Chapter 15

15.1

For $R_a = d/\mathrm{u}$

$$K_p = \frac{\partial R_a}{\partial u} = -\frac{d}{u^2}$$

which can vary more than K_p in Eq. 15-2, because the new K_p depends on both d and u.

15.2

By definition, the ratio station sets

$$(u_m - u_{m0}) = K_R\,(d_m - d_{m0})$$

Thus $$K_R = \frac{u_m - u_{m0}}{d_m - d_{m0}} = \frac{K_2 u^2}{K_1 d^2} = \frac{K_2}{K_1}\left(\frac{u}{d}\right)^2 \qquad (1)$$

For constant gain K_R, the values of u and d in Eq. 1 are taken to be at the desired steady state so that $u/d = R_d$, the desired ratio. Moreover, the transmitter gains are

$$K_1 = \frac{(20-4)\text{mA}}{S_d^{\,2}} \quad , \quad K_2 = \frac{(20-4)\text{mA}}{S_u^{\,2}}$$

Substituting for K_1, K_2 and u/d into (1) gives:

$$K_R = \frac{S_u^{\,2}}{S_d^{\,2}} R_d^{\,2} = \left(R_d \frac{S_d}{S_u}\right)^2$$

15.3

(a) The block diagram is the same as in Fig. 15.11 where $Y \equiv H_2$, $Y_m \equiv H_{2m}$, $Y_{sp} \equiv H_{2sp}$, $D \equiv Q_1$, $D_m \equiv Q_{1m}$, and $U \equiv Q_3$.

b) (A steady-state mass balance on both tanks gives

$$0 = q_1 - q_3 \quad \text{or} \quad Q_1 = Q_3 \text{ (in deviation variables)} \qquad (1)$$

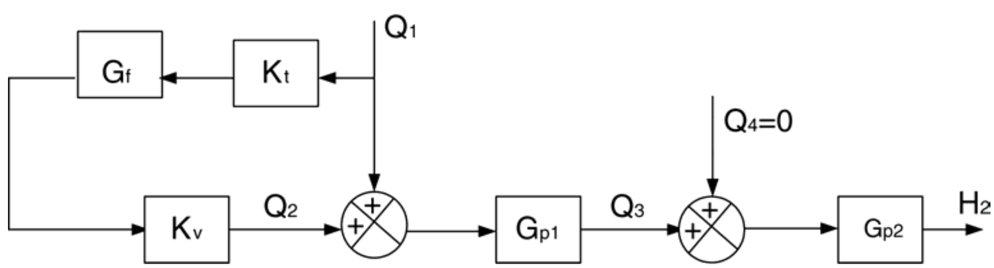
From the block diagram, at steady state:

$$Q_3 = K_v K_f K_t Q_1$$

From (1) and (2), $K_f = \dfrac{1}{K_v K_t}$ (2)

c) (No, because Eq. 1 above does not involve q_2.

15.4



(b) From the block diagram, exact feedforward compensation for Q_1 would result when

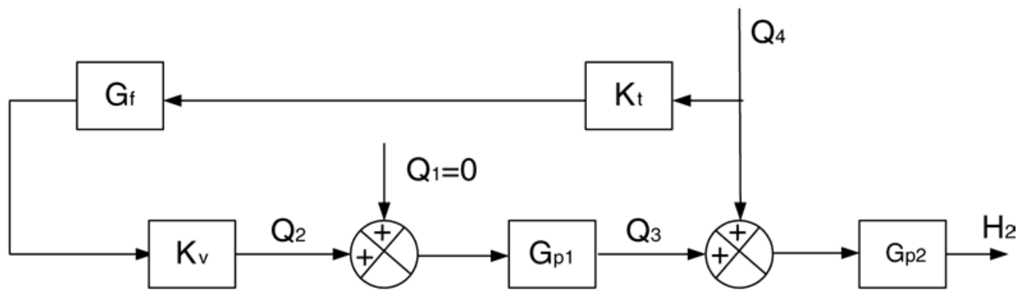
$$Q_1 + Q_2 = 0$$

Substituting $Q_2 = K_V G_f K_t Q_1$,

$$G_f = -\frac{1}{K_v K_t}$$

(c) Same as part (b), because the feedforward loop does not have any dynamic elements.

