Chapter 13

13.1

$$AR = |G(j\omega)| = \frac{3|G_1(j\omega)|}{|G_2(j\omega)||G_3(j\omega)|}$$

$$= \frac{3\sqrt{(-\omega)^2+1}}{\omega\sqrt{(2\omega)^2+1}} = \frac{3\sqrt{\omega^2+1}}{\omega\sqrt{4\omega^2+1}}$$

From the statement, we know the period P of the input sinusoid is 0.5 min and, thus,

$$\omega = \frac{2\pi}{P} = \frac{2\pi}{0.5} = 4\pi \text{ rad/min}$$

Substituting the numerical value of the frequency:

$$\hat{A} = AR \times A = \frac{3\sqrt{16\pi^2+1}}{4\pi\sqrt{64\pi^2+1}} \times 2 = 0.12 \times 2 = 0.24^\circ$$

Thus the amplitude of the resulting temperature oscillation is 0.24 degrees.

13.2

First approximate the exponential term as the first two terms in a truncated Taylor series

$$e^{-\theta s} \approx 1 - \theta s$$

Then $G(j\omega) = 1 - j\omega$

and $AR_{two\ term} = \sqrt{1+(-\omega\theta)^2} = \sqrt{1+\omega^2\theta^2}$

$$\phi_{two\ term} = \tan^{-1}(-\omega\theta) = -\tan^{-1}(\omega\theta)$$

For a first-order Pade approximation

$$e^{-\theta s} \approx \frac{1-\frac{\theta s}{2}}{1+\frac{\theta s}{2}}$$

from which we obtain

$$AR_{Pade} = 1$$
$$\phi_{Pade} = -2\tan^{-1}\left(\frac{\omega\theta}{2}\right)$$

Both approximations represent the original function well in the low frequency region. At higher frequencies, the Padé approximation matches the amplitude ratio of the time delay element exactly (AR_{Pade} = 1), while the two-term approximation introduces amplification ($AR_{two\ term}$ >1). For the phase angle, the high-frequency representations are:

$$\phi_{two\ term} \to -90^\circ$$
$$\phi_{Pade} \to -180^\circ$$

Since the angle of $e^{-j\omega\theta}$ is negative and becomes unbounded as $\omega \to \infty$, we see that the Pade representation also provides the better approximation to the time delay element's phase angle, matching ϕ of the pure time delay element to a higher frequency than the two-term representation.

13.3

Nominal temperature $\overline{T} = \dfrac{127\ ^\circ\text{F} + 119\ ^\circ\text{F}}{2} = 123\ ^\circ\text{F}$

$$\hat{A} = \frac{1}{2}(127\ ^\circ\text{F} - 119\ ^\circ\text{F}) = 4\ ^\circ\text{F}$$

$\tau = 4.5\,\text{sec.}, \quad \omega = 2\pi(1.8/60\,\text{sec}) = 0.189$ rad/s

Using Eq. 13-2 with K=1,

$$A = \hat{A}\left(\sqrt{\omega^2\tau^2 + 1}\right) = 4\sqrt{(0.189)^2(4.5)^2 + 1} = 5.25\ ^\circ\text{F}$$

Actual maximum air temperature = $\overline{T} + A = 128.25\ ^\circ\text{F}$

Actual minimum air temperature = $\overline{T} - A = 117.75\ ^\circ\text{F}$

13.4

$$\frac{T_m'(s)}{T'(s)} = \frac{1}{0.2s+1}$$

$$T'(s) = (0.2s+1)T_m'(s)$$

amplitude of T' =3.464 $\sqrt{(0.2\omega)^2+1} = 3.467$

phase angle of T' = φ + tan^{-1}(0.2ω) = φ + 0.04

Since only the maximum error is required, set φ = 0 for the comparison of T' and T_m'. Then

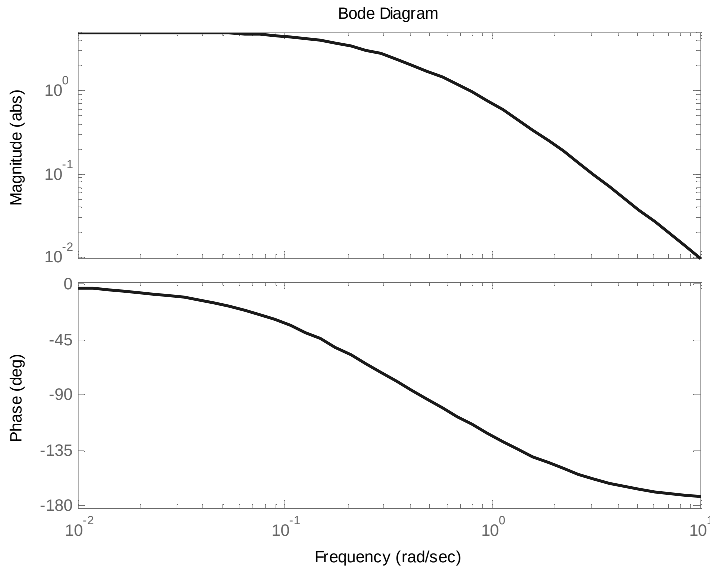
Error = $T_m' - T'$ =3.464 sin (0.2*t*) – 3.467sin(0.2*t* + 0.04)
= 3.464 sin(0.2*t*) –3.467[sin(0.2*t*) cos 0.04 + cos(0.2*t*)sin 0.04]
= 0.000 sin(0.2*t*) – 0.1386 cos(0.2*t*)

Since the maximum absolute value of cos(0.2*t*) is 1,

maximum absolute error = 0.1386

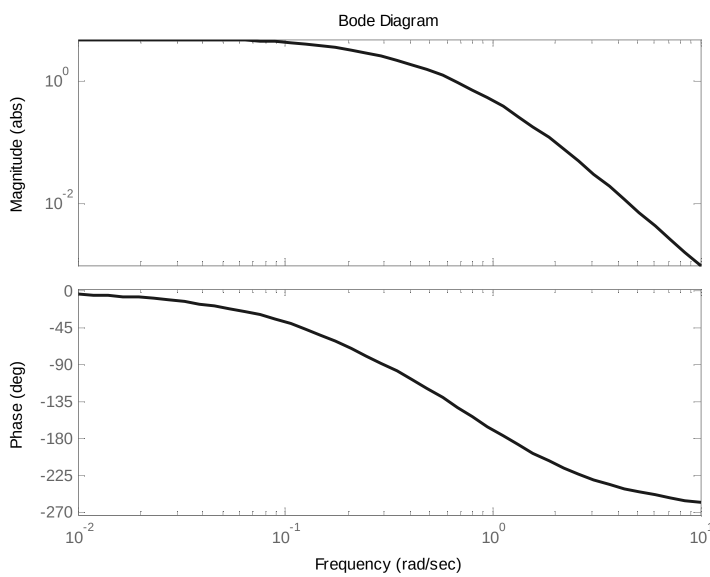
13.5

a)



