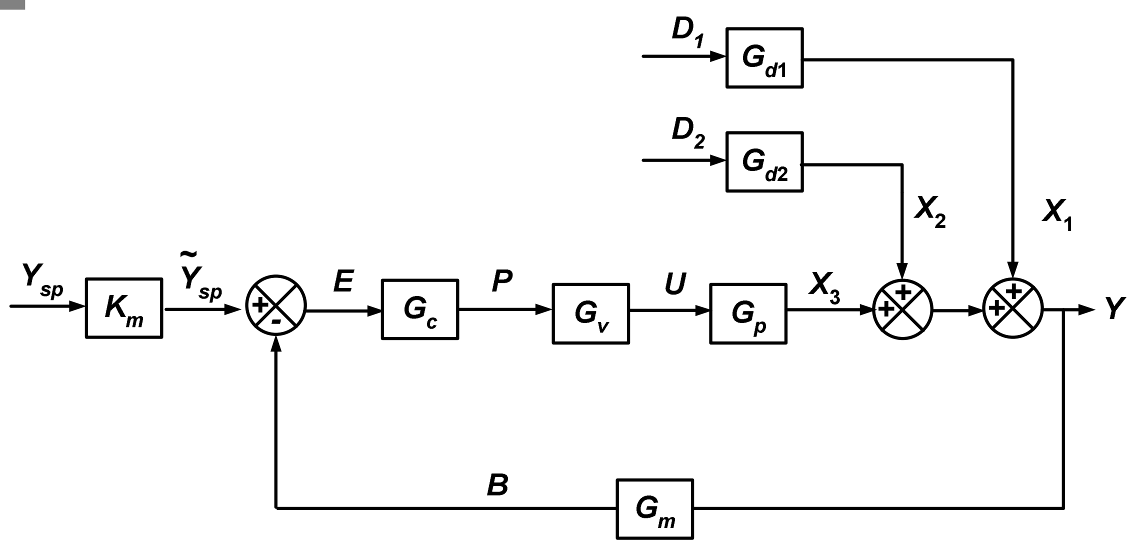
Chapter 11

11.1



11.2

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

The closed-loop transfer function for set-point changes is given by Eq. 11-36 with K_c replaced by $K_c\left(1+\frac{1}{\tau_I s}\right)$,

$$\frac{H'(s)}{H'_{sp}(s)} = \frac{K_c K_v K_p K_m\left(1+\frac{1}{\tau_I s}\right)\frac{1}{(\tau s+1)}}{1+K_c K_v K_p K_m\left(1+\frac{1}{\tau_I s}\right)\frac{1}{(\tau s+1)}}$$

$$\frac{H'(s)}{H'_{sp}(s)} = \frac{(\tau_I s+1)}{\tau_3 s^2 + 2\zeta_3\tau_3 s+1}$$

where ζ_3, τ_3 are defined in Eqs. 11-62, 11-63 , $K_p = R = 1.0$ min/ft^2 , and $\tau = RA = 3.0$ min

$$K_{OL} = K_c K_v K_p K_m = (4)\left(0.2\frac{\text{ft}^3/\text{min}}{\text{psi}}\right)\left(1.0\frac{\text{min}}{\text{ft}^2}\right)\left(1.7\frac{\text{psi}}{\text{ft}}\right) = 1.36$$

$$\tau_3^{\,2} = \frac{\tau\,\tau_I}{K_{OL}} = \frac{(3\text{ min})(3\text{ min})}{1.36} = 6.62\text{ min}^2$$

$$2\,\zeta_3\,\tau_3 = \left(\frac{1+K_{OL}}{K_{OL}}\right)\tau_I = \frac{2.36}{1.36}\times 3 = 5.21\text{ min}$$

$$\frac{H'(s)}{H'_{sp}(s)} = \frac{3s+1}{(3.0s+1)+(2.21s+1)} = \frac{1}{2.21s+1}$$

For $H'_{sp}(s) = \dfrac{(3-2)}{s} = \dfrac{1}{s}$

$$h'(t) = 1 - e^{-t/2..21}$$

$$t = -2.21\ln[1-h'(t)]$$

$h(t) = 2.5$ ft $\quad h'(t) = 0.5$ ft $\quad t = 1.53$ min

$h(t) = 3.0$ ft $\quad h'(t) = 1.0$ ft $\quad t \to \infty$

Therefore,

$$h(t = 1.53\text{ min}) = 2.5\text{ft}$$

$$h(t \to \infty) = 3.0\text{ ft}$$

11.3

$G_c(s) = K_c = 5$ ma/ma

Assume $\tau_m = 0$, $\tau_v = 0$, and $K_1 = 1$, in Fig 11.7.

a) Offset = $T'_{sp}(\infty) - T'(\infty) = 5^\circ F - 4.14^\circ F = 0.86^\circ F$

b) $$\frac{T'(s)}{T'_{sp}(s)} = \frac{K_m K_c K_{IP} K_v \left(\dfrac{K_2}{\tau s + 1}\right)}{1 + K_m K_c K_{IP} K_v \left(\dfrac{K_2}{\tau s + 1}\right)}$$

Using the standard current range of 4-20 ma,

$$K_m = \frac{20 \text{ ma} - 4 \text{ ma}}{50^\circ F} = 0.32 \text{ ma}/^\circ\text{F}$$

$K_v = 1.2$, $K_{IP} = 0.75$ psi/ma , $\tau = 5$ min , $T'_{sp}(s) = \dfrac{5}{s}$

