## **Chapter 9**

9.1

a) Flowrate pneumatic transmitter:

$$q_m(\text{psig}) = \left(\frac{15 \text{ psig} - 3 \text{ psig}}{400 \text{ gpm} - 0 \text{ gpm}}\right) (q \text{ gpm} - 0 \text{ gpm}) + 3 \text{ psig}$$
$$= \left(0.03 \frac{\text{psig}}{\text{gpm}}\right) q(\text{gpm}) + 3 \text{ psig}$$

Pressure current transmitter:

$$P_m(\text{mA}) = \left(\frac{20 \text{ mA} - 4 \text{ mA}}{30 \text{ in.Hg} - 10 \text{ in.Hg}}\right) (p \text{ in.Hg} - 10 \text{ in.Hg}) + 4 \text{ mA}$$
$$= \left(0.8 \frac{\text{mA}}{\text{in.Hg}}\right) p(\text{in.Hg}) - 4 \text{ mA}$$

Level voltage transmitter:

$$h_m(VDC) = \left(\frac{5 \text{ VDC} - 1 \text{ VDC}}{20 \text{ m} - 0.5 \text{ m}}\right) (h(\text{m}) - 0.5\text{m}) + 1 \text{ VDC}$$
$$= \left(0.205 \frac{\text{VDC}}{\text{m}}\right) h(\text{m}) + 0.897 \text{ VDC}$$

Concentration transmitter:

$$C_m(\text{VDC}) = \left(\frac{10 \text{ VDC} - 1 \text{ VDC}}{20 \text{ g/L} - 2 \text{ g/L}}\right) (C(\text{g/L}) - 2 \text{ g/L}) + 1 \text{ VDC}$$
$$= \left(0.5 \frac{\text{VDC}}{\text{g/L}}\right) C(\text{g/L})$$

b) The gains, zeros and spans are:

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	PNEUMATIC	CURRENT	VOLTAGE	VOLTAGE
GAIN	0.03psig/gpm	0.8mA/in.Hg	0.205 VDC/m	0.5VDC/g/L
ZERO	0gal/min	10 in.Hg	0.5m	2g/L
SPAN	400gal/min	20 in.Hg	19.5m	18g/L

<sup>\*</sup>The gain is a constant quantity

9.2

a) The safest conditions are achieved by the lowest temperatures and pressures in the flash vessel.

VALVE 1.- Fail close

VALVE 2.- Fail open

VALVE 3.- Fail open

VALVE 4.- Fail open

VALVE 5.- Fail close

Setting valve 1 as fail close prevents more heat from going to flash drum and setting valve 3 as fail open to allow the steam chest to drain. Setting valve 3 as fail open prevents pressure build up in the vessel. Valve 4 should be fail-open to evacuate the system and help keep pressure low. Valve 5 should be fail-close to prevent any additional pressure build-up.

b) Vapor flow to downstream equipment can cause a hazardous situation

VALVE 1.- Fail close

VALVE 2.- Fail open

VALVE 3.- Fail close

VALVE 4.- Fail open

VALVE 5.- Fail close

Setting valve 1 as fail close prevents more heat from entering flash drum and minimizes future vapor production. Setting valve 2 as fail open will allow the steam chest to be evacuated, setting valve 3 as fail close prevents vapor from escaping the vessel. Setting valve 4 as fail open allows liquid to leave, preventing vapor build up. Setting valve 4 as fail-close prevents pressure buildup.

c) Liquid flow to downstream equipment can cause a hazardous situation

VALVE 1.- Fail close

VALVE 2.- Fail open

VALVE 3.- Fail open

VALVE 4.- Fail close

VALVE 5.- Fail close

Set valve 1 as fail close to prevent all the liquid from being vaporized (This would cause the flash drum to overheat). Setting valve 2 as fail open will allow the steam chest to be evacuated. Setting valve 3 as fail open prevents pressure buildup in drum. Setting valve 4 as fail close prevents liquid from escaping. Setting valve 5 as fail close prevents liquid build-up in drum