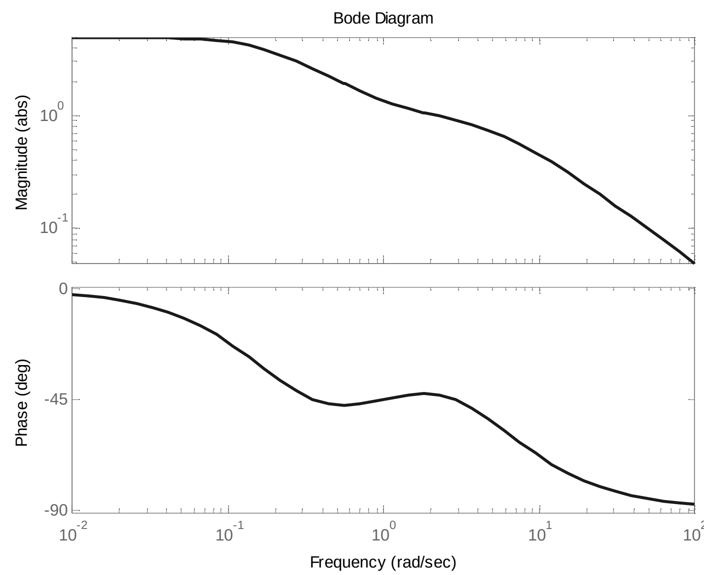
ω	AR (absolute)	φ
0.1	4.44	-32.4°
1	0.69	-124°
10	0.005	-173°

b)



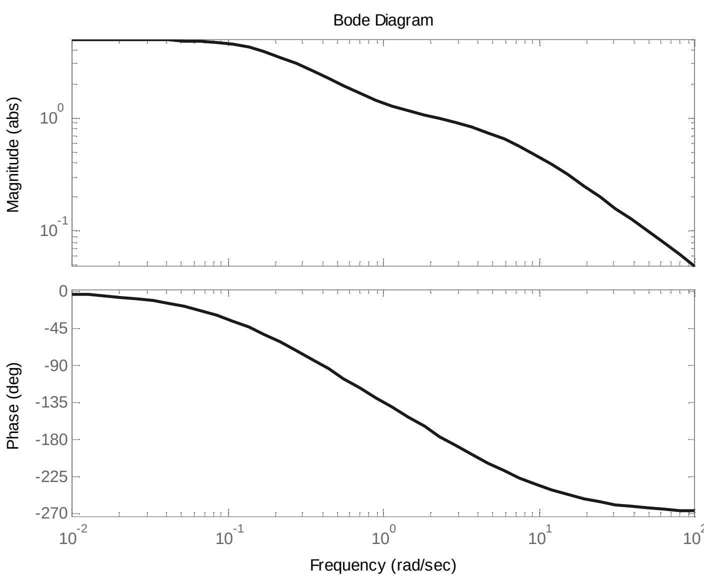
ω	AR (absolute)	φ
0.1	4.42	-38.2°
1	0.49	-169°
10	0.001	-257°

c)



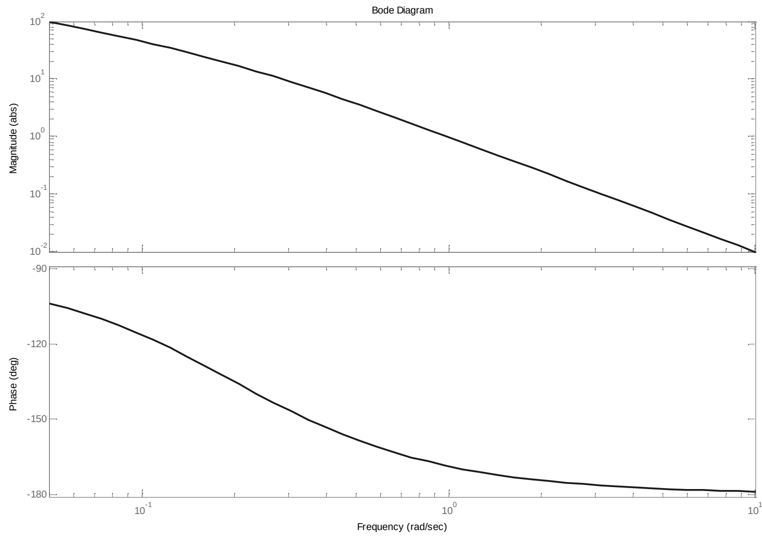
ω	AR (absolute)	φ
0.1	4.48	-22.1°
1	2.14	-44.9°
10	0.003	-87.6°

d)



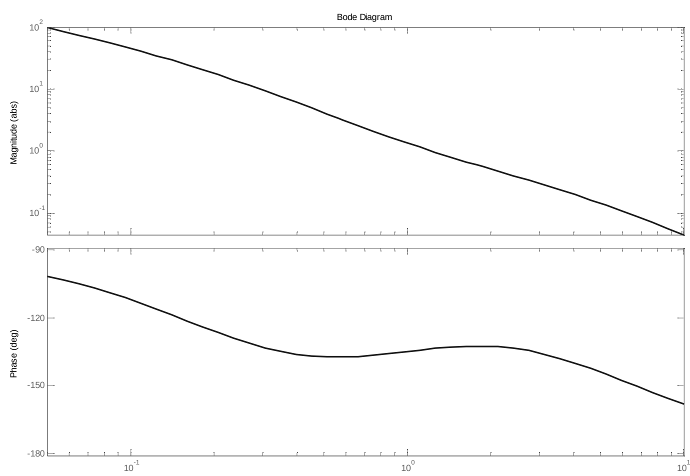
ω	AR (absolute)	φ
0.1	4.48	-33.6°
1	1.36	-136°
10	0.04	-266°

e)



ω	AR (absolute)	φ
0.1	44.6	-117°
1	0.97	-169°
10	0.01	-179°

f)



ω	AR (absolute)	φ
0.1	44.8	-112°
1	1.36	-135°
10	0.04	-158°

13.6

a) Multiply the AR in Eq. 13-41a by $\sqrt{\omega^2\tau_a^{\ 2}+1}$. Add to the value of φ in Eq. 13-41b the term $+\ \tan^{-1}(\omega\tau_a)$.

$$|G(j\omega)| = K\ \sqrt{\omega^2\tau_a^{\ 2}+1}\Big/\sqrt{(1-\omega^2\tau^2)^2+(0.4\omega\tau)^2}$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{-0.4\omega\tau}{1-\omega^2\tau^2}\right) +\ \tan^{-1}(\omega\tau_a).$$

b)

