Chapter 12

12.1

For $K = 1.0$, $\tau_1=10$, $\tau_2=5$, the PID controller settings are obtained using Eq.(12-14):

$$K_c = \frac{1}{K}\frac{\tau_1+\tau_2}{\tau_c} = \frac{15}{\tau_c} \qquad , \qquad \tau_I = \tau_1+\tau_2=15 \qquad ,$$

$$\tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2} = 3.33$$

The characteristic equation for the closed-loop system is

$$1+\left[K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\right]\left[\frac{1.0+\alpha}{(10s+1)(5s+1)}\right]=0$$

Substituting for K_c, τ_I, and τ_D, and simplifying gives

$$\tau_c s+(1+\alpha)=0$$

Hence, for the closed-loop system to be stable,

$$\tau_c > 0$$

and $(1+\alpha) > 0$ or $\alpha > -1$.

(a) Closed-loop system is stable for $\alpha > -1$

(b) Choose $\tau_c > 0$

(c) The choice of τ_c does not affect the robustness of the system to changes in α. For $\tau_c \leq 0$, the system is unstable regardless of the value of α. For $\tau_c > 0$, the system is stable in the range $\alpha > -1$ regardless of the value of τ_c.

12.2

$$G = G_v G_p G_m = \frac{-1.6(1-0.5s)}{s(3s+1)}$$

The process transfer function contains a zero at s = +2. Because the controller in the Direct Synthesis method contains the inverse of the process model, the controller will contain an unstable pole. Thus, Eqs. (12-4) and (12-5) give:

$$G_c = \frac{1}{G}\frac{1}{\tau_c s} = -\frac{(3s+1)}{2\tau_c(1-0.5s)}$$

Modeling errors and the unstable controller pole at s = +2 would render the closed-loop system unstable.

Modify the specification of Y/Y_{sp} such that G_c will not contain the offending (1-0.5s) factor in the denominator. The obvious choice is

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{1-0.5s}{\tau_c s+1}$$

Then using Eq.(12-3b),

$$G_c = -\frac{3s+1}{2\tau_c+1}$$

which is not physically realizable because it requires ideal derivative action. Modify Y/Y_{sp},

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{1-0.5s}{(\tau_c+1)^2}$$

Then Eq.(12-3b) gives

$$G_c = -\frac{3s+1}{2\tau_c^2 s+4\tau_c+1}$$

which is physically realizable.

12.3

$K = 2, \quad \tau = 1, \quad \theta = 0.2$

(a) Using Eq.(12-11) for $\tau_c = 0.2$

$K_c = 1.25 \quad , \quad \tau_I = 1$

(b) Using Eq.(12-11) for $\tau_c = 1.0$

$K_c = 0.42 \quad , \quad \tau_I = 1$

(c) From Table 12.3 for a disturbance change

$KK_c = 0.859(\theta/\tau)^{-0.977}$ or $K_c = 2.07$
$\tau/\tau_I = 0.674(\theta/\tau)^{-0.680}$ or $\tau_I = 0.49$

(d) From Table 12.3 for a setpoint change

$KK_c = 0.586(\theta/\tau)^{-0.916}$ or $K_c = 1.28$
$\tau/\tau_I = 1.03 - 0.165(\theta/\tau)$ or $\tau_I = 1.00$

(e) Conservative settings correspond to low values of K_c and high values of τ_I. Clearly, the Direct Synthesis method ($\tau_c = 1.0$) of part (b) gives the most conservative settings; ITAE of part (c) gives the least conservative settings.

(f) A comparison for a unit step disturbance is shown in Fig. S12.3.

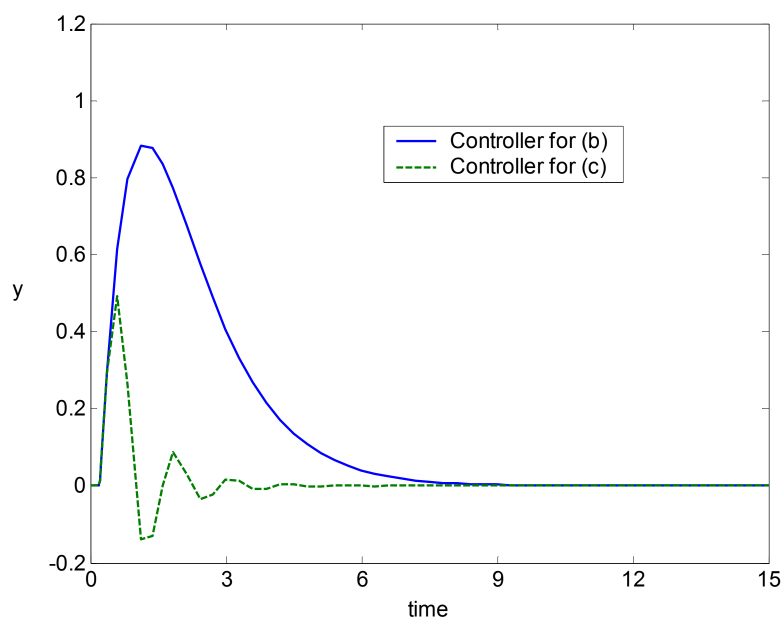


Fig S12.3. *Comparison of part (e) PI controllers for unit step disturbance.*

12.4

The process model is,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{s} \tag{1}$$

Approximate the time delay by Eq. 12-24b,

$$e^{-\theta s} = 1 - \theta s \tag{2}$$

Substitute into (1):

$$\tilde{G}(s) = \frac{K(1-\theta s)}{s} \tag{3}$$

Factoring (3) gives $\tilde{G}_+(s) = 1 - \theta s$ and $\tilde{G}_-(s) = K/s$.