9.3

a) Assume that the differential-pressure transmitter has the standard range of 3 psig to 15 psig for flow rates of 0 gpm to  $q_m(\text{gpm})$ . Then, the pressure signal of the transmitter is

$$P_{T} = 3 + \left(\frac{12}{q_{m}^{2}}\right)q^{2}$$

$$K_{T} = \frac{dP_{T}}{dq} = \left(\frac{24}{q_{m}^{2}}\right)q$$

$$K_{T} = \begin{cases} 2.4/q_{m} & , & q = 10\% \text{ of } q_{m} \\ 12/q_{m} & , & q = 50\% \text{ of } q_{m} \\ 18/q_{m} & , & q = 75\% \text{ of } q_{m} \\ 21.6/q_{m} & , & q = 90\% \text{ of } q_{m} \end{cases}$$

b) Eq. 9-2 gives

$$q = C_{v} f(\ell) \left( \frac{\Delta P_{v}}{g_{s}} \right)^{1/2} = q_{m} f(\ell)$$

For a linear valve,

 $f(\ell) = \ell = \alpha P$ , where  $\alpha$  is a constant.

$$K_V = \frac{dq}{dP} = q_m \alpha$$

Hence, linear valve gain is same for all flowrates

For a <u>square-root valve</u>,

$$f(\ell) = \sqrt{\ell} = \sqrt{\alpha P}$$

$$K_{V} = \frac{dq}{dP} = q_{m}\sqrt{\alpha} \frac{1}{2\sqrt{p}} = \frac{q_{m}\alpha}{2} \frac{1}{\sqrt{\ell}} = \frac{q_{m}\alpha}{2} \frac{q_{m}}{q}$$

$$\begin{cases}
5q_{m}\alpha & , & q = 10\% \text{ of } q_{m} \\
q_{m}\alpha & , & q = 50\% \text{ of } q_{m} \\
0.67q_{m}\alpha & , & q = 75\% \text{ of } q_{m} \\
0.56q_{m}\alpha & , & q = 90\% \text{ of } q_{m}
\end{cases}$$

For an equal-percentage valve,

$$f(\ell) = R^{\ell-1} = R^{\alpha P-1}$$

$$K_{V} = \frac{dq}{dP} = q_{m} \alpha R^{\ell-1} \ln R = q_{m} \alpha \ln R \left(\frac{q}{q_{m}}\right)$$

$$K_{V} = \begin{cases} 0.1q_{m} \alpha \ln R &, & q = 10\% \text{ of } q_{m} \\ 0.5q_{m} \alpha \ln R &, & q = 50\% \text{ of } q_{m} \\ 0.75q_{m} \alpha \ln R &, & q = 75\% \text{ of } q_{m} \\ 0.9q_{m} \alpha \ln R &, & q = 90\% \text{ of } q_{m} \end{cases}$$

c) The overall gain is

$$K_{TV} = K_T K_V$$

Using results in parts a) and b)

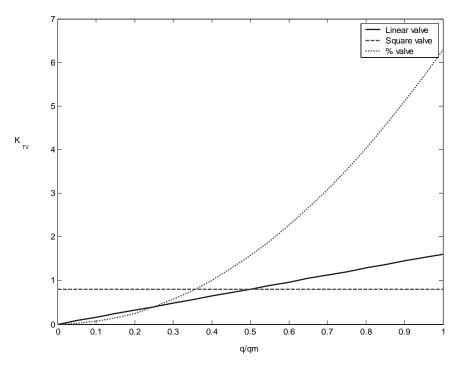
For a linear valve

## For a <u>square-root valve</u>

$$K_{TV} = 12\alpha$$
 for all values of q

For an equal-percentage valve

The combination with a square-root valve gives linear characteristics over the full range of flow rate. For R = 50 and  $\alpha = 0.067$  values, a graphical comparison is shown in Fig. S9.3



**Fig. S9.3.-** *Graphical comparison of the gains for the three valves* 

d) In a real situation, the square-root valve combination will not give an exactly linear form of the overall characteristics, but it will still be the combination that gives the most linear characteristics.

