Chapter 14

14.1

Let $G_{OL}(j\omega_c) = R + jI$

where ω_c is the critical frequency. Then, according to the Bode stability criterion

$$| G_{OL}(j\omega_c)| = 1 = \sqrt{R^2 + I^2}$$
$$\angle G_{OL}(j\omega_c) = -\pi = \tan^{-1}(I/R)$$

Solving for R and I: $R = -1$ and $I = 0$

Substituting $s = j\omega_c$ into the characteristic equation gives,

$$1 + G_{OL}(j\omega_c) = 0$$

$$I + R + jI = 0 \quad \text{or} \quad R = -1 \quad , \quad I = 0$$

Hence, the two approaches are equivalent.

14.2

Because sustained oscillations occur at the critical frequency

$$\omega_c = \frac{2\pi}{10 \text{ min}} = 0.628 \text{ min}^{-1}$$

(a) Using Eq. 14-7,

$$1 = (K_c)(0.5)(1)(1.0) \quad \text{or} \quad K_c = 2$$

(b) Using Eq. 14-8,

$$-\pi = 0 + 0 + (-\theta\omega_c) + 0$$
$$\text{or} \;\; \theta = \frac{\pi}{\omega_c} = 5 \text{ min}$$

14.3

(a) From inspection of the Bode diagrams in Tables 13.4 and 13.5, the transfer function is selected to be of the following form

$$G(s) = \frac{K(\tau_a s+1)}{s(\tau_1 s+1)(\tau_2 s+1)}$$

where τ_a, τ_1, τ_2 correspond to frequencies of ω = 0.1, 2, 20 rad/min, respectively.

Therefore, $\tau_a = 1/0.1 = 10$ min

$\tau_1 = 1/2 = 0.5$ min

$\tau_2 = 1/20 = 0.05$ min

For low frequencies, AR $\approx |K/s| = K/\omega$

At $\omega = 0.01$, AR = 3.2, so that $K = (\omega)(\text{AR}) = 0.032$

Therefore,

$$G(s) = \frac{0.032(10s+1)}{s(0.5s+1)(0.05s+1)}$$

(b) Because the phase angle does not cross -180°, the concept of GM is meaningless.

14.4

The following process transfer can be derived in analogy with Eq. 6-71:

$$\frac{H_1(s)}{Q_1(s)} = \frac{R_1}{(A_1R_1A_2R_2)s^2 + (A_1R_1 + A_2R_1 + A_2R_2)s + 1}$$

For $R_1 = 0.5$, $R_2 = 2$, $A_1 = 10$, $A_2 = 0.8$:

$$G_p(s) = \frac{0.5}{8s^2 + 7s + 1} \qquad (1)$$

For $R_2 = 0.5$: $$G_p(s) = \frac{0.5}{2s^2 + 5.8s + 1} \qquad (2)$$

(a) **For $R_2 = 2$**

$$\angle G_p = \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^2}\right] \quad , \quad |G_p| = \frac{0.5}{\sqrt{(1-8\omega_c^2)^2 + (7\omega_c)^2}}$$

For $G_v = K_v = 2.5$, $\varphi_v = 0$, $|G_v| = 2.5$

For $G_m = \frac{1.5}{0.5s+1}$, $\varphi_m = -\tan^{-1}(0.5\omega)$, $|G_m| = \frac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}$

K_{cu} and ω_c are obtained using Eqs. 14-7 and 14-8:

$$-180° = 0 + 0 + \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^2}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, $\omega_c = 1.369$ rad/min.

$$1 = (K_{cu})(2.5)\left(\frac{0.5}{\sqrt{(1-8\omega_c^2)^2 + (7\omega_c)^2}}\right)\left(\frac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}\right)$$

Substituting $\omega_c = 1.369$ rad/min, $K_{cu} = 10.96$, $\omega_c K_{cu} = 15.0$

For R_2=0.5

$$\angle G_p = \tan^{-1}\left[\frac{-5.8\omega_c}{1-2\omega_c^2}\right] \quad , \quad |G_p| = \left(\frac{0.5}{\sqrt{(1-2\omega_c^2)^2 + (5.8\omega_c)^2}}\right)$$

$$-180° = 0 + 0 + \tan^{-1}\left[\frac{-5.8\omega_c}{1-2\omega_c^2}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, $\omega_c = 2.51$ rad/min.

Substituting $\omega_c = 2.51$ rad/min, $K_{cu} = 15.93$, $\omega_c K_{cu} = 40.0$

(a) From part (a), for R_2=2,

$$\omega_c = 1.369 \text{ rad/min}, \quad K_{cu} = 10.96$$

$$P_u = \frac{2\pi}{\omega_c} = 4.59 \text{ min}$$

Using Table 12.6, the Ziegler-Nichols PI settings are

$$K_c = 0.45\, K_{cu} = 4.932 \quad , \quad \tau_I = P_u/1.2 = 3.825 \text{ min}$$

Using Eqs. 13-63 and 13-62 ,

$$\varphi_c = -\tan^{-1}(-1/3.825\omega)$$

$$|G_c| = 4.932\sqrt{\left(\frac{1}{3.825\omega}\right)^2 + 1}$$

Then, from Eq. 14-7

$$-180° = \tan^{-1}\left[\frac{-1}{3.825\omega_c}\right] + 0 + \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^{\,2}}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, ω_c = 1.086 rad/min.

Using Eq. 14-8,

$$A_c = \text{AR}_{OL}|_{\omega=\omega c} =$$

$$= \left(4.932\sqrt{\left(\frac{1}{3.825\omega_c}\right)^2 + 1}\right)(2.5)\left(\frac{0.5}{\sqrt{(1-8\omega_c^{\,2})^2 + (7\omega_c)^2}}\right)\left(\frac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}\right)$$

$$= 0.7362$$

Therefore, gain margin $GM = 1/A_c$ = 1.358.

Solving Eq.(14-16) for ω_g

$$\text{AR}_{OL}|_{\omega=\omega c} = 1 \quad \text{at} \quad \omega_g = 0.925$$

Substituting into Eq. 14-7 gives $\varphi_g = \varphi|_{\omega=\omega g} = -172.7°$.

Therefore, phase margin $PM = 180 + \varphi_g = 7.3°$.

14.5

(a) $K=2$, $\tau = 1$, $\theta = 0.2$, $\tau_c=0.3$

Using Eq. 12-11, the PI settings are

$$K_c = \frac{1}{K}\frac{\tau}{\theta+\tau_c} = 1 \quad , \quad \tau_I = \tau = 1 \text{ min,}$$

Using Eq. 14-8 ,

$$-180° = \tan^{-1}\left(\frac{-1}{\omega_c}\right) - 0.2\omega_c - \tan^{-1}(\omega_c) = -90° - 0.2\omega_c$$

or $\quad \omega_c = \dfrac{\pi/2}{0.2} = 7.85$ rad/min

Using Eq. 14-7,

$$A_c = \text{AR}_{OL}\big|_{\omega=\omega_c} = \sqrt{\frac{1}{\omega_c^{\,2}}+1}\left(\frac{2}{\sqrt{\omega_c^{\,2}+1}}\right) = \frac{2}{\omega_c} = 0.255$$

From Eq. 14-11, $GM = 1/A_c = 3.93$.

