/\text{min}} = -11.9\,\frac{\text{K}}{\text{m}^3/\text{min}}$$

$y(0) = 609.5$ K, $\quad y(\infty) = 584.5$ K; $\;$ thus $\Delta y = 609.5 - 584.5 = -25$ K

$y_{63.2} = 609.5 + (0.632)(-25) = 593.7$ K

From the figure, $t_{63.2} = 19.5 - 14 - 4 = 5.5$ min. Thus, the transfer function model is:

$$\frac{Y(s)}{U(s)} = \frac{-11.9}{5.5s+1}$$

b) Fit a second-order model:

Use Smith's method to find τ_1 and τ_2.

$$y_{20} = y(0) + (0.2)(\Delta y) = 609.5 + (0.2)(-25) = 604.5 \text{ K}$$

$$y_{60} = y(0) + (0.6)\,(\Delta y) = 609.5 + (0.6)(-25) = 594.6 \text{ K}$$

From inspection of the data,

$$t_{20} \approx 19.5 - 10 = 1.5 \text{ min}$$

Similarly,

$$t_{60} = 23 - 18 = 5 \text{ min}$$

Therefore,

$$\frac{t_{20}}{t_{60}} = \frac{1.5}{5} = 0.3$$

From Fig. 7.7:

$$\zeta = 2.2$$

and

$$\frac{t_{60}}{\tau} = 4 \quad \Rightarrow \quad \tau = \frac{t_{60}}{4} = \frac{5}{4} = 1.25\,\text{min}$$

Thus the second-order transfer function can be written in standard form as:

$$\frac{T_{HC}(s)}{F_A(s)} = \frac{1.5e^{-2s}}{5.66\,s^2 + 5.71\,s + 1}$$

$$\frac{T_{HC}(s)}{F_A(s)} = \frac{-11.9e^{-4s}}{(1.25)^2\, s^2 + 2(2.2)(1.25)s + 1}$$

From (5-45) and (5-46) or by factoring (e.g., using MATLAB command *roots*) gives:

$$\frac{T_{HC}}{F_A} = \frac{-11.9e^{-4s}}{(5.2\,s + 1)\,(0.3\,s + 1)}$$

c) Simulations

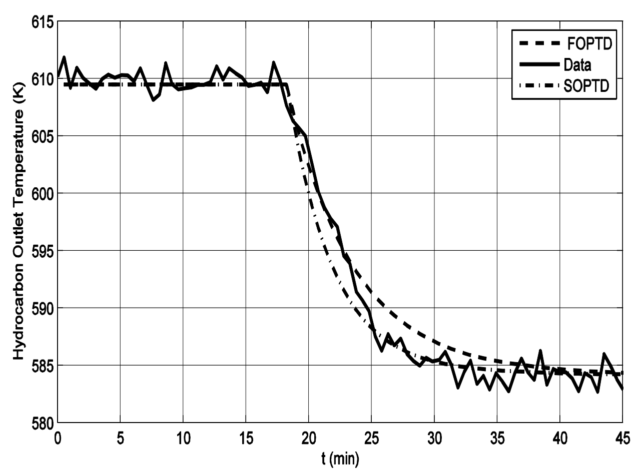


Figure S7.17b *Comparison of furnace step response data and model responses.*

d) Discussion

The model comparisons in Fig. S7.17b indicate that the two models are very similar and reasonably accurate. However, the low-order transfer function models fail to capture the higher order dynamics of the physical furnace model that was used to generate the step response data. The first-order model has a lower value of the least squares index, S:

First order model: $S = 1.71 \times 10^4$
Second-order model: $S = 2.01 \times 10^4$

7.18

(a) Fit a FOPTD model to the column step response data:

Let y = distillate MeOH composition, x_D
u = reflux ratio, R

The step response data is shown in Fig. S7.18a with the step change in u from 1.75 to 2.0 occurring at t = 3950 s. By inspection of the noisy data, the time delay is $\theta \approx 50$ s.

The following information can be obtained from the step response data:

$$x_D(0) = 0.85, \qquad x_D(\infty) = 0.88;$$

thus

$$\Delta x_D = 0.88 - 0.85 = 0.03$$

and

$$K = \frac{\Delta y}{\Delta u} = \frac{0.03}{0.25} = 0.12$$

Also,

$$y_{63.2} = 0.85 + (0.632)(0.03) = 0.869$$

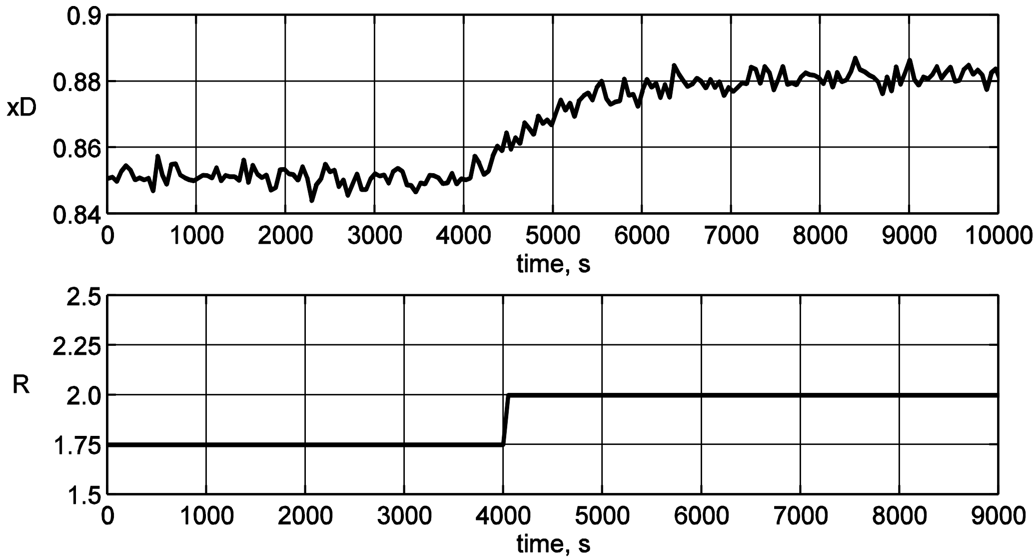


Figure S7.18a *Step response data for the column module.*

From the figure, $\tau = t_{63.2} - t(0) = 5050 - 3950 - 50 \approx 1050$ s. Thus, one estimate of the time constant is τ = 1050 s. A second estimate can be obtained from the settling time, $t_s \approx 7600 - 3950 = 3650$ s. Thus, $\tau \approx t_{s/4} = 912$ s. Averaging these two estimates gives:

$$\tau_{ave} = \frac{1050 + 912}{2} = 981 \text{ s}$$

Thus the identified transfer function is,

$$\frac{x_D(s)}{R(s)} = \frac{0.12\, e^{-50s}}{981s + 1}$$

(b) SOPTD model:

Use Smith's method:

y_{20} = 0.85 + (0.2)(0.03) = 0.856

y_{60} = 0.85 + (0.6)(0.03) = 0.868

From the step response data:

$t_{20} \approx 4280 - 3950 - 50 = 280$ s

$t_{60} \approx 4900 - 3950 - 50 = 900$ s

Thus,

$$\frac{t_{20}}{t_{60}} = \frac{280}{900} = 0.31$$

From Fig. 7.7:

$$\frac{t_{60}}{\tau} = 4 \quad \Rightarrow \quad \tau = \frac{t_{60}}{4} = 225 \text{ s}$$

and

$$\zeta = 2.0$$

The SOPTD model can be written as:

$$\frac{x_D(s)}{R(s)} = \frac{Ke^{-\theta s}}{\tau^2 s^2 + 2\zeta t s + 1} = \frac{0.12\, e^{-50s}}{4\, x 10^4\, s^2 + 800s + 1}$$

which can be factored using (5-45) and (5-46):

$$\frac{x_D(s)}{R(s)} == \frac{0.12\, e^{-50s}}{(769s+1)(54s+1)}$$

c) Simulations

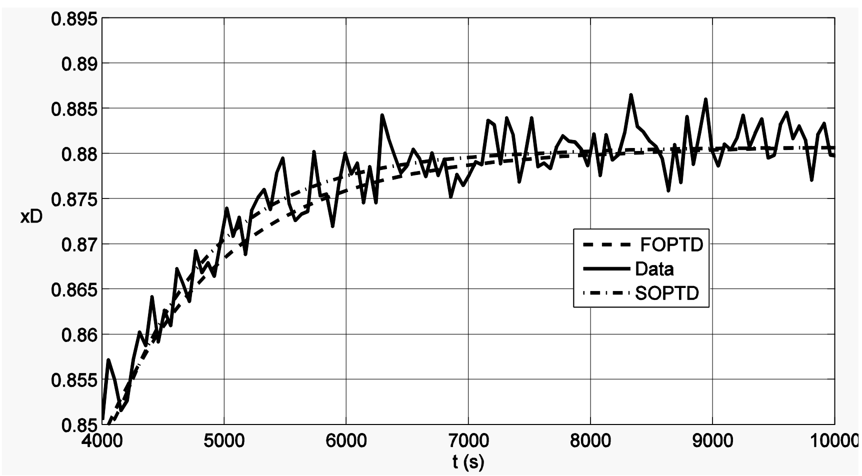


Figure 7.18b *Comparison of column step response data and model responses*.

d) Discussion

The model comparisons in Fig. S7.18b indicate that both models are reasonably accurate. However, the second-order model is more accurate as indicated visually and by its slightly lower *S* value:

First order model: $S = 9.014 \times 10^{-4}$

Second-order model: $S = 9.012 \times 10^{-4}$

Chapter 8 ©

Many of the problems in this chapter require determining whether a controller should be direct-acting or reverse-acting. The following chart can help guide the thinking process for these problems when also considering the style of the valve. Note that the chart assumes all unmentioned gains are positive (measurement, I/P, etc.).

Table S8.1: Chart for determining if controller should be direct-acting or reverse-acting.

<table>
<tr><th>If the process gain is:</th><th>And the valve is:</th><th>Then the controller should be:</th></tr>
<tr><td rowspan="2">Positive
MV ↑, CV ↑
K_p +

For control, if CV ↑,
want MV ↓</td><td>Fail Close (Air-to-Open)
K_v +
For MV ↓, want p ↓</td><td>Reverse-Acting
Kc +
For CV ↑, want p ↓</td></tr>
<tr><td>Fail Open (Air-to-Close)
K_v –
For MV ↓, want p ↑</td><td>Direct Acting
Kc –
For CV ↑, want p ↑</td></tr>
<tr><td rowspan="2">Negative
MV ↑, CV ↓
K_p –

For control, if CV ↑,
want MV ↑</td><td>Fail Close (Air-to-Open)
K_v +
For MV ↑, want p ↑</td><td>Direct Acting
Kc –
For CV ↑, want p ↑</td></tr>
<tr><td>Fail Open (Air-to-Close)
K_v –
For MV ↑, want p ↓</td><td>Reverse-Acting
Kc +
For CV ↑, want p ↓</td></tr>
</table>

8.1

The response of a PI controller to a unit step change in set point at t = 0 is shown in Fig. 8.6. The instantaneous change at t = 0 is K_c and the slope of the response is K_c/τ_I. Now consider a more general step change in the set point of magnitude M.

$$\frac{P(s)}{E(s)} = K_c(1+\frac{1}{\tau_I s}) = K_c + \frac{K_c}{\tau_I s}$$

$$E(s) = \frac{M}{s}$$

$$P(s) = \frac{K_c M}{s} + \frac{K_c M}{\tau_I s^2}$$

$$p(t) = K_c M + \frac{K_c M}{\tau_I} t \text{ for } t \geq 0$$

The instantaneous change at t = 0 is K_cM and the slope of the response is K_cM/τ_I From the data given in the table, the initial instantaneous change is -1.3 mA and the slope is -0.0335 mA/s for a step change of M = 2.5 mA. Thus,

$$K_C = \frac{-1.3 \text{ mA}}{M} = \frac{-1.3\text{mA}}{2.5\text{mA}} = -0.52$$

$$\frac{K_C M}{\tau_I} = -0.0335 \text{ mA/s}$$

$$\tau_I = \frac{K_C M}{-0.0335 \text{ mA/s}} = \frac{-0.52(2.5\text{mA})}{-0.05 \text{ mA/s}} = 26 \text{ s}$$

Because K_C is negative, we classify this controller as **direct acting**.

8.2

a) $$\frac{P'(s)}{E(s)} = \frac{K_1}{\tau_1 s+1} + K_2 = \frac{K_1 + K_2\tau_1 s + K_2}{\tau_1 s+1} = (K_1+K_2)\left[\frac{\frac{K_2\tau_1}{K_1+K_2}s+1}{\tau_1 s+1}\right]$$

b) $K_c = K_1 + K_2 \quad \rightarrow \quad K_2 = K_c - K_1$

$$\tau_1 = \alpha\tau_D$$

$$\tau_D = \frac{K_2\tau_1}{K_1+K_2} = \frac{K_2\alpha\tau_D}{K_1+K_2}$$

or $$1 = \frac{K_2\alpha}{K_1 + K_2}$$

$$K_1 + K_2 = K_2\alpha$$

$$K_1 = K_2\alpha - K_2 = K_2(\alpha - 1)$$

Substituting,

$$K_1 = (K_c - K_1)(\alpha - 1) = (\alpha - 1)K_c - (\alpha - 1)K_1$$

Then,

$$K_1 = \left(\frac{\alpha - 1}{\alpha}\right)K_c$$

c) If $K_c = 3$, $\tau_D = 2$, $\alpha = 0.1$ then,

$$K_1 = \frac{-0.9}{0.1} \times 3 = -27$$

$$K_2 = 3 - (-27) = 30$$

$$\tau_1 = 0.1 \times 2 = 0.2$$

Hence

$$K_1 + K_2 = -27 + 30 = 3$$

$$\frac{K_2\tau_1}{K_1 + K_2} = \frac{30 \times 0.2}{3} = 2$$

$$G_c(s) = 3\left(\frac{2s + 1}{0.2s + 1}\right)$$

8.3

a) From Eq. 8-14, the parallel form of the PID controller is :

$$G_i(s) = K_c'\left[1 + \frac{1}{\tau_I' s} + \tau_D' s\right]$$

From Eq. 8-15, for $\alpha \to 0$, the series form of the PID controller is:

$$G_a(s) = K_c\left[1+\frac{1}{\tau_I s}\right]\left[\tau_D s+1\right]$$

$$= K_c\left[1+\frac{\tau_D}{\tau_I}+\frac{1}{\tau_I s}+\tau_D s\right]$$

$$= K_c\left(1+\frac{\tau_D}{\tau_I}\right)\left[1+\frac{1}{\left(1+\frac{\tau_D}{\tau_I}\right)\tau_I s}+\frac{\tau_D s}{\left(1+\frac{\tau_D}{\tau_I}\right)}\right]$$

Comparing $G_a(s)$ with $G_i(s)$

$$K_c' = K_c\left(1+\frac{\tau_D}{\tau_I}\right)$$

$$\tau_I' = \tau_I\left(1+\frac{\tau_D}{\tau_I}\right)$$

$$\tau_D' = \frac{\tau_D}{1+\frac{\tau_D}{\tau_I}}$$

b) Since $\left(1+\frac{\tau_D}{\tau_I}\right) \geq 1$ for all τ_D, τ_I, therefore

$$K_c \leq K_c' \quad , \quad \tau_I \leq \tau_I' \text{ and } \tau_D \geq \tau_D'$$

c) For $K_c = 4$, τ_I=10 min , τ_D =2 min

$$K_c' = 4.8 \quad , \quad \tau_I' = 12\,\text{min} \quad , \quad \tau_D' = 1.67\,\text{min}$$

d) Considering only first-order effects, a non-zero value α will dampen all responses, making them slower.

8.4

a) System I (air-to-open valve): as the signal to the control valve increases, the flow through the valve increases $\Rightarrow$ $K_v > 0$.

System II (air-to-close valve): as the signal to the control valve increases, the flow through the valve decreases $\Rightarrow$ $K_v < 0$.

b) System I: Flow rate too high $\Rightarrow$ need to close valve $\Rightarrow$ decrease controller output $\Rightarrow$ reverse acting controller

Or: Process gain +
Valve gain +
Controller gain must be + (which means reverse acting)

System II: Flow rate too high $\Rightarrow$ need to close valve $\Rightarrow$ increase controller output $\Rightarrow$ direct acting controller.

Or: Process gain +
Valve gain –
Controller gain must be – (which means direct acting)

c) System I: $K_c > 0$.
System II: $K_c < 0$.

8.5

a) From Eqs. 8-1 and 8-2,

$$p(t) = \bar{p} + K_c\left[y_{sp}(t) - y_m(t)\right] \tag{1}$$

The liquid-level transmitter relation is

$$y_m(t) = K_T h(t) \tag{2}$$

where:

h is the liquid level

$K_T > 0$ is the gain of the direct acting transmitter.

The control-valve relation is

$$q(t) = K_v p(t) \tag{3}$$

where

q is the manipulated flow rate

K_v is the gain of the control valve.

(a) Configuration (a) in Fig. E8.5:
As h increases, we want to decrease q_i, the inlet flow rate. For an air-to-close control valve, the controller output p should increase. Thus as h increases p decreases $\Rightarrow$ a *direct-acting controller*.

Configuration (b):

As h increases, we want to increase q, the exit flow rate. For an air-to-close control valve, the controller output should decrease. Thus as h increases p decreases $\Rightarrow$ a *reverse-acting controller*.

(b) Configuration (a) in Fig. E8.5:

As h increases, we want to decrease q_i, the inlet flow rate. For an air-to-open control valve, the controller output p should decrease. Thus as h increases p decreases $\Rightarrow$ a *reverse-acting controller*.

Configuration (b):

As h increases, we want to increase q, the exit flow rate. For an air-to-open control valve, the controller output should increase. Thus as h increases p increases $\Rightarrow$ a *direct-acting controller*.

8.6

For PI control

$$p(t) = \bar{p} + K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)dt^* \right)$$

$$p'(t) = K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)dt^* \right)$$

Since

$$e(t) = y_{sp} - y_m = 0 - 2 = -2$$

Then

$$p'(t) = K_c\left(-2 + \frac{1}{\tau_I}\int_0^t (-2)dt^* \right) = K_c\left(-2 - \frac{2}{\tau_I}t \right)$$

The initial response at $t = 0$ is $-2\,K_c$

The slope of the response is $-\dfrac{2K_c}{\tau_I}$

Substitute the numerical values of the initial response and slope from Fig. E8.6:

$-2\,K_c = 6 \quad \Rightarrow \quad K_c = -3$

$-\dfrac{2K_c}{\tau_I} = 1.2\ \text{min}^{-1} \quad \Rightarrow \quad \tau_I = 5\ \text{min}$

8.7

(a) The error signal can be described by:

$$e(t) = 0.5t$$
$$E(s) = \frac{0.5}{s^2}$$

The PID controller transfer function is given by (Eq. 8-14):

$$\frac{P'(s)}{E(s)} = K_C\left[1 + \frac{1}{\tau_I s} + \tau_D s\right]$$

Substituting gives the controller output:

$$P'(s) = \frac{0.5K_C}{s^2}\left[1 + \frac{1}{\tau_I s} + \tau_D s\right]$$
$$p'(t) = 0.5K_C\left[t + \frac{1}{2\tau_I}t^2 + \tau_D S(t)\right]$$

Substituting numerical values and adding $\bar{p} = 12$ mA gives:

$$p_{PID}(t) = 12 + \frac{1}{3}t^2 + t + 0.5S(t)$$

(b) The equation for a PI controller is obtained by setting τ_D to zero.

$$p_{PI}(t) = 12 + \frac{1}{3}t^2 + t$$

(c) The plot of the controller response for both controllers is shown in Fig. S8.7.

The two controllers have similarly-shaped responses. The difference is the sudden jump at t=0 that occurs with the PID controller as a result of the derivative term. When the set point begins to change with a constant slope, there is a step change in the error derivative from 0 to 0.5. The derivative term in the controller gives it a jumpstart right when the setpoint begins to change that the PID controller does not have.

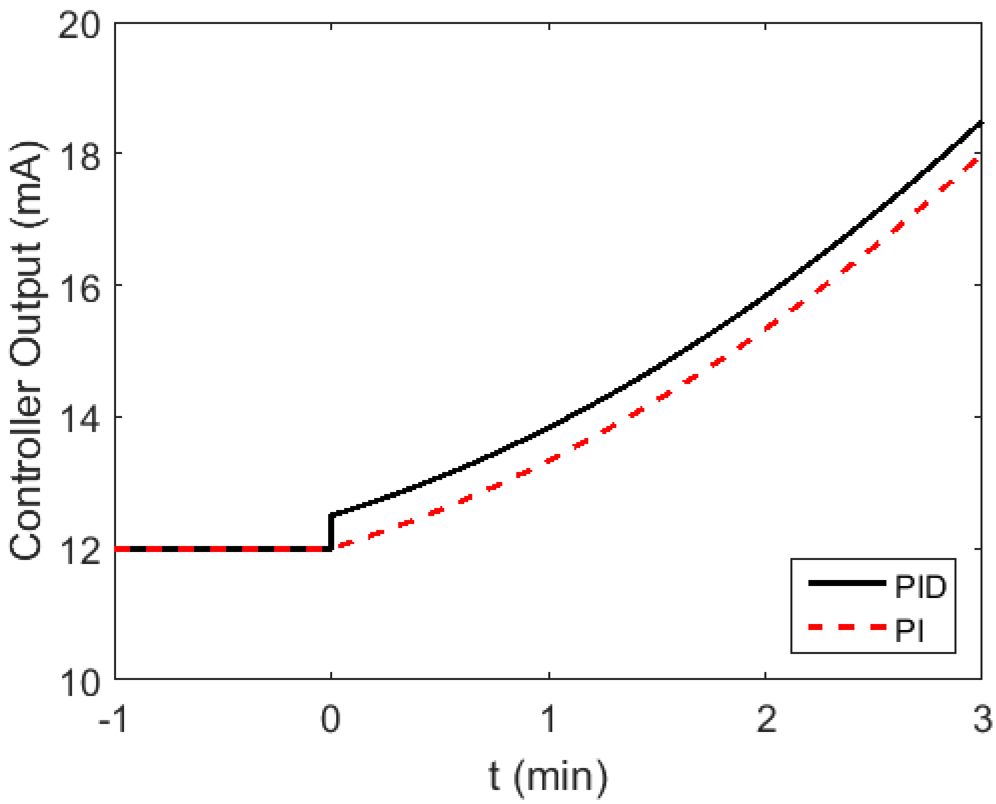


Figure S8.7: PID controller output response

8.8

From inspection of Eq. 8-25, the derivative kick = $K_c \dfrac{\tau_D}{\Delta t}\Delta r$

a) Proportional kick = $K_c \Delta r$

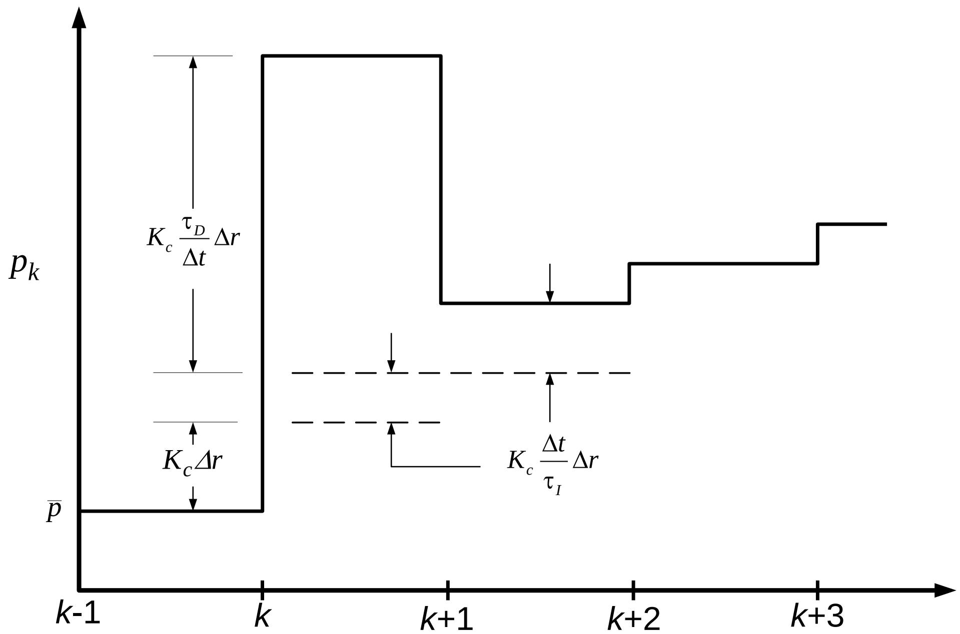
b) $e_1 = e_2 = e_3 = ... = e_{k-2} = e_{k-1} = 0$

$e_k = e_{k+1} = e_{k+2} = ... = \Delta r$

$$p_{k-1} = \overline{p}$$

$$p_k = \overline{p} + K_c\left[\Delta r + \frac{\Delta t}{\tau_I}\Delta r + \frac{\tau_D}{\Delta t}\Delta r\right]$$

$$p_{k+i} = \bar{p} + K_c\left[\Delta r + (1+i)\frac{\Delta t}{\tau_I}\Delta r\right] \quad , \quad i = 1, 2, \ldots$$



a) To eliminate derivative kick, replace $(e_k - e_{k-1})$ in Eq. 8-25 by - $(y_k - y_{k-1})$. (Note the minus sign.)

8.9

a) Let the constant set point be denoted by $\bar{y}_{sp}$. The digital velocity P algorithm is obtained by setting $1/\tau_I = \tau_D = 0$ in Eq. 8-27:

$$\Delta p_k = K_c(e_k - e_{k-1})$$

$$= K_c\left[(\bar{y}_{sp} - y_k) - (\bar{y}_{sp} - y_{k-1})\right]$$

$$= K_c\left[y_{k-1} - y_k\right]$$

The digital velocity PD algorithm is obtained by setting $1/\tau_I = 0$ in Eq. 8-27:

$$\Delta p_k = K_c\left[(e_k - e_{k-1}) + \frac{\tau_D}{\Delta t}(e_k - 2e_{k-1} + e_{k-2})\right]$$

$$= K_c\left[(-y_k + y_{k-1}) + \frac{\tau_D}{\Delta t}(-y_k - 2y_{k-1} + y_{k-2})\right]$$

In both cases, Δp_k does not depend on $\bar{y}_{sp}$.

b) For both these algorithms $\Delta p_k = 0$ if $y_{k-2} = y_{k-1} = y_k$. Thus a steady state is reached with a value of y that is independent of the value of $\bar{y}_{sp}$. Use of these control algorithms is inadvisable if offset is a concern.

c) If the integral mode is present, then Δp_k contains the term $K_c \dfrac{\Delta t}{\tau_I}(\bar{y}_{sp} - y_k)$.

Thus, at steady state, $\Delta p_k = 0$ and $y_{k-2} = y_{k-1} = y_k$, $y_k = \bar{y}_{sp}$, and the offset problem is eliminated.

8.10

a)
$$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha\tau_D s + 1}\right)$$

$$= K_c \frac{\left(\tau_I s(\alpha\tau_D s + 1) + \alpha\tau_D s + 1 + \tau_D s \tau_I s\right)}{\tau_I s(\alpha\tau_D s + 1)}$$

$$= K_c\left[\frac{1 + (\tau_I + \alpha\tau_D)s + (1+\alpha)\tau_I\tau_D s^2}{\tau_I s(\alpha\tau_D s + 1)}\right]$$

Cross- multiplying

$$(\alpha\tau_I\tau_D s^2 + \tau_I s)\; P'(s) = K_c\left(1 + (\tau_I + \alpha\tau_D)s + (1+\alpha)\tau_I\tau_D s^2\right)E(s)$$

Taking inverse Laplace transforms gives,

$$\alpha\tau_I\tau_D \frac{d^2 p'(t)}{dt^2} + \tau_I \frac{dp'(t)}{dt} = K_c\left(e(t) + (\tau_I + \alpha\tau_D)\frac{de(t)}{dt} + (1+\alpha)\tau_I\tau_D \frac{d^2 e(t)}{dt^2}\right)$$

b)
$$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)\left(\frac{\tau_D s}{\alpha\tau_D s + 1}\right)$$

Cross-multiplying

$$\tau_I s^2(\alpha\tau_D s + 1)\; P'(s) = K_c\left((\tau_I s + 1)(\tau_D s + 1)\right)E(s)$$

$$\alpha\tau_I\tau_D \frac{d^2p'(t)}{dt^2} + \tau_I \frac{dp'(t)}{dt} = K_c\left(e(t) + (\tau_I + \tau_D)\frac{de(t)}{dt} + \tau_I\tau_D \frac{d^2e(t)}{dt^2} \right)$$

c) The simulation is performed for the following parameter values:

$$K_c = 2 \quad , \quad \tau_I = 3\,, \quad \tau_D = 0.5 \quad , \quad \alpha = 0.1 \quad , \quad M = 1$$

The Simulink-MATLAB results are shown in Figure S8.10.:

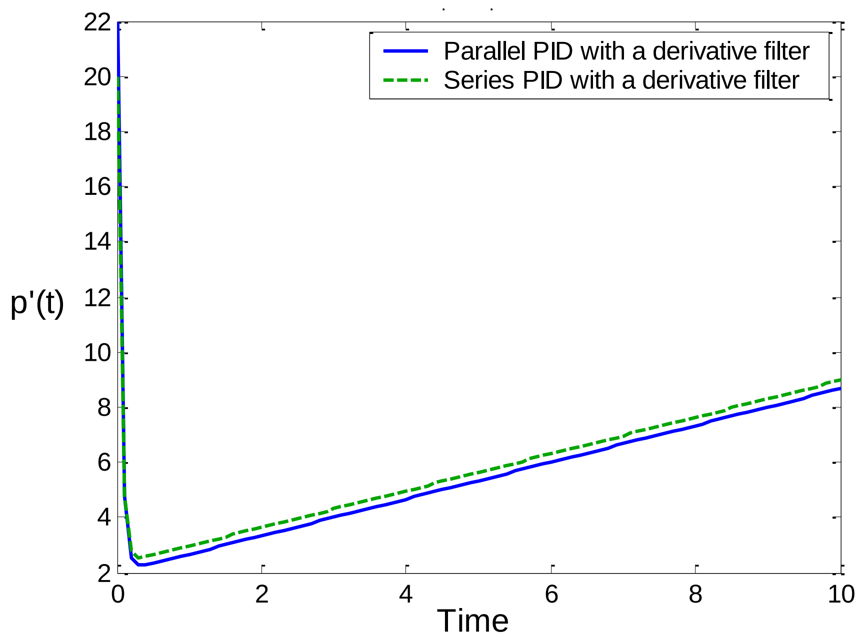


Figure S8.10. *Step responses for both parallel and series PID controllers with a derivative filter.*

8.11

The integral component of the controller action is determined by integrating the error between the measurement and the set point over time. As long as the sign on the error stays the same (i.e., if the measurement does not cross the set point), the integral component will continue to change monotonically. If the measurement crosses the set point, the error term will change sign and the integral component will begin to change in the other direction. Thus, it will no longer be monotonic.

8.12

a) **False**. The controller output could saturate or the controller could be in the manual mode.
b) **False**. Even with integral control action, offset can occur if the controller output saturates. Or the controller could be in the manual mode.

8.13

First consider qualitatively how h_2 responds to a change in q_2. From physical considerations, it is clear that if q_2 increases, h_2 will increase. Thus, if h_2 is increasing, we want q_2 to decrease, and vice versa. Since the q_2 control valve is air-to-open, the level controller output p should decrease in order to have q_2 decrease. In summary, if h_2 increases we want p to decrease; thus a reverse-acting controller is required.

8.14

First consider qualitatively how solute mass fraction x responds to a change in steam flow rate, S. From physical considerations, it is clear that if S increases, x will also increase. Thus, if x is increasing, we want S to decrease, and vice versa. For a fail-open (air-to-close) control valve, the controller output p should increase in order to have S decrease. In summary, if x increases we want S to decrease, which requires an increase in controller output p; thus a direct-acting controller is required.

8.15

First consider qualitatively how exit temperature T_{h2} responds to a change in cooling water flow rate, w_c. From physical considerations, it is clear that if w_c decreases, T_{h2} will increase. Thus, if T_{h2} is decreasing, we want w_c to decrease, and vice versa. But in order to specify the controller action, we need to know if the control valve is fail open or fail close. Based on safety considerations, the control valve should be fail open (air-to-close). Otherwise, the very hot liquid stream could become even hotter and cause problems (e.g., burst the pipe or generate a two phase flow).

For an air-to-close control valve, the temperature controller output p should increase in order to have w_c decrease. In summary, if T_{h2} decreases we want w_c to decrease, which requires controller output p to increase; thus a reverse-acting controller is required.

8.16

Two pieces of information are needed to specify controller action:

i) Is the control valve fail open or fail close

ii) Is $x_1 > x_2$ or $x_1 < x_2$

If $x_1 > x_2$, then the mass balance is:

$$x_1 w_1 + x_2 w_2 = xw = x(w_1 + w_2)$$
$$x_1 > x_2$$
$$\therefore x_1 = x_2 + \Delta$$
$$(x_2 + \Delta)w_1 + x_2 w_2 = x(w_1 + w_2)$$
$$x_2 w_1 + \Delta w_1 + x_2 w_2 = x(w_1 + w_2)$$
$$x_2(w_1 + w_2) + \Delta w_1 = x(w_1 + w_2)$$
$$x = x_2 + \frac{\Delta w_1}{(w_1 + w_2)}$$

Since all the variables in the equation are positive, then $x > x_2$. The only way to decrease x is to increase w_2 (but x can never be less than x_2). Therefore, w_2 should be increased when x increases, in order to have x decrease. If the control valve is fail open (air-to-close), then the composition controller output signal p should decrease. Thus a reverse-acting controller should be selected. Conversely, for a fail close (air-to-open) control valve, a direct-acting controller should be used.

If $x_1 < x_2$, then

$$x_1 w_1 + x_2 w_2 = xw = x(w_1 + w_2)$$
$$x_1 < x_2$$
$$\therefore x_1 = x_2 - \Delta$$
$$(x_2 - \Delta)w_1 + x_2 w_2 = x(w_1 + w_2)$$
$$x_2 w_1 - \Delta w_1 + x_2 w_2 = x(w_1 + w_2)$$
$$x_2(w_1 + w_2) - \Delta w_1 = x(w_1 + w_2)$$
$$x = x_2 - \frac{\Delta w_1}{(w_1 + w_2)}$$

Since all the variables are positive, then $x < x_2$. If x increases, the controller will need to decrease it to bring it back to the set point. The only way to decrease x is to decrease w_2 (although x can never be smaller than x_1). If the control valve is fail open (air-to-close), then the composition controller output signal p should increase in order to reduce w_2. Thus the composition controller should be direct acting. Conversely, for a fail close control valve, a reverse acting controller should be used.

Chapter 9 ©

9.1

a) Flow rate transmitter:

$$q_m(\text{psig})=\left(\frac{15\text{ psig - 3 psig}}{400\text{ gpm-0 gpm}}\right)(q\text{ gpm - 0 gpm})+3\text{ psig}$$

$$=\left(0.03\frac{\text{psig}}{\text{gpm}}\right)q(\text{gpm})+3\text{ psig}$$

Pressure transmitter:

$$P_m(\text{mA})=\left(\frac{20\text{ mA - 4 mA}}{30\text{ in.Hg - 10 in.Hg}}\right)(p\text{ in.Hg}-10\text{ in.Hg})+4\text{ mA}$$

$$=\left(0.8\frac{\text{mA}}{\text{in.Hg}}\right)p(\text{in.Hg})-4\text{ mA}$$

Level transmitter:

$$h_m(\text{VDC})=\left(\frac{5\text{ VDC - 1 VDC}}{10\text{ m - 0.5 m}}\right)(h(\text{m})\text{ - 0.5m})+1\text{ VDC}$$

$$=\left(0.421\frac{\text{VDC}}{\text{m}}\right)h(\text{m})+0.789\text{ VDC}$$

Concentration transmitter:

$$C_m(\text{VDC})=\left(\frac{10\text{ VDC - 1 VDC}}{20\text{ g/L - 3 g/L}}\right)(C(\text{g/L})\text{-3 g/L})+1\text{ VDC}$$

$$=\left(0.529\frac{\text{VDC}}{\text{g/L}}\right)C(\text{g/L})-0.59\text{VDC}$$

b) The gains, zeros and spans are:

	Flow	Pressure	Level	Concentration
Gain	0.03 psig/gpm	0.8 mA/in.Hg	0.421 VDC/m	0.529 VDC/g/L
Zero	0 gpm	10 in.Hg	0.5 m	3 g/L
Span	400 gpm	20 in.Hg	9.5 m	17 g/L

9.2

a) The safest conditions are achieved by the lowest temperatures and pressures in the flash vessel.

VALVE 1.- Fail close (air-to-open)
VALVE 2.- Fail open (air-to-close)
VALVE 3.- Fail open (air-to-close)
VALVE 4.- Fail open (air-to-close)
VALVE 5.- Fail close (air-to-open)

Setting valve 1 as fail close prevents more heat from going to flash drum and setting valve 3 as fail open to allow the steam chest to drain. Setting valve 3 as fail open prevents pressure build up in the vessel. Valve 4 should be fail-open to evacuate the system and help keep pressure low. Valve 5 should be fail-close to prevent any additional pressure build-up.

b) Vapor flow to downstream equipment can cause a hazardous situation

VALVE 1.- Fail close (air-to-open)
VALVE 2.- Fail open (air-to-close)
VALVE 3.- Fail close (air-to-open)
VALVE 4.- Fail open (air-to-close)
VALVE 5.- Fail close (air-to-open)

Setting valve 1 as fail close (air-to-open) prevents more heat from entering flash drum and minimizes future vapor production. Setting valve 2 as fail open (air-to-close) will allow the steam chest to be evacuated, setting valve 3 as fail close (air-to-open) prevents vapor from escaping the vessel. Setting valve 4 as fail open (air-to-close) allows liquid to leave, preventing vapor build up. Setting valve 4 as fail close (air-to-open) prevents pressure buildup.

c) Liquid flow to downstream equipment can cause a hazardous situation

VALVE 1.- Fail close (air-to-open)
VALVE 2.- Fail open (air-to-close)
VALVE 3.- Fail open (air-to-close)
VALVE 4.- Fail close (air-to-open)
VALVE 5.- Fail close (air-to-open)

Set valve 1 as fail close to prevent all the liquid from being vaporized (This would cause the flash drum to overheat). Setting valve 2 as fail open will allow the steam chest to be evacuated. Setting valve 3 as fail open prevents pressure buildup in drum. Setting valve 4 as fail close prevents liquid from escaping. Setting valve 5 as fail close prevents liquid build-up in drum

9.3

Note: This exercise is best understood after the material in Ch. 11 has been considered.

a) Changing the span of the temperature transmitter will change its steady-state gain, according to Eq. 9-1. Because the performance of the closed-loop system depends on the gains of each individual element (cf. Chapter 11), closed-loop stability could be adversely affected.

b) Changing the zero of a transmitted does not affect its gain. Thus, this change will not affect closed-loop stability.

c) Changing the control valve trim will change the (local) steady-state gain of the control valve, d*q*/d*p*. Because the performance of the closed-loop system depends on the gains of each individual element (cf. Chapter 11), closed-loop stability could be adversely affected

d) For this process, changing the feed flow rate could affect both its steady-state gain and its dynamic characteristics (e.g., time constant and time delay). Because the performance of the closed-loop system depends on the gains of each individual element (cf. Chapter 11), closed-loop stability could be adversely affected.

9.4

Starting from Eq. 9-7:

$$C_v = \frac{q}{Nf(l)\sqrt{\dfrac{\Delta P_v}{g_s}}} \qquad (1)$$

The pressure drop in the valve is:

$$\Delta P_v = \Delta P - \Delta P_s \qquad (2)$$

where

$$\Delta P_s = Kq^2 \qquad (3)$$

Solve for K by plugging in the nominal values of q and ΔP_s. First, convert the nominal value of q into units of m^3/h to match the metric units version of N (the parameter N = 0.0865 $m^3/h(Kpa)^{1/2}$ when q has units of m^3/h and pressure has units of KPa).

$$q_d = 0.6 \text{ m}^3/\text{min} = 36 \text{ m}^3/\text{h}$$

$$\Delta P_{sd} = 200 \text{ kPa}$$

$$K = \frac{\Delta P_{sd}}{q_d{}^2} = \frac{200 \text{ kPa}}{36^2 \ (\text{m}^3/\text{h})^2} = 0.154 \text{ kPa/(m}^3\text{/h)}^2$$

Now substitute (3) into (2) to get an expression for ΔP_v in terms of q.

$$\Delta P_v = \Delta P - Kq^2 \tag{4}$$

Substitute (4) into (1) to get:

$$C_v = \frac{q}{Nf(l)\sqrt{\dfrac{\Delta P - Kq^2}{g_s}}} \tag{5}$$

The problem specifies that q_d should be 2/3 of q_{max} (where q_{max} is the flow rate through the valve when the valve is fully open).

$$q_d = \frac{2}{3} q_{\max}$$

$$q_{\max} = \frac{3}{2} q_d = \frac{3}{2} 36 \text{ m}^3/\text{h}$$

$$q_{\max} = 54 \text{m}^3/\text{h}$$

Now find the C_v that will give q_{max} = 54 m^3/h. Substitute $q = q_{max}$ and $f(l)$=1 (valve fully open) into (5).

$$C_v = \frac{q_{\max}}{N\sqrt{\dfrac{\Delta P - Kq_{\max}{}^2}{g_s}}}$$

Now that all of the variables on the right hand side of the equation are known, plug in to solve for C_v.

$$\Delta P = 450 \text{ kPa}, \quad K = 0.154 \frac{\text{kPa}}{(\text{m}^3/h)^2}, \quad N = 0.0865 \frac{\text{m}^3}{\text{h(kPa)}^{1/2}},$$

$$g_s = 1.2, \quad q_{\max} = 54 \text{ m}^3/\text{h}$$

$$C_v = \frac{54 \dfrac{\text{m}^3}{\text{h}}}{0.0865 \dfrac{\text{m}^3}{\text{h(kPa)}^{1/2}} \sqrt{\dfrac{450\text{kPa} - 0.154 \dfrac{\text{kPa}}{(\text{m}^3/\text{h})^2} 54^2 (\text{m}^3/\text{h})^2}{1.2}}}$$

$$= \frac{54 \dfrac{\text{m}^3}{\text{h}}}{0.0865 \dfrac{\text{m}^3}{\text{h(kPa)}^{1/2}} \left(0.88 (\text{kPa})^{1/2}\right)} = \frac{54 \dfrac{\text{m}^3}{\text{h}}}{0.076 \dfrac{\text{m}^3}{\text{h}}}$$

$$C_v = 710.5$$

9.5

Let $\Delta P_v/\Delta P_s$ = 0.33 at the nominal $\bar{q} = 320$ gpm

$$\Delta P_s = \Delta P_{b+} \Delta P_o = 40 + 1.953\times 10^{-4}\, q^2$$

$$\Delta P_v = P - \Delta P_s = (1 - 2.44\times 10^{-6}\, q^2)P_{DE} - (40 + 1.953\times 10^{-4}\, q^2)$$

$$\frac{(1 - 2.44\times 10^{-6}\ \times 320^2)\mathrm{P_{DE}} - (40 + 1.953\times 10^{-4}\ \times 320^2)}{(40 + 1.953\times 10^{-4}\ \times 320^2)} = 0.33$$

$$P_{DE} = 106.4 \text{ psi}$$

Let $q_d = \bar{q} = 320$ gpm

For the rated C_v, the valve is completely open at 110% q_d i.e., at 352 gpm or the upper limit of 350 gpm

$$C_v = q\left(\frac{\Delta p_v}{g_s}\right)^{-\frac{1}{2}}$$

$$C_v = 350\left[\frac{(1 - 2.44\times 10^{-6}\times 350^2)106.4 - (40 + 1.953\times 10^{-4}\times 350^2)}{0.9}\right]^{-\frac{1}{2}}$$

Then using Eq. 9-27,

$$l = 1 + \frac{\ln\left[\frac{q}{101.6}\left(\frac{66.4 - 4.55\times 10^{-4}\, q^2}{0.9}\right)^{-1/2}\right]}{\ln 50}$$

The plot of the valve characteristic is shown in Figure S9.5. From the plot of the valve characteristic for the rated C_v of 101.6, it is evident that the characteristic is reasonably linear in the operating region $250 \le q \le 350$.

The pumping cost could be further reduced by lowering P_{DE} to a value that would make $\Delta P_v/\Delta P_s$ = 0.25 at $\bar{q} = 320$ gpm. Then P_{DE} = 100 and for q_d = 320 gpm, the rated C_v = 133.5. However, as the plot shows, the valve characteristic for this design is only slightly more nonlinear in the operating region. Hence, the selected valve coefficient is C_v = 133.5.

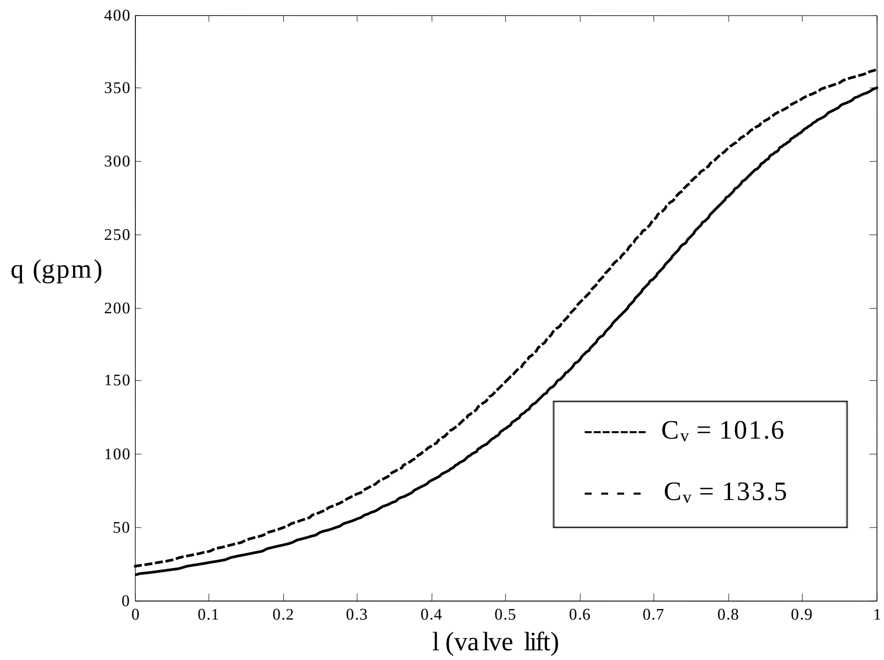


Figure S9.5. *Control valve characteristics.*

9.6

a) There are three control valves. The selection of air-to-close vs. air-to-open is based on safety considerations:

i. *Steam control valve*: Air-to-open to prevent overheating of the evaporator.

ii. *Level control valve* (that adjusts liquid flow rate *B*): Air-to-open to prevent the steam coils from being exposed to the vapor space, which could lead the coils to being burned out.

iii. *Pressure control valve* (that adjusts solvent flow rate *D*): Air-to-close to prevent over-pressurization of the evaporator.

b) For the three controllers:

i. *Concentration controller*: As the product concentration x_B *increases*, we want the steam pressure, P_s to increase. Since the steam valve is air-to-open, this means that the controller output signal to the control valve (via the I/P) should *increase*. Thus, the controller should be *direct-acting*.

ii. *Level controller*: As the liquid level *h increases*, we want the product flow rate *B* to increase. Since the control valve is air-to-open, this means that the controller output signal to the control valve (via the I/P) should *increase*. Thus, the controller should be *direct-acting*.

iii. *Pressure controller*: As the pressure *P increases*, we want the solvent flow rate *D* to increase. Since the control valve is air-to-close, this means that the controller output signal to the control valve (via the I/P) should *decrease*. Thus, the controller should be *reverse-acting*.

9.7

Because the system dynamic behavior would be described using deviation variables, the dynamic characteristic can be analyzed by considering that the input terms (not involving *x*) can be considered to be constant, and thus deviations are zero. The starting form is the linear homogeneous ODE:

$$\frac{M}{g_c}\frac{d^2x}{dt^2} + R\frac{dx}{dt} + Kx = 0$$

Taking the Laplace transform gives,

$$X(s)\left(\frac{M}{g_c}s^2 + Rs + K\right) = 0$$

$$X(s)\left(\frac{M}{Kg_c}s^2 + \frac{R}{K}s + 1\right) = 0$$

Calculate τ and ζ by comparing this equation to the standard form of the second-order model in (5-39) (keeping in mind that g_c = 32.174 lbm ft/(lbf s^2)).

$$\tau = \sqrt{\frac{M}{Kg_c}} = 0.00965s$$

$$2\varsigma\tau = 2\varsigma\sqrt{\frac{M}{Kg_c}} = \frac{R}{K}$$

$$\varsigma = \frac{R}{2}\sqrt{\frac{g_c}{KM}} = 155.3$$

The valve characteristics are highly overdamped and can be accurately approximated by a first-order model obtained by neglecting the d^2x/dt^2 term.

9.8

Configuration I: This series configuration will not be very effective because a large flow rate has to pass through a small control valve. Thus, the pressure drop will be very large and flow control will be ineffective.

Configuration II: This parallel configuration will be effective because the large control valve can be adjusted to provide the nominal flow rate, while the small control valve can be used to regulate the flow rate. If the small valve reaches its maximum or minimum value, the large valve can be adjusted slightly so that the small valve is about half open, thus allowing it to regulate flow again.

9.9

First write down the time-domain step response for a step change of 10°C. Then solve the equation to find when $y(t)$ is equal to 5 (since the variables are in deviation variables, this represents when T_M will reach 30°C).

$$y_m(t) = KM(1 - e^{-t/\tau})$$

$$\text{where } M = 10^{o}\text{C}, \quad K = 1, \quad \text{and} \quad \tau = 10\text{s}$$

$$y_m(t) = 10(1 - e^{-t/10})$$

$$5 = 10(1 - e^{-t_a/10})$$

$$t_a = 6.93\text{s}$$

Therefore, the alarm will sound 6.93 seconds after 1:10PM.

9.10

- precision = $\dfrac{0.1\text{ psig}}{20\text{ psig}} = 0.5\%$ of full scale
- accuracy is unknown since the "true" pressure in the tank is unknown
- resolution = $\dfrac{0.1\text{ psig}}{20\text{ psig}} = 0.5\%$ of full scale
- repeatability = $\dfrac{\pm 0.1\text{ psig}}{20\text{ psig}} = \pm 0.5\%$ of full scale

9.11

Assume that the gain of the sensor/transmitter is unity (i.e. there is no steady-state measurement error). Then,

$$\frac{T'_m(s)}{T'(s)} = \frac{1}{(s+1)(0.1s+1)}$$

where T is the temperature being measured and T_m is the measured value. For the ramp temperature change:

$$T'(t) = 0.3t \ \ (°\text{C/s}) \quad , \quad T'(\text{s}) = \frac{0.3}{s^2}$$

$$T'_m(s) = \frac{1}{(s+1)(0.1s+1)} \times \frac{0.3}{s^2}$$

$$T'_m(t) = -0.00333e^{-10t} + 0.333e^{-t} + 0.3t - 0.33$$

The maximum error occurs as $t \to \infty$:

$$\text{Maximum error} = |0.3t \ - (0.3t - 0.33)| = 0.33 \text{ °C}$$

If the smaller time constant is neglected, the time domain response is slightly different for small values of t, although the maximum error ($t \to \infty$) does not change.

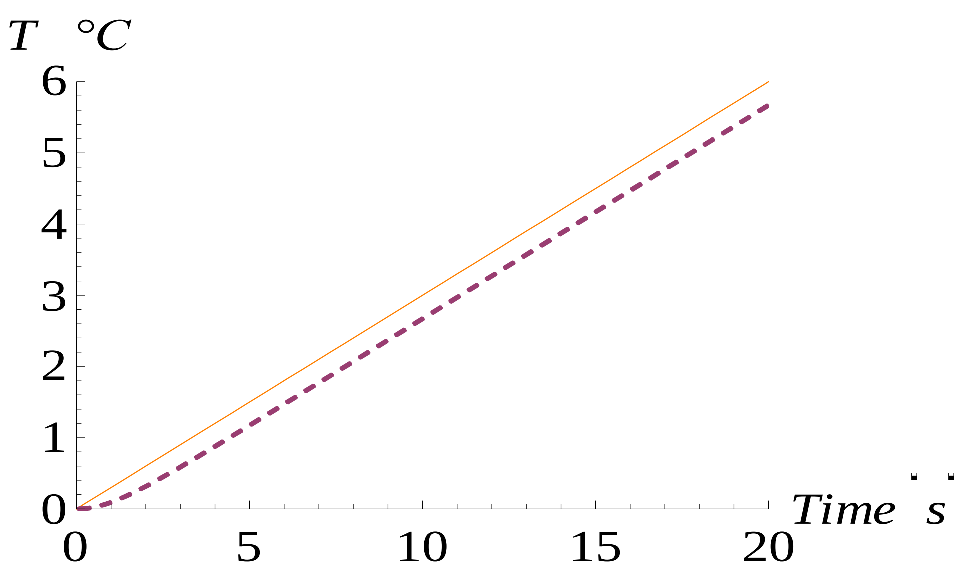


Figure S9.11. *Response for process temperature sensor/transmitter. Orange solid line is T'(t), and purple dashed line is $T'_m(t)$.*

Chapter 10 ©

10.1

Assumptions:

1. Incompressible flow.
2. Chlorine concentration does not affect the air sample density.
3. T and P are approximately constant.

The detection time, t_d, depends on the transportation time delay, θ, and the response time of the analyzer, $t_r = 10\ s$:

$$t_d = \theta + t_r \qquad (1)$$

Time delay θ can be calculated as the ratio of the volume of the tubing V divided by the volumetric flow rate of chlorine q:

$$\theta = \frac{V}{q} \qquad (2)$$

where $q = 10\ \text{cm}^3/\text{s}$ and,

$$V = \frac{\pi D_i^2 L}{4} \qquad (3)$$

where the inside diameter D_i is:

$$D_i = 6.35\ \text{mm} - 2(0.762\ \text{mm}) = 4.83\ \text{mm} = 4.83 \times 10^{-3}\ \text{m}$$

Substitute D_i and $L = 60$ m into (3):

$$V = 1.10 \times 10^{-3}\ \text{m}^3$$

Substitute D_i into (2):

$$\theta = \frac{V}{q} = \left(\frac{1.10\, x 10^{-3}\,\text{m}^3}{10\ \text{cm}^3/\text{s}}\right)\left(\frac{100\,\text{cm}}{1\ \text{m}}\right)^3 = 110\ \text{s}$$

Substitute into (1):

$$t_d = \theta + t_r = 110 + 10 = 120\ \text{s} = 2\ \text{min}$$

Carbon monoxide (CO) is one of the most widely occurring toxic gases, especially for confined spaces. High concentrations of carbon monoxide can saturate a person's blood in matter of minutes and quickly lead to respiratory problems or even death. Therefore, the long detection time would not be acceptable if the hazardous gas is CO.

10.2

(a) Start with a mass balance on the tank. Then solve the equation to find how much time it takes for the height to decrease from 1 m to 0.25 m.

$$\frac{dV(t)}{dt} = -C\sqrt{h(t)}$$

$$\frac{Adh(t)}{dt} = -C\sqrt{h(t)}$$

$$\frac{dh}{dt} = -\frac{C}{A}h^{0.5}$$

$$h^{-0.5}dh = -\frac{C}{A}dt$$

$$\int_{1}^{0.25} h^{-0.5}dh = \int_{0}^{t_f} -\frac{C}{A}dt$$

$$2(\sqrt{0.25[m]} - \sqrt{1[m]}) = -\frac{C}{A}(t_f - 0)$$

$$-1m^{0.5} = -\frac{C}{A}t_f$$

$$t_f = \frac{A}{C}1[m^{0.5}]$$

$$t_f = \frac{\pi(0.5)^2[m^2]}{0.065[m^{2.5}/\text{min}]}[m^{0.5}]$$

$$t_f = 12.1[\text{min}]$$

Therefore, the alarm will sound at 5:12:06AM

(b) To find how much liquid has leaked out of the tank, calculate the difference in volume between the starting level and the alarm level.

$$\Delta V = V_{h=1m} - V_{h=0.25m} = \pi\left(\frac{1}{2}[m]\right)^2\left(1[m] - 0.25[m]\right) = 0.59m^3$$

$0.59m^3$ of liquid has leaked out when the alarm sounds.

10.3

The key safety concerns include:

1. Early detection of leaks to the surroundings
2. Over-pressurizing the flash drum
3. Maintain enough liquid level so that the pump does not cavitate.
4. Avoid having liquid entrained in the gas.

These concerns can be addressed by the following instrumentation.

1. Leak detection: sensors for hazardous gases should be located in the vicinity of the flash drum.
2. Over pressurization: Use a high pressure switch (PSH) to shut off the feed when a high pressure occurs.
3. Liquid inventory: Use a low level switch (LSL) to shut down the pump if a low level occurs.
4. Liquid entrainment: Use a high level alarm to shut off the feed if the liquid level becomes too high.

This SIS system is shown in Fig. S10.3 with conventional control loops for pressure and liquid level.

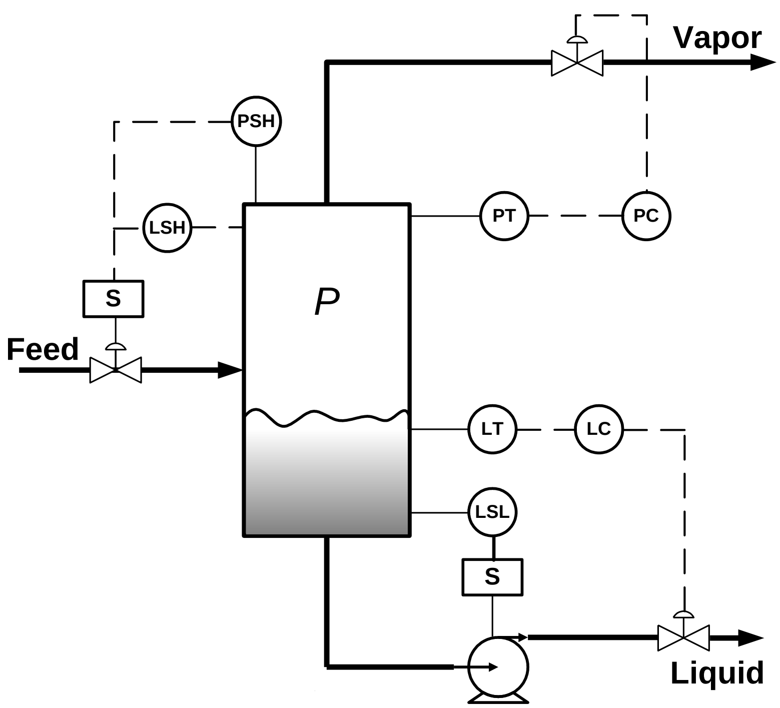


Figure S10.3: *SIS system*

10.4

The proposed alarm/SIS system is shown in Figure S10.4:

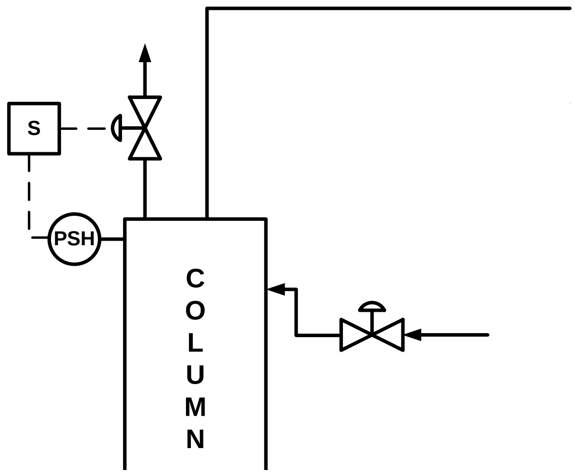


Figure S10.4: *Proposed alarm/SIS system*

The solenoid-operated valve is normally closed. But if the pressure in the column exceeds a specified limit, the high pressure switch (PSH) activates an alarm (PAH) and causes the valve to open fully, thus reducing the pressure in the tank.

10.5

Define k as the number of sensors that are working properly. We wish to calculate the probability that $k \geq 2$, $P(k \geq 2)$.

Because k = 2 and k = 3 are mutually exclusive events (cf. Appendix F),

$$P(k \geq 2) = P(k = 2) + P(k = 3) \qquad (1)$$

These probabilities can be calculated from the binomial distribution [1]

$$P(k = 2) = \binom{3}{2}(0.05)^1 (0.95)^2 = 0.135$$

$$P(k = 3) = \binom{3}{3}(0.05)^0 (0.95)^3 = 0.857$$

where the notation, $\begin{pmatrix} n \\ r \end{pmatrix}$, refers to the number of combinations of *n* objects taken *r* at a time, when the order of the *r* objects is not important. Thus $\begin{pmatrix} 3 \\ 2 \end{pmatrix} = 3$ and $\begin{pmatrix} 3 \\ 3 \end{pmatrix} = 1$. From Eq. 1,

$$P(k \geq 2) = 0.135 + 0.857 = \boxed{0.992}$$

[1] See any standard probability or statistics book, e.g., Montgomery D.C and G.C. Runger, *Applied Statistics and Probability for Engineers, 6h edition*, Wiley, New York, 2013.

10.6

Solenoid switch: $\mu_S = 0.01$

Level switch: $\mu_{LS} = 0.45$

Level alarm: $\mu_A = 0.3$

Notation:

P_S = the probability that the solenoid switch fails

P_{LS} = the probability that the level switch fails

P_A = the probability that the level alarm fails

P_I = the probability that the interlock system (solenoid & level switch fails)

We wish to determine,

P = the probability that both safety systems fail (i.e., the original system and the additional level alarm)

Because the interlock and level alarm systems are independent, it follows that (cf. Appendix F):

$$P = P_I \, P_A \qquad (1)$$

From the failure rates, the following table can be constructed, in analogy with Example 10.4:

Component	**μ**	***R***	***P* = 1 – *R***
Solenoid:	0.01	0.990	0.010
Level switch:	0.45	0.638	0.362
Level alarm	0.3	0.741	0.259

Assume that the switch and solenoid are independent. Then,

$$P_I = P_S + P_{SW} - P_S P_{SW}$$
$$P_I = 0.01 + 0.362 – (0.01)(0.362)$$
$$P_I = 0.368$$

Substitute into (1):

$$P = P_I P_A = (0.368)(0.259) = \boxed{0.095}$$

Mean time between failures, *MTBF*:

From (10-6) through (10-8):

$$R = 1 – P = 1 – 0.095 = 0.905$$
$$\mu = - \ln (0.905) = 0.0998$$
$$MTBF = \frac{1}{\mu} = \boxed{10.0 \text{ years}}$$

10.7

Let P_2 = the probability that neither D/P flowmeter is working properly. Then P_2 and the related reliability, R_2, can be calculated as (cf. Appendix F):

$$P_2 = (0.82)^2 = 0.672$$
$$R_2 = 1 - P_2 = 1 - 0.672 = 0.33$$

To calculate the overall system reliability, substitute R_2 = 0.33 for the reliability value for a single D/P flowmeter, 0.18, in the *R* calculation of Example 10.4:

$$R = \prod_{i=1}^{5} R_i = (0.33)(0.95)(0.61)(0.55)(0.64)$$
$$R = 0.067$$

Thus, the addition of the second D/P flowmeter has increased the overall system reliability from 0.037 (for Example 10.4) to 0.067.

10.8

Let P_3 = the probability that none of the 3 D/P flowmeters are working properly. Then P_3 and the related reliability, R_3, can be calculated as (cf. Appendix F):

$$P_3 = (0.82)^3 = 0.551$$

$$R_3 = 1 - P_3 = 1 - 0.551 = 0.449$$

To calculate the overall system reliability, substitute $R_3 = 0.449$ for the reliability value for two D/P flowmeters (R_2=0.33) in the R calculation from Exercise 10.7:

$$R = \prod_{i=1}^{5} R_i = (0.449)(0.95)(0.61)(0.55)(0.64)$$

$$R = 0.092$$

Thus, the addition of the third D/P flowmeter has increased the overall system reliability from 0.067 (for Exercise 10.7) to 0.092.

10.9

Assume that the switch and solenoid are independent. From the failure rate data, the following table can be constructed, in analogy with Example 10.4:

Component	**μ**	***R***	***P* = 1 – *R***
Pressure switch	0.34	0.712	0.288
Solenoid switch/valve:	0.42	0.657	0.343

Assume that the switch and solenoid are independent. Then, the overall reliability of the interlock system is,

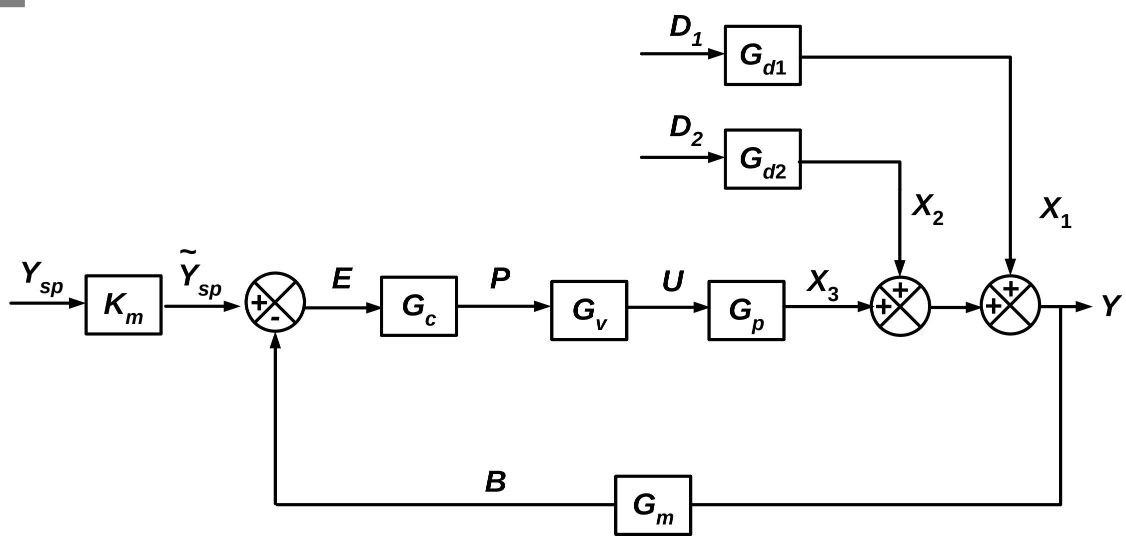
$$R = (0.712)(0.657) = \boxed{0.468}$$

$$\mu = -\ln(0.468) = 0.760$$

$$MTBF = \frac{1}{\mu} = \boxed{1.32 \text{ years}}$$

Chapter 11

11.1



11.2

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

The closed-loop transfer function for set-point changes is given by Eq. 11-36 with K_c replaced by $K_c\left(1+\frac{1}{\tau_I s}\right)$,

$$\frac{H'(s)}{H'_{sp}(s)} = \frac{K_c K_{IP} K_v K_p K_m \left(1+\frac{1}{\tau_I s}\right)\frac{1}{(\tau s+1)}}{1+K_c K_{IP} K_v K_p K_m \left(1+\frac{1}{\tau_I s}\right)\frac{1}{(\tau s+1)}}$$

where $K_p = R = 1.0$ min/ft^2 ,
and $\tau = RA = 3.0$ min. Note also that $\tau_I = \tau = 3.0$ min.

$$K_{OL} = K_c K_{IP} K_v K_p K_m = (5.33)\left(0.75\frac{\text{psi}}{\text{mA}}\right)\left(0.2\frac{\text{ft}^3/\text{min}}{\text{psi}}\right)\left(1.0\frac{\text{min}}{\text{ft}^2}\right)\left(4\frac{\text{mA}}{\text{ft}}\right) = 3.2$$

$$\frac{H'(s)}{H'_{sp}(s)} = \frac{3.2\left(\frac{3s+1}{3s}\right)\left(\frac{1}{3s+1}\right)}{1+3.2\left(\frac{3s+1}{3s}\right)\left(\frac{1}{3s+1}\right)} = \frac{3.2}{3s+3.2} = \frac{1.0}{0.94s+1}$$

For $H'_{sp}(s) = \dfrac{(3-2)}{s} = \dfrac{1}{s}$

$$h'(t) = 1 - e^{-1.07\tau}$$

$$t = -0.94\ln[1 - h'(t)]$$

$h(t) = 2.5$ ft $\quad h'(t) = 0.5$ ft $\quad t = 0.65$ min

$h(t) = 3.0$ ft $\quad h'(t) = 1.0$ ft $\quad t \to \infty$

Therefore,

$$h(t = 0.65\,\text{min}) = 2.5\text{ft}$$

$$h(t \to \infty) = 3.0\text{ ft}$$

11.3

$G_c(s) = K_c = 5$ ma/ma

Assume $\tau_m = 0$, $\tau_v = 0$, and $K_1 = 1$, in Fig 11.7.

a) Offset = $T'_{sp}(\infty) - T'(\infty) = 5^\circ F - 4.14^\circ F = 0.86^\circ F$

b) $$\frac{T'(s)}{T'_{sp}(s)} = \frac{K_m K_c K_{IP} K_v \left(\frac{K_2}{\tau s+1}\right)}{1 + K_m K_c K_{IP} K_v \left(\frac{K_2}{\tau s+1}\right)}$$

Using the standard current range of 4-20 ma,

$$K_m = \frac{20 \text{ ma} - 4 \text{ ma}}{50^\circ F} = 0.32 \text{ ma}/^\circ\text{F}$$

$$K_v = 1.2 \quad , \quad K_{IP} = 0.75 \text{ psi/ma} \quad , \quad \tau = 5 \text{ min} \; , \quad T'_{sp}(s) = \frac{5}{s}$$

$$T'(s) = \frac{7.20K_2}{s(5s + 1 + 1.440K_2)}$$

$$T'(\infty) = \lim_{s\to 0} sT'(s) = \frac{7.20K_2}{(1 + 1.440K_2)}$$

$$T'(\infty) = 4.14^\circ \text{F} \qquad K_2 = 3.34^\circ \text{F/psi}$$

c) From Fig. 11.7, since $T_i' = 0$

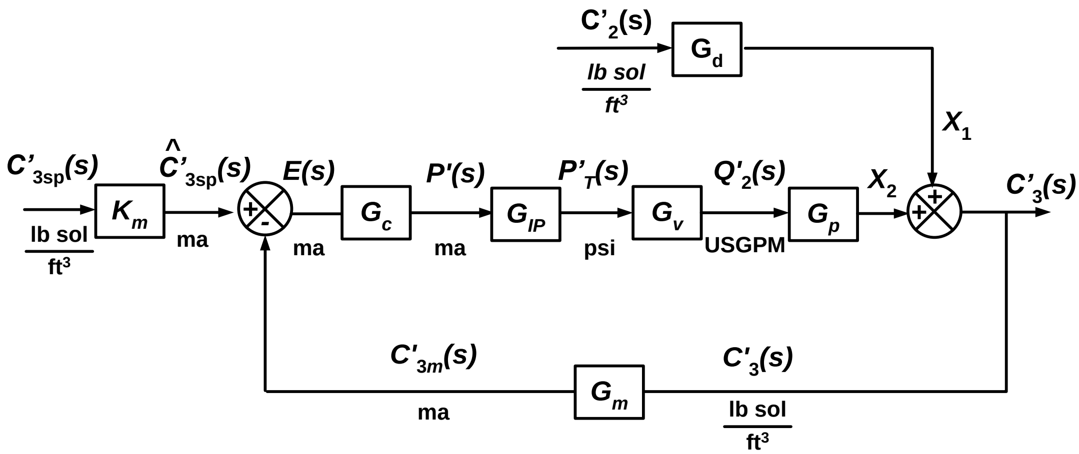
$$P_t'(\infty)K_vK_2 = T'(\infty) \quad , \qquad P_t'(\infty) = 1.03 \text{ psi}$$

and $\quad \overline{P}_t'K_vK_2 + \overline{T}_iK_1 = \overline{T} \quad , \quad \overline{P}_t = 3.74 \text{ psi}$

$$P_t(\infty) = \overline{P}_t - P_t'(\infty) = 4.77 \text{ psi}$$

11.4

(a) Controlled variable: c_3
Manipulated variable: q_2
Disturbance variable: c_2 (note: q_1 and c_1 are kept constant.)
If c_2 changes, then q_2 must be adjusted to keep c_3 at the set point.



(b)

$G_m(s) = K_m e^{-\theta_m s}$ assuming $\tau_m = 0$

$$G_m(s) = \frac{(20-4)\text{ma}}{(9-3)\frac{\text{lb sol}}{\text{ft}^3}} e^{-2s} = \left(2.67 \frac{\text{ma}}{\text{lb sol/ft}^3}\right) e^{-2s}$$

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

$$G_{IP}(s) = K_{IP} = 0.3 \text{ psi/ma}$$

$$G_v(s) = K_v = \frac{(10-20)\text{ USGPM}}{(12-6)\ psi} = -1.67 \frac{\text{USGPM}}{\text{psi}}$$

For process and disturbance transfer function:

Overall material balance for the tank,

$$\left(7.481 \frac{\text{USgallons}}{\text{ft}^3}\right) A \frac{dh}{dt} = q_1 + q_2 - q_3$$

As *h* is held constant at 4 ft by the overflow pipe:

$$0 = 10 + 15 - \bar{q}_3 \qquad (1)$$

Thus $\boxed{\bar{q}_3 = 25}$

Component balance for the solute,

$$7.481\,Ah\frac{d(c_3)}{dt} = q_1c_1 + q_2c_2 - q_3c_3 \qquad (2)$$

Linearize each term on the right hand side of Eq. 2 as described in Section 4.3:

$$\begin{aligned} q_1c_1 &= \bar{q}_1\bar{c}_1 + \bar{q}_1c'_1 + q'_1\,\bar{c}_1 \\ q_2c_2 &= \bar{q}_2\bar{c}_2 + \bar{q}_2c'_2 + q'_2\,\bar{c}_2 \\ q_3c_3 &= \bar{q}_3\bar{c}_3 + \bar{q}_3c'_3 + q'_3\,\bar{c}_3 \end{aligned} \qquad (3)$$

At steady state:

$$0 = \bar{q}_1\bar{c}_1 + \bar{q}_2\bar{c}_2 - \bar{q}_3\bar{c}_3 \qquad (4)$$

Put (2) into deviation variable by considering (3) and (4):

$$7.481\,Ah\frac{dc'_3}{dt} = \bar{q}_2c'_2 + q'_2\,\bar{c}_2 - \bar{q}_3c'_3 - q'_3\,\bar{c}_3$$

As q_1 is constant, $q'_3 = q'_2$:

$$7.481\,Ah\frac{dc'_3}{dt} = \bar{q}_2c'_2 + q'_2\,\bar{c}_2 - \bar{q}_3c'_3 - q'_2\,\bar{c}_3$$

$$7.481\,Ah\frac{dc'_3}{dt} = \bar{q}_2c'_2 + q'_2\,(\bar{c}_2 - \bar{c}_3) - \bar{q}_3c'_3 \qquad (5)$$

Taking Laplace transform and rearranging gives

$$\boxed{C'_3(s) = \frac{K_1}{\tau s+1}Q'_2(s) + \frac{K_2}{\tau s+1}C'_2(s)} \qquad (6)$$

where $K_1 = \frac{\bar{c}_2 - \bar{c}_3}{\bar{q}_3} = 0.08\frac{\text{lb sol/ft}^3}{\text{USGPM}}$, $K_2 = \frac{\bar{q}_2}{\bar{q}_3} = 0.6$ and $\tau = \frac{7.481Ah}{\bar{q}_3} = 15\text{ min}$

since $A = \pi D^2/4 = 12.6\text{ ft}^2$, and $h = 4\ ft$.

Therefore, $\boxed{G_p(s) = \frac{0.08}{15s+1}}$ and $\boxed{G_d(s) = \frac{0.6}{15s+1}}$

(c)

The closed-loop responses for disturbance changes and for setpoint changes can be obtained using block diagram algebra for the block diagram in part (a).

Therefore, these responses will change only if any of the transfer functions in the blocks of the diagram change.

i. $\bar{c}_2$ changes. Then block transfer function $G_p(s)$ changes due to K_1. Hence $G_c(s)$ does need to be changed, and retuning is required.

ii. K_m changes. The close loop transfer functions changes, hence G_c(s) needs to be adjusted to compensate for changes in G_m and K_m. The PI controller should be retuned.

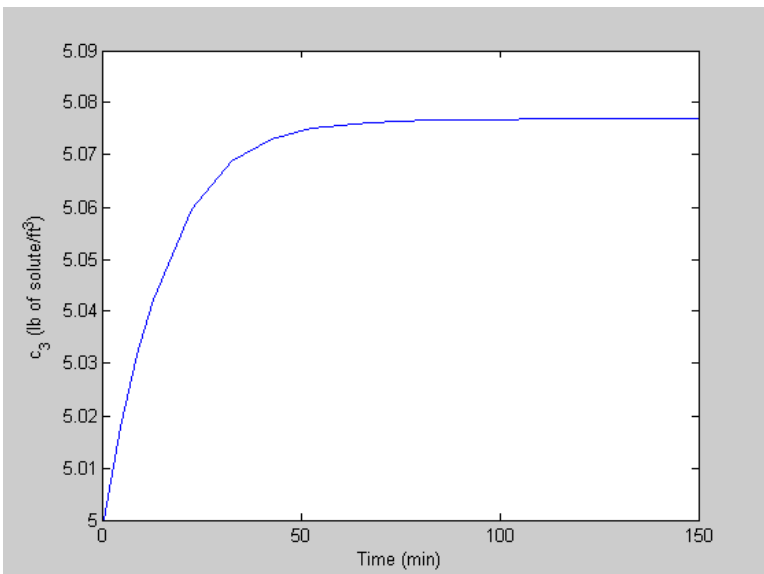
iii. K_m remains unchanged when zero is adjusted. The controller does not need to be retuned.

To verify the linearization results, the nonlinear model is used:

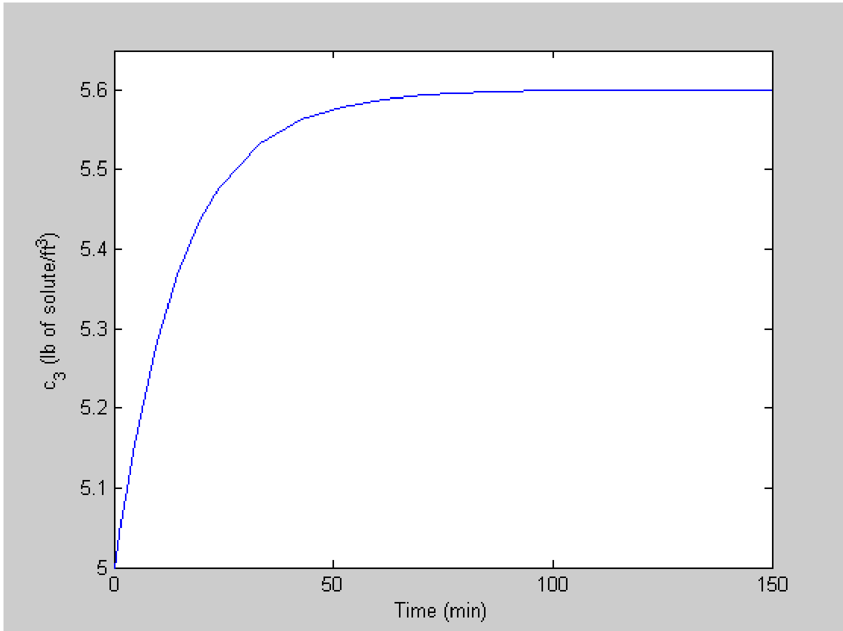
$$7.481\, Ah \frac{d(c_3)}{dt} = q_1 c_1 + q_2 c_2 - q_3 c_3$$

$$q_1 + q_2 = q_3$$

Step response of c_3 to q_2: (Gain 0.077 compared with linearized gain (K_p) 0.08 in Eq. 6)



Step response of c_3 to c_2: (Gain 0.6 compared with linearized gain (K_d) 0.6 in Eq. 6)



The results agree with linearization.

11.5

(a)

From Eq. 11-26 we get the closed loop transfer function for set point changes

$$\frac{Y}{Y_{sp}} = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m}$$

Substituting the information from the problem gives

$$\frac{Y}{Y_{sp}} = \frac{\frac{4}{s(s+4)}}{1 + \frac{4}{s(s+4)}} = \frac{4}{s(s+4)+4} = \frac{4}{s^2+4s+4}$$

Or in standard form (Eq. 5-40), with $\tau = \frac{1}{2}$ and $\zeta = 1$

$$\frac{Y}{Y_{sp}} = \frac{1}{\frac{1}{4}s^2 + s + 1}$$

(b)

Given a unit step change in set point we obtain

$$Y(s) = \frac{4}{s(s^2 + 4s + 4)}$$

Using the Final Value Theorem we get

$$\lim_{s \to 0} sY(s) = \frac{4}{s^2 + 4s + 4} = \frac{4}{4} = 1$$

Therefore $y(\infty) = 1$

(c) As the step change is a unit step change, and we have shown that $y(\infty) = 1$, we can say that offset = 0. This is consistent with the fact that the gain of the overall transfer function is 1, so no offset will occur. Normally proportional control does not eliminate offset, but it does for this integrating process.

(d) Using Eq. 5-50 or taking the inverse Laplace transform of the response given above we get

$$y(t) = 1 - (1 + 2t)e^{-2t}$$

Substituting the value of 0.5 for t gives

$$y(t) = 0.264$$

(e) We can tell from the response derived above that the response will not be oscillatory, since $\zeta = 1$.

11.6

For proportional controller, $G_c(s) = K_c$

Assume that the level transmitter and the control valve have negligible dynamics. Then,

$$G_m(s) = K_m$$
$$G_v(s) = K_v$$

The block diagram for this control system is the same as in Fig.11.8. Hence Eqs. 11-26 and 11-29 can be used for closed-loop responses to setpoint and load changes, respectively.

The transfer functions $G_p(s)$ and $G_d(s)$ are as given in Eqs. 11-66 and 11-67, respectively.

a) Substituting for G_c, G_m, G_v, and G_p into Eq. 11-26 gives

$$\frac{Y}{Y_{sp}} = \frac{K_m K_c K_v\left(-\frac{1}{As}\right)}{1+K_c K_v\left(-\frac{1}{As}\right)K_m} = \frac{1}{\tau s+1}$$

where $\tau = -\dfrac{A}{K_c K_v K_m}$ (1)

For a step change in the setpoint, $Y_{sp}(s) = M/s$

$$Y(t\to\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\left[\frac{M/s}{\tau s+1}\right] = M$$

Offset = $Y_{sp}(t\to\infty) - Y(t\to\infty) = M - M = 0$

b) Substituting for G_c, G_m, G_v, G_p , and G_d into (11-29) gives

$$\frac{Y(s)}{D(s)} = \frac{\left(\frac{1}{As}\right)}{1+K_c K_v\left(-\frac{1}{As}\right)K_m} = \frac{\left(\frac{-1}{K_c K_v K_m}\right)}{\tau s+1}$$

where τ is given by Eq. 1.
For a step change in the disturbance, $D(s) = M/s$

$$Y(t\to\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\left[\frac{-M/(K_c K_v K_m)}{s(\tau s+1)}\right] = \frac{-M}{K_c K_v K_m}$$

Offset = $Y_{sp}(t\to\infty) - Y(t\to\infty) = 0 - \left(\dfrac{-M}{K_c K_v K_m}\right) \neq 0$

Hence, offset is not eliminated for a step change in disturbance.

11.7

Using block diagram algebra

$$Y = G_d D + G_p U \tag{1}$$

$$U = G_c\left[Y_{sp} - \left(Y - \tilde{G}_p U\right)\right] \tag{2}$$

From (2), $U = \dfrac{G_c Y_{sp} - G_c Y}{1 - G_c \tilde{G}_p}$

Substituting for U in Eq. 1

$$\left[1 + G_c(G_p - \tilde{G}_p)\right]Y = G_d(1 - G_c\tilde{G}_p)D + G_p G_c Y_{sp}$$

Therefore,

$$\frac{Y}{Y_{sp}} = \frac{G_p G_c}{1 + G_c(G_p - \tilde{G}_p)}$$

and

$$\frac{Y}{D} = \frac{G_d(1 - G_c\tilde{G}_p)}{1 + G_c(G_p - \tilde{G}_p)}$$

11.8

The available information can be translated as follows

1. The outlets of both the tanks have flow rate q_0 at all times.

2. $T_o(s) = 0$

3. Since an energy balance would indicate a first-order transfer function between T_1 and Q_0 ,

$$\frac{T'(t)}{T'(\infty)} = 1 - e^{-t/\tau_1} \quad \text{or} \quad \frac{2}{3} = 1 - e^{-12/\tau_1}$$, τ_1 = 10.9 min

Therefore

$$\frac{T_1(s)}{Q_0(s)} = \frac{3^\circ F / (-0.75 gpm)}{10.9s+1} = -\frac{4}{10.9s+1}$$

$$\frac{T_3(s)}{Q_0(s)} = \frac{(5-3)^\circ F / (-0.75 gpm)}{\tau_2 s+1} = -\frac{2.67}{\tau_2 s+1} \qquad \text{for} \quad T_2(s) = 0$$

4. $$\frac{T_1(s)}{V_1(s)} = \frac{(78-70)^\circ F / (12-10)V}{10s+1} = \frac{4}{10s+1}$$

$$\frac{T_3(s)}{V_2(s)} = \frac{(90-85)^\circ F / (12-10)V}{10s+1} = \frac{2.5}{10s+1}$$

5. $5\tau_2$ =50 min or τ_2 = 10 min

Since inlet and outlet flow rates for tank 2 are q_0 and the volumes of the tanks are equal,

$$\frac{T_3(s)}{T_2(s)} = \frac{\bar{q}_0 / \bar{q}_0}{\tau_2 s+1} = \frac{1}{10.0s+1}$$

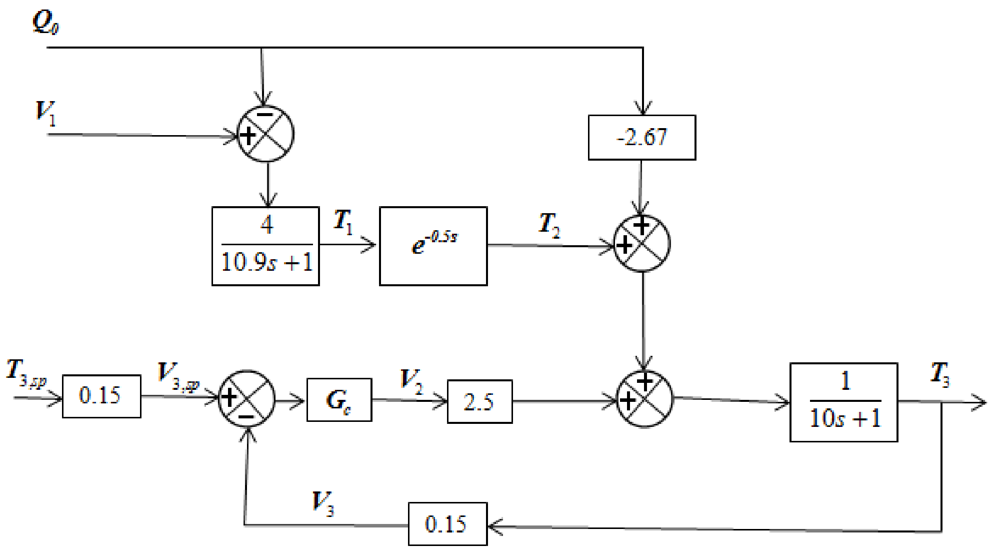
6. $$\frac{V_3(s)}{T_3(s)} = 0.15$$

7. $$T_2(t) = T_1\left(t - \frac{30}{60}\right) = T_1(t-0.5)$$

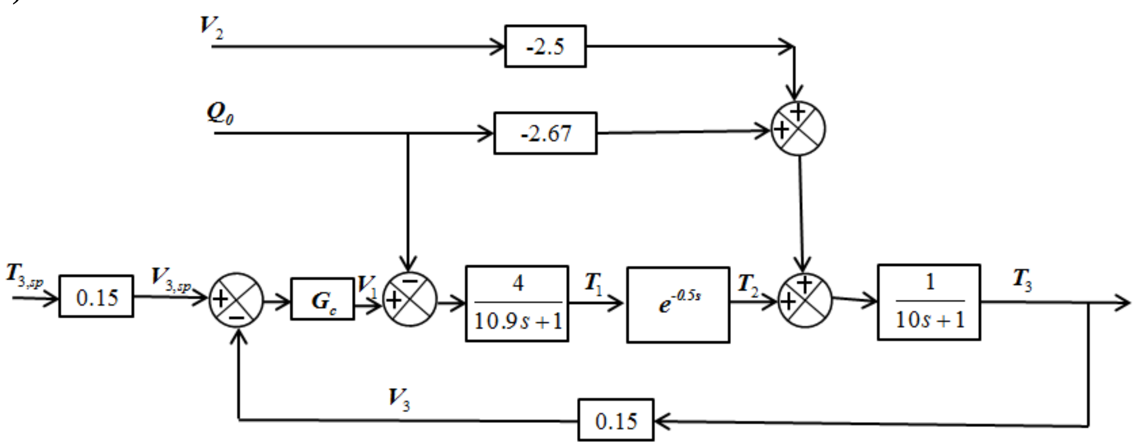
$$\frac{T_2(s)}{T_1(s)} = e^{-0.5s}$$

Using these transfer functions, the block diagrams are as follows.

a)



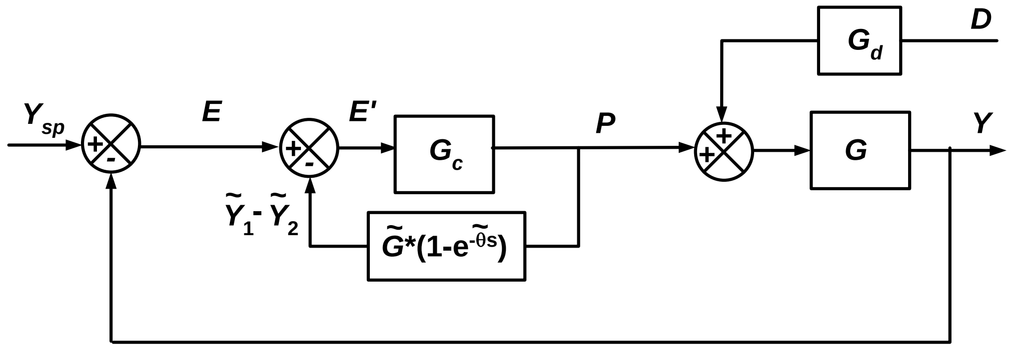
b)



c) The control configuration in part a) will provide the better control. As is evident from the block diagrams above, the feedback loop contains, in addition to G_c, only a first-order process in part a), but a second-order-plus-time-delay process in part b). Hence the controlled variable responds faster to changes in the manipulated variable for part a).

11.9

The given block diagram is equivalent to



For the inner loop, let

$$\frac{P}{E} = G_c' = \frac{G_c}{1 + G_c \tilde{G}^* (1 - e^{-\tilde{\theta}s})}$$

In the outer loop, we have

$$\frac{Y}{D} = \frac{G_d G}{1 + G_c' G}$$

Substitute for G_c',

$$\frac{Y}{D} = \frac{G_d G}{1 + \dfrac{G_c G}{1 + G_c \tilde{G}^* (1 - e^{-\tilde{\theta}s})}}$$

$$\frac{Y}{D} = \frac{G_d G \left(1 + G_c \tilde{G}^* (1 - e^{-\tilde{\theta}s})\right)}{1 + G_c \tilde{G}^* (1 - e^{-\tilde{\theta}s}) + G_c G}$$

11.10

a) Derive CLTF:

$$Y = Y_3 + Y_2 = G_3 Z + G_2 P$$

$$Y = G_3(D + Y_1) + G_2 K_c E$$

$$Y = G_3 D + G_3 G_1 K_c E + G_2 K_c E$$

$$Y = G_3 D + (G_3 G_1 K_c + G_2 K_c)E \qquad E = -K_m Y$$

$$Y = G_3 D - K_c (G_3 G_1 + G_2) K_m Y$$

$$\frac{Y}{D} = \frac{G_3}{1 + K_c (G_3 G_1 + G_2) K_m}$$

b) Characteristic Equation:

$$1 + K_c (G_3 G_1 + G_2) K_m = 0$$
$$1 + K_c \left[\frac{5}{s-1} + \frac{4}{2s+1} \right] = 0$$

$$1 + K_c \left[\frac{5(2s+1) + 4(s-1)}{(s-1)(2s+1)} \right] = 0$$

$$(s-1)(2s+1) + K_c [5(2s+1) + 4(s-1)] = 0$$

$$2s^2 - s - 1 + K_c (10s + 5 + 4s - 4) = 0$$

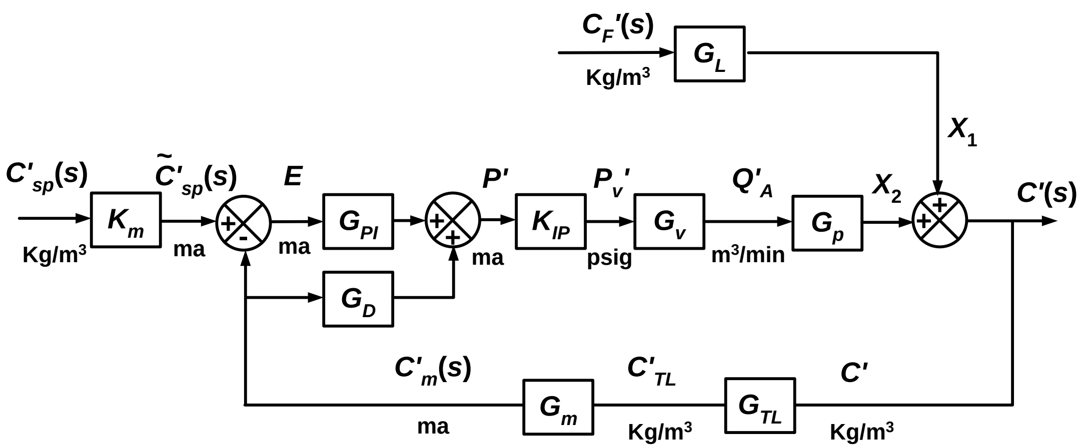
$$2s^2 + (14K_c - 1)s + (K_c - 1) = 0$$

Necessary conditions: $K_c > 1/14$ and $K_c > 1$

For a 2nd order characteristic equation, these conditions are also sufficient. Therefore, $K_c > 1$ for closed-loop stability.

11.11

a)



b)

Transfer Line:

Volume of transfer line = $\pi/4\,(0.5\text{ m})^2(20\text{m}) = 3.93\text{ m}^3$

Nominal flow rate in the line = $\bar{q}_A + \bar{q}_F = 7.5\text{ m}^3/\text{min}$

Time delay in the line = $\dfrac{3.93\text{ m}^3}{7.5\text{ m}^3/\text{min}} = 0.52\,\text{min}$

$$G_{TL}(s) = e^{-0.52s}$$

Composition Transmitter:

$$G_m(s) = K_m = \frac{(20-4)\text{ ma}}{(200-0)\text{ kg/m}^3} = 0.08\frac{\text{ma}}{\text{kg/m}^3}$$

Controller

From the ideal controller in Eq. 8-14

$$P'(s) = K_c\left(1+\frac{1}{\tau_I s}\right)E(s) + K_c\tau_D s\left[\tilde{C}'_{sp}(s) - C'_m(s)\right]$$

In the above equation, set $\tilde{C}'_{sp}(s) = 0$ in order to get the derivative on the process output only. Then,

$$G_{PI}(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

$$G_D(s) = -K_c \tau_D s$$

with K_c >0 as the controller should be reverse-acting, since $P(t)$ should increase when $C_m(t)$ decreases.

<u>I/P transducer</u>

$$K_{IP} = \frac{(15-3)\text{ psig}}{(20-4)\text{ ma}} = 0.75\frac{\text{psig}}{\text{ma}}$$

<u>Control valve</u>

$$G_v(s) = \frac{K_v}{\tau_v s + 1}$$

$$5\tau_v = 1 \qquad , \qquad \tau_v = 0.2 \text{ min}$$

$$K_v = \left.\frac{dq_A}{dp_v}\right|_{p_v=\bar{p}_v} = 0.03(1/12)(\ln 20)(20)^{\frac{\bar{p}_v-3}{12}}$$

$$q_A = 0.5 = 0.17 + 0.03(20)^{\frac{\bar{p}_v-3}{12}}$$

$$0.03(20)^{\frac{\bar{p}_v-3}{12}} = 0.5 - 0.17 = 0.33$$

$$K_v = (1/12)(\ln 20)(0.33) = 0.082\frac{\text{m}^3/\text{min}}{\text{psig}}$$

$$G_v(s) = \frac{0.082}{0.2s+1}$$

<u>Process</u>

Assume c_A is constant for pure A. Material balance for A:

$$V\frac{dc}{dt} = q_A\bar{c}_A + \bar{q}_F c_F - (q_A + \bar{q}_F)c \qquad (1)$$

Linearizing and writing in deviation variable form

$$V\frac{dc'}{dt} = \bar{c}_A q'_A + \bar{q}_F c'_F - (\bar{q}_A + \bar{q}_F)c' - \bar{c}q'_A$$

Taking Laplace transform

$$[Vs + (\bar{q}_A + \bar{q}_F)]C'(s) = (\bar{c}_A - \bar{c})Q'_A(s) + \bar{q}_F C'_F(s) \qquad (2)$$

From Eq. 1 at steady state, $dc/dt = 0$,

$$\bar{c} = (\bar{q}_A\bar{c}_A + \bar{q}_F\bar{c}_F)/(\bar{q}_A + \bar{q}_F) = 100 \text{ kg/m}^3$$

Substituting numerical values in Eq. 2,

$$[5s + 7.5]C'(s) = 700\,Q'_A(s) + 7\,C'_F(s)$$

$$[0.67s + 1]C'(s) = 93.3Q'_A(s) + 0.93C'_F(s)$$

$$G_p(s) = \frac{93.3}{0.67s+1}$$
$$G_d(s) = \frac{0.93}{0.67s+1}$$

11.12

The stability limits are obtained from the characteristic Eq. 11-83. Hence if an instrumentation change affects this equation, then the stability limits will change and vice-versa.

a) The transmitter gain, K_m, changes as the span changes. Thus $G_m(s)$ changes and the characteristic equation is affected. Stability limits would be expected to change.

b) The zero on the transmitter does not affect its gain K_m. Hence $G_m(s)$ remains unchanged and stability limits do not change.

c) Changing the control valve trim changes $G_\gamma(s)$. This affects the characteristic equation and the stability limits would be expected to change as a result.

11.13

(a) $1+\frac{K_c}{(5s+1)(s+1)}=0 \Leftrightarrow 5s^2+6s+1+K_c=0$

Applying the quadratic formula yields the roots:

$$s=\frac{-6\pm\sqrt{36-20(1+K_c)}}{10}$$

To have a stable system, both roots of the characteristic equation must have negative real parts. Thus, $20(1+K_c)>0 \Rightarrow K_c>-1$

(b) $1+\frac{K_c\left(1+\frac{1}{\tau_I s}\right)}{(5s+1)(s+1)}=0 \Leftrightarrow \tau_I\left(5s^3+6s^2+s+K_cs\right)+K_c=0$

- When $\tau_I=0.1$, $0.5s^3+0.6s^2+0.1(1+K_c)s+K_c=0$

Using direct substitution, and set $s=j\omega$:

$$\left(-0.5\omega^3+0.1(1+K_c)\omega\right)j-0.6\omega^2+K_c=0$$

Re: $-0.6\omega^2+K_c=0$ (1)

Im: $-0.5\omega^3+0.1(1+K_c)\omega=0$ (2)

$$\omega\neq 0: K_{cm}=0.136$$

To have a stable system, we have:

$0<K_c<0.136$

- When $\tau_I=1$, $5s^3+6s^2+(1+K_c)s+K_c=0$

Set $s=j\omega$:

$$\left(-5\omega^3+(1+K_c)\omega\right)j-6\omega^2+K_c=0$$

Re: $-6\omega^2+K_c=0$ (1)

Im: $-5\omega^3+(1+K_c)\omega=0$ (2)

$$\omega\neq 0: K_{cm}=-6$$

To have a stable system, we have:

$K_c>0$

- When $\tau_I = 10$, $50s^3 + 60s^2 + 10(1+K_c)s + K_c = 0$

Set $s = j\omega$:

$$\left(-50\omega^3 + 10(1+K_c)\omega\right)j - 60\omega^2 + K_c = 0$$

Re: $-60\omega^2 + K_c = 0$ (1)

Im: $-50\omega^3 + 10(1+K_c)\omega = 0$ (2)

$$\omega \neq 0 : K_{cm} = -1.09$$

To have a stable system, we have:

$K_c > 0$

(c) Adding larger amounts of integral weighting (decreasing τ_I) will destabilize the system

11.14

From the block diagram, the characteristic equation is obtained as

$$1 + K_c\left[\frac{(1)\left(\frac{2}{s+3}\right)}{1+(1)\left(\frac{2}{s+3}\right)}\right]\left[\frac{2}{s-1}\right]\left[\frac{1}{s+10}\right] = 0$$

that is,

$$1 + K_c\left[\frac{2}{s+5}\right]\left[\frac{2}{s-1}\right]\left[\frac{1}{s+10}\right] = 0$$

Simplifying,

$$s^3 + 14s^2 + 35s + (4K_c - 50) = 0$$

Set $s = j\omega$:

$$\left(-\omega^3 + 35\omega\right)j - 14\omega^2 + \left(4K_c - 50\right) = 0$$

Re: $-14\omega^2 + 4K_c - 50 = 0$ (1)

Im: $(j) - \omega^3 + 35\omega = 0$ (2)

$$\omega \neq 0 : K_{cm} = 135$$

$$\omega = 0 : K_{cm} = 12.5$$

$$12.5 < K_c < 135$$

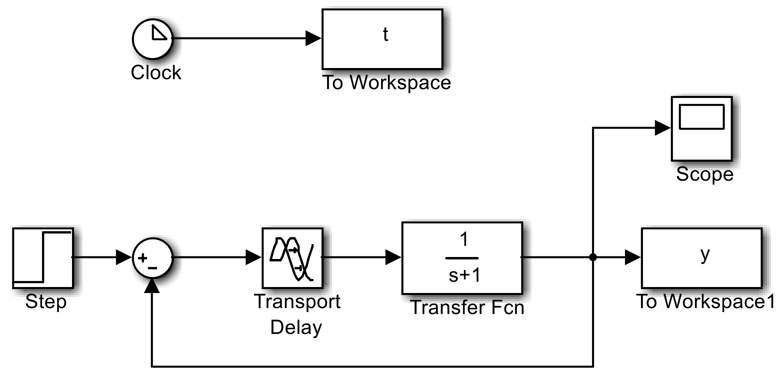
11.15

Substituting the transfer functions into the characteristic equation in (11-81) gives:

$$\frac{Y}{Y_{sp}} = \frac{K_m G_c G_v G_p}{1+G_c G_v G_p G_m} = \frac{K_c K_v e^{-\theta s} \dfrac{K_p}{\tau_p s+1}}{1+K_c K_v e^{-\theta s} \dfrac{K_p}{\tau_p s+1}} = \frac{K_c K_v K_p e^{-\theta s}}{\tau_p s+1+K_c K_v K_p e^{-\theta s}}$$

Let $K_c = K_v = K_p = \tau_p = \theta = 1$, we have $\dfrac{Y}{Y_{sp}} = \dfrac{e^{-s}}{s+1+e^{-s}}$; thus, $G_{OL} = \dfrac{e^{-s}}{1+s}$

Simulate the above relation through MATLAB, we have:



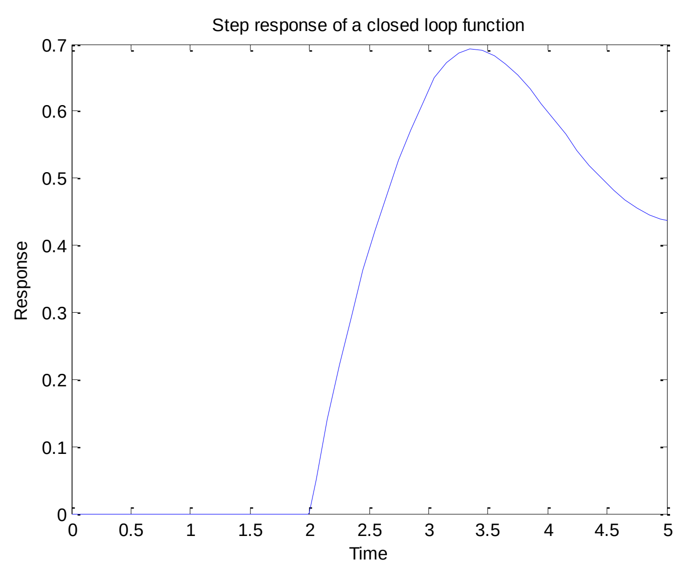


Figure S11.15 *Step response of a closed loop function*

As shown in the figure, the time delay will not lead to an inverse response.