$$T'(s) = \frac{7.20 K_2}{s(5s + 1 + 1.440 K_2)}$$

$$T'(\infty) = \lim_{s \to 0} sT'(s) = \frac{7.20 K_2}{(1 + 1.440 K_2)}$$

$T'(\infty) = 4.14^\circ$ F $\qquad K_2 = 3.34^\circ$ F / psi

c) From Fig. 11-7, since $T_i' = 0$

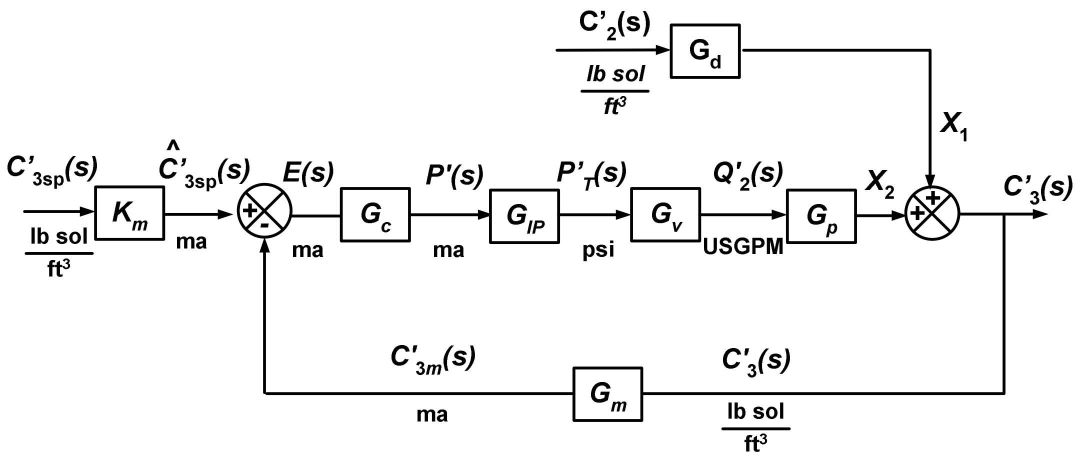
$P_t'(\infty) K_v K_2 = T'(\infty)$, $\qquad P_t'(\infty) = 1.03$ psi

and $\bar{P}_t' K_v K_2 + \bar{T}_i K_1 = \bar{T}$, $\bar{P}_t = 3.74$ psi

$P_t(\infty) = \bar{P}_t - P_t'(\infty) = 4.77$ psi

11.4

a)



b) $G_m(s) = K_m e^{-\theta_m s}$ assuming $\tau_m = 0$

$$G_m(s) = \frac{(20-4)\text{ma}}{(9-3)\dfrac{\text{lb sol}}{\text{ft}^3}} e^{-2s} = \left(2.67\frac{\text{ma}}{\text{lb sol/ft}^3}\right)e^{-2s}$$

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

$$G_{IP}(s) = K_{IP} = 0.3 \text{ psi/ma}$$

$$G_v(s) = K_v = \frac{(10-20)\text{ USGPM}}{(12-6)\ psi} = -1.67\frac{\text{USGPM}}{\text{psi}}$$

Overall material balance for the tank,

$$\left(7.481\frac{\text{USgallons}}{\text{ft}^3}\right)A\frac{dh}{dt} = q_1 + q_2 - C_v\sqrt{h} \qquad (1)$$

Component balance for the solute,

$$7.481\ A\frac{d(hC_3)}{dt} = q_1c_1 + q_2c_2 - (C_v\sqrt{h})c_3 \qquad (2)$$

Linearizing (1) and (2) gives

$$7.481\, A\frac{dh'}{dt} = q_2' - \left(\frac{C_v}{2\sqrt{\bar{h}}}\right)h' \qquad (3)$$

$$7.481\, A\left(\bar{c}_3\frac{dh'}{dt} + \bar{h}\frac{dc_3'}{dt}\right) = \bar{c}_2 q_2' + \bar{q}_2 c_2' - \bar{c}_3\left(\frac{C_v}{2\sqrt{\bar{h}}}\right)h' - \left(C_v\sqrt{\bar{h}}\right)c_3'$$

Subtracting (3) times $\bar{c}_3$ from the above equation gives

$$7.481\, A\bar{h}\frac{dc_3'}{dt} = (\bar{c}_2 - \bar{c}_3)\; q_2' + \bar{q}_2 c_2' - \left(C_v\sqrt{\bar{h}}\right)c_3'$$

Taking Laplace transform and rearranging gives

$$C_3'(s) = \frac{K_1}{\tau s+1}Q_2'(s) + \frac{K_2}{\tau s+1}C_2'(s)$$

where

$$K_1 = \frac{\bar{c}_2 - \bar{c}_3}{C_v\sqrt{\bar{h}}} = 0.08\frac{\text{lb sol/ft}^3}{\text{USGPM}}$$

$$K_2 = \frac{\bar{q}_2}{C_v\sqrt{\bar{h}}} = 0.6$$

$$\tau = \frac{7.481A\sqrt{\bar{h}}}{C_v} = 15 \text{ min}$$

since $A = \pi D^2/4 = 12.6 \text{ ft}^2$, and

$$\bar{h} = \left(\frac{\bar{q}_3}{C_v}\right)^2 = \left(\frac{\bar{q}_1 + \bar{q}_2}{C_v}\right)^2 = 4\ ft$$

Therefore,

$$G_p(s) = \frac{0.08}{15s+1}$$

$$G_d(s) = \frac{0.6}{15s+1}$$

c) The closed-loop responses for disturbance changes and for setpoint changes can be obtained using block diagram algebra for the block diagram in part (a). Therefore, these responses will change only if any of the transfer functions in the blocks of the diagram change.

i. $\bar{c}_2$ changes. Then block transfer function $G_p(s)$ changes due to K_1. Hence $G_c(s)$ does need to be changed, and retuning is required.

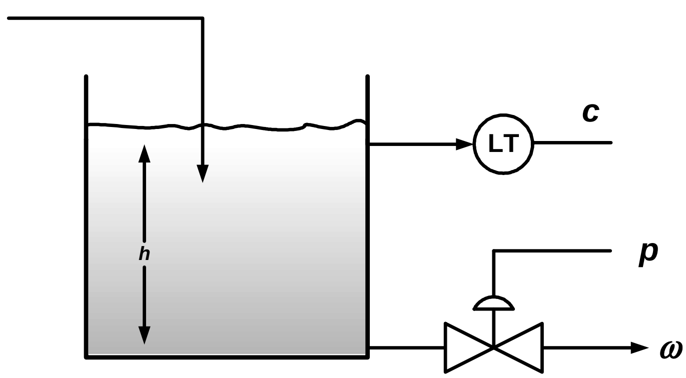
ii. K_m changes. Block transfer functions do change. Hence G_c(s) needs to be adjusted to compensate for changes in block transfer functions. The PI controller should be retuned.

iii. K_m remains unchanged. No block transfer function changes. The controller does not need to be retuned.

