Chapter 8

8.1

a) For step response,

input is $u'(t) = M$, $U'(s) = \frac{M}{s}$

$$Y_a'(s) = K_c\left[\frac{\tau_D s+1}{\alpha\tau_D s+1}\right]U'(s) \quad = \quad K_c M\left[\frac{\tau_D s+1}{s(\alpha\tau_D s+1)}\right]$$

$$Y_a'(s) = \frac{K_c M\tau_D}{\alpha\tau_D s+1} + \frac{K_c M}{s(\alpha\tau_D s+1)}$$

Taking inverse Laplace transform

$$y_a'(t) = \frac{K_c M}{\alpha}e^{-t/(\alpha\tau_D)} + K_c M(1-e^{-t/(\alpha\tau_D)})$$

As $\alpha \to 0$

$$y_a'(t) = K_c M\delta(t)\int_{t=0}^{\infty}\frac{e^{-t/(\alpha\tau_D)}}{\alpha}dt \quad + \quad K_c M$$

$$y_a'(t) = K_c M\delta(t)\tau_D \quad + \quad K_c M$$

Ideal response,

$$Y_i'(s) = G_i(s)U'(s) \quad = \quad K_c M\left[\frac{\tau_D s+1}{s}\right] \; = \; K_c M\tau_D + \frac{K_c M}{s}$$

$$y_i'(t) = K_c M\tau_D\delta(t) \quad + \quad K_c M$$

Hence $y_a'(t) \to y_i'(t)$ as $\alpha \to 0$

For ramp response,

input is $u'(t) = Mt$, $U'(s) = \frac{M}{s^2}$

$$Y_a'(s) = K_c\left[\frac{\tau_D s+1}{\alpha\tau_D s+1}\right]U'(s) \quad = \quad K_c M\left[\frac{\tau_D s+1}{s^2(\alpha\tau_D s+1)}\right]$$

$$Y_a'(s) = \frac{K_c M\tau_D}{s(\alpha\tau_D s+1)} + \frac{K_c M}{s^2(\alpha\tau_D s+1)}$$

$$= K_c M\tau_D\left[\frac{1}{s} - \frac{\alpha\tau_D}{\alpha\tau_D s+1}\right] + K_c M\left[\frac{-\alpha\tau_D}{s} + \frac{1}{s^2} + \frac{(\alpha\tau_D)^2}{\alpha\tau_D s+1}\right]$$

Taking inverse Laplace transform

$$y_a'(t) = K_c M\tau_D\left[1 - e^{-t/(\alpha\tau_D)}\right] + K_c M\left[t + \alpha\tau_D(e^{-t/(\alpha\tau_D)} - 1)\right]$$

As $\alpha \to 0$

$$y_a'(t) = K_c M\tau_D \quad + \quad K_c M t$$

Ideal response,

$$Y_i'(s) = \; K_c M\left[\frac{\tau_D s+1}{s^2}\right] \; = \; \frac{K_c M\tau_D}{s} + \frac{K_c M}{s^2}$$

$$y_i'(t) = K_c M\tau_D \quad + \quad K_c M t$$

Hence $y_a'(t) \to y_i'(t)$ as $\alpha \to 0$

b) It may be difficult to obtain an accurate estimate of the derivative for use in the ideal transfer function.

c) Yes. The ideal transfer function amplifies the noise in the measurement by taking its derivative. The approximate transfer function reduces this amplification by filtering the measurement.