(d)



For exact feedforward compensation

$$Q_4 + Q_3 = 0 \tag{1}$$

From the block diagram, $Q_2 = K_V G_f K_t Q_4$ (2)

Using <u>steady-state analysis</u>, a mass balance on tank 1 for no variation in q_1 gives

$$Q_2 - Q_3 = 0 \tag{3}$$

Substituting for Q_3 from (3) and (2) into (1) gives

$$Q_4 + K_V G_f K_t Q_4 = 0$$

or $$G_f = -\frac{1}{K_v K_t}$$

For <u>dynamic analysis</u>, find G_{p1} from a mass balance on tank 1,

$$A_1 \frac{dh_1}{dt} = q_1 + q_2 - C_1\sqrt{h_1}$$

Linearizing (4), noting that $q_1' = 0$, and taking Laplace transforms:

$$A_1 \frac{dh'}{dt} = q_2' - \frac{C_1}{2\sqrt{\bar{h}_1}} h_1'$$

or
$$\frac{H_1'(s)}{Q_2'(s)} = \frac{(2\sqrt{\bar{h}_1}/C_1)}{(2A_1\sqrt{\bar{h}_1}/C_1)s+1} \tag{5}$$

Since
$$q_3 = C_1\sqrt{h_1}$$
$$q_3' = \frac{C_1}{2\sqrt{\bar{h}_1}} h_1' \quad \text{or} \quad \frac{Q_3'(s)}{H_1'(s)} = \frac{C_1}{2\sqrt{\bar{h}_1}} \tag{6}$$

From (5) and (6),

$$\frac{Q_3'(s)}{Q_2'(s)} = \frac{1}{(2A_1\sqrt{\bar{h}_1}/C_1)s+1} = G_{P_1} \tag{7}$$

Substituting for Q_3 from (7) and (2) into (1) gives

$$Q_4 + \left(\frac{1}{(2A_1\sqrt{\bar{h}_1}/C_1)s+1}\right)K_v G_f K_t Q_4 = 0$$

or
$$G_f = -\frac{1}{K_v K_t}[(2A_1\sqrt{\bar{h}_1}/C_1)s+1]$$

15.5

(a) For a steady-state analysis:

$G_p=1$, $G_d=2$, $G_v = G_m = G_t =1$

From Eq.15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{-2}{(1)(1)(1)} = -2$$

(b) Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{\dfrac{-2}{(s+1)(5s+1)}}{(1)(1)\left(\dfrac{1}{s+1}\right)} = \frac{-2}{5s+1}$$

(c) Using Eq. 12-19,

$$\tilde{G} = G_v G_p G_m = \frac{1}{s+1} = \tilde{G}_+ \tilde{G}_-$$

where $\tilde{G}_+ = 1, \; \tilde{G}_- = \dfrac{1}{s+1}$

For τ_c=2, and r=1, Eq. 12-21 gives

$$f = \frac{1}{2s+1}$$

From Eq. 12-20

$$G_c^* = \tilde{G}_-^{-1} f = (s+1)\left(\frac{1}{2s+1}\right) = \frac{s+1}{2s+1}$$

From Eq. 12-16

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} = \frac{\dfrac{s+1}{2s+1}}{1 - \dfrac{1}{2s+1}} = \frac{s+1}{2s}$$

(d) For feedforward control only, G_c=0. For a unit step change in disturbance, $D(s) = 1/s$.

Substituting into Eq. 15-20 gives

$$Y(s) = (G_d + G_t G_f G_v G_p)\frac{1}{s}$$

For the controller of part (a)

$$Y(s) = \left[\frac{2}{(s+1)(5s+1)} + (1)(-2)(1)\left(\frac{1}{s+1}\right)\right]\frac{1}{s}$$

$$Y(s) = \frac{-10}{(s+1)(5s+1)} = \frac{5/2}{s+1} + \frac{-25/2}{5s+1} = \frac{2.5}{s+1} - \frac{-2.5}{s+1/5}$$

or $y(t) = 2.5\,(e^{-t} - e^{-t/5})$

For the controller of part (b)

$$Y(s) = \left[\frac{2}{(s+1)(5s+1)} + (1)\left(\frac{-2}{5s+1}\right)(1)\left(\frac{1}{s+1}\right)\right]\frac{1}{s} = 0$$

or $y(t) = 0$

The plots are shown in Fig. S15.5a below.

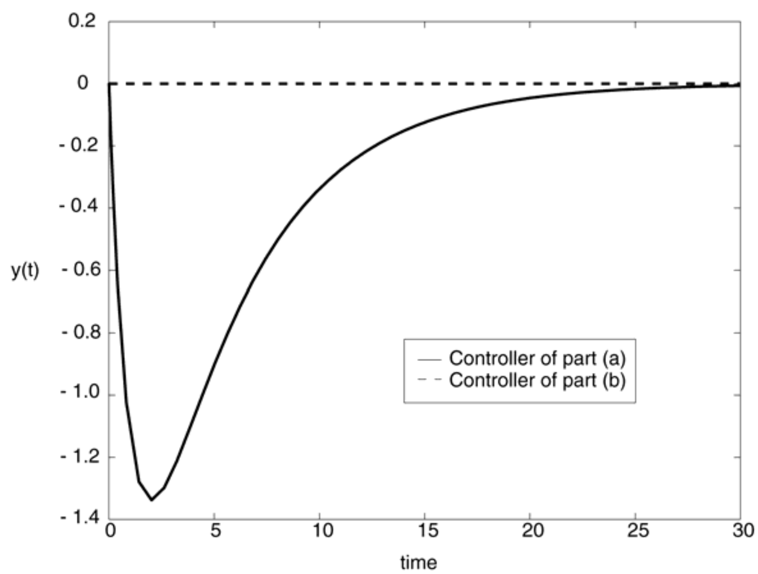


Figure S15.5a. *Closed-loop response using feedforward control only.*

(e) Using Eq. 15-20:

For the controller of parts (a) and (c),

$$Y(s) = \left[\frac{\dfrac{2}{(s+1)(5s+1)} + (1)(-2)(1)\left(\dfrac{1}{s+1}\right)}{1+\left(\dfrac{s+1}{2s}\right)(1)\left(\dfrac{1}{s+1}\right)(1)}\right]\frac{1}{s}$$

or $$Y(s) = \frac{-20s}{(s+1)(5s+1)(2s+1)} = \frac{5}{s+1} + \frac{25/3}{5s+1} + \frac{-40/3}{2s+1}$$

$$= \frac{5}{s+1} - \frac{20/3}{s+1/2} + \frac{5/3}{s+1/5}$$

or $$y(t) = 5e^{-t} - \frac{20}{3}\ e^{-t/2} + \frac{5}{3}e^{-\,t/5}$$

and for controllers of parts (b) and (c)

$$Y(\text{s}) = \left[\frac{\dfrac{2}{(s+1)(5s+1)} + (1)\left(\dfrac{-2}{5s+1}\right)(1)\left(\dfrac{1}{s+1}\right)}{1+\left(\dfrac{s+1}{2s}\right)(1)\left(\dfrac{1}{s+1}\right)(1)}\right]\frac{1}{s} = 0$$

or $y(t) = 0$

The plots of the closed-loop responses are shown in Fig. S15.5b.

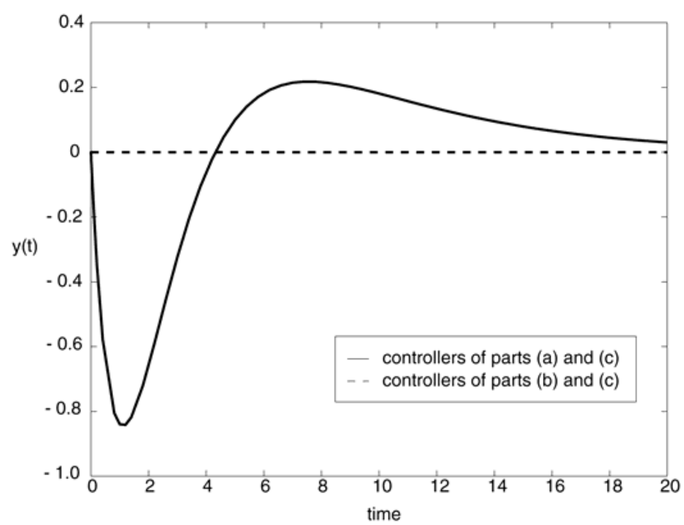


Figure S15.5b. *Closed-loop response for feedforward-feedback control.*

15.6

(a) The steady-state energy balance for both tanks takes the form

$$0 = w_1 C\, T_1 + w_2 C\, T_2 - w\, C\, T_4 + Q$$

where Q is the power input of the heater
C is the specific heat of the fluid.

Solving for Q and replacing unmeasured temperatures and flow rates by their nominal values,

$$Q = C\,(\overline{w}_1 T_1 + \overline{w}_2 \overline{T}_2 - w\overline{T}_4) \qquad (1)$$

Neglecting heater and transmitter dynamics,

$$Q = K_h p \qquad (2)$$

$$T_{1m} = T_{1m}^{\ 0} + K_T(T_1 - T_1^{\ 0}) \qquad (3)$$

$$w_m = w_m^{\ 0} + K_w(w - w^0) \qquad (4)$$

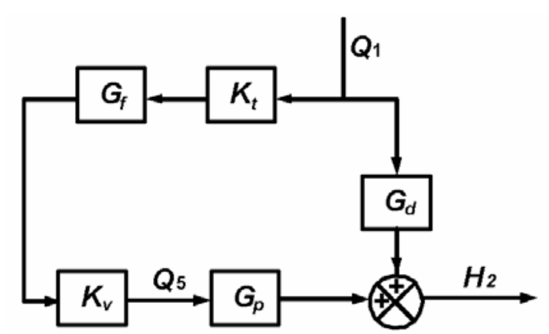
Substituting into (1) for Q, T_1, and w from (2), (3), and (4), gives

$$p = \frac{C}{K_h}[\overline{w_1}(T_1^{\ 0} + \frac{1}{K_T}(T_{1m} - T_{1m}^{\ 0})) + \overline{w_2}\,\overline{T_2} - \overline{T_4}(w^0 + \frac{1}{K_w}(w_m - w_m^{\ 0}))]$$

(b) Dynamic compensation is desirable because the process transfer function $G_p = T_4(s)/P(s)$ is different from each of the disturbance transfer functions, $G_{d1} = T_4(s)/T_1(s)$, and $G_{d2} = T_4(s)/w(s)$; this is more so for G_{d1} which has a higher order.