11.5

a)

One example of a negative gain process that we have seen is the liquid level process with the outlet stream flow rate chosen as the manipulated variable



With an "air-to-open" valve, w increases if p increases. However, h decreases as w increases. Thus K_p <0 since $\Delta h/\Delta w$ is negative.

b) K_cK_p must be positive. If K_p is negative, so is K_c. See (c) below.

c) If h decreases, p must also decrease. This is a direct acting controller whose gain is negative [$p'(t) = K_c(r'(t) - h'(t))$]

11.6

For proportional controller, $G_c(s) = K_c$

Assume that the level transmitter and the control valve have negligible dynamics. Then,

$$G_m(s) = K_m$$
$$G_v(s) = K_v$$

The block diagram for this control system is the same as in Fig.11.8. Hence Eqs. 11-26 and 11-29 can be used for closed-loop responses to setpoint and load changes, respectively.

The transfer functions $G_p(s)$ and $G_d(s)$ are as given in Eqs. 11-66 and 11-67, respectively.

a) Substituting for G_c, G_m, G_v, and G_p into Eq. 11-26 gives

$$\frac{Y}{Y_{sp}} = \frac{K_m K_c K_v\left(-\dfrac{1}{As}\right)}{1 + K_c K_v\left(-\dfrac{1}{As}\right)K_m} = \frac{1}{\tau s + 1}$$

where $\tau = -\dfrac{A}{K_c K_v K_m}$ (1)

For a step change in the setpoint, $Y_{sp}(s) = M/s$

$$Y(t\to\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\left[\frac{M/s}{\tau s + 1}\right] = M$$

Offset = $Y_{sp}(t\to\infty) - Y(t\to\infty) = M - M = 0$

b) Substituting for G_c, G_m, G_v, G_p , and G_d into (11-29) gives

$$\frac{Y(s)}{D(s)} = \frac{\left(\dfrac{1}{As}\right)}{1 + K_c K_v\left(-\dfrac{1}{As}\right)K_m} = \frac{\left(\dfrac{-1}{K_c K_v K_m}\right)}{\tau s + 1}$$

where τ is given by Eq. 1.

For a step change in the disturbance, $D(s) = M/s$

$$Y(t \to \infty) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s\left[\frac{-M/(K_c K_v K_m)}{s(\tau s+1)}\right] = \frac{-M}{K_c K_v K_m}$$

$$\text{Offset} = Y_{sp}(t\to\infty) - Y(t\to\infty) = 0 - \left(\frac{-M}{K_c K_v K_m}\right) \neq 0$$

Hence, offset is not eliminated for a step change in disturbance.

11.7

Using block diagram algebra

$$Y = G_d D + G_p U \qquad (1)$$

$$U = G_c\left[Y_{sp} - \left(Y - \tilde{G}_p U\right)\right] \qquad (2)$$

From (2), $$U = \frac{G_c Y_{sp} - G_c Y}{1 - G_c \tilde{G}_p}$$

Substituting for U in Eq. 1

$$\left[1 + G_c(G_p - \tilde{G}_p)\right]Y = G_d(1 - G_c\tilde{G}_p)D + G_p G_c Y_{sp}$$

Therefore,

$$\frac{Y}{Y_{sp}} = \frac{G_p G_c}{1 + G_c(G_p - \tilde{G}_p)}$$

and

$$\frac{Y}{D} = \frac{G_d(1 - G_c\tilde{G}_p)}{1 + G_c(G_p - \tilde{G}_p)}$$

11.8

The available information can be translated as follows

1. The outlets of both the tanks have flow rate q_0 at all times.

2. $T_o(s) = 0$

3. Since an energy balance would indicate a first-order transfer function between T_1 and Q_0 ,

$$\frac{T'(t)}{T'(\infty)} = 1 - e^{-t/\tau_1} \qquad \text{or} \qquad \frac{2}{3} = 1 - e^{-12/\tau_1} \quad , \tau_1 = 10 \text{ min}$$

Therefore

$$\frac{T_1(s)}{Q_0(s)} = \frac{3^\circ F/(-0.75 gpm)}{10s+1} = -\frac{4}{10s+1}$$

$$\frac{T_3(s)}{Q_0(s)} = \frac{(5-3)^\circ F/(-0.75 gpm)}{\tau_2 s+1} = -\frac{2.67}{\tau_2 s+1} \qquad \text{for} \quad T_2(s) = 0$$

4. $$\frac{T_1(s)}{V_1(s)} = \frac{(78-70)^\circ F/(12-10)V}{10s+1} = \frac{4}{10s+1}$$

$$\frac{T_3(s)}{V_2(s)} = \frac{(90-85)^\circ F/(12-10)V}{10s+1} = \frac{2.5}{10s+1}$$

5. $5\tau_2$ =50 min or τ_2 = 10 min

Since inlet and outlet flow rates for tank 2 are q_0

$$\frac{T_3(s)}{T_2(s)} = \frac{\overline{q}_0/\overline{q}_0}{\tau_2 s+1} = \frac{1}{10.0s+1}$$