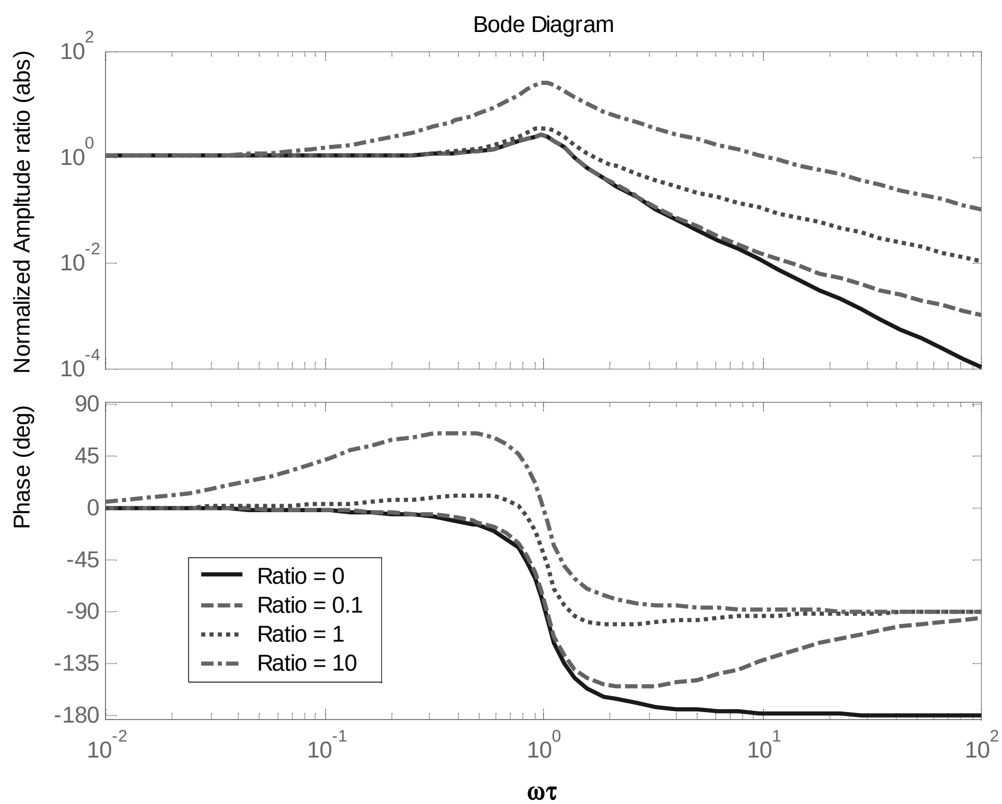


Figure S13.6. *Frequency responses for different ratios τ_a/τ*

13.7

Using MATLAB

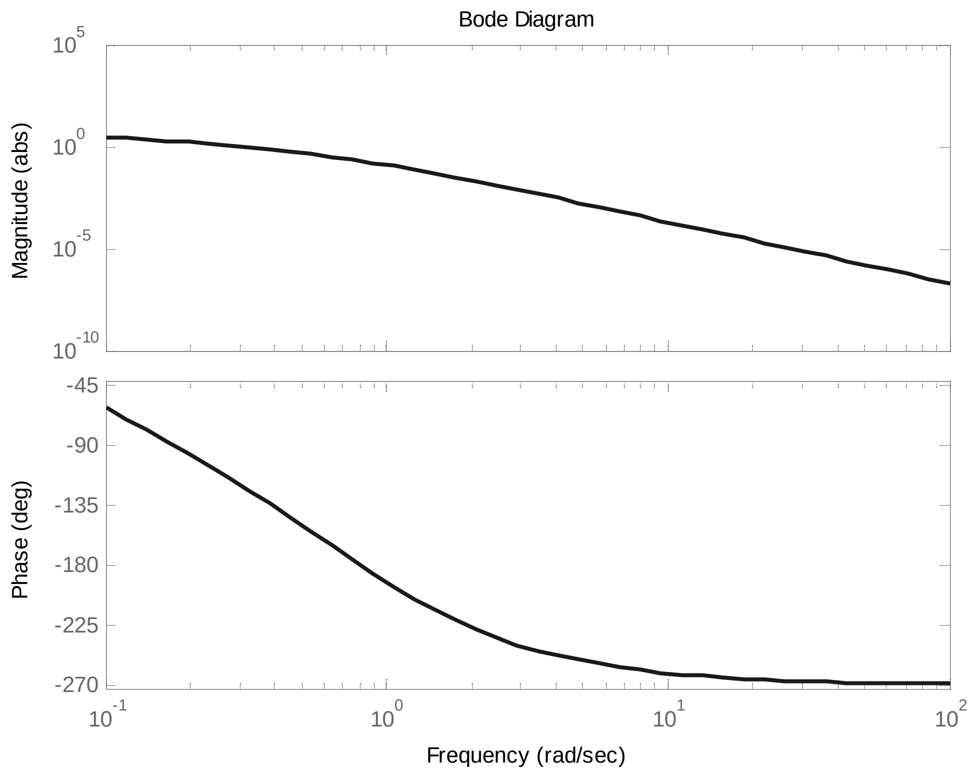


Figure S13.7. *Bode diagram of the third-order transfer function.*

The value of ω that yields a -180° phase angle and the value of AR at that frequency are:

ω = 0.807 rad/sec
AR = 0.202

13.8

Using MATLAB,

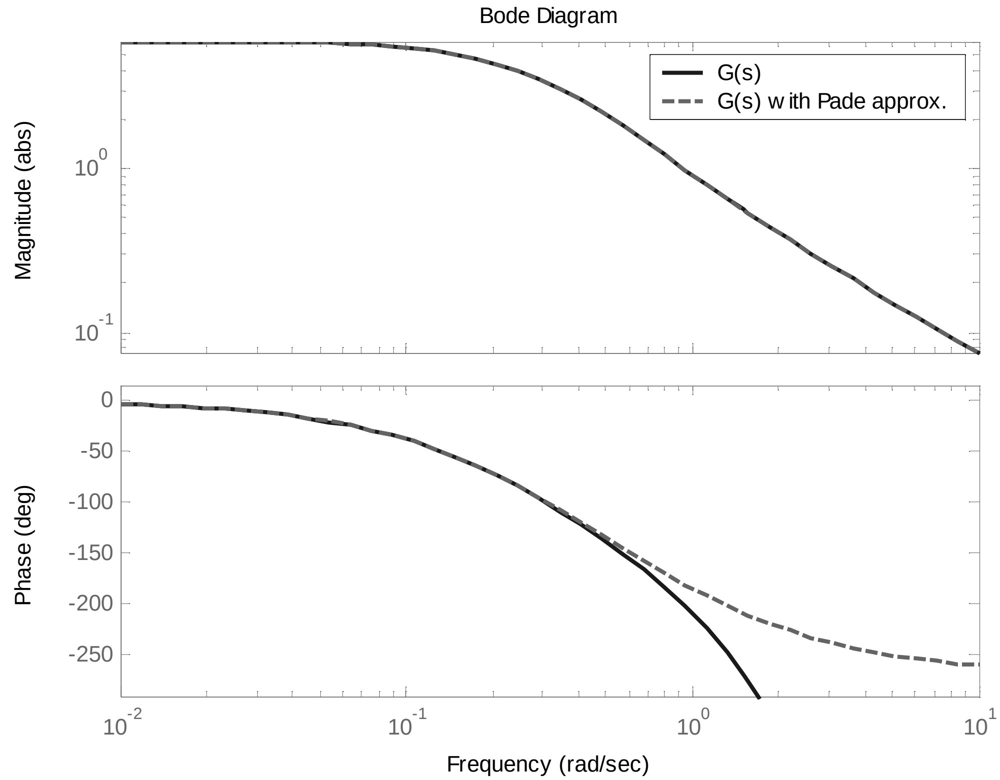


Figure S13.8. *Bode diagram for G(s) and G(s) with Pade approximation.*

13.9

$\omega = 2\pi f$ where f is in cycles/min

For the standard thermocouple, using Eq. 13-20b

$$\varphi_1 = -\tan^{-1}(\omega\tau_1) = \tan^{-1}(0.15\omega)$$

Phase difference $\Delta\varphi = \varphi_1 - \varphi_2$

Thus, the phase angle for the unknown unit is

$$\varphi_2 = \varphi_1 - \Delta\varphi$$

and the time constant for the unknown unit is

$$\tau_2 = \frac{1}{\omega}\tan(-\varphi_2)$$

using Eq. 13-20b . The results are tabulated below

f	ω	φ_1	Δφ	φ_2	τ_2
0.05	0.31	-2.7	4.5	-7.2	0.4023
0.1	0.63	-5.4	8.7	-14.1	0.4000
0.2	1.26	-10.7	16	-26.7	0.4004
0.4	2.51	-26.6	24.5	-45.1	0.3995
0.8	6.03	-37	26.5	-63.5	0.3992
1	6.28	-43.3	25	-68.3	0.4001
2	12.57	-62	16.7	-78.7	0.3984
4	25.13	-75.1	9.2	-84.3	0.3988