8.2

a) $$\frac{P'(s)}{E(s)} = \frac{K_1}{\tau_1 s+1} + K_2 = \frac{K_1 + K_2\tau_1 s + K_2}{\tau_1 s+1} \; = (K_1+K_2)\left[\frac{\frac{K_2\tau_1}{K_1+K_2}s+1}{\tau_1 s+1}\right]$$

b) $K_c = K_1 + K_2 \quad \rightarrow \quad K_2 = K_c - K_1$

$$\tau_1 = \alpha\tau_D$$

$$\tau_D = \frac{K_2\tau_1}{K_1 + K_2} = \frac{K_2\alpha\tau_D}{K_1 + K_2}$$

or $\quad 1 = \dfrac{K_2\alpha}{K_1 + K_2}$

$$K_1 + K_2 = K_2\alpha$$

$$K_1 = K_2\alpha - K_2 = K_2(\alpha - 1)$$

Substituting,

$$K_1 = (K_c - K_1)(\alpha - 1) = (\alpha - 1)K_c - (\alpha - 1)K_1$$

Then,

$$K_1 = \left(\frac{\alpha - 1}{\alpha}\right)K_c$$

c) If $K_c = 3$, $\tau_D = 2$, $\alpha = 0.1$ then,

$$K_1 = \frac{-0.9}{0.1} \times 3 = -27$$

$$K_2 = 3 - (-27) = 30$$

$$\tau_1 = 0.1 \times 2 = 0.2$$

Hence

$$K_1 + K_2 = -27 + 30 = 3$$

$$\frac{K_2\tau_1}{K_1 + K_2} = \frac{30 \times 0.2}{3} = 2$$

$$G_c(s) = 3\left(\frac{2s + 1}{0.2s + 1}\right)$$

8.3

a) From Eq. 8-14, the parallel form of the PID controller is :

$$G_i(s) = K_c'\left[1 + \frac{1}{\tau_I' s} + \tau_D' s\right]$$

From Eq. 8-15, for $\alpha \rightarrow 0$, the series form of the PID controller is:

$$G_a(s) = K_c\left[1 + \frac{1}{\tau_I s}\right][\tau_D s + 1]$$

$$= K_c\left[1 + \frac{\tau_D}{\tau_I} + \frac{1}{\tau_I s} + \tau_D s\right]$$

$$= K_c\left(1 + \frac{\tau_D}{\tau_I}\right)\left[1 + \frac{1}{\left(1 + \frac{\tau_D}{\tau_I}\right)\tau_I s} + \frac{\tau_D s}{\left(1 + \frac{\tau_D}{\tau_I}\right)}\right]$$

Comparing $G_a(s)$ with $G_i(s)$

$$K_c' = K_c\left(1 + \frac{\tau_D}{\tau_I}\right)$$

$$\tau_I' = \tau_I\left(1 + \frac{\tau_D}{\tau_I}\right)$$

$$\tau_D' = \frac{\tau_D}{1 + \frac{\tau_D}{\tau_I}}$$

b) Since $\left(1 + \frac{\tau_D}{\tau_I}\right) \geq 1$ for all τ_D, τ_I, therefore

$K_c \leq K_c'$, $\tau_I \leq \tau_I'$ and $\tau_D \geq \tau_D'$

c) For $K_c = 4$, τ_I=10 min , τ_D =2 min

$K_c' = 4.8$, $\tau_I' = 12\,\text{min}$, $\tau_D' = 1.67\,\text{min}$

d) Considering only first-order effects, a non-zero α will dampen all responses, making them slower.

8.4

Note that parts a), d), and e) require material from Chapter 9 to work.

a) System I (air-to-open valve) : K_v is positive.

System II (air-to-close valve) : K_v is negative.

b) System I : Flowrate too high $\rightarrow$ need to close valve $\rightarrow$ decrease controller output $\rightarrow$ reverse acting

System II: Flow rate too high $\rightarrow$ need to close valve $\rightarrow$ increase controller output $\rightarrow$ direct acting.

c) System I : K_c is positive
System II : K_c is negative

d)

	K_c	K_v	K_p	K_m
System I :	+	+	+	+
System II :	–	–	+	+

K_c and K_v must have same signs

e) Any negative gain must have a counterpart that "cancels" its effect. Thus, the rule:

# of negative gains to have negative feedback = 0 , 2 or 4.
# of negative gains to have positive feedback = 1 or 3.

8.5

a) From Eqs. 8-1 and 8-2,

$$p(t) = \bar{p} + K_c\left[y_{sp}(t) - y_m(t)\right] \tag{1}$$

The liquid-level transmitter characteristic is

$$y_m(t) = K_T\, h(t) \tag{2}$$

where h is the liquid level
$K_T > 0$ is the gain of the direct acting transmitter.

