Chapter 20

20.1

a) The unit step response is

$$Y(s) = G_p(s)U(s) = \left(\frac{2e^{-s}}{(10s+1)(5s+1)}\right)\left(\frac{1}{s}\right) = 2e^{-s}\left[\frac{1}{s} + \frac{5}{5s+1} - \frac{20}{10s+1}\right]$$

Therefore,

$$y(t) = 2S(t-1)\left[1 + e^{-(t-1)/5} - 2e^{-(t-1)/10}\right]$$

For $\Delta t = 1.0$,

$$S_i = y(i\Delta t) = y(i) = \{0,\ \ 0.01811,\ \ 0.06572,\ \ 0.1344,\ \ 0.2174,\ \ 0.3096...\}$$

b) From the expression for $y(t)$ in part (a) above

$y(t) = 0.95\ (2)$ at $t = 37.8$, by trial and error.

Hence $N = 38$, for 95% complete response.

20.2

Note that $G(s) = G_v(s)G_p(s)G_m(s)$. From Figure 12.2,

a)
$$\frac{Y_m(s)}{P(s)} = G(s) = \frac{2(1-9s)}{(15s+1)(3s+1)} \qquad (1)$$

For a unit step change, $P(s) = 1/s$, and (1) becomes:

$$Y_m(s) = \frac{1}{s}\frac{2(1-9s)}{(15s+1)(3s+1)}$$

Partial Fraction Expansion:

$$Y_m(s) = \frac{A}{s} + \frac{B}{(15s+1)} + \frac{C}{(3s+1)} = \frac{1}{s}\frac{2(1-9s)}{(15s+1)(3s+1)} \qquad (2)$$

where

$$A = \left.\frac{2(1-9s)}{(15s+1)(3s+1)}\right|_{s=0} = 2$$

$$B = \left.\frac{2(1-9s)}{s(3s+1)}\right|_{s=-\frac{1}{15}} = -60$$

$$C = \left.\frac{2(1-9s)}{s(15s+1)}\right|_{s=-\frac{1}{3}} = 6$$

Substitute into (2) and take inverse Laplace transform:

$$y_m(t) = 2 - 4e^{-t/15} + 2e^{-t/3} \qquad (3)$$

b) The new steady-state value is obtained from (3) to be $y_m(\infty)=2$

For $t = t_{99}$, $y_m(t)=0.99y_m(\infty) = 1.98$. Substitute into (3)

$$1.98 = 2 - 4e^{-t_{99}/15} + 2e^{-t_{99}/3} \qquad (4)$$

Solving (4) for t_{99} by trial and error gives $t_{99} \approx 79.5$ min

Thus, we specify that Δt =79.5 min/40 ≈ 2 min

Sample No	S_i	Sample No	S_i	Sample No	S_i
1	-0.4739	16	1.5263	31	1.9359
2	-0.5365	17	1.5854	32	1.9439
3	-0.4106	18	1.6371	33	1.9509
4	-0.2076	19	1.6824	34	1.9570
5	0.0177	20	1.7221	35	1.9624
6	0.2393	21	1.7568	36	1.9671
7	0.4458	22	1.7871	37	1.9712
8	0.6330	23	1.8137	38	1.9748
9	0.8022	24	1.8370	39	1.9779
10	0.9482	25	1.8573	40	1.9807
11	1.0785	26	1.8751		
12	1.1931	27	1.8907		
13	1.2936	28	1.9043		
14	1.3816	29	1.9163		
15	1.4587	30	1.9267		

Table S20.2. *Step response coefficients*

20.3

From the definition of matrix $\boldsymbol{S}$, given in Eq. 20-20, for P=5, M=1, with S_i obtained from Exercise 20.1,

$$\boldsymbol{S} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.01811 \\ 0.06572 \\ 0.1344 \\ 0.2174 \end{bmatrix}$$

From Eq. 20-58

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{S})^{-1}\boldsymbol{S}^T$$

$$\boldsymbol{K}_c = \begin{bmatrix} 0 & 0.2589 & 0.9395 & 1.9206 & 3.1076 \end{bmatrix} = \boldsymbol{K}_{c1}^T$$

Because $\boldsymbol{K}_{c1}^{\mathrm{T}}$ is defined as the first row of $\boldsymbol{K}_c$.

Using the given analytical result,

$$\begin{aligned} \boldsymbol{K}_{c1}^T &= \frac{1}{\sum_{i=1}^{5}(S_i^{\,2})}\begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 \end{bmatrix} \\ &= \frac{1}{0.06995}\begin{bmatrix} 0 & 0.01811 & 0.06572 & 0.1344 & 0.2174 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0.2589 & 0.9395 & 1.9206 & 3.1076 \end{bmatrix} \end{aligned}$$

which is the same as the answer obtained above using (20-58)

20.4

The step response is obtained from the analytical unit step response as in Example 20.1. The feedback matrix $\boldsymbol{K}_c$ is obtained using Eq. 20-57 as in Example 20.5. These results are not reported here for sake of brevity. The closed-loop response for set-point and disturbance changes are shown below for each case. MATLAB *MPC Toolbox* was used for the simulations.

i) For this model horizon, the step response is over 99% complete as in Example 20.5; hence the model is good. The set-point and disturbance responses shown below are non-oscillatory and have long settling times