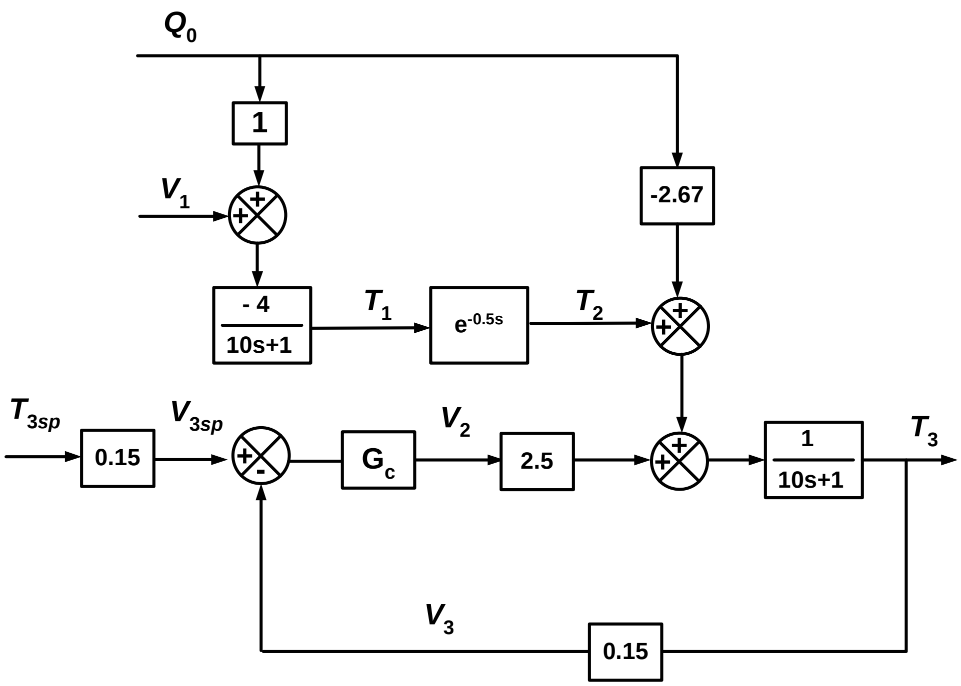
6. $$\frac{V_3(s)}{T_3(s)} = 0.15$$

7. $$T_2(t) = T_1\left(t - \frac{30}{60}\right) = T_1(t-0.5)$$

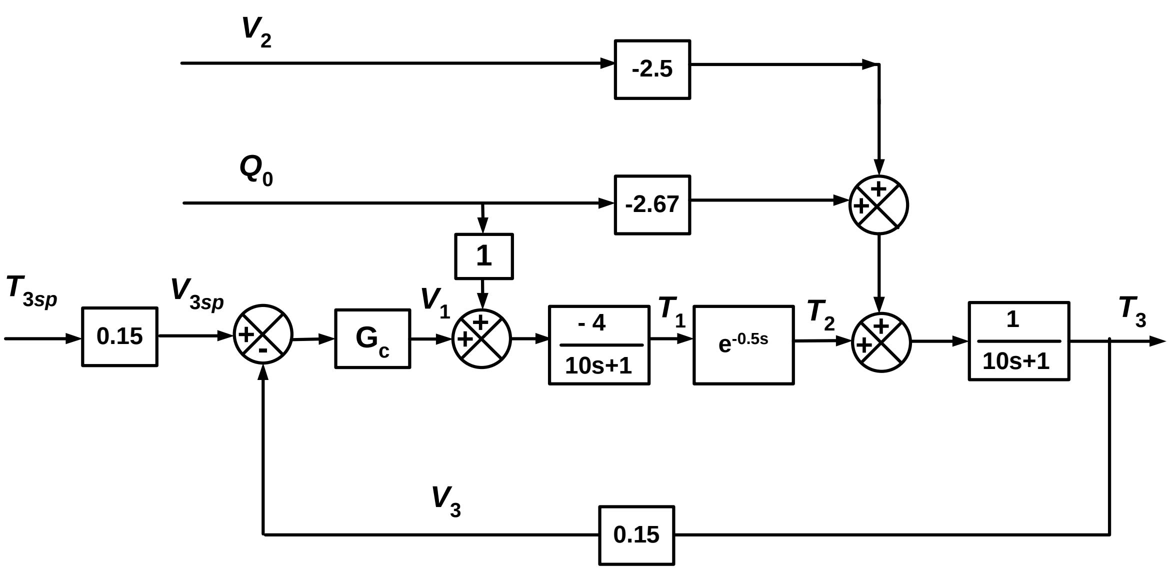
$$\frac{T_2(s)}{T_1(s)} = e^{-0.5s}$$

Using these transfer functions, the block diagrams are as follows.

a)



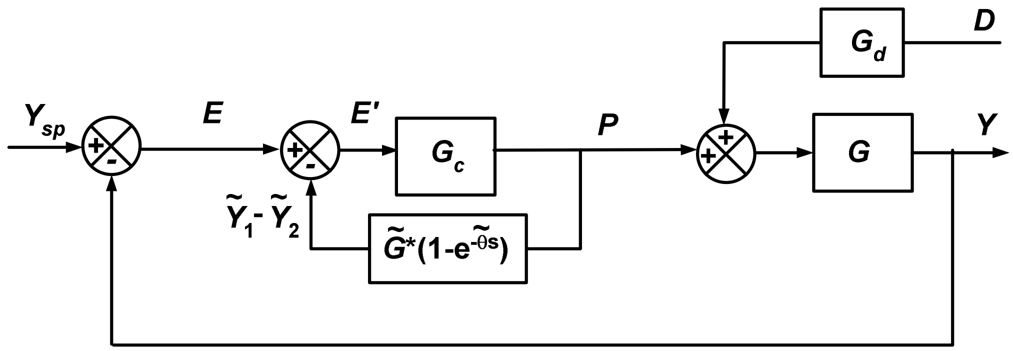
b)



c) The control configuration in part a) will provide the better control. As is evident from the block diagrams above, the feedback loop contains, in addition to G_c, only a first-order process in part a), but a second-order-plus-time-delay process in part b). Hence the controlled variable responds faster to changes in the manipulated variable for part a).

11.9

The given block diagram is equivalent to



For the inner loop, let

$$\frac{P}{E} = G_c' = \frac{G_c}{1 + G_c\tilde{G}^*(1 - e^{-\tilde{\theta}s})}$$

In the outer loop, we have

$$\frac{Y}{D} = \frac{G_d G}{1 + G_c' G}$$

Substitute for G_c',

$$\frac{Y}{D} = \frac{G_d G}{1 + \dfrac{G_c G}{1 + G_c\tilde{G}^*(1 - e^{-\tilde{\theta}s})}}$$

$$\frac{Y}{D} = \frac{G_d G\left(1 + G_c\tilde{G}^*(1 - e^{-\tilde{\theta}s})\right)}{1 + G_c\tilde{G}^*(1 - e^{-\tilde{\theta}s}) + G_c G}$$