The control-valve characteristic is

$$q(t) = K_v p(t) \qquad (3)$$

where q is the manipulated flow rate
K_v is the gain of the control valve.

From Eqs. 1, 2, and 3

$$q(t) - \bar{q} = K_v\left[p(t) - \bar{p}\right] = K_V K_c\left[y_{sp}(t) - K_T h(t)\right]$$

$$K_V K_c = \frac{q(t) - \bar{q}}{y_{sp} - K_T h(t)}$$

For inflow manipulation configuration, $q(t) > \bar{q}$ when $y_{sp}(t) > K_T h(t)$. Hence $K_v K_c > 0$

then for "air-to-open" valve (K_v>0), K_c>0 : reverse acting controller
and for "air-to-close" valve (K_v<0), K_c<0 : direct acting controller

For outflow manipulation configuration, $K_v K_c < 0$

then for "air-to-open" valve, K_c<0 : direct acting controller
and for "air-to-close" valve, K_c>0 : reverse acting controller

b) See part(a) above

8.6

For PI control

$$p(t) = \bar{p} + K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)dt^*\right)$$

$$p'(t) = K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)dt^*\right)$$

Since
$e(t) = y_{sp} - y_m$ and $y_m = 2$

Then

$e(t) = -2$

$$p'(t) = K_c\left(-2 + \frac{1}{\tau_I}\int_0^t (-2)dt^*\right) = K_c\left(-2 - \frac{2}{\tau_I}t\right)$$

Initial response = $-2K_c$

Slope of early response = $-\dfrac{2K_c}{\tau_I}$

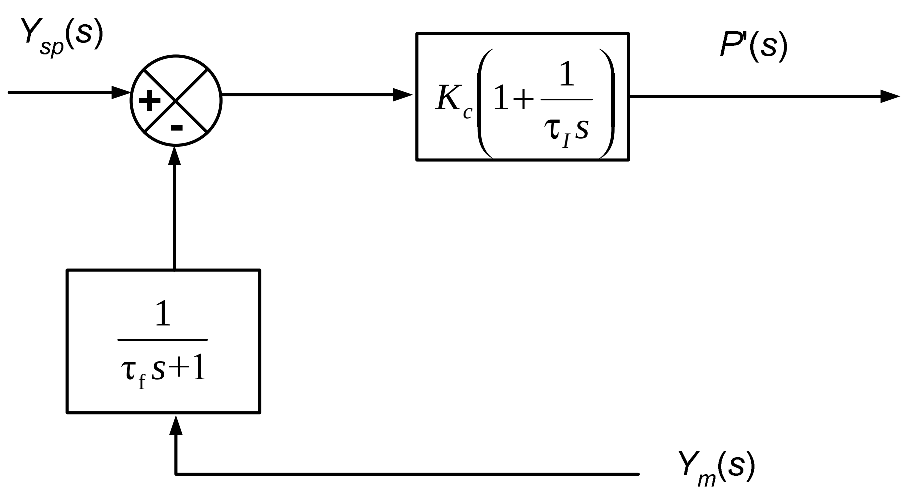
$-2K_c = 6 \quad \rightarrow \quad K_c = -3$

$-\dfrac{2K_c}{\tau_I} = 1.2 \text{ min}^{-1} \quad \rightarrow \quad \tau_I = 5 \text{ min}$