11.16

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

$$G_v(s) = \frac{K_v}{(10/60)s+1} = \frac{-1.3}{0.167s+1}$$

$$G_p(s) = -\frac{1}{As} = -\frac{1}{22.4s} \qquad \text{since} \quad A = 3\ \text{ft}^2 = 22.4\frac{\text{gal}}{\text{ft}}$$

$$G_m(s) = K_m = 4$$

Characteristic equation is

$$1+K_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{-1.3}{0.167s+1}\right)\left(\frac{-1}{22.4s}\right)(4) = 0$$

$$(3.73\tau_I)s^3 + (22.4\tau_I)s^2 + (5.2K_c\tau_I)s + (5.2K_c) = 0$$

Use direct substitution, and set $s = j\omega$:

$$\left(-3.73\tau_I\omega^3 + 5.2K_c\tau_I\omega\right) j - 22.4\tau_I\omega^2 + 5.2K_c = 0$$

Re: $\quad -22.4\tau_I\omega^2 + 5.2K_c = 0 \qquad (1)$

Im: $\quad (j) - 3.73\tau_I\omega^3 + 5.2K_c\tau_I\omega = 0 \qquad (2)$

$$\omega \neq 0 : \tau_{cm} = 0.167$$

To have a stable system, we have:

$K_c > 0, \tau_I > 0.167$

11.17

$$G_{OL}(s) = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)\left(\frac{5}{(10s+1)^2}\right) = \frac{N(s)}{D(s)}$$

$$D(s) + N(s) = \tau_I s(100s^2 + 20s + 1) + 5K_c(\tau_I s + 1) = 0$$

$$= 100\tau_I s^3 + 20\tau_I s^2 + (1+5K_c)\tau_I s + 5K_c = 0$$

Set $s = j\omega$, we have:

$$\left[-100\tau_I\omega^3 + (1+5K_c)\tau_I\omega\right] j - 20\tau_I\omega^2 + 5K_c = 0$$

Re: $$-20\tau_I\omega^2 + 5K_c = 0 \qquad (1)$$

Im: $$(j) - 100\tau_I\omega^3 + (1+5K_c)\tau_I\omega = 0 \ (2)$$

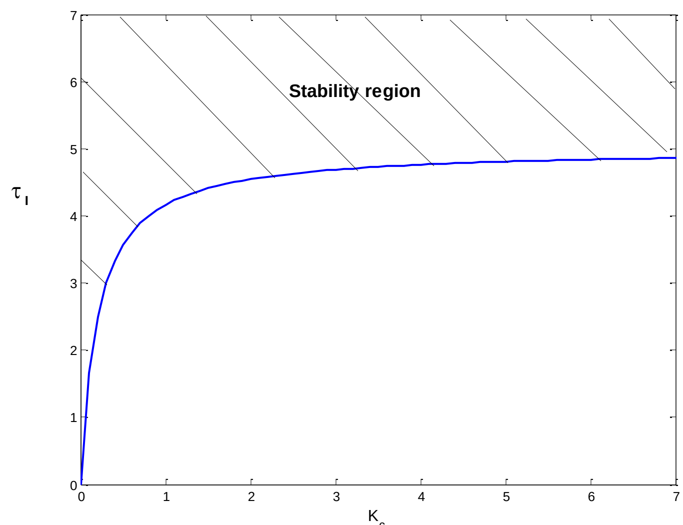
$$\omega \neq 0 : \tau_{\text{Im}} = \frac{25K_c}{1+5K_c}$$

$$\omega = 0 : K_{cm} = 0$$

To have a stable system, we have:

$$K_c > 0, \tau_I > \frac{25K_c}{1+5K_c}$$

The stability region is shown in the figure below:



c) Find τ_I as $K_c \to \infty$

$$\lim_{K_c\to\infty}\left[\frac{25K_c}{1+5K_c}\right] = \lim_{K_c\to\infty}\left[\frac{25}{1/K_c + 5}\right] = 5$$

$\therefore$ $\tau_I > 5$ guarantees stability for any value of K_c. Appelpolscher is wrong yet again.

11.18

$$G_c(s) = K_c$$

$$G_V(s) = \frac{K_V}{\tau_V s + 1}$$

$$K_v = \left.\frac{dw_s}{dp}\right|_{p=12} = \frac{0.6}{2\sqrt{12-4}} = 0.106\,\frac{\text{lbm/s}}{\text{ma}}$$

$$5\tau_v = 20\text{ s} \qquad \tau_v = 4\text{ s}$$

$$G_p(s) = \frac{2.5e^{-s}}{10s+1}$$

$$G_m(s) = K_m = \frac{(20-4)\text{ ma}}{(160-120)^\circ F} = 0.4\,\frac{\text{ma}}{^\circ F}$$

Characteristic equation is

$$1 + (K_c)\left(\frac{0.106}{4s+1}\right)\left(\frac{2.5e^{-s}}{10s+1}\right)(0.4) = 0 \qquad (1)$$

a) Substituting s=jω in (1) and using Euler's identity

$$e^{-j\omega} = \cos\omega - j\sin\omega$$

gives

$$-40\omega^2 + 14j\omega + 1 + 0.106\,K_c(\cos\omega - j\sin\omega) = 0$$

Thus

$$-40\omega^2 + 1 + 0.106\,K_c\cos\omega = 0 \qquad (2)$$

and $$14\omega - 0.106K_c\sin\omega = 0 \qquad (3)$$

From (2) and (3),

$$\tan\omega = \frac{14\omega}{40\omega^2 - 1} \qquad (4)$$

Solving (4), $\omega = 0.579$ by trial and error.

Substituting for ω in (3) gives

$$K_c = 139.7 = K_{cm}$$

Frequency of oscillation is 0.579 rad/sec

b) Substituting the Pade approximation into (1) gives:

$$e^{-s} \approx \frac{1-0.5s}{1+0.5s}$$

$$20s^3 + 47s^2 + (14.5 - 0.053K_c)s + (1 + 0.106K_c) = 0$$

Substituting s=jω in above equation, we have:

$$-47\omega^2 + 1 + 0.106K_c + \left[-20\omega^3 + 14.5\omega - 0.053K_c\omega\right] j = 0$$

Thus, we have:

$$\begin{cases} -47\omega^2 + 1 + 0.106\,\mathrm{K}_c = 0 \\ -20\omega^3 + 14.5\omega - 0.053K_c\omega = 0 \end{cases} \Rightarrow \begin{cases} \omega = 0.587 \\ K_c = 143.46 \end{cases}$$

Therefore, the maximum gain, K_c = 143.46, is a satisfactory approximation of the true value of 139.7 in (a) above

11.19

a) $$G(s) = \frac{4(1-5s)}{(25s+1)(4s+1)(2s+1)}$$

$$G_c(s) = K_c$$

$$D(s) + N(s) = (25s+1)(4s+1)(2s+1) + 4K_c(1-5s) = 0$$

$$200s^3 + 158s^2 + (31 - 20K_c)s + 1 + 4K_c = 0$$

Substituting s=jω in above equation, we have:

$$-158\omega^2 + 1 + 4K_c + \left[-200\omega^3 + 31\omega - 20K_c\omega\right] j = 0$$

Thus, we have:

$$\begin{cases} -158\omega^2 + 1 + 4\,\mathrm{K}_c = 0 \\ -200\omega^3 + 31\omega - 20K_c\omega = 0 \end{cases} \Rightarrow \begin{cases} \omega = 0.191 \\ K_{cm} = 1.19 \end{cases}$$

b) $$(25s+1)(4s+1)(2s+1) + 4K_c = 0$$

$$200s^3 + 158s^2 + 31s + (1 + 4K_c) = 0$$

Substituting s=jω in above equation, we have:

$$-158\omega^2 + 1 + 4K_c + \left[-200\omega^3 + 31\omega\right] j = 0$$

Thus, we have:

$$\begin{cases} -158\omega^2 + 1 + 4\,\mathrm{K}_c = 0 \\ -200\omega^3 + 31\omega = 0 \end{cases} \Rightarrow \begin{cases} \omega = 0.394 \\ K_{cm} = 5.873 \end{cases}$$

c) Because K_c can be much higher without the RHP zero being present, the process can be made to respond faster.

11.20

The characteristic equation is

$$1+\frac{0.5K_c e^{-3s}}{10s+1}=0 \qquad (1)$$

a) Using the Pade approximation

$$e^{-3s}\approx\frac{1-(3/2)s}{1+(3/2)s}$$

in (1) gives

$$15s^2+(11.5-0.75K_c)s+(1+0.5K_c)=0$$

Substituting s=jω in above equation, we have:

$$-15\omega^2+1+0.5K_c+\left[11.5\omega-0.75K_c\omega\right]j=0$$

Thus, we have:

$$\begin{cases}-15\omega^2+1+0.5\mathrm{K}_c=0\\ 11.5\omega-0.75K_c\omega=0\end{cases}\Rightarrow\begin{cases}\omega=0.760\\ K_{cm}=15.33\end{cases}$$

b) Substituting $s = j\omega$ in (1) and using Euler's identity.

$$e^{-3j\omega}=\cos(3\omega)-j\sin(3\omega)$$

gives

$$10j\omega+1+0.5K_c\left[\cos(3\omega)-j\sin(3\omega)\right]=0$$

Then,

$$1+0.5K_c\cos(3\omega)=0 \qquad (2)$$

and $$10\omega-0.5K_c\sin(3\omega)=0 \qquad (3)$$

From (2) and (3)

$$\tan(3\omega) = -10\omega \qquad (4)$$

Eq. 4 has infinite number of solutions. The solution for the range $\pi/2 < 3\omega < 3\pi/2$ is found by trial and error to be $\omega = 0.5805$.
Then from Eq. 2, $K_c = 11.78$
The other solutions for the range $3\omega > 3\pi/2$ occur at values of ω for which $\cos(3\omega)$ is smaller than $\cos(3\times5.805)$. Thus, for all other solutions of ω, Eq. 2 gives values of K_c that are larger than 11.78. Hence, stability is ensured when

$$0 < K_c < 11.78$$

To solve Eqs. 2 and 3, another way is to use Newton's method. With initial guess $K_c = 5$, $\omega = 0$ (steady state), the solution to Eqs. 2 and 3 is:

$$K_c = -2,\ \omega = 0$$

With a different initial guess (e.g., $K_c = 5$, $\omega = 5$), the solution is:

$$K_c = 11.78,\ \omega = 0.5805$$

Again, $\omega_c = 0.5805$ and the stability is ensured when

$$0 < K_c < 11.78$$

(c).

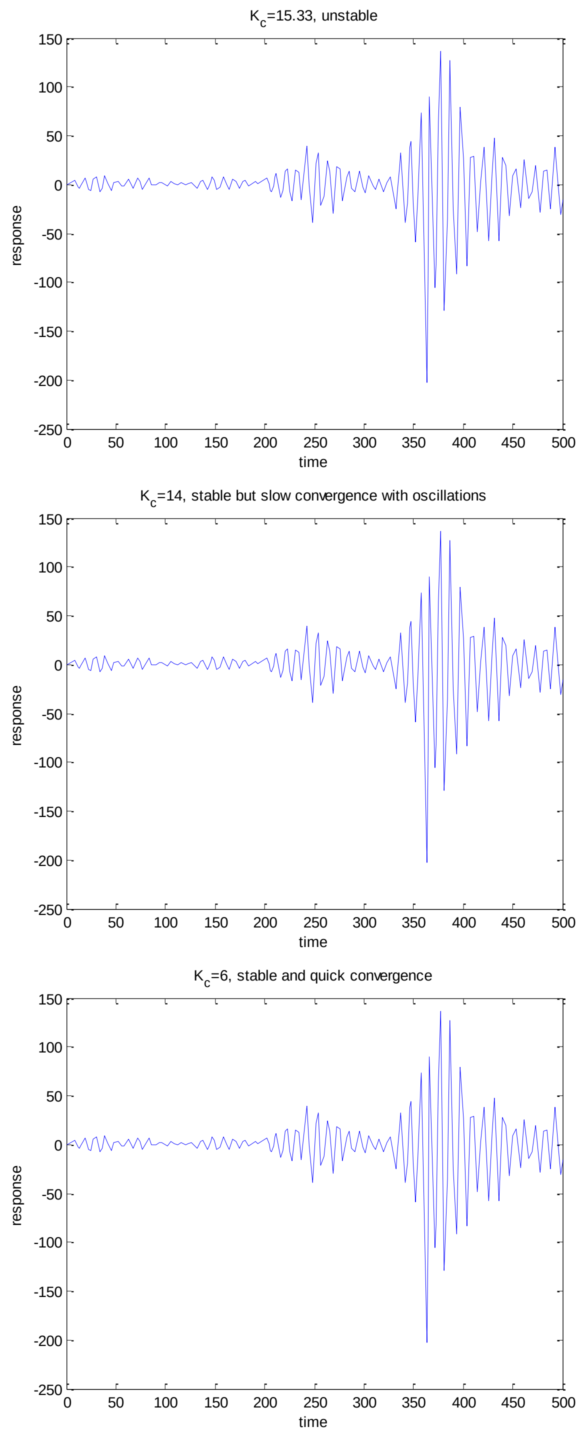


Figure S11.20 *Simulation results of different K_c settings*

11.21

a) To approximate G_{OL}(s) by a FOPTD model, the Skogestad approximation technique in Chapter 6 is used.
Initially,

$$G_{OL}(s) = \frac{3K_c e^{-(1.5+0.3+0.2)s}}{(60s+1)(5s+1)(3s+1)(2s+1)} = \frac{3K_c e^{-2s}}{(60s+1)(5s+1)(3s+1)(2s+1)}$$

Skogestad approximation method to obtain a FOPTD model:

Time constant ≈ 60 + (5/2)

Time delay ≈ 2 +(5/2) + 3 + 2 =9.5

Then

$$G_{OL}(s) \approx \frac{3K_c e^{-9.5s}}{62.5s+1}$$

(b) The characteristic equation is

$$1 + \frac{3K_c e^{-9.5s}}{62.5s+1} = 0 \qquad (1)$$

Substituting $s = j\omega$ in (1) and using Euler's identity.

$$e^{-9.5j\omega} = \cos(9.5\omega) - j\sin(9.5\omega)$$

gives

$$3K_c\cos\left(9.5\omega\right) + 1 + \left[62.5\omega - 3K_c\sin(9.5\omega)\right]j = 0$$

Then,

$$1 + 3K_c\cos(9.5\omega) = 0 \qquad (2)$$

and $$62.5\omega - 3K_c\sin(9.5\omega) = 0 \qquad (3)$$

From (2) and (3)

$$\tan\left(9.5\omega\right) = -62.5\omega \qquad (4)$$

Eq. 4 has infinite number of solutions. The solution for the range $\pi/2 < 9.5\omega < 3\pi/2$ (to make sure K_c is positive) is found by trial and error to be $\omega = 0.1749$.
Then from Eq. 2, $K_c = 3.678$
Hence, stability is ensured when

$$0 < K_c < 3.678$$

c) Conditional stability occurs when $K_c = K_{cu} = 3.678; \omega = 0.1749$

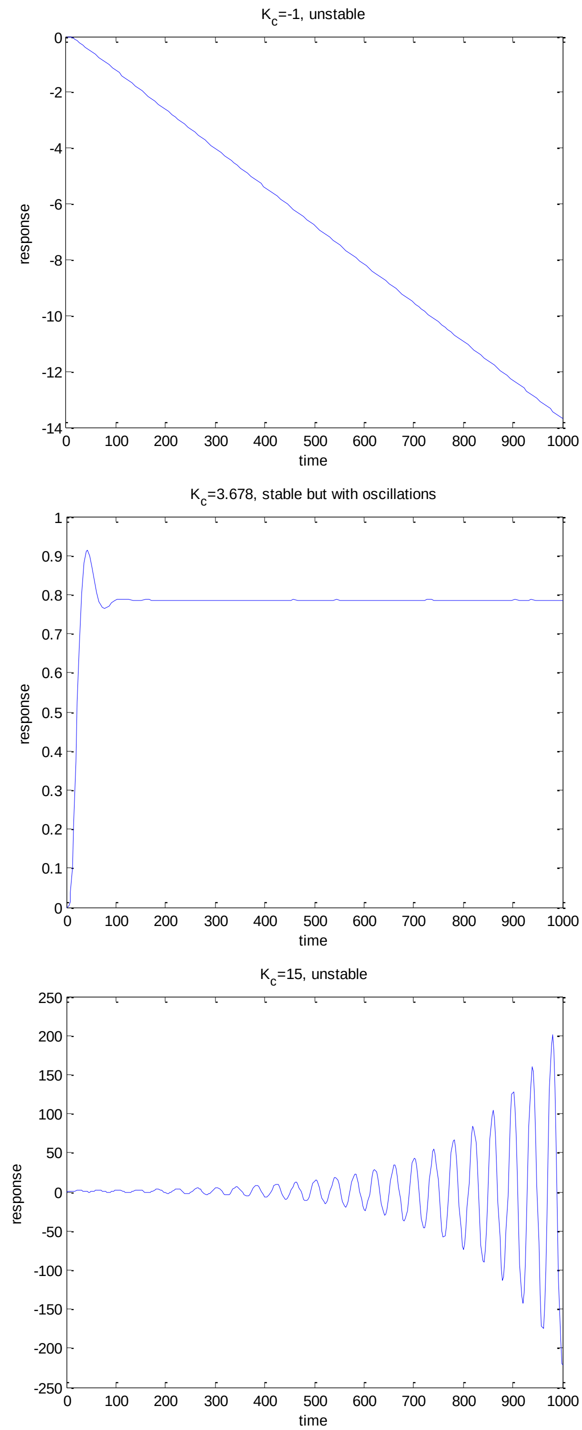


Figure S11.21 *Simulation results of different K_c settings*

11.22

Characteristic equation is:

$$1+G_cG_pG_vG_m = 1+K_c\,\frac{5-as}{(s+1)^3} = \frac{s^3+3s^2+3s+1+5K_c-aK_cs}{(s+1)^3}$$

$$= \frac{s^3+3s^2+(3-aK_c)s+1+5K_cs}{(s+1)^3}$$

A necessary condition for stability is all the coefficients of the numerator are positive.
When $a < 3/K_c$ $(K_c > 0)$, the coefficient of s becomes negative so the control system becomes unstable.

11.23

(a)
Offset = $h_{ss} - h_{final}$ = 22.00 – 21.92 = 0.08 ft
(b)

$$K_m = \frac{20-4 \quad mA}{10 \quad ft} = 1.6 \quad mA/\,ft$$

$$K_{IP} = \frac{15-3}{20-4} = 0.75 \quad psi/mA$$

$K_V = 0.4 \quad cfm/\,psi$ and $K_c = 5$

We have:

$$K_{OL} = K_mK_cK_{IP}K_VG_p = 1.6\times5\times0.75\times0.4K_p = 2.4K_p$$

Offset equals to:

$$offset = \frac{M}{1+K_{OL}} = \frac{22-20}{1+2.4K_p} = 0.08$$

$$K_p = 10\,ft/\,cfm$$

(c)
Add integral action to eliminate offset.

11.24

The open loop process transfer function is:

$$G_p = \frac{K_p}{(\tau_1 s+1)(\tau_2 s+1)} = \frac{2}{(4s+1)(s+1)}$$

The controller transfer function is:

$$G_c = K_c(1+\frac{1}{\tau_I s}) = 2+\frac{1}{2s}$$

(a) According to Eq. 11-26, the closed loop transfer function for set point tracking is:

$$\frac{Y}{Y_{sp}} = \frac{K_m G_p G_c G_v}{1+G_m G_p G_c G_v} = \frac{\frac{2}{(4s+1)(s+1)}\times(2+\frac{1}{2s})}{1+\frac{2}{(4s+1)(s+1)}\times(2+\frac{1}{2s})}$$

$$\frac{Y}{Y_{sp}} = \frac{K_m G_p G_c G_v}{1+G_m G_p G_c G_v} = \frac{\frac{2}{(4s+1)(s+1)}\times(2+\frac{1}{2s})}{1+\frac{2}{(4s+1)(s+1)}\times(2+\frac{1}{2s})} = \frac{1}{s^2+s+1}$$

The closed loop transfer function is:

$$\frac{Y}{Y_{sp}} = \frac{1}{s^2+s+1}$$

(b)
The characteristic equation is the denominator of the closed loop transfer function, which is underdamped (ζ = 0.5):

$$s^2+s+1$$

(c)
For stability analysis, $G_c = K_c(1+\frac{1}{4s})$ is substituted into $1+G_m G_p G_c G_v$ and we get:

$$1+G_m G_p G_c G_v = 1+\frac{2}{(4s+1)(s+1)}\times 4(1+\frac{1}{4s})$$

$$= \frac{2s(s+1)+Kc}{2s(s+1)} = \frac{2s^2+2s+Kc}{2s(s+1)}$$

To find the stability region, the roots of the numerator polynomial should be on the right half plane. For this 2nd order polynomial, this means:

$$K_c > 0$$

K_c can be arbitrarily large for this PI controlled second order system and still maintain stability.

11.25

First we use Eq. 11-26 to get the closed loop transfer function

$$\frac{Y}{Y_{sp}} = \frac{\frac{10}{(s+1)(2s+1)}}{1+\frac{10}{(s+1)(2s+1)}} = \frac{10}{(s+1)(2s+1)+10} = \frac{10}{2s^2+3s+11}$$

Or in standard form

$$\frac{Y}{Y_{sp}} = \frac{10/11}{\frac{2}{11}s^2+\frac{3}{11}s+1} \qquad \zeta = \frac{3\sqrt{22}}{44} \qquad \tau = \frac{\sqrt{22}}{11}$$

The time at which the maximum occurs is given by Eq. 5-52

$$t_p = \pi\tau \Big/ \sqrt{1-\zeta^2} \qquad t_p = 1.41$$

(b)

The response is given by

$$Y(s) = \frac{20}{s(2s^2+3s+11)}$$

The Final Value Theorem gives the steady state value as

$$y(\infty) = \frac{20}{11}$$

Subtracting the steady state value from the set point change gives offset as

$$\text{offset} = \frac{2}{11}$$

(c)

The period of oscillation is given by Eq. 5-55

$$P = \frac{2\pi\tau}{\sqrt{1-\zeta^2}} \qquad\qquad P = 2.83$$

(d)

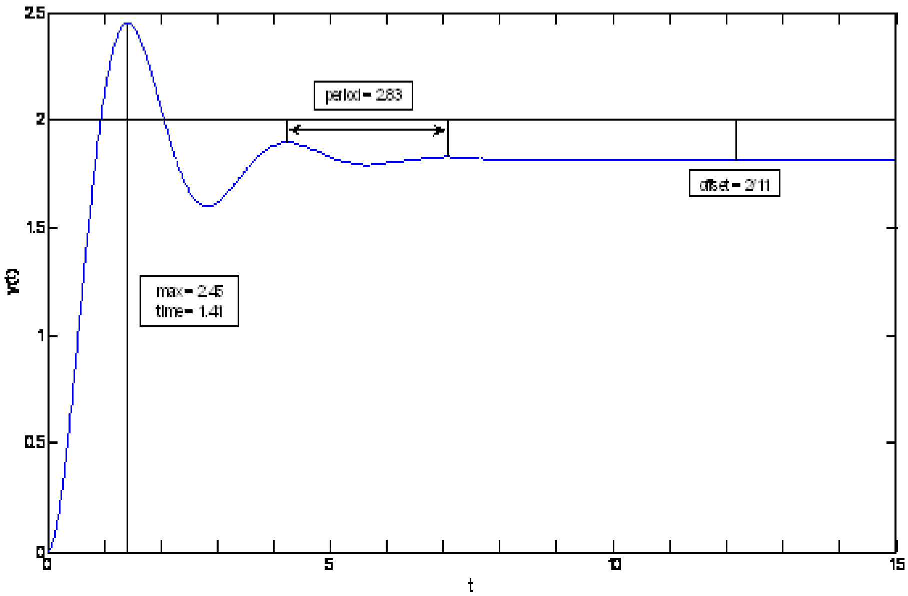


Figure S11.25 *y(t) responses as a function of time.*

Hint: You do not need to obtain the analytical response *y*(*t*) to answer the above questions. Use the standard second order model expressed in terms of ζ and τ (see Chapter 5).

11.26

The closed loop transfer function for a set point change (Eq. 11-26), is given by

$$\frac{Y}{Y_{sp}} = \frac{K_m G_c G_v G_p}{1 + G_c G_v G_p G_m}$$

Substituting the values from above and in part (a), we get,

$$\frac{Y}{Y_{sp}} = \frac{K_c E}{1 + K_c E}$$

Multiplying by a unit step change in set point gives

$$Y(s) = \frac{K_c E}{1 + K_c E}\frac{1}{s} \Longrightarrow y(t) = \frac{K_c E}{1 + K_c E}$$

A sketch might look like this (the step change at *t* = *5*)

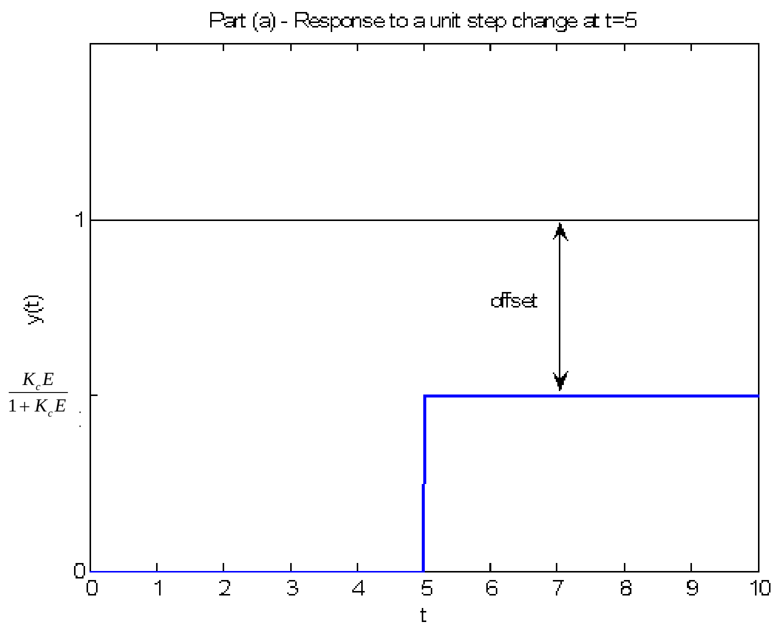


Figure S11.26a *Step response to unit step change with proportional control.*

As evidenced by the sketch, there is **offset** for this controller.

For part (b), we substitute the values into Eq. 11-26 to get

$$\frac{Y}{Y_{sp}} = \frac{E}{\tau_I s + E} = \frac{1}{\frac{\tau_I}{E}s + 1}$$

Multiplying by a unit step change in set point gives

$$Y(s) = \frac{1}{\frac{\tau_I}{E}s + 1}\frac{1}{s} \Longrightarrow y(t) = 1 - \exp\left(-\frac{E}{\tau_I}t\right)$$

A sketch would look like this

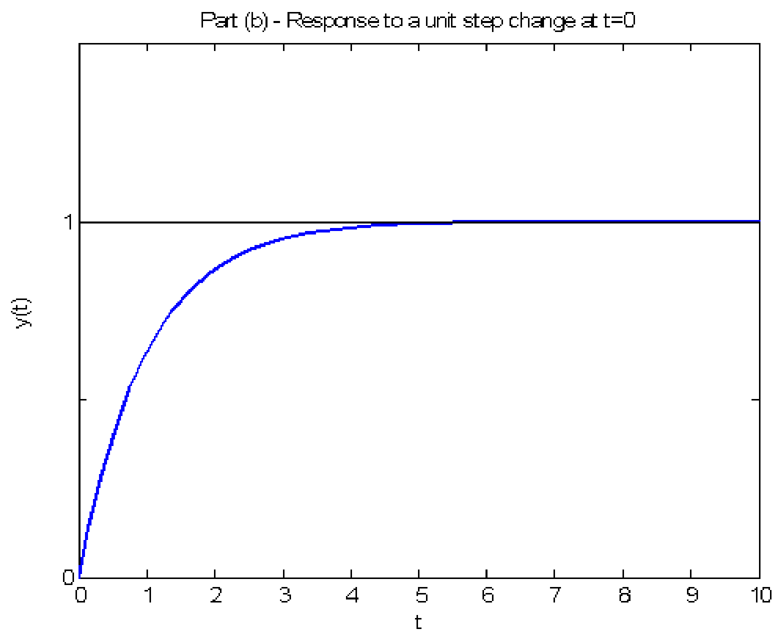


Figure S11.26b *Step response to unit step change with integral control*

As evident in the sketch, there is **no offset** for this controller.

11.27

$$\frac{Y}{D} = \frac{G_d}{1+G_cG_vG_pG_m} = \frac{\dfrac{8}{(s+2)^3}}{1+K_c \times 1 \times \dfrac{8}{(s+2)^3} \times 1} = \frac{8}{s^3+6s^2+12s+8+8K_c}$$

The characteristic equation for above is shown as:

$s^3+6s^2+12s+8+8K_c = 0$

Substituting s=jω in above equation, we have:

$$-6\omega^2+8+8K_c+\left[12\omega-\omega^3\right]j = 0$$

Thus, we have:

$$\begin{cases} -6\omega^2+8+8\mathrm{K}_c = 0 \\ 12\omega-\omega^3 = 0 \end{cases} \Rightarrow \begin{cases} \omega = 2\sqrt{3} \\ K_{cm} = 8 \end{cases}$$

So $K_c = 1$ is stable; $K_c = 8$ is marginally stable, and $K_c = 27$ is unstable

For a step change $D = \frac{1}{s}$, applying the final value theorem:

Offset: $\lim_{t\to\infty} y(t) = \lim_{s\to 0}\left[s\dfrac{8\frac{1}{s}}{s^3+6s^2+12s+8+8K_c}\right] = \dfrac{1}{1+K_c}$

(b)

$$\frac{Y}{D} = \frac{G_d}{1+G_cG_vG_pG_m} = \frac{\frac{8}{(s+2)^3}}{1+K_c\left(1+\frac{1}{\tau_I s}\right)\times 1\times\frac{8}{(s+2)^3}\times 1} = \frac{8\tau_I s}{\tau_I s(s+2)^3 + K_c(\tau_I s+1)}$$

For a step change $D=\frac{1}{s}$, applying the final value theorem:

Offset: $\lim_{t\to\infty} y(t) = \lim_{s\to 0}\left[s\dfrac{8\frac{1}{s}\tau_I s}{\tau_I s(s+2)^3+K_c(\tau_I s+1)}\right] = 0$

So there is no offset for PI controller.

11.28

The closed loop transfer function for set point changes is given by

$$\frac{Y}{Y_{sp}} = \frac{K_mG_cG_vG_p}{1+G_cG_vG_pG_m}$$

Substituting the information in the problem gives

$$\frac{Y}{Y_{sp}} = \frac{K_c(s+3)}{(s+1)(0.5s+1)(s+3)+3K_c} = \frac{K_c(s+3)}{0.5s^3+3s^2+5.5s+3+3K_c}$$

So the characteristic equation is

$$0.5s^3+3s^2+5.5s+3+3K_c=0$$

Substituting s=jω in above equation, we have:

$$-3\omega^2+3+3K_c+\left[5.5\omega-0.5\omega^3\right]j=0$$

Thus, we have:

$$\begin{cases}-3\omega^2+3+3\mathrm{K}_c=0\\ 5.5\omega-0.5\omega^3=0\end{cases} \Rightarrow \begin{cases}\omega=\sqrt{11}\\ K_{c,\max}=10\end{cases}$$

So the answers to parts (a)-(c) are:

(a) stable

(b) unstable

(c) unstable

The following plot shows the Simulink responses and confirms the above answers:

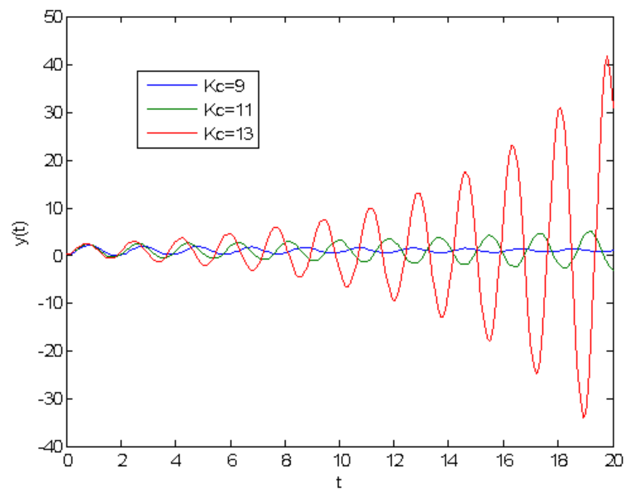


Figure S11.28 *y(t) responses with different K_c*

11.29

a) Proportional controller:

We derive the transfer function as follows:

$$\frac{Y}{Y_{SP}} = \frac{K_m G_c G_v G_p}{1 + G_m G_c G_v G_p}$$

$$\frac{Y}{Y_{SP}} = \frac{K_c \frac{1}{(s+1)^3}}{1 + K_c \frac{1}{(s+1)^3}} = \frac{K_c}{(s+1)^3 + K_c} = \frac{K_c}{s^3 + 3s^2 + 3s + 1 + K_c} \tag{1}$$

The characteristic equation of (1) is the following:

$$s^3 + 3s^2 + 3s + 1 + K_c = 0 \tag{2}$$

Substituting *s*=jω in above equation, we have:

$$-3\omega^2 + 1 + K_c + \left[3\omega - \omega^3\right] j = 0$$

Thus, we have:

$$\begin{cases} -3\omega^2 + 1 + K_c = 0 \\ 3\omega - \omega^3 = 0 \end{cases} \Rightarrow \begin{cases} \omega = \sqrt{3} \\ K_{c,\max} = 8 \end{cases}$$

We conclude that the system will be stable if

$$K_c < 8 \qquad (3)$$

Simulation results are in **Figure** S11.29a.

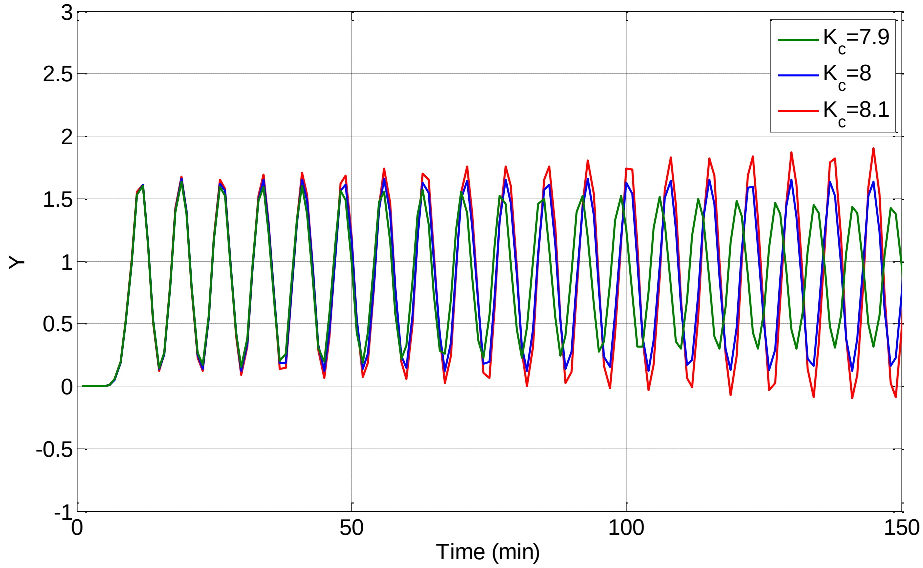


Figure S11.29a: *System response to a unit step setpoint change. Note that the system is stable at K_c=7.9, marginally stable at K_c=8, and unstable at K_c=8.1.*

b) PD controller:

We derive the transfer function as follows:

$$G_c = K_c\left(1 + \tau_D s\right)$$

$$\frac{Y}{Y_{SP}} = \frac{K_m G_c G_v G_p}{1 + G_m G_c G_v G_p}$$

$$\frac{Y}{Y_{SP}} = \frac{K_c\left(1 + \tau_D s\right)\dfrac{1}{\left(s+1\right)^3}}{1 + K_c\left(1 + \tau_D s\right)\dfrac{1}{\left(s+1\right)^3}}$$

$$= \frac{K_c\left(\tau_D s + 1\right)}{s^3 + 3s^2 + 3s + 1 + K_c + \tau_D K_c s}, \qquad (4)$$

where K_c=10.

The characteristic equation of (4) is the following:

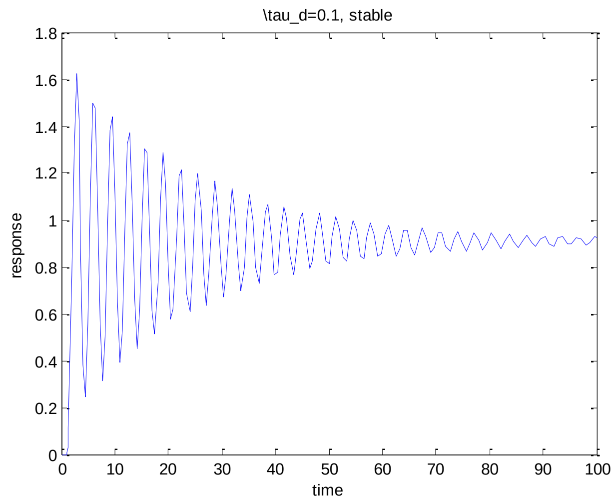
$$s^3 + 3s^2 + (10\tau_D + 3)s + 11 = 0 \tag{5}$$

Substituting s=jω in above equation, we have:

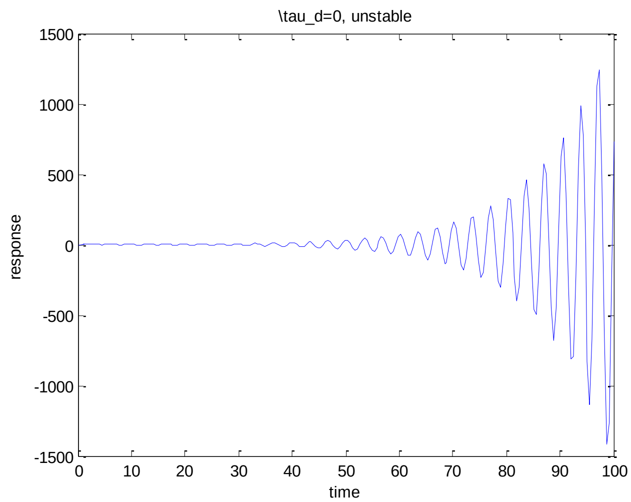
$$-3\omega^2 + 11 + \left[(10\tau_D + 3)\omega - \omega^3\right] j = 0$$

Thus, we have:

$$\begin{cases} -3\omega^2 + 11 = 0 \\ (10\tau_D + 3)\omega - \omega^3 = 0 \end{cases} \Rightarrow \tau_{D,\min} = \frac{1}{15} = 0.0667$$



(i) $\tau_D > \tau_{D,min}$



(ii) $\tau_D < \tau_{D,min}$

Figure S11.29b *Simulation results of different τ_D settings*

Chapter 12 ©

12.1

For $K = 1$, $\tau_1=10$, $\tau_2=5$, and $\theta=0$, the PID controller settings are obtained using Eq.12-14 as

$$K_c = \frac{1}{K}\frac{\tau_1+\tau_2}{\tau_c} = \frac{15}{\tau_c} \quad , \quad \tau_I = \tau_1+\tau_2=15 \quad , \quad \tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2} = 3.33$$

The characteristic equation for the closed-loop system is

$$1+\left[K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\right]\left[\frac{1.0+\alpha}{(10s+1)(5s+1)}\right]=0$$

Substituting for K_c, τ_I, and τ_D, and simplifying gives

$$\tau_c s+(1+\alpha)=0$$

In order for the closed loop system to be stable, the coefficients of this first-order polynomial in *s* must be positive. Thus,

$$\tau_c > 0$$

and

$$(1+\alpha) > 0 \quad \Rightarrow \quad \alpha > -1.$$

Results:

a) The closed loop system is stable for $\alpha > -1$

b) Choose $\tau_c > 0$

c) The choice of τ_c does not affect the robustness of the system to changes in α. For $\tau_c \leq 0$, the system is unstable regardless of the value of α. For $\tau_c > 0$, the system is stable if $\alpha > -1$, regardless of the value of τ_c .

12.2

$$G = G_v G_p G_m = \frac{4(1-s)}{s}$$

a) Let $\tilde{G} = G$. Factor the model as

$$\tilde{G} = \tilde{G}_+ \tilde{G}_-$$

with:

$$\tilde{G}_+ = 1 - s, \qquad \tilde{G}_- = \frac{4}{s}$$

The controller design equation in (12-20) is:

$$G^*_c = \frac{1}{\tilde{G}_-} f$$

with a given first-order "filter",

$$f == \frac{1}{\tau_c s + 1}$$

Substitute,

$$G^*_c = \frac{1}{4} \frac{s}{\tau_c s + 1}$$

b) The equivalent controller in the classical feedback control configuration in Fig. 12.6(a) is:

$$G_c = \frac{G^*_c}{1 - G^*_c \tilde{G}}$$

Substitute to give,

$$G_c = \frac{1}{4(\tau_c + 1)}$$

Thus G_c is a proportional-only controller.

12.3

For the FOPTD model, $K = 2$, $\tau = 1$, and $\theta = 0.2$.

a) Using entry G in Table 12.1 for $\tau_c = 0.2$

$$K_c = \frac{\tau}{K(\tau_c + \theta)} = \frac{1}{2(0.2 + 0.2)} = 1.25$$

$$\tau_I = \tau = 1$$

b) Using entry G in Table 12.1 for $\tau_c = 1$

$$K_c = \frac{\tau}{K(\tau_c + \theta)} = \frac{1}{2(1+0.2)} = 0.42$$
$$\tau_I = \tau = 1$$

c) From Table 12.4 for a disturbance change

$KK_c = 0.859(\theta/\tau)^{-0.977}$ or $K_c = 2.07$

$\tau/\tau_I = 0.674(\theta/\tau)^{-0.680}$ or $\tau_I = 0.49$

d) From Table 12.4 for a set-point change

$KK_c = 0.586(\theta/\tau)^{-0.916}$ or $K_c = 1.28$

$\tau/\tau_I = 1.03 - 0.165(\theta/\tau)$ or $\tau_I = 1.00$

e) Conservative settings correspond to low values of K_c and high values of τ_I. Clearly, the IMC method (τ_c = 1.0) of part (b) gives the more conservative settings; the ITAE method of part (c) gives the least conservative settings. The controller setting for (a) and (d) are essentially identical.

f) A comparison for a unit step disturbance is shown in Fig. S12.3

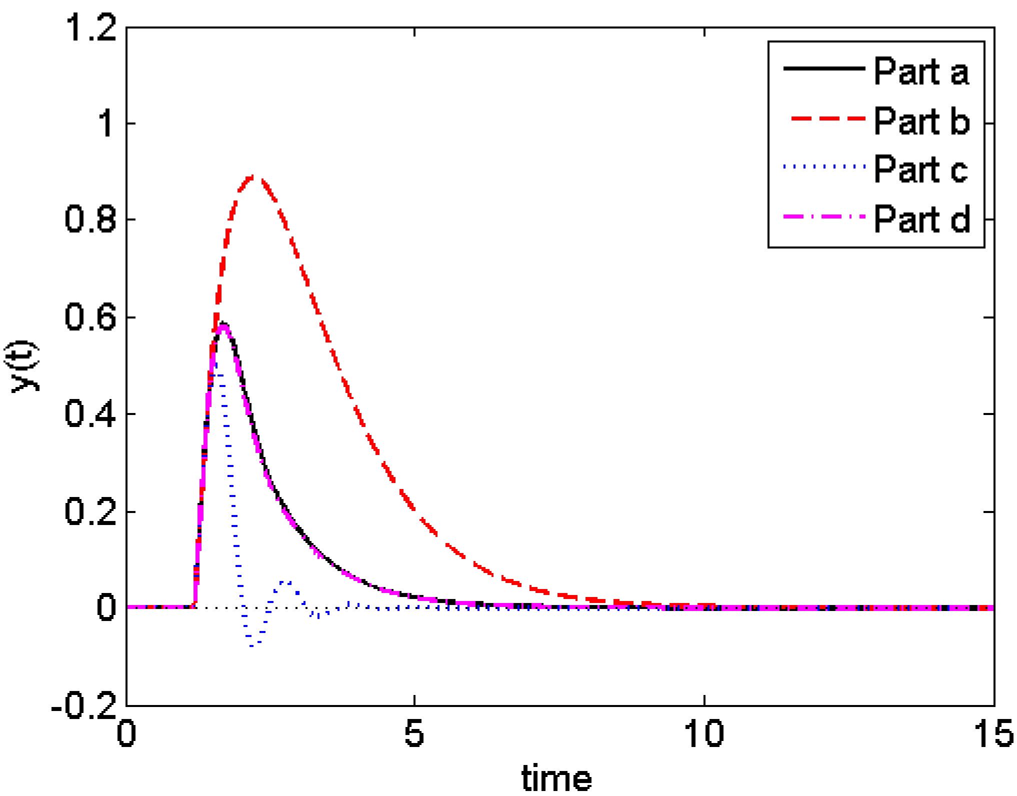


Figure S12.3. *Comparison of PI controllers for a unit step disturbance.*

12.4

The process model is,

$$G(s) = \frac{4e^{-3s}}{s} \quad \text{(assume the time delay has units of minutes)} \qquad (1)$$

(a) Proportional only control, $G_c(s) = K_c$. The characteristic equation is:

$$1 + K_c G(s) = 0$$

Substitute and rearrange,

$$s + 4K_c e^{-3s} = 0$$

Substitute the stability limit conditions from Section 11.4.3: $s = j\omega$, $\omega = \omega_u$, and $K_c = K_{cu}$:

$$j\omega_u + 4K_{cu} e^{-3\omega_u j} = 0 \qquad (2)$$

Apply Euler's identity, $e^{-\theta j} = \cos(\theta) - j\sin(\theta)$:

$$e^{-3\omega_u j} = \cos(3\omega_u) - j\sin(3\omega_u)$$

Substitute into (2),

$$j\omega_u + 4K_{cu}[\cos(3\omega_u) - j\sin(3\omega_u)] = 0$$

Collect terms for the real and imaginary parts:

$$4K_{cu}\cos(3\omega_u) = 0 \qquad (3)$$

$$\omega_u - 4K_{cu}\sin(3\omega_u) = 0 \qquad (4)$$

For (3), because $K_{cu} \neq 0$, it follows that:

$$\cos(3\omega_u) = 0 \quad \Rightarrow \quad 3\omega_u = \frac{\pi}{2} \qquad (5)$$

$$\Rightarrow \quad \omega_u = \frac{\pi}{6} = 0.5236 \text{ rad/min} \qquad (6)$$

From (4) – (6),

$$0.5236 - 4K_{cu}\sin\left(\frac{\pi}{2}\right) = 0 \quad \Rightarrow \quad K_{cu} = \frac{0.5236}{4} = \boxed{0.130}$$

(b) Controller settings using AMIGO method

The model parameters are: $K = 4,\ \theta=3$
For this model, use the right-hand column of Table 12.5.

$$K_c = \frac{0.35}{K\theta} = \frac{0.35}{12} = 0.029$$
$$\tau_I = 13.4\theta = 13.4(3) = 40.2$$

12.5

Assume that the process can be modeled adequately by the first-order-plus-time delay-model in Eq. 12-10. The step response data and the tangent line at the inflection point for the slope-intercept identification method of Chapter 7 are shown in Fig. S12.5.

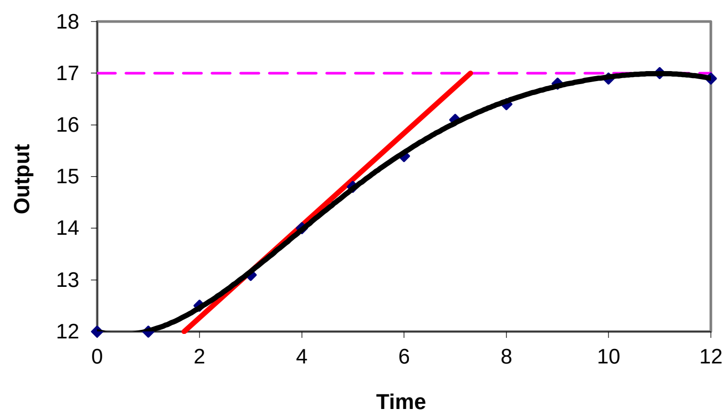


Figure S12.5. *Step response data and tangent line at the inflection point.*

This estimated model parameters are:

$$K = K_{IP}\,K_v(K_pK_m) = \left(0.75\,\frac{\text{psi}}{\text{mA}}\right)\left(0.9\,\frac{\text{psi}}{\text{psi}}\right)\left(\frac{16.9-12.0}{20-18}\,\frac{\text{mA}}{\text{psi}}\right) = 1.65$$

$\theta = 1.7$ min

$\theta + \tau = 7.2$ min $\Rightarrow$ $\tau = 5.5$ min

a) Since $\theta/\tau > 0.25$, a conservative choice of $\tau_c = \frac{1}{2}\tau$ is used. Thus, $\tau_c = 2.75$ min. From Case H in Table 12.1:

$$K_c = \frac{1}{K}\,\frac{\tau + \dfrac{\theta}{2}}{\tau_c + \dfrac{\theta}{2}} = 1.76$$

$$\tau_I = \tau + \frac{\theta}{2} = 6.35\,\text{min}, \qquad \tau_D = \frac{\tau\theta}{2\tau + \theta} = 0.736\,\text{min}$$

b) From Table 12.6, the AMIGO tuning parameters are:

$$K_c = \frac{1}{K}\left(0.2 + 0.45\frac{\tau}{\theta}\right) = \frac{1}{1.65}\left(0.2 + 0.45\frac{5.5}{1.7}\right) = 1$$

$$\tau_I = \frac{0.4\theta + 0.8\tau}{\theta + 0.1\tau}\theta = \frac{0.4(1.7) + 0.8(5.5)}{1.7 + 0.1(5.5)}(1.7) = 3.8\,\text{min}$$

$$\tau_D = \frac{0.5\theta\tau}{0.3\theta + \tau} = \frac{0.5(1.7)(5.5)}{0.3(1.7) + 5.5} = 0.78\,\text{min}$$

c) From Table 12.4, the ITAE PID settings for a step disturbance are

$KK_c = 1.357(\theta/\tau)^{-0.947}$ or $K_c = 2.50$
$\tau/\tau_I = 0.842\,(\theta/\tau)^{-0.738}$ or $\tau_I = 2.75$ min
$\tau_D/\tau = 0.381\,(\theta/\tau)^{0.995}$ or $\tau_D = 0.65$ min

d) The most aggressive controller is the one from part c, which has the highest value of K_c and smallest value of τ_I

12.6

The model for this process has K=5, τ=4, and θ=3. The PI controller parameters for an FOPDT model using IMC tuning are given by entry G in Table 12.1:

$$K_c = \frac{\tau}{K(\tau_c + \theta)} = \frac{4}{5(3+3)} = 0.13$$

$$\tau_I = \tau = 4$$

The parameters for a PID controller are given by entry H in Table 12.1:

$$K_c = \frac{\tau + \frac{\theta}{2}}{K(\tau_c + \frac{\theta}{2})} = \frac{4 + \frac{3}{2}}{5(3 + \frac{3}{2})} = 0.24$$

$$\tau_I = \tau + \frac{\theta}{2} = 4 + \frac{3}{2} = 5.5$$

$$\tau_D = \frac{\tau\theta}{2\tau + \theta} = \frac{4(3)}{2(4) + 3} = 1.1$$

The simulated process for a step change in the set point is plotted below for both the PI and PID controllers. Note that the PID controller was implemented in the proper form to eliminate derivative kick (see chapter 8).

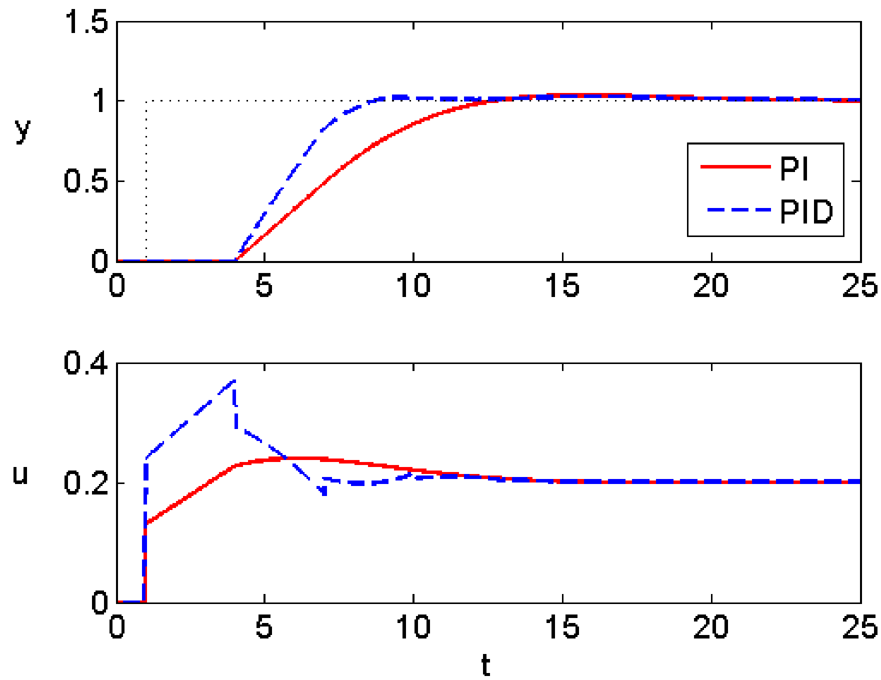


Figure S12.6: Responses to a step change in the set point at t = 1for PI and PID controllers.

The PID controller allows the controlled variable to reach the new set point more quickly than the PI controller, due to its larger K_c value. This large K_c allows an initially larger response from the controller during times from 1 to 4 minutes. The reason that the K_c can be larger is that, after the controlled variable begins to change and move toward the set point, the derivative term can “put on the brakes” and slow down the aggressive action so the controlled variable lands nicely at the set point.

12.7

a.i) The model reduction approach of Skogestad gives the following approximate model:

$$G(s) = \frac{e^{-0.028s}}{(s+1)(0.22s+1)}$$

Since $\theta/\tau < 0.25$, an aggressive choice of $\tau_c = \theta = 0.028$ is made. From Case I in Table 12.1 with $\tau_3 = 0$, the IMC settings are:

$$K_c = \frac{1}{K}\frac{\tau_1+\tau_2}{\tau_c+\theta} = 21.8$$

$$\tau_I = \tau_1+\tau_2 = 1.22, \qquad \tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2} = 0.180$$

a.ii) To use the AMIGO tuning relations in Table 12.6, the model reduction method of Skogestad can be used to reduce the model to a FOPDT model. The time constant in the resulting FOPDT model is the largest time constant in the full-order model plus one half of the next biggest time constant, 1 + 0.5(0.2) = 1.1. The time delay in the resulting FOPDT model is half of the second-biggest time constant in the full-order model, 0.5(0.2) = 0.1. The other smaller time constants are neglected.

$$G(s) = \frac{e^{-0.1s}}{1.1s+1}$$

The AMIGO rules for a PID controller in Table 12.6 give:

$$K_c = \frac{1}{K}\left(0.2+0.45\frac{\tau}{\theta}\right) = \left(0.2+0.45\frac{1.1}{0.1}\right) = 5.15$$

$$\tau_I = \frac{0.4\theta+0.8\tau}{\theta+0.1\tau}\theta = \frac{0.4(0.1)+0.8(1.1)}{0.1+0.1(1.1)}(0.1) = 0.44$$

$$\tau_D = \frac{0.5\theta\tau}{0.3\theta+\tau} = \frac{0.5(0.1)(1.1)}{0.3(0.1)+1.1} = 0.049$$

b) The simulation results shown in Figure S12.7 indicate that the IMC controller is superior for a step disturbance due to its smaller maximum deviation and lack of oscillations. This result makes sense, given that we made an aggressive choice for τ_C for the IMC controller.

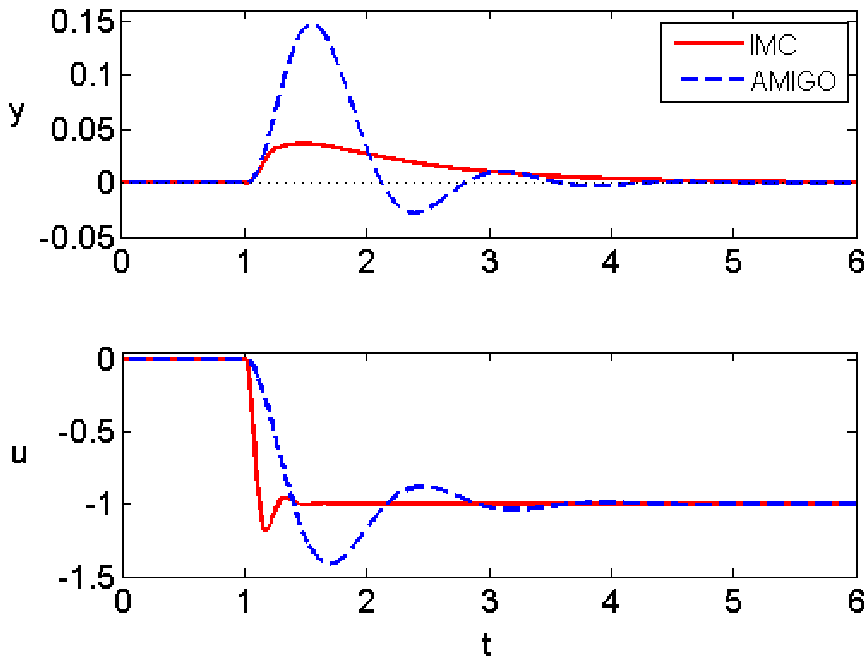


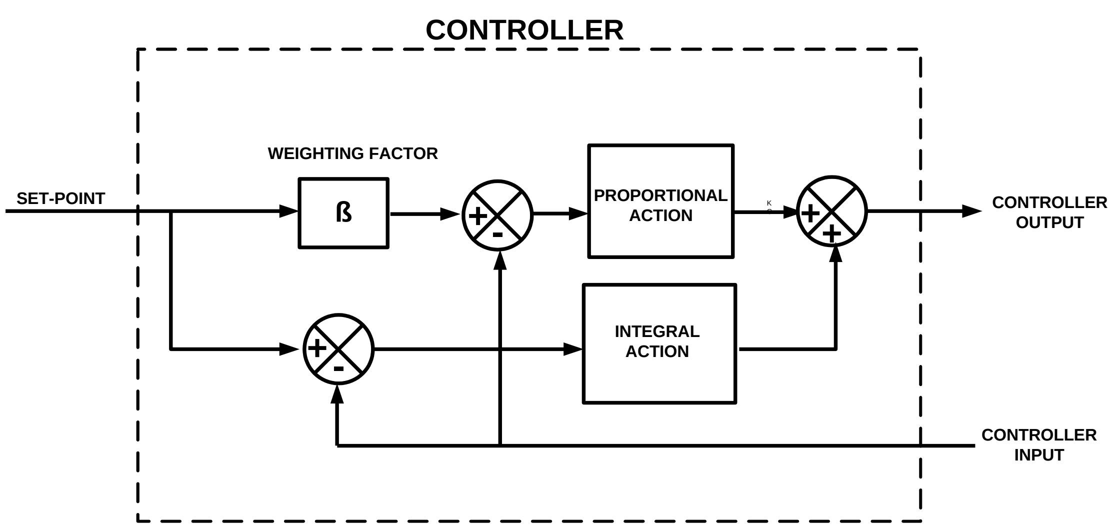
Figure S12.7. *Closed-loop responses to a unit step disturbance at t=1.*

12.8

From Eq. 12-40 (with γ=0):

$$p(t) = \bar{p} + K_c\left[\beta y_{sp}(t) - y_m(t)\right] + K_c\left[\frac{1}{\tau_I}\int e(t^*)dt^* - \tau_D\frac{dy_m}{dt}\right]$$

This control law can be implemented with Simulink as follows:



Closed-loop responses are compared for four values of β: 1, 0.7, 0.5 & 0.3.

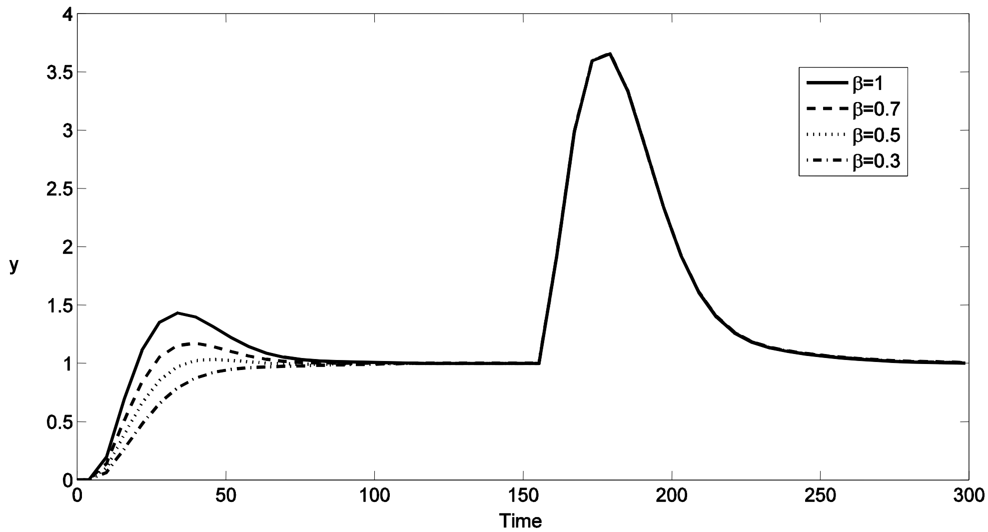


Figure S12.8. *Closed-loop responses for different values of β.*

As shown in Figure S12.8, as β increases the set-point response becomes faster but exhibits more overshoot. The value of β =0.5 seems to be a good choice. The disturbance response is independent of the value of β.

12.9

a) From Table 12.2, the controller settings for the series form are:

$$K_c = K_c'\left(1+\frac{\tau_D'}{\tau_I'}\right) = 0.971$$

$$\tau_I = \tau_I' + \tau_D' = 26.52$$

$$\tau_D = \frac{\tau_I' \tau_D'}{\tau_I' + \tau_D'} = 2.753$$

Closed-loop responses generated from Simulink are shown in Fig. S12.9. The series form results in more oscillatory responses; thus, it produces more aggressive control action for this example.

b) By changing the derivative term in the controller block, the Simulink results show that the system becomes more oscillatory as τ_D increases. For the parallel form, the closed-loop system becomes unstable for $\tau_D \geq 5.4$; for the series form, it becomes unstable for $\tau_D \geq 4.5$.

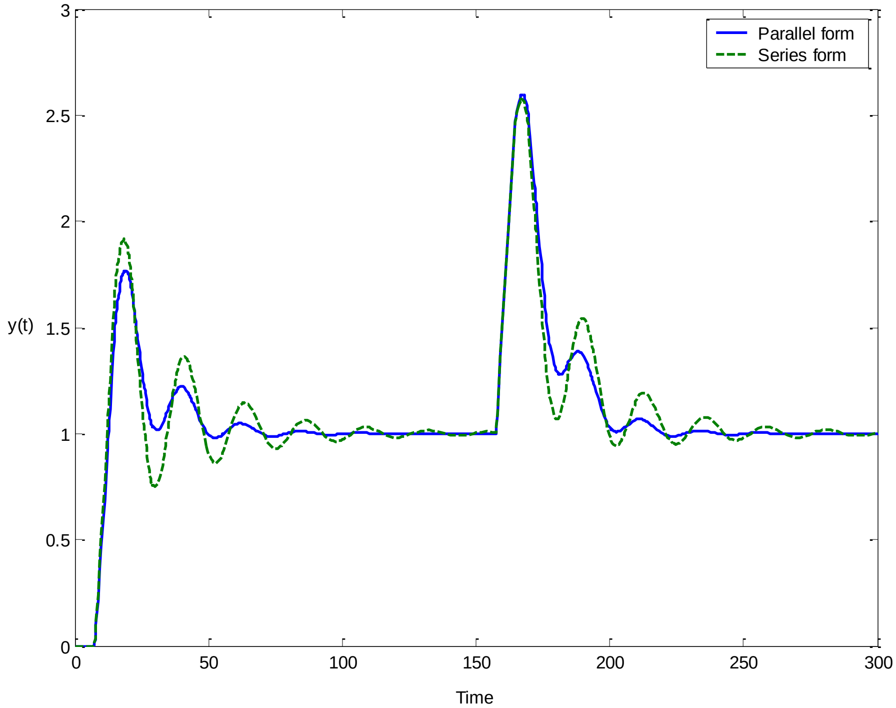
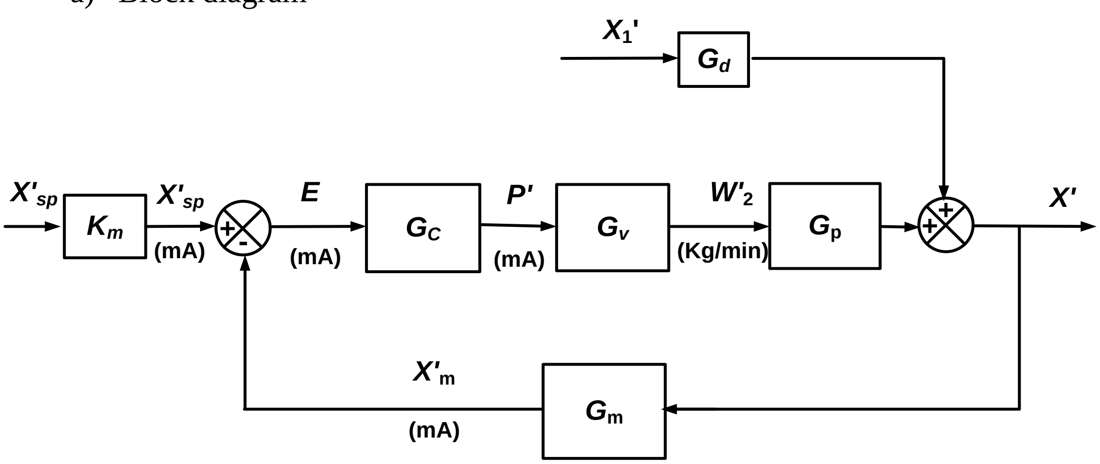


Figure S12.9. *Closed-loop responses for parallel and series controller forms.*

12.10

a) Block diagram



b) <u>Process and disturbance transfer functions</u>:

Overall material balance:

$$w_1 + w_2 - w = 0 \qquad (1)$$

Solute balance:

$$w_1 x_1 + w_2 x_2 - wx = \rho V \frac{dx}{dt} \qquad (2)$$

Substituting (1) into (2) and putting into deviation variables:

$$w_1 x_1' + w_2' x_2 - w_1 x' - \overline{w}_2 x - w_2' \overline{x} = \rho V \frac{dx'}{dt}$$

Taking the Laplace transform:

$$w_1 X_1'(s) + (x_2 - \overline{x}) W_2'(s) = (w_1 + \overline{w}_2 + \rho V s) X'(s)$$

Finally:

$$G_p(s) = \frac{X'(s)}{W_2'(s)} = \frac{x_2 - \overline{x}}{w_1 + \overline{w}_2 + \rho V s} = \frac{\dfrac{x_2 - \overline{x}}{w_1 + \overline{w}_2}}{1 + \tau s}$$

$$G_d(s) = \frac{X'(s)}{X_1'(s)} = \frac{w_1}{w_1 + \overline{w}_2 + \rho V s} = \frac{\dfrac{w_1}{w_1 + \overline{w}_2}}{1 + \tau s}$$

where $\tau = \dfrac{\rho V}{w_1 + \overline{w}_2}$

Substituting numerical values:

$$G_p(s) = \frac{2.6 \times 10^{-4}}{1 + 4.71 s}$$

$$G_d(s) = \frac{0.65}{1 + 4.71 s}$$

Composition measurement transfer function:

$$G_m(s) = \frac{20 - 4}{0.5} e^{-s} = 32 e^{-s}$$

Final control element transfer function:

$$G_v(s) = \frac{15-3}{20-4} \times \frac{300/1.2}{0.0833s+1} = \frac{187.5}{0.0833s+1}$$

Controller:

Let $$G = G_v G_p G_m = \frac{187.5}{0.0833s+1} \frac{2.6\times 10^{-4}}{1+4.71s} 32e^{-s}$$

then $$G = \frac{1.56e^{-s}}{(4.71s+1)(0.0833s+1)}$$

For process with a dominant time constant, $\tau_c = \tau_{dom}/3$ is recommended. Hence . $\tau_c = 1.57$ min. From Table 12.1

K_c = 1.92 and τ_I = 4.71 min

c) Simulink results:

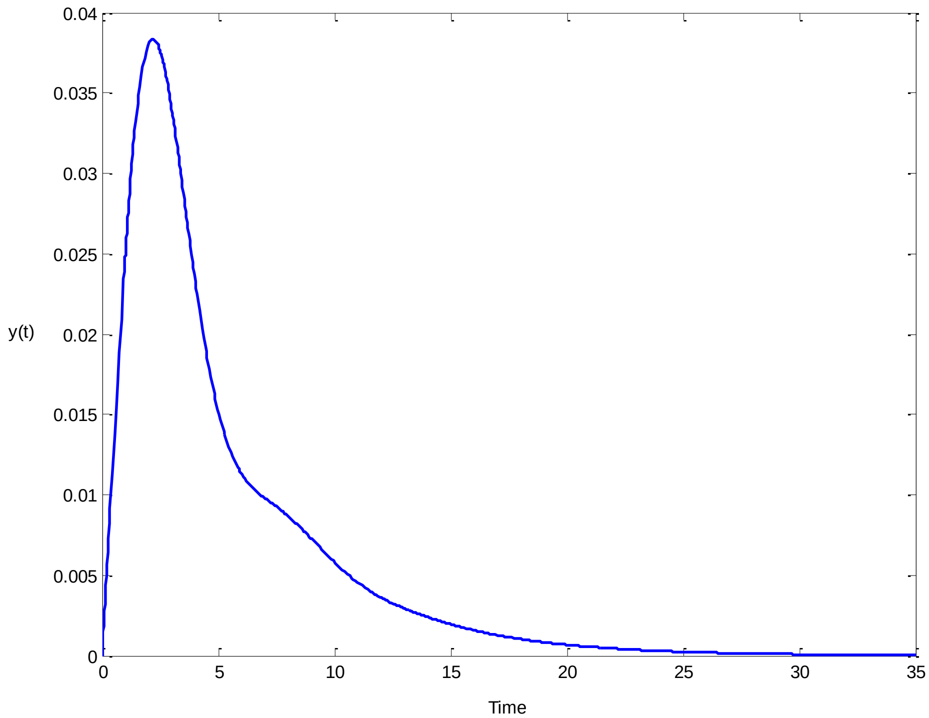


Figure S12.10c. *Closed-loop response for the step disturbance.*

d) Figure S12.10d indicates that $\tau_c = 1.57$ min gives very good results.

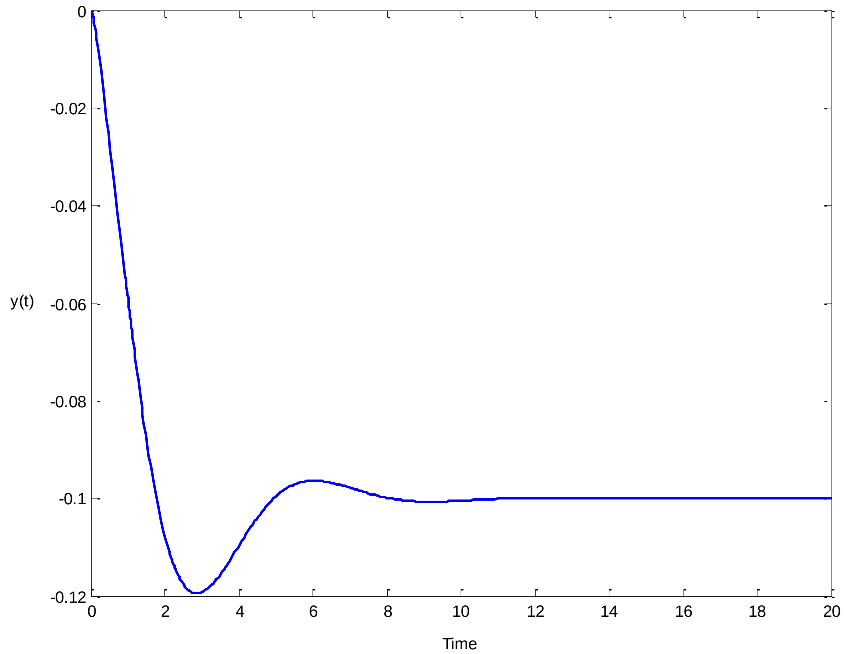


Figure S12.10d. *Closed-loop response for set-point change.*

e) Improved control can be obtained by adding derivative action: $\tau_D = 0.4$ min.

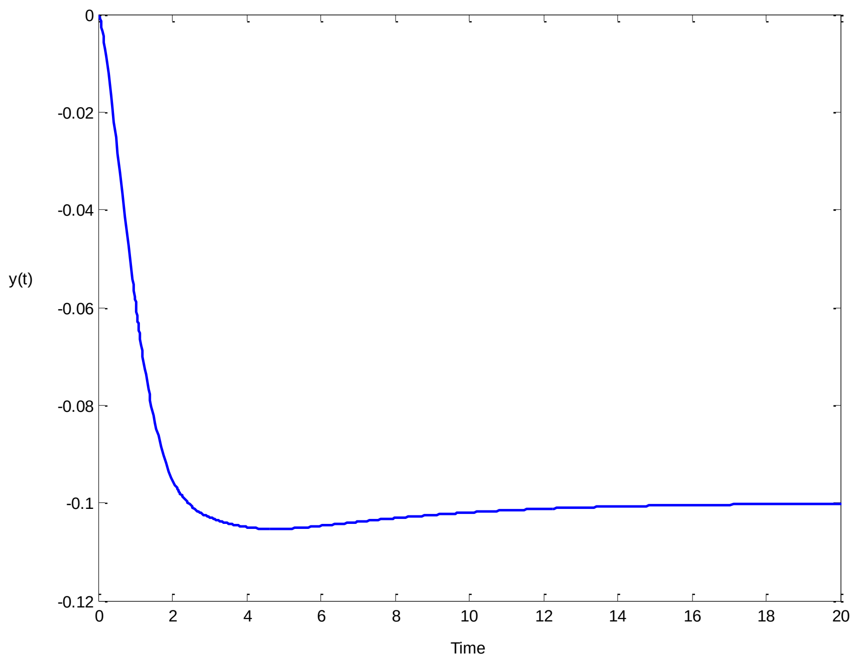


Figure S12.10e. *Closed-loop response after adding derivative action.*

f) For θ =3 min, the closed-loop response becomes unstable. It is well known that the presence of a large time delay in a feedback control loop limits its performance. In fact, a time delay adds phase lag to the feedback loop, which adversely affects closed-loop stability (cf. Ch 13). Consequently, the controller gain must be reduced below the value that could be used if a smaller time delay were present.

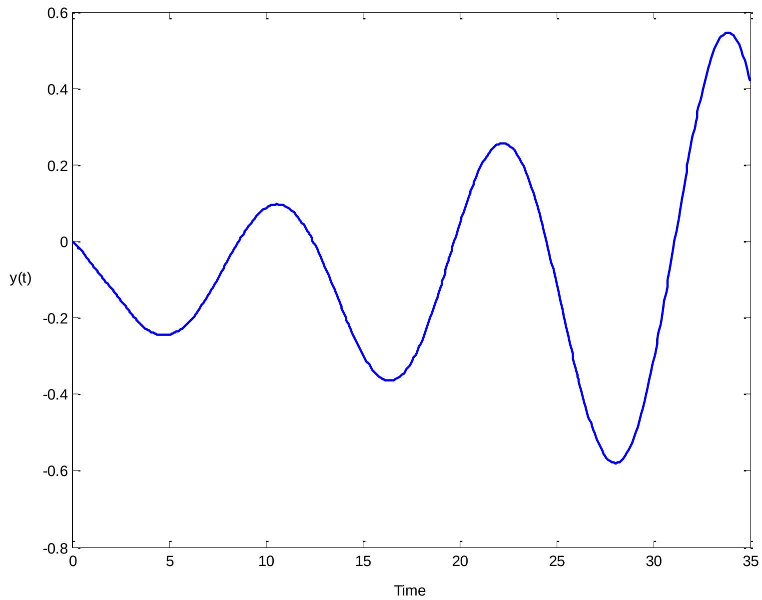


Figure S12.10f. *Closed-loop response for* θ = 3 min.

12.11

The controller retuning decision is based on the characteristic equation, which takes the following form for the standard feedback control system.

$$1 + G_c G_{I/P} G_v G_p G_m = 0$$

The PID controller may have to be retuned if any of the transfer functions, $G_{I/P}$, G_v, G_p or G_m, change.

a) G_m changes. The controller may have to be retuned.

b) The zero does not affect G_m. Hence the controller does not require retuning.

c) G_v changes. Retuning may be necessary.

d) G_p changes. The controller may have to be retuned.

12.12

The process model is given as: $G(s) = \dfrac{2e^{-s}}{3s+1}$

a) From Table 12.1, the IMC settings are:

$$K_c = \frac{1}{K}\frac{\tau}{\tau_c + \theta} = 0.75$$

$$\tau_I = \tau = 3\,\text{min}$$

b) Cohen-Coon tuning relations:

$$K_c = \frac{1}{K}\frac{\tau}{\theta}[0.9 + \theta/12\tau] \; = \; 1.39$$

$$\tau_I = \frac{\theta[30 + 3(\theta/\tau)]}{9 + 20(\theta/\tau)} \; = \; 1.98\,\text{min}$$

The IMC settings are more conservative because they have a smaller K_c value and a larger τ_I value.

c) The Simulink simulation results are shown in Fig. S12.12. Both controllers are rather aggressive and produce oscillatory responses. The IMC controller is less aggressive (that is, more conservative).

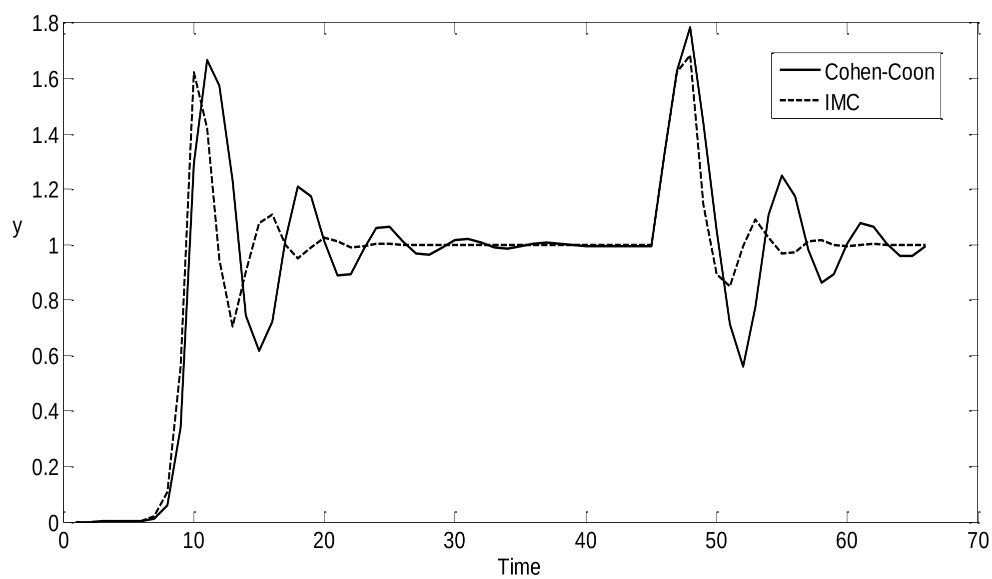


Figure S12.12. *Controller comparison*.

12.13

From the solution to Exercise 12.5, the process reaction curve method yields,

$K = 1.65$, $\theta = 1.7$ min, $\tau = 5.5$ min

a) IMC method:

From Table 12.1, Controller G with $\tau_c = \tau/3$:

$$K_c = \frac{1}{K}\frac{\tau}{\tau_c + \theta} = \frac{1}{1.65}\frac{5.5}{(5.5/3)+1.7} = 0.94$$

$\tau_I = \tau = 5.5$ min

b) Ziegler-Nichols settings:

$$G(s) = \frac{1.65e^{-1.7s}}{5.5s+1}$$

First, determine the stability limits; the characteristic equation is:

$$1 + G_cG = 0$$

Substitute the Padé approximation,

$$e^{-s} \approx \frac{1-0.85s}{1+0.85s}$$

into the characteristic equation:

$$0 = 1 + G_cG = 1 + \frac{1.65K_c(1-0.85s)}{4.675s^2+6.35s+1}$$

Rearrange,

$$4.675s^2 + (6.35 - 1.403K_c)s + 1 + 1.65K_c = 0$$

Substitute $s = j\omega_u$ at $K_c = K_{cu}$:

$$-4.675\,\omega_u^2 + j(6.35 - 1.403K_{cu})\,\omega_u + 1 + 1.65K_{cu} = 0 + j0$$

Equate real and imaginary coefficients,

$$(6.35 - 1.403K_{cu})\omega_u = 0\ ,$$

$$1 + 1.65K_{cu} - 4.675\,\omega_u^2 = 0$$

Ignoring $\omega_u = 0$, the approximate values are:

$K_{cu} = 4.53$ and $\omega_u = 1.346$ rad/min

$$P_u = \frac{2\pi}{\omega_u} = 4.67 \text{ min}$$

The Z-N PI settings from Table 12.7 are:

$K_c = 2.04$ and $\tau_I = 3.89$ min (approximate)

Note that the values of K_{cu} and ω_u are approximate due to the Padé approximation. By using Simulink, more accurate values can be obtained by trial and error. For this case, no Padé approximation is needed and:

$K_{cu} = 3.76$ $P_u = 5.9$ min

The Z-N PI settings from Table 12.7 are:

$K_c = 1.69$ $\tau_I = 4.92$ min (more accurate)

Compared to the Z-N settings, the IMC method setting gives a smaller K_c and a larger τ_I, and therefore provides more conservative controller settings.

12.14

Eliminate the effect of the feedback loop by opening the loop. That is, operate temporarily in an open loop mode by switching the controller to the manual mode. This change provides a constant controller output and a constant manipulated input. If oscillations persist, they must be due to external disturbances. If the oscillations vanish, they were caused by the feedback loop.

12.15

The sight glass has confirmed that the liquid level is rising. Because the controller output is saturated, the controller is working fine. Hence, either the feed flow is higher than recorded, or the liquid flow is lower than recorded, or both. Because the flow transmitters consist of orifice plates and differential pressure transmitters, a plugged orifice plate could lead to a higher recorded flow. Hence, the liquid-flow-transmitter orifice plate would be the prime suspect.

12.16

a) IMC design:

From Table 12.1, Controller *H* with $\tau_c = \tau/2 = 3.28$ min is:

$$K_c = \frac{1}{K}\frac{\tau+\theta/2}{\tau_c+\theta/2} = \frac{1}{220}\frac{6.5+2/2}{3.25+2/2}$$

$$K_c = 0.00802$$

$$\tau_I = \tau + \frac{\theta}{2} = 6.5 + \frac{2}{2} = 7.5\,\text{min}$$

$$\tau_D = \frac{\tau\theta}{2\tau+\theta} = \frac{(6.5)(2)}{2(6.5)+2} = 0.867\,\text{min}$$

b) Relay auto tuning (RAT) controller

From the documentation for the RAT results, it follows that:

$a = 54, \qquad d = 0.5$

From (12-46),

$$K_{cu} = \frac{4d}{\pi a} = \frac{4(0.5)}{\pi(54)} = 0.0118$$

$$P_u = 14\,\text{min}$$

From Table 12.7, the Ziegler-Nichols controller settings are:

$$K_c = 0.6K_{cu} = 0.0071$$

$$\tau_I = \frac{P_u}{2} = 7\text{ min}, \qquad \tau_D = \frac{P_u}{8} = 1.75\text{ min}$$

c) Simulation results

The closed-loop responses for the IMC and RAT controller settings and a step change in feed composition from 0.5 to 0.55 are shown in Figs. S12.16a and S12.16b, respectively.

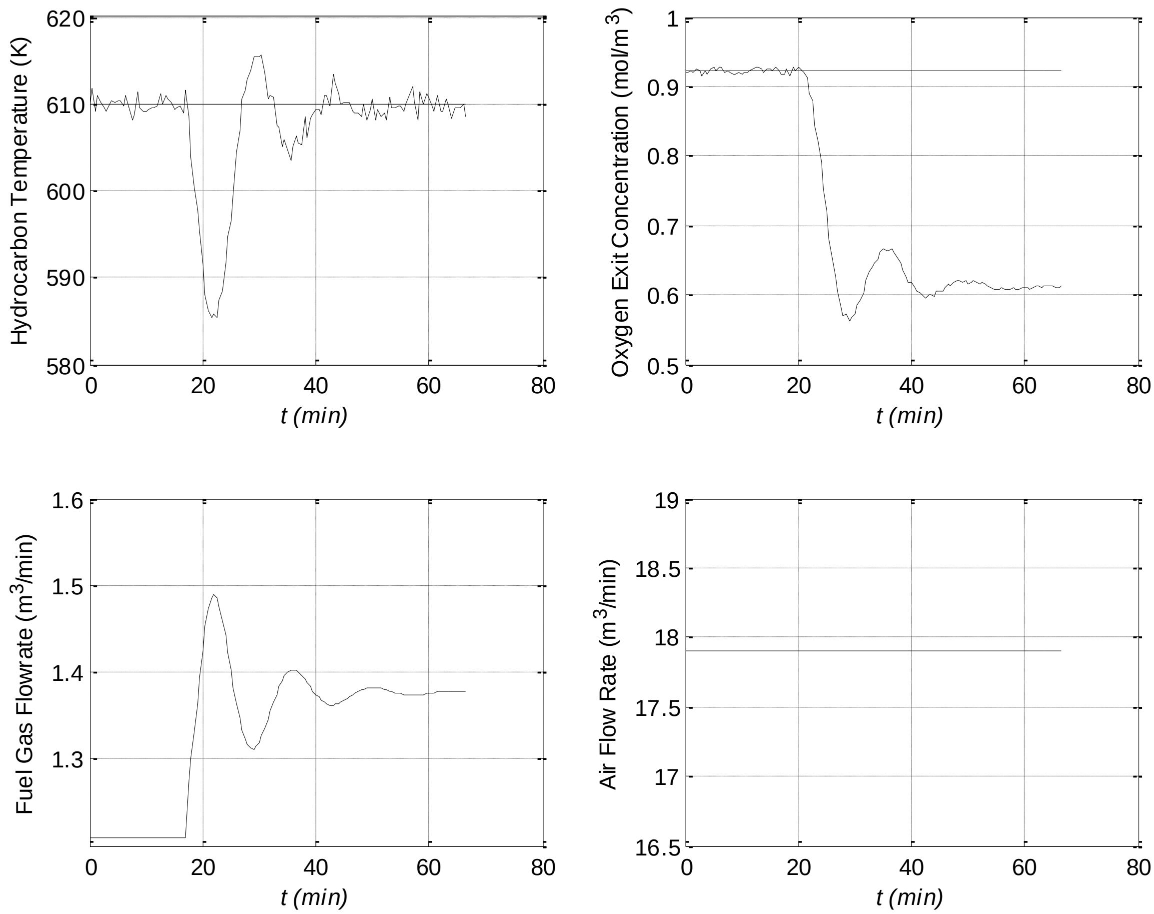


Fig. S12.16a. *Performance of the IMC-PID controller for a step change in hydrocarbon flow rate from 0.035 to 0.040 m^3/min.*

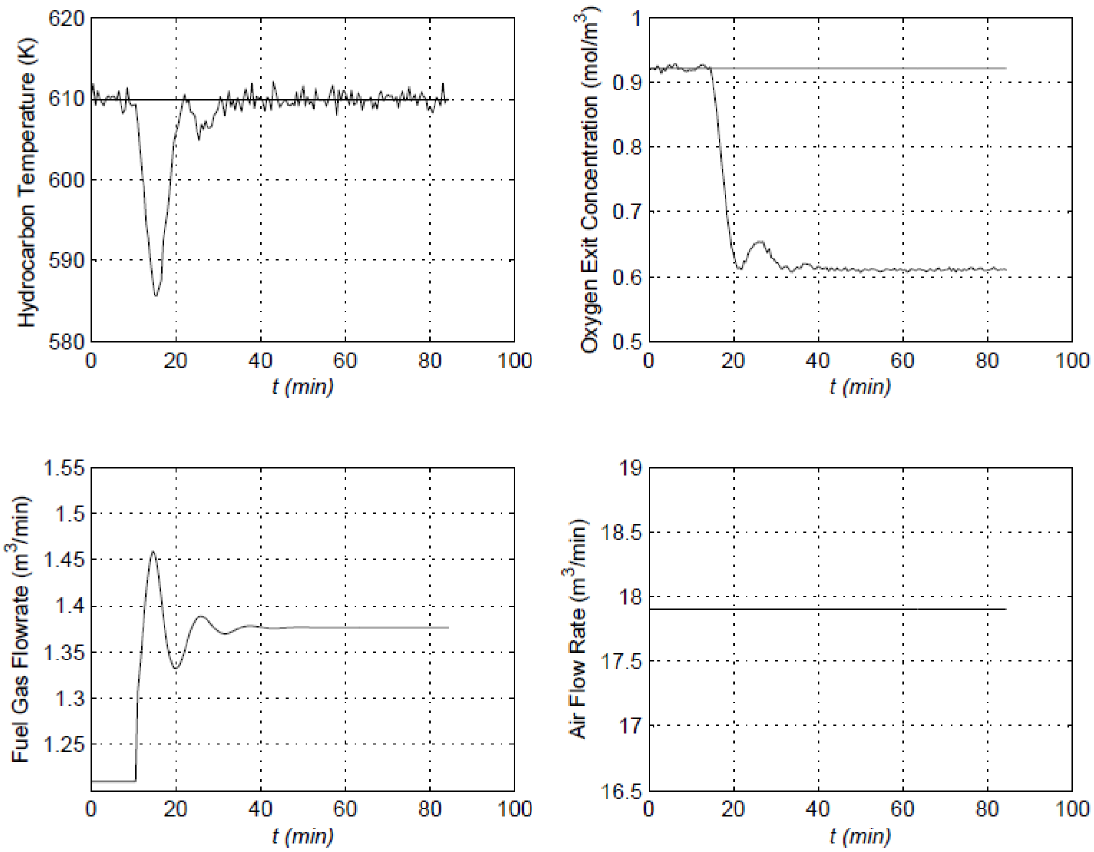


Fig. S12.16b. *Performance of the RAT controller for a step change in hydrocarbon flow rate from 0.035 to 0.040 m^3/min.*

The RAT controller is superior due to its smaller maximum deviation and shorter settling time.

d) Due to the high noise level for the x_D response, it is difficult to obtain improved controller settings. The RAT settings are considered to be satisfactory.

12.17

a) IMC design:

From Table 12.1, Controller *H* with τ_c = τ/2 =381 is:

$$K_c = \frac{1}{K}\frac{\tau+\theta/2}{\tau_c+\theta/2} = \frac{1}{0.126}\frac{762+138/2}{381+138/2}$$

$$K_c = 14.7$$

$$\tau_I = \tau + \frac{\theta}{2} = 762 + \frac{138}{2} = 831\,\text{min}$$

$$\tau_D = \frac{\tau\theta}{2\tau+\theta} = \frac{(762)(138)}{2(762)+138} = 63.3\,\text{min}$$

b) Relay auto tuning (RAT) controller

The distillation column model includes an RAT option for the x_B control loop, but not the x_D control loop. Thus, the Simulink diagram must be modified by copying the RAT loop for x_B and adding it to the x_D portion of the diagram. Also, the parameters for the *relay* block must be changed. The new Simulink diagram and appropriate relay settings are shown in Fig. S12.17a. The results from the RAT are shown in Fig.S12.17b.

From the documentation from the RAT results, it follows that:

$a = 5.55 \text{ x } 10^{-3}$, $d = 0.2$

From (12-46),

$$K_{cu} = \frac{4d}{\pi a} = \frac{4(0.2)}{\pi(5.55x10^{-3})} = 45.9$$

$$P_u = 950\text{ s}$$

From Table 12.7, the Ziegler-Nichols controller settings are:

$$K_c = 0.6K_{cu} = 27.5$$

$$\tau_I = \frac{P_u}{2} = 425 \text{ s}, \quad \tau_D = \frac{P_u}{8} = 119 \text{ s}$$

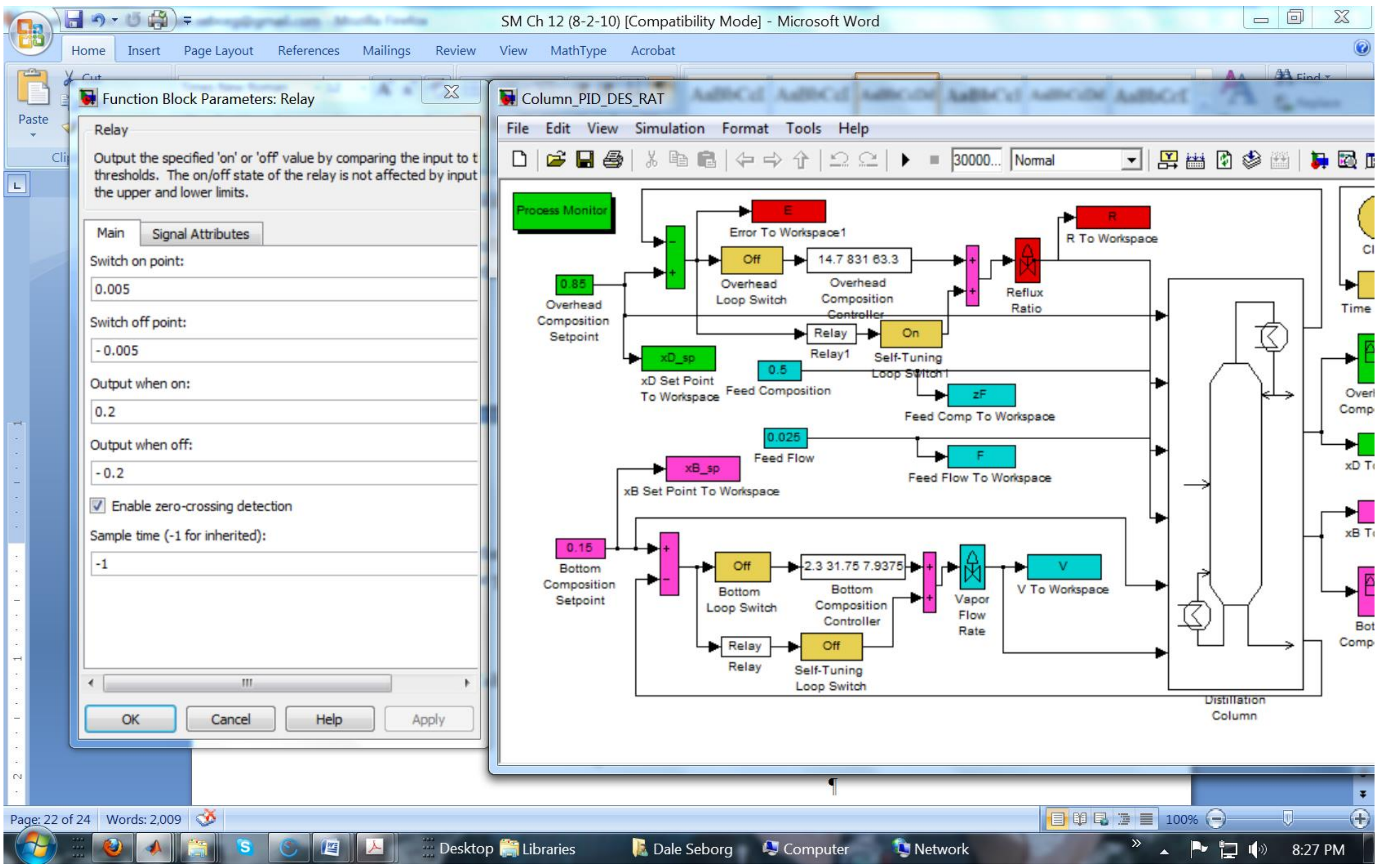


Fig. S12.17a. *Modified RAT Simulink diagram and relay settings.*

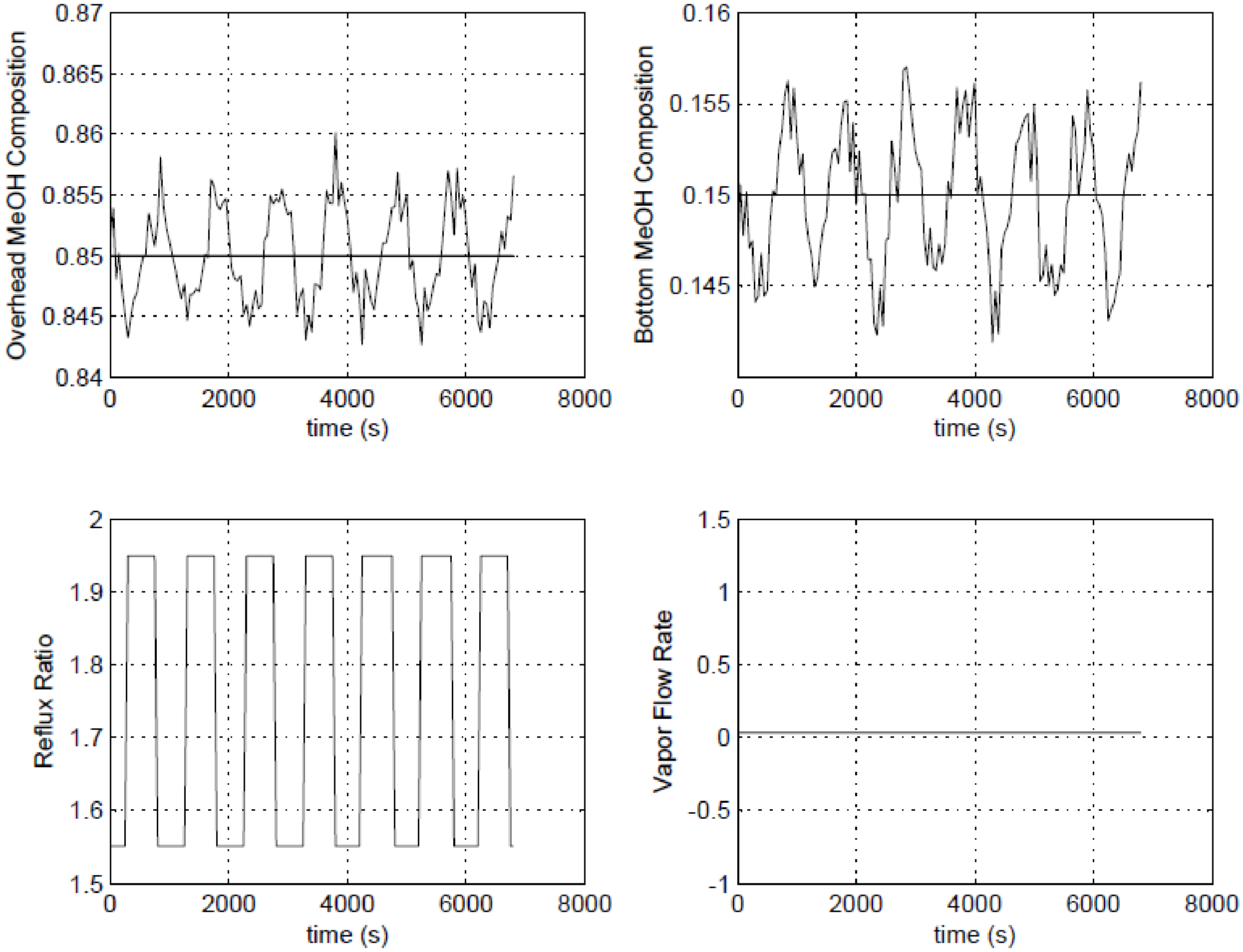


Fig. S12.17b. *Results from RAT.*

c) Simulation results

The closed-loop responses for the IMC and RAT controller settings and a step change in feed composition from 0.5 to 0.55 are shown in Figs. S12.17c and S12.17d, respectively.

The RAT controller provides a somewhat better response with a smaller maximum deviation and a shorter settling time.

d) Due to the high noise level for the x_D response, it is difficult to obtain improved controller settings. The RAT settings are considered to be satisfactory.