9.4

Nominal pressure drop over the condenser is 30 psi

$$\Delta P_c = Kq^2$$

$$30 = K (200)^2 \qquad , \qquad K = \frac{3}{4000} \frac{\text{psi}}{\text{gpm}^2}$$

$$\Delta P_c = \frac{3}{4000} q^2$$

Let  $\Delta P_{\nu}$  be the pressure drop across the valve and  $\overline{\Delta P}_{\nu}$ ,  $\overline{\Delta P}_{c}$  be the nominal values of  $\Delta P_{\nu}$ ,  $\Delta P_{c}$ , respectively. Then,

$$\Delta P_{\nu} = \left(\Delta \overline{P}_{\nu} + \Delta \overline{P}_{c}\right) - \Delta P_{c} = \left(30 + \Delta \overline{P}_{\nu}\right) - \frac{3}{4000}q^{2} \tag{1}$$

Using Eq. 9-2

$$q = C_{v} f(\ell) \left( \frac{\Delta P_{v}}{g_{s}} \right)^{1/2}$$
 (2)

and

$$C_{v} = \frac{\overline{q}}{f(\overline{l})} \left(\frac{\Delta \overline{P}_{v}}{g_{s}}\right)^{-1/2} = \frac{200}{0.5} \left(\frac{\Delta \overline{P}_{v}}{1.11}\right)^{-1/2}$$
(3)

Substituting for  $\Delta P_{\nu}$  from (1) and  $C_{\nu}$  from (3) into (2),

$$q = 400 \left(\frac{\Delta \overline{P}_{v}}{1.11}\right)^{-1/2} f(\ell) \left(\frac{30 + \Delta \overline{P}_{v} - \frac{3}{4000} q^{2}}{1.11}\right)^{-1/2}$$
(4)

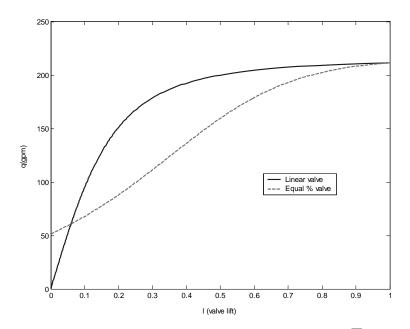
a) 
$$\Delta \overline{P}_{v} = 5$$

Linear valve:  $f(\ell) = \ell$ , and Eq. 4 becomes

$$l = \frac{q}{188.5} \left( \frac{35 - 0.00075q^2}{1.11} \right)^{-1/2}$$

Equal % valve:  $f(\ell) = R^{\ell-1} = 20^{\ell-1}$  assuming R=20

$$l = 1 + \frac{\ln\left[\frac{q}{188.5} \left(\frac{35 - 0.00075q^2}{1.11}\right)^{-1/2}\right]}{\ln 20}$$



**Figure S9.4a.** Control valve characteristics for  $\Delta \overline{P}_v = 5$ 

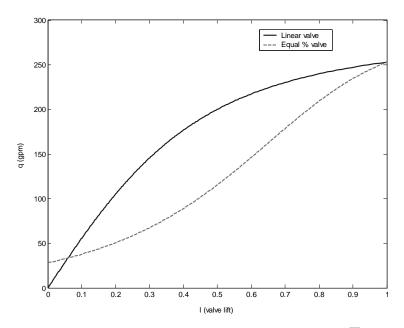
b) 
$$\Delta \overline{P}_v = 30$$

Linear valve:  $f(\ell) = \ell$  , and Eq. 4 becomes

$$l = \frac{q}{76.94} \left( \frac{60 - 0.00075q^2}{1.11} \right)^{-1/2}$$

Equal % valve:  $f(\ell) = 20^{\ell-1}$ ; Eq. 4 gives

$$l = 1 + \frac{\ln \left[ \frac{q}{76.94} \left( \frac{60 - 0.00075q^2}{1.11} \right)^{-1/2} \right]}{\ln 20}$$



**Figure S9.4b.** Control valve characteristics for  $\Delta \overline{P}_{\nu} = 30$ 

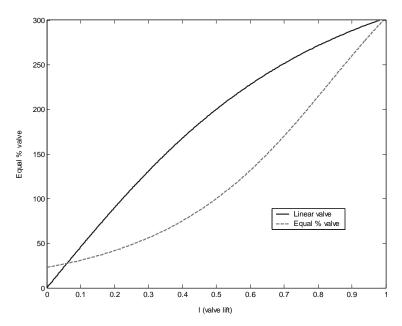
c) 
$$\Delta \overline{P}_{v} = 90$$

Linear valve:  $f(\ell) = \ell$ , and Eq. 4 becomes

$$l = \frac{q}{44.42} \left( \frac{120 - 0.00075q^2}{1.11} \right)^{-1/2}$$

Equal % valve:  $f(\ell) = 20^{\ell-1}$ ; Eq. 4 gives

$$l = 1 + \frac{\ln\left[\frac{q}{44.42} \left(\frac{120 - 0.00075q^2}{1.11}\right)^{-1/2}\right]}{\ln 20}$$