8.7

a) To include a process noise filter within a PI controller, it would be placed in the feedback path

b)



c) The TF between controller output $P'(s)$ and feedback signal $Y_m(s)$ would be

$$\frac{P'(s)}{Y_m(s)} = \frac{-K_c(\tau_I s+1)}{\tau_I s(\tau_f s+1)}$$ Negative sign comes from comparator

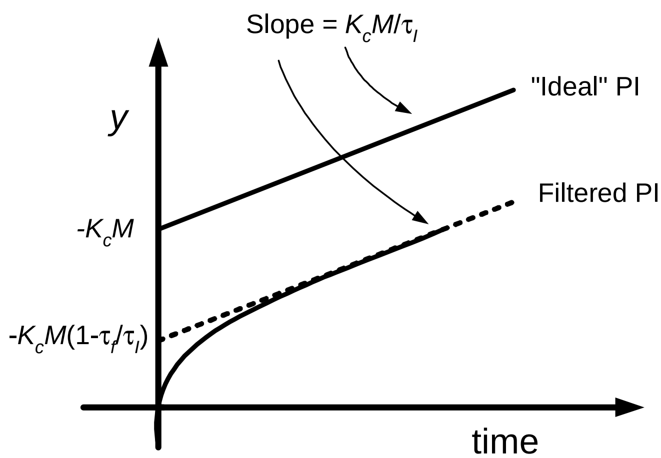
For $Y_m(s) = \frac{M}{s}$

$$P'(s) = \frac{-K_c M}{\tau_I}\left[\frac{\tau_I s+1}{s^2(\tau_f s+1)}\right] = \frac{-K_c M}{\tau_I}\left[\frac{A}{s^2} + \frac{B}{s} + \frac{C}{\tau_f s+1}\right]$$

The $\frac{C}{\tau_f s+1}$ term gives rise to an exponential.

To see the details of the response, we need to obtain B (= τ_I - τ_f) and A(=1) by partial fraction expansion.

The response, shown for a *negative* change in Y_m, would be



Note that as $\frac{\tau_f}{\tau_I} \rightarrow 0$, the two responses become the same.

d) If the measured level signal is quite noisy, then these changes might still be large enough to cause the controller output to jump around <u>even after filtering.</u>

One way to make the digital filter more effective is to filter the process output at a higher sampling rate (e.g., 0.1 sec) while implementing the controller algorithm at the slower rate (e.g., 1 sec).

A well-designed digital computer system will do this, thus eliminating the need for analog (continuous) filtering.

8.8

a) From inspection of Eq. 8-26, the derivative kick = $K_c \dfrac{\tau_D}{\Delta t}\Delta r$

b) Proportional kick = $K_c \Delta r$

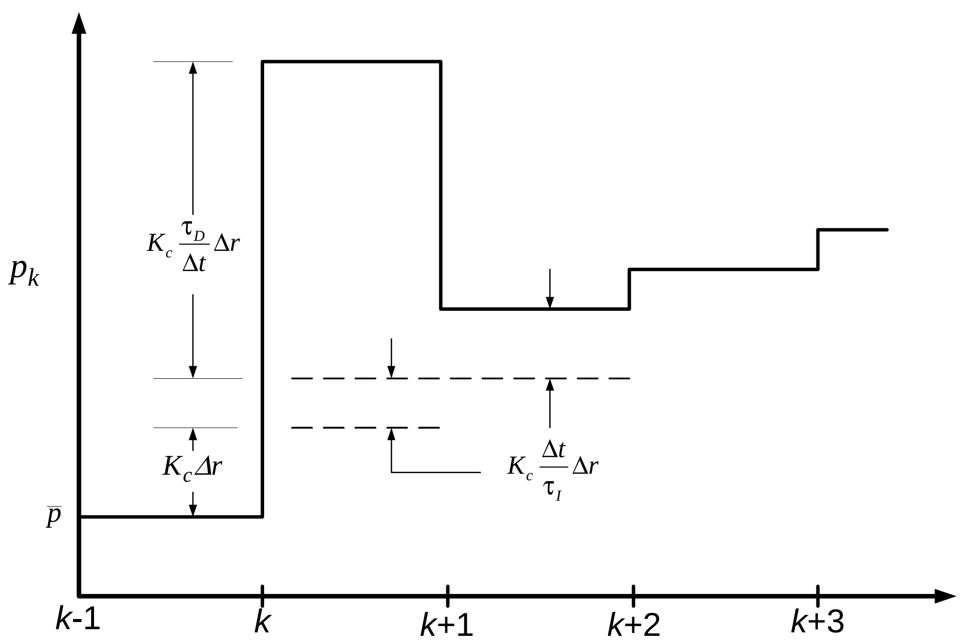
c) $e_1 = e_2 = e_3 = ... = e_{k\text{-}2} = e_{k\text{-}1} = 0$