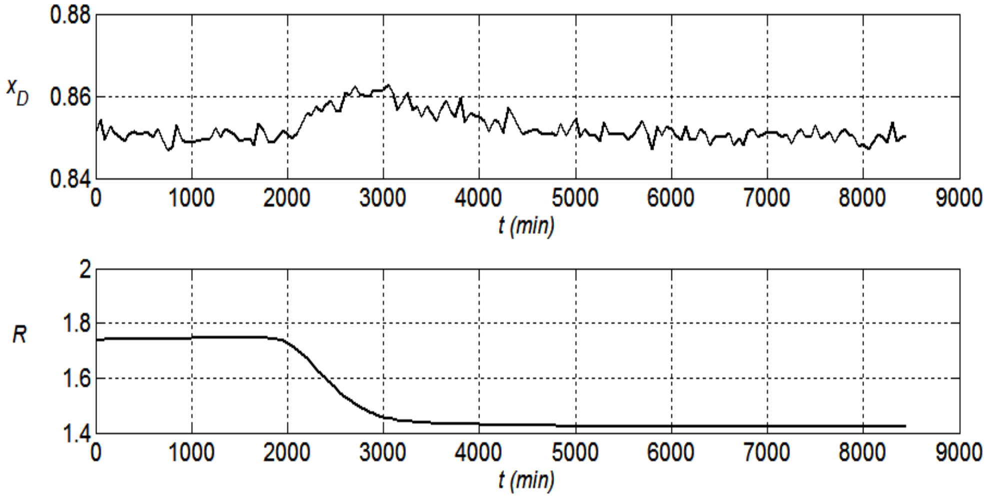


Fig. S12.17c. *Performance of the IMC-PID controller for a step change in feed composition from 0.5 to 0.55.*

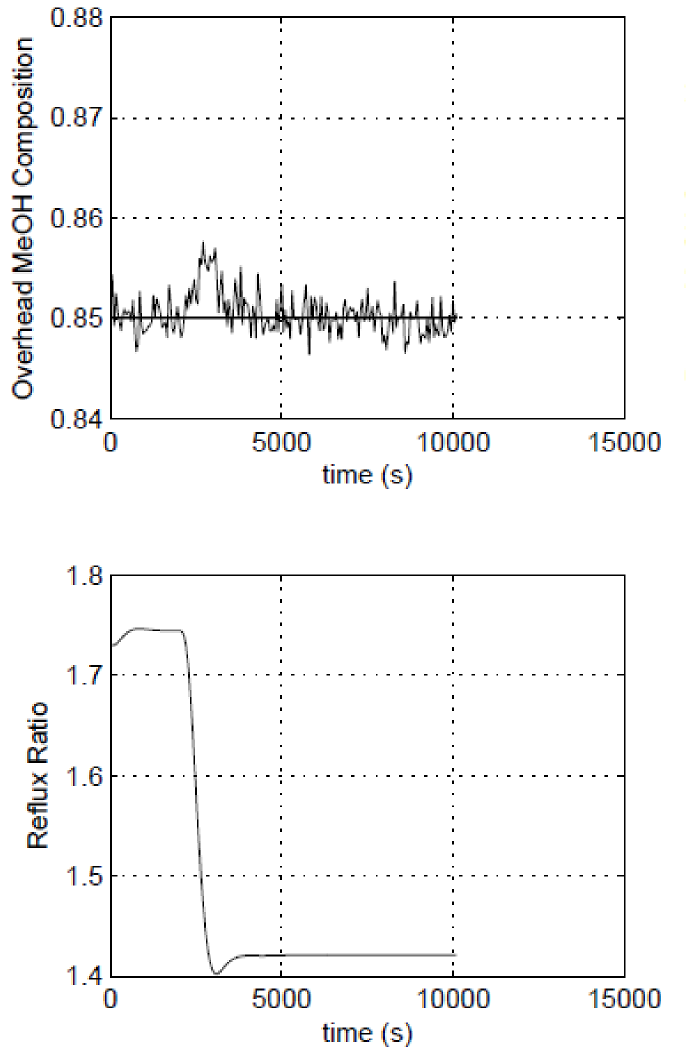


Fig. S12.17d. *Performance of the RAT controller for a step change in feed composition from 0.5 to 0.55.*

Chapter 13 ©

13.1

According to Guideline 6, the manipulated variable should have a large effect on the controlled variable. Clearly, it is easier to control a liquid level by manipulating a large exit stream, rather than a small stream. Because R/D >1, the reflux flow rate R is the preferred manipulated variable.

13.2

Exit flow rate w_4 has no effect on x_3 or x_4 because it does not change the relative amounts of materials that are blended. The bypass fraction f has a dynamic effect on x_4 but has no steady-state effect because it also does not change the relative amounts of materials that are blended. Thus, w_2 is the best choice.

13.3

Both the steady-state and dynamic behaviors need to be considered. From a steady-state perspective, the reflux stream temperature T_R would be a poor choice because it is insensitive to changes in x_D, due to the small nominal value of 5 ppm. For example, even a 100% change in from 5 to 10 ppm would result in a negligible change in T_R. Similarly, the temperature of the top tray would be a poor choice. An intermediate tray temperature would be more sensitive to changes in the tray composition but may not be representative of x_D. Ideally, the tray location should be selected to be the highest tray in the column that still has the desired degree of sensitivity to composition changes.

The choice of an intermediate tray temperature offers the advantage of early detection of feed disturbances and disturbances that originate in the stripping (bottom) section of the column. However, it would be slow to respond to disturbances originating in the condenser or in the reflux drum. But on balance, an intermediate tray temperature is the best choice.

13.4

For the flooded condenser in Fig. E13.4, the area available for heat transfer changes as the liquid level changes. Consequently, pressure control is easier when the liquid level is low and more difficult when the level is high. By contrast, for the conventional process design in Fig. 13.2, the liquid level has a very small effect on the pressure control loop. Thus, the flooded condenser is more difficult to control because the level and pressure control loops are more interacting, than they are for the conventional process design in Fig. 13.2.

13.5

(a) The larger the tank, the more effective it will be in "damping out" disturbances in the reactor exit stream. A large tank capacity also provides a large feed inventory for the distillation column, which is desirable for periods where the reactor is shut down. Thus a large tank is preferred from a process control perspective. However a large tank has a high capital cost, so a small tank is appealing from a steady-state, design perspective. Thus, the choice of the storage tank size involves a tradeoff of control and design objectives.

(b) After a set-point change in reactor exit composition occurs, it would be desirable to have the exit compositions for both the reactor and the storage tank change to the new values as soon as possible. But concentration in the storage tank will change gradually due to its liquid inventory. The time constant for the storage tank is proportional to the mass of liquid in the tank (cf. blending system models in Chapters 2 and 4). Thus, a large storage tank will result in sluggish responses in its exit composition, which is not desirable when frequent set-point changes are required. In this situation, the storage tank size should be smaller than for case (a).

13.6

Variables : $q_1, q_2, \ldots q_6, h_1, h_2$ $\quad N_v = 8$

Equations :

Three flow-head relations: $q_3 = C_{V1}\sqrt{h_1}$

$$q_5 = C_{V2}\sqrt{h_2}$$

$$q_4 = f(h_1, h_2)$$

Two conservation of mass equations:

$$\rho A_1 \frac{dh_1}{dt} = \rho(q_1 + q_6 - q_3 - q_4)$$

$$\rho A_2 \frac{dh_2}{dt} = \rho(q_2 + q_4 - q_5)$$

Conclude: $N_E = 5$

Degrees of freedom: $= N_F = N_V - N_E = 8 - 5 = 3$

Disturbance variable: $q_6 \;\Rightarrow\; N_D = 1$

$$N_F = N_{FC} + N_D$$

$$N_{FC} = 3 - 1 = 2$$

13.7

Consider the following energy balance assuming a reference temperature of $T_{ref} = 0$:

Heat exchanger:

$$C_c(1-f)w_c(T_{c0} - T_{c1}) = C_h w_h(T_{h1} - T_{h2}) \qquad (1)$$

Overall:

$$C_c w_c(T_{c2} - T_{c1}) = C_h w_h(T_{h1} - T_{h2}) \qquad (2)$$

Mixing point:

$$w_c = (1-f)w_c + fw_c \qquad (3)$$

Thus,

$N_E = 3$, $N_V = 8$ $(f, w_c, w_h, T_{c1}, T_{c2}, T_{c0}, T_{h1}, T_{h2})$

$N_F = N_V - N_E = 8 - 3 = 5$

$N_{FC} = 2$ (f, w_h)

also

$N_D = N_F - N_{FC} = 3$ (w_c, T_{c1}, T_{c2})

The degree of freedom analysis is identical for both cocurrent and countercurrent flow because the mass and energy balances are the same for both cases.

13.8

The dynamic model consists of the following balances:

Mass balance on the tank:

$$\rho A \frac{dh}{dt} = (1 - f)w_1 + w_2 - w_3 \tag{1}$$

Component balance on the tank:

$$\rho A \frac{d(hx_3)}{dt} = (1 - f)x_1 w_1 + x_2 w_2 - x_3 w_3 \tag{2}$$

Mixing point balances:

$$w_4 = w_3 + fw_1 \tag{3}$$
$$x_4 w_4 = x_3 w_3 + fx_1 w_1 \tag{4}$$

Thus,

$N_E = 4$ (Eqs. 1- 4)

$N_V = 9$ $(h, f, w_2, w_3, w_4, x_1, x_2, x_3, x_4)$

$N_F = N_V - N_E = 5$

Since three flow rates $(fw_1, w_2 \; and \; w_3)$ can be independently adjusted, it would appear that there are three control degrees of freedom. But the bypass flow rate, fw_1, has no steady-state effect on x_4. To confirm this assertion, consider the overall steady-state component balance for the tank and the mixing point:

$$x_1 w_1 + x_2 w_2 = x_4 w_4 \tag{5}$$

This balance does not depend on the fraction bypassed, *f*, either directly or indirectly,

Conclusion: $N_{FC} = 2$ (w_2 and w_4)

13.9

(a) In order to analyze this situation, consider a steady-state analysis.

Assumptions:

1. Steady-state conditions with w, T_h, *and* T_c at their nominal values.
2. Constant heat capacities
3. No heat losses
4. Perfect mixing

Steady-state balances:

$$w_c + w_h = w \tag{1}$$

$$w_c T_c + w_h T_h = wT \tag{2}$$

Assume that $T = T_{sp}$, where T_{sp} is the set point.

$$w_c + w_h = w \tag{3}$$

$$w_c T_c + w_h T_h = wT_{sp} \tag{4}$$

Equations (3) and (4) are two independent equations with two unknown variables, w_h and w_c. For any arbitrary value of T_{sp}, these equations have a unique solution. Thus the proposed multiloop control strategy is feasible.

This simple analysis does not prove that the liquid level *h* can also be controlled to an arbitrary set point h_{sp}. However, this result can be demonstrated by a more complicated theoretical analysis or by simulation studies.

(b) Consider the steady-state model in (1) and (2). Substituting (1) into (2) and solving for T gives:

$$T = \frac{w_c T_c + w_h T_h}{w_c + w_h} \tag{5}$$

Since w does not appear in (5), it has no steady-state effect on T. Consequently, the proposed multiloop control strategy is not feasible.

13.10

(a) Model degrees of freedom, N_F

$$N_F = N_V - N_E \tag{13-1}$$

$N_V = 11$ $(x_F, T_F, F, w_L, L, w_V, V, T, P, h, V_T)$

where T_F is the feed temperature and V_T is the volume of the flash separator.

$N_V = 7$:

- Mass balance
- Component balance
- Energy balance
- Vapor-liquid equilibrium relation
- Valve relations (2)
- Ideal gas law

Thus, $N_F = 11 - 7 = \boxed{4}$

(b) Control degrees of freedom, N_{FC}

$$N_F = N_{FC} + N_D \tag{13-2}$$

Typically, some knowledge of the feed conditions would be available. We consider two cases:

<u>Case 1: x_F and T_F are disturbance variables</u>

Here N_D =2 and:

$$N_{FC} = N_F - N_D = 4 - 2 = \boxed{2}$$

The two degrees of freedom can be utilized by manipulating two of the three flow rates, for example, *V* and *L*, or *F* and *V*.

<u>Case 2: x_F, T_F, and *F* are disturbance variables</u>

Here N_D =3 and:

$$N_{FC} = N_F - N_D = 4 - 3 = \boxed{1}$$

The single degree of freedom could be utilized by manipulating one of the exit flow rates, either *V* or *L*.

Chapter 14

14.1

$$AR = |G(j\omega)| = \frac{3|G_1(j\omega)|}{|G_2(j\omega)||G_3(j\omega)|}$$

$$= \frac{3\sqrt{(-\omega)^2+1}}{\omega\sqrt{(2\omega)^2+1}} = \frac{3\sqrt{\omega^2+1}}{\omega\sqrt{4\omega^2+1}}$$

From the statement, we know the period P of the input sinusoid is 0.5 min and, thus,

$$\omega = \frac{2\pi}{P} = \frac{2\pi}{0.5} = 4\pi \text{ rad/min}$$

Substituting the numerical value of the frequency:

$$\hat{A} = AR \times A = \frac{3\sqrt{16\pi^2+1}}{4\pi\sqrt{64\pi^2+1}} \times 2 = 0.12 \times 2 = 0.24^\circ$$

Thus the amplitude of the resulting temperature oscillation is 0.24 degrees.

14.2

First approximate the exponential term as the first two terms in a truncated Taylor series

$$e^{-\theta s} \approx 1 - \theta s$$

Then $G(j\omega) = 1 - j\omega$

and $AR_{two\ term} = \sqrt{1+(-\omega\theta)^2} = \sqrt{1+\omega^2\theta^2}$

$$\phi_{two\ term} = \tan^{-1}(-\omega\theta) = -\tan^{-1}(\omega\theta)$$

For a first-order Pade approximation

$$e^{-\theta s} \approx \frac{1-\dfrac{\theta s}{2}}{1+\dfrac{\theta s}{2}}$$

from which we obtain

$$AR_{Pade} = 1$$

$$\phi_{Pade} = -2\tan^{-1}\left(\frac{\omega\theta}{2}\right)$$

Both approximations represent the original function well in the low frequency region. At higher frequencies, the Padé approximation matches the amplitude ratio of the time delay element exactly ($AR_{Pade} = 1$), while the two-term approximation introduces amplification ($AR_{two\ term} > 1$). For the phase angle, the high-frequency representations are:

$$\phi_{two\ term} \to -90^\circ$$

$$\phi_{Pade} \to -180^\circ$$

Since the angle of $e^{-j\omega\theta}$ is negative and becomes unbounded as $\omega \to \infty$, we see that the Pade representation also provides the better approximation to the time delay element's phase angle, matching ϕ of the pure time delay element to a higher frequency than the two-term representation.

14.3

Nominal temperature $\bar{T} = \dfrac{128\ ^\circ\text{F} + 120\ ^\circ\text{F}}{2} = 124\ ^\circ\text{F}$

$$\hat{A} = \frac{1}{2}(128\ ^\circ\text{F} - 120\ ^\circ\text{F}) = 4\ ^\circ\text{F}$$

$\tau = 5\text{sec.},\qquad \omega = 2\pi(1.8/60\text{sec}) = 0.189\ \text{rad/s}$

Using Eq. 13-2 with K=1,

$$A = \hat{A}\left(\sqrt{\omega^2\tau^2+1}\right) = 4\sqrt{(0.189)^2(5)^2+1} = 5.50\ ^\circ\text{F}$$

Actual maximum air temperature = $\bar{T} + A = 129.5\ ^\circ\text{F}$

Actual minimum air temperature = $\bar{T} - A = 118.5\ ^\circ\text{F}$

14.4

$$\frac{T_m'(s)}{T'(s)} = \frac{1}{0.1s+1}$$

$T'(s) = (0.1s+1)T_m'(s)$

amplitude of T' =3.464 $\sqrt{(0.1\omega)^2+1} = 3.465$

phase angle of T' = φ + tan^{-1}(0.1ω) = φ + 0.02

Since only the maximum error is required, set φ = 0 for the comparison of T' and T_m'. Then

Error = T_m' – T' =3.464 sin (0.2*t*) – 3.465sin(0.2*t* + 0.02)
= 3.464 sin(0.2*t*) –3.465[sin(0.2*t*) cos 0.02 + cos(0.2*t*)sin 0.02]
= 0.000 sin(0.2*t*) – 0.0693 cos(0.2*t*)

Since the maximum absolute value of cos(0.2*t*) is 1,

maximum absolute error = 0.0693

14.5

(a) No, cannot make 1st order closed-loop system unstable.

(b) No, cannot make 2nd order overdamped system unstable for closed-loop.

(c) Yes, 3rd order system can be made unstable.

(d) Yes, anything with time delay can be made unstable.

14.6

Engineer A is correct.
Second order overdamped process cannot become unstable with a proportional controller.
FOPTD model can become unstable with a large K_c due to the time delay.

14.7

Using MATLAB

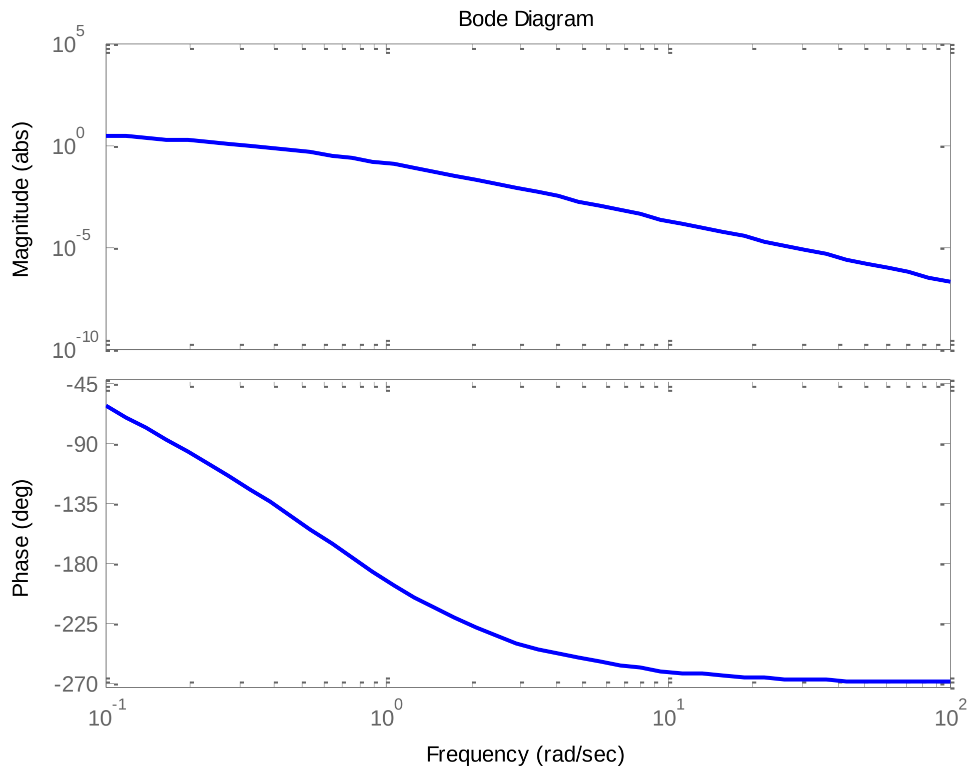


Figure S14.7. *Bode diagram of the third-order transfer function.*

The value of ω that yields a -180° phase angle and the value of AR at that frequency are:

ω = 0.807 rad/sec
AR = 0.202

14.8

Using MATLAB,

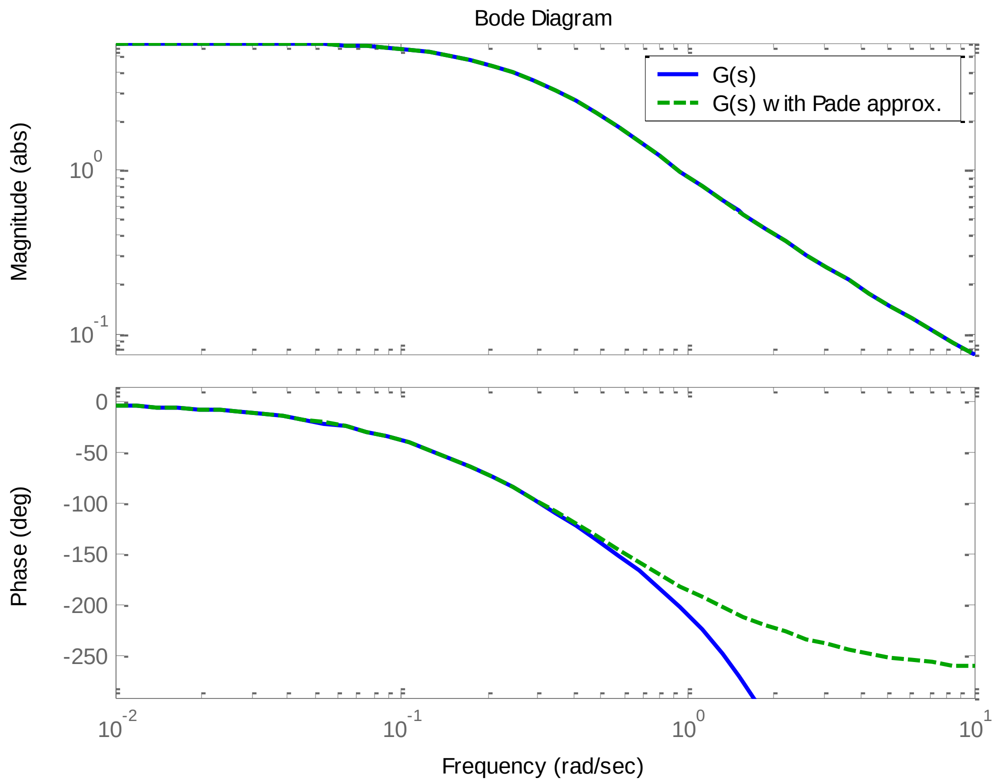


Figure S14.8. *Bode diagram for G(s) and G(s) with Pade approximation.*

As we can see from the figures, the accuracy of Pade approximation does not change as frequency increases in magnitude plot, but it will be compromised in the phase plot as frequency goes higher.

14.9

$\omega = 2\pi f$ where f is in cycles/min

For the standard thermocouple, using Eq. 14-13b

$$\varphi_1 = -\tan^{-1}(\omega\tau_1) = \tan^{-1}(0.15\omega)$$

Phase difference $\Delta\varphi = \varphi_1 - \varphi_2$

Thus, the phase angle for the unknown unit is

$$\varphi_2 = \varphi_1 - \Delta\varphi$$

and the time constant for the unknown unit is

$$\tau_2 = \frac{1}{\omega}\tan(-\varphi_2)$$

using Eq. 14-13b . The results are tabulated below

f	ω	φ_1	Δφ	φ_2	τ_2
0.05	0.31	-2.7	4.5	-7.2	0.4023
0.1	0.63	-5.4	8.7	-14.1	0.4000
0.2	1.26	-10.7	16	-26.7	0.4004
0.4	2.51	-26.6	24.5	-45.1	0.3995
0.8	6.03	-37	26.5	-63.5	0.3992
1	6.28	-43.3	25	-68.3	0.4001
2	12.57	-62	16.7	-78.7	0.3984
4	25.13	-75.1	9.2	-84.3	0.3988

That the unknown unit is first order is indicated by the fact that $\Delta\varphi \to 0$ as $\omega \to \infty$, so that $\varphi_2 \to \varphi_1 \to -90°$ and $\varphi_2 \to -90°$ for $\omega \to \infty$ implies a first-order system. This is confirmed by the similar values of τ_2 calculated for different values of ω, implying that a graph of $\tan(-\varphi_2)$ versus ω is linear as expected for a first-order system. Then using linear regression or taking the average of above values, τ_2 = 0.40 min.

14.10

From the solution to Exercise 5-19, for the two-tank system

$$\frac{H_1'(s)/h'_{1\max}}{Q_{1i}'(s)} = \frac{0.01}{1.32s+1} = \frac{K}{\tau s+1}$$

$$\frac{H_2'(s)/h'_{2\max}}{Q_{1i}'(s)} = \frac{0.01}{(1.32s+1)^2} = \frac{K}{(\tau s+1)^2}$$

$$\frac{Q_2'(s)}{Q_{1i}'(s)} = \frac{0.1337}{(1.32s+1)^2} = \frac{0.1337}{(\tau s+1)^2}$$

and for the one-tank system

$$\frac{H'(s)/h'_{\max}}{Q_{1i}'(s)} = \frac{0.01}{2.64s+1} = \frac{K}{2\tau s+1}$$

$$\frac{Q'(s)}{Q_{1i}'(s)} = \frac{0.1337}{2.64s+1} = \frac{0.1337}{2\tau s+1}$$

For a sinusoidal input $q_{1i}'(t) = A\sin\omega t$, the amplitudes of the heights and flow rates are

$$\hat{A}[h'/h'_{max}] = KA/\sqrt{4\omega^2\tau^2+1} \qquad (1)$$

$$\hat{A}[q'] = 0.1337A/\sqrt{4\omega^2\tau^2+1} \qquad (2)$$

for the one-tank system, and

$$\hat{A}[h'_1/h'_{1max}] = KA/\sqrt{\omega^2\tau^2+1} \qquad (3)$$

$$\hat{A}[h'_2/h'_{2max}] = KA/\sqrt{(\omega^2\tau^2+1)^2} \qquad (4)$$

$$\hat{A}[q'_2] = 0.1337A/\sqrt{(\omega^2\tau^2+1)^2} \qquad (5)$$

for the two-tank system.

Comparing (1) and (3), for all ω

$$\hat{A}[h'_1/h'_{1max}] \geq \hat{A}[h'/h'_{max}]$$

Hence, for all ω, the first tank of the two-tank system will overflow for a smaller value of A than will the one-tank system. Thus, from the overflow consideration, the one-tank system is better for all ω. However, if A is small enough so that overflow is not a concern, the two-tank system will provide a smaller amplitude in the output flow for those values of ω that satisfy

$$\hat{A}[q'_2] \leq \hat{A}[q']$$

or $$\frac{0.1337A}{\sqrt{(\omega^2\tau^2+1)^2}} \leq \frac{0.1337A}{\sqrt{4\omega^2\tau^2+1}}$$

or $\omega \geq \sqrt{2}/\tau = 1.07$

Therefore, the two-tank system provides better damping of a sinusoidal disturbance for $\omega \geq 1.07$ if and only if

$$\hat{A}[h'_1/h'_{1max}] \leq 1 \text{ , that is, } A \leq \frac{\sqrt{1.32^2\omega^2+1}}{0.01}$$

14.11

Using Eqs. 14-28 , 14-13, and 14-17,

$$AR = \frac{2\sqrt{\omega^2 \tau_a^{\ 2} + 1}}{\sqrt{100\omega^2 + 1}\sqrt{4\omega^2 + 1}}$$

$$\phi = \tan^{-1}(\omega\tau_a) - \tan^{-1}(10\omega) - \tan^{-1}(2\omega)$$

The Bode plots shown below indicate that

i) AR does not depend on the sign of the zero.
ii) AR exhibits resonance for zeros close to origin.
iii) All zeros lead to ultimate slope of –1 for AR.
iv) A left-plane zero yields an ultimate ϕ of -90°.
v) A right-plane zero yields an ultimate ϕ of -270°.
vi) Left-plane zeros close to origin can give phase lead at low ω.
vii) Left-plane zeros far from the origin lead to a greater lag (i.e., smaller phase angle) than the ultimate value. $\phi_u = -90°$ with a left-plane zero present.

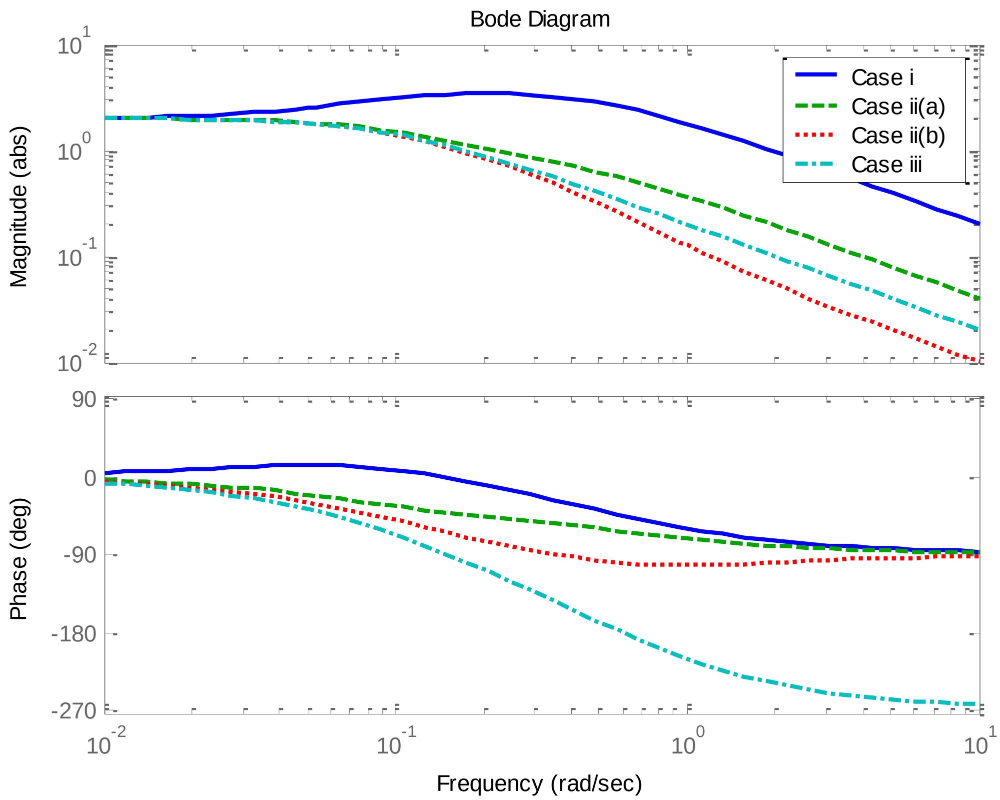


Figure S14.11. *Bode plot for each of the four cases of numerator dynamics.*

14.12

a) From Eq. 8-14 with $\tau_I = 4\tau_D$

$$G_c(s) = K_c \frac{(4\tau_D s + 1 + 4\tau_D{}^2 s^2)}{4\tau_D s} = K_c \frac{(2\tau_D s + 1)^2}{4\tau_D s}$$

$$|G_c(j\omega)| = K_c \frac{\left(\sqrt{4\tau_D{}^2\omega^2 + 1}\right)^2}{4\tau_D\omega} = K_c \frac{4\tau_D{}^2\omega^2 + 1}{4\tau_D\omega}$$

b) From Eq. 8-15 with $\tau_I = 4\tau_D$ and $\alpha = 0.1$

$$G_c(s) = K_c \frac{(4\tau_D s + 1)(\tau_D s + 1)}{4\tau_D s(0.1\tau_D s + 1)}$$

$$|G_c(j\omega)| = K_c \frac{\left(\sqrt{16\tau_D{}^2\omega^2 + 1}\right)\left(\sqrt{\tau_D{}^2\omega^2 + 1}\right)}{4\tau_D\omega\sqrt{0.01\tau_D{}^2\omega^2 + 1}}$$

The differences are significant for $0.25 < \omega\tau_D < 1$ by a maximum of $0.5\ K_c$ at $\omega\tau_D = 0.5$, and for $\omega\tau_D > 10$ by an amount increasing with $\omega\tau_D$.

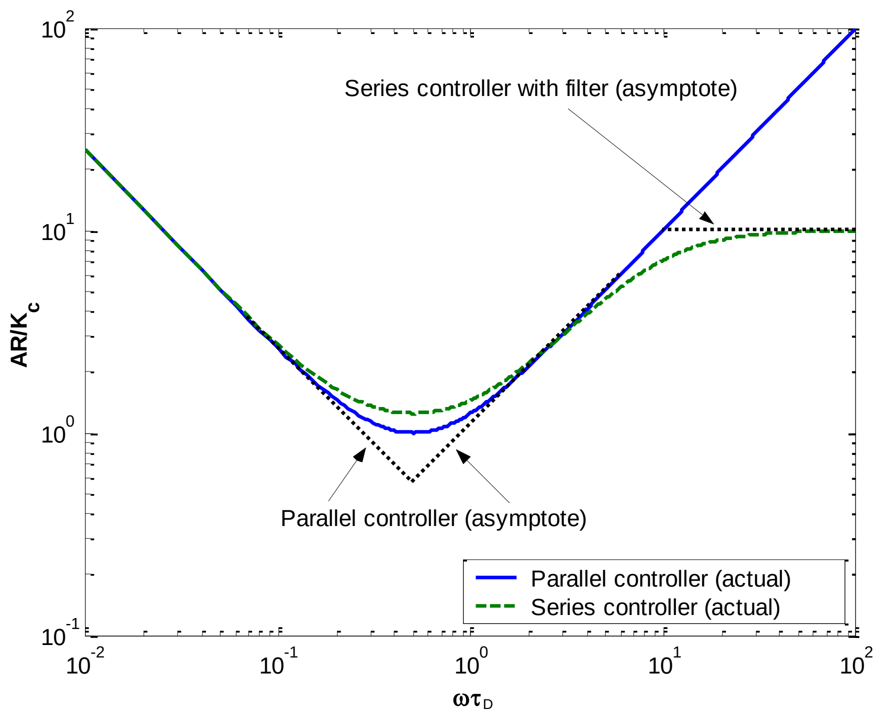


Figure S14.12. *Nominal amplitude ratio for parallel and series controllers*

14.13

$$1+G_{OL}=1+G_mG_pG_cG_v=1+\frac{2}{\tau s+1}\frac{0.6}{50s+1}\frac{4}{2s+1}K_c$$

Characteristic equation:

$$(\tau s+1)(50s+1)(2s+1)+K_c(4)(2)(0.6)=0 \qquad (1)$$

For a third order process, a K_c can always be chosen to make the process unstable. A stability analysis would verify this but was not required.

Substitute $s=j\omega$ into Eq. (1), we have:

$$(j\omega\tau+1)(50j\omega+1)(2j\omega+1)+K_c(4)(2)(0.6)=0$$

For $\tau = 1$, we have:

$$(-100\omega^3+53\omega)j+(1+4.8K_c-152\omega^2)=0 \qquad (2)$$

Thus, we have $\omega_c = 0.53$ and $K_{cu} = 16.58$.

For $\tau = 0.4$, we have:

$$(-40\omega^3+52.4\omega)j+(1+4.8K_c-120.8\omega^2)=0 \qquad (3)$$

So we have:

$\omega_c = 1.31$ and $K_{cu} = 41.28$

The second measurement is preferred because of a larger stability region of Kc.

14.14

(a) Always true. Increasing the gain does speed up the response for a set point change. Care must be taken to not increase the gain too much or oscillations will result.

(b) False. If the open loop system is first order, increasing K_c cannot result in oscillation.

(c) Generally true. Increasing the controller gain can cause real part of the roots of the characteristic polynomial to turn positive. However, for first or second order processes, increasing K_c will not cause instability.

(d) Always true. Increasing the controller gain will decrease offset. However, if the gain is increased too much, oscillations may occur. Even with the oscillations the offset will continue to decrease until the system becomes unstable.

14.15

(a)

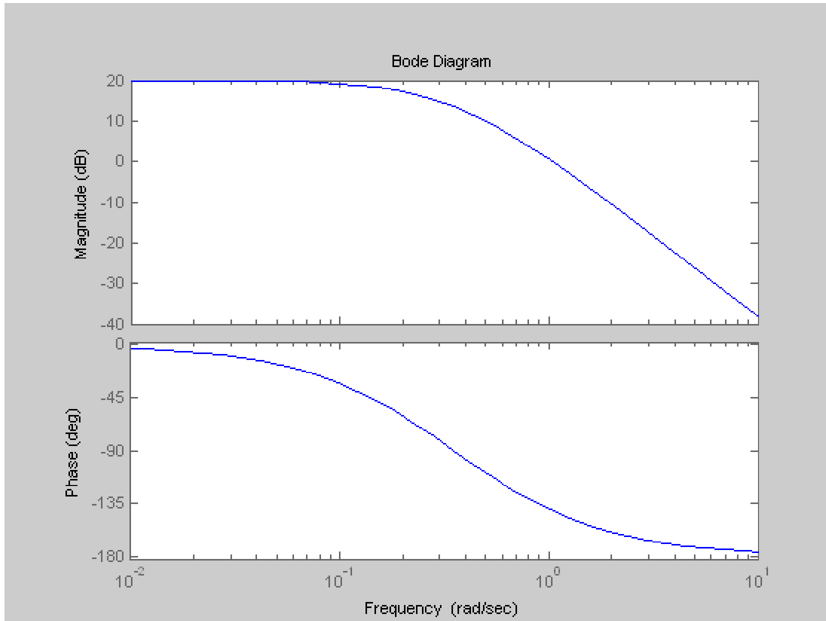


Figure S14.15a *Bode plot of G_{OL}. (K_c = 10)*

$$G_{OL} = GG_c = \frac{1}{(4s+1)(2s+1)} K_c$$

Cannot become unstable – max phase angle 2nd order overdamped process (G_{OL}) is -180 degree.

(b)

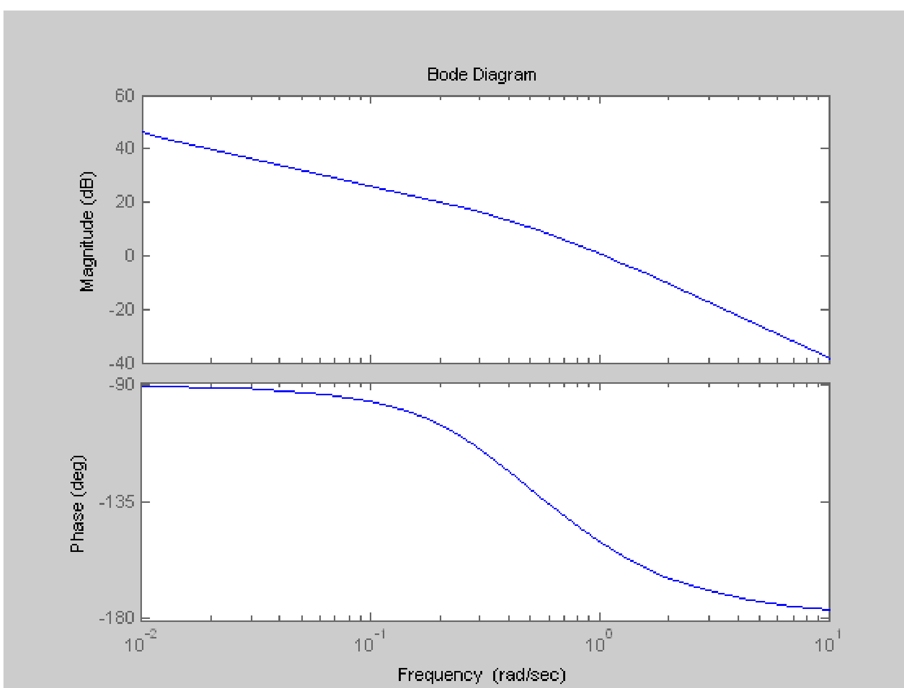


Figure S14.15b *Bode plot of G_{OL}. (K_c = 10)*

$$G_{OL} = GG_c = \frac{1+1/5s}{(4s+1)(2s+1)} K_c = \frac{(5s+1)K_c}{5s(4s+1)(2s+1)}$$

Cannot become unstable – max phase angle (G_{OL}) is -180 degree while at low frequency the integrator has -90 degree phase angle.

(c)

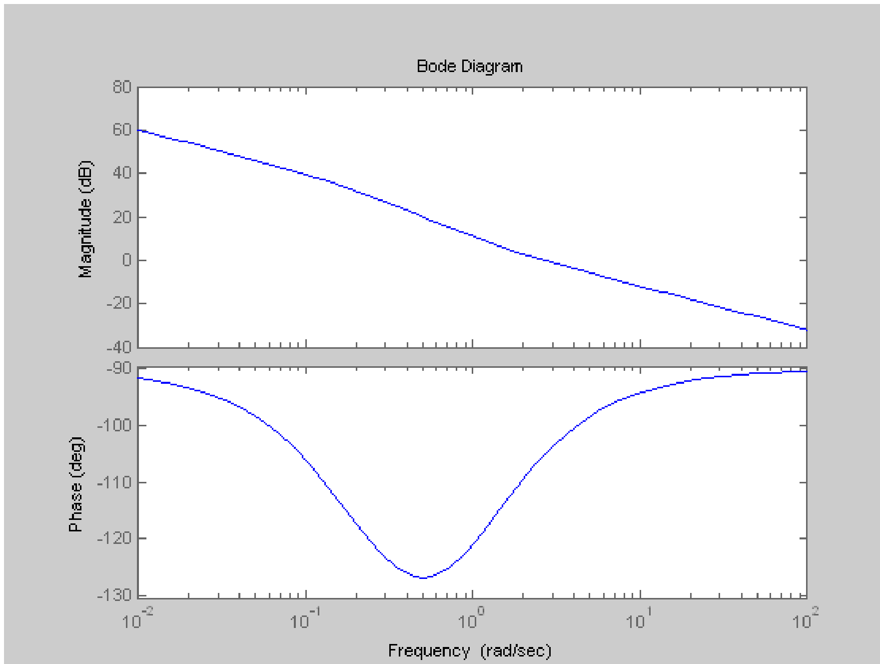


Figure S14.15c *Bode plot of G_{OL}. (K_c = 10)*

$$G_{OL} = GG_c = \frac{s+1}{(4s+1)(2s+1)} K_c \frac{2s+1}{s} = \frac{(s+1)K_c}{s(4s+1)}$$

Cannot become unstable – lead lag unit has phase lag larger than -90, integrator contributes -90 degree; the total phase angel is larger than -180.

(d)

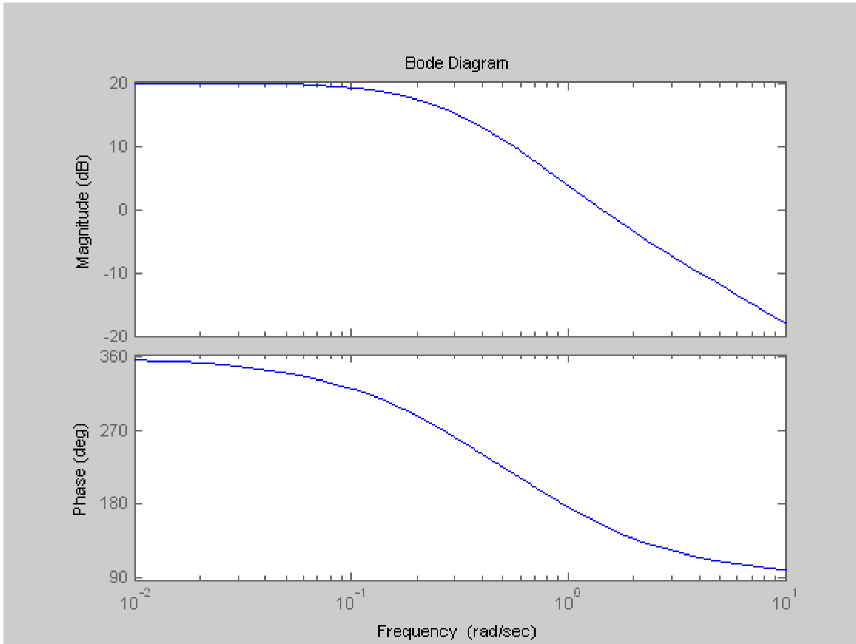


Figure S14.15d *Bode plot of G_{OL}. (K_c = 10)*

$$G_{OL} = GG_c = \frac{1-s}{(4s+1)(2s+1)}K_c = \frac{(1-s)K_c}{(4s+1)(2s+1)}$$

Can become unstable – max phase angle (G_{OL}) is -270 degree.

(e)

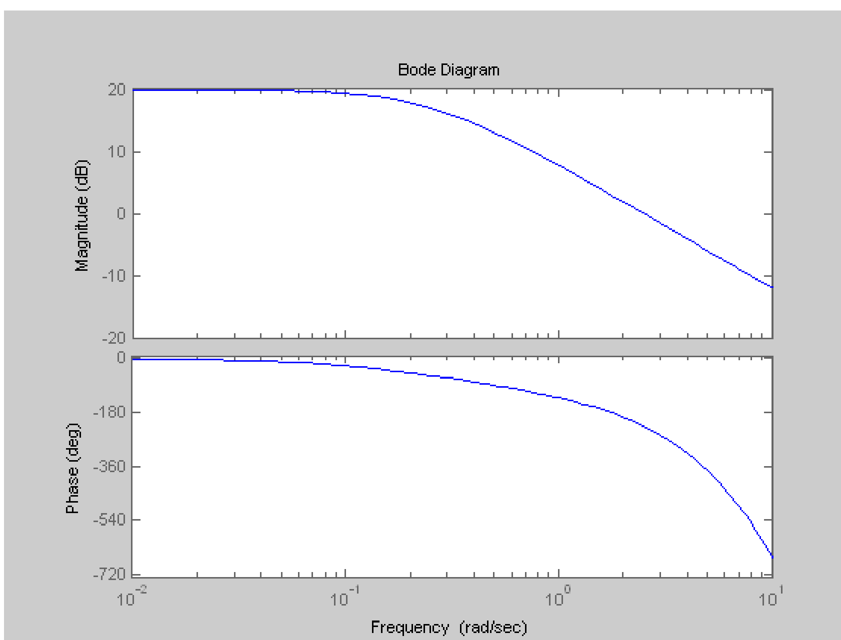


Figure S14.15e *Bode plot of G_{OL}. (K_c = 10)*

$$G_{OL} = GG_c = \frac{e^{-s}}{(4s+1)} K_c$$

Can become unstable due to time delay at high frequency.

14.16

By using MATLAB,

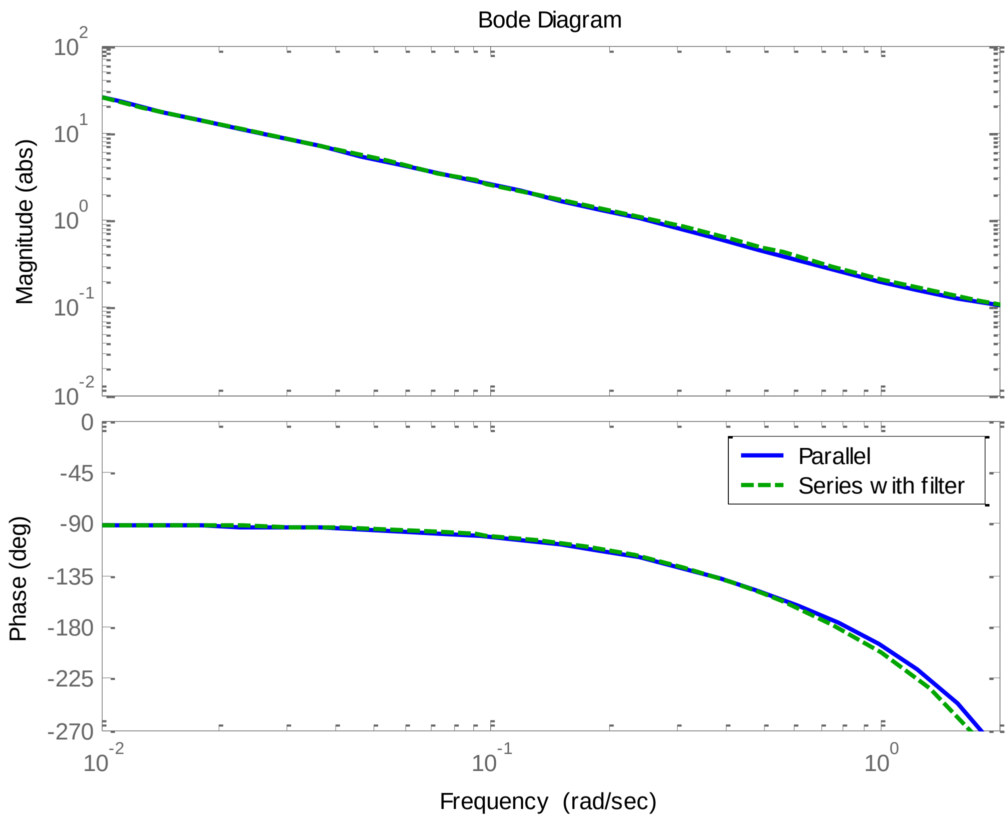


Figure S14.16 *Bode plot for Exercise 13.8 Transfer Function multiplied by PID Controller Transfer Function. Two cases: a)Parallel b) Series with Deriv. Filter (α=0.2).*

.

Amplitude ratios:

Ideal PID controller: AR= 0.246 at ω = 0.80
Series PID controller: AR=0.294 at ω = 0.74

There is 19.5% difference in the AR between the two controllers.

14.17

a) <u>Method discussed in Section 6.3</u>:

$$\hat{G}_1(s) = \frac{12e^{-0.3s}}{(8s+1)(2.2s+1)}$$

<u>Method discussed in Section 7.2.1</u>:

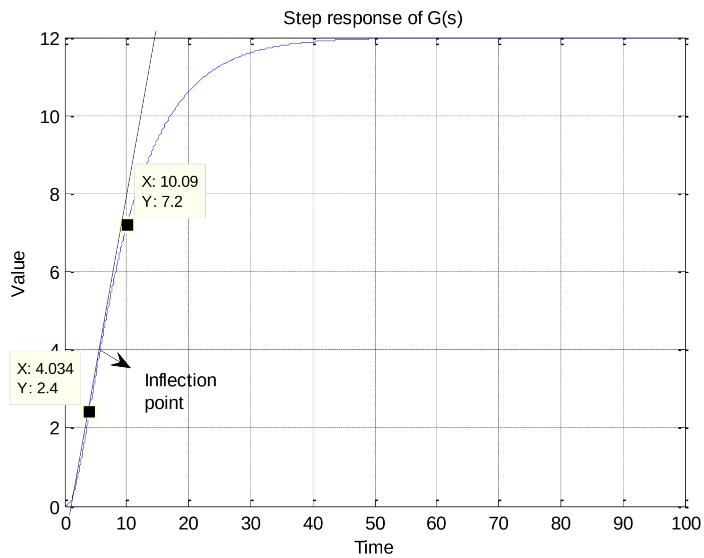


Figure S14.17a *Step response of G(s)*

Based on Figure S14.17, we can obtain the time stamps corresponding to 20% and 60% response: $t_{20} = 4.034; t_{60} = 10.09; {}^{t_{20}}/_{t_{60}} = 0.4$. Based on Figure 7.7, we have $\frac{t_{60}}{\tau} = 2.0; \zeta = 1.15$, so we have $\tau = 5.045$.Using the slope of the inflection point we can estimate the time delay to be 0.8. So we have:

$$\hat{G}_2(s) = \frac{12e^{-0.8s}}{25.45s^2 + 11.60s + 1}$$

b) Based on Figure S14.17a, we can obtain $\theta = 0.8; \tau = 15 - 0.8 = 14.2$

$$\hat{G}_3(s) = \frac{12e^{-0.8s}}{14.2s + 1}$$

Comparison of three estimated models and the exact model in the frequency domain using Bode plots:

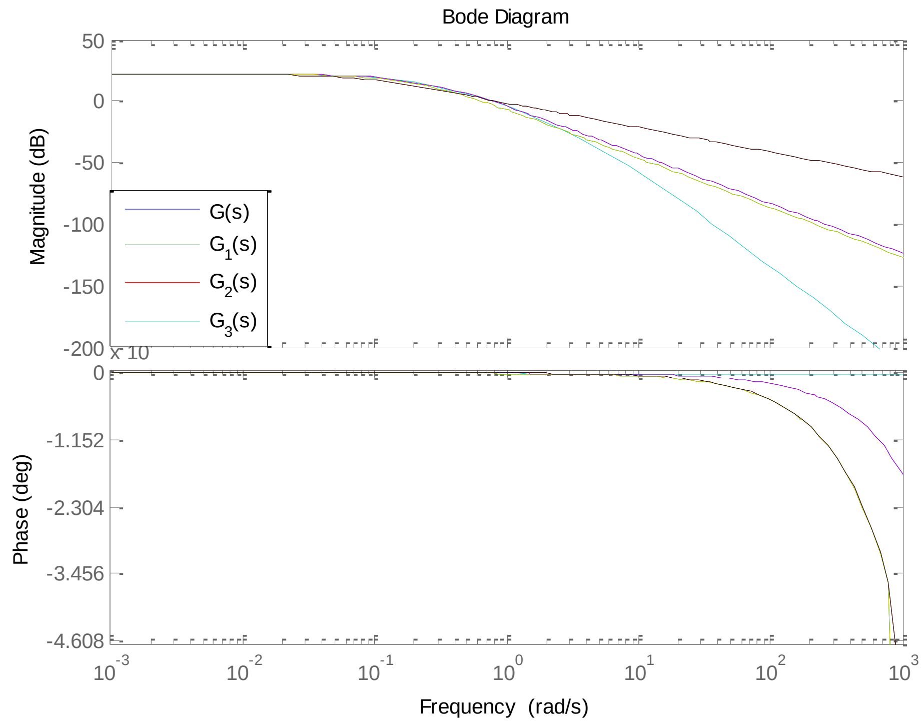


Figure S14.17b *Bode plots for the exact and approximate models.*

14.18

The original transfer function is

$$G(s) = \frac{10(2s+1)e^{-2s}}{(20s+1)(4s+1)(s+1)}$$

The approximate transfer function obtained using Section 6.3 is:

$$G^{'}(s) = \frac{10(2s+1)e^{-5s}}{(22s+1)}$$

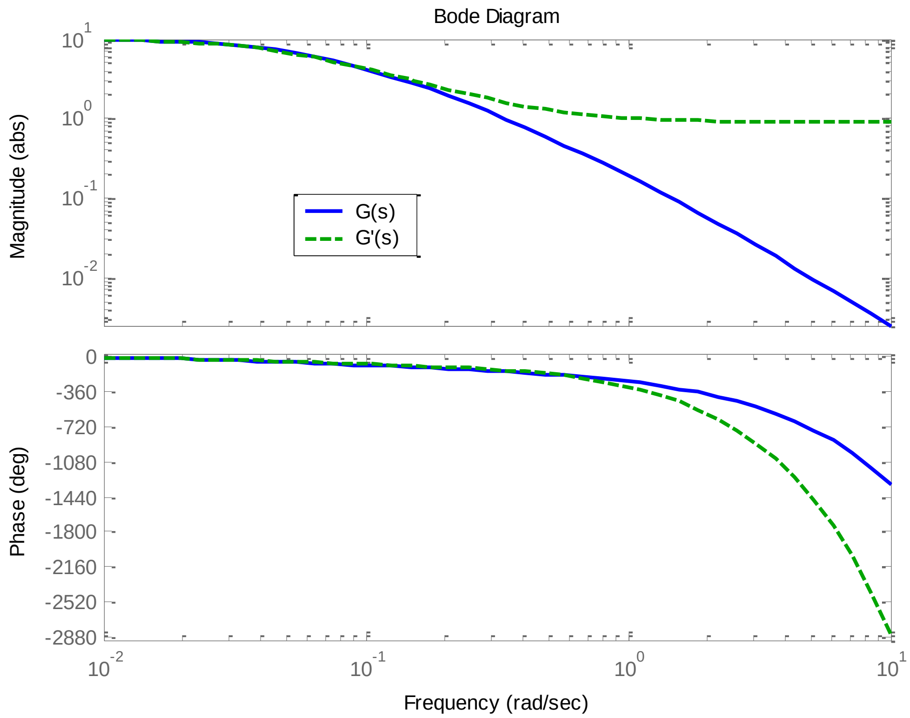


Figure S14.18 *Bode plots for the exact and approximate models.*

As seen in Fig.S14.18, the approximation is good at low frequencies, but not that good at higher frequencies.

14.19

(a)

$$G = G_p G_v G_m = \frac{2e^{-1.5s}}{(60s+1)(5s+1)} \frac{0.5e^{-0.3s}}{3s+1} \frac{3e^{-0.2s}}{2s+1} = \frac{3e^{-2s}}{(60s+1)(5s+1)(3s+1)(2s+1)}$$

ω_c occurs where φ = -180:

$\omega_c = 0.152 \qquad AR(\omega_c) = 0.227$

$$K_{cu} = \frac{1}{AR(\omega_c)} = 4.41$$

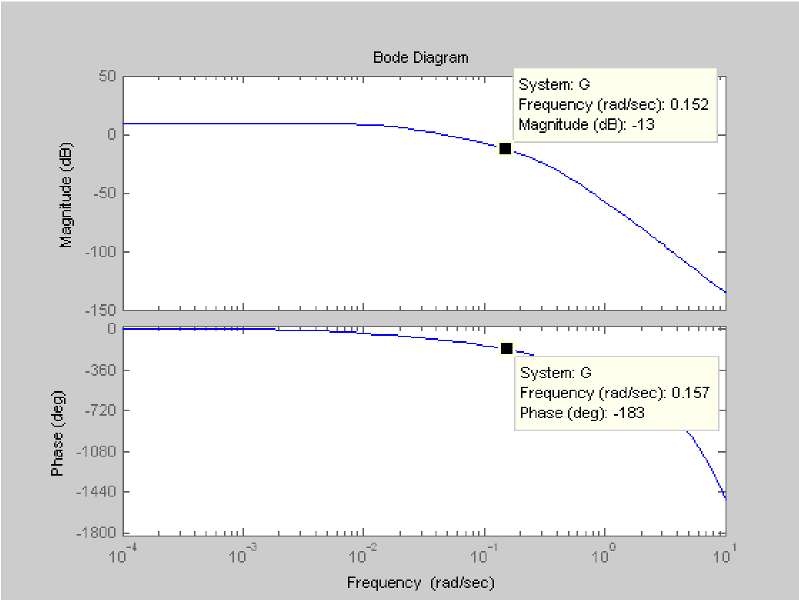


Figure S14.19a *Bode plot of to find ω_c.*

Simulation results with different *Kc* are shown in Fig. S14.19b. *Kc* > *Kcu*, the system becomes unstable as expected.

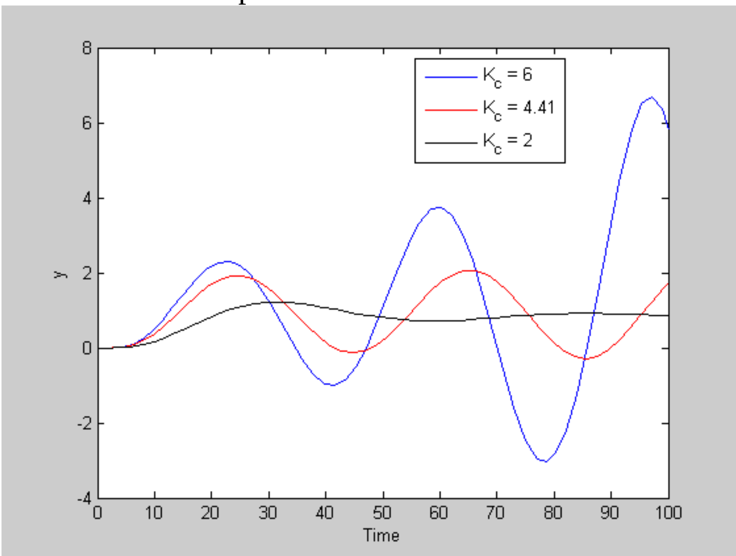


Figure S14.19b *Step response of closed loop system with different K_c.*

(b)

Use Skogestad's half rule

$\tau = 60 + 0.5 \times 5 = 62.5$

$\theta = 2.5 + 3 + 2 + 2 = 9.5$

The approximated FOPTD model:

$$G = \frac{3e^{-9.5s}}{62.5s+1}$$

Using Table 12.3, $K_c = 0.586(9.5/62.5)^{-0.916}/3 = 1.10$;

$$\tau_I = \frac{62.5}{-0.165(9.5/62.5)+1.03} = 62.19$$

Then,

$$G_c = 1.10(1+\frac{1}{62.19s}) \;, \quad G_{OL} = GG_c$$

ω_c occurs where φ = -180:

$\omega_c = 0.153 \qquad AR(\omega_c) = 0.249$

$$K_{cu} = \frac{1}{AR(\omega_c)} = 4.02$$

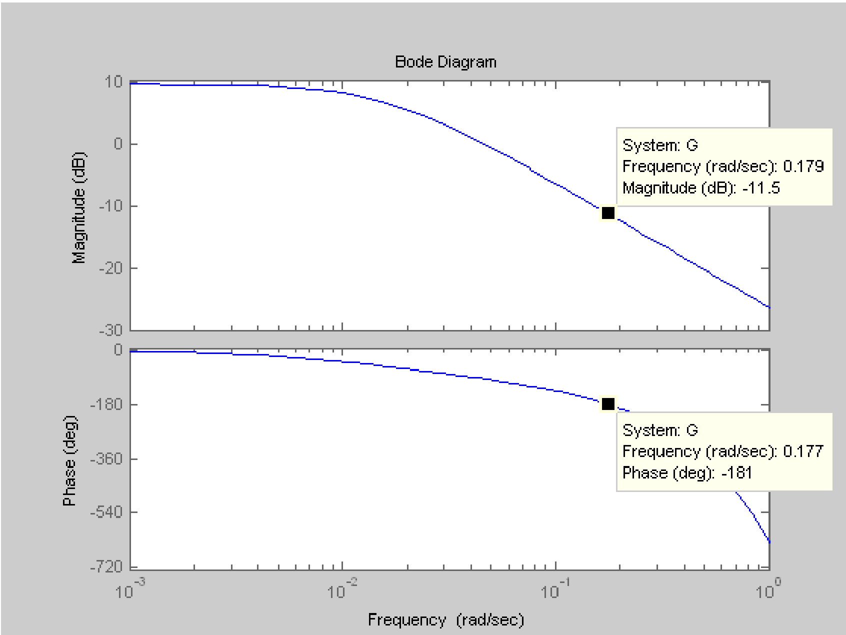


Figure S14.19c *Bode plot of FOPTD model.*

14.20

Using the Bode plot, at a phase angle of -180 °, we require that $K_c K_V K_p K_m < 1$

$G_p(s) = e^{-\theta s} \qquad G_V = 0.5 \qquad G_m = 1.0$

The gain of G_p = 1.0 for all ω.

At the critical frequency (ω_c), a sine wave is formed with period

$$P_m = 10 \min s = \frac{2\pi}{\omega_c}, so \quad \omega_c = \frac{2\pi}{10} \frac{rad}{\min} = 0.628 \frac{rad}{\min}$$

(a) The critical gain is easily found from

$K_c K_V K_p K_m = 1$ at $\omega = \omega_c$

$K_{cu}(0.5)(1)(1) = 1$, or $K_{cu} = 2.0$

(b) The phase angle of $G_c G = G_{cu}$ = phase angle of $e^{-\theta s}$, or $\Phi = -\omega\theta$ (rad) (Eq. 14-33)

when $\Phi = -180° = -\pi = -\omega_c \theta$

Because $\omega_c = \frac{2\pi}{10} \frac{\text{rad}}{\min}$ then $\theta = \frac{10}{2} = 5 \min$.

14.21

a) Using Eqs. 14-56 and 14-57

$$\text{AR}_{OL} = \left(K_c \sqrt{\frac{1}{25\omega^2} + 1} \right) \left(\frac{5}{\sqrt{100\omega^2 + 1}} \right) \left(\frac{1}{\sqrt{\omega^2 + 1}} \right) (1.0)$$

$\varphi = \tan^{-1}(-1/5\omega) + 0 + (-2\omega - \tan^{-1}(10\omega)) + (-\tan^{-1}(\omega))$

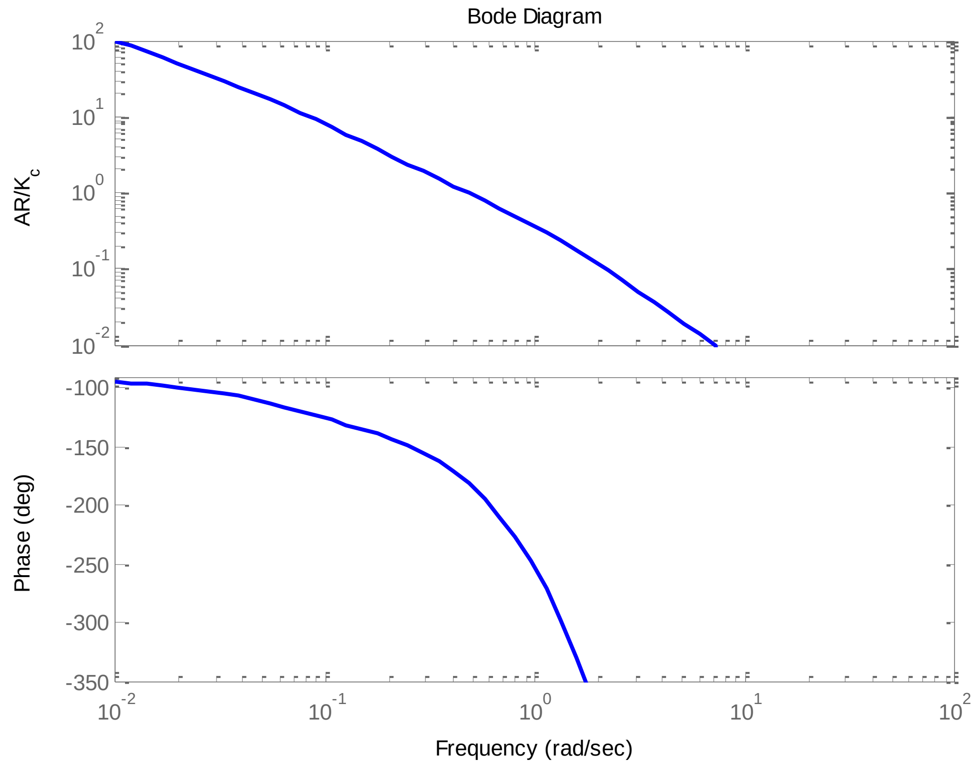


Figure S14.21a *Bode plot*

b) Set φ = 180° and solve for ω to obtain ω_c = 0.4695

Then $\text{AR}_{OL}\big|_{\omega=\omega_c} = 1 = K_{cu}(1.025)$

Therefore, K_{cu} = 1/1.025 = 0.976

System is stable for $K_c \leq 0.976$

c) For K_c = 0.2, set AR_{OL} = 1 and solve for ω to obtain ω_g = 0.1404

Then $\varphi_g = \varphi\big|_{\omega=\omega_g} = -133.6°$

From Eq. 14-61, $PM = 180° + \varphi_g = 46.4°$

d) From Eq. 14-60

$$GM = 1.7 = \frac{1}{A_c} = \frac{1}{\text{AR}_{OL}\big|_{\omega=\omega_c}}$$

From part b), $\text{AR}_{OL}\big|_{\omega=\omega_c} = 1.025\,K_c$

Therefore $1.025\,K_c = 1/1.7$ or $K_c = 0.574$

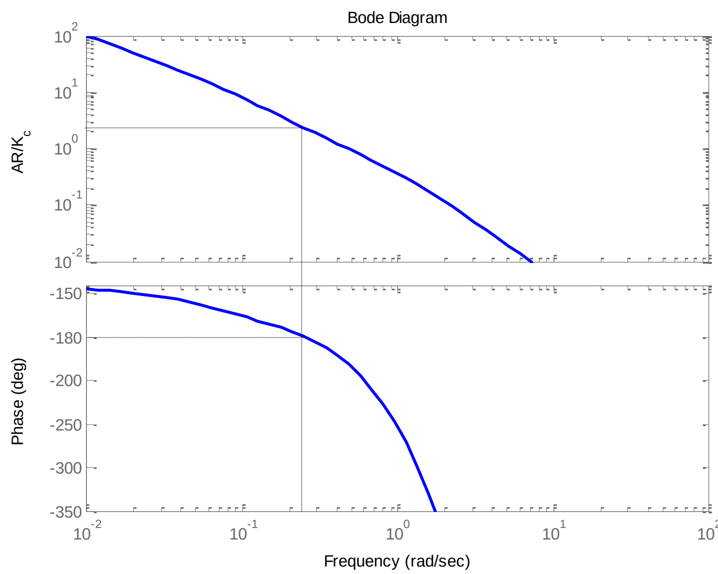


Figure S14.21b *Solution for part b) using Bode plot*

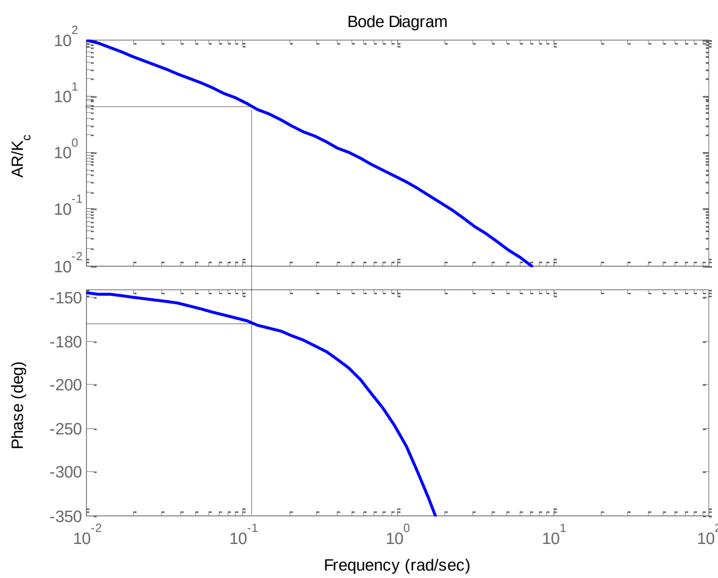


Figure S14.21c *Solution for part c) using Bode plot*

14.22

From modifying the solution to the two tanks in Section 6.4, which have a slightly different configurations,

$$G_p(s) = \frac{R_1}{(A_1R_1A_2R_2)s^2 + (A_1R_1 + A_2R_1 + A_2R_2)s + 1}$$

For R_1=0.5, R_2 = 2, A_1 = 10, A_2 = 0.8

$$G_p(s) = \frac{0.5}{8s^2 + 7s + 1} \qquad (1)$$

For R_2 = 0.5, $$G_p(s) = \frac{0.5}{2s^2 + 5.8s + 1} \qquad (2)$$

a) For R_2=2

$$\angle G_p = \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^{\,2}}\right] \quad , \quad |G_p| = \left(\frac{0.5}{\sqrt{(1-8\omega_c^{\,2})^2 + (7\omega_c)^2}}\right)$$

K_{cu} and ω_c are obtained using Eqs. 14-7 and 14-8:

$$-180° = 0 + 0 + \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^{\,2}}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, ω_c = 1.369 rad/min

$$1 = (K_{cu})(2.5)\left(\frac{0.5}{\sqrt{(1-8\omega_c^{\,2})^2 + (7\omega_c)^2}}\right)\left(\frac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}\right)$$

Substituting ω_c = 1.369 rad/min, K_{cu} = 10.96, $\omega_c K_{cu}$ = 15.0

For R_2=0.5

$$\angle G_p = \tan^{-1}\left[\frac{-5.8\omega_c}{1-2\omega_c^{\,2}}\right] \quad , \quad |G_p| = \left(\frac{0.5}{\sqrt{(1-2\omega_c^{\,2})^2 + (5.8\omega_c)^2}}\right)$$

For $G_v = K_v$ = 2.5, φ_v=0, $|G_v|$ = 2.5

For $G_m = \frac{1.5}{0.5s+1}$, φ_m= $-\tan^{-1}(0.5\omega)$, $|G_m| = \frac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}$

$$-180° = 0 + 0 + \tan^{-1}\left[\frac{-5.8\omega_c}{1-2\omega_c^{\,2}}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, $\omega_c = 2.51$ rad/min

Substituting $\omega_c = 2.51$ rad/min, $K_{cu} = 15.93$, $\omega_c K_{cu} = 40.0$

b) From part a), for R_2=2,

$\omega_c = 1.369$ rad/min, $K_{cu} = 10.96$

$P_u = \dfrac{2\pi}{\omega_c} = 4.59$ min

Using Table 12.4, the Ziegler-Nichols PI settings are

K_c = 0.45 K_{cu} = 4.932 , $\tau_I = P_u/1.2 = 3.825$ min

Using Eqs. 13.63 and 13-62 ,

$\varphi_c = -\tan^{-1}(-1/3.825\omega)$

$$|G_c| = 4.932\sqrt{\left(\frac{1}{3.825\omega}\right)^2 + 1}$$

Then, from Eq. 14-56

$$-180° = \tan^{-1}\left[\frac{-1}{3.825\omega_c}\right] + 0 + \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^{\,2}}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, $\omega_c = 1.086$ rad/min

Using Eq. 14-57

$A_c = \text{AR}_{OL}|_{\omega=\omega c} =$

$$= \left(4.932\sqrt{\left(\frac{1}{3.825\omega_c}\right)^2 + 1}\right)(2.5)\left(\frac{0.5}{\sqrt{(1-8\omega_c^{\,2})^2 + (7\omega_c)^2}}\right)\left(\frac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}\right)$$

= 0.7362

Therefore, gain margin $GM = 1/A_c$ = 1.358

Solving Eq.(14-16) for ω_g

$AR_{OL}|_{\omega=\omega c} = 1$ at $\omega_g = 0.925$

Substituting into Eq. 14-57 gives $\varphi_g=\varphi|_{\omega=\omega g} = -172.7°$

Therefore, phase margin $PM = 180 + \varphi_g = 7.3°$

14.23

a) K=2 , τ = 1 , θ = 0.2 , τ_c=0.3

Using Eq. 12-11, the PI settings are

$$K_c = \frac{1}{K}\frac{\tau}{\theta+\tau_c} = 1 \qquad , \qquad \tau_I = \tau = 1 \text{ min,}$$

Using Eq. 14-58 ,

$$-180° = \tan^{-1}\left(\frac{-1}{\omega_c}\right) - 0.2\omega_c - \tan^{-1}(\omega_c) = -90° - 0.2\omega_c$$

or $\omega_c = \dfrac{\pi/2}{0.2}$ = 7.85 rad/min

Using Eq. 14-57,

$$A_c = AR_{OL}\big|_{\omega=\omega_c} = \sqrt{\frac{1}{\omega_c^{\ 2}}+1}\left(\frac{2}{\sqrt{\omega_c^{\ 2}+1}}\right) = \frac{2}{\omega_c} = 0.255$$

From Eq. 14-60, $GM = 1/A_c$ = 3.93

b) Using Eq. 14-61,

$\varphi_g = PM - 180° = -140° = \tan^{-1}(-1/0.5\omega_g) - 0.2\omega_g - \tan^{-1}(\omega_g)$

Solving, ω_g = 3.04 rad/min

$$\text{AR}_{OL}\Big|_{\omega=\omega_g} = 1 = K_c \sqrt{\left(\frac{1}{0.5\omega_g}\right)^2 + 1}\ \left(\frac{2}{\sqrt{\omega_g{}^2 + 1}}\right)$$

Substituting for ω_g gives K_c = 1.34. Then from Eq. 14-8

$$-180° = \tan^{-1}\left(\frac{-1}{0.5\omega_c}\right) - 0.2\omega_c - \tan^{-1}(\omega_c)$$

Solving, ω_c =7.19 rad/min
From Eq. 14-56,

$$A_c = \text{AR}_{OL}\Big|_{\omega=\omega_c} = 1.34\sqrt{\left(\frac{1}{0.5\omega_c}\right)^2 + 1}\ \left(\frac{2}{\sqrt{\omega_c{}^2 + 1}}\right) = 0.383$$

From Eq. 14-60, $GM = 1/A_c$ = 2.61

c) By using Simulink-MATLAB, these two control systems are compared for a unit step change in the set point.

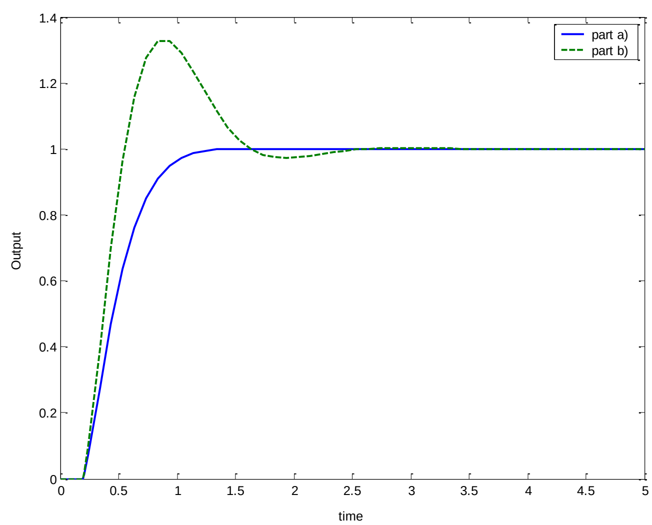


Figure S14.23 *Close-loop response for a unit step change in set point.*

The controller designed in part a) (Direct Synthesis) provides better performance giving a first-order response. Part b) controller yields a large overshoot.

14.24

a) Using Eqs. 14-56 and 14-57

$$\mathrm{AR}_{OL} = \frac{Y_{sp}}{Y_m} = \left(K_c \frac{\sqrt{4\omega^2+1}}{\sqrt{0.01\omega^2+1}} \right) \left(\frac{2}{\sqrt{0.25\omega^2+1}} \right) \left(\frac{0.4}{\omega\sqrt{25\omega^2+1}} \right) (1.0)$$

$$\varphi = \tan^{-1}(2\omega) - \tan^{-1}(0.1\omega) - \tan^{-1}(0.5\omega) - (\pi/2) - \tan^{-1}(5\omega)$$

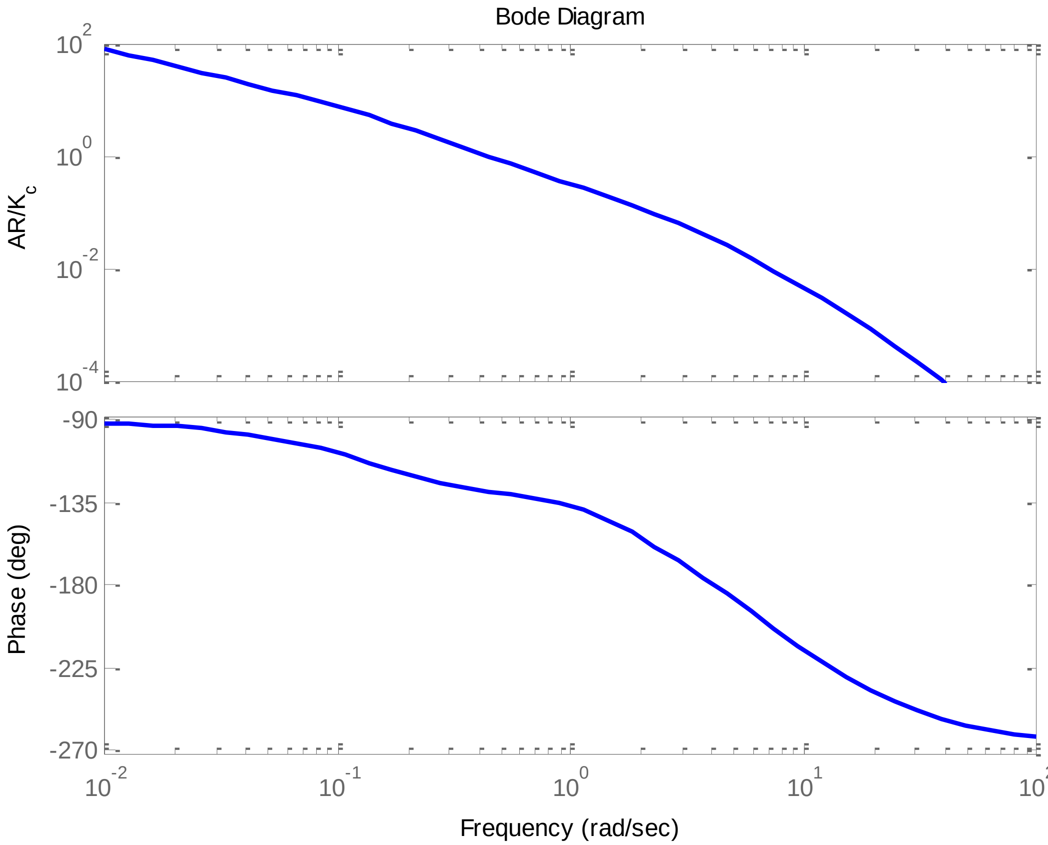


Figure S14.24a *Bode plot*

b) Using Eq.14-61

$\varphi_g = PM - 180° = 30° - 180° = -150°$

From the plot of φ vs. ω, $\varphi_g = -150°$ at $\omega_g = 1.72$ rad/min

From the plot of $\frac{AR_{OL}}{K_c}$ vs ω, $\left.\frac{AR_{OL}}{K_c}\right|_{\omega=\omega_g} = 0.144$

Since $AR_{OL}\big|_{\omega=\omega_g} = 1$, $K_c = \frac{1}{0.144} = 6.94$

c) From the plot of φ vs. ω, $\varphi = -180°$ at $\omega_c = 4.05$ rad/min

From the plot of $\frac{AR_{OL}}{K_c}$ vs ω, $\left.\frac{AR_{OL}}{K_c}\right|_{\omega=\omega_c} = 0.0326$

$A_c = AR_{OL}\big|_{\omega=\omega_c} = 0.326$

From Eq. 14-60, $GM = 1/A_c = 3.07$

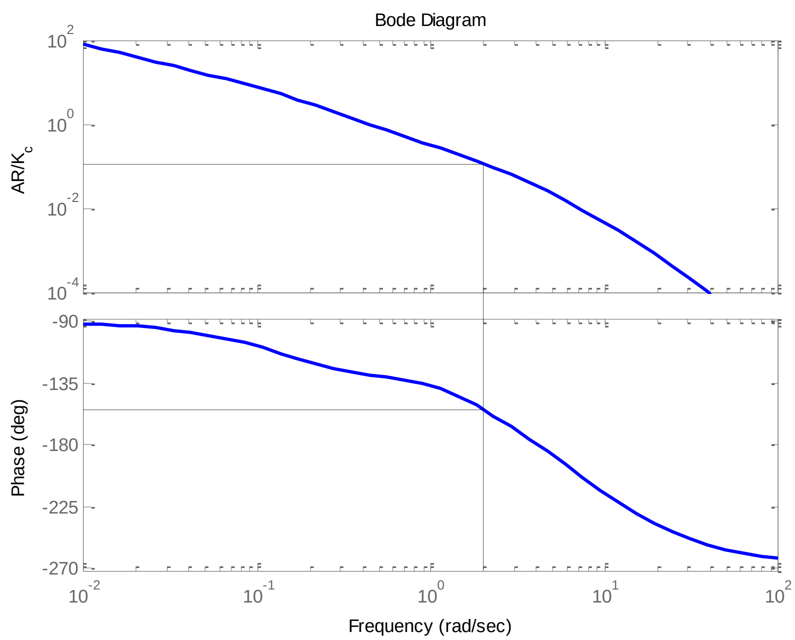


Figure S14.24b *Solution for part b) using Bode plot*

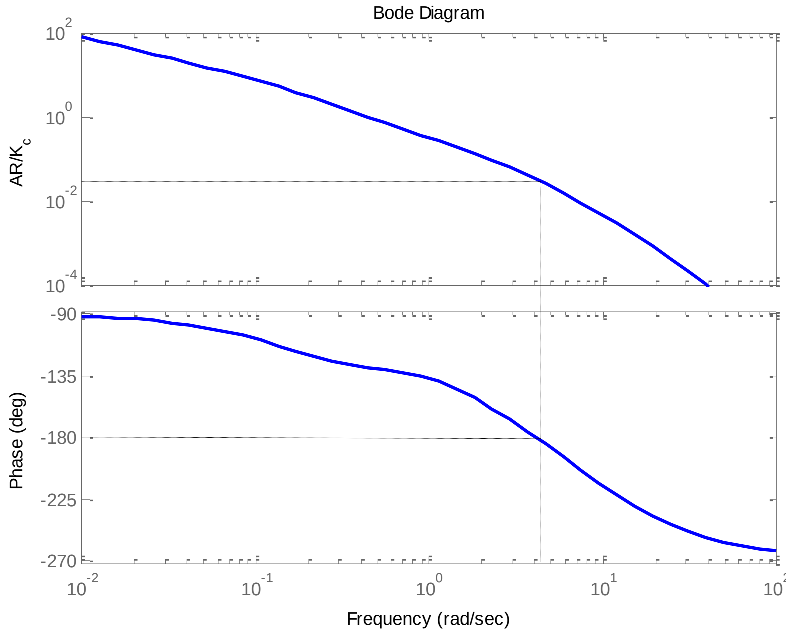
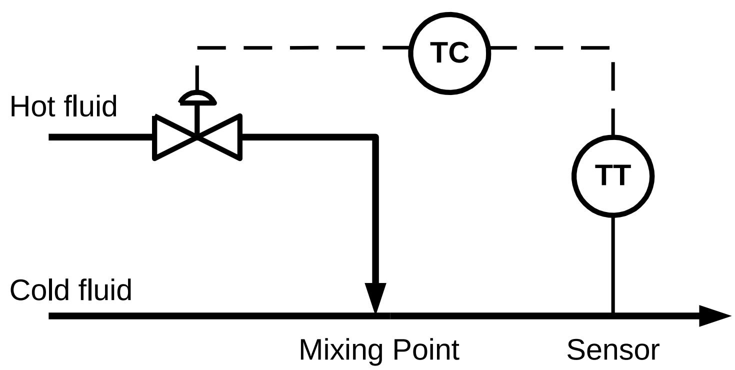


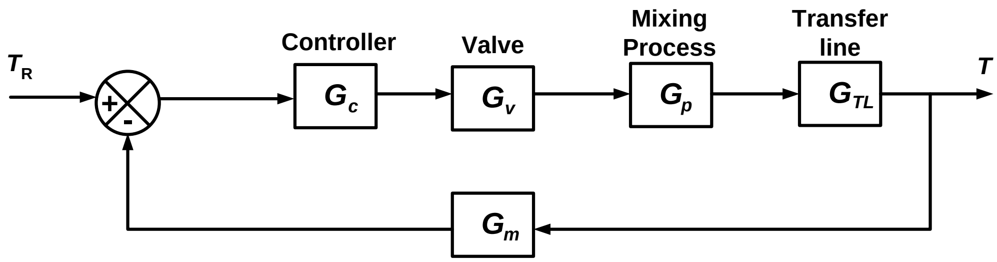
Figure S14.24c *Solution for part c) using Bode plot*

14.25

a) Schematic diagram:



Block diagram:



b) $G_vG_pG_m = K_m = 6$ ma/ma

$G_{TL} = e^{-8s}$

$G_{OL} = G_vG_pG_mG_{TL} = 6e^{-8s}$

If $G_{OL} = 6e^{-8s}$

$|G_{OL}(j\omega)| = 6$

$\angle G_{OL}(j\omega) = -8\omega$ [rad]

Find ω_c: The critical frequency corresponds to an open-loop phase angle of $-180°$ phase angle = $-\pi$ radians

$$-8\omega_c = -\pi \quad \text{or} \quad \omega_c = \pi/8 \text{ rad/s}$$

Find P_u: $P_u = \dfrac{2\pi}{\omega_c} = \dfrac{2\pi}{\pi/8} = 16\,\text{s}$

Find K_{cu}: $K_{cu} = \dfrac{1}{|G_p(j\omega_c)|} = \dfrac{1}{6} = 0.167$

[Note that for this unusual process, the process AR is independent of frequency]

Ziegler-Nichols ¼ decay ratio settings:

PI controller:

$K_c = 0.45\ K_{cu} = (0.45)(0.167) = 0.075$
$\tau_I = P_u/1.2 = 16$ s/1.2 = 13.33 s

PID controller:

$K_c = 0.6\ K_{cu} = (0.6)(0.167) = 0.100$
$\tau_I = P_u/2 = 16/2 = 8$ s

$\tau_D = P_u/8 = 16/8 = 2$ s

c)

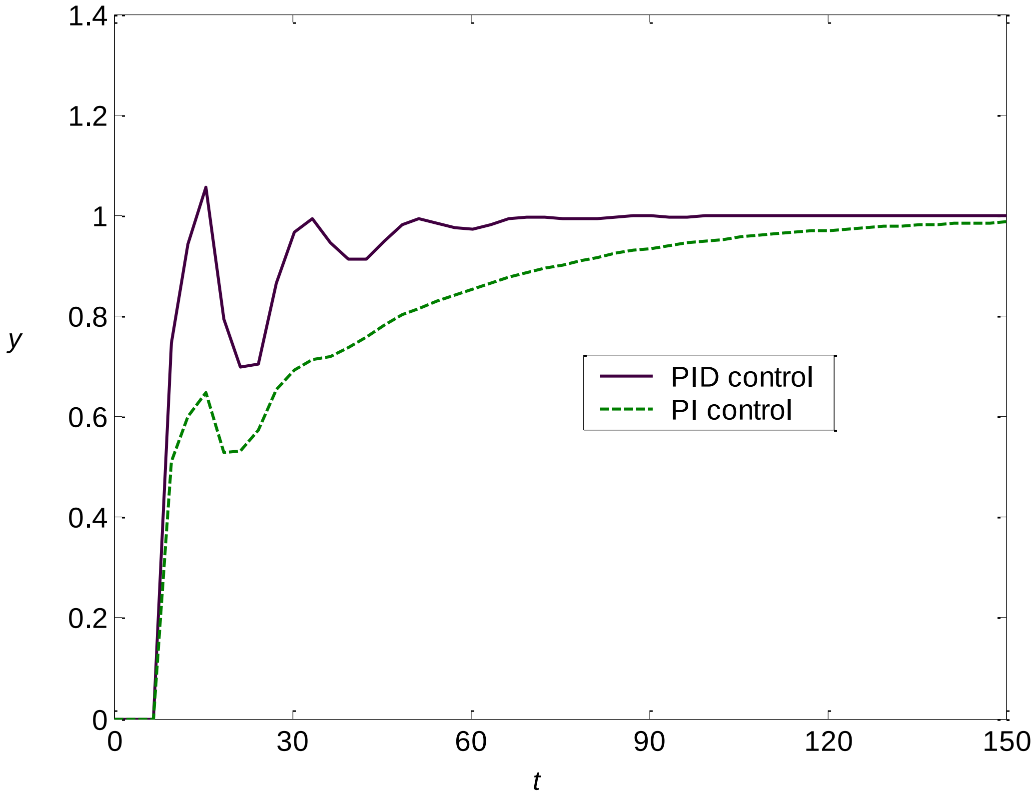


Figure S14.25 *Set-point responses for PI and PID control.*

Note: The MATLAB version of PID control uses the following controller settings: $ki = K_c/\tau_I$ and $kd = K_c\tau_D$.

d) Derivative control action improves the closed-loop response by reducing the settling time, at the expense of a more oscillatory response.

14.26

K_{cu} and ω_c are obtained using Eqs. 14-56 and 14-57. Including the filter G_F into these equations gives

$$-180° = 0 + [-0.2\omega_c - \tan^{-1}(\omega_c)] + [-\tan^{-1}(\tau_F\omega_c)]$$

Solving,

$\omega_c = 8.443$ for $\tau_F = 0$
$\omega_c = 5.985$ for $\tau_F = 0.1$

Then, from Eq. 14-57,

$$1 = (K_{cu})\left(\frac{2}{\sqrt{\omega_c{}^2+1}}\right)\left(\frac{1}{\sqrt{\tau_F{}^2\omega_c{}^2+1}}\right)$$

Solving for K_{cu} gives,

$K_{cu} = 4.251$ for $\tau_F = 0$
$K_{cu} = 3.536$ for $\tau_F = 0.1$

Therefore,

$\omega_c K_{cu} = 35.9$ for $\tau_F = 0$
$\omega_c K_{cu} = 21.2$ for $\tau_F = 0.1$

Since $\omega_c K_{cu}$ is lower for $\tau_F = 0.1$, filtering the measurement results in worse control performance.

14.27

a) $$G_v(s) = \frac{0.047}{0.083s+1}\times 112 = \frac{5.264}{0.083s+1}$$

$$G_p(s) = \frac{2}{(0.432s+1)(0.017s+1)}$$

$$G_m(s) = \frac{0.12}{(0.024s+1)}$$

Using Eq. 14-61

$$-180° = 0 - \tan^{-1}(0.083\omega_c) - \tan^{-1}(0.432\omega_c) - \tan^{-1}(0.017\omega_c) - \tan^{-1}(0.024\omega_c)$$

Solving , $\omega_c = 18.19$ rad/min

Using Eq. 14-60

$$1 = (K_{cu})\left(\frac{5.624}{\sqrt{(0.083\omega_c)^2+1}}\right)\cdot\left(\frac{2}{\sqrt{(0.432\omega_c)^2+1}\,\sqrt{(0.017\omega_c)^2+1}}\right)$$

$$\cdot\left(\frac{0.12}{\sqrt{(0.024\omega_c)^2+1}}\right)$$

Substituting ω_c=18.19 , K_{cu} = 12.97

$P_u = 2\pi/\omega_c = 0.345$ min

Using Table 12.4, the Ziegler-Nichols PI settings are

$K_c = 0.45\ K_{cu} = 5.84$, $\tau_I = P_u/1.2 = 0.288$ min

b) Using Eqs.14-39 and 14-40

$$\varphi_c = \angle G_c = \tan^{-1}(-1/0.288\omega) = -(\pi/2) + \tan^{-1}(0.288\omega)$$

$$|G_c| = 5.84\ \sqrt{\left(\frac{1}{0.288\omega}\right)^2+1}$$

Then, from Eq. 14-57,

$$-\pi = -(\pi/2) + \tan^{-1}(0.288\omega_c) - \tan^{-1}(0.083\omega_c) - \tan^{-1}(0.432\omega_c)$$

$$- \tan^{-1}(0.017\omega_c) - \tan^{-1}(0.024\omega_c)$$

Solving, ω_c = 15.11 rad/min.

Using Eq. 14-56

$$A_c = \mathrm{AR}_{OL}\big|_{\omega=\omega c} = \left[5.84\sqrt{\left(\frac{1}{0.288\omega_c}\right)^2+1}\right]\cdot\left[\frac{5.264}{\sqrt{(0.083\omega_c)^2+1}}\right]$$

$$\cdot\left[\frac{2}{\sqrt{(0.432\omega_c)^2+1}\,\sqrt{(0.017\omega_c)^2+1}}\right]\cdot\left[\frac{0.12}{\sqrt{(0.024\omega_c)^2+1}}\right]$$

= 0.651

Using Eq. 14-60, $GM = 1/A_c = 1.54$

Solving Eq. 14-56 for ω_g gives

$$\text{AR}_{OL}\big|_{\omega=\omega_g} = 1 \qquad \text{at} \quad \omega_g = 11.78 \text{ rad/min}$$

Substituting into Eq. 14-57 gives

$$\varphi_g = \varphi\big|_{\omega=\omega_g} = -(\pi/2) + \tan^{-1}(0.288\omega_g) - \tan^{-1}(0.083\omega_g) - \tan^{-1}(0.432\omega_g)$$

$$- \tan^{-1}(0.017\omega_g) - \tan^{-1}(0.024\omega_g) = -166.8°$$

Using Eq. 14-61

$$PM = 180° + \varphi_g = 13.2\ °$$

14.28

a) From Exercise 14.28,

$$G_v(s) = \frac{5.264}{0.083s+1}$$
$$G_p(s) = \frac{2}{(0.432s+1)(0.017s+1)}$$
$$G_m(s) = \frac{0.12}{(0.024s+1)}$$

The PI controller is $G_c(s) = 5\left(1 + \frac{1}{0.3s}\right)$

Hence the closed-loop transfer function is

$$G_{OL} = G_c G_v G_p G_m$$

Rearranging,

$$G_{OL} = \frac{6.317s + 21.06}{1.46\times10^{-5}s^5 + 0.00168s^4 + 0.05738s^3 + 0.556s^2 + s}$$

By using MATLAB, the Nyquist diagram for this open-loop system is

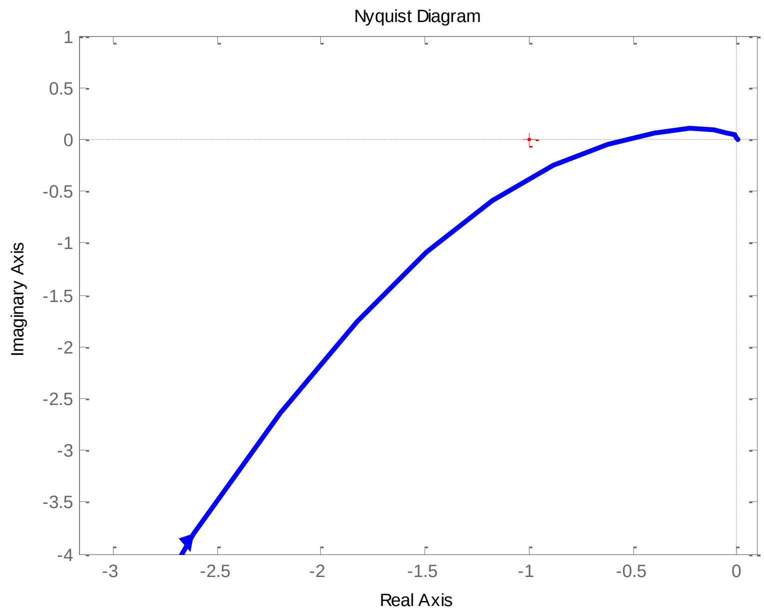


Figure S14.28a *The Nyquist diagram for the open-loop system.*

b) Gain margin = $GM = \dfrac{1}{\mathrm{AR}_c}$

where AR_c is the value of the open-loop amplitude ratio at the critical frequency ω_c. By using the Nyquist plot

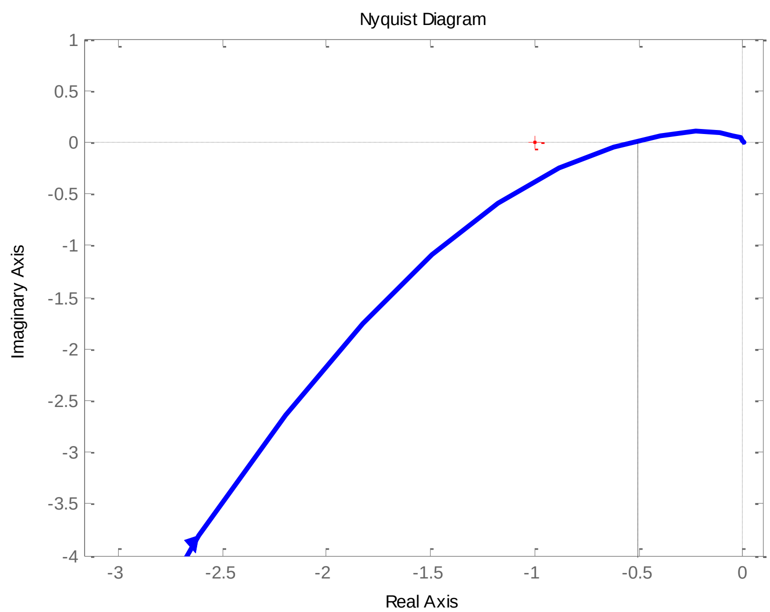


Figure S14.28b *Graphical solution for part b)*

$\theta = -180 \quad \Rightarrow \quad AR_c = |\, G(j\omega_c)| = 0.5$

Therefore the gain margin is $GM = 1/0.5 = 2$

Chapter 15 ©

15.1

For $R_a = d/\text{u}$

$$K_p = \frac{\partial R_a}{\partial u} = -\frac{d}{u^2}$$

which can vary more than K_p in Eq. 15-2, because the new K_p depends on both d and u.

15.2

By definition, the ratio station sets

$$u_m = u_{m0} + K_R (d_m - d_{m0})$$

Thus $$K_R = \frac{u_m - u_{m0}}{d_m - d_{m0}} = \frac{K_2 u^2}{K_1 d^2} = \frac{K_2}{K_1}\left(\frac{u}{d}\right)^2 \qquad (1)$$

For constant gain K_R, the values of u and d in Eq. 1 are the desired steady-state values so that $u/d = R_d$, the desired ratio. Moreover, the transmitter gains are

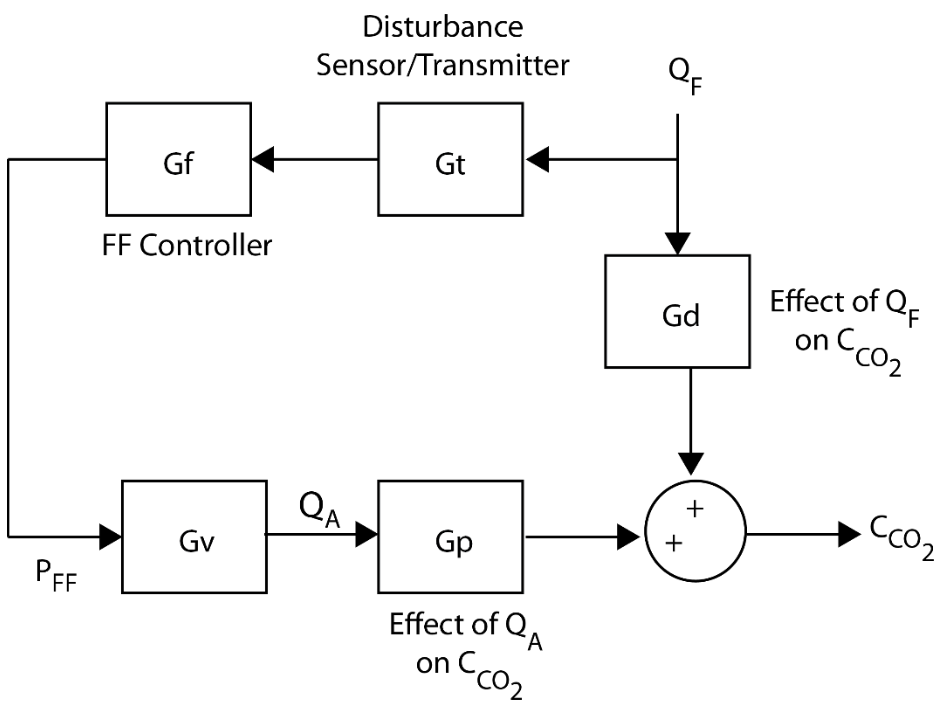
$$K_1 = \frac{(15-3)\,\text{mA}}{S_d^{\,2}} \qquad , \qquad K_2 = \frac{(15-3)\,\text{mA}}{S_u^{\,2}}$$

Substituting for K_1, K_2 and u/d into (1) gives,

$$K_R = \frac{S_u^{\,2}}{S_d^{\,2}} R_d^{\,2} = \left(R_d \frac{S_d}{S_u}\right)^2$$

15.3

(a) Block diagram of the feedforward control system



(b) Feedforward design based on a steady-state analysis

The starting point in feedforward controller design is Eq. 15-21. For a design based on a steady-state analysis, the transfer functions in (15-21) are replaced by their corresponding steady-state gains:

$$G_F(s) = -\frac{K_d}{K_t K_v K_p} \tag{1}$$

From the given information,

$$K_t = 0.08 \frac{\text{mA}}{\text{L/min}}$$

$$K_v = 4 \frac{\text{gal/min}}{\text{mA}}$$

Next, calculate K_p and K_d from the given data. Linear regression gives:

$$K_p = -2.1 \frac{\text{ppm}}{\text{gal/min}}$$

$$K_d = 0.235 \frac{\text{ppm}}{\text{L/min}}$$

Substitute these gains into (1) to get:

$$G_F(s) = -\frac{0.235 \frac{\text{ppm}}{\text{L/min}}}{\left(0.08 \frac{\text{mA}}{\text{L/min}}\right)\left(4 \frac{\text{gal/min}}{\text{mA}}\right)\left(-2.1 \frac{\text{ppm}}{\text{gal/min}}\right)}$$

$$G_F(s) = 0.35$$

15.4

(TBA)

15.5

a) Using steady-state gains

G_p=1, G_d=2, $G_v = G_m = G_t$ =1

From Eq.15-21

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{-2}{(1)(1)(1)} = -2$$

b) Using Eq. 15-21

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{\frac{-2}{(s+1)(4s+1)}}{(1)(1)\left(\frac{1}{s+1}\right)} = \frac{-2}{4s+1}$$

c) Using Eq. 12-19

$$\tilde{G} = G_v G_p G_m = \frac{1}{s+1} = \tilde{G}_+ \tilde{G}_-$$

where $\tilde{G}_+ = 1, \quad \tilde{G}_- = \dfrac{1}{s+1}$

For τ_c=3, and r=1, Eq. 12-21 gives,

$$f = \frac{1}{3s+1}$$

From Eq. 12-20,

$$G_c^* = \tilde{G}_-^{-1} f = (s+1)\left(\frac{1}{3s+1}\right) = \frac{s+1}{3s+1}$$

From Eq. 12-16,

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} = \frac{\dfrac{s+1}{3s+1}}{1 - \dfrac{1}{3s+1}} = \frac{s+1}{3s}$$

d) For feedforward control only, $G_c = 0$ for a unit step change in disturbance, $D(s) = 1/s$

Substituting into Eq. 15-20 gives

$$Y(s) = (G_d + G_t G_f G_v G_p)\frac{1}{s}$$

For the <u>controller of part (a)</u>

$$Y(s) = \left[\frac{2}{(s+1)(4s+1)} + (1)(-2)(1)\left(\frac{1}{s+1}\right)\right]\frac{1}{s}$$

$$Y(s) = \left[\frac{-8}{(s+1)(4s+1)}\right] = \frac{8/3}{s+1} - \frac{32/3}{4s+1} = \frac{8/3}{s+1} - \frac{8/3}{s+1/4}$$

Taking inverse Laplace transforms,

$$y(t) = \frac{8}{3}(e^{-t} - e^{-t/4})$$

For the <u>controller of part (b)</u>

$$Y(s) = \left[\frac{2}{(s+1)(4s+1)} + (1)\left(\frac{-2}{4s+1}\right)(1)\left(\frac{1}{s+1}\right)\right]\frac{1}{s} = 0$$

or $\quad y(t) = 0$

The step responses are shown in Fig. S15.5 (left panel).

e) Using Eq. 15-20

For the controller of parts (a) and (c),

$$Y(s) = \left[\frac{\dfrac{2}{(s+1)(4s+1)} + (1)(-2)(1)\left(\dfrac{1}{s+1}\right)}{1+\left(\dfrac{s+1}{3s}\right)(1)\left(\dfrac{1}{s+1}\right)(1)}\right]\frac{1}{s}$$

or

$$Y(s) = \frac{-24s}{(s+1)(4s+1)(3s+1)} = \frac{-36}{3s+1} + \frac{32}{4s+1} + \frac{4}{s+1}$$

$$= \frac{-12}{s+1/3} + \frac{8}{s+1/4} + \frac{4}{s+1}$$

Thus,

$$y(t) = -12e^{-t/3} + 8e^{-t/4} + 4e^{-t}$$

and for controllers of parts (b) and (c)

$$Y(s) = \left[\frac{\dfrac{2}{(s+1)(4s+1)} + (1)\left(\dfrac{-2}{4s+1}\right)(1)\left(\dfrac{1}{s+1}\right)}{1+\left(\dfrac{s+1}{3s}\right)(1)\left(\dfrac{1}{s+1}\right)(1)}\right]\frac{1}{s} = 0$$

Thus, $y(t) = 0$

The closed-loop responses are shown in Fig. S15.5 (right panel).