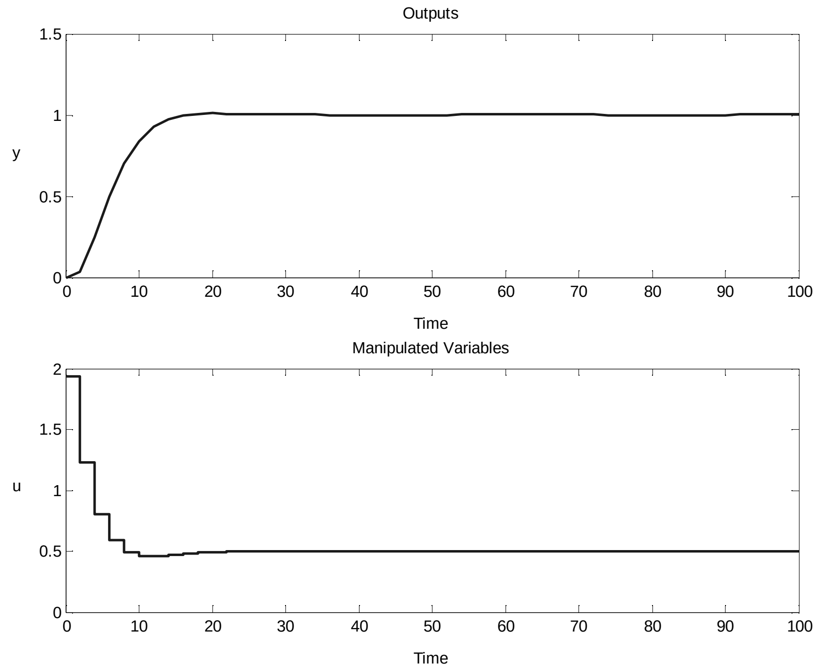

Figure S20.4a. *Controller i); set-point change.*

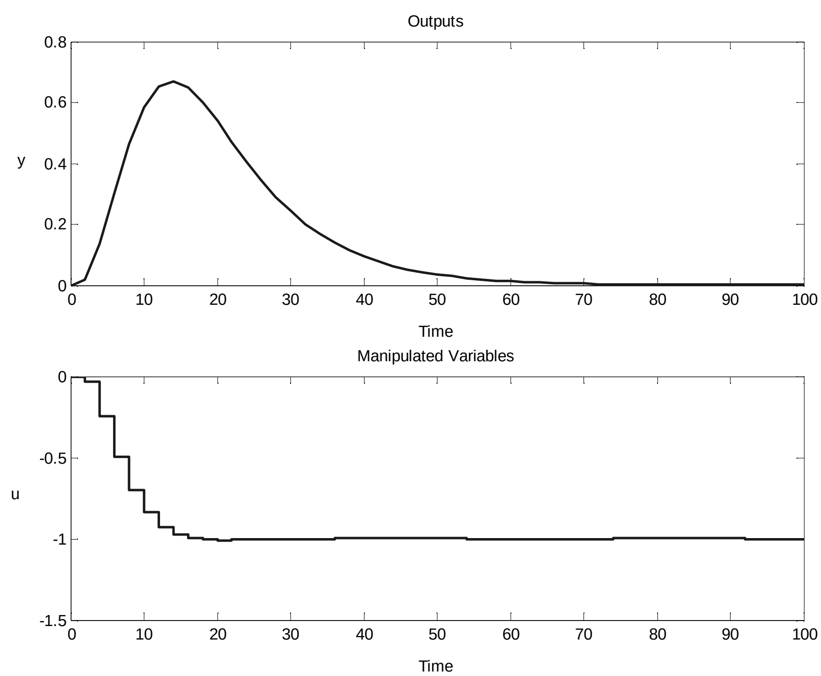

Figure S20.4b. *Controller i); disturbance change.*

ii) The set-point response shown below exhibits same overshoot, smaller settling time and undesirable "ringing" in *u* compared to part i). The disturbance response shows a smaller peak value, a lack of oscillations, and faster settling of the manipulated input.