$e_k = e_{k+1} = e_{k+2} = \ldots = \Delta r$

$$p_{k-1} = \overline{p}$$

$$p_k = \overline{p} + K_c\left[\Delta r + \frac{\Delta t}{\tau_I}\Delta r + \frac{\tau_D}{\Delta t}\Delta r\right]$$

$$p_{k+i} = \overline{p} + K_c\left[\Delta r + (1+i)\frac{\Delta t}{\tau_I}\Delta r\right] \quad , \quad i = 1, 2, \ldots$$



c) To eliminate derivative kick, replace ($e_k - e_{k\text{-}1}$) in Eq. 8-26 by (y_k-$y_{k\text{-}1}$).

8.9

a) The digital velocity P algorithm is obtained by setting $1/\tau_I = \tau_D = 0$ in Eq. 8-28 as

$$\Delta p_k = K_c(e_k - e_{k-1})$$

$$= K_c\left[(\bar{y}_{sp} - y_k) - (\bar{y}_{sp} - y_{k-1})\right]$$

$$= K_c\left[y_{k-1} - y_k\right]$$

The digital velocity PD algorithm is obtained by setting $1/\tau_I = 0$ in Eq. 8-28 as

$$\Delta p_k = K_c\left[(e_k - e_{k-1}) + \frac{\tau_D}{\Delta t}(e_k - 2e_{k-1} + e_{k-2})\right]$$

$$= K_c\left[(-y_k + y_{k-1}) + \frac{\tau_D}{\Delta t}(-y_k - 2y_{k-1} + y_{k-2})\right]$$

In both cases, Δp_k does not depend on $\bar{y}_{sp}$.

b) For both these algorithms $\Delta p_k = 0$ if $y_{k-2} = y_{k-1} = y_k$. Hence steady state is reached with a value of y that is independent of the value of $\bar{y}_{sp}$. Use of these algorithms is inadvisable if offset is a concern.

c) If the integral mode is present, then Δp_k contains the term $K_c \dfrac{\Delta t}{\tau_I}(\bar{y}_{sp} - y_k)$.

Thus, at steady state when $\Delta p_k = 0$ and $y_{k-2} = y_{k-1} = y_k$, $y_k = \bar{y}_{sp}$ and the offset problem is eliminated.

8.10

a)

$$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha\tau_D s + 1}\right)$$

$$= K_c \frac{(\tau_I s(\alpha\tau_D s + 1) + \alpha\tau_D s + 1 + \tau_D s \tau_I s)}{\tau_I s(\alpha\tau_D s + 1)}$$

$$= K_c\left[\frac{1 + (\tau_I + \alpha\tau_D)s + (1+\alpha)\tau_I\tau_D s^2}{\tau_I s(\alpha\tau_D s + 1)}\right]$$

