Chapter 19

19.1

From definition of x_c, $0 \le x_c \le 1$

$$f(x) = 5.3\, x\, e^{(-3.6x + 2.7)}$$

Let three initial points in [0,1] be 0.25, 0.5 and 0.75. Calculate x_4 using Eq. 19-8,.

x_1	f_1	x_2	f_2	x_3	f_3	x_4
0.25	8.02	0.5	6.52	0.75	3.98	0.0167

For next iteration, select x_4, and x_1 and x_2 since f_1 and f_2 are the largest among f_1, f_2, f_3. Thus successive iterations are

x_1	f_1	x_2	f_2	x_3	f_3	x_4
0.25	8.02	0.5	6.52	0.017	1.24	0.334
0.25	8.02	0.5	6.52	0.334	7.92	0.271
0.25	8.02	0.334	7.92	0.271	8.06	0.280
0.25	8.02	0.271	8.06	0.280	8.06	not needed

x^{opt} **= 0.2799** 7 function evaluations

19.2

As shown in the drawing, there is both a minimum and maximum value of the air/fuel ratio such that the thermal efficiency is non- zero. If the ratio is too low, there will not be sufficient air to sustain combustion. On the other hand, problems in combustion will appear when too much air is used.

The maximum thermal efficiency is obtained when the air/fuel ratio is stoichiometric. If the amount of air is in excess, relatively more heat will be “absorbed” by the air (mostly nitrogen). However if the air is not sufficient to sustain the total combustion, the thermal efficiency will decrease as well.

19.3

By using Excel-Solver, this optimization problem is quickly solved. The selected starting point is (1,1):

	X_1	X_2
Initial values	1	1
Final values	0.776344	0.669679
max *Y*=	0.55419	
Constraints		
$0 \le X_1 \le 2$		
$0 \le X_2 \le 2$		

Table S19.3. *Excel solution*

Hence the optimum point is $(X_1^*, X_2^*) = (0.776, 0.700)$

and the maximum value of Y is $Y_{max} = 0.554$

19.4

Let N be the number of batches/year. Then $NP \ge 300{,}000$
Since the objective is to minimize the cost of annual production, only the required amount should be produced annually and no more. That is,

$$NP = 300{,}000 \qquad (1)$$

a) Minimize the total annual cost,

$$\min TC = 400{,}000 \left(\frac{\$}{\text{batch}}\right) + 2\,P^{0.4}\left(\frac{\text{hr}}{\text{batch}}\right) 50\left(\frac{\$}{\text{hr}}\right)\ N\left(\frac{\text{batch}}{\text{yr}}\right) + 800\,P^{0.7}\left(\frac{\$}{yr}\right)$$

Substituting for N from (1) gives

$$\min TC = 400{,}000 + 3\text{x}10^7\,P^{-0.6} + 800\,P^{0.7}$$

b) There are three constraints on P

i) $P \geq 0$

ii) N is integer. That is,

$(300{,}000/P) = 0, 1, 2, \ldots$

iii) Total production time is 320 x 24 hr/yr

$$(2\,P^{0.4} + 14)\left(\frac{\text{hr}}{\text{batch}}\right) \times N\left(\frac{\text{batch}}{\text{yr}}\right) \leq 7680$$

Substituting for N from (1) and simplifying

$$6\times10^{5}P^{-0.6} + 4.2\times10^{6}P^{-1} \leq 7680$$

c)
$$\frac{d(TC)}{dP} = 0 = 3\times10^{7}(-0.6)P^{-1.6} + 800(0.7)P^{-0.3}$$

$$P^{opt} = \left[\frac{3\times10^{7}(-0.6)}{-800(0.7)}\right]^{1/1.3} = 2931\frac{\text{lb}}{\text{batch}}$$

$$\frac{d^2(TC)}{dP^2} = 3\times10^{7}(-0.6)(-1.6)P^{-2.6} + 800(0.7)(-0.3)P^{-1.3}$$

$$\left.\frac{d^2(TC)}{dP^2}\right|_{P=P^{opt}} = 2.26\times10^{-2}\rangle 0 \quad \text{hence minimum}$$

$N^{opt} = 300{,}000/P^{opt} = 102.35$ not an integer.

Hence check for $N^{opt} = 102$ and $N^{opt} = 103$

For $N^{opt} = 102$, $P^{opt} = 2941.2$, and $TC = 863207$

For $N^{opt} = 103$, $P^{opt} = 2912.6$, and $TC = 863209$

Hence optimum is 102 batches of 2941.2 lb/batch.