That the unknown unit is first order is indicated by the fact that $\Delta\varphi \to 0$ as $\omega \to \infty$, so that $\varphi_2 \to \varphi_1 \to$-90° and $\varphi_2 \to$-90° for $\omega \to \infty$ implies a first-order system. This is confirmed by the similar values of τ_2 calculated for different values of ω, implying that a graph of tan(-φ_2) versus ω is linear as expected for a first-order system. Then using linear regression or taking the average of above values, τ_2 = 0.40 min.

13.10

From the solution to Exercise 5-19, for the two-tank system

$$\frac{H_1'(s)/h_{1\max}'}{Q_{1i}'(s)} = \frac{0.01}{1.32s+1} = \frac{K}{\tau s+1}$$

$$\frac{H_2'(s)/h_{2\max}'}{Q_{1i}'(s)} = \frac{0.01}{(1.32s+1)^2} = \frac{K}{(\tau s+1)^2}$$

$$\frac{Q_2'(s)}{Q_{1i}'(s)} = \frac{0.1337}{(1.32s+1)^2} = \frac{0.1337}{(\tau s+1)^2}$$

and for the one-tank system

$$\frac{H'(s)/h_{\max}'}{Q_{1i}'(s)} = \frac{0.01}{2.64s+1} = \frac{K}{2\tau s+1}$$

$$\frac{Q'(s)}{Q_{1i}'(s)} = \frac{0.1337}{2.64s+1} = \frac{0.1337}{2\tau s+1}$$

For a sinusoidal input $q'_{1i}(t) = A\sin\omega t$, the amplitudes of the heights and flow rates are

$$\hat{A}[h'/h'_{\max}] = KA/\sqrt{4\omega^2\tau^2+1} \quad (1)$$

$$\hat{A}[q'] = 0.1337A/\sqrt{4\omega^2\tau^2+1} \quad (2)$$

for the one-tank system, and

$$\hat{A}[h'_1/h'_{1\max}] = KA/\sqrt{\omega^2\tau^2+1} \quad (3)$$

$$\hat{A}[h'_2/h'_{2\max}] = KA/\sqrt{(\omega^2\tau^2+1)^2} \quad (4)$$

$$\hat{A}[q'_2] = 0.1337A/\sqrt{(\omega^2\tau^2+1)^2} \quad (5)$$

for the two-tank system.

Comparing (1) and (3), for all ω

$$\hat{A}[h'_1/h'_{1\max}] \geq \hat{A}[h'/h'_{\max}]$$

Hence, for all ω, the first tank of the two-tank system will overflow for a smaller value of A than will the one-tank system. Thus, from the overflow consideration, the one-tank system is better for all ω. However, if A is small enough so that overflow is not a concern, the two-tank system will provide a smaller amplitude in the output flow for those values of ω that satisfy

$$\hat{A}[q'_2] \leq \hat{A}[q']$$

or $$\frac{0.1337A}{\sqrt{(\omega^2\tau^2+1)^2}} \leq \frac{0.1337A}{\sqrt{4\omega^2\tau^2+1}}$$

or $\omega \geq \sqrt{2}/\tau = 1.07$

Therefore, the two-tank system provides better damping of a sinusoidal disturbance for $\omega \geq 1.07$ if and only if

$$\hat{A}[h'_1/h'_{1\max}] \leq 1 \quad \text{, that is, } A \leq \frac{\sqrt{1.32^2\omega^2+1}}{0.01}$$

13.11

Using Eqs. 13-48 , 13-20, and 13-24,

$$AR = \frac{2\sqrt{\omega^2 \tau_a^2 + 1}}{\sqrt{100\omega^2 + 1}\sqrt{4\omega^2 + 1}}$$

$$\phi = \tan^{-1}(\omega\tau_a) - \tan^{-1}(10\omega) - \tan^{-1}(2\omega)$$

The Bode plots shown below indicate that

i) AR does not depend on the sign of the zero.
ii) AR exhibits resonance for zeros close to origin.
iii) All zeros lead to ultimate slope of –1 for AR.
iv) A left-plane zero yields an ultimate ϕ of -90°.
v) A right-plane zero yields an ultimate ϕ of -270°.
vi) Left-plane zeros close to origin can give phase lead at low ω.
vii) Left-plane zeros far from the origin lead to a greater lag (i.e., smaller phase angle) than the ultimate value. $\phi_u = -90°$ with a left-plane zero present.