(b) Using Eq. 14-12,

$$\varphi_g = PM - 180° = -140° = \tan^{-1}(-1/0.5\omega_g) - 0.2\omega_g - \tan^{-1}(\omega_g)$$

Solving by trial and error, $\omega_g = 3.04$ rad/min

$$\text{AR}_{OL}\big|_{\omega=\omega_g} = 1 = K_c\sqrt{\left(\frac{1}{0.5\omega_g}\right)^2+1}\left(\frac{2}{\sqrt{\omega_g^{\,2}+1}}\right)$$

Substituting for ω_g gives $K_c = 1.34$. Then from Eq. 14-8

$$-180° = \tan^{-1}\left(\frac{-1}{0.5\omega_c}\right) - 0.2\omega_c - \tan^{-1}(\omega_c)$$

Solving by trial and error, ω_c =7.19 rad/min.

From Eq. 14-7,

$$A_c = \text{AR}_{OL}\big|_{\omega=\omega_c} = 1.34\sqrt{\left(\frac{1}{0.5\omega_c}\right)^2+1}\left(\frac{2}{\sqrt{\omega_c^{\ 2}+1}}\right) = 0.383$$

From Eq. 14-11, $GM = 1/A_c = 2.61$

(c) By using Simulink-MATLAB, these two control systems are compared for a unit step change in the set point.

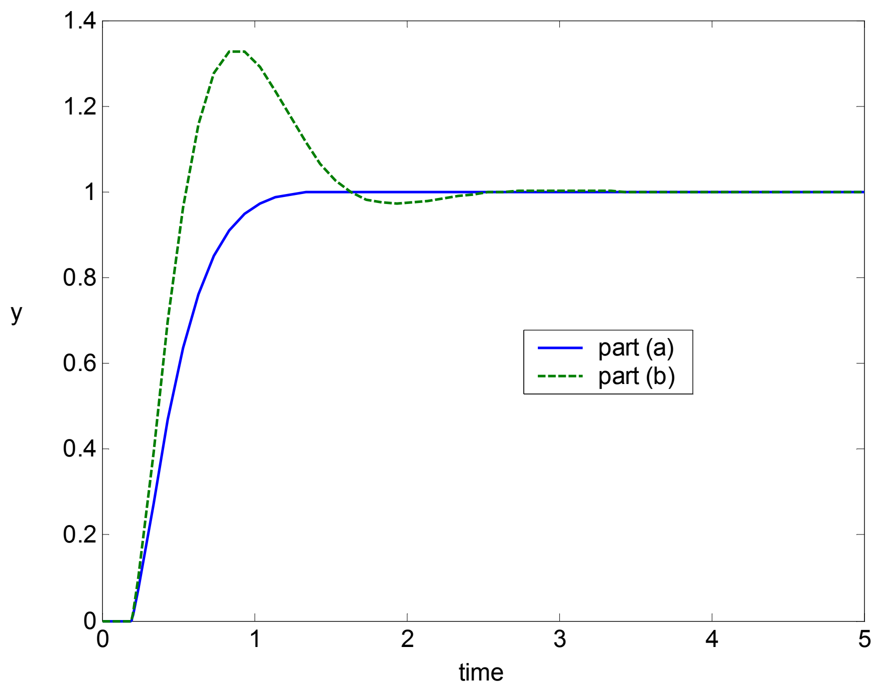


Fig S14.5. *Closed-loop response for a unit step change in set point.*

The controller designed in part a) (Direct Synthesis) provides better performance giving a first-order response. Part b) controller yields a large overshoot.

14.6

(a) Using Eqs. 14-7 and 14-8,

$$\text{AR}_{OL} = \frac{Y_m}{Y_{sp}} = \left(K_c \frac{\sqrt{4\omega^2+1}}{\sqrt{0.01\omega^2+1}} \right) \left(\frac{2}{\sqrt{0.25\omega^2+1}} \right) \left(\frac{0.4}{\omega\sqrt{25\omega^2+1}} \right) (1.0)$$

$$\varphi = \tan^{-1}(2\omega) - \tan^{-1}(0.1\omega) - \tan^{-1}(0.5\omega) - (\pi/2) - \tan^{-1}(5\omega)$$

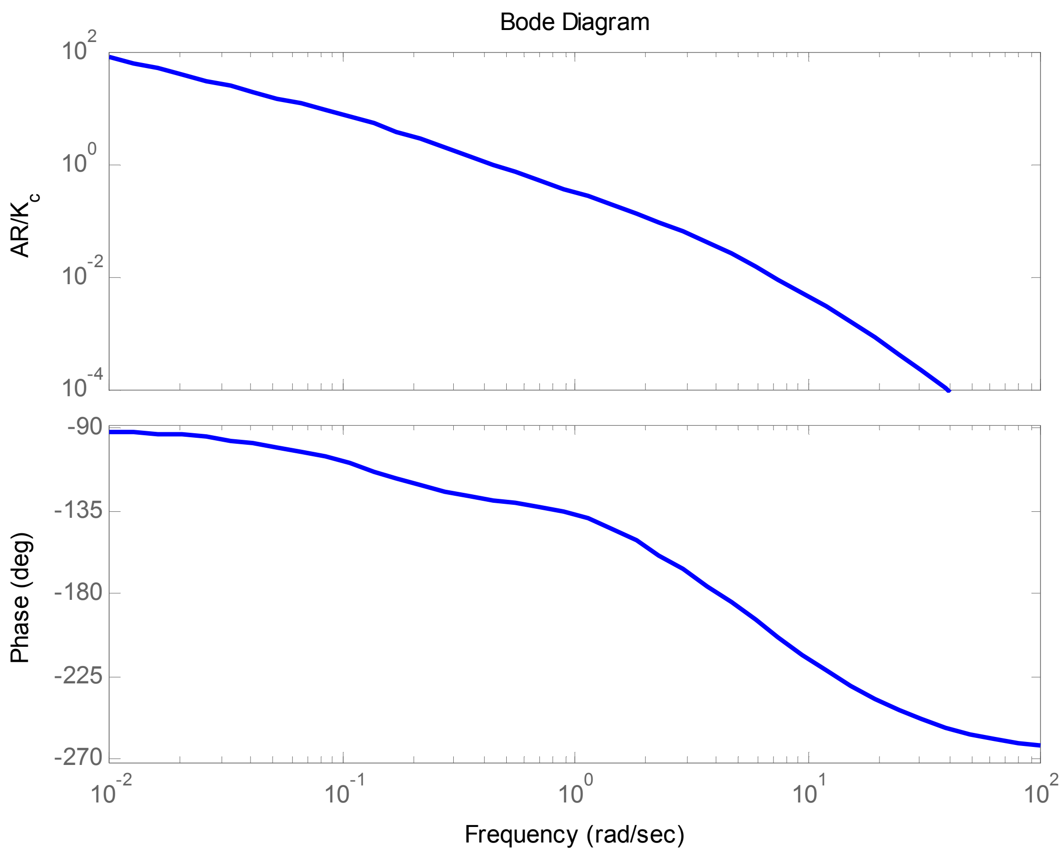


Figure S14.6a. *Bode plot*

(b) Using Eq.14-12

$$\varphi_g = PM - 180° = 30° - 180° = -150°$$

From the plot of φ vs. ω: $\varphi_g = -150°$ at $\omega_g = 1.72$ rad/min

From the plot of $\frac{\text{AR}_{OL}}{K_c}$ vs ω: $\left.\frac{\text{AR}_{OL}}{K_c}\right|_{\omega=\omega_g} = 0.144$

Because $\text{AR}_{OL}\big|_{\omega=\omega_g} = 1$, $K_c = \frac{1}{0.144} = 6.94$

(c) From the phase angle plot:

$\varphi = -180°$ at $\omega_c = 4.05$ rad/min

From the plot of $\frac{\text{AR}_{OL}}{K_c}$ vs ω, $\left.\frac{\text{AR}_{OL}}{K_c}\right|_{\omega=\omega_c} = 0.0326$

$A_c = \text{AR}_{OL}\big|_{\omega=\omega_c} = 0.326$

From Eq. 14-11, $GM = 1/A_c = 3.07$.