**Figure S9.4c.** Control valve characteristics for  $\Delta \overline{P}_v = 90$ 

Conclusions from the above plots:

1) Linearity of the valve

For  $\Delta \overline{P}_{v} = 5$ , the linear valve is not linear and the equal % valve is linear over a narrow range.

For  $\Delta \overline{P}_{\nu} = 30$ , the linear valve is linear for very low  $\ell$  and equal % valve is linear over a wider range of  $\ell$ .

For  $\Delta \overline{P}_v = 90$ , the linear valve is linear for  $\ell < 0.5$  approx., equal % valve is linear for  $\ell > 0.5$  approx.

- 2) Ability to handle flowrates greater than nominal increases as  $\Delta \overline{P}_{\nu}$  increases, and is higher for the equal % valve compared to that for the linear valve for each  $\Delta \overline{P}_{\nu}$ .
- 3) The pumping costs are higher for larger  $\Delta \overline{P}_{\nu}$ . This offsets the advantage of large  $\Delta \overline{P}_{\nu}$  in part 1) and 2)

Let  $\Delta P_v/\Delta P_s = 0.33$  at the nominal q = 320 gpm

$$\Delta P_s = \Delta P_{B+} \Delta P_o = 40 + 1.953 \times 10^{-4} q^2$$

$$\Delta P_v = P_D - \Delta P_s = (1 - 2.44 \times 10^{-6} q^2) P_{DE} - (40 + 1.953 \times 10^{-4} q^2)$$

$$(1.244 \times 10^{-6} \times 320^2) P_{DE} - (40 + 1.953 \times 10^{-4} \times 320^2)$$

$$\frac{(1-2.44\times10^{-6}\times320^2)P_{DE}-(40+1.953\times10^{-4}\times320^2)}{(40+1.953\times10^{-4}\times320^2)}=0.33$$

$$P_{DE} = 106.4 \text{ psi}$$

Let 
$$q_{des} = \overline{q} = 320 \text{ gpm}$$

For rated  $C_{\nu}$ , valve is completely open at 110%  $q_{des}$  i.e., at 352 gpm or the upper limit of 350 gpm

$$C_{v} = q \left( \frac{\Delta p_{v}}{q_{s}} \right)^{\frac{1}{2}}$$

$$= 350 \left[ \frac{(1 - 2.44 \times 10^{-6} \times 350^{2})106.4 - (40 + 1.953 \times 10^{-4} \times 350^{2})}{0.9} \right]^{-\frac{1}{2}}$$

Then using Eq. 9-11

$$l = 1 + \frac{\ln \left[ \frac{q}{101.6} \left( \frac{66.4 - 4.55 \times 10^{-4} q^2}{0.9} \right)^{-1/2} \right]}{\ln 50}$$

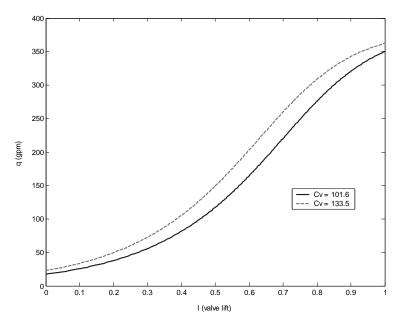


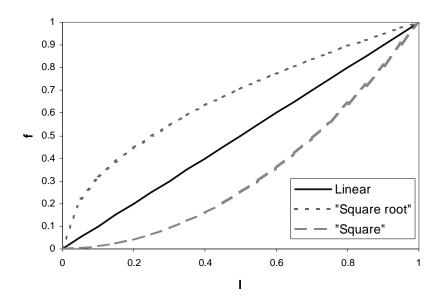
Figure S9.5. Control valve characteristics

From the plot of valve characteristic for the rated  $C_v$  of 101.6, it is evident that the characteristic is reasonably linear in the operating region  $250 \le q \le 350$ .