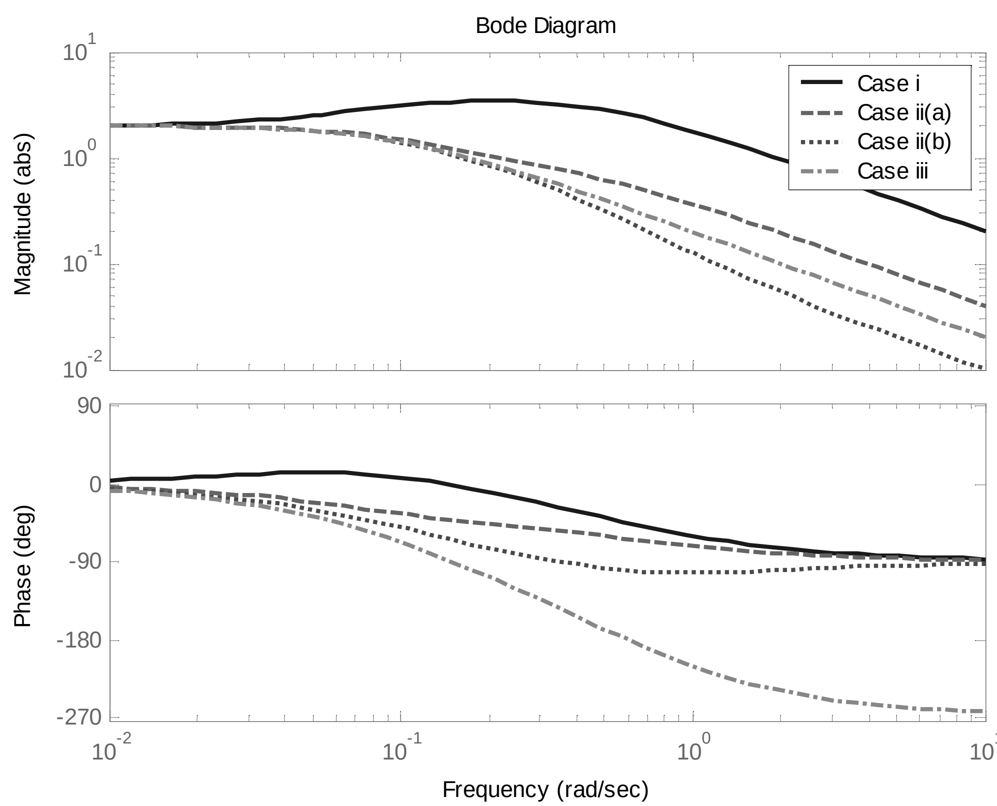


Figure S13.11. *Bode plot for each of the four cases of numerator dynamics.*

13.12

a) From Eq. 8-14 with $\tau_I = 4\tau_D$

$$G_c(s) = K_c \frac{(4\tau_D s + 1 + 4\tau_D{}^2 s^2)}{4\tau_D s} = K_c \frac{(2\tau_D s + 1)^2}{4\tau_D s}$$

$$|G_c(j\omega)| = K_c \frac{\left(\sqrt{4\tau_D{}^2\omega^2 + 1}\right)^2}{4\tau_D \omega} = K_c \frac{4\tau_D{}^2\omega^2 + 1}{4\tau_D \omega}$$

b) From Eq. 8-15 with $\tau_I = 4\tau_D$ and $\alpha = 0.1$

$$G_c(s) = K_c \frac{(4\tau_D s + 1)(\tau_D s + 1)}{4\tau_D s(0.1\tau_D s + 1)}$$

$$|G_c(j\omega)| = K_c \frac{\left(\sqrt{16\tau_D{}^2\omega^2 + 1}\right)\left(\sqrt{\tau_D{}^2\omega^2 + 1}\right)}{4\tau_D \omega \sqrt{0.01\tau_D{}^2\omega^2 + 1}}$$

The differences are significant for $0.25 < \omega\tau_D < 1$ by a maximum of $0.5\ K_c$ at $\omega\tau_D = 0.5$, and for $\omega\tau_D > 10$ by an amount increasing with $\omega\tau_D$.

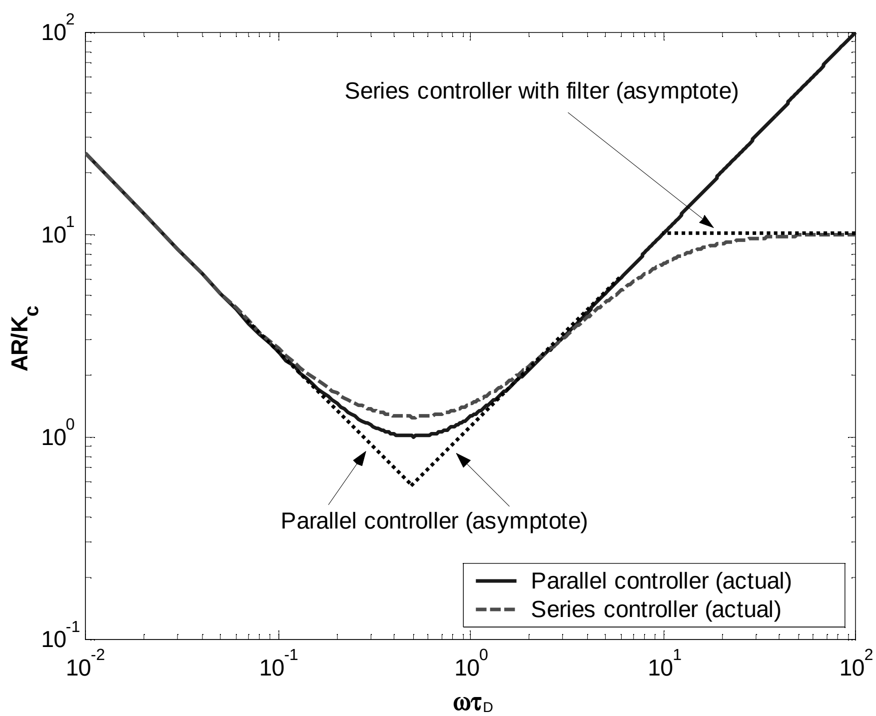


Figure S13.12. *Nominal amplitude ratio for parallel and series controllers.*

13.13

MATLAB does not allow the addition of transfer functions with different time delays. Hence the denominator time delay needs to be approximated if a MATLAB program is used. However, the use of Mathematica or even Excel to evaluate derived expressions for the AR and angle, using various values of omega, and to make the plots will yield exact results:

MATLAB - Padé approximation:

Substituting the 1/1 Padé approximation gives:

$$G(s) \approx \frac{K}{\tau s + \left(\dfrac{2-\theta s}{2+\theta s}\right) + 1} = \frac{K(\theta s + 2)}{\theta \tau s^2 + 2\tau s + 4} \tag{1}$$

By using MATLAB,

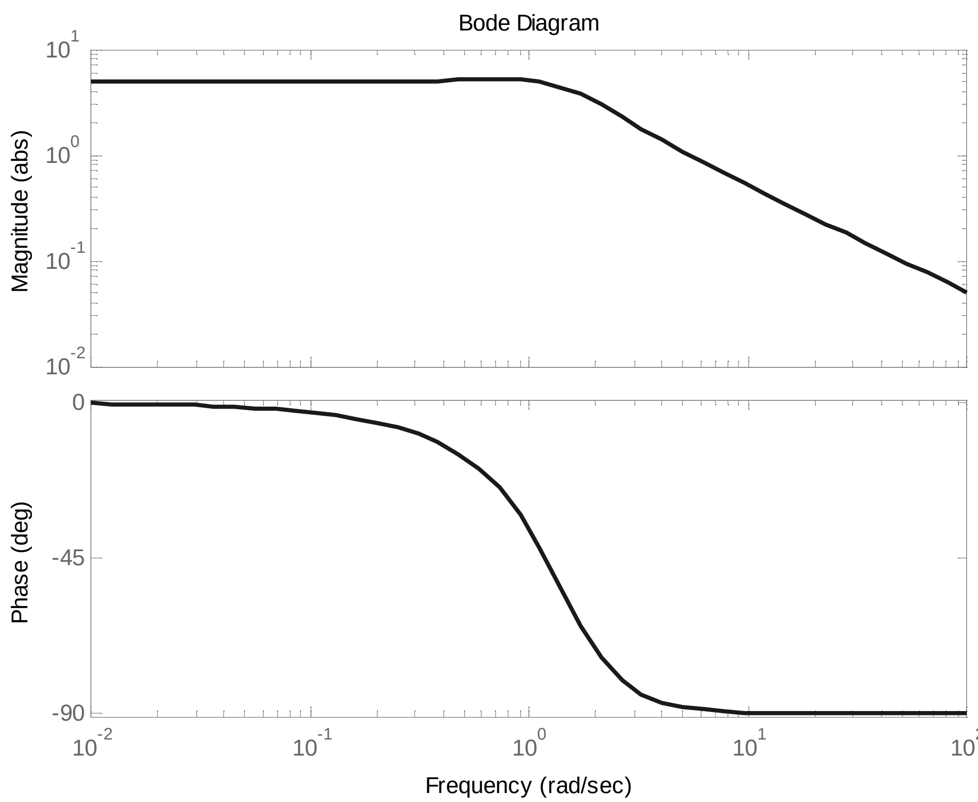


Figure S13.13. *Bode plot by using Padé approximation.*

13.14

$$\omega = 600\frac{\text{rotations}}{\text{min}} \times 4\frac{\text{cycles}}{\text{rotation}} \times 2\pi\frac{\text{radians}}{\text{cycle}} = 15080\frac{\text{rad}}{\text{min}}$$