15.7

(a)



(b) A steady-state material balance for both tanks gives,

$$0 = q_1 + q_2 + q_4 - q_5$$

Because $q_2' = q_4' = 0$, the above equation gives

$$0 = q_1' - q_5' \quad \text{or} \quad 0 = Q_1 - Q_5 \tag{1}$$

From the block diagram,

$$Q_5 = K_v\, G_f\, K_t\, Q_1$$

Substituting for Q_5 into (1) gives

$$0 = Q_1 - K_v\, G_f\, K_t\, Q_1 \qquad \text{or} \qquad G_f = \frac{1}{K_v K_t}$$

(c) To find G_d and G_p, the mass balance on tank 1 is

$$A_1 \frac{dh_1}{dt} = q_1 + q_2 - C_1\sqrt{h_1}$$

where A_1 is the cross-sectional area of tank 1.

Linearizing and setting $q_2' = 0$ leads to

$$A_1 \frac{dh_1'}{dt} = q_1' - \frac{C_1}{2\sqrt{\bar{h}_1}} h_1'$$

Taking the Laplace transform,

$$\frac{H_1'(s)}{Q_1'(s)} = \frac{R_1}{A_1 R_1 s + 1} \qquad \text{where} \quad R_1 \equiv \frac{2\sqrt{\bar{h}_1}}{C_1} \tag{2}$$

Linearizing $q_3 = C_1\sqrt{h_1}$ gives

$$q_3' = \frac{1}{R_1} h_1' \quad \text{or} \quad \frac{Q_3'(s)}{H_1'(s)} = \frac{1}{R_1} \tag{3}$$

Mass balance on tank 2 is

$$A_2 \frac{dh_2}{dt} = q_3 + q_4 - q_5$$

Using deviation variables, setting $q_4' = 0$, and taking Laplace transform

$$A_2 s H_2'(s) = Q_3'(s) - Q_5'(s)$$

$$\frac{H_2'(s)}{Q_3'(s)} = \frac{1}{A_2 s} \qquad (4)$$

and

$$\frac{H_2'(s)}{Q_5'(s)} = -\frac{1}{A_2 s} = G_p(s)$$

$$G_d(s) = \frac{H_2'(s)}{Q_1'(s)} = \frac{H_2'(s)}{Q_3'(s)} \frac{Q_3'(s)}{H_1'(s)} \frac{H_1'(s)}{Q_1'(s)} = \frac{1}{A_2 s(A_1 R_1 s + 1)}$$

upon substitution from (2), (3), and (4).

Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_t G_v G_p} = \frac{-\dfrac{1}{A_2 s(A_1 R_1 s + 1)}}{K_t K_v (-1/A_2 s)}$$
$$= +\frac{1}{K_v K_t} \frac{1}{A_1 R_1 s + 1}$$

15.8

For the process model in Eq. 15-22 and the feedforward controller in Eq. 15-29, the correct values of τ_1 and τ_2 are given by Eq. 15-42 and (15-43).

Therefore,

$$\tau_1 - \tau_2 = \tau_p - \tau_L \qquad (1)$$

for a unit step change in d, and no feedback controller, set $D(s)=1/s$, and $G_c = 0$ in Eq. 15-20 to obtain

$$Y(s) = \left[G_d + G_t G_f G_v G_p\right]\frac{1}{s}$$

Setting $G_t = G_v = 1$, and using Eqs. 15-22 and 15-29,

$$Y(s) = \left[\frac{K_d}{\tau_d s+1} + (1)\left(\frac{-K_d / K_P(\tau_1 s+1)}{\tau_2 s+1}\right)(1)\left(\frac{K_p}{\tau_p s+1}\right)\right]\frac{1}{s}$$

$$= K_d\left[\frac{1}{s} - \frac{\tau_d}{\tau_d s+1} - \frac{1}{s} - \frac{\tau_2(\tau_1-\tau_2)}{(\tau_2-\tau_p)}\frac{1}{\tau_2 s+1} - \frac{(\tau_1-\tau_p)\tau_p}{\tau_p-\tau_2}\frac{1}{\tau_p s+1}\right]$$