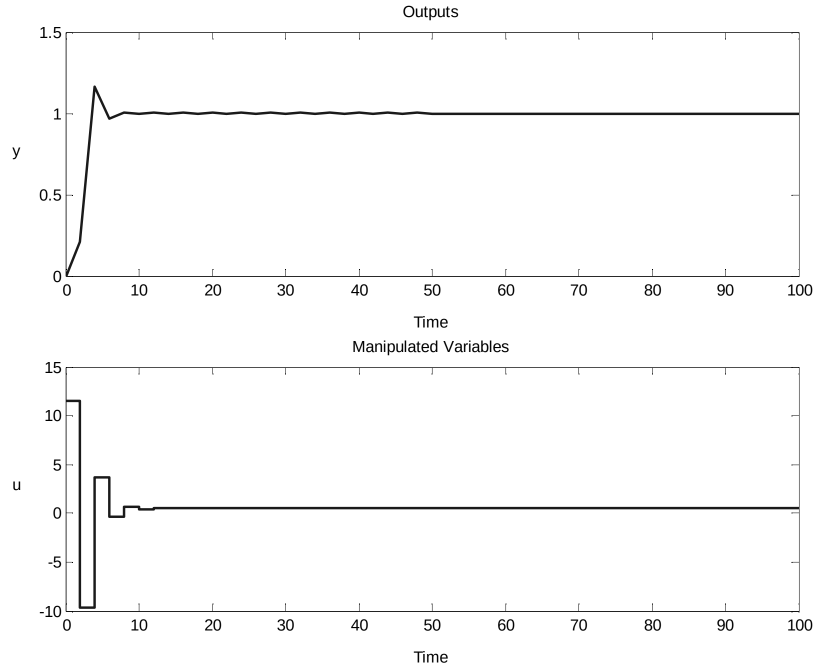

Figure S20.4c. *Controller ii); set-point change.*

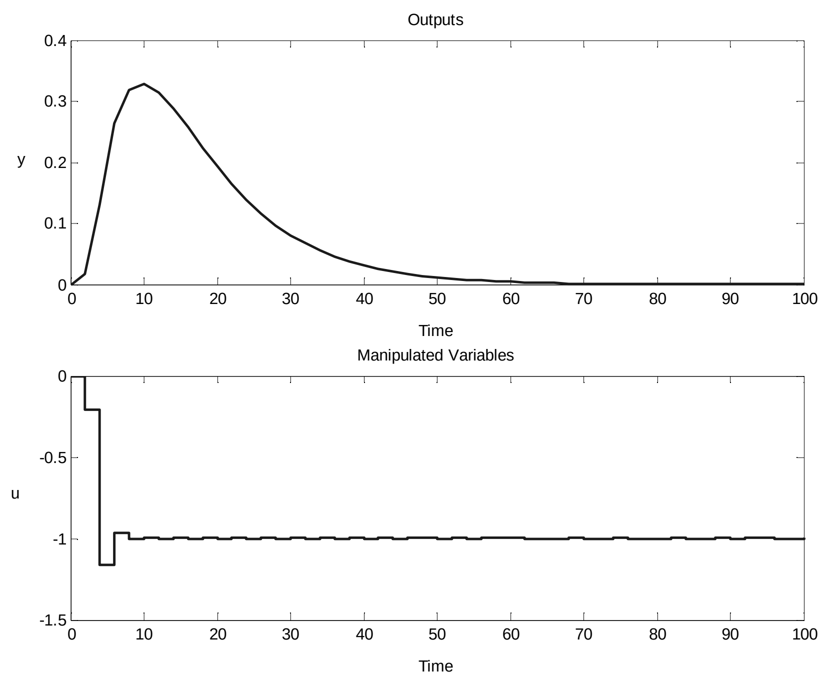

Figure S20.4d. *Controller ii); disturbance change.*

iii) The set-point and disturbance responses shown below show the same trends as in part i).

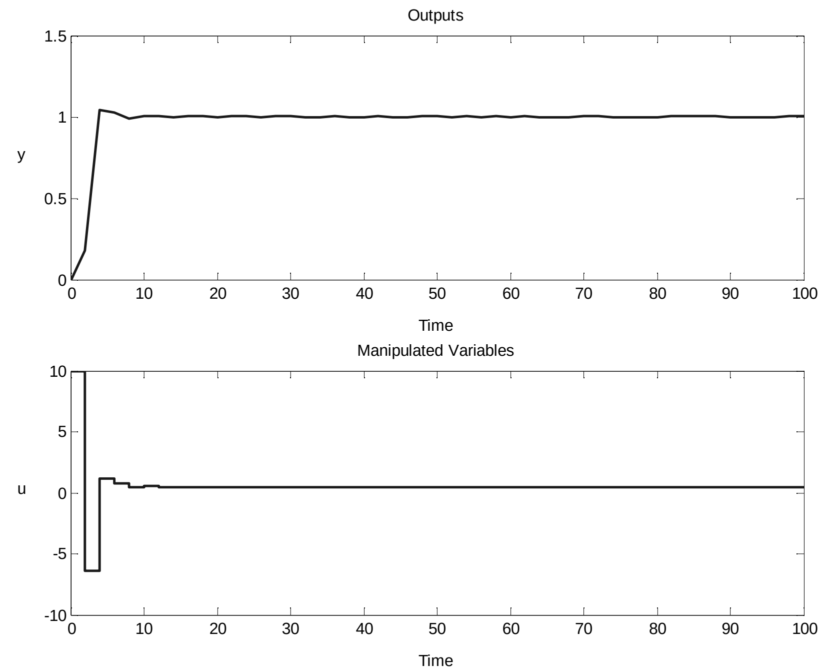

Figure S20.4e. *Controller iii); set-point change.*

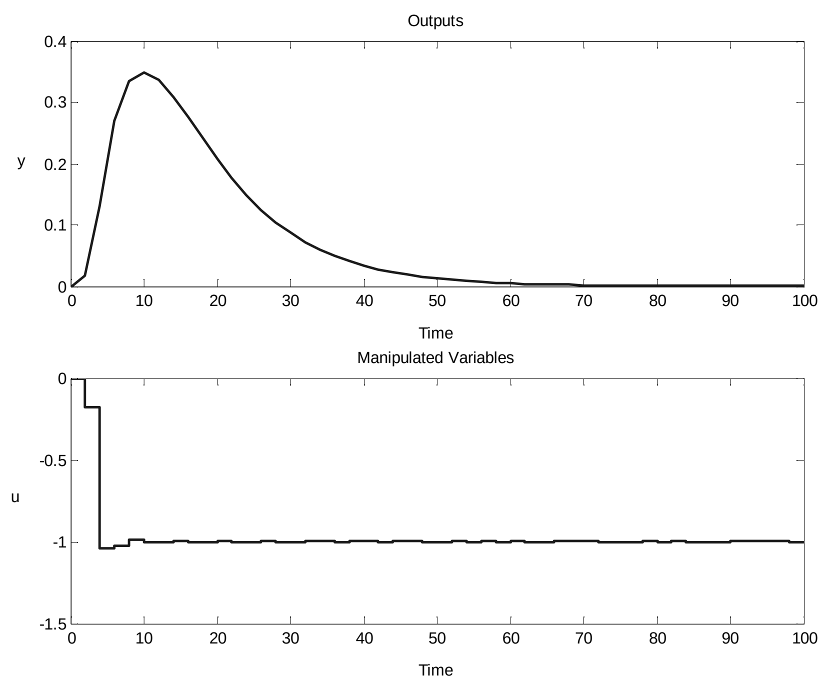

Figure S20.4f. *Controller iii); disturbance change.*

iv) The set-point and load responses shown below exhibit the same trends as in parts (i) and (ii). In comparison to part (iii), this controller has a larger penalty on the manipulated input and, as a result, leads to smaller and less oscillatory input effort at the expense of larger overshoot and settling time for the controlled variable.