$A = 2$ psig $\qquad\qquad \hat{A} = 0.02$ psig

$\text{AR} = \hat{A}/A = 0.01$

Volume of the pipe connecting the compressor to the reactor is

$$V_{pipe} = 20 \text{ ft} \times \frac{\pi}{4}\left(\frac{3}{12}\right)^2 \text{ft}^2 = 0.982 \text{ ft}^3$$

<u>Two-tank surge system</u>

Using the figure and nomenclature in Exercise 2.5, the 0.02 psig variation in $\hat{A}$ refers to the pressure before the valve R_c, namely the pressure P_2. Hence the transfer function $P_2'(s)/P_d'(s)$ is required in order to use the value of AR. Mass balance for the tanks is (referring to the solution for Exercise 2.5.

$$\frac{V_1 M}{RT_1}\frac{dP_1}{dt} = w_a - w_b \qquad (1)$$

$$\frac{V_2 M}{RT_2}\frac{dP_2}{dt} = w_b - w_c \qquad (2)$$

where the ideal-gas assumption has been used. For linear valves,

$$w_a = \frac{P_d - P_1}{R_a} \quad , \quad w_b = \frac{P_1 - P_2}{R_b} \quad , \quad w_c = \frac{P_2 - P_f}{R_c}$$

At nominal conditions,

$P_d = 200$ psig

$w_a = w_b = w_c = 6000 \text{ lb/hr} = 100 \text{ lb/min}$

$$P_d - P_1 = P_1 - P_2 = \frac{0.1 P_d}{2} = 10 \text{ psig}$$

$$R_a = \frac{P_d - P_1}{w_a} = \frac{10\text{ psig}}{100\text{ lb/min}} = 0.1\frac{\text{psig}}{\text{lb/min}} = \frac{P_1 - P_2}{w_b} = R_b$$

Assume $R_c = R_a = R_v$
Assume $T_2 = T_1 = 300\ ^\circ\text{F} = 792\ ^\circ\text{R}$
Given $V_1 = V_2 = V$

Then equations (1) and (2) become

$$\left(\frac{VM}{RT}R_v\right)\frac{dP_1}{dt} = P_d - P_1 - (P_1 - P_2) = P_d - 2P_1 + P_2$$

$$\left(\frac{VM}{RT}R_v\right)\frac{dP_2}{dt} = P_1 - P_2 - (P_2 - P_f) = P_1 - 2P_2 - P_f$$

Taking deviation variables, Laplace transforming, and noting that P_f' is zero since P_f is constant, gives

$$\tau s P_1'(s) = \frac{1}{2}P_d'(s) - P_1'(s) + \frac{1}{2}P_2'(s) \qquad (3)$$

$$\tau s P_2'(s) = \frac{1}{2}P_1'(s) - P_2'(s) \qquad (4)$$

where

$$\tau = \frac{1}{2}\left(\frac{VM}{RT}R_v\right)$$

$$= \frac{1}{2}\left(V\text{ ft}^3\right)\left(28\frac{\text{lb}}{\text{lb mole}}\right)\left(0.1\frac{\text{psig}}{\text{lb/min}}\right)\Big/\left(10.731\frac{\text{ft}^3\text{psig}}{\text{lb mole}^\circ\text{R}}\right)\left(792\ ^\circ\text{R}\right)$$

$$= (1.647\times10^{-4}V)\text{ min}$$

From Eq. 3

$$P_1'(s) = \frac{1}{2(\tau s + 1)}P_d'(s) + \frac{1}{2(\tau s + 1)}P_2'(s)$$

Substituting for $P_1'(s)$ into Eq. 4

$$(\tau s+1)P_2'(s)=\frac{1}{4(\tau s+1)}P_d'(s)+\frac{1}{4(\tau s+1)}P_2'(s)$$

or

$$\frac{P_2'(s)}{P_d'(s)}=\frac{1}{4(\tau s+1)^2-1}=\frac{1}{4\tau^2 s^2+8\tau s+3}$$

$$\frac{P_2'(j\omega)}{P_d'(j\omega)}=\frac{1}{(3-4\omega^2\tau^2)+j8\omega\tau}$$

$$\text{AR}=\frac{1}{\sqrt{(3-4\omega^2\tau^2)^2+64\omega^2\tau^2}}=\frac{1}{\sqrt{16\omega^4\tau^4+40\omega^2\tau^2+9}}$$

Setting AR = 0.01 gives

$$16\omega^4\tau^4+40\omega^2\tau^2+9=10000$$

$$16\omega^4\tau^4+40\omega^2\tau^2-9991=0$$

$$\omega^2\tau^2=\frac{1}{2\times 16}\left(-40+\sqrt{40^2+4\times 16\times 9991}\right)=23.77$$

$$\tau=\frac{\sqrt{23.77}}{\omega}=\frac{4.875}{\omega}=3.233\times 10^{-4}\text{ min}$$

$$V=\frac{\tau}{1.647\times 10^{-4}}=1.963\text{ ft}^3$$

Total surge volume $V_{surge}=2V=3.926\text{ ft}^3$

Letting the connecting pipe provide part of this volume, the volume of each tank = $\frac{1}{2}(V_{surge}-V_{pipe})=1.472\text{ ft}^3$

Single-tank system

In the figure for the two-tank system, remove the second tank and the valve before it (R_b). Now, $\hat{A}$ refers to P_1 and AR refers to $P_1'(s)/P_d'(s)$.

Mass balance for the tank is

$$\frac{V_1 M}{RT_1}\frac{dP_1}{dt} = w_a - w_c$$

where $\qquad w_a = \dfrac{P_d - P_1}{R_a} \qquad , \qquad w_c = \dfrac{P_1 - P_f}{R_c}$

At nominal conditions

$$P_d - P_1 = 0.1\, P_d = 20 \text{ psig}$$

$$R_a = \frac{P_d - P_1}{w_a} = \frac{20 \text{ psig}}{100 \text{ lb/min}} = 0.2\frac{\text{psig}}{\text{lb/min}}$$

Assume $R_c = R_a = R_v$

Then Eq. 1 becomes

$$\left(\frac{V_1 M}{RT_1}R_v\right)\frac{dP_1}{dt} = P_d - P_1 - (P_1 - P_7) = P_d - 2P_1 + P_7$$

Using deviation variables and taking the Laplace transform

$$\frac{P_1'(s)}{P_d'(s)} = \frac{1/2}{\tau s + 1}$$

where

$$\tau = \frac{1}{2}\left(\frac{V_1 M}{RT_1}R_v\right) = (3.294\times10^{-4} V_1)\text{ min}$$