The DS and IMC design methods give identical controllers if,

$$\left(\frac{Y}{Y_{sp}}\right)_d = \tilde{G}_+ f \tag{12-23}$$

For integrating process, *f* is specified by Eq. 12-32:

$$C = \left.\frac{d\tilde{G}_+}{ds}\right|_{s=0} = -\theta \tag{4}$$

$$f = \frac{(2\tau_c - C)s + 1}{(\tau_c s + 1)^2} = \frac{(2\tau_c + \theta)s + 1}{(\tau_c s + 1)^2} \tag{5}$$

Substitute $\tilde{G}_+$ and *f* into (12-23):

$$\left(\frac{Y}{Y_{sp}}\right)_d = (1-\theta s)\left[\frac{(2\tau_c + \theta)s + 1}{(\tau_c s + 1)^2}\right] \tag{6}$$

The Direct Synthesis design equation is:

$$G_c = \frac{1}{\widetilde{G}}\left[\frac{\left(\dfrac{Y}{Y_{sp}}\right)_d}{1-\left(\dfrac{Y}{Y_{sp}}\right)_d}\right] \tag{12-3b}$$

Substitute (3) and (6) into (12-3b):

$$G_c = \left[\frac{s}{K(1-\theta s)}\right]\frac{(1-\theta s)\left[\dfrac{(2\tau_c+\theta)s+1}{(\tau_c s+1)^2}\right]}{1-(1-\theta s)\left[\dfrac{(2\tau_c+\theta)s+1}{(\tau_c s+1)^2}\right]} \tag{7}$$

or

$$G_c = \frac{s}{K}\frac{(2\tau_c+\theta)s+1}{(\tau_c s+1)^2-(1-\theta s)\left[(2\tau_c+\theta)s+1\right]} \tag{8}$$

Rearranging,

$$G_c = \frac{1}{Ks}\frac{(2\tau_c+\theta)s+1}{\tau_c{}^2+2\tau_c\theta s+\theta^2} = \frac{1}{Ks}\frac{(2\tau_c+\theta)s+1}{(\tau_c+\theta)^2} \tag{9}$$

The standard PI controller can be written as

$$G_c = K_c\frac{\tau_I s+1}{\tau_I s} \tag{10}$$

Comparing (9) and (10) gives:

$$\tau_I = 2\tau_c+\theta \tag{11}$$

$$\frac{K_c}{\tau_I} = \frac{1}{K}\frac{1}{(\tau_c+\theta)^2} \tag{12}$$

Substitute (11) into (12) and rearrange gives:

$$K_c = \frac{1}{K}\frac{2\tau_c+\theta}{(\tau_c+\theta)^2} \tag{13}$$

Controller M in Table 12.1 has the PI controller settings of Eqs. (11) and (13).

12.5

Assume that the process can be modeled adequately by a first-order-plus-time-delay model as in Eq. 12-10. Then using the given step response data, the model fitted graphically is shown in Fig. S12.5,

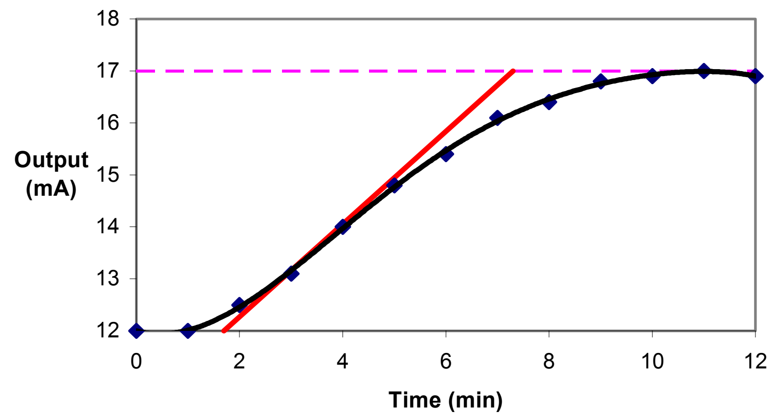


Figure S12.5 *Process data; first order model estimation.*

This gives the following model parameters:

$$K = K_{IP} K_v K_p K_m = \left(0.75 \frac{\text{psi}}{\text{mA}}\right)\left(0.9 \frac{\text{psi}}{\text{psi}}\right)\left(\frac{16.9-12.0}{20-18} \frac{\text{mA}}{\text{psi}}\right) = 1.65$$

$\theta = 1.7$ min

$\theta + \tau = 7.2$ min or $\tau = 5.5$ min

(a) Because θ/τ is greater than 0.25, a conservative choice of $\tau_c = \tau/2$ is used. Thus $\tau_c = 2.75$ min.

Settling $\theta_c = \theta$ and using the approximation $e^{-\theta s} \approx 1 - \theta s$, Eq. 12-11 gives

$$K_c = \frac{1}{K}\frac{\tau}{\theta + \tau_c} = 0.75\ , \qquad \tau_I = \tau = 5.5 \text{ min}, \qquad \tau_D = 0$$

(b) From Table 12.3 for PID settings for set-point change,

$KK_c = 0.965(\theta/\tau)^{-0.85}$ or $K_c = 1.58$

$\tau/\tau_I = 0.796 - 0.1465\,(\theta/\tau)$ or $\tau_I = 7.33$ min

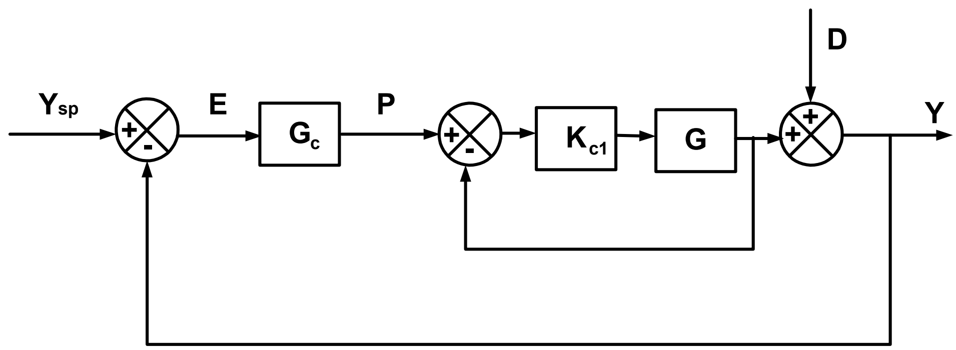
$\tau_D/\tau = 0.308\,(\theta/\tau)^{0.929}$ or $\tau_D = 0.57$ min