11.10

a) Derive CLTF:

$$Y = Y_3 + Y_2 = G_3 Z + G_2 P$$

$$Y = G_3(D + Y_1) + G_2 K_c E$$

$$Y = G_3 D + G_3 G_1 K_c E + G_2 K_c E$$

$$Y = G_3 D + (G_3 G_1 K_c + G_2 K_c) E \qquad E = -K_m Y$$

$$Y = G_3 D - K_c (G_3 G_1 + G_2) K_m Y$$

$$\frac{Y}{D} = \frac{G_3}{1 + K_c (G_3 G_1 + G_2) K_m}$$

b) Characteristic Equation:

$$1 + K_c (G_3 G_1 + G_2) K_m = 0$$
$$1 + K_c \left[\frac{5}{s-1} + \frac{4}{2s+1} \right] = 0$$

$$1 + K_c \left[\frac{5(2s+1) + 4(s-1)}{(s-1)(2s+1)} \right] = 0$$

$$(s-1)(2s+1) + K_c [5(2s+1) + 4(s-1)] = 0$$

$$2s^2 - s - 1 + K_c (10s + 5 + 4s - 4) = 0$$

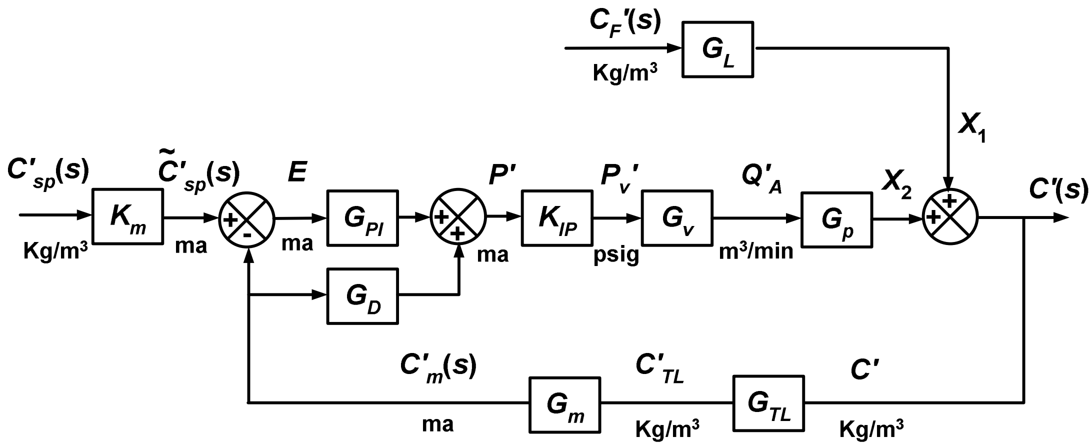
$$2s^2 + (14K_c - 1)s + (K_c - 1) = 0$$

Necessary conditions: $K_c > 1/14$ and $K_c > 1$

For a 2nd order characteristic equation, these conditions are also sufficient. Therefore, $K_c > 1$ for closed-loop stability.

11.11

a)



c) Transfer Line:

Volume of transfer line = $\pi/4\,(0.5\text{ m})^2(20\text{m}) = 3.93\text{ m}^3$

Nominal flow rate in the line = $\bar{q}_A + \bar{q}_F = 7.5\text{ m}^3/\text{min}$

Time delay in the line = $\dfrac{3.93\text{ m}^3}{7.5\text{ m}^3/\text{min}} = 0.52\,\text{min}$

$$G_{TL}(s) = e^{-0.52s}$$

Composition Transmitter:

$$G_m(s) = K_m = \frac{(20-4)\text{ ma}}{(200-0)\text{ kg/m}^3} = 0.08\frac{\text{ma}}{\text{kg/m}^3}$$

Controller

From the ideal controller in Eq. 8.14

$$P'(s) = K_c\left(1+\frac{1}{\tau_I s}\right)E(s) + K_c\tau_D s\left[\tilde{C}'_{sp}(s) - C'_m(s)\right]$$

In the above equation, set $\tilde{C}'_{sp}(s) = 0$ in order to get the derivative on the process output only. Then,

$$G_{PI}(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

$$G_D(s) = -K_c \tau_D s$$

with K_c >0 as the controller should be reverse-acting, since $P(t)$ should increase when $C_m(t)$ decreases.

I/P transducer

$$K_{IP} = \frac{(15-3)\text{ psig}}{(20-4)\text{ ma}} = 0.75\frac{\text{psig}}{\text{ma}}$$

Control valve

$$G_v(s) = \frac{K_v}{\tau_v s+1}$$

$$5\tau_v = 1 \qquad , \quad \tau_v = 0.2 \text{ min}$$

$$K_v = \left.\frac{dq_A}{dp_v}\right|_{p_v=\bar{p}_v} = 0.03(1/12)(\ln 20)(20)^{\frac{\bar{p}_v-3}{12}}$$

$$q_A = 0.5 = 0.17 + 0.03(20)^{\frac{\bar{p}_v-3}{12}}$$

$$0.03(20)^{\frac{\bar{p}_v-3}{12}} = 0.5 - 0.17 = 0.33$$

$$K_v = (1/12)(\ln 20)(0.33) = 0.082\frac{\text{m}^3/\text{min}}{\text{psig}}$$

$$G_v(s) = \frac{0.082}{0.2s+1}$$

Process

Assume c_A is constant for pure A. Material balance for A:

$$V\frac{dc}{dt} = q_A\bar{c}_A + \bar{q}_F c_F - (q_A + \bar{q}_F)c \qquad (1)$$

Linearizing and writing in deviation variable form

$$V\frac{dc'}{dt} = \bar{c}_A q'_A + \bar{q}_F c'_F - (\bar{q}_A + \bar{q}_F)c' - \bar{c} q'_A$$

Taking Laplace transform

$$[Vs + (\bar{q}_A + \bar{q}_F)]C'(s) = (\bar{c}_A - \bar{c})Q'_A(s) + \bar{q}_F C'_F(s) \qquad (2)$$

From Eq. 1 at steady state, $dc/dt = 0$,

$$\bar{c} = (\bar{q}_A \bar{c}_A + \bar{q}_F \bar{c}_F)/(\bar{q}_A + \bar{q}_F) = 100 \text{ kg/m}^3$$

Substituting numerical values in Eq. 2,

$$[5s + 7.5]C'(s) = 700\, Q'_A(s) + 7\, C'_F(s)$$

$$[0.67s + 1]C'(s) = 93.3 Q'_A(s) + 0.93 C'_F(s)$$

$$G_p(s) = \frac{93.3}{0.67s+1}$$
$$G_d(s) = \frac{0.93}{0.67s+1}$$

11.12

The stability limits are obtained from the characteristic Eq. 11-83. Hence if an instrumentation change affects this equation, then the stability limits will change and vice-versa.

a) The transmitter gain, K_m, changes as the span changes. Thus $G_m(s)$ changes and the characteristic equation is affected. Stability limits would be expected to change.

b) The zero on the transmitter does not affect its gain K_m. Hence $G_m(s)$ remains unchanged and stability limits do not change.

c) Changing the control valve trim changes $G_v(s)$. This affects the characteristic equation and the stability limits would be expected to change as a result.