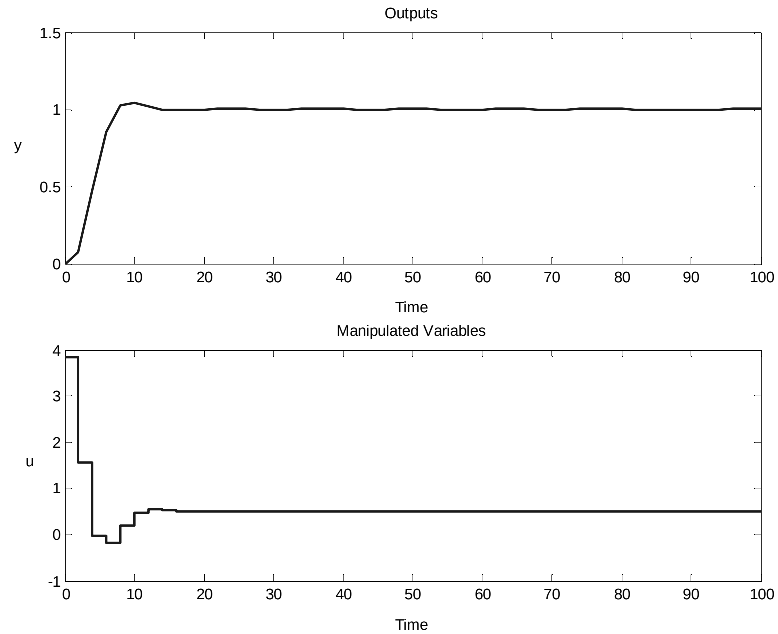

Figure S20.4g. *Controller iv); set-point change.*

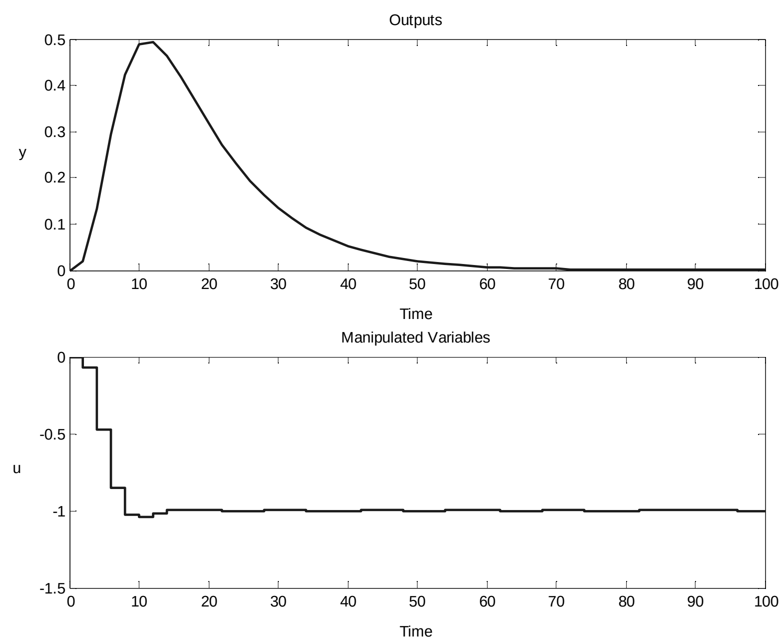

Figure S20.4h. *Controller iv); disturbance change.*

20.5

There are many sets of values of *M*, *P* and *R* that satisfy the given constraint for a unit load change. One such set is *M*=3, *P*=10, *R*=0.01 as shown in Exercise 20.4(iii). Another set is *M*=3, *P*=10, *R*=0.1 as shown in Exercise 20.4(iv). A third set of values is *M*=1, *P*=5, *R*=0 as shown in Exercise 20.4(i).

20.6

(Use MATLAB *Model Predictive Control Toolbox)*

As shown below, controller a) gives a better disturbance response with a smaller peak deviation in the output and less control effort. However, controller (a) is poorer for a set-point change because it leads to undesirable "ringing" in the manipulated input.