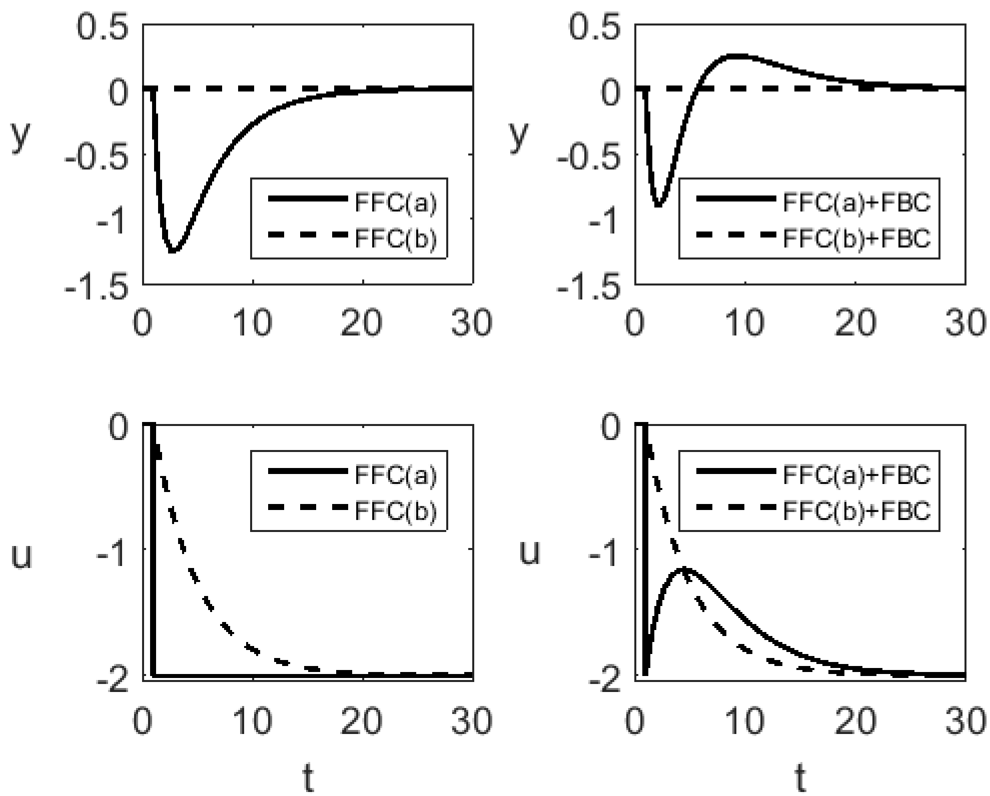


Figure S15.5. *Closed-loop responses for feedforward-only control (FFC, left panel) and feedforward-feedback control (FFC+FBC, right panel).*

15.6

a) The steady-state energy balance for both tanks takes the form

$$0 = w_1 C\, T_1 + w_2 C\, T_2 - w\, C\, T_4 + Q$$

where:

Q is the power input of the heater.
C is the specific heat of the fluid.

Solving for Q and replacing unmeasured temperatures and flow rates by their nominal values,

$$Q = C\,(\overline{w}_1 T_1 + \overline{w}_2 \overline{T}_2 - w\overline{T}_4) \qquad (1)$$

Neglecting heater and transmitter dynamics,

$$Q = K_h\, p \qquad (2)$$

$$T_{1m} = T_{1m}^0 + K_T(T_1 - T_1^0) \qquad (3)$$

$$w_m = w_m^0 + K_w(w - w^0) \qquad (4)$$

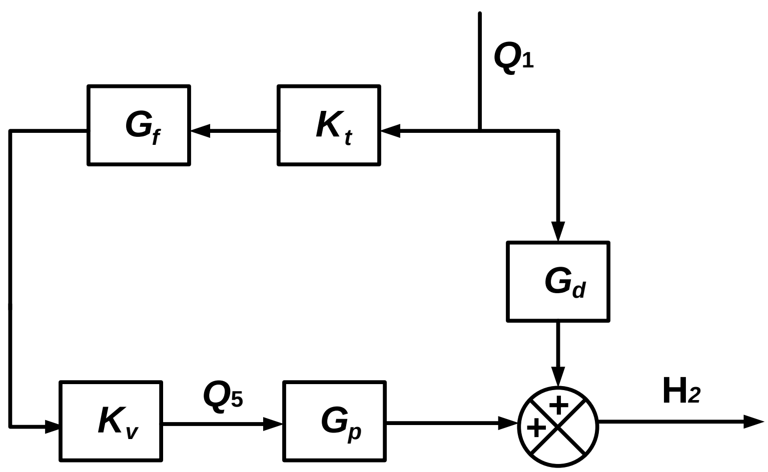
Substituting into (1) for Q,T_1, and w from (2),(3), and (4), gives

$$P = \frac{C}{K_h}[\overline{w_1}(T_1^{\ 0} + \frac{1}{K_T}(T_{1m} - T_{1m}^{\ \ 0})) + \overline{w_2}\,\overline{T_2} - \overline{T_4}(w^0 + \frac{1}{K_w}(w_m - w_m^{\ \ 0}))]$$

b) Dynamic compensation is desirable because the process transfer function $G_p = T_4(s)/P(s)$ is different from each of the disturbance transfer functions $G_{d1} = T_4(s)/T_1(s)$, and $G_{d2} = T_4(s)/w(s)$; especially for G_{d1} which has a higher order.

15.7

a)



b) A steady-state material balance for both tanks gives,

$$0 = q_1 + q_2 + q_4 - q_5$$

Because $q_2' = q_4' = 0$, the above equation in deviation variables is:

$$0 = q_1' - q_5' \qquad (1)$$

From the block diagram (which uses deviation variables),

$$Q_5(s) = K_v\, G_f\, K_t\, Q_1(s)$$

Substituting for $Q_5(s)$ into (1) gives

$$0 = Q_1(s) - K_v\, G_f(s)\, K_t\, Q_1(s) \qquad \text{or}$$

Thus

$$G_f = \frac{1}{K_v K_t}$$

c) To find G_d and G_p, the mass balance on tank 1 is

$$A_1 \frac{dh_1}{dt} = q_1 + q_2 - C_1\sqrt{h_1}$$

where A_1 is the cross-sectional area of tank 1.

Linearizing and setting $q_2' = 0$ leads to

$$A_1 \frac{dh_1'}{dt} = q_1' - \frac{C_1}{2\sqrt{\bar{h}_1}} h_1'$$

Taking Laplace transform,

$$\frac{H_1(s)}{Q_1(s)} = \frac{R_1}{A_1 R_1 s + 1} \qquad \text{where} \quad R_1 \equiv \frac{2\sqrt{\bar{h}_1}}{C_1} \qquad (2)$$

Linearizing $q_3 = C_1\sqrt{h_1}$ gives

$$q_3' = \frac{1}{R_1} h_1'$$

Thus

$$\frac{Q_3(s)}{H_1(s)} = \frac{1}{R_1} \qquad (3)$$

Mass balance on tank 2 is

$$A_2 \frac{dh_2}{dt} = q_3 + q_4 - q_5$$

Using deviation variables, setting $q_4' = 0$, and taking the Laplace transforms gives:

$$A_2\, sH_2(s) = Q_3(s) \;\; Q_5(s)$$

$$\frac{H_2(s)}{Q_3(s)} = \frac{1}{A_2 s} \tag{4}$$

and

$$\frac{H_2(s)}{Q_5(s)} = -\frac{1}{A_2 s} = G_p(s)$$

Substitution from (2), (3), and (4) yields,

$$G_d(s) = \frac{H_2(s)}{Q_1(s)} = \frac{H_2(s)}{Q_3(s)}\frac{Q_3(s)}{H_1(s)}\frac{H_1(s)}{Q_1(s)} = \frac{1}{A_2 s(A_1 R_1 s + 1)}$$

Using Eq. 15-21

$$G_f = \frac{-G_d}{G_t G_v G_p} = \frac{-\dfrac{1}{A_2 s(A_1 R_1 s + 1)}}{K_t K_v(-1/A_2 s)}$$

$$G_f = \frac{1}{K_v K_t}\frac{1}{A_1 R_1 s + 1}$$

15.8

a) Feedforward controller design

A dynamic model will be developed based on the following assumptions:

1. Perfect mixing
2. Isothermal operation
3. Constant volume

Component balances:

$$V\frac{dc_A}{dt} = q(c_{Ai} - c_A) - V(k_1 c_A - k_2 c_B)$$

$$V\frac{dc_B}{dt} = -qc_B + V(k_1 c_A - k_2 c_B)$$

Linearize,

$$V\frac{dc'_A}{dt} = a_{11}c'_A + a_{12}c'_B + b_1 q' + dc'_{Ai} \tag{1}$$

$$V\frac{dc'_B}{dt} = a_{21}c'_A + a_{22}c'_B + b_2 q' \tag{2}$$

where:

$$a_{11} = -\frac{\bar{q}}{V} - k_1, \qquad a_{12} = k_2$$

$$a_{21} = k_1, \qquad a_{22} = -\frac{\bar{q}}{V} - k_2 \tag{3}$$

$$d = \frac{\bar{q}}{V}, \quad b_1 = \frac{\bar{c}_{Ai} - \bar{c}_A}{V}, \quad b_2 = -\frac{\bar{c}_B}{V}$$

$c'_A = c_A - \bar{c}_A$ and $\bar{c}_A$ denotes the nominal steady-state value

Take Laplace transforms and solve, after substituting the first equation for $C'_A(s)$ into the second equation. The result is:

$$C'_B(s) = G_p(s)Q'(s) + G_d(s)C'_{Ai}(s) \tag{4}$$

where:

$$G_p(s) = \frac{a_{21}b_1 + b_2(s - a_{11})}{\Delta(s)} \tag{5}$$

$$G_d(s) = \frac{a_{21}d_1}{\Delta(s)}$$

$$\Delta(s) = (s - a_{22})(s - a_{11}) - a_{21}a_{12}$$

$c'_A = c_A - \bar{c}_A$ and $\bar{c}_A$ denotes the nominal steady-state value

Feedforward controller design equation (based on Eq. 5-21):

$$G_f(s) = -\frac{G_d(s)}{K_t K_v G_p(s)} \tag{6}$$

Substitute for $G_d(s)$ and $G_p(s)$:

$$G_f(s) = -\left[\frac{a_{21}d}{a_{21}b_1 + b_2(s - a_{11})}\right]\left(\frac{1}{K_t K_v}\right) \tag{7}$$

Rearrange and substitute from (3):

$$\boxed{G_f(s) = \frac{K}{\tau s + 1}}$$

where:

$$K = \left(\frac{1}{K_t K_v}\right)\left[\frac{k_1 \bar{q} V}{k_1 V(\bar{c}_A - \bar{c}_{Ai}) + \bar{q}\,\bar{c}_B + \bar{c}_B k_1 V}\right]$$

$$\tau = \frac{\bar{c}_B V}{k_1 V(\bar{c}_A - \bar{c}_{Ai}) + \bar{q}\,\bar{c}_B + \bar{c}_B k_1 V}$$

b) Reverse or direct acting controller?
From Ch. 11, we know that in order for the closed-loop system to be stable,

$K_c K_v K_p K_m > 0$

The available information indicates that $K_v > 0$ and $K_m > 0$, assuming that q is still the manipulated variable. Thus K_c should have the same sign as K_p and we need to determine the sign of K_p.

From (5) K_p can be calculated as:

$$K_p = \lim_{s \to 0} G_p(s) = \frac{a_{21} b_1 - b_2 a_{11}}{a_{11} a_{22} - a_{21} a_{12}}$$

Substitute from (3) and simplify to get:

$$K_p = \left(\frac{1}{K_t K_v}\right)\left[\frac{k_1 \bar{q} V(\bar{c}_{Ai} - \bar{c}_A) + \bar{c}_B V^2(\bar{q} + V k_2)}{\bar{q}^2 + V(k_1 + k_2)}\right] \qquad (8)$$

Because both the numerator and denominator terms of (8) are positive, $K_p > 0$. Thus K_c should be positive.

Conclusion: *The feedback controller should be reverse acting.*

c) The advantages of using a steady-state controller are that the calculations are quite simple and a detailed process model is not required. The disadvantage is that the control system may not perform well during transient conditions.

To decide whether or not to add dynamic compensation, we would need to know whether controlled variable c_B is affected more rapidly, or more slowly, by the disturbance variable c_{Ai} than it is by the manipulated variable, q. If the response times are quite different, then dynamic compensation could be beneficial. An unsteady-state model (or experimental data) would be required to resolve this issue. Even then, if tight control of c_B is not essential, it might be decided to use the simpler design method based on the steady-state analysis.

15.9

The block diagram for the feedforward-feedback control system is shown in Fig. 15.12.

(a) Not required

(b) Feedforward controllers

From Example 15.5,

$$G_{IP} = K_{IP} = 0.75 \text{ psi/mA},$$

$$G_v(s) = \frac{K_v}{\tau_v s+1} = \frac{250}{0.0833s+1}$$

Since the measurement time delay is now 0.1 min, it follows that:

$$G_t(s) = G_m(s) = K_t e^{-\theta s} = 32e^{-0.1s}$$

The process and disturbance transfer functions are:

$$\frac{X'(s)}{W_2'(s)} = \frac{2.6x10^{-4}}{4.71s+1}, \qquad \frac{X'(s)}{X_1'(s)} = \frac{0.65}{4.71s+1}$$

The ideal dynamic feedforward controller is given by Eq. 15-21:

$$G_f = -\frac{G_d}{K_{IP}G_tG_vG_p} \tag{15-21}$$

Substituting the individual transfer functions into Eq. 15-21 gives,

$$G_f(s) = -0.417(0.0833s+1)e^{+0.1s} \tag{1}$$

The static (or steady-state) version of the controller is simply a gain, K_f:

$$K_f = - 0.417 \tag{2}$$

Note that $G_f(s)$ in (1) is physically unrealizable. In order to derive a physically realizable dynamic controller, the unrealizable controller in (1) is approximated by a lead-lag unit, in analogy with Example 15.5:

$$G_f(s) = -0.417\frac{0.1833s+1}{0.01833s+1} \qquad (3)$$

Equation 3 was derived from (1) by: (i) omitting the time delay term, (ii) adding the time delay of 0.1 min to the lead time constant, and (iii) introducing a small time constant of α x 0.1833= 0.01833 for α = 0.1.

(c) Feedback controller

Define G as,

$$G = G_{IP}G_vG_pGm = (0.75)\left(\frac{25}{0.0833s+1}\right)\left(\frac{2.6x10^{-4}}{4.71s+1}\right)\left(32e^{-0.1s}\right)$$

First, approximate G as a FOPTD model, $\tilde{G}$ using Skogestad's half-rule method in Section 6.3:

τ = 4.71 + 0.5(0.0833) = 4.75 min

θ = 0.1 + 0.5(0.0833) = 0.14 min

Thus,

$$\tilde{G} = \frac{0.208e^{-0.14s}}{4.75s+1}$$

The ITAE controller settings are calculated as:

$$K\,K_c = 0.859\left(\frac{\theta}{\tau}\right)^{-0.977} = 0.859\left(\frac{0.14}{4.752}\right)^{-0.977} \quad \Rightarrow\; K_c = 134$$

$$\frac{\tau}{\tau_I} = 0.874\left(\frac{\theta}{\tau}\right)^{-0.680} = 0.674\left(\frac{0.14}{4.752}\right)^{-0.680} \quad \Rightarrow\; \tau_I = 0.642 \text{ min}$$

(d) Combined feedforward-feedback control

This control system consists of the dynamic feedforward controller of part (b) and the PI controller of part (c).

The closed-loop responses to a +0.2 step change in x_1 for the two feedforward controllers are shown in Fig. S15.9a. The dynamic feedforward controller is superior to the static feedforward controller because both the maximum deviation from the set point and the settling time are smaller. Figure S15.9b shows that the combined feedforward-feedback control system provides the best control and is superior to the PI controller. A comparison of Figs. S15.9a and S15.9b shows that the addition of feedback control significantly reduces the settling time due to the very large value of K_c that can be employed because the time delay is very small. (Note that $\theta/\tau = 0.14/4.75 = 0.0029$.)

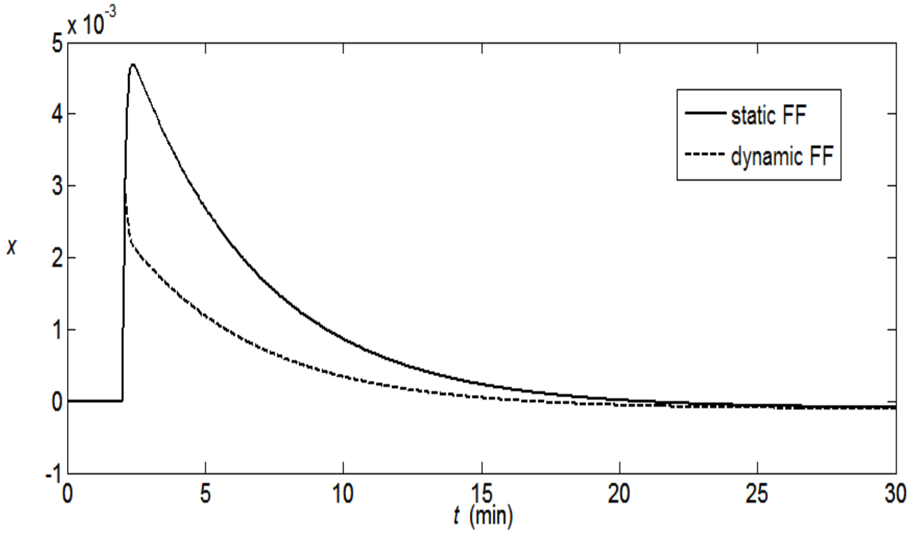


Fig. S15.9a. *Comparison of static and dynamic feedforward controllers for a step disturbance of +0.2 in x_1 at t =2 min.*

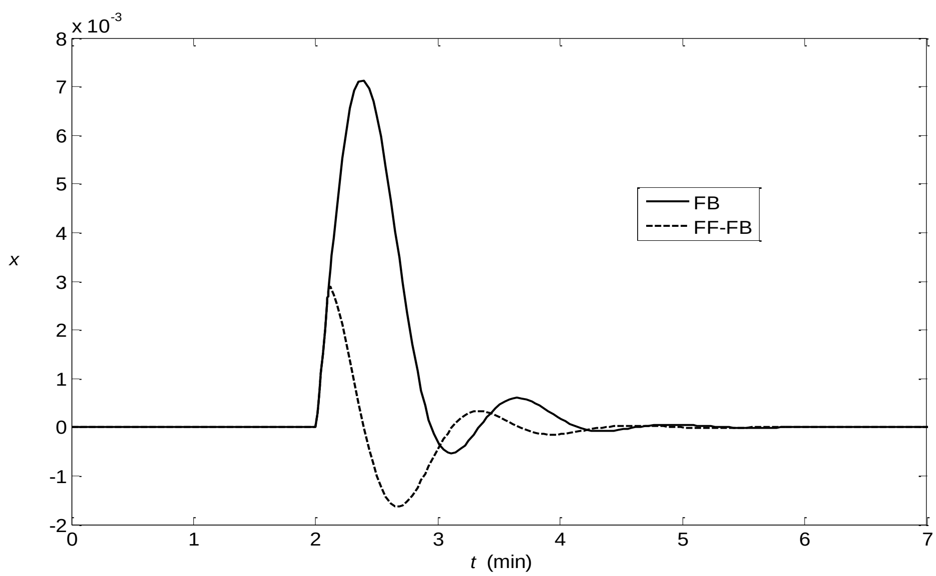


Fig. S15.9b. *Comparison of feedback and feedforward-feedback controllers for a step disturbance of +0.2 in x_1 at t =2 min.*

15.10

a) For steady-state conditions,

$$G_p = K_p, \quad G_d = K_L, \quad G_v = G_m = G_t = 1$$

Using Eq. 15-21

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{-0.5}{(1)(1)(2)} = -0.25$$

b) From Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{\dfrac{-0.5e^{-30s}}{60s+1}}{(1)(1)\left(\dfrac{2e^{-20s}}{95s+1}\right)} = -0.25\frac{(95s+1)}{(60s+1)}e^{-10s}$$

c) Using Table 12.1, a PI controller is obtained from item *G*,

$$K_c = \frac{1}{K_p}\frac{\tau}{\tau_c+\theta} = \frac{1}{2}\frac{95}{(30+20)} = 0.95$$

$$\tau_i = \tau = 95$$

d) As shown in Fig.S15.10a, the dynamic controller provides significant improvement.

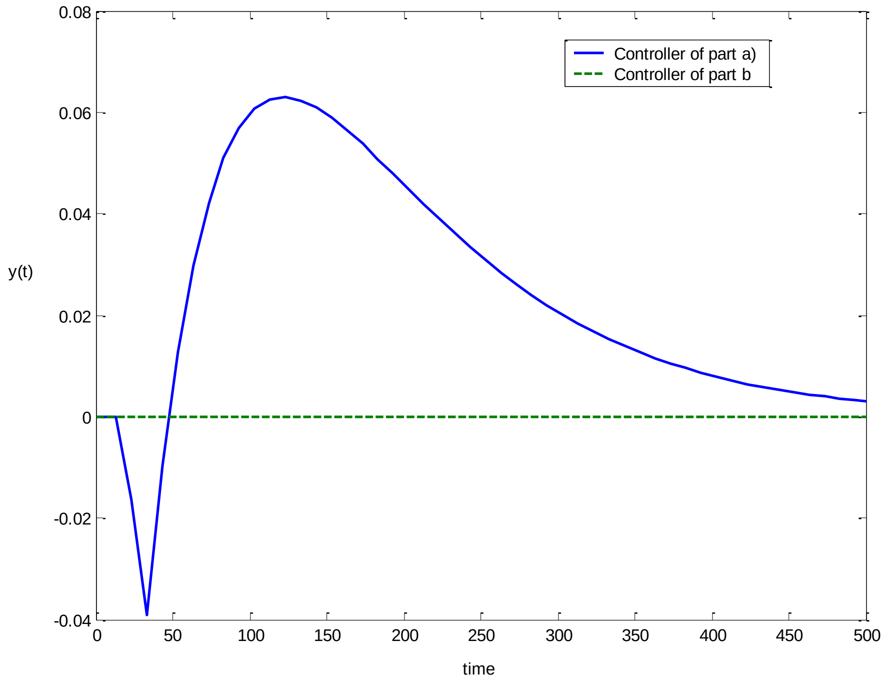


Figure S15.10a. *Closed-loop response using feedforward control only.*

e)

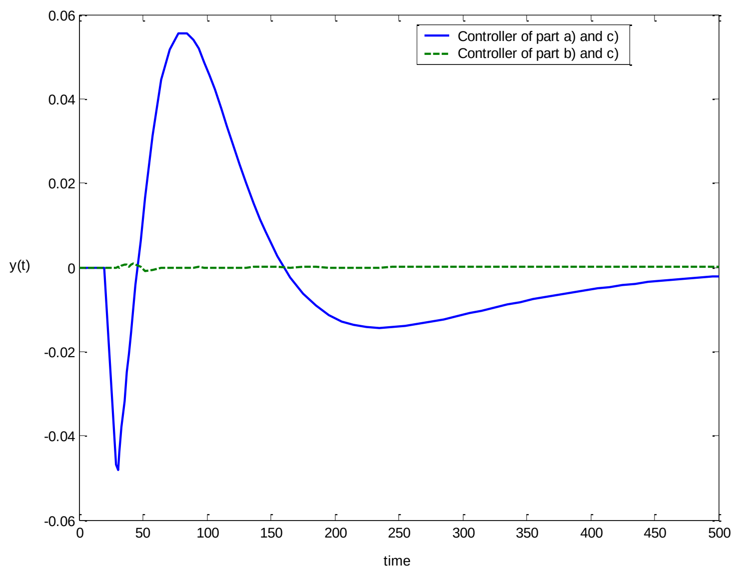


Figure S15.10b. *Closed-loop response for the feedforward-feedback control.*

f) As shown in Fig. S15.10b, the feedforward-feedback configuration with the dynamic controller provides the best control.

15.11

Energy Balance:

$$\rho VC\frac{dT}{dt} = wC(T_i - T) - U(1+q_c)A(T - T_c) - U_L A_L (T - T_a) \qquad (1)$$

Expanding the RHS,

$$\rho VC\frac{dT}{dt} = wC(T_i - T) - UA(T - T_c)$$
$$- UAq_c T + UAq_c T_c - U_L A_L (T - T_a) \qquad (2)$$

Linearizing the nonlinear term,

$$q_c T \approx \bar{q}_c \bar{T} + \bar{q}_c T' + \bar{T} q_c' \qquad (3)$$

Substituting (3) into (2), subtracting the steady-state equation, and introducing deviation variables,

$$\rho VC\frac{dT'}{dt} = wC(T_i' - T') - UAT' - UA\bar{T}q_c' - UA\bar{q}_c T'$$
$$+ UAT_c q_c' - U_L A_L T' \qquad (4)$$

Taking the Laplace transform and assuming steady-state at t = 0 gives,

$$\rho VCsT'(s) = wCT_i'(s) + UA(T_c - T')q_c'(s)$$

$$-(wC + UA + UA\bar{q}_c + U_L A_L)T'(s) \qquad (5)$$

Rearranging,

$$T'(s) = G_L(s)T_i'(s) + G_p(s)q_c'(s) \qquad (6)$$

where:

$$G_d(s) = \frac{K_d}{\tau s + 1}$$
$$G_p(s) = \frac{K_p}{\tau s + 1}$$

$$K_d = \frac{wC}{K} \tag{7}$$

$$K_p = \frac{UA(T_c - \overline{T})}{K}$$

$$\tau = \frac{\rho VC}{K}$$

$$K = wC + UA + UA\overline{q}_c + U_L A_L$$

The ideal FF controller design equation is given by,

$$G_F = \frac{-G_d}{G_t G_v G_p} \tag{15-21}$$

But, $G_t = K_t e^{-\theta s}$ and $G_v = K_v$ (8)

Substituting (7) and (8) gives,

$$G_F = \frac{-wCe^{+\theta s}}{K_t K_v UA(T_c - \overline{T})} \tag{9}$$

In order to have a physically realizable controller, ignore the $e^{+\theta s}$ term,

$$G_F = \frac{-wC}{K_t K_v UA(T_c - \overline{T})} \tag{10}$$

15.12

Note: The disturbance transfer function is incorrect in the first printing. It should be:

$$\frac{c_{O_2}}{FG} = -\frac{-2.82e^{-4s}}{4.3s+1}$$

(a) The feedforward controller design equation is (15-21):

$$G_f = \frac{-G_d}{G_v G_t G_p} = -\frac{\dfrac{-2.82e^{-4s}}{4.3s+1}}{(1)(1)\left(\dfrac{0.14e^{-4s}}{4.2s+1}\right)} = 20.1\,\frac{4.2s+1}{4.3s+1}$$

$$G_f \approx 20.1 \text{ m}^3/\text{min}$$

b) Using Item G in Table 12.1, a PI controller is obtained from for $G = G_v G_p G_m$, Assume that $\tau_c = \tau/2 = 2.1$ min.

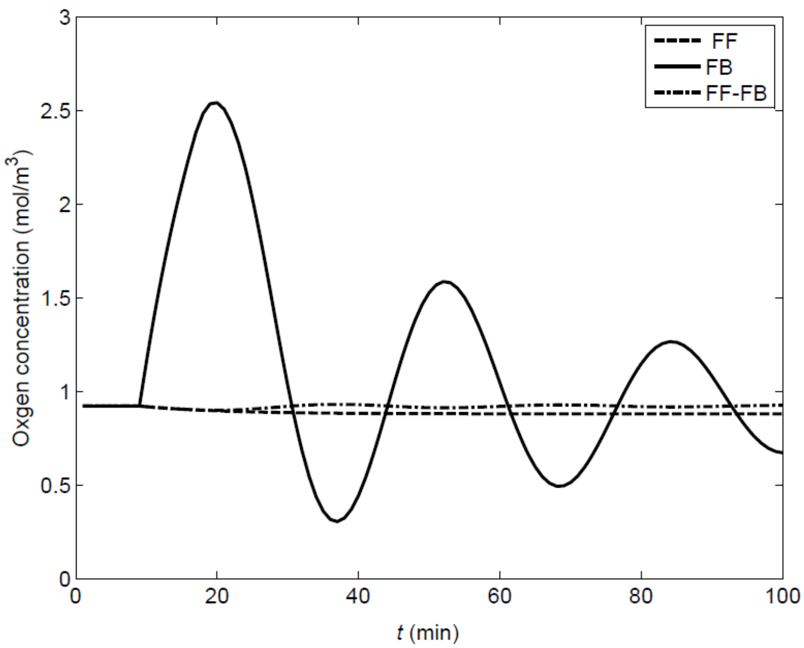
$$K_c = \frac{1}{K}\,\frac{\tau + \theta/2}{\tau_c + \theta/2} = \frac{1}{0.14}\,\frac{4.2+2}{(2.1+2)} = 10.8$$

$$\tau_I = \tau + \theta/2 = 4.2 + 2 = 6.2 \text{ min}$$

$$\tau_D = \frac{\tau\theta}{2\tau + \theta} = \frac{(4.2)(4)}{2(4.2)+4} = 1.35 \,\text{min}$$

$$\tau_i = \tau = 95$$

c) As shown in Fig.S15.12a, the FF-FB controller provides the best control with a small maximum deviation and no offset. The oscillation due to the feedback controller can be damped by using a larger value of design parameter, τ_c.



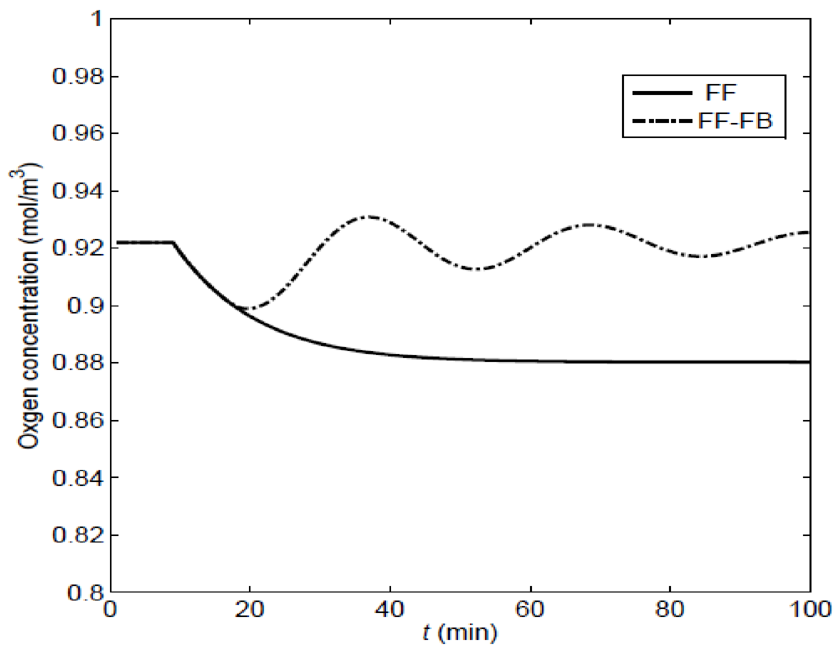


Figure S15.12a: *Controller Comparison for step change in fuel gas purity from 1.0 to 0.9 at t = 0. Top: full scale; Bottom: expanded scale.*

15.13

Steady-state balances:

$$0 = \bar{q}_5 + \bar{q}_1 - \bar{q}_3 \tag{1}$$

$$0 = \bar{q}_3 + \bar{q}_2 - \bar{q}_4 \tag{2}$$

$$0 = \bar{x}_5\bar{q}_5 + \cancelto{0}{\bar{x}_1\bar{q}_1} - \bar{x}_3\bar{q}_3 \tag{3}$$

$$0 = \bar{x}_3\bar{q}_3 + \bar{x}_2\bar{q}_2 - \bar{x}_4\bar{q}_4 \tag{4}$$

Solve (4) for $\bar{x}_3\bar{q}_3$ and substitute into (3),

$$0 = \bar{x}_5\bar{q}_5 + \bar{x}_2\bar{q}_2 - \bar{x}_4\bar{q}_4 \tag{5}$$

Rearrange,

$$\bar{q}_2 = \frac{\bar{x}_4\bar{q}_4 - \bar{x}_5\bar{q}_5}{\bar{x}_2} \tag{6}$$

In order to derive the feedforward control law, let

$$\bar{x}_4 \to x_{4sp} \qquad \bar{x}_2 \to x_2(t) \qquad \bar{x}_5 \to x_5(t) \qquad \text{and} \quad \bar{q}_2 \to q_2(t)$$

Thus,

$$q_2(t) = \frac{x_{4sp}\bar{q}_4 - x_5(t)q_5(t)}{\bar{x}_2} \tag{7}$$

Substitute numerical values:

$$q_2(t) = \frac{(3400)x_{4sp} - x_5(t)q_5(t)}{0.990} \tag{8}$$

or

$$q_2(t) = 3434x_{4sp} - 1.01x_5(t)q_5(t) \tag{9}$$

Note: If the transmitter and control valve gains are available, then an expression relating the feedforward controller output signal, $p(t)$, to the measurements , $x_{5m}(t)$ *and* $q_{5m}(t)$, can be developed.

Dynamic compensation: It will be required because of the extra dynamic lag introduced by the tank on the left hand side. The stream 5 disturbance affects x_3 while q_3 does not.

15.14

The three x_D control strategies are compared in Figs. S15.14a-b for the step disturbance in feed composition. The FF-FB controller is slightly superior because it minimizes the maximum deviation from set point. Note that the PCM feedforward controller design ignores the two time delays, which are quite different. Thus, the feedforward controller overcorrects and is not effective as it could be.

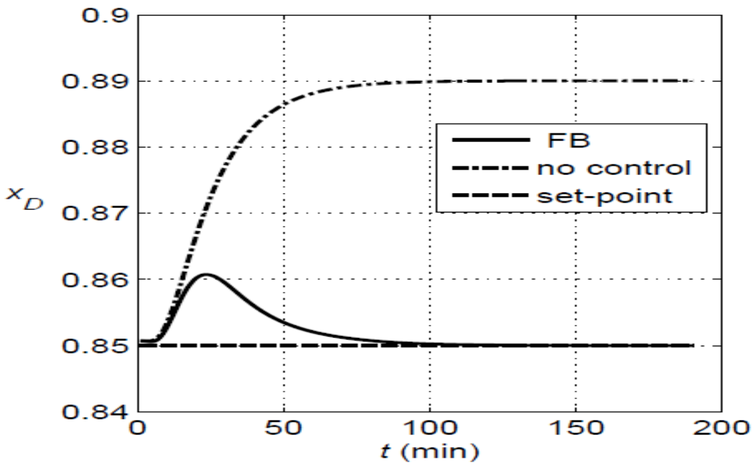


Fig. S15.14a. *Comparison of feedback control and no control for a step change in feed composition from 0.5 to 0.55 at t = 0.*

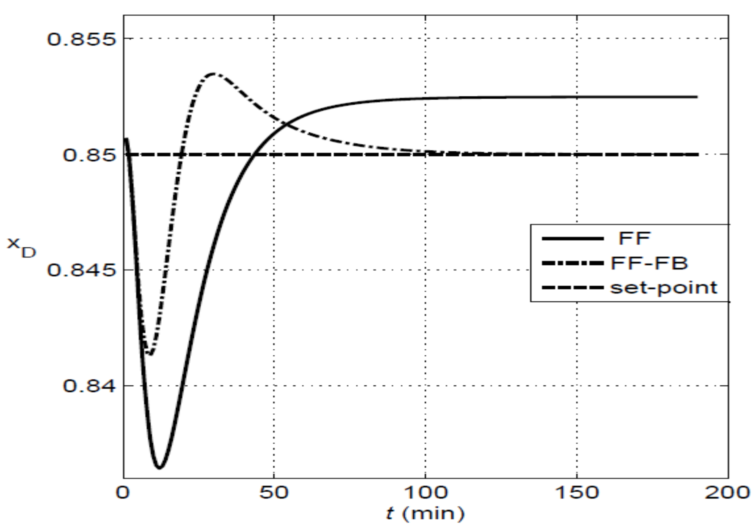


Fig. S15.14b. *Comparison of feedforward and feedforward-feedback control for a step change in feed composition from 0.5 to 0.55 at t = 0.*

Chapter 16

16.1

The difference between systems A and B lies in the dynamic lag in the measurement elements G_{m1} (primary loop) and G_{m2}(secondary loop). With a faster measurement device in A, better control action is achieved. In addition, for a cascade control system to function properly, the response of the secondary control loop should be faster than the primary loop. Hence System A should be faster and yield better closed-loop performance than B.

Because G_{m2} in system B has an appreciable lag, cascade control has the potential to improve the overall closed-loop performance more than for system A. Little improvement in system A can be achieved by cascade control versus conventional feedback.

Comparisons are shown in Figs. S16.1a/b. PI controllers are used in the outer loop. The PI controllers for both System A and System B are designed based on Table 12.1 ($\tau_c = 3$). P controllers are used in the inner loops. Because of different dynamics the proportional controller gain of System B is about one-fourth as large as the controller gain of System A

System A: $K_{c2} = 1$ K_{c1}=0.5 τ_I=15
System B: $K_{c2} = 0.25$ K_{c1}=2.5 τ_I=15

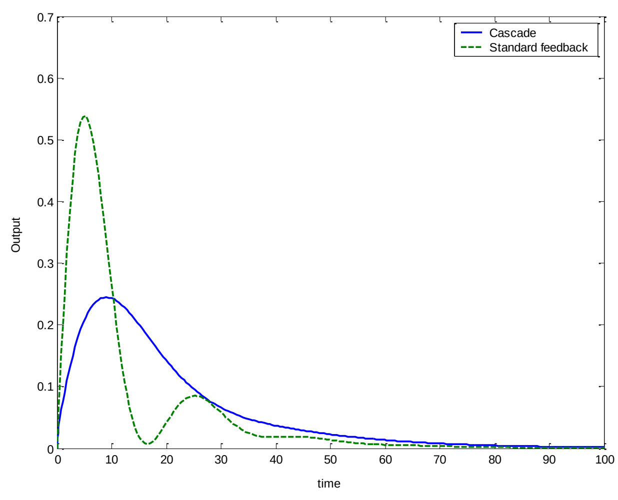


Figure S16.1a *System A. Comparison of D_2 responses (D_2=1/s) for cascade control and conventional PI control.*

In comparing the two figures, it appears that the standard feedback results are essentially the same, but the cascade response for system A is much faster and has much less absolute error than for the cascade control of B

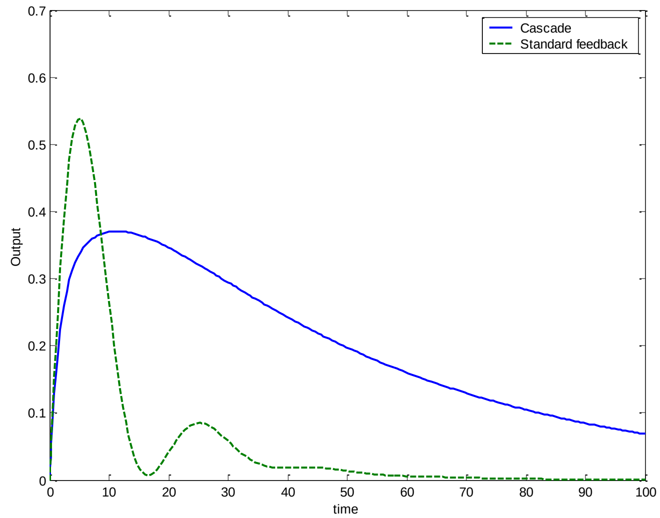


Figure S16.1b *System B .Comparison of D_2 responses (D_2=1/s) for cascade control and conventional PI control.*

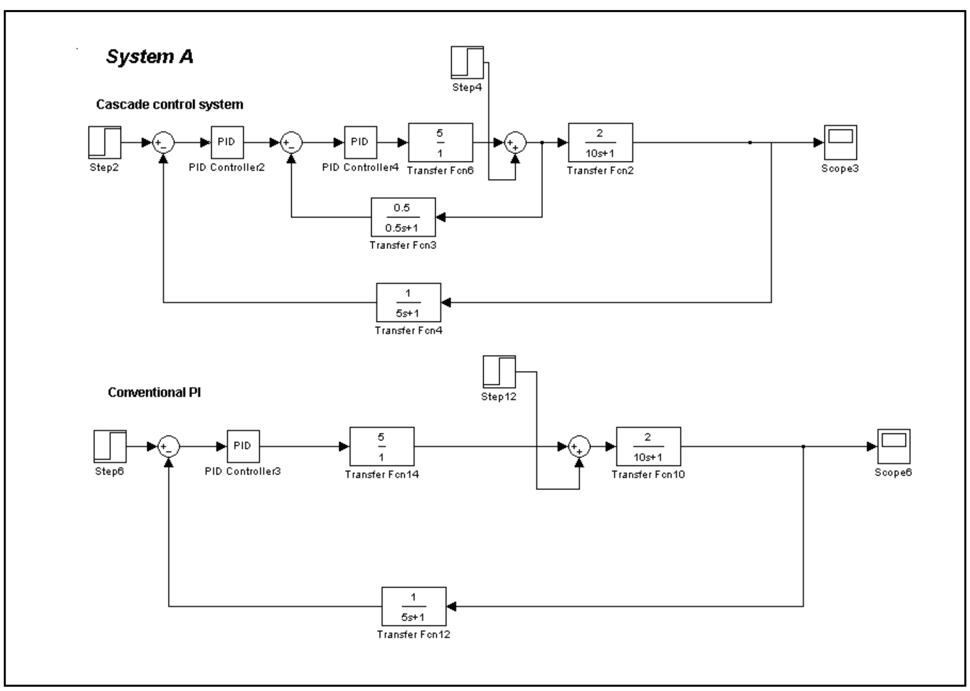


Figure S16.1c *Block diagram for System A*

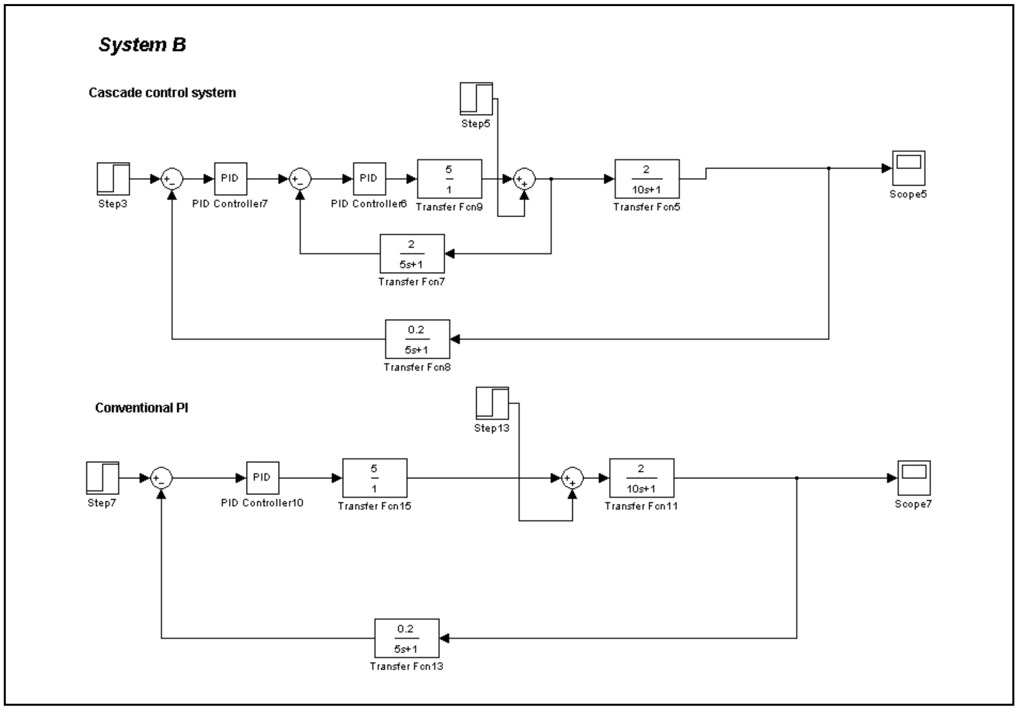


Figure S16.1d *Block diagram for System B*

16.2

a) The transfer function between Y_1 and D_1 is

$$\frac{Y_1}{D_1} = \frac{G_{d1}}{1 + G_{c1}\left(\dfrac{G_{c2}G_v}{1 + G_{c2}G_vG_{m2}}\right)G_pG_{m1}}$$

and that between Y_1 and D_2 is

$$\frac{Y_1}{D_2} = \frac{G_pG_{d2}}{1 + G_{c2}G_vG_{m2} + G_{c2}G_vG_{m1}G_{c1}G_p}$$

using $G_v = \dfrac{5}{s+1}$, $G_{d2} = 1$, $G_{d1} = \dfrac{1}{3s+1}$,

$$G_p = \frac{4}{(2s+1)(4s+1)} \quad , G_{m1} = 0.05 \quad , \quad G_{m2} = 0.2$$

For $G_{c1} = K_{c1}$ and $G_{c2} = K_{c2}$, we obtain

$$\frac{Y_1}{D_1} = \frac{8s^3 + (14+8K_{c2})s^2 + (7+6K_{c2})s + K_{c2} + 1}{24s^4 + (50+24K_{c2})s^3 + [10+K_{c2}(9+3K_{c1})]s + (35+26K_{c2})s^2 + K_{c2}(1+K_{c1}) + 1}$$

$$\frac{Y_1}{D_2} = \frac{4(s+1)}{8s^3 + (14+8K_{c2})s^2 + (7+6K_{c2})s + K_{c2}(1+K_{c1}) + 1}$$

The figures below show the step load responses for K_{c1}=43.3 and for K_{c2}=25. Note that both responses are stable. You should recall that the critical gain for K_{c2}=5 is K_{c1}=43.3. Increasing K_{c2} stabilizes the controller, as is predicted.

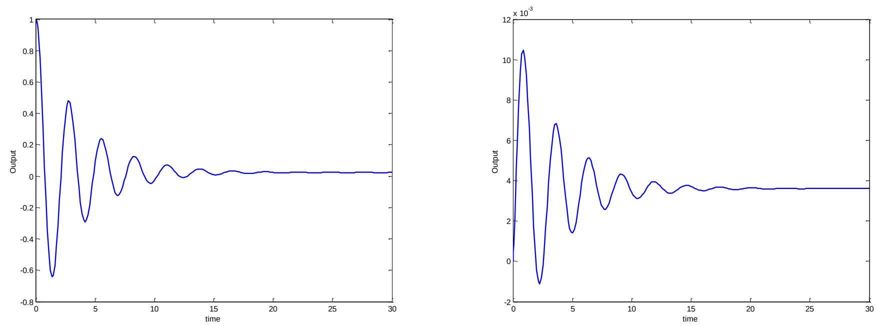


Figure S16.2a *Responses for unit load change in D_1 (left) and D_2 (right)*

b) The characteristic equation for this system is

$$1 + G_{c2}G_vG_{m2} + G_{c2}G_vG_{m1}G_{c1}G_p = 0 \qquad (1)$$

Let G_{c1}=K_{c2} and G_{c2}=K_{c2}. Then, substituting all the transfer functions into (1), we obtain

$$8s^3 + (14+8K_{c2})s^2 + (7+6K_{c2})s + K_{c2}(1+K_{c1}) + 1 = 0 \qquad (2)$$

Now we can use the direct substitution:

$$\left[-8\omega^3 + \left(7+6K_{c2}\right)\omega\right]j - (14+8K_{c2})\omega^2 + K_{c2}(1+K_{c1}) + 1 = 0 \qquad (3)$$

$$j: -8\omega^3 + \left(7+6K_{c2}\right)\omega = 0$$

$$-\left(14+8K_{c2}\right)\omega^2 + K_{c2}\left(1+K_{c1}\right) + 1 = 0$$

Hence, for normal (positive) values of K_{c1} and K_{c2},

$$K_{c1,u} = \frac{24K_{c2}^{\ 2} + 66K_{c2} + 45}{4K_{c2}}$$

The results are shown in the table and figure below. Note the nearly linear variation of K_{c1} ultimate with K_{c2}. This is because the right hand side is very nearly 6 K_{c2}+16.5. For larger values of K_{c2}, the stability margin on K_{c1} is higher. There don't appear to be any nonlinear effects of K_{c2} on K_{c1}, especially at high K_{c2}.

There is no theoretical upper limit for K_{c2}, except that large values may cause the valve to saturate for small set-point or load changes.

K_{c2}	$K_{c1,u}$
1	33.75
2	34.13
3	38.25
4	43.31
5	48.75
6	54.38
7	60.11
8	65.91
9	71.75
10	77.63
11	83.52
12	89.44
13	95.37
14	101.30
15	107.25
16	113.20
17	119.16
18	125.13
19	131.09
20	137.06

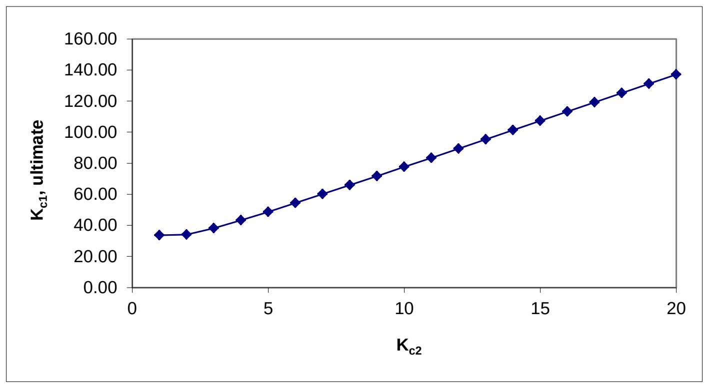


Figure S16.2b *Effect of K_{c2} on the critical gain of K_{c1}*

c) With integral action in the inner loop,

$$G_{c1} = K_{c1}$$

$$G_{c2} = 5\left(1 + \frac{1}{5s}\right)$$

Substitution of all the transfer functions into the characteristic equation yields

$$1 + 5\left(1 + \frac{1}{5s}\right)\frac{5}{s+1}(0.2) + 5\left(1 + \frac{1}{5s}\right)\frac{5}{s+1}(0.05)K_{c1}$$

$$\frac{4}{(4s+1)(2s+1)} = 0$$

Rearrangement gives

$$8s^4 + 54s^3 + 45s^2 + (12 + 5K_{c1})s + K_{c1} + 1 = 0$$

Now we can use the direct substitution:

$$\left[-54\omega^3 + \left(12 + 5K_{c1}\right)\omega\right] j + 8\omega^4 - 45\omega^2 + K_{c1} + 1 = 0$$

$$j : -54\omega^3 + \left(12 + 5K_{c1}\right)\omega = 0$$

$$8\omega^4 - 45\omega^2 + K_{c1} + 1 = 0$$

Solve the equations above, and we obtain:

$$K_{c1,u} = 44.2$$

The ultimate K_{c1} is 44.2, which is close to the result as for proportional only control of the secondary loop.

With integral action in the outer loop only,

$$G_{c1} = K_{c1}\left(1 + \frac{1}{5s}\right)$$

$$G_{c2} = 5$$

Substituting the transfer functions into the characteristic equation.

$$1 + 5\frac{5}{s+1}(0.2) + 5\frac{5}{s+1}(0.05)K_{c1}\left(1 + \frac{1}{5s}\right)\frac{4}{(4s+1)(2s+1)} = 0$$

$$\therefore\ 8s^4 + 54s^3 + 37s^2 + (6 + 5K_{c1})s + K_{c1} = 0$$

Now we can use the direct substitution:

$$\left[-54\omega^3 + \left(6 + 5K_{c1}\right)\omega\right] j + 8\omega^4 - 37\omega^2 + K_{c1} = 0$$

$$j : -54\omega^3 + \left(6 + 5K_{c1}\right)\omega = 0$$

$$8\omega^4 - 37\omega^2 + K_{c1} = 0$$

Solve the equations above, and we obtain:

$$K_{c1,u} = 34.66$$

Hence, K_{c1}<34.66 is the limiting constraint. Note that due to integral action in the primary loop, the ultimate controller gain is reduced.

Calculation of offset:

For $\quad G_{c1} = K_{c1}\left(1 + \frac{1}{\tau_{I1} s}\right) \quad , \quad G_{c2} = K_{c2} \quad , \quad (\tau_{I2} = \infty)$

$$\frac{Y_1}{D_1} = \frac{G_{d1}(1+K_{c2}G_vG_{m2})}{1+K_{c2}G_vG_{m2}+K_{c2}G_vG_{m1}K_{c1}\left(1+\frac{1}{\tau_{I1}s}\right)G_p}$$

$$\frac{Y_1}{D_1}(s=0)=0$$

Since G_{c1} contains integral action, a step-change in D_1 does not produce an offset in Y_1.

$$\frac{Y_1}{D_2} = \frac{G_pG_{d2}}{1+K_{c2}G_vG_{m2}+K_{c2}G_vG_{m1}K_{c1}\left(1+\frac{1}{\tau_{I1}s}\right)G_p}$$

$$\frac{Y_1}{D_2}(s=0)=0$$

Thus, for the same reason as before, a step-change in D_2 does not produce an offset in Y_1.

For $\quad G_{c1}=K_{c1} \quad$ (ie. $\tau_{I1}=\infty$) $\quad , \quad G_{c2}=K_{c2}\left(1+\frac{1}{\tau_{I2}s}\right)$

$$\frac{Y_1}{D_1} = \frac{G_{d1}\left(1+K_{c2}\left(1+\frac{1}{\tau_{I2}s}\right)G_vG_{m2}\right)}{1+K_{c2}\left(1+\frac{1}{\tau_{I2}s}\right)G_vG_{m2}+K_{c2}G_vG_{m1}K_{c1}\left(1+\frac{1}{\tau_{I2}s}\right)G_p}$$

$$\frac{Y_1}{D_1}(s=0)\neq 0$$

Therefore, when there is no integral action in the outer loop, a primary disturbance produces an offset.

Thus, there is no offset for a step-change in the secondary disturbance.

.

$$\frac{Y_1}{D_2} = \frac{G_pG_{d2}}{1+K_{c2}\left(1+\frac{1}{\tau_{I2}s}\right)G_vG_{m2}+K_{c2}G_vG_{m1}K_{c1}\left(1+\frac{1}{\tau_{I2}s}\right)G_p}$$

$$\frac{Y_1}{D_2}(s=0)=0$$

Thus, there is no offset for a step-change in the secondary disturbance.

16.3

For the inner controller (Slave controller), IMC tuning rules are used

$$G_{c2}{*}=\frac{1}{G_2^-}=\frac{(2s+1)(5s+1)(s+1)}{(\tau_{c2}s+1)^3}$$

Closed-loop responses for different values of τ_{c2} are shown below. A τ_{c2} value of 3 yields a good response.

For the Master controller,

$$G_{c1}{*}=\frac{1}{G_1^-} \qquad \text{where} \qquad G_1^-=\frac{(2s+1)(5s+1)(s+1)}{(\tau_{c1}s+1)^3}\,\frac{1}{(10s+1)}$$

This higher-order transfer function is approximated by first order plus time delay using a step test:

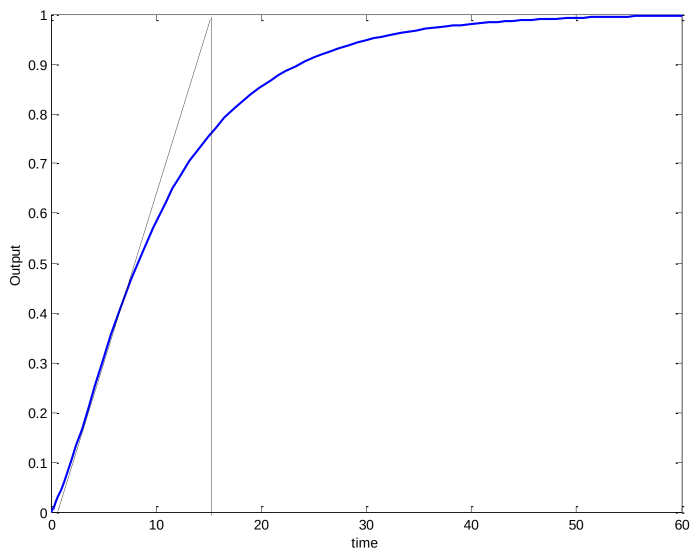
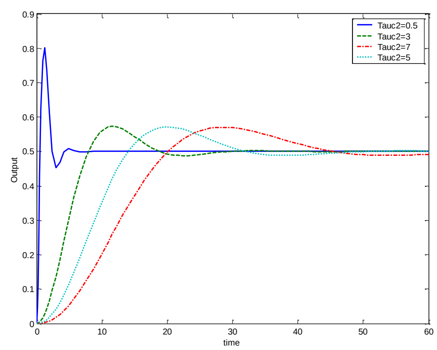


Figure S16.3a *Reaction curve for the higher order transfer function*

Hence $G_1^- \approx \dfrac{e^{-0.38s}}{(15.32s+1)}$

From Table 12.1: (PI controller, Case G): $K_c = \dfrac{15.32}{\tau_{c1}+0.38}$ and $\tau_i = 15.32$

Closed-loop responses are shown for different values of τ_{c1}. A τ_{c1} value of 7 yields a good response.



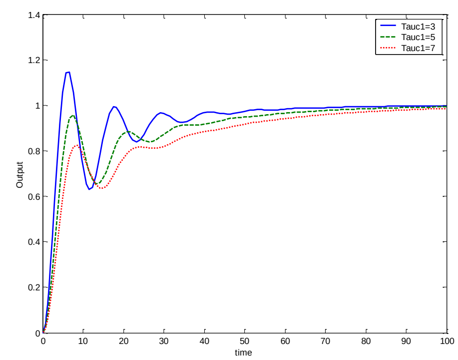


Figure S16.3b *Closed-loop response for τ_{c2}* **Figure S16.3c** *Closed-loop response for τ_{c1}*

Hence for the master controller, $K_c = 2.07$ and $\tau_I = 15.32$

16.4

(a) The single control loop configuration is shown as in Figure S16.4a:

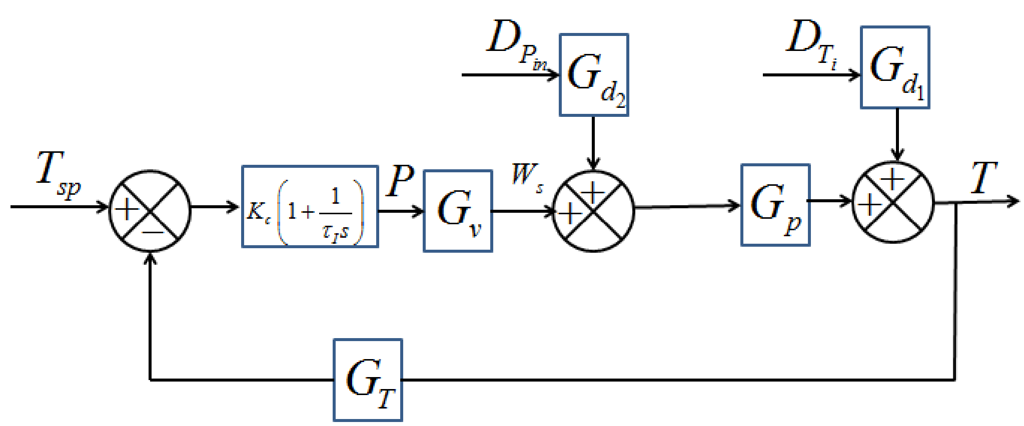


Figure S16.4a *Single control loop configuration*

Assuming $T_{sp} = 0$, the closed-loop transfer function for temperature output is shown as follows:

$$T=\frac{K_c\left(1+\frac{1}{\tau_I s}\right)G_vG_p}{1+K_c\left(1+\frac{1}{\tau_I s}\right)G_TG_vG_p}T_{sp}+\frac{G_{d_2}G_p}{1+K_c\left(1+\frac{1}{\tau_I s}\right)G_TG_vG_p}D_{P_{in}}+\frac{G_{d_1}}{1+K_c\left(1+\frac{1}{\tau_I s}\right)G_TG_vG_p}D_{T_{in}}$$

The characteristic equation of above equation is:

$$1+K_c\left(1+\frac{1}{\tau_I s}\right)G_TG_vG_p=0$$

Or :

$$15s^4+23s^3+9s^2+(K_c+1)s+5K_c=0$$

Set $s=j\omega$:

$$\left(-23\omega^3+(K_c+1)\omega\right)j+15\omega^4-9\omega^2+5K_c=0$$

$$\text{Re: } 15\omega^4-9\omega^2+5K_c=0$$

$$\text{Im: } -23\omega^3+(K_c+1)\omega=0$$

$$K_{cm}=0.08$$

To have a stable system, we have:

$0<K_c<0.08$

(b). The cascade control loop configuration is shown as in Figure S16.4b:

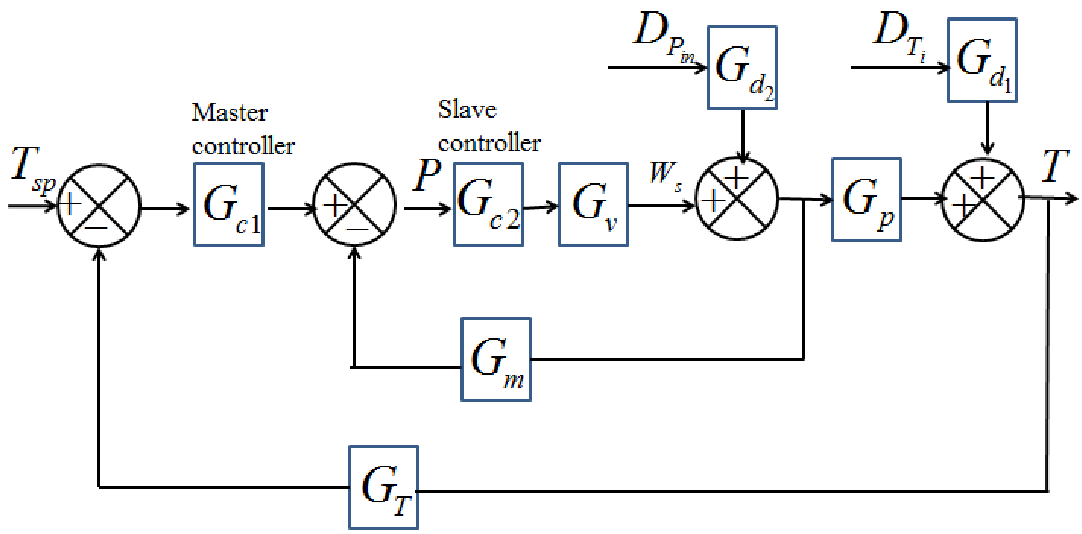


Figure S16.4b *Cascade control loop configuration*

(c) From (a) we can derive the closed-loop transfer function with the standard PI controller for a disturbance in steam pressure:

$$\frac{T(s)}{D_{P_{in}}(s)} = \frac{G_{d_2} G_p}{1 + K_c \left(1 + \frac{1}{\tau_I s}\right) G_T G_v G_p}$$

Assuming $T_{sp} = D_{T_i} = 0$, based on Figure S16.4b, we can derive the closed-loop transfer function with the cascade controller for a disturbance in steam pressure:

$$\frac{T(s)}{D_{P_{in}}(s)} = \frac{G_{d_2} G_p}{1 + G_{c1} G_{c2} G_T G_v G_p + G_m G_{c2} G_v}$$

Set $K_{c1} = K_c = 3; K_{c2} = 2; \tau_I = 5$ in a Simulink diagram, and we obtain results shown in Figure S16.4c: the cascade control system improves stability characteristics by dampening aggressive control responses.

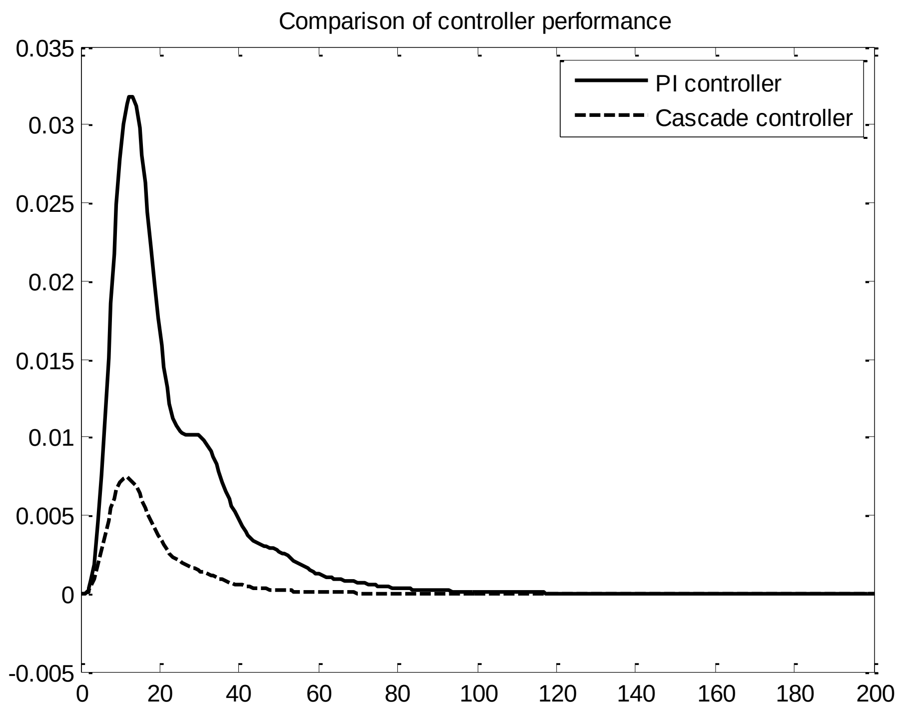


Figure S16.4c *Comparison of closed-loop response with PI controller and cascade controller*

16.5

a) The T_2 controller (TC-2) adjusts the set-point, T_{1sp}, of the T_1 controller (TC-1). Its output signal is added to the output of the feedforward controller.

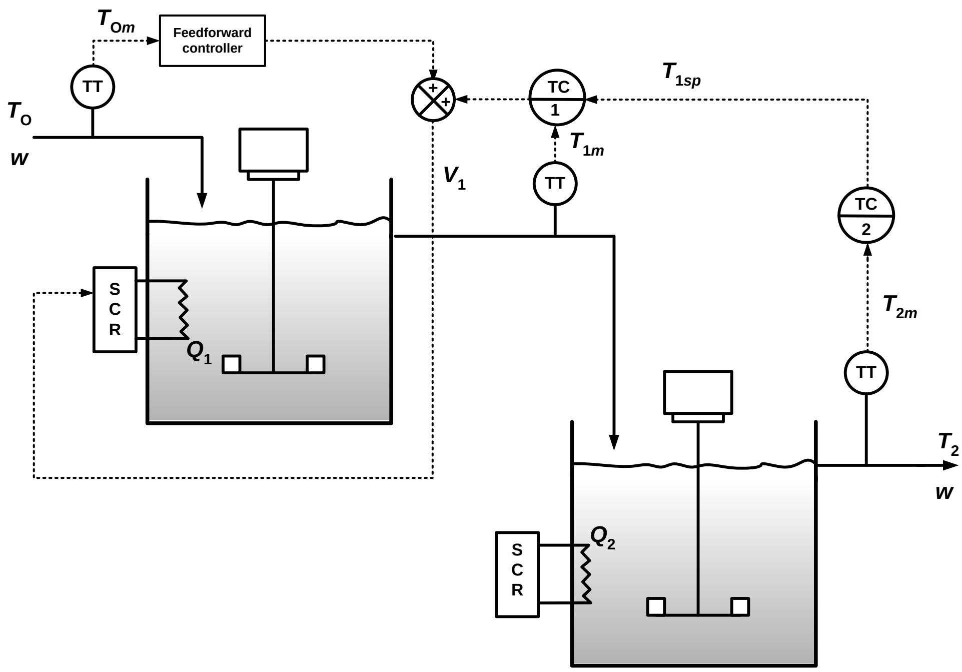


Figure S16.5a *Schematic diagram for the control system*

b) This is a cascade control system with a feedforward controller being used to help control T_1. Note that T_1 is an intermediate variable rather than a disturbance variable since it is affected by V_1.

c) Block diagram:

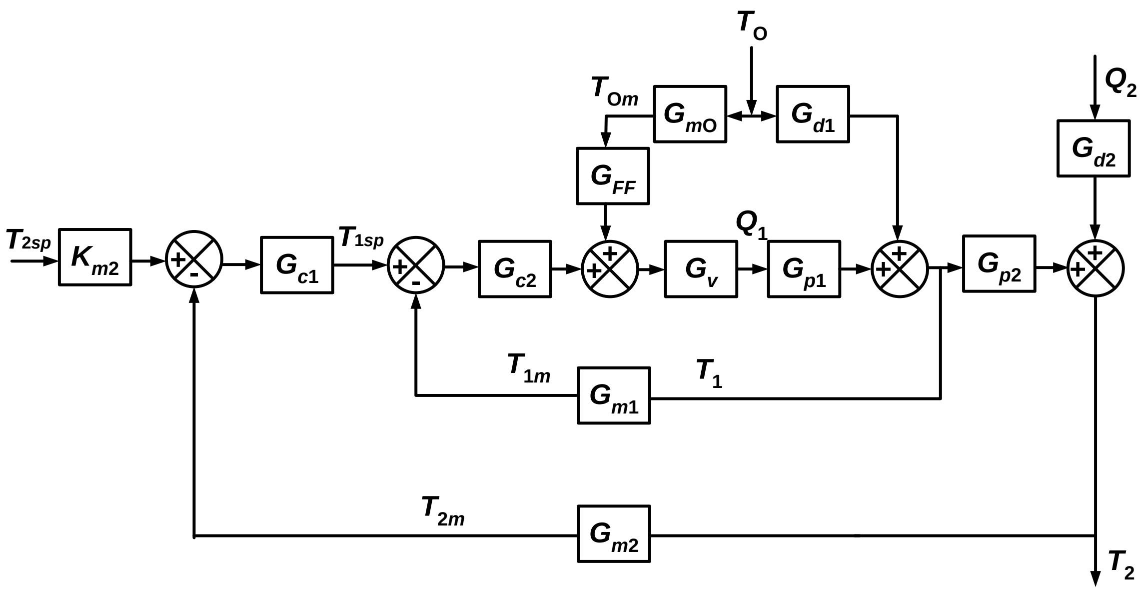


Figure S16.5b *Block diagram for the control system in Exercise 16.5.*

16.6

(a)

FF control can be more beneficial in treating D_2. D_1 can be compensated by feedback loop right after the sensor G_m detected. D_2 needs to go through G_{p1} first where significant time delay may exist before being measured and corrected. Thus, FF control on D_2 can cancel out the disturbance much faster.

(b)

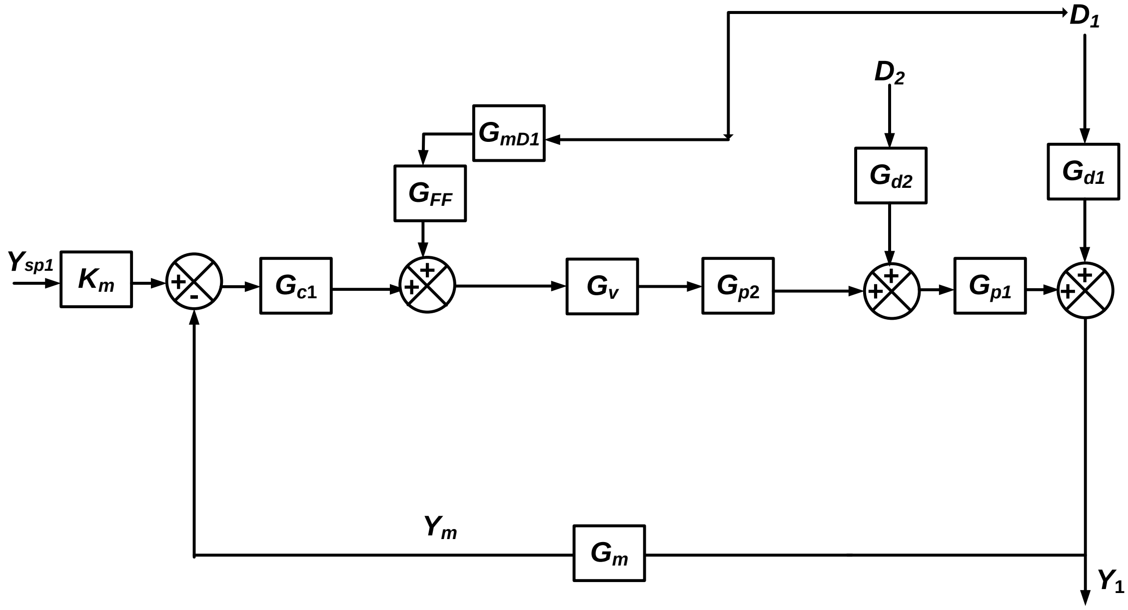


Fig. S16.6. Block diagram of a feedforward control system.

(c)
Cold oil temperature sensor is required.

16.7

Using MATLAB-Simulink, the block diagram for the closed-loop system is shown below.

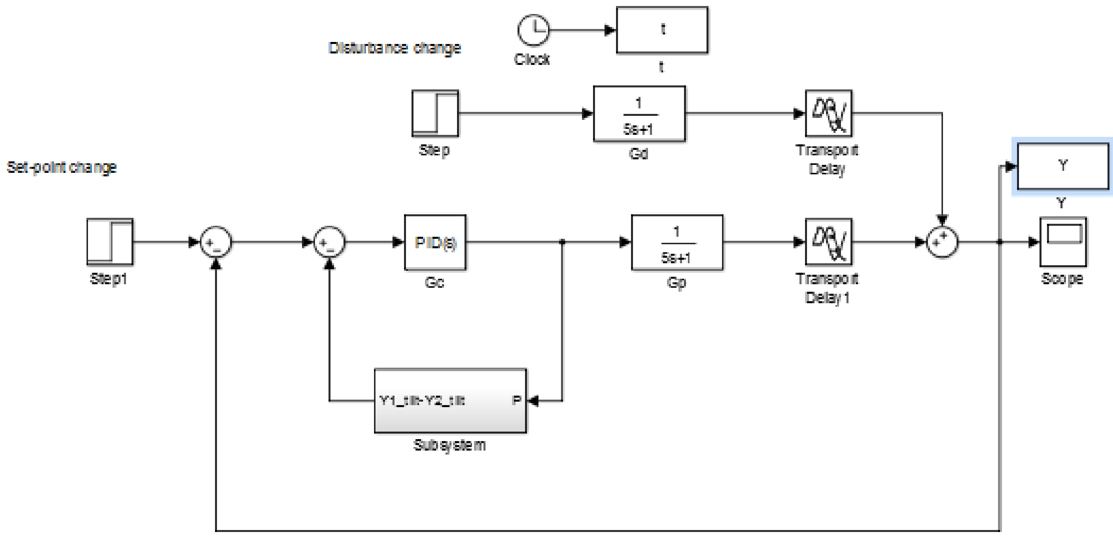
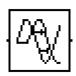


Figure S16.7a *Block diagram for Smith predictor*

where the block  represents the time-delay term $e^{-\theta s}$.

The closed-loop response for unit set-point and disturbance changes are shown below. Consider a PI controller designed by using Table 12.1(Case A) with $\tau_c = 3$ and set $G_d = G_p$. Note that no offset occurs,

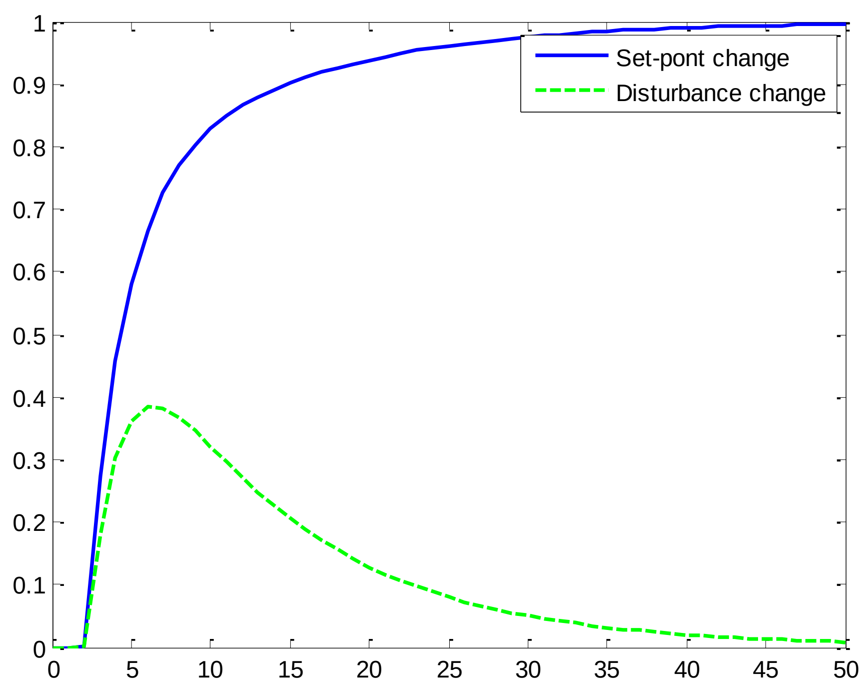


Figure S16.7b *Closed-loop response for setpoint and disturbance changes.*

16.8

The block diagram for the closed-loop system is

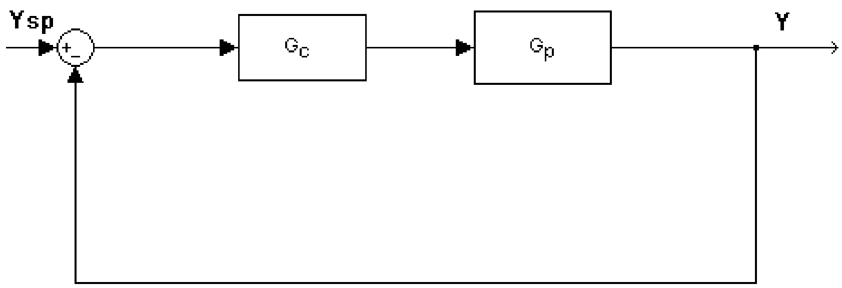


Figure S16.8 *Block diagram for the closed-loop system*

where $G_c = K_c\left(\dfrac{1+\tau_I s}{1+\tau_I s - e^{-\theta s}}\right)$ *and* $G_p = \dfrac{K_p e^{-\theta s}}{1+\tau s}$

a)

$$\frac{Y}{Y_{sp}} = \frac{G_c G_p}{1+G_c G_p} = \frac{K_c K_p\left(\dfrac{1+\tau_I s}{1+\tau_I s - e^{-\theta s}}\right)\dfrac{e^{-\theta s}}{1+\tau s}}{1+K_c K_p\left(\dfrac{1+\tau_I s}{1+\tau_I s - e^{-\theta s}}\right)\dfrac{e^{-\theta s}}{1+\tau s}}$$

Since $K_c = \dfrac{1}{K_p}$ and $\tau_I = \tau$

$$\frac{Y}{Y_{sp}} = \frac{\left(\dfrac{e^{-\theta s}}{1+\tau_I s - e^{-\theta s}}\right)}{1+\left(\dfrac{e^{-\theta s}}{1+\tau_I s - e^{-\theta s}}\right)} = \frac{e^{-\theta s}}{1+\tau_I s - e^{-\theta s} + e^{-\theta s}}$$

Hence dead-time is eliminated from characteristic equation:

$$\frac{Y}{Y_{sp}} = \frac{e^{-\theta s}}{1+\tau_I s}$$

b) The closed-loop response will not exhibit overshoot, because it is a first order plus dead-time transfer function.

16.9

For a first-order process with time delay, use of a Smith predictor and proportional control should make the process behave like a first-order system, i.e., no oscillation. In fact, if the model parameters are accurately known, the controller gain can be as large as we want, and no oscillations will occur.

Appelpolscher has verified that the process is linear, however it may not be truly first-order. If it were second-order (plus time delay), proportional control would yield oscillations for a well-tuned system. Similarly, if there are errors in the model parameters used to design the controller even when the actual process is first-order, oscillations can occur.

16.10

a) Analyzing the block diagram of the Smith predictor

$$\frac{Y}{Y_{sp}} = \frac{G_c G_p' e^{-\theta s}}{1 + G_c \tilde{G}_p' (1 - e^{-\tilde{\theta} s}) + G_c G_p' e^{-\theta s}}$$

$$= \frac{G_c G_p' e^{-\theta s}}{1 + G_c \tilde{G}_p' + G_c G_p' e^{-\theta s} - G_c \tilde{G}_p' e^{-\tilde{\theta} s}}$$

Note that the last two terms of the denominator can when $G_p' = \tilde{G}_p'$ and $\theta = \tilde{\theta}$

The characteristic equation is

$$= 1 + G_c \tilde{G}_p' + G_c G_p' e^{-\theta s} - G_c \tilde{G}_p' e^{-\tilde{\theta} s} = 0$$

b) The closed-loop responses to step set-point changes are shown below for the various cases.

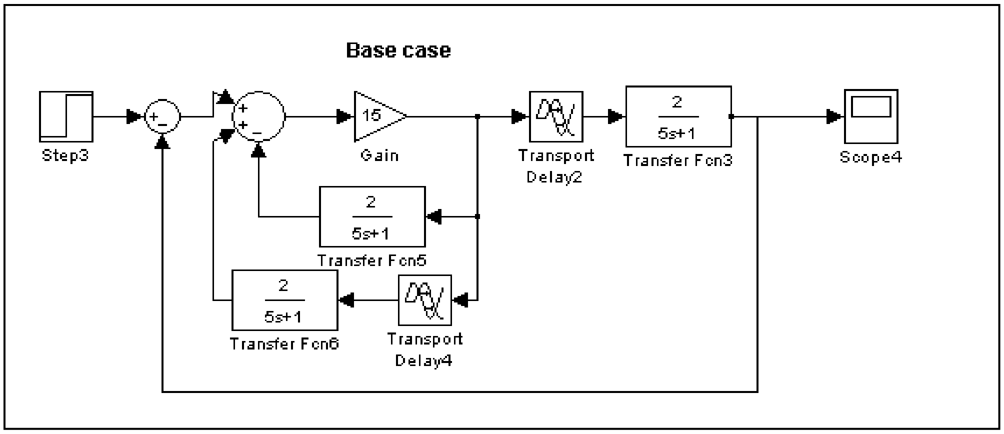


Figure S16.10a *Simulink diagram block; base case*

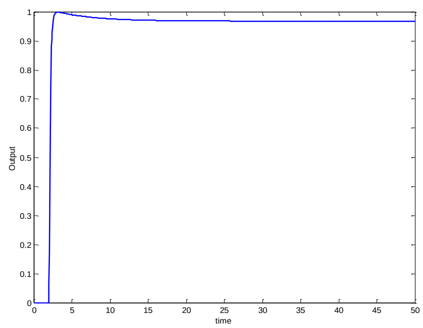


Figure S16.10b *Base case*

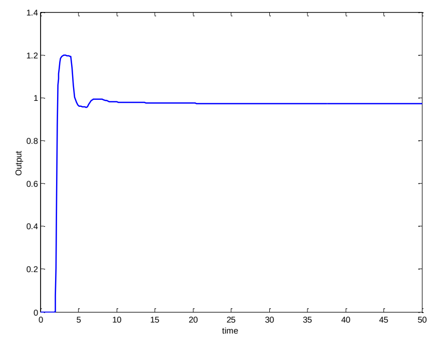


Figure S16.10c $K_p = 2.4$

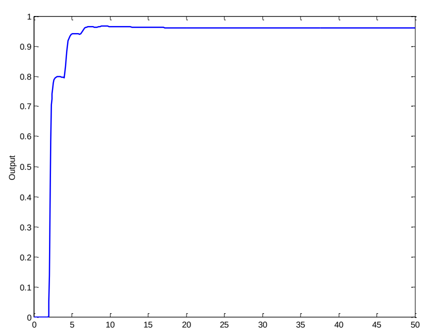


Figure S16.10d $K_p = 1.6$

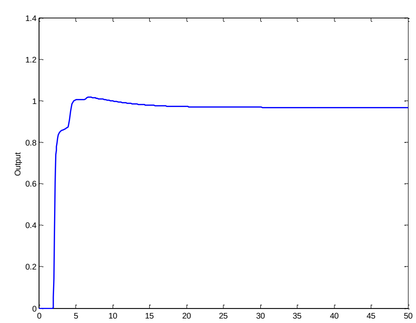


Figure S16.10e $\tau = 6$

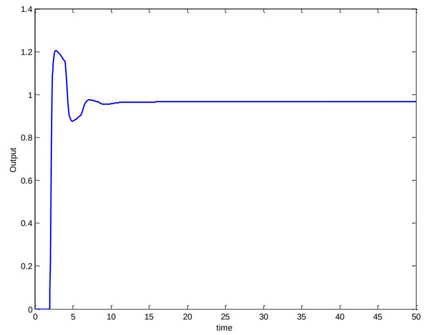


Figure S16.10f $\tau = 4$

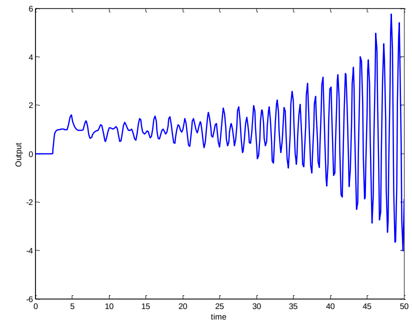


Figure S16.10g θ=2.4

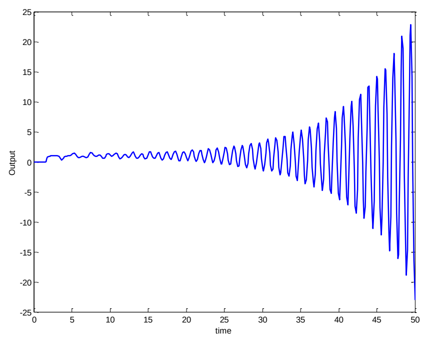
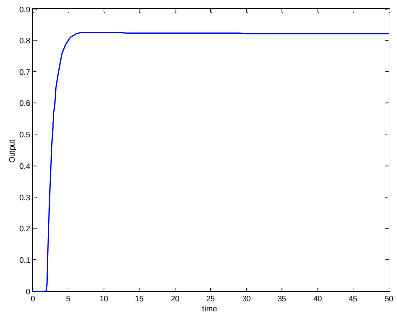


Figure S16.10h $\theta = 1.6$

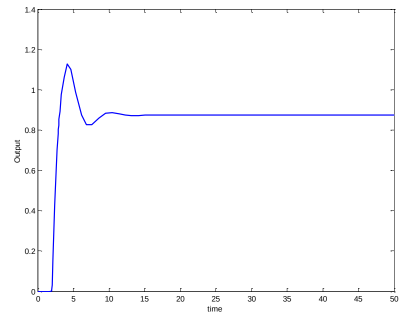
It is immediately evident that errors in time-delay estimation are the most serious. This is because the terms in the characteristic equation which contain dead-time do not cancel, and cause instability at high controller gains.

When the actual process time constant is smaller than the model time constant, the closed–loop system may become unstable. In our case, the error is not large enough to cause instability, but the response is more oscillatory than for the base (perfect model) case. The same is true if the actual process gain is larger than that of the model. If the actual process has a larger time constant, or smaller gain than the model, there is no significant degradation in closed loop performance (for the magnitude of the error, ± 20% considered here). Note that in all the above simulations, the model is considered to be $\frac{2e^{-2s}}{5s+1}$ and the actual process parameters have been assumed to vary by ± 20% of the model parameter values.

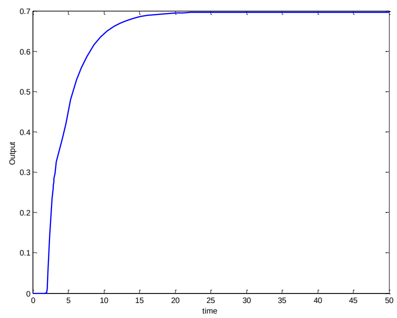
c) The proportional controller was tuned so as to obtain a gain margin of 2.0. This resulted in K_c = 2.3. The responses for the various cases are shown below



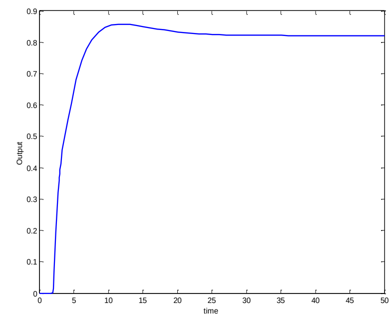
Base case



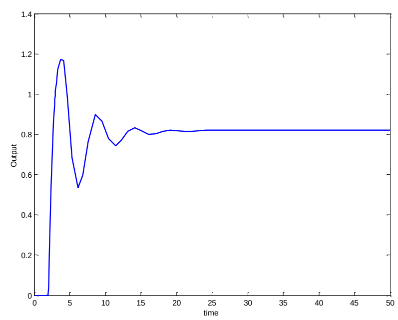
$K_p = 3$



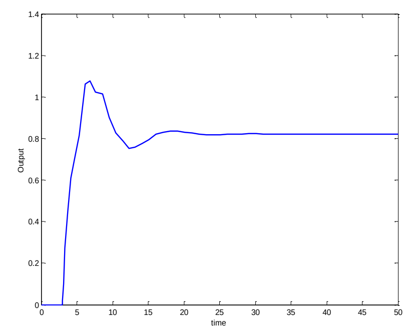
$K_p = 1$



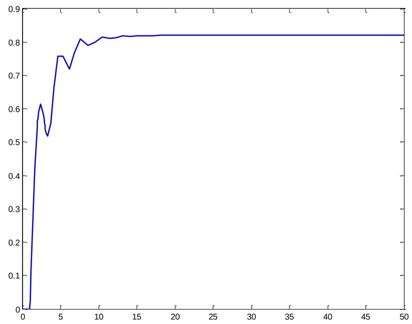
$\tau = 1$



$\tau = 2.5$



$\theta = 3$



$\theta = 1$

Nyquist plots were prepared for different values of K_p, τ and θ, and checked to see if the stability criterion was satisfied. The stability regions when the three parameters are varied one to time are.

$K_p \leq 4.1 \quad (\tau = 5 \quad , \quad \theta = 2)$

$\tau \quad \geq 2.4 \quad (K_p = 2 \quad , \quad \theta = 2)$

$\theta \quad \leq 0.1$ and $1.8 \leq \theta \leq 2.2 \quad (K_p = 2 \quad , \quad \tau = 5)$

16.11

From Eq. 16-24,

$$\frac{Y}{D} = \frac{G_d\left(1 + G_c G^*\left(1 - e^{-\theta s}\right)\right)}{1 + G_c G^*}$$

that is,

$$\frac{Y}{D} = \frac{\frac{2}{s}e^{-3s}\left(1 + \frac{K_c + K_c\tau_I s}{\tau_I s}\frac{2}{s}\left(1 - e^{-3s}\right)\right)}{1 + \frac{K_c + K_c\tau_I s}{\tau_I s}\frac{2}{s}}$$

Using the final value theorem for a step change in D:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

then

$$\lim_{s\to 0} sY(s) = \lim_{s\to 0} s \frac{\frac{2}{s}e^{-3s}\left(1+\frac{K_c+K_c\tau_I s}{\tau_I s}\frac{2}{s}\left(1-e^{-3s}\right)\right)}{1+\frac{K_c+K_c\tau_I s}{\tau_I s}\frac{2}{s}}\frac{1}{s}$$

$$= \lim_{s\to 0} \frac{\frac{2}{s}e^{-3s}\left(\tau_I s+(K_c+K_c\tau_I s)\frac{2}{s}\left(1-e^{-3s}\right)\right)}{\tau_I s+(K_c+K_c\tau_I s)\frac{2}{s}}$$

Multiplying both numerator and denominator by s^2,

$$= \lim_{s\to 0} \frac{2e^{-3s}\left(\tau_I s^2+(K_c+K_c\tau_I s)2\left(1-e^{-3s}\right)\right)}{\tau_I s^3+(K_c+K_c\tau_I s)2s}$$

Applying L'Hopital's rule:

$$= \lim_{s\to 0} \quad \frac{-6e^{-3s}\left(\tau_I s^2+(K_c+K_c\tau_I s)2\left(1-e^{-3s}\right)\right)}{3\tau_I s^2+2(K_c+2K_c\tau_I s)}$$
$$+ \quad \frac{2e^{-3s}(2\tau_I s+6K_c e^{-3s}+2K_c\tau_I-2K_c\tau_I e^{-3s}+6K_c\tau_I s e^{-3s})}{3\tau_I s^2+2(K_c+2K_c\tau_I s)} \;=\; 6$$

Therefore

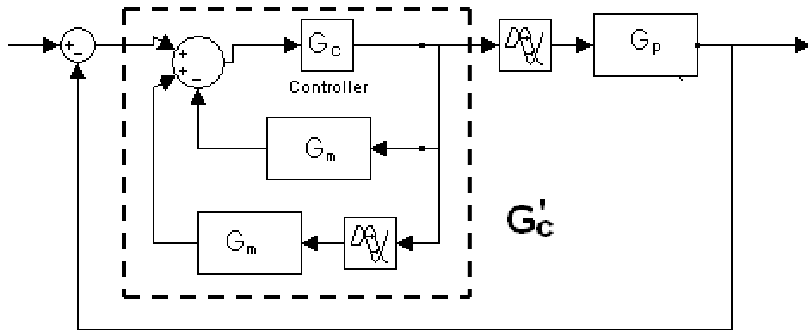
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \;=\; 6$$

and the PI control will not eliminate offset.

16.12

For a Smith predictor, we have the following system

Figure S16.12. *Smith Predictor diagram block*



where the process model is $G_p(s) = Q(s)\, e^{-\theta s}$

For this system,

$$\frac{Y}{Y_{sp}} = \frac{G_c' G_p}{1 + G_c' G_p}$$

where G_c' is the transfer function for the system in the dotted box.

$$G_c' = \frac{G_c}{1 + G_c Q(1 - e^{-\theta s})}$$

$$\therefore \frac{Y}{Y_{sp}} = \frac{\dfrac{G_c G_p}{1 + G_c Q(1 - e^{-\theta s})}}{1 + \dfrac{G_c G_p}{1 + G_c Q(1 - e^{-\theta s})}}$$

Simplification gives

$$\frac{Y}{Y_{sp}} = \frac{G_c Q e^{-\theta s}}{1 + G_c Q} = P(s) e^{-\theta s}$$

where $\quad P(s) = \dfrac{G_c Q}{1 + G_c Q}$

If $P(s)$ is the desired system performance (after the time delay has elapsed) under feedback control, then we can solve for G_c in terms of $P(s)$.

$$G_c = \frac{P(s)}{Q(s)(1 - P(s))}$$

The IMC controller requires that we define

$$\tilde{G}_+ = e^{-\theta s}$$

$$\tilde{G}_- = Q(s) \qquad \text{(the invertible part of } G_p\text{)}$$

Let the filter for the controller be $f(s) = \dfrac{1}{\tau_F s + 1}$

Therefore, the controller is

$$G_c = \tilde{G}_-^{-1} f(s) = \frac{f(s)}{Q(s)}$$

The closed-loop transfer function is

$$\frac{Y}{Y_{sp}} = G_c G_p = \frac{e^{-\theta s}}{1+\tau_F s} = \tilde{G}_+ f$$

Note that this is the same closed-loop form as analyzed in part (a), which led to a Smith Predictor type of controller. Hence, the IMC design also provides time-delay compensation.

16.13

Referring to Example 4.8, if flow rate q and inlet temperature T_i are constant, then (4-88) is the starting point for the derivation:

$$(s - a_{22})T'(s) = a_{21}C_A'(s) + b_2 T_c'(s) \qquad \text{(4-88)}$$

Rearranging gives,

$$C_A'(s) = \frac{s - a_{22}}{a_{21}} T'(s) - \frac{b_2}{a_{21}} T_c'(s)$$

Replacing $C'_A(s)$ by its estimate, $\hat{C}'_A(s)$, provides an inferential estimate of exit composition from T and T_c. However, the first term on the right hand side is not realizable, consequently, a small time constant τ is added to the denominator to provide a lead-lag unit that is physically realable:

$$\hat{C}'_A(s) = \frac{1}{a_{21}}\left(\frac{s - a_{22}}{\tau s + 1}\right)T'(s) - \frac{b_2}{a_{21}}T'_c(s)$$

Thus, inferential control of concentration based on T and T_c temperature is feasible. If q and T_i measurements were available, these variables could be included in the linearized model of Example 4.8. Then, in an analogous manner, C_A can be inferred from the available measurements: T, T_c, q and T_i.

16.14

One possible solution would be to use a split range valve to handle the $100 \leq p \leq 200$ and higher pressure ranges. Moreover, a high-gain controller with set-point = 200 psi can be used for the vent valve. This valve would not open while the pressure is less than 200 psi, which is similar to how a selector operates.

Stephanopoulos (Chemical Process Control, Prentice-Hall, 1989) has described many applications for this so-called split-range control. A typical configuration consists of 1 controller and 2 final control elements or valves.

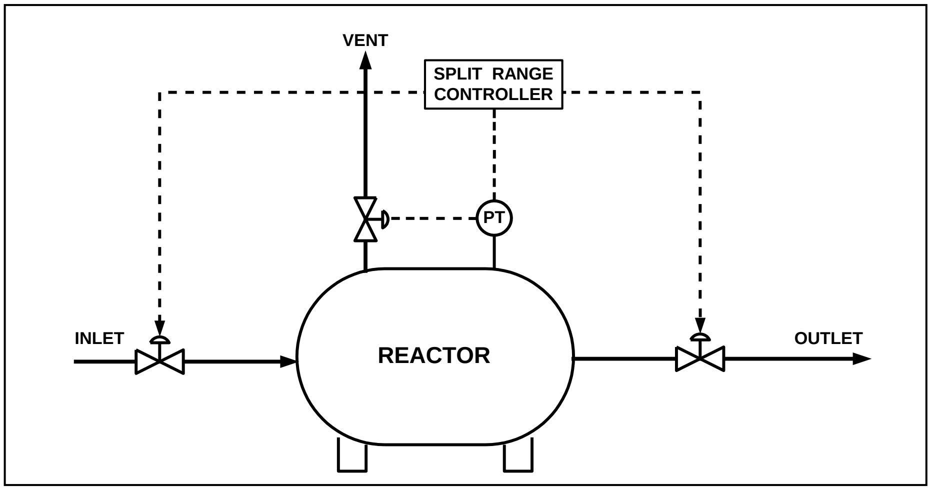


Figure S16.14. *Process instrumentation diagram*

16.15

The amounts of air and fuel are changed in response to the steam pressure. If the steam pressure is too low, a signal is sent to increase both air and fuel flowrates, which in turn increases the heat transfer to the steam. Selectors are used to prevent the possibility of explosions (low air-fuel ratio). If the air flowrate is too low, the low selector uses that measurement as the set-point for the fuel flow rate controller. If the fuel flowrate is too high, its measurement is selected by the high selector as the set-point for the air flow controller. This also protects against dynamic lags in the set-point response.

16.16

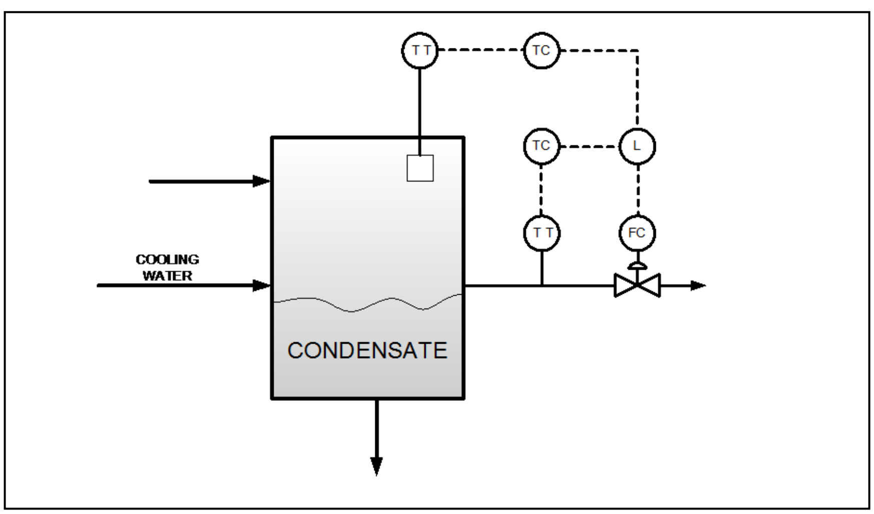


Figure S16.16. *Control condensate temperature in a reflux drum*

16.17

Supposing a first-order plus dead time process, the closed-loop transfer function is

$$G_{CL}(s)=\frac{G_cG_p}{1+G_cG_p} \quad \therefore \quad G_{CL}(s)=\frac{K_cK_p\dfrac{\left(1+\dfrac{1}{\tau_I s}+\tau_D s\right)e^{-\theta s}}{(\tau_p s+1)}}{1+K_cK_p\dfrac{\left(1+\dfrac{1}{\tau_I s}+\tau_D s\right)e^{-\theta s}}{(\tau_p s+1)}}$$

Notice that K_c and K_p always appear together as a product. Hence, if we want the process to maintain a specified performance (stability, decay ratio specification, etc.), we should adjust K_c such that it changes inversely with K_p; as a result, the product K_cK_p is kept constant. Also note, that since there is a time delay, we should adjust K_c based upon the future estimate of K_p:

$$K_c(t)=\frac{\bar{K}_c\bar{K}_p}{\hat{K}_p(t+\theta)}=\frac{\bar{K}_c\bar{K}_p}{a+\dfrac{b}{\hat{M}(t+\theta)}}$$

where $\hat{K}_p(t+\theta)$ is an estimate of K_p θ time units into the future.

16.18

This is an application where self-tuning control would be beneficial. In order to regulate the exit composition, the manipulated variable (flowrate) must be adjusted. Therefore, a transfer function model relating flowrate to exit composition is needed. The model parameters will change as the catalyst deactivates, so some method of updating the model (e.g., periodic step tests) will have to be derived. The average temperature can be monitored to determine a significant change in activation has occurred, thus indicating the need to update the model.