Time constraint is

$$6\times10^{5}P^{-0.6} + 4.2\times10^{6}P^{-1} = 6405.8 \leq 7680 \text{, satisfied}$$

19.5

Let x_1 be the daily feed rate of Crude No.1 in bbl/day
x_2 be the daily feed rate of Crude No.2 in bbl/day

Objective is to maximize profit

$$\max P = 2.00\, x_1 + 1.40\, x_2$$

Subject to constraints

gasoline : $0.70\, x_1 + 0.31\, x_2 \leq 6000$
kerosene: $0.06\, x_1 + 0.09\, x_2 \leq 2400$
fuel oil: $0.24\, x_1 + 0.60\, x_2 \leq 12{,}000$

By using Excel-Solver,

	x_1	x_2
Initial values	1	1
Final values	0	19354.84

max *P* = 27096.77

Constraints	
$0.70\, x_1 + 0.31\, x_2$	6000
$0.06\, x_1 + 0.09\, x_2$	1741.935
$0.24\, x_1 + 0.60\, x_2$	11612.9

Table S19.5. *Excel solution*

Hence the optimum point is (0, 19354.8)

Crude No.1 = 0 bbl/day Crude No.2 = 19354.8 bbl/day

19.6

Objective function is to maximize the revenue,

$$\max R = -40x_1 + 50x_3 + 70x_4 + 40x_5 - 2x_1 - 2x_2 \qquad (1)$$

*Balance on column 2

$$x_2 = x_4 + x_5 \qquad (2)$$

* From column 1,

$$x_1 = \frac{1.0}{0.60} x_2 = 1.667(x_4 + x_5) \qquad (3)$$

$$x_3 = \frac{0.4}{0.60} x_2 = 0.667(x_4 + x_5) \qquad (4)$$

Inequality constraints are

$$x_4 \geq 200 \qquad (5)$$
$$x_4 \leq 400 \qquad (6)$$
$$x_1 \leq 2000 \qquad (7)$$
$$x_4 \geq 0 \quad x_5 \geq 0 \qquad (8)$$

The restricted operating range for column 2 imposes additional inequality constraints. Medium solvent is 50 to 70% of the bottoms; that is

$$0.5 \leq \frac{x_4}{x_2} \leq 0.7 \quad \text{or} \quad 0.5 \leq \frac{x_4}{x_4 + x_5} \leq 0.7$$

Simplifying,

$$x_4 - x_5 \geq 0 \qquad (9)$$
$$0.3\,x_4 - 0.7x_5 \leq 0 \qquad (10)$$

No additional constraint is needed for the heavy solvent. That the heavy solvent will be 30 to 50% of the bottoms is ensured by the restriction on the medium solvent and the overall balance on column 2.

By using Excel-Solver,

	x_1	x_2	x_3	x_4	x_5
Initial values	1	1	1	1	1
Final values	1333.6	800	533.6	400	400
max R =	**13068.8**				
Constraints					
$x_2 - x_4 - x_5$	0				
$x_1 - 1.667x_2$	7.467E-10				
$x_3 - 0.667x_2$	-1.402E-10				
x_4	400				
x_4	400				
$x_1 - 1.667x_2$	1333.6				
$x_4 - x_5$	0				
$0.3x_4 - 0.7x_5$	-160				

Table S19.6. *Excel solution*

Thus the optimum point is x_1 =1333.6, x_2 =800; x_3=533.6, x_4 = 400 and x_5 = 400.

Substituting into (5), the maximum revenue is 13,068 \$/day, and the percentage of output streams in column 2 is 50 % for each stream.

19.7

The objective is to minimize the sum of the squares of the errors for the material balance, that is,

$$\min E = (w_A + 11.1 - 92.4)^2 + (w_A + 10.8 - 94.3)^2 + (w_A + 11.4 - 93.8)^2$$

Subject to $w_A \geq 0$

Solve analytically,

$$\frac{dE}{dw_A} = 0 = 2(w_A + 11.1 - 92.4) + 2(w_A + 10.8 - 94.3) + 2(w_A + 11.4 - 93.8)$$

Solving for w_A... $\qquad w_A^{opt} = 82.4$ Kg/hr

Check for minimum,

$$\frac{d^2E}{dw_A^2} = 2 + 2 + 2 = 6 > 0, \quad \text{hence minimum}$$

19.8

a) Income = 50 (0.1 +0.3x_A + 0.0001S – 0.0001 $x_A S$)