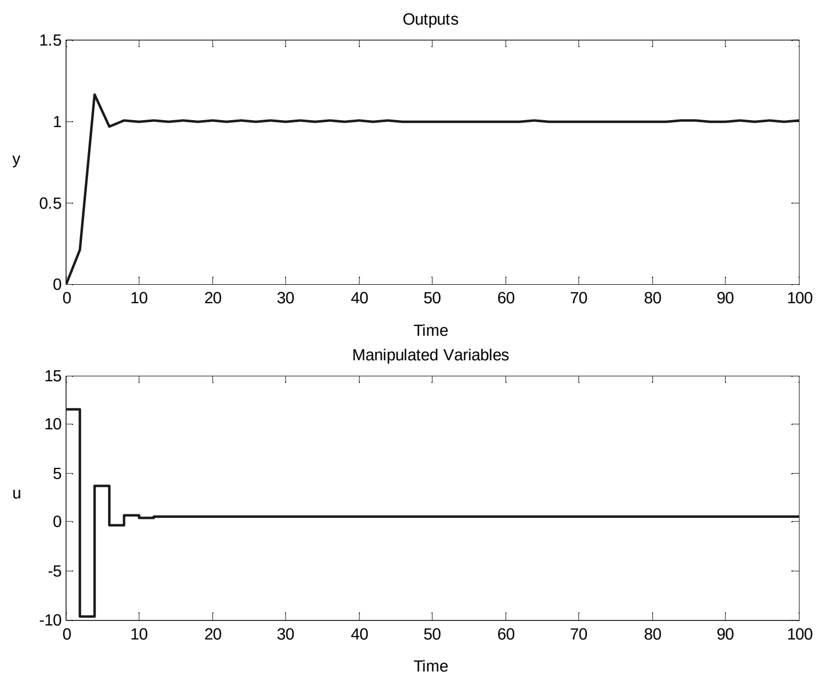

Figure S20.6a. *Controller a); set-point change*

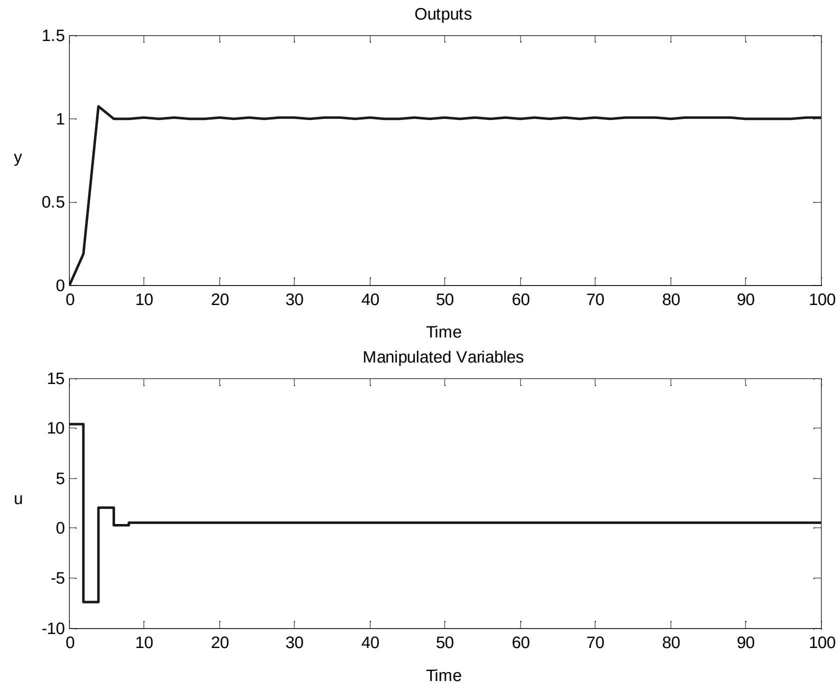

Figure S20.6b. *Controller a); disturbance change.*

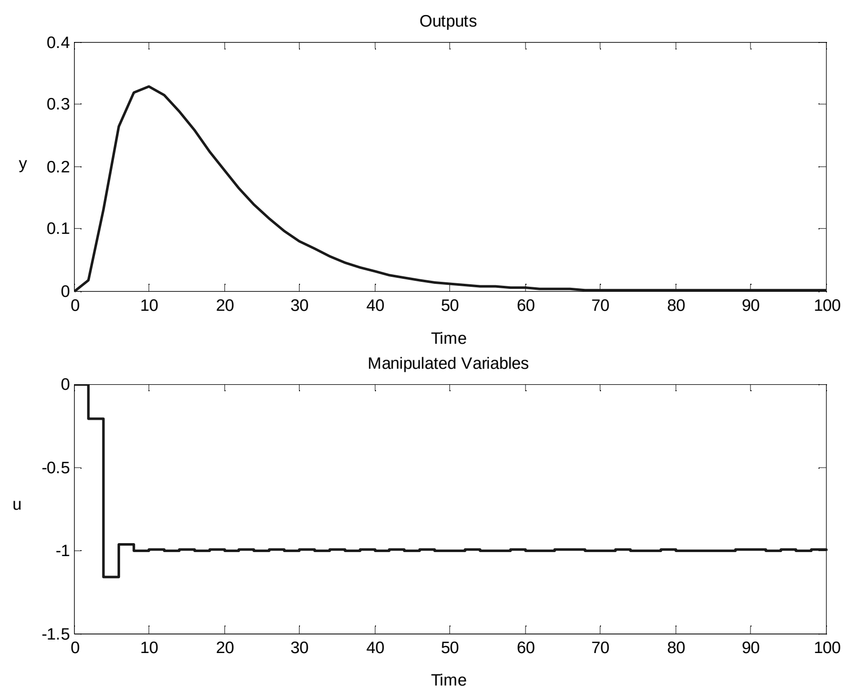

Figure S20.6c. *Controller b); set-point change.*

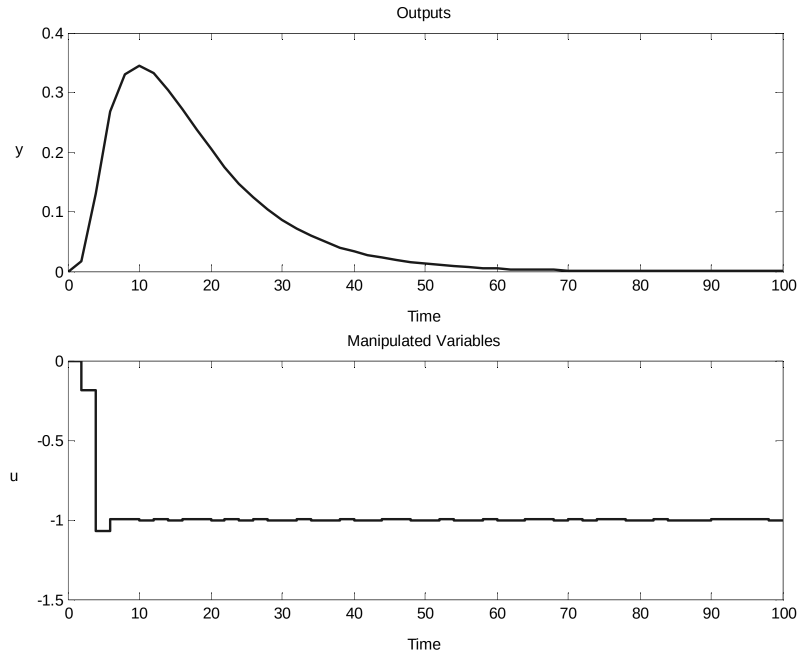

Figure S20.6d. *Controller b); disturbance change.*

20.7

The unconstrained MPC control law has the controller gain matrix:

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{QS}+\boldsymbol{R})^{-1}\boldsymbol{S}^T\boldsymbol{Q}$$

For this exercise, the parameter values are:
$m = r = 1$ (SISO), $\boldsymbol{Q}$=I, $\boldsymbol{R}$=1 and M=1

Thus (20-57) becomes

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{QS}+\boldsymbol{R})^{-1}\boldsymbol{S}^T\boldsymbol{Q}$$

Which reduces to a row vector: $\boldsymbol{K}_c = \dfrac{[S_1\ S_2\ S_3 \ldots S_P]}{\sum_{i=1}^{P} S_i^{\,2} + 1}$

20.8

Inequality constraints on the manipulated variables are usually satisfied if the instrumentation and control hardware are working properly. However the constraints on the controlled variables are applied to the predicted outputs. If the predictions are inaccurate, the actual outputs could exceed the constraints even though the predicted values do not.

20.9

(Use MATLAB *Model Predictive Control Toolbox)*

a) M=5 vs. M=2

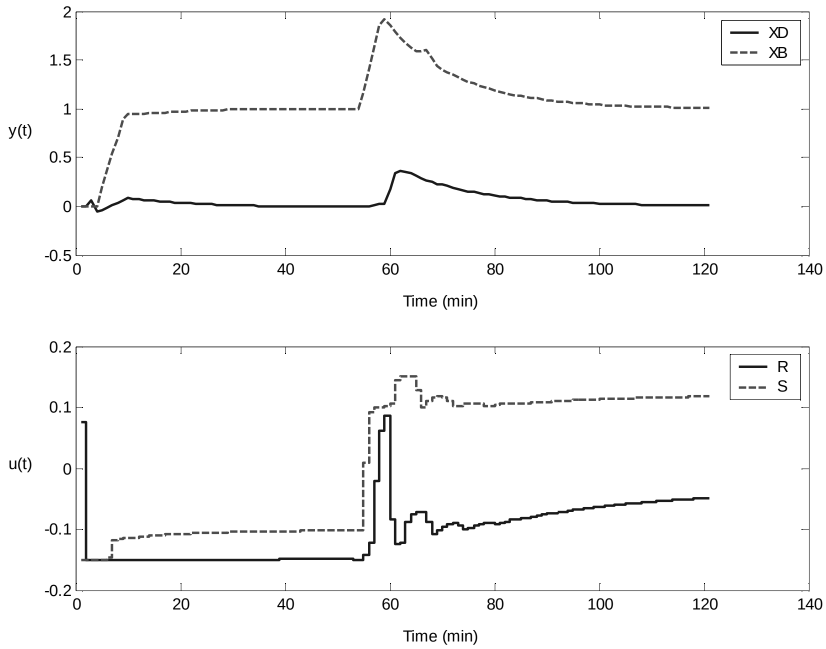

Figure S20.9a1. *Simulations for P=10 , M=5 and **R**=0.1**I**.*

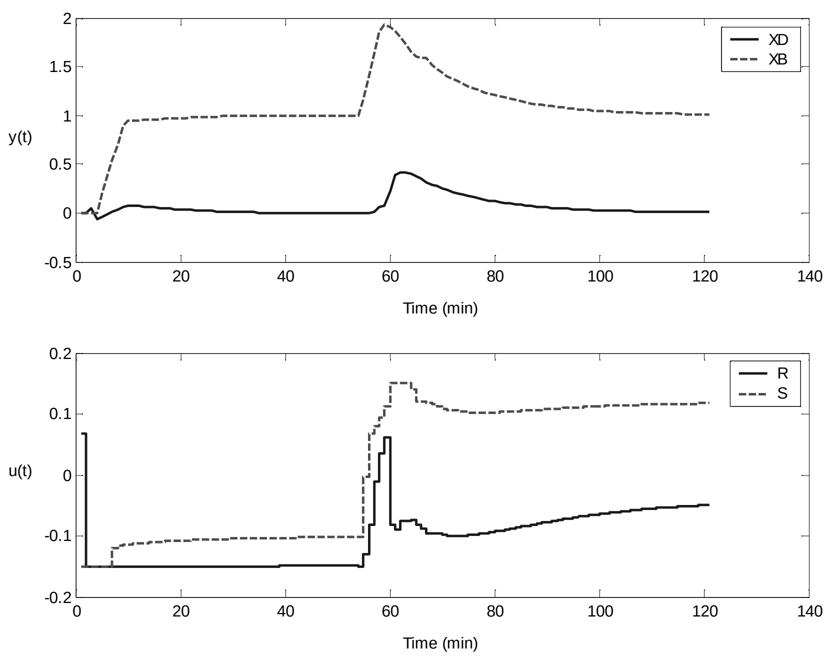