16.19

a) $$\frac{G_cG_p}{1+G_cG_p}=\frac{1}{\tau_c s+1} \qquad \therefore \quad G_c=\frac{\dfrac{1}{\tau_c s+1}}{G_p\left(1-\dfrac{1}{\tau_c s+1}\right)}=\frac{1}{G_p}\frac{1}{\tau_c s}$$

Substituting for G_p

$$G_c(s)=\frac{1}{\tau_c s}\frac{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1}{K_p}=\frac{1}{K_p\tau_c}\left[(\tau_1+\tau_2)+\tau_1\tau_2 s+\frac{1}{s}\right]$$

Thus, the PID controller tuning constants are

$$K_c=\frac{(\tau_1+\tau_2)}{K_p\tau_c}$$

$$\tau_I=\tau_1+\tau_2$$

$$\tau_D=\frac{\tau_1\tau_2}{\tau_1+\tau_2}$$

(See Eq. 12-14 for verification)

b) For $\tau_1 = 3$ and $\tau_2 = 5$ and $\tau_c = 1.5$, we have

$K_c = 5.333$ $\tau_I = 8.0$ and $\tau_D = 1.875$

Using this PID controller, the closed-loop response will be first order when the process model is known accurately. The closed-loop response to a unit step-change in the set-point when the model is known exactly is shown above. It is assumed that τ_c was chosen such that the closed loop response is reasonable, and the manipulated variable does not violate any bounds that are imposed. An approximate derivative action is used by Simulink-MATLAB, namely $\frac{\tau_D s}{1+\beta s}$ when β=0.01

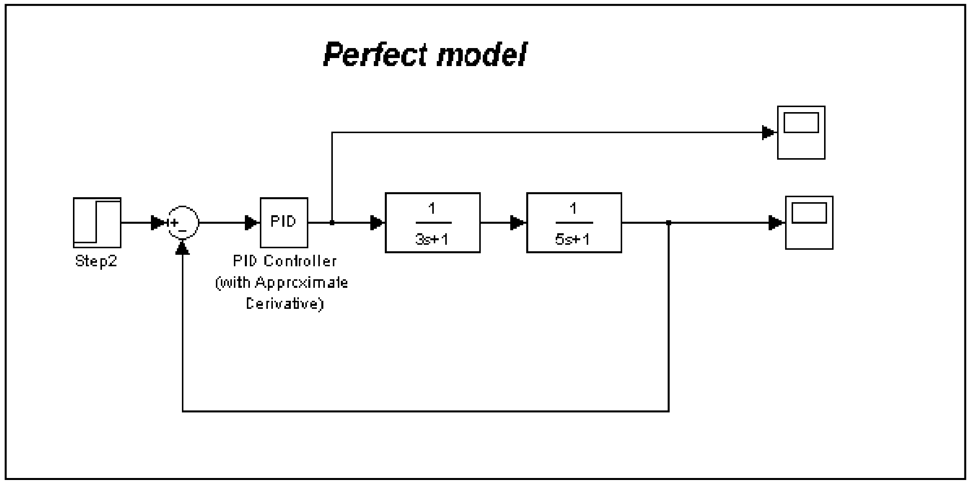


Figure S16.19a. *Simulink block diagram.*

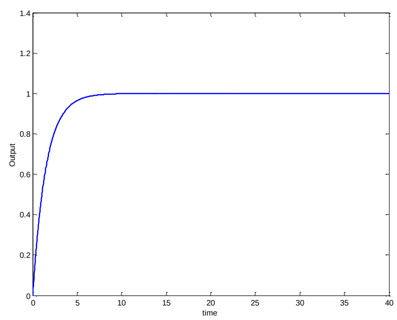


Figure S16.19b. *Output (no model error)*

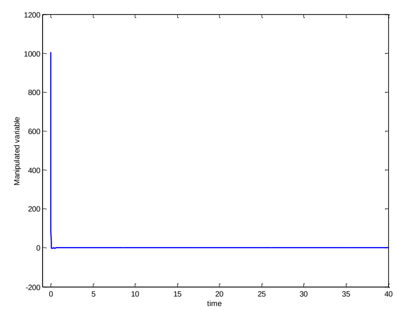


Figure S16.19c. *Manipulated variable (no model error)*

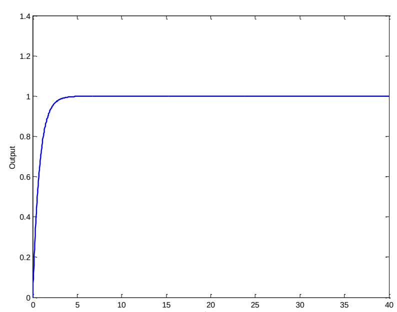


Figure S16.19d. *Output (K_p = 2)*

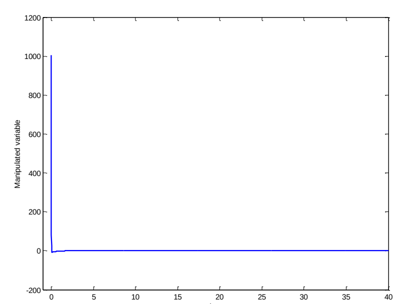


Figure S16.19e. *Manipulated variable (K_p = 2)*

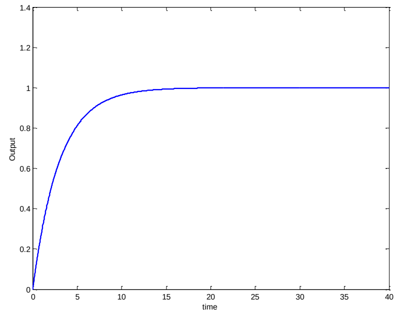


Figure S16.19f. *Output (K_p = 0.5)*

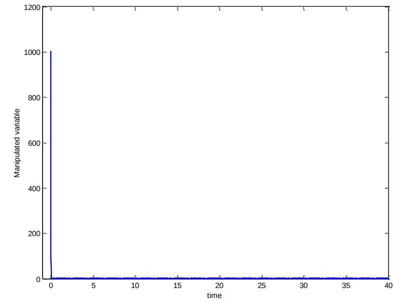


Figure S16.19g. *Manipulated variable (K_p =0.5)*

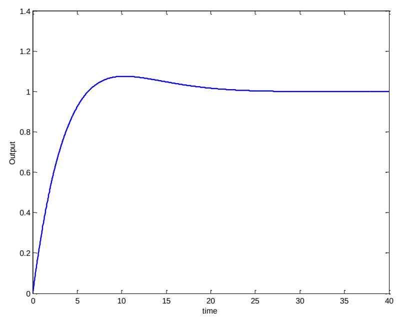


Figure S16.19h. *Output (τ_2 = 10)*

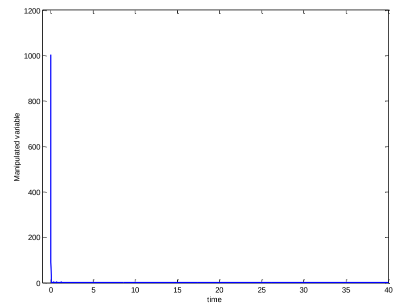


Figure S16.19i. *Manipulated variable (τ_2 = 10)*

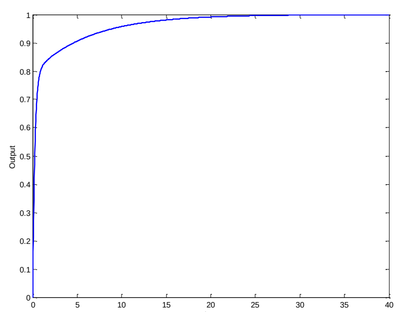


Figure S16.9 j. *Output (τ_2 = 1)*

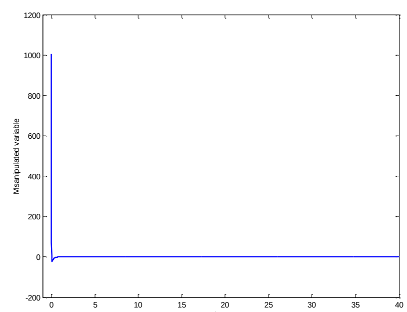


Figure S16.9 k. *Manipulated variable (τ_2 = 1)*

(1) The closed-loop response when the actual K_p is 2.0 is shown above. The controlled variable reaches its set-point much faster than for the base case (exact model), but the manipulated variable assumes values that are more negative (for some period of time) than the base case. This may violate some bounds.

(2) When $K_p = 0.5$, the response is much slower. In fact, the closed-loop time constant seems to be about 3.0 instead of 1.5. There do not seem to be any problems with the manipulated variable.

(3) If ($\tau_2 = 10$), the closed-loop response is no longer first-order. The settling time is much longer than for the base case. The manipulated variable does not seem to violate any bounds.

(4) Both the drawbacks seen above are observed when $\tau_2 = 1$. The settling time is much longer than for the base case. Also the rapid initial increase in the controlled variable means that the manipulated variable drops off sharply, and is in danger of violating a lower bound.

16.20

Based on discussions in Chapter 12, increasing the gain of a controller makes it more oscillatory, increasing the overshoot (peak error) as well as the decay ratio. Therefore, if the quarter-decay ratio is a goal for the closed-loop response (e.g., Ziegler-Nichols tuning), then the rule proposed by Appelpolscher should be satisfactory from a qualitative point of view. However, if the controller gain is increased, the settling time is also decreased, as is the period of oscillation. Integral action influences the response characteristics as well. In general, a decrease in τ_I gives comparable results to an increase in K_c. So, K_c can be used to influence the peak error or decay ratio, while τ_I can be used to speed up the settling time (a decrease in τ_I decreases the settling time). See Chapter 8 for typical response for varying K_c and τ_I.

16.21

SELECTIVE CONTROL

Selectors are quite often used in forced draft combustion control system to prevent an imbalance between air flow and fuel flow, which could result in unsafe operating conditions.

For this case, a flow controller adjusts the air flowrate in the heater. Its set-point is determined by the High Selector, which chooses the higher of the two input signals:

.- Signal from the fuel gas flowrate transmitter (when this is too high)

.- Signal from the outlet temperature control system.

Similarly, if the air flow rate is too low, its measurement is selected by the low selector as the set-point for the fuel-flow rate.

CASCADE CONTROLLER

The outlet temperature control system can be considered the master controller that adjusts the set-point of the fuel/air control system (slave controller). If a disturbance in fuel or air flow rate exists, the slave control system will act very quickly to hold them at their set-points.

FEED-FORWARD CONTROL

The feedforward control scheme in the heater provides better control of the heater outlet temperature. The feed flowrate and temperature are measured and sent to the feedback control system in the outflow. Hence corrective action is taken before they upset the process. The outputs of the feedforward and feedback controller are added together and the combined signal is sent to the fuel/air control system.

16.22

ALTERNATIVE A.

Since the control valves are "air to close", each K_v is positive (cf. Chapter 9). Consequently, each controller must be reverse acting (K_c>0) for the flow control loop to function properly.

Two alternative control strategies are considered:

Method 1: use a default feed flowrate when P_{cc} > 80%

Let : P_{cc} = output signal from the composition controller (%)
$\tilde{F}_{sp}$ = (internal) set point for the feed flow controller (%)

Control strategy:

$$\text{If } P_{cc} > 80\% \;,\; \tilde{F}_{sp} = \tilde{F}_{sp,low}$$

where $\tilde{F}_{sp,low}$ is a specified default flow rate that is lower than the normal value, $\tilde{F}_{sp\;nom}$.

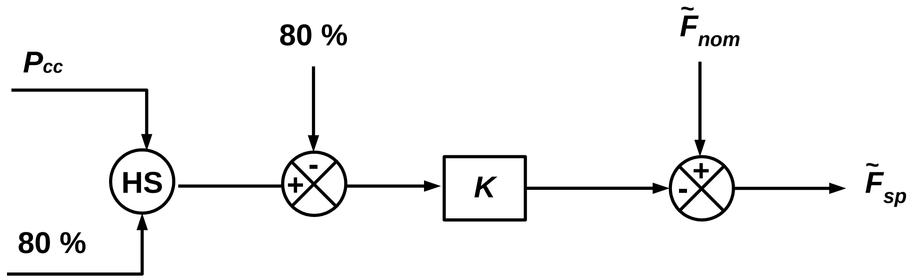
<u>Method 2</u>: <u>Reduce the feed flow when P_{cc} > 80%</u>

Control strategy:

$$\text{If } P_{cc} < 80\%, \;\; \tilde{F}_{sp} = \tilde{F}_{sp\;nom} - K(P_{cc} - 80\%)$$

where K is a tuning parameter ($K > 0$)

Implementation:



Note: A check should be made to ensure that $0 \le \tilde{F}_{sp} \le 100\%$

ALTERNATIVE B.-

A selective control system is proposed:

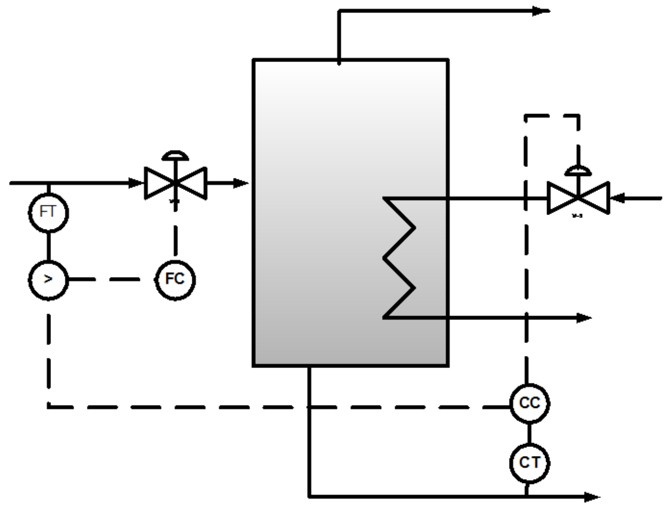


Figure S16.22. *Proposed selective control system*

Both control valves are A-O and transmitters are "direct acting", so the controller have to be "reverse acting".

When the output concentration decreases, the controller output increases. Hence this signal cannot be sent directly to the feed valve (it would open the valve). Using a high selector that chooses the higher of these signals can solve the problem

.- Flow transmitter
.- Output concentration controller

Therefore when the signal from the output controller exceeds 80%, the selector holds it and sends it to the flow controller, so that feed flow rate is reduced.

16.23

ALTERNATIVE A.-
Time delay.- Use time delay compensation, e.g., Smith Predictor

Variable waste concentration.- Tank pH changes occurs due to this unpredictable changes. Process gain changes also (c,f. literature curve for strong acid-strong base)

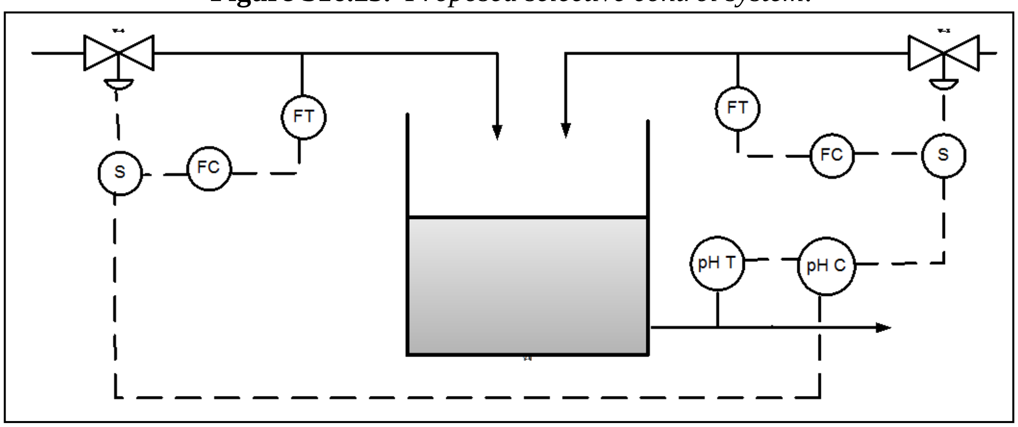
Variable waste flow rate.- Use FF control or ratio q_{base} to q_{waste}.

Measure q_{base} .- This suggests you may want to use cascade control to compensate for upstream pressure changes, etc

ALTERNATIVE B.-

Several advanced control strategies could provide improved process control. A selective control system is commonly used to control pH in wastewater treatment .The proposed system is shown below (pH T = pH sensor; pH C = pH controller)

Figure S16.23. *Proposed selective control system*.



where S represents a selector (< or >, to be determined)

In this scheme, several manipulated variables are used to control a single process variable. When the pH is too high or too low, a signal is sent to the selectors in either the waste stream or the base stream flowrate controllers. The exactly configuration of the system depends on the transmitter, controller and valve gains.

In addition, a Smith Predictor for the pH controller is proposed due to the large time delay. There would be other possibilities for this process such as an adaptive control system or a cascade control system. However the scheme above may be good enough

Necessary information:

.- Descriptions of measurement devices, valves and controllers; direct action or reverse action.

.- Model of the process in order to implement the Smith Predictor

16.24

For setpoint change, the closed-loop transfer function with an integral controller and steady state process ($G_p = K_p$) is:

$$Y/Y_{sp} = \frac{G_C G_P}{1+G_C G_P} = \frac{1/\tau_I s \, K_P}{1+ 1/\tau_I s \, K_P} = \frac{K_P}{\tau_I s + K_P} = \frac{1}{\tau_I/K_P \, s+1}$$

Hence a first order response is obtained and satisfactory control can be achieved.

For disturbance change ($G_d = G_p$):

$$Y/D = \frac{G_d}{1+G_C G_P} = \frac{K_P}{1+ 1/\tau_I s \, K_P} = \frac{K_P(\tau_I s)}{\tau_I s + K_P} = \frac{\tau_I s}{\tau_I/K_P \, s+1}$$

Therefore a first order response is also obtained for disturbance change.

16.25

MV: insulin pump flow rate
CV: body sugar level
DV: food intake (sugar or glucose)

The standard PID control algorithm could be used to provide a basic control level. However, it may be subject to saturation in order to keep the blood glucose within the stated bounds. Feedforward control could be used if the effect of the meal intake (disturbance) can be quantified according to its glucose level. Then the insulin injection can anticipate the effect of the meal by taking preventative actions before the change in blood glucose is sensed. A pitfall of a FF/FB control could be that high insulin pump flow rates may be required in order to keep the blood glucose within the desired range, and the pump flow rate may saturate. Another enhancement would be adaptive control, which would allow the controller to be automatically tuned for a given human in order to obtain a better response (every person's body chemistry is different). A drawback of adaptive control is that it may be too aggressive and cause rapid changes in blood glucose. A less aggressive adaptive controller could employ gain scheduling, where a higher controller gain is used when the blood glucose level goes too high or too low.

16.26

In the event that the feed temperature is too high, the slave controller will sense the increase in temperature and increase the signal to the coolant valve, which will increase the flow of coolant to reduce the temperature of the feed. The master controller will sense a slight increase in temperature in the reactor and will increase the set point of the slave controller, which will in turn increase the flow rate of the coolant a second time. In this case, **both the slave and the master controller work together** to counteract the disturbance. As a result, the disturbance is dealt with quickly and the reactor temperature is only affected slightly.

In the event that the feed flow rate is too high, the temperature of the feed exiting the heat exchanger will increase. The slave controller will sense this and will act as above by increasing the coolant flow rate. The increased flow rate of higher temperature feed in the reactor will most likely increase the reactor temperature, and the master controller will alter the set point of the slave controller accordingly. Again **the master controller and slave controller work together** to counteract the disturbance.

16.27

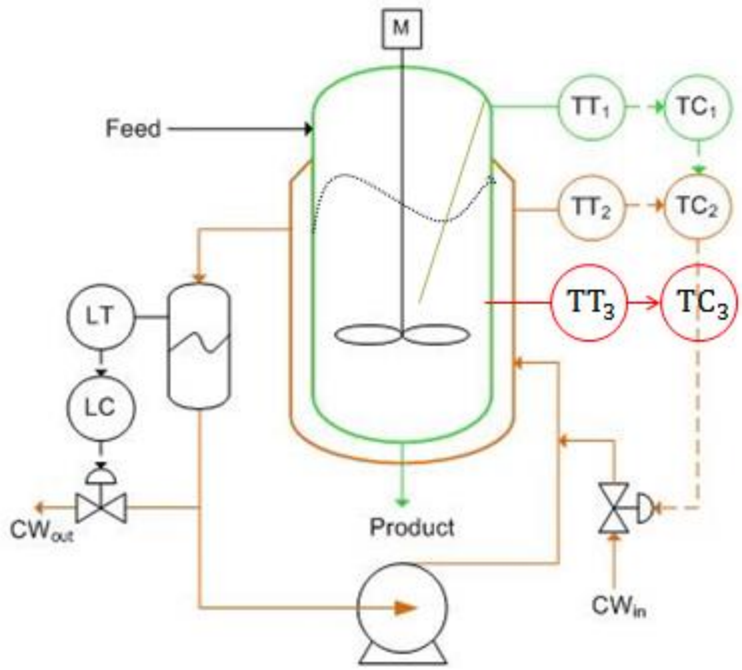


Figure S16.7a *Cascade control of an exothermic chemical reactor*

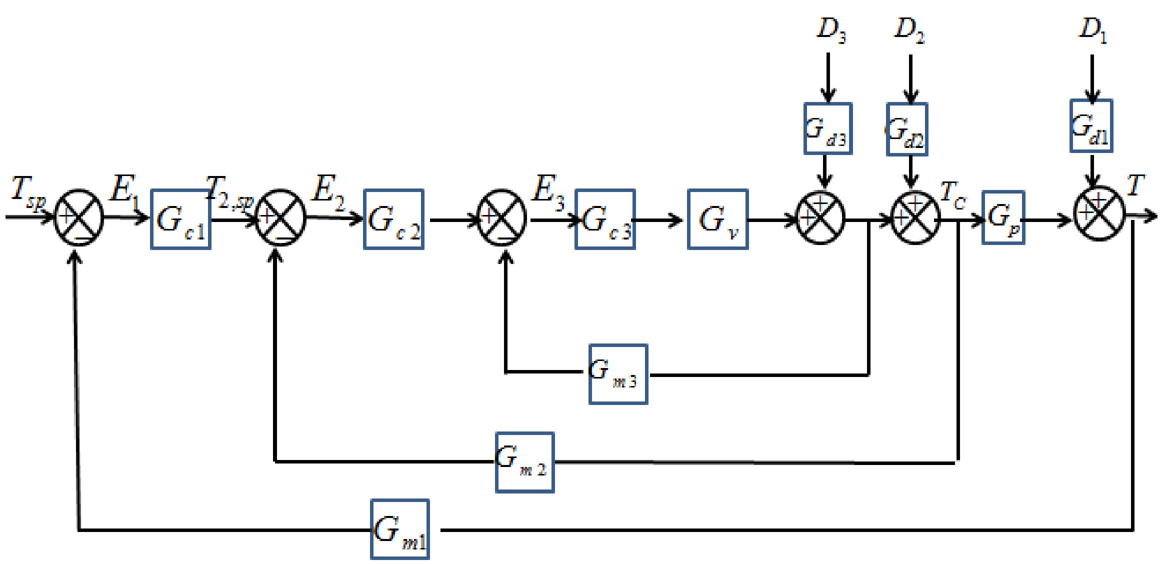


Figure S16.7b *Block diagram of Cascade control of an exothermic chemical reactor*

D_1 : Reactor temperature

D_2 : Cooling water

D_3 : Temperature of the reactor wall

The control system measures the temperature of the reactor wall to gather information on the temperature gradients in the tank contents, compares to a set point, and adjusts the cooling water makeup. The principal advantage of the new cascade control strategy is that the reactor wall temperature is located close to a potential disturbance of temperature gradients in the tank contents and its associated feedback loop can react quickly, thus improving the closed-loop response.

16.28

For a one-input-two-output linear algebraic model shown in Eqs. (1)~(2):

$$y_1 = K_{12}u_1 + b_1 \qquad (1)$$

$$y_2 = K_{21}u_2 + b_2 \qquad (2)$$

The output y_1 can reach the set-point y_1^{sp} by tuning u_1 based on Eq. (3):

$$u_1 = \frac{y_1^{sp} - b_1}{K_{12}} \quad (3)$$

But for output y_2, it is determined by combining Eqs. (2) and (3), and cannot be specified arbitrarily leading to offset.

Chapter 17

17.1

Using Eq. 17-9, the filtered values of x_D are shown in Table S17.1

time(min)	$\alpha = 1$	$\alpha = 0.8$	$\alpha = 0.5$
0	0	0	0
1	0.495	0.396	0.248
2	0.815	0.731	0.531
3	1.374	1.245	0.953
4	0.681	0.794	0.817
5	1.889	1.670	1.353
6	2.078	1.996	1.715
7	2.668	2.534	2.192
8	2.533	2.533	2.362
9	2.908	2.833	2.635
10	3.351	3.247	2.993
11	3.336	3.318	3.165
12	3.564	3.515	3.364
13	3.419	3.438	3.392
14	3.917	3.821	3.654
15	3.884	3.871	3.769
16	3.871	3.871	3.820
17	3.924	3.913	3.872
18	4.300	4.223	4.086
19	4.252	4.246	4.169
20	4.409	4.376	4.289

Table S17.1. *Unfiltered and filtered data.*

To obtain the analytical solution for x_D, set $F(s) = \frac{1}{s}$ in the given transfer function, so that

$$X_D(s) = \frac{5}{10s+1}F(s) = \frac{5}{s(10s+1)} = 5\left(\frac{1}{s} - \frac{1}{s+1/10}\right)$$

Taking inverse Laplace transform

$$x_D(t) = 5\,(1 - e^{-t/10})$$

A graphical comparison is shown in Fig. S17.1

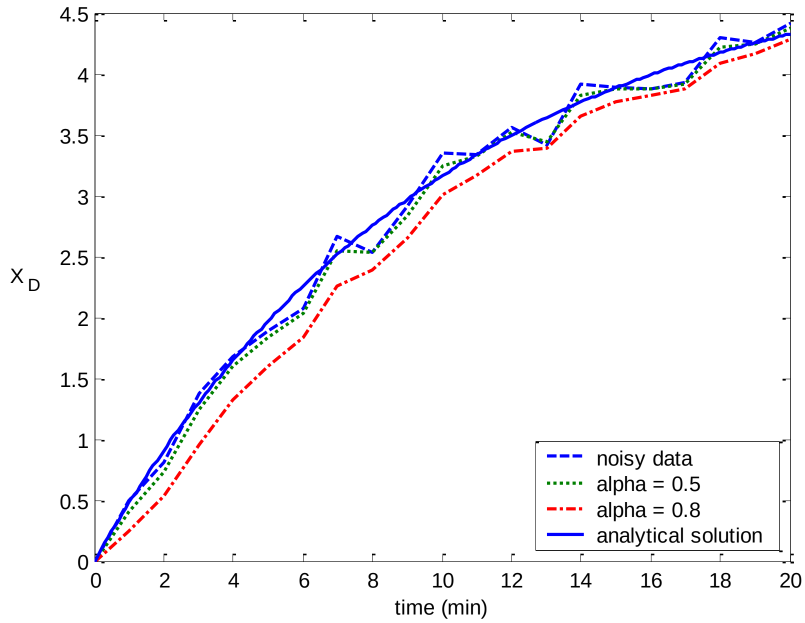


Fig S17.1. *Graphical comparison for noisy data, filtered data and analytical solution.*

As α decreases, the filtered data give a smoother curve compared to the no-filter (α=1) case, but this noise reduction is traded off with an increase in the deviation of the curve from the analytical solution.

17.2

The exponential filter output in Eq. 17-9 is

$$y_F(k) = \alpha y_m(k) + (1-\alpha) y_F(k-1) \tag{1}$$

Replacing *k* by *k*-1 in Eq. 1 gives

$$y_F(k-1) = \alpha y_m(k-1) + (1-\alpha) y_F(k-2) \tag{2}$$

Substituting for $y_F(k-1)$ from (2) into (1) gives

$$y_F(k) = \alpha y_m(k) + (1-\alpha)\alpha y_m(k-1) + (1-\alpha)^2 y_F(k-2)$$

Successive substitution of $y_F(k-2)$, $y_F(k-3)$,… gives the final form

$$y_F(k) = \sum_{i=0}^{k-1} (1-\alpha)^i \alpha y_m(k-i) + (1-\alpha)^k y_F(0)$$

17.3

Table S17.3 lists the unfiltered output and, from Eq. 17-9, the filtered data for sampling periods of 1.0 and 0.1. Notice that for sampling period of 0.1, the unfiltered and filtered outputs were obtained at 0.1 time increments, but they are reported only at intervals of 1.0 to preserve conciseness and facilitate comparison.

The results show that for each value of Δt, the data become smoother as α decreases, but at the expense of lagging behind the mean output $y(t)=t$. Moreover, lower sampling period improves filtering by giving smoother data and less lagg for the same value of α.

		Δ*t*=1			Δ*t*=0.1		
t	**α=1**	**α=0.8**	**α=0.5**	**α=0.2**	**α=0.8**	**α=0.5**	**α=0.2**
0	0	0	0	0	0	0	0
1	1.421	1.137	0.710	0.284	1.381	1.261	0.877
2	1.622	1.525	1.166	0.552	1.636	1.678	1.647
3	3.206	2.870	2.186	1.083	3.227	3.200	2.779
4	3.856	3.659	3.021	1.637	3.916	3.973	3.684
5	4.934	4.679	3.977	2.297	4.836	4.716	4.503
6	5.504	5.339	4.741	2.938	5.574	5.688	5.544
7	6.523	6.286	5.632	3.655	6.571	6.664	6.523
8	8.460	8.025	7.046	4.616	8.297	8.044	7.637
9	8.685	8.553	7.866	5.430	8.688	8.717	8.533
10	9.747	9.508	8.806	6.293	9.741	9.749	9.544
11	11.499	11.101	10.153	7.334	11.328	11.078	10.658
12	11.754	11.624	10.954	8.218	11.770	11.778	11.556
13	12.699	12.484	11.826	9.115	12.747	12.773	12.555
14	14.470	14.073	13.148	10.186	14.284	14.051	13.649
15	14.535	14.442	13.841	11.055	14.662	14.742	14.547
16	15.500	15.289	14.671	11.944	15.642	15.773	15.544
17	16.987	16.647	15.829	12.953	16.980	16.910	16.605
18	17.798	17.568	16.813	13.922	17.816	17.808	17.567
19	19.140	18.825	17.977	14.965	19.036	18.912	18.600
20	19.575	19.425	18.776	15.887	19.655	19.726	19.540

Table S17.3. *Unfiltered and filtered output for sampling periods of 1.0 and 0.1*

Graphical comparison:

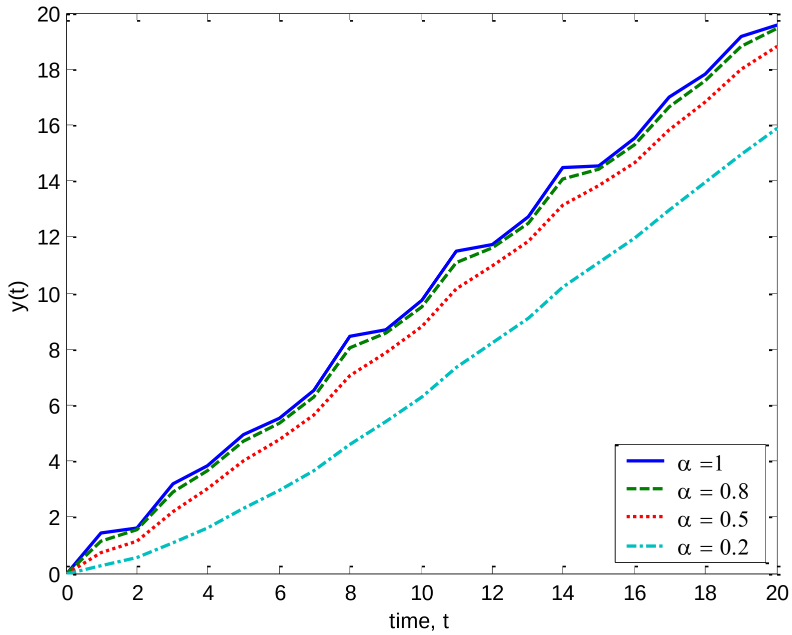


Figure S17.3a. *Graphical comparison for Δt = 1.0*

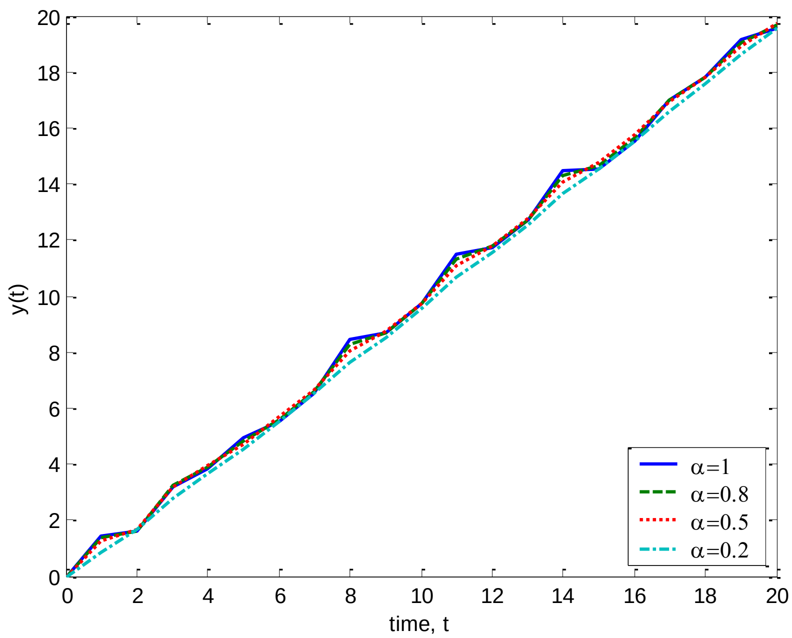


Figure S17.3b. *Graphical comparison for Δt = 0.1*

17.4

Using Eq. 17-9 for α = 0.2 and α = 0.5, Eq. 17-18 for N^* = 4, and Eq. 17-19 for Δy=0.5, the results are tabulated and plotted below.

t	α=1	(a) α=0.2	(a) α=0.5	(b) N*=4	(c) Δy=0.5
0	0	0	0	0	0
1	1.50	0.30	0.75	0.38	0.50
2	0.30	0.30	0.53	0.45	0.30
3	1.60	0.56	1.06	0.85	0.80
4	0.40	0.53	0.73	0.95	0.40
5	1.70	0.76	1.22	1.00	0.90
6	1.50	0.91	1.36	1.30	1.40
7	2.00	1.13	1.68	1.40	1.90
8	1.50	1.20	1.59	1.68	1.50

Table S17.4. *Unfiltered and filtered data.*

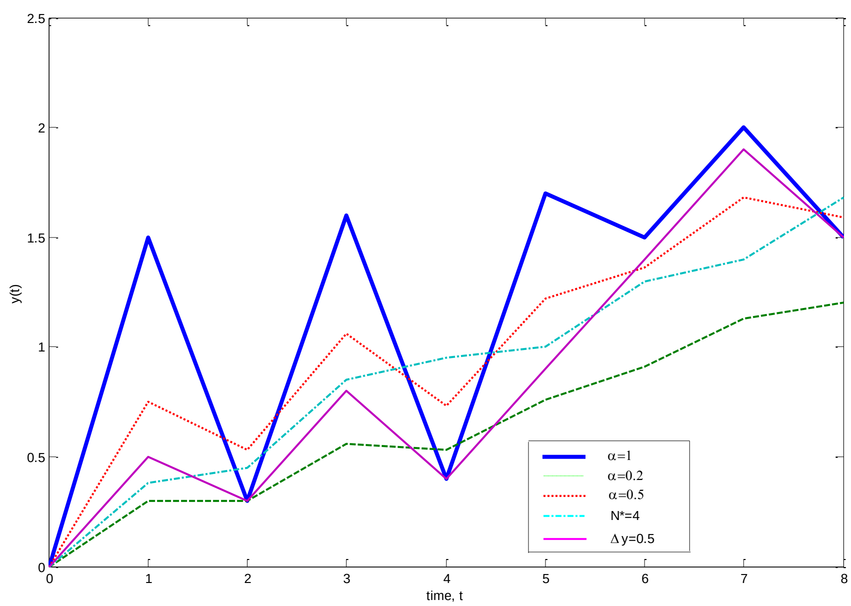


Figure S17.4. *Graphical comparison for filtered data and the raw data.*

17.5

Parameter setting:

$G_p = \frac{1}{2s+1}$; $d(t) = 1 + 0.2\sin(t)$; $\tau_F = 0$ (no filtering) or 3

To do this problem, build the Simulink diagrams below. Note that the filter is represented by a first order transfer function with time constant of τ_F minutes. This can be shown by performing the Laplace transform of equation 17.4 in the book.

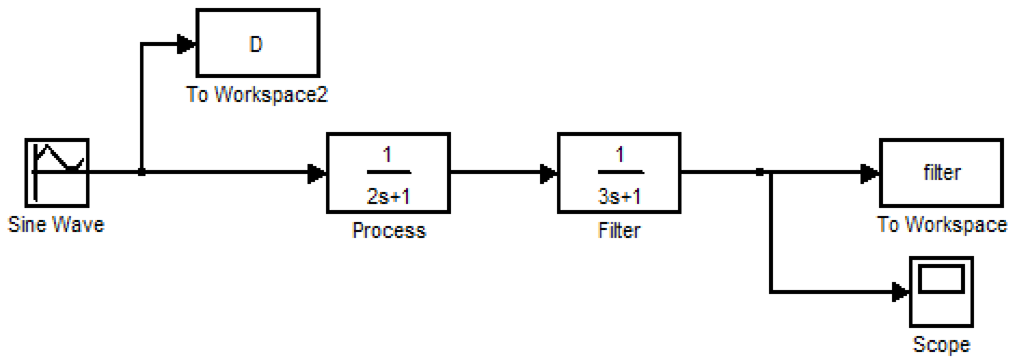


Figure S17.5a. *Block diagram when a filter is used on the output with time constant of 3 minutes. A sine wave of frequency 1 and amplitude 0.2 is the input.*

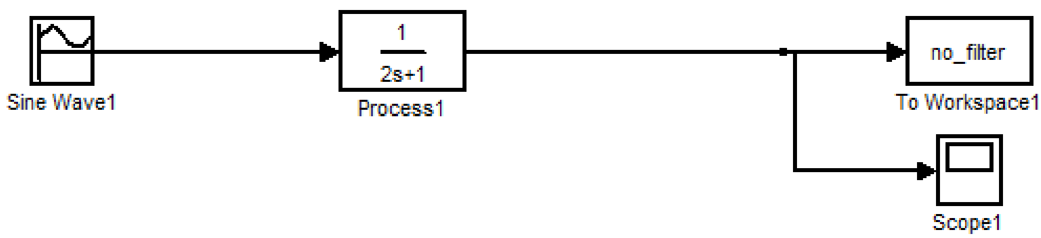


Figure S17.5b. *Block diagram when no filter is used on the output. A sine wave of frequency 1 and amplitude 0.2 is the input.*

Simulating the diagram for 50 mins:

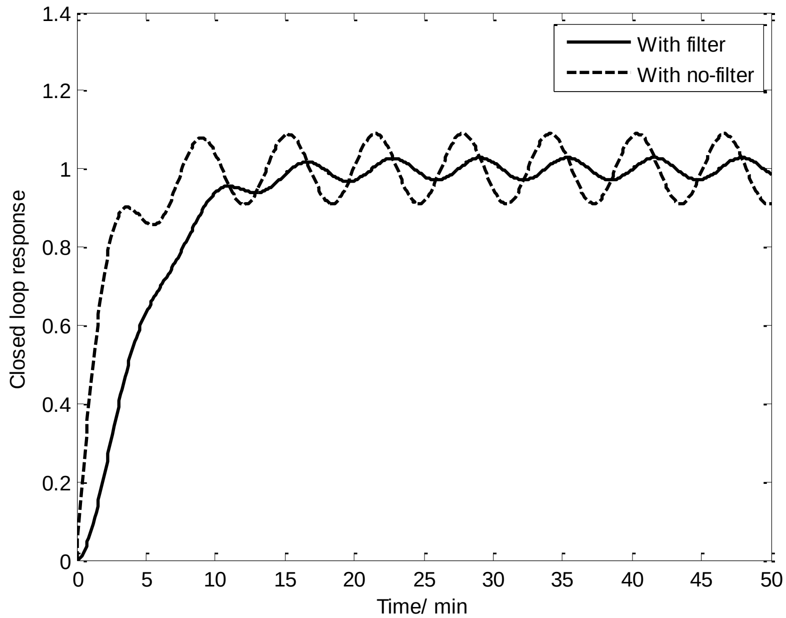


Figure S17.5c. *First-order process response to a disturbance, d(t)=1+0.2sin(t), with and without an exponential filter.*

From Figure S17.5c, we can see that the filter will significantly dampen the oscillation at the cost of inducing a time lag in the first 10 minutes.

17.6

$$Y(s) = \frac{1}{s+1}X(s) = \frac{1}{s+1}\frac{1}{s} \quad , \qquad \text{then} \quad y(t) = 1 - e^{-t}$$

For noise level of ± 0.05 units, several different values of α are tried in Eq. 17-9 as shown in Fig. S17.6a. While the filtered output for α = 0.7 is still quite noisy, that for α = 0.3 is too sluggish. Thus α = 0.4 seems to offer a good compromise between noise reduction and lag addition. Therefore, the designed first-order filter for noise level ± 0.05 units is α = 0.4, which corresponds to τ_F = 1.5 according to Eq. 17-8a.

<u>Noise level</u> = ± 0.05

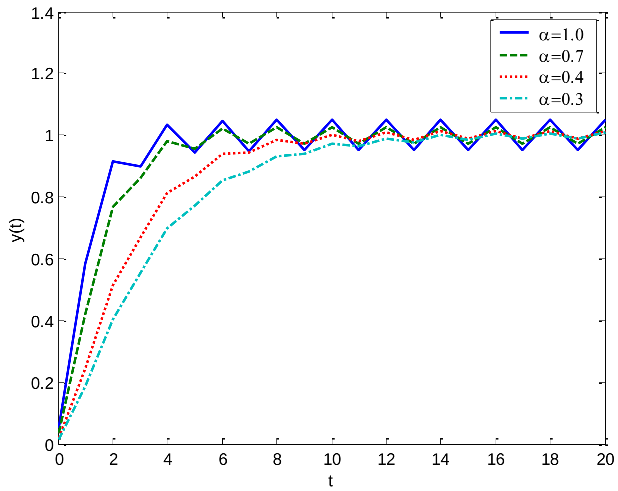


Figure S17.6a. *Digital filters for noise level = ±0.05*

Noise level = ± 0.1

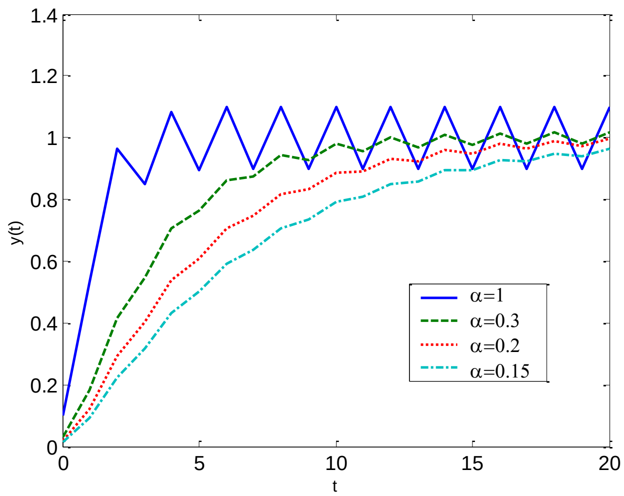


Figure S17.6b. *Digital filters for noise level = ±0.1*

Noise level = ± 0.01

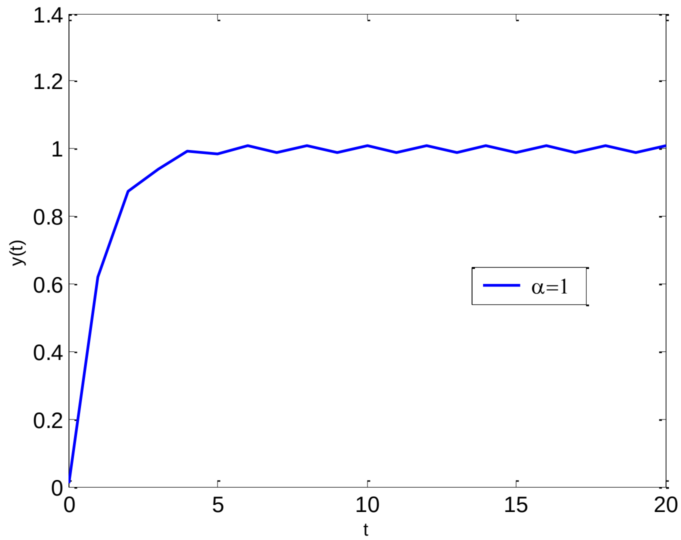


Figure S17.6c. *Response for noise level = ± 0.01; no filter needed.*

Similarly, for noise level of ± 0.1 units, a good compromise is α =0.2 or τ_F = 4.0 as shown in Fig. S17.6b. However, for noise level of ±0.01 units, no filter is necessary as shown in Fig. S17.6c. thus α=1.0, τ_F = 0

17.7

$$y(k) = y(k\text{-}1) - 0.21\, y(k\text{-}2) + u(k\text{-}2)$$

k	*u*(*k*)	*u*(*k*-1)	*u*(*k*-2)	*y*(*k*)
0	1	0	0	0
1	0	1	0	0
2	0	0	1	1.00
3	0	0	0	1.00
4	0	0	0	0.79
5	0	0	0	0.58
6	0	0	0	0.41
7	0	0	0	0.29
8	0	0	0	0.21
9	0	0	0	0.14
10	0	0	0	0.10
11	0	0	0	0.07
12	0	0	0	0.05
13	0	0	0	0.03
14	0	0	0	0.02
15	0	0	0	0.02
16	0	0	0	0.01
17	0	0	0	0.01
18	0	0	0	0.01
19	0	0	0	0.00

Plotting this results

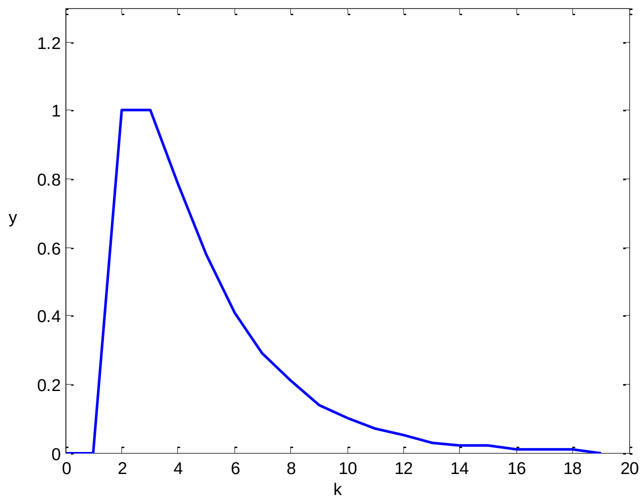


Figure S17.7. *Graphical simulation of the difference equation*

The steady state value of *y* is zero.

17.8

a) By using Simulink and STEM routine to convert the continuous signal to a series of pulses,

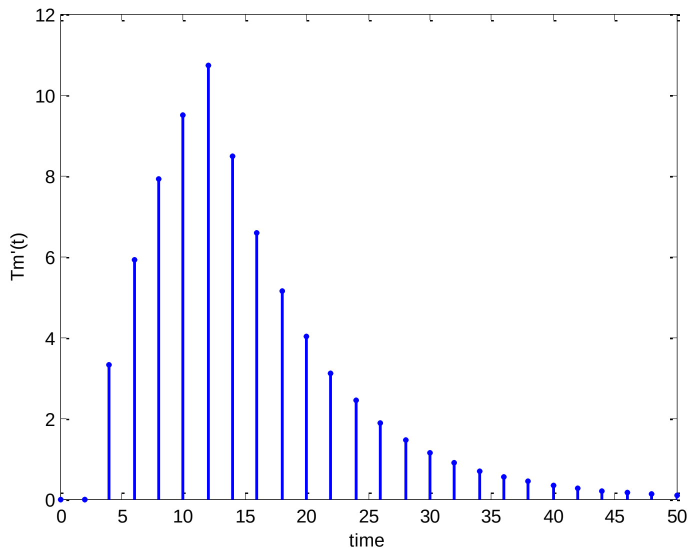


Figure S17.8. *Discrete time response for the temperature change.*

Hence the maximum value of the logged temperature is 80.7° C.
This maximum point is reached at t = 12 min.

17.9

a)

$$\frac{Y(z)}{U(z)} = \frac{2.7z^{-1}(z+3)}{z^2 - 0.5z + 0.06} = \frac{2.7 + 8.1z^{-1}}{z^2 - 0.5z + 0.06}$$

Dividing both numerator and denominator by z^2

$$\frac{Y(z)}{U(z)} = \frac{2.7z^{-2} + 8.1z^{-3}}{1 - 0.5z^{-1} + 0.06z^{-2}}$$

Then $Y(z)(1 - 0.5z^{-1} + 0.06z^{-2}) = U(z)(2.7z^{-2} + 8.1z^{-3})$

or $y(k) = 0.5y(k\text{-}1) - 0.06y(k\text{-}2) + 2.7u(k\text{-}2) + 8.1u(k\text{-}3)$

The simulation of the difference equation yields

k	***u*(*k*)**	***u*(*k*-2)**	***u*(*k*-3)**	***y*(*k*)**
0	1	0	0	0
1	1	0	0	0
2	1	1	0	2.70
3	1	1	1	12.15
4	1	1	1	16.71
5	1	1	1	18.43
6	1	1	1	19.01
7	1	1	1	19.20
8	1	1	1	19.26
9	1	1	1	19.28
10	1	1	1	19.28
11	1	1	1	19.28
12	1	1	1	19.29
13	1	1	1	19.29
14	1	1	1	19.29
15	1	1	1	19.29
16	1	1	1	19.29
17	1	1	1	19.29
18	1	1	1	19.29
19	1	1	1	19.29
20	1	1	1	19.29

b)

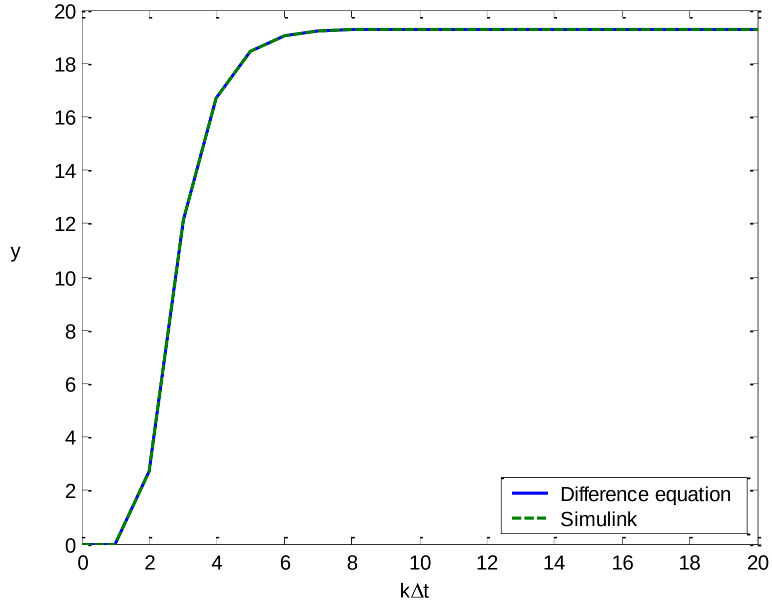


Figure S17.9. *Simulink response to a unit step change in u*

c) The steady state value of y can be found be setting z =1. In doing so,

$$y = 19.29$$

This result is in agreement with data above.

17.10

$$G_c(s) = 2\left(1 + \frac{1}{8s}\right)$$

Substituting $s \cong (1\text{-}z^{-1})/\Delta t$ and accounting for Δt=1

$$G_c(z) = 2\left(1 + \frac{1}{8(1 - z^{-1})}\right) = \frac{2.25 - 2z^{-1}}{(1 - z^{-1})}$$

By using Simulink-MATLAB, the simulation for a unit step change in the controller error signal $e(t)$ is shown in Fig. S17.10

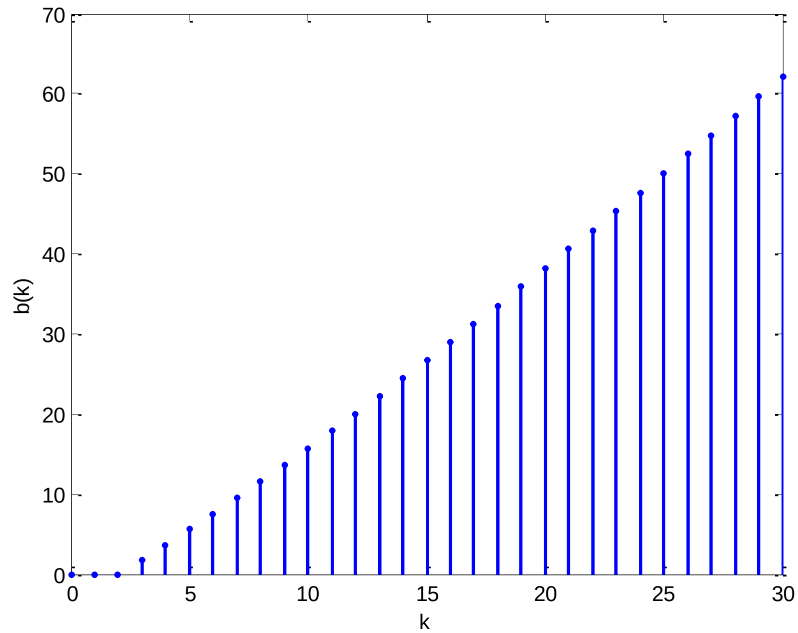


Figure S17.10. *Open-loop response for a unit step change*

17.11

a) $$\frac{Y(z)}{U(z)} = \frac{5(z+0.6)}{z^2 - z + 0.41}$$

Dividing both numerator and denominator by z^2

$$\frac{Y(z)}{U(z)} = \frac{5z^{-1} + 3z^{-2}}{1 - z^{-1} + 0.41z^{-2}}$$

Then $Y(z)(1 - z^{-1} + 0.41z^{-2}) = U(z)(5z^{-1} + 3z^{-2})$

or $y(k) = y(k\text{-}1) - 0.41y(k\text{-}2) + 5u(k\text{-}1) + 3u(k\text{-}2)$

b) The simulation of the difference equation yields

k	***u*(*k*)**	***u*(*k*-1)**	***u*(*k*-2)**	***y*(*k*)**
1	1	1	0	5
2	1	1	1	13.00
3	1	1	1	18.95
4	1	1	1	21.62
5	1	1	1	21.85
6	1	1	1	20.99
7	1	1	1	20.03
8	1	1	1	19.42
9	1	1	1	19.21
10	1	1	1	19.25
11	1	1	1	19.37
12	1	1	1	19.48
13	1	1	1	19.54
14	1	1	1	19.55
15	1	1	1	19.54
16	1	1	1	19.52
17	1	1	1	19.51
18	1	1	1	19.51
19	1	1	1	19.51

c) By using Simulink-MATLAB, the simulation for a unit step change in *u* yields

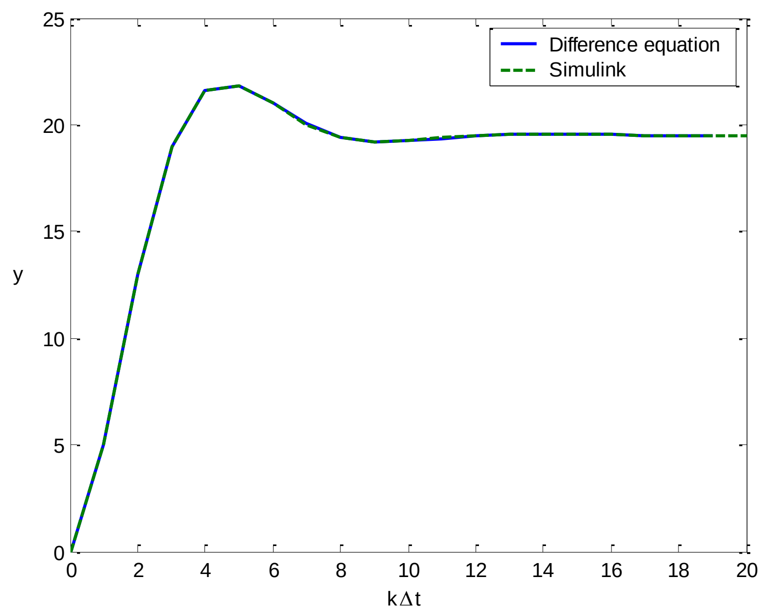


Figure S17.11. *Simulink response to a unit step change in u*

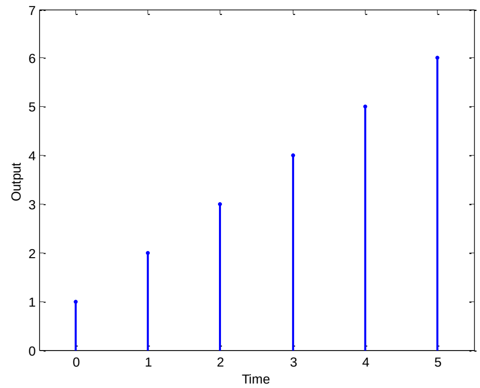
d) The steady state value of *y* can be found be setting *z* =1. In doing so,

$$y = 19.51$$

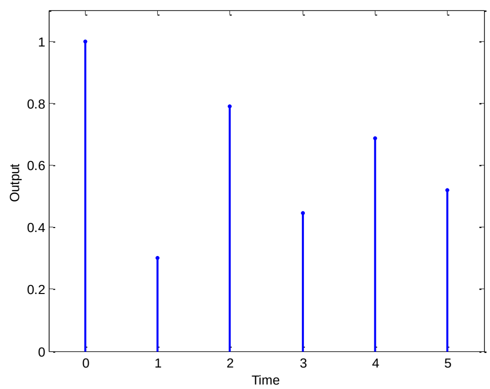
This result is in agreement with data above.

17.12

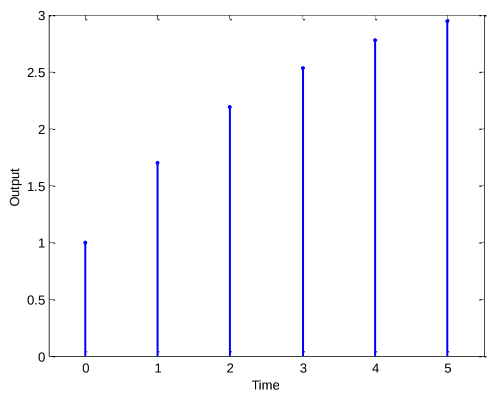
a) $\dfrac{1}{1-z^{-1}}$



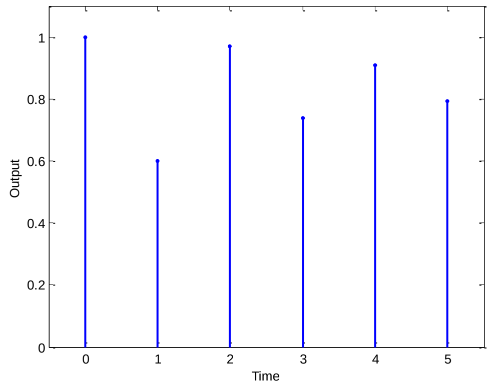
b) $\dfrac{1}{1+0.7z^{-1}}$



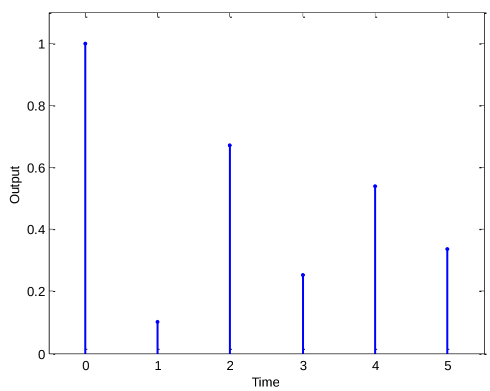
c) $\dfrac{1}{1-0.7z^{-1}}$



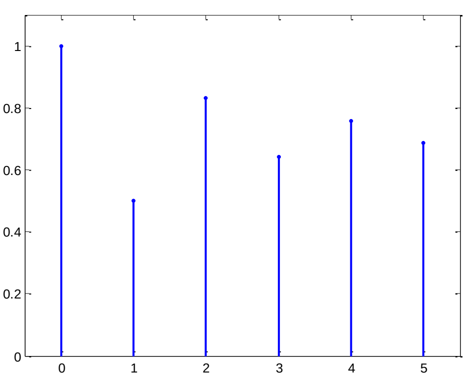
d) $$\frac{1}{(1+0.7z^{-1})(1-0.3z^{-1})}$$



e) $$\frac{1-0.5z^{-1}}{(1+0.7z^{-1})(1-0.3z^{-1})}$$



f) $$\frac{1-0.2z^{-1}}{(1+0.6z^{-1})(1-0.3z^{-1})}$$



Conclusions:

.- A pole at $z = 1$ causes instability.

.- Poles only on positive real axis give oscillation free response.

.- Poles on the negative real axis give oscillatory response.

.- Poles on the positive real axis dampen oscillatory responses.

..- Zeroes on the positive real axis increase oscillations.

.- Zeroes closer to $z = 0$ contribute less to the increase in oscillations.

17.13

By using Simulink, the response to a unit set-point change is shown in Fig. S17.13a

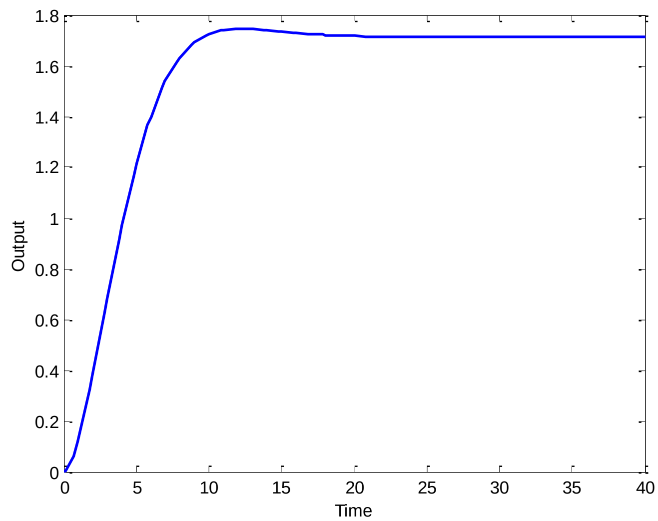


Figure S17.13a. *Closed-loop response to a unit set-point change ($K_c = 1$)*

Therefore the <u>controlled system is stable</u>.

The ultimate controller gain for this process is found by trial and error

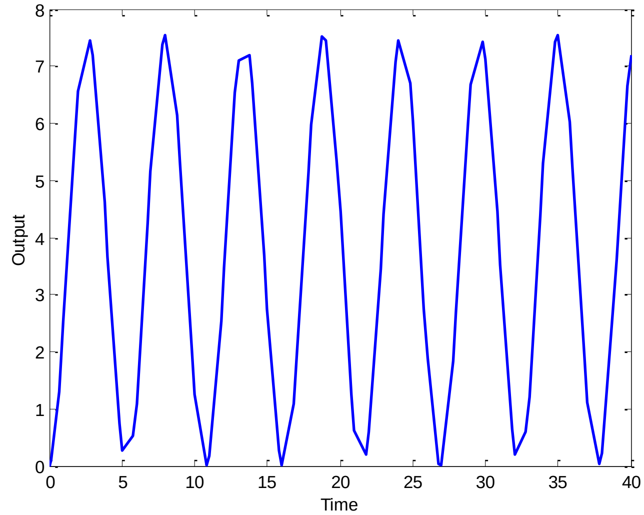


Figure S17.13b. *Closed-loop response to a unit set-point change (K_c =21.3)*

Then $K_{cu} = 21.3$

17.14

By using Simulink-MATLAB, these ultimate gains are found:

Δt = 0.01

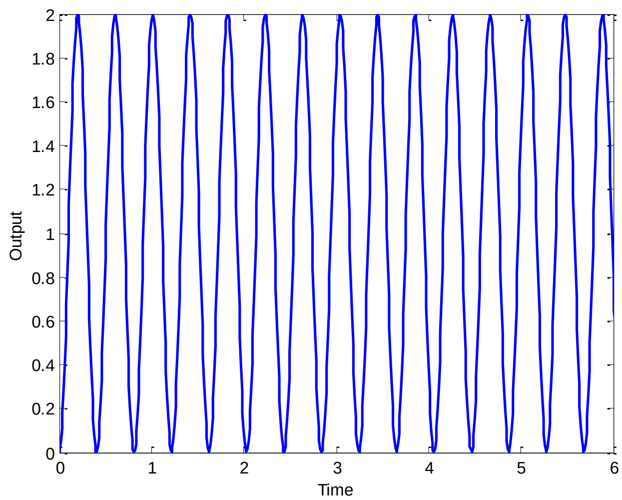


Figure S17.14a. *Closed-loop response to a unit set-point change (K_c =1202)*

Δt = 0.1

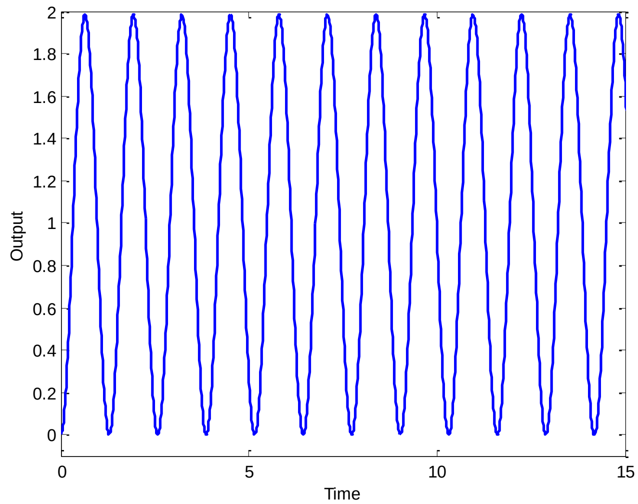


Figure S17.14b. *Closed-loop response to a unit set-point change (K_c =122.5)*

Δt = 0.5

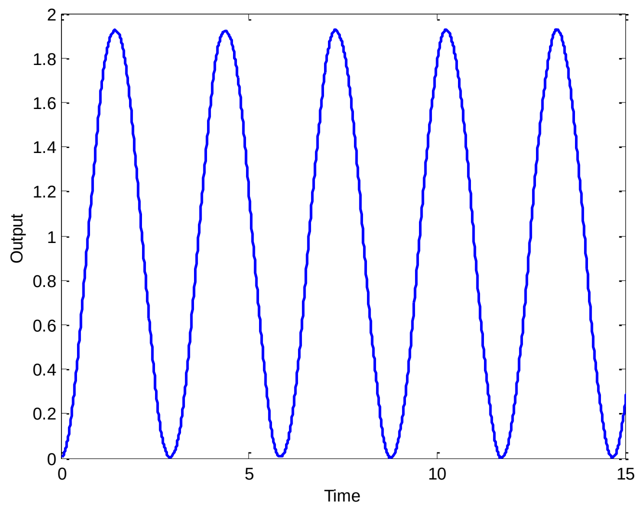


Figure S17.14c. *Closed-loop response to a unit set-point change (K_c =26.7)*

Hence

$\Delta t = 0.01$	$K_{cu} = 1202$
$\Delta t = 0.1$	$K_{cu} = 122.5$
$\Delta t = 0.5$	$K_{cu} = 26.7$

As noted above, decreasing the sampling time makes the allowable controller gain increases. For small values of Δt, the ultimate gain is large enough to guarantee wide stability range.

17.15

By using Simulink-MATLAB

***K*$_c$ = 1**

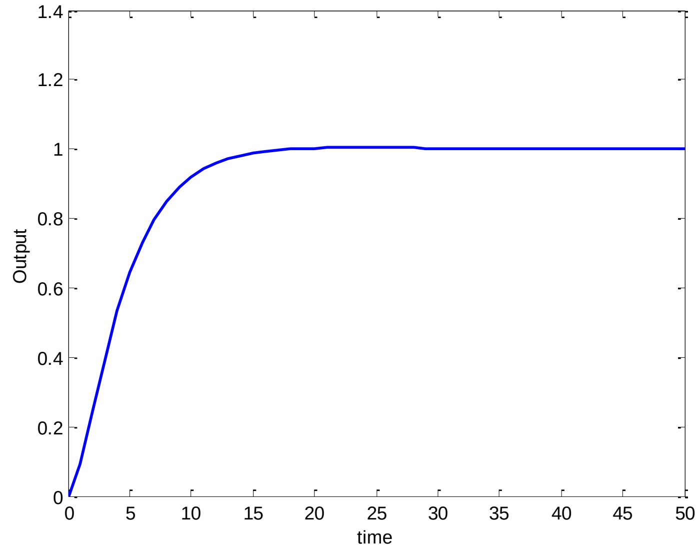


Figure S17.15a. *Closed-loop response to a unit set-point change (K_c =1)*

***K*$_c$ = 10**

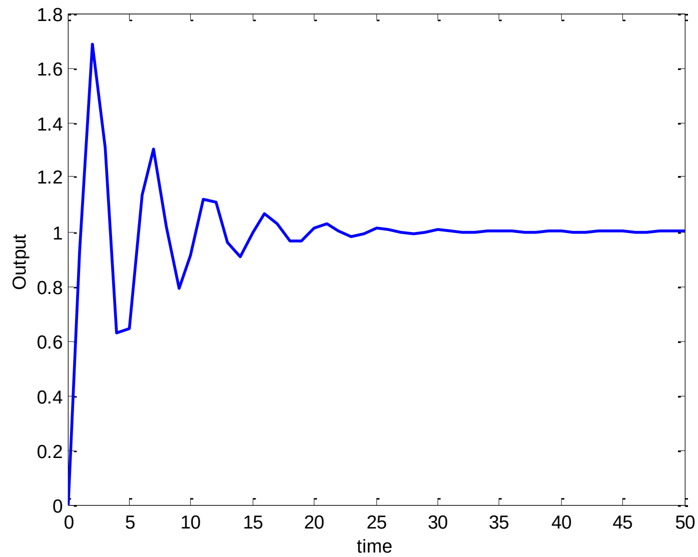


Figure S17.15b. *Closed-loop response to a unit set-point change (K_c =10)*

K_c = 17

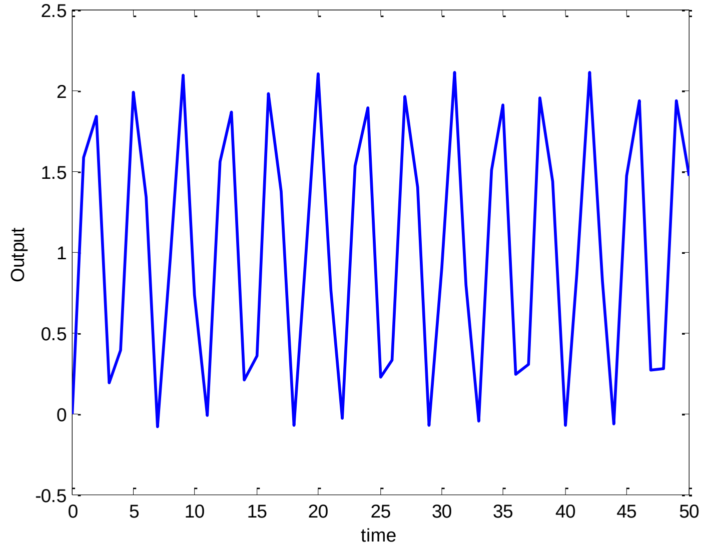


Figure S17.15c. *Closed-loop response to a unit set-point change (K_c =17)*

Thus the maximum controller gain is

$$K_{cm} = 17$$

17.16

$G_v(s) = K_v = 0.1\ \text{ft}^3 / (\text{min})(\text{ma})$

$$G_m(s) = \frac{4}{0.5s+1}$$

In order to obtain $G_p(s)$, write the mass balance for the tank as

$$A\frac{dh}{dt} = q_1 + q_2 - q_3$$

Using deviation variables and taking Laplace transform

$$AsH'(s) = Q_1'(s) + Q_2'(s) - Q_3'(s)$$

Therefore,

$$G_p(s) = \frac{H'(s)}{Q_3'(s)} = \frac{-1}{As} = \frac{-1}{12.6s}$$

By using Simulink-MATLAB,

***K_c* = -10**

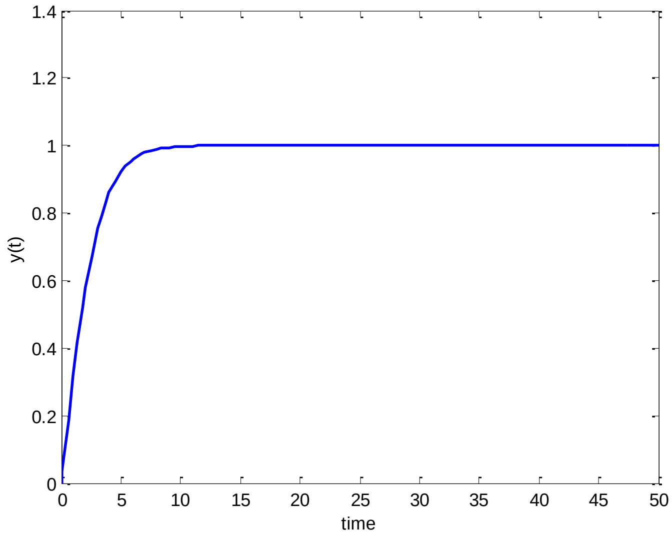


Figure S17.16a. *Closed-loop response to a unit set-point change (K_c = -10)*

***K_c* = -50**

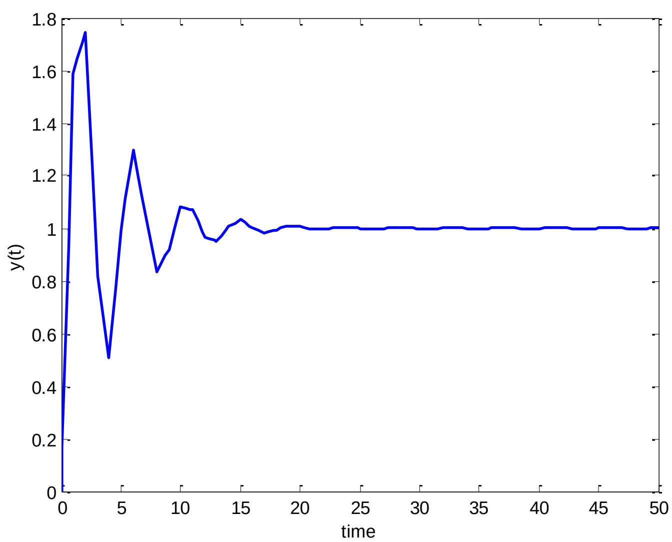


Figure S17.16b. *Closed-loop response to a unit set-point change (K_c = -50)*

K_c = -92

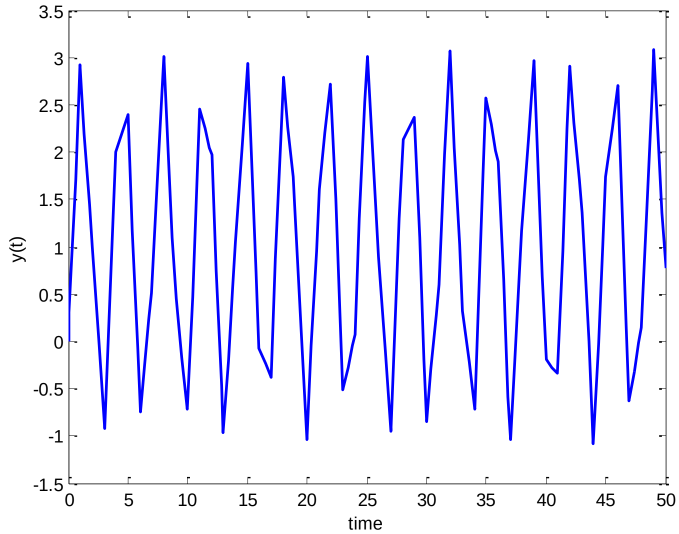


Figure S17.16c. *Closed-loop response to a unit set-point change (K_c = -92)*

Hence the closed loop system is stable for

$$-92 < K_c < 0$$

As noted above, offset occurs after a change in the setpoint.

17.17

a) The closed-loop response for set-point changes is

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{G_c G(s)}{1 + G_c G(s)} \quad \text{then} \quad G_c(z) = \frac{1}{G}\frac{(Y/Y_{sp})}{1-(Y/Y_{sp})}$$

We want the closed-loop system exhibits a first order plus dead time response,

$$(Y/Y_{sp}) = \frac{e^{-hs}}{\lambda s + 1} \quad \text{or} \quad (Y/Y_{sp}) = \frac{(1-A)z^{-N-1}}{1-Az^{-1}} \qquad \text{where } A = e^{-\Delta t/\lambda}$$

Moreover,

$$G(s) = \frac{e^{-2s}}{3s+1} \qquad \text{or} \qquad G(z) = \frac{0.284z^{-3}}{1-0.716z^{-1}}$$

Thus, the resulting digital controller is the Dahlin's controller Eq. 17-66.

$$G_c(z) = \frac{(1-A)}{1-Az^{-1}-(1-A)z^{-N-1}} \frac{1-0.716z^{-1}}{0.284} \qquad (1)$$

If a value of λ=1 is considered, then A = 0.368 and Eq. 1 is

$$G_c(z) = \frac{0.632}{1-0.368z^{-1}-0.632z^{-3}} \frac{1-0.716z^{-1}}{0.284} \qquad (2)$$

b) $(1\text{-}z^{-1})$ is a factor of the denominator in Eq. 2, indicating the presence of integral action. Then no offset occurs.

c) From Eq. 2, the denominator of $G_c(z)$ contains a non-zero z^{-0} term. Hence the controller is physically realizable.

d) First adjust the process time delay for the zero-order hold by adding $\Delta t/2$ to obtain a time delay of 2 + 0.5 = 2.5 min. Then obtain the continuos PID controller tuning based on the ITAE (setpoint) tuning relation in Table 12.3 with K = 1, τ=3, θ = 2.5. Thus

$KK_c = 0.965(2.5/3)^{-0.85}$, $K_c = 1.13$

$\tau/\tau_I = 0.796 + (-0.1465)(2.5/3)$, , $\tau_I = 4.45$

$\tau_D/\tau = 0.308(2.5/3)^{0.929}$, $\tau_D = 0.78$

Using the position form of the PID control law (Eq. 8-26 or 17-55)

$$G_c(z) = 1.13\left[1+0.225\left(\frac{1}{1-z^{-1}}\right)+0.78(1-z^{-1})\right]$$

$$= \frac{2.27-2.89z^{-1}+0.88z^{-2}}{1-z^{-1}}$$

By using Simulink-MATLAB, the controller performance is examined:

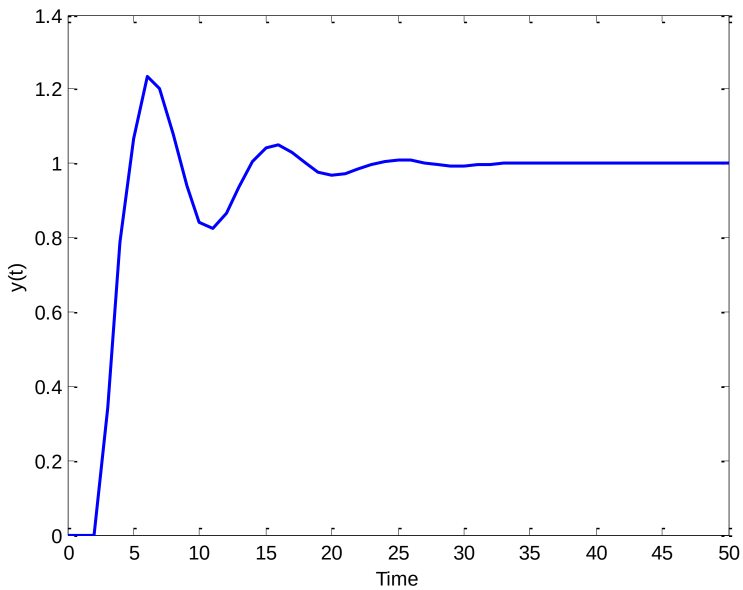
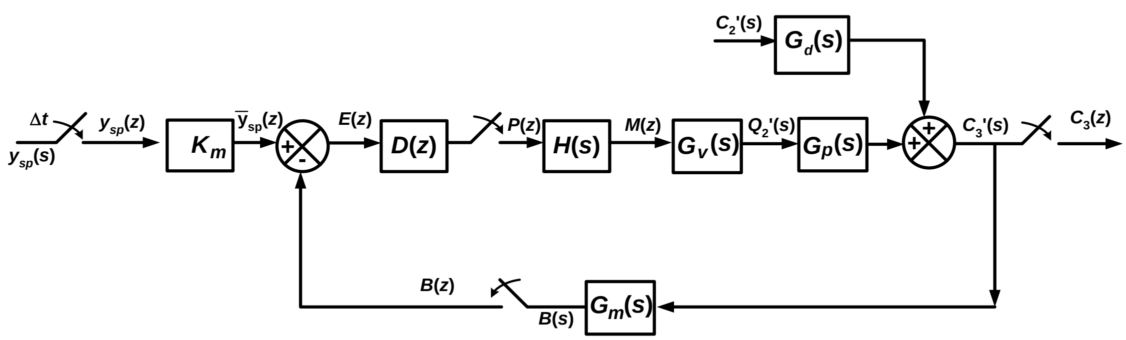


Figure S17.17. *Closed-loop response for a unit step change in set point.*

Hence performance shows 21% overshoot and also oscillates.

17.18

a)



The transfer functions in the various blocks are as follows.

K_m = 2.5 ma / (mol solute/ft^3)

$G_m(s) = 2.5e^{-s}$

$H(s) = \dfrac{1 - e^{-s}}{s}$

$G_v(s) = K_v = 0.1$ ft^3/min.ma

To obtain $G_p(s)$ and $G_d(s)$, write the solute balance for the tank as

$$V \frac{dc_3}{dt} = q_1 c_1 + q_2(t) c_2(t) - q_3 c_3(t)$$

Linearizing and using deviation variables

$$V \frac{dc_3'}{dt} = \bar{q}_2 c_2' + \bar{c}_2 q_2' - q_3 c_3'$$

Taking Laplace transform and substituting numerical values

$$30s C_3'(s) = 1.5 Q_2'(s) + 0.1 C_2'(s) - 3 C_3'(s)$$

Therefore,

$$G_p(s) = \frac{C_3'(s)}{Q_2'(s)} = \frac{1.5}{30s+3} = \frac{0.5}{10s+1}$$

$$G_d(s) = \frac{C_3'(s)}{C_2'(s)} = \frac{0.1}{30s+3} = \frac{0.033}{10s+1}$$

b) $$G_p(z) = \frac{C_3(z)}{Q_2(z)} = \frac{0.05}{1 - 0.9z^{-1}}$$

A proportional-integral controller gives a first order exponential response to a unit step change in the disturbance C_2. This controller will also give a first order response to setpoint changes. Therefore, the desired response could be specified as

$$(Y / Y_{sp}) = \frac{1}{\lambda s + 1}$$

17.19

$$\frac{Y}{Y_{sp}} = \frac{HG_p(z)K_m G_c(z)}{1 + HG_p G_m(z) G_c(z)}$$

Solving for $G_c(z)$

$$G_c(z) = \frac{\dfrac{Y}{Y_{sp}}}{HG_p(z)K_m - HG_p G_m(z)\dfrac{Y}{Y_{sp}}} \qquad (1)$$

Since the process has no time delay, N = 0. Hence

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{(1-A)z^{-1}}{1-Az^{-1}}$$

Moreover

$$HG_p(z) = \frac{z^{-1}}{1-z^{-1}}$$

$$HG_p G_m(z) = \frac{z^{-2}}{1-z^{-1}}$$

$K_m = 1$

Substituting into (1) gives

$$G_c(z) = \frac{\dfrac{(1-A)z^{-1}}{1-Az^{-1}}}{\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-2}}{1-z^{-1}}\dfrac{(1-A)z^{-1}}{1-Az^{-1}}}$$

Rearranging,

$$G_c(z) = \frac{(1-A)-(1-A)z^{-1}}{1-Az^{-1}-(1-A)z^{-2}}$$

By using Simulink-MATLAB, the closed-loop response is shown for different values of A (actually different values of λ) :

$\lambda = 3 \qquad A = 0.716$

$$\lambda = 1 \qquad A = 0.368$$
$$\lambda = 0.5 \qquad A = 0.135$$

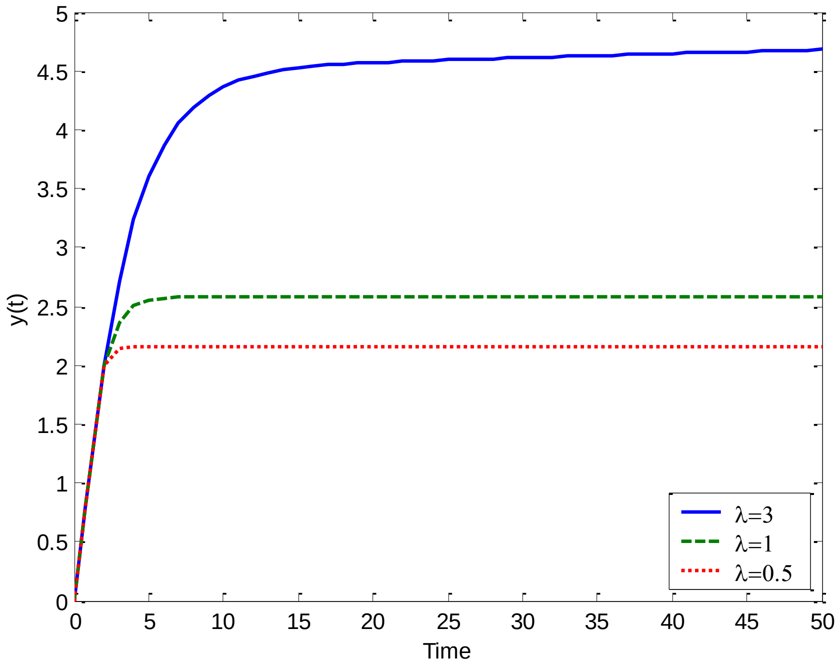


Figure S17.19. *Closed-loop response for a unit step change in disturbance.*

17.20

The closed-loop response for a setpoint change is

$$\frac{Y}{Y_{sp}} = \frac{HG(z)K_v G_c(z)}{1 + HG(z)K_v K_m(z) G_c(z)}$$

Hence

$$G_c(z) = \frac{1}{HG(z)} \frac{\dfrac{Y}{Y_{sp}}}{K_v - K_v K_m \dfrac{Y}{Y_{sp}}}$$

The process transfer function is

$$G(s) = \frac{2.5}{10s + 1} \qquad \text{or} \qquad HG(z) = \frac{0.453z^{-1}}{1 - 0.819z^{-1}} \quad (\theta = 0 \ \text{ so } N = 0)$$

Minimal prototype controller implies λ = 0 (i.e., $A \to 0$). Then, $\dfrac{Y}{Y_{sp}} = z^{-1}$

Therefore the controller is

$$G_c(z) = \frac{1-0.819z^{-1}}{0.453z^{-1}} \frac{z^{-1}}{0.2-(0.2)(0.25)z^{-1}}$$

Simplifying,

$$G_c(z) = \frac{z^{-1}-0.819z^{-2}}{0.091z^{-1}-0.023z^{-2}} = \frac{1-0.819z^{-1}}{0.091-0.023z^{-1}}$$

17.21

a) From Eq. 17-71, the Vogel-Edgar controller is

$$G_{VE} = \frac{(1+a_1z^{-1}+a_2z^{-2})(1-A)}{(b_1+b_2)(1-Az^{-1})-(1-A)(b_1+b_2z^{-1})z^{-N-1}}$$

where $A = e^{-\Delta t/\lambda} = e^{-1/5} = 0.819$

Using z-transforms, the discrete-time version of the second-order transfer function yields

a_1 = -1.625
a_2 = 0.659
b_1 = 0.0182
b_2 = 0.0158

Therefore

$$G_{VE} = \frac{(1-1.625z^{-1}+0.659z^{-2})0.181}{(0.0182+0.0158)(1-0.819z^{-1})-0.181(0.0182+0.0158z^{-1})z^{-1}}$$

$$= \frac{0.181-0.294z^{-1}+0.119z^{-2}}{0.034-0.031z^{-1}-0.003z^{-2}}$$

By using Simulink-MATLAB, the controlled variable $y(k)$ and the controller output $p(k)$ are shown for a unit step change in y_{sp}.

Controlled variable *y*(*k*):

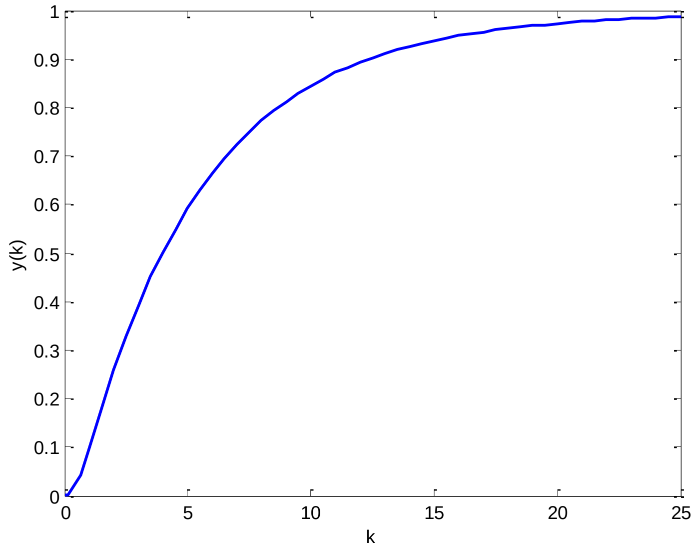


Figure S17.21a. *Controlled variable y(k) for a unit step change in* y_{sp}.

Controller output *p*(*k*):

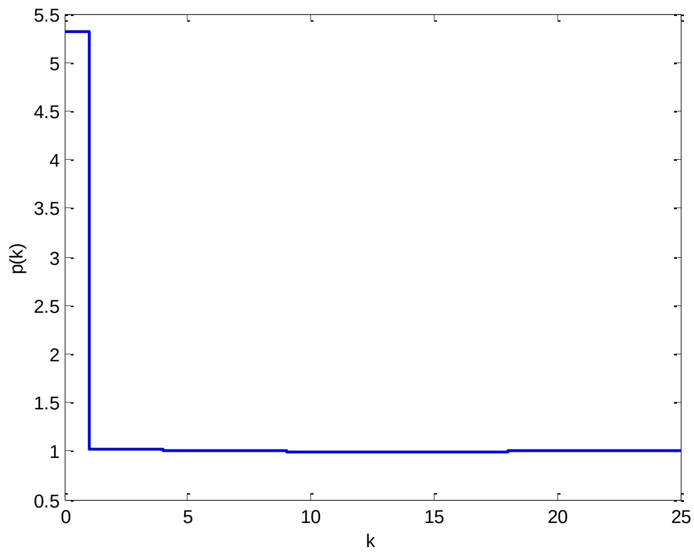


Figure S17.21b. *Controlled output p(k) for a unit step change in* y_{sp}.

17.22

Dahlin's controller

From Eq. 17-66 with a_1 = $e^{-1/10}$=0.9, N=1, and A=$e^{-1/1}$ = 0.37, the Dahlin controller is

$$G_{DC}(z) = \frac{(1-0.37)}{1-0.37z^{-1}-(1-0.37)z^{-2}}\frac{1-0.9z^{-1}}{2(1-0.9)}$$
$$= \frac{3.15-2.84z^{-1}}{1-0.37z^{-1}-0.63z^{-2}} = \frac{3.15-2.84z^{-1}}{(1-z^{-1})(1+0.63z^{-1})}$$

By using Simulink, controller output and controlled variable are shown below:

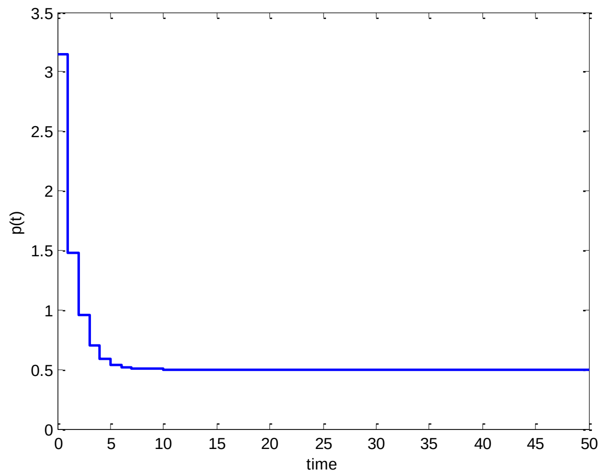


Figure S17.22a. *Controller output for Dahlin controller.*

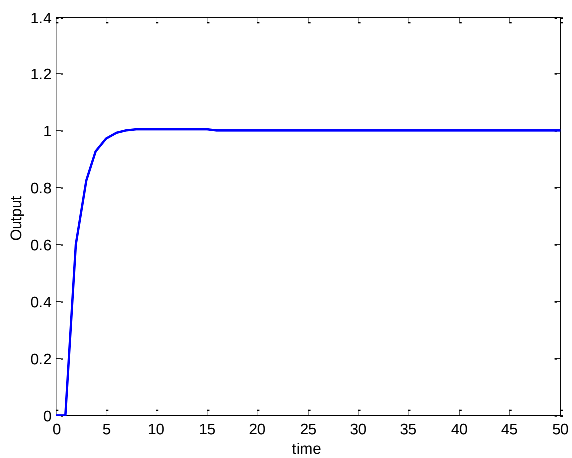


Figure S17.22b. *Closed-loop response for Dahlin controller.*

Thus, there is no ringing (this is expected for a first-order system) and no adjustment for ringing is required.

PID (ITAE setpoint)

For this controller, adjust the process time delay for the zero-order hold by adding $\Delta t/2$, and K=2, τ=10, θ=1.5 obtain the continuous PID controller tunings from Table 12.4 as

$KK_c = 0.965(1.5/10)^{-0.85}$, $K_c = 2.42$

$\tau/\tau_I = 0.796 + (-0.1465)(1.5/10)$, , $\tau_I = 12.92$

$\tau_D/\tau = 0.308(1.5/10)^{0.929}$, $\tau_D = 0.529$

Using the position form of the PID control law (Eq. 8-25 or 17-55)

$$G_c(z) = 2.42\left[1 + \frac{1}{12.92}\left(\frac{1}{1-z^{-1}}\right) + 0.529(1-z^{-1})\right]$$

$$= \frac{3.89 - 4.98z^{-1} + 1.28z^{-2}}{1-z^{-1}}$$

By using Simulink,

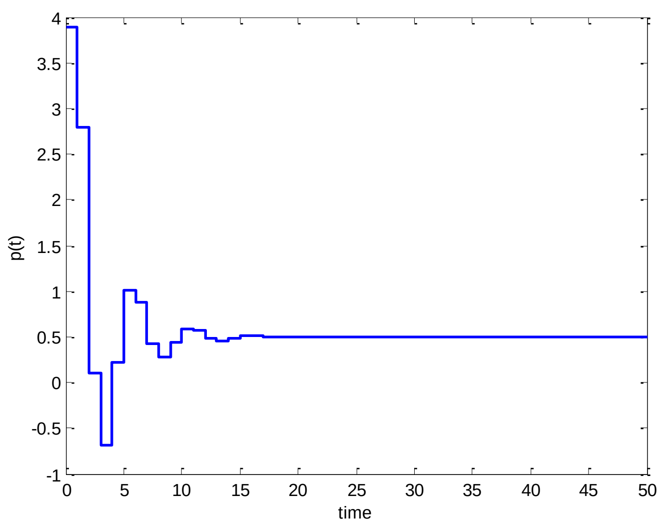


Figure S17.22c. *Controller output for PID (ITAE) controller*

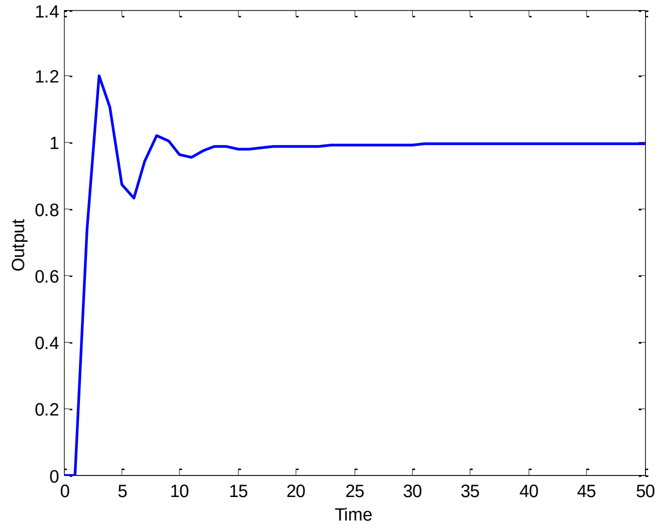


Figure S17.22d. *Closed-loop response for PID (ITAE) controller.*

Dahlin's controller gives better closed-loop performance than PID because it includes time-delay compensation.

17.23

From Eq. 17-66 with a_1 = $e^{-1/5}$=0.819, N=5, and A=$e^{-1/1}$ = 0.37, the Dahlin controller is

$$G_{DC}(z) = \frac{(1-0.37)}{1-0.37z^{-1}-(1-0.37)z^{-6}}\frac{1-0.819z^{-1}}{1.25(1-0.819)}$$

$$= \frac{2.78-2.28z^{-1}}{(1-0.37z^{-1}-0.63z^{-6})}$$

By using Simulink-MATLAB, the controller output is shown in Fig. S17.23

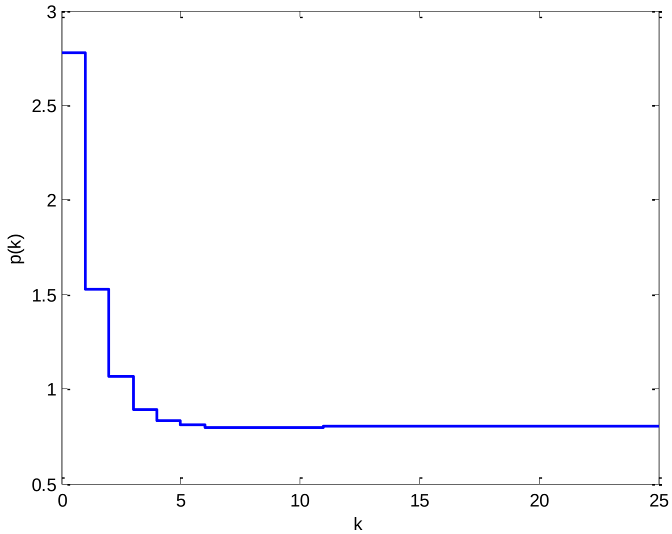


Figure S17.23. *Controller output for Dahlin controller.*

As noted in Fig.S17.23, ringing does not occur. This is expected for a first-order system.

17.24

Dahlin controller

Using Table 17.1 with K=0.5 , r =1.0, p =0.5,

$$G(z) = \frac{0.1548z^{-1} + 0.0939z^{-2}}{1 - 0.9744z^{-1} + 0.2231z^{-2}}$$

From Eq. 17-64, with $\lambda = \Delta t = 1$, Dahlin's controller is

$$G_{DC}(z) = \frac{(1 - 0.9744z^{-1} + 0.2231z^{-2})}{0.1548z^{-1} + 0.0939z^{-2}} \frac{0.632z^{-1}}{1 - z^{-1}}$$

$$= \frac{0.632 - 0.616z^{-1} + 0.141z^{-2}}{(1 - z^{-1})(0.1548 + 0.0939z^{-1})}$$

From Eq. 17-63,

$$\frac{Y(z)}{Y_{sp}(z)} = \frac{0.632z^{-1}}{1-0.368z^{-1}}$$

$y(k) = 0.368\, y(k-1) + 0.632\, y_{sp}(k-1)$

Since this is first order, no overshoot occurs.

By using Simulink-MATLAB, the controller output is shown:

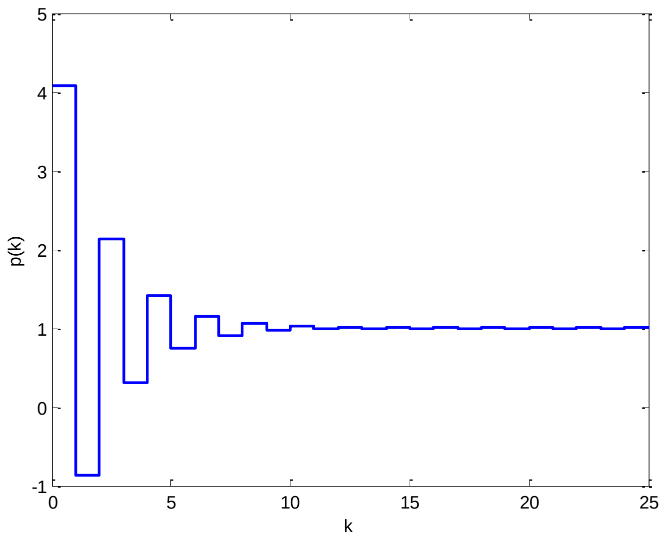


Figure S17.24a. *Controller output for Dahlin controller.*

As noted in Fig. S17.24 a, ringing occurs for Dahlin's controller.

<u>Vogel-Edgar controller</u>

From Eq. 17-71, the Vogel-Edgar controller is

$$G_{VE}(z) = \frac{2.541 - 2.476z^{-1} + 0.567z^{-2}}{1 - 0.761z^{-1} - 0.239z^{-2}}$$

Using Eq. 17-70 and simplifying,

$$\frac{Y(z)}{Y_{sp}(z)} = \frac{(0.393z^{-1} + 0.239z^{-2})}{1-0.368z^{-1}}$$

$y(k) = 0.368\, y(k-1) + 0.393\, y_{sp}(k-1) + 0.239\, y_{sp}(k-2)$

Again no overshoot occurs since $y(z)/y_{sp}(z)$ is first order.

By using Simulink-MATLAB, the controller output is shown below:

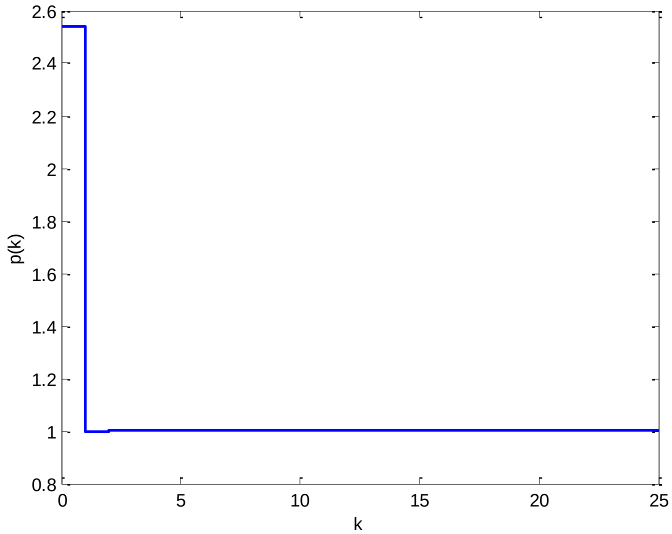


Figure S17.24b. *Controller output for Vogel-Edgar controller.*

As noted in Fig. S17.24 b, the V-E controller does not ring.