Costs = 2.0 + 10x_A + 20 x_A^2 + 1.0 + 0.003 S + 2.0x10$^{-6}S^2$

f = 2.0 +5x_A + 0.002S – 20x_A^2 – 2.0x10$^{-6}S^2$ – 0.005$x_A S$

b) Using analytical method

$$\frac{\partial f}{\partial x_A} = 0 = 5 - 40x_A - 0.005S$$
$$\frac{\partial f}{\partial S} = 0 = 0.002 - 4.0\times10^{-6} - 0.005x_A$$

Solving simultaneously, x_A = 0.074 , S = 407 which satisfy the given constraints.

19.9

By using Excel-Solver

		τ_1	τ_2
	Initial values	1	0.5
	Final values	2.991562	1.9195904

TIME	EQUATION	DATA		SQUARE ERROR
0	0.000	0.000		0.00000000
1	0.066	0.058		0.00005711
2	0.202	0.217		0.00022699
3	0.351	0.360		0.00007268
4	0.490	0.488		0.00000403
5	0.608	0.600		0.00006008
6	0.703	0.692		0.00012252
7	0.778	0.772		0.00003428
8	0.835	0.833		0.00000521
9	0.879	0.888		0.00008640
10	0.911	0.925		0.00019150
			SUM=	0.00086080

Hence the optimum values are τ_1=3 and τ_2=1.92. The obtained model is compared with that obtained using MATLAB.

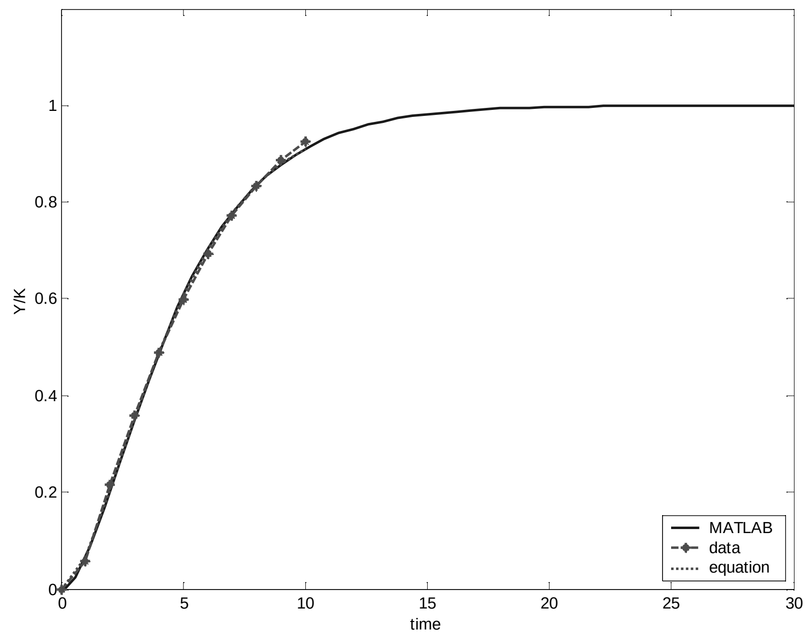

Figure S19.9. *Comparison between the obtained model with that obtained using MATLAB*

19.10

Let x_1 be gallons of suds blended
x_2 be gallons of premium blended
x_3 be gallons of water blended

Objective is to minimize cost

$$\min C = 0.25x_1 + 0.40x_2 \quad (1)$$

Subject to

$$x_1 + x_2 + x_3 = 10{,}000 \quad (2)$$

$$0.035\, x_1 + 0.050\, x_2 = 0.040 \times 10{,}000 \quad (3)$$

$$x_1 \geq 2000 \quad (4)$$

$$x_1 \leq 9000 \quad (5)$$

$$x_2 \geq 0 \qquad (6)$$

$$x_3 \geq 0 \qquad (7)$$

The problem given by Eqs. 1, 2, 3, 4, 5, 6, and 7 is optimized using Excel-Solver,

	x_1	x_2	x_3
Initial values	1	1	1
Final values	6666.667	3333.333	0
min C =	3000		

Constraints	
$x_1+x_2+x_3-10000$	0
$0.035x_1+0.050x_2-400$	0.0E+00
x_1-2000	4666.667
x_1-9000	-2333.333
x_2	3333.333
x_3	0