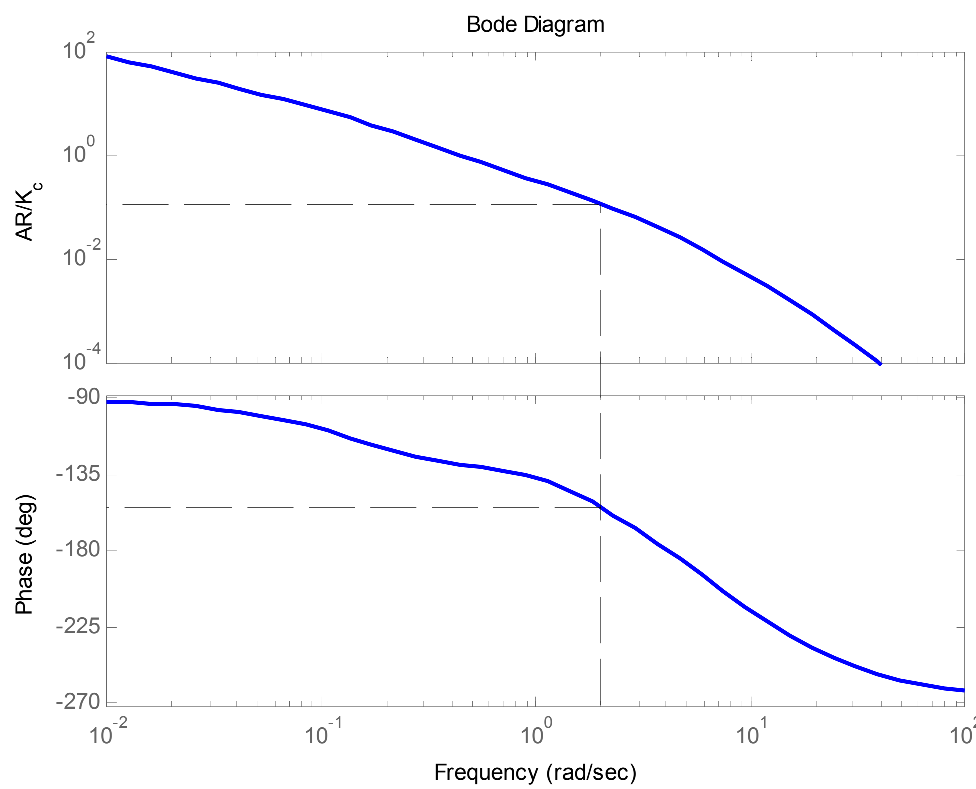


Figure S14.6b. *Solution for part (b) using Bode plot.*

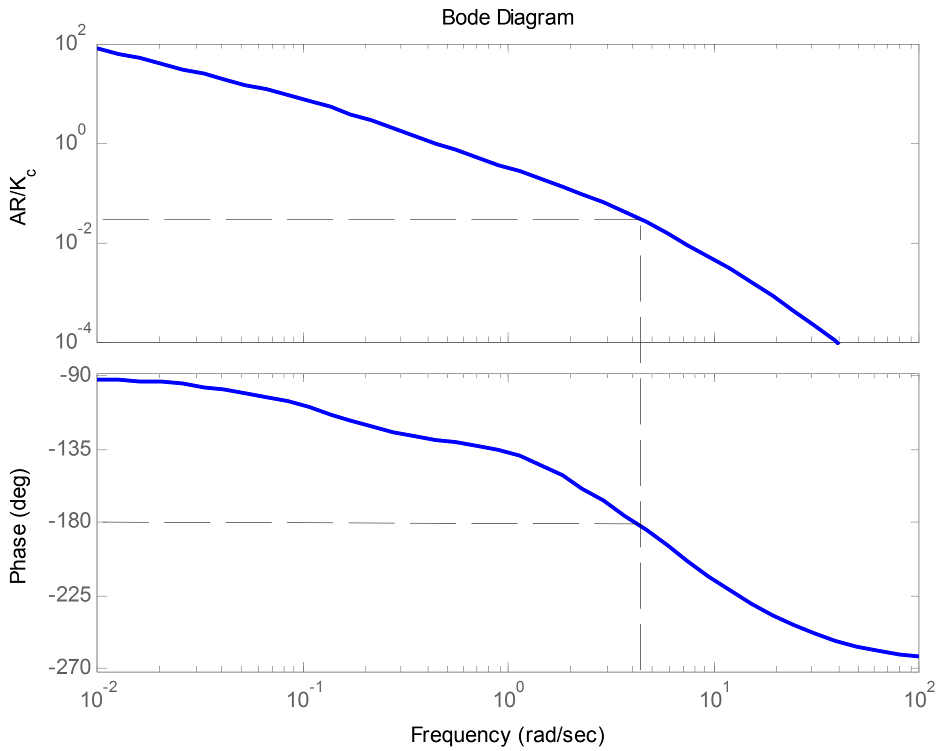


Figure S14.6c. *Solution for part (c) using Bode plot.*

14.7

(a) For a PI controller, the $|G_c|$ and $\angle G_c$ from Eqs. 13.62 and 13.63 need to be included in the AR and φ given for $G_vG_pG_m$ to obtain AR_{OL} and φ_{OL}. The results are tabulated below

ω	AR	$\|Gc\|/K_c$	AR_{OL}/Kc	φ	$\angle G_c$	φ_{OL}
0.01	2.40	250	600	-3	-89.8	-92.8
0.10	1.25	25.020	31.270	-12	-87.7	-99.7
0.20	0.90	12.540	11.290	-22	-85.4	-107.4
0.50	0.50	5.100	2.550	-41	-78.7	-119.7
1.00	0.29	2.690	0.781	-60	-68.2	-128.2
2.00	0.15	1.601	0.240	-82	-51.3	-133.3
5.00	0.05	1.118	0.055	-122	-26.6	-148.6
10.00	0.02	1.031	0.018	-173	-14.0	-187.0
15.00	0.01	1.014	0.008	-230	-9.5	-239.5

From Eq. 14-12, $\varphi_g = PM - 180° = 45° - 180° = -135°$.