Figure S20.9a2. *Simulations for P=10 , M=2 and **R**=0.1**I**.*

b) ***R**=0.1**I*** *.vs* ***R**=**I***

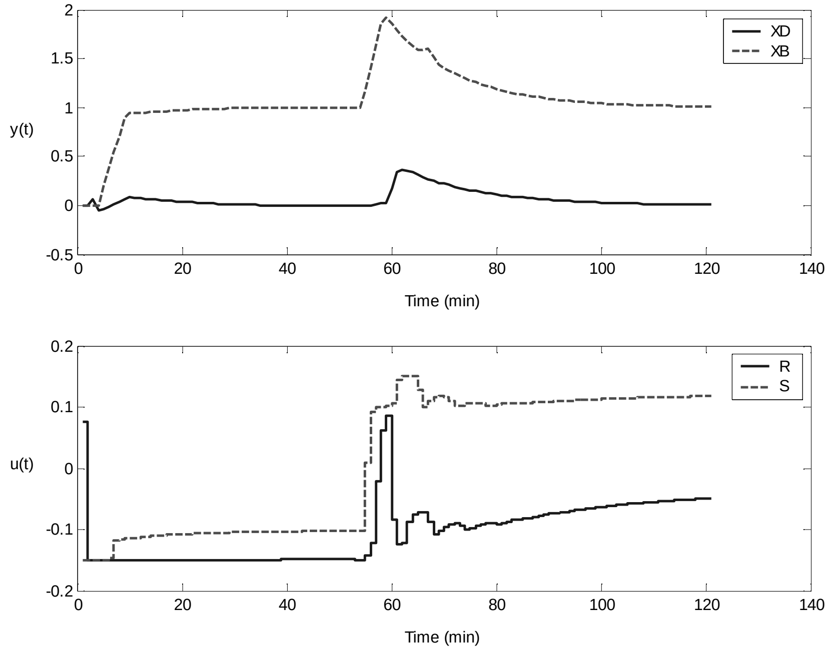

Figure S20.9b1. *Simulations for P=10 , M=5 and **R**=0.1**I**.*

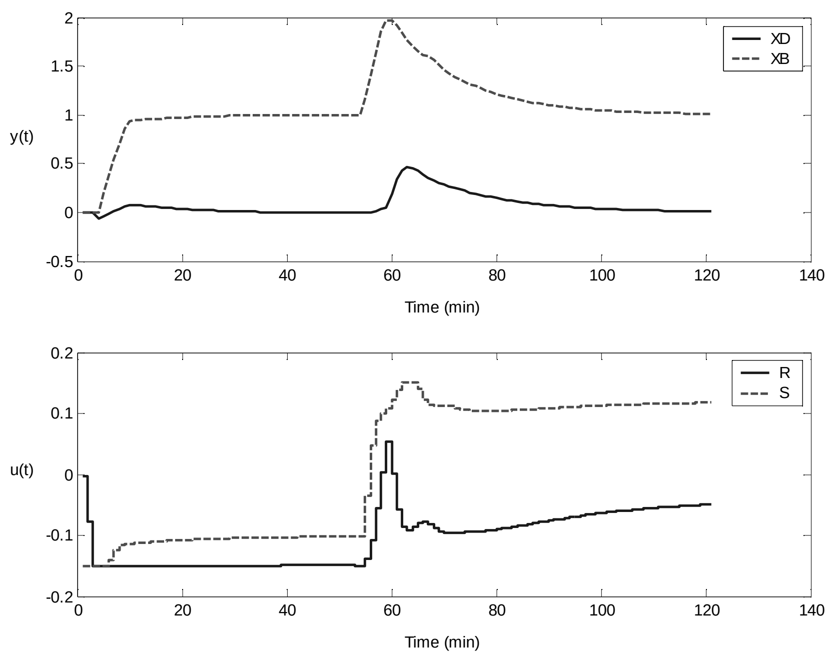

Figure S20.9b2. *Simulations for P=10 , M=5 and **R**=**I**.*

Notice that the larger control horizon *M* and the smaller input weighting *R*, the more control effort is needed.

20.10

The open-loop unit step response of $G_p(s)$ is

$$y(t) = \mathcal{L}^{-1}\left(\frac{e^{-6s}}{10s+1}\frac{1}{s}\right) = \mathcal{L}^{-1}\left(e^{-6s}\left(\frac{1}{s}-\frac{10}{10s+1}\right)\right) = S(t-6)\left[1-e^{-(t-6)/10}\right]$$

By trial and error, $y(34) < 0.95$, $y(36) > 0.95$.

Therefore $N\Delta t = 36$ or $N = 18$

The coefficients $\{S_i\}$ are obtained from the expression for $y(t)$ and the predictive controller is obtained following the procedure of Example 20.5. The closed-loop responses for a unit set-point change are shown below for the three controller tunings