Table S19.10. *Excel solution*

Thus the optimum point is $x_1 = 6667$, $x_2 = 3333$ and $x_3 = 0$.
The minimum cost is \$3000

19.11

Let x_A be bbl/day of A produced
x_B be bbl/day of B produced

Objective is to maximize profit

$$\max P = 10x_A + 14x_B \qquad (1)$$

Subject to

Raw material constraint: $120x_A + 100x_B \leq 9{,}000$ (2)

Warehouse space constraint: $0.5\,x_A + 0.5\,x_B \leq 40$ (3)

Production time constraint: $(1/20)x_A + (1/10)x_B \leq 7$ (4)

	x_A	x_B
Initial values	1	1
Final values	20	60
max *P* =	1040	
Constraints		
$\mathbf{120}x_A\mathbf{+ 100}x_B$	8400	
$\mathbf{0.5}\ x_A \mathbf{+ 0.5}\ x_B$	40	
$\mathbf{(1/20)}x_A \mathbf{+ (1/10)}x_B$	7	

Table S19.11. *Excel solution*

Thus the optimum point is $x_A = 20$ and $x_B = 60$
The maximum profit = $1040/day

19.12

PID controller parameters are usually obtained by using either process model, process data or computer simulation. These parameters are kept constant in many cases, but when operating conditions vary, supervisory control could involve the optimization of these tuning parameters. For instance, using process data, K_c ,τ_I and τ_D can be automatically calculated so that they maximize profits. Overall analysis of the process is needed in order to achieve this type of optimum control.

Supervisory and regulatory control are complementary. Of course, supervisory control may be used to adjust the parameters of either an analog or digital controller, but feedback control is needed to keep the controlled variable at or near the set-point.

19.13

Assuming steady state behavior, the optimization problem is,

$$\max\ f = D\,e$$

Subject to

$$0.063\,c - D\,e = 0 \qquad (1)$$
$$0.9\,s\,e - 0.9\,s\,c - 0.7\,c - D\,c = 0 \qquad (2)$$

$$-0.9\, s\, e + 0.9\, s\, c + 10D - D\, s = 0 \quad (3)$$

$$D, e, s, c \geq 0$$

where $f = f(D, e, c, s)$

Excel-Solver is used to solve this problem,

	c	*D*	*e*	*s*
Initial values	1	1	1	1
Final values	0.479031	0.045063	0.669707	2.079784

max *f* =	0.030179
Constraints	
0.063 *c* –*D e*	2.08E-09
0.9 *s e* – 0.9 *s c* – 0.7 *c* – *Dc*	-3.1E-07
-0.9 *s e* + 0.9 *s c* + 10*D* – *Ds*	2.88E-07

Table S19.13. *Excel solution*

Thus the optimum value of D is equal to 0.045 h^{-1}

19.14

Material balance:

Overall : $F_A + F_B = F$

Component B: $F_B\, C_{BF} + VK_1C_A - VK_2C_B = F\, C_B$

Component A: $F_A\, C_{AF} + VK_2C_B - VK_1C_A = FC_A$

Thus the optimization problem is:

$$\max\ (150 + F_B)\, C_B$$

Subject to:

$$0.3\, F_B + 400C_A - 300C_B = (150 + F_B)C_B$$

$$45 + 300\, C_B - 400\, C_A = (150 + F_B)\, C_A$$

$$F_B \leq 200$$

$C_A, C_B, F_B \geq 0$

By using Excel- Solver, the optimum values are

F_B = 200 l/hr

C_A = 0.129 mol A/l

C_B = 0.171 mol B/l

19.15

Material balance:

Overall : $F_A + F_B = F$

Component B: $F_B C_{BF} + VK_1C_A - VK_2C_B = F C_B$

Component A: $F_A C_{AF} + VK_2C_B - VK_1C_A = FC_A$

Thus the optimization problem is:

$$\max (150 + F_B) C_B$$

Subject to:

$$0.3 F_B + 3\times10^6 e^{(-5000/T)}C_A V - 6\times10^6 e^{(-5500/T)}C_B V = (150 + F_B)C_B$$

$$45 + 6\times10^6 e^{(-5500/T)}C_B V - 3\times10^6 e^{(-5000/T)} C_A V = (150 + F_B) C_A$$

$F_B \leq 200$
$300 \leq T \leq 500$
$C_A, C_B, F_B \geq 0$

By using Excel- Solver, the optimum values are

F_B = 200 l/hr

C_A = 0.104 molA/l

C_B = 0.177 mol B/l

T = 311.3 K