Interpolating the above table, $\varphi_{OL} = -135°$ at $\omega_g = 2.5$ rad/min and

$$\left.\frac{\text{AR}_{OL}}{K_c}\right|_{\omega=\omega_g} = 0.165$$

Because $\text{AR}_{OL}\big|_{\omega=\omega_g} = 1$, $K_c = \dfrac{1}{0.165} = 6.06$

(b) From the table above,

$$\varphi_{OL} = -180° \text{ at } \omega_c = 9.0 \text{ rad/min and } \left.\frac{\text{AR}_{OL}}{K_c}\right|_{\omega=\omega_c} = 0.021$$

$$A_c = \text{AR}_{OL}\big|_{\omega=\omega_c} = 0.021 \qquad K_c = 0.127$$

From Eq. 14-11,

$$GM = 1/A_c = 1/0.127 = 7.86$$

(c) From the table in part (a),

$$\varphi_{OL} = -180° \text{ at } \omega_c = 10.5 \text{ rad/min and } \text{AR}\big|_{\omega=\omega_c} = 0.016.$$

Therefore, $P_u = \dfrac{2\pi}{\omega_c} = 0.598$ min and $K_{cu} = \dfrac{1}{AR\big|_{\omega=\omega_c}} = 62.5.$

Using Table 12.6, the Ziegler-Nichols PI settings are

$$K_c = 0.45\, K_{cu} = 28.1, \quad \tau_I = P_u/1.2 = 0.50 \text{ min}$$

Tabulating AR_{OL} and φ_{OL} as in part (a) and the corresponding values of M using Eq. 14-18 gives:

ω	$\|G_c\|$	$\angle G_c$	AR_{OL}	φ_{OL}	M
0.01	5620	-89.7	13488	-92.7	1.00
0.10	563.0	-87.1	703	-99.1	1.00
0.20	282.0	-84.3	254	-106.3	1.00
0.50	116.0	-76.0	57.9	-117.0	1.01
1.00	62.8	-63.4	18.2	-123.4	1.03
2.00	39.7	-45.0	5.96	-127.0	1.10
5.00	30.3	-21.8	1.51	-143.8	1.64
10.00	28.7	-11.3	0.487	-184.3	0.94
15.00	28.3	-7.6	0.227	-237.6	0.25

Therefore, the estimated value is M_p =1.64.

14.8

K_{cu} and ω_c are obtained using Eqs. 14-7 and 14-8. Including the filter G_F into these equations gives

$$-180° = 0 + [-0.2\omega_c - \tan^{-1}(\omega_c)]+[-\tan^{-1}(\tau_F\omega_c)]$$

Solving,

$\omega_c = 8.443$ for $\tau_F = 0$
$\omega_c = 5.985$ for $\tau_F = 0.1$

Then from Eq. 14-8,

$$1 = (K_{cu})\left(\frac{2}{\sqrt{\omega_c^{\,2}+1}}\right)\left(\frac{1}{\sqrt{\tau_F^{\,2}\omega_c^{\,2}+1}}\right)$$

Solving for K_{cu} gives,

$K_{cu} = 4.251$ for $\tau_F = 0$
$K_{cu} = 3.536$ for $\tau_F = 0.1$

Therefore,

$\omega_c K_{cu} = 35.9$ for $\tau_F = 0$
$\omega_c K_{cu} = 21.2$ for $\tau_F = 0.1$

Because $\omega_c K_{cu}$ is lower for $\tau_F = 0.1$, filtering the measurement results in worse control performance.

14.9

(a) Using Eqs. 14-7 and 14-8,

$$AR_{OL} = \left(K_c\sqrt{\frac{1}{25\omega^2}+1}\right)\left(\frac{5}{\sqrt{100\omega^2+1}}\right)\left(\frac{1}{\sqrt{\omega^2+1}}\right)(1.0)$$

$$\varphi = \tan^{-1}(-1/5\omega) + 0 \;\; + (-2\omega - \tan^{-1}(10\omega)) + (-\tan^{-1}(\omega))$$

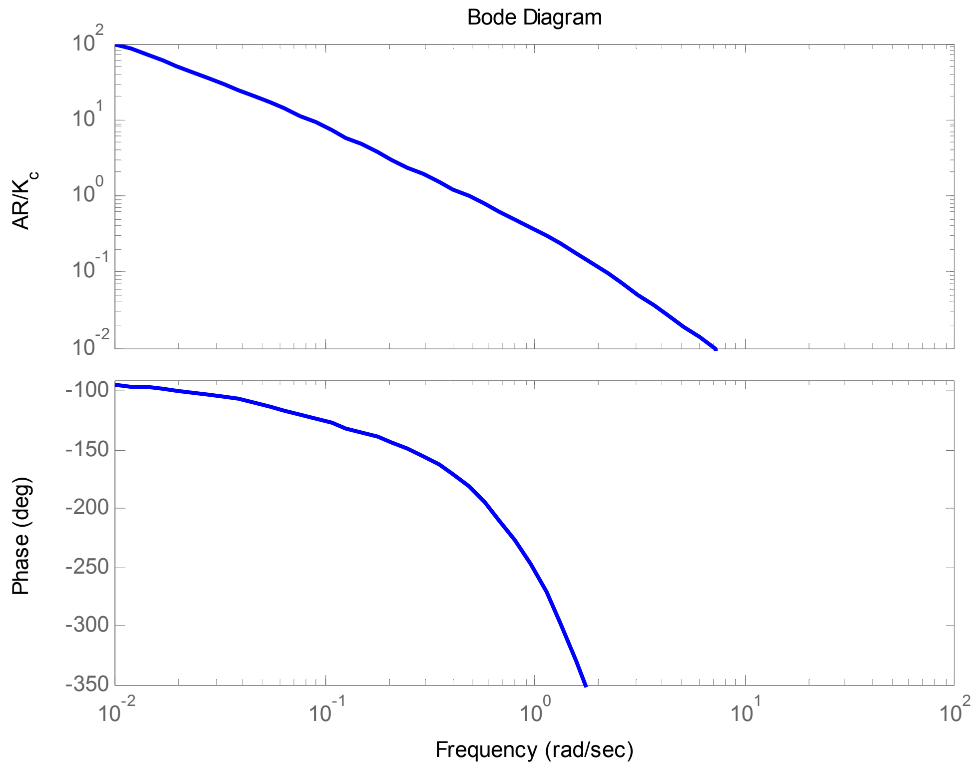


Figure S14.9a. *Bode plot*

(b) Set $\varphi = 180°$ and solve for ω to obtain $\omega_c = 0.4695$.

Then $AR_{OL}\big|_{\omega=\omega_c} = 1 = K_{cu}(1.025)$

Therefore, $K_{cu} = 1/1.025 = 0.976$ and the closed-loop system is stable for $K_c \leq 0.976$.

(c) For $K_c = 0.2$, set $AR_{OL} = 1$ and solve for ω to obtain $\omega_g = 0.1404$.