(c) From Table 12.3 for PID settings for disturbance input,

$$KK_c = 1.357(\theta/\tau)^{-0.947} \quad \text{or} \quad K_c = 2.50$$
$$\tau/\tau_I = 0.842\,(\theta/\tau)^{-0.738} \quad \text{or} \quad \tau_I = 2.75 \text{ min}$$
$$\tau_D/\tau = 0.381\,(\theta/\tau)^{0.995} \quad \text{or} \quad \tau_D = 0.65 \text{ min}$$

12.6

Let G be the open-loop unstable process. First, stabilize the process by using proportional-only feedback control, as shown below.



Then,

$$\frac{Y}{Y_{sp}} = \frac{G_c \dfrac{K_{c1}G}{1+K_{c1}G}}{1+G_c \dfrac{K_{c1}G}{1+K_{c1}G}} = \frac{G_c G'}{1+G_c G'}$$

where $G' = \dfrac{K_{c1}G}{1+K_{c1}G}$

Then G_c is designed using the Direct Synthesis approach for the stabilized, modified process G' .

12.7

(a.i) The model reduction approach of Skogestad gives the following approximate model:

$$G(s) = \frac{e^{-0.028s}}{(s+1)(0.22s+1)}$$

Applying the controller settings of Table 12.5 (notice that $\tau_1 \geq 8\theta$)

$K_c = 35.40$
$\tau_I = 0.444$
$\tau_D = 0.111$

(a.ii) By using Simulink, the ultimate gain and ultimate period are found:

$K_{cu} = 30.24$
$P_u = 0.565$

From Table 12.6:

$K_c = 0.45K_{cu} = 13.6$
$\tau_I = 2.2P_u = 1.24$
$\tau_D = P_u/6.3 = 0.089$

(b)

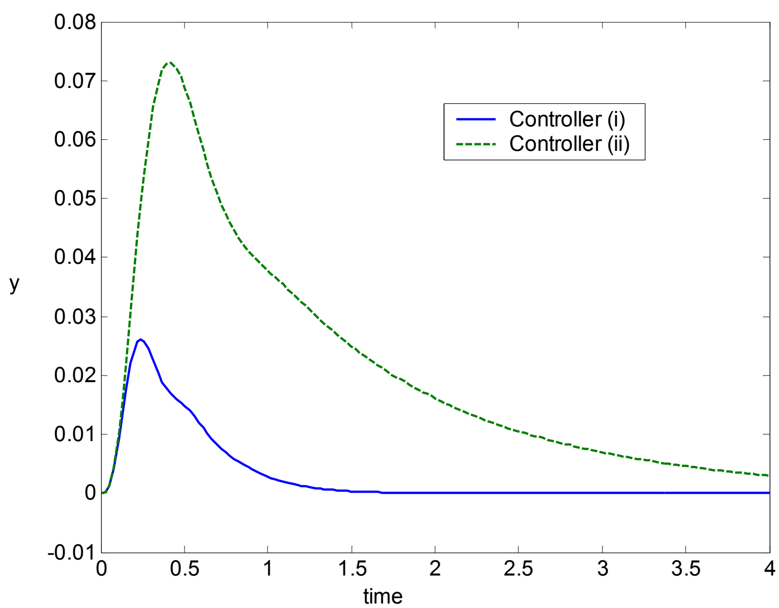


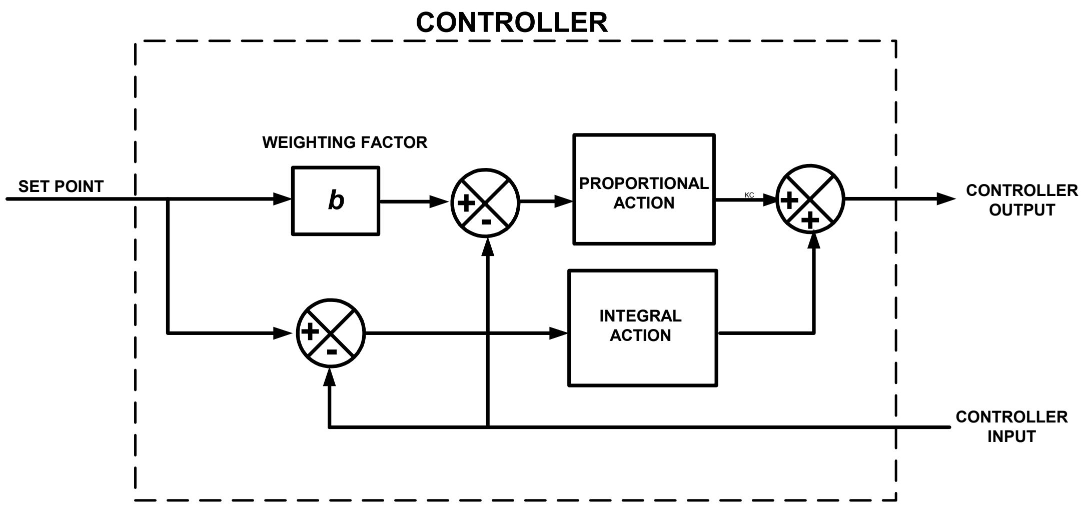
Figure S12.7. *Closed-loop responses to a unit step change in a disturbance.*