or $\quad y(t) = K_d\left[-e^{-t/\tau} - \frac{(\tau_1-\tau_2)}{\tau_2-\tau_p}e^{-t/\tau_2} - \frac{\tau_1-\tau_p}{\tau_p-\tau_2}e^{-t/\tau_p}\right]$

$$\int_0^\infty e(t)dt = \int_0^\infty y(t)dt = -K_d\left[\tau_d + \frac{\tau_2(\tau_1-\tau_2)}{\tau_2-\tau_p} + \frac{\tau_p(\tau_1-\tau_p)}{\tau_p-\tau_2}\right]$$

$$= \frac{-K_d}{\tau_2-\tau_p}\left[\tau_d\tau_2 - \tau_d\tau_p + \tau_2\tau_1 - \tau_2^2 - \tau_p\tau_1 + \tau_p^2 + (\tau_p\tau_2 - \tau_p\tau_2)\right]$$

$$= -K_d\left[(\tau_1-\tau_2) - (\tau_p-\tau_d)\right]$$

$= 0$ when (1) holds.

15.9

(a) For steady-state conditions

G_p=1, G_d=2, $G_v = G_m = G_t$=1

Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{-2}{(1)(1)(1)} = -2$$

(b) Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{\dfrac{-2e^{-s}}{(s+1)(5s+1)}}{(1)(1)\left(\dfrac{1}{s+1}\right)e^{-s}} = \frac{-2}{5s+1}$$

(c) Using Eq. 12-19,

$$\widetilde{G} = G_v G_p G_m = \frac{e^{-s}}{s+1} = \widetilde{G}_+ \widetilde{G}_-$$

where $\widetilde{G}_+ = e^{-s}$, $\widetilde{G}_- = \dfrac{1}{s+1}$

For τ_c=2, and r = 1, Eq. 12-21gives

$$f = \frac{1}{2s+1}$$

From Eq. 12-20

$$G_c^* = \frac{1}{\widetilde{G}_-} f = (s+1)\frac{1}{2s+1} = \frac{s+1}{2s+1}$$

From Eq. 12-16

$$G_c = \frac{G_c^*}{1 - G_c^* \widetilde{G}} = \frac{\dfrac{s+1}{2s+1}}{1 - \dfrac{1}{2s+1}} = \frac{s+1}{2s}$$

(d) For feedforward control only, G_c=0. For a unit step disturbance, $D(s) = 1/s$.

Substituting into Eq. 15-20 gives

$$Y(s) = (G_d + G_t G_f G_v G_p)\frac{1}{s}$$

For the <u>controller of part (a)</u>

$$Y(s) = \left[\frac{2e^{-s}}{(s+1)(5s+1)} + (1)(-2)(1)\left(\frac{e^{-s}}{s+1}\right)\right]\frac{1}{s}$$

$$= \frac{-10e^{-s}}{(s+1)(5s+1)}$$

or $y(t) = 2.5\,(e^{-(t-1)} - e^{-(t-1)/5})S(t-1)$

For the <u>controller of part (b)</u>

$$Y(\text{s}) = \left[\frac{2e^{-s}}{(s+1)(5s+1)} + (1)\left(\frac{-2}{5s+1}\right)(1)\left(\frac{e^{-s}}{s+1}\right)\right]\frac{1}{s} = 0$$

or $y(t) = 0$

The plots are shown in Fig. S15.9a below.

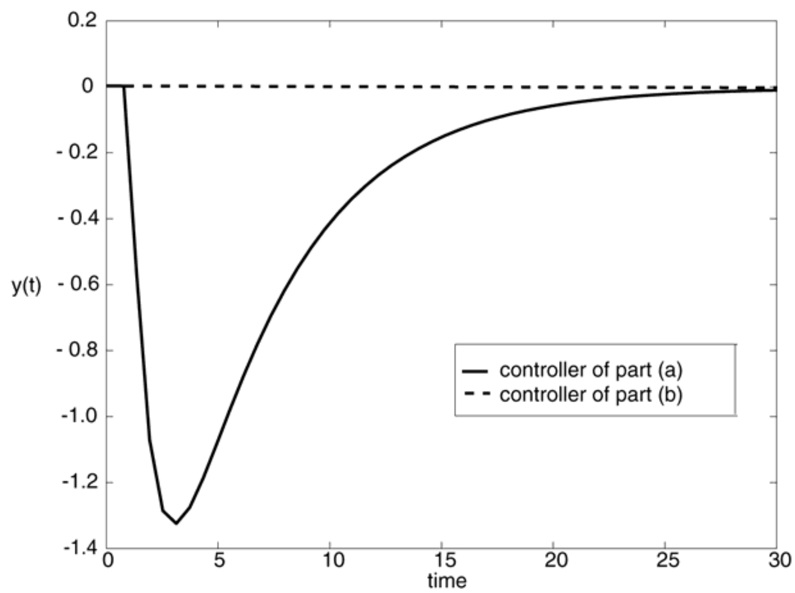


Figure S15.9a. *Closed-loop response using feedforward control only.*

(e) Using Eq. 15-20:

For the controllers of parts (a) and (c),

$$Y(s) = \left[\frac{\dfrac{2e^{-s}}{(s+1)(5s+1)} + (1)(-2)(1)\left(\dfrac{e^{-s}}{s+1}\right)}{1+\left(\dfrac{s+1}{2s}\right)(1)\left(\dfrac{e^{-s}}{s+1}\right)(1)}\right]\frac{1}{s}$$

and for the controllers of parts (b) and (c),

$$Y(s) = \left[\frac{\dfrac{2}{(s+1)(5s+1)} + (1)\left(\dfrac{-2}{5s+1}\right)(1)\left(\dfrac{1}{s+1}\right)}{1 + \left(\dfrac{s+1}{2s}\right)(1)\left(\dfrac{1}{s+1}\right)(1)} \right] \frac{1}{s} = 0$$

or $\quad y(t) = 0$

The plots of the closed-loop responses are shown in Fig. S15.9b.

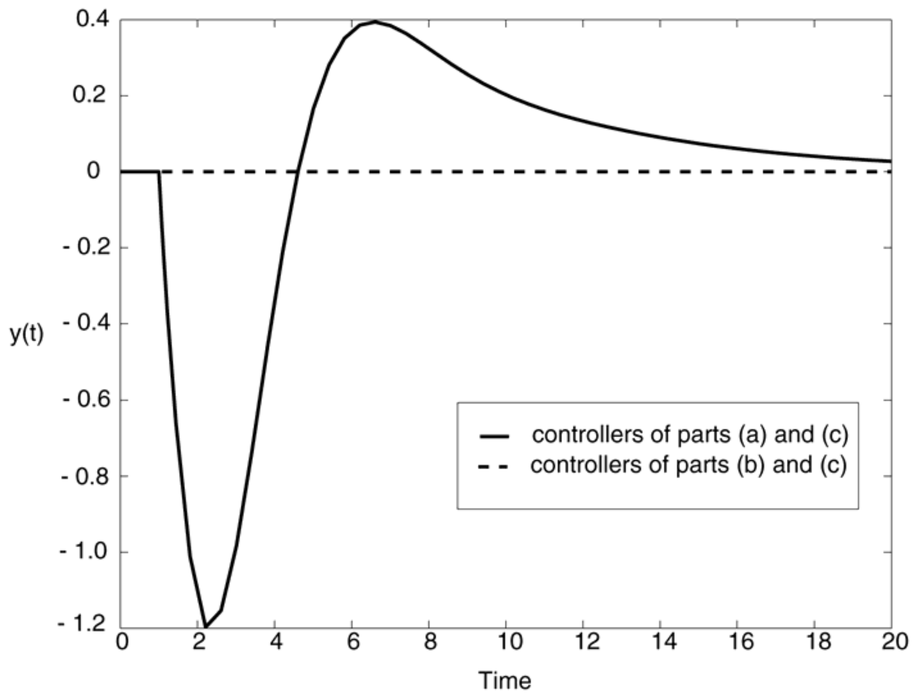


Figure S15.9b. *Closed-loop response for the feedforward-feedback control.*

15.10

(a) For steady-state conditions

$$G_p = K_p, \quad G_d = K_d, \quad G_v = G_m = G_t = 1$$

Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{-0.5}{(1)(1)(2)} = -0.25$$

(b) Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{\dfrac{-0.5e^{-30s}}{60s+1}}{(1)(1)\left(\dfrac{2e^{-20s}}{95s+1}\right)} = -0.25\frac{(95s+1)}{(60s+1)}e^{-10s}$$

(c) Using Table 12.1, a PI controller is obtained from equation G,

$$K_c = \frac{1}{K_p}\frac{\tau}{\tau_c+\theta} = \frac{1}{2}\frac{95}{(30+20)} = 0.95$$

$$\tau_I = \tau = 95$$

(d) As shown in Fig.S15.10a, the dynamic controller provides significant improvement.