The pumping cost could be further reduced by lowering the  $P_{DE}$  to a value that would make  $\Delta P_v/\Delta P_s = 0.25$  at q = 320 gpm. Then  $P_{DE} = 100.0$  and for  $q_{des} = 320$  gpm, the rated  $C_v = 133.5$ . However, as the plot shows, the valve characteristic for this design is more nonlinear in the operating region. Hence the selected valve is  $C_v = 101.6$ 



a)



The "square" valve appears similar to the equal percentage valve in Fig. 9.8

Valve	Gain ( $d\!f$ $/$ $d\ell$ )	ℓ =0	ℓ <b>=</b> 0.5	ℓ =1
Quick open	$1/2\sqrt{\ell}$	8	0.707	0.5
Linear	1	1	1	1
Slow open	$2\ell$	0	1	2

The largest gain for quick opening is at  $\ell = 0$  (gain =  $\infty$ ), while largest for slow opening is at  $\ell = 1$  (gain = 2). A linear valve has constant gain.

c) 
$$q = C_v f(\ell) \sqrt{\frac{\Delta P_v}{g_s}}$$

For 
$$g_s = 1$$
 ,  $\Delta P_v = 64$  ,  $q = 1024$ 

 $C_v$  is found when  $f(\ell)=1$  (maximum flow):

$$C_v = \frac{q}{\sqrt{\Delta P_v / g_s}} = \frac{1024 \text{ gal/min}}{\sqrt{64 \text{ lb/in}^2}} = \frac{1024}{8} = 128 \frac{\text{gal.in}}{\text{min.(lb)}^{1/2}}$$

d)  $\ell$  in terms of applied pressure

$$\ell = 0$$
 when  $p = 3$  psig  $\ell = 1$  when  $p = 15$  psig

Then 
$$\ell = \frac{(1-0)}{(15-3)}(p-3) = \frac{1}{12}p - 0.25$$

e) 
$$q = 128 \ell^2 \sqrt{\Delta P_{\nu}}$$
 for slow opening ("square") valve  
 $= 128 \sqrt{\Delta P_{\nu}} \left( \frac{1}{12} p - 0.25 \right)^2$   
 $= \frac{128}{144} \sqrt{\Delta P_{\nu}} (p-3)^2 = 0.8889 \sqrt{\Delta P_{\nu}} (p-3)^2$ 

$$p=3$$
 ,  $q=0$  for all  $\Delta P_v$ 

$$p=15$$
 ,  $q=128\sqrt{\Delta P_{v}}$  
$$=0 \text{ for } \Delta P_{v}=0$$
 
$$=1024 \text{ for } \Delta P_{v}=64$$

looks O.K

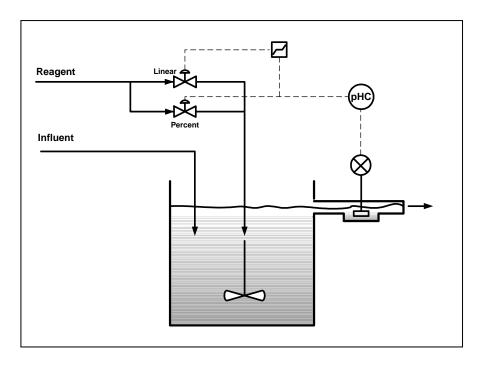
9.7

Because the system dynamic behavior would be described using deviation variables, all that is important are the terms involving x, dx/dt and  $d^2x/dt^2$ . Using the values for M, K and R and solving the homogeneous o.d.e:

$$0.3\frac{d^2x}{dt^2} + 15,000\frac{dx}{dt} + 3600x = 0$$

This yields a strongly overdamped solution, with  $\zeta$ =228, which can be approximated by a first order model by ignoring the  $d^2x/dt^2$  ter

A control system can incorporate valve sequencing for wide range along with compensation for the nonlinear curve (Shinskey, 1996). It features a small equal-percentage valve driven by a proportional pH controller. The output of the pH controller also operates a large linear valve through a proportional-plus-reset controller with a dead zone. The system is shown in Fig. E9.8



**Figure S9.8.** *Schematic diagram for pH control* 

Equal-percentage valves have an exponential characteristic, similar to the pH curve. As pH deviates from neutrality, the gain of the curve decreases; but increasing deviation will open the valve farther, increasing its gain in a compensating manner. As the output of the proportional controller drives the small valve to either of its limits, the dead zone of the two-mode controller is exceeded. The large valve is moved at a rate determined by the departure of the control signal from the dead zone and by the values of proportional and reset. When the control signal reenters the dead zone, the large valve is held in its last position. The large valve is of linear characteristic, because the process gain does not vary with flow, as some gains do.