12.8

From Eq.12-39:

$$p(t) = \bar{p} + K_c\left[by_{sp}(t) - y_m(t)\right] + K_c\left[\frac{1}{\tau_I}\int_0^t e(t^*)dt^* - \tau_D\frac{dy_m}{dt}\right]$$

This control law can be implemented with Simulink as follows:



Closed-loop responses are compared for $b = 1$, $b = 0.7$, $b = 0.5$ and $b = 0.3$:

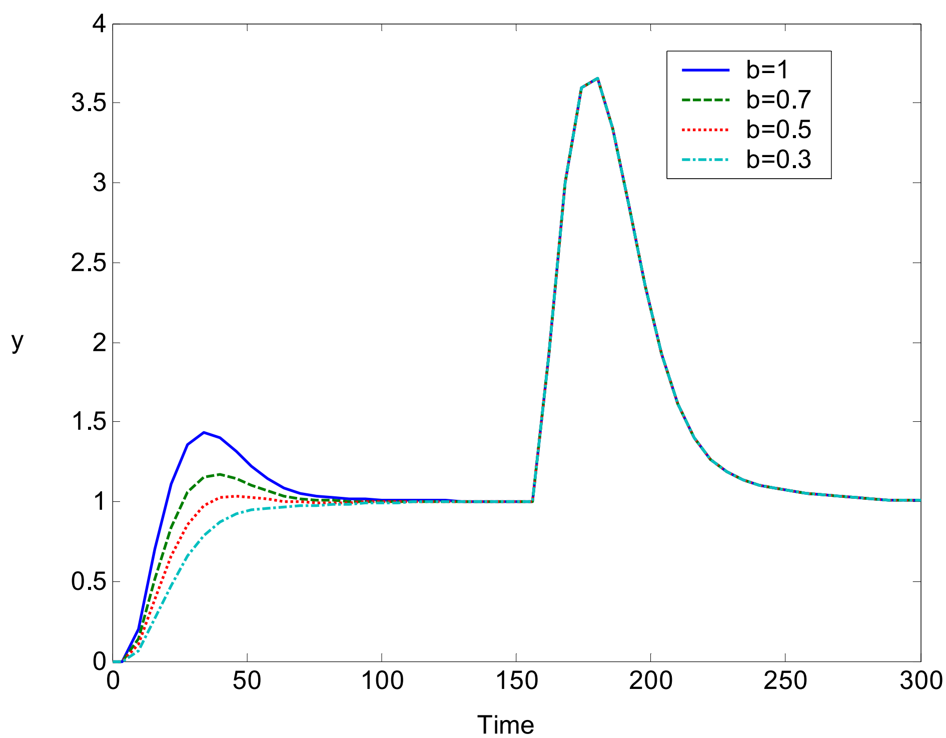


Figure S12.8. *Closed-loop responses for different values of b.*

As shown in Figure E12.8, as b increases, the set-point response becomes faster but exhibits more overshoot. The value of $b = 0.5$ seems to be a good choice. The disturbance response is independent of the value of b.

12.9

In order to implement the series form using the standard Simulink form of PID control (the expanded form in Eq. 8-16), we first convert the series controller settings to the equivalent parallel settings.

(a) From Table 12.2, the controller settings for series form are:

$$K_c = K_c'\left(1+\frac{\tau_D'}{\tau_I'}\right) = 0.971$$

$$\tau_I = \tau_I' + \tau_D' = 26.52$$

$$\tau_D = \frac{\tau_I'\tau_D'}{\tau_I' + \tau_D'} = 2.753$$

By using Simulink, closed-loop responses are shown in Fig. S12.9:

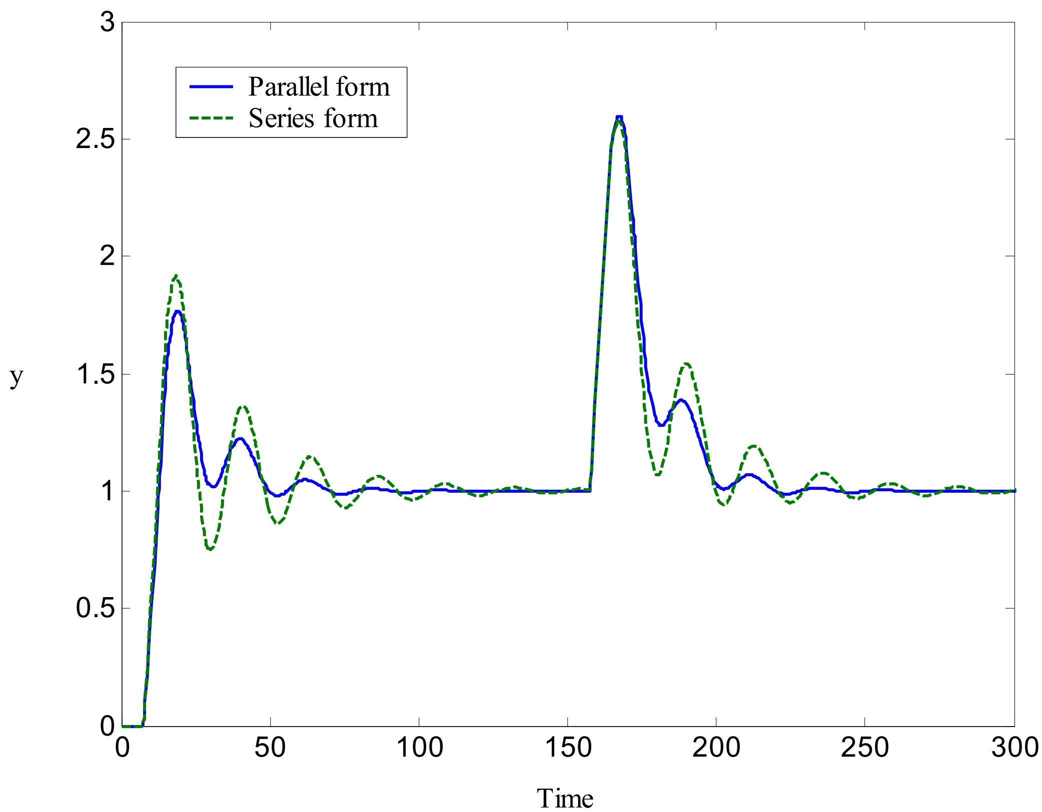


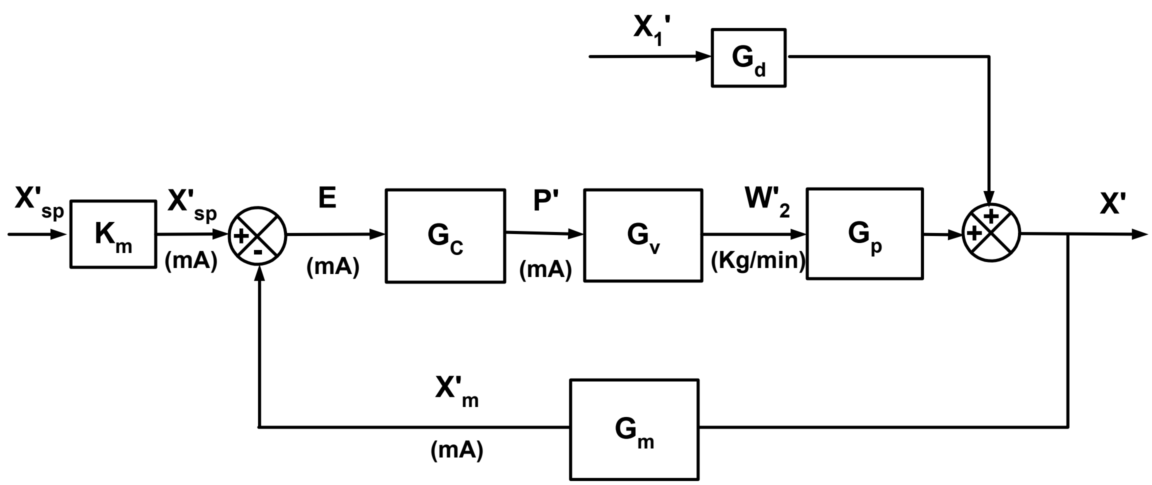
Figure S12.9. *Closed-loop responses for parallel and series form.*

The closed-loop responses to the set-point change are significantly different. On the other hand, the responses to the disturbance are slightly closer.

(b) By changing the derivative term in the controller block, Simulink shows that the system becomes more oscillatory as τ_D increases. For the parallel form, system becomes unstable for $\tau_D \geq 5.4$; for the series form, system becomes unstable for $\tau_D \geq 4.5$.

12.10

(a)



(b) <u>Process and disturbance transfer functions</u>:

Overall material balance: $w_1 + w_2 - w = 0$ (1)

Component material balance: $w_1 x_1 + w_2 x_2 - wx = \rho V \dfrac{dx}{dt}$ (2)

Substituting (1) into (2) and introducing deviation variables:

$$w_1 x_1' + w_2' x_2 - w_1 x' - \overline{w}_2 x - w_2' \overline{x} = \rho V \frac{dx'}{dt}$$

Taking the Laplace transform,

$$\mathrm{w_1 X_1'(s) + (x_2 - \overline{x}) W_2'(s) = (w_1 + \overline{w}_2 + \rho V s) X'(s)}$$

Finally:

$$G_p(s) = \frac{X'(s)}{W_2'(s)} = \frac{x_2 - \overline{x}}{w_1 + \overline{w}_2 + \rho V s} = \frac{\dfrac{x_2 - \overline{x}}{w_1 + \overline{w}_2}}{1 + \tau s}$$

$$G_d(s) = \frac{X'(s)}{X_1'(s)} = \frac{w_1}{w_1 + \overline{w}_2 + \rho V s} = \frac{\dfrac{w_1}{w_1 + \overline{w}_2}}{1 + \tau s}$$

$$\text{where} \quad \tau \triangleq \frac{\rho V}{w_1 + \overline{w}_2}$$

Substituting numerical values:

$$G_p(s) = \frac{2.6 \times 10^{-4}}{1 + 4.71s}$$

$$G_d(s) = \frac{0.65}{1 + 4.71s}$$

<u>Composition measurement transfer function</u>:

$$G_m(s) = \frac{20 - 4}{0.5} e^{-s} = 32 e^{-s}$$

<u>Final control element transfer function</u>:

$$G_v(s) = \frac{15 - 3}{20 - 4} \times \frac{300/1.2}{0.0833s + 1} = \frac{187.5}{0.0833s + 1}$$

<u>Controller</u>:

Let $\quad G = G_v G_p G_m = \dfrac{187.5}{0.0833s + 1} \dfrac{2.6 \times 10^{-4}}{1 + 4.71s} 32 e^{-s}$

then $$G = \frac{1.56e^{-s}}{(4.71s+1)(0.0833s+1)}$$

For a process with a dominant time constant, $\tau_c = \tau_{dom}/3$ is recommended.