(e)

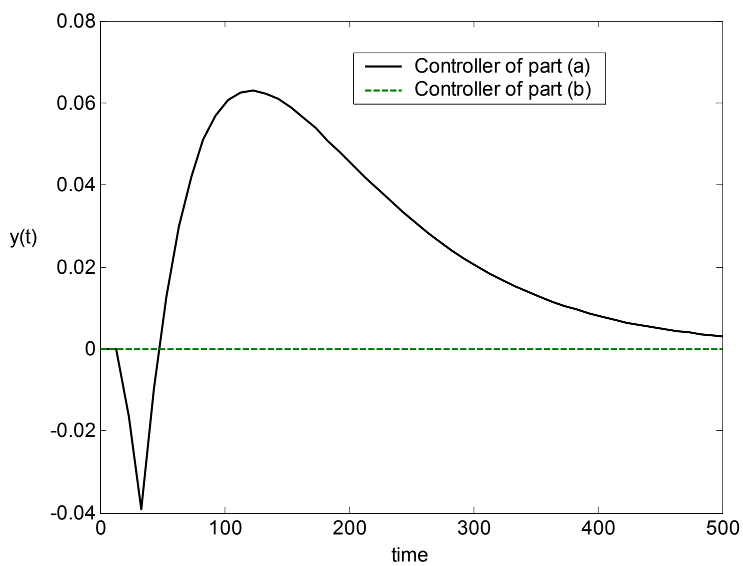


Figure S15.10a. *Closed-loop response using feedforward control only.*

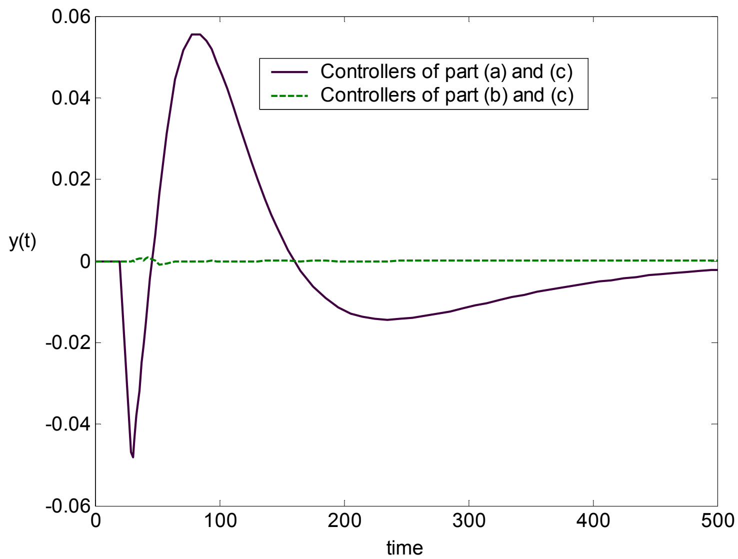


Figure S15.10b. *Closed-loop response for feedforward-feedback control.*

f) As shown in Fig. S15.10b, the feedforward configuration with the dynamic controller provides the best control.

15.11

Energy Balance:

$$\rho VC\frac{dT}{dt} = wC(T_i - T) - U(1+q_c)A(T - T_c) - U_L A_L (T - T_a) \quad (1)$$

Expanding the right hand side,

$$\rho VC\frac{dT}{dt} = wC(T_i - T) - UA(T - T_c) - UAq_cT + UAq_cT_c - U_L A_L (T - T_a) \quad (2)$$

Linearizing,

$$q_cT \approx \overline{q}_c\overline{T} + \overline{q}_cT' + \overline{T}q_c' \quad (3)$$

Substituting (3) into (2), subtracting the steady-state equation, and introducing deviation variables,

$$\rho VC\frac{dT'}{dt} = wC(T_i' - T') - UAT' - UA\overline{T}q_c' - UA\overline{q}_c T'$$
$$+ UAT_c q_c' - U_L A_L T' \qquad (4)$$

Taking the Laplace transform and assuming steady-state at $t = 0$ gives,

$$\rho VCsT'(s) = wCT_i'(s) + UA(T_c - T^{-})Q_c'(s)$$

$$-(wC + UA + UA\overline{q}_c + U_L A_L)T'(s) \qquad (5)$$

Rearranging,

$$T'(s) = G_L(s)T_i'(s) + G_p(s)Q_c'(s) \qquad (6)$$

where:

$$G_d(s) = \frac{K_L}{\tau s + 1}$$
$$G_p(s) = \frac{K_p}{\tau s + 1}$$
$$K_d = \frac{wC}{K} \qquad (7)$$
$$K_p = \frac{UA(T_c - \overline{T})}{K}$$
$$\tau = \frac{\rho VC}{K}$$
$$K = wC + UA + UA\overline{q}_c + U_L A_L$$

The ideal FF controller design equation is given by,

$$G_F = \frac{-G_d}{G_t G_v G_p} \qquad (17\text{-}27)$$

But, $G_t = K_t e^{-\theta s}$ and $G_v = K_v$ (8)