11.13

a) $$G_a(s) = \frac{K_c K}{(\tau s+1)(s+1)}$$

b) $$G_b(s) = \frac{K_c K(\tau_I s+1)}{\tau_I s(\tau s+1)(s+1)}$$

For a)

$$D(s)+N(s) = (\tau s+1)(s+1)+K_c K = \tau s^2 + (\tau+1)s + 1 + K_c K_p$$

Stability requirements:

$1 + K_c K_p > 0$ or $\infty > K_c K_p > -1$

For b)

$$D(s)+N(s) = \tau_I(\tau s+1)(s+1)+K_c K(\tau_I s+1)$$

$$= \tau_I \tau s^3 + \tau_I(\tau+1)s^2 + \tau_I(1+K_c K_p)s + K_c K_p$$

Necessary condition: $K_c K_p > 0$

Sufficient conditions (Routh array):

$\tau_I \tau$ $\quad\quad$ $\tau_I(1+K_c K_p)$

$\tau_I(\tau+1)$ $\quad\quad$ $K_c K_p$

$$\frac{\tau_I{}^2(\tau+1)(1+K_c K_p) - \tau_I \tau K_c K_p}{\tau_I(\tau+1)}$$

$K_c K_p$

Additional condition is:

$$\tau_I(\tau+1)(1+K_c K_p) - \tau(K_c K_p) > 0$$

(since τ_I and τ are both positive)

$$\tau_I(\tau+1)+\tau_I(\tau+1)K_cK_p-\tau K_cK_p>0$$

$$[\tau_I(\tau+1)-\tau]K_cK_p>-\tau_I(\tau+1)$$

Note that RHS is negative for all positive τ_I and τ

($\therefore$ RHS is always negative)

Case 1:

If $\quad \tau_I(\tau+1)-\tau>0 \quad \left[i.e., \quad \tau_I>\dfrac{\tau}{\tau+1}\right]$

then $\quad K_cK_p > 0 > \left[\dfrac{-\tau_I(\tau+1)}{\tau_I(\tau+1)-\tau}\right]$

In other words, this condition is less restrictive than $K_cK_p > 0$ and doesn't apply.

Case 2:

If $\quad \tau_I(\tau+1)-\tau<0 \quad \left[i.e., \quad \tau_I<\dfrac{\tau}{\tau+1}\right]$

then $\quad K_cK_p < \left[\dfrac{-\tau_I(\tau+1)}{\tau_I(\tau+1)-\tau}\right]$

In other words, there would be an upper limit on K_cK_p so the controller gain is bounded on both sides

$$0 < K_cK_p < \frac{-\tau_I(\tau+1)}{\tau_I(\tau+1)-\tau}$$

c) Note that, in either case, the addition of the integral mode decreases the range of stable values of K_c.

11.14

From the block diagram, the characteristic equation is obtained as

$$1+K_c\left[\frac{(0.5)\left(\frac{4}{s+3}\right)}{1+(0.5)\left(\frac{4}{s+3}\right)}\right]\left[\frac{2}{s-1}\right]\left[\frac{1}{s+10}\right]=0$$

that is,

$$1+K_c\left[\frac{2}{s+5}\right]\left[\frac{2}{s-1}\right]\left[\frac{1}{s+10}\right]=0$$

Simplifying,

$$s^3+14s^2+35s+(4K_c-50)=0$$

The Routh Array is

1	35
14	$4K_c$-50
$\dfrac{490-(4K_c-50)}{14}$	
$4K_c-50$	

For the system to be stable,

$$\frac{490-(4K_c-50)}{14}>0 \qquad \text{or} \qquad K_c<135$$

and $4K_c-50>0$ or $K_c>12.5$

Therefore $12.5 < K_c < 135$

11.15

a) $$\frac{Y(s)}{Y_{sp}(s)} = \frac{\dfrac{K_cK}{1-\tau s}}{1+\dfrac{K_cK}{1-\tau s}} = \frac{K_cK}{1-\tau s + K_cK} = \frac{K_cK/(1+K_cK)}{-\dfrac{\tau}{1+K_cK}s+1}$$

For stability $$-\frac{\tau}{1+K_cK} > 0$$

Since τ is positive, the denominator must be negative, i.e.,

$$1+K_cK < 0$$

$$K_cK < -1$$

$$K_c < -1/K$$

Note that $$K_{CL} = \frac{K_cK}{1+K_cK}$$

b) If $K_cK < -1$ and $1+K_cK$ is negative,

then CL gain is positive. $\therefore$ it has the proper sign.

c) $K = 10$ and $\tau = 20$

and we want $$-\frac{\tau}{1+K_cK} = 10$$

or
$$-20 = 10 + (10)(10)K_c$$
$$-30 = 100K_c$$
$$K_c = -0.3$$

Offset: $$K_{CL} = \frac{(-0.3)(10)}{1+(-0.3)(10)} = \frac{-3}{-2} = 1.5$$

$\therefore$ Offset = +1 – 1.5 = – 50% (Note this result implies overshoot)

d) $$\frac{Y(s)}{Y_{sp}(s)} = \frac{\dfrac{K_c K}{(1-\tau s)(\tau_m s+1)}}{1+\dfrac{K_c K}{(1-\tau s)(\tau_m s+1)}} = \frac{K_c K}{(1-\tau s)(\tau_m s+1)+K_c K}$$

$$= \frac{K_c K}{-\tau\tau_m s^2 + (\tau_m - \tau)s + 1 + K_c K}$$

$$= \frac{K_c K/(1+K_c K)}{-\dfrac{\tau\tau_m}{1+K_c K}s^2 + \dfrac{\tau_m - \tau}{1+K_c K}s + 1} \quad \text{(standard form)}$$

For stability,

(1) $-\dfrac{\tau\tau_m}{1+K_c K} > 0$ (2) $\dfrac{\tau_m - \tau}{1+K_c K} > 0$

From (1) Since $1+K_c K < 0$

$$K_c K < -1$$

$$K_c < -\frac{1}{K}$$

From (2) Since $1+K_c K < 0$

$$\tau_m - \tau < 0$$

$$-\tau < -\tau_m$$

$$\tau > \tau_m$$

For $K = 10$, $\tau = 20$, $K_c = -0.3$, $\tau_m = 5$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{1.5}{-\dfrac{(20)(5)}{1-3}s^2 + \dfrac{(5-20)}{(1-3)}s + 1} = \frac{1.5}{50s^2 + 2.5s + 1}$$

Underdamped but stable.