AR = 0.01= $0.5\big/\sqrt{\omega^2\tau^2 + 1}$, $\tau = 3.315\times10^{-3}\text{ min}$, $V_1 = 10.06\text{ ft}^3$

Volume of single tank = $(V_1 - V_{pipe}) = 9.084\text{ ft}^3 > 4\times1.472\text{ ft}^3$
Hence, recommend two surge tanks, each with volume 1.472 ft^3

13.15

By using MATLAB

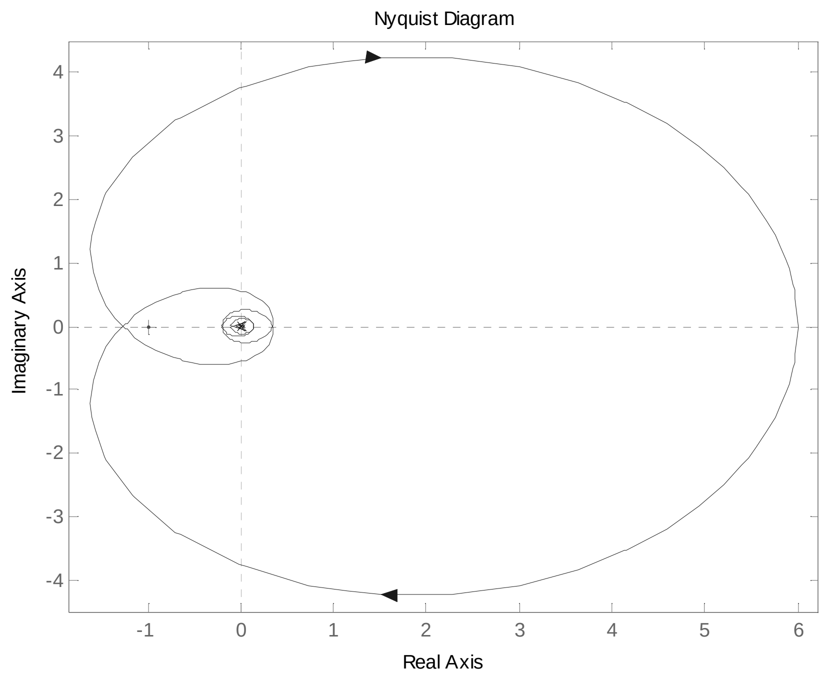


Figure S13.15a. *Nyquist diagram.*

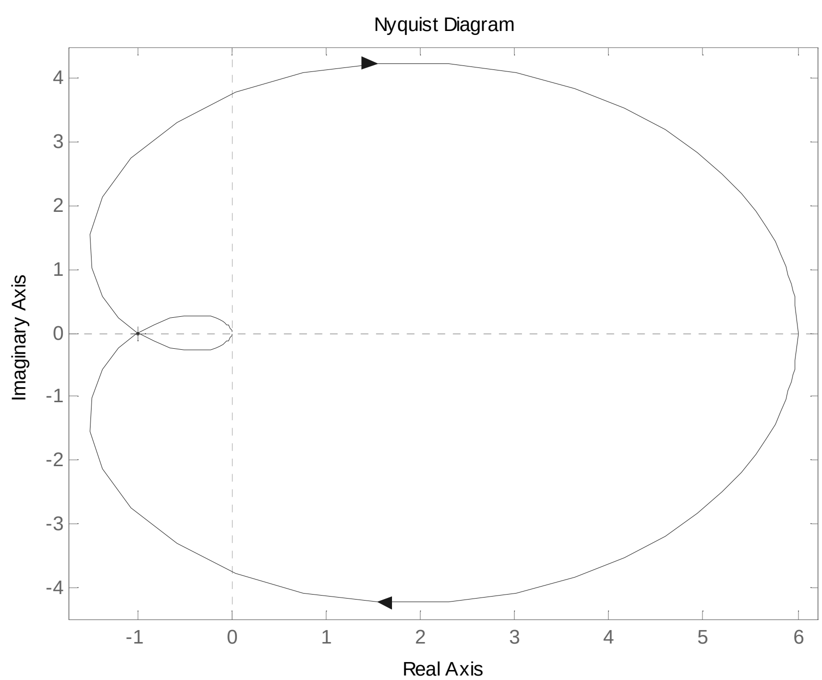


Figure S13.15b. *Nyquist diagram by using Pade approximation.*

The two plots are very different in appearance for large values of ω. The reason for this is the time delay. If the transfer function contains a time delay in addition to poles and zeros, there will be an infinite number of encirclements of the origin. This result is a consequence of the unbounded phase shift for the time delay.

A subtle difference in the two plots, but an important one for the Nyquist design methods of Chapter 14, is that the plot in S13.5a "encircles" the -1, 0 point while that in S13.5b passes through it exactly.

13.16

By using MATLAB,

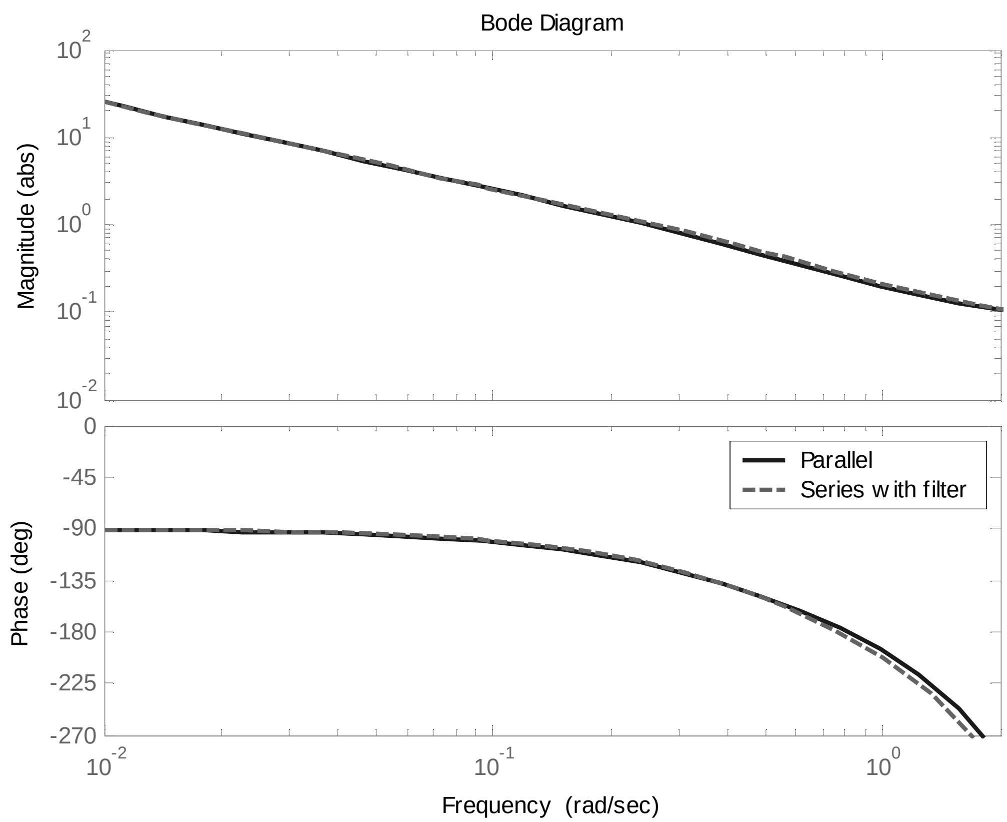


Figure S3.16. *Bode plot for Exercise 13.8 Transfer Function multiplied by PID Controller Transfer Function. Two cases: a)Parallel b) Series with Deriv. Filter (α=0.2).*

.