Then $\varphi_g = \varphi\big|_{\omega=\omega_g} = -133.6°$

From Eq. 14-12, $PM = 180° + \varphi_g = 46.4°$

(d) From Eq. 14-11

$$GM = 1.7 = \frac{1}{A_c} = \frac{1}{\mathrm{AR}_{OL}\big|_{\omega=\omega_c}}$$

From part (b), $\mathrm{AR}_{OL}\big|_{\omega=\omega_c} = 1.025\ K_c$

Therefore, $1.025\ K_c = 1/1.7$ or $K_c = 0.574$

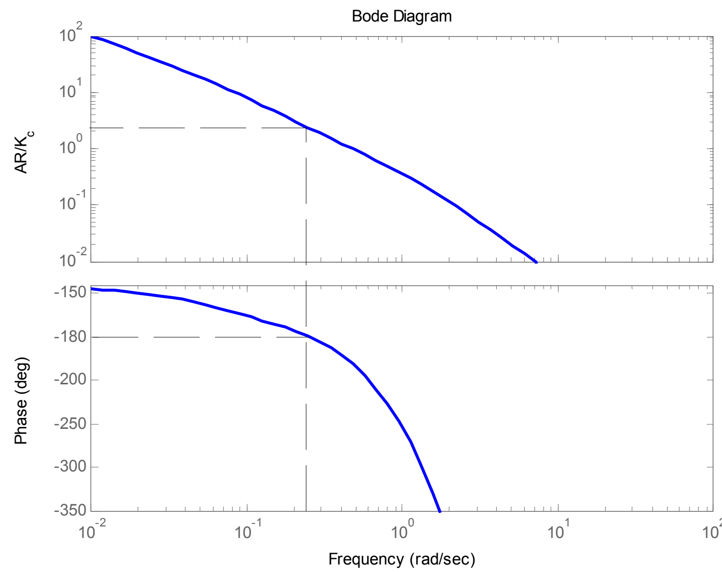


Figure S14.9b. *Solution for part b) using Bode plot.*

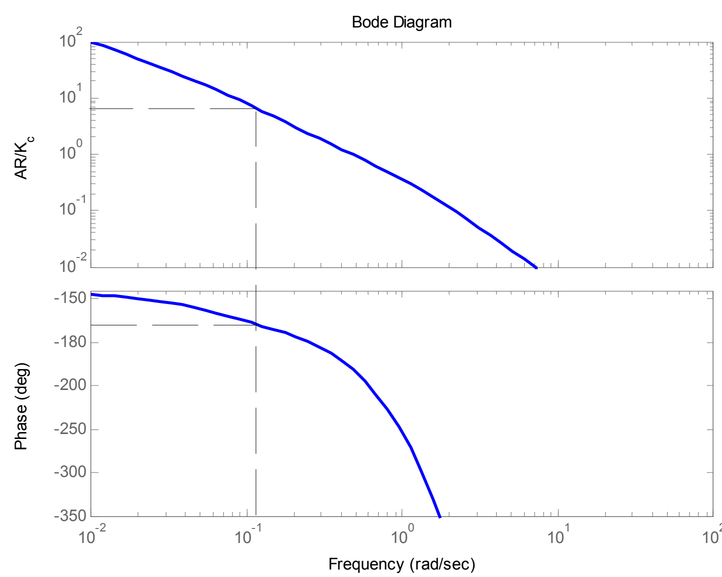


Figure S14.9c. *Solution for part c) using Bode plot.*

14.10

(a) $$G_v(s) = \frac{0.047}{0.083s+1} \times 112 = \frac{5.264}{0.083s+1}$$

$$G_p(s) = \frac{2}{(0.432s+1)(0.017s+1)}$$

$$G_m(s) = \frac{0.12}{0.024s+1}$$

Using Eq. 14-8

$$-180° = 0 - \tan^{-1}(0.083\omega_c) - \tan^{-1}(0.432\omega_c) - \tan^{-1}(0.017\omega_c) - \tan^{-1}(0.024\omega_c)$$

Solving by trial and error, ω_c = 18.19 rad/min.

Using Eq. 14-7,

$$1 = (K_{cu})\left(\frac{5.624}{\sqrt{(0.083\omega_c)^2+1}}\right)\cdot\left(\frac{2}{\sqrt{(0.432\omega_c)^2+1}\sqrt{(0.017\omega_c)^2+1}}\right) \times \left(\frac{0.12}{\sqrt{(0.024\omega_c)^2+1}}\right)$$

Substituting ω_c=18.19 rad/min, K_{cu} = 12.97.

$P_u = 2\pi/\omega_c = 0.345$ min

Using Table 12.6, the Ziegler-Nichols PI settings are

$K_c = 0.45\ K_{cu} = 5.84$, $\tau_I = P_u/1.2 = 0.288$ min

(b) Using Eqs.13-62 and 13-63

$$\varphi_c = \angle G_c = \tan^{-1}(-1/0.288\omega) = -(\pi/2) + \tan^{-1}(0.288\omega)$$

$$|G_c| = 5.84\sqrt{\left(\frac{1}{0.288\omega}\right)^2 + 1}$$

Then, from Eq. 14-8,

$-\pi = -(\pi/2) + \tan^{-1}(0.288\omega_c) - \tan^{-1}(0.083\omega_c) - \tan^{-1}(0.432\omega_c)$

$- \tan^{-1}(0.017\omega_c) - \tan^{-1}(0.024\omega_c)$

Solving by trial and error, $\omega_c = 15.11$ rad/min.

Using Eq. 14-7,

$$A_c = \text{AR}_{OL}\Big|_{\omega=\omega c} = \left[5.84\sqrt{\left(\frac{1}{0.288\omega_c}\right)^2+1}\right]\cdot\left[\frac{5.264}{\sqrt{(0.083\omega_c)^2+1}}\right]$$

$$\times\left[\frac{2}{\sqrt{(0.432\omega_c)^2+1}\,\sqrt{(0.017\omega_c)^2+1}}\right]\cdot\left[\frac{0.12}{\sqrt{(0.024\omega_c)^2+1}}\right]$$

$= 0.651$

Using Eq. 14-11, $GM = 1/A_c = 1.54$.

Solving Eq. 14-7 for ω_g gives

$$\text{AR}_{OL}\Big|_{\omega=\omega_g} = 1 \qquad \text{at} \quad \omega_g = 11.78 \text{ rad/min}$$

Substituting into Eq. 14-8 gives

$$\varphi_g = \varphi\Big|_{\omega=\omega_g} = -(\pi/2) + \tan^{-1}(0.288\omega_g) - \tan^{-1}(0.083\omega_g) -$$
$$\tan^{-1}(0.432\omega_g) - \tan^{-1}(0.017\omega_g) - \tan^{-1}(0.024\omega_g) = -166.8°$$

Using Eq. 14-12,

$$PM = 180° + \varphi_g = 13.2°$$

14.11

(a)

$$|G| = \left(\frac{10}{\sqrt{\omega^2+1}}\right)\left(\frac{1.5}{\sqrt{100\omega^2+1}}\right)(1)$$