Note: in the book's second printing, the transient response in this problem will be modified by adding 5 minutes to the time at which each temperature reading was taken.

We wish to find the model:

$$\frac{T_m'(s)}{T'(s)} = \frac{K_m}{\tau_m s + 1}$$

where  $T_m$  is the measurement T is liquid temperature

From Eq. 9-1,

$$K_m = \frac{\text{range of instrument output}}{\text{range of instrument input}} = \frac{20 \text{ mA} - 4 \text{ mA}}{400^{\circ}\text{C} - 0^{\circ}\text{C}} = \frac{16 \text{ mA}}{400^{\circ}\text{C}} = 0.04 \frac{\text{mA}}{^{\circ}\text{C}}$$

From Fig. 5.5,  $\tau$  can be found by plotting the thermometer reading vs. time and the transmitter reading vs. time and drawing a horizontal line between the two ramps to find the time constant. This is shown in Fig. S9.9.

Hence,  $\Delta \tau = 1.33 \text{ min} = 80 \text{ sec}$ 

To get  $\tau$ , add the time constant of the thermometer (20 sec) to  $\Delta \tau$  to get

$$\tau = 100 \text{ sec.}$$

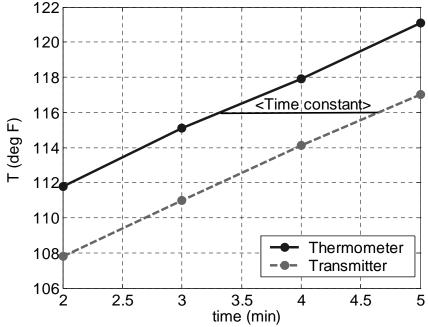


Figure S9.9. Data test from the Thermometer and the Transmitter

9.10

precision = 
$$\frac{0.1 \text{ psig}}{20 \text{ psig}} = 0.5\%$$
 of full scale

accuracy is unknown since the "true" pressure in the tank is unknown

resolution = 
$$\frac{0.1 \text{ psig}}{20 \text{ psig}} = 0.5\%$$
 of full scale

repeatability = 
$$\frac{\pm 0.1 \text{ psig}}{20 \text{ psig}} = \pm 0.5\%$$
 of full scale

9.11

Assume that the gain of the sensor/transmitter is unity. Then,

$$\frac{T'_m(s)}{T'(s)} = \frac{1}{(s+1)(0.1s+1)}$$

where T is the quantity being measured  $T_m$  is the measured value

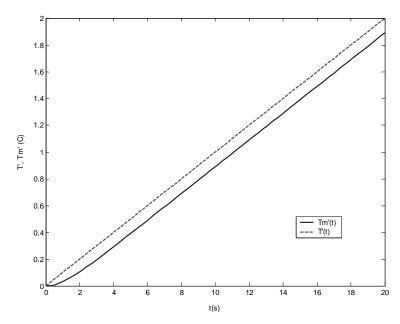
$$T'(t) = 0.1 \ t \ ^{\circ}\text{C/s} \ , \ T'(s) = \frac{0.1}{s^2}$$

$$T'_m(s) = \frac{1}{(s+1)(0.1s+1)} \times \frac{0.1}{s^2}$$

$$T'_m(t) = -0.0011e^{-10t} + 0.111e^{-t} + 0.1t - 0.11$$

Maximum error occurs as  $t \rightarrow \infty$  and equals |0.1t - (0.1t - 0.11)| = 0.11 °C

If the smaller time constant is neglected, the time domain response is a bit different for small values of time, although the maximum error  $(t\rightarrow\infty)$  doesn't change.



**Figure S9.11.** Response for process temperature sensor/transmitter