Amplitude ratios:

Ideal PID controller: AR= 0.246 at $\omega = 0.80$
Series PID controller: AR=0.294 at $\omega = 0.74$

There is 19.5% difference in the AR between the two controllers.

13.17

a) Method discussed in Section 6.3:

$$\hat{G}_1(s) = \frac{12e^{-0.3s}}{(8s+1)(2.2s+1)}$$

Visual inspection of the frequency responses:

$$\hat{G}_2(s) = \frac{12e^{-0.4s}}{(5.64s+1)(2.85s+1)}$$

b) Comparison of three models:

Bode plots:

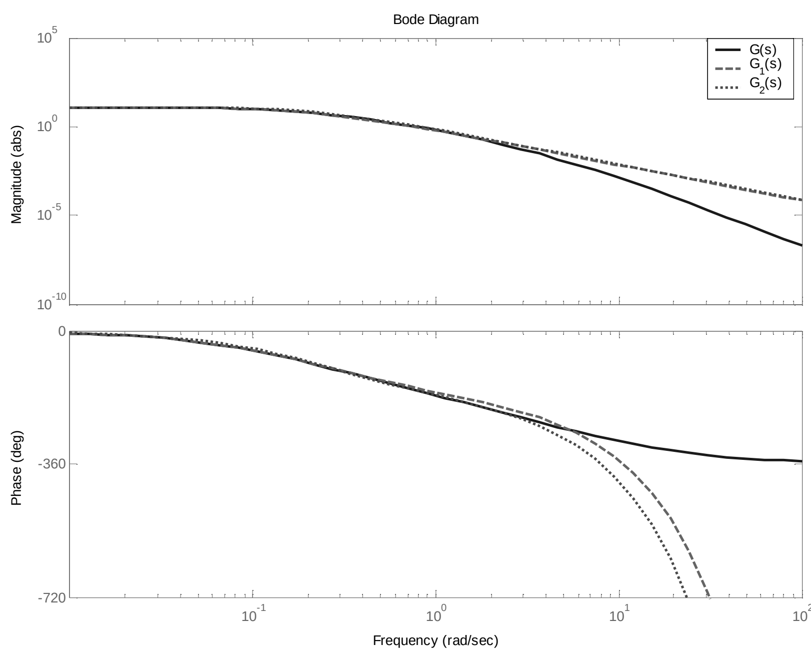


Figure S13.17a. *Bode plots for the exact and approximate models.*

Impulse responses:

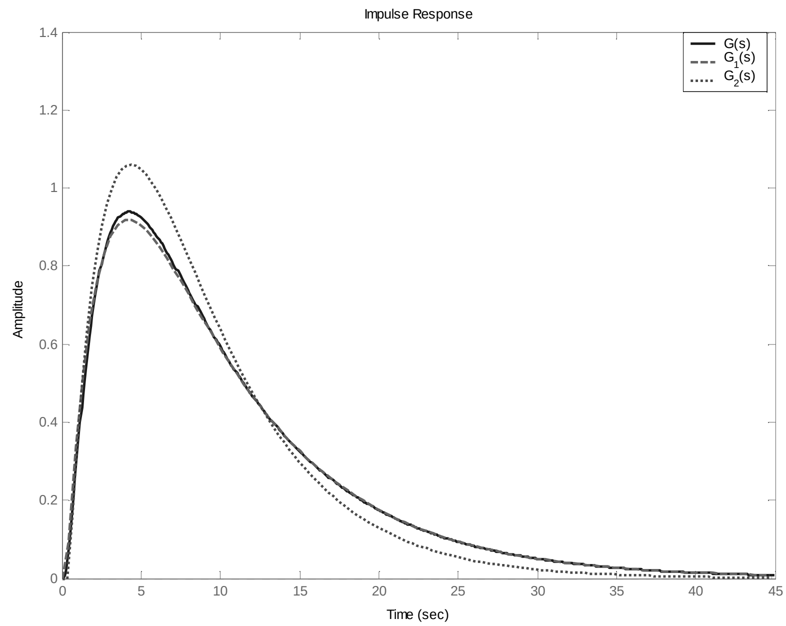


Figure S13.17b. *Impulse responses for the exact and approximate models*

13.18

The original transfer function is

$$G(s) = \frac{10(2s+1)e^{-2s}}{(20s+1)(4s+1)(s+1)}$$

The approximate transfer function obtained using Section 6.3 is:

$$G'(s) = \frac{10(2s+1)e^{-5s}}{(22s+1)}$$

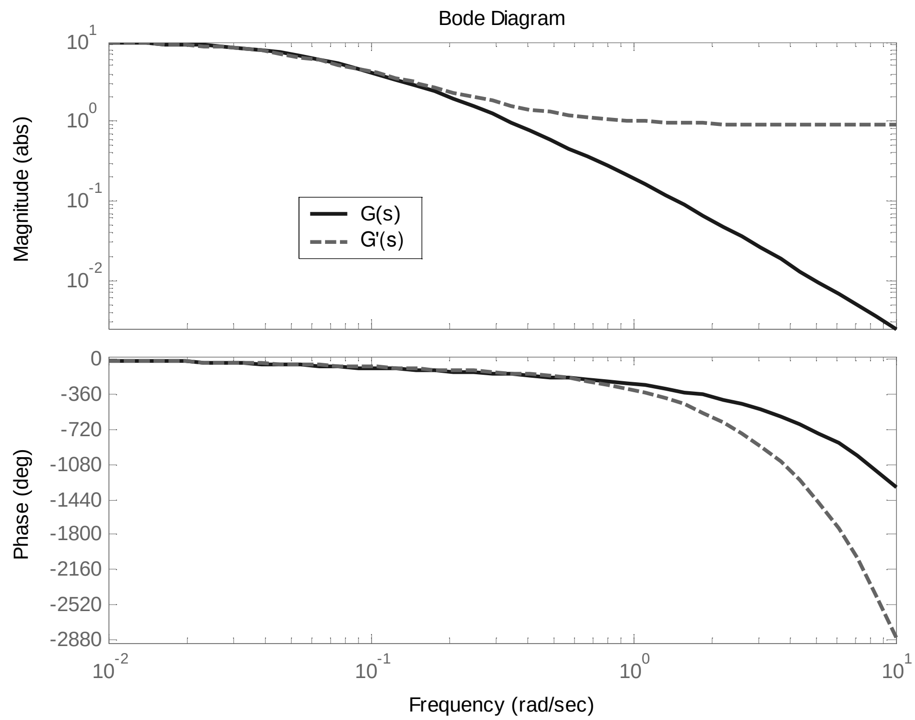


Figure S13.18. *Bode plots for the exact and approximate models.*

As seen in Fig.S13.18, the approximation is good at low frequencies, but not that good at higher frequencies.