11.16

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

$$G_v(s) = \frac{K_v}{(10/60)s+1} = \frac{-1.3}{0.167s+1}$$

$$G_p(s) = -\frac{1}{As} = -\frac{1}{22.4s} \qquad \text{since} \quad A = 3\ \text{ft}^2 = 22.4\frac{\text{gal}}{\text{ft}}$$

$$G_m(s) = K_m = 4$$

Characteristic equation is

$$1+K_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{-1.3}{0.167s+1}\right)\left(\frac{-1}{22.4s}\right)(4) = 0$$

$$(3.73\tau_I)s^3 + (22.4\tau_I)s^2 + (5.2K_c\tau_I)s + (5.2K_c) = 0$$

The Routh Array is

$3.73\tau_I$	$5.2K_c\tau_I$
$22.4\tau_I$	$5.2K_c$
$5.2K_c\tau_I - 0.867K_c$	
$5.2K_c$	

For stable system,

$\tau_I > 0$, $\qquad 5.2K_c\tau_I - 0.867K_c > 0 \qquad K_c > 0$

That is,

$K_c > 0$

$\tau_I > 0.167\ \text{min}$

11.17

$$G_{OL}(s) = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)\left(\frac{5}{(10s+1)^2}\right) = \frac{N(s)}{D(s)}$$

$$D(s)+N(s) = \tau_I s(100s^2+20s+1)+5K_c(\tau_I s+1) = 0$$

$$= 100\tau_I s^3 + 20\tau_I s^2 + (1+5K_c)\tau_I s + 5K_c = 0$$

a) Analyze characteristic equation for necessary and sufficient conditions

Necessary conditions:

$$5K_c > 0 \qquad \rightarrow \qquad K_c > 0$$

$$(1+5K_c)\tau_I > 0 \quad \rightarrow \quad \tau_I > 0 \quad \text{and} \quad K_c > -\frac{1}{5}$$

Sufficient conditions obtained from Routh array

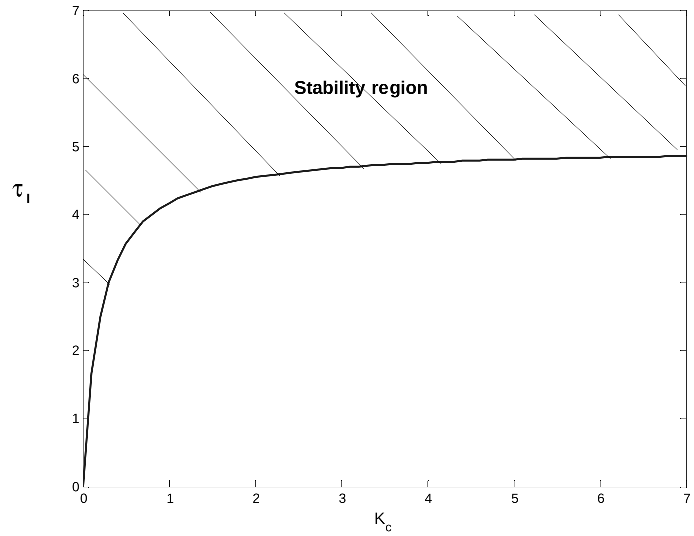
$100\tau_I$	$(1+5K_c)\tau_I$
$20\tau_I$	$5K_c$
$\dfrac{20\tau_I{}^2(1+5K_c)-500\tau_I K_c}{20\tau_I}$	
$5K_c$	

Then,

$$20\tau_I{}^2(1+5K_c)-500\tau_I K_c > 0$$

$$\tau_I(1+5K_c) > 25K_c \qquad \text{or} \qquad \tau_I > \frac{25K_c}{1+5K_c}$$

b) Sufficient condition is appropriate. Plot is shown below.



c) Find τ_I as $K_c \rightarrow \infty$

$$\lim_{Kc\to\infty}\left[\frac{25K_c}{1+5K_c}\right] = \lim_{Kc\to\infty}\left[\frac{25}{1/K_c+5}\right] = 5$$

$\therefore \quad \tau_I > 5$ guarantees stability for any value of K_c. Appelpolscher is wrong yet again.

11.18

$$G_c(s) = K_c$$

$$G_V(s) = \frac{K_V}{\tau_V s + 1}$$

$$K_v = \left.\frac{dw_s}{dp}\right|_{p=12} = \frac{0.6}{2\sqrt{12-4}} = 0.106\,\frac{\text{lbm/sec}}{\text{ma}}$$

$$5\tau_v = 20 \text{ sec} \qquad \tau_v = 4 \text{ sec}$$

$$G_p(s) = \frac{2.5e^{-s}}{10s+1}$$

$$G_m(s) = K_m = \frac{(20-4)\text{ ma}}{(160-120)^\circ F} = 0.4\,\frac{\text{ma}}{^\circ F}$$

Characteristic equation is

$$1+(K_c)\left(\frac{0.106}{4s+1}\right)\left(\frac{2.5e^{-s}}{10s+1}\right)(0.4)=0 \qquad (1)$$

a) Substituting s=jω in (1) and using Euler's identity

$$e^{-j\omega}=\cos\omega - j\sin\omega$$

gives

$$-40\omega^2 + 14j\omega + 1 + 0.106\,K_c(\cos\omega - j\sin\omega)=0$$

Thus

$$-40\omega^2 + 1 + 0.106\,K_c\cos\omega = 0 \qquad (2)$$

and $$14\omega - 0.106K_c\sin\omega = 0 \qquad (3)$$

From (2) and (3),

$$\tan\omega = \frac{14\omega}{40\omega^2 - 1} \qquad (4)$$

Solving (4), $\omega = 0.579$ by trial and error.