17.25

a) Material Balance for the tanks,

$$A_1\frac{dh_1}{dt} = q_1 - q_2 - \frac{1}{R}(h_1 - h_2)$$
$$A_2\frac{dh_2}{dt} = \frac{1}{R}(h_1 - h_2)$$

where $A_1 = A_2 = \pi/4(2.5)^2$=4.91 ft^2

Using deviation variables and taking Laplace transform

$$A_1 sH_1'(s) = Q_1'(s) - Q_2'(s) - \frac{1}{R}H_1'(s) + \frac{1}{R}H_2'(s) \qquad (1)$$
$$A_2 sH_2'(s) = \frac{1}{R}H_1'(s) - \frac{1}{R}H_2'(s) \qquad (2)$$

From (2)

$$H_2'(s) = \frac{1}{A_2Rs+1} H_1'(s)$$

Substituting into (1) and simplifying

$$\left[(A_1A_2R)s^2 + (A_1 + A_2)s\right]H_1'(s) = \left[A_2Rs+1\right]\left[Q_1'(s) - Q_2'(s)\right]$$

$$G_p(s) = \frac{H_1'(s)}{Q_2'(s)} = \frac{-(A_2Rs+1)}{(A_1A_2R)s^2 + (A_1+A_2)s} = \frac{-0.204(s+0.102)}{s(s+0.204)}$$

$$G_d(s) = \frac{H_1'(s)}{Q_1'(s)} = \frac{A_2Rs+1}{(A_1A_2R)s^2 + (A_1+A_2)s} = \frac{0.204(s+0.102)}{s(s+0.204)}$$

Using Eq. 17-64, with N =0, A=$e^{-\Delta t/\lambda}$ and $HG(z)$ = $K_tK_vHG_p(z)$, Dahlin's controller is

$$G_{DC}(z) = \frac{1}{HG}\frac{(1-A)z^{-1}}{(1-z^{-1})}$$

Using z-transforms,

$$HG(z)=K_tK_vHG_p(z)= \frac{-0.202z^{-1} + 0.192z^{-2}}{(1-z^{-1})(1-0.9z^{-1})}$$

Then,

$$G_{DC}(z) = \frac{(1-z^{-1})(1-0.9z^{-1})}{(-0.202z^{-1} + 0.192z^{-2})} \cdot \frac{(1-A)z^{-1}}{(1-z^{-1})}$$

$$= \frac{(1-A)(1-0.9z^{-1})}{-0.202+0.192z^{-1}}$$

b) $$G_{DC} = \frac{(1-A)(1-0.9z^{-1})}{-0.202+0.192z^{-1}}$$

By using Simulink-MATLAB,

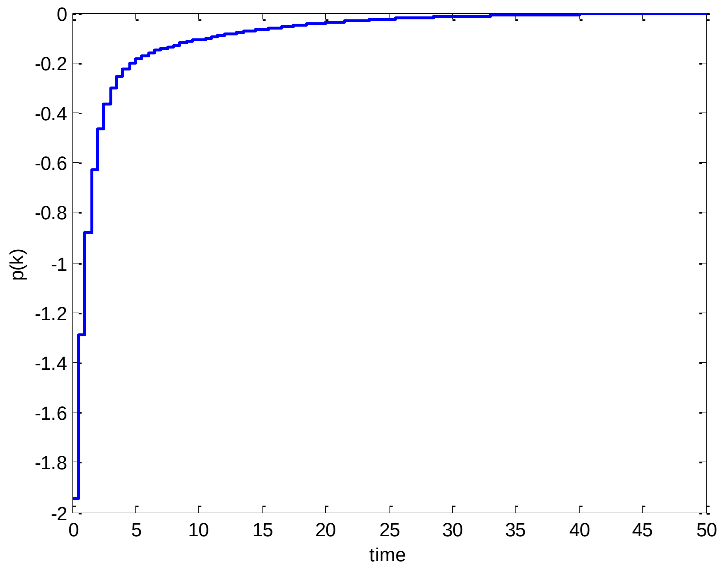


Figure S17.25. *Controller output for Dahlin's controller.*

As noted in Fig. S17.25, the controller output doesn't oscillate.

c) This controller is physically realizable since the z^{-0} coefficient in the denominator is non-zero. Thus, controller is physically realizable for all values of λ.

d) λ is the time constant of the desired closed-loop transfer function. From the expression for $G_p(s)$ the open-loop dominant time constant is 1/0.204 = 4.9 min.

A conservative initial guess for λ would be equal to the open-loop time constant, i.e., λ = 4.9 min. If the model accuracy is reliable, a more bold guess would involve a smaller λ, say 1/3 rd of the open-loop time constant. In that case, the initial guess would be $\lambda = (1/3)\times 4.9 = 1.5$ min.

17.26

$$G_f(s) = \frac{K(\tau_1 s+1)}{\tau_2 s+1} = \frac{P(s)}{E(s)}$$

Substituting $s \cong (1-z^{-1})\ /\ \Delta t$ into equation above:

$$G_f(z) = K\frac{\tau_1(1-z^{-1})/\Delta t+1}{\tau_2(1-z^{-1})/\Delta t+1} = K\frac{\tau_1(1-z^{-1})+\Delta t}{\tau_2(1-z^{-1})+\Delta t} = K\frac{(\tau_1+\Delta t)-\tau_1 z^{-1}}{(\tau_2+\Delta t)-\tau_2 z^{-1}}$$

Then,

$$G_f(z) = \frac{b_1 + b_2 z^{-1}}{1 + a_1 z^{-1}} = \frac{P(z)}{E(z)}$$

where $b_1 = \dfrac{K(\tau_1 + \Delta t)}{\tau_2 + \Delta t}$, $b_2 = \dfrac{-K\tau_1}{\tau_2 + \Delta t}$ and $a_1 = \dfrac{-\tau_2}{\tau_2 + \Delta t}$

Therefore,

$$(1 + a_1 z^{-1})P(z) = (b_1 + b_2 z^{-1})E(z)$$

Converting the controller transfer function into a difference equation form:

$$p(k) = -a_1 p(k-1) + b_1 e(k) + b_2 e(k-1)$$

Using Simulink-MATLAB, discrete and continuous responses are compared : (Note that b_1=0.5 , b_2 = –0.333 and a_1= –0.833)

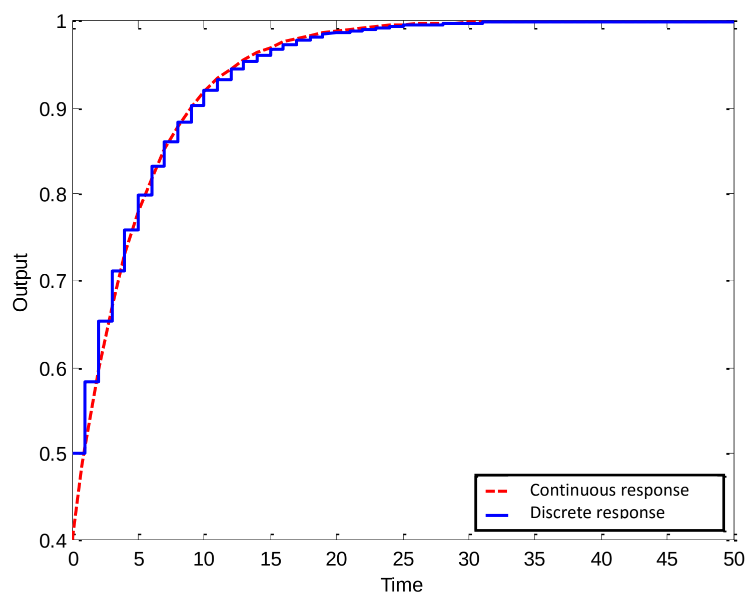


Figure S17.26. *Comparison between discrete and continuous controllers*.

Chapter 18

18.1

McAvoy has reported the PI controller settings shown in Table S18.1 and the set-point responses of Fig. S18.1a and S18.1b. When both controllers are in automatic with Z-N settings, undesirable damped oscillations result due to the control loop interactions. The multiloop tuning method results in more conservative settings and more sluggish responses.

Controller Pairing	Tuning Method	K_c	τ_I(min)
$T_{17} – R$	Single loop/Z-N	-2.92	3.18
T_4 - S	Single loop/Z-N	4.31	1.15
$T_{17} – R$	Multiloop	-2.59	2.58
T_4 - S	Multiloop	4.39	2.58

Table S18.1. *Controller Settings for Exercise 18.1*

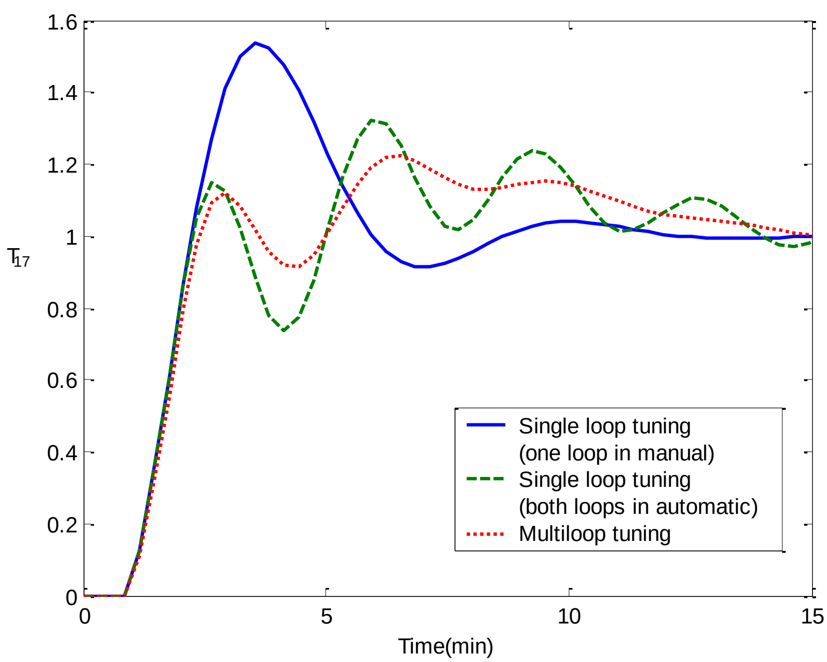


Figure S18.1a. *Set point responses for Exercise 18.1. Analysis for T_{17}*

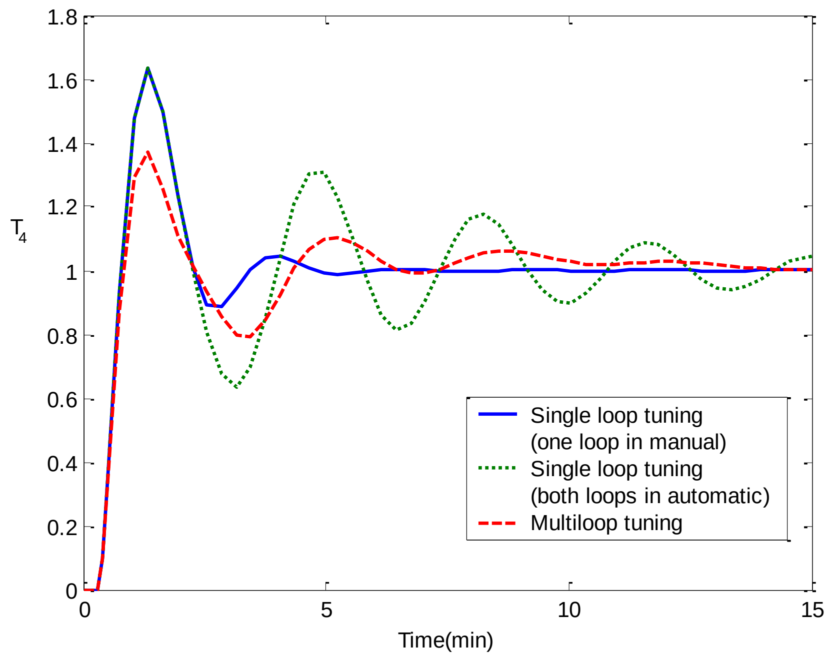


Figure S18.1b. *Set point responses for Exercise 18.1. Analysis for T_4*

18.2

The characteristic equation is found by determining any one of the four transfer functions $Y_1(s)/Y_{sp1}(s)$, $Y_1(s)/Y_{sp2}(s)$, $Y_2(s)/Y_{sp1}(s)$ and $Y_2(s)/Y_{sp2}(s)$, and setting its denominator equal to zero.

In order to determine, say, $Y_1(s)/Y_{sp1}(s)$, set $Y_{sp2} = 0$ in Fig 18.3*b* and use block diagram algebra to obtain

$$C_1(s) = G_{P_{12}} G_{C_1} [R_1(s) - C_1(s)] + G_{P_{11}} M_1(s) \tag{1}$$

$$M_1(s) = G_{C_2} (-[G_{P_{21}} M_1(s) + G_{P_{22}} G_{C_1} [R_1(s) - C_1(s)]]) \tag{2}$$

Simplifying (2),

$$M_1(s) = \frac{-G_{C_2} G_{P_{22}} G_{C_1}}{1 + G_{C_2} G_{P_{21}}} [R_1(s) - C_1(s)] \tag{3}$$

Substituting (3) into (1) and simplifying gives

$$\frac{C_1(s)}{R_1(s)} = \frac{(G_{C_1} G_{P_{12}})(1+G_{C_2} G_{P_{21}}) - G_{C_1} G_{C_2} G_{P_{11}} G_{P_{22}}}{(1+G_{C_1} G_{P_{12}})(1+G_{C_2} G_{P_{21}}) - G_{C_1} G_{C_2} G_{P_{11}} G_{P_{22}}}$$

Therefore characteristic equations is

$$(1 + G_{c1} G_{p12})(1 + G_{c2} G_{p21}) - G_{c1} G_{c2} G_{p11} G_{p22} = 0$$

If either G_{p11} or G_{p22} is zero, this reduces to

$(1 + G_{c1} G_{p12}) = 0$ or $(1 + G_{c2} G_{p21}) = 0$

So that the stability of the overall system merely depends on the stability of the two individual feedback control loops in Fig. 18.3*b* since the third loop containing G_{p11} and G_{p22} is broken.

18.3

Consider the block diagram for the 1-1/2-2 control scheme in Fig.18.3*a* but including a sensor and valve transfer function (G_{v1}, G_{v2}) ,(G_{m1}, G_{m2}) for each output (y_1, y_2). The following expressions are easily derived,

$\boldsymbol{Y}(s) = \boldsymbol{G}_p(s)\, \boldsymbol{U}(s)$

or $$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{p11}(s) & G_{p12}(s) \\ G_{p21}(s) & G_{p22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \qquad (1)$$

$\boldsymbol{U}(s) = \boldsymbol{G}_c(s)\, \boldsymbol{G}_v(s)\, \boldsymbol{E}(s)$

or $$\begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} = \begin{bmatrix} G_{c1}(s)G_{v1}(s) & 0 \\ 0 & G_{c2}(s)G_{v2}(s) \end{bmatrix} \begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} \qquad (2)$$

$\boldsymbol{E}(s) = \boldsymbol{Y}_{sp}(s) - \boldsymbol{G}_m(s)\boldsymbol{Y}(s)$

or $$\begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} = \begin{bmatrix} Y_{sp1}(s) \\ Y_{sp2}(s) \end{bmatrix} - \begin{bmatrix} G_{m1}(s) & 0 \\ 0 & G_{m2}(s) \end{bmatrix} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} \qquad (3)$$

If Eqs. 1 through 3 are solved for the response of the output to variations of set points, the result is

$$\boldsymbol{Y}(s) = \boldsymbol{G}_p(s)\boldsymbol{G}_c(s)\,\boldsymbol{G}_v(\mathrm{s})\;[\boldsymbol{I} + \boldsymbol{G}_p(s)\boldsymbol{G}_c(s)\boldsymbol{G}_m(s)]^{-1}\,\boldsymbol{Y}_{sp}(s) =$$

where $\boldsymbol{I}$ is the identity matrix.

In terms of the component transfer function the matrix

$$\boldsymbol{V} = \boldsymbol{I} + \boldsymbol{G}_p(s)\boldsymbol{G}_c(s\,\boldsymbol{G}_v(\mathrm{s}))\boldsymbol{G}_m(s) = \begin{bmatrix} 1+h_{11}(s) & h_{12}(s) \\ h_{21}(s) & 1+h_{22}(s) \end{bmatrix}$$

where

$$h_{11}(\mathrm{s}) = G_{p11}(\mathrm{s})\, G_{c1}(s)\, G_{v1}(s)\, G_{m1}(\mathrm{s})$$
$$h_{12}(\mathrm{s}) = G_{p12}(\mathrm{s})\, G_{c2}(s)\, G_{v2}(s)\, G_{m2}(\mathrm{s})$$
$$h_{21}(\mathrm{s}) = G_{p21}(\mathrm{s})\, G_{c1}(s)\, G_{v1}(s)\, G_{m1}(\mathrm{s})$$
$$h_{22}(\mathrm{s}) = G_{p22}(\mathrm{s})\, G_{c2}(s)\, G_{v2}(s)\, G_{m2}(\mathrm{s})$$

The inverse of $\boldsymbol{V}$, if it exists, is $\boldsymbol{V}^{-1} = \dfrac{1}{\Delta}\begin{bmatrix} 1+h_{22}(s) & -h_{12}(s) \\ -h_{21}(s) & 1+h_{11}(s) \end{bmatrix}$

where $\Delta = (1+h_{11}(\mathrm{s}))(1+h_{22}(\mathrm{s}))-h_{12}(\mathrm{s})h_{21}(\mathrm{s})$

By accounting for $\boldsymbol{Y}(s) = [\boldsymbol{G}_p(s)\boldsymbol{G}_c(s)\,\boldsymbol{G}_v(s)\,\boldsymbol{V}^{-1}(s)]\;\boldsymbol{Y}_{sp}(s)$, the closed-loop transfer functions are (see book notation):

$$T_{11}(s) = \frac{1}{G_{m1}(s)\Delta}\left[h_{11}(s)(1+h_{22}(s)) - h_{12}(s)h_{21}(s)\right]$$

$$T_{12}(s) = \frac{h_{12}(s)}{G_{m2}(s)\Delta}$$

$$T_{21}(s) = \frac{h_{21}(s)}{G_{m1}(s)\Delta}$$

$$T_{22}(s) = \frac{1}{G_{m2}(s)\Delta}\left[h_{22}(s)(1+h_{11}(s)) - h_{21}(s)h_{12}(s)\right]$$

18.4

From Eqs. 6-91 and 6-92 and from physical reasoning, it is evident that although h is affected by both the manipulated variables, T is affected only by w_h and is independent of w. Hence, T can be paired only with w_h. Thus, the pairing based on this reasoning for the control scheme is T-w_h, h-w.

18.5

System transfer function matrix:

$$G_p(s) = \begin{bmatrix} \dfrac{2}{10s+1} & \dfrac{1.5}{s+1} \\ \dfrac{1.5}{s+1} & \dfrac{2}{10s+1} \end{bmatrix} \tag{1}$$

- $K_{c1} = 1;\ K_{c2} = -1$: the pairing is unstable

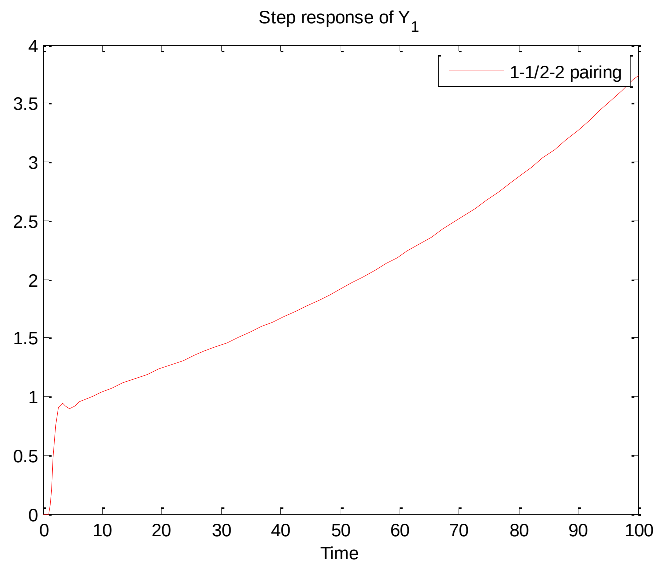


Figure S18.5a. *Step response of Y_1*

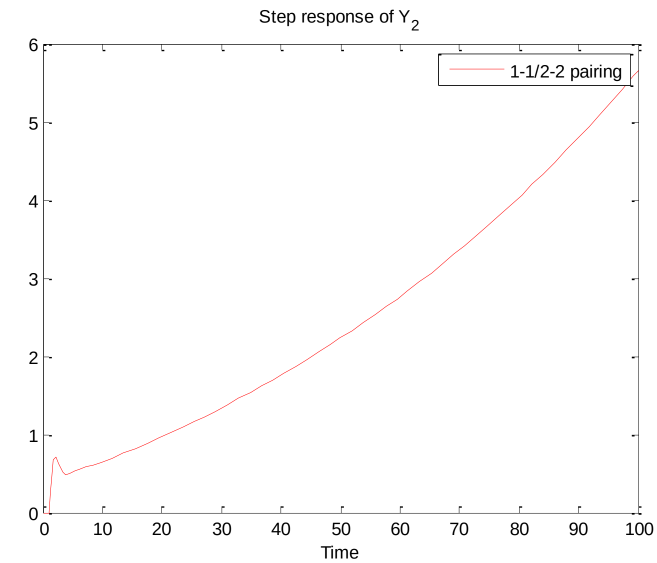


Figure S18.5b *Step response of* Y_2

- $K_{c1} = 1;\ K_{c2} = 0$: the paring is stable

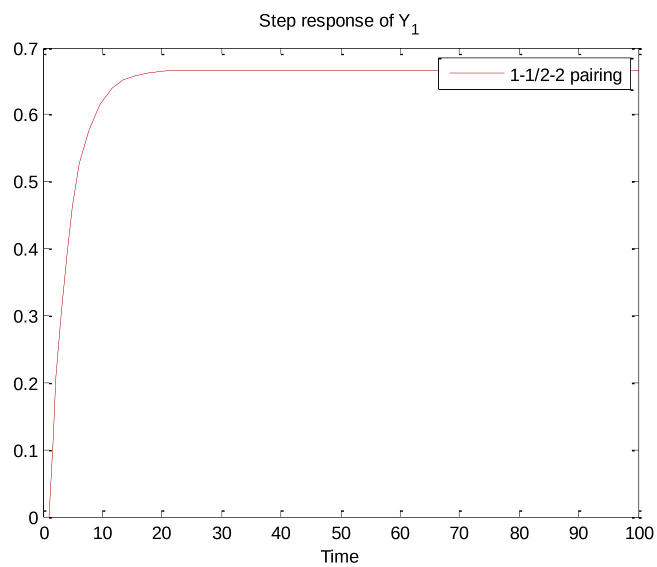


Figure S18.5c. *Step response of* Y_1

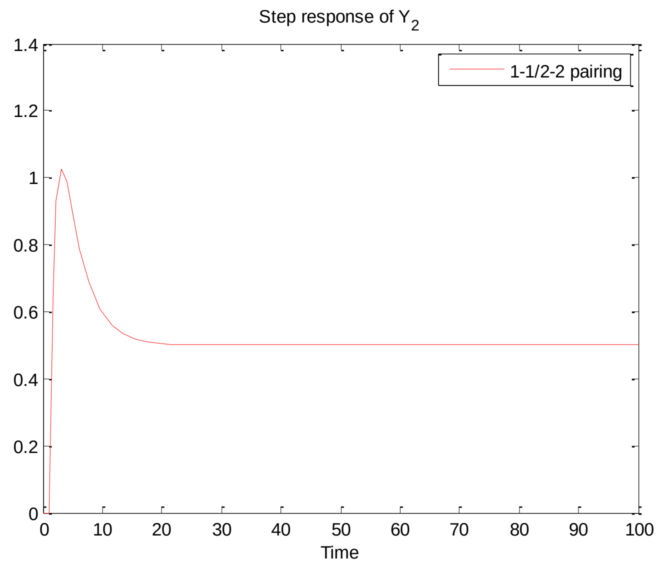


Figure S18.5d. *Step response of* Y_2

- $K_{c1} = 1$; $K_{c2} = 2$: the pairing is unstable

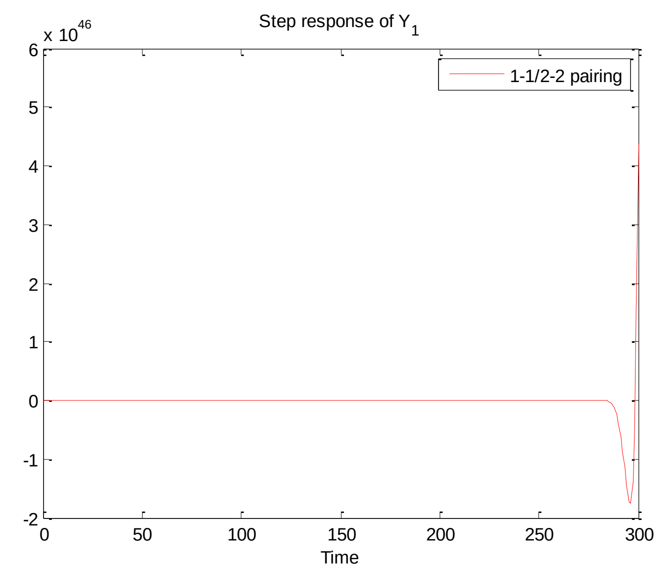


Figure S18.5e. *Step response of* Y_1

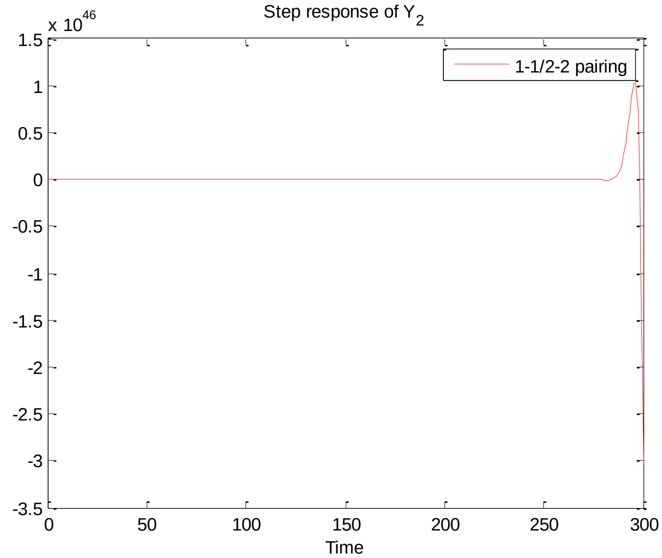


Figure S18.5f. *Step response of Y_2*

18.6

i) Calculate the steady-state gains as

$$K_{11} = \left(\frac{\Delta X_D}{\Delta R}\right)_S = \frac{0.97-0.93}{(125-175)\text{ lb/min}} = -8\times10^{-4}\,\text{min/lb}$$

$$K_{12} = \left(\frac{\Delta X_D}{\Delta S}\right)_R = \frac{0.96-0.94}{(24-20)\text{ lb/min}} = +5\times10^{-3}\,\text{min/lb}$$

$$K_{21} = \left(\frac{\Delta X_B}{\Delta R}\right)_S = \frac{0.06-0.04}{(175-125)\text{lb/min}} = +4\times10^{-4}\,\text{min/lb}$$

$$K_{22} = \left(\frac{\Delta X_B}{\Delta S}\right)_R = \frac{0.04-0.06}{(24-20)lb/\text{min}} = -5\times10^{-3}\text{ min}/\,lb$$

Substituting into Eq. 18-34,

$$\lambda = \frac{1}{1 - \frac{(5*10^{-3})(4*10^{-4})}{(-8*10^{-4})(-5*10^{-3})}} = 2$$

Thus the RGA is

$$\begin{matrix} & R & S \\ x_D & 2 & -1 \\ x_B & -1 & 2 \end{matrix}$$

Pairing for positive relative gains requires X_D-R, X_B-S.

ii) This pairing seems appropriate from dynamic considerations as well; because of the lag in the column, R affects X_D sooner than X_B, and S affects X_B sooner that X_D.

18.7

a) The corresponding steady-state gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} 12.8 & -18.9 \\ 6.6 & -19.4 \end{bmatrix}$$

Using the formula in Eq. 18-34 , we obtain $\lambda_{11} = 2.0$

Thus the RGA is

$$\boldsymbol{\Lambda} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Pairing for positive relative gains requires X_D-R and X_B-S.

b) The same pairing is recommended based on dynamic considerations. The transfer functions between X_D and R contains a smaller dead time and a smaller time constant, so X_D will respond very fast to changes in R. For the pair X_B-S, the time constant is not favorable but the dead time is significantly smaller and the response will be fast as well.

18.8

a) From Eq. 6-105

$$G_{p_{11}}(s) = \frac{(\overline{T}_h - \overline{T})/\overline{w}}{\tau s + 1}\ , \qquad G_{p_{12}}(s) = \frac{(\overline{T}_c - \overline{T})/\overline{w}}{\tau s + 1}$$

$$G_{p_{21}}(s) = \frac{1/AP}{s}\ , \qquad G_{p_{22}}(s) = \frac{1/AP}{s}$$

Thus $K_{11} = \dfrac{\overline{T}_h - \overline{T}}{\overline{w}}, \qquad K_{12} = \dfrac{\overline{T}_c - \overline{T}}{\overline{w}}$

and since G_{p21}, G_{p22} contain integrating elements,

$$K_{21} = \lim_{s\to 0} sG_{P_{21}}(s) = \frac{1}{AP}$$

$$K_{22} = \lim_{s\to 0} sG_{P_{22}}(s) = \frac{1}{AP}$$

Substituting into Eq. 18-34,

$$\lambda = \frac{1}{1 - \dfrac{\overline{T}_c - \overline{T}}{\overline{T}_h - \overline{T}}} = \frac{\overline{T}_h - \overline{T}}{\overline{T}_h - \overline{T}_c}$$

Hence $0 \le \lambda \le 1$, and the choice of pairing depends on whether $\lambda > 0.5$ or not. The RGA is

$$\begin{array}{c} \\ T \\ \\ h \end{array} \begin{array}{c} \begin{array}{cc} w_h & w_c \end{array} \\ \begin{bmatrix} \dfrac{\overline{T}_h - \overline{T}}{\overline{T}_h - \overline{T}_c} & \dfrac{\overline{T} - \overline{T}_c}{\overline{T}_h - \overline{T}_c} \\ \dfrac{\overline{T} - \overline{T}_c}{\overline{T}_h - \overline{T}_c} & \dfrac{\overline{T}_h - \overline{T}}{\overline{T}_h - \overline{T}_c} \end{bmatrix} \end{array}$$

b) Assume that $\lambda \ge 0.5$ so that the pairing is T-w_h, h-w_c. Assume valve gains to be unity. Then the ideal decoupling control system will be as in Fig.18.9 where $Y_1 \equiv T$, $Y_2 \equiv h$, $U_1 \equiv w_h$, $U_2 \equiv w_c$, and using Eqs. 18-78 and 18-80,

$$T_{21}(s) = -\frac{(1/AP)s}{(1/AP)s} = -1$$

$$T_{12}(s) = -\frac{[(\overline{T}_c - \overline{T})/\overline{w}]/(\tau s+1)}{[(\overline{T}_h - \overline{T})/\overline{w}]/(\tau s+1)} = \frac{\overline{T} - \overline{T}_c}{\overline{T}_h - \overline{T}}$$

c) The above decouplers are physically realizable.

18.9

OPTION A: Controlled variable: T_{17}, T_{24}
Manipulated variables: u_1, u_2

The corresponding steady-state gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} 1.5 & 0.5 \\ 2 & 1.7 \end{bmatrix}$$

Using the formula in Eq.18-34, we obtain $\lambda_{11} = 1.65$
Thus the RGA is

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1.65 & -0.65 \\ -0.65 & 1.65 \end{bmatrix}$$

OPTION B: Controlled variable: T_{17}, T_{30}
Manipulated variables: u_1, u_2

The corresponding steady-state gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} 1.5 & 0.5 \\ 3.4 & 2.9 \end{bmatrix}$$

Using the formula in Eq.18-34, we obtain $\lambda_{11} = 1.64$

Thus the RGA is

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1.64 & -0.64 \\ -0.64 & 1.64 \end{bmatrix}$$

OPTION C: Controlled variable: T_{24}, T_{30}
Manipulated variables: u_1, u_2

The corresponding steady-state gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} 2 & 1.7 \\ 3.4 & 2.9 \end{bmatrix}$$

Using the formula in Eq.18-34, we obtain $\lambda_{11} = 290$

Thus the RGA is

$$\boldsymbol{\Lambda} = \begin{bmatrix} 290 & -289 \\ -289 & 290 \end{bmatrix}$$

Hence options A and B yield approximately the same results. Option C is the least desirable to multi-loop control configuration because it will be difficult to change the outputs without very large changes in the two inputs.

18.10

a) Material balance for each of the two tanks is

$$A_1 \frac{dh_1}{dt} = q_1 + q_6 - \frac{\sqrt{h_1}}{R_1} - K(h_1 - h_2) \qquad (1)$$

$$A_2 \frac{dh_2}{dt} = q_2 - \frac{\sqrt{h_2}}{R_2} + K(h_1 + h_2) \qquad (2)$$

where A_1, A_2 are cross-sectional areas of tanks 1, 2, respectively. Linearizing, putting in deviation variable form, and taking Laplace transform,

$$A_1 sH_1'(s) = Q_1'(s) + Q_6'(s) - (\frac{1}{2R_1\sqrt{\bar{h}_1}})H_1'(s) - K[H_1'(s) - H_2'(s)]$$

$$A_2 sH_2'(s) = Q_2'(s) - (\frac{1}{2R_2\sqrt{\bar{h}_2}})H_2'(s) + K[H_1'(s) - H_2'(s)]$$

Let $K_1 \equiv \dfrac{1}{2R_1\sqrt{\bar{h}_1}}$ and $K_2 \equiv \dfrac{1}{2R_2\sqrt{\bar{h}_2}}$, and

Solve the above equations simultaneously to get,

$$[(A_1 s+K_1+K)(A_2 s+K_2+K)-K^2]H_1'(s) = (A_2 s+K_2+K)[Q_1'(s)+Q_6'(s)]+KQ_2'(s) \quad (3)$$

$$[(A_1 s+K_1+K)(A_2 s+K_2+K)-K^2]H_2'(s) = K[Q_1'(s)-Q_6'(s)]+(A_1 s+K_1+K)Q_2'(s) \quad (4)$$

The four steady-state process gains are determined using Eqs. 3 and 4 as

$$K_{11} = \lim_{s\to 0}\left[\frac{H_1'(s)}{Q_1'(s)}\right] = \frac{K_2+K}{K_1K_2+K(K_1+K_2)}$$

$$K_{12} = \lim_{s\to 0}\left[\frac{H_1'(s)}{Q_2'(s)}\right] = \frac{K}{K_1K_2+K(K_1+K_2)}$$

$$K_{21} = \lim_{s\to 0}\left[\frac{H_2'(s)}{Q_1'(s)}\right] = \frac{K}{K_1K_2+K(K_1+K_2)}$$

$$K_{22} = \lim_{s\to 0}\left[\frac{H_2'(s)}{Q_2'(s)}\right] = \frac{K_1+K}{K_1K_2+K(K_1+K_2)}$$

Substituting into Eq. 18-34

$$\lambda = \frac{1}{1-\dfrac{K^2}{(K_2+K)(K_1+K)}} = \frac{(K_2+K)(K_1+K)}{K_1K_2+K(K_1+K_2)}$$

Thus RGA is

$$\begin{array}{c} \\ \dfrac{1}{K_1K_2+K(K_1+K_2)} \end{array}\begin{array}{c} \begin{array}{cc} q_1 & \qquad\qquad q_2 \end{array} \\ \begin{bmatrix} (K_1+K)(K_2+K) & -K^2 \\ -K^2 & (K_1+K)(K_2+K) \end{bmatrix} \end{array}\begin{array}{c} \\ h_1 \\ h_2 \end{array}$$

b) Substituting the given numerical values, the RGA is

$$\begin{array}{c} \\ h_1 \\ h_2 \end{array}\begin{array}{c} \begin{array}{cc} q_1 & \quad q_2 \end{array} \\ \begin{bmatrix} 2.50 & -1.50 \\ -1.50 & 2.50 \end{bmatrix} \end{array}$$

For the relative gains to be positive, the preferred pairing is h_1-q_1, h_2-q_2.

18.11

a) Let

$$\underline{Y}(s)=\begin{bmatrix} H_1'(s) \\ H_2'(s) \end{bmatrix}, \underline{U}(s)=\begin{bmatrix} Q_1'(s) \\ Q_2'(s) \end{bmatrix}, D(s)=Q_6'(s)$$

Then by inspection of Eqs. (3) and (4) in the solution to Exercise 18-10,

$$\underline{\underline{G_P}}(s)=\frac{1}{(A_1s+K_1+K)(A_2s+K_2+K)-K^2}\begin{bmatrix} A_2s+K_2+K & K \\ K & A_1s+K_1+K \end{bmatrix}$$

and

$$\underline{G_d}(s)=\frac{1}{(A_1s+K_1+K)(A_2s+K_2+K)-K^2}\begin{bmatrix} A_2s+K_2+K \\ -K \end{bmatrix}$$

where A_1, A_2, K_1, K_2 are as defined in the solution to Exercise 18.10.

b) The block diagram for h_1-q_1 / h_2-q_2 pairing is identical to Fig.18.3*a* with the addition of the load. Thus the signal $D(s)$ passes through a block G_{d1} whose output is added to the summer with output Y_1. Similarly, the summer leading to Y_2 is influenced by the signal D(s) that passes through block G_{d2}.

18.12

$F = 20\ u_1\,(P_0 - P_1)$ (1)
$F = 30\ u_2\,(P_1 - P_2)$ (2)

Taking P_0 and P_2 to be constant, Eq. 1 gives

$$\left(\frac{\partial F}{\partial u_1}\right)_{u_2} = 20(P_0 - P_1) - 20u_1\left(\frac{\partial P_1}{\partial u_1}\right)_{u_2} \tag{3}$$

and

$$\left(\frac{\partial F}{\partial u_1}\right)_{P_2} = 20(P_0 - P_1) \tag{4}$$

and Eq. 2 gives

$$\left(\frac{\partial F}{\partial u_1}\right)_{u_2} = 30M_2\left(\frac{\partial P_1}{\partial u_1}\right)_{u_2} \tag{5}$$

Substituting for $\left(\frac{\partial P_1}{\partial M_1}\right)_{M_2}$ from (5) into (3) and simplifying

$$\left(\frac{\partial F}{\partial u_1}\right)_{u_2} = \frac{20(P_0 - P_1)}{1 + \frac{20u_1}{30u_2}} \tag{6}$$

Using Eq. 18-24,

$$\lambda_{11} = \frac{(\partial F / \partial u_1)_{u_2}}{(\partial F / \partial u_1)_{P_2}} = \frac{1}{1 + \frac{20u_1}{30u_2}} \tag{7}$$

At nominal conditions

$$u_1 = \frac{F}{20(P_0 - P_1)} = 1/2 \qquad , \qquad u_2 = \frac{F}{30(P_1 - P_2)} = 2/3$$

Substituting into (7), $\lambda_{11} = 2/3 > 0.5$. Hence, the best controller pairing is F-u_1, P_1-u_2.

18.13

a) Material balances for the tank,

$$A\frac{dh}{dt} = q_1 + q_2 - q_3 \tag{1}$$

$$\frac{d(Ahc_3)}{dt} = c_1 q_1 + c_2 q_2 - c_3 q_3 \tag{2}$$

Substituting for d*h*/d*t* from (1) into (2) and simplifying

$$Ah\frac{dC_3}{dt} = (c_1 - c_3)q_1 + (c_2 - c_3)q_2 \tag{3}$$

Linearizing, using deviation variables, and taking the Laplace transform

$$A\bar{h}sC_3'(s) = (\bar{c}_1 - \bar{c}_3)Q_1'(s) - \bar{q}_1 C_3'(s) + (\bar{c}_2 - \bar{c}_3)Q_2'(s) - \bar{q}_2 C_3'(s)$$

Since $\bar{q}_1 + \bar{q}_2 = \bar{q}_3$, this becomes

$$\left[\left(\frac{A\bar{h}}{\bar{q}_3}\right)s + 1\right]C_3'(s) = \left(\frac{\bar{c}_1 - \bar{c}_3}{\bar{q}_3}\right)Q_1'(s) + \left(\frac{\bar{c}_2 - \bar{c}_3}{\bar{q}_3}\right)Q_2'(s) \tag{4}$$

Similarly from (1),

$$AsH'(s) = Q_1'(s) + Q_2'(s) - Q_3'(s) \tag{5}$$

Therefore,

$$\underline{\underline{G}}(s) = \begin{bmatrix} \dfrac{H'(s)}{Q_1'(s)} & \dfrac{H'(s)}{Q_3'(s)} \\ \dfrac{C_3'(s)}{Q_1'(s)} & \dfrac{C_3'(s)}{Q_3'(s)} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{As} & -\dfrac{1}{As} \\ \dfrac{(\bar{c}_1 - \bar{c}_3)/\bar{q}_3}{\left(\dfrac{A\bar{h}}{\bar{q}_3}\right)s + 1} & 0 \end{bmatrix}$$

Substituting numerical values

$$\underline{\underline{G}}(s) = \begin{bmatrix} \dfrac{0.1415}{s} & \dfrac{-0.1415}{s} \\ \dfrac{0.0075}{1.06s + 1} & 0 \end{bmatrix}$$

For the control valves

$$G_v(s) = \frac{0.15}{\left(\frac{10}{60}\right)s+1} = \frac{0.15}{0.167s+1} \quad (6)$$

Thus,

$$\underline{\underline{G_P}}(s) = G_v(s)\underline{\underline{G}}(s) = \begin{bmatrix} \dfrac{0.0212}{s(0.167s+1)} & \dfrac{-0.0212}{s(0.167s+1)} \\ \dfrac{0.0011}{(1.06s+1)(0.167s+1)} & 0 \end{bmatrix}$$

b) Since $C_3'(s)/Q_3'(s) = 0$, c_3 is not affected by q_3 and must be paired with q_1. Thus, the pairing that should be used is h-q_3, c_3-q_1.

c) For the pairing determined above, Fig.18.9 can be used with $Y_1 \equiv H'$, $Y_2 \equiv C_3'$, $U_1 \equiv Q_3'$, $U_2 \equiv Q_1'$. Notice that this pairing requires $G_p(s)$ above the switch columns. Then using Eqs. 18-78 and 18-80,

$$T_{21}(s) = -\frac{G_{P_{21}}(s)}{G_{P_{22}}(s)} = -\frac{0}{\left[\dfrac{0.0011}{(1.06s+1)(0.167s+1)}\right]} = 0$$

$$T_{12}(s) = -\frac{G_{P_{12}}(s)}{G_{P_{11}}(s)} = -\frac{0.0212/\left[s(0.167s+1)\right]}{-0.0212/\left[s(0.167s+1)\right]} = 1$$

18.14

In this case, an RGA analysis is not needed. The manipulated and controlled variables are:

Controlled variables: F_1, P_1 and I
Manipulated variables: m_1, m_2, m_3

Basically, the pairing could be done based on dynamic considerations, so that the time constants and dead times in the response must be as low as possible.

The level of the interface "I" may be easily controlled with m_3 so that any change in the set-point is controlled by opening or closing the valve in the bottom of the decanter.

The manipulated variable m_1 could be used to control the inflow rate F_1. If F_1 is moved away from its set-point, the valve will respond quickly to control this change.

The decanter overhead pressure P_1 is controlled by manipulating m_2. That way, pressure changes will be quickly treated. This control configuration is also used in distillation columns.

18.15

<u>OPTION A:</u> Controlled variable: Y_1, Y_2
Manipulated variables: U_1, U_2

The corresponding steady-state gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} 3 & -0.5 \\ -10 & 2 \end{bmatrix}$$

Using the formula in Eq.18-34, we obtain $\lambda_{11} = 6$

Thus the RGA is

$$\boldsymbol{\Lambda} = \begin{bmatrix} 6 & -5 \\ -5 & 6 \end{bmatrix}$$

<u>OPTION B</u>: Controlled variable: Y_1, Y_2
Manipulated variables: U_1, U_3

The corresponding steady-state gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} 3 & 1/2 \\ -10 & 4 \end{bmatrix}$$

Using the formula in Eq.18-34, we obtain $\lambda_{11} = 0.71$

Thus the RGA is

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0.71 & 0.29 \\ 0.29 & 0.71 \end{bmatrix}$$

<u>OPTION C</u>: Controlled variable: Y_1, Y_2

Manipulated variables: U_2, U_3

The corresponding steady-state gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} -0.5 & 1/2 \\ 2 & 4 \end{bmatrix}$$

Using the formula in Eq.18-34, we obtain $\lambda_{11} = 0.67$

Thus the RGA is

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0.67 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

By accounting for Bristol's original recommendation, the controlled and manipulated variables are paired so that the corresponding relative gains are positive and as close to one as possible. Thus, OPTION B leads to the best control configuration.

18.16

The process scheme is shown below

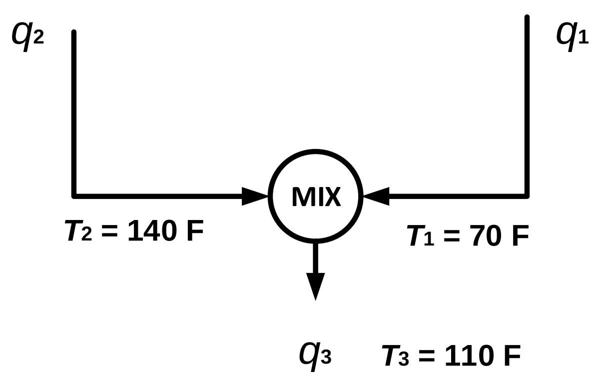


Figure S18.16. *Process scheme*

a) Steady state material balance:

$$q_1 + q_2 = q_3 \tag{1}$$

Steady state energy balance:

$$q_1C(T_1\text{-}T_{ref})+q_2C(T_2\text{-}T_{ref}) = q_3C(T_3\text{-}T_{ref}) \tag{2}$$

By substituting (1) in (2) and solving:

q_1 = 9/7 gpm
q_2 = 12/7 gpm

b) The steady-state gain matrix ***K*** must be calculated :

$$\begin{bmatrix} T_3' \\ q_3' \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} q_1' \\ q_2' \end{bmatrix} \tag{3}$$

From (1), it follows that K_{21}=K_{22}=1. From (2),

$$q_3T_3 = q_1T_1 + q_2T_2 \tag{4}$$

Substitute (1) and rearrange,

$$T_3 = \frac{q_1}{q_1+q_2}(T_1+T_2) \tag{5}$$

$$K_{11} = \left(\frac{\partial T_3}{\partial q_1}\right)_{q_2} = (T_1+T_2)\left[\frac{(q_1+q_2)-q_1}{(q_1+q_2)^2}\right] = \frac{(T_1+T_2)q_2}{(q_1+q_2)^2}$$

$$K_{12} = \left(\frac{\partial T_3}{\partial q_2}\right)_{q_1} = (T_1+T_2)\left[-\frac{q_1}{(q_1+q_2)^2}\right]$$

RGA analysis:

$$\lambda_{11} = \frac{1}{1-\dfrac{K_{12}K_{21}}{K_{11}K_{22}}} = \frac{1}{1-\left(-\dfrac{q_1}{q_2}\right)} = \frac{q_2}{q_2+q_1} \qquad \rightarrow \qquad \lambda_{12} = 1-\lambda_{11} = \frac{q_1}{q_2+q_1}$$

Thus the RGA is,

$$\mathbf{\Lambda} = \begin{array}{c} \\ T_3 \\ q_3 \end{array} \begin{array}{cc} q_1 & q_2 \\ \left(\begin{array}{cc} \dfrac{q_2}{q_2+q_1} & \dfrac{q_1}{q_2+q_1} \\ \\ \dfrac{q_1}{q_2+q_1} & \dfrac{q_2}{q_2+q_1} \end{array} \right) \end{array}$$

Substitute numerical values for numerical conditions,

$$\mathbf{\Lambda} = \begin{array}{c} \\ T_3 \\ q_3 \end{array} \begin{array}{cc} q_1 & q_2 \\ \left(\begin{array}{cc} \dfrac{4}{7} & \dfrac{3}{7} \\ \\ \dfrac{3}{7} & \dfrac{4}{7} \end{array} \right) \end{array}$$

Pairing: $T_3 - q_2 \;/\; q_3 - q_1$

18.17

a) <u>Dynamic Model</u>:

Mass Balance:

$$\rho A \frac{dh}{dt} = (1-f)w_1 + w_2 - w_3 \tag{1}$$

Energy Balance: ($T_{ref} = 0$)

$$\rho C_p A \frac{d(hT_3)}{dt} = C_p(1-f)w_1T_1 + C_p w_2 T_2 - C_p w_3 T_3 - UA_c(T_3 - T_c) \tag{2}$$

Mixing Point:

$$w_4 = w_3 + f w_1 \tag{3}$$

Energy Balance on Mixing Point:

$$C_p w_4 T_4 = C_p w_3 T_3 + C_p f w_1 T_1 \tag{4}$$

Control valves:

$$U = C_3 X_c \tag{5}$$

$$w_3 = x_3 (C_1 h - C_2 f w_1) \tag{6}$$

b) <u>Degrees of freedom</u>:

Variables: 14

h, w_1, w_2, w_3, w_4, T_1, T_2, T_3, T_4, T_c, x_c, x_3, f, U

Equations: 6

Degrees of freedom = $N_V – N_E$ = 8

Specified by the environment: 4 (T_c, w_1, T_1, T_2)

Manipulated variables: 4 (f, w_2, x_c, x_3)

c) Controlled variables:

h Guidelines #2 and 5 (i.e., G2 and G5)

T_4 G3 and G5

w_4 G3 and G5

w_2 (or T_3)   G4 and G5 (or G2 and G5)

d) RGA

At steady state, (1) and (2) become:

$$0 = (1 - f) w_1 + w_2 - w_3 \tag{7}$$

$$0 = C_p (1 - f) w_1 T_1 + C_p w_2 T_2 - C_3 w_3 T_3 - UA_c (T_3 - T_c) \tag{8}$$

Rearrange (8) and substitute (5),

$$T_3 = \frac{C_p(1-f)w_1 + C_p w_2 T_2 - C_3 x_c A_c T_c}{C_3 w_3 + C_3 x_c A_c} \tag{9}$$

Rearrange (7)

$$w_3 = (1-f)w_1 + w_2 \tag{10}$$

Substitute (10) into (9),

$$T_3 = \frac{C_p(1-f)w_1 + C_p w_2 T_2 + C_3 x_c A_c T_c}{C_3(1-f)w_1 + C_3 w_2 + C_3 x_c A_c} \tag{11}$$

Substitute (10), (3) and (11) into (4),

$$(w_3 + f w_1)T_4 = w_3 T_3 + f w_1 T_1 \tag{12}$$

or

$$\left[(1-f)w_1 + w_2 + f w_1\right]T_4 = f w_1 T_1 +$$

$$+ \left[(1-f)w_1 + w_2\right]\left[\frac{C_p(1-f)w_1 + C_p w_2 T_2 - C_3 x_c A_c T_c}{C_3(1-f)w_1 + C_3 w_2 + C_3 x_c A_c}\right] \tag{13}$$

Rearrange,

$$T_4 = \frac{f w_1 T_1}{w_1 + w_2} + \left[\frac{(1-f)w_1 + w_2}{w_1 + w_2}\right]\left[\frac{C_p(1-f)w_1 + C_p w_2 T_2 - C_3 x_c A_c T_c}{C_3(1-f)w_1 + C_3 w_2 + C_3 x_c A_c}\right] \tag{14}$$

Rearrange (6),

$$h = \frac{w_3 + x_3 C_2 f w_1}{x_3 C_1} \tag{15}$$

Substitute (10) into (15),

$$h = \frac{(1-f)w_1 + w_2 + x_3 C_2 f w_1}{x_3 C_1} \tag{16}$$

Rewrite (14) as,

$$T_4 = \frac{f w_1 T_1}{w_1 + w_2} + \left[\frac{E_1 + E_8 f + w_2}{w_1 + w_2}\right]\left[\frac{E_2 f + E_3 w_2 + E_4}{E_5 f + E_6 w_2 + E_7}\right] \tag{17}$$

where:

$$\left.\begin{array}{lll} E_1 = w_1 & E_2 = -C_p w_1 & E_3 = C_p T_2 \\ E_4 = C_3 X_c A T_c + C_p w_1 & & E_5 = -C_3 w_1 \\ E_6 = C_3 & E_7 = C_3 X_c A + C_3 w_1 & E_8 = -w_1 \end{array}\right\} \quad (18)$$

Can write (17) as,

$$T_4 = \frac{f w_1 T_1}{w_1 + w_2} +$$

$$+ \frac{\overbrace{E_8 E_2 f^2 + (E_3 E_8 + E_2) f w_2 + (E_1 E_3 + E_4) w_2 + (E_1 E_2 + E_8 E_4) f + E_1 E_4}^{F_1}}{\underbrace{E_6 w_2{}^2 + (w_1 E_6 + E_7) w_2 + w_1 E_5 f + E_5 w_2 f + E_7 w_1}_{F_2}}$$

$$(19)$$

Thus

$$\frac{\partial T_4}{\partial f} = K_{11} = \frac{w_1 T_1}{w_1 + w_2} + \frac{2 E_B E_2 f + (E_3 E_8 + E_2) w_2 + E_1 E_2 + E_8 E_4}{[F_2]} - \frac{(F_1)\left[w_1 E_5 + E_5 w_2\right]}{F_2{}^2} \quad (20)$$

Similarly

$$\frac{\partial T_4}{\partial f} = K_{12}$$

From (16)

$$\frac{\partial h}{\partial f} = K_{21} = \frac{x_3 C_2 w_1 - w_1}{x_3 C_1}$$

$$\frac{\partial h}{\partial w_2} = K_{22} = \frac{1}{x_3 C_1}$$

Then

$$\Lambda = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}$$

where

$$\lambda = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}$$

e) It will be difficult to control T_4 because neither x_3 nor f has a large steady-state effect on T_4.

18.18

(a) Mass balance:

$$F = F_1 + F_2$$
$$Fw = F_2 w_2 = 0.4F_2$$

CV: w, F, MV: F_1 and F_2.

Linearize the process at operation point as described in Section 18.2.2.

$$K_{11} = \left(\frac{\partial F}{\partial F_1}\right)_{F_2} = 1 \qquad K_{12} = \left(\frac{\partial F}{\partial F_2}\right)_{F_1} = 1$$

$$K_{21} = \left(\frac{\partial w}{\partial F_1}\right)_{F_2} = \frac{-0.4F_2}{F^2} = -0.025 \qquad K_{22} = \left(\frac{\partial w}{\partial F_2}\right)_{F_1} = \frac{0.4F - 0.4F_2}{F^2} = 0.025$$

(b) RGA:

$$\lambda_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}} = \frac{1}{1 - \frac{-0.4F_2/F^2}{(0.4F - 0.4F_2)/F^2}} = \frac{F_1}{F_1 + F_2} = \frac{F - F_2}{F}$$

Thus the RGA array is

$$\begin{bmatrix} \frac{F-F_2}{F} & \frac{F_2}{F} \\ \frac{F_2}{F} & \frac{F-F_2}{F} \end{bmatrix}$$

(c)

$$\lambda_{11} = \frac{1}{1-\frac{K_{12}K_{21}}{K_{11}K_{22}}} = \frac{1}{1+0.025/0.025} = 0.5$$

Thus， the RGA array becomes: $\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$, in this case, either pairing is recommended.

18.19

a)

i) Static considerations:

Pairing according to RGA elements closest to +1:

$H_1 - Q_3$, pH_1-Q_1, H_2-Q_4, $pH_2 - Q_6$

ii) Dynamic considerations:

The some pairing results in the smallest time constants for tank 1. It is also dynamically best for tank 2 because it avoids the large θ/τ ratio of 0.8.

iii) Physical considerations

The proposed pairing makes sense because the controlled variables for each tank are paired with the inlet flows for that some tank.

Because pH is more important than level, we might use the pairing, $H_1 - Q_1$ / pH_1-Q_3 , for the first tank to provide better pH control due to the smaller time delay (0.5 vs. 1.0 min).

b) The new gain matrix for the 2×2 problem is

$$\boldsymbol{K} = \begin{bmatrix} 0.42 & 0.41 \\ -0.32 & 0.32 \end{bmatrix}$$

From Eq. 18-34,

$$\lambda_{11} = \frac{1}{1 - \dfrac{(0.41)(-0.32)}{(0.42)(0.32)}} = 0.506$$

Thus

$$\boldsymbol{\Lambda} = \begin{bmatrix} 0.506 & 0.494 \\ 0.494 & 0.506 \end{bmatrix}$$

RGA pairing: $H_2 - Q_4$ / pH$_2$-Q_6. The pairing also avoids the large delay of 0.8 min.

18.20

Since level is tightly controlled, there is a no accumulation, and a material balance yields:

Overall: $w_F - E\, w_S - w_P \approx 0$ (1)

Solute: $w_F x_F$ - $w_P x_P$ ≈ 0 (2)

Controlled variable: x_P', w_F'

Manipulated variables: w_P', w_s'

From (1):

$$w_F = w_S\, E + w_P$$

From (2):

$$x_P = \frac{x_F}{w_P} w_F = \frac{x_F}{w_P}(w_s E + w_P) \qquad (3)$$

Using deviation variables:

$$w'_F = w'_s E + w'_P$$

Linearizing (3):

$$x_P = \bar{x}_P + \left.\frac{\partial x_P}{\partial w_P}\right|_{\bar{w}_P,\bar{w}_s}(w'_P) + \left.\frac{\partial x_P}{\partial w_s}\right|_{\bar{w}_P,\bar{w}_s}(w'_s)$$

$$x'_P = \left(\frac{-x_F E\bar{w}_s}{\bar{w}_P^{\ 2}}\right)w'_P + \left(\frac{x_F E}{\bar{w}_P}\right)w'_s \qquad (5)$$

Then the steady-state gain matrix is

$$\begin{array}{c c} & \begin{array}{cc} w'_P & w'_s \end{array} \\ \begin{array}{c} x'_P \\ \\ w'_F \end{array} & \begin{pmatrix} \left(\dfrac{-x_F E\bar{w}_s}{\bar{w}_P^{\ 2}}\right) & \left(\dfrac{x_F E}{\bar{w}_P}\right) \\ \\ 1 & \text{E} \end{pmatrix} \end{array}$$

By using the formula in Eq.18-34, we obtain

$$\lambda_{11} = \frac{1}{1+\dfrac{\bar{w}_P}{E\bar{w}_s}} = \frac{E\bar{w}_s}{E\bar{w}_s + \bar{w}_p} = \lambda_{22}$$

$$\lambda_{12} = \lambda_{21} = 1-\lambda_{11} = \frac{\bar{w}_p}{E\bar{w}_s + \bar{w}_p}$$

So the RGA is

$$\mathbf{\Lambda} = \begin{bmatrix} \dfrac{E\bar{w}_s}{E\bar{w}_s + \bar{w}_p} & \dfrac{\bar{w}_P}{E\bar{w}_s + \bar{w}_p} \\ \\ \dfrac{\bar{w}_P}{E\bar{w}_s + \bar{w}_p} & \dfrac{E\bar{w}_s}{E\bar{w}_s + \bar{w}_p} \end{bmatrix}$$

So, if $E\bar{w}_s > \bar{w}_P$, the pairing should be x'_P - w'_P / w'_F - w'_s

So, if $E\bar{w}_s < \bar{w}_P$, the pairing should be x'_P - w'_s / w'_F - w'_P

18.21

a) The corresponding steady-state gain matrix is

$$\boldsymbol{K} = \begin{bmatrix} -0.04 & -0.0005 \\ 0.22 & -0.02 \end{bmatrix}$$

Using the formula in Eq. 18-34, we obtain $\lambda_{11} = 1.16$
Thus the RGA is

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1.16 & -0.16 \\ -0.16 & 1.16 \end{bmatrix}$$

b) Pairing for positive relative gains requires y_1-u_1 and y_2-u_2.

18.22

For higher-dimension process (n>2) the RGA can be calculated from the expression

$$\lambda_{ij} = K_{ij}\, H_{ij}$$

where H_{ij} is the (i,j) element of $H = (\boldsymbol{K}^{-1})^T$

By using MATLAB,

$$\boldsymbol{K}^{-1} = \begin{bmatrix} 62.23 & -122.17 & 58.02 \\ -84.47 & 170.83 & -83.43 \\ 1.95 & -14.85 & 13.09 \end{bmatrix}$$

$$\boldsymbol{H} = \begin{bmatrix} 62.23 & -84.47 & 1.95 \\ -122.17 & 170.83 & -14.85 \\ 58.02 & -83.43 & 13.09 \end{bmatrix}$$

Thus the RGA is

$$\mathbf{\Lambda} = \begin{bmatrix} 210.34 & -211.18 & 1.89 \\ -390.95 & 406.58 & -14.642 \\ 181.60 & -194.39 & 13.80 \end{bmatrix}$$

This RGA analysis shows the control difficulties for this process because of the control loop interactions. For instance, if the pairings are 1-3, 2-2, 3-1, the third loop will experience difficulties in closed-loop operation. But other options not be better.

SVA analysis:

Determinant of $\boldsymbol{K} = |\boldsymbol{K}| = 0.0034$
The condition number = CN = 1845

Since the determinant is small, the required adjustments in U will be very large, resulting in excessive control actions. In addition, this example shows the $\boldsymbol{K}$ matrix is poorly conditioned and very sensitive to small variations in its elements.

18.23

Applying SVA analysis:

Determinant of $\mathbf{K} = |\boldsymbol{K}| = -6.76$
The condition number = CN = 542.93

The large condition number indicates poor conditioning. Therefore this process will require large changes in the manipulated variables in order to influence the controlled variables. Some outputs or inputs should be eliminated to achieve better control, and singular value decomposition (SVD) can be used to select the variables to be eliminated.

By using the MATLAB command SVD, singular values of matrix $\boldsymbol{K}$ are:

$$\sum = \begin{bmatrix} 21.3682 & & & \\ & 6.9480 & & \\ & & 1.1576 & \\ & & & 0.0394 \end{bmatrix}$$

Note that $\sigma_3/\sigma_4 > 10$, then the last singular value can be neglected. If we eliminate one input and one output variable, there are sixteen possible pairing shown in Table S18.23, along with the condition number CN.

Pairing number	Controlled variables	Manipulated variables	CN
1	y_1,y_2,y_3	u_1,u_2,u_3	114.29
2	y_1,y_2,y_3	u_1,u_2,u_4	51.31
3	y_1,y_2,y_3	u_1,u_3,u_4	398.79
4	y_1,y_2,y_3	u_2,u_3,u_4	315.29
5	y_1,y_2,y_4	u_1,u_2,u_3	42.46
6	y_1,y_2,y_4	u_1,u_2,u_4	30.27
7	y_1,y_2,y_4	u_1,u_3,u_4	393.20
8	y_1,y_2,y_4	u_2,u_3,u_4	317.15
9	y_1,y_3,y_4	u_1,u_2,u_3	21.21
10	y_1,y_3,y_4	u_1,u_2,u_4	16.14
11	y_1,y_3,y_4	u_1,u_3,u_4	3897.2
12	y_1,y_3,y_4	u_2,u_3,u_4	693.25
13	y_2,y_3,y_4	u_1,u_2,u_3	24.28
14	y_2,y_3,y_4	u_1,u_2,u_4	20.62
15	y_2,y_3,y_4	u_1,u_3,u_4	1332.7
16	y_2,y_3,y_4	u_2,u_3,u_4	868.34

Table S18.23. *CN for different 3x3 pairings.*

Based on having minimal condition number, pairing 10 (y_1-u_1,y_3-u_2,y_4-u_4) is recommended. The RGA for the reduced variable set is

$$\mathbf{\Lambda} = \begin{bmatrix} 1.654 & -0.880 & 0.226 \\ -0.785 & 3.742 & -1.957 \\ 0.1312 & -1.8615 & 2.7304 \end{bmatrix}$$

18.24

1-2/2-1 controller pairing has a larger stability region compared with 1-1/2-2.

RGA:

$$\lambda_{11} = \frac{1}{1-\dfrac{K_{12}K_{21}}{K_{11}K_{22}}} = \frac{1}{1-\dfrac{1.5\times 1.5}{2\times 2}} = 2.28$$

$$\begin{bmatrix} 2.28 & -1.28 \\ -1.28 & 2.28 \end{bmatrix}$$

Based on RGA, controller pairing should be 1-1/2-2 to avoid negative values.

Stability analysis is based on dynamic effects and employs the numerical region of controller gain to get a stable closed-loop response. RGA is based on static process gain (K_{ij}) analysis, which only show the open loop steady state behavior.

For this problem, 1-2/2-1 pairing has a larger stability region, which means choice of K_{c1} and K_{c2} has a larger margin with guaranteed stability. However, around the steady state, the negative RGA indicates control loop “fighting”, which may be vulnerable to process noise. Thus, 1-2/2-1 pairing should be avoided in this case.

Chapter 19

19.1

From definition of x_c, $0 \le x_c \le 1$

$$f(x) = 5.3\, x\, e^{(-3.6x + 2.7)}$$

Let three initial points in [0,1] be 0.25, 0.5 and 0.75. Calculate x_4 using Eq. 19-8,.

x_1	f_1	x_2	f_2	x_3	f_3	x_4
0.25	8.02	0.5	6.52	0.75	3.98	0.0167

For next iteration, select x_4, and x_1 and x_2 since f_1 and f_2 are the largest among f_1, f_2, f_3. Thus successive iterations are

x_1	f_1	x_2	f_2	x_3	f_3	x_4
0.25	8.02	0.5	6.52	0.017	1.24	0.334
0.25	8.02	0.5	6.52	0.334	7.92	0.271
0.25	8.02	0.334	7.92	0.271	8.06	0.280
0.25	8.02	0.271	8.06	0.280	8.06	not needed

x^{opt} = **0.2799**   7 function evaluations

19.2

As shown in the drawing, there is both a minimum and maximum value of the air/fuel ratio such that the thermal efficiency is non- zero. If the ratio is too low, there will not be sufficient air to sustain combustion. On the other hand, problems in combustion will appear when too much air is used.

The maximum thermal efficiency is obtained when the air/fuel ratio is stoichiometric. If the amount of air is in excess, relatively more heat will be “absorbed” by the air (mostly nitrogen). However, if the air is not sufficient to sustain the total combustion, the thermal efficiency will decrease as well.

19.3

By using Excel-Solver, this optimization problem is quickly solved. The selected starting point is (1,1):

	X_1	X_2
Initial values	1	1
Final values	0.776344	0.669679
max *Y*=	0.55419	
Constraints		
$0 \le X_1 \le 2$		
$0 \le X_2 \le 2$		

Table S19.3. *Excel solution*

Hence the optimum point is $(X_1^*, X_2^*) = (0.776, 0.700)$

and the maximum value of Y is $Y_{max} = 0.554$

19.4

Let N be the number of batches/year. Then $NP \ge 300{,}000$
Since the objective is to minimize the cost of annual production, only the required amount should be produced annually and no more. That is,

$$NP = 300{,}000 \qquad (1)$$

a) Minimize the total annual cost,

$$\min TC = 400{,}000 \left(\frac{\$}{\text{batch}}\right) + 2\,P^{0.4}\left(\frac{\text{hr}}{\text{batch}}\right) 50\left(\frac{\$}{\text{hr}}\right)\; N\left(\frac{\text{batch}}{\text{yr}}\right) + 800\,P^{0.7}\left(\frac{\$}{yr}\right)$$

Substituting for N from (1) gives

$$\min TC = 400{,}000 + 3\text{x}10^{7}\,P^{-0.6} + 800\,P^{0.7}$$

b) There are three constraints on P

i) $P \geq 0$

ii) N is integer. That is,

$(300{,}000/P) = 0, 1, 2, \ldots$

iii) Total production time is 320 x 24 hr/yr

$$(2\,P^{0.4} + 14)\left(\frac{\text{hr}}{\text{batch}}\right) \times N\left(\frac{\text{batch}}{\text{yr}}\right) \leq 7680$$

Substituting for N from (1) and simplifying

$$6\times10^{5}P^{-0.6} + 4.2\times10^{6}P^{-1} \leq 7680$$

c)
$$\frac{d(TC)}{dP} = 0 = 3\times10^{7}(-0.6)P^{-1.6} + 800(0.7)P^{-0.3}$$

$$P^{opt} = \left[\frac{3\times10^{7}(-0.6)}{-800(0.7)}\right]^{1/1.3} = 2931\frac{\text{lb}}{\text{batch}}$$

$$\frac{d^{2}(TC)}{dP^{2}} = 3\times10^{7}(-0.6)(-1.6)P^{-2.6} + 800(0.7)(-0.3)P^{-1.3}$$

$$\left.\frac{d^{2}(TC)}{dP^{2}}\right|_{P=P^{opt}} = 2.26\times10^{-2} \rangle 0 \quad \text{hence minimum}$$

$N^{opt} = 300{,}000/P^{opt} = 102.35$ not an integer.

Hence check for $N^{opt} = 102$ and $N^{opt} = 103$

For $N^{opt} = 102$, $P^{opt} = 2941.2$, and $TC = 863207$

For $N^{opt} = 103$, $P^{opt} = 2912.6$, and $TC = 863209$

Hence optimum is 102 batches of 2941.2 lb/batch.

Time constraint is

$$6\times10^{5}P^{-0.6} + 4.2\times10^{6}P^{-1} = 6405.8 \leq 7680, \text{ satisfied}$$

19.5

Let x_1 be the daily feed rate of Crude No.1 in bbl/day
x_2 be the daily feed rate of Crude No.2 in bbl/day

Objective is to maximize profit

$$\max P = 3.00\, x_1 + 2.0\, x_2$$

Subject to constraints

gasoline : $0.70\, x_1 + 0.41\, x_2 \leq 6000$
kerosene: $0.06\, x_1 + 0.09\, x_2 \leq 2400$
fuel oil: $0.24\, x_1 + 0.50\, x_2 \leq 12{,}000$

By using Excel-Solver,

	x_1	x_2
Initial values	1	1
Final values	0	14634.15

max *P* = 29268.3

Constraints	
$0.70\, x_1 + 0.31\, x_2$	6000
$0.06\, x_1 + 0.09\, x_2$	1317
$0.24\, x_1 + 0.60\, x_2$	7317

Table S19.5. *Excel solution*

Hence the optimum point is (0, 14634.15)

Crude No.1 = 0 bbl/day Crude No.2 = 14634.15 bbl/day

19.6

Objective function is to maximize the revenue,

$$\max R = -40x_1 + 50x_3 + 70x_4 + 40x_5 - 2x_1 - 2x_2 \quad (1)$$

*Balance on column 2

$$x_2 = x_4 + x_5 \quad (2)$$

* From column 1,

$$x_1 = \frac{1.0}{0.60} x_2 = 1.667(x_4 + x_5) \quad (3)$$

$$x_3 = \frac{0.4}{0.60} x_2 = 0.667(x_4 + x_5) \quad (4)$$

Inequality constraints are

$$x_4 \geq 200 \quad (5)$$
$$x_4 \leq 400 \quad (6)$$
$$x_1 \leq 2000 \quad (7)$$
$$x_4 \geq 0 \quad x_5 \geq 0 \quad (8)$$

The restricted operating range for column 2 imposes additional inequality constraints. Medium solvent is 50 to 70% of the bottoms; that is

$$0.5 \leq \frac{x_4}{x_2} \leq 0.7 \quad \text{or} \quad 0.5 \leq \frac{x_4}{x_4 + x_5} \leq 0.7$$

Rewriting in linear form,

$$0.5\, x_2 \leq x_4 \leq 0.7 \quad \text{or} \quad 0.5\,(x_4 + x_5) \leq x_4 \leq 0.7\,(x_4 + x_5)$$

Simplifying,

$$x_4 - x_5 \geq 0 \quad (9)$$
$$0.3\, x_4 - 0.7x_5 \leq 0 \quad (10)$$

No additional constraint is needed for the heavy solvent. That the heavy solvent will be 30 to 50% of the bottoms is ensured by the restriction on the medium solvent and the overall balance on column 2.

By using Excel-Solver,

	x_1	x_2	x_3	x_4	x_5
Initial values	1	1	1	1	1
Final values	1333.6	800	533.6	400	400
max *R* =	**13068.8**				
Constraints					
$x_2 - x_4 - x_5$	0				
$x_1 - 1.667x_2$	7.467E-10				
$x_3 - 0.667x_2$	-1.402E-10				
x_4	400				
x_4	400				
$x_1 - 1.667x_2$	1333.6				
$x_4 - x_5$	0				
$0.3x_4 - 0.7x_5$	-160				

Table S19.6. *Excel solution*

Thus the optimum point is $x_1 = 1333.6$, $x_2 = 800$; $x_3 = 533.6$, $x_4 = 400$ and $x_5 = 400$.

Substituting into (5), the maximum revenue is 13,068 \$/day, and the percentage of output streams in column 2 is 50 % for each stream.

19.7

The objective is to minimize the sum of the squares of the errors for the material balance, that is,

$$\min E = (w_A + 11.3 - 92.1)^2 + (w_A + 10.9 - 94.2)^2 + (w_A + 11.6 - 93.6)^2$$

Subject to $w_A \geq 0$

Solve analytically,

$$\frac{dE}{dw_A} = 0 = 2\,(w_A + 11.3 - 92.1) + 2(w_A + 10.9 - 94.2) + 2(w_A + 11.6 - 93.6)$$

Solving for w_A... $w_A^{opt} = 82.0$ Kg/hr

Check for minimum,

$$\frac{d^2E}{dw_A{}^2} = 2+2+2 = 6 > 0, \quad \text{hence minimum}$$

19.8

The reactor equations are:

$$\frac{dx_1}{dt} = -k_1x_1 \quad (1)$$

$$\frac{dx_2}{dt} = k_1x_1 - k_2x_2 \quad (2)$$

Where $k_1 = 1.335*10^{10}e^{-75000/(8.31*T)}; k_2 = 1.149*10^{17}e^{-125000/(8.31*T)}$

By using MATLAB , this differential equation system can be solved using the command "ode45". Furthermore, we need to apply the command "fminsearch" ino order to optimize the temperature. In doing so, the results are:

$T_{op} = 360.92K; x_{2,\max} = 0.343$

MATLAB code:

```
%% Exercise 19.8
function y = Exercise_19_8(T)
   k10 = 1.335*10^10; % min^(-1)
   k20 = 1.149*10^17; % min^(-1)
   E1 = 75000; % J/(g.mol)
   E2 = 125000; % J/(g.mol)
   R = 8.31; % J/(g.mol.K)
   x10 = 0.7; % mol/L
   x20 = 0; % mol/L
   k1 =k10*exp(-E1/(R*T));
   k2 = k20*exp(-E2/(R*T));
   time = [0,6];     % Time period;
   initial_val = [x10, x20];
   options = odeset('RelTol',1e-4,'AbsTol',[1e-4 1e-4]);
   [~,X] = ode45(@reactor, time,initial_val, options);
   y = -X(end,2);    % Because of fminsearch, has to be opposite

    function dx = reactor(t,x)
        dx = zeros(2,1); % A column vector
        dx(1) = -k1*x(1);
        dx(2) = k1*x(1)-k2*x(2);
    end
end

%% Exercise 22.10 main
clear all;clc; close all;
T_range = [200, 500];
T = fminsearch(@Exercise_19_8, 200);
x2_max =-Exercise_19_8(T);
```

19.9

By using Excel-Solver:

	τ_1	τ_2	
Initial values	1	0	
Final values	2.907801325	1.992609	
Time	**Equation**	**Data**	**Square Error**
0	0	0	0
1	0.065457105	0.0583	5.12241E-05
2	0.200864506	0.2167	0.000250763
3	0.350748358	0.36	8.55929E-05
4	0.489635202	0.488	2.67388E-06
5	0.607853765	0.6	6.16816E-05
6	0.703626108	0.692	0.000135166
7	0.778766524	0.772	4.57858E-05
8	0.836422873	0.833	1.17161E-05
9	0.879953971	0.888	6.47386E-05
10	0.912423493	0.925	0.000158169
11	0.936416639	0.942	3.11739E-05
		SUM=	0.000898685

Hence the optimal values are $\tau_1 = 2.9; \tau_2 = 1.99$..

19.10

Let x_1 be gallons of suds blended
x_2 be gallons of premium blended
x_3 be gallons of water blended

Objective is to minimize cost

$$\min C = 0.3x_1 + 0.40x_2 \quad (1)$$

Subject to

$$x_1 + x_2 + x_3 = 10{,}000 \quad (2)$$

$$0.03\, x_1 + 0.060\, x_2 = 0.050 \times 10{,}000 \quad (3)$$

$x_1 \geq 2000$ (4)

$x_1 \leq 9000$ (5)

$x_2 \geq 0$ (6)

$x_3 \geq 0$ (7)

The problem given by Eqs. 1, 2, 3, 4, 5, 6, and 7 is optimized using Excel-Solver,

	x1	x2	x3
Initial values	1	0	0
Final values	2000	7333.333	666.6666667
Objective function	3533.333333		
Constraints			
x1+x2+x3	10000	=	10000
0.03x1+0.06x2	500	=	500
x1	2000	>=	2000
x1	2000	<=	9000
x2	7333.333333	>=	0
x3	666.6666667	>=	0

We obtain: suds = 2000 gallons; premium = 7333.3 gallons; water= 666.7 gallons, with the minimum cost of $3533.3.

19.11

Let x_A be bbl/day of A produced
x_B be bbl/day of B produced

Objective is to maximize profit

$$\max P = 10x_A + 14x_B \quad (1)$$

Subject to

Raw material constraint: $120x_A + 100x_B \leq 9{,}000$ (2)

Warehouse space constraint: $0.5\, x_A + 0.5\, x_B \leq 40$ (3)

Production time constraint: $(1/20)x_A + (1/10)x_B \leq 7$ (4)

	x_A	x_B
Initial values	1	1
Final values	20	60
max *P* =	1040	
Constraints		
$120x_A + 100x_B$	8400	
$0.5\,x_A + 0.5\,x_B$	40	
$(1/20)x_A + (1/10)x_B$	7	

Table S19.11. *Excel solution*

Thus the optimum point is $x_A = 20$ and $x_B = 60$
The maximum profit = $1040/day

19.12

PID controller parameters are usually obtained by using either process model, process data or computer simulation. These parameters are kept constant in many cases, but when operating conditions vary, supervisory control could involve the optimization of these tuning parameters. For instance, using process data, K_c ,τ_I and τ_D can be automatically calculated so that they maximize profits. Overall analysis of the process is needed in order to achieve this type of optimum control.

Supervisory and regulatory control are complementary. Of course, supervisory control may be used to adjust the parameters of either an analog or digital controller, but feedback control is needed to keep the controlled variable at or near the set-point.

19.13

Assuming steady state behavior, the optimization problem is,

$$\max \; f = D\,e$$

Subject to

$$0.063\,c - D\,e = 0 \qquad (1)$$
$$0.9\,s\,e - 0.9\,s\,c - 0.7\,c - D\,c = 0 \qquad (2)$$

$$-0.9\, s\, e + 0.9\, s\, c + 10D - D\, s = 0 \qquad (3)$$
$$D, e, s, c \geq 0$$

where $f = f(D, e, c, s)$

Excel-Solver is used to solve this problem,

	c	*D*	*e*	*s*
Initial values	1	1	1	1
Final values	0.479031	0.045063	0.669707	2.079784
max *f* =	0.030179			
Constraints				
0.063 *c* –*D e*	2.08E-09			
0.9 *s e* – 0.9 *s c* – 0.7 *c* – *Dc*	-3.1E-07			
-0.9 *s e* + 0.9 *s c* + 10*D* – *Ds*	2.88E-07			

Table S19.13. *Excel solution*

Thus the optimum value of D is equal to 0.045 h^{-1}

19.14

Material balance:

Overall : $F_A + F_B = F$

Component B: $F_B\, C_{BF} + VK_1C_A - VK_2C_B = F\, C_B$

Component A: $F_A\, C_{AF} + VK_2C_B - VK_1C_A = FC_A$

Thus the optimization problem is:

$$\max\ (150 + F_B)\, C_B$$

Subject to:

$$0.3\, F_B + 400C_A - 300C_B = (150 + F_B)C_B$$

$$45 + 300\, C_B - 400\, C_A = (150 + F_B)\, C_A$$

$$F_B \leq 200$$

$$C_A, C_B, F_B \geq 0$$

By using Excel- Solver, the optimum values are

$F_B = 200$ l/hr

$C_A = 0.129$ mol A/l

$C_B = 0.171$ mol B/l

19.15

Material balance:

Overall : $F_A + F_B = F$

Component B: $F_B C_{BF} + VK_1C_A - VK_2C_B = F C_B$

Component A: $F_A C_{AF} + VK_2C_B - VK_1C_A = FC_A$

Thus the optimization problem is:

$$\max (150 + F_B) C_B$$

Subject to:

$$0.3 F_B + 3\times 10^6 e^{(-5000/T)} C_A V - 6\times 10^6 e^{(-5500/T)} C_B V = (150 + F_B)C_B$$

$$45 + 6\times 10^6 e^{(-5500/T)} C_B V - 3\times 10^6 e^{(-5000/T)} C_A V = (150 + F_B) C_A$$

$$F_B \leq 200$$
$$300 \leq T \leq 500$$
$$C_A, C_B, F_B \geq 0$$

By using Excel- Solver, the optimum values are

$F_B = 200$ l/hr

$C_A = 0.104$ molA/l

$C_B = 0.177$ mol B/l

$T = 311.3$ K

Chapter 20

20.1

a) The unit step response is

$$Y(s) = G_p(s)U(s) = \left(\frac{3e^{-2s}}{(15s+1)(10s+1)}\right)\left(\frac{1}{s}\right) = 3e^{-2s}\left[\frac{1}{s} - \frac{45}{15s+1} + \frac{20}{10s+1}\right]$$

Therefore,

$$y(t) = 3S(t-2)\left[1 + 2e^{-(t-2)/10} - 3e^{-(t-2)/15}\right]$$

For $\Delta t = 1$,

$$S_i = y(i\Delta t) = y(i) = \{0,\ 0,\ 0.0095,\ 0.036,\ 0.076,\ 0.13...\}$$

b) Evaluate the expression for $y(t)$ in part (a)

$y(t) = 0.99\ (3) \approx 2.97$ at $t = 87$.

Thus, $N = 87$, for 99% complete response.

20.2

a) Note that $G(s) = G_v(s)G_p(s)G_m(s)$. From Figure 12.2,

$$\frac{Y_m(s)}{P(s)} = G(s) = \frac{4(1-3s)}{(15s+1)(5s+1)} \qquad (1)$$

For a unit step change, $P(s) = 1/s$, and (1) becomes:

$$Y_m(s) = \frac{1}{s}\frac{4(1-3s)}{(15s+1)(5s+1)}$$

Partial Fraction Expansion:

$$Y_m(s) = \frac{A}{s} + \frac{B}{(15s+1)} + \frac{C}{(5s+1)} = \frac{1}{s}\frac{4(1-3s)}{(15s+1)(5s+1)} \qquad (2)$$

where:

$$A = \left.\frac{4(1-3s)}{(15s+1)(5s+1)}\right|_{s=0} = 4$$

$$B = \left.\frac{4(1-3s)}{s(5s+1)}\right|_{s=-\frac{1}{15}} = -108$$

$$C = \left.\frac{4(1-3s)}{s(15s+1)}\right|_{s=-\frac{1}{5}} = 16$$

Substitute into (2) and take the inverse Laplace transform:

$$y_m(t) = 4 - \frac{36}{5}e^{-t/15} + \frac{16}{5}e^{-t/5} \qquad (3)$$

b) The new steady-state value is obtained from (3) to be $y_m(\infty)=4$.

For $t = t_{99}$, $y_m(t)=0.99y_m(\infty) = 3.96$. Substitute into (3)

$$3.96 = 4 - \frac{36}{5}e^{-t_{99}/15} + \frac{16}{5}e^{-t_{99}/5} \qquad (4)$$

Solving (4) for t_{99} gives $t_{99} \approx 77.9$ min

Thus, we specify that $\Delta t = 77.9/30 \approx 3$ min

Table S20.2. *Step response coefficients*

k	t (min)	S_i	k	t (min)	S_i	k	t (min)	S_i
1	3	-0.139	11	33	3.207	21	63	3.892
2	6	0.138	12	36	3.349	22	66	3.912
3	9	0.578	13	39	3.467	23	69	3.928
4	12	1.055	14	42	3.563	24	72	3.941
5	15	1.511	15	45	3.642	25	75	3.951
6	18	1.919	16	48	3.707	26	78	3.960
7	21	2.272	17	51	3.760	27	81	3.967
8	24	2.573	18	54	3.803	28	84	3.973
9	27	2.824	19	57	3.839	29	87	3.978
10	30	3.034	20	60	3.868	30	90	3.982

20.3

From the definition of matrix $\boldsymbol{S}$, given in Eq. 20-28, for P=5, M=1, with S_i obtained from Exercise 20.1,

$$\boldsymbol{S} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.01811 \\ 0.06572 \\ 0.1344 \\ 0.2174 \end{bmatrix}$$

From Eq. 20-65:

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{S})^{-1}\boldsymbol{S}^T$$

$$\boldsymbol{K}_c = \begin{bmatrix} 0 & 0.2589 & 0.9395 & 1.9206 & 3.1076 \end{bmatrix} = \boldsymbol{K}_{c1}^T$$

Because $\boldsymbol{K}_{c1}^{\mathrm{T}}$ is defined as the first row of $\boldsymbol{K}_c$, Using the given analytical result,

$$\boldsymbol{K}_{c1}^T = \frac{1}{\sum_{i=1}^{5}(S_i^2)}\begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 \end{bmatrix}$$

$$\boldsymbol{K}_{c1}^T = \frac{1}{0.06995}\begin{bmatrix} 0 & 0.01811 & 0.06572 & 0.1344 & 0.2174 \end{bmatrix}$$

$$\boldsymbol{K}_{c1}^T = \begin{bmatrix} 0 & 0.2589 & 0.9395 & 1.9206 & 3.1076 \end{bmatrix}$$

which is the same as the answer that was obtained above using (20-65).

20.4

The step response is obtained from the analytical unit step response as in Example 20.1. The feedback matrix $\boldsymbol{K}_c$ is obtained using Eq. 20-65 as in Example 20.5. These results are not reported here for sake of brevity. The closed-loop response for set-point and disturbance changes are shown below for each case. The MATLAB *MPC Toolbox* was used for the simulations.

i) For this model horizon, the step response is over 99% complete as in Example 20.5; hence the model is good. The set-point and disturbance responses shown below are non-oscillatory and have long settling times

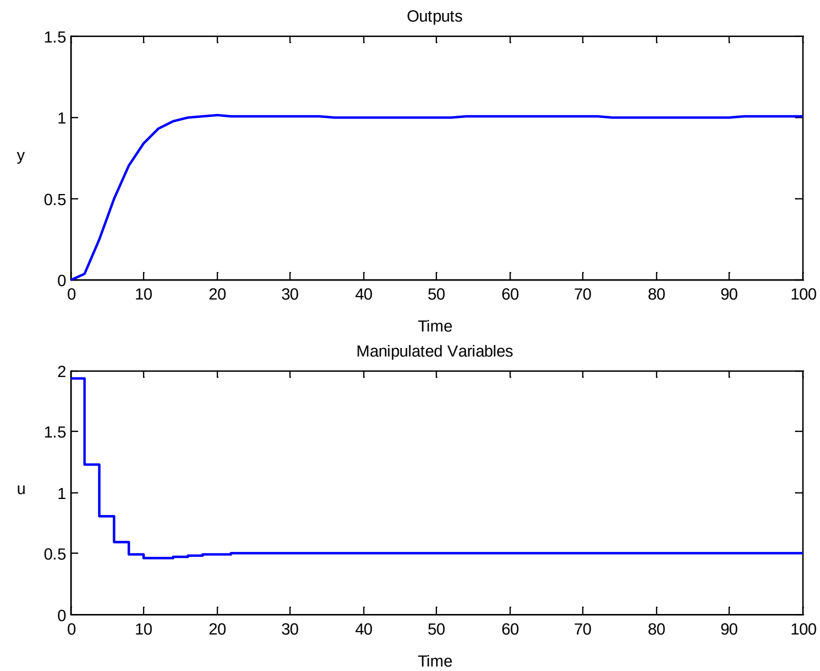


Figure S20.4a. *Controller i); set-point change.*

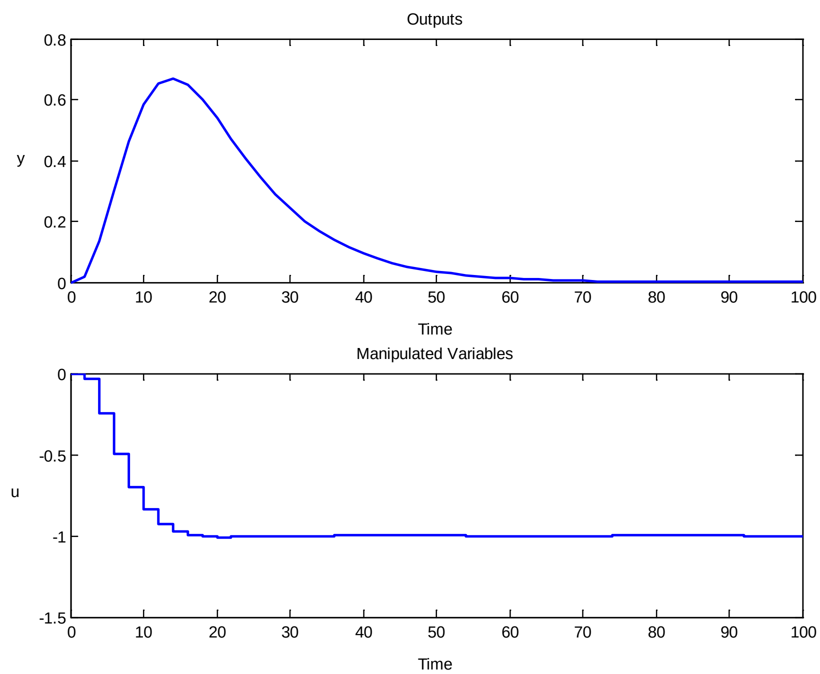


Figure S20.4b. *Controller i); disturbance change.*

ii) The set-point response shown below exhibits same overshoot, smaller settling time and undesirable "ringing" in *u* compared to part i). The disturbance response shows a smaller peak value, a lack of oscillations, and faster settling of the manipulated input.

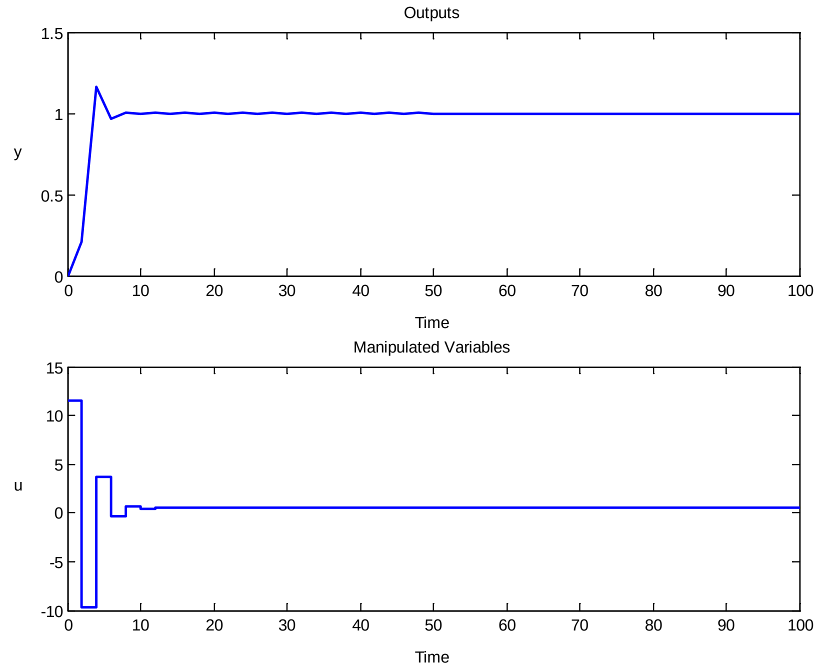


Figure S20.4c. *Controller ii); set-point change.*

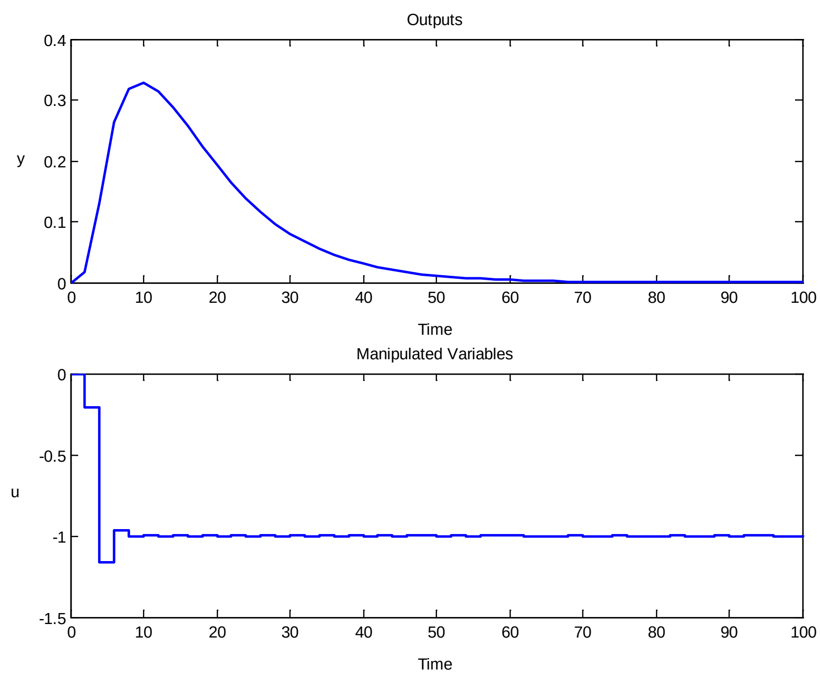


Figure S20.4d. *Controller ii); disturbance change.*

iii) The set-point and disturbance responses shown below show the same trends as in part i).

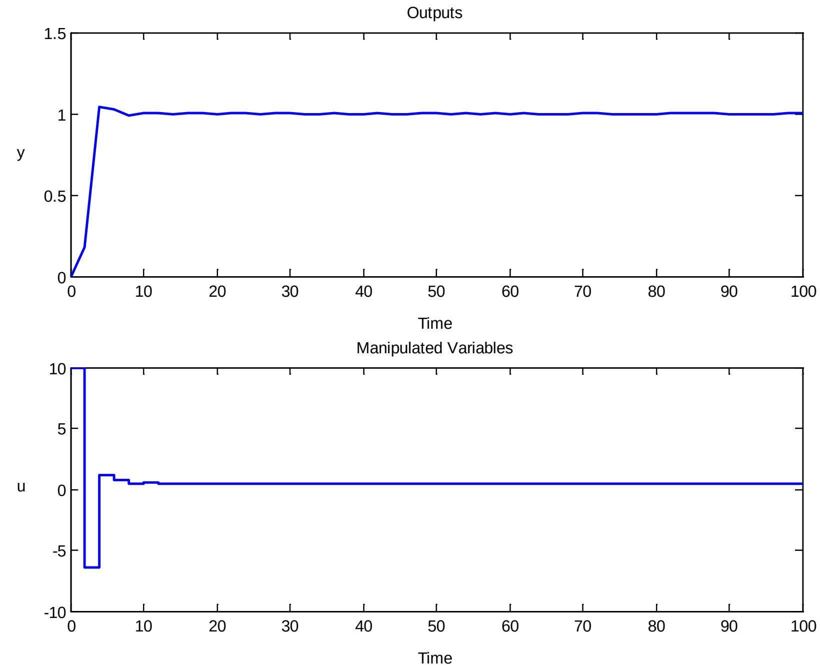


Figure S20.4e. *Controller iii); set-point change.*

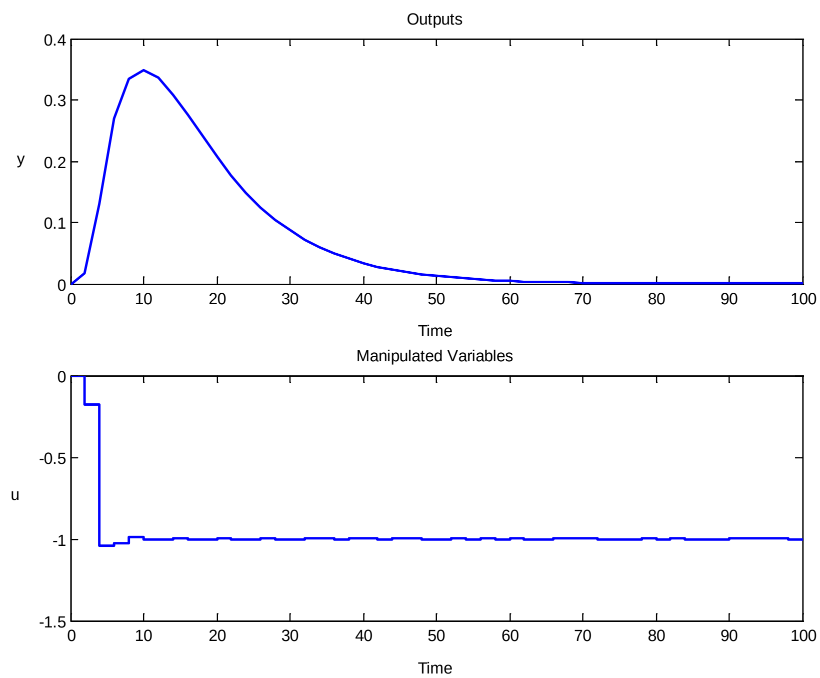


Figure S20.4f. *Controller iii); disturbance change.*

iv) The set-point and load responses shown below exhibit the same trends as in parts (i) and (ii). In comparison to part (iii), this controller has a larger penalty on the manipulated input and, as a result, leads to smaller and less oscillatory input effort at the expense of larger overshoot and settling time for the controlled variable.

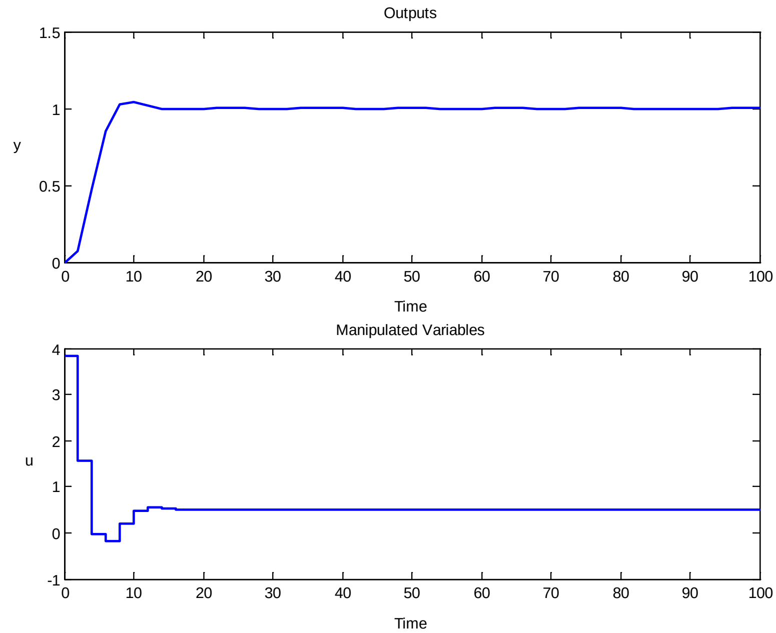


Figure S20.4g. *Controller iv); set-point change.*

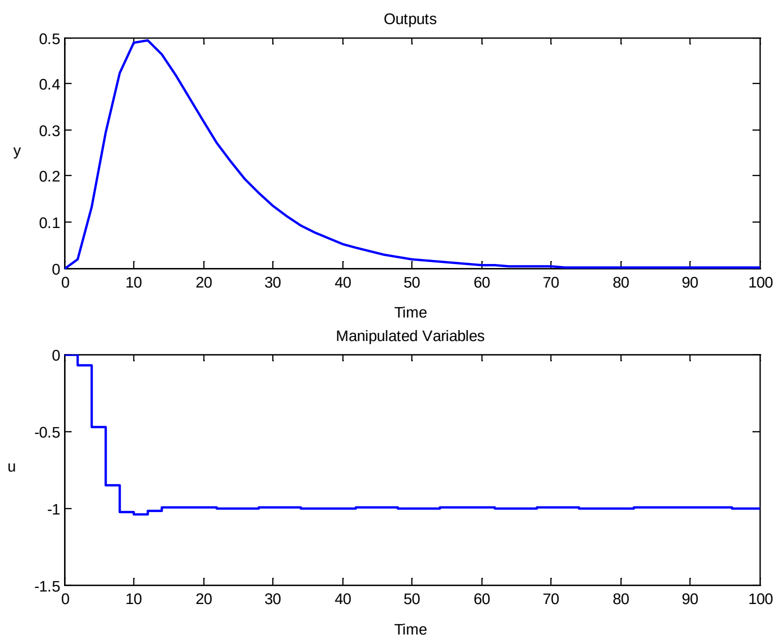


Figure S20.4h. *Controller iv); disturbance change.*

20.5

There are many sets of values of *M*, *P* and *R* that satisfy the given constraint for a unit load change. One such set is *M*=3, *P*=10, *R*=0.01 as shown in Exercise 20.4(iii). Another set is *M*=3, *P*=10, *R*=0.1 as shown in Exercise 20.4(iv). A third set of values is *M*=1, *P*=5, *R*=0 as shown in Exercise 20.4(i).

20.6

(Use MATLAB *Model Predictive Control Toolbox)*

As shown below, controller a) gives a better disturbance response with a smaller peak deviation in the output and less control effort. However, controller (a) is poorer for a set-point change because it leads to undesirable "ringing" in the manipulated input.