$$\varphi = -\tan^{-1}(\omega) - \tan^{-1}(10\omega) - 0.5\omega$$

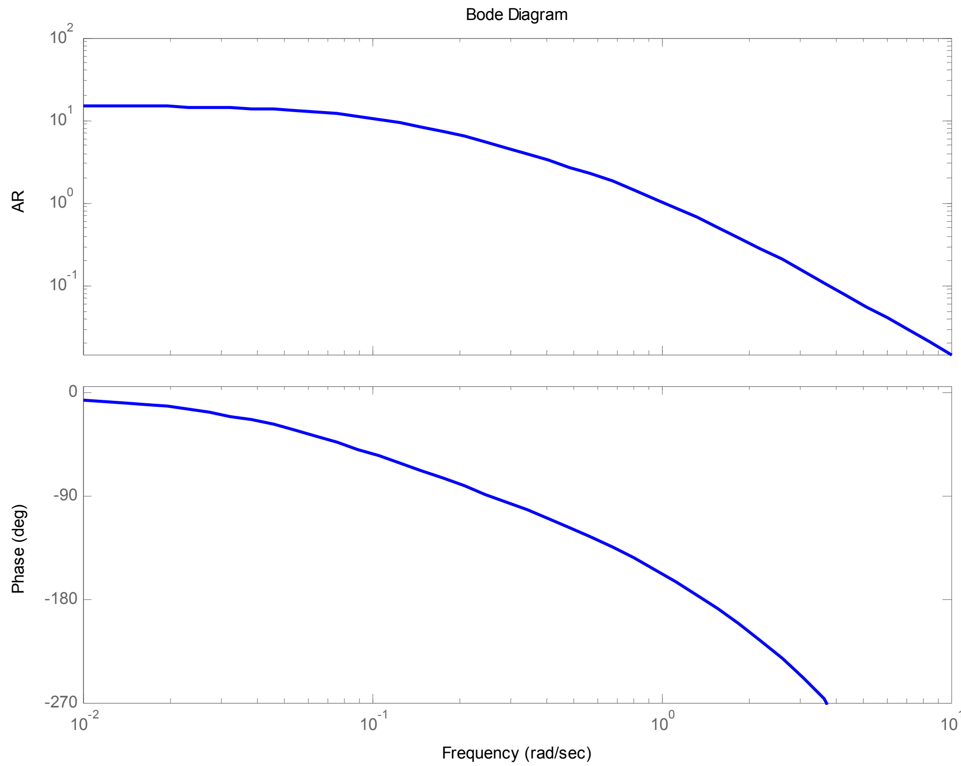


Figure S14.11a. *Bode plot for the transfer function $G=G_vG_pG_m$.*

(b) From the plots in part (a)

$$\angle G = -180° \text{ at } \omega_c = 1.4 \quad \text{and} \quad |G|_{\omega=\omega c} = 0.62$$

$$\mathrm{AR}_{OL}\big|_{\omega=\omega_c} = 1 = (-K_{cu})\,|G|_{\omega=\omega c}$$

Therefore, $K_{cu} = -1/0.62 = -1.61$ and

$$P_u = 2\pi/\omega_c = 4.49$$

Using Table 12.6, the Ziegler-Nichols PI-controller settings are:

$$K_c = 0.45K_{cu} = -0.72 \quad , \quad \tau_I = P_u/1.2 = 3.74$$

Including the $|G_c|$ and $\angle G_c$ from Eqs. 13-62 and 13-63 into the results of part (a) gives

$$\mathrm{AR}_{OL} = 0.72\sqrt{\left(\frac{1}{3.74\omega}\right)^2 + 1}\left(\frac{15}{\sqrt{\omega^2+1}\sqrt{100\omega^2+1}}\right)$$

$$= \frac{2.89\sqrt{14.0\omega^2 + 1}}{\sqrt{\omega^2 + 1}\sqrt{100\omega^2 + 1}\ \ \omega}$$

$$\varphi = \tan^{-1}(-1/3.74\omega) - \tan^{-1}(\omega) - \tan^{-1}(10\omega) - 0.5\omega$$

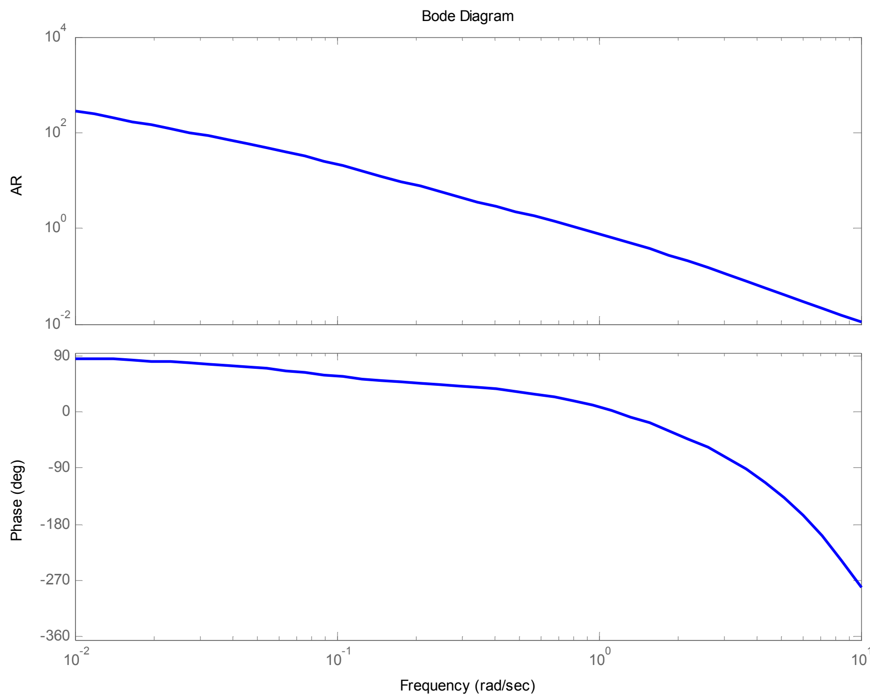


Figure S14.11b. *Bode plot for the open-loop transfer function* $G_{OL}=G_cG$.

(c) From the graphs in part (b),

$$\varphi = -180° \text{ at } \omega_c = 1.15$$

$$AR_{OL}\Big|_{\omega=\omega_c} = 0.63 < 1$$

Hence, the closed-loop system is stable.

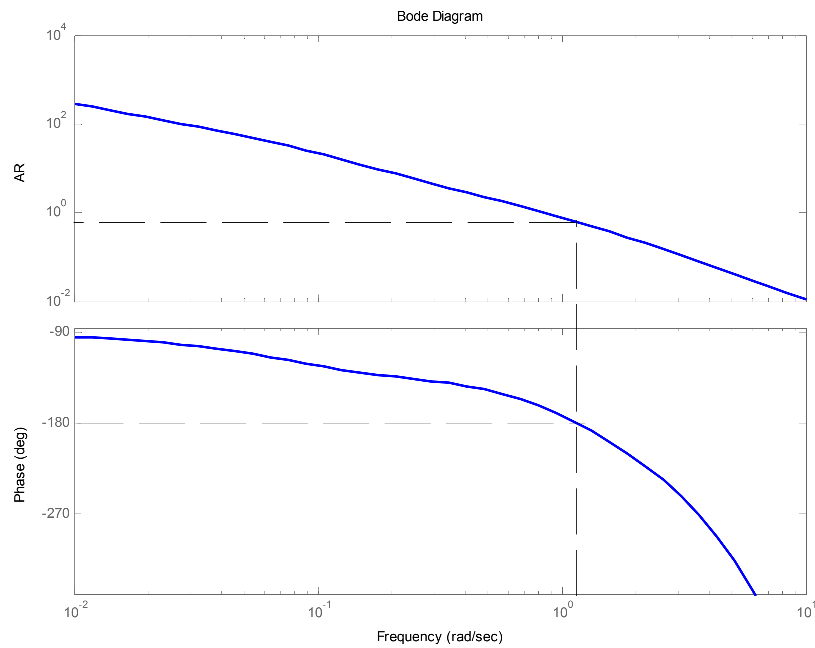


Figure S14.11c. *Solution for part (c) using Bode plot.*

(d) From the graph in part b),

$$AR_{OL}\big|_{\omega=0.5} = 2.14 = \frac{\text{amplitude of } y_m(t)}{\text{amplitude of } y_{sp}(t)}$$

Therefore, the amplitude of y_m(t) = $2.14 \times 1.5 = 3.21$.