Hence $\tau_c = 1.57$. From Table 12.1,

$K_c = 1.92$
$\tau_I = 4.71$

(c) By using Simulink,

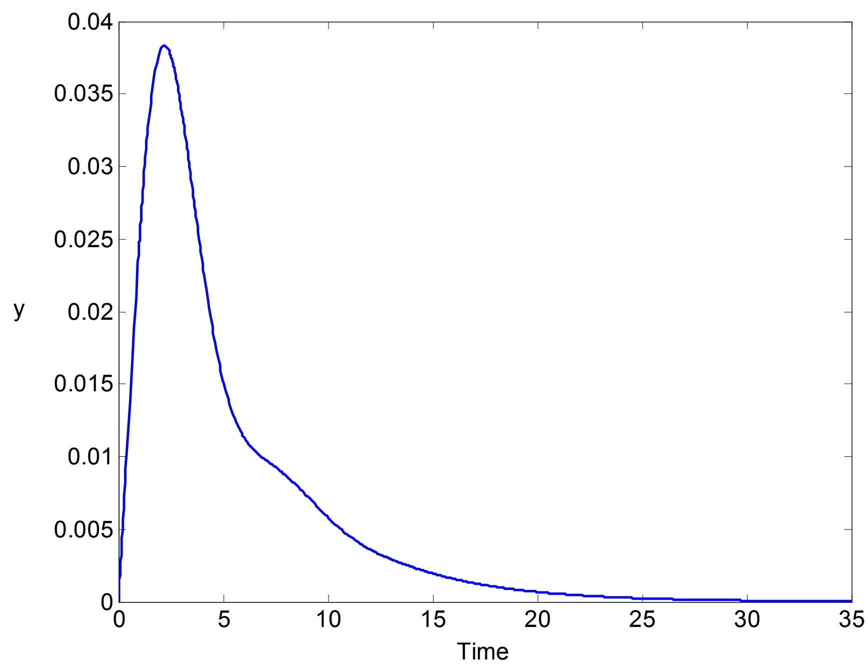


Figure S12.10c. *Closed-loop response for step disturbance.*

(d) By using Simulink

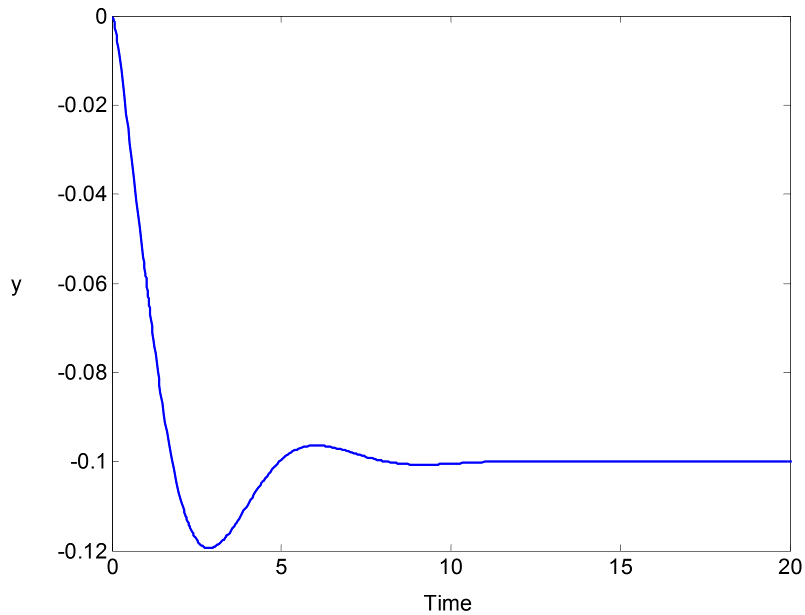


Figure S12.10d. *Closed-loop response for a set-point change.*

The recommended value of $\tau_c = 1.57$ gives very good results.

(e) Improved control can be obtained by adding derivative action: $\tau_D = 0.4$.

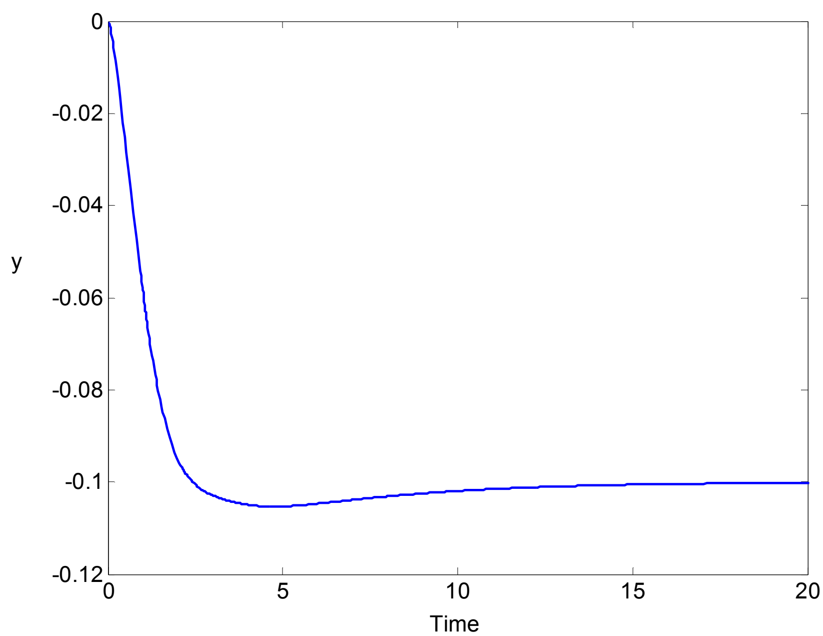


Figure S12.10e. *Closed-loop response by adding derivative action.*

(e) For θ =3 min, the closed-loop response becomes unstable. It's well known that the presence of a large process time delay limits the performance of a conventional feedback control system. In fact, a time delay adds phase lag to the feedback loop which adversely affects closed-loop stability (cf. Ch. 14). Consequently, the controller gain must be reduced below the value that could be used if smaller time delay were present.