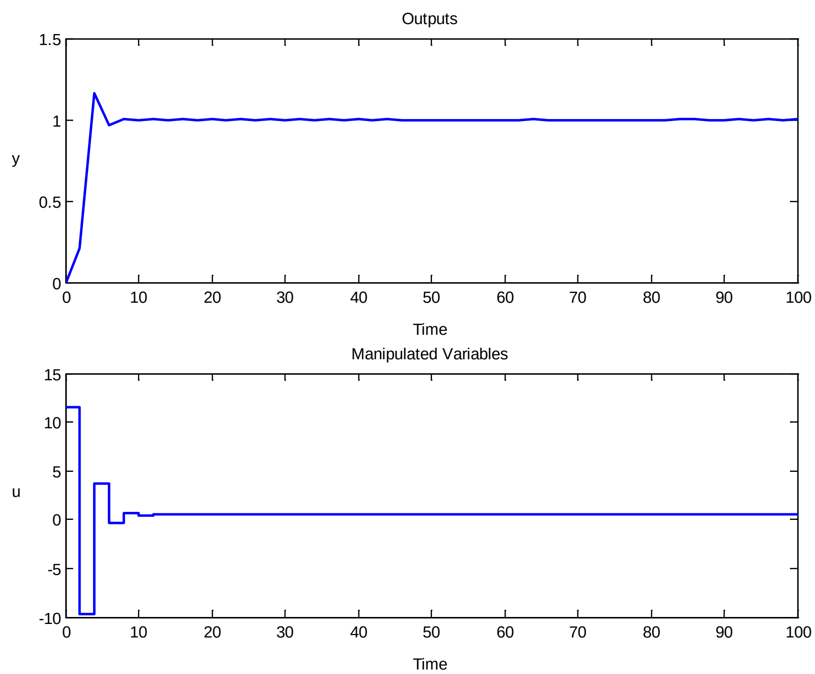


Figure S20.6a. *Controller a); set-point change.*

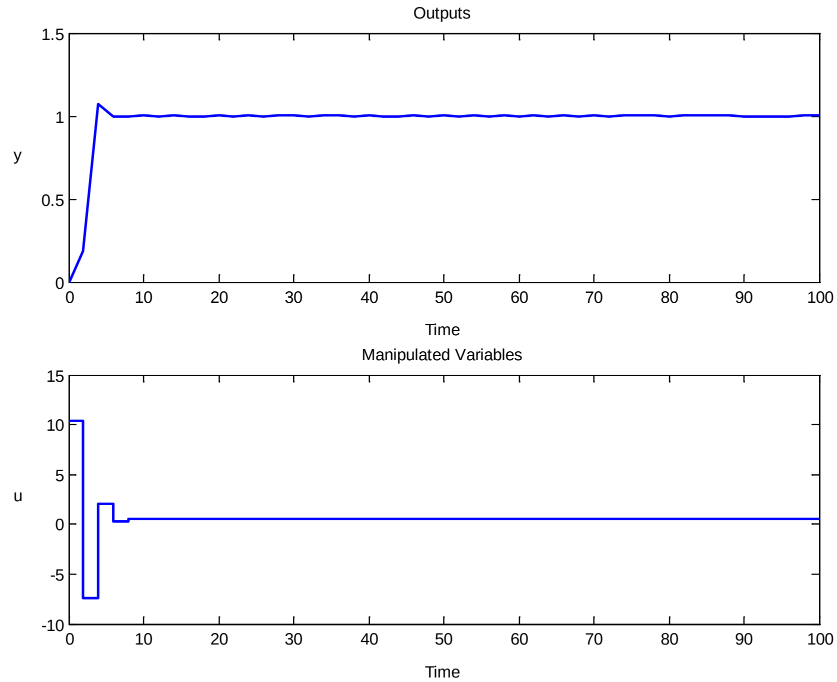


Figure S20.6b. *Controller a); disturbance change.*

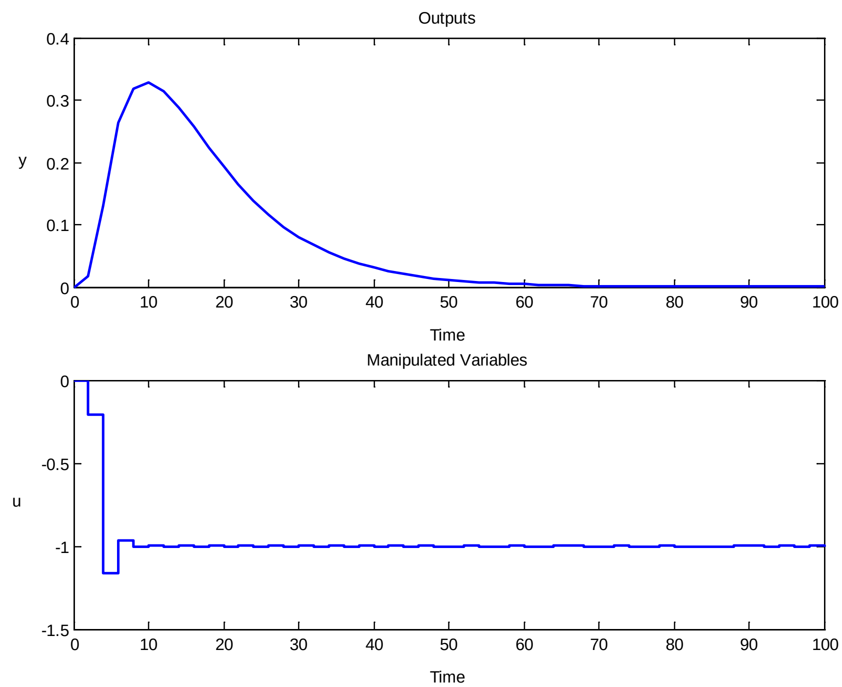


Figure S20.6c. *Controller b); set-point change.*

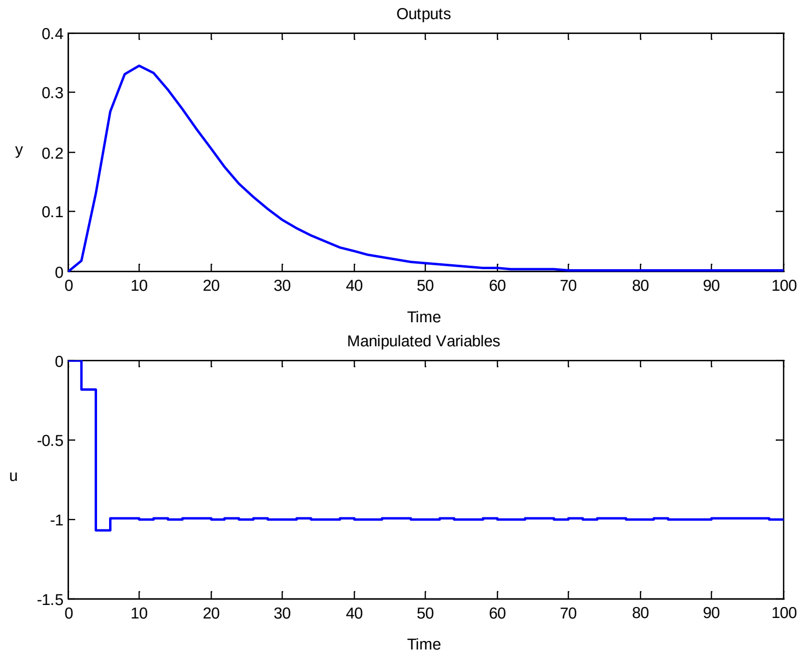


Figure S20.6d. *Controller b); disturbance change.*

20.7

The unconstrained MPC control law has the controller gain matrix:

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{QS}+\boldsymbol{R})^{-1}\boldsymbol{S}^T\boldsymbol{Q}$$

For this exercise, the parameter values are:
$m = r = 1$ (SISO), $\boldsymbol{Q}$=I, $\boldsymbol{R}$=1 and M=1

Thus (20-65) becomes

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{QS}+\boldsymbol{R})^{-1}\boldsymbol{S}^T\boldsymbol{Q}$$

Which reduces to a row vector: $\boldsymbol{K}_c = \dfrac{[S_1\ S_2\ S_3 \ldots S_P]}{\sum_{i=1}^{P} S_i^{\,2} + 1}$

20.8

(Use MATLAB *Model Predictive Control Toolbox)*

a) M=5 vs. M=2

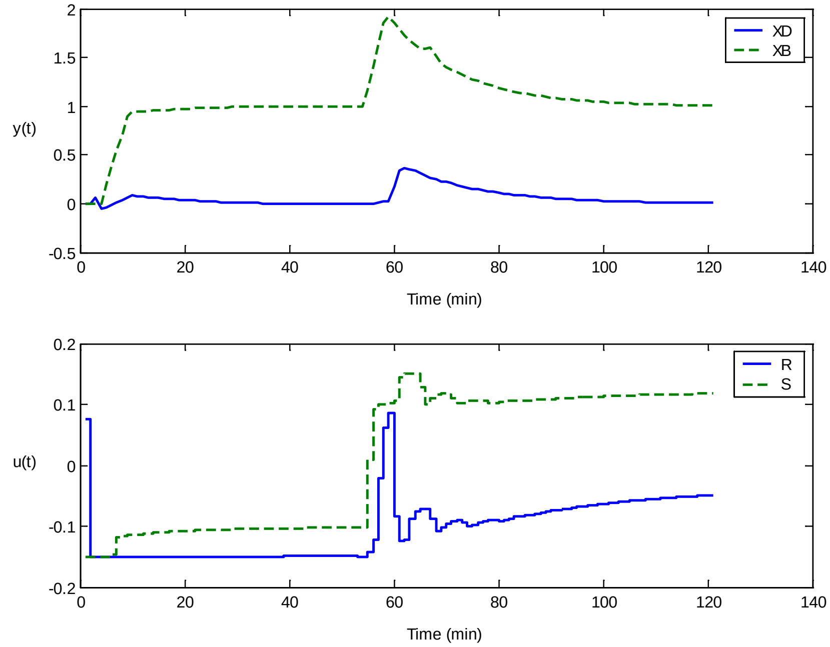


Figure S20.8a1. *Simulations for P=10, M=5 and* ***R****=0.1****I****.*

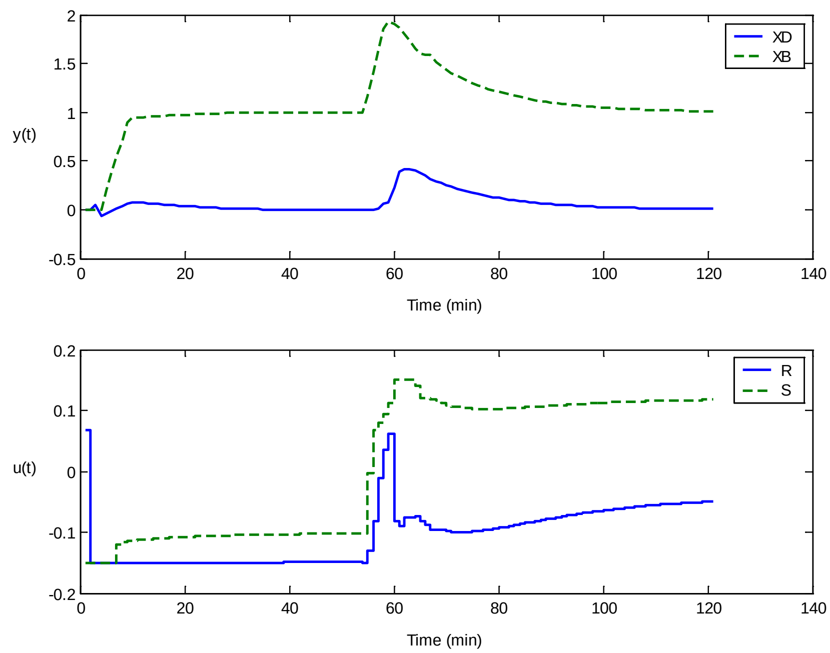


Figure S20.8a2. *Simulations for P=10, M=2 and* ***R****=0.1****I****.*

b) ***R**=0.1**I*** *.vs* ***R**=**I***

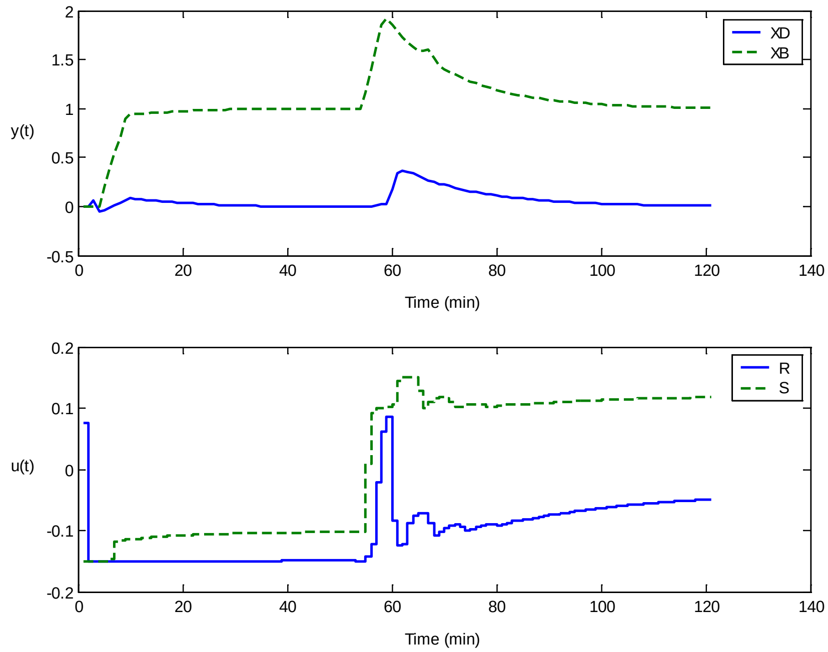


Figure S20.8b1. *Simulations for P=10, M=5 and **R**=0.1**I**.*

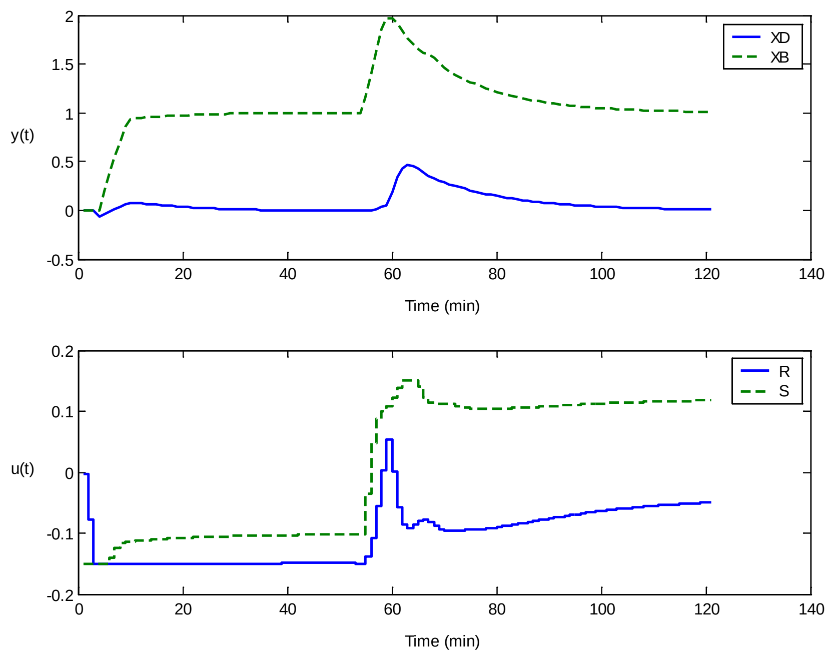


Figure S20.8b2. *Simulations for P=10, M=5 and **R**=**I**.*

Notice that the larger control horizon *M* and the smaller input weighting *R*, the more control effort is needed.

20.9

The open-loop unit step response of $G_p(s)$ is

$$y(t) = \mathcal{L}^{-1}\left(\frac{e^{-6s}}{10s+1}\frac{1}{s}\right) = \mathcal{L}^{-1}\left(e^{-6s}\left(\frac{1}{s} - \frac{10}{10s+1}\right)\right) = S(t-6)\left[1 - e^{-(t-6)/10}\right]$$

By trial and error, $y(34) < 0.95$, $y(36) > 0.95$.

Therefore $N\Delta t = 36$ or $N = 18$.

The coefficients $\{S_i\}$ are obtained from the expression for $y(t)$ and the predictive controller is obtained following the procedure of Example 20.5. The closed-loop responses for a unit set-point change are shown below for the three sets of controller design parameters.

20.10

Note: These results were generated using the PCM Furnace Module, MPC option

c) C_{O_2} *Set*-point change

The set-point responses in Figs. S20.10a and . S20.10b demonstrate that increasing the elements of the ***R*** matrix makes the controller more conservative and results in more sluggish responses.

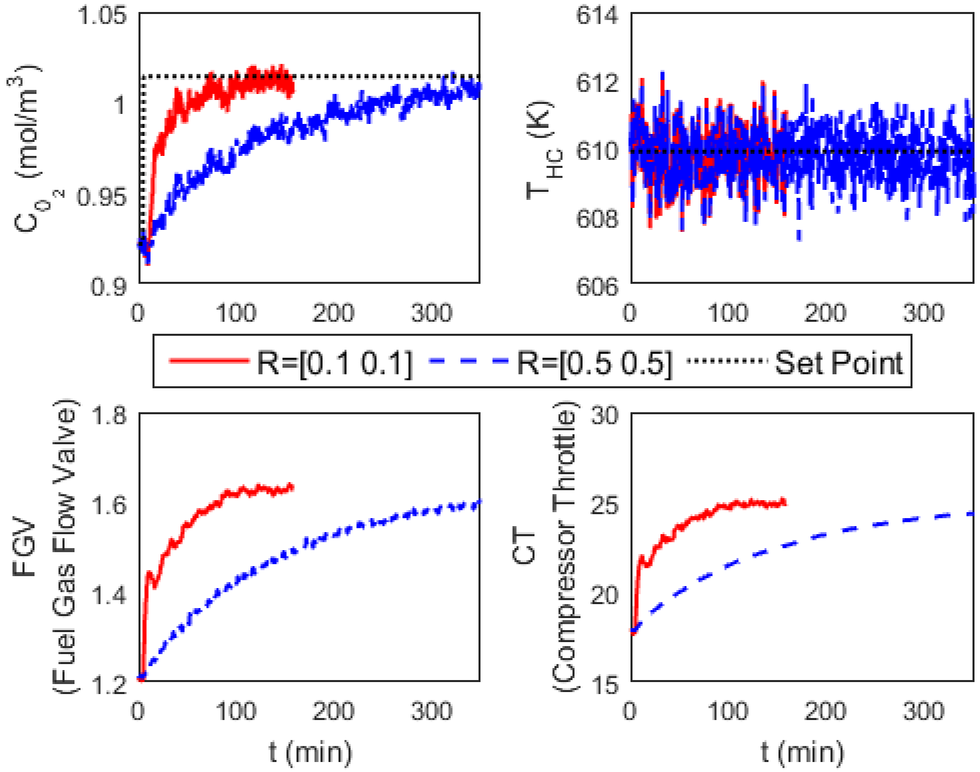


Figure S20.10a. *C_{O_2} Set-point change from 0.922 to 1.0143 for P=20, M=1, and **Q** = diag [0.1, 1]. The two series represent **R** = diag [0.1, 0.1]* and ***R** = diag [0.5, 0.5].*

d) Step disturbance in hydrocarbon flow rate

The disturbance responses in Fig. S20.10b are sluggish after an initial oscillatory period, and the two MVs change very slowly. When the diagonal elements of the ***R*** matrix are increased to 0.5, the disturbance responses are even more sluggish.

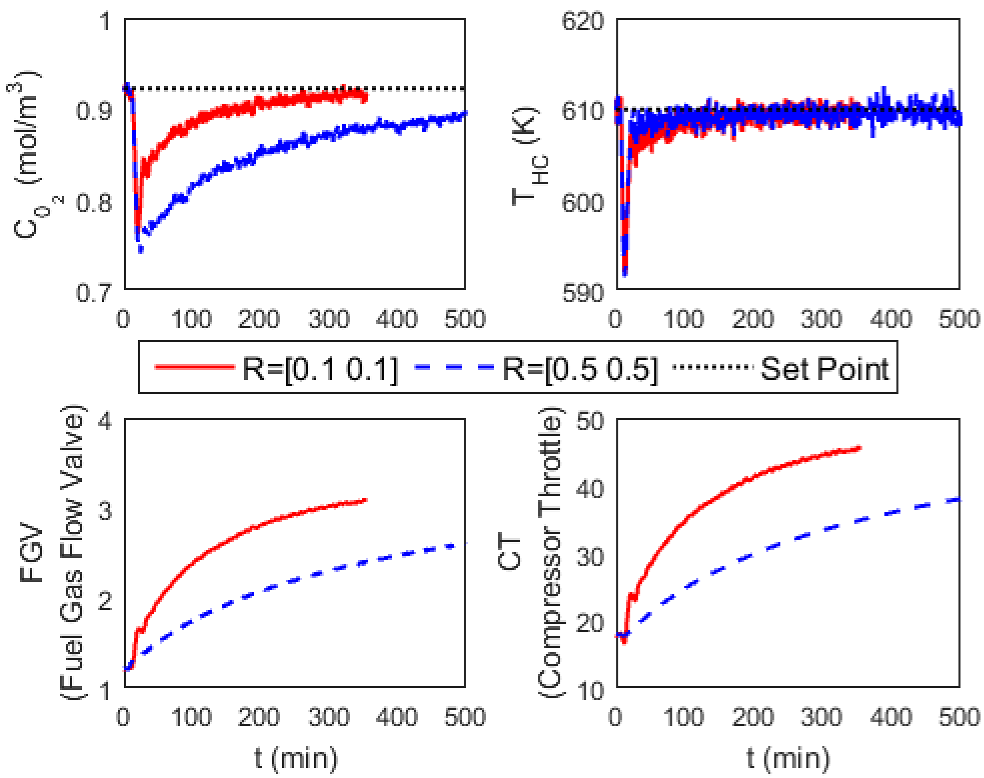


Figure S20.10b. *The two series represent* ***R*** *= diag [0.1, 0.1]* and ***R*** *= diag [0.5, 0.5]*.

20.11

We repeat 20.10 for R [0.1 0.1], Q = [0.1 1] and (a) M=1 and (b) M=4

First we evaluate the controller response to a step change in the oxygen concentration setpoint from 0.922 to 1.0143.

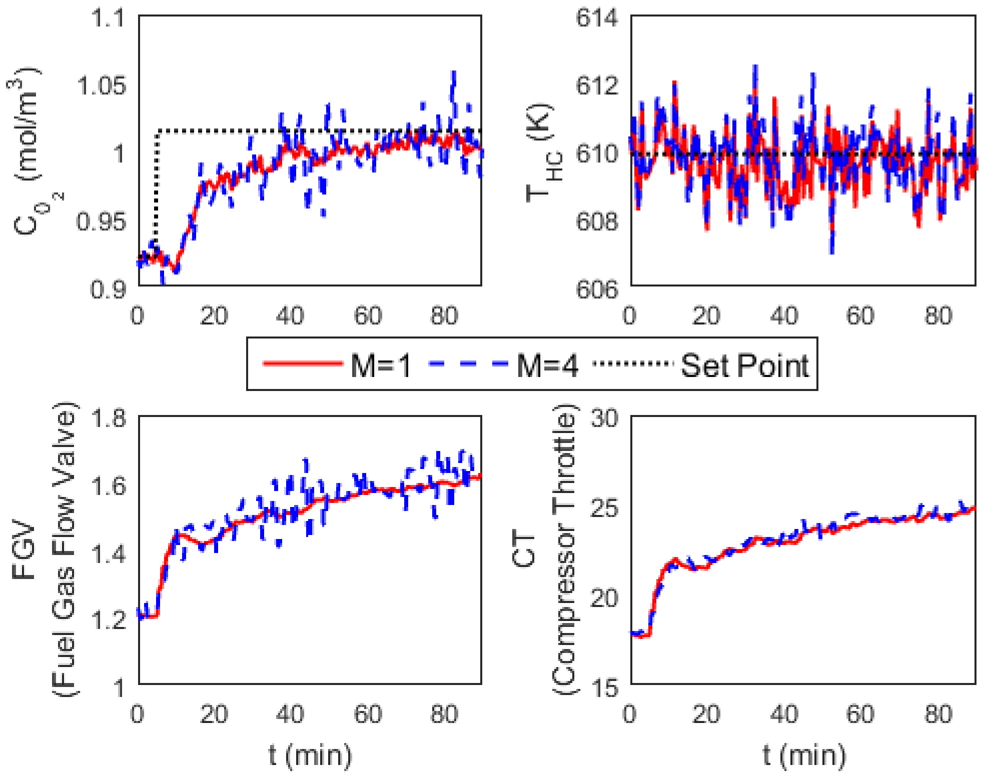


Figure S20.11a. *Step change in oxygen concentration setpoint for P=20,* $\mathbf{Q}$ = *diag [0.1, 1],* $\mathbf{R}$ = *diag [0.1, 0.1], and M=1 or M=4.*

Next we test a step change in the hydrocarbon flow rate from 0.035m^3/min to 0.038m^3/min.

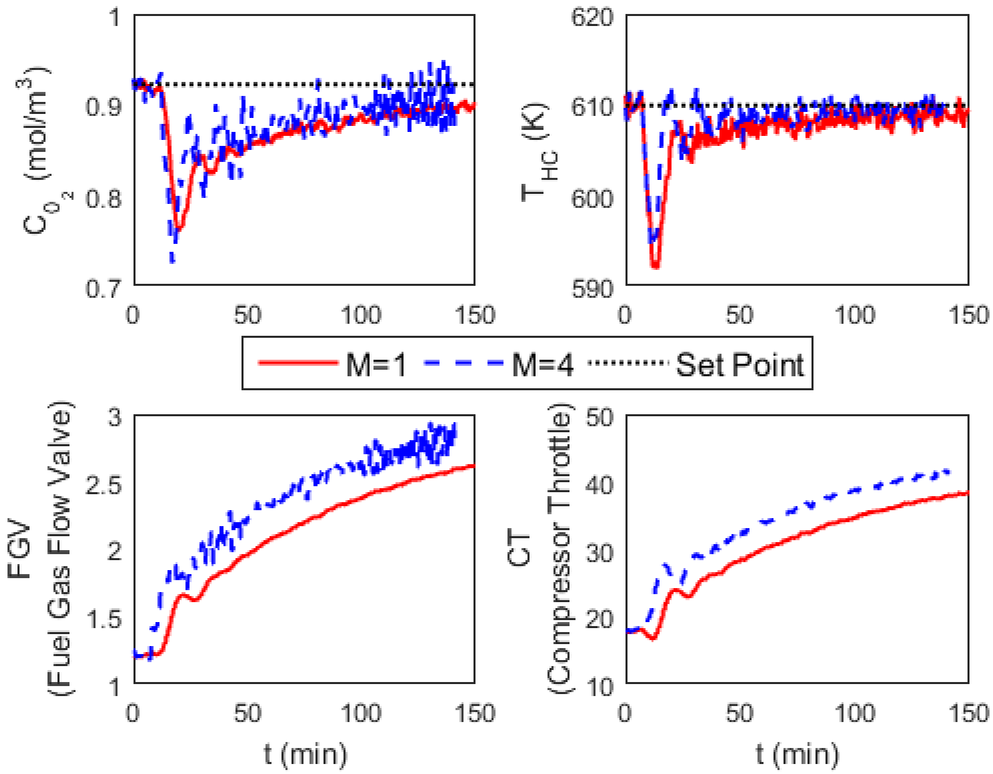


Figure S20.11b. *Step change in fuel gas flow rate for P=20,* ***Q*** *= diag [0.1, 1],* ***R*** *= diag [0.1, 0.1], and M=1 or M=4.*

20.12

Note: These results were generated using the PCM Distillation Column Module, MPC option

For parts (a) and (b), the step response for the models were generated in the workspace. Then the PCM distillation column module was opened. The controller parameters were entered into the MPC controller as specified in parts (a) and (b). Then, the tests described in parts (c)-(e) were carried out for each controller. The results are shown below.

(c) Step change in xD setpoint from 0.85 to 0.8

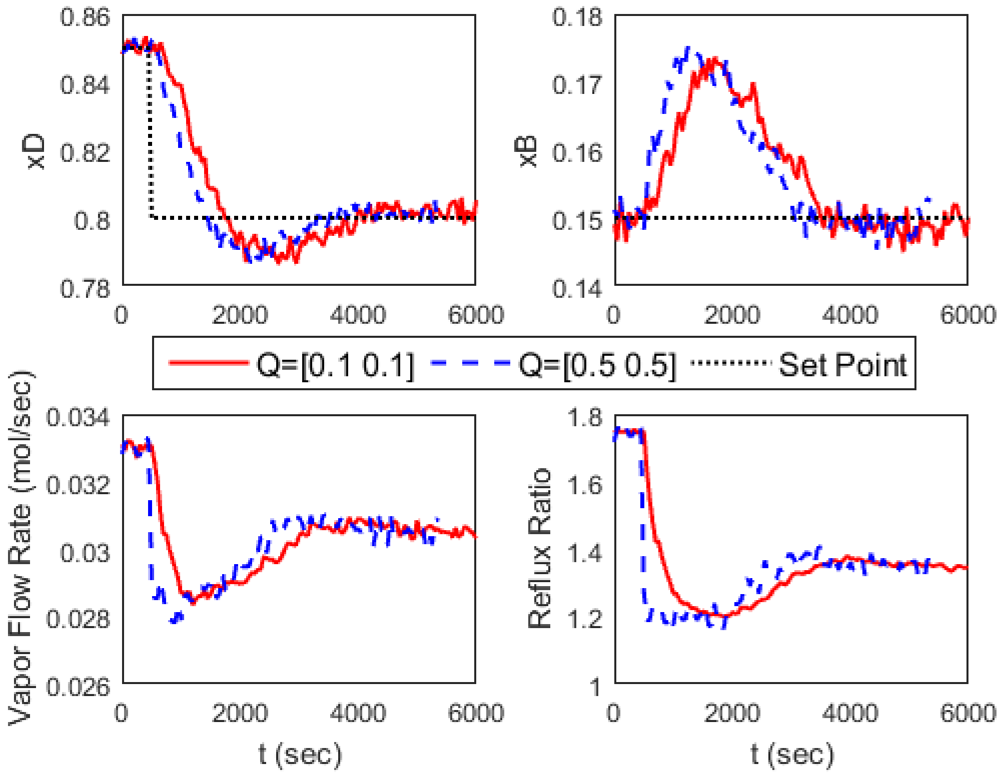


Figure S20.12a. *Step change in xD setpoint from 0.85 to 0.8 for Q=diag [0.1 0.1] and Q=diag [0.5 0.5]*.

(d) Step change in xB setpoint from 0.15 to 0.20

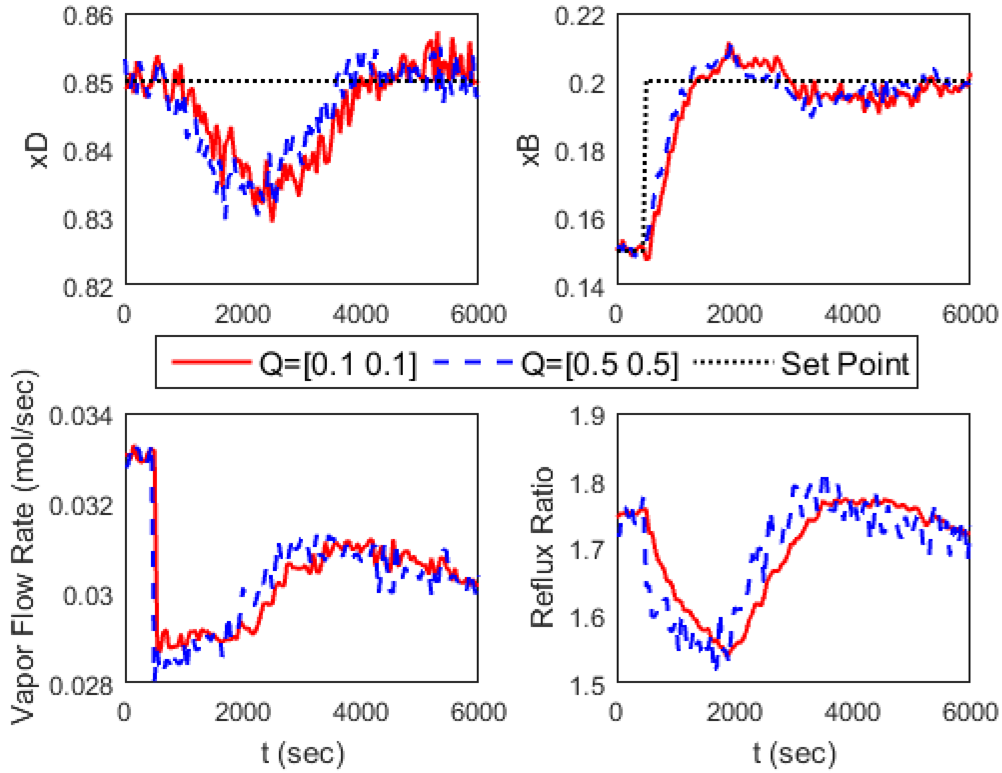


Figure S20.12b. *Step change in xB setpoint from 0.15 to 0.2 for Q=diag [0.1 0.1] and Q=diag [0.5 0.5].*

(e) Step change in column feed flow rate from 0.025 to 0.03

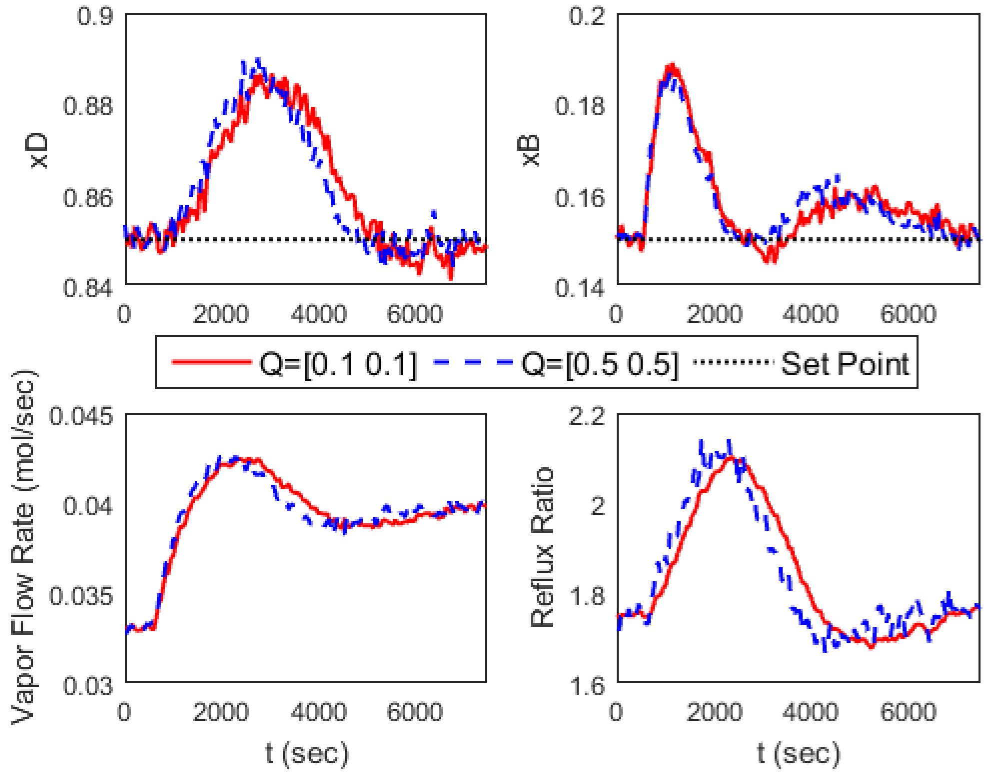


Figure S20.12a. *Step change in column feed flow rate from 0.025 to 0.03 for Q=diag [0.1 0.1] and Q=diag [0.5 0.5].*

20.13

We repeat problem 20.12, but this time we look at the case where R = [0.1 1], Q = [0.1 0.1], and M=1 or M=5. The same three tests are repeated from 20.12.

(c) Step change in xD from 0.85 to 0.8

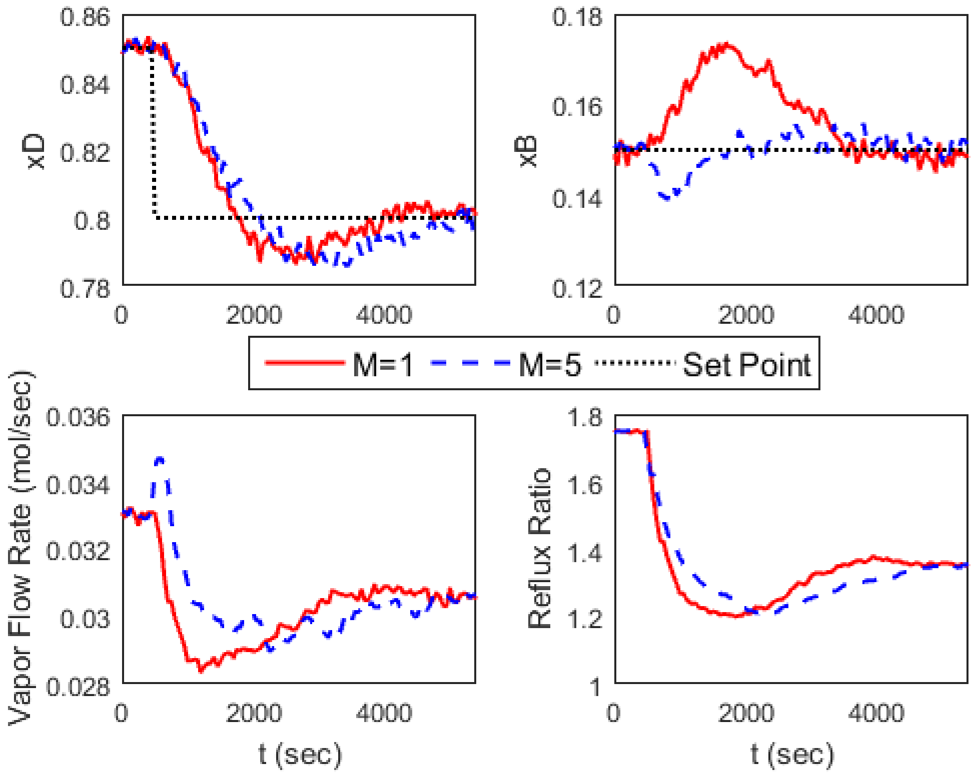


Figure S20.13a. *Step change in xD setpoint from 0.85 to 0.8 for M=1 and M=5.*

(d) Step change in xB setpoint from 0.15 to 0.20

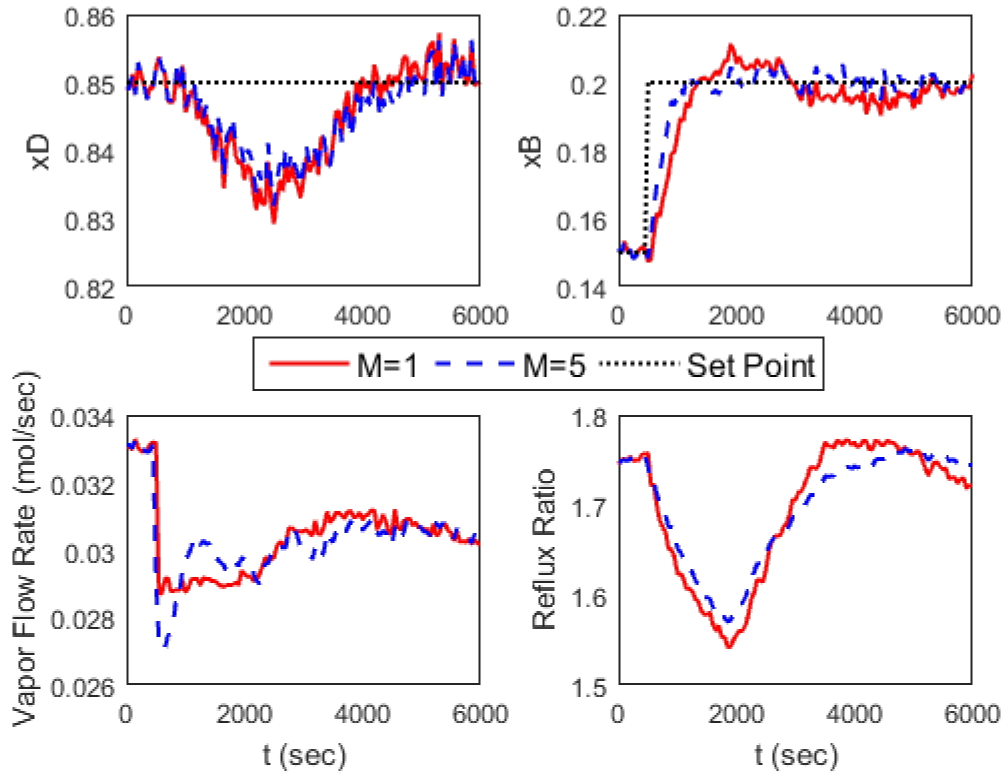


Figure S20.13b. *Step change in xB setpoint from 0.15 to 0.2 for M=1 and M=5.*

(e) Step change in column feed flow rate from 0.025 to 0.03

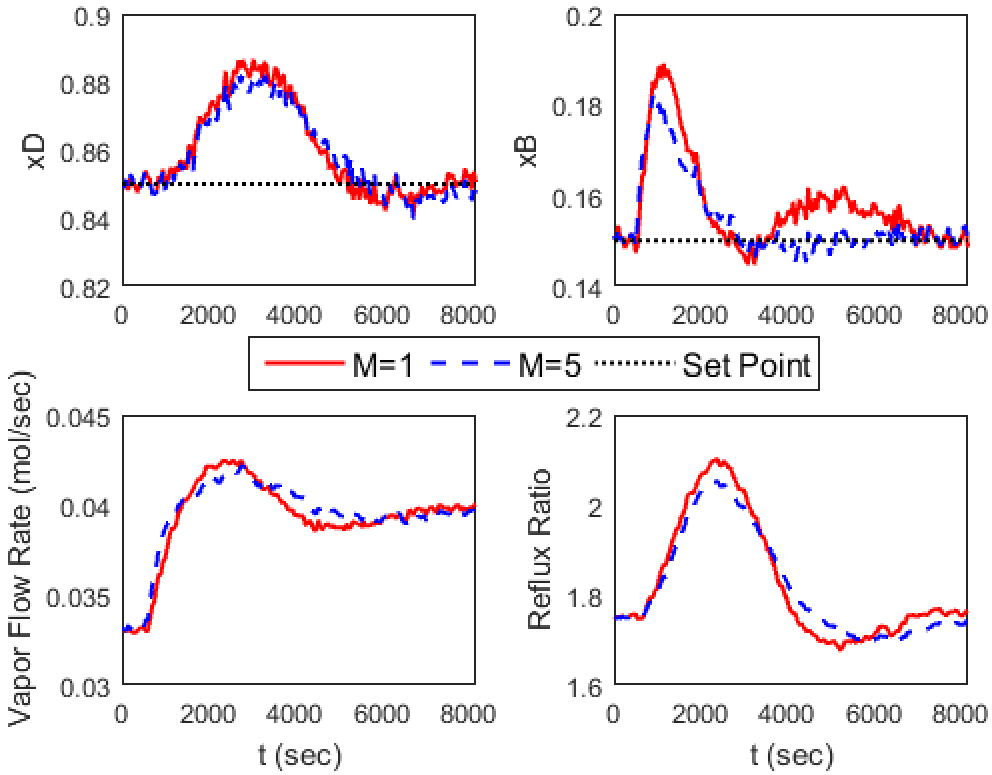


Figure S20.13c. *Step change in column feed flow rate from 0.025 to 0.03 for M=1 and M=5.*

Chapter 21 ©

21.1

No. It is desirable that the minimum value of the output signal be greater than zero, in order to readily detect instrument failures. Thus, for a conventional electronic instrument, an output signal of 0 mA indicates that a malfunction has occurred such as a power failure. If the instrument range were 0-20 mA, instead of 4-20 mA, the output signal could be zero during normal operation. Thus, instrument failures would be more difficult to detect.

21.2

The difference between a measurement of 6.0 and the sample mean, 5.75, is 0.25 pH units. Because the standard deviation is s = 0.05 pH units, this measurement is five standard deviations from the mean. If the pH measurement is normally distributed (an assumption), then Fig. 21.7 indicates that the probability that the measurement is less than or equal to five standard deviations from the mean is 0.99999943. Thus, the probability p of a measurement being greater than five standard deviations from the mean is only $p = 1 - 0.99999943 = 5.7 \times 10^{-7}$. Consequently, the probability that a measurement will be larger than five standard deviations from the mean is half of this value, p/2, or 2.85×10^{-7}. A very small value!

21.3

Make the usual SPC assumption that the temperature measurement is normally distributed. According to Eq. 21-6, the probability that the measurement is within three standard deviations from the mean is 0.9973 Thus, the probability that a measurement is beyond these limits, during routine operation is p = 1- 0.9973 = 0.0027. From Eq. 21-19, the average run length *ARL* between false positives is,

$$ARL = \frac{1}{p} \approx 366 \text{ samples}$$

Thus for a sampling period of one minute, on average we would expect a false positive every 366 min. Consequently, for an eight hour period, the expected number of false alarms N is given by:

$$N = \frac{(8\text{ h})(60\text{ samples / h})}{366\text{ samples/false alarm}} = 1.31 \approx \boxed{1\text{ false alarm}}$$

21.4

Let p denote the desired probability.

(a) $p = (0.95)^3 = 0.857$

(b) $p = (0.05)^3 = 1.25 \times 10^{-4}$

(c) A much better approach is available. The median of the three measurements is much less sensitive to a sensor failure. Thus, it should be used instead of the average.

21.5

A plot of the data in Figure S21.5 does not indicate any abnormal behavior.

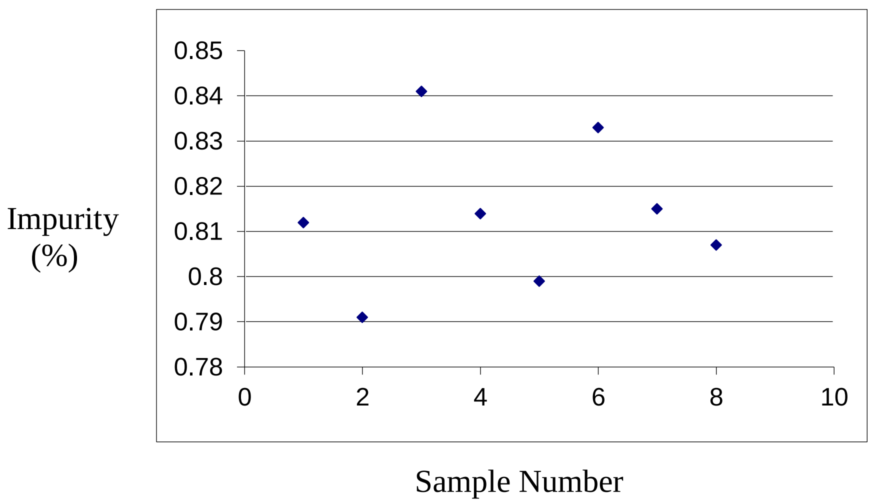


Figure S21.5. *Impurity data for Exercise 21.5.*

The following statistics and chart limits can be calculated from the data:

$$UCL = T + 3\sigma = 0.8 + 3(0.021) = 0.863\ \%$$
$$LCL = T - 3\sigma = 0.8 - 3(0.021) = 0.737\ \%$$

Figure S21.5 indicates that all eight data points are within the Shewhart chart limits.

A standard CUSUM chart (k=0.5, h=5) also does not exhibit any chart violations since the CUSUM chart limit is $h = 5\sigma = 0.105$ and neither C^+ or C^- calculated from Eq. 21-21 and 21-22 exceed this limit. The CUSUM calculations are shown in Table S21.5.

Table S21.5. CUSUM calculations for Exercise 21.5

Day	Impurity (%)	Deviation from Target	CUSUM +	CUSUM -
1	0.812	0.012	0.0015	0
2	0.791	-0.009	0.0015	0
3	0.841	0.041	0.0320	0
4	0.814	0.014	0.0355	0
5	0.799	-0.001	0.0355	0
6	0.833	0.033	0.0580	0
7	0.815	0.015	0.0625	0
8	0.807	0.007	0.0625	0

21.6

(a) The Shewhart chart for the rainfall data is shown in Fig. S21.6a. The following items were calculated from the data for 1870-1919:

s = 7.74 in. UCL = 41.9 in.
$\bar{x}$ =18.6 in. LCL = - 4.71 in. (actually zero)

The rainfall exceeded a chart limit for only one year, 1941.

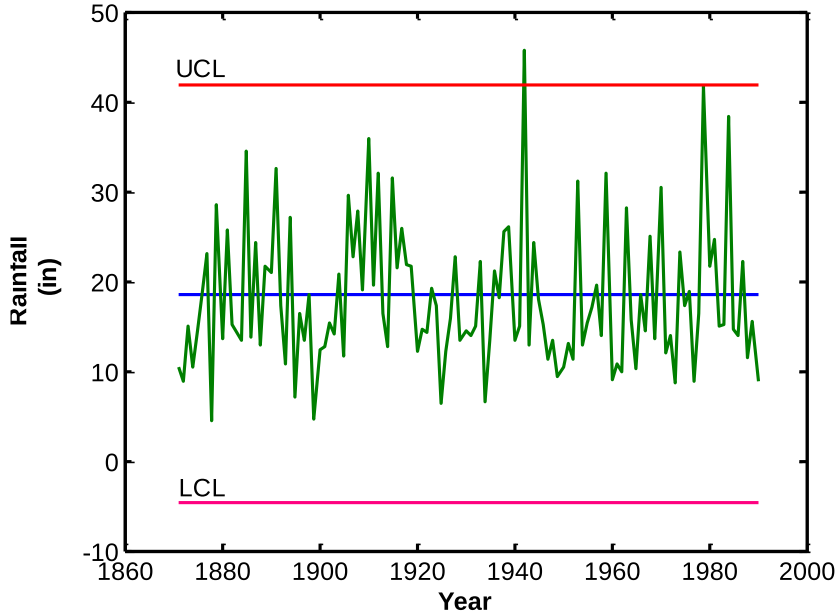


Figure S21.6a. *Shewhart chart for rainfall data.*

(b) The control chart for the standard deviation of the subgroup data (for each decade) is shown in Fig. S21.6b. The following items were calculated for the sub-group data prior to 1940:

$$\bar{s} = 6.87 \text{ in.}$$

$$UCL = B_4\bar{s} = (1.716)(6.87 \text{ in}) = 11.8 \text{ in.}$$

$$LCL = B_3\bar{s} = (0.284)(6.87 \text{ in}) = 1.95 \text{ in.}$$

The sub-group data does not violate the chart limits for 1940-1990.

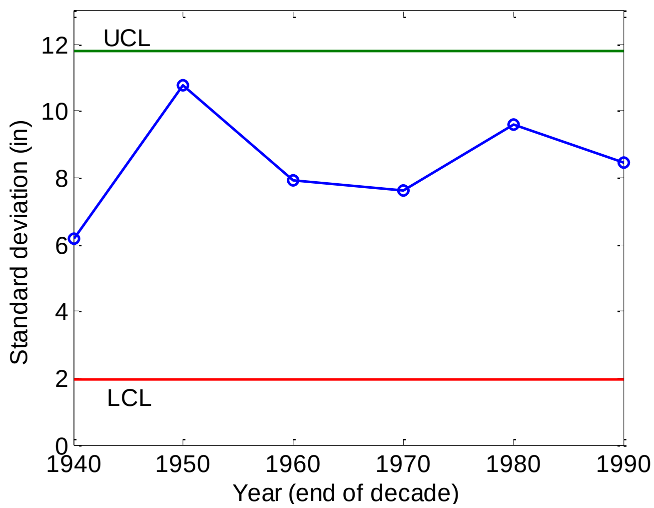


Figure S21.6b. *Standard deviations for sub-groups.*

21.7

The CUSUM and EWMA control charts for the period 1900-1960 are shown in Figure S21.7. The Shewhart chart and the data are also shown in the top portion, for the sake of comparison. The following statistics and chart limits were calculated from the data for 1900 through 1929:

s = 7.02 in. $\bar{x}$ =19.2 in.

Control Chart	**UCL (in.)**	**LCL (in.)**
Shewhart	40.2	- 1.9 (actually zero)
CUSUM	35.1	0
EWMA	27.1	11.2

The rainfall exceeded a Shewhart chart limit for only one year, 1941 the wettest year in the dataset. The CUSUM chart has both high (C^+) and low (C^-) chart violations during the initial period, 1900-1929. Two subsequent low limit violations occurred after 1930. After each CUSUM violation, the corresponding sum was reset to zero. No chart violations occur for the EWMA chart and the entire dataset.

The CUSUM chart indicate that the period from 1930 to 1950 had two dry spells while the Shewhart chart identifies one wet spell. The rainfall during the 1950s was quite normal.

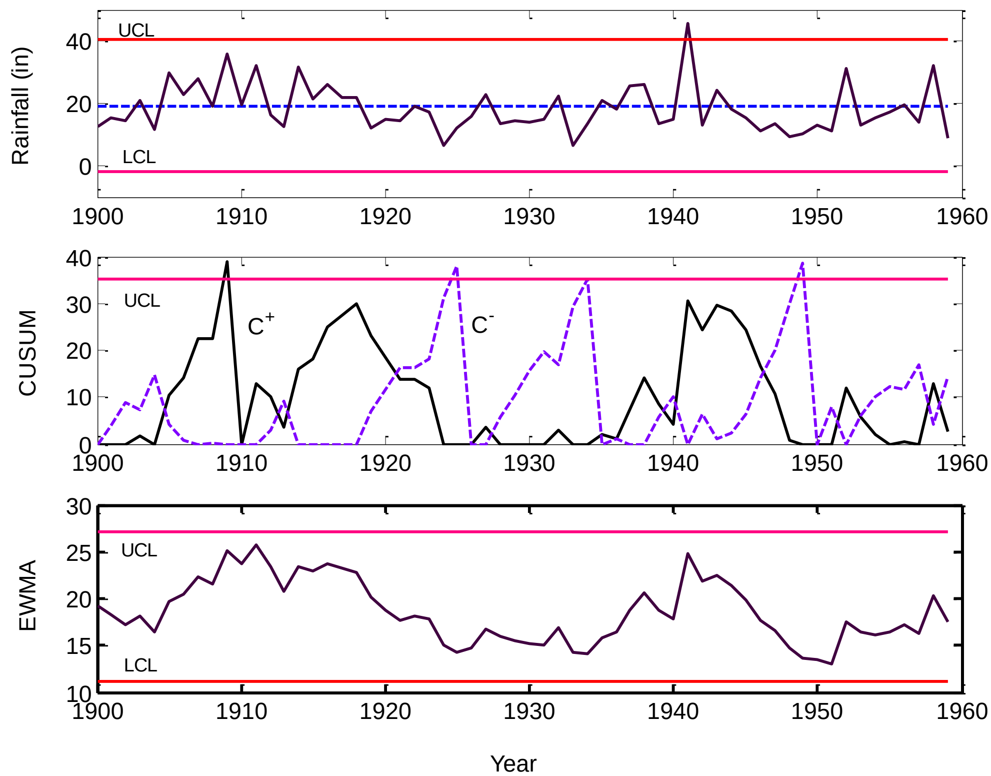


Fig. S21.7. *Control charts for Rainfall Data.*

21.8

In general, it is preferable to plot unfiltered measurements because they contain the most information. However, it is important to be consistent. Thus, if the control chart limits were calculated based on unfiltered data, unfiltered measurements should be plotted for subsequent monitoring. Conversely, if the chart limit calculations were based on filtered data, filtered measurements should be plotted.

21.9

The control charts in Fig. S21.9 do not exhibit any control chart violations. Thus, the process performance is considered to be normal. The CUSUM chart was designed using the default values of $K = 0.5\hat{\sigma} = 0.5s$ and $H = 5\hat{\sigma} = 5s$ where s is the sample standard deviation. The EWMA chart was designed using λ=0.25.

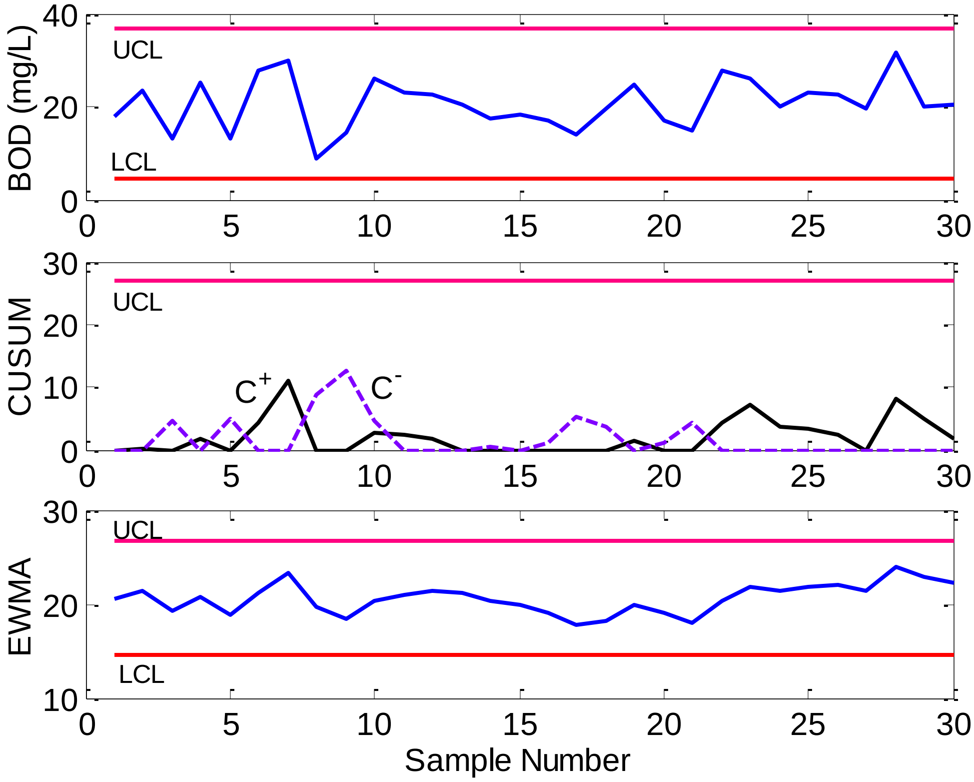


Figure S21.9. *Control charts for the BOD data of Example 21.5.*

21.10

By definition,

$$C_p \triangleq \frac{USL - LSL}{6\sigma} \qquad (21-25)$$

Because the population standard deviation σ is not known, it must be replaced by an estimate, $\hat{\sigma}$. Let $\hat{\sigma} = s$ where s is the sample standard deviation. The standard deviation of the BOD data is s = 5.41 mg/L. Substitution gives,

$$C_p = \frac{35-5}{6(5.41)} = \boxed{0.924}$$

Capability index C_{pk} is defined as:

$$C_{pk} \triangleq \frac{\min[\bar{x} - LSL, USL - \bar{x}]}{3\sigma} \qquad (21-26)$$

The sample mean for the BOD data is $\bar{x}$ = 20.6 mg/L. Substituting numerical values into (21-26) gives:

$$C_{pk} = \frac{\min[20.6 - 5, 35 - 20.6]}{3(5.41)} = \boxed{0.887}$$

Because both capability indices are less than one, the product specifications are not being met and process is considered to be performing poorly.

21.11

By definition,

$$C_p \triangleq \frac{USL - LSL}{6\sigma} \qquad (21-25)$$

Because the population standard deviation σ is not known, it must be replaced by an estimate, $\hat{\sigma}$. Let $\hat{\sigma}$ = *s* where s is the sample standard deviation. The standard deviation of the solids data is *s* = 56.3 mg/L. Substitution gives,

$$C_p = \frac{1600 - 1200}{6(56.3)} = \boxed{1.18}$$

Capability index C_{pk} is defined as:

$$C_{pk} \triangleq \frac{\min[\bar{x} - LSL, USL - \bar{x}]}{3\sigma} \qquad (21-26)$$

The sample mean for the solids data is $\bar{x}$ = 1413 mg/L. Substituting numerical values into (21-26) gives:

$$C_{pk} = \frac{\min[1413 - 1200, 1600 - 1413]}{3(56.3)} = \boxed{1.26}$$

Because both capability indices are well below the acceptable value of 1.5, the process is considered to be performing poorly.

21.12

The new data are plotted on a T^2 chart in Fig. S21.12. A chart violation occurs for the second data point. Because one of the six measurements is beyond the chart limit, it appears that the process behavior could be abnormal. However, this measurement may be an "outlier" and thus further investigation is advisable. Also, additional data should be collected before concluding that the process operation is abnormal.

Note that the previous control chart limit of 11.63 from Example 21.6 is also used in this exercise.

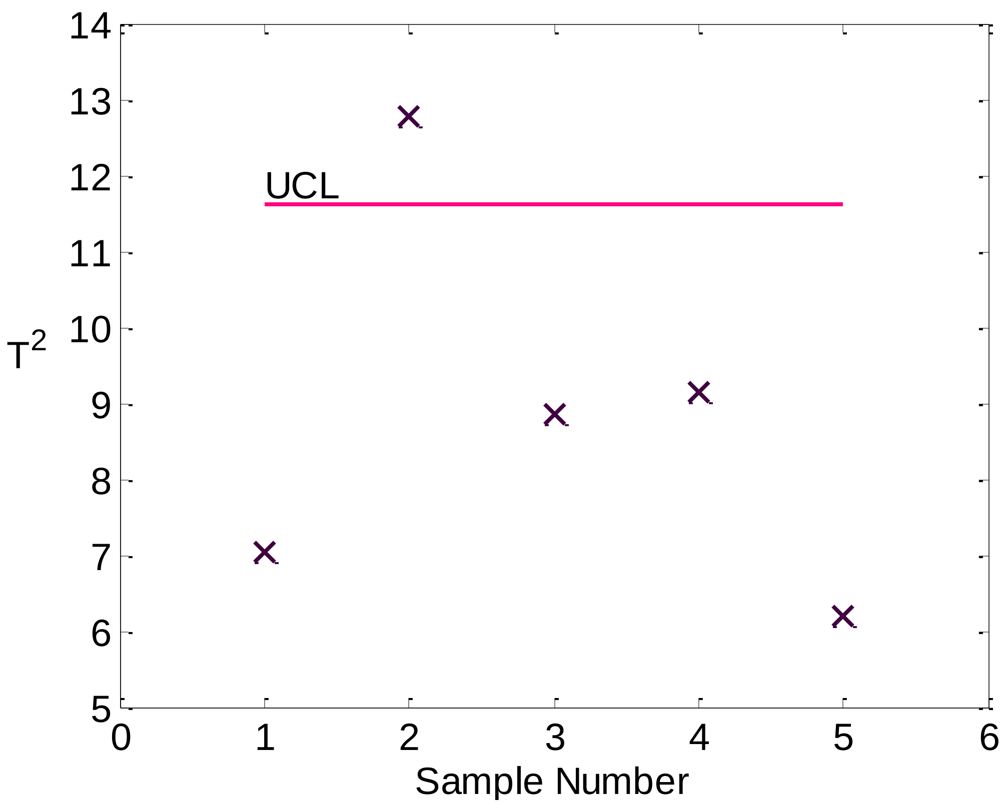


Figure S21.12. *T^2 Control chart and new wastewater data.*

Chapter 22

22.1

Microwave Operating States

Condition	Fan	Light	Timer	Rotating Base	Microwave Generator	Door Switch
Open the door Place the food inside	OFF	ON	OFF	OFF	OFF	ON
Close the door	OFF	OFF	OFF	OFF	OFF	OFF
Set the time	OFF	OFF	OFF	OFF	OFF	OFF
Heat up food	ON	ON	ON	ON	ON	OFF
Cooking complete	OFF	OFF	OFF	OFF	OFF	OFF

Safety Issues:

- Door switch is always OFF before the microwave generator is turned ON.
- Fan always ON when microwave generator is ON.

22.2

Input Variables:

ON
STOP
EMERGENCY

Output Variables:

START (1)
STOP (0)

Truth Table

ON	STOP	EMERGENCY	START/STOP
1	1	1	0
0	1	1	0
1	0	1	0
0	0	1	0
1	1	0	0
0	1	0	0
1	0	0	1
0	0	0	0

The truth state table is used to find the logic law that relates inputs with outputs:

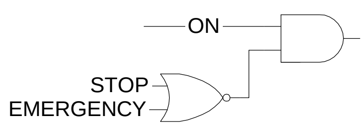
$$ON \bullet \overline{STOP} \bullet \overline{EMERGENCY}$$

Applying Boolean Algebra we can obtain an equivalent expression:

$$ON \bullet (\overline{STOP} \bullet \overline{EMERGENCY}\ ) = ON \bullet (\overline{STOP + EMERGENCY}\ )$$

Finally the binary logic and ladder logic diagrams are given in Figure S22.2:

<u>Binary Logic Diagram:</u>



<u>Ladder Logic Diagram</u>

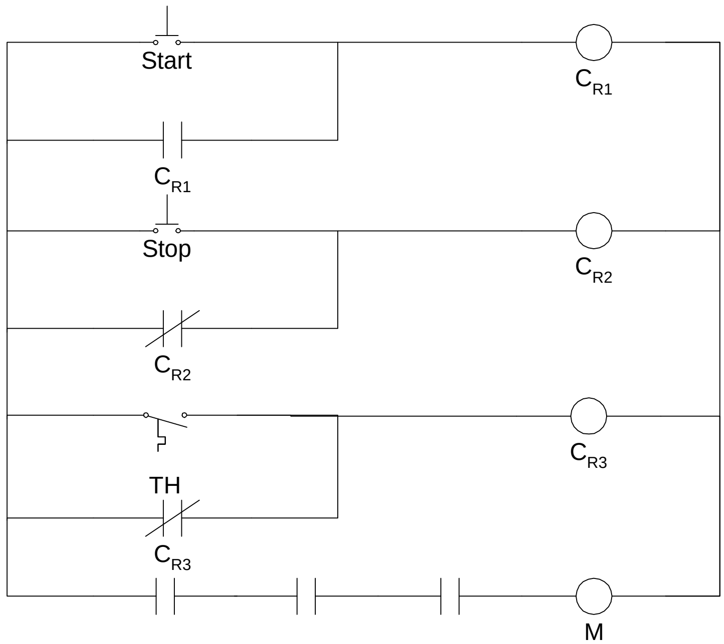


Figure S22.2.

22.3

A	B	Y
0	0	1
1	0	1
0	1	0
1	1	1

From the truth table it is possible to find the logic operation that gives the desired result,

$$\overline{\overline{A} \bullet B}$$

Since a NAND gate is equivalent to an OR gate with two negated inputs, our expression reduces to: $\overline{\overline{A} \bullet B} = A + \overline{B}$

Finally the binary logic diagram is given in Figure S22.3.

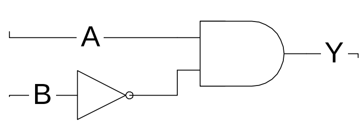
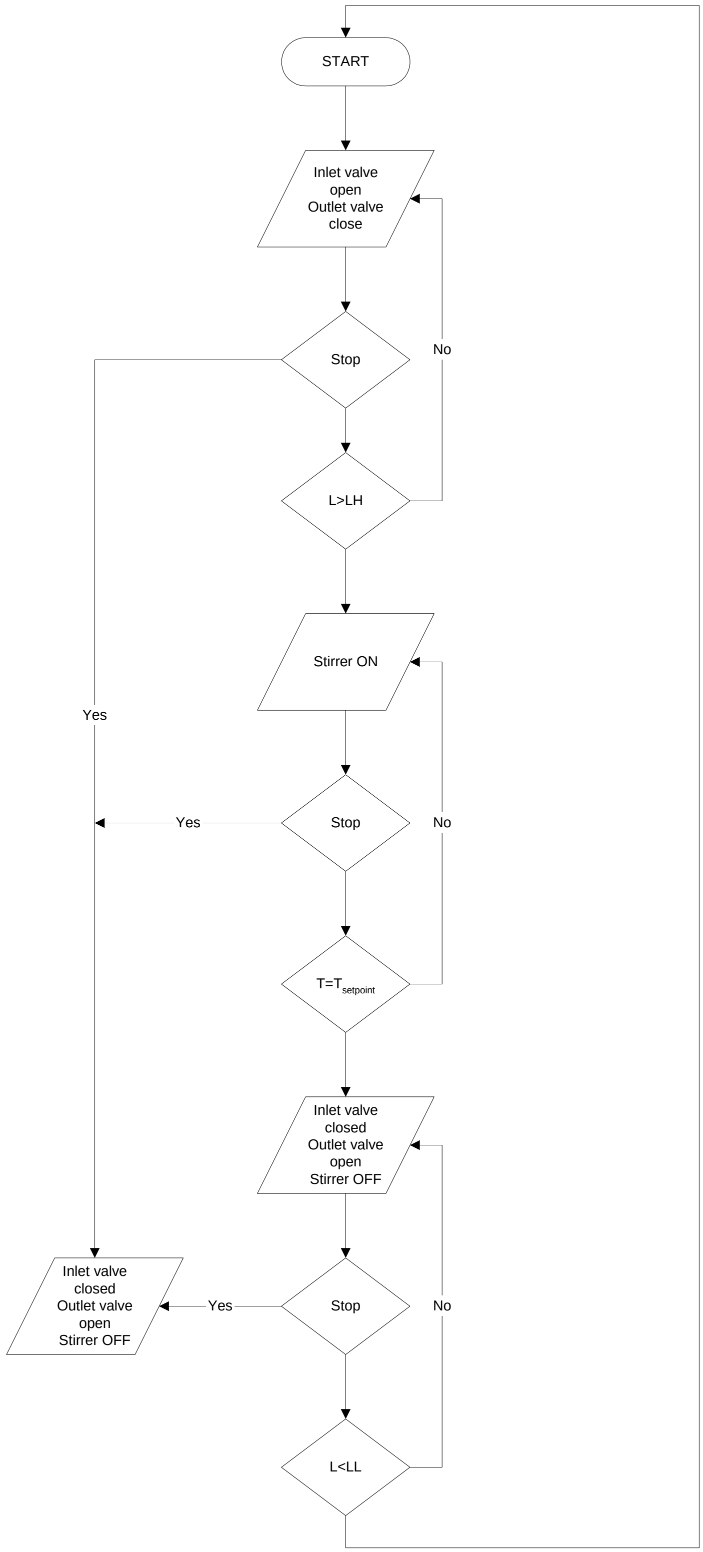


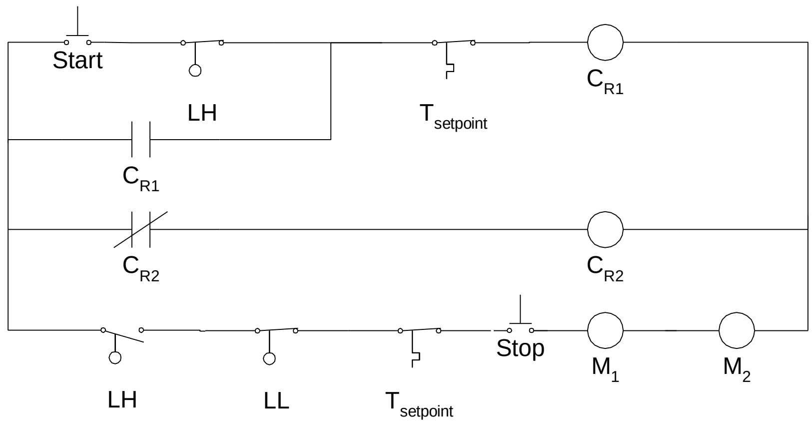
Figure S22.3.

22.4

Information Flow Diagram



Ladder Logic Diagram



Sequential Function Chart

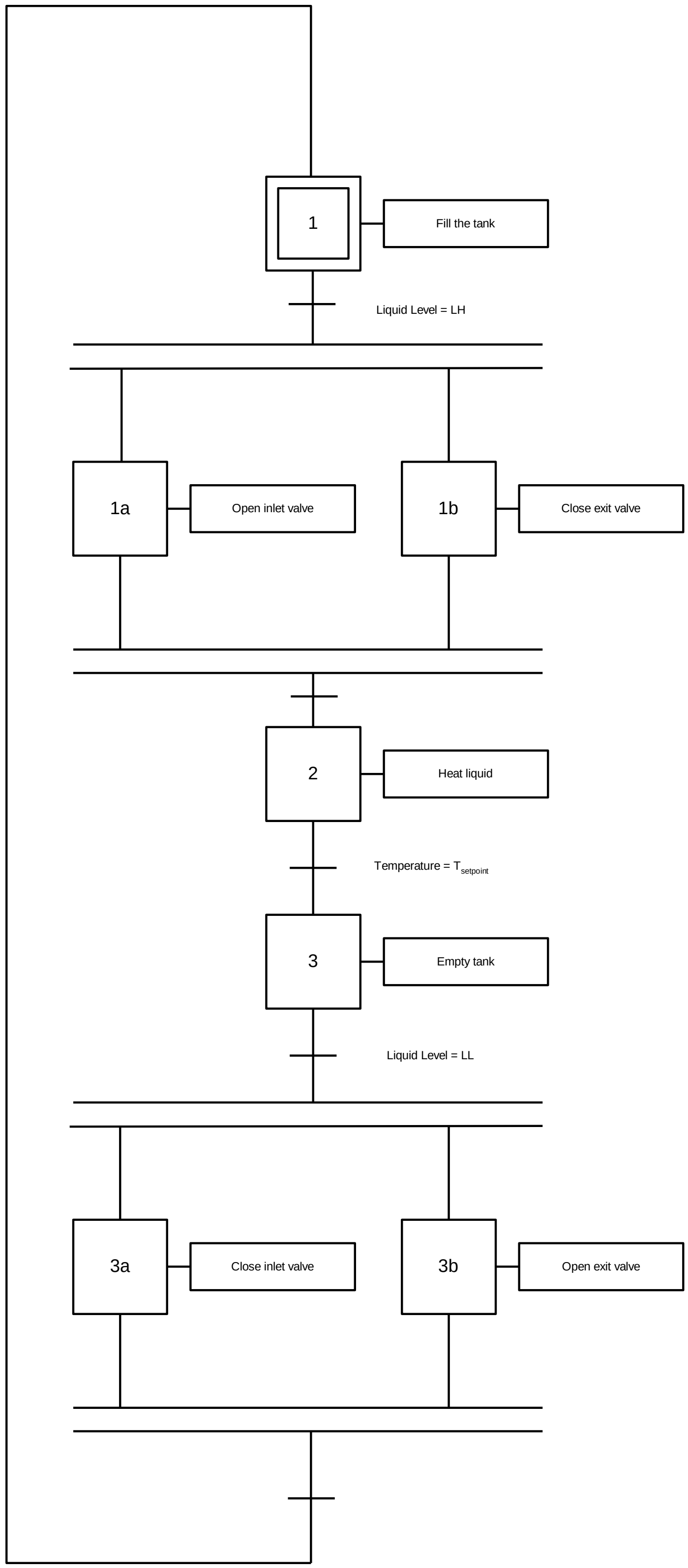
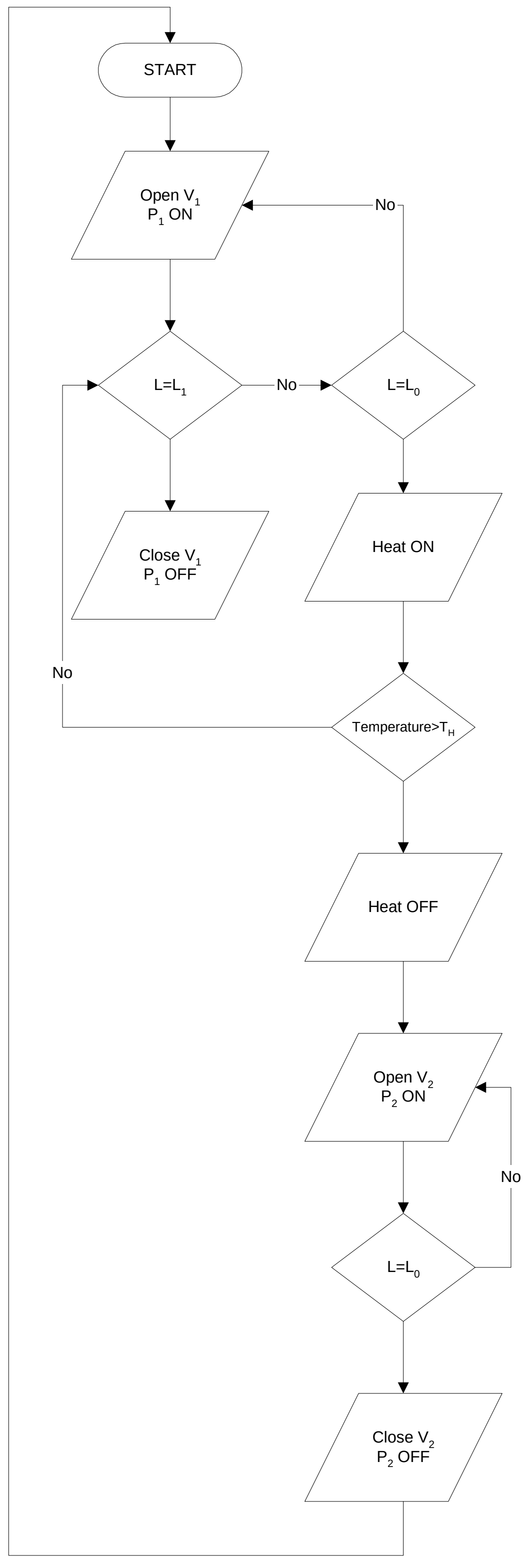

1
Fill the tank
Liquid Level = LH
1a
Open inlet valve
1b
Close exit valve
2
Heat liquid
Temperature = $T_{setpoint}$
3
Empty tank
Liquid Level = LL
3a
Close inlet valve
3b
Open exit valve

Figure S22.4.

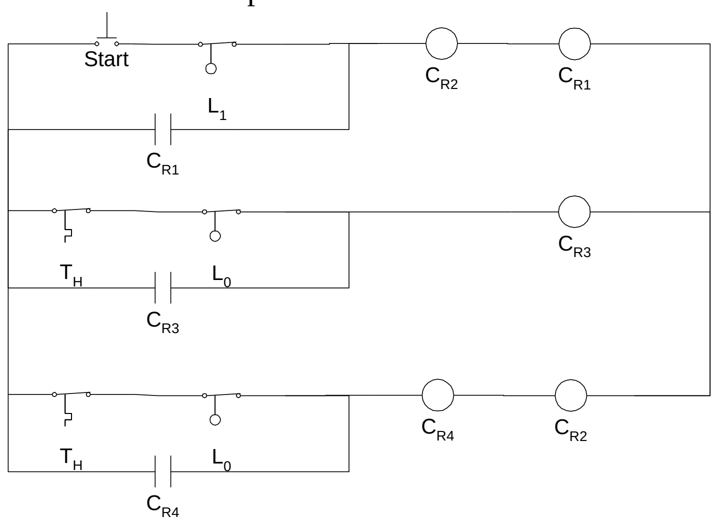
22.5

<u>Information Flow Diagram</u>



Ladder Logic Diagram:

R1= Pump 1   R2= Valve 2   R3= Heater   R4= Pump 2



Sequential Function Chart:

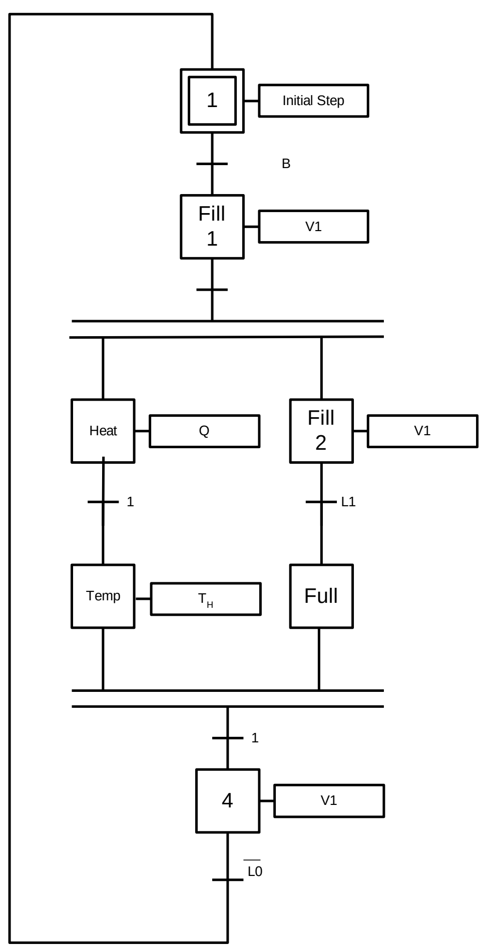
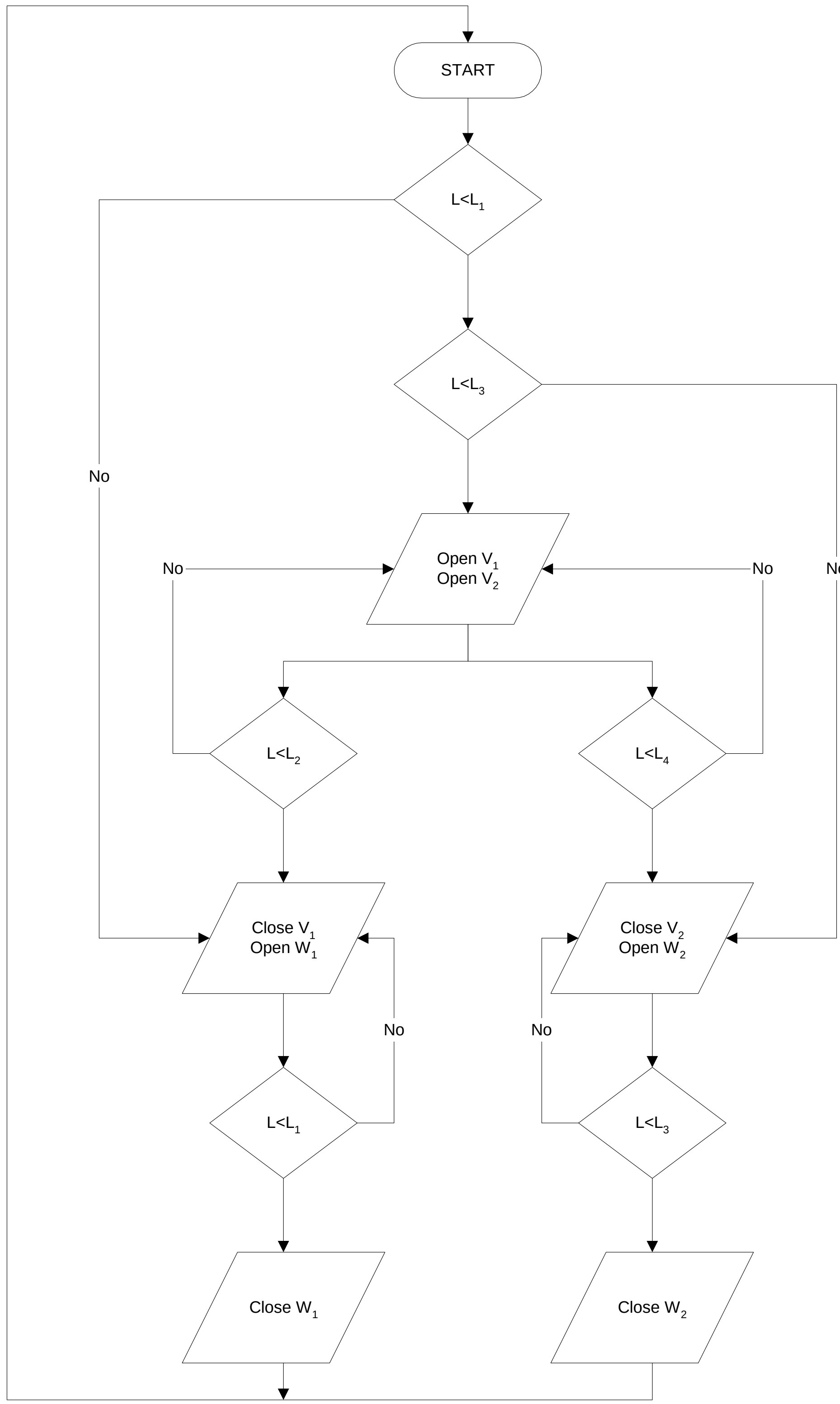


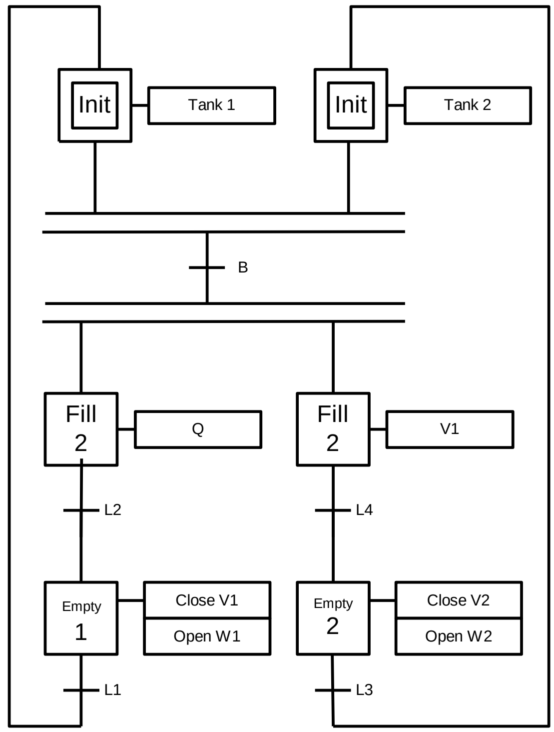
Figure S22.5.

22.6

Information Flow Diagram:



Sequential Function Chart:



Ladder Logic Diagram:

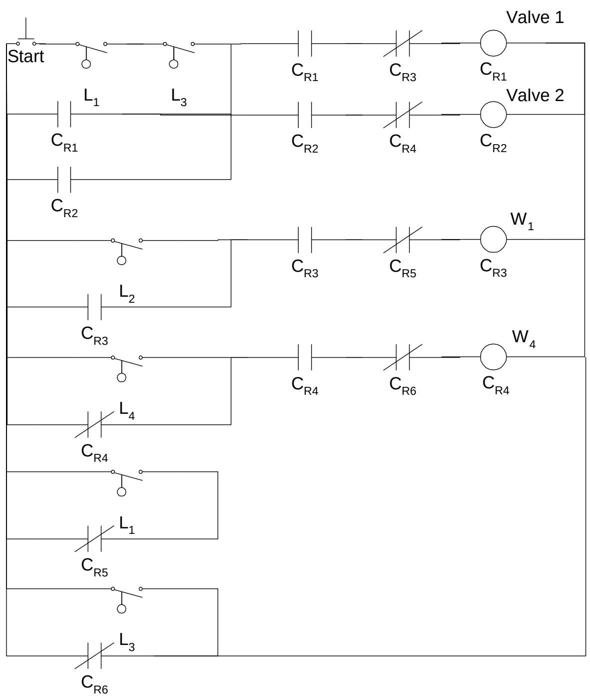
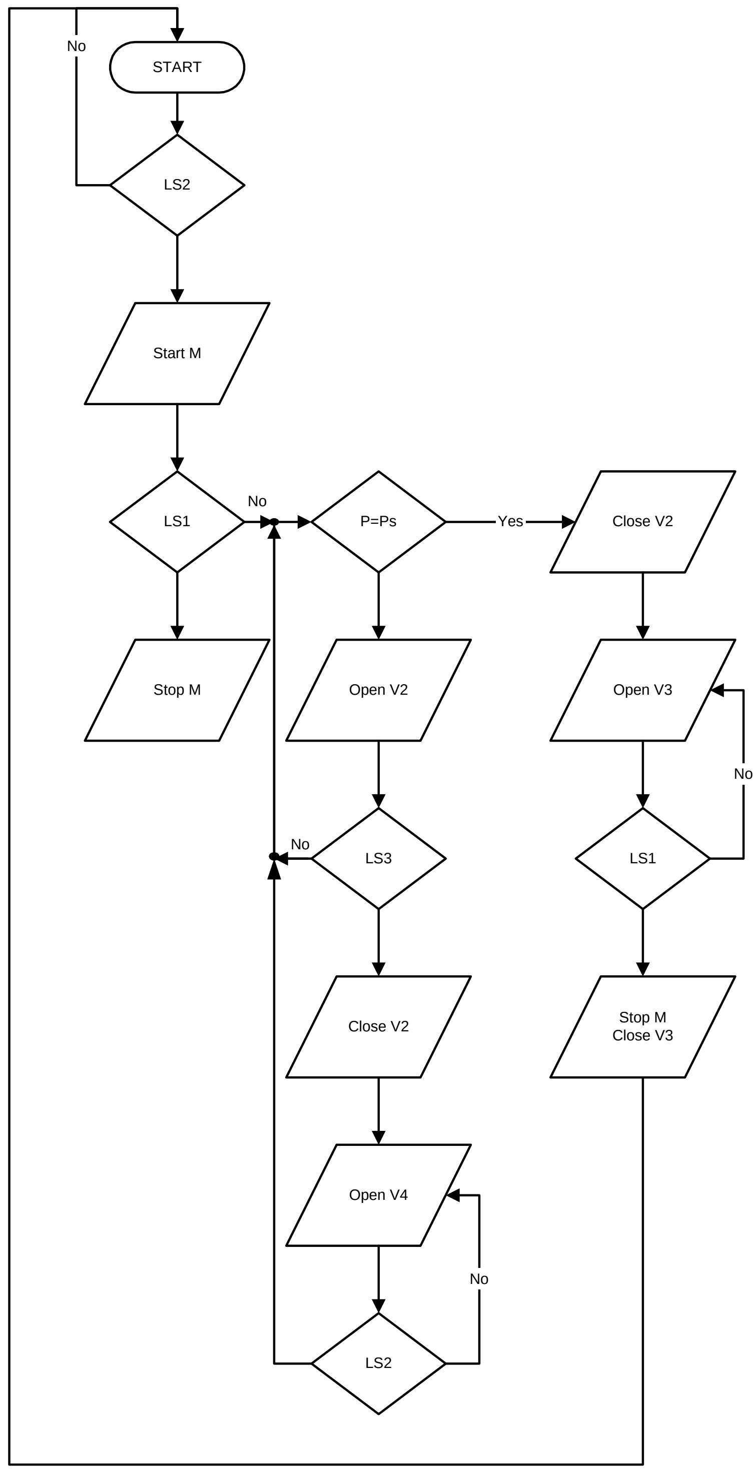


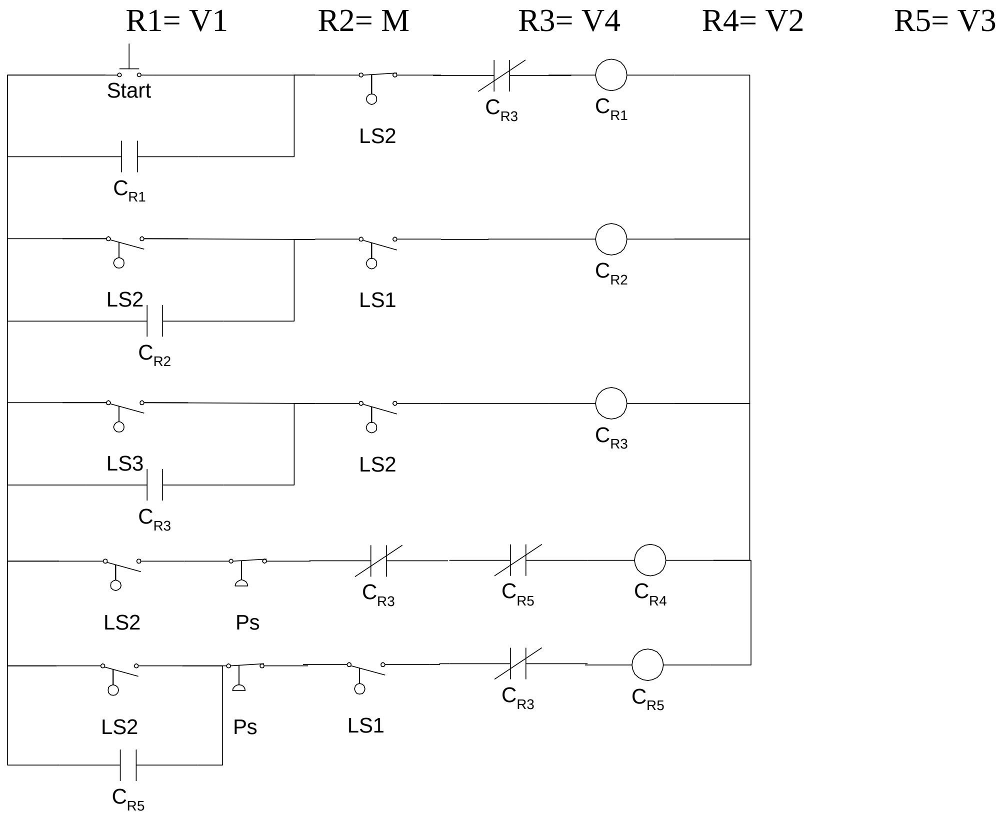
Figure S22.6.

22.7

Information Diagram:



Ladder Logic Diagram:



Sequential Function Chart:

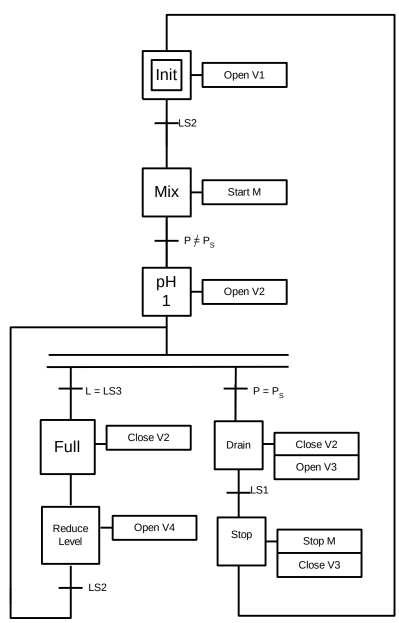


Figure S22.7.

22.8

In batch processing, a sequence of one or more steps is performed in a defined order, yielding a finished product of a specific quantity. Equipment must be properly configured in unit operations in order to be operated and maintained in a reasonable manner.

The discrete steps necessary to carry out this operation could be:

.- Open exit valve in tank car.

.- Turn on pump 1

.- Empty the tank car by using the pump and transfer the chemical to the storage tank (assume the storage tank has larger capacity than the tank car)

.- Turn off pump 1

.- Close tank car valve (to prevent backup from storage tank)

.- Open exit valve in storage tank.

.- Transfer the chemical to the reactor by using the second pump

.- Close the storage tank exit valve and turn off pump 2.

.- Wait for the reaction to reach completion.

.- Open the exit valve in the reactor.

.- Discharge the resulting product

Safety concerns:

Because a hazardous chemical is to be handled, several safety issues must be considered:

.- Careful and appropriate transportation of the chemical, based on safety regulation for that type of product.

.- Appropriate instrumentation must also be used. Liquid level indicators could be installed so that pumps are turned off based on level.

.- Chemical leak testing, detection, and emergency shut-down

.- Emergency escape plan.

Therefore, care should be exercised when transporting and operating hazardous chemicals. First of all, tanks and units should be vented prior to charging. Generally, materials should be stored in a cool dry, well-ventilated location with low fire risk. In addition, outside storage tanks must be located at minimum distances from property lines.

Pressure, level, flow and temperature control could be utilized in all units. Hence, they must be equipped with instrumentation to monitor these variables. For instance, tank levels can be measured accurately with a float-type device, and storage temperatures could be maintained with external heating pads operated by steam or electricity. It is possible for a leak to develop between the tank car and storage tank, which could cause high flow rates, so a flow rate upper limit may be desirable.

Valves and piping should have standard connections. Enough valves are required to control flow under normal and emergency conditions. Centrifugal pumps are often preferred for most hazardous chemicals. In any case, the material of construction must take into account product chemical properties.

Don't forget that batch process control often requires a considerable amount of logic and sequencing for their operation. Besides, interlocks and overrides are usually considered to analyze and treat possible failure modes.

22.9

1.- Because there is no steady state for a batch reactor, a new linearization point is selected at $t = 0$. Then,

Linearization point for batch reactor: $t = 0 \equiv t^*$

2.- Available information:

$$k = 2.4 \times 10^{15} e^{-20000/T} \text{ (min}^{-1}) \quad \text{where } T \text{ is in } {}^{\circ}\text{R}$$

$C = 0.843 \dfrac{\text{BTU}}{\text{lb}^\circ\text{F}}$ $\qquad V = 1336 \text{ ft}^3$

$\rho = 52 \dfrac{\text{lb}}{\text{ft}^3}$ $\qquad q = 26 \dfrac{\text{ft}^3}{\text{min}}$

$(-\Delta H) = 500 \dfrac{\text{kJ}}{\text{mol}}$ $\qquad C_{Ai} = 0.8 \dfrac{\text{mol}}{\text{ft}^3}$

$T_i = 150^\circ\text{F}$ $\qquad T_s = 25^\circ\text{C}$

$UA = 142.03 \dfrac{\text{kJ}}{\text{min}^\circ\text{F}}$

For continuous reactor, $\bar{T} = 150^\circ\text{F}$

Physical properties are assumed constant.

Problem solution:

A stirred batch reactor has the following material and energy balance equations:

$$-kC_A = \frac{dC_A}{dt} \tag{1}$$

$$(-\Delta H)kVC_A + UA(T_s - T) = V\rho C \frac{dT}{dt} \tag{2}$$

where $k = k_0 e^{-E/RT}$

From Eqs. 1 and 2, linearization gives:

$$-\left[k^* C_A{}^* + k^* C'_A + C_A{}^* k_0 e^{-E/RT^*} \frac{E}{RT^{*2}} T'\right] = \frac{dC'_A}{dt} \tag{3}$$

$$(-\Delta H)V \left[k^* C_A{}^* + k^* C'_A + C_A{}^* k_0 e^{-E/RT^*} \frac{E}{RT^{*2}} T'\right]$$

$$+UA(T'_s - T') = V\rho C \frac{dT'}{dt} \tag{4}$$

Rearranging, the following equations are obtained:

$$b_{11}C'_A + b_{12}T' = \frac{dC'_A}{dt} \tag{5}$$

$$b_{21}C_A + b_{22}T' + b_{23}T'_s = \frac{dT'}{dt} \tag{6}$$

where

$$b_{11} = -k_0 e^{-E/RT^*} = -13.615$$

$$b_{12} = -k_0 e^{-E/RT^*} C_A{}^* \left(\frac{E}{RT^{*2}} \right) = -0.586$$

$$b_{21} = \frac{(-\Delta H) k_0 e^{-E/RT^*}}{\rho C} = 155.30$$

$$b_{22} = \frac{1}{\rho C} (-\Delta H) k_0 e^{-E/RT^*} C_A{}^* \left(\frac{E}{RT^{*2}} \right) - \frac{UA}{\rho VC} = 6.66$$

$$b_{23} = \frac{UA}{\rho VC} = 2.43 \times 10^{-3}$$

From Example 4.8, substituting values for continuous reactor

$$a_{11} = -13.636$$

$$a_{12} = -8.35 \times 10^{-4}$$

$$a_{21} = 155.27$$

$$a_{22} = -0.0159$$

$$b_2 = 2.43 \times 10^{-3}$$

(Note that , from material balance, $\bar{C}_A = 0.00114$)

Hence the transfer functions relating the steam jacket temperature $T'_s(s)$ and the tank outlet concentration $C'_A(s)$ are:

Continuous reactor:

$$\frac{C'_A(s)}{T'_s(s)} = \frac{-2.03\times10^{-6}}{s^2+13.651s+0.3464} = \frac{-5.86\times10^{-6}}{2.887s^2+39.4s+1}$$

then $\tau_{dom} \approx 35\,\text{min}$

Batch reactor:

$$\frac{C'_A(s)}{T'_s(s)} = \frac{-1.424\times10^{-3}}{s^2+6.931s+0.26} = \frac{-5.47\times10^{-3}}{3.84s^2+26.65s+1}$$

then $\tau_{dom} \approx 25\,\text{min}$

As noted in transfer functions above, the time constant for the batch is smaller than the time constant for the continuous reactor, but the gain is much larger.

22.10

The reactor equations are:

$$\frac{dx_1}{dt} = -k_1 x_1 \qquad (1)$$

$$\frac{dx_2}{dt} = k_1 x_1 - k_2 x_2 \qquad (2)$$

where $k_1 = 1.335\times10^{10}e^{-75{,}000/(8.31\times T)}$ and $k_2 = 1.149\times10^{17}e^{-125{,}000/(8.31\times T)}$

By using MATLAB, this differential equation system can be solved using the command "*ode45*". Furthermore we need to apply the command "*fminsearch*" in order to optimize the temperature. In doing so, the results are:

a) Isothermal operation to maximize conversion ($x_2(8)$):

$T_{op} = 357.8$ K and $x_{2max} = 0.3627$

b) Cubic temperature profile: the values of the parameters in $T=a_0 + a_1 t + a_2 t^2 + a_3 t^3$ that maximize $x_2(8)$ are:

$$\begin{cases} a_0 = 372.78 \\ a_1 = -10.44 \\ a_2 = 2.0217 \\ a_3 = -0.1316 \end{cases} \quad \text{and} \quad x_{2max} = 0.3699$$

The optimum temperature profile and the optimum isothermal operation are shown in Fig. S22.10.

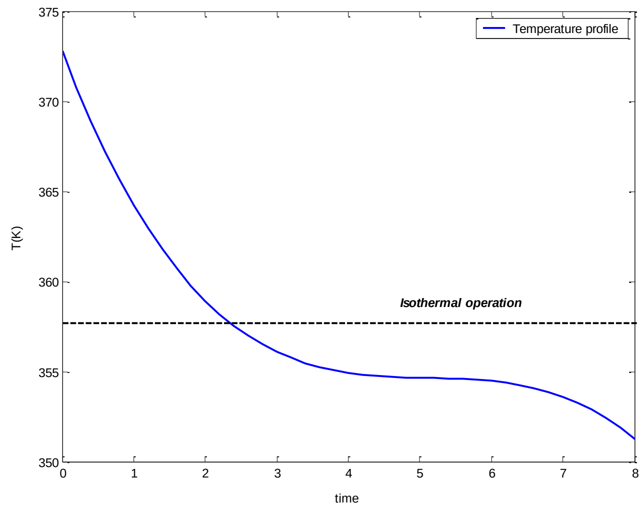


Figure S22.10. *Optimum temperature for the batch reactor.*

MATLAB simulation:

a) Constant temperature (First declare **Temp** as global variable)

1.- Define the differential equation system in a file called **batchreactor**.

```
function dx_dt=batchreactor(time_row,x)
global Temp
dx_dt(1,1)=-1.335e10*x(1)*exp(-75000/8.31/Temp);
dx_dt(2,1)=1.335e10*x(1)*exp(-75000/8.31/Temp) -
1.149e17*x(2)*exp(-125000/8.31/Temp);
```

2.- Define a function called **conversion** that gives the final value of x_2 (given a value of the temperature)

```
function x2=conversion(T)
global Temp
Temp=T;
x_0=[0.7,0];
[time_row, x] = ode45('batchreactor', [0 8], x_0 );
x2=-(x(length(x),2));
```

3.- Find the optimum temperature by using the command fminsearch

```
[T,negative_x2max]=fminsearch('conversion', To)
```

where T_o is our initial value to find the optimum temperature.

b) Temperature profile (First declare **a0 a1 a2 a3** as global variables)

1.- Define the differential equation system in a file called **batchreactor2**.

```
function dx_dt=batchreactor2(time_row,x)
global a0 a1 a2 a3
Temp=a0+a1*time_row+a2*time_row^2+a3*time_row^3;
dx_dt(1,1)=-1.335e10*x(1)*exp(-75000/8.31/Temp);
dx_dt(2,1)=1.335e10*x(1)*exp(-75000/8.31/Temp) -
1.149e17*x(2)*exp(-125000/8.31/Temp);
```

2.- Define a function called **conversion2** that gives the final value of x_2 (given the values of the temperature coefficients)

```
function x2b=conversion(a)
global a0 a1 a2 a3
a0=a(1);a1=a(2);a2=a(3);a3=a(4);x_0=[0.7,0];
[time_row, x] = ode45('batchreactor2', [0 8], x_0 );
x2b=-x(length(x),2);
```

3.- Find the optimum temperature profile by using the command fminserach

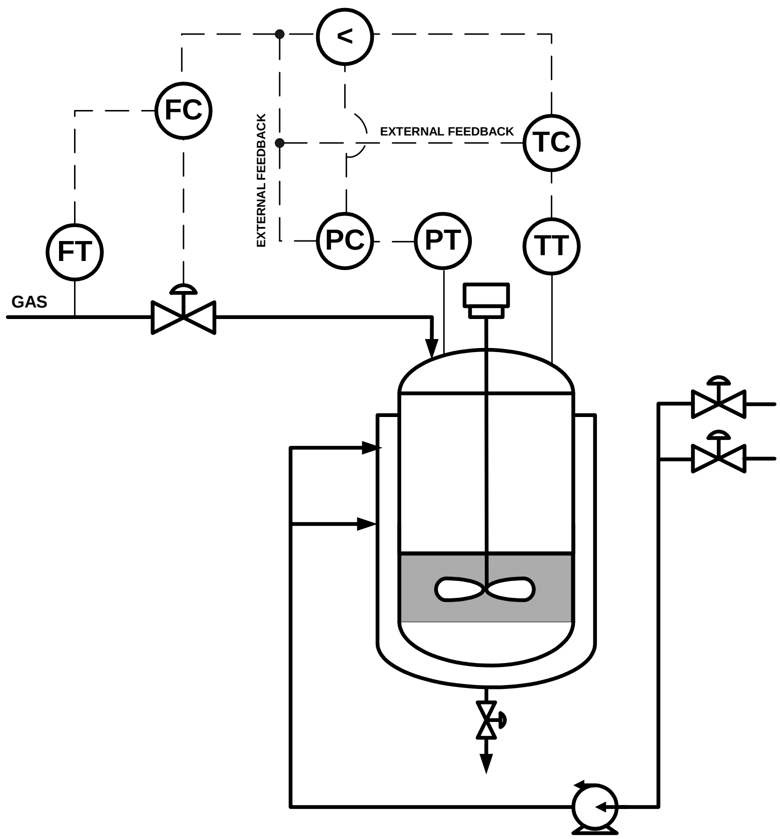
```
[T,negative_x2max]=fminsearch('conversion2', ao)
```

where a_o is the vector of initial values to find the optimum temperature profile.