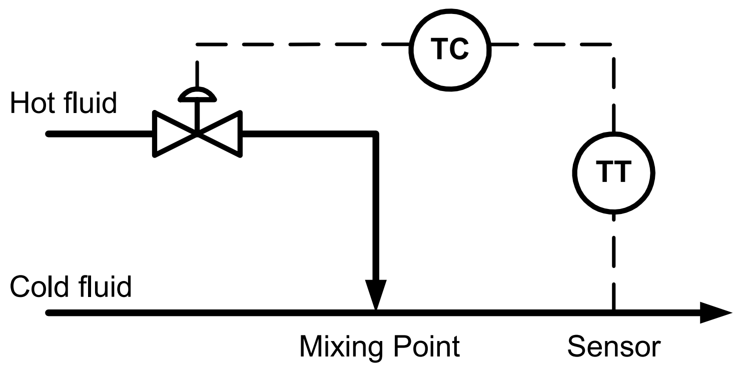
(e) From the graphs in part (b), $AR_{OL}\big|_{\omega=0.5} = 2.14$ and $\varphi\big|_{\omega=0.5} = -147.7°$.

Substituting into Eq. 14-18 gives $M = 1.528$. Therefore, the amplitude of $y(t)$ = 1.528×1.5 = 2.29 which is the same as the amplitude of y_m(t) because G_m is a time delay.

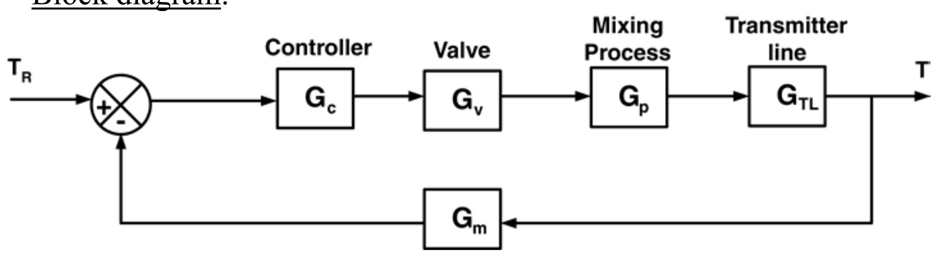
(f) The closed-loop system produces a slightly smaller amplitude for $\omega = 0.5$. As ω approaches zero, the amplitude approaches one due to the integral control action.

14.12

(a) Schematic diagram:



Block diagram:



(b) $G_vG_pG_m = K_m = 6$ mA/mA

$G_{TL} = e^{-8s}$

$G_{OL} = G_vG_pG_mG_{TL} = 6e^{-8s}$

If $G_{OL} = 6e^{-8s}$,

$| G_{OL}(j\omega) | = 6$

$\angle G_{OL}(j\omega) = -8\omega$ rad

Find ω_c: Crossover frequency generates – 180° phase angle = – π radians

$-8\omega_c = -\pi$ or $\omega_c = \pi/8$ rad/s

Find P_u: $P_u = \dfrac{2\pi}{\omega_c} = \dfrac{2\pi}{\pi/8} = 16\,\text{s}$

Find K_{cu}: $K_{cu} = \dfrac{1}{|G_p(j\omega_c)|} = \dfrac{1}{6} = 0.167$

Ziegler-Nichols ¼ decay ratio settings:

PI controller:

$K_c = 0.45\ K_{cu} = (0.45)(0.167) = 0.075$
$\tau_I = P_u/1.2 = 16/1.2 = 13.33$ sec

PID controller:

$K_c = 0.6\ K_{cu} = (0.6)(0.167) = 0.100$
$\tau_I = P_u/2 = 16/2 = 8$ s
$\tau_D = P_u/8 = 16/8 = 2$ s

(c)

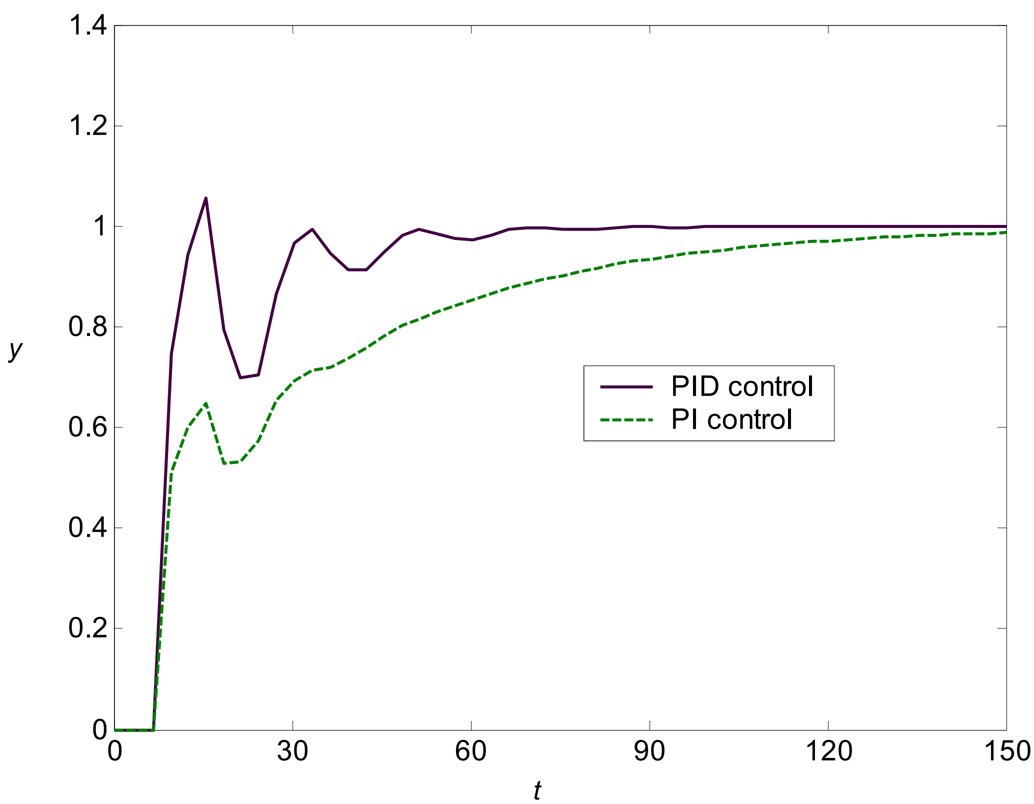


Fig. S14.12. *Set-point responses for PI and PID control.*

(d) Derivative control action reduces the settling time but results in a more oscillatory response.