Substituting (7) and (8) gives,

$$G_F = \frac{-wCe^{+\theta s}}{K_t K_v UA(T_c - \overline{T})} \qquad (9)$$

In order to have a physically realizable controller, ignore the $e^{+\theta s}$ term,

$$G_F = \frac{-wC}{K_t K_v UA(T_c - \overline{T})} \tag{10}$$

15.12

a) A component balance in A gives:

$$V\frac{dc_A}{dt} = qc_{Ai} - qc_A - Vkc_A \tag{1}$$

At steady state,

$$0 = \overline{q}\,\overline{c}_{Ai} - \overline{q}\,\overline{c}_A - Vk\overline{c}_A \tag{2}$$

Solve for $\overline{q}$,

$$\overline{q} = \frac{kV\overline{C}_A}{\overline{C}_{Ai} - \overline{C}_A} \tag{3}$$

For an ideal FF controller, replace $\overline{C}_{Ai}$ by C_{Ai}, $\overline{q}$ by q_1 and $\overline{C}_A$ by C_{Asp}:

$$q = \frac{kVC_{Asp}}{C_{Ai} - C_{Asp}}$$

b) Linearize (1):

$$V\frac{dc_A}{dt} = \overline{q}\,\overline{c}_{Ai} + \overline{q}c'_{Ai} + \overline{c}_{Ai}q' - \overline{q}\,\overline{c}_A - \overline{q}c'_A - \overline{c}_A q' - Vkc_A$$

Subtract (2),

$$V\frac{dc'_A}{dt} = \overline{q}c'_{Ai} + \overline{c}_{Ai}q' - \overline{q}c'_A - \overline{c}_A q' - Vkc'_A$$

Take the Laplace transform,

$$sVc'_A(s) = \overline{q}c'_{Ai}(s) + \overline{c}_{Ai}Q'(s) - \overline{q}c'_A(s) - \overline{c}_A Q'(s) - Vkc'_A(s)$$

Rearrange,

$$C'_A(s) = \frac{\bar{q}}{sV + \bar{q} + Vk} C'_{Ai}(s) + \frac{\bar{c}_{Ai} - \bar{c}_A}{sV + \bar{q} + Vk} Q'(s) \tag{6}$$

or

$$C'_A(s) = G_d(s) C'_{Ai}(s) + G_p(s) Q'(s) \tag{7}$$

The ideal FF controller design equation is,

$$G_F(s) = -\frac{G_d(s)}{G_v(s) G_p(s) G_t(s)} \tag{8}$$

Substitute from (6) and (7) with $G_v(s)$=K_v and $G_t(s)$=K_t :

$$G_F(s) = -\frac{\bar{q}}{K_v(\bar{c}_{Ai} - \bar{c}_A) K_t} \tag{9}$$

Note: $G_F(s) = P'(s)/C'_{Aim}(s)$ where P is the controller output and c_{Aim} is the measured value of c_{Ai}.

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(a) Steady-state balances:

$$0 = \bar{q}_5 + \bar{q}_1 - \bar{q}_3 \tag{1}$$

$$0 = \bar{q}_3 + \bar{q}_2 - \bar{q}_4 \tag{2}$$

$$0 = \bar{x}_5 \bar{q}_5 + \cancelto{0}{\bar{x}_1 \bar{q}_1} - \bar{x}_3 \bar{q}_3 \tag{3}$$

$$0 = \bar{x}_3 \bar{q}_3 + \bar{x}_2 \bar{q}_2 - \bar{x}_4 \bar{q}_4 \tag{4}$$

Solve (4) for $\bar{x}_3 \bar{q}_3$ and substitute into (3),

$$0 = \bar{x}_5 \bar{q}_5 + \bar{x}_2 \bar{q}_2 - \bar{x}_4 \bar{q}_4 \tag{5}$$

Rearrange,

$$\overline{q}_2 = \frac{\overline{x}_4\overline{q}_4 - \overline{x}_5\overline{q}_5}{\overline{x}_2} \tag{6}$$

In order to derive the feedforward control law, let

$\overline{x}_4 \rightarrow x_{4sp}$, $\overline{x}_2 \rightarrow x_2(t)$, $\overline{x}_5 \rightarrow x_5(t)$, and $\overline{q}_2 \rightarrow q_2(t)$

Thus,

$$q_2(t) = \frac{x_{4sp}\overline{q}_4 - x_5(t)q_5(t)}{\overline{x}_2} \tag{7}$$

Substitute numerical values:

$$q_2(t) = \frac{(3400)x_{4sp} - x_5(t)q_5(t)}{0.990} \tag{8}$$

or

$$q_2(t) = 3434x_{4sp} - 1.01x_5(t)q_5(t) \tag{9}$$

Note: If transmitter and control valve gains are available, then an expression relating the feedforward controller output signal, $p(t)$, to the measurements , $x_{5m}(t)$ and $q_{5m}(t)$, can be developed.

(b) Dynamic compensation: It will be required because of the extra dynamic lag preceding the tank on the left hand side. The stream 5 disturbance affects x_3 while q_3 does not.