20.11

(Use MATLAB *Model Predictive Control Toolbox)*

a) M=5 vs. M=2

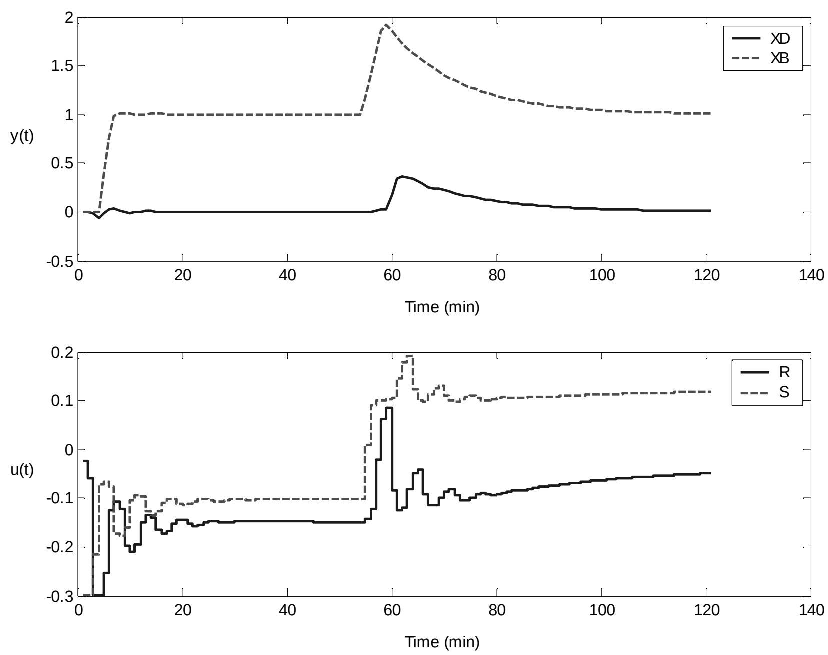

Figure S20.11a1. *Simulations for P=10 , M=5 and* ***R****=0.1****I****.*

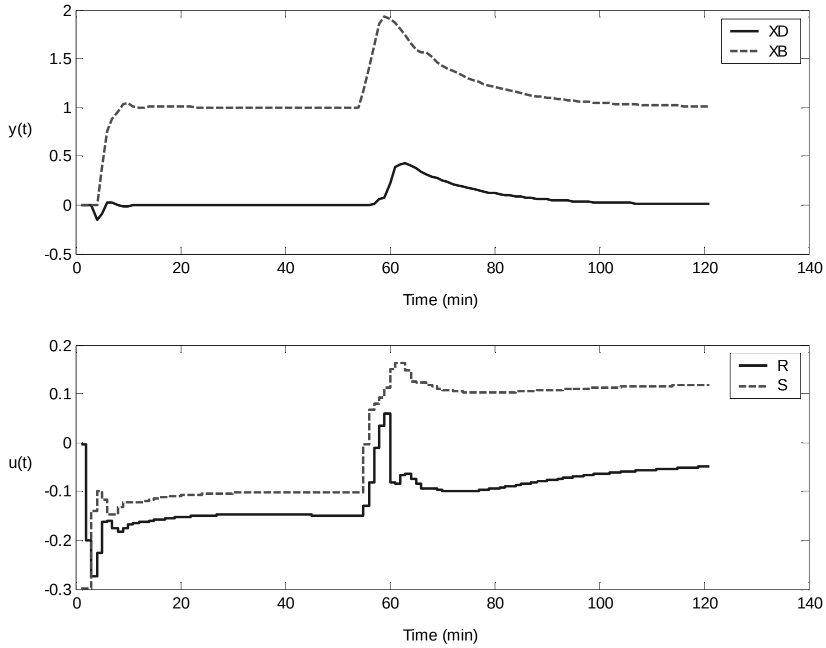

Figure S20.11a2. *Simulations for P=10 , M=2 and* ***R****=0.1****I****.*

b) ***R****=0.1****I*** *.vs* ***R****=****I***

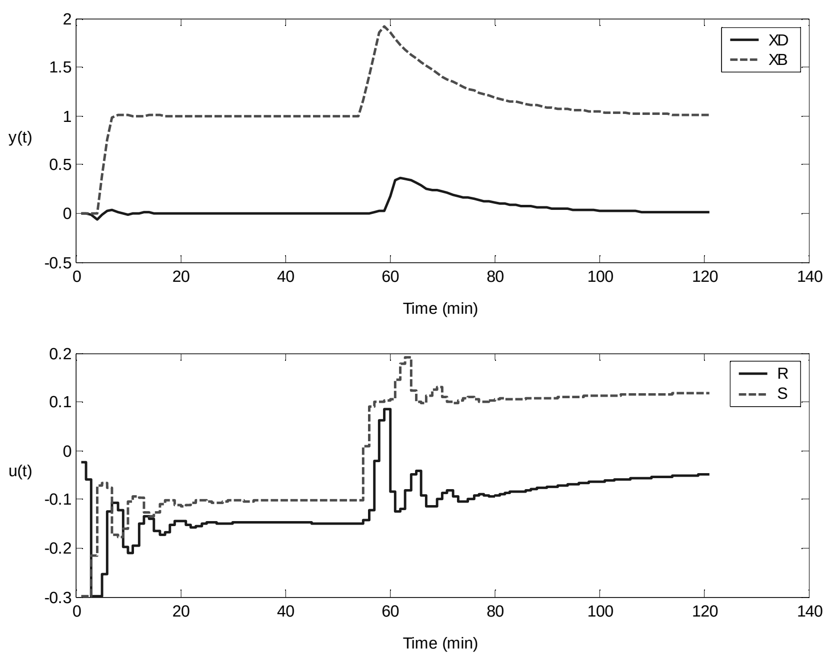

Figure S20.11b1. *Simulations for P=10 , M=5 and* ***R****=0.1****I****.*

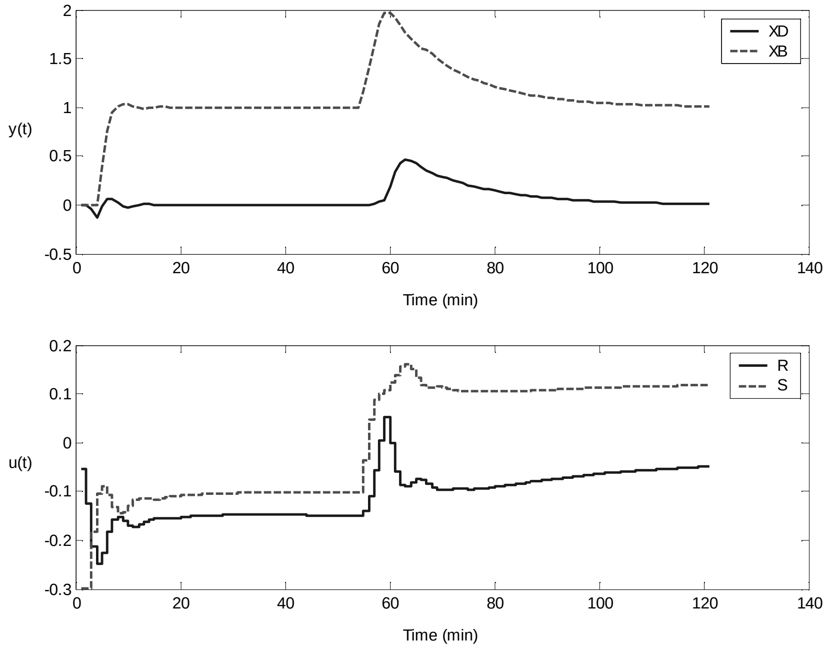

Figure S20.11b2. *Simulations for P=10 , M=5 and* ***R****=****I****.*.