Cross- multiplying

$$(\alpha\tau_I\tau_D s^2 + \tau_I s)\ P'(s) = K_c\left(1 + (\tau_I + \alpha\tau_D)s + (1+\alpha)\tau_I\tau_D s^2\right) E(s)$$

$$\alpha\tau_I\tau_D \frac{d^2 p'(t)}{dt^2} + \tau_I \frac{dp'(t)}{dt} = K_c\left(e(t) + (\tau_I + \alpha\tau_D)\frac{de(t)}{dt} + (1+\alpha)\tau_I\tau_D \frac{d^2 e(t)}{dt^2}\right)$$

b) $$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)\left(\frac{\tau_D s}{\alpha\tau_D s + 1}\right)$$

Cross-multiplying

$$\tau_I s^2(\alpha\tau_D s + 1)\ P'(s) = K_c\left((\tau_I s + 1)(\tau_D s + 1)\right) E(s)$$

$$\alpha\tau_I\tau_D \frac{d^2 p'(t)}{dt^2} + \tau_I \frac{dp'(t)}{dt} = K_c\left(e(t) + (\tau_I + \tau_D)\frac{de(t)}{dt} + \tau_I\tau_D \frac{d^2 e(t)}{dt^2}\right)$$

c) We need to choose parameters in order to simulate:

e.g., $K_c = 2$, $\tau_I = 3$, $\tau_D = 0.5$, $\alpha = 0.1$, $M = 1$

By using Simulink-MATLAB

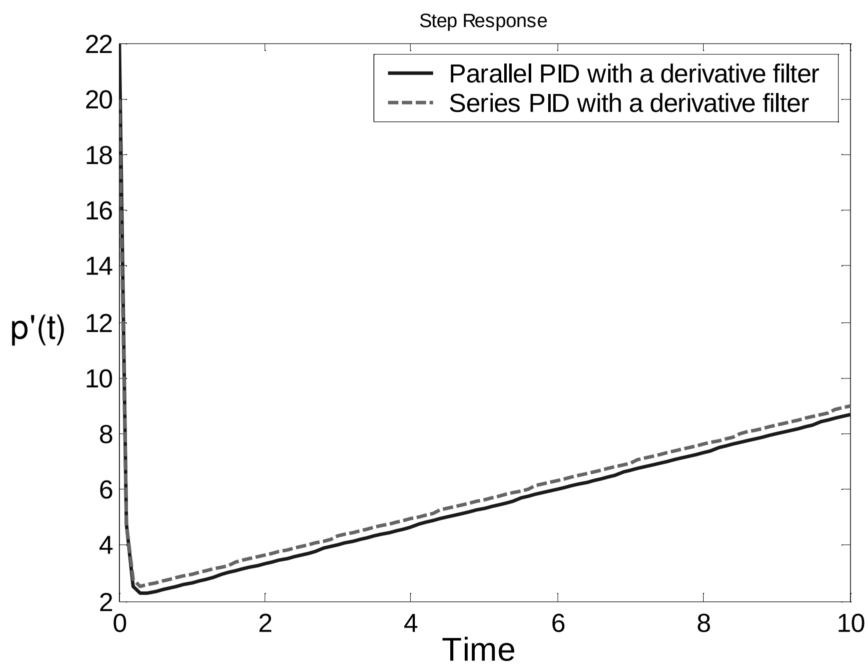


Figure S8.10. *Step responses for both parallel and series PID controllers with derivative filter.*

8.11

a) $$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)(\tau_D s+1)$$

$$\tau_I s\; P'(s) = K_c\big((\tau_I s+1)(\tau_D s+1)\big)\,E(s)$$

$$\frac{dp'(t)}{dt} = \frac{K_c}{\tau_I}\left(e(t)+(\tau_I+\tau_D)\frac{de(t)}{dt}+\tau_I\tau_D\frac{d^2e(t)}{dt^2}\right)$$

b) With the derivative mode active, an impulse response will occur at $t = 0$. Afterwards, for a unit step change in $e(t)$, the response will be a ramp with slope = $K_c(\tau_I+\tau_D)/\tau_I$ and intercept = K_c/τ_I for $t>0$.