22.11

The intention is to run the reactor at the maximum feed rate of the gas to minimize the time cycle, but the reactor is also cooling-limited. Therefore, if the pressure controller calls for a gas flow that exceeds the cooling capability of the reactor, the temperature will start to rise. The reaction temperature is not critical, but it must not exceed some maximum temperature. The temperature controller will then take over control of the feed valve and reduce the feed rate. The output of the selector sets the setpoint of a flow controller. The flow controller minimizes the effects of supply pressure changes on the gas flow rate. So this is a cascade type control system, with the primary controller being an override control system.

In an override control system, one of the controllers is always in a standby condition, which will cause that controller to saturate. Reset windup can be prevented by feeding back the selector relay output to the setpoint of each controller. Because the reset actions of both controllers have the same feedback signal, control will transfer when both controllers have no error. Then the outputs of both controllers will be equal to the signal in the reset sections. Because neither controller has any error, the outputs of both controllers will be the same. Particular attention must be paid to make sure that at least one controller in an override control system will always be in control. If not, then one of the controllers can wind up, and reset windup protection is necessary.



22.12

Material balance:

$$(-r_A) = -\frac{dC_A}{dt} = kC_{A0}{}^2(1-X)(\Theta_B - 2X)$$

Since

$$C_A = C_{A0}(1-X)$$

then

$$\frac{dX}{dt} = -\frac{1}{C_{A0}}\frac{dC_A}{dt}$$

Therefore

$$\boxed{\frac{dX}{dt} = kC_{A0}(1-X)(\Theta_B - 2X)} \qquad (1)$$

<u>Energy balance</u>:

$$\boxed{\frac{dT}{dt} = \frac{Q_g - Q_r}{NC_p}} \qquad (2)$$

where $Q_g = kC_{AO}{}^2(1-X)(\Theta_B - 2X)V(\Delta H_{RX})$

$Q_r = UA(T-298)$

Eqs. 1 and 2 constitute a differential equation system. By using MATLAB, this system can be solved as long as the initial conditions are specified. Command "*ode45*" is suggested.

A.- ISOTHERMAL OPERATION UP TO 45 MINUTES

We will first carry out the reaction isothermally at 175 °C up to the time the cooling was turned off at 45 min.

Initial conditions : $X(0) = 0$ and $T(0) = 448$ K

Figure S22.12a shows the isothermal behavior for these first 45 minutes.

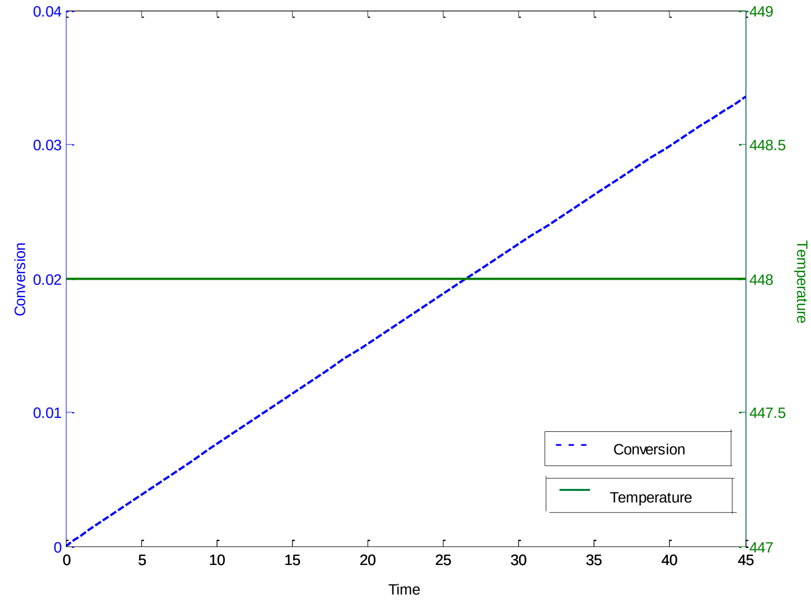


Figure S22.12a. *Isothermal behavior for the first 45 minutes*

B.- ADIABATIC OPERATION FOR 10 MINUTES

The cooling is turned off for 45 to 55 min. We will now use the conditions at the end of the period of isothermal operation as our initial conditions for adiabatic operation period between 45 and 55 minutes.

$$t = 45 \text{ min} \quad X = 0.033 \quad T = 448$$

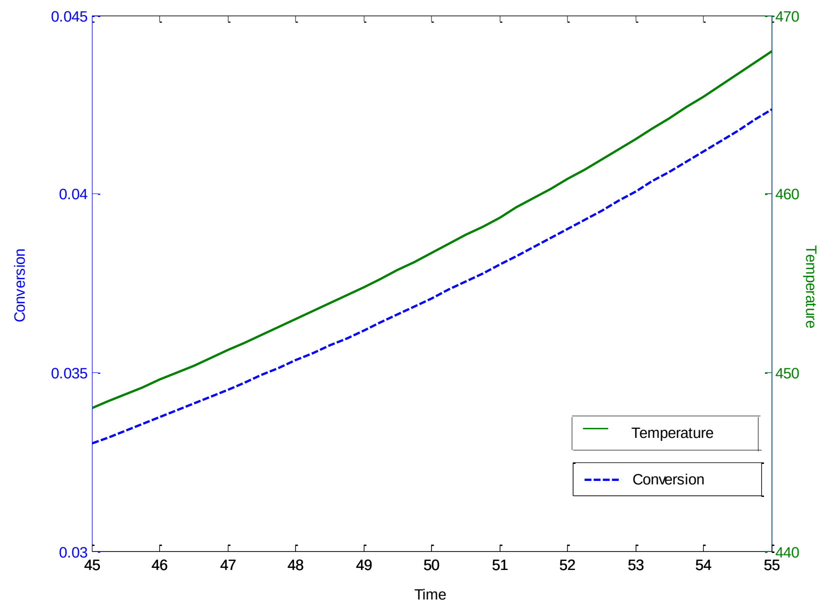


Figure S22.12b. *Adiabatic operation when the cooling is turned off.*

C.- BATCH OPERATION WITH HEAT EXCHANGE

Return of the cooling occurs at 55 min. The values at the end of the period of adiabatic operation are:

$t = 55$ $\quad T = 468$ K $\quad X = 0.0423$

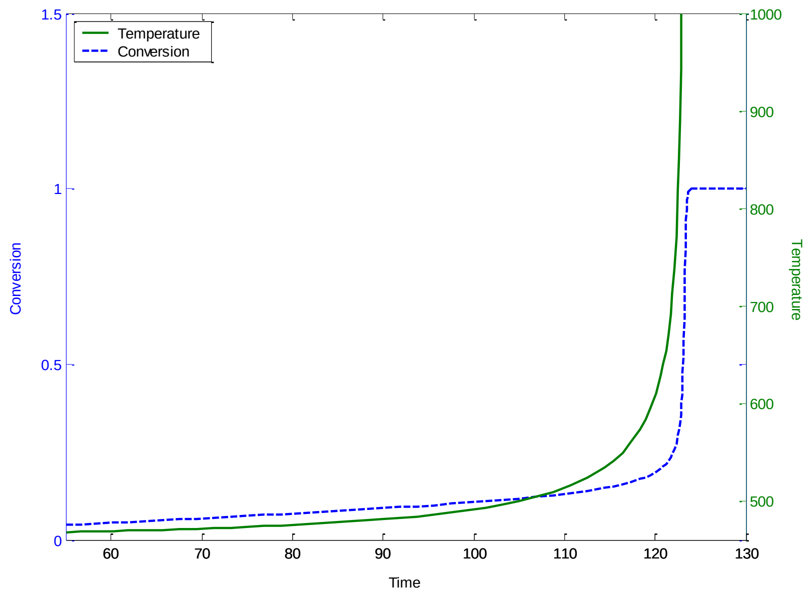


Figure S22.12c. *Batch operation with Heat Exchange; temperature runaway.*

As shown in Fig. S22.12c, the temperature runaway is finally unavoidable under new conditions:

. Feed composition = 9.044 kmol of ONCB, 33.0 kmol of NH_3, and 103.7 kmol of H_20

. Shut off cooling to the reactor at 45 minutes and resume cooling reactor at 55 minutes.

MATLAB simulation:

1.- Let's define the differential equation system in a file called reactor.

```
function dx_dt=reactor(t,x)

dx_dt(1,1)=((17e-5*exp(11273/1.987*(1/461-
1/x(2))))*1.767*(1-x(1))*(3.64-2*x(1)));

dx_dt(2,1)=((-(17e-5*exp(11273/1.987*(1/461-
1/x(2))))* 122*(1-x(1))*(3.64-2*x(1))*5.119*(-5.9e5)
-  35.85*(x(2)-298))/2504 );
```

where `dx_dt(2,1)` the must be equal to 0 for the isothermal operation

2.- By using the command "ode45", system above can be solved

```
[times_row,x]=ode45('reactor',[tₒ, t_f],[X₀,T₀]);
plot(times_row,x(:,1),times_row,x(:,2));
```

where t_o, t_f, X_0 and T_0 must be specified for each interval.

22.13

T_r = Reactor temperature profile
T_{jsp} = Jacket set-point temperature profile (manipulated variable)

a) PID controller:

K_c = 26.5381
τ_I = 2.8658
τ_D = 0.4284

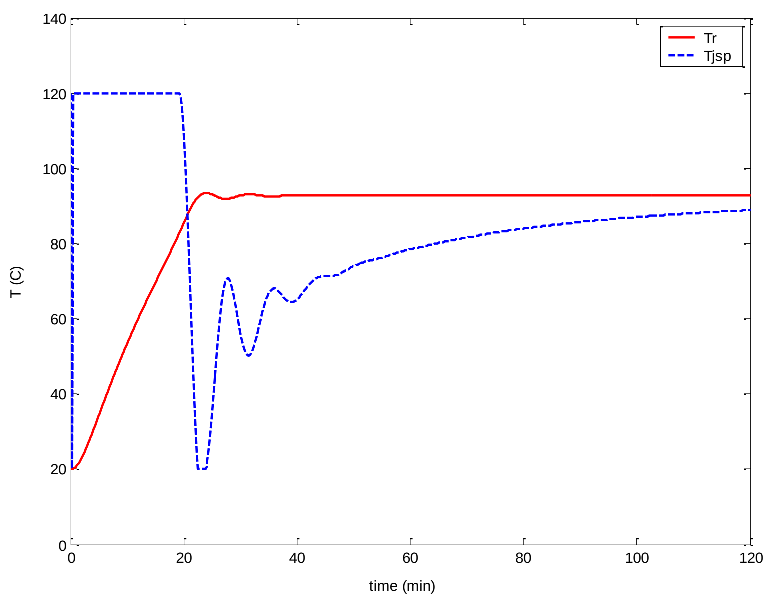


Figure S22.13a. *Numerical simulation for PID controller.*

b) Batch unit

$K_c = 10.7574$
$\tau_I = 53.4882$

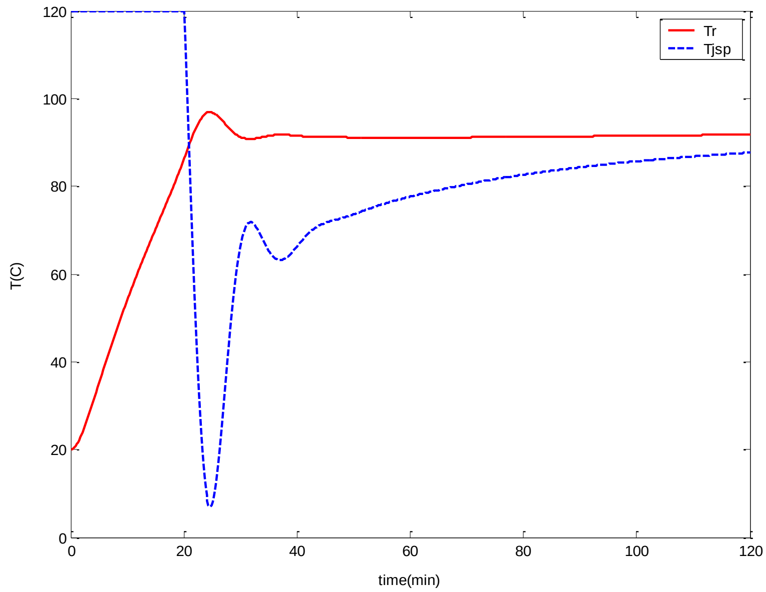


Figure S22.13b. *Numerical simulation for batch unit.*

c) Batch unit with preload

$K_c = 10.7574$
$\tau_I = 53.4882$

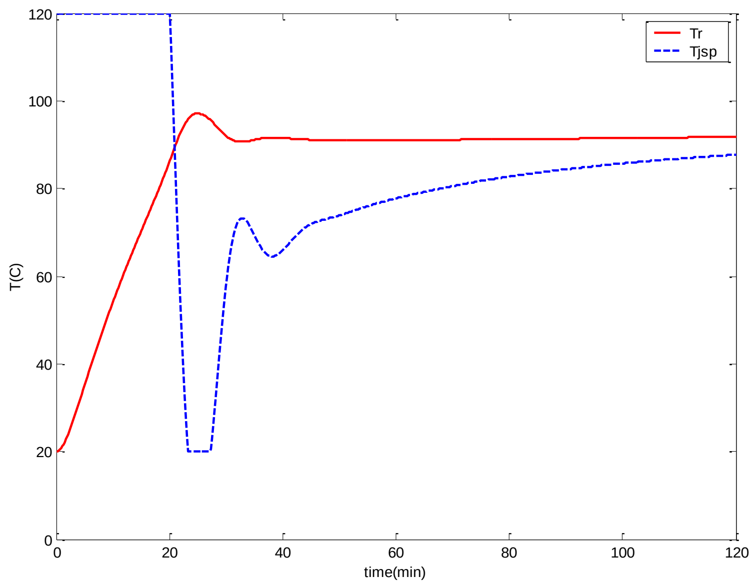


Figure S22.13c. *Numerical simulation for batch unit with preload.*

d) Dual mode controller

1.- Full heating is applied until the reactor temperature is within 5% of its set point temperature.
2.- Full cooling is then applied for 2.8 min
3.- The jacket temperature set point T_{jsp} of controller is then set to the preload temperature (46 °C) for 2.4 min.

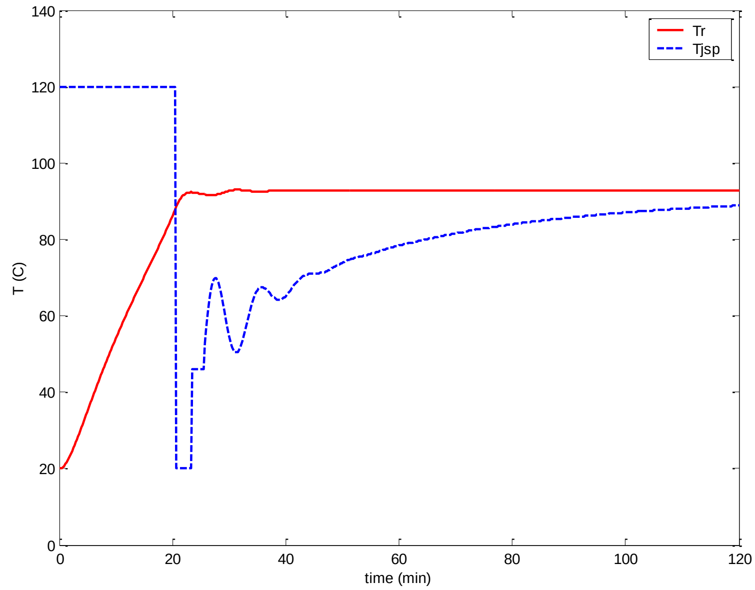


Figure S22.13d. *Numerical simulation for dual-mode controller.*

MATLAB simulation:

1.- Define a file called brxn:

```
function dy=brxn(t,y)
%
% Batch reactor example
%  Cott & Machietto (1989); "Temperature control
%  of exothermic batch reactors using generic model
%  control", I&EC Research, 28, 1177
%

% Parameters
 cpa=18.0; cpb=40.0; cpc=52.0; cpd=80.0;
 cp=0.45; cpj=0.45;
 dh1=-10000.0; dh2=6000.0;
 uxa=9.76*6.24;
 rhoj=1000.0;
 k11=20.9057; k12=10000;
 k21=38.9057; k22=17000;
 vj=0.6921;
```

```
 tauj=3.0;
 wr=1560.0;
 dy=zeros(7,1);
 ma=y(1); mb=y(2); mc=y(3); md=y(4); tr=y(5);
tj=y(6);
 tjsp=y(7);

 k1=exp(k11-k12/(tr+273.15));
 k2=exp(k21-k22/(tr+273.15));
 r1=k1*ma*mb;
 r2=k2*ma*mc;
 qr=-dh1*r1-dh2*r2;
 mr=ma+mb+mc+md;
 cpr=(cpa*ma+cpb*mb+cpc*mc+cpd*md)/mr;
 qj=uxa*(tj-tr);

 dy(1)=-r1-r2;
 dy(2)=-r1;
 dy(3)=r1-r2;
 dy(4)=r2;
 dy(5)=(qr+qj)/(mr*cpr);
 dy(6)=(tjsp-tj)/tauj-qj/(vj*rhoj*cpj);
 dy(7)=0;
```

Note: The error between the reactor temperature and its set-point (e=cvsp-cv) is computed at each sampling time. That is, control actions are computed in the discrete-time. For the integral action, error is simply summed (*se* = *se*+*e*). Controller output is estimated by *mv*=*Kc***e*+*Kc*/*taui***se***st*, where *Kc* = proportional gain, *taui*=integral time, *e*=error, *se*=summation of error and *st*=sampling time

2.- PID controller simulation

```
clear
clf
%
%  batch reactor control system
%  PID controller (velocity form)
%

% process initial values
ma=12.0; mb=12.0; mc=0; md=0; tr=20.0; tj=20.0;
tjsp=20.0;

y0=[ma,mb,mc,md,tr,tj,tjsp];

% controller initial values
kc=26.5381; taui=2.8658; taud=0.4284;
en=0; enn=0;
cvsp=92.83; mv=20;

% simulation
st=0.2;
t0=0; tfinal=120;
```

```
ntf=round(tfinal/st)+1;
cvt=zeros(1,ntf); mvt=zeros(1,ntf);

for it=1:ntf
[tt,y]=ode45('brxn',[(it-1)*st it*st],y0);
 y0=y(length(y(:,1)),:);

 cv=y0(5);

 % PID control calculation

 e=cvsp-cv;
 mv=mv+kc*(e*st/taui+(e-en)+taud*(e-2*en+enn)/st);
 if mv>120, mv=120; elseif mv<20, mv=20; end
 enn=en; en=e;

 y0(7)=mv;

 cvt(it)=cv; mvt(it)=mv;
 end

 t=(1:it)*st;
 plot(t,cvt,'-r',t,mvt,'--g')
```

3.- Batch unit simulation

```
% controller
 kc=10.7574; taui=53.4882;
 mh=120; ml=20; mq=46;

 mv=20;
 cvsp=92.83;

 % simulation
 st=0.2;
 z=ml; al=exp(-st/taui);
 t0=0; tfinal=120;
 ntf=round(tfinal/st)+1;

 for it=1:ntf
 [tt,y]=ode45('brxn',[(it-1)*st,it*st],y0);
 y0=y(length(y(:,1)),:);

 cv=y0(5);

 e=cvsp-cv;
 m=kc*e+z;

 if m>mh, m=mh;
 end
 f=m
 z=al*z+(1-al)*f; [f z m]

y0(7)=m;
```

```
 cvt(it)=cv;
mvt(it)=m;
end

t=(1:it)*st;
plot(t,cvt,'-r',t,mvt,'-g');
```

4.- Batch unit with preload simulation

```
% controller
kc=10.7574; taui=53.4882;
mh=120; ml=20; mq=46;
mv=20;
cvsp=92.83;

 % simulation
 st=0.2;
 z=ml; al=exp(-st/taui);
 t0=0; tfinal=120;
 ntf=round(tfinal/st)+1;

 for it=1:ntf
 [tt,y]=ode45('brxn',[(it-1)*st,it*st],y0);
 y0=y(length(y(:,1)),:);
 cv=y0(5);
 e=cvsp-cv;
 m=kc*e+z;

 if m>mh, m=mh; else if m<ml, m=ml
 end
 end
 f=m
 z=al*z+(1-al)*f; [f z m]

 y0(7)=m;

 cvt(it)=cv;
 mvt(it)=m;
 end

 t=(1:it)*st;
 plot(t,cvt,'-r',t,mvt,'-g');
```

5.- Dual-mode simulation

```
clear
clf
%
%  batch reactor control system
%  dual-mode controller
%
```

```
% initial values
 ma=12.0; mb=12.0; mc=0; md=0; tr=20.0; tj=20.0;
 tjsp=20.0;

 y0=[ma,mb,mc,md,tr,tj,tjsp];

% controller initial values
 kc=26.5381; taui=2.8658; taud=0.4284;
en=0; enn=0;
cvsp=92.83;
td1=2.8; td2=2.4; pl=46; Em=0.95;
mv=20;
is=0;

% simulation
st=0.2;
t0=0; tfinal=120;
ntf=round(tfinal/st)+1;
cvt=zeros(1,ntf); mvt=zeros(1,ntf);

for it=1:ntf
[tt,y]=ode45('brxn',[(it-1)*st it*st],y0);
y0=y(length(y(:,1)),:);

cv=y0(5);

if is==0    %   heat up stage
 if cv<Em*cvsp
  mv=120;
 else
  is=1;
 tcool=it*st;
end
end

if is==1    %   cooling stage
 if it*st<tcool+td1
  mv=20;
 else
  is=2;
  tpre=it*st;
 end
end

if is==2   %   preload stage
 if it*st<tpre+td2
  e=cvsp-cv;
  mv=pl;
 else
  is=3;
 end
  enn=en; en=e;
 end

if is==3   %   control stage
  e=cvsp-cv;
```

```
 mv=mv+kc*(e*st/taui+(e-en)+taud*(e-
2*en+enn)/st);
  if mv>120, mv=120; elseif mv<20, mv=20; end
  enn=en; en=e;
end

 y0(7)=mv;

cvt(it)=cv;
mvt(it)=mv;
end
t=(1:it)*st;
plot(t,cvt,'-r',t,mvt,'-g')
```

Chapter 23

23.1

(a) Use IMC-tuning-based PI: identify open loop model as τ=4.5h, K=44 (average of high and low open-loop step changes), pick τ_c as 1/3 of τ. PI tuning: Kc=-.07 L/g-h, τ_I=4.5h. Closed-loop responses are given in the following figure:

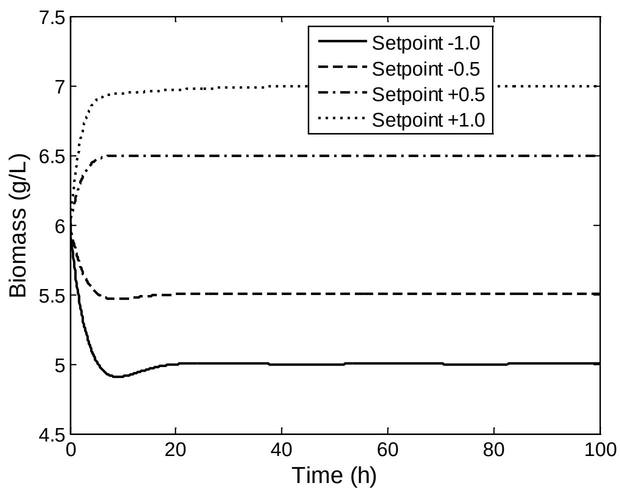


Figure S23.1a. Biomass closed-loop response for setpoint change

(b) Closed-loop simulation for a -12.5% step change in the maximum growth rate (μ_m):

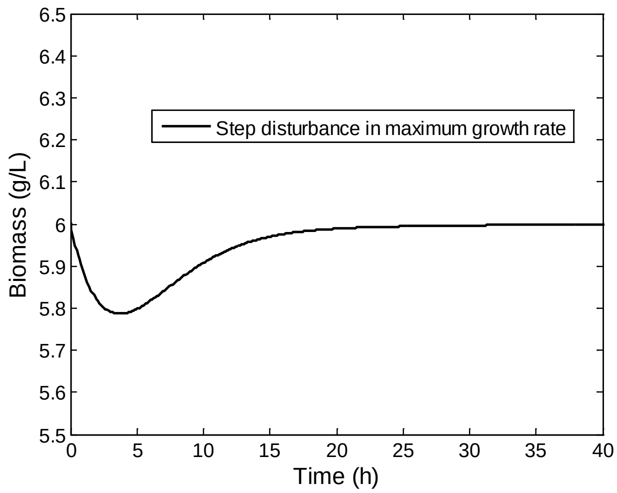


Figure S23.1b. Biomass closed-loop response for disturbance change

(c) From setpoint response, get slightly underdamped response on negative setpoint changes – corresponding to strong open-loop nonlinearity observed in Figure 23.2.

(d) Major difference is new gain (with opposite sign), and different time constant. Gain is smaller, time constant is larger, suggesting larger τ_I, and larger controller gain.

23.2

(a) Sample code for MPC design provided below

```
P=40;                                  %Prediction Horizon
M=2;                                   %Control Horizon
Weights=[0,0,0];                       %Manipulated Variables Weights
(Default = 0,0,0)
Penalize=[5,1,1];                      %[Bulk Weight, d5 Weight, d90
Weight]
Nominals=[180,180,180,40,400,1600];   %[Flow Rates 1-3, Bulk density, d5,
d90]
Constrains=[105,345];                  %Lower and Upper Flow Rates
NominalModelFlag=1;                    % 1=Nominal Model, Otherwise ->
Actual plant model
SimTime=[0,50];                        %Simulation time [Start,End]
StepTarget=[90,400,1600];              %Simulated step change in physical
units: [Bulk, d5, d90]
StepTime=1;                            %Time of Simulated step change;

[tsim,ysim_rescaled]=MPCSim(P,M,Weights,Penalize,Nominals,Constrains,Nomin
alModelFlag,SimTime,StepTarget,StepTime);
plotsimresults(tsim,ysim_rescaled,Constrains,'Nominal Controller');
```

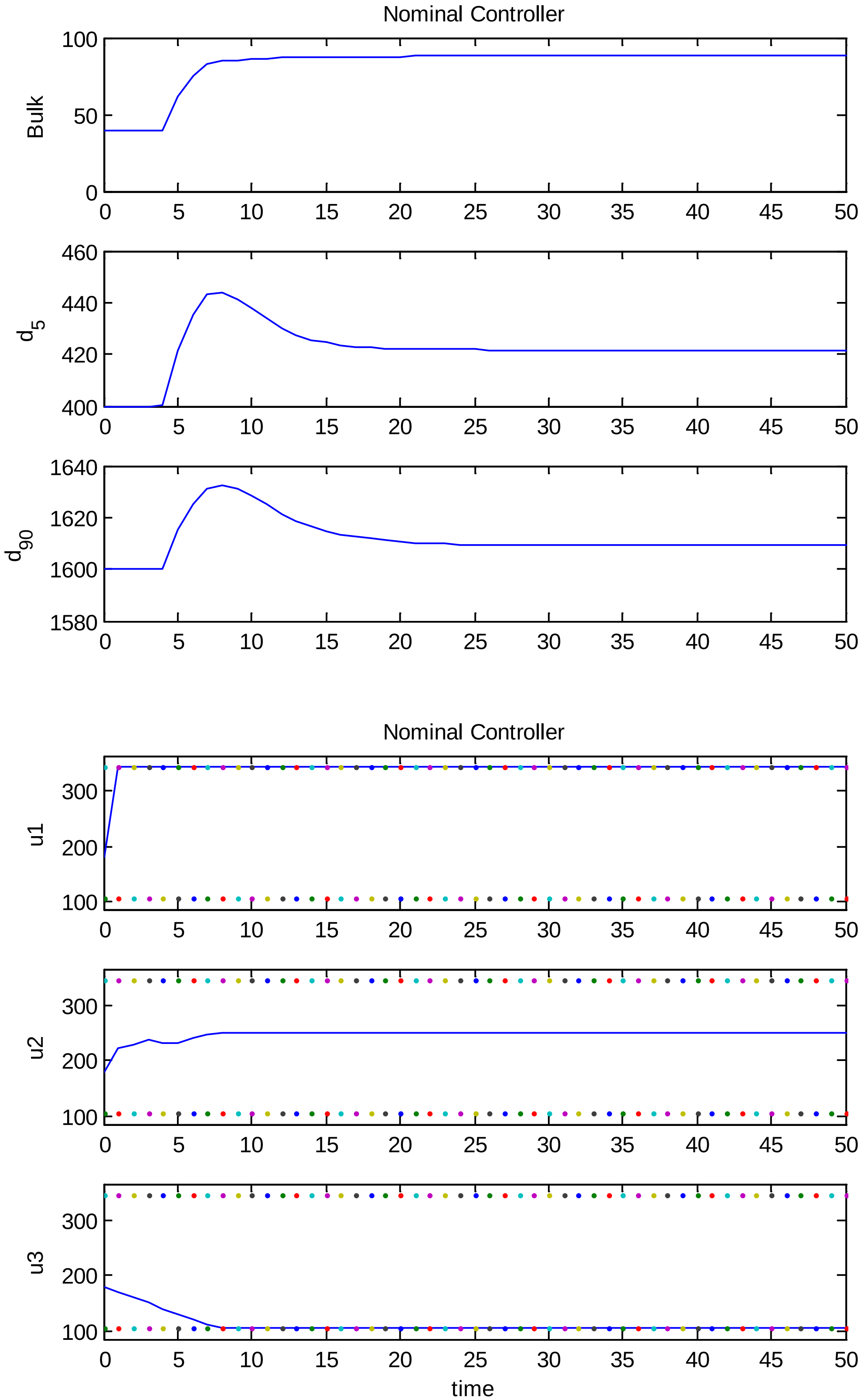
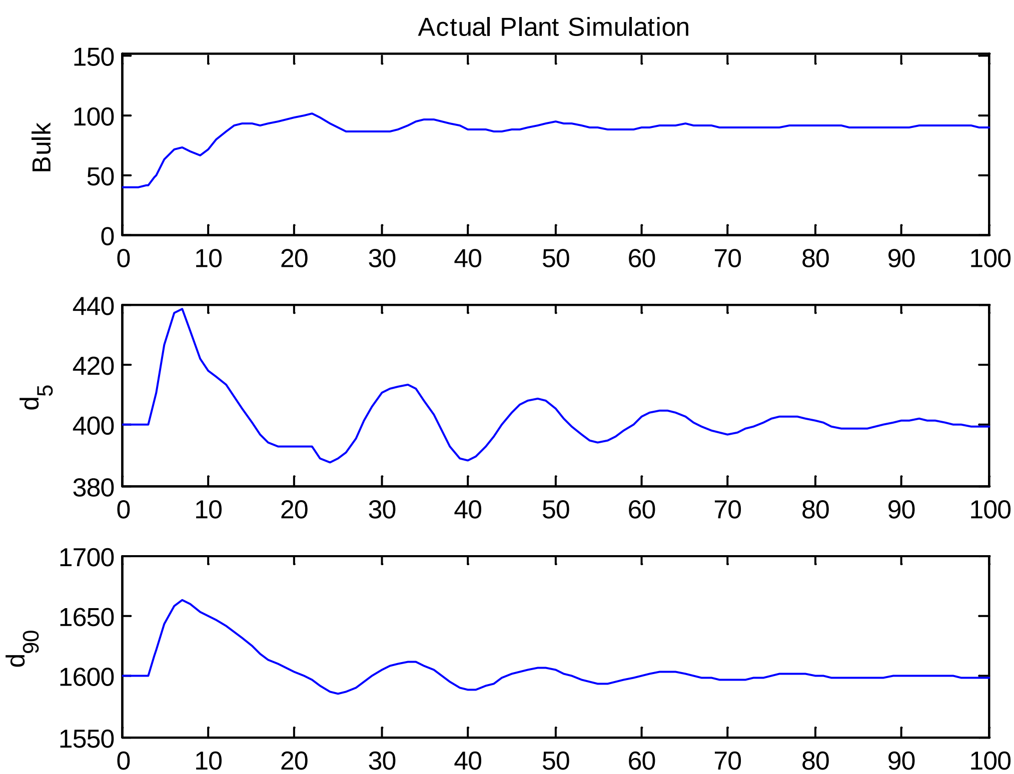


Figure S23.2a. Setpoint response for closed-loop granulation system under MPC control (nominal case)

(b) Sample code for MPC design provided below

```
P=40;                                  % Prediction Horizon
M=2;                                   % Control Horizon
Weights=[0.1,0,0];                       % Manipulated Variables Weights
(Default = 0,0,0)
Penalize=[2,1,1];                      % [Bulk Weight, d5 Weight, d90
Weight]
Nominals=[180,180,180,40,400,1600];    % [Flow Rates 1-3, Bulk density, d5,
d90]
Constrains=[105,345];                  % Lower and Upper Flow Rates
NominalModelFlag=0;                    % 1=Nominal Model, Otherwise ->
Actual plant model
SimTime=[0,100];                       % Simulation time [Start,End]
StepTarget=[90,400,1600];              % Simulated step change in physical
units: [Bulk, d5, d90]
StepTime=1;                            % Time of Simulated step change;

[tsim,ysim_rescaled]=MPCSim(P,M,Weights,Penalize,Nominals,Constrains,Nomin
alModelFlag,SimTime,StepTarget,StepTime);
plotsimresults(tsim,ysim_rescaled,Constrains,'Actual Plant Simulation');
```



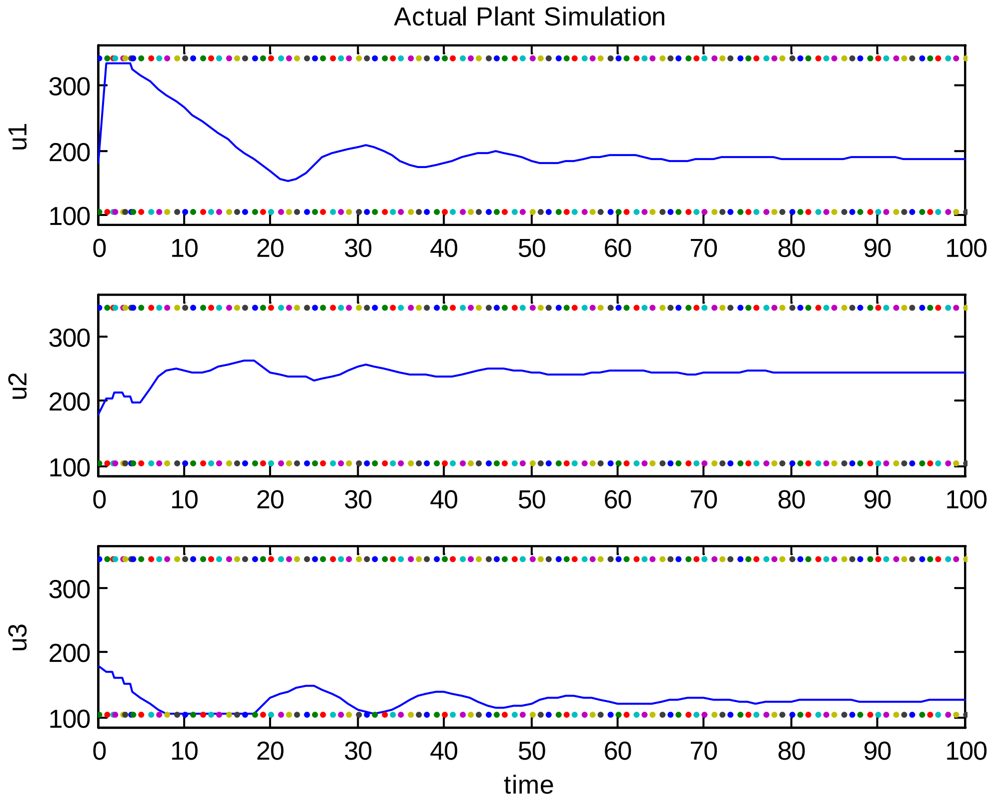


Figure S23.2b. Setpoint response for closed-loop granulation system under MPC control (uncertain case)

(c) Sample code provided for each scenario below:

i) step change in bulk density from 40 to 90

```
P=40;                                  % Prediction Horizon
M=2;                                   % Control Horizon
Weights=[0.1,0,0];                       % Manipulated Variables Weights
(Default = 0,0,0)
Penalize=[1,1,1.5];                      % [Bulk Weight, d5 Weight, d90
Weight]
Nominals=[175,175,245,40,400,1620];    % [Flow Rates 1-3, Bulk density, d5,
d90]
Constrains=[100,340];                  % Lower and Upper Flow Rates
NominalModelFlag=0;                    % 1=Nominal Model, Otherwise ->
Actual plant model
SimTime=[0,100];                       % Simulation time [Start,End]
StepTarget=[90,400,1620];              % Simulated step change in physical
units: [Bulk, d5, d90]
StepTime=1;                            % Time of Simulated step change;

[tsim,ysim_rescaled]=MPCSim(P,M,Weights,Penalize,Nominals,Constrains,Nomin
alModelFlag,SimTime,StepTarget,StepTime);
plotsimresults(tsim,ysim_rescaled,Constrains,'Scenario c - 1');
```

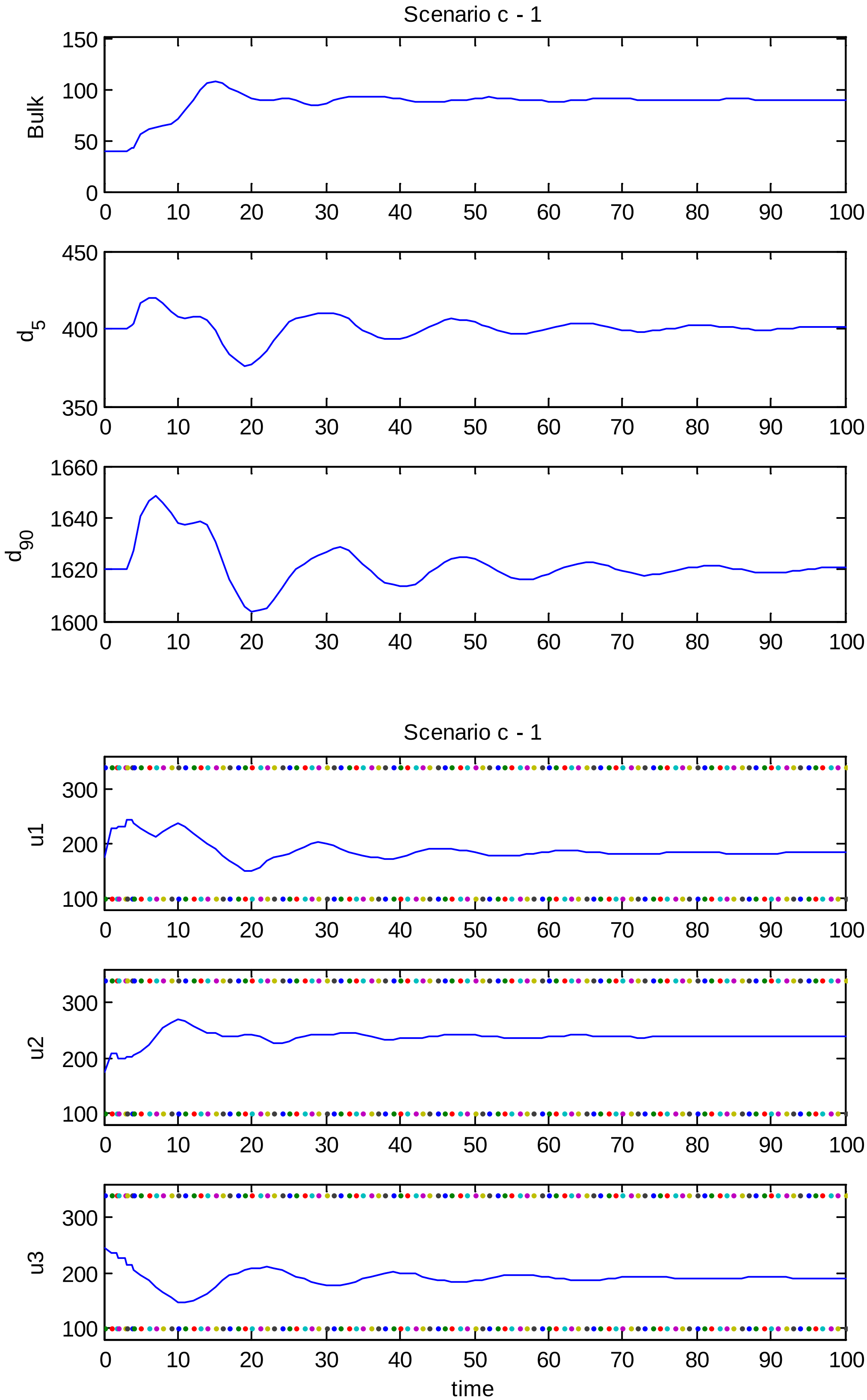


Figure S23.2c.i Closed-loop response for bulk density change

ii) simultaneous change in 5th percentile from 400 to 375, and 90th percentile from 1620 to 1630

```
P=40;                               % Prediction Horizon
M=2;                                % Control Horizon
Weights=[0.2,0.1,0.1];              % Manipulated Variables Weights
(Default = 0,0,0)
Penalize=[3,2,2];                   % [Bulk Weight, d5 Weight, d90
Weight]
Nominals=[175,175,245,40,400,1620]; % [Flow Rates 1-3, Bulk density, d5,
d90]
Constrains=[100,340];               % Lower and Upper Flow Rates
NominalModelFlag=0;                 % 1=Nominal Model, Otherwise ->
Actual plant model
SimTime=[0,100];                    % Simulation time [Start,End]
StepTarget=[40,375,1630];           % Simulated step change in physical
units: [Bulk, d5, d90]
StepTime=1;                         % Time of Simulated step change;

[tsim,ysim_rescaled]=MPCSim(P,M,Weights,Penalize,Nominals,Constrains,Nomin
alModelFlag,SimTime,StepTarget,StepTime);
plotsimresults(tsim,ysim_rescaled,Constrains,'Scenario c - 2');
```

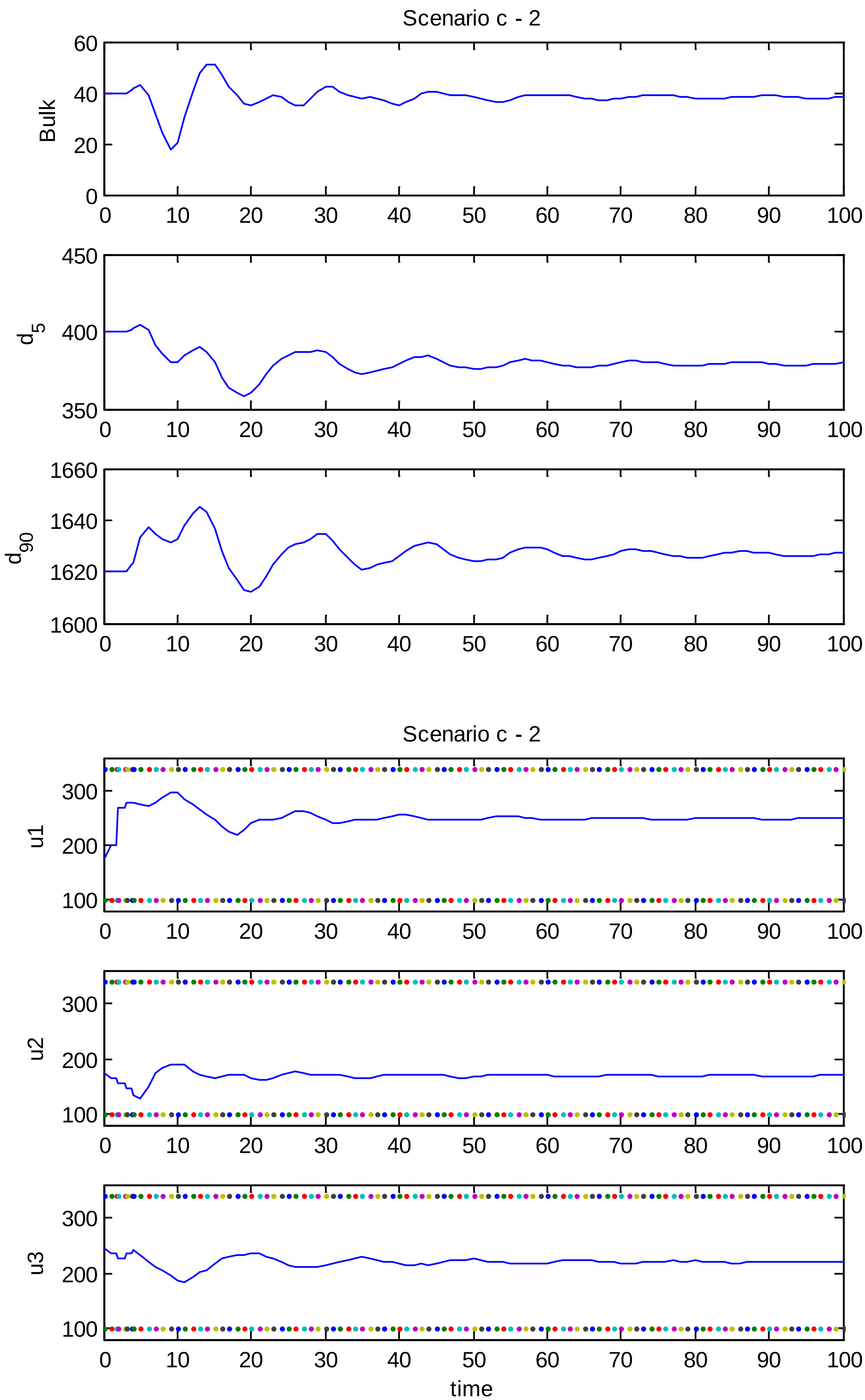


Figure S23.2c.ii Closed-loop response for d5 and d90 changes

23.3

(a) τ_c =1 corresponds to an aggressively tuned controller (fast response). Pick τ_c as 1/3 of tau: K_c=1.43, τ_I =10.1

(b)

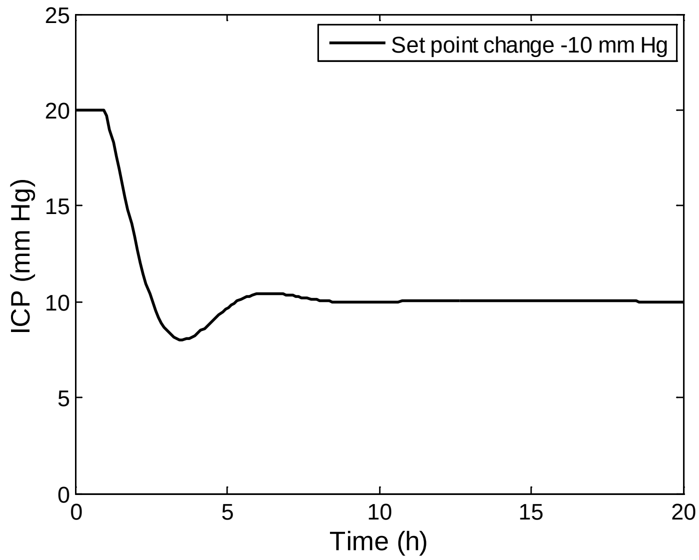


Figure S23.3.b Closed-loop response for ICP setpoint change

Undershoot=2/10=.2

Minimum = 8 mm Hg

Settling time = 4.85 h

(c) Overshoot is modest, the settling time is a bit long, but possibly acceptable for a delay system. Smith predictor and/or MPC would make good sense.

23.4

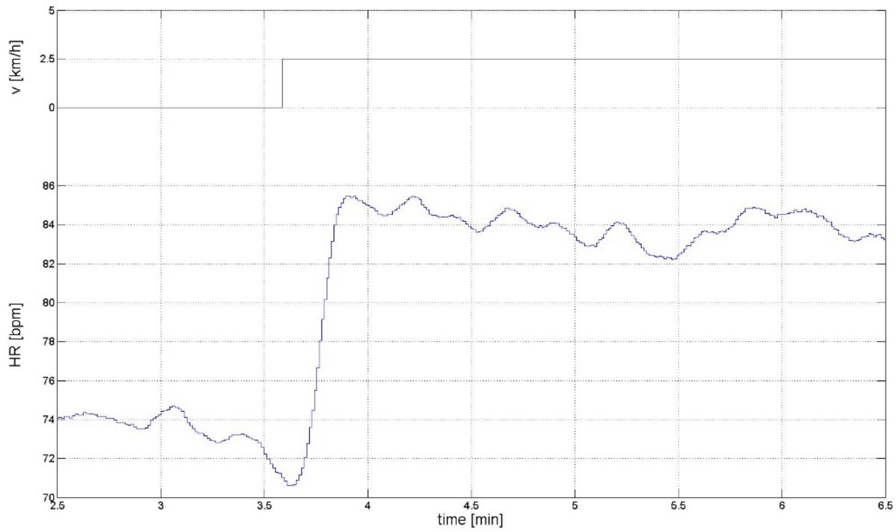


Figure E23.4

(a) Assume the change was made at t=3.5 min

Delay =0.5 min

Tau=(1/3)*.5=.167min

Gain=(84-72)/2.5=4.8

(b)

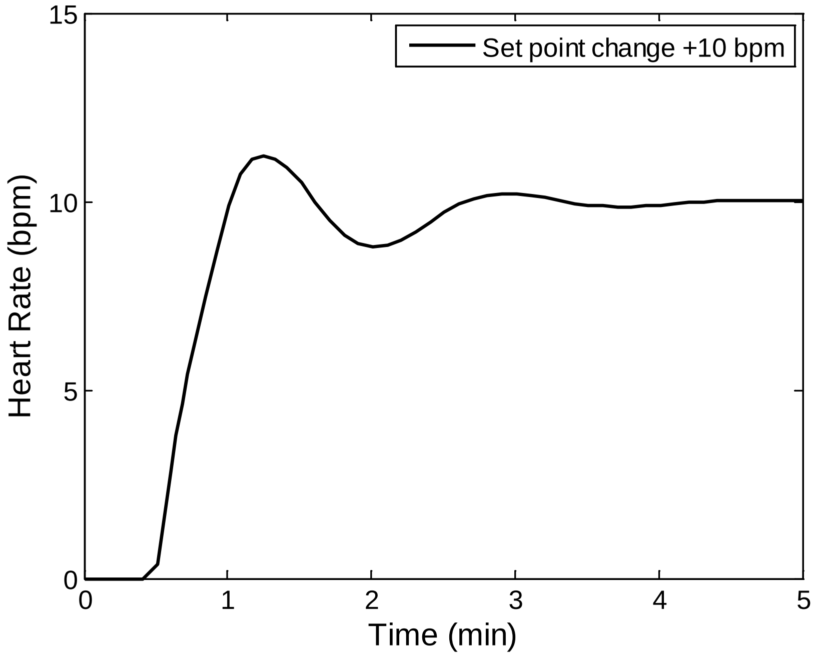


Figure S23.4.b Closed-loop response for Heart Rate setpoint change

Rise time = ~1.02 min

Overshoot = 11.22-10/10=.112 (11.2%)

Settling time =~2.4 min

(c) Improved response might be possible with multiple step changes, larger step changes, second-order model

23.5

(a) RGA calculated below:

RGA =

1.4209	0.0467	-0.4676
-0.1508	0.9643	0.1864
-0.2701	-0.0111	1.2812

This suggests a diagonal pairing of MVs and CVs

(b) All first order processes:

IMC tuning rules for PI controllers:

$$Kc = \frac{\tau}{K\tau_c}$$

$$\tau_I = \tau$$

	1-1 Loop	2-2 Loop	3-3 Loop
Kc	$\frac{3.7}{2.7\cdot 10^5\tau_c}$	$-\frac{7.5}{1.13\tau_c}$	$\frac{3.7}{6.3\cdot 10^4\tau_c}$
τ_I	3.7	7.5	3.7

Naïve choice for τ_c would be $\tau_c = \tau/3$

(c) Following are step test for setpoint change of 10% of steady-state values for each of the controlled variables (using $\tau_c = \tau/3$) . The fourth step test is a combined change in both variables 2 & 3.

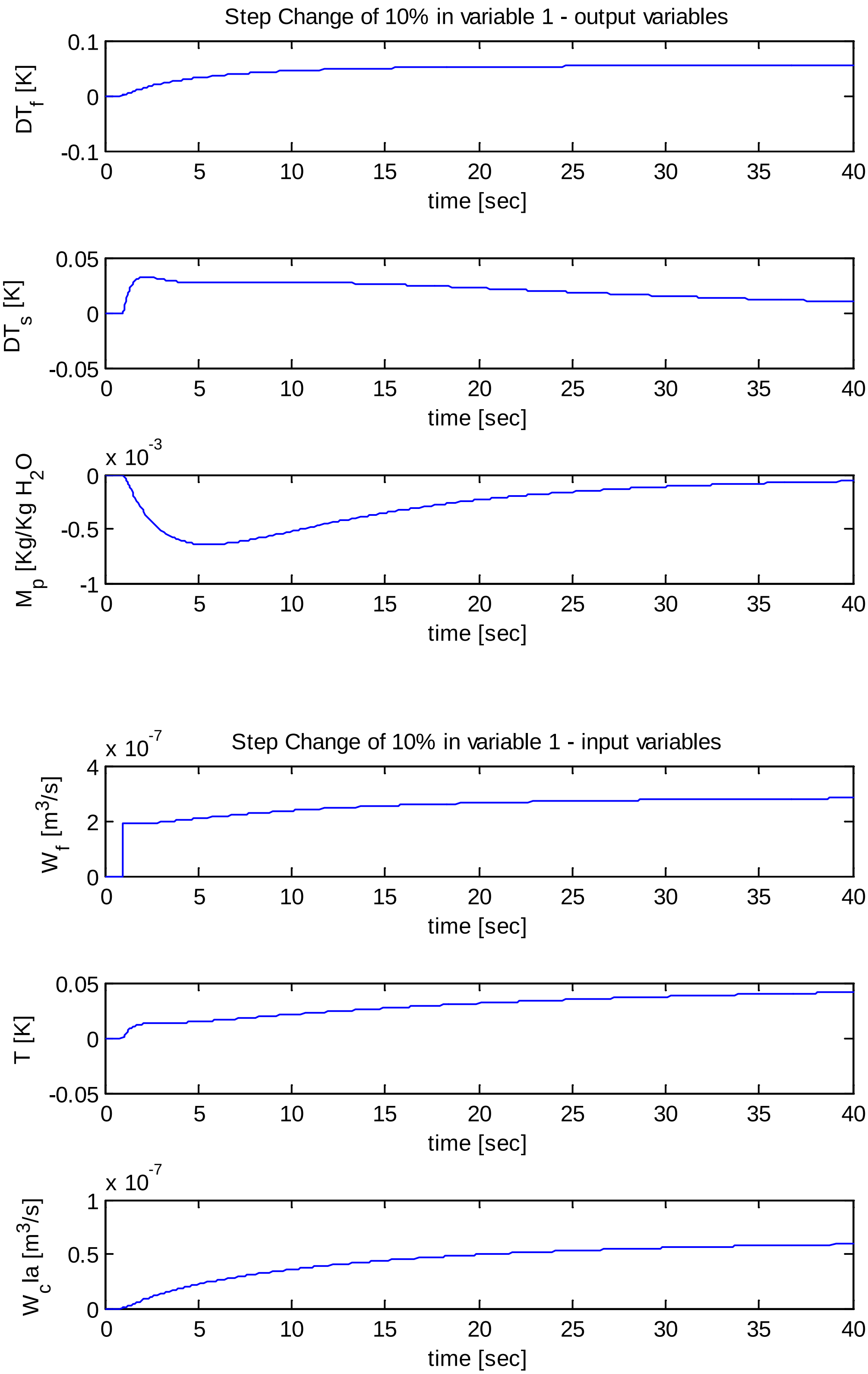


Figure S23.5.c.i Closed-loop response for y_1 setpoint change

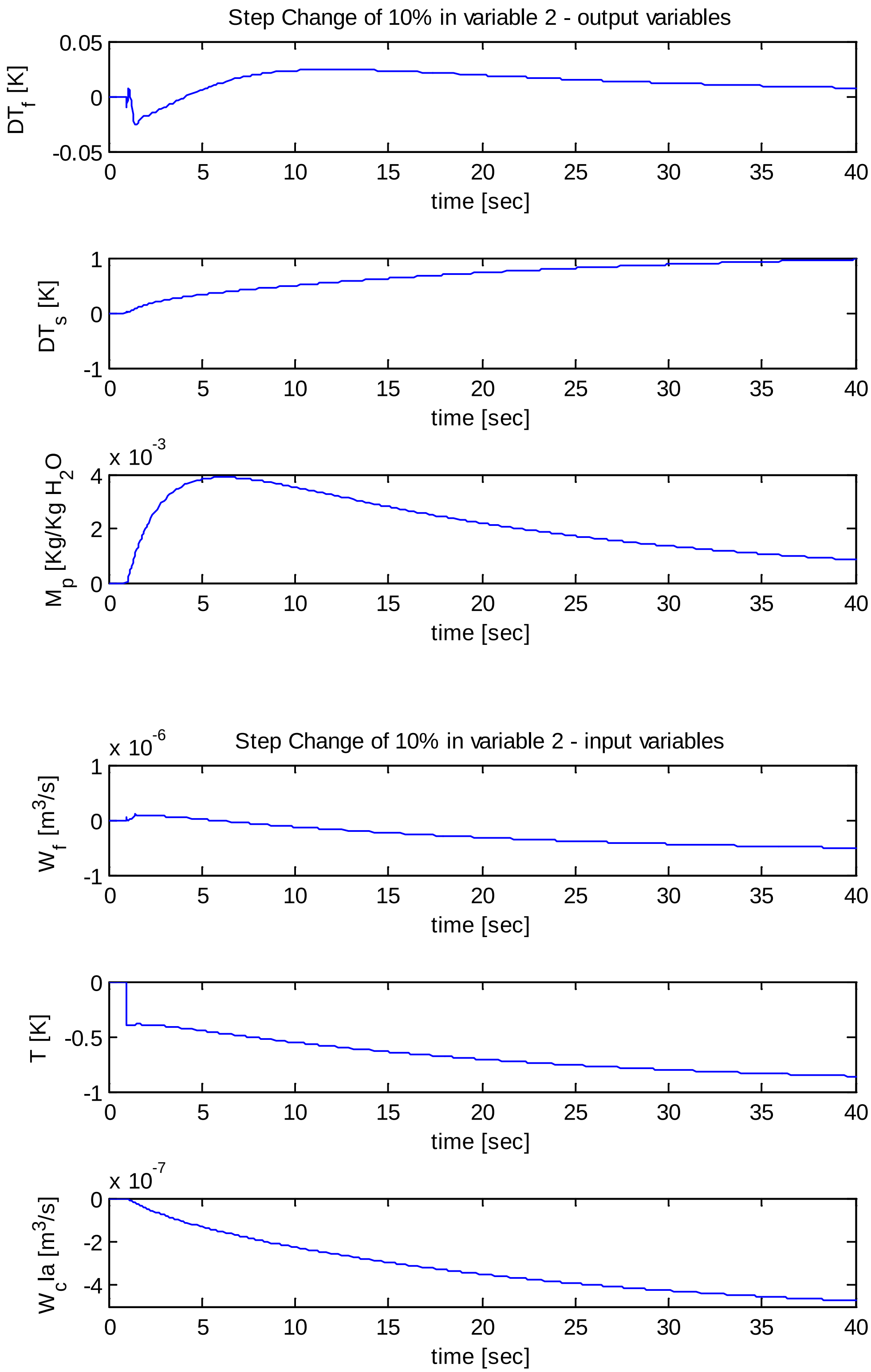


Figure S23.5.c.ii Closed-loop response for y_2 setpoint change

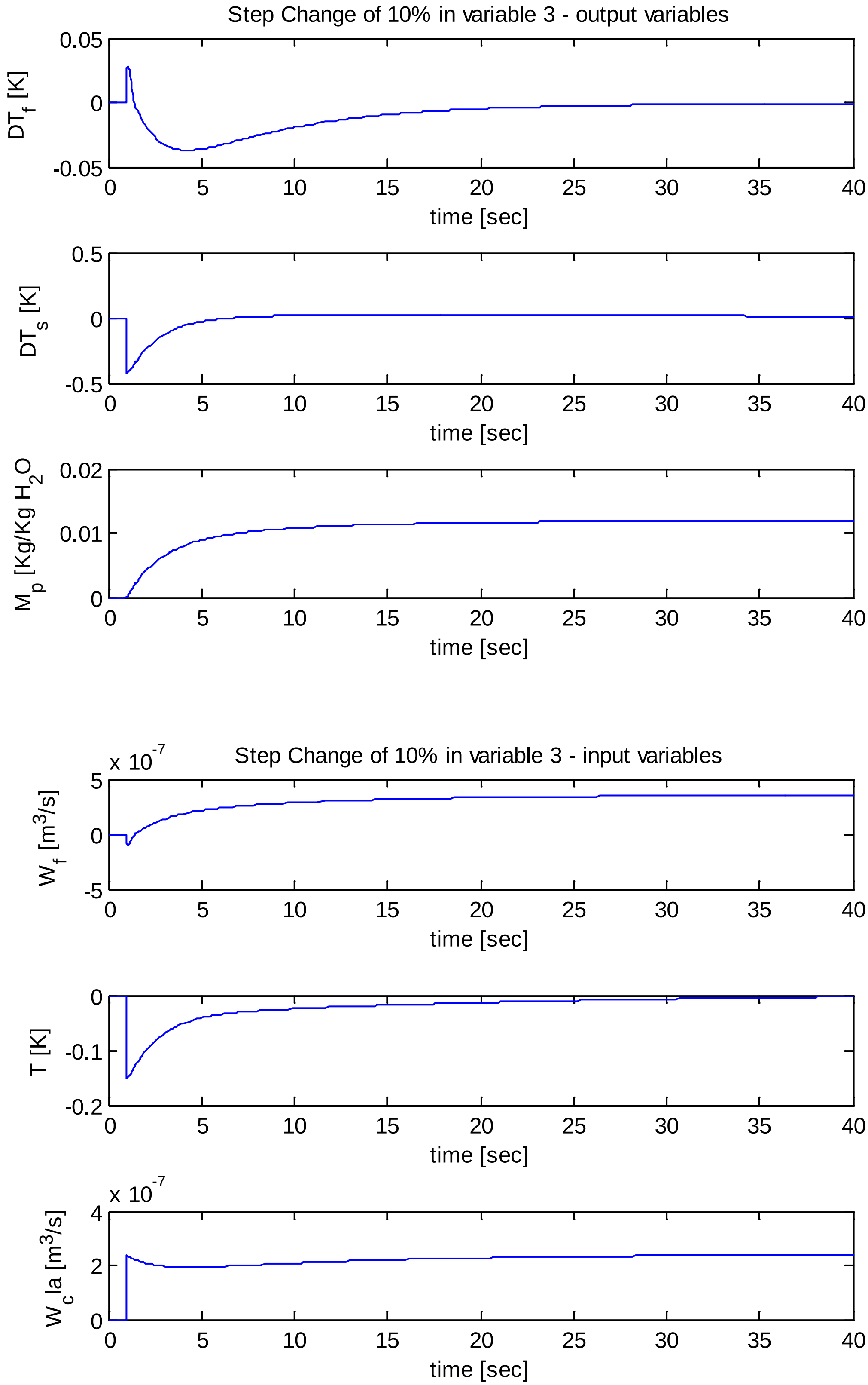


Figure S23.5.c.iii Closed-loop response for y_3 setpoint change

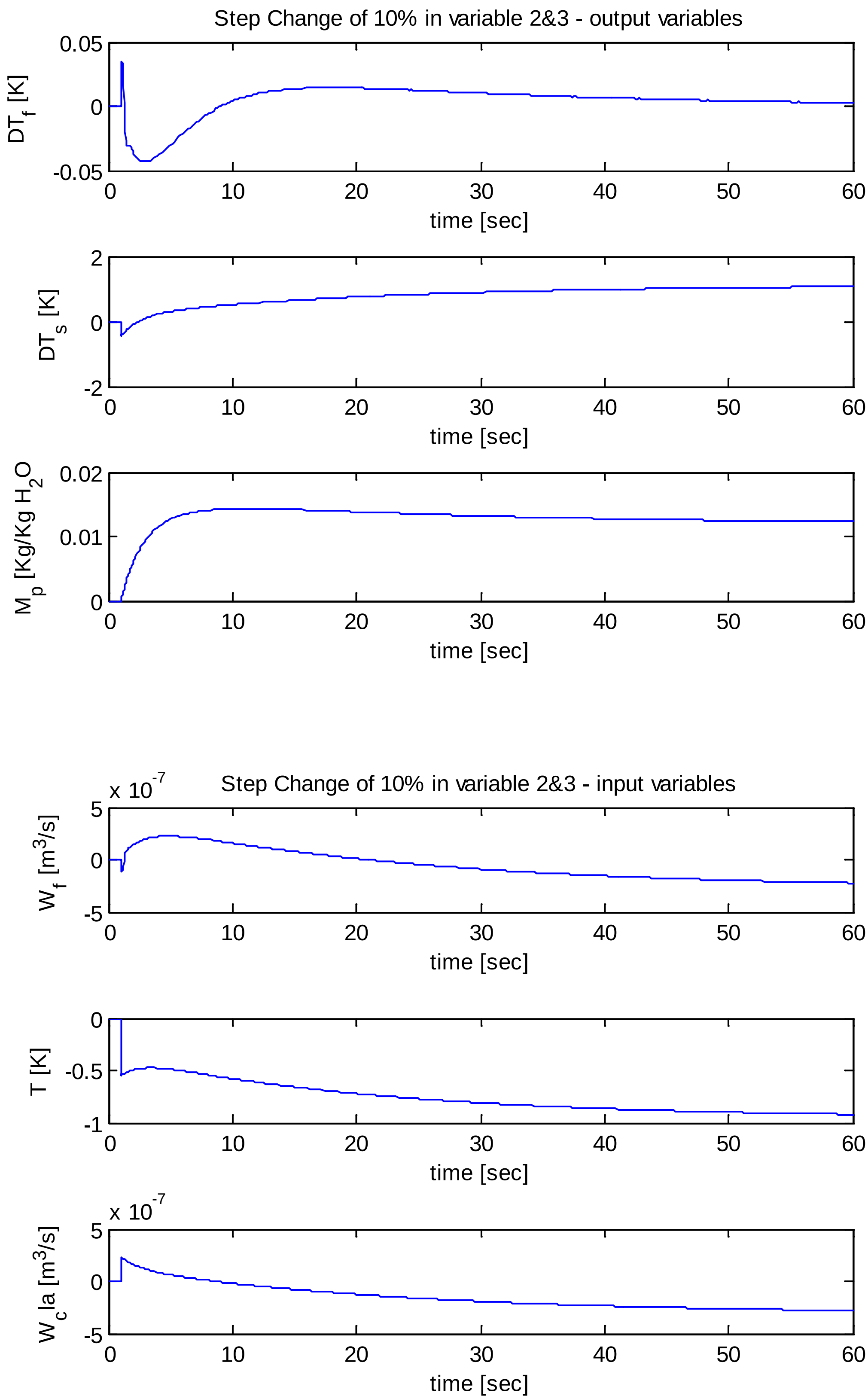


Figure S23.5.c.iv Closed-loop response for y_2 and y_3 setpoint changes

23.6

(a) To calculate an approximate second-order insulin-glucose model for the patient we shall set the disturbance, D, to zero. A step of one mU/min shall be introduced to the system. One simulates the response for 400 min with constant insulin injection of 15 mU/min to reach a steady state. Then introduce a step change. Using Smith's method one can identify from the figurer below a second- order model of the form:

$$G(s) = \frac{K}{\tau^2 s^2 + 2\xi\tau s + 1}$$

K=-5.16
$t20/t60 \cong 0.5 \rightarrow \xi \cong 0.48$ and $\tau \cong 59$min

$$G(s) = \frac{-5.16}{59^2 s^2 + 2 \cdot 0.48 \cdot 59s + 1}$$

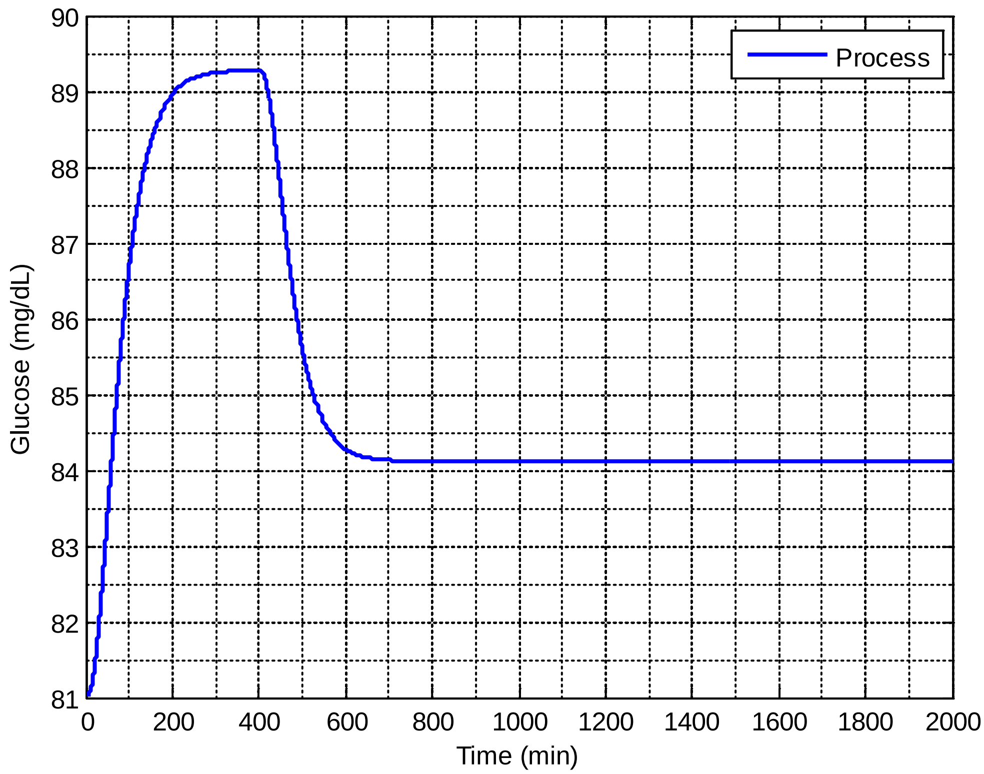


Figure S23.6.a.i Open-loop step response for change in insulin

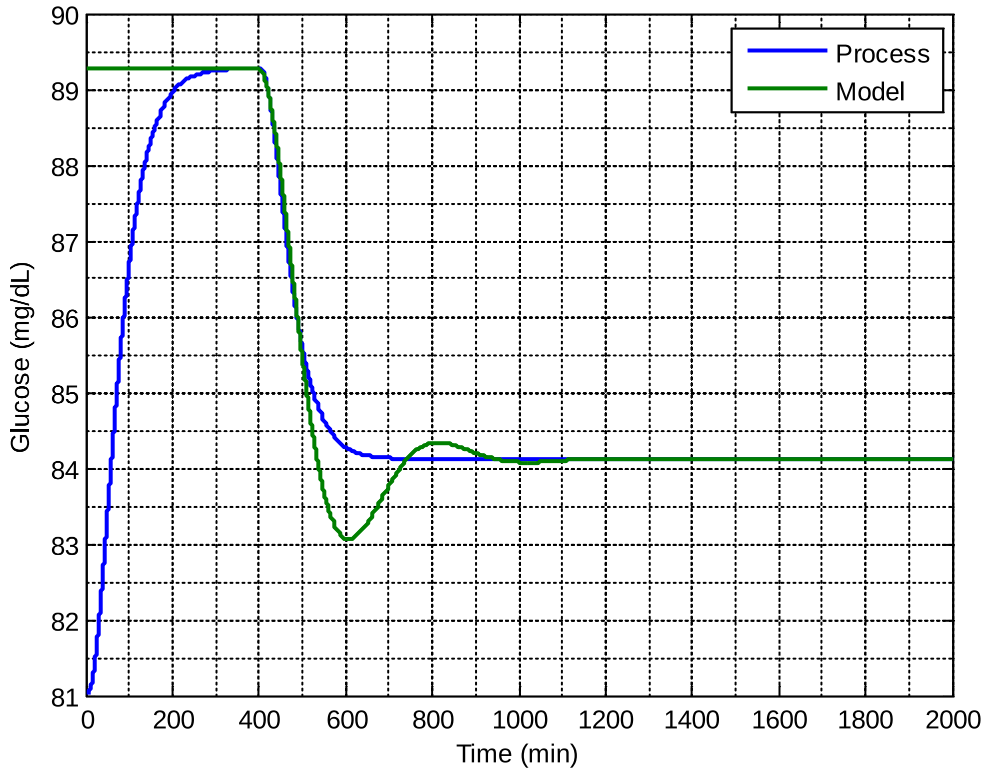


Figure S23.6.a.ii Comparison of model and process

(b) Using IMC- Based PID controller settings for G_c for a second-order model:

$Kc=2^* \xi * \tau /K/\tau c=-10.29/\tau c$
$\tau_i=2^* \xi * \tau=53.1$
$\tau_d= \tau /2/ \xi=65.56$

(c) Simulation results of the closed-loop system response to a step setpoint change in blood glucose of -20 mg/dl. As can be seen from figure below, one can tune τ_c to improve the transient response.

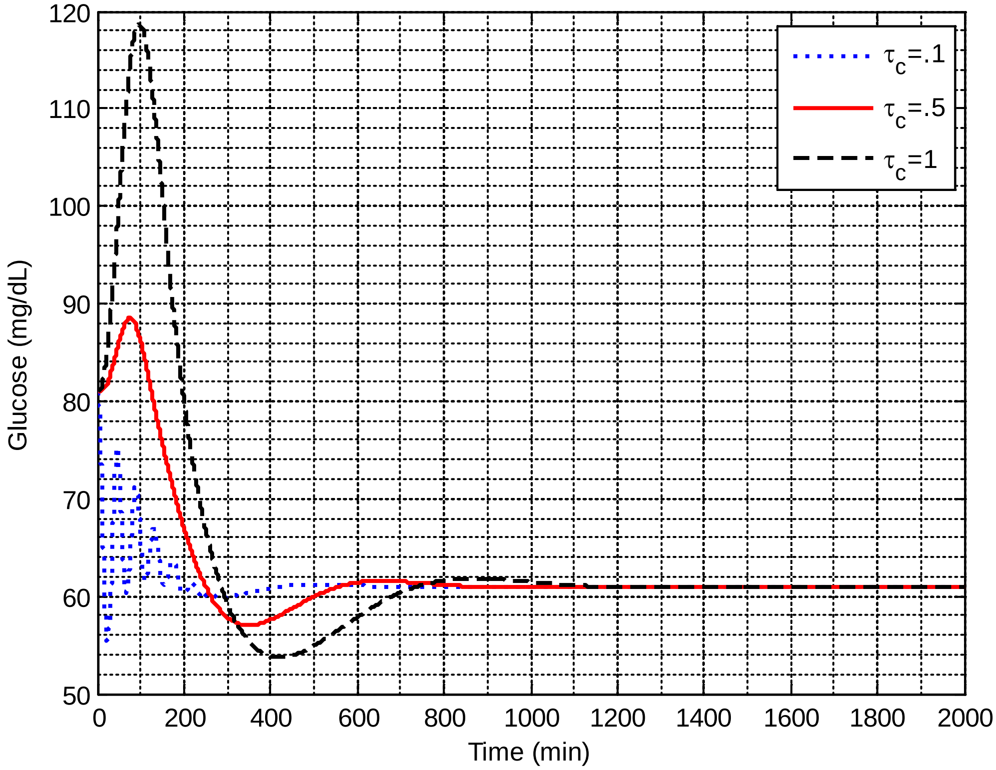


Figure S23.6.c Influence of controller tuning on closed-loop response

(d) With τ_c=0.5 as can be seen in figure below, one can maintain the hypoglycemic boundary but one still violates the upper constraint with maximum glucose of 159 mg/dL

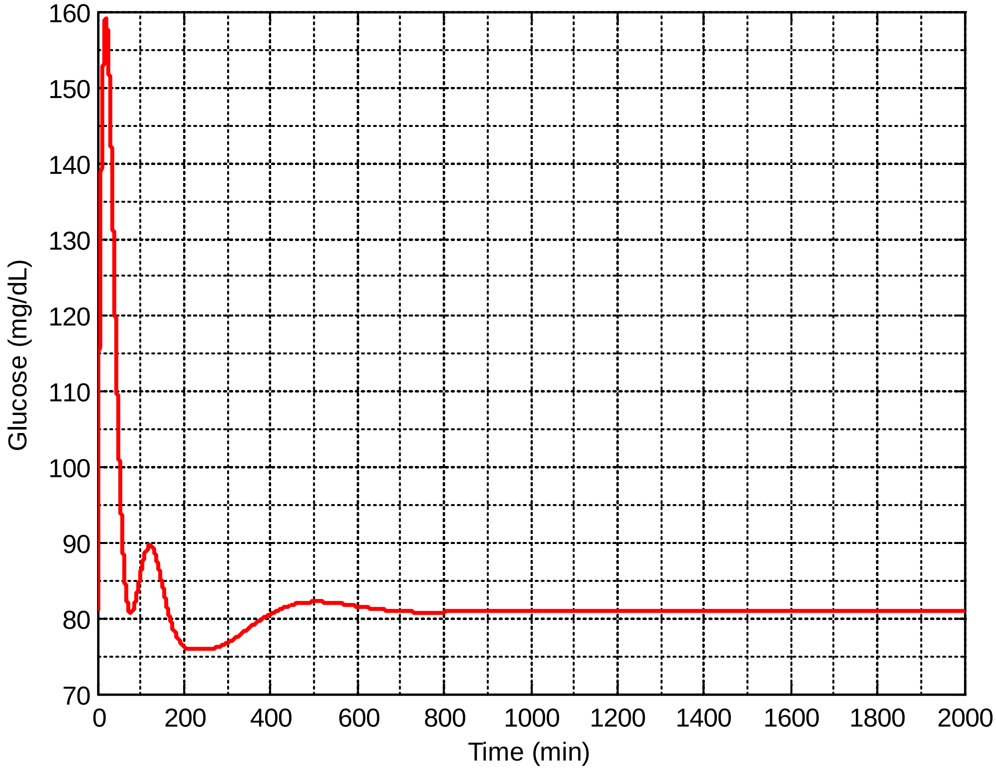


Figure S23.6.d Closed-loop response to meal disturbance

(e) With 10 min sensor delay the response is sluggish and one violates the upper constraint. The response will become unstable if one tries to tune τ_c to a lower value

23.7

(a) On the basis of the transfer function characteristics, the glucagon pump has more favorable qualities for use as a manipulated variable.

- The time delay is smaller, meaning the MV will have an effect on the CV more quickly
- The glucagon pump has simpler (first order versus second order) dynamics and a smaller time constant. Overall the dynamics of the glucagon pump are faster.
- The glucagon pump has a larger gain, meaning it will take less glucagon to have the same magnitude of effect on the blood glucose.
- There are fewer safety concerns with a glucagon pump (the insulin pump has a risk of overdosing insulin and causing death, whereas the glucagon pump does not

have the same type of risk). Also, the glucagon pump has the ability to correct hypoglycemia, while the insulin pump does not.

Note that if the glucagon pump is used as the MV, insulin would still need to be delivered either by the patient manually or by a set pattern on an insulin pump. Insulin is necessary for survival. Also, glucagon alone cannot be used to lower the BG following the meal disturbance.

(b) For the insulin pump, the process transfer function parameters are given as follow:

K = -1.5
τ_1 = 20 min
τ_2 = 25 min
θ = 30 min

Using IMC tuning rules, the PID controller parameters are given by the following expressions from Table 12.1, row I (with τ_C=20min and τ_3=0):

$$K_C = \frac{\tau_1 + \tau_2}{K(\tau_C + \theta)} = \frac{20+25}{-1.5(20+30)} = -0.6$$

$$\tau_I = \tau_1 + \tau_2 = 20 + 25 = 45 \text{min}$$

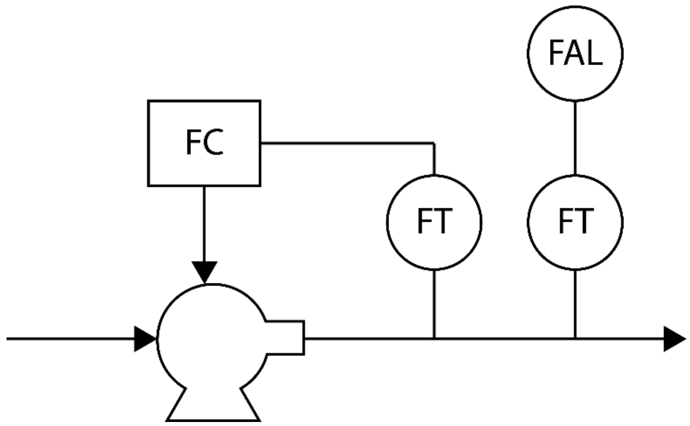
$$\tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} = \frac{20(25)}{20+25} = 11.1 \text{min}$$

(c) $Gf = -\frac{G_d}{G_p} = \frac{(20s+1)(25s+1)}{(30s+1)} \frac{5}{1.5} e^{20s}$

This controller is not realizable due to the positive 20 min delay (requiring knowledge from the future) and the fact that the numerator order is greater than the denominator order.

To make the controller realizable, you could set the delay to zero (remove the delay term from the controller) and introduce a filter to increase the order of the denominator.

(d) See the diagram below



23.8

(a) and (b) Reading from the graph, we can generate the following readings:

Sensor	%<70	%<180	%<80	%<140
BG	0.6	67.3	2.2	42.9
CGM1	1.9	69.1	4.1	48.4
CGM2	2.6	68.1	4.8	46.1

Using these readings, we can calculate the percentage time from 70-180 mg/dL and from 80-140 mg/dL as determined by each sensor.

	BG	CGM1	CGM2
70-180	66.7	67.2	65.5
80-140	40.7	44.3	41.3

According to the BG measurement, the algorithm kept the BG between 70 and 180mg/dL for 66.7% of the time. The BG spent 0.6% of time below 70mg/dL and 32.7% of time above 180mg/dL. This means that the BG was in the desired range for about 2/3 of the total time. Most of the time that was not spent within 70-180mg/dL was spent above 180mg/dL. Very little time was spent below 70mg/dL, which is good for safety. The time spent below 70mg/dL should be minimized, although in reality it is very difficult to completely eliminate hypoglycemia. According to the graph, very little time was spent with BG above 300mg/dL, which is also good for safety.

(c) The two CGMs overestimated the time that was spent below 70 mg/dL. In fact, from the graph we see that the CGMs overestimated the time spent below glucose levels up to about 200mg/dL. This difference indicates a bias of the sensor to read below the actual BG. The differences could also be due to lags and delays in the CGMs. Generally the two CGMs had similar readings, meaning they are fairly precise. The resolution is to the ones place in mg/dL.

Chapter 24

24.1

[problem adapted from Alon, **Introduction to Systems Biology**, Chapman & Hall]

a)

$$\frac{dM}{dt} = G_0 - k_d^{mRNA} M$$ where G_0 is the input. Solve steady-state balance:

$$\frac{G_0}{k_d^{mRNA}} = M_{ss}$$

b)

$$\frac{dP}{dt} = k_T M - k_d^P M$$

c) step change in G_0 from basal value to G_1, mRNA has first order response with time constant and gain both equal:

$$\tau_1 = K_1 = \frac{1}{k_d^{mRNA}}$$

Protein has first order response to mRNA with gain and time constant:

$$\tau_2 = \frac{1}{k_d^P}$$

$$K_2 = \frac{k_T}{k_d^P}$$

Analytical expression for deviation protein concentration (P') is given by the expression for two first-order systems in series:

$$P' = \frac{k_T}{k_d^P} \bullet \frac{1}{k_d^{mRNA}} \left(1 + \frac{A_1}{\tau_1} e^{-t/\tau_1} + \frac{A_2}{\tau_2} e^{-t/\tau_2} \right)$$

Where (using partial fraction expansion):

$$A_1 = \lim_{s \to -1/\tau_1} \left(\frac{(\tau_1 s + 1) g(s)}{Ks} \right) = \frac{1}{(-1/\tau_1)((-\tau_2/\tau_1)+1)}$$

$$A_1 = \lim_{s \to -1/\tau_1} \left(\frac{(\tau_2 s + 1) g(s)}{Ks} \right) = \frac{1}{(-1/\tau_2)((-\tau_1/\tau_2)+1)}$$

24.2

(i-a)

$$y = P_1 P_2 P_3 u$$

(i-b)

Algebra here follows:

$$y = P_3(C_3 y + P_2(C_2 y + P_1(u + C_1 y)))$$
$$= P_3 C_3 y + P_3 P_2 C_2 y + P_3 P_2 P_1 u + P_3 P_2 P_1 C_1 y$$

$$y = \frac{P_1 P_2 P_3}{1 - C_3 P_3 - C_2 P_2 P_3 - C_1 P_1 P_2 P_3} u$$

(i-c)

$$y = \frac{P_1 P_2 P_3}{1 - C_1 P_1 P_2 P_3} u$$

(ii)

$$y = \frac{P_1 P_2 P_3}{1 - K_{c3} P_3 - K_{c2} P_2 P_3 - K_{c1} P_1 P_2 P_3} u$$

(iii)

$$y = \frac{P_1 P_2 P_3}{1 - K_{c1} P_1 P_2 P_3} u$$

(iv) More attenuation possibilities in case (b) since there are more control loops that can regulate the process

24.3

(a)

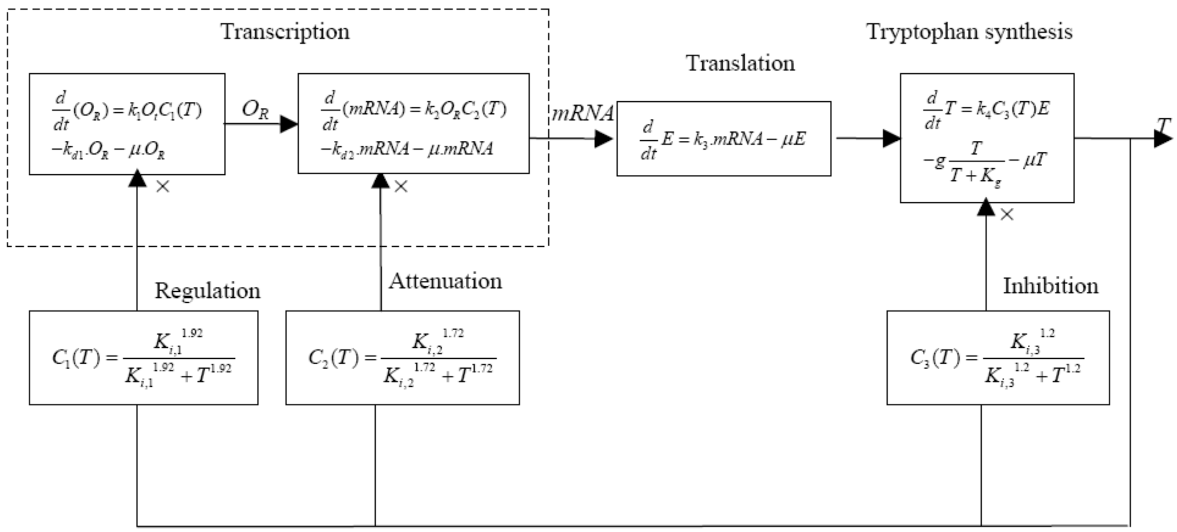
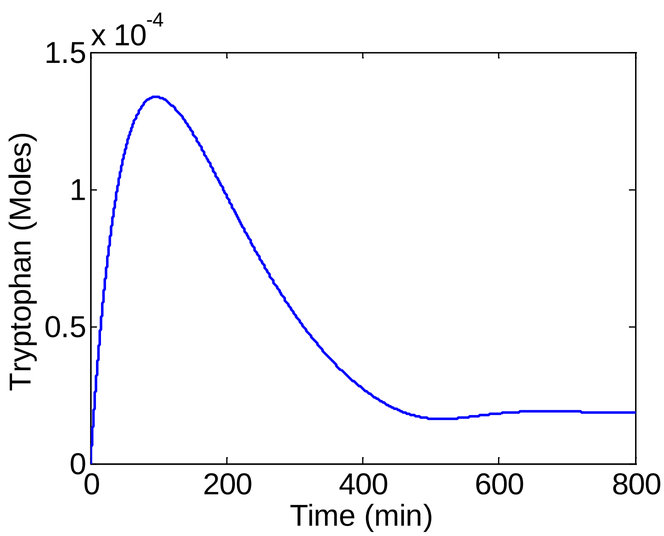


Figure S24.3a. Block diagram for tryptophan process

In this figure, four states (synthesis of free operator, mRNA transcription, translation and tryptophan synthesis) are represented as each block. Also, controllers (regulation, Attenuation and inhibition) are connected to the specific states. This block diagram is exactly the same as in Exercise 24.2, excluding one less state.



(b)

Figure S24.3b.i Tryptophan response for g=0

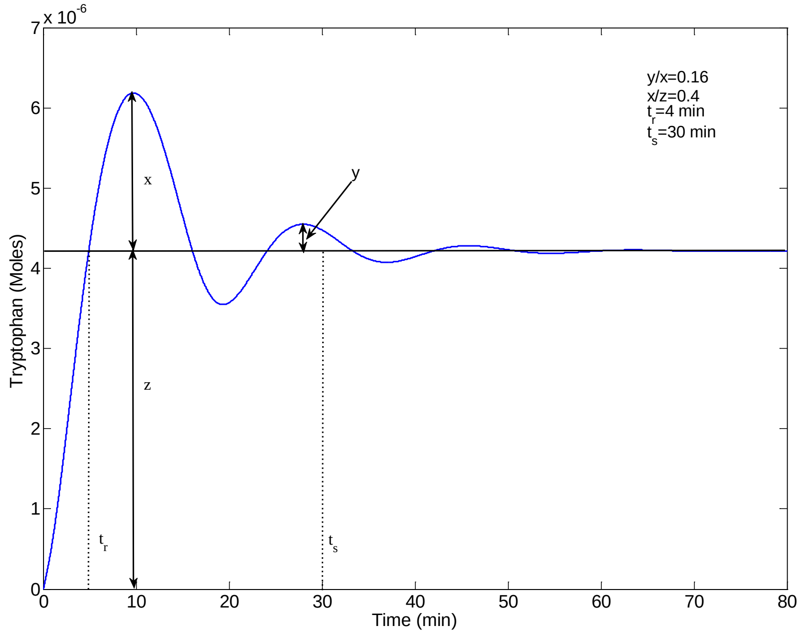


Figure S24.3b.ii Tryptophan response for g=25

(c) Rise time=4 mins, overshoot=0.4, decay ratio=0.16, settling time=30 mins

(d) Response of tryptophan after deleting the two feedback loops (red curve; blue curve is with all the feedback loops). Here, system is sluggish taking almost 50 mins to reach steady state without any overshoot.

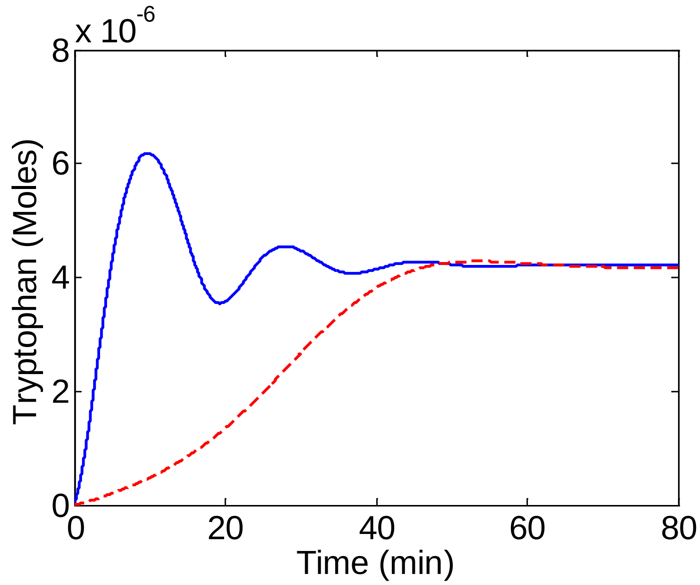


Figure S24.3d Tryptophan response for two feedback loop case

24.4

(a) Algebra for derivation follows (recall that numerator of the first term involves an additional differentiation wrt *s* compared to the example derived in the chapter):

$$T_{dur} = \sqrt{\frac{\int_{t=0}^{\infty} t^2 y(t)dt}{\int_{t=0}^{\infty} y(t)dt} - T_{sig}^2}$$

$$= \sqrt{\frac{-\frac{d}{ds}\left(\lambda\alpha^4(s+\lambda)^{-2}(s+\beta)^{-4} + 4\lambda\alpha^4(s+\lambda)^{-1}(s+\beta)^{-5}\right)_{s=0}}{\frac{\alpha^4}{\beta^4}} - \left(\frac{1}{\lambda}+\frac{4}{\beta}\right)^2}$$

$$\sqrt{\frac{\lambda\alpha^4\left(\frac{8}{\lambda^2\beta^5}+\frac{2}{\beta^4\lambda^3}+\frac{20}{\lambda\beta^6}\right)}{\frac{\alpha^4}{\beta^4}}-\left(\frac{1}{\lambda}+\frac{4}{\beta}\right)^2}$$

$$=\sqrt{\frac{1}{\lambda^2}+\frac{4}{\beta^2}}$$

(b) Algebra, combining results from part (a) with results in chapter:

$$A=\frac{\int_{t=0}^{\infty} y(t)dt}{2T_{dur}}=\frac{\frac{\alpha^4}{\beta^4}}{2\sqrt{\frac{1}{\lambda^2}+\frac{4}{\beta^2}}}=\frac{\frac{\lambda\alpha^4}{2\beta^4}}{\sqrt{1+\frac{4\lambda^2}{\beta^2}}}$$

24.5

(a) Equation defining variables in the loop

$$y = k\left(u - \frac{y}{s}\right)$$

Transfer function has a pole located at *s*=-*k*, therefore if *k* is positive, loop is stable. With integrator in loop, require zero activity at steady state

(b)

Solving transfer function:

$$\frac{y}{u} = \frac{ks}{s+k}$$

(c)

Receptor activity always resets to zero, always capable of full range of action

24.6

[Adapted from problem described in Goldbeter & Koshland, *PNAS*, 78, 6840-6844, 1981]

(a) Laying out the relevant mass balances:

$$\frac{d[P]}{dt} = -a_1[P][E_1] + d_1[PE_1] + k_2[P^*E_2]$$

$$\frac{d[P_1]}{dt} = a_1[P][E_1] - (d_1 + k_1)[PE_1]$$

$$\frac{d[P^*]}{dt} = -a_2[P^*][E_2] + d_2[P^*E_2] + k_1[PE_1]$$

$$\frac{d[P^*E_2]}{dt} = a_2[P^*][E_2] - (d_2 + k_2)[P^*E_2]$$

Invoking assumption about fixed total amounts, and dropping concentration notation ([.]):

$$P + P^* + PE_1 + P^*E_2 = P_T$$
$$E_{1T} = E_1 + +PE_1$$
$$E_{2T} = E_2 + P^*E_2$$

Algebra leads to

$$\frac{P^*}{P_T} = \frac{\left(\frac{V_1}{V_2} - 1\right) - K_2\left(\frac{V_1}{V_2} + \frac{K_1}{K_2}\right) + \left(\left[\frac{V_1}{V_2} - 1 - K_2\left(\frac{V_1}{V_2} + \frac{K_1}{K_2}\right)\right]^2 + 4K_2\left(\frac{V_1}{V_2} - 1\right)\left(\frac{V_1}{V_2}\right)\right)^{1/2}}{2\left(\frac{V_1}{V_2} - 1\right)}$$

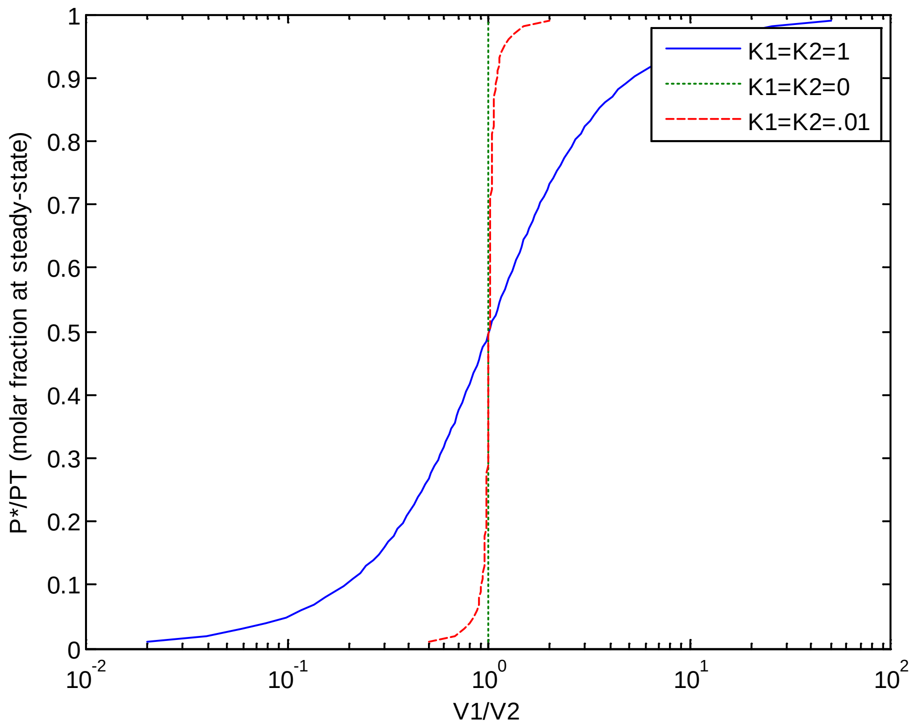
Where the following variables are used:

$$V_1 = k_1 E_{1T}$$
$$V_1 = k_2 E_{2T}$$
$$K_1 = \frac{d_1 + k_1}{a_1 P_T}$$
$$K_2 = \frac{d_2 + k_2}{a_2 P_T}$$

Invoking the conservation balance on P at steady-state, and assuming that V_1 and V_2 are not equal, one can derive the following simplified expression:

$$\frac{V_1}{V_2} = \frac{\frac{P^*}{P_T}\left(1-\frac{P^*}{P_T}+K_1\right)}{\left(1-\frac{P^*}{P_T}\right)\left(\frac{P^*}{P_T}+K_2\right)}$$

(b) and (c) [combined plot, also included K_1=K_2=.01 for illustration]



(d) For small values of K_1 and K_2, the response approaches a switch-like shape. Larger values lead to more sigmoidal response profiles. Hence, this biochemical network consisting of two antagonistic enzymes can be tuned (or regulated) to give switch like behavior under appropriate conditions. In some texts this is referred to as "zero order ultra-sensitivity".

24.6

(a) G_a = transcription (DNA to RNA)
G_b = translation (RNA to protein)
G_c = protein activation
Y = activated protein

(b) Inner Loop:

$$\frac{G_b}{1+G_b}$$

Inner two loops:

$$\frac{G_c \frac{G_b}{1+G_b}}{1+G_c \frac{G_b}{1+G_b}}$$

All three loops:

$$\Omega = \frac{G_c \frac{G_b}{1+G_b}}{1+G_c \frac{G_b}{1+G_b}}$$

$$\frac{Y(s)}{X(s)} = \frac{G_a\Omega}{1+G_a\Omega}$$

$$= \frac{G_a \dfrac{G_c \frac{G_b}{1+G_b}}{1+G_c \frac{G_b}{1+G_b}}}{1+G_a \dfrac{G_c \frac{G_b}{1+G_b}}{1+G_c \frac{G_b}{1+G_b}}}$$

Simplifying this expression gives:

$$\frac{Y(s)}{X(s)} = \frac{G_a G_b G_c}{1 + G_b + G_b G_c + G_a G_b G_c}$$

(c) Now we can substitute the given values for the biological processes:

$$G_a = 5$$

$$G_b = \frac{1}{2s}$$

$$G_c = \frac{3}{s}$$

$$\frac{Y(s)}{X(s)} = \frac{5\frac{1}{2s}\frac{3}{s}}{1 + \frac{1}{2s} + \frac{1}{2s}\frac{3}{s} + 5\frac{1}{2s}\frac{3}{s}}$$

$$= \frac{\frac{15}{2s^2}}{1 + \frac{1}{2s} + \frac{3}{2s^2} + \frac{15}{2s^2}}$$

$$= \frac{15}{2s^2 + s + 18}$$

The roots of the characteristic equation are

$$s_1 = -\frac{1}{4} i\sqrt{143} - \frac{1}{4}$$

$$s_2 = \frac{1}{4} i\sqrt{143} - \frac{1}{4}$$

Since the real part of both roots is negative, the system is stable.

24.7

$$G_{transcription} = G_1 = \frac{5}{s+1}$$

$$G_{translation-\deg} = G_2 = Ke^{-\theta s}$$

(a) We calculate the frequency response measures as follows:

$$AR(G_1) = \frac{5}{\sqrt{\omega^2 + 1}}$$
$$\phi(G_1) = -\tan^{-1}(\omega)$$
$$AR(G_2) = K$$
$$\phi(G_2) = -\theta\omega$$

(b) During circadian rhythms, we require:

$$\omega_{circ} = \frac{1 \text{ cycle}}{24 \text{ hr}} = \frac{2\pi \text{ rad}}{24 \text{ hr}}$$

For stable oscillations,

$$-\pi = -\theta\omega_{circ} - \tan^{-1}(\omega_{circ})$$
$$\theta = \frac{\pi - \tan^{-1}(\omega_{circ})}{\omega_{circ}}$$
$$\theta = 11.0 \text{ hr}$$

(c) Now we want to find the gain, K, of the translation/degradation process.

$AR(G_2) = K$ at all frequencies, so need to calculate
G_1 amplitude at ω_{circ}

$$AR(G_1(\omega = \omega_{circ})) = \frac{5}{\sqrt{\omega_{circ}^2 + 1}} = 4.84$$

Need overall gain equal to 1, so therefore:

$$K = \frac{1}{AR(G_1(\omega_{circ}))} = 0.207$$