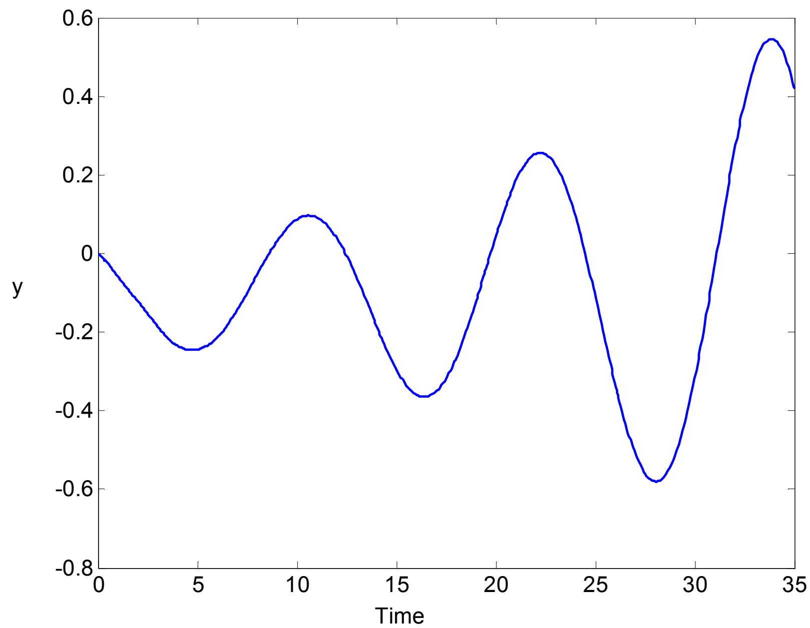


Figure S12.10f. *Closed-loop response for* θ *=3min.*

12.11

The controller tuning is based on the characteristic equation for standard feedback control.

$$1 + G_c G_{I/P} G_v G_p G_m = 0$$

Thus, the PID controller will have to be retuned only if any of the transfer functions, $G_{I/P}$, G_v, G_p *or* G_m, change.

(a) K_m changes. The controller may have to be retuned.

(b) The zero does not affect G_m. Thus, the controller does not require retuning.

(c) K_v changes. Retuning may be necessary.

(d) G_p changes. Controller may have to be retuned.

12.12

(a) Using Table 12.4,

$$K_c = \frac{0.14}{K} + \frac{0.28\tau}{K\theta}$$

$$\tau_I = 0.33\theta + \frac{6.8\theta}{10\theta+\tau}$$

(b) Comparing to the Z-N settings, the H-A settings give much smaller K_c and slightly smaller τ_I, and are therefore more conservative.

(c) The Simulink responses for the two controllers are compared in Fig. S12.12. The controller settings are:

H-A: $K_c = 0.49$, τ_I=1.90
Cohen-Coon: $K_c = 1.39$, τ_I=1.98

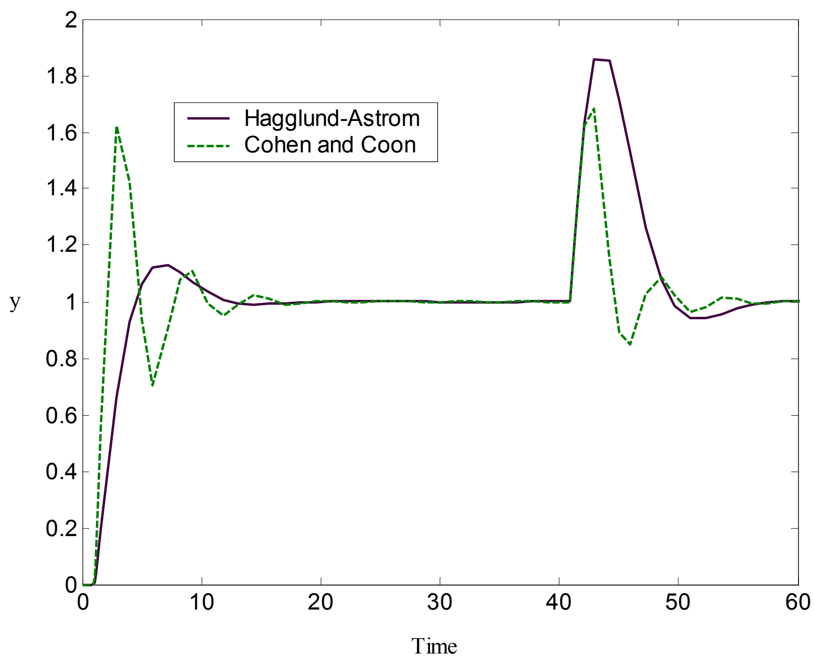


Fig. S12.12. *Comparison of Häggland-Åström and Cohen-Coon controller settings.*

From Fig. S12.12, it is clear that the H-A parameters provide a better set-point response, although they produce a more sluggish disturbance response.

12.13

From the solution to Exercise 12.5, the process reaction curve method yields

$K = 1.65$
$\theta = 1.7$ min
$\tau = 5.5$ min

(a) Direct Synthesis method:

From Table 12.1, Controller G:

$$K_c = \frac{1}{K}\frac{\tau}{\tau_c + \theta} = \frac{1}{1.65}\frac{5.5}{(5.5/3)+1.7} = 0.94$$

$$\tau_I = \tau = 5.5 \text{ min}$$

(b) Ziegler-Nichols settings:

$$G(s) = \frac{1.65e^{-1.7s}}{5.5s+1}$$

In order to find the stability limits, consider the characteristic equation

$$1 + G_cG = 0$$

Substituting the Padé approximation, $e^{-s} \approx \dfrac{1-0.85s}{1+0.85s}$, gives:

$$1 + G_cG = 1 + \frac{1.65K_c(1-0.85s)}{4.675s^2 + 6.35s + 1}$$

or

$$4.675s^2 + (6.35 - 1.403K_c)s + 1 + 1.65K_c = 0$$

Substitute $s = j\omega_u$ and $K_c = K_{cu}$,

$$-4.675\,\omega_u^2 + j(6.35 - 1.403K_{cu})\omega_u + 1 + 1.65K_{cu} = 0 + j0$$

Equating real and imaginary coefficients gives,

$$(6.35 - 1.403K_{cu})\omega_u = 0 \ , \ 1 + 1.65K_{cu} - 4.675\,\omega_u^2 = 0$$