Substituting for ω in (3) gives

$$K_c = 139.7 = K_{cu}$$

Frequency of oscillation is 0.579 rad/sec

b) Substituting the Pade approximation

$$e^{-s} \approx \frac{1-0.5s}{1+0.5s}$$

into (1) gives

$$20s^3 + 47s^2 + (14.5 - 0.053K_c)s + (1 + 0.106K_c) = 0$$

The Routh Array is

20	$14.5 - 0.053\,K_c$
47	$1 + 0.106\,K_c$
$14.07 - 0.098\,K_c$	
$1 + 0.106\,K_c$	

For stability,

$$14.07 - 0.098K_c > 0 \qquad \text{or} \quad K_c < 143.4$$

and $\quad 1 + 0.106K_c > 0 \qquad$ or $\quad K_c > -9.4$

Therefore, the maximum gain, K_{cu} = 143.4, is a satisfactory approximation of the true value of 139.7 in (a) above.

11.19

a) $$G(s) = \frac{4(1-5s)}{(25s+1)(4s+1)(2s+1)}$$

$$G_c(s) = K_c$$

$$D(s) + N(s) = (25s+1)(4s+1)(2s+1) + 4K_c(1-5s) = 0$$

$$100s^2 + 29s + 1$$
$$2s + 1$$

$$200s^3 + 58s^2 + 2s$$
$$100s^2 + 29s + 1$$

$$200s^3 + 158s^2 + 31s + 1 + 4K_c - 20K_c s = 0$$
$$200s^3 + 158s^2 + (31 - 20K_c)s + 1 + 4K_c = 0$$

Routh array:

200	31-20 K_c
158	1+4 K_c

$$\frac{158(31\text{-}20K_c)\text{-}200(1+4K_c)}{158} = \frac{4898 - 3160K_c - 200 - 800K_c}{158}$$

1+ 4 K_c

$\therefore \quad 4698 - 3960\ K_c > 0 \qquad$ or $\qquad K_c < 1.2$

b) $(25s+1)(4s+1)(2s+1)+4K_c=0$

Routh array:

$200s^3+158s^2+31s+(1+4K_c)=0$

200 31

158 $1 + 4\, K_c$

$158\,(31) - 200(1{+}4K_c) \;=\; 4898 - 200 - 800K_c$

$1{+}\,4\,K_c$

$\therefore\;\; 4698 - 800\,K_c > 0$ or $K_c < 5.87$

c) Because K_c can be much higher without the RHP zero present, the process can be made to respond faster.

11.20

The characteristic equation is

$$1+\frac{0.5K_c e^{-3s}}{10s+1}=0 \qquad (1)$$

a) Using the Pade approximation

$$e^{-3s}\approx\frac{1-(3/2)s}{1+(3/2)s}$$

in (1) gives

$$15s^2+(11.5-0.75K_c)s+(1+0.5K_c)=0$$

For stability,

$11.5-0.75K_c>0$ or $K_c<15.33$

and $1+0.5K_c>0$ or $K_c>-2$

Therefore $-2 < K_c < 15.33$

b) Substituting $s = j\omega$ in (1) and using Euler's identity.

$$e^{-3j\omega} = \cos(3\omega) - j\sin(3\omega)$$

gives

$$10j\omega + 1 + 0.5K_c\left[\cos(3\omega) - j\sin(3\omega)\right] = 0$$

Then,

$$1 + 0.5K_c\cos(3\omega) = 0 \qquad (2)$$

and $$10\omega - 0.5K_c\sin(3\omega) = 0 \qquad (3)$$

From (3), one solution is $\omega = 0$, which gives $K_c = -2$

Thus, for stable operation $K_c > -2$

From (2) and (3)

$$\tan(3\omega) = -10\omega$$

Eq. 4 has infinite number of solutions. The solution for the range $\pi/2 < 3\omega < 3\pi/2$ is found by trial and error to be $\omega = 0.5805$.

Then from Eq. 2, $K_c = 11.78$

The other solutions for the range $3\omega > 3\pi/2$ occur at values of ω for which $\cos(3\omega)$ is smaller than $\cos(3\times5.805)$. Thus, for all other solutions of ω, Eq. 2 gives values of K_c that are larger than 11.78. Hence, stability is ensured when

$$-2 < K_c < 11.78$$

11.21

a) To approximate $G_{OL}(s)$ by a FOPTD model, the Skogestad approximation technique in Chapter 6 is used.

Initially,

$$G_{OL}(s) = \frac{3K_c e^{-(1.5+0.3+0.2)s}}{(60s+1)(5s+1)(3s+1)(2s+1)} = \frac{3K_c e^{-2s}}{(60s+1)(5s+1)(3s+1)(2s+1)}$$

Skogestad approximation method to obtain a 1st -order model:

Time constant ≈ 60 + (5/2)

Time delay ≈ 2 +(5/2) + 3 + 2 =9.5

Then

$$G_{OL}(s) \approx \frac{3K_c e^{-9.5s}}{62.5s+1}$$

b) The only way to apply the Routh method to a FOPTD transfer function is to approximate the delay term.

$$e^{-9.5s} \approx \frac{-4.75s+1}{4.75s+1}$$ (1st order Pade-approximation)

Then

$$G_{OL}(s) \approx \frac{N(s)}{D(s)} \approx \frac{3K_c(-4.75s+1)}{(62.5s+1)(4.75s+1)}$$

The characteristic equation is:

$$D(s)+N(s) = (62.5s+1)(4.75s+1)+3K_c(-4.75s+1)$$

$$297s^2 + 67.3s + 1 - 14.3K_c s + 3K_c = 0$$

$$297s^2 + (67.3 - 14.3K_c)s + (1+3K_c) = 0$$

Necessary conditions:

$$67.3 - 14.3K_c > 0 \qquad\qquad 1 + 3K_c > 0$$
$$-14.3K_c > -67.3 \qquad\qquad 3K_c > -1$$
$$K_c < 4.71 \qquad\qquad K_c > -1/3$$

Range of stability: $-1/3 < K_c < 4.71$

c) Conditional stability occurs when $K_c = K_{cu} = 4.71$

With this value the characteristic equation is:

$$297s^2 + (67.3 - 14.3 \times 4.71)s + (1 + 3 \times 4.71) = 0$$

$$297s^2 + 15.13 = 0$$

$$s^2 = \frac{-15.13}{297}$$

We can find ω by substituting $j\omega \rightarrow s$

$\omega = 0.226$ at the maximum gain.