14.13

(a) From Exercise 14.10,

$$G_v(s) = \frac{5.264}{0.083s+1}$$

$$G_p(s) = \frac{2}{(0.432s+1)(0.017s+1)}$$

$$G_m(s) = \frac{0.12}{(0.024s+1)}$$

The PI controller is $G_c(s) = 5\left(1+\frac{1}{0.3s}\right)$

Hence the open-loop transfer function is

$$G_{OL} = G_c G_v G_p G_m$$

Rearranging,

$$G_{OL} = \frac{6.317s+21.06}{1.46\times10^{-5}s^5 + 0.00168s^4 + 0.05738s^3 + 0.556s^2 + s}$$

By using MATLAB, the Nyquist diagram for this open-loop system is

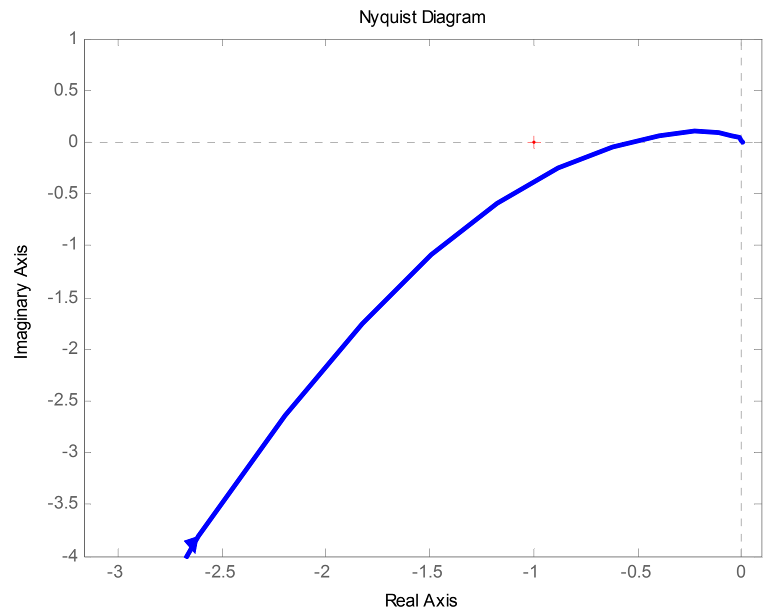


Figure S14.13a. *The Nyquist diagram for the open-loop system.*

(b) Gain margin = $GM = \dfrac{1}{AR_c}$

where AR_c is the value of the open-loop amplitude ratio at the critical frequency ω_c. By using the Nyquist plot,

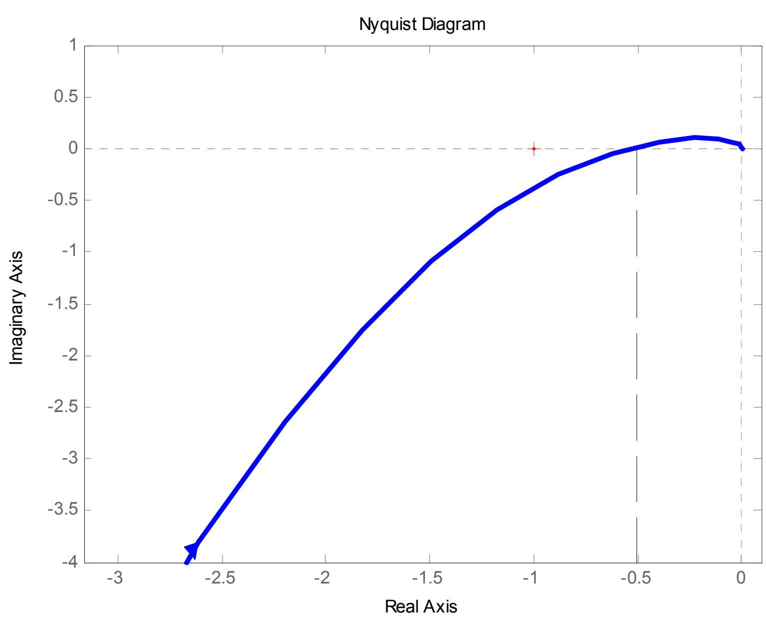


Figure S14.13b. *Graphical solution for part (b).*

$\theta = -180 \quad \Rightarrow \quad AR_c = |G(j\omega_c)| = 0.5$

Therefore the gain margin is $GM = 1/0.5 = 2$.

14.14

To determine $\max_{\omega} |e_m| < \frac{1}{M_p}$, we must calculate M_p based on the CLTF with IMC controller design. In order to determine a reference M_p, we assume a perfect process model (i.e. $G - \widetilde{G} = 0$) for the IMC controller design.

$$\therefore \quad \frac{C}{R} = G_c^* G$$

Factoring,

$$\widetilde{G} = \widetilde{G}_+ \widetilde{G}_-$$

$$\widetilde{G}_+ = e^{-s} \quad , \quad \widetilde{G}_- = \frac{10}{2s+1}$$

$$\therefore \quad G_c^* = \frac{2s+1}{10} f$$

Filter Design: Because $\tau = 2$ s, let $\tau_c = \tau/3 = 2/3$ s.

$$\Rightarrow \quad f = \frac{1}{2/3\,s + 1}$$

$$\therefore \quad G_c^* = \frac{2s+1}{10} \frac{1}{2/3\,s+1} = \frac{2s+1}{20/3\,s+10}$$

$$\therefore \quad \frac{C}{R} = G_c^* G = \left(\frac{2s+1}{20/3\,s+10}\right)\left(\frac{10e^{-s}}{2s+1}\right) = \frac{10e^{-s}}{20/3s+10}$$

$$\therefore \quad M_p = 1$$

The relative model error with K as the actual process gain is:

$$\therefore \qquad e_m = \frac{G-\widetilde{G}}{\widetilde{G}} = \frac{\left[\dfrac{Ke^{-s}}{2s+1}\right]-\left[\dfrac{10e^{-s}}{2s+1}\right]}{\dfrac{10e^{-s}}{2s+1}} = \frac{K-10}{10}$$

Since $M_p = 1$, $\max_{\omega} |e_m| = \left|\dfrac{K-10}{10}\right| < 1$

$$\Rightarrow \qquad \frac{K-10}{10} < 1 \qquad \Rightarrow \qquad K < 20$$

$$\frac{K-10}{10} > -1 \qquad \Rightarrow \qquad K > 0$$

$\therefore$ $\boxed{0 < K < 20}$ for guaranteed closed-loop stability.

14.15

Denote the process model as,

$$\widetilde{G} = \frac{2e^{-0.2s}}{s+1}$$

and the actual process as:

$$G = \frac{2e^{-0.2s}}{\tau s+1}$$

The relative model error is:

$$\therefore \qquad \Delta(s) = \frac{G(s)-\widetilde{G}(s)}{\widetilde{G}(s)} = \frac{(1-\tau)s}{\tau s+1}$$

Let $s = j\omega$. Then,

$$\therefore \qquad |\Delta| = \left|\frac{(1-\tau)j\omega}{\tau j\omega+1}\right| = \frac{|(1-\tau)\omega|}{|\tau j\omega+1|} \qquad (1)$$

or

$$|\Delta| = \frac{|(1-\tau)|\,\omega}{\sqrt{\tau^2\omega^2+1}}$$

Because | Δ | in (1) increases monotonically with ω,

$$\max_{\omega} |\Delta| = \lim_{\omega\to\infty} |\Delta| = \frac{|1-\tau|}{\tau} \qquad (2)$$

Substituting (2) and M_p = 1.25 into Eq. 14-34 gives:

$$\frac{|1-\tau|}{\tau} < 0.8$$

This inequality implies that

$$\frac{1-\tau}{\tau} < 0.8 \qquad \Rightarrow \qquad 1 < 1.8\tau \qquad \Rightarrow \qquad \tau > 0.556$$

and

$$\frac{\tau-1}{\tau} < 0.8 \qquad \Rightarrow \qquad 0.2\tau < 1 \qquad \Rightarrow \qquad \tau < 5$$

Thus, closed-loop stability is guaranteed if

$$0.556 < \tau < 5$$