Ignoring $\omega_u = 0$, $K_{cu} = 4.526$ and $\omega_u = 1.346$ rad/min. Thus,

$$P_u = \frac{2\pi}{\omega_u} = 4.67 \text{ min}$$

ThePI settings from Table 12.6 are:

	K_c	τ_I (min)
Ziegler-Nichols	2.04	3.89

The ultimate gain and ultimate period can also be obtained using Simulink. For this case, no Padé approximation is needed and the results are:

$K_{cu} = 3.76$ $\quad$ $P_u = 5.9$ min

The PI settings from Table 12.6 are:

	K_c	τ_I (min)
Ziegler-Nichols	1.69	4.92

Compared to the Z-N settings, the Direct Synthesis settings result in smaller K_c and larger τ_I. Therefore, they are more conservative.

12.14

$$G_v G_p G_m = \frac{2e^{-s}}{5s+1}$$

To find stability limits, consider the characteristic equation:

$$1 + G_c G_v G_p G_m = 0$$

or

$$1 + \frac{2K_c(1-0.5s)}{2.5s^2 + 5.5s + 1} = 0$$

Substituting a Padé approximation, $e^{-s} \approx \frac{1-0.5s}{1+0.5s}$, gives:

$$2.5s^2 + (5.5 - K_c)s + 1 + 2K_c = 0$$

Substituting $s = j\omega_u$ and $K_c = K_{cu}$.

$$-2.5\,\omega_u^2 + j(5.5 - K_{cu})\omega_u + 1 + 2K_{cu} = 0 + j0$$

Equating real and imaginary coefficients,

$$(5.5 - K_{cu})\omega_u = 0 \;\;,\;\; 1 + 2K_{cu} - 2.5\,\omega_u^2 = 0$$

Ignoring $\omega_u = 0$, $K_{cu} = 5.5$ and $\omega_u = 2.19$. Thus,

$$P_u = \frac{2\pi}{\omega_u} = 2.87$$

Controller settings (for the Padé approximation):

	K_c	τ_I	τ_D
Ziegler-Nichols	3.30	1.43	0.36
Tyreus-Luyben	2.48	6.31	0.46

The ultimate gain and ultimate period could also be found using Simulink. For this approach, no Padé approximation is needed and:

$$K_u = 4.26 \qquad P_u = 3.7$$

Controller settings (exact method):

	K_c	τ_I	τ_D
Ziegler-Nichols	2.56	1.85	0.46
Tyreus-Luyben	1.92	8.14	0.59

The set-point responses of the closed-loop systems for these controller settings are shown in Fig. S12.14.

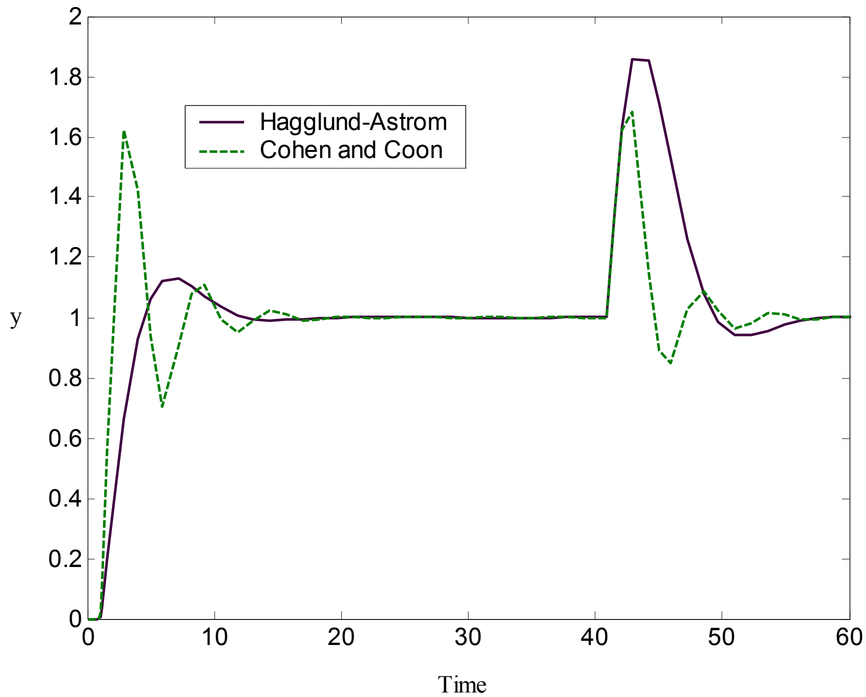


Figure S12.14. *Closed-loop responses for a unit step change in the set point.*

12.15

Eliminate the effect of the feedback control loop by opening the loop. That is, operate temporarily in open loop by switching the controller to the manual mode. This action provides a constant controller output signal. If oscillations persist, they must be due to external disturbances. If the oscillations vanish, they were caused by the feedback loop.

12.16

The sight glass observation confirms that the liquid level is actually rising. Since the controller output is saturated in response to the rising level, the controller is working properly. Thus, either the actual feed flow is higher than recorded, or the actual liquid flow is lower than recorded, or both. Because the flow transmitters consist of orifice plates and differential pressure transmitters, a plugged orifice plate could lead to a higher recorded flow. Thus, the liquid-flow-transmitter orifice plate